Asynchronous iteration and Monte Carlo method for solution system of linear equations

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Iterations methods for the solution linear and nonlinear equations are widely used because of their simplicity, fault tolerance, ease of parallelization. Historically, iterative algorithms were created and studied for use single processor computers. In multiprocessor computers the parallel application of iterative methods often shows poor scaling and less than optimal parallel efficiency. As opposed to ordinary iterative asynchronous method often have much better parallel efficiency as they almost never need to wait to communicate between possessors.

Our investigation concerns to study [see:1,2,3,4,5,6] of asynchronous algorithms with Monte Carlo approach and presenting a mathematical description of this computational process to the multiprocessor environment.

Let's given system of linear equations in operator form as $\psi=F(\psi)$ with the dimensions NxN. Let $\{J_n\}_{n=1}^{\infty}$ be a sequence of non-empty subsets of set {1,2,...,N} which is a called chaotic sequence of sets. Initial vector be $\psi(0)$. We will construct the sequence $\{\psi(n)\}_{n=1}^{\infty}$ of iterations by the following way

$$\psi(n+1) = F_{J_{n+1}}(\psi(n)), \quad n = 0,1,2... \qquad (1)$$

The method of asynchronous iterations (1) is constructed with the following rule. Let $\psi(0)$ be given, then

$$\psi_i(n) = \begin{cases} \psi_i(n-1) & if \quad i \notin J_n \\ f_i(\psi_1(S_1(n)),...,\psi_N(S_N(n))) & if \quad i \in J_n \end{cases}. \qquad (2)$$

Here in (2) ψ_i is a component of the vector ψ, and $\{S_i(n)\}_{n=1}^{\infty}, i = 1,N$ is a sequence of sets of non-negative integers, satisfying the following conditions, for any $i = 1,N$

$$S_i(n) \le n-1, \; n = 1,2,..., \quad i = 1,...,N \qquad (3)$$

$$S_i(n) \to \infty, \; n \to \infty; \qquad i = 1,...,N \;, \qquad (4)$$

and every element i occurs infinitely many often in the sets J_n n=1,2,... . The $S_i(n)$, i=1,N are called delays or lag. The condition (3) says that only components of previous iterates can be used in the evaluation of a new iterate. The condition (4) eventually say that values of an early iterate cannot be used any more in further evaluations, and more and more recent values of the components have to be used instead. The last condition "every element i occurs infinitely many often in the sets J_n n=1,2,... guarantees that no component is abandoned forever.

The main difference between asynchronous iterative and the other iterative methods in parallel is the chaotic behavior of the vector components, which is expressed by the set of chaotic sequences J_n . The chaotic iterative process has the following two main advantages:

a) it is possible to calculate each coordinate of the iteration vector independently from the others (like the Monte Carlo method),
b) the convergence rate is higher, because this method sometimes essentially becomes an implicit iterative method like Gauss-Seidel method.

The asynchronous iterative method converges to the fixed point slower than the simple Jacobi iterative methods; however, the efficiency of asynchronous iterative process is better. The results of our simple Monte Carlo computational experiments show the convergence and efficiency of asynchronous iterative processes for considered problems.

References

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