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S² S³...

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S² S³

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1,2,i ,n.

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j

(« ») $P_{ij,k}$,

$$p_{ij,k} \geq 0, \quad \sum_{k=1}^n p_{ij,k} = 1 \quad i, j, k$$

$x = (x_1, x_2, \dots, x_n)$ $i \quad j$ $x_i x_j$,

$$x'_k = \sum_{i,j=1}^n P_{ij,k} x_i x_j \quad (1)$$

$$S^{n-1} = \{x = (x_1, x_2, \dots, x_n) : x_i \geq 0, \quad i=1,2,\dots,n, \quad \sum_{i=1}^n x_i = 1\}$$

$$n-1 - \quad , \quad \sum_{k=1}^n x'_k = 1 \quad x'_k \geq 0,$$

(1),

S^{n-1}

1.1.

1,2,í ,n.

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(ö) $P_{ij,k}$,

:

$$P_{ij,k} \geq 0, \quad \sum_{k=1}^n P_{ij,k} = 1, \quad i,j,k.$$

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$x = (x_1, x_2, \dots, x_n)$, . . . x_i

i

$x = (x_1, x_2, \dots, x_n)$ i j

$x_i x_j$, ,

$$x'_k = \sum_{i,j=1}^n P_{ij,k} x_i x_j \quad (1.1.1)$$

k

$S^{n-1} = \{x = (x_1, x_2, \dots, x_n) : x_i \geq 0, i=1,2,\dots,n, \sum_{i=1}^n x_i = 1\}$ n-1

$$\sum_{k=1}^n x'_k = 1 \quad x'_k \geq 0 \quad (1.1.1)$$

S^{n-1}

(,L)

L-

$$\sigma: \Lambda \rightarrow \Phi$$

$$S(\Lambda, \Phi)$$

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$$S(\Lambda, \Phi)$$

{ i }-

(, L) ,

i=1,2,í n.

$$\sigma_1, \sigma_2 \in \Omega$$

$$A(\sigma_1, \sigma_2) = \{x \in \Lambda : \sigma_1(x) = \sigma_2(x)\}$$

и

$$\tilde{A}(\sigma_1, \sigma_2) = \bigcup_{j: \Lambda(\sigma_1, \sigma_2) \cap \Lambda_j \neq \emptyset} \Lambda_j$$

$$\tilde{A}(\sigma_1, \sigma_2) \neq \emptyset,$$

$$\Omega(\Lambda, \tilde{A}(\sigma_1, \sigma_2)) = \{(\sigma \in \Omega : \sigma = \sigma_1 \in \tilde{A}(\sigma_1, \sigma_2) \cdot \cdot \sigma_2 = \sigma \in \tilde{A}(\sigma_1, \sigma_2))\}$$

$$\tilde{A}(\sigma_1, \sigma_2) = \emptyset,$$

$$\Omega(\Lambda, \tilde{A}(\sigma_1, \sigma_2)) = \{(\sigma \in \Omega : \sigma = \sigma_1 \in \Lambda_i \cdot \cdot \sigma = \sigma_2 \in \Lambda_i \cdot \cdot \cdot i = 1 \dots n)\}$$

$$\mu \in S(\Lambda, \Phi) -$$

á

$$\mu(\sigma) > 0$$

$\sigma \in \Omega$

$$P_{\sigma_1 \sigma_2, \sigma}$$

:

$$P_{\sigma_1 \sigma_2, \sigma} = \left\{ \begin{array}{l} \frac{\mu(\sigma)}{\mu(\Omega(\Lambda, \tilde{A}(\sigma_1, \sigma_2)))}, \cdot \sigma \in \Omega(\Lambda, \tilde{A}(\sigma_1, \sigma_2)) \\ 0, \cdot \cdot \cdot \cdot \end{array} \right\} \quad (1.1.2)$$

