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Quantum interference effects in conformal Weyl gravity

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We investigate the effects of conformal gravity as a phase shift by quantum interference and alternate approach of Sagnac effect which is based on the anisotropy of the coordinate speed of light in the fourth-order theory of conformal Weyl space–time. In the nonrelativistic approximation, it has been shown that the phase shift of the interfering particle in neutron interferometer includes the potential terms with the Weyl parameter of the conformal fourth-order theory. Comparing the results of the measurement of the gravitational redshift by the interferometer in the gravitational field of the earth with our theoretical prediction, it has been obtained upper limit for the Weyl parameter as $\gamma \leq 2 \cdot 10^{-20}$ cm^{-1}.

Keywords: Conformal Weyl gravity; quantum interference; Sagnac effect.

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1. Introduction

Motivations for the more recent alternatives to general relativity (GR) are almost cosmological, associated with or replacing such constructs as "inflation," "dark matter" and "dark energy." In the last century, scientists were surprised with the discovery of unexpected rotation curves[1,2] for galaxies. Could there be more mass in the universe than we are aware of, or is the theory of gravity itself wrong? The consensus now is that the missing mass is "cold dark matter," which could be detected only by its gravity. Among many theories of gravity, the theory of conformal Weyl gravity[3–5] alternative to the standard second-order Einstein theory of gravity, is a possible solution to current cosmological puzzles such as dark matter and dark energy. Weyl gravity is a theory that is invariant under local conformal transformations

$$g_{\mu\nu}(x) \to \Omega^2(x) g_{\mu\nu}(x)\,, \tag{1}$$

where $\Omega(x)$ is a function on space–time. This leads to the Weyl gravitational action given by

$$\begin{aligned} I_W &= -\alpha_g \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \\ &= -2\alpha_g \int d^4x(-g)^{1/2} \left[R_{\mu\kappa} R^{\mu\kappa} - \frac{1}{3}(R^{\nu}{}_{\nu})^2 \right], \end{aligned} \tag{2}$$

where α_g is dimensionless and the conformal Weyl tensor[6] $C_{\lambda\mu\nu\kappa}$ defined by

$$\begin{aligned} C_{\lambda\mu\nu\kappa} &= R_{\lambda\mu\nu\kappa} - \frac{1}{2}\big(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}\big) \\ &\quad + \frac{1}{6}R^{\alpha}{}_{\alpha}\big(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}\big)\,. \end{aligned} \tag{3}$$

From the variational principle, the action (2) leads to

$$(-g)^{1/2} g_{\mu\alpha} g_{\nu\beta} \frac{\delta I_W}{\delta g_{\alpha\beta}} = -2\alpha_g W = -\frac{1}{2} T_{\mu\nu}\,, \tag{4}$$

where $T_{\mu\nu}$ is energy–momentum tensor, and

$$W_{\mu\nu} = 2C^{\alpha}{}_{\mu\nu}{}^{\beta}{}_{;\beta\alpha} + C^{\alpha}{}_{\mu\nu}{}^{\beta} R_{\alpha\beta}\,. \tag{5}$$

In the fourth-order theory, there have been found exact solutions[7–10] of conformal Weyl gravity which generalize Kerr–Newman and cylindrical solutions of GR. The vacuum solution of static spherically symmetric source for conformal gravity is given by the metric[3]

$$ds^2 = -B(r)dt^2 + \frac{dr^2}{B(r)}(r) + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,, \tag{6}$$

where

$$B(r) = 1 - \frac{\beta(2 - 3\beta\gamma)}{r} - 3\beta\gamma + \gamma r - kr^2\,, \tag{7}$$

and β, γ and k are integration constants and defined as follows: $\beta = \frac{GM}{c^2}$(cm) is geometrized mass, where M is the mass of the (spherically symmetric) source and G is the universal gravitational constant; and others $\gamma(\text{cm}^{-1})$ and $k(\text{cm}^{-2})$ are required by conformal gravity. The theory comes to the standard Schwarzschild solution ($\gamma = 0 = k$), and to the Schwarzschild–de Sitter ($\gamma = 0$), as well as the term $-kr^2$, means a background de Sitter space–time, which is important only over cosmological distances, while k has a very small value, the term γr becomes significant over galactic distance scales.

