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«BIRLIK DOIRADA CHEGARALANGAN GOLOMORF FUNKSIYALAR SINFINING BA'ZI XOSSALARI »

mavzusidagi

BITIRUV MALAKAVIY ISHI

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IMZONI TASDIQLAYMAN
KADRLAR BO'LIMI BOSHLIG'I

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Mundarija

Kirish

Aytaylik $\Delta=\{z\in C:\ |z|<1\}$ – birlik doira bo`lib, $H^{\infty}(\Delta)$ – $\Delta$ da golomorf va chegaralangan funksiyalar fazosi bo`lsin.

Agar $f_1,f_2,...,f_m\in H^{\infty}$ $\sup_k|f_k(z)|>\delta>0$

bo`lsa, u holda

$$\sum_{k=1}^{m} f_k g_k = 1$$

shartni qanoatlantiruvchi

$$g_1, g_2, ..., g_m \in H^{\infty}(\Delta)$$

funksiyalar mavjudmi?

Bu masala Karona masalasi deyiladi. Uni birinchi bo`lib, 1962 yili Karleson to'lik yechgan, ya'ni shu shartni qanoatlantiruvchi $H^{\infty}(\Delta)$ fazodan bo`lgan funksiyalarni mavjudligini isbotlagan.

Karlesonni isboti juda murakkab bo`lib, xammaga tushinarli va soddarok isbotiga zarurat paydo bo`lgan. 1979 - yilda Volf, 1986 - Berntson va Ransford uni soddarok isbotini keltirgan. Undan keyin ham bir qancha olimlar turli xil usullarda bu masalani yechishga xarakat qilishgan. Muammo shundan iboratki masala n> 1 xolatda yechilmagan.

Ushbu bitiruv malakaviy ishida quyidagi teoremalar o`rganilgan.

teorema. Aytaylik $X\subset T\times C$ – kompakt qism to`plam va har bir $\xi\in T$ uchun $X_{\xi}=\{z\in C,\ (\xi,z)\in X\}$ – qavariq to`plam bo`lsin. $(0,z_0)$, $(z_0 \in C)$ nuqtani o`zida saqlovchi X ning polinomial qobig`ini $\hat{X}$ bilan belgilaylik. U holda shunday $h \in H^{\infty}(\Delta)$ funksiya mavjudki, har qanday $\xi \in T$ uchun $h^{*}(\xi) \in X_{\xi}$ bo`ladi. Bunda $h^{*}(\xi)-h(z)$ funksiya radial limit qiymati deb ataladi.

Teorema. Aytaylik $f_1, f_2, ..., f_m \in H^{\infty}$ bo'lib,

$$1 > |f|^2 = \sum_{j=1}^{m} |f_j|^2 > \delta^2 > 0$$

shartni Δ da qanoatlantirsin.

U holda shunday

$$g_1, g_2, ..., g_m \in H^{\infty}(\Delta)$$

funksiyalar mavjudki,

$$\sum_{j=1}^{m} f_j g_j = 1$$

bo`ladi.

Ushbu bitiruv malakaviy ishi uchta paragrafdan iborat bo`lib, birinchi paragrafda golomorf funksiyalar va ularning xossalari keltirilgan. Ikkinchi paragrafda analitik multifunksiyalar va ularning chegaraviy xossalari urganilgan. Uchinchi paragrafda esa asosiy teoremani isboti keltirilgan.

1-§. Golomorf funksiyalar.

Ta'rif. $e:\mathbf{C}^2 \to \mathbf{C}$ funksiya, $\mathbf{R}$ chiziqli deyiladi, agar u quyidagi shartlarni qanoatlantirsa:

a) $e(z'+z'') = e(z') + e(z''),\ \forall z', z'' \in \mathbf{C}^n$

b) $e(\lambda z) = \lambda e(z),\ \forall z \in \mathbf{C}^2, \forall \lambda \in \mathbf{R}$

agar b) shart $e(\lambda z) = \lambda e(z), \forall \lambda \in \mathbf{C}$ shart bilan almashtirilsa, u xolda bunday chiziqli funksiyaga, $\mathbf{C}$ chiziqli funksiya deyiladi. $\mathbf{C}^2$ da ixtiyoriy $\mathbf{R}$ chiziqli funksiya

$$e(z) = \sum_{\nu=1}^{2} (a_\nu z_\nu + b_\nu \bar{z}_\nu),\quad a_\nu, b_\nu \in \mathbf{C} \tag{1}$$

ko`rinishga ega. $\mathbf{C}$ chiziqli funksiya esa

$$e(z) = \sum_{\nu=1}^{2} a_\nu z_\nu, \qquad a_\nu \in \mathbf{C} \tag{2}$$

ko`rinishga ega.

