$M|G|1|N$ XIZMAT KO‘RSATISH TARMOG‘I NOSTATSIONAR NAVBAT UZUNLIGINING TAQSIMOTI HAQIDA

О РАСПРЕДЕЛЕНИИ НЕСТАЦИОНАРНОЙ ДЛИНА ОЧЕРЕДИ СИСТЕМА ОБСЛУЖИВАНИЯ $M|G|1|N$

ON THE DISTRIBUTION OF NON STATSIONARY QUEUE LENGTH OF THE QUEUE SYSTEM $M|G|1|N$

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Annotatsiya. Ishda kutish joylari chegaralangan xizmat ko‘rsatish tarmog‘i nostatsionar navbat uzunligi taqsimotining Laplas almashtirishi topilgan. Ushbu ifodadan statsionar navbat uzunligi taqsimotini aniqlovchi formula keltirib chiqarilgan.

Аннотация. В работе найдено преобразование Лапласа для расределения нестационарной длины очереди система с ограниченный очередью. Из этого виражения выводится формула, определающая распределения стационарной длина очереди.

Annotation. Laplace’s transformation for distribution of non stationary length of turn system with limited turn is found in work. The formula defining distributions stationary turn length is brought out of this exspression.

Kalit so‘zlar. Xizmat ko‘rsatish tarmog‘i, nostatsionar navbat uzunligi, statsionar navbat uzunligi, bandlik davri.

Ключовые слова.Система обслуживания, нестационарная длина очереди, стационарная длина очереди, период занятости.

Keywork. Queue systems, non stationary queue size (length), stationary queue length (size), busy period.

Bitta xizmat ko‘rsatish qurilmasidan iborat tarmoqga λ parametrli Puasson oqimini tashkil etuvchi talablar kelib tushayotgan bo‘lsin. Talablarga ularning kelish tartibida xizmat ko‘rsatiladi va xizmat ko‘rsatish vaqti uzunliklari o‘zaro bog‘liq bo‘lmagan tasodifiy miqdorlar bo‘lib, bir xil $B(x)$ $[B(+0) = 0]$ taqsimot funksiyasiga ega. Kutish joylar soni N ga $(N \geq 1)$, ya’ni tarmoqda bir vaqtning o‘zida qurilmadagi bilan birga $N+1$ ta talab bo‘lishi mumkin. Qaralayotgan ushbu tarmoq $M|G|1|N$ orqali belgilanadi.

Xizmat jarayoni $t=0$ momentda tarmoqda j ta $(j>0)$ talab mavjud bo‘lgan holda, ulardan birining xizmat ko‘rsatish qurilmasiga tushishi bilan boshlansin. Quyidagi belgilashlarni kiritamiz:

$\xi_N^{(j)}(t)-t$ momentda tarmoqda mavjud bo‘lgan talablar soni,

$\xi_N^{(j)}-$ boshlang‘ich bandlik davri, ya’ni $t=0$ momentdan tarmoq birinchi marta talablardan bo‘shagan momentgacha bo‘lgan vaqt uzunligi. Bu yerda va bundan keyin qavsga olingan yuqori indeks $t=0$ momentda sistemada mavjud bo‘lgan talablar sonini bildiradi.

$$P^{(j)}(k,t)=P\left(\xi_N^{(j)}(t)=k, \varsigma_N^{(j)}\geq t\right), \qquad k=\overline{1,N+1}\ ,$$

$$\overline{P}^{(j)}(k,s)=\int_0^\infty e^{-st}\,P^{(j)}(k,t)dt\,,\quad k=\overline{1,N+1}\,,\quad Res>0\,,$$

$$\hat{\overline{P}}^{(j)}(v,s)=\sum_{k=1}^{N+1} v^k\,\overline{P}^{(j)}(k,s)\,,\quad |v|\leq 1\,,$$

$$g_N^{(j)}(t)=\frac{d}{dt}P\left(\varsigma_N^{(j)}<t\right),$$

$$\overline{g}_N^{(j)}(s)=\int_0^\infty e^{-st}\,g_N^{(j)}(t)dt\,,\ \ Res>0$$

$$\overline{b}(s)=\int_0^\infty e^{-st}\,dB(t),$$

$$\Pi^{(j)}(k,t)=P\left(\xi_N^{(j)}(t)=k\right),$$

$$\overline{\Pi}^{(j)}(k,s)=\int_0^\infty e^{-st}\,\Pi^{(j)}(k,t)dt\,,\ \ Res>0\,,$$

$$\hat{\overline{\Pi}}^{(j)}(v,s)=\sum_{k=0}^{N+1} v^k\,\overline{\Pi}^{(j)}(k,s)\,,\quad |v|\leq 1\,.$$

