INFLUENCE OF MAGNETIC FIELDS ON MAGNETIZED PARTICLE MOTION AROUND NON SCHWARZSCHILD BLACK HOLE

Rayimbaev J.R.[1] , Nazrilloyev B.M.[2]

[1]*Ulugh Beg Astronomical Institute of the Uzbek Academy of Sciences*
[2]*National University of Uzbekistan*

The motion of a magnetized particle orbiting around non-Schwarzschild black hole immersed in an external uniform magnetic field is considered. The influence of deformation parameter *h* to effective potential of the radial motion of the magnetized particle around non-Schwarzschild black hole using Hamilton-Jacobi formalism is studied.

<u>Introduction.</u>

In the literature magnetized particles motion and the relevant important properties have been investigated for the Schwarzschild black hole [1], the Kerr black hole [2], black hole in braneworld [3], and other spherically symmetric black holes [4] where authors have accomplished area around compact objects where magnetized particles motion is allowed. Here, we plan to extend these results to the case of non Schwarzschild black hole immersed in external magnetic field. In this work we have focused on to describe a magnetized

particle motion and to show the influence of space-time deformation and external magnetic fields on this motion.

<u>Main section.</u>

The spacetime of a black hole is described by non-Schwarzschild metric with deformation coefficient $\boldsymbol{h}$ in a spherical coordinates (t,r,θ,φ) which can be written as following [5]

$$ds^2 = -f(1+\boldsymbol{h})dt^2 + f^{-1}(1+\boldsymbol{h})dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2 \qquad (1)$$

here $f = 1 - 2\,{}^{M}/_{r}$ is the lapse function, $\boldsymbol{h} = \epsilon\,{}^{M^3}/_{r^3}$ is deformation parameter and ϵ is deformation coefficient of the spacetime which can be either positive or negative, namely, $\epsilon > 0$ or $\epsilon < 0$. If there is no deformation, i.e. $\epsilon = 0$, the metric form (1) becomes the Schwarzschild one. The four-vector potential *Aα* of the electromagnetic field can be written through the strength of uniform magnetic field B_0 as it is given in [6] $A_\alpha = \frac{1}{2\delta_\alpha^\varphi B}\sin^2\theta.$ One can easily obtain the nonvanishing components of the electromagnetic field tensor as $F_{r\varphi} = B_0 r\sin^2\theta,\ F_{\theta\varphi} = B_0 r^2 \sin\theta\cos\theta$.

The Hamilton-Jacobie quation for the magnetized particle is

$$g^{\alpha\beta}\frac{\partial S}{\partial x^\alpha}\frac{\partial S}{\partial x^\beta} + m^2 - mD^{\alpha\beta}F_{\alpha\beta} = 0 \qquad (2)$$

It is clear that the scalar product $D \cdot F$ in (2) is responsible for the interaction between magnetic field and magnetized particle, according to [7] $D_{\alpha\beta}$ is given as $D_{\alpha\beta} = \varepsilon^{\alpha\beta\sigma\tau}u_\sigma\mu_\tau,\quad D^{\alpha\beta}u_\beta = 0$, where μ^σ and u^σ are the magnetic moment four-vectors and four-velocities of the particle, respectively. Using nonvanishing components of the electromagnetic field tensor it is possible to calculate the scalar product $D \cdot F = 2\mu B_0\, Z[\gamma_{\tilde\alpha}]$.

It is clear that at the equatorial plane i.e. $\theta = \frac{\pi}{2}$, θ components of the four-momentum $p_\theta = 0$, but other components are conserved. The spacetime symmetries are preserved by the axial symmetric configuration of the magnetic field and, therefore, they still allow for two conserved quantities: $p_\varphi = L$ and $p_t = E$. These are energy and angular momentum of particle, respectively. From (2) using the scalar product $D \cdot F$ one can get the expression for effective potential of magnetized particle moving around non-Schwarzschild black hole immersed in an external magnetic field as

$$V(\rho,\gamma,e,\beta,\epsilon) = \left(1 - \frac{1}{\rho}\right)\left(1 + \frac{\epsilon}{8\rho^3}\right)\left(1 + \frac{\gamma^2}{\rho^2} - \beta\, Z[\gamma_{\tilde{a}}]\right). \quad (3)$$

