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UMUMLASHGAN KASR TARTIBLI INTEGRO-DIFFERENSIAL OPERATORLAR VA ULARNING TATBIQLARI

5A130101 –Matematika (differensial tenglamalar) mutaxassisligi bo’yicha magistr akademik darajasini olish uchun

MAGISTRLIK DISSERTATSIYASI

Ilmiy rahbar: fizika - matematika fanlari nomzodi, dotsent X.N.Qosimov

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MUNDARIJA:

KIRISH

Birinchi prezidentimiz "Yuksak ma'naviyat – yengilmas kuch" asarida shunday degan edilar: "Vatanimizning kelajagi, xalqimizning ertangi kuni, mamlakatimizning jahon hamjamiyatidagi obro'–e'tibori, avvalambor, farzandlarimizning unib – o'sib, ulg'ayib, qanday inson bo'lib hayotga kirib borishiga bog'liqdir. Biz bunday haqiqatni hech qachon unutmasligimiz kerak". Qolaversa, Birinchi prezidentimiz Islom Karimov ta'kidlaganidek, «Hammamizga teran bir haqiqat ayon bo'lishi kerak – biz yurtimizning ertangi rivoji yo'lida qanday chuqur o'ylangan dasturlarni tuzmaylik, bu rejalarni bajarish uchun qanday moddiy baza va imkoniyatlarni yaratmaylik, buning uchun qanday ko'p sarmoya safarbar etmaylik, ularning barchasini amalga oshiradigan, ro'yobga chiqaradigan qudratli bir omil borki, u ham bo'lsa, yuqori malakali ishchi kuchi va yurtimizning ertangi kuni, taraqqiyoti uchun mas'uliyatni o'z zimmasiga olishga qodir bo'lgan yetuk mutaxassis yoshlarimiz, desak, o'ylaymanki, hech xato bo'lmaydi». 1

O'zbekiston mustaqillikka erishgan kundan boshlab o'tgan qisqa vaqt ichida o'zbek xalqi siyosiy – ijtimoiy, iqtisodiy va madaniy sohalarda katta yutuqlarga erishdi. O'z tarixiga yangicha tafakkur asosida yondoshish, ulug' ajdodlar qoldirgan boy madaniy, ma'naviy merosni o'rganish sharafiga muyassar bo'ldi, milliy g'ururi qayta tiklandi. Respublikada ilm-fan, jumladan, matematika fani taraqqiyot bosqichiga ko'tarilmoqda. O'tmishdagi riyoziyot (matematika) daholarining shuxratini tiklash, ularning g'oyalarini xalq hayotiga tatbiq etishdek ulug' ishlar amalga oshirilmoqda. Hozirgi kunga kelib, matematika va uning turli shahobchalari shunchalik taraqqiy etib rivojlanib bormoqdaki, bu fanning eng kamida asoslarini mukammal bilmay turib, inson hayotida o'z o'rnini topishga qiynaladi. "Kardlar tayyorlash milliy dasturi"da ko'zda tutilgan kadrlar tayyorlashning milliy modeli va uni ta'minlovchi ta'lim tizimida ana shu nuqtai nazar hisobga olingan. 2

Shu maqsadda, Birinchi prezidentimiz Islom Karimov tashabbusi bilan ishlab chiqilgan “Kadrlar tayyorlash Milliy dasturi” ning hayotga tatbiq etilishi tufayli uzluksiz ta’lim tizimi muntazam yangilanib va takomillashib, ta’lim muassasalarining zamonaviy moddiy-texnik va o‘quv bazasini shakllantirish va mustahkamlash, ta’lim-tarbiya jarayoniga yangi standartlar, ilg‘or pedagogik va axborot texnologiyalarini joriy etish borasida keng ko‘lamli ishlar amalga oshirilmoqda.