[1] ishda $\hat{\hat{\Pi}}^{(0)}(v,s)$ funksiyani aniqlovchi formulalar keltirib chiqarilgan. Shuningdek, ushbu ishda $\lim\limits_{s\to 0} s\cdot\hat{\hat{\Pi}}^{(0)}(v,s)$ mavjudligi ham isbotlangan.

Qaralayotgan ishda $\Pi^{(j)}(v,s)$ funksiya $j>0$ bo‘lgan hol uchun aniqlanadi va [1] ishda olingan natija boshqa usulda keltirib chiqariladi. Shuni qayd etib o‘tamizki, [2] ishda $\hat{\hat{P}}^{(j)}(v,s)$ funksiya aniq ko‘rinishda topilgan.

1-Teorema. $|v|\leq 1$ va $Res\geq 0$ da barcha $j=\overline{1,N+1}$ lar uchun quyidagi formula o‘rinli:

$$\hat{\hat{\Pi}}^{(j)}(v,s)=\frac{\overline{g}_N^{(j)}(s)}{s+\lambda-\lambda\overline{g}_N^{(j)}(s)}\cdot\left\{1+\lambda\hat{\hat{P}}^{(1)}(v,s)+\hat{\hat{P}}^{(j)}(v,s)\right\},$$

bu yerda $\overline{g}_N^{(j)}(s)$ [2] ishda aniqlangan, ya’ni

$$\overline{g}_N^{(j)}(s)=\frac{\Delta_{N-j}(s)}{\Delta_N(s)},$$

$$\sum_{k=0}^{\infty}v^k\Delta_k=\frac{v\overline{b}(s)-\overline{b}(s+\lambda-\lambda v)}{(1-v)(v-\overline{b}(s+\lambda-\lambda v))}.$$

Isbot. $\Pi^{(j)}(k,t)$ va $P^{(j)}(k,t)$ ehtimollarni bir-biriga quyidagicha bog‘lash mumkin. Avval $k\geq 1$ deb hisoblaylik. U holda

$$\Pi^{(j)}(k,t)=P\left(\xi_N^{(j)}(t)=k\right)=P\left(\xi_N^{(j)}(t)=k,\varsigma_N^{(j)}\geq t\right)+$$
$$+P\left(\xi_N^{(j)}(t)=k,\varsigma_N^{(j)}<t\right). \qquad (1)$$

$k\geq 1$ da

$$P\left(\xi_N^{(j)}(t)=k,\varsigma_N^{(j)}<t\right)=P\left(\xi_N^{(j)}(t)=k,\varsigma_N^{(j)}+æ_1<t\right),$$

bu yerda $æ_1-$ tarmoqning birinchi bo‘sh holatdagi davri. U holda (1) ushbu ko‘rinishga keladi:

$$\Pi^{(j)}(k,t)=P^{(j)}(k,t)+P\left(\xi_N^{(j)}(t)=k,\varsigma_{1N}^{(j)}+æ_1<t\right). \qquad (2)$$

$\varsigma_{1N}^{(j)}$, $\varsigma_2^{(1)}$, $\varsigma_3^{(1)},\dots$ tarmoqning bandlik davrlari va $æ_1,æ_2,\dots$ tarmoqning bo‘sh holatdagi davrlari ketma-ketligi bo‘lsin.

