ANIQ FANLAR

***F.M.Almurotov***

***D.Sh.Mirubaydova***

*Samarqand davlat universiteti 1-bosqich magistrantlari*

# QO'ZG'ALISHLI BA'ZI MODEL OPERATORLARINING DISKRET SPEKTRI

**Annotatsiya:** Mazkur ishda

$$\mathrm{H}_{\mu\lambda} = \mathrm{H}_0 - \mu V_1 - \lambda \mathrm{V}_2, \qquad \mu\lambda \geq 0,$$

ko'rinishdagi operatorlarning xos qiymatlari hamda ularning joylashgan o'rni tavsiflangan.

***Kalit so'zlar:*** *Operator, tor, spektr, Veyl teoremasi, Hilbert fazosi.*

**Abstract:** In the present work, the number and location of discrete spectrum of model operators are studied.

$$\mathrm{H}_{\mu\lambda} = \mathrm{H}_0 - \mu V_1 - \lambda \mathrm{V}_2, \qquad \mu\lambda \geq 0,$$

***Key words:*** *Operator, tor, spectrum, Veil's theorem, Hilbert space.*

**Аннотация:** В настоящей работе изучается число и местоположения собственных значений модельного оператора.

$$\mathrm{H}_{\mu\lambda} = \mathrm{H}_0 - \mu V_1 - \lambda \mathrm{V}_2, \qquad \mu\lambda \geq 0,$$

***Ключевые слова:*** *Оператор, тор, спектр, теорема Вейля, Гильбертово пространство.*

$T = (-\pi; \pi]$ — bir o'lchamli tor bo'lsin. Eslatib o'tamizki, torda qo'shish va songa ko'paytirish amallari haqiqiy sonlarni $2\pi$ modul bo'yicha qo'shish va songa ko'paytirish sifatida kiritiladi. [1]. $L^2(T)$ orqali T da aniqlangan, Haar ma'nosida o'lchovli va kvadrati bilan integrallanuvchi funksiyalarning Hilbert fazosi ni qaraymiz. Bu fazoda aniqlangan quydagi operatorni qaraymiz:

$$H_{\mu\lambda} = H_0 - \mu V_1 - \lambda V_2$$

Bunda $\mu, \lambda \in R$ va $H_0 - L^2(T)$ da $cos\, p$ funksiyaga ko'paytirish operatori

$$(H_0 f)(p) = cosp\, f(p)$$

$V_1$ va $V_2$ lar esa quydagi musbat operatorlar:

$$(V_1 f)(p) = \int_{-\pi}^{\pi} f(q)dq, \quad (V_2 f)(p) = cosp \int_{-\pi}^{\pi} cosq\, f(q)dq.$$

**Eslatma 1.** $H_{\mu\lambda}$ operator $L^2(T)$ da chiziqli, chegaralangan va o'z-o'ziga qo'shma operatordir.

**2.** Muhim spektri haqidagi Veyl teoremasi [2] ga asosan $H_{\mu\lambda}$ ning muhim spektri

$$\sigma_{ess}(H_{\mu\lambda}) = \sigma(H_0) = [-1,1]$$

dan iborat.

**3.** Minimaks prinspi [3] ga asosan $H_{\mu\lambda}$ ning muhim spekrtdan tashqarida ko'pi bilan 2 ta xos qiymati mavjud.

Quydagi teoremalar mazkur ishning asosiy natijasidir.

**Teorema 1.** ***a)*** *$H_{\mu 0} \mu > 0$ va $H_{0\lambda}$ $\lambda > 0$ operatorlar $(-\infty; -1)$ da mos ravishda yagona $E_1(\mu)$ $va$ $E_1(\lambda)$xos qiymatlarga ega.*

***b)*** *$H_{\mu 0}$, $\mu < 0$ va $H_{0\lambda}$, $\lambda < 0$ operatorlar $(1; +\infty)$ da mos ravishda yagona $E_2(\mu)$ va $E_2(\lambda)$ xos qiymatlarga ega.*

Quydagi to'plamlarni kiritamiz.

$$G_1^+ := \{ (\mu,\lambda) \in R^+ \times R^+ \quad 2\pi\mu\lambda - \mu - \lambda > 0 \}$$

$$G_2^+ := \{ (\mu,\lambda) \in R^+ \times R^+ \quad 2\pi\mu\lambda - \mu - \lambda < 0 \}$$

$$G_3^+ := \{ (\mu,\lambda) \in R^+ \times R^+ \quad 2\pi\mu\lambda - \mu - \lambda = 0 \}$$

$$G_1^- := \{ (\mu,\lambda) \in R^- \times R^- \quad - 2\pi\mu\lambda - \mu - \lambda > 0 \}$$

$$G_2^- := \{ (\mu,\lambda) \in R^- \times R^- \quad - 2\pi\mu\lambda - \mu - \lambda < 0 \}$$

$$G_3^- := \{ (\mu,\lambda) \in R^- \times R^- \quad - 2\pi\mu\lambda - \mu - \lambda = 0 \}$$

**Teorema 2.** *Quydagi tasdiqlar o'rinli:*

***a).*** *Agar $(\mu,\lambda)\epsilon G_1^+$ bo'lsin. U holda $H_{\mu\lambda}$ operator -1 dan chapda ikkita $z_1(\mu,\lambda)$ va $z_2(\mu,\lambda)$ xos qiymatga ega va bu xos qiymatlar uchun*

$$z_1(\mu,\lambda) < \min\{E_1(\mu), E_1(\lambda)\} < z_2(\mu,\lambda) < \max\{E_1(\mu), E_1(\lambda)\} < -1$$

*tengsizliklar o'rinli.*

***b).*** *Agar $(\mu,\lambda)\epsilon G_2^+ \cup G_3^+$ bo'lsin. U holda $H_{\mu\lambda}$ operator -1 dan chapda yagona $z(\mu,\lambda)$ xos qiymatga ega. Shuningdek*

$$z(\mu,\lambda) < \min\{E_1(\mu), E_1(\lambda)\}$$

***c).*** *Agar $(\mu,\lambda)\epsilon G_1^-$ bo'lsin. U holda $H_{\mu\lambda}$ operator 1 dan o'ngda ikkita $u_1(\mu,\lambda)$ va $u_2(\mu,\lambda)$ xos qiymatga ega va bu xos qiymatlar uchun*

$$1 < u_1(\mu,\lambda) < \min\{E_2(\mu), E_2(\lambda)\} < u_2(\mu,\lambda) < \max\{E_2(\mu), E_2(\lambda)\}$$

*tengsizliklar o'rinli.*

***d).*** *Agar* $(\mu,\lambda)\epsilon G_2^- \cup G_3^-$ *bo'lsin. U holda* $H_{\mu\lambda}$ *operator 1 dan o'ngda yagona* $u(\mu,\lambda)$ *xos qiymatga ega. Shuningdek*

$$\max\{E_2(\mu), E_2(\lambda)\} < u(\mu,\lambda).$$

FOYDALANILGAN ADABIYOTLAR.


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