The pioneer work of observing the quantum-mechanical phase shift of neutron beams caused by the interaction with earth's gravitational field has been done by Overhauser and Colella.[11,12] After that, there were found other effects, associated with the phase shift of interfering particles. Among them, there are effects due to the rotation of the earth,[13,14] which is the quantum mechanical analog of the Sagnac effect, and the Lense–Thirring effect[15] which is a general relativistic one due to the dragging of the reference frames. The influence of the spin of the neutron to the phase shift is negligible in the gravitational field of the earth compared to the phase shifts which come out with other effects, so we do not take into account the neutron spin in this work. In our previous works[16,17] we have studied effects of quantum interference in alternative theories of gravity.

Different explanations for the Sagnac effect have been discussed in Ref. 18. It has been shown that the Sagnac effect is a consequence of the relativistic law of velocity composition. The authors of Refs. 19 and 20 showed formal analogy of deriving the Sagnac effect in flat space–time as a gravitomagnetic Aharonov–Bohm effect. Here, another derivation of the Sagnac effect has been discussed, which is based on the anisotropy of the coordinate velocity of light. We extend the results to the case of slowly rotating gravitating object in conformal Weyl gravity.

In this paper, we study quantum interference effects in particular the phase shift effect and Sagnac effect in a neutron interferometer in conformal Weyl gravity model. This paper is organized as follows. The phase shift of the neutron beams in the rotating interferometer in the gravitational field of Weyl space–time has been studied by Klein–Gordon equation in Sec. 2. In Sec. 3, we study synchronization of clocks as interpretation of Sagnac effect in rotating reference frame in Weyl gravity. Section 4 is devoted to conclusions.

Throughout the paper, we use a space-like signature $(-,+,+,+)$. Greek indices run from 0 to 3 and Latin ones from 1 to 3.

2. The Phase Shift

We assume that the external gravitational field of gravitating object in conformal fourth-order gravity is described by the rotating metric,[8] namely

$$ds^2 = (bf - \bar{c}e)\left(\frac{dx^2}{a} + \frac{dy^2}{d}\right) + \frac{1}{bf - \bar{c}e}\left[d(b\,d\phi - \bar{c}c\,dt)^2 - a(e\,d\phi - f\,dt)^2\right], \tag{8}$$

which is the canonical Carter form of the metric, where

$$\begin{aligned} a(x) &= j^2 + ux + px^2 + vx^3 - kx^4\,, \\ d(y) &= 1 + \bar{r}y - py^2 + sy^3 - j^2ky^4\,, \\ b = j^2 + x^2\,, &\quad e = j(1 - y^2)\,, \quad \bar{c} = j\,, \quad f = 1\,, \end{aligned} \tag{9}$$

and u, p, v, k, $\bar{r}$ and s are constants satisfying the constraint $uv - \bar{r}s = 0$. The rotation parameter of the source is represented by j. The fourth-order Kerr solution includes the Kerr solution ($u = -2MG/c^2$, $p = 1$, $v = \bar{r} = s = k = 0$) and the Kerr–de Sitter solution ($u = 2MG/c^2$, $p = 1 - kj^2$, $v = \bar{r} = s = 0$, $k = \Lambda/3$; Λ being the cosmological constant) as special cases.

Substituting for the functions in (9) with the choice $u = -2MG/c^2, p = 1, v = \gamma$ in the metric (8) and then considering the weak field ($u/r \ll 1$) approximation and slow rotation ($j/r \ll 1$) limit, the above metric (8) in the Boyer–Lindquist coordinates ($x = r, y = \cos\theta$) takes the following form:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r} + \gamma r\right)c^2\,dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r} + \gamma r} + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2 - 2(\omega r^2 - j\gamma r)\sin^2\theta c\,dt\,d\phi\,, \tag{10}$$

where $\omega = 2GMj/c^2 r^3$ is the angular velocity of dragging inertial frames, $j = J/Mc$ is the specific angular momentum being equal to the total momentum J of the gravitating object per unit mass.