Quyidagi belgilashni kiritamiz:

$$\mathcal{E}_1 = \varsigma_{1N}^{(j)} + æ_1 \,,$$

$$\mathcal{E}_n = \varsigma_n^{(1)} + æ_n \,, \qquad n \geq 2 \,.$$

Ravshanki, $\mathcal{E}_1, \mathcal{E}_2, \dots$ ketma-ketlik qayta tiklanish jarayonini tashkil qiladi. Keluvchi talablar Puasson oqimini tashkil etganligi sababli $\mathcal{E}_1, \mathcal{E}_2, \dots$ lar o‘zaro bir-biriga bog‘liq bo‘lmaydi va $j > 1$ da

$$P(\mathcal{E}_2 < x) = P(\mathcal{E}_3 < x) = \cdots \ , \ x \geq 0$$

va $j = 1$ da

$$P(\mathcal{E}_1 < x) = P(\mathcal{E}_2 < x) = \cdots \ , \ x \geq 0$$

Agar $\mathcal{E}_1$ qayta tiklanish momenti ekanligi e’tiborga olinsa, u holda

$$P\left(\xi_N^{(j)}(t) = k, \ \varsigma_N^{(j)} + æ_1 < t\right) = P\left(\xi_N^{(j)}(t) = k, \ \mathcal{E}_1 \ < t\right) =$$

$$= \int_0^t P\left(\xi_N^{(j)}(t) = k, \ u \leq \mathcal{E}_1 < u + du\right) =$$

$$= \int_0^t P\left(\xi_N^{(1)}(t-u) = k\right) \cdot P(\,u \leq \mathcal{E}_1 < u + du\Big) =$$

$$= \int_0^t \Pi^{(1)}(k, t-u) dP(\mathcal{E}_1 < u).$$

Shunday qilib, quyidagi tenglikni hosil qilamiz:

$$\Pi^{(j)}(k,t) = P^{(j)}(k,t) + \int_0^t \Pi^{(1)}(k, t-u) dP(\mathcal{E}_1 < u).$$

Bu yerdan ($P(æ_1 \geq x) = e^{-\lambda x}\,, x > 0$, ekanligini e’tiborga olib) Laplas almashtirishlariga o‘tamiz va ushbu munosabatni hosil qilamiz:

$$\overline{\Pi}^{(j)}(k,s) = \overline{P}^{(j)}(k,s) + \frac{\lambda}{\lambda + s}\overline{g}_N^{(j)}(s)\overline{\Pi}^{(1)}(k,s)\,. \tag{3}$$

Endi $k = 0$ bo‘lsin. U holda

$$P\left(\xi_N^{(j)}(t) = 0\right) = P\left(\xi_N^{(j)}(t) = 0, \ \mathcal{E}_1 \geq t) + P(\xi_N^{(j)}(t) = 0, \mathcal{E}_1 < t\right) =$$

$$P(\varsigma_N^{(j)} + æ_1 \geq t) + \int_0^t \Pi^{(1)}(0, t-u) dP(\mathcal{E}_1 < u) =$$

$$\int_0^t P(æ_1 > t-u) dP\left(\varsigma_N^{(j)}(t) < u\right) + \int_0^t \Pi^{(1)}(0, t-u) dP(\mathcal{E}_1 < u) .$$

Bu yerdan Laplas almashtirishlariga o‘tib, quyidagi tenglikni hosil qilamiz:

$$\overline{\Pi}^{(j)}(0,s) = \overline{g}_N^{(j)}(s) \frac{1}{\lambda+s} + \overline{g}_N^{(j)}(s) \cdot \frac{\lambda \overline{\Pi}^{(1)}(0,s)}{\lambda+s} . \qquad (4)$$

(3) va (4) tenglikilardan hosil qiluvchi funksiyaga o‘tib, ushbu munosabatga ega bo‘lamiz:

$$\hat{\overline{\Pi}}^{(j)}(v,s) = \hat{\overline{P}}^{(j)}(v,s) + \overline{g}_N^{(j)}(s) \frac{1}{\lambda+s} + \overline{g}_N^{(j)}(s) \frac{\lambda}{\lambda+s} \hat{\overline{\Pi}}^{(1)}(v,s) . \qquad (5)$$

Bu tenglikdan $j = 1$ da $\hat{\overline{\Pi}}^{(1)}(v,s)$ ni topamiz:

$$\hat{\overline{\Pi}}^{(1)}(v,s) = \frac{\overline{g}_N^{(1)}(s)}{s+\lambda-\lambda\overline{g}_N^{(1)}(s)} + \frac{(s+\lambda)\hat{\overline{P}}^{(1)}(v,s)}{s+\lambda-\lambda\overline{g}_N^{(1)}(s)} . \qquad (6)$$

Bunga ko‘ra, (5) dan teoremaning tasdig‘i kelib chiqadi.

