

ELASTIK PLASTINKA ERKIN TEBRANISHLARI

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In work the task about movement of points is considered continuous plates from system equalizes fluctuations of a three-layer elastic plate. For this purpose you understand the system of the differential equation in the private derivatives describing fluctuations of a three-layer elastic plate, and then, in that specific case, behind analytical comments and continuations.

В работе рассмотрена задача вывода перемещение точек сплошной пластин из системы уравнены колебания трехслойной упругой пластины. С этой целью выведении система дифференциального уравнения в частных производных, описывающие колебания трехслойной упругой пластины, а затем, в частном случае получены анализированы продольный и поперечный колебания сплошной пластины при отсутствие внешних нагрузок.

Kirish. Bugungi kunga kelib texnikaning turli sohalarida va qurilishda, ko'p qatlamli xususan, uch qatlamli plastinkalar keng qo'llanilmoqda. Bunga sabab uch qatlamli plastinkalarning turli xildagi tebranishlar jarayonida mustahkamligi yuqori darajada qolishi va iqtisodiy muammolarning oson yechilishi bo'lmoqda. Shuning uchun uch qatlamli plastinkalar ustida ko'plab tadqiqot ishlari olib borilmoqda. Bular qatoriga [1, 2] ishlarni misol qilib ko'rsatish mumkin. Bunda juda ko'p hollarda plastinkalarning dinamik hisobi klassik nazariyaga tayangan holda olib boriladi [3]. Ba'zi hollarda dinamik hisoblar ko'ndalang siljish deformatsiyasi va aylanish inersiyasini hisobga oluvchi aniqlashtirilgan S.P.Timoshenko tipidagi tenglamalarga asoslanadi [4].

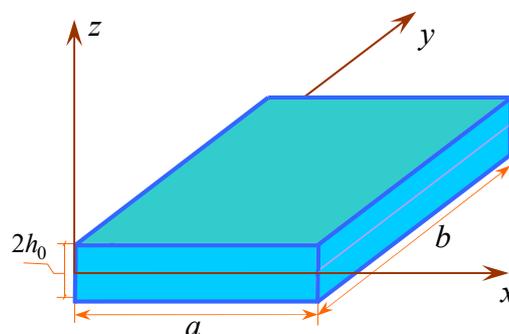
Keyingi yilliklarda aniq yechimlar usuliga asoslangan plastinkalar nazariyalari ishlab chiqilgan. Xususan ushbu usul bilan professor I.G.Filippov [6] va uning o'quvchilari tomonidan simmetrik strukturaga ega bo'lgan uch qatlamli plastinkalar tebranish nazariyalari yaratilgan.

Ushbu maqolada bir qatlamli elastik plastinkaning tebranish tenglamalari masala tekis masala deb qaralgan hol uchun keltirib chiqarilgan. Tebranish tenglamalari bilan bir qatorda plastinka ixtiyoriy kesimidagi ko'chish va kuchlanishlarni aniqlashga imkon beruvchi algoritm ishlab chiqilgan.

Masalaning qo'yilishi. Dekart koordinatalar sistemasida bir qatlamli plastinkani qaraymiz. Plastinka tekis deformatsiya holatida deb unga 1-rasmdagi holatdagidek Oxz to'g'ri burchakli koordinata o'qlarini o'tkazamiz. Bunda Ox o'qini ko'ndalang kesimning o'rta chizig'i bo'ylab yo'naltiramiz, Oz - o'qini esa unga tik ravishda yuqoriga yo'naltiramiz.

Qatlam qalinligini $2h_0$ orqali, qatlam materiali zichligini ρ_0 orqali, qatlam materiali elastik harakteristikalarini μ_0 va λ_0 orqali belgilaymiz.

Bu bir qatlamli plastinka tebranish tenglamasini xususiy holda [5] ishda keltirilgan uch qatlamli plastinka tebranish tenglamasidan keltirib chiqarish hamda



1-rasm

qaralayotgan bir qatlamli plastinkada yuzaga keladigan ko'chish va kuchlanishlarni topish masalasini qaraymiz.

