# Dynamical gluon mass at non-zero temperature in instanton vacuum model

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## Abstract

In the framework of the instanton liquid model (ILM), we consider thermal modifications of the gluon properties in different scenarios of temperature $T$ dependence of the average instanton size $\bar{\rho}(T)$ and the instanton density $n(T)$ known from the literature [7, 8]. Due to interactions with instantons, the gluons acquire the dynamical temperature dependent "electric" gluon mass $M_{el}(q,T)$. We found that at small momenta and zero temperature $M_{el}(0,0) \approx 362\,\mathrm{MeV}$, however the $T$-dependence of the mass is very sensitive to the temperature dependence of the instanton vacuum parameters $\bar{\rho}(T)$, $n(T)$: it is very mild in case of the lattice-motivated dependence [8] and decreases steeply in the whole range with theoretical parametrization [7]. The inclusion of one-loop thermal gluon corrections gives a rising with temperature contribution $M_{pert,el}(0,T) \sim T$ and allows to reproduce lattice results for the dynamical gluon mass [12, 13].

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## I. INTRODUCTION

The gluodynamics at non-zero temperature $T(\equiv 1/\beta)$ is described by the partition function

$$Z = \int DA_\mu \exp\left\{-\frac{1}{2g^2}\int_0^\beta dx_4 \int d^3x \,\mathrm{tr}\, F_{\mu\nu}F_{\mu\nu}\right\}, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, and the gauge field $A_\mu$ satisfies the periodic condition $A_\mu(\vec{x}, x_4 + \beta) = A_\mu(\vec{x}, x_4)$. The extension of the zero-temperature instanton solution [1], caloron, found in [2] has the form

$$A^I_\mu = \Pi\bar\eta^a_{\mu\nu}(\tau_a/2i)\partial_\nu\Pi^{-1}, \; F_{\mu\nu} = \frac{1}{2}\Pi(\tau\partial)\bar\eta^a_{\mu\nu}(\tau_a/2i)(\tau^+\partial)\Pi^{-1}, \; \Pi^{-1}\partial^2\Pi = 0, \tag{2}$$

$$\Pi(r,t) = 1 + \frac{\pi\rho^2}{\beta r}\sinh\frac{2\pi r}{\beta}/(\cosh\frac{2\pi r}{\beta} - \cos\frac{2\pi t}{\beta}) = 1 + \sum_{n=-\infty}^{\infty}\frac{\rho^2}{r^2 + (t - n\beta)^2} \tag{3}$$

where $r = |\vec{x}|$, $t = x_4$ and $\tau_\mu = (\vec{\tau}, i)$. At small distances $r, t \ll \beta$ the profile $\Pi(x)$ may be approximated as

$$\Pi(x) \approx \Pi_0(x) = \left(1 + \frac{\lambda^2}{3}\right) + \rho^2/x^2, \tag{4}$$

where $\lambda = \pi\rho/\beta$, so the gluon field has an instanton-like shape with modified instanton size,

$$A^{I,a}_\mu = \frac{2\rho'^2}{x^2}\frac{\bar\eta^a_{\mu\nu}x_\nu}{(x^2 + \rho'^2)}, \; \rho'^2 = \rho^2/\left(1 + \frac{\lambda^2}{3}\right). \tag{5}$$

In fact, the accuracy of the approximation (4) is about one per cent up to $r, t \sim \beta$. The extension of the instanton vacuum liquid model (ILM) [5, 6] to non-zero temperature in this regime is straightforward and might be encoded in the temperature dependencies of main parameters of the model, the average instanton size $\bar\rho(T)$ and average instanton density $n(T) = NT/V_3 = 1/R^4(T)$, where $N$ is the total number of instantons [7]. Both $\bar\rho(T)$ and $n(T)$ in ILM are homogeneously decreasing functions of $T$.

But the simplified approximation (4) does not describe nontrivial phase transition near the critical temperature $T_c \sim \Lambda_{QCD}$. Indeed, for $T < T_c$ all color objects are bound into colorless hadrons. The heat bath predominantly consists of weakly interacting pions, so the $T$-dependence of instanton density $n(T)$ should be rather mild, which agrees with expectation of almost constant $T$-dependence $n(T) = n_0(1 + O(T^2/(6f_\pi^2)))$. However, this behavior changes during phase transition, and the expected instanton density $n(T)$ should be exponentially suppressed at large temperature [6].