We use the Klein–Gordon equation

$$\nabla^\mu \nabla_\mu \Phi - \left(\frac{mc}{\hbar}\right)^2 \Phi = 0\,, \tag{11}$$

for interfering particles with mass m and with the wave function Φ as

$$\Phi = \Psi \exp\left(-i\frac{mc^2}{\hbar}t\right), \tag{12}$$

where Ψ is the nonrelativistic wave function (see Ref. 21).

In the case of slowly rotating space–time here we can neglect higher order terms of parameters GM/rc^2 and j/r considering they are sufficiently small. Therefore, to the first order in M, γ and neglecting the terms of $O((v/c)^2)$, the Klein–Gordon equation in slowly rotating Weyl gravity becomes:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{L^2}{r^2\hbar^2}\right]\Psi - \frac{mc^2}{2}\left(\frac{2GM}{c^2 r} - \gamma r\right)\Psi + \frac{jc}{r^2}\left(\frac{2GM}{c^2 r} - \gamma r\right)L_z\Psi\,, \tag{13}$$

where the following notations are used:

$$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right], \tag{14}$$

$$L_z = -i\hbar\frac{\partial}{\partial\phi}\,, \tag{15}$$

here L and L_z are the orbital angular momentum operators of the particle with respect to the center of the earth, respectively.

For a static observer on the earth, we set the coordinate transformation as $\phi \to \phi + \Omega t$, where $\Omega = \Omega_\oplus$ is the angular velocity of the earth. Then, Eq. (13) would be in the following form:

$$i\hbar\Psi_t = H_0\Psi + H_1\Psi + H_2\Psi + H_3\Psi\,, \tag{16}$$

here

$$H_0 = -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{L^2}{2mr^2} \tag{17}$$

is the Hamiltonian for a freely propagating particle,

$$H_1 = -\frac{GMm}{r} + \frac{mc^2}{2}\gamma r \tag{18}$$

is the Weyl gravitational potential energy, where the first term is the Newtonian gravitational term and

$$H_2 = -\Omega L_z \tag{19}$$

is concerned with rotation related to the Sagnac effect and the last Hamiltonian

$$H_3 = \left(\frac{2GMj}{cr^3} - \frac{cj\gamma}{r}\right)L_z \tag{20}$$

is responsible for the effect of dragging of inertial frames, i.e. Lense–Thirring effect.[22]

We suppose the solution for the total Hamiltonian H may be in the following form

$$\Psi = \Psi_0\exp[-i(\beta_1 + \beta_2 + \beta_3)]\,, \tag{21}$$

where Ψ_0 is a solution for the Hamiltonian H_0, β_i denotes the phase shift caused by $H_i (i = 1 - 3)$. The quasi-classical approximation of the phase shift due to the gravitational potential is given by the integration along a classical trajectory $\beta_1 = \hbar^{-1}\int H_1 dt$. Thus, for a beam of neutrons which splits into two alternative paths and then recombines as shown in Fig. 1 the phase difference $\beta_{W\,\mathrm{grav}}$ is

$$\beta_{W\,\mathrm{grav}} = \beta_{1(ABD)} - \beta_{1(ACD)} \simeq \beta_{\mathrm{Newtonian}}\left(1 + \frac{c^2\gamma}{2g}\right), \tag{22}$$

$$\beta_{\mathrm{Newtonian}} = \frac{m^2 gS\lambda}{2\pi\hbar^2}\sin\chi\,, \tag{23}$$

where $\beta_{\mathrm{Newtonian}}$ is the phase shift due to the Newtonian gravitational potential, g is the acceleration of gravity, $S = d_1 d_2$ is the area of the parallelogram, and λ is the de Broigle wavelength. The phase shift term due to H_2, i.e. due to the rotation of the interferometer, which is in our case caused by the earth's rotation is the phase shift of the Sagnac effect is

$$\beta_{\mathrm{Sag}} = \beta_{2(ABD)} - \beta_{2(ACD)} \simeq \frac{2m\mathbf{\Omega}\cdot\mathbf{S}}{\hbar}\,, \tag{24}$$