Masalaning yechimi. Shunday qilib bir qatlamli plastinkaning tebranishlari haqidagi masalani yechish uchun uch qatlamli plastinkaning [5] ishda keltirilgan tebranish tenglamalaridan foydalanamiz. Bizga ma'lumki [5] ishda ikki izlanuvchi funksiyalarga nisbatan uch qatlamli plastinkaning amaliy masalalarni yechishda qo'llash mumkin bo'lgan tebranish tenglamalari sistemasi ba'zi almashtirishlardan keyin quyidagi ko'rinishga ega bo'ladi.

$$\begin{aligned}
& \left\{ A_{11} \frac{\partial^4}{\partial t^4} + A_{12} \frac{\partial^4}{\partial x^2 \partial t^2} + A_{13} \frac{\partial^4}{\partial x^4} + A_{14} \frac{\partial^2}{\partial t^2} + A_{15} \frac{\partial^2}{\partial x^2} + A_{16} \right\} \frac{\partial}{\partial x} W_0^{(0)} + \\
& + \left\{ B_{11} \frac{\partial^4}{\partial t^4} + B_{12} \frac{\partial^4}{\partial x^2 \partial t^2} + B_{13} \frac{\partial^4}{\partial x^4} + B_{14} \frac{\partial^2}{\partial t^2} + B_{15} \frac{\partial^2}{\partial x^2} \right\} U_0^{(0)} = \\
& = \left\{ S_{11} \frac{\partial^4}{\partial t^4} + S_{12} \frac{\partial^4}{\partial x^2 \partial t^2} + S_{13} \frac{\partial^4}{\partial x^4} + S_{14} \frac{\partial^2}{\partial t^2} + S_{15} \frac{\partial^2}{\partial x^2} + S_{16} \right\} f_x^{(1)}(k, p); \quad (1) \\
& \left\{ A_{21} \frac{\partial^4}{\partial t^4} + A_{22} \frac{\partial^4}{\partial x^2 \partial t^2} + A_{23} \frac{\partial^4}{\partial x^4} + A_{24} \frac{\partial^2}{\partial t^2} + A_{25} \frac{\partial^2}{\partial x^2} + A_{26} \right\} W_0^{(0)} + \\
& + \left\{ B_{21} \frac{\partial^4}{\partial t^4} + B_{22} \frac{\partial^4}{\partial x^2 \partial t^2} + B_{23} \frac{\partial^4}{\partial x^4} + B_{24} \frac{\partial^2}{\partial t^2} + B_{25} \frac{\partial^2}{\partial x^2} + B_{26} \right\} \frac{\partial U_0^{(0)}}{\partial x} = \\
& = \left\{ S_{21} \frac{\partial^4}{\partial t^4} + S_{22} \frac{\partial^4}{\partial x^2 \partial t^2} + S_{23} \frac{\partial^4}{\partial x^4} + S_{24} \frac{\partial^2}{\partial t^2} + S_{25} \frac{\partial^2}{\partial x^2} + S_{26} \right\} f_z^{(2)}(k, p).
\end{aligned}$$