$\mathbf{R}$- chiziqli funksiya, $\mathbf{C}$ chiziqli funksiya bo`ladi, faqat shu xoldaki,

$$e(iz) = ie(z) \tag{3}$$

bo`lsa.

f funksiya z nuqtada, $(z \in \mathbf{C}^2)$ $R-$ differensiallanuvchi funksiya deyiladi, agar u

$$f(z+h) = f(z) + e(h) + o(h) \tag{4}$$

ko`rinishda ifodalansa, bunda $e(h)$ biror **R** chiziqli funksiya.

Agar (4) dagi $e(h)$ biror **C** chiziqli funksiya bo`lsa, $f$ funksiya $z$ nuqtada **C** differensiallanuvchi deyiladi. **R** differensiallanuvchi funksiya **C** differensiallanuvchi bo`lishi uchun

$$\bar{\partial} f = 0 \qquad (5)$$

bo`lishi zarur va yetarli.

Ta'rif. $f(z)=f(z_1,z_2)$ funksiya z nuqtada golomorf deyiladi, agar u shu nuqtaning biror atrofida **C** differensiallanuvchi bo`lsa. Ochiq to`plamda, **C** differensiallanuvchanlik va golomorflik ustma -ust tushadi.

f funksiya $D \in \mathbf{C}^2$ soxada golomorf deyiladi, agar uning barcha nuktalarida golomorf bo'lsa. D soxada golomorf funksiyalar sinfi

$$D = H(D) = A(D)$$

kabi belgilanadi.

Golomorf funksiyalarning asosiy xossalari.

I. Koshining karrali integral formulasi. Agar $f(z)$ funksiya biror

$$f(z) \in O(U) \cap C(\overline{U})$$

bo`lsa, u xolda

$$f(z) = \frac{1}{(2\pi i)^2} \int_{\Gamma} \frac{f(\xi_1, \xi_2)}{(\xi_1 - z_1)(\xi_2 - z_2)} d\xi_1 d\xi_2, \quad z = (z_1, z_2) \qquad (6)$$

tenglik o'rinli.

Bu yerda

$$U=\{z\in \mathbf{C}^2:|z_\nu-a_\nu|\le r_\nu,\ \nu=\overline{1,2}\},\Gamma$$
$$\Gamma=\{Z\in \subset^{2},|z_1-a_1|=r_1\,|z_2-a|=r_2\}$$

II. Teylor qatori. Agar

$$f(z)\in O(U)\cap C(\overline{U})$$

bo'lsa, u xolda uni quyidagi ko'rinishdagi karrali darajali qatorga yoyish mumkin

$$f(z)=\sum_{|k|=0}^{\infty}c_k(z-a)^k \tag{7}$$

bu yerda

$$c_k=\frac{1}{(2\pi i)^k}\int_{\Gamma}\frac{f(\xi)}{(\xi-a)^{k+1}}d\xi \tag{8}$$

bunda

$k=(k_1,k_2)$ multindeks, $k_\nu\ge 0$, $k_\nu=1,2$,

$$|k|=k_1+k_2,\quad c_k=c_{(k_1,k_2)}$$
$$(z-a)^k=(z_1-a_1)^{k_1}(z_2-a_2)^{k_2}$$
$$(\xi-a)^{k+1}=(\xi_1-a_1)^{k_1+1}(\xi_2-a_2)^{\kappa_2+1}.$$

III. Agar $f(z)\in O(U)$ bo'lsa, u xolda ixtiyoriy $z\in U$ nuqtada bu funksiya barcha tartibli xususiy xosilalarga ega va bu xususiy xosilalar U da golomorf.

IV. Agar $f(z)$ funksiya $a\in \mathbf{C}^2$ nuqtada golomorf bulsa va u (7) ko'rinishdagi qatorga yoyilsa, u xolda bu qatorning koeffisentlari Teylor formulasi buyicha aniqlanadi.

$$c_k = \frac{1}{k_1!k_2!}, \frac{\partial^{k_1+k_2} f}{\partial_{z_1}{}^{k_1} \partial_{z_2}{}^{k_2}}\Bigg|_{z=a} = \frac{1}{k!}\frac{\partial^{|k|} f}{\partial z^k}\Bigg|_{z=a} \qquad (9)$$

Agar a nuqtada f funksiya golomorf bo'lsa, shunday $U(a,\varepsilon)$ topiladiki,

$$\frac{\partial^{|k|} f(z)}{\partial z^K}\Bigg|_{z=a} = \frac{k!}{(2\pi i)^2}\int_{\gamma}\frac{f(\xi)}{(\xi-a)^{k+1}}d\xi$$

o'rinli.