The extension of ILM which is able to describe the phase transition from confined to deconfined phase near the critical temperature $T_c$ is the so-called dyon-instanton liquid model (DLM) [10]. The authors of [10] concluded later [11] that at very low temperature, the semi-classical description of the Yang-Mills state reconciles the instanton liquid model without confinement, with the t'Hooft-Mandelstam proposal of confinement. In the former, the low temperature thermal state is composed of a liquid of instanton and anti-instantons, while in the latter it is a superfluid of monopoles and anti-monopoles.

The temperature dependence of the QCD vacuum model might be tested comparing with lattice simulations. For example, for the "electric" gluon mass $M_{el}\,(T)$ in the framework of lattice QCD [12, 13] it was found that at $T \geq T_c$ $M_{el}(T) \sim T$ is consistent with Debye screening, has a minimum at $T \sim T_c$, whereas for $T \leq T_c$ $M_{el}(T)$ is a rather slowly decreasing function of temperature $T$. This behavior might be naturally explained in the framework of ILM, which predicts the temperature dependence like $M_{el} \sim (packing\ parameter(T))^{1/2}\bar{\rho}^{-1}(T) = \bar{\rho}(T)n^{1/2}(T)$, a decreasing function of temperature at $T \leq T_c$. Combined with perturbative one-loop thermal gluon contribution to the gluon propagator, which is rising with temperature as $M_{pert,el}(0,T) \sim T$, this model is able to reproduce lattice results for the dynamical gluon mass [12, 13].

The two major technical challenges in calculation of the gluon propagator in ILM framework: the account of zero-modes (fluctuations along of instanton collective coordinates) and the averaging over the collective coordinates of all instantons. We address the former using the approach of [17], while for the latter we extend Pobylitsa's approach [14], applied earlier by us for the gluons at $T = 0$ [15], and consider in this paper its further extension for the ILM averaged gluon propagator at $T \neq 0$.

**Variational estimates in ILM at $T \neq 0$**

The application of the Feynman variational principle to the QCD vacuum filled with instanton gas led to the ILM [4]. Further it was generalized to non-zero temperatures in [7]. Main variational ingredients of this approach are the instanton size distribution function

$\mu(\rho,\, T,\, n)$ and the density of instantons $n(T)$ [6, 7, 9]. ILM instanton size distribution function is closely related to the thermal single instanton one-loop distribution function $d(\rho, T) = C\rho^{b-5}\exp(-A_{N_c}T^2)$ [3]

$$\mu(\rho,\, T,\, n) = C\rho^{b-5}\exp\left[-\Phi(n, T)\rho^2\right], \tag{6}$$

$$\Phi(n,\, T) = \frac{1}{2}A_{N_c}T^2 + \left[\frac{1}{4}A_{N_c}^2 T^4 + \nu\bar{\beta}\gamma^2 n\right]^{1/2}. \tag{7}$$

where $A_{N_c} = 1/3\,[11/6\,N_c - 1]\pi^2$, $b = 11/3\,N_c$, $\nu = \frac{b-4}{2}$, $\bar{\beta} = -b\log(\Lambda\bar{\rho})$, $\gamma^2 = \frac{27\pi^2 N_c}{4(N_c^2-1)}$, and

$$\bar{\rho}^2(T, n) = \frac{1}{\mu_0(T, n)}\int_0^\infty d\rho\mu(\rho, T, n)\rho^2,\ \ \mu_0(T, n) = \int_0^\infty d\rho\mu(\rho, T, n). \tag{8}$$

The content of variational ILM partition function [7] is given by

$$Z \geq Z_1\exp(-\langle E - E_1\rangle), \tag{9}$$

$$Z_1 = \frac{1}{(N/2\,!)^2}\left(\frac{2\mu_0(T, n)V}{N}\right)^N,\ \ \langle E - E_1\rangle = -\frac{N}{2}\bar{\beta}\gamma^2 n\bar{\rho^2}^2 \tag{10}$$

The right hand side of (9) is a trial partition function which depends on variational parameters $\mu$ and $n$. Its maximization with respect to parameter $\mu$ leads to the above-given result (6) for the density dependence. The minimization of the free energy $F = -T/V_3\log Z$ by variation over $n$ leads to the equation for the density

$$n(T) = 2\mu_0(n, T) \tag{11}$$

The variational estimates demonstrated that $\bar{\rho}(T)$ and $n(T)$ are decreasing functions of temperature $T$ (see fig.1) due to the exponential factor in $d(\rho, T) \sim A_{N_c}$. On the other hand, lattice data show that instanton density $n$ is not modified by temperature up to critical temperature $T_c$ [8]. In numerical simulations of ILM [6] it was suggested to interpolate between no suppression ($A_{N_c} = 0$ in Eq.(7)) below $T_c$ and full suppression ($A_{N_c} \neq 0$ in Eq.(7)) above $T_c \sim 150MeV$ , with a width $\Delta T = 0.3T_c$ to be in the correspondence with lattice result [8]. We are following this suggestion and are repeating the calculations with the modification in the Eq.(7) as $A_{N_c} \to A_{N_c}\Theta_{\Delta x}(x - x_c)$, where $x = \bar{\rho}_0 T$, $x_c = \bar{\rho}_0 T_c =$