bu yerda

$$\begin{aligned}
A_{11} &= -\left(\frac{q_0}{a_1^2 b_1^2} + \frac{1-q_1}{a_0^2 a_1^2} \right) \frac{z_1 h_0^4}{12} - \left(\frac{2q_1}{a_1^2 b_0^2} + \frac{3(1+q_1-3q_0 q_1)}{a_1^2 b_1^2} + \frac{2q_0 q_1}{a_0^2 a_1^2} + \frac{3q_0(1-q_1)}{a_1^4} + \frac{1+q_1}{b_0^2 b_1^2} + \frac{q_0(1+q_1)}{a_0^2 b_1^2} \right) \frac{z_1^3 h_0^2}{36}; \\
A_{12} &= \left(\frac{1-q_1+2q_0}{a_1^2} + \frac{q_0}{b_1^2} + \frac{1-q_1}{a_0^2} \right) \frac{z_1 h_0^4}{12} + \left(\frac{3+5q_1+7q_0-13q_0 q_1}{a_1^2} + \frac{1+3q_1}{b_0^2} + \frac{4+4q_1+q_0-8q_0 q_1}{b_1^2} \right) \frac{z_1^3 h_0^2}{36}; \\
A_{13} &= -(1-q_1+2q_0) \frac{z_1 h_0^4}{12} - (4+6q_1+4q_0-9q_0 q_1) \frac{z_1^3 h_0^2}{36}; \quad A_{15} = (4-2q_1+4q_0-2q_0 q_1) \frac{z_1 h_0^2}{6} + (1+3q_1) \frac{z_1^3}{6}; \\
A_{14} &= -\left(\frac{1+q_1}{b_0^2} + \frac{3(1-q_1)(1+q_0)}{a_1^2} + \frac{q_0(1+q_1)}{a_0^2} \right) \frac{z_1 h_0^2}{6} - \left(\frac{2q_1}{a_1^2} + \frac{1+q_1}{b_1^2} \right) \frac{z_1^3}{6}; \quad A_{16} = -(1+q_1) z_1; \\
A_{21} &= \frac{1-q_2}{a_0^2 b_2^2} \frac{h_0^4}{12} + \left(\frac{1-q_2}{a_2^2} + \frac{3(1-q_2)}{a_2^2 b_2^2} + \frac{(1-q_2)q_0}{a_0^2 a_2^2} \right) \frac{h_0^2 z_2^2}{12}; \quad A_{25} = -(4-10q_2+6q_0+2q_0 q_2) \frac{h_0^2}{6} - (1+q_2) \frac{z_2^2}{2}; \\
A_{22} &= \left(\frac{1-3q_2}{a_0^2} + \frac{2q_0}{a_2^2} + \frac{1-q_2}{b_2^2} \right) \frac{h_0^4}{12} - \left(\frac{(5+3q_0)(1-q_2)}{a_2^2} + \frac{1+q_2}{a_0^2} + \frac{3(1-q_2)}{b_2^2} + \frac{q_0(1+q_2)}{a_0^2} \right) \frac{h_0^2 z_2^2}{12}; \quad A_{26} = (1-q_2); \\
A_{23} &= (1-3q_2+2q_0) \frac{h_0^4}{12} + (5-3q_2+3q_0-q_0 q_2) \frac{h_0^2 z_2^2}{12}; \quad A_{24} = \left(\frac{3(1-q_2)}{b_2^2} + \frac{1-q_2}{b_0^2} + \frac{q_0(1-q_2)}{a_0^2} \right) \frac{h_0^2}{6} + \frac{1-q_2}{a_2^2} \frac{z_2^2}{2}; \\
B_{11} &= -\xi \left(\frac{3(1-q_1)(q_0-1)}{a_0^2} - \frac{1}{b_1^2} \right) \frac{1}{a_1^2} \frac{z_1 h_0^2}{6} - \xi \frac{q_1-1}{a_1^2 b_1^2} \frac{z_1^3}{6}; \quad B_{13} = -\xi(4-2q_1)(q_0-1) \frac{z_1 h_0^2}{6} - \xi(1+q_1) \frac{z_1^3}{6}; \\
B_{12} &= \xi \left(\frac{3q_0-2-3q_0 q_1+4q_1}{a_0^2} + \frac{3q_0-4-3q_0 q_1+2q_1}{a_1^2} - \frac{1}{b_1^2} \right) \frac{z_1 h_0^2}{6} - \xi \left(\frac{3q_1-1}{a_1^2} + \frac{q_1-1}{b_1^2} \right) \frac{z_1^3}{6}; \quad B_{14} = -\xi \left(\frac{q_1-1}{a_1^2} \right) z_1; \\
B_{15} &= \xi(q_1-1) z_1; \quad B_{21} = \xi \left(\frac{2(q_0-q_2)}{a_2^2} - \frac{1+q_2}{b_2^2} \right) \frac{1}{a_0^2} \frac{h_0^4}{12} + \xi \left(\frac{3(q_2-1)}{a_0^2 a_2^2} - \frac{1+q_2}{a_2^2 b_2^2} + \frac{2q_0}{a_0^2 b_2^2} \right) \frac{h_0^2 z_2^2}{12};
\end{aligned}$$