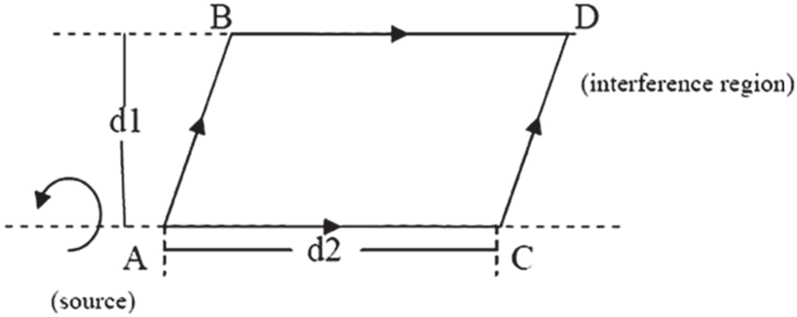

Fig. 1. Schematic illustration of alternate paths separated in the vertical direction in a neutron interferometer.

where $\mathbf{\Omega} = (0,0,\Omega)$ and $\mathbf{S}$ is the area vector of the ACDB. The third term of the phase shift is responsible for the dragging of the reference frames, the so-called Lense–Thirring effect will be

$$\beta_{W\,\mathrm{drag}} = \frac{2Gm}{\hbar c^2 R^3}\mathbf{J}\left[\mathbf{S} - 3\left(\frac{\mathbf{R}}{R}\cdot\mathbf{S}\right)\frac{\mathbf{R}}{R}\right] - \frac{m\gamma}{M\hbar R}\mathbf{J}\left[\mathbf{S} - \left(\frac{\mathbf{R}}{R}\cdot\mathbf{S}\right)\frac{\mathbf{R}}{R}\right], \quad (25)$$

where $\mathbf{J} = (0,0,J)$ is the angular momentum vector of the earth, $\mathbf{R}$ is the position vector of the instrument from the center of the earth. We suppose that the earth is a sphere with radius R, then $\mathbf{J} = 2MR^2\mathbf{\Omega}/5$ and

$$\beta_{W\,\mathrm{drag}} = \frac{1}{5}\beta_{\mathrm{Sag}}\left(\frac{r_g}{r} - r\gamma\right),$$

if $\mathbf{R}$ is perpendicular to $\mathbf{S}$ or

$$\beta_{W\,\mathrm{drag}} = -\frac{2}{5}\frac{r_{\mathrm{g}}}{R}\beta_{\mathrm{Sag}}\,,$$

if $\mathbf{R}$ is parallel to $\mathbf{S}$, where $r_{\mathrm{g}} = 2GM/c^2$ is the Schwarzschild radius of the earth.

The precise measurement of the gravitational redshift by the interference of matter waves in the gravitational field of the earth was described in Ref. 23. In Ref. 23, the atom interferometry experiment on Cesium has been described. Despite here we studied the neutron interferometry, neglecting the structure properties we can get rough constraint on γ parameter. Using the relative precision 2×10^{-9} we can compare the correction due to γ parameter to the phase shift and get rough estimation. For this purpose, we will use the error of the measurement of the phase shift in the experiment[23] being equal to $\approx 1\%$. Supposing that the correction due to Weyl gravity lies in the error bar one can easily find the rough estimation of the upper limit of the Weyl parameter. Comparison the experimental results of Ref. 23 with our theoretical ones will help to obtain the rough estimation of the upper limit for the Weyl parameter as $\gamma \leq 2\cdot10^{-20}$ cm^{-1}.

3. Alternate Approach to Sagnac Effect

Knowing the space–time metric is not enough for a full description of the physical picture of the world. Separation of four-dimensional world into the physical time and three-dimensional physical space of simultaneous events is a useful tool to describe the physics of space–time. In addition to introducing the concept of proper time measured by the clock on reference frame and a single standard equipping of clocks of their own time, it is also necessary to synchronize these clocks to be able to determine the temporal relationship of any events and proper positions in physical space.

Let P_1 and P_3 be the events on the worldline γ of some clock being at rest in a given reference system. The events correspond to the emission and registration of a light signal reflected (P' event) from the mirror, which is at a short distance from the first one together with the other clocks reference system (see Fig. 2). Each of these events correspond to the points of proper time of $\tau(P_1)$, $\tau(P_2)$ and $\tau(P')$. The events P_1, P' and P_3 are causally connected to each other, so they are in a certain time relation: P_3 event happened later than P' event, and the P' event later than P_1. Events in the worldline γ which happens after P_3 and before P_1 are also in a certain time relations with P'.

