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Chegaralangan sohada birinchi tartibli elliptik sistema uchun Koshi masalasi

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Annotatsiya. Bu ishda chegaralangan yarim sharda birinchi tartibli elliptik sistema uchun Koshi masalasi yechimi keltirilgan.

Kalit so'zlar: Chegaralangan soha, Gel'mgol's tenglamasi, integral formula, Koshi masalasi.

Задача Коши Для эллиптической системы первого порядка в ограниченной области

Аннотация. В данной работе приведено решение задачи Коши для эллиптической системы первого порядка в ограниченном **пространстве** полу шара.

Ключевые слова: Ограниченная область, уравнения Гельмгольца, Интегральная формула, задача Коши.

The Cauchy problem for the first order elliptic equation system in a bounded domain

Abstract. In this work we give a solution of the Cauchy problem for the first order elliptic system in bounded of half-ball.

Keywords: Bounded domain, the Helmholtz equation, integral formula, Cauchy problem.

Bu ishda birinchi tartibli elliptik tipli Ge'lmgol's operatori bilan faktorizatsiyalanuvchi tenglamalar sistemasi uchun Koshi masalasi qaralgan.

Ma'lumki, Koshi masalasi elliptik tipli tenglamalar sistemasi uchun korrekt bo'lmagan masalalar qatoriga kiradi. Bunday masalalarning yechimi turg'un bo'lmaydi. Uning turg'unlik darajasi xuddi Laplas tenglamasi uchun qo'yilgan Koshi masalasidek bo'ladi. Bunga misol sifatida Adamar misolini keltirish mumkin. Agar masalaning yechimini kompakt to'plamgacha qisqartirsak u holda masalani yechimi shartli korrekt masalalar hisoblanadi. Bunday masalalarni yechish uchun Karleman funksiyasini tuzish lozim, bu funksiyaga Koshining berilganlaridagi soha chegarasining qolgan qismida o‘zi va uning hosilasi cheksiz kichik bo‘lishi lozim. Bunday funksiyalar tuzilsa yechim integral formada oshkor ko‘rinishda ifodalanadi.

Analitik funksiyalar uchun Koshi masalasini Adamar, Goluzin, Krilovlar tomonidan yechilgan. Laplas tenglamasi uchun qo‘yigan Koshi masalasini
V.K. Ivanov, M.M. Lavrent’ev, L.A. Ayzenberg, Sh. Yarmuxamedov, N.N. Tarxanov va boshqalar tomonidan yechilgan. Prof. Sh. Yarmuxamedov tomonidan oshkor ko‘rinishida Karleman funksiyasini tuzish yo‘lidan foydalanib elliptik tipli tenglamalar sistemasi uchun Koshi masalasining yechimini keltirilgan.

Faraz qilaylik $G \subset R^3$ uch o‘lchovli Evklid fazosidan olingan chegaralangan soha bo‘lsin.

Quyidagi belgilashlarni kiritamiz:

$$x=(x_1,x_2,x_3), y=(y_1,y_2,y_3)\in R^3, x^{'}=(x_1,x_2,0), y^{'}=(y_1,y_2,0)\in R^2$$
$$\alpha^2=(y_1-x_1)^2+(y_2-x_2)^2, r^2=\alpha^2+(y_3-x_3)^2=\left|y^{'}-x^{'}\right|$$

$$E(t)=\begin{pmatrix} t_1 & 0 & 0 & . \\ 0 & t_2 & 0 & . \\ . & . & . & . \\ 0 & 0 & . & t_n \end{pmatrix}$$

Berilgan $E(t)$ dioganal matritsa, $u^0 -$ birlik vektor $u^0 \in R^n, n \geq 3.$

Faraz qilaylik G soha R^3 fazodan olingan chegaralangan, chegarasi silliq sirtdan iborat bo‘lgan soha bo’lsin.

$A_{l\times n}(x)$ orqali shunday $D(x^T)$ matritsalar sinfini belgilaymizki, bu matritsaning har bir elementi chiziqli funksiyadan iborat bo‘lib, koeffitsiyentlari haqiqiy yoki kompleks sonlardan tashkil topib, quydagi sharni qanoatlantirsin.

$$D^*(x^T)D(x^T)=E[(x^2+\lambda^2)u^0]$$

Bu yerda $D^*(x^T)$ matritsa $D(x^T)$ matritsaga ermitli qo‘shma bo‘lgan matritsadan iborat.

Faraz qilaylik, G soha yarim shardan ya’ni

$$G=\{y_1^2+y_2^2+y_3^2<R^2, y_3>0\} \tag{1}$$

dan iborat bo'lsin. G sohaning chegarasi yarim sfera S sirtidan va $y_3 = 0$ tekislikning bir qismidan iborat bo'lsin.