Yuqori malakali pedagog kadrlar tayyorlash va qayta tayyorlashga alohida e’tibor berib kelinmoqda, chunki kadrlar tayyorlashning sifati, erkin fikrlovchi shaxs-fuqoroni kamol toptirishiga ertaga sinf xonalari va auditoriyalarda kimlar dars va saboq berishiga bog‘liq.

Ma’lumki, oliy ta’limni ikki bosqichli qilib tashkil etilganligi, ya’ni bakalavriat va magistratura bosqichlaridan iboratligi oliy malakali mutaxassislar tayyorlash sifatini ko‘tarishga xizmat qilmoqda. Haqiqatdan ham, bakalavriatda ma’lum yo‘nalish bo‘yicha u yoki bu kasbni egallash uchun zarur bo‘lgan fundamental fanlar puxta o‘rganilsa, magistraturada shu yo‘nalishga mos biron bir mutaxassislikni egallash uchun zarur bo‘lgan fanlarni chuqur o‘zlashtirib, bu fanlar rivojining xozirgi zamon darajasigacha o‘rganish imkoni mavjuddir. Mana shu nuqtai nazardan qaraganda, magistraturani tugatayotgan har bir mutaxassis o‘z yo‘nalishidagi asosiy fanlari bo‘yicha zamonaviy bilimlarga ega bo‘lishi va shu soha bo‘yicha ilmiy tadqiqot olib borish qobilyatiga ega bo‘lishi zarur .

Fanni malakali kadrlar bilan ta’minlash, xodimlarning professional bilimdonligi darajasini oshirish, ularning qobiliyatlarini ro‘yobga chiqarish uchun barcha sharoitlarni yaratish ilmiy jarayonni jadallashtirishning asosiy omilidir...

Fan sohasidagi kadrlarni jadal ko‘paytirish va yoshartirish uchun, O‘zbekiston intellektual imkoniyatlarini keskin darajada oshirish uchun respublika rahbariyati hozir zarur mablag‘larini ajratib va tegishli tashkiliy masalalarni hal qilib kelmoqda.

Keltirilgan fikrlarga e’tibor qaratadigan bo‘lsak, mamlakatimizning rivojlangan davlatlar qatoridan o‘z o‘rnini egallashiga erishishning asosiy omillaridan biri yuqori malakali kadrlar tayyorlashdir.

Mavzuning dolzarbligi. Hozirgi zamon texnikasining tez rivojlanishi differensial tenglamalarga xususan giperbolik turdagi tenglamalarni turli ko’rinishdagi siljishli chegaraviy masalalar qo’yilishi va o’rganishni qilmoqda.

Shuning uchun ham ikkita buzilish chizig’iga ega bo’lgan giperbolik tipdagi tenglama uchun yangi siljishli chegaraviy masalalar mavzusi dolzarb hisoblanadi.

Ishning maqsadi. Ushbu magistrlik dissertatsiyasida ikkita buzilish chizig’iga ega bo;lgan giperbolik tipdagi tenglama uchun chegaraviy shartida umumlashgan kasr tartibli integro-differensial operatorlar qatnashgan siljishli chegaraviy masalalarni o’rganishga bag’ishlangan.

Tekshirish usullari. Qo’yilgan masalalarni tekshirish umumlashgan kasr tartibli integro-differensial operatorlar nazariyasidan foydalanib ikkinchi tur Volterra integral tenglamalariga keltirilgan.

Ilmiy yangiligi, nazariy va amaliy ahamiyati. Ikkita buzilish chizig’iga ega bo’lgan giperbolik tipdagi uchun yangi siljishli masalalar o’rganilgan. Funksiyadan funksiya bo’yicha olingan kasr tartibli integro-differensial operatorlarning xossalaridan hamda integral tenglamalar nazariyasidan foydalanib, qo’yilgan masalalar yechimining mavjudligi va yagonaligi isbotlangan.

Olingan natijalar ko’proq nazariy ahamiyatga ega bo’lib, ulardan differensial tenglamalar uchun chegaraviy masalalar yo’nalishini rivojlantirishda foydalanish mumkin.

