SAMARQAND DAVLAT UNIVERSITETI

MAGISTRANTLARNING XVIII-ILMIY KONFERENSIYASI MATERIALLARI

(ANIQ, TABBIIY VA IJTIMOIY-GUMANITAR FANLAR)

O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

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*SamDU Ilmiy texnik
kengashida muhokama qilingan
va nashrga tavsiya etilgan
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u vaqtda

$$\frac{1}{a_k}\sum_{k=1}^{\infty} b_k x_k \to x.$$

Lemma 2. Faraz qilaylik $\sum_{k=1}^{\infty} x_k$ yaqinlashuvchi, $b_k \uparrow \infty$. U vaqtda

$$\frac{1}{a_k}\sum_{k=1}^{\infty} b_k x_k \to x \to 0.$$

Teorema 2. Faraz qilaylik $\{X_n\}$, $\{g_n(x)\}$ va, $\{a_n\}$ lar 1- teoremaning barcha shartlarini qanoatlantiruvchi ketma-ketlik bo'lsin va $a_k \uparrow \infty$ bo'lsin. Agar

$$\sum_{n=1}^{\infty} \frac{M^{F_1} g_n(x)}{g_n(a_n)} < \infty$$

bo'lsa, $\frac{S_n}{a_n} \to 0$ deyarli yaqinlashadi.

Bu teoremaning isboti teorema 1 va lemma 2 dan kelib chiqadi [2].

Bu teoremada hamma n lar uchun $g_n(x) = g(x)$ desak $g(x) = |x|^p$,p>0 deb olsak, quydagi teoremaga kelamiz.

Teorema 3. Faraz qilaylik $\{X_n\}$ matematek kutilmasi nolga teng, o'zaro bog'liq bo'lmagan tasodifiy miqdorlar ketma-ketligi bo'lsin. Agar $a_k \uparrow \infty$ bo'lsa va

$$\sum_{k=1}^{n} \frac{M^{F_1}|x_n|^p}{a_p^n} < \infty$$

Qandaydir p, $1 < p < 2$ lar uchun yaqinlashuvchi bo'lsa, u vaqtda $\frac{S_n}{a_n}$ deyarli yaqinlashuvchi .

ADABIYOTLAR

1. Я. Х. Кучкаров Вероятностные расределения со значениями в просиданствах измеремых функций . – Ташкент , ФАН , 2007 г. – 176 стр .
2. A.Abdushukurov, T.Zuparov. "Ehtimollar nazariyasi va matematik statis-tika" Darslik. Tafakkur-Bo'stoni.Toshkent, 2015.

DISKRET HARDI TENGSIZLIGI

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Mutaxassislik: Ehtimollar nazariyasi va matematik statistika (1-2-bosqich)

Annotatsiya: *Ushbu maqolada Hardi tipidagi tengsizlikning yadrosi o'rta vazn matritsa bo'lganda o'rinli bo'lishi ko'rsatilgan.*

Mashhur Hardi tengsizligi quyidagi ko'rinishga ega:

$$\sum_{n=1}^{\infty}\left|\frac{1}{n}\sum_{k=1}^{n} a_k\right|^p \le \left(\frac{p}{p-1}\right)^p \sum_{k=1}^{\infty}|a_k|^p, \qquad (1.1)$$

bunda $p > 1$, $\{a_k\}$ –ixtiyoriy sonli ketma-ketlik. [1] da bu tengsizlik va uning ba'zi umumlashmalari to'g'risida batafsil natijalar keltirilgan. Bundan tashqari hozirgi zamon matematikasida ushbu ko'rinishdagi tengsizlikni

$$\sum_{j=1}^{\infty}\left|\sum_{k=1}^{n} c_{j,k} a_k\right|^p \le C \sum_{n=1}^{\infty}|a_k|^p \qquad (1.2)$$

o'rganish zaruriyati paydo bo'lmoqda. Bunda $\left(c_{j,k}\right)_{j,k=1}^{\infty}$ -haqiqiy cheksiz o'lchovli matritsadir. (1.2) tengsizlik ba'zi operatorlarning spektral xossalari haqida ham muhim xulosalar chiqarishda ishlatilishi mumkin. Biz bu ishda (1.2) tengsizlikning xususiy hollarda o‘rinli bo‘lish masalasi bilan shug‘ullanamiz.