$0.25 \sim T_c = 150\, MeV$, $\Delta x = \bar{\rho}_0 \Delta T_c = 0.075 \sim \Delta T = 0.3 T_c$ and step-like function

$$\Theta_{\Delta x}(x - x_c) = 1/2[1 + \tanh((x - x_c)/\Delta x)]. \tag{12}$$

Then, we have $\bar{\rho^2}(x)/\bar{\rho^2}(0)$, $n(x)/n(0)$ given by Fig. (1).

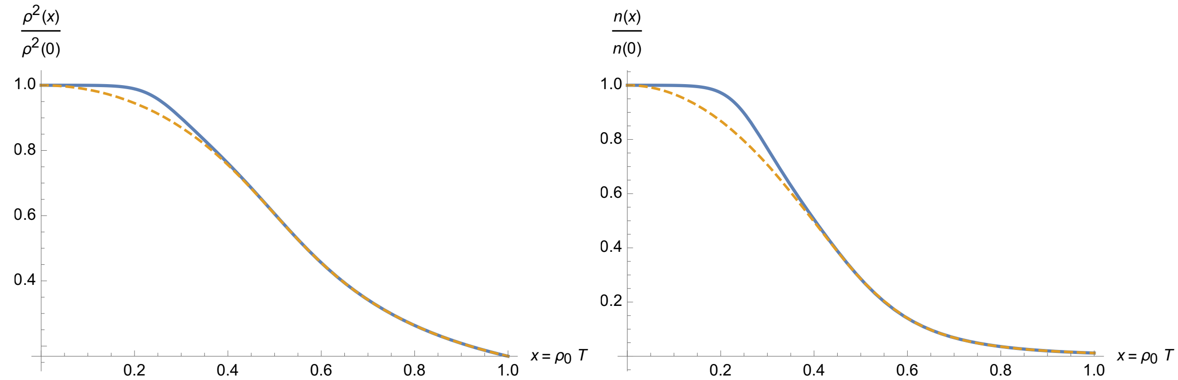


FIG. 1: Left figure represents the ratio of instanton sizes $\bar{\rho}^2(x)/\bar{\rho}^2(0)$, while the right plot illustrates the ratio of instanton densities $n(x)/n(0)$ as functions of $x = \bar{\rho}_0 T$ corresponding to the variational estimates from Refs. [6, 7, 9] at the phenomenological values of $\bar{\rho}(0) = 1/3\, fm$ and $n(0) = 1\, fm^{-4}$. Full line corresponds the modification $A_{N_c} \to A_{N_c} \Theta_{\Delta x}(x - x_c)$ (see Eq.(7) and $\Theta_{\Delta x}(x - x_c)$ is the step-like function Eq.((12))) to interpolate between no suppression below $T_c$ and full suppression above $T_c = 150 MeV$ , with a width $\Delta T = 0.3 T_c$ [6]. Dashed lines correspond to the full suppression at the whole region of $T$.

## II. SCALAR "GLUON" PROPAGATOR AT NON-ZERO TEMPERATURE

We start from the scalar massless field $\phi$ belonging to the adjoint representation, the same as a physical gluon. We have to find its propagator in the external classical gluon field in the ILM $A_\mu = \sum_I A^I_\mu(\gamma_I)$, where $A^I_\mu(\gamma_I)$ is a generic notation for the QCD (anti-) instanton, and $\gamma_I$ stands for all the relevant collective coordinates: the position in Euclidean 4D space $z_I$, the size $\rho_I$ and the $SU(N_c)$ color orientation $U_I$ ($4N_c$ collective coordinates in total). The averaging over the instanton collective coordinates contains the integral over the instanton position $\int d^4 z \equiv \int_{V_3} d^3 z_I \int_0^\beta dz_{I,4}$. In view of periodicity of the fields $\phi(\vec{x}, t + \beta) = \phi(\vec{x}, t)$ at nonzero temperature, we may restrict the integration over $t$ to the period $\beta$, so the effective action takes a form