Ishning muhokamasi. Magistrlik dissertatsiyasida olingan ilmiy natijalar Farg’ona davlat universiteti matematik analiz va differensial tenglamalar kafedrasi qoshidagi f.m.f.doktori, professor A.Q.O’rinov rahbarligidagi “Differensial tenglamalar va unga turdosh matematik sohalarning dolzarb muammolari” nomli ilmiy seminarida ma’ruza qilinib, muhokamaga o’tkazilgan.

Chop etilgan ishlar. Dissertatsiyada olingan ilmiy natijalar "Об одной краевой задаче со смешением для уравнения гиперболического типа с двумя линиями вырождения", "Об одной краевое задаче со смешением для гиперболического уравнения типа с двумя линиями вырождения, вырождающегося внутри области", " О некоторых задачах со смешением для уравнения гиперболического типа с двумя линиями вырождения" nomli ilmiy maqolalar sifatida chop etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yhatidan iborat.

Dissertatsiyaning mazmuni.

Elliptik va giperbolik hamda aralash tipdagi tenglamalar nazariyasi amaliy xarakterdagi masalalarni yechishga tadbiq etilayotganligi uchun ham hozirgi kunda tez rivojlanib bormoqda. Hozirgi kunda bu nazariya differensial tenglamalar nazariyasining asosiy nazariyalardan biriga aylandi.

Bu nazariyaga quyidagi ishlarda asos solingan: Ф.Трикоми, С.Геллерстедта, А.В.Бицадзе [7], Ф.И. Франкля, К.И.Бабенко [4].

Giperbolik va aralash tipdagi tenglamalar uchun chegaraviy masalalar nazariyasining tez rivojlanib borayotgan yo'nalishlardan biri siljishli chegaraviy masalalar nazariyasidir. Bu sohada А.В.Бицадзе [7], Л. Берса [6], M.S.Salohiddinov [17], T.D. Jo'rayev, М.М.Смирнова, A.Q. O'rinov [22] va ularning o'quvchilari ishlarini ta'kidlash lozim.

Giperbolik va aralash tipdagi tenglamalar uchun lokal va nolokal chegaraviy masalalarni o'rganishda kasr tartibli integro-differensial operatorlar kompozitsiyasi xossalari o'rganish zaruriyati kelib chiqadi.

Bu dissertatsiyada umulashgan kasr tartibli integro-differensial operatorlar kompozoitsiyasi xossalari va uning chegaraviy masalalar yechishga tadbiqi o'rganiladi.

Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yhatidan iborat.

Dissertatsiyaning kirish qismida mavzuning dolzarbligi, o'rganilganlik darajasi berilgan.Birinchi bob uchta paragrafdan iborat. Birinchi bobning birinchi paragrafida umumlashgan kasr tartibli integro-differensial operatorlarga ta'rif beriladi.

$$F_{kx}\begin{bmatrix} a & ,b \\ c & ;x \end{bmatrix} f(x) \equiv \begin{cases} \dfrac{sign(x-k)}{\Gamma(c)} \int\limits_k^x |x-t|^{c-1} F\left(a,b,c;\dfrac{x-t}{x}\right) f(t)dt, \ c>0; \\ f(x), \qquad c=0; \\ sign(x-k)x^a \dfrac{d}{dx} x^{-a} F_{kx}\begin{bmatrix} a & ,b+1 \\ c+1 & ;x \end{bmatrix} f(x), \ -1<c<0. \end{cases} \qquad (1.1.1)$$

(1.1.1) operatorlar *kasr tartibli umumlashgan integro-differensial operatorlar* deb atalib, ulardan hususiy holda Riman-Luivillning integro-diffrensial operatorlari kelib chiqadi, ya'ni

$$F_{kx}\begin{bmatrix} 0 & ,b \\ c & ;x \end{bmatrix} \equiv F_{kx}\begin{bmatrix} a & ,0 \\ c & ;x \end{bmatrix} \equiv D_{kx}^{-c}, \ c>0;$$

$$F_{kx}\begin{bmatrix} 0 & ,b \\ c & ;x \end{bmatrix} \equiv F_{kx}\begin{bmatrix} a & ,0 \\ c & ;x \end{bmatrix} \equiv \frac{d}{dx} D_{kx}^{-(c+1)} \equiv D_{kx}^{-c}, \ -1<c<0.$$