V. Koshi tengsizligi. Agar $f(z)\in O(U)\cap C(\overline{U})$ bo`lsa va Γ ostovda $|f|\le M$ bo'lsa, u xolda f funksiyaning a nuqtadagi Teylor qatoriga yoyilmasining koeffisientlari quyidagi tengsizlikni qanoatlantiradi.

$$|c_k| \le \frac{M}{r^k}, \qquad (10)$$

bunda $r^k = r_1^{k_1} r_2^{k2}$ ga teng bo'ladi.

VI. Yagonalik teoremasi. Agar $f(z)$ funksiya D soxada golomorf va $a\in D\subset \mathbf{C}^2$ nuqtada o'zinining barcha (tartibdagi) xususiy xosilalari bilan birga nolga teng bo'lsa, ya'ni $\forall k$ da

$$\left|\frac{\partial^k f}{\partial z^k}\right|_{z=a} = 0$$

u holda D da $f\equiv 0$ bo`ladi.

VII. Modulning maksimum prinsipi. Agar $f \in O(D)$ va f ning moduli biror $z \in D$ nuqtada maksimumga erishsa, u xolda D да $f = const$ bo`ladi

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Plyurigarmonik funksiyalar.

Agar $f = u + iv$ funksiya $z \in \mathbf{C}^2$ nuqtada golomorf bulsa, u xolda $\bar{f} = u - iv$ funksiya shu nuqtaning atrofida $\mathbf{R}$ differensiallanuvchi va ixtiyoriy $v = 1{,}2$ uchun

$$\frac{\partial \bar{f}}{\partial z_v} = \frac{1}{2}\left(\frac{\partial \bar{f}}{\partial x_v} - i\frac{\partial \bar{f}}{\partial y_v}\right) = \frac{1}{2}\overline{\left(\frac{\partial f}{\partial x_v} + \frac{\partial f}{\partial y_v}\right)} = \overline{\left(\frac{\partial f}{\partial \bar{z}_v}\right)} = 0 \qquad (11)$$

bo`ladi, bundan kelib chiqadiki $\bar{f}$ - antigolomorf funksiya ekan. $f$ funksiya $z \in \mathbf{C}^2$ nuqtada golomorf bo`lsin. U xolda yuqoridagiga ko'ra uning xaqiqiy qismi $u = \frac{1}{2}(f + \bar{f})$ uchun z nuqtaning atrofida quyidagiga egamiz:

$$\frac{\partial u}{\partial z_v} = \frac{1}{2}\left(\frac{\partial f}{\partial z_v} + \frac{\partial \bar{f}}{\partial z_v}\right) = \frac{1}{2}\frac{\partial f}{\partial z_v}, \quad v = 1{,}2.$$

Golomof funksiyaning xususiy xosilasi xam golomorfligidan $\mu, v = \overline{1{,}2}$ uchun

$$\frac{\partial}{\partial \bar{z}_\mu}\left(\frac{\partial u}{\partial z_v}\right) = \frac{\partial^2 u}{\partial \bar{z}_\mu \partial z_v} = \frac{1}{2}\frac{\partial}{\partial \bar{z}_\mu}\left(\frac{\partial f}{\partial z_v}\right) = \frac{1}{2}\frac{\partial}{\partial z_v}\left(\frac{\partial f}{\partial \bar{z}_\mu}\right) = 0 \qquad (12)$$

bundan kelib chiqadi: $$\frac{\partial^2 u}{\partial \bar{z}_\mu \partial z_\nu} = 0, \quad \mu,\nu = \overline{1,2}$$

Ta'rif: $\mathbf{C}^2$ sinfdan olingan u funksiya $D \subset \mathbf{C}^2$ soxada plyurigarmonik deyladi, agar D ning xar bir nuqtasida (12) shart bajarilsa. (12) dan quyidagi munosabatni xosil qilamiz:

$$\frac{\partial}{\partial \bar{z}_\mu}\left(\frac{\partial u}{\partial z_\nu}\right) = \frac{1}{4}\left[\frac{\partial}{\partial x_\mu}\left(\frac{\partial u}{\partial x_\nu} + \frac{1}{i}\frac{\partial u}{\partial y_\nu}\right) - \frac{1}{i}\frac{\partial}{\partial y_\mu}\left(\frac{\partial u}{\partial x_\nu} + \frac{1}{i}\frac{\partial u}{\partial y_\nu}\right)\right] =$$

$$= \frac{1}{4}\left[\frac{\partial^2 u}{\partial x_\mu \partial x_\nu} + \frac{1}{i}\frac{\partial^2 u}{\partial x_\mu \partial y_\nu} - \frac{1}{i}\frac{\partial^2 u}{\partial y_\mu \partial x_\nu} + \frac{\partial^2 u}{\partial y_\nu \partial y_\mu}\right] = 0$$

$$\frac{\partial}{\partial \bar{z}_\mu} = \frac{1}{2}\left(\frac{\partial}{\partial x_\mu} - \frac{1}{i}\frac{\partial}{\partial y_\mu}\right), \frac{\partial}{\partial z_\nu} = \frac{1}{2}\left(\frac{\partial}{\partial x_\nu} + \frac{1}{i}\frac{\partial}{\partial y_\nu}\right)$$

$$\begin{cases} \dfrac{\partial^2 u}{\partial x_\mu \partial x_\nu} + \dfrac{\partial^2 u}{\partial y_\nu \partial y_\mu} = 0 \\ \dfrac{\partial^2 u}{\partial x_\mu \partial y_\nu} - \dfrac{\partial^2 u}{\partial y_\nu \partial x_\mu} = 0 \end{cases}$$