2-Teorema. $|v| \leq 1$ va $Res \geq 0$ da ushbu tenglik o‘rinli:

$$\hat{\overline{\Pi}}^{(0)}(v,s) = (s+\lambda)^{-1} + \lambda(s+\lambda)^{-1}\hat{\overline{\Pi}}^{(1)}(v,s) .$$

Isbot. $\Pi^{(0)}(k,t)$ va $\Pi^{(1)}(k,t)$ $(k = \overline{0, N+1})$ lar o‘rtasida quyidagi munosabatni o‘rnatish mumkin:

$$\Pi^{(0)}(k,t) = \int_0^t \Pi^{(1)}(k, t-u) dP(æ_0 < u) , \ \ k = \overline{1, N+1}$$

$$\Pi^{(0)}(0,t) = P(æ_0 \geq t) + \int_0^t \Pi^{(1)}(0, t-u)\, dP(æ_0 < u). \qquad (7)$$

Bu yerda $æ_0$ – tarmoqning boshlang‘ich bo‘sh holatdagi davri. Ushbu tengliklarni 1-teorema isbotidagi usulda oson hosil qilish mumkin.

(7) dan hosil qiluvchi va Laplas almashtirishiga o‘tib,

$$\hat{\hat{\Pi}}^{(0)}(v,s) = (s+\lambda)^{-1} + \lambda(s+\lambda)^{-1}\hat{\hat{\Pi}}^{(1)}(v,s)$$

tenglikni hosil qilamiz. Bu yerda $\hat{\hat{\Pi}}^{(1)}(v,s)$ (6) tenglikda aniqlangan funksiya.

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$M\,|G\,|1\,|N$ XIZMAT KO‘RSATISH TARMOG‘I NOSTATSIONAR NAVBAT UZUNLIGI TAQSIMOTIGA OID AYRIM MUNOSABATLAR

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(Sam DU)

Birkanalli xizmat ko‘rsatish tarmog‘iga kelib tushayotgan talablar $\boldsymbol{\lambda}$ parametrli Puasson oqimini tashkil etsin. Talablarga ularning kelish tartibida xizmat ko‘rsatiladi va xizmat ko‘rsatish vaqti uzunliklari o‘zaro bog‘liq bo‘lmagan tasodifiy miqdorlar bo‘lib, bir xil $B(x)$ $[B(+0)=0]$ taqsimot funksiyasiga va μ^{-1} o‘rta qiymatga ega. Kutish joylar soni $N(N\geq 1)$ ga teng, ya’ni bir paytning o‘zida tarmoqda qurilmadagi bilan birga $N+1$ ta talab bo‘lishi mumkin. Tarmoqda $N+1$ ta talab bo‘lgan holda kelib tushgan talab qabul qilinmaydi va keying xizmat jarayoniga ta’sir ko‘rsatmaydi. Xizmat jarayoni $t=0$ momentda tarmoqda j ta $(j>0)$ talab mavjud bo‘lgan holda, ulardan birining xizmat ko‘rsatish qurilmasiga tushishi bilan boshlanadi. Quyidagi belgilashlarni kiritamiz:

$\xi_N^{(j)}(t) - t$ momentda tarmoqda mavjud bo‘lgan talablar soni,

$\varsigma_N^{(j)} -$ boshlang‘ich bandlik davri, ya’ni $t=0$ momentdan tarmoq birinchi marta talablardan bo‘shagan momentgacha bo‘lgan vaqt uzunligi. Bu yerda va bundan keyin qavsga olingan yuqori indeks $t=0$ momentda tarmoqda mavjud bo‘lgan talablar sonini bildiradi.