$$\begin{aligned}
B_{22} &= -\xi \left(\frac{2q_0 - 3q_2 - 1}{a_0^2} + \frac{2(q_0 - q_2)}{a_2^2} - \frac{1 + q_2}{b_2^2} \right) \frac{h_0^4}{12} - \xi \left(\frac{3q_2 + 2q_0 - 3}{a_0^2} + \frac{2q_2 + 2q_0 - 4}{a_2^2} - \frac{1 + q_2}{b_2^2} \right) \frac{h_0^2 z_2^2}{12}; \\
B_{23} &= \xi (2q_0 - 3q_2 - 1) \frac{h_0^4}{12} + \xi (2q_2 + 2q_0 - 4) \frac{h_0^2 z_2^2}{12}; \quad B_{25} = -\xi (q_2 - 1) \frac{z_2^2}{2} - \xi (2q_0 q_2 + 4q_0 - 2q_2) \frac{h_0^2}{6}; \\
B_{24} &= \xi \left(\frac{3 - q_2}{b_2^2} + \frac{2q_0 q_2 + 4q_0 - 3q_2 - 3}{a_0^2} + \frac{2q_2}{a_2^2} \right) \frac{h_0^2}{6} + \xi \frac{q_2 - 1}{a_2^2} \frac{z_2^2}{2}; \quad B_{26} = -\xi (1 + q_2); \\
S_{i1} &= \xi \mu_i^{-1} \frac{1}{a_i^2 b_i^2} \frac{h_0^4}{12}; \quad S_{i2} = -\xi \mu_i^{-1} \left(\frac{1}{a_i^2} + \frac{1}{b_i^2} \right) \frac{h_0^4}{12}; \quad S_{i3} = \xi \mu_i^{-1} \frac{h_0^4}{12}; \\
S_{i4} &= \xi \mu_i^{-1} \left(\frac{3 - 2q_i}{a_i^2} + \frac{1}{b_i^2} \right) \frac{h_0^2}{6}; \quad S_{i5} = -\xi \mu_i^{-1} (4 - 2q_i) \frac{h_0^2}{6}; \quad S_{i6} = \xi \mu_i^{-1};
\end{aligned}$$

bunda ($i=1,2$); $z_1 = h_0 + h_1$; $z_2 = h_0 + h_2$; $q_m = 1 - \frac{\lambda_m}{\mu_m}$; a_m - plastinka materiallaridagi bo'ylama to'lqin tarqalish tezligi va b_m - ko'ndalang to'lqin tarqalish tezligi; m - qatlamlar nomeri.

Xususiy holda (1) tenglamalar sistemasida $h_0 = 0$, $h_2 = h_1$, $q_2 = q_1$ va $a_2 = a_1$ deb olsak va ta'sir etuvchi kuchlarni nolga tenglashtirsak erkin holdagi bir qatlamli plastinka tebranish tenglamasi kelib chiqadi.

$$\begin{aligned}
&\left(c_{11} \frac{\partial^2}{\partial t^2} + c_{12} \frac{\partial^2}{\partial x^2} + c_{13} \right) \frac{\partial}{\partial x} W_0^{(0)} + \\
&+ \left(d_{11} \frac{\partial^4}{\partial t^4} + d_{12} \frac{\partial^4}{\partial x^2 \partial t^2} + d_{13} \frac{\partial^4}{\partial x^4} + d_{14} \frac{\partial^2}{\partial t^2} + d_{15} \frac{\partial^2}{\partial x^2} \right) U_0^{(0)} = 0 \\
&\left(c_{21} \frac{\partial^2}{\partial t^2} + c_{22} \frac{\partial^2}{\partial x^2} + c_{23} \right) W_0^{(0)} + \left(d_{21} \frac{\partial^2}{\partial t^2} + d_{22} \frac{\partial^2}{\partial x^2} + d_{23} \right) \frac{\partial U_0^{(0)}}{\partial x} = 0
\end{aligned} \quad (2)$$