$$S_\phi = \int_{V_3} d^3 x \int_0^\beta dt \phi^+(\vec{x}, t) P^2 \phi(\vec{x}, t) \tag{13}$$

where $P_\mu = p_\mu + A_\mu$ (in the coordinate representation $p_\mu = i\partial_\mu$). The scalar "gluon" propagator is given by

$$\Delta = (p+A)^{-2} = (p^2 + \sum_i (\{p, A_i\} + A_i^2) + \sum_{i\neq j} A_i A_j)^{-1}, \;\; \Delta_0 = p^{-2}, \tag{14}$$
$$\tilde{\Delta} = (p^2 + \sum_i (\{p, A_i\} + A_i^2))^{-1}, \;\; \Delta_i = P_i^{-2} = (p^2 + \{p, A_i\} + A_i^2)^{-1}.$$

There are no zero modes in $\Delta_i^{-1} = P_i^2$ and $\Delta^{-1} = P^2$, which means the existence of the inverse operators $\Delta_i$ and $\Delta$. Our aim is to find the propagator averaged over instanton collective coordinates $\bar{\Delta} \equiv \langle \Delta \rangle = \int D\gamma\, \Delta$. It is obvious, that the propagator $\Delta$ and free propagator $\Delta_0$ must be periodical functions of time with period $\beta$. In order to proceed, we need to define a few useful notations. The Euclidean time operator $\hat{t}$, defined as $\hat{t}|t\rangle = t|t\rangle$, is hermitian, for this reason its eigenfunctions form a complete ortho-normalized set,

$$\sum_t |t\rangle\langle t| = 1, \;\; \langle t'|t\rangle = \delta(t'-t). \tag{15}$$

If we define the step-operator, $\langle t'|\Theta|t\rangle = \Theta(t'-t)$, in view of $\frac{d}{dt'}\Theta(t'-t) = \delta(t'-t)$, we may conclude that $\Theta^{-1} \equiv \frac{d}{dt}$. The time-periodic state with period $\beta$ may be represented in terms of eigenvectors $|t\rangle$ as

$$|t_\beta\rangle \equiv \sum_{n=-\infty}^{\infty} |t - n\beta\rangle, \;\; \langle t'|t_\beta\rangle = \delta(t - t_\beta) \equiv \sum_{n=-\infty}^{\infty} \delta(t' - (t - n\beta)). \tag{16}$$

Now the evaluation of the propagator $\Delta = (P^2)^{-1}$is straightforward. On the other hand, since $< t'|P^2|t_\beta >= \delta(t'-t_\beta)(\vec{P}^2 + (i\frac{\partial}{\partial t} + A_4)^2) = \delta(t'-t_\beta)P^2$, we have the equation in the form

$$P^2 < t'|\Delta|t_\beta >= \delta(t'-t_\beta) \Rightarrow < \vec{x}', t'|\Delta|\vec{x}, t_\beta >\equiv \sum_{n=-\infty}^{\infty} \Delta(\vec{x}', t'|\vec{x}, t - n\beta), \tag{17}$$

where $\Delta(\vec{x}', t'|\vec{x}, t - n\beta)$ is a usual zero-temperature ($T = 0$) aperiodical propagator.

Our aim is to find the propagator averaged over instanton collective coordinates $\bar{\Delta} = < \Delta > = \int D\gamma\, \Delta$. We are following our previous paper [15], where the approach [14], derived for the quark correlators, was extended for the gluon case. We start first from $\bar{\tilde{\Delta}} =< \tilde{\Delta} >$ (see Eq. (14)). Since Pobylitsa Eqs. [14, 15] are written in operator form, they can be easily extended to $T \neq 0$ case just by calculating of matrix elements of propagator $\tilde{\Delta}$ with periodical states $|t_\beta\rangle$ on the right side.

We can derive the solution of Pobylitsa Eqs. in the ILM by expanding it with respect to the instanton density $n$, since the actual dimensionless expansion parameter is in fact the packing parameter $\rho^4 n \sim (1/3)^4 = 0.012$. The expansion of the inverse propagator up the first-order $\mathcal{O}(n)$ terms has a form

$$\bar{\bar{\Delta}}^{-1} - \Delta_0^{-1} =< \sum_i \{\Delta_0 + (\Delta_i^{-1} - \Delta_0^{-1})^{-1}\}^{-1} >= N\Delta_0^{-1}(\bar{\Delta}_I - \Delta_0)\Delta_0^{-1} + \mathcal{O}(n^2), \quad (18)$$

where $\bar{\Delta}_I = \int d\gamma_I \, \Delta_I$. It is obvious that in this order of expansion we may approximate $\bar{\Delta}^{-1} = \bar{\bar{\Delta}}^{-1} = p^2 + M_s^2$, where we introduced squared dynamical scalar "gluon" mass operator $M_s^2$. Then, we have

$$< t'|M_s^2|t_\beta > \delta_{ab} = N\, p^2(< t'|\bar{\Delta}_I^{ab}|t_\beta > - < t'|\Delta_0^{ab}|t_\beta >)p^2. \quad (19)$$

