Heavy quarks potential

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Instanton liquid is represented as the sum of the fields of separate pseudoparticles(instantons and anti-instantons)

$$A_\mu(x) = \sum_I A_{I\mu}(x). \tag{1}$$

Instanton liquid is characterized with the average size of pseudoparticles $\bar{\rho}$ and their density N/V, where N is the total number of pseudoparticles($N/2$ instantons and $N/2$ anti-instantons) in the four dimensional volume V.

At large number of colours N_c the influence of quarks on the gluon sector of QCD vanishes. This is called "quenched approximation" and we can use this approximation treating instanton liquid as a given classical background field. In this approximation the quark propagator is $\bar{S}$ is

$$\bar{S} = \left\langle (i\hat{\partial} + im + \sum_I \hat{A}_I)^{-1} \right\rangle. \tag{2}$$

Here the brackets denote averaging over positions, sizes, and orientations of all pseudoparticles. In Refs. [1, 2] a quark propagator approximated in the background field of a single instanton by the sum of the free propagator and of zero mode contribution

$$\langle x|(i\hat{\partial} + \hat{A}_I + im)^{-1}|y\rangle \cong \langle x|i\hat{\partial} + im)^{-1}|y\rangle + \frac{\psi_I(x)\psi_I^\dagger(y)}{im} \tag{3}$$

In the Euclidean QCD the quark propagator can be represented as the functional integral

$$\langle\psi(x)\psi^\dagger(y)\rangle = Z^{-1}\int DA\int D\psi\int D\psi^\dagger exp\left[\int d^4z\psi^\dagger(i\hat{\partial} + \hat{A}_I + im)\psi + S(A)\right], \tag{4}$$

where $S(A)$ is a Yang-Mills action. Integrating over the quark fields $\psi, \psi^\dagger$ and considering th limit $N_c \to \infty$ we obtain

$$\langle\psi(x)\psi^\dagger(y)\rangle \overset{N_c\to\infty}{=} Z^{-1}\int DA \exp(S(A))\langle x|(-i\hat{\partial} - \hat{A}_I - im)^{-1}|y\rangle. \tag{5}$$

Introducing free quark propagator

$$S_0 = -(i\hat{\partial} + im)^{-1} \tag{6}$$

and the quark propagator in the background field of a single pseudoparticle

$$S_I = -(i\hat{\partial} + \hat{A}_I + im)^{-1} \tag{7}$$

one can get the inverse Dirac operator as a power series in A_I averaged over collective coordinates

$$-\left\langle (i\hat{\partial} + \sum_I \hat{A}_I + im)^{-1} \right\rangle = S_0 + \sum_I \langle S_I - S_0\rangle + \sum_{I\neq J}\langle S_I - S_0\rangle S_0^{-1}\langle S_J - S_0\rangle$$
$$+ \sum_{I\neq J\neq K}\langle S_I - S_0\rangle S_0^{-1}\langle S_J - S_0\rangle S_0^{-1}\langle S_K - S_0\rangle$$

$$+\sum_{I\neq J}\langle (S_I - S_0)S_0^{-1}\langle S_J - S_0\rangle_J S_0^{-1}(S_K - S_0)\rangle_I \cdots. \qquad (8)$$

Next using diagram expansion technique which was developed in Ref.[1] we get the expression for the difference of inverse propagators

$$\bar{S}^{-1} - S_0^{-1} = \frac{N}{2}\left\langle (\bar{S} - \hat{A}_I^{-1}\right\rangle + \frac{N}{2}\left\langle (\bar{S} - \hat{A}_{\bar{I}}^{-1}\right\rangle \qquad (9)$$

Here A_I is an instanton field, and $A_{\bar{I}}$ is an anti-instanton field. In Eq.(9) we need to average over only one pseudoparticle, but not the whole instanton medium. The operator $(\bar{S} - \hat{A}_I^{-1})^{-1}$ should be averaged over the instanton pozition z_I, its orientaion matrix U_I, and its size ρ_I. The result is

$$\bar{S}^{-1} = S_0^{-1} - \frac{N}{2VN_c}\cdot Tr_{colour}\left(\int d^4 z_I S_0^{-1}(S_I - S_0)S_0^{-1} + (I \to \bar{I})\right) + O((\frac{N}{VN_c})^2). \quad (10)$$

The potential between heavy quarks can be obtained from the calculation of Wilson loop along a rectangular contour $T\times R$ with $T\to\infty$. In this limit one can neglect the short sides of the rectangle. One has to calculate two P-exponents along two oppositely directed straight lines $L_{1,2}$ parallel to the x_4 axis and separated by a distance R. We have

$$\mathrm{Tr}W(L_1L_2) = -\left\langle\left\langle \mathrm{Tr}\left[\langle T|\left(ddt - \sum_I A_I^{(1)} + 0\right)^{-1}|0\rangle\langle 0|\left(ddt - \sum_I A_I^{(2)} - 0\right)^{-1}|T\rangle\right]\right\rangle\right\rangle.$$

(11)

Here $A_I^{(1,2)}$ are the gauge fields projected onto the lines $L_{1,2}$. Averaged Eq.(11) in the leading order in the density N/VN_c the potential can be expressed through one-instanton P-exponents:

$$V(R) = N2VN_c\int d^3 z_I \mathrm{Tr}_{colour}\left[1 - P\exp(i\int_{L_1} dx_4 A_{I4})P\exp(-i\int_{L_2} dx_4 A_{I4})\right]_{zI4} + (I \to \bar{I}) \quad (12)$$

The expression (12) is called the Dirac contribution for the heavy quarks potential.

REFERENCES

1. D. I. Diakonov, V. Yu. Petrov, P. V. Pobylitsa, Phys. Lett.B **226**,p.372 (1989)
2. P. V. Pobylitsa, Phys. Lett.B **226**, p.387 (1989)