bu yerda

$$\begin{aligned}
c_{11} &= -\left(\frac{2q_1}{a_1^2} + \frac{1 + q_1}{b_1^2} \right) \frac{h_1^3}{6}; \quad c_{12} = (1 + 3q_1) \frac{h_1^3}{6}; \quad c_{13} = -(1 + q_1) h_1; \quad c_{21} = \frac{1 - q_1}{a_1^2} \frac{h_1^2}{2}; \quad c_{22} = -(1 + q_1) \frac{h_1^2}{2}; \\
c_{23} &= (1 - q_1); \quad d_{11} = -\xi \frac{q_1 - 1}{a_1^2 b_1^2} \frac{h_1^3}{6}; \quad d_{12} = -\xi \left(\frac{3q_1 - 1}{a_1^2} + \frac{q_1 - 1}{b_1^2} \right) \frac{h_1^3}{6}; \quad d_{13} = -\xi (1 + q_1) \frac{h_1^3}{6}; \quad d_{15} = \xi (q_1 - 1) h_1; \\
d_{14} &= -\xi \left(\frac{q_1 - 1}{a_1^2} \right) h_1; \quad d_{21} = \xi \frac{q_1 - 1}{a_1^2} \frac{h_1^2}{2}; \quad d_{22} = -\xi (q_1 - 1) \frac{h_1^2}{2}; \quad d_{23} = -\xi (1 + q_1);
\end{aligned}$$

Bir qatlamli plastinka tebranish tenglamalar sistemasini (2) ni o'lchovsiz holatga keltiramiz. Buning uchun quyidagi ko'rinishdagi o'lchovsiz kattaliklarni kiritamiz.

$$\begin{aligned}
x^* &= \frac{x}{l}, \quad z^* = \frac{z}{l}, \quad h_m^* = \frac{h_m}{l}, \quad t^* = \frac{t \cdot b_0}{l}, \quad a_m^* = \frac{a_m}{b_0}, \quad b_m^* = \frac{b_m}{b_0}, \quad \mu_m^* = \frac{\mu_m}{\mu_0}, \\
U_0^{(0)*} &= \frac{U_0^{(0)}}{l}, \quad W_0^{(0)*} = \frac{W_0^{(0)}}{l}, \quad \xi^* = \frac{\xi}{l}.
\end{aligned} \quad (3)$$

(3) o'lchovsiz kattaliklarni (2) tenglamalar sistemasiga kiritib so'ngra bir qancha matematik soddalashtirishlarni bajarib quyidagi tenglamalar sistemasiga ega bo'lamiz

$$\begin{aligned}
&\left(c_{11}^* \frac{\partial^2}{\partial t^{*2}} + c_{12}^* \frac{\partial^2}{\partial x^{*2}} + c_{13}^* \right) \frac{\partial}{\partial x^*} W_0^{(0)*} + \\
&+ \left(d_{11}^* \frac{\partial^4}{\partial t^{*4}} + d_{12}^* \frac{\partial^4}{\partial x^{*2} \partial t^{*2}} + d_{13}^* \frac{\partial^4}{\partial x^{*4}} + d_{14}^* \frac{\partial^2}{\partial t^{*2}} + d_{15}^* \frac{\partial^2}{\partial x^{*2}} \right) U_0^{(0)*} = 0 \\
&\left(c_{21}^* \frac{\partial^2}{\partial t^{*2}} + c_{22}^* \frac{\partial^2}{\partial x^{*2}} + c_{23}^* \right) W_0^{(0)*} + \left(d_{21}^* \frac{\partial^2}{\partial t^{*2}} + d_{22}^* \frac{\partial^2}{\partial x^{*2}} + d_{23}^* \right) \frac{\partial U_0^{(0)*}}{\partial x^*} = 0
\end{aligned} \quad (4)$$

bu yerda

$$c_{11}^* = -\left(\frac{2q_1}{a_1^{*2}} + \frac{1+q_1}{b_1^{*2}}\right)\frac{h_1^{*3}}{6}; \quad c_{12}^* = (1+3q_1)\frac{h_1^{*3}}{6}; \quad c_{13}^* = -(1+q_1)h_1^*; \quad c_{21}^* = \frac{1-q_1}{a_1^2}\frac{h_1^{*2}}{2}; \quad c_{22}^* = -(1+q_1)\frac{h_1^{*2}}{2};$$