$\mu = \nu$ bo`lganda :

$$\frac{\partial^2 u}{\partial x_\mu^2} + \frac{\partial^2 u}{\partial y_\mu^2} = 0, \ \Delta = 0, \ \mu = \overline{1,2}$$,

bu shartda μ bo'yicha yig'indi olsak

$$\Delta U(x,y) = \sum_{\mu=1}^{n}\left(\frac{\partial^2 u}{\partial x_\mu^2} + \frac{\partial^2 u}{\partial y_\mu^2}\right) = 0, \quad \mu = \nu$$

bo`lganda golomorf funksiyaning xaqiqiy qismi $\mathbf{R}^4$ da garmonik funksiyani beradi. $z$ nuqtada golomorf bo`lgan

$f(z)$ funksiyaning xaqiqiy va mavxum qismlari plyurigarmonik funksiya bo`ladi. Xaqiqiy qism uchun bu o'rinli, $-if \in O(D)$ va $\operatorname{Im} f = \operatorname{Re}(-if)$ bu esa $f(z)$ funksiyaning mavxum qismi xam plyurigarmonik ekanini ko`rsatadi.

2–§. Analitik multifunksiyalar.

$\Delta = \{\lambda \in C, \ |\lambda| < 1\}$ birlik doira bo`lib,

$$T = \{\lambda \in C, \ |\lambda| = 1\}$$

esa uning chegarasi bo`lsin.

1- teorema. Aytaylik $X \subset T \times C$ – kompakt qism to`plam va har bir $\xi \in T$ uchun $X_\xi = \{z \in C, \ (\xi, z) \in X\}$ – qavariq to`plam bo`lsin. $(0, z_0)$, $(z_0 \in C)$ nuqtani o'zida saqlovchi $X$ ning polinomial qobig`ini $\hat{X}$ bilan belgilaylik. U holda shunday $h \in H^\infty(\Delta)$ funksiya mavjudki, har qanday $\xi \in T$ uchun $h^*(\xi) \in X_\xi$ bo`ladi. Bunda $h^*(\xi) - h(z)$ funksiya radial limit qiymati.

Isbot. P ko`p had bo`lsin. U holda $f(p) = p(o, z_0)$ – chiziqli uzluksiz funksional uchun tashuvchisi X da yotgan shunday μ o`lchov mavjudki,

$$f(p) = \int_X p(\xi, z) d\mu(\xi, z)$$

bo`ladi. Bundan esa

$$P(o, z_0) = \int_X p(\xi, z) d\mu(\xi, z) \qquad (1)$$

tenglik o`rinlidir. Xar bir $\lambda \in \Delta$ uchun

$$h(\lambda) = \int_X \frac{1 - |\lambda|^2}{|1 - \lambda\xi|^2} z d\mu(\xi, z) \qquad (2)$$

belgilash qilamiz.

Endi istalgan $\lambda \in \Delta$ va $\xi \in T$ uchun

$$\frac{1-|\lambda|^2}{|1-\lambda\xi|^2}=\frac{1}{2}\left(\frac{1+\lambda\bar{\xi}}{1-\lambda\bar{\xi}}+\frac{1+\bar{\lambda}\xi}{1-\bar{\lambda}\xi}\right)$$

tenglikni ko`rsatamiz.

$$\frac{1}{2}\left(\frac{1+\lambda\bar{\xi}}{1-\lambda\bar{\xi}}+\frac{1+\bar{\lambda}\xi}{1-\bar{\lambda}\xi}\right)=\frac{1}{2}\left(\frac{(1+\lambda\bar{\xi})(1-\bar{\lambda}\xi)+(1+\bar{\lambda}\xi)(1-\lambda\bar{\xi})}{(1-\lambda\bar{\xi})(1-\bar{\lambda}\xi)}\right)=$$

$$=\frac{1}{2}\left(\frac{1-\bar{\lambda}\xi+\lambda\bar{\xi}-|\lambda|^2+1-\lambda\bar{\xi}+\bar{\lambda}\xi-|\lambda|^2}{|1-\lambda\xi|^2}\right)=\frac{1-|\lambda|^2}{|1-\lambda\xi|^2};$$

isbotlangan tenglikni (2) ga qo`yamiz.