Ta’rif. Agar $A = \left(c_{j,k}\right)_{j,k=1}^{\infty}$ matritsaning elementlari barcha $k > j$ uchun $c_{j,k} \geq 0, k \leq j, \sum_{k=1}^{j} c_{j,k} = 1$ shartlarni qanoatlantirsa A matritsani jamlanuvchi deb ataymiz. Agar A jamlanuvchi matritsaning elementlari ushbu ko‘rinishga ega bo‘lsa, ya’ni shunday $\{\lambda_i\}_{i=1}^{\infty}$ ketma-ketlik mavjudki, bunda $\lambda_1 > 0$ va $\lambda_i \geq 0,\ (i \geq 2)$

$$c_{j,k} = \frac{\lambda_k}{\Lambda_j}, 1 \leq k \leq j, \qquad \Lambda_j = \sum_{i=1}^{j} \lambda_j,$$

bo‘lsa u holda A matrisaga o‘rta vaznli matritsa deyiladi.

Teorema. *(1.2) tengsizlik ixtiyoriy o‘rta vaznli A matritsa uchun o‘rinli.*

Izoh. (1.2) tengsizlikni $\left(c_{j,k}\right)_{j,k=1}^{\infty}$ o‘rta vaznli matritsa bilan ehtimollar nazariyasidagi katta sonlar qonuni uchun ba’zi natijalarni olishda foydalanish mumkin.

Teoremaning isboti. Faraz qilaylik, (1.2) tengsizlik o‘rinli bo‘lsin va faraz qilaylik $\{w_i\}_{i=1}^{\infty}$ ixtiyoriy musbat ketma-ketlik bo‘lsin. U holda ushbu Gyolder tengsizligi o‘rinli

$$\left(\sum_{k=1}^{n} \lambda_k |a_k|\right)^p = \left(\sum_{k=1}^{n} \lambda_k |a_k| w_k^{-\frac{1}{p^*}} \cdot w_k^{\frac{1}{p^*}}\right)^p$$

$$\leq \left(\sum_{k=1}^{n} \lambda_k{}^p |a_k|^p w_k{}^{-(p-1)}\right)\left(\sum_{j=1}^{n} w_j\right)^{p-1}.$$

Bundan va (1.2) dan foydalanib quyidagini hosil qilamiz

$$\sum_{n=1}^{\infty}\left|\frac{1}{\Lambda_n}\sum_{k=1}^{n}\lambda_k a_k\right|^p \leq \sum_{n=1}^{\infty}\frac{1}{\Lambda_n^p}\left(\sum_{k=1}^{n}\lambda_k{}^p |a_k|^p w_k{}^{-(p-1)}\right)\left(\sum_{j=1}^{n} w_j\right)^{p-1}$$

$$= \sum_{k=1}^{\infty} w_k{}^{-(p-1)} \lambda_k{}^p \left(\sum_{n=k}^{\infty}\frac{1}{\Lambda_n^p}\left(\sum_{j=1}^{n} w_j\right)^{p-1}\right)|a_k|^p.$$

Faraz qilaylik, har bir $p > 1$ uchun musbat o‘zgarmas C mavjud bo‘lsin va w ketma-ketlik w_n^{p-1}/λ_n^p hadlari 0 ga intilsin. U holda barcha $n \geq 1$ butun sonlar uchun

$$(w_1 + \cdots + w_n)^{p-1} < C\Lambda_n^p\left(\frac{w_n^{p-1}}{\lambda_n^p} - \frac{w_{n+1}^{p-1}}{\lambda_{n+1}^p}\right),$$

o‘rinli bo‘ladi. Bundan oxirgi bahoni quyidagicha davom ettirish mumkin

$$\sum_{n=1}^{\infty}\left|\frac{1}{\Lambda_n}\sum_{k=1}^{n}\lambda_k a_k\right|^p \leq \sum_{k=1}^{\infty} w_k{}^{-(p-1)}\lambda_k{}^p\left(\sum_{n=k}^{\infty}\frac{1}{\Lambda_n^p}C\Lambda_n^p\left(\frac{w_n^{p-1}}{\lambda_n^p} - \frac{w_{n+1}^{p-1}}{\lambda_{n+1}^p}\right)\right)|a_k|^p$$

$$= C\sum_{k=1}^{\infty} w_k{}^{-(p-1)}\lambda_k{}^p \frac{w_k^{p-1}}{\lambda_k^p}|a_k|^p = C\sum_{k=1}^{\infty}|a_k|^p.$$

Teorema isbot bo‘ldi.

Adabiyotlar

1. A.Kufner, L.Maligranda, L-E.Persson. The Hardy inequality - About its history and some Related Serults. Vydavatelský Servis, Plzeń (2007).
2. A.Kufner, L-E.Persson. Weighted Inequlities of Hardy Type. World Scientific Publishing Co., Inc., River Edge, Nj (2003).