Bu sohada quydagi tenglamalar sistemasini qaraymiz:

$$D\left(\frac{\partial}{\partial x}\right)u(x) = 0 \tag{2}$$

Bu sistemaning xarakteristik matritsasi $D(x^T) \in A_{l\times n}(x)$ iborat bo'lgan matritsa, (2) sistema koeffitsiyentlari o'zgarmas bo'lgan Ge'lmgol's tenglamasi bilan faktorizatsiyalanuvchi elliptik sistema bo'ladi.

Masalaning qo'yilishi: G sohada shunday $u(x)-$ vektor funksiyani topish lozimki, $u(x)$ vektor funksiya G soha ichida (2) sistemani qanoatlantirib $G \cup \partial G$ uzluksiz bo'lib, quydagi chegaraviy shartni qanoatlantirsin.

$$u(x)|_s = f(x) \tag{3}$$

(bu yerda $f(x)$ uzluksiz berilgan vektor funksiyadan iborat), $u(x)$ vektor funksiyani G sohada tiklash masalasiga (2) sistema uchun Koshi masalasi deyiladi.

Agar $u(x) \in C^{'}(G) \cap C(\overline{G})$ sinfdan olingan vektor funksiya bo'lib, (2) sistemani qanoatlantirsa, u holda quydagi integral formula o'rinli bo'ladi [3]:

$$\begin{aligned} u(x) &= \int_{\partial G} M(x,y)u(y)ds, x \in G \\ M(x,y) &= E\left(4\pi e^{-ix\lambda}\frac{1}{r}\right)D^*\left(\frac{\partial}{\partial x}\right)D(t^T) \end{aligned} \tag{4}$$

$t = (\cos\alpha, \cos\beta, \cos\gamma)$ sohaning chegarasida o'tkazilgan tashqi birlik normalidan iborat.

Faraz qilaylik, $K(w), w = u + iv$ butun funksiya bo'lib w ning haqiqiy qiymatlarida haqiqiy qiymat qabul qilsin.

$$K(u) \neq 0, \sup_{v \geq 1}\left|v^p k^p(u + iv)\right| = M(u) < \infty$$

(2), (3)- Koshi masalasini, yechish uchun Sh.Yarmuxamedov [5] usulidan foydalanib $\Phi(y, x)$ funksiyani quydagicha tanlab olamiz.

$$C_3K(x_3)\Phi(y,x)=\int_0^\infty \operatorname{Im}\frac{K(w)}{iv+y_3-x_3}\frac{u\cos\lambda u}{\sqrt{u^2+\alpha^2}}du \qquad (5)$$
$$v^2=u^2+\alpha^2$$

[5] ishda $\Phi(y,x)$ funksiyani quydagi ko'rnishda ifodalash ko'rsatilgan:

$$\Phi(y,x)=\frac{1}{4\pi}\frac{e^{ix\lambda}}{r}+g(y,x)$$

Bu yerda $g(y,x)$

$$\Delta g+\lambda^2 g=0, \lambda^2=c>0$$

tenglamani qanoatlantiradi.

(5) formulada $K(w)=e^{\sigma w^2}$ ko'rnishda tanlab olamiz, u holda $\Phi_\sigma(y,x)$ quydagicha aniqlanadi

$$\Phi_\sigma(y,x)=\frac{1}{4\pi}e^{-\sigma x_3^2}\int_0^\infty \operatorname{Im}\frac{e^{\sigma w^2}}{w-x_3}\frac{u\cos\lambda u}{\sqrt{u^2+\alpha^2}}du \qquad (6)$$

Buni hisobga olsak (4) formula quyidagicha ko'rnishni oladi.

$$u_\sigma(x)=\int_S M_\sigma(x,y)f(y)ds, x\in G \qquad (7)$$

bu yerda

$$M_\sigma(x,y)=E(\Phi_\sigma(y,x)D^*(\frac{\partial}{\partial x})D(t^T))$$

Quydagi teorema o'rinli.

Teorema. Agar $u(y)$ vektor funksiya $C^{'}(G)\cap C(\overline{G})$ sinflardan iborat bo'lib (1) sistemani va

$$|u(y)|\le 1, y_3=0 \qquad (8)$$

shartni qanoatlantirsa, u holda

$$|u(x)-u_\sigma(x)|\le C(x)\sigma e^{-\sigma x_3^2}, x\in G \qquad (9)$$

tengsizlik o'rinlidir.

Isbot: Teoremaning isboti

$$u(x)-u_\sigma(y)=\int_{y_3=0} M_\sigma(x,y)u(y)ds, x\in G$$

ni baholashdan kelib chiqadi.