V,

$$S(\Lambda, \Phi)$$

(1.1.2),

$\lambda \in S(\Lambda, \Phi)$

:

$$V\lambda = \lambda' \in S(\Lambda, \Phi)$$

$$\lambda'(\sigma) = \sum_{\sigma_1, \sigma_2 \in \Omega} P_{\sigma_1 \sigma_2, \sigma} \lambda(\sigma_1) \lambda(\sigma_2) \quad (1.1.3)$$

$$\sigma \in \Omega.$$

,

:

$$P_{\sigma_1\sigma_2,\sigma} \geq 0, \quad \sum_{\sigma \in \Omega} P_{\sigma_1\sigma_2,\sigma} = 1 \quad \text{è} \quad P_{\sigma_1\sigma_2,\sigma} = P_{\sigma_2\sigma_1,\sigma} \quad \sigma_1, \sigma_2, \sigma \in \Omega$$

(1.1.3)

$$(\Lambda, L), \quad (\quad) \quad \mu.$$

(1.1.1)

$$P_{ij,k} = \begin{cases} \neq 0 & i=j \quad j=k \\ 0 & \end{cases}$$

$$1.1.1. \quad |\Phi| > 1 \quad |\Lambda > 1|$$

(1.1.3.)

$$(\Lambda, L) - .$$

(\Lambda, L)

$$A(\sigma_1, \sigma_2) = \Lambda,$$

$$, \quad \Omega(\Lambda, \tilde{A}(\sigma_1, \sigma_2)) = \{\sigma_1\} \quad A(\sigma_1, \sigma_2) \neq \Lambda, \quad A(\sigma_1, \sigma_2) \neq \emptyset,$$

$$\tilde{A}(\sigma_1, \sigma_2) = \Lambda \quad \Omega(\Lambda, \tilde{A}(\sigma_1, \sigma_2)) = \{\sigma_1, \sigma_2\}. \quad , \quad \tilde{A}(\sigma_1, \sigma_2) \neq \emptyset$$

$$\Omega(\Lambda, \tilde{A}(\sigma_1, \sigma_2)) = \{\sigma_1, \sigma_2\} \quad ,$$

$$P_{\sigma_1\sigma_2,\sigma} = 0 \quad \sigma \neq \sigma_1 \cdot \sigma \neq \sigma_2, \quad \dots \quad \acute{o}$$

(1.1.3.)

\mu

\sigma_1, \sigma_2

$$\Omega(\Lambda, \tilde{A}(\sigma_1, \sigma_2)) = \{\sigma_1, \sigma_2\}, \quad (\Lambda, L).$$

\sigma(x)

[3],

[4] ,

\tilde{o} \quad \ddot{o}, \quad \tilde{o} \quad \ddot{o}

, \quad \alpha.

$$\Phi = \{A, \alpha\}$$

1.2.

$$|\Lambda|=n \quad \Phi = \{A, \alpha\}. \quad \sigma \in \Omega \quad n_A(\sigma) -$$

$$\sigma \quad \tilde{\sigma} \quad \ddot{\sigma} \quad \mu_\alpha \quad \Omega$$

$$\mu_\alpha(\sigma) = p^{n_A(\sigma)} q^{n-n_A(\sigma)}$$

$$p \geq 0, \quad q \geq 0 \quad p+q=1 \quad p/q = \alpha. \quad p=q, \quad \dots \quad \alpha=1, \quad \mu_1$$

$$\Omega.$$

$$(\Lambda, L)-, \quad \dots \quad \Lambda$$

$$\alpha_1 \text{ è } \alpha_2$$

$$n_A(\sigma_1) = n_A(\sigma_2)$$

$$\Omega = \{\tilde{\sigma}_1, \tilde{\sigma}_2, \dots, \tilde{\sigma}_n, \quad \tilde{\sigma}_i- \quad n_A(.) = i\}$$

$$\mu_\alpha(\tilde{\sigma}_i) = c_n^i p^i q^{n-1} \quad (1.2.1)$$

$$\tilde{V}_\alpha-$$

(1.2.1)

$$S(\Omega) = \{x = (x_1, x_2, \dots, x_n) : x_i \geq 0, \quad i=\overline{0, n}, \quad \sum_{i=1}^n x_i = 1\}$$