$$h(\lambda)=\frac{1}{2}\int_X\frac{1+\lambda\bar{\xi}}{1-\lambda\bar{\xi}}\,zd\mu(\xi,z)+\frac{1}{2}\int_X\frac{1+\bar{\lambda}\xi}{1-\bar{\lambda}\xi}zd\mu(\xi,z)=$$

$$=\frac{1}{2}\int_X\frac{1+\lambda\bar{\xi}}{1-\lambda\bar{\xi}}zd\mu(\xi,z)+\frac{1}{2}z_0$$

Demak, $h(z)\in H^{\infty}(\Delta)$. Xar bir $\xi\in T$ uchun

$$\alpha(\xi)=\sup\{\operatorname{Re}z;\ z\in X_{\xi}\}$$

bo`lsin. U holda

$$\operatorname{Re}h(z)\leq\int_X\frac{1-|\lambda|^2}{(1-\lambda\xi)^2}\alpha(\xi)d\mu(\xi,z).$$

Bundan esa

$$\overline{\lim_{\substack{\lambda\to\xi\\ \lambda\in\Delta}}}\operatorname{Re}h\leq\alpha(\xi)$$

bo`ladi

Har bir $\xi\in T$ uchun X_{ξ} qavariq bo`lganligi sababli

$$\rho\left(h(\lambda), X_\xi\right) \to 0, \quad \lambda \to \xi.$$

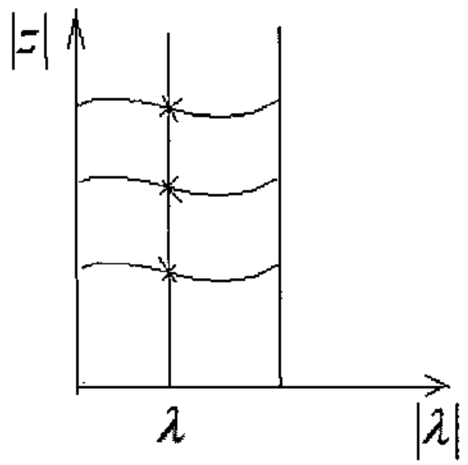
Demak, $$h^*(\xi) \in X_\xi, \qquad (\forall \xi \in T)$$

$K \subset C$ to`plam quyidagi shartlarni qanoatlantirsin:

1) har bir $\lambda \in \overline{\Delta}$ uchun unga mos bo`sh bo`lmagan $K(\lambda) \subset K$ kompakt qism to`plam mavjud.

2) $\Gamma(\mathrm{K}) = \{(\lambda, z) \in \overline{\Delta} \times C; \quad z \in K(\lambda)\}$ -graf kompakt to`plamdan iborat bo`lsin.

**Ta`rif.** Agar $U = (\Delta \times C) \setminus \Gamma(\mathrm{K})$– psevdoqavarik soha bo`lsa, K multifunksiya Δ da analitik deyiladi.



Xossa. Agar K multifunksiya Δ da analitik bo`lsa, u holda $\Gamma(K)$ to`plamni polinomial qobig`i $\Gamma(K) \cap (T \times C)$ dan iborat bo`ladi.

2-teorema. $K - \overline{\Delta}$ aniqlangan multifunksiya bo`lib, $\Delta$ da analitik va $\Gamma(K)-$ kompakt bo`lsin. Agar har bir $\xi \in T$ ucun $K(\xi)-$ qavariq bo`lsa, u holda shunday $h \in H^\infty(\Delta)$ funksiya mavjudki, $h^*(\xi) \in K(\xi)$ bo`ladi.

Isbot. $\Gamma(K)\bigcap(T\times C)=X-$ kompakt, har bir $\xi\in T$ uchun $X_{\xi}=\{z\in C: \ (\xi,z)\in X\}=K(\xi)$ qavariq bo`ladi. Yuqoridagi xossaga ko'ra $X$ to`plamning polinomial qobig`i $\Gamma(K)\bigcap(T\times C)$ dan iborat. Demak, 1-teoremaga ko`ra shunday $h\in H^{\infty}(\Delta)$ funksiya mavjudki $\forall\xi\in T$ uchun $h^{*}(\xi)\in K(\xi)$ bo`ladi. Isbotlangan teoremadan Karona teoremasini isbotlashda foydalaniladi.

$G=D\times C=C_z\times C_w$ soxa va $S\subset G-$ undagi yopiq to`plam bo`lsin. S_{z^0} bilan S to`plamni $\{z=z^0\}$ kompleks to'g`ri chiziq kesishmasini belgilaymiz.

Ta'rif: $F:z\to S_z \ \ z\in D$ to`plam qiymatli funksiya multifunksiya deyiladi. Agar S- (G da) psevdobotiq to`plam bo`lsa, F(z)- analitik multifunksiya deyiladi.