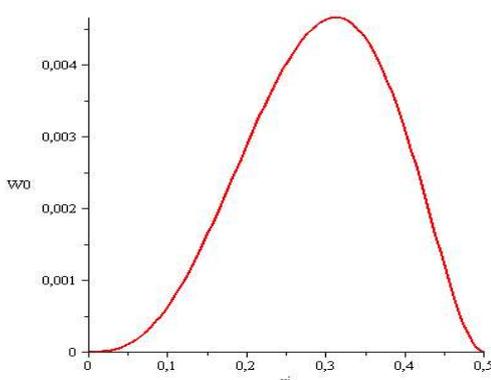
$$c_{23}^* = 1-q_1; \quad d_{11}^* = -\xi^* \frac{q_1-1}{a_1^{*2}b_1^{*2}}\frac{h_1^{*3}}{6}; \quad d_{12}^* = -\xi^* \left(\frac{3q_1-1}{a_1^{*2}} + \frac{q_1-1}{b_1^{*2}}\right)\frac{h_1^{*3}}{6}; \quad d_{13}^* = -\xi^*(1+q_1)\frac{h_1^{*3}}{6};$$

$$d_{15}^* = \xi^*(q_1-1)h_1^*; \quad d_{14}^* = -\xi^* \left(\frac{q_1-1}{a_1^{*2}}\right)h_1^*; \quad d_{21}^* = \xi^* \frac{q_1-1}{a_1^{*2}}\frac{h_1^{*2}}{2}; \quad d_{22}^* = -\xi^*(q_1-1)\frac{h_1^{*2}}{2}; \quad d_{23}^* = -\xi^*(1+q_1);$$

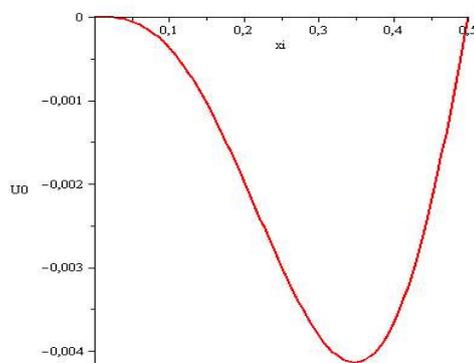
Bu (4) tenglamalar sistemasini “Maple 12” dasturi yordamida yechib izlanuvchi funksiyalarni grafik ko’rinishida quyidagicha ekanligini topib olamiz. 2-rasmda $w_0^{(0)*}$ izlanuvchi funksiyaning grafigi tasvirlangan bo’lsa, 3-rasmda $U_0^{(0)*}$ izlanuvchi funksiyaning grafigi keltirilgan.

bunda,

$$\xi = 1; \quad l = 1; \quad h = 0.05; \quad \rho_{alyu \min} = 2700 \text{ kg/m}^3; \quad E_{alyu \min} = 7 \cdot 10^{10} \text{ Pa}; \quad \nu_{alyu \min} = 0.34. \quad (5)$$



2-Rasm



3-Rasm

Shundan so’ng bir qatlamli plastinkada yuzaga keladigan ko’chishlarni topilgan bu izlanuvchi funksiyalar orqali o’lchovsiz holatda quyidagi ko’rinishda tasvirlaymiz.