Accordingly[3] periodic scalar "gluon" propagator in instanton field (3) is

$$\Delta_I^{ab}(x,y) = \Delta_0^{ab}(x,y) + \Delta_1^{ab}(x,y) + \Delta_2^{ab}(x,y) \quad (20)$$

$$\Delta_0^{ab}(x,y) = 1/2\, \mathrm{tr} \frac{\tau_a F(x,y) \tau_b F(y,x)}{\Pi(x) 4\pi^2 (x-y)^2 \Pi(y)} \quad (21)$$

$$F(x,y) = 1 + \sum_m \frac{\rho^2 (\tau x_m)(\tau^+ y_m)}{x_m^2 y_m^2}, \ \ (x_m \equiv x - m\beta\hat{t}, \ y_m \equiv y - m\beta\hat{t}) \quad (22)$$

$$\Delta_1^{ab}(x,y) = 1/2\, \mathrm{tr} {\sum_m}' \frac{\tau_a F(x,y_m) \tau_b F(y_m,x)}{\Pi(x) 4\pi^2 (x-y_m)^2 \Pi(y)} \quad (23)$$

$$\Delta_2^{ab}(x,y) = \sum_m \frac{C^{ab}(x,y_m)}{\Pi(x) 4\pi^2 \Pi(y)}, \quad (24)$$

$$C^{ab}(x,y) = \sum_{r\neq s} \frac{2\Phi_{rs}^a(x)\Phi_{rs}^b(y)}{\beta^2(r-s)^2} - \sum_{r\neq s}\sum_{t\neq u} \frac{\rho^2 \Phi_{rs}^a(x)}{\beta^2(r-s)^2} \frac{\Phi_{tu}^b(y)}{\beta^2(t-u)^2} h_{rs,tu}, \ \ \Phi_{rs}^a(x) = \frac{\rho^2 \beta (r-s) x^a}{x_r^2 x_s^2}$$

$$\sum_m C^{ab}(x,y_m) = \sum_m \sum_{r\neq s} \frac{2\Phi_{rs}^a(x)\Phi_{rs}^b(y_m)}{\beta^2(r-s)^2} = \sum_{r\neq s} \frac{\rho^2 x^a}{x_r^2 x_s^2} \sum_m \frac{\rho^2 y^b}{y_{r+m}^2 y_{s+m}^2}$$

Then

$$\Delta_2^{ab}(x,y) = \sum_{r\neq s} \frac{\rho^2 x^a}{x_r^2 x_s^2} \sum_m \frac{\rho^2 y^b}{y_{r+m}^2 y_{s+m}^2} \frac{1}{\Pi(x) 4\pi^2 \Pi(y)} \quad (25)$$

Let's consider the region $r \sim t \leq \beta$. Then caloron field becomes instanton-like (5) with the modification of the instanton radius $\rho \to \rho^{'}$, where $\rho^{'2} = \rho^2/(1 + 1/3\,\lambda^2)$, and $\lambda = \pi\rho/\beta$. In this region

$$\Delta^{ab}_{I,0} = \frac{1}{2}\mathrm{tr}\frac{\tau_a F_0(x,y)\tau_b F_0(y,x)}{4\pi^2(x-y)^2\Pi_0(x)\Pi_0(y)}, \quad \Pi_0(x) = \frac{x^2+\rho^{'2}}{x^2}, \tag{26}$$

$$\tau_\mu = (\vec{\tau}, i), \;\; \tau^+_\mu = (\vec{\tau}, -i), \;\; \tau_\mu\tau^+_\nu = \delta_{\mu\nu} + i\bar{\eta}_{a\mu\nu}\tau_a, \tag{27}$$

$$F_0(x,y) = 1 + \rho^{'2}\frac{(\tau x)(\tau^+ y)}{x^2y^2} = 1 + \rho^{'2}\frac{(xy)}{x^2y^2} + \rho^{'2}\frac{i\bar{\eta}_{a\mu\nu}\tau_a x_\mu y_\nu}{x^2y^2}, \tag{28}$$