Bizni $F:z\to S_z$ funksiyani analitik bo`lishi qiziqtiradi. F funksiyani analitikligini plyurisubgarmonik funksiyalar bilan bog`laymiz.

$$V(z,w)=-\ln\rho(w,S_Z)$$

Bunda $\rho(w,S_Z)$-w nuqtadan S_Z to`plamgacha bo`lgan masofa. Agar S- psevdobotiq to`plam bo`lsa, V(z,w)- plyurisubgarmonik funksiya bo`ladi.([3] ga qarang).

Slodkovskiy ([6]ga qarang) ishlarida chegaralangan multifunksiyalar uchun quyidagi to'rta xolatni ekvivallent ekanini isbotlagan.

I. $F:z\to S_z$ - analitik multifunksiya.

II. $-\ln p(w,s_z)\in psh(G)$

III. Xar qanday $U\subset D$ va $(U\times C)\cap C$ to`plamning atrofida plyurisubargonik. $\psi(z,w)$ funksiya uchun $\varphi(z)=\max_{w\in S_z}\psi(z,w)$ funksiya U da suborganik.

IV. Xar qanday $\rho(z,w)$ ko'pxadni $|\rho(z,w)|_S$ izi o'zining eng katta qiymatiga S da erishadi.

Slodkovskiyning bu natijasi $G=D\times C\subset C^2$ bo`lgan xolatdagina o'rinli. $G=D\times C\subset C_z^n\times C_w$- soxalar uchun o'rinli emas. Bunday soxalar uchun quyidagi teoremalar o'rinli ([5]ga qarang)

3-teorema: Aytaylik $S\subset D\times C$- shunday multifunksiyaki, S_Z to`plam xar qanday $z\in D$ da ichki nuqtaga ega emas.U xolda S- da ichki nuqtaga ega emas.U xolda,

$$V(z,w)=-\ln\rho(w,s_z)\in psh(G\,|\,S)$$

bo`lsa.

4-teorema: Agar $S\subset G$ -chegaralangan plyuripolyar to`plam bo`lib,

$$V(z,w)=-\ln\rho(w,s_z)\in psh(G\,|\,S)$$

bo`lsa u xolda S- analitik multifunksiya bo`ladi.

5-teorema: Aytaylik $F:z\to S_z$- chegaralangan multifunksiya bo`lib $S_z(z\in D)$ to`plam ichki nuqtaga ega emas.

Agar xar bir $e\subset C^n$, $e\cap D\neq\varnothing$ kompleks chiziqda $F:z\to S_z,\ z\in e\cap D$ funksiya analitik bo`lsa, u xolda bu funksiya analitik multifunksiya bo`ladi.

3 §. H^∞ - fazoni ba'zi xossalari .

Agar $f_1, f_2, ..., f_m \in H^\infty$ va $\sup_k |f_k(z)| > \delta > 0$

bo`lsa, u holda

$$\sum_{k=1}^{n} f_k g_k = 1$$

shartni qanoatlantiruvchi

$$g_1, g_2, ..., g_m \in H^\infty$$

funksiyalar mavjudmi?

Bu masala Karona masalasi deyiladi. Uni birinchi bo`lib, 1962 yili Karleson yechgan, ya'ni shu shartni qanoatlantiruvchi funksiyalarni mavjudligini isbotlagan. Karlesonni isboti juda murakkab bo`lgan. 1979 yilda Volf uni soddarok isbotini keltirgan. Undan keyin xam bir qancha olimlar turli xil usullarda Karona muammosini hal qilishgan. Ularning hammasini bir umumiy tarafi bu isbotlarning barchasini C^n ga analogi o'tmaydi. Karona muammosi hozirgi vaqtgacha ko`p o'lchamli fazo uchun ochiq holatda turibdi. Bu paragrafda Karona masalasini Berntson va Ransford tamonidan qilingan isboti keltirilgan.

6-teorema(Karona). Aytaylik, $f_1, f_2, ..., f_m \in H^\infty$ bo`lib,

$$1 > |f|^2 = \sum_{j=1}^{m} |f_j|^2 > \delta^2 > 0$$

shartni Δ da qanoatlantirsin.

U holda shunday $g_1, g_2, ..., g_m \in H^\infty(\Delta)$

funksiyalar mavjudki,

$$\sum_{j=1}^{m} f_j g_j = 1$$

bo`ladi.