Demak, (1.1.1) Riman-Luivill ma'nosidagi D_{kx}^{α} integro-differensial operatorlarni $F\ a,b,c;x$ -Gauss gipergeometrik funksiyasi yordamidagi umumlashmasi ekan.

Keyingi satrlarda $k<x$ da $F_{kx}=F_{kx}$, $k>x$ da esa $F_{kx}=F_{xk}$ kabi yozishga kelishib olamiz.

1.1.1.Teorema. *Agar* $f(x)\in C(m,n)\cap L_1(m,n)$ *bo'lsa, ixtiyoriy* $a,b\in R, c\in(0,1)$ *va* $x,k\in(m,n)$ *uchun*

$$x^a F_{kx}\begin{bmatrix} -a & ,b-c \\ -c & ;x \end{bmatrix} x^{-a} F_{kx}\begin{bmatrix} a & ,b \\ c & ;x \end{bmatrix} f(x) = f(x) \qquad (1.1.2)$$

tenglik o'rinli bo'ladi

1.1.2.Teorema. *Agar* $f(x)\in C^{\,0,\delta}(0,1)\cap L_1(0,1)$ *bo'lsa, ixtiyoriy* $a,b\in R, c\in(0,1)$ $\delta\in\ 0,1\ , x\in\ 0,1$ *uchun*

$$x^{a}F_{0x}\begin{bmatrix}-a\ ,b-c\\ -c\ ;x\end{bmatrix}x^{-a}F_{x1}\begin{bmatrix}a\ ,b\\ c\ ;x\end{bmatrix}f(x)=$$

$$=\cos(c\pi)f(x)+\frac{\lambda_1}{\pi}\int_0^1\left(\frac{t}{x}\right)^{c-a}\frac{f(t)dt}{t-x}+\frac{\lambda_2}{\pi}\int_0^1\left(\frac{t}{x}\right)^{c-b}\frac{f(t)dt}{t-x}$$

tenglik o'rinli bo'ladi, bu yerda

$$\lambda_1=\frac{\sin(b\pi)\sin\ (c-a)\pi}{\sin\ (b-a)\pi},\qquad \lambda_2=\frac{\sin(a\pi)\sin\ (c-b)\pi}{\sin\ (a-b)\pi}.$$

1.1.3.Teorema. *Faraz qilaylik,* $-1<b<c<a<0$ *yoki* $-1<a<c<b<0,$ $l\geq 0;\omega(x),\ \tau(x)\in C\ 0,1$; $\omega(x)$*-kamaymaydigan musbat funksiya va* $x=x_0\,(0<x_0<1)$ *nuqtaning qisqa atrofida* $\omega(x)\tau(x)$ *ko'paytma* $\gamma>(-c)$ *tartibli Gyolder shartini qanoatlantirsin. Agar* $0,x_0$ *oraliqda* $\tau(x)$ *funksiya eng katta musbat (eng kichik manfiy) qiymatga* $x=x_0$ *nuqtada erishsa, u holda*

$$F_{0x}\begin{bmatrix}a\ ,b\\ c\ ;x\end{bmatrix}x^{l}\omega(x)\tau(x)\Big|_{x=x_0}>0\qquad(<0)\qquad(1.1.3)$$

tengsizlik o'rinli bo'ladi.