where $\bar{\eta}_{a\mu\nu} = -\bar{\eta}_{a\nu\mu}$ is the 'tHooft symbol. In Eq.(26) it is assumed the position of the instanton $z = 0$ and the orientation $U = 1$. In order to average over the position $z$, we have to change $x \to x - z$, $y \to y - z$ and perform integration $\int_0^\beta dz_4 \int_{V_3} d^3z$. To perform a color orientation averaging, we introduce the orientation factor $O^{ab} = \mathrm{tr}(U^+t^aU\tau^b)$, where $t_a$ are $SU(N_c)$- matrices, change $\Delta^{ab}_I$ to $O^{ab}O^{a'b'}\Delta^{bb'}_I$, and carry out integration over $\int dO$. Here $\int dO O^{ab}O^{ab'} = \delta_{bb'}$, $\int dO O^{ab}O^{a'b'} = (N_c^2-1)^{-1}\delta_{aa'}\delta_{bb'}$. Also, $\int dO O^{ab}\bar{\eta}_{b\mu\nu}O^{a'b'}\bar{\eta}_{b'\mu'\nu'} = (N_c^2-1)^{-1}\delta_{aa'}(\delta_{\mu\mu'}\delta_{\nu\nu'} - \delta_{\mu\nu'}\delta_{\nu\mu'})$.

The contribution of the $\Delta^{ab}_{I,0}$ to the scalar "gluon" dynamical mass operator is given by

$$M^2_{s,0}\delta_{ab} = N\,p^2\left(\bar{\Delta}^{ab}_{I,0} - \Delta^{ab}_0\right)p^2. \tag{29}$$

In the coordinate representation

$$\begin{aligned}
&\bar{\Delta}^{aa'}_{I,0}(x,y) - \Delta^{aa'}_0(x,y) \\
&= \int d^4z dO O^{ac}O^{a'c'}(\Delta^{cc'}_{I,0}(x',y') - \Delta^{cc'}_0(x',y')) \;\; (x' \equiv x - z, \;\; y' \equiv y - z), \\
&= \delta_{aa'}\int d^4z[\frac{3\rho^{'2}}{4\pi^2(N_c^2-1)}f_1(x')f_1(y') + \frac{2\rho^{'4}}{N_c^2-1}f_2(x')g(x'-y')f_2(y')], \\
&f_1(x) = \frac{1}{(x^2+\rho^{'2})}, \;\; f_2(x) = \frac{(x_\mu x_\nu, ix^2)}{x^2(x^2+\rho^{'2})}, \;\; g(x-y) = \frac{1}{4\pi^2(x-y)^2}.
\end{aligned} \tag{30}$$

Dynamical scalar "gluon" mass $M^2_s(\vec{q}, n)$ is naturally defined in 3-momentum $\vec{q}$, Matsubara modes $n$ ($\omega_n = 2\pi nT$) representation. In fact, we have to calculate so called "electric" dynamical scalar "gluon" mass $M^2_s(\vec{q}, n=0) \equiv M^2_s(\vec{q}, T)$.

Define Fourier transformation as

$$f_1(x-z) = \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \exp[i\vec{p}(\vec{x}-\vec{z})] \exp(2\pi n(x-z)_4/\beta) f_1(\vec{p},n). \tag{31}$$

Then, the contribution of the first term of Eq. (30)

$$M^2_{s,0,1}(q,T) \sim q^4 f_1(\vec{q},n=0) f_1(-\vec{q},-n=0) = q^4 f_1^2(q,0),$$

where

$$q^2 f_1(q,0) = q^2\rho'^2 \int_{-\beta/2\rho'}^{\beta/2\rho'} dx_4 \int_{-\infty}^{\infty} dx_2 dx_3 \pi \exp(-q\rho'(\sum_{i=2}^{4} x_i{}^2+1)^{1/2})) \frac{1}{(\sum_{i=2}^4 x_i{}^2+1)^{1/2}}$$
$$\leq q^2\rho'^2 \int_{-\infty}^{\infty} dx_4 dx_2 dx_3 \pi \frac{\exp(-q\rho'(\sum_{i=2}^4 x_i{}^2+1)^{1/2}))}{(\sum_{i=2}^4 x_i{}^2+1)^{1/2}} = 4\pi^2 q\rho' K_1(q\rho'), \tag{32}$$