Agar $|u(y)|\le 1$ tengsizlikdan foydalansak u holda $M_\sigma(y,x)$ ni baholash yetarlidir. $M_\sigma(y,x)$esa $\Phi_\sigma(y,x)$va $\dfrac{\partial \Phi}{\partial y_i},(i=1,2,3)$ larni baholashga olib keladi.

Teoremaning isboti quydagi tenglikdan kelib chiqadi:

$$u(x)-u_\sigma(x)=\int_{\partial G} M_\sigma(y,x)u(y)ds_y-\int_S M_\sigma(y,x)u(y)ds$$

Bu tenglikni baholashda

$$|u(y)|\le 1<\infty$$

dan foydalanamiz.

Bunda $\Phi_\sigma(y,x)$ni quydagicha aniqlaymiz.

$$\Phi_\sigma(y,x)=\frac{1}{4\pi}e^{-\sigma x_3^2}\int_0^\infty \operatorname{Im}\frac{e^{\sigma(i\sqrt{u^2+\alpha^2}+y_3)^2}}{(i\sqrt{u^2+\alpha^2}+y_3-x_3)}\frac{u\cos\lambda u}{\sqrt{u^2+\alpha^2}}du=$$

$$=\frac{1}{4\pi}e^{-\sigma x_3^2}\int_0^\infty \operatorname{Im}\frac{e^{\sigma(y_3^2-u^2-\alpha^2)}(\cos 2\sigma y_3\sqrt{u^2+\alpha^2}+i\sin 2\sigma y_3\sqrt{u^2+\alpha^2})}{(i\sqrt{u^2+\alpha^2}+y_3-x_3)}\frac{u\cos\lambda u}{\sqrt{u^2+\alpha^2}}du=$$

$$=\frac{1}{4\pi}e^{-\sigma x_3^2}\int_0^\infty e^{\sigma(y_3^2-u^2-\alpha^2)}\Big(-\frac{\cos 2\sigma y_3\sqrt{u^2+\alpha^2}}{u^2+r^2}\Big)u\cos\lambda u du+$$

$$\frac{1}{4\pi}e^{-\sigma x_3^2}\int_0^\infty e^{\sigma(y_3^2-u^2-\alpha^2)}\frac{(y_3-x_3)\sin 2\sigma y_3\sqrt{u^2+\alpha^2}}{(u^2+r^2)\sqrt{u^2+\alpha^2}}u\cos\lambda u du$$

$$\int_T|\Phi(y,x)dx|\le\frac{1}{4\pi}e^{-\sigma x_3^2}\int_T\int_0^\infty\frac{e^{-\sigma(u^2+\alpha^2)}}{u^2+r^2}ududs$$

Shu yo‘l bilan

$$\left|\int_T\frac{\partial\Phi_\sigma}{\partial y_3}dx\right|\le\frac{1}{4\pi}e^{-\sigma x_3^2}\int_T\int_0^\infty\left(\frac{2\sigma x_3}{u^2+\alpha^2+x_3^2}+\frac{x_3}{(u^2+\alpha^2+x_3^2)^2}\right)e^{-\sigma(u^2+\alpha^2)}duds$$

$$\int_T\frac{\partial\Phi}{\partial y_i}ds=\int_T\frac{1}{4\pi}e^{-\sigma x_3^2}\left[\int_0^\infty e^{-\sigma(u^2+\alpha^2)}\frac{2\sigma(y_1-x_1)}{u^2+r^2}u\cos\lambda u du+\int_0^\infty e^{-\sigma(u^2+\alpha^2)}\frac{2(y_1-x_1)^2}{(u^2+r^2)^2}u\cos\lambda u du\right]ds$$

$$\int_T \left(\left| \Phi_\sigma(y,x) \right| + \left| \frac{\partial \Phi_\sigma}{\partial y_3} \right| \right) ds \le \frac{e^{-\sigma x_3^2}}{2\sqrt{\pi}} \left(\frac{1}{\sqrt{\sigma}} + \frac{2+\pi}{2\sqrt{\pi}} \right).$$

Yuqori tenglikdan foydalanib quyidagi integrallarni baholaymiz.

$$\int_T \left| \frac{\partial \Phi}{\partial y_i} \right| ds = \frac{1}{4\pi} e^{-\sigma x_3^2} C(x)$$

Bu integrallarni baholashdan teoremaning isboti kelib chiqadi.

Bu tengsizliklarni $\Phi_\sigma(y,x)$ va $\dfrac{\partial \Phi}{\partial y_i}$ larga qo‘llab,

$$\left| u(x) - u_\sigma(y) \right| \le C(x_3) \sigma e^{-\sigma x_3^2}, x \in G, \sigma \ge 1$$

ni hosil qilamiz.

Natija: Quydagi limit

$$\lim_{\sigma \to \infty} u_\sigma(x) = u(x), x \in G.$$

G sohaning har bir kompakt qismida tekis yaqinlashadi.

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