1.1.4. Teorema. *Faraz qilaylik,* $-1<c<a<0<b<1+c$ *yoki* $-1<a<c<b<0;$ $\omega\ x\ ,\tau\ x\ \in C\ 0,1$; $\omega\ z$ *-o'smaydigan musbat funksiya va* $x=x_0\ \ 0<x_0<1$ *nuqtaning qisqa atrofida* $\omega\ x\ \tau\ x$ *ko'paytma* $\gamma\left[>\ -c\ \right]$ *ko'rsatkichli Gyolder shartini qanoatlantirsin. Agar* $x_0,1$ *oraliqda* $\tau\ x$ *funksiya eng katta musbat (eng kichik manfiy) qiymatga* $x=x_0$ *nuqtada erishsa, u holda*

$$F_{x1}\begin{bmatrix}a,\ b\\ c;\ x\end{bmatrix}\omega\ x\ \tau\ x\ \Big|_{x=x_0}>0\ \ <0\qquad(1.1.8)$$

tengsizlik o'rinli bo'ladi.

Birinchi bobning ikkinchi paragrafida funksiyadan funksiya bo'yicha olingan kasr tartibli hosila va integrallarni o'rganishga bag'ishlangan.

Umumlashgan Abel integral tenglamasi. Ushbu

$$\frac{1}{\Gamma(\alpha)}\int_a^x \frac{\varphi(t)dg(t)}{\left(g(x)-g(t)\right)^{1-\alpha}} = f(x), \quad 0<\alpha<1 \tag{1.2.1}$$

ko'rinishdagi integral tenglama *umumlashgan Abel integral tenglamasi* deyiladi. Bu yerda $g(x)$ uzluksiz differensiallanuvchi, o'suvchi va $g'(x)\neq 0$ funksiya.

Ta'rif. *$\varphi(x)\in L_1(a,b)$ $(a<b<+\infty)$ va $g(x)$ uzluksiz differensiallanuvchi, o'suvchi (kamayuvchi) funksiya bo'lsin. Ushbu*

$$D^{-\alpha}_{a,x,g(x)}\varphi(x)=\frac{1}{\Gamma(\alpha)}\int_a^x \left(g(x)-g(t)\right)^{\alpha-1}\varphi(t)dg(t), \qquad \alpha>0, \tag{1.2.7}$$

$$\left[D^{-\alpha}_{a,b,g(x)}\varphi(x)=\frac{1}{\Gamma(\alpha)}\int_x^b \left(g(t)-g(x)\right)^{\alpha-1}\varphi(t)dg(t), \qquad \alpha>0\right] \tag{1.2.8}$$

ko'rinishdagi ifodalar $\varphi(x)$ funksiyaning $g(x)$ funksiya bo'yicha olingan α (kasr) tartbli (Riman-Luivill ma'nosida) integrallari deyiladi.

Ta'rif. *$\varphi(x)$ funksiya $[a,b]$ kesmada aniqlangan va $g(x)$ uzluksiz differensiallanuvchi, o'suvchi (kamayuvchi) bo'lib, $g'(x)\neq 0$ bo'lsin.*

$$D^{\alpha}_{a,x;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dg(x)}\int_a^x \frac{\varphi(t)dg(t)}{[g(x)-g(t)]^{\alpha}}, \quad 0<\alpha<1, \tag{1.2.10}$$

$$\left[D^{\alpha}_{x,b;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dg(x)}\int_x^b \frac{\varphi(t)dg(t)}{[g(x)-g(t)]^{\alpha}}, \quad 0<\alpha<1\right] \tag{1.2.11}$$

ko'rinishdagi ifodalar $\varphi(x)$ funksiyadan $g(x)$ funksiya bo'yicha olingan α (kasr) tartibli (Luivill ma'nosidagi) hosilalari deyiladi.