$$P^{(j)}(k,t) = P\left(\xi_N^{(j)}(t)=k, \varsigma_N^{(j)} > t\right), \qquad k=\overline{1,N+1}\ ,$$

$$\overline{P}^{(j)}(k,s) = \int_0^\infty e^{-st} P^{(j)}(k,t)dt\,, \quad k=\overline{1,N+1}\,, \quad Res \geq 0\,,$$

$$\hat{\overline{P}}^{(j)}(v,s) = \sum_{k=1}^{N+1} v^k\, \overline{P}^{(j)}(k,s)\,, \quad |v|\leq 1\,,$$

$$\bar{g}_N^{(j)}(t) = \frac{d}{dt} P\left(\varsigma_N^{(j)} < t\right),$$

$$\overline{g}_N^{(j)}(s)=\int_0^\infty e^{-st}\, g_N^{(j)}(t)dt\,,\ \ Res\geq 0$$

$$\overline{b}(s)=\int_0^\infty e^{-st}\, dB(t),\qquad Res\geq 0$$

$$\Pi^{(j)}(k,t)=P\left(\xi_N^{(j)}(t)=k\right),k=\overline{1,N+1}\ ,$$

$$\overline{\Pi}^{(j)}(k,s)=\int_0^\infty e^{-st}\,\Pi^{(j)}(k,t)dt\,,\ \ Res\geq 0\,,$$

$$\widehat{\overline{\Pi}}^{(j)}(v,s)=\sum_{k=0}^{N+1} v^k\,\overline{\Pi}^{(j)}(k,s)\,,\quad |v|\leq 1\,.$$

Ushbu ishda $\overline{\Pi}^{(j)}(k,s)$ funksiyalarni aniqlovchi munosabatlar keltirib chiqariladi. Bu munosabatlar [1] ishda olingan natijalarning nostatsionar holatlar uchun umumlashmasi hisoblanadi, ya'ni $s\to 0$ da [1] ishdagi statsionar holat uchun olingan natijalar kelib chiqadi. Shuningdek, [2] da $\widehat{\overline{P}}^{(j)}(v,s)$ funksiyani aniqlovchi munosabatlar topilgan.

<u>Teorema</u>. $Re\, s\geq 0$ da barcha $j=\overline{1,N}$ lar uchun quyidagi munosabatlar o'rinli:

$$\overline{\Pi}^{(j)}(0,s)=\frac{\Delta_{N-j}}{(s+\lambda)\Delta_N-\lambda\Delta_{N-1}}\,,$$

$$\overline{\Pi}^{(j)}(1,s)=(s+\boldsymbol{\lambda})\boldsymbol{f_1}\overline{\Pi}^{(j)}(0,s)$$

$$\overline{\Pi}^{(j)}(k,s)=\overline{\Pi}^{(j)}(0,s)\{(s+\lambda)f_k-\lambda f_{k-1}\},\quad 2\leq k\leq j\,,$$

$$\overline{\Pi}^{(j)}(k,s)=\overline{\Pi}^{(j)}(0,s)\{(s+\lambda)f_k-\lambda f_{k-1}\}-f_{k-j}\,,\quad j\leq k\leq N\,,$$

$$\overline{\Pi}^{(j)}(N+1,s)=\frac{1-s\overline{\Pi}^{(j)}(0,s)}{s}-\overline{\Pi}^{(j)}(0,s)\{(s+\lambda)\varphi_N-\lambda\varphi_{N-1}\}-\varphi_{N-j}\ \ ,$$

Bu yerda

$$\varphi_n = \sum_{k=1}^{n} f_k \ , \quad f_k = f_k(s) \ \ \boldsymbol{va} \ \Delta_{\boldsymbol{k}} = \Delta_{\boldsymbol{k}}(\boldsymbol{s})$$

lar quyidagi tengliklardan aniqlanadi:

$$f(v,s) = \sum_{k=0}^{\infty} v^k f_k(s) = \frac{v}{s+\lambda-\lambda v} \cdot \frac{1-\bar{b}(s+\lambda-\lambda v)}{\bar{b}(s+\lambda-\lambda v)-v} \ , \tag{1}$$

$$\Delta(v,s) = \sum_{k=0}^{\infty} v^k \Delta_k(s) = \frac{v\bar{b}(s)-\bar{b}(s+\lambda-\lambda v)}{(1-v)(v-\bar{b}(s+\lambda-\lambda v))} \ . \tag{2}$$