[4].

$$p=q=1/2$$

$$\Lambda, \quad |\Lambda|=n$$

1. $n=1.$

1971 [4]

$P_{AA,A}$ -

A

$$p_{AA,A} -$$

A

$$p_{aa,A} -$$

A

$$p_{\dots,a} = 1 - p_{\dots,A} .$$

$$(x_1, x_2) \in S^1 \quad \begin{matrix} x_1 & x_2 & \acute{o} & A & \alpha & . & , \\ & & & & & & , \end{matrix} \quad (1.1.1)$$

$$\begin{cases} x_1' = p_{AA,A}x_1^2 + 2p_{Aa,A}x_1x_2 + p_{aa,A}x_2^2 \\ x_2' = p_{AA,a}x_1^2 + 2p_{Aa,a}x_1x_2 + p_{aa,a}x_2^2 \end{cases} \quad (1.2.3)$$

$$\begin{matrix} p_{AA,A} = 1 & p_{Aa,A} = 1/2 & p_{aa,A} = 0 \\ p_{AA,a} = 0 & p_{Aa,a} = 1/2 & p_{aa,a} = 1 \end{matrix} \quad (1.2.4)$$

$$(1.2.4) \quad (1.2.3),$$

$$\begin{cases} x_1' = x_1^2 + x_1x_2 \\ x_2' = x_2^2 + x_1x_2 \end{cases}$$

$$, \dots x_1 + x_2 = 1$$

$$\begin{cases} x_1' = x_1 \\ x_2' = x_2 \end{cases} \quad (1.2.5)$$

$$(1.2.1).$$

$$p_{Aa,A} = b, \quad p_{aa,A} = c, \quad p_{AA,A} = a,$$

$$x = ax^2 + 2bx(1-x) + c(1-x)^2,$$

$$x = (a - 2b + c)x^2 + 2(b - c)x + c$$

$$\begin{cases} a - 2b + c = 0 \\ 2(b - c) = 1 \\ c = 0 \end{cases}$$

$$a = 1, \quad b = 1/2, \quad c = 0$$

(1.2.1).

$$(1.2.5) \quad V(S^1) = S^1.$$

2. $n = 2.$ $x = (x_1, x_2, x_3) \in S^2 -$

AA, Aa, aa

$$\{p_{ij,k}\}_{i,j,k=1}^3$$

$$\begin{pmatrix} 1 & 1/4 & 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 1 & 1/2 \\ 0 & 1/4 & 1 & 0 & 0 & 1/2 \end{pmatrix}$$

(1.1.1).

(1.1.1)

$$\begin{cases} x_1' = x_1^2 + x_1x_2 + 1/4x_2^2 \\ x_2' = x_1x_2 + 1/2x_2^2 + 2x_1x_3 + x_2x_3 \\ x_3' = 1/4x_2^2 + x_2x_3 + x_3^2 \end{cases}$$

$$\begin{cases} x_1' = (x_1 + 1/2x_2)^2 \\ x_2' = 2(x_1 + 1/2x_2)(x_3 + 1/2x_2) \\ x_3' = (x_3 + 1/2x_2)^2 \end{cases} \quad (1.2.6)$$

, (1.2.6)

$$x_1', x_2', x_3' \quad x_1, x_2, x_3, \dots$$

$$\begin{cases} x_1'' = (x_1' + 1/2x_2'')^2 \\ x_2'' = 2(x_1' + 1/2x_2'')(x_3' + 1/2x_2'') \\ x_3'' = (x_3' + 1/2x_2'')^2 \end{cases} \quad (1.2.7)$$

(1.2.7) (1.2.6),

$$\begin{cases} x_1'' = (x_1 + 1/2x_2)^2 \\ x_2'' = 2(x_1 + 1/2x_2)(x_3 + 1/2x_2) \\ x_3'' = (x_3 + 1/2x_2)^2 \end{cases}$$