Lemma. *Agar $\varphi(x)$ funksiya kesmada absolyut uzluksiz bo'lsa, $[a,b]$ kesmaning deyarli barcha nuqtalarida $\varphi(x)$ funksiyaning kasr tartibli hosilalari mavjud bo'lib, quyidagi formula o'rinli bo'ladi:*

$$D^{\alpha}_{a,x;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\left[\frac{\varphi(a)}{[g(x)-g(a)]^{\alpha}}+\int_a^x \frac{\varphi'(t)dg(t)}{[g(x)-g(t)]^{\alpha}}\right], \quad 0<\alpha<1$$

$$D^{\alpha}_{x,b;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\left[\frac{\varphi(b)}{[g(b)-g(x)]^{\alpha}}+\int_x^b \frac{\varphi'(t)dg(t)}{[g(x)-g(t)]^{\alpha}}\right], \quad 0<\alpha<1$$

Teorema. *$\alpha>0$ bo'lsin. U holda*

$$D^{\alpha}_{a,x;g(x)}D^{-\alpha}_{a,x;g(x)}\varphi(x)=\varphi(x)\ , \qquad D^{\alpha}_{x,b;g(x)}D^{-\alpha}_{x,b;g(x)}\varphi(x)=\varphi(x) \tag{1.2.12}$$

tengliklar barcha $\varphi(x)\in L_1(a,b)$ *funksiyalar uchun,*

$$D^{-\alpha}_{a,x;g(x)}D^{\alpha}_{a,x;g(x)}\varphi(x)=\varphi(x), \qquad D^{-\alpha}_{x,b;g(x)}D^{\alpha}_{x,b;g(x)}\varphi(x)=\varphi(x) \tag{1.2.13}$$

tengliklar mos ravishda barcha

$$\varphi(x)\in D^{-\alpha}_{a,x;g(x)}(L_1), \qquad \varphi(x)\in D^{-\alpha}_{x,b;g(x)}(L_1)$$

funksiyalar uchun bajariladi.

Yuqoridagi $\varphi(x)$ funksiyaning $g(x)$ funksiya bo'yicha olingan kasr tartibli hosilasi va integrali yordamida quyidagi integro-differensial operatorlarni kiritamiz.

$$D^{\alpha}_{a,x;g(x)}\varphi(x)=\begin{cases}\dfrac{1}{\Gamma(-\alpha)}\displaystyle\int_a^x[g(x)-g(t)]^{-\alpha-1}\varphi(t)dg(t), \ agar\ \alpha<0\ \ bo'lsa,\\ \dfrac{d^n}{d[g(x)]^n}D^{\alpha-n}_{a,x;g(x)}\varphi(x), \ \ agar\ \alpha>0\ \ bo'lsa,\end{cases}$$

$$D^{\alpha}_{x,b;g(x)}\varphi(x)=\begin{cases}\dfrac{1}{\Gamma(-\alpha)}\displaystyle\int_x^b[g(x)-g(t)]^{-\alpha-1}\varphi(t)dg(t), \ agar\ \alpha<0\ \ bo'lsa,\\ (-1)^n\dfrac{d^n}{d[g(x)]^n}D^{\alpha-n}_{x,b;g(x)}\varphi(x), \ \ agar\ \alpha>0\ \ bo'lsa,\end{cases}$$

bu yerda $n=[\alpha]+1.$

Birinchi bobning uchinchi paragrafida quyidagi ko'rinishdagi kasr tartibli integro-differensial operatorlar kompozitsiyasining ba'zi xossalari haqida so'z yuritilgan:

$$A\big[\varphi\ x\ \big]\equiv D^{l_1}_{ax}\ \ x-a\ ^{l_2}D^{l_2}_{ax}\ \ x-a\ ^{l_4}D^{l_5}_{ax}\varphi\ \ x\ \ . \tag{1.3.1}$$