Here $K_1(x)$ is a modified Bessel function of the second kind, and $\lim_{x\to 0} x\, K_1(x) = 1$. Since, temperature mildly affects dynamical mass form-factor, we may neglect this modification at small temperatures $T \leq T_c$. Careful analysis shows that second term in Eq. (30) and all of other terms including $\Delta_1^{ab}$ and $\Delta_2^{ab}$ give zero or negligible contribution, so we finally obtain

$$M_s(q,T) \approx M_{s,0,1}(q,T) = \left[\frac{3\bar{\rho}'^2(T)n(T)}{(N_c^2-1)} 4\pi^2\right]^{1/2} F(q,T), \tag{33}$$
$$F(0,0)=1,\ F(q,T) \leq F(q,0) = q\bar{\rho}K_1(q\bar{\rho}).$$

## III. REAL GLUON PROPAGATOR AT NON-ZERO TEMPERATURE

We have to extend the calculations of averaged full gluon propagator $\bar{G}^{ab}_{\mu\nu}$, considered in previous paper [15], to non-zero temperature case. The most essential point here is the lack of the relativistic covariance, since Euclidian time is limited to interval $0 \leq x_4 \leq \beta$, and all of the bosonic fields ( background $A_\mu$, the fluctuations $a_\mu$ and zero-modes $\phi_\mu$) must be time-periodic functions with period $\beta$, $A_\mu(\vec{x}, x_4+\beta) = A_\mu(\vec{x}, x_4)$. The result for the gluon propagator can be written in the operator form [15] , using the framework of evaluations of zero-modes suggested in [17]. This significantly simplifies our problem, since at the end we have just to calculate the matrix element of the operators between time state $|t'\rangle$ and periodical state $|t_\beta\rangle$ defined in Eq (16). Then we can obtain for the propagators written in terms of three-momenta $\vec{k}$ and Matsubara modes $n$ ($\omega_n = 2\pi nT$)

$$\bar{G}^{ab}_{\mu\nu}(\vec{k},\omega_n) = [(k^2+\omega_n^2)\delta_{ab}\delta_{\mu\nu} + \Pi^{ab}_{\mu\nu}(\vec{k},\omega_n)]^{-1},$$

where from now we take the gauge fixing parameter $\xi = 1$. On the other hand, in the first order in density $n$, we have the solution of the Pobylitsa Eq. for the polarization operator in the form

$$\Pi_{\rho\nu} = N{S^0}^{-1}_{\rho\sigma}(\bar{S}^I_{\sigma\mu} - S^0{}_{\sigma\mu}){S^0}^{-1}_{\mu\nu}. \tag{34}$$

where $S^0{}_{\mu\nu} = \delta_{\mu\nu}/p^2$ is the free gluon propagator and the single instanton gluon propagator [16] is given as

$$S^I_{\mu\nu} = q_{\mu\nu\rho\sigma}P^I_\rho\Delta^2_I P^I_\sigma, \tag{35}$$

where $q^I_{\mu\nu\rho\sigma} = \delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho} + \epsilon_{\mu\nu\rho\sigma}$. The "electric" gluon mass is defined by Eq. $M^2_{el}(\vec{k},T)\delta_{ab} = \Pi^{ab}_{44}(\vec{k},n=0)$.

We expect that the most slowly decreasing part of the matrix elements of $S^I_{\nu\mu} - S^0{}_{\nu\mu}$ will only contribute to $M_{el}$. In coordinate space comparing the effects from $i\partial_\mu$ with $A^I_\mu$, we conclude from Eq. (35) that the the most slowly decreasing part part of the $S^I_{\nu\mu} - S^0{}_{\nu\mu}$ in Eq. (34) comes from

$$p_\rho(\textit{the most slowly decreasing part of } (\Delta_I - \Delta_0)\Delta_0 + \Delta_0(\Delta_I - \Delta_0))p_\sigma$$

and only this term will contribute to $M_{el}$. Comparing it with Eq. (29), we conclude that $M^2_{el}(\vec{k},T) = 2M^2_s(\vec{k},T)$, where $T$ and $q$ dependencies are represented by Fig.2. Using the phenomenological values of $\bar{\rho}$ and $n$ at $T=0$, we obtain $M_{el}(0,0) = 362\,MeV$.