1.2.2. ()

(1.2.6) (0;1;0)

3. $n = 3$. $x = (x_1, x_2, x_3, x_4) \in S^3 -$

AAA, AAa, Aaa, aaa

AAA, AAa, Aaa, aaa 1, 2, 3, 4

$(P_{ij,k})_{i,j,k=1}^4$

:

$$\begin{pmatrix} 1 & 2/9 & 0 & 0 & 1/2 & 1/6 & 0 & 1/18 & 0 & 0 \\ 0 & 5/9 & 2/9 & 0 & 1/2 & 2/3 & 1/2 & 4/9 & 1/6 & 0 \\ 0 & 2/9 & 5/9 & 0 & 0 & 1/6 & 1/2 & 4/9 & 2/3 & 1/2 \\ 0 & 0 & 2/9 & 1 & 0 & 0 & 0 & 1/18 & 1/6 & 1/2 \end{pmatrix}$$

(1.1.3).

(1.1.3)

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$$\begin{cases} \dot{x}_1 = x_1^2 + x_1x_2 + 2/9x_2^2 + 1/3x_1x_3 + 1/9x_2x_3 \\ \dot{x}_2 = x_1x_2 + 5/9x_2^2 + 4/3x_1x_3 + 8/9x_2x_3 + 2/9x_3^2 + x_1x_4 + 1/3x_2x_4 \\ \dot{x}_3 = x_1x_4 + 5/9 + 4/3x_4x_2 + 8/9x_2x_3 + 2/9x_2^2 + x_3x_4 + 1/3x_1x_3 \\ \dot{x}_4 = 2/9x_3^2 + x_4^2 + 1/9x_2x_3 + 1/3x_2x_4 + x_3x_4 \end{cases}$$

,

$$\begin{cases} \dot{x}_1 = (x_1 + 1/3x_2)(x_1 + 2/3x_2 + 1/3x_3) \\ \dot{x}_2 = (x_1 + 1/3x_2)(x_4 + 2/3x_3 + 1/3x_2) + 2/3(x_2 + x_3)(x_1 + 2/3x_2 + 1/3x_3) \\ \dot{x}_3 = (x_4 + 1/3x_3)(x_1 + 2/3x_2 + 1/3x_3) + 2/3(x_2 + x_3)(x_4 + 2/3x_3 + 1/3x_2) \\ \dot{x}_4 = (x_4 + 1/3x_3)(x_4 + 2/3x_3 + 1/3x_2) \end{cases} \quad (1.2.8)$$

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(1.2.8)

$\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4$

x_1, x_2, x_3, x_4

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...

$$\begin{cases} \ddot{x}_1 = (\dot{x}_1 + 1/3\dot{x}_2)(\dot{x}_1 + 2/3\dot{x}_2 + 1/3\dot{x}_3) \\ \ddot{x}_2 = (\dot{x}_1 + 1/3\dot{x}_2)(\dot{x}_4 + 2/3\dot{x}_3 + 1/3\dot{x}_2) + 2/3(\dot{x}_2 + \dot{x}_3)(\dot{x}_1 + 2/3\dot{x}_2 + 1/3\dot{x}_3) \\ \ddot{x}_3 = (\dot{x}_4 + 1/3\dot{x}_3)(\dot{x}_1 + 2/3\dot{x}_2 + 1/3\dot{x}_3) + 2/3(\dot{x}_2 + \dot{x}_3)(\dot{x}_4 + 2/3\dot{x}_3 + 1/3\dot{x}_2) \\ \ddot{x}_4 = (\dot{x}_4 + 1/3\dot{x}_3)(\dot{x}_4 + 2/3\dot{x}_3 + 1/3\dot{x}_2) \end{cases} \quad (1.2.9)$$

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(1.2.9)