1.3.1-Lemma. Quyidagi shartlar bajarilsa

1) $\varphi\ \ x\ \in L(a,b), \quad -\infty<a<b<\infty;$

2) $l_1<-l_3-l_5,\ l_2>l_3-l_4+l_5,\ l_3<-l_5,\ l_4>l_5, l_5<0,$

(a,b) ning hamma yerida quyidagi formula o'rinli bo'ladi:

$$A[\varphi\ x\ ]\equiv\ x-a\ ^{l_2-l_1-l_3+l_4-l_5-1}\times$$
$$\times\int_a^x G_{33}^{03}\left(\frac{z-a}{x-a}\middle|\begin{matrix} l_4-l_3-l_5,-l_5,l_2-l_1-l_3+l_4-l_5 \\ l_4-l_5,\ \ 0\ \ ,l5-l_3+l_4-l_5\end{matrix}\right)\varphi(z)dz, \tag{1.3.2}$$

bu yerda $G_{33}^{03}\ \ y|...$ - Meyer funksiyasi [3]

1.3.2-Lemma. $\varphi^{(k+1)}\ (x)\in L(a,b)$,

$l_1<-l_3-l_5,\ l_2>l_3-l_4+l_5,\ l_3<-l_5,\ l_4>l_5,l_5<0.$

U holda deyarli barcha (a,b) uchun quyidagi munosabat bajariladi:

$$A[\varphi\ x\ ]=\frac{d^{k+1}}{dx^{k+1}}\ x-a\ ^{l_2-l_1-l_3+l_4-l_5+k}\times$$
$$\times\int_a^x G_{33}^{30}\left(\frac{z-a}{x-a}\middle|\begin{matrix} l_4-l_3-l_5,\ l_2-l_1-l_3+l_4-l_5k+1 \\ l_4-l_5,\ \ 0\ ,\ l_2-l_3+l_4-l_5\end{matrix}\right)\varphi(z)dz, \tag{1.3.29}$$

bu yerda k - $l_1+l_3+l_5$ sonining butun qismi.

Dissertatsiyaning ikkinchi bobi ikkita buzilish chizig'iga ega bo'lgan giperbolik tipdagi tenglamalar uchun chegaraviy masalalarni o'rganishga bag'ishlangan.

Ikkinchi bobning birinchi paragrafida sohaning chegarasida buziladigan giperbolik tipdagi tenglamalar uchun chegaraviy masalalar o'rganiladi.

$$-y\ ^m U_{xx}-x^n U_{yy}=0,\qquad m,n=const>0. \tag{2.1.1}$$

tenglamani Ω sohada ko'ramiz.

Bu yerda Ω- (2.1.1) tenglamaning $A(0,0)$, $B(h_1,0)$ nuqtadan chiquvchi xarakteristikalari

$$AC:\frac{1}{q}x^q-\frac{1}{p}(-y)^p=0,\ \ BC:\frac{1}{q}x^q+\frac{1}{p}(-y)^p=1$$

va $y=0$ to'g'ri chiziqning AB kesmasi bilan chegaralangan bir bog'lamli soha.

Bu yerda $2p=m+2,\ 2q=n+2,\ h_1=q^{\frac{1}{q}}$

Quyidagi belgilashni kiritamiz:

$$J=\{(x,y):0<x<h_1,\ y=0\},$$

$$\theta(x)=\left[\frac{x^q}{2}\right]^{1/q}-i\left[\frac{p}{q}\cdot\frac{x^q}{2}\right]^{1/p}, \qquad (2.1.2)$$

$$2\alpha=n/(n+2),\quad 2\beta=m/(m+2)$$

Bu yerda $\theta(x)$-(2.1.1) tenglamaning xarakteristikalari bilan kesishgan nuqtasi affiksi.

2.1.1. Masala. Quyidagi xossalarga ega bo'lgan $U(x,y)$ funksiya topilsin.

1) $U(x,y)\in C(\overline{\Omega})\cap C^1(\Omega\cup J)$,

$$\int_0^1\left|U_y(x^{1/q},0)\right|x(1-x)^{-\beta}dx<\infty;$$

2) $U(x,y)$ (2.1.1) tenglamaning Ω sohadagi regulyar yechimi;

3) $U(x,y)$ quyidagi shartlarni qanoatlantiradi:

$$U(x,0)=\tau(x),\quad x\in\overline{J}, \qquad (2.1.3)$$

$$D^{l_1}_{0x^{2x}}(x^{2q})^{l_2}u[\theta(x)]=a_1(x)u_y(x,0)+b_1(x),\quad x\in J, \qquad (2.1.4)$$

bu yerda l_1 va l_2-xaqiqiy sonlar; $\tau(x)$, $a_1(x)$ va $b_1(x)$-berilgan funksiyalar.