## IV. SUMMARY AND DISCUSSION

We extended the calculations of the dynamical gluon mass in ILM [15] to non-zero temperature. In this case we are interested in the so-called "electric" gluon mass $M_{el}(q,T)$, which corresponds to $\Pi_{44}$-component of polarization operator. We analyzed the temperature $T$ dependence of the main parameters of the ILM, the average instanton size $\bar{\rho}(T)$ and

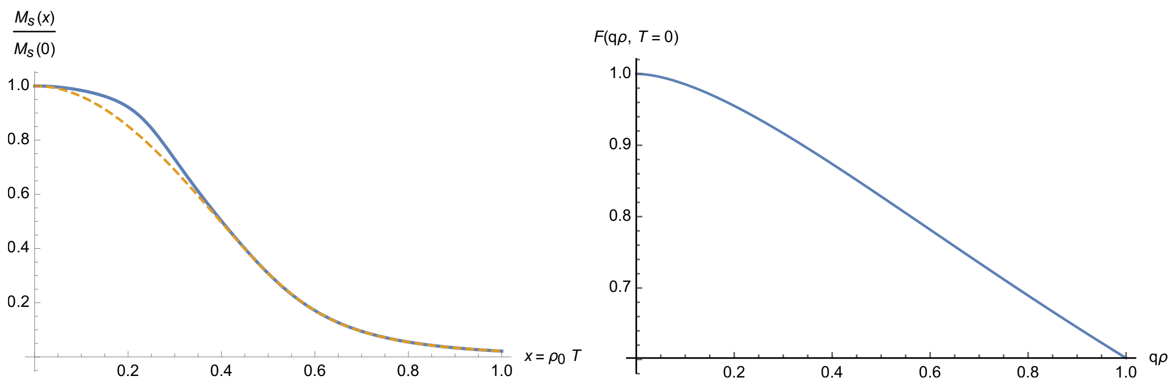


FIG. 2: Full line on the left plot: temperature dependencies of "electric" and "scalar" gluon dynamical masses $M_{el}(0,T)/M_{el}(0,0) = M_s(0,T)/M_s(0,0)$ (since $M_{el}(q,T) = 2^{1/2}M_s(q,T)$) with modification $A_{N_c} \to A_{N_c}\Theta_{\Delta x}(x - x_c)$ (see Eqs.((7),(12))) to interpolate between no suppression below and full suppression above $T_c = 150\ MeV$ , with a width $\Delta T = 0.3\,T_c$ [6]. At small $T \leq T_c$ full line correspond to the $M_{el}(0,T)/M_{el}(0,0) = \bar{\rho}'(T)/\bar{\rho}(T) = (1 - 1/6\,\pi^2\bar{\rho}_0^2T^2)$. Dashed line here corresponds to the full suppression at the whole region of $T$ ($A_{N_c}$ is not modified). Here $M_{el}(0,0) = 2^{1/2}M_s(0,0) = 362\,MeV$ at the phenomenological values of $\bar{\rho}(0) = 1/3\,fm$ and $n(0) = 1\,fm^{-4}$. Right plot: form-factor of dynamical mass $F(q,0)$, Eq. (33).

instanton density $n(T)$. . We found they are homogeneously decreasing functions of temperature due to influence of thermal gluon fluctuations [7]. These findings agree with lattice investigations [8], which demonstrated that $\bar{\rho}(T),\ n(T)$ are decreasing rapidly for $T \geq T_c$, where $T_c$ is the critical temperature . For temperatures below the critical temperature $T_c$ these functions are almost constant, and we took into account this scenario by neglecting the contributions of thermal gluon fluctuations at low temperature $T \leq T_c$ [6]. The comparison of both of these scenarios is presented at the Fig. 1.

In order to find gluon propagator in the ILM background field at nonzero temperature $T \neq 0$, we have to solve the gluon zero-modes problem and to average full gluon propagator over collective coordinates of all instantons. It was done in the framework of our previous paper [15], extended to non-zero temperature case. First, we evaluated the three-momentum $\vec{q}$ and temperature $T$ dependent "electric" scalar "gluon" dynamical mass $M_s(q,T)$. The solution of zero-modes problem leads to the relation $M_{el}^2(q,T) = 2M_s^2(q,T)$. The final results for the "electric" gluon dynamical mass $M_{el}(q,T)$ are presented in the Fig.2.

It is interesting to compare our result for the dynamical "electric" gluon mass $M_{el}$ with the result of lattice calculations, which observed that $M_{el}(0,T)$ is a decreasing function of $T$ for $T \leq T_c$ , and is an increasing function of $T$ above the confinement-deconfinement phase transition [12, 13]. The grows of $M_{el}(0,T)$ for $T \geq T_c$ may be explained by perturbative thermal gluon correction and is expected to have an almost linear functional dependence $M_{pert,el}(0,T) \sim T$. Since thermal gluons are incorporated in our framework, it is easy to reproduce within ILM model the lattice measurements of the dynamical "electric" gluon mass.

We assume to apply our result to the calculations of temperature dependencies of the heavy quarkonium properties.

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