Isbot. Biz $m=2$ bo`lgan sodda holni ko'rib chiqamiz. Faraz qilamiz $f_1(z)$ va $f_2(z)$ funksiyalar $\overline{\Delta}$ da golomorf bo`lsin.

$$f_1(rz),\ f_2(rz),\qquad 0<r<1$$

funksiya $\overline{\Delta}$ da golomorf). g_1, g_2 funksiyalar $\delta>0$ soniga bog'liq bo`ladi. Faraz qilaylik shunday

$$h_1(z),\ h_2(z)\in H^{\infty}(\overline{\Delta})$$

funksiyalar mavjud bo`lib,

$$f_1h_1+f_2h_2=1 \qquad (1)$$

bo`lsin. U holda

$$\begin{aligned} g_1&=h_1-hf_2,\\ g_2&=h_2+hf_1 \end{aligned} \qquad (2)$$

funksiyalar ham (1) tenglikni qanoatlantiradi. Haqiqatan,

$$\begin{aligned} &f_1g_1+f_2g_2=f_2(h_1-hf_2)+f_2(h_2+hf_1)=\\ &f_1h_1-hf_1f_2+h_2f_2+hf_1f_2=1. \end{aligned}$$

(2) munosabatda $h(z)$ funksiya Δ da golomorf. Endi biz $h(z)$ tanlashimiz va $g_1(z)$ va $g_2(z)$ larni δ orqali baholashimiz kerak.

Ixtiyoriy $\lambda\in\overline{\Delta}$ uchun

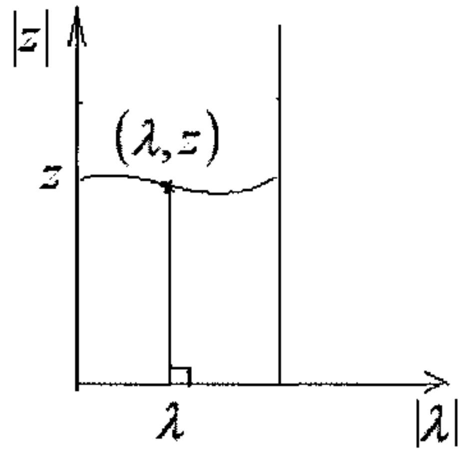
$$K(\lambda)=\{z\in C:\ |h_1(\lambda)-zf_2(z)|^2+|h_2(\lambda)+zf_1(z)|^2\le r(\lambda)\} \quad (3)$$

bo`lsin. Bunda $r(\lambda)$ $T$ da aniqlangan musbat silliq funksiya bo`lib, uni aniqlash kerak.

Agar $r(\lambda)$ ni $K-$ analitik funksiya bo`ladigan qilib tanlasak, u holda 2-teoremaga ko'ra (1) va (2) tengliklarni qanoatlantiruvchi $h$ funksiya mavjud bo`ladi. Agar K analitik multifunksiya bo`lsa, u holda

har qanday $q_0 \in \Gamma(K) \cap (\Delta \times C)$ nuqta uchun shu nuqtadan o'tuvchi va $\Gamma(k)$ ichida yotgan analitik truba mavjud. Bu esa har bir $q=(\lambda,z)$ Karona masalasi yechimini beradi, ya'ni shunday β_1,β_2 sonlar mavjudki, $f_1(\lambda)\beta_1+f_2(\lambda)\beta_2=1$ bo`ladi.
Bunda

$$|\beta_1|^2+|\beta_2|^2 \le r(\lambda)$$



Endi K analitik bo`lishi uchun $r(\lambda)$ ga zarur va yetarli bo`lgan shartni keltiramiz.

Lemma ([4]). Aytaylik K (3) formula bilan aniqlangan Δ dagi bo'sh bo'lmagan multifunksiya bo`lsin. $K$ multifunksiya $\Delta$ da analitik bo`lishi uchun $u=\ln r(\lambda)$ funksiya

$$\sqrt{u_{\lambda\bar{\lambda}}}-\sqrt{v_{\lambda\bar{\lambda}}} \ge \frac{|u_\lambda+v_\lambda|}{\sqrt{e^{u+v}-1}} \qquad (4)$$

tengsizlikni qanoatlantirishi zarur va yetarli.
Bunda

$$v=\ln|f|^2=\ln\left(|f_1|^2+|f_2|^2\right)$$

(4) fo'rmula uchun h_1, h_2 lar ixtiyoriy va o'zaro bog'liq emas, ammo f_1 va f_2 lar fiksirlangan. Agar $r(\lambda) = A|f|^2$ deb tanlasak $\ln r(\lambda) = \ln A + \ln|f|^2$ bo`lib, $u = v + \ln A$ bo`ladi. (A musbat o'zgarmas son). Ammo $u = v + \ln A$ funksiya (4) tengsizlikni qanoatlantirmaydi. Shuning uchun

$$u = v + F(v) \qquad (-\gamma < v < 0) \tag{5}$$

bo`lsin. Bunda

$$v = \ln|f|^2, \ \gamma = \ln\left(\frac{1}{\delta^2}\right).$$

(5)formuladagi $F(v)$ funksiya quyidagi shartlarni qanoatlantirsin.