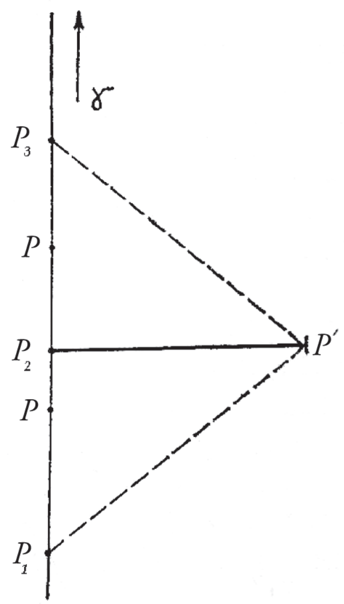

Fig. 2. P_1 and P_3 are the events on the worldline γ, the event P' belongs to light lines P_1, P' and P', P_3 (dashed lines). The event P_2 is the middle of the interval $[P_1, P_3]$ and at the same time, simultaneous with the event P', according to Einstein. This is true when the speed of light does not depend on directions. If the speed of light is bigger in the direction from γ to P', then the event P, simultaneous with P', lies in the range (P_1, P_2), otherwise, in the interval (P_2, P_3).

According to Einstein's definition, the simultaneity of events is based on the assumption of independence of the local speed of light from directions of propagations.

The Einstein definition. The simultaneous event P' is the middle event P_2 of the interval (P_1, P_3) (see Fig. 2). The clocks are synchronized if the moments $\tau(P_1)$, $\tau(P_3)$ and $\tau(P')$ of proper time satisfy the condition

$$\tau(P') = \tau(P_2) = \frac{1}{2}[\tau(P_1) + \tau(P_3)]\,. \tag{26}$$

Following the Einstein definition, one can find that if an event $P_a(x^0, x^i)$ belongs to some worldline γ and $P_b(x^0 + dx^0, x^i + dx^i)$ is an event close to it but does not belong to γ then the time interval between these events can be expressed by the following formula

$$d\tau = \sqrt{-g_{00}}\left(dx^0 + \frac{g_{0i}}{g_{00}}\,dx^i\right), \tag{27}$$

$$dl^2 = h_{ik}\,dx^i\,dx^k\,, \tag{28}$$

here

$$h_{ik} = g_{ik} - \frac{g_{0i}g_{0k}}{g_{00}}\,. \tag{29}$$

If the events are simultaneous, then the condition for their coordinate is

$$dx^0 + \frac{g_{0i}}{g_{00}}\,dx^i = 0\,. \tag{30}$$

Then the time differential is given in a simple formula as

$$dt = e^{\lambda}\sqrt{-g_{00}}\left(dx^0 + \frac{g_{0i}}{g_{00}}dx^i\right), \tag{31}$$

here λ is the scalar function which can be calculated by the following integral

$$\lambda = \int_{x_0^i}^{(x^i)} \rho_k(t = \mathrm{const}, x^n)dx^k\,, \tag{32}$$

where ρ relates to the rotation.[24,25]

Taking Eqs. (29) and (31) into account, one can represent the interval ds in canonical form:

$$ds^2 = -e^{-2\lambda}\,dt^2 + dl^2\,. \tag{33}$$

The physical meaning of the function $\lambda(t, x^i, x_0^i)$ consists in the fact that it is equal to the logarithm, taken with the opposite sign, of the coordinate velocity of light v_c at the moment of time t at the point of space x^i, measured with respect to a clock located at the point x_0^i. The local velocity of light, measured with respect to a local clock, is always equal to unity.