$$U_0^* = \frac{1-q_0}{a_0^{*2}} \frac{z^{*2}}{2} \frac{\partial^2 U_0^{(0)*}}{\partial t^{*2}} - (1-q_0) \frac{z^{*2}}{2} \frac{\partial^2 U_0^{(0)*}}{\partial x^{*2}} + U_0^{(0)*} - \frac{1}{\xi^*} q_0 \frac{z^{*2}}{2} \frac{\partial}{\partial x^*} W_0^{(0)*};$$

$$W_0^* = \frac{1}{\xi^*} \left(1 + \frac{q_0}{a_0^{*2}}\right) \frac{z^{*3}}{6} \frac{\partial^2 W_0^{(0)*}}{\partial t^{*2}} - (1+q_0) \frac{z^{*3}}{6} \frac{1}{\xi^*} \frac{\partial^2 W_0^{(0)*}}{\partial x^{*2}} + \frac{1}{\xi^*} z^* W_0^{(0)*} +$$

$$+ \frac{q_0}{a_0^{*2}} \frac{z^{*3}}{6} \frac{\partial^3 U_0^{(0)*}}{\partial t^{*2} \partial x^*} - q_0 \frac{z^{*3}}{6} \frac{\partial^3 U_0^{(0)*}}{\partial x^{*3}};$$

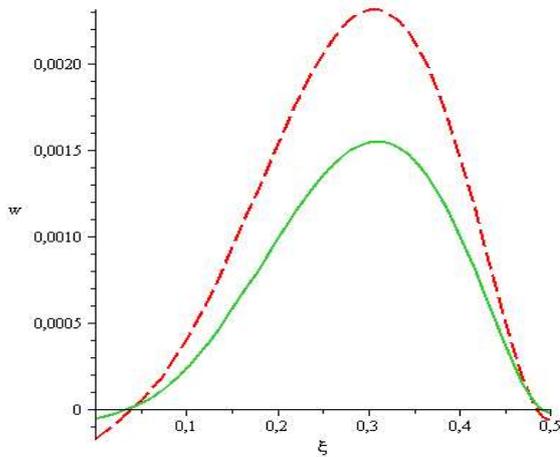
(6)

Bu (6) tenglamalar sistemasi bir qatlamli plastinkada yuzaga keladigan ko’chishlarni ifodalaydi.

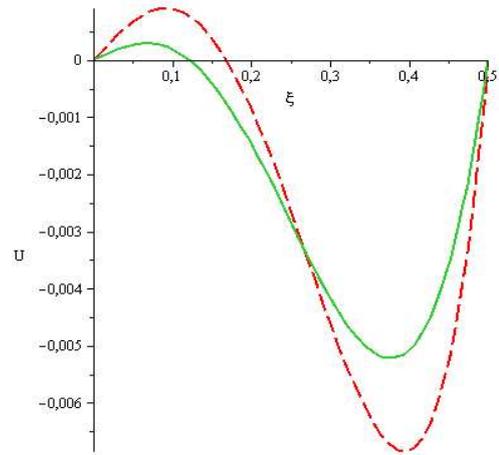
Olingan natijalar. Bu (6) ko’rinishdagi ko’chishlar ifodalarini topilgan izlanuvchi funksiyalardan foydalangan holda grafikda tasvirlash uchun bir qatlamli plastinka geometrik va mexanik parametrlarni yana (5) ko’rinishda kiritamiz va (6) ifodalarni “Maple 12” dasturi yordamida yechib quyidagi grafiklarni olamiz.

Bunda 4-rasmda bir qatlamli plastinka materiali nuqtalarining z o’qi bo’ylab yo’nalgan w_0^* ko’chishi grafigi tasvirlangan bo’lsa, 5-rasmda esa bir qatlamli plastinka materiali nuqtalarining x o’qi bo’ylab yo’nalgan U_0^* ko’chishi grafigi tasvirlangan. Iki

holatda ham plastinka materiali alyuminiydan iborat. Faqatgina 4- va 5 - rasmlardagi uzlukli chiziqlar (6) ifodalardagi z^* o'rniga $h/2$ qiymatni qo'ygan vaqti



4-Rasm



5-Rasm

kelib chiqqan bo'lsa, uzluksiz chiziqlar esa (6) ifodalardagi z^* o'rniga $h/3$ qiymatni qo'ygan vaqti kelib chiqqan.

Adabiyotlar

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