$$F'(v) > 0, \quad F''(v) > 0 \ (-\gamma < v < 0) \tag{6}$$

$$F(-\gamma) > 2\gamma \tag{7}$$

(6) va (7) munosabatlardan foydalinib, $r(\lambda)$ funksiyani quyidan baholaymiz.

$$r(\lambda) = e^u = e^{v+F(v)} \geq e^{-v+F(-v)} \geq e^{-v+2v} = e^v = \left(|f_1|^2 + |f_2|^2\right) > \frac{1}{\delta^2}$$

(5) munosabatdan quyidagilarga ega bo`lamiz.

$$u_\lambda = v_\lambda + F' v_\lambda \tag{8}$$

$$u_{\lambda\bar{\lambda}} = v_{\lambda\bar{\lambda}} + F''|v_\lambda|^2 + F' v_{\lambda\bar{\lambda}} \tag{9}$$

Endi (4) tengsizlikning chap tomonidagi $\sqrt{v_{\lambda\bar{\lambda}}}$ ifodani o'ng tomonga o'tkazamiz va kvadratga oshiramiz

$$\sqrt{u_{\lambda\bar{\lambda}}} \geq \frac{|u_\lambda + v_\lambda|^2}{\sqrt{e^{u+v}-1}} + \sqrt{v_{\lambda\bar{\lambda}}}$$

$$u_{\lambda\bar{\lambda}} \geq \frac{|u_\lambda + v_\lambda|^2}{e^{u+v}-1} + 2\frac{|u_\lambda + v_\lambda|}{\sqrt{e^{u+v}-1}} + v_{\lambda\bar{\lambda}}$$

(8) va (9) munosabatlardan foydalanamiz.

$$F' v_{\lambda\bar{\lambda}} + F''\left|v_\lambda{}^2\right| \geq \frac{|v_\lambda|^2 (2+F')^2}{e^{2v}e^{F(v)}-1} + 2\sqrt{v_{\lambda\bar{\lambda}}}\,\frac{|2+F'||v_\lambda|^2}{\sqrt{e^{2v}e^{F(v)}-1}} \qquad (10)$$

Belgilash kiritamiz:

$$y = \frac{\left(2+F'(u)\right)^2 |v_\lambda|^2)}{e^{2v}e^{F(v)}-1}$$

u xolda (10) tengsizlik quyidagi ko'rinishga keladi.

$$F'(v)v_{\lambda\bar{\lambda}} + F''(v)|v_\lambda|^2 \geq y + 2\sqrt{v_{\lambda\bar{\lambda}}}\sqrt{y}.$$ Ikkita son uchun o'rta arifmetikni geometrikdan kichik emasligini e'tiborga olsak, quyidagi tengsizlikga ega bo`lamiz.

$$2\sqrt{v_{\lambda\bar{\lambda}}}\sqrt{y} = 2\sqrt{F' v_{\lambda\bar{\lambda}}}\sqrt{\frac{y}{F'}} \leq F' v_{\lambda\bar{\lambda}} + \frac{y}{F'(v)}$$

endi (10) tengsizlikni yechish o`rniga

$$F''(u)|v_\lambda|^2 \geq \frac{y}{F'(u)} + y$$

tengsizlikni yechamiz.

$$F''(v)\cdot|v_\lambda|^2 \geq \frac{(2+F')^2}{e^{2v}e^F-1}\cdot\frac{1+F'}{F'}\cdot|v_\lambda|^2 e^{2v}e^{F(u)} \geq$$
$$\geq \frac{(2+F'(u))^2}{F'(v)\cdot F''(v)}\cdot(1+F('v))+1 \qquad (11)$$

Bu tengsizlikni qanoatlantiruvchi $F(v)$ funksiyani tanlaymiz.

Bevosita tekshirib ko`rish mumkin

$$F(v)=\left(1+\frac{v}{\gamma}\right)^{3/2}+2\gamma+3\ln\gamma+B$$

funksiya (11) tengsizlikni qanoatlantiradi. (3) munosabatni qanoatlantiruvchi (g_1,g_2) lar uchun quyidagi baholashni olamiz

$$|g_1|^2+|g_2|^2 \leq \sup\{r(\xi),\xi\in T\}\leq$$
$$\leq e^{\sup(v+F(v))}\leq e^{F(0)+0}=e^{2\gamma+3\ln\gamma+B}\leq$$
$$\leq e^{2\ln\frac{1}{\delta^2}+3\ln\ln\frac{1}{\delta^2}+B}=e^{B\frac{1}{\delta^4}\left(\ln\frac{1}{\delta^2}\right)^3}=\frac{c\left(\ln\frac{1}{\delta^2}\right)^3}{\delta^4};$$

Demak, $g_1,\ g_2\in H^\infty(\Delta)$ va Karona masalasini yechimi bo`ladi.

Adabiyotlar.

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