From Einstein's definition, it follows the constancy of the local speed of light in synchronous frames of reference. On the contrary, the well-known optical experience of Sagnac indicates to the dependence of the local velocity of light on the direction of propagation in the rotating reference frame, which is relevant to the class of nonsynchronous ones. But if the coordinate speed of light is anisotropic then simultaneous events cannot satisfy Einstein's definition. If it is different in opposite directions then the event P simultaneous with P' (Fig. 2) divides the interval (P_1, P_3) into unequal parts. Therefore, simultaneous to the event P' is the event P whose moment of proper time satisfies the condition[26]

$$\tau(P) = \frac{1}{2}[\tau(P_1) + \tau(P_3)] + \frac{1}{2}[\tau(P_1) - \tau(P_3)] \times \frac{a_n\, dx^n}{\sqrt{\left(g_{ik} - \frac{g_{0i}g_{0k}}{g_{00}}\right) dx^i\, dx^k}}\,, \tag{34}$$

here $dx^i = x^i(P) - x^i(P')$. Clocks are synchronized if $\tau(P') = \tau(P)$. The condition (34) differs from the Einstein definition of simultaneity (26) with terms proportional to the projection of the metric vector $\vec{a}$, direction of which is given by simultaneous events P and P' (see Fig. 2). This term defines the degree of anisotropy of the coordinate speed of light in a given direction in a given reference system. In a synchronous reference system, the speed of light does not depend on the direction of propagation, so the simultaneity condition (34) reduces to the Einstein condition (26). The distance between them is defined by the following formula[24,25]

$$dl^2 = h_{ik}\, dx^i\, dx^k\,, \tag{35}$$

where

$$h_{ik} = g_{ik} - \frac{g_{0i}g_{0k}}{g_{00}} - a_i a_k\,. \tag{36}$$

Using Eq. (34) one can obtain the expression for the interval of the proper time between two arbitraries but close events expressed by the formula

$$d\tau = \sqrt{-g_{00}}\left(dx^0 + \frac{g_{0i}}{g_{00}} dx^i\right) + a_i\, dx^i\,, \tag{37}$$

while the differential of the time is expressed as

$$dt = e^{\lambda}\left[\sqrt{-g_{00}}\left(dx^0 + \frac{g_{0i}}{g_{00}} dx^i\right) + a_i dx^i\right]. \tag{38}$$

From the expressions (38) and (35), the interval ds can be written as

$$ds^2 = -e^{-2\lambda}\, dt^2 + 2e^{-\lambda} a_i\, dx^i\, dt + dl^2\,, \tag{39}$$

where a_n are the covariant components and a is the module of the metric vector can be defined as

$$a_n = \frac{g_{0n}}{\sqrt{-g_{00}}}\,, \quad a = \sqrt{\frac{g^{ik}a_i a_k}{1 - g^{mj}a_m a_j}}\,. \tag{40}$$

From the expression (39), quadratic form of the interval comes out of the formula for the module of the coordinate speed of light in the direction to the metric vector and having an angle α with it:

$$v_c = e^{-\lambda}\left(-a\cos\alpha + \sqrt{1 + a^2\cos^2\alpha}\right). \tag{41}$$

So, at a given point, the coordinate speed of light depends on the direction of propagation. It has the smallest value toward the metric vector and the highest value in the opposite direction. The speed of light is constant in all directions forming the circular conical surface with apex in the given point and the axis of symmetry along the metric vector. From Eq. (41), one can see that the product of absolute values of the velocity of light in any two directions is constant in a given point, and following equation comes out

$$e^{2\lambda} \cdot v_c \bar{v}_c = 1\,, \tag{42}$$

where $\bar{v}_c$ is the coordinate velocity of light propagating in the opposite direction.

The difference in time of light rays propagating in the same closed loop in opposite directions can be calculated by the curvilinear integral so that

$$\Delta t = \oint \left(\frac{1}{v_c} - \frac{1}{\bar{v}_c}\right) dl\,, \tag{43}$$

where dl is the path element.