(1.2.8)

$$\left\{ \begin{array}{l} x_1'' = 1/3(x_1 + 1/3x_2)(x_1 + 2/3x_2 + 1/3x_3) + 2/3(x_1 + 2/3x_2 + 1/3x_3)^3 \\ x_2'' = 1/3(x_1 + 1/3x_2)(x_4 + 2/3x_3 + 1/3x_2) + 2/9(x_2 + x_3)(x_1 + 2/3x_2 + 1/3x_3) + \\ \quad + 2(x_1 + 2/3x_2 + 1/3x_3)^2(x_4 + 2/3x_3 + 1/3x_2) \\ x_3'' = 1/3(x_4 + 1/3x_3)(x_1 + 2/3x_2 + 1/3x_3) + 2/9(x_2 + x_3)(x_4 + 2/3x_3 + 1/3x_2) + \\ \quad + 2(x_1 + 2/3x_2 + 1/3x_3)(x_4 + 2/3x_3 + 1/3x_2)^2 \\ x_4'' = 1/3(x_4 + 1/3x_3)(x_4 + 2/3x_3 + 1/3x_2) + 2/3(x_4 + 2/3x_3 + 1/3x_2)^3 \end{array} \right.$$

n

$$\left\{ \begin{array}{l} x_1^{(n+1)} = 1/3^n(x_1 + 1/3x_2)(x_1 + 2/3x_2 + 1/3x_3) + \\ (3^n - 1)/3^n(x_1 + 2/3x_2 + 1/3x_3)^3 \\ x_2^{(n+1)} = 1/3^n(x_1 + 1/3x_2)(x_4 + 2/3x_3 + 1/3x_2) + 2/3^{n-1}(x_2 + x_3)(x_1 + 2/3x_2 + 1/3x_3) + \\ \quad + (3^n - 1)/3^{n-1}(x_1 + 2/3x_2 + 1/3x_3)^2(x_4 + 2/3x_3 + 1/3x_2) \\ x_3^{(n+1)} = 1/3^n(x_4 + 1/3x_3)(x_1 + 2/3x_2 + 1/3x_3) + 2/3^{n-1}(x_2 + x_3)(x_4 + 2/3x_3 + 1/3x_2) + \\ \quad + (3^n - 1)/3^{n-1}(x_1 + 2/3x_2 + 1/3x_3)(x_4 + 2/3x_3 + 1/3x_2)^2 \\ x_4^{(n+1)} = 1/3^n(x_4 + 1/3x_3)(x_4 + 2/3x_3 + 1/3x_2) + (3^n - 1)/3^n(x_4 + 2/3x_3 + 1/3x_2)^3 \end{array} \right.$$

$n \rightarrow \infty$,

$$\lim_{n \rightarrow \infty} x_1^{(n)} = (x_1 + 2/3x_2 + 1/3x_3)^3$$

$$\lim_{n \rightarrow \infty} x_2^{(n)} = 3(x_1 + 2/3x_2 + 1/3x_3)^3(x_4 + 2/3x_2 + 1/3x_3)$$

$$\lim_{n \rightarrow \infty} x_3^{(n)} = 3(x_1 + 2/3x_2 + 1/3x_3)(x_4 + 2/3x_3 + 1/3x_3)^2$$

$$\lim_{n \rightarrow \infty} x_4^{(n)} = (x_4 + 2/3x_3 + 1/3x_2)^3$$

$$(02) \quad , \quad (0,1,0,0) \quad (0,0,1,0)$$

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1.2.1.

\tilde{V}_α ,

$\alpha = 1$

$n = 1$

$n = 2,$

$n = 3$

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, $x^{(k+1)}$, , ,

1.2.2. \tilde{V}_α $\alpha=1$ $n=3$
 , ... $x^{(k)}$ $k \rightarrow \infty$.

1.2.3. \tilde{V}_α $\alpha=1$ $n=1$
 , $n=2$ $n=3$.

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2. 1969.
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3.
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4. // . . .,1987 .300 .