EHTIMOLLARNI QO‘SHISH TEOREMASINING UMUMLASHMASI

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Mutaxassislik: 5A130102-Ehtimollar nazariyasi va matematik statistika (1-bosqich)

Annotatsiya. *Ushbu ishda ixtiyoriy tajribalar uchun polinomial taqsimot umumlashtiriladi va uning xususiy holi sifatida bog‘liqsiz tajribalar ketma-ketligi qaralgan holda polinomial taqsimot keltirib chiqariladi.*

Quyidagi hodisalar sistemasini qaraymiz.

$$\left\{\begin{array}{ll} A_1^{(1)}, A_2^{(1)}, \dots, A_N^{(1)} & (1') \\ A_1^{(2)}, A_2^{(2)}, \dots, A_N^{(2)} & (1'') \\ \text{---------------} & \\ A_1^{(k-1)}, A_2^{(k-1)}, \dots, A_N^{(k-1)} & (1^{k-1}) \end{array}\right. \qquad (1)$$

Faraz qilaylik, biror tajribada ushbu hodisalardan ko‘pi bilan N tasi ro‘y berishi mumkin bo‘lsin, agar ξ_i $(i=\overline{1,k-1})$ ro‘y bergan $(1^{(i)})$ tipdagi hodisalar soni bo‘lsa, ushbu shartlar bajarilsin:

$$\begin{cases} 0 \le \xi_i \le N, & i=\overline{1,k-1} \\ 0 \le \xi_1+\xi_2+\dots+\xi_{k-1} \le N \end{cases}$$

Teorema. Tajribada m_1 ta $(1')$ tipdagi, m_2 ta $(1'')$ tipdagi va h.k. m_{k-1} *ta* $(1^{(k-1)})$ tipdagi hodisalarning ro‘y berish ehtimoli quyidagi formula bilan aniqlanadi:

$$P_N(m_1,\dots,m_{k-1}) = \sum_{r=0}^{m_k} (-1)^r \sum_{i_1=0}^{r} \sum_{i_2=0}^{r-i_1} \dots \sum_{i_{k-2}}^{r-i_1-m-i_{k-3}} S_r(m_1,\dots,m_{k-1},i_1,\dots,i_{k-2})$$

$$\cdot \prod_{j=1}^{k-r} C_{m_j+i_j}^{m_j} C_{m_{k-1}+r-i_1-\dots-i_{k-2}}^{m_{k-1}} \qquad (2)$$

bu yerda $m_k = N - m_1 - \dots - m_{k-1}$,

$$S_r(m_1,\dots,m_{k-1},i_1,\dots,i_{k-2}) =$$
$$= \sum_{j^{(-1)},j^{(k-1)}} PA_{j_1^{(1)}} \cdot \dots \cdot A_{j_{m_1-i}^{(1)}} \cdot \dots \cdot A_{j_1^{(k-1)}}^{(k-1)} \cdot \dots \cdot A_{j_m^{(k-1)}}^{(k-1)}(k-1)+r-i_1-\dots-i_{k-2}$$

(1.2) munosabatdan $k=2$ da [1] ishning 4-bo‘limida isbotlangan (1.3.1) formula, k=3 da esa [3] ishda isbotlangan formula kelib chiqadi.

Agar (1) hodisalar o‘zaro bog’liq bo‘lmasa va $A_r^{(i)} \cdot A_r^{(j)} = \emptyset, i \ne j,$ $i,j=\overline{1,k-1}$ hamda $P(A_1^{(i)}) = P(A_2^{(i)}) = \dots = P(A_N^{(i)}) = P_i, i=\overline{1,k-1},$ shartlar bajarilsa, (2) formuladan quyidagi taqsimlot kelib chiqadi:

$P_N(m_1,m_2,\dots,m_{k-1}) = \frac{N!}{m_1!m_2!\dots m_k!} P_1^{m_1} P_2^{m_2} \dots P_k^{m_k}$ bu yerda $P_k = 1 - P_1 - \dots - P_{k-1}$.

Teoremaning isboti. Teoremani qo‘shib va chiqarib tashlash metodi yordamida isbot qilamiz. $\Omega = \{\omega\}$ elmentar hodisalar fazosi va

$G = \{\xi_1 = m_1, \xi_2 = m_2, \dots, \xi_{k-1} = m_{k-1}\}$

bo‘lsin. U holda ehtimolning xossasiga ko‘ra