Isbot. [2] ishda $\hat{\bar{P}}^{(j)}(v,s)$ funksiya uchun quyidagi ko‘rinish topilgan:

$$\hat{\bar{P}}^{(j)}(v,s) = \frac{v^{j+1} - v\bar{g}_N^{(j)}(s) + \lambda(1-v)v^{N+1}\bar{D}^{(j)}(v,s)}{v-\bar{b}(s+\lambda-\lambda v)} \times$$

$$\times \frac{1-\bar{b}(s+\lambda-\lambda v)}{s+\lambda-\lambda v} \ , \tag{3}$$

bu yerda $|v| \le 1$ va $Res \ge 0$ da $\left|\bar{D}^{(j)}(v,s)\right| < \infty$. (1) dan foydalanib, (3) ni ushbu ko‘rinishda yozib olamiz:

$$\hat{\bar{P}}^{(j)}(v,s) = \sum_{k=1}^{N+1} v^k \bar{P}^{(j)}(k,s) =$$

$$= \left[\bar{g}_N^{(j)}(s) - v^j\right] f(v,s) + \lambda(1-v)v^{N+1}\bar{D}^{(j)}(v,s)f(v,s) \tag{4}$$

Agar $(1-v)v^{N+1}\bar{D}^{(j)}(v,s)f(v,s)$ funksiyani v ning darajalari bo‘yicha qatorga yoysak, ushbu qatorda daraja ko‘rsatkichlari N dan katta bo‘lgan qo‘shiluvchilar ishtirok etadi. Shu sababli, $\bar{P}^{(j)}(k,s)$, $k = \overline{1,N}$ funksiyalarni aniqlash uchun

$$\left[\bar{g}_N^{(j)}(s) - v^j\right] f(v,s)$$

funksiyani qatorga yoyish yetarli. Darajali qatorlarning tenglik shartiga ko‘ra (4)dan ushbu munosabatlarga ega bo‘lamiz:

$$\bar{P}^{(j)}(k,s) = \bar{g}_N^{(j)}(s) f_k \ , \qquad 1 \le k \le j \ , \tag{5}$$

$$\bar{P}^{(j)}(k,s) = \bar{g}_N^{(j)}(s) f_k - f_{k-j} \ , \ \ j < k \le N \ .$$

$v = 1$ da (3) dan

$$\hat{\overline{P}}^{(j)}(1,s) = \sum_{k=1}^{N+1} \overline{P}^{(j)}(k,s) = \frac{1-\overline{g}_N^{(j)}(s)}{s}$$

yoki

$$\overline{P}^{(j)}(N+1,s) = \frac{1-\overline{g}_N^{(j)}(s)}{s} - \sum_{k=1}^{N} \overline{P}^{(j)}(k,s) \qquad (6)$$

tenglikni hosil qilamiz.

Endi [3] ishda $\hat{\overline{\Pi}}^{(j)}(v,s)$ funksiya uchun aniqlangan munosabatni ushbu ko‘rinishda yozib olamiz:

$$\sum_{k=0}^{N+1} v^k \overline{\Pi}^{(j)}(k,s) =$$

$$= \sum_{k=1}^{N+1} v^k \overline{P}^{(j)}(k,s) + \frac{\overline{g}_N^{(j)}(s)}{s+\lambda-\lambda\overline{g}^{(1)}(s)} \cdot \left[1 + \lambda \sum_{k=1}^{N+1} v^k \overline{P}^{(1)}(k,s)\right].$$

Bu yerdan darajali qatorlar tenglik shartiga ko‘ra quyidagilarni hosil qilamiz:

$$\overline{\Pi}^{(j)}(0,s) = \frac{\overline{g}^{(j)}(s)}{s+\lambda-\lambda\overline{g}^{(1)}(s)} \ , \qquad (7)$$

$$\overline{\Pi}^{(j)}(k,s) = \overline{P}^{(j)}(k,s) + \frac{\lambda\overline{g}^{(j)}(s)}{s+\lambda-\lambda\overline{g}^{(1)}(s)} \overline{P}^{(1)}(k,s) \, , \qquad k = \overline{1, N+1}$$

[4] ishda quyidagi formula keltirilgan:

$$\overline{g}^{(j)}(s) = \frac{\Delta_{N-j}}{\Delta_N} \, . \qquad (8)$$

(5) , (6) va (8) tengliklarga ko‘ra (7) dan teoremaning isboti kelib chiqadi.

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