In axial symmetric space–times, metric vector is perpendicular to the axis of symmetry. For the light rays, counter-propagating in the plane $\theta = \pi/2$, one can choose the angle α being equal to 0 and π and

$$v_c(\alpha = \pi, 0) = e^{-\lambda}\left(\pm a + \sqrt{1 + a^2}\right). \tag{44}$$

Using Eqs. (42) and (44), we can write expression for the time difference (43) in this way

$$\Delta t = \oint \frac{\bar{v}_c - v_c}{v_c \bar{v}_c}\, dl = \oint \frac{(\bar{v}_c - v_c)}{-g_{00}}\, dl = 2\oint e^{\lambda} a\, dl\,. \tag{45}$$

Now using above expressions, one can easily calculate time delay in slowly rotating space–time metric (10). After the coordinate transformation $\phi \to \phi + \Omega t$ and in equatorial plane $\theta = \pi/2$, the line element of the metric (10) will look as

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2GM}{c^2 r} + \gamma r - \Omega^2 r^2 + 2\Omega(\omega r^2 - j\gamma r)\right) c^2\, dt^2 \\ &+ \left(1 - \frac{2GM}{c^2 r} + \gamma r\right) dr^2 + r^2\, d\phi^2 + 2\big((\Omega - \omega) r^2 + j\gamma r\big) d\phi\, c\, dt\,, \end{aligned} \tag{46}$$

where Ω is the angular velocity of the gravitating object.

One can find only one nonvanishing component of the metric vector (40) of the above metric (46) as

$$a_3 = \frac{\frac{\Omega - \omega}{c} r^2 + j\gamma r}{\sqrt{1 - \frac{2GM}{c^2 r} + \gamma r - \frac{\Omega^2 r^2}{c^2} + 2\frac{\Omega}{c}\left(\frac{\omega r^2}{c} - j\gamma r\right)}}\,, \tag{47}$$

and its absolute value in the linear approximation takes the form

$$a = \frac{\Omega - \omega}{c} r + j\gamma\,. \tag{48}$$

Finally using the expression (48), we can calculate time delay (45) between two counter-propagating rays in the rotating Weyl space–time as

$$\Delta t = \frac{4\pi R}{c} \frac{\frac{\Omega-\omega}{c} R + j\gamma}{\sqrt{1 - \frac{2GM}{c^2 R} + \gamma R - \frac{\Omega^2 R^2}{c^2} + 2\frac{\Omega}{c}\left(\frac{\omega R^2}{c} - j\gamma R\right)}}\,. \tag{49}$$

The expression (49) is obtained using an alternate approach of interpretation of Sagnac effect which is based on anisotropy of the velocity of light.

Following Ref. 20, one can find a critical angular velocity $\bar{\Omega}$ as

$$\bar{\Omega} = \frac{jc}{R^2}\left(\frac{2GM}{c^2 R} - \gamma R\right) = \omega - \frac{\gamma j c}{R}\,, \tag{50}$$

which corresponds to zero time delay $\Delta T = 0$. $\bar{\Omega}$ is the angular velocity of the locally nonrotating observers (LNRO).

As we can see, the term with γ-parameter presents a subtraction to this velocity, in other words, parameter $\bar{\Omega}$ in Weyl space–time becomes smaller when compared to that in Kerr one.

4. Conclusion

In this paper, we studied quantum interference effects in the gravitational field of the slowly rotating object in conformal fourth-order Weyl gravitational theory and found that they can be affected by the γ integration constant of the theory which is responsible for the dark matter in galaxies.[9,10]

It has been found that the phase shift of neutron beams which is responsible to the gravitational potential of the earth has additional linear term γr and has been studied its feasibility of detection with the help of interferometer (Fig. 1).

We obtained the phase shift and time delay in alternate approach of Sagnac effect which can be affected by new parameter γ of the Weyl gravity.

To get the estimation for the value of Weyl parameter γ, one should compare the observational results with the theoretical results. In Ref. 3, one has obtained expression for integration constant in the theory as $\gamma \leq 10^{-28}$ cm^{-1} which was in good agreement with the observed galactic rotation curves without the need for dark matter. Regarding solar system experiments, in Ref. 27, there has been used CASSINI Doppler Datas and got an upper bound for Weyl parameter as $\gamma \sim 10^{-21}$ cm^{-1}. Recently the authors of Ref. 28 from the correction to geodesic effect obtained $\gamma \leq 1.5 \times 10^{-20}$ cm^{-1}. Here, we have estimated lower limit for parameter γ as $\gamma \leq 2 \times 10^{-20}$ cm^{-1} using the experimental results of Ref. 23 which was set on the earth as central body on the precise measurement of the gravitational redshift by the interference of matter waves.

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