Here we introduce the notices following [7] $\rho = \frac{r}{2M}, \gamma = \frac{L}{2mM}, e = \frac{E}{m}, \beta = \frac{2\mu B_0}{m}$. Now it is possible to calculate circular orbits following the standard procedure:

$$\frac{d\rho}{d\tau} = 0, \frac{\partial(\rho,\gamma,e,\beta,\epsilon)}{\partial\rho} = 0. \quad (4)$$

From these equations one can easily get relations between the parameter β and other particle and spacetime parameters which are given in (4) in the following form:

$$\beta(\rho,\gamma,e,\epsilon) = \left(\frac{8\rho^4}{(\rho-1)(8\rho^3+\epsilon)} - \frac{\gamma^2}{\rho^2 e^2}\right)^{\frac{1}{2}}\left(1 + \frac{\gamma^2}{\rho^2} - \frac{8\rho^4 e^2}{(\rho-1)(8\rho^3+\epsilon)}\right)$$

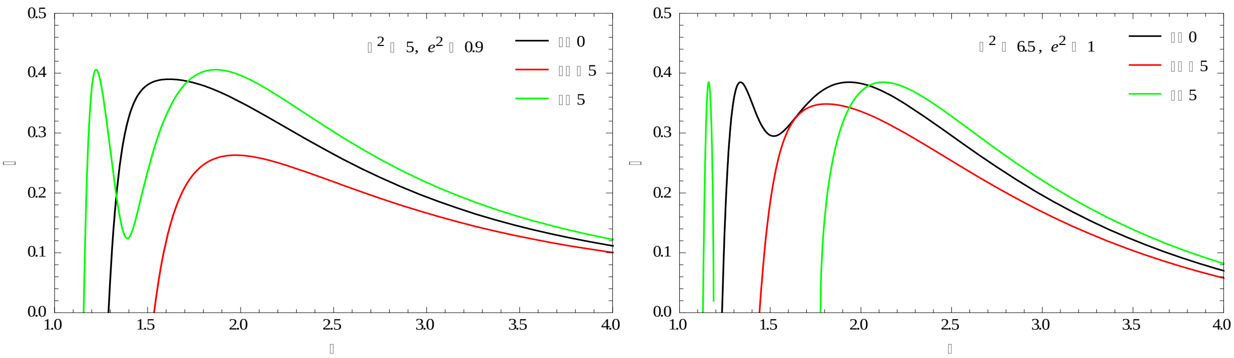

Figure 1 The dependence of magnetic coupling β from the radial coordinate ρ for the different values of the deformation parameters ϵ, γ and e

Defining the effective potential for magnetized particles as $\beta(\rho,\gamma,e,\epsilon)$, one can study the radial motion of magnetized particle. Figure 1 demonstrates the radial dependence of the effective potential of radial magnetized particles motion around non-Schwarzschild black hole. Moreover, it is possible to conclude from the plots of Fig. 1 that for positive (negative) values of the deformation parameter the circular orbits of the magnetized particles become unstable.

References

1. Abdujabbarov, A., Ahmedov, B., Rahimov, O., Salikhbaev, U.: Phys. Scr. **89**, 084008 (2014)
2. de Felice, F., Sorge, F., Zilio, S.: Class. Quantum Gravity **21**, 961(2004)
3. Rahimov, O.G.: Mod. Phys. Lett. A **26**, 399 (2011). 1012.1481
4. Preti, G.: Class. Quantum Gravity **21**, 3433 (2004)
5. Johannsen, T., Psaltis, D.: Phys. Rev. D **83**, 124015 (2011). 1105.3191
6. Abdujabbarov, A.A., Ahmedov, B.J., Jurayeva, N.B.: Phys. Rev. D **87**, 064042 (2013)
7. de Felice, F., Sorge, F.: Class. Quantum Gravity **20**, 469 (2003)