2.1.1.Teorema. Quyidagi shartlar bajarilganda 2.1. masala yagona yechimga ega bo'ladi:

1. $l_1<1-\beta,\quad l_2>\max\left\{\frac{\alpha+\beta-2}{2},l_1+\frac{2\beta-1}{2}\right\}$;

2. $a_1(x)\neq 0,\ \forall x\in\overline{J}$, $a_1(x),b_1(x)\in C^1(\overline{J})$;

3. $\tau(x)\in C^{K_8}(\overline{J})\cap C^{K_8+2}(J)$, $K_8=\max(0,K_7+1)$,(K_7-$l_1-\beta$ ning butun qismi).

2.1.2. Masala. Ω sohada quyidagi shartlarni qanoatlantiruvchi $u(x,y)$ funksiyasi topilsin.

1. $u(x,y) \in C\left(\overline{\Omega}\right)$;

2. $u(x,y)$ -(2.1.1) tenglamaning regulyar yechimi;

3. $u_y(x,0) = \nu(x), \quad x \in J,$ (2.1.19)

$$D_{0x}^{l_1}(x^{2q})^{l_2} u[\theta(x)] = a_2(x)u(x,0) + b_2(x), \quad x \in J, \tag{2.1.20}$$

bu yerda $\nu(x)$, $a_2(x)$ va $b_2(x)$ berilgan funksiyalar; l_1 va l_2 -haqiqiy sonlar.

2.1.2. Teorema. Agar quyidagi shartlar bajarilsa

1) $l_1 < \beta, \quad l_2 > \max\left\{\dfrac{\alpha+\beta-2}{2}, l_1\right\}$;

2) $a_2(x) \neq 0 \ \forall x \in \bar{J}$, $a_2(x), b_2(x) \in C\left(\bar{J}\right) \cap C^2\left(J\right)$;

3) agar $\nu(x) \in C^2\left(J\right)$ va $\nu(x)$ funksiya J ning chetki nuqtalarida $(1-2\beta)/1-2\alpha$ tartibli maxsuslikka ega. U holda 2.1.2. masalaning yagona yechimi mavjud bo'ladi.

Ta'kidlaymizki, 1.1.1 va 1.1.2. masalalar $l_1 = 1-\beta, \quad l_2 = 0; l_1 = \beta, \quad l_2 = 2\beta - 1$ va $l_1 < 1-\beta, \quad l_2 = 0$ bo'lganda bir chiziqda buziluvchi giperbolik tipdagi tenglamalar uchun bir qator ishlarda o'rganilgan.

Ikkinchi bobning ikkinchi paragrafi (2.2.1) tenglama uchun chegaraviy masalalar yechishga umumlashgan kasr tartibli integro-differensial operatorlar qo'llanilishiga bag'ishlangan.

$$-y^m U_{xx} - x^n U_{yy} = 0, \tag{2.2.1}$$

Bu yerda Ω- (2.2.1) tenglamaning nuqtadan chiquvchi xarakteristikalari

$$AC: \frac{1}{q}x^q - \frac{1}{p}(-y)^p = 0, \qquad\qquad BC: \frac{1}{q}x^q + \frac{1}{p}(-y)^p = 1$$

va $y=0$ to’g’ri chiziqning AB kesmasi bilan chegaralangan bir bog’lamli soha. bu yerda $2p = m+2$, $2q = n+2$, $m,n = const$, $m > n > 0$.

Umumlashgan kasr tartibli integro-differensial operatorni quyidagi ko’rinishda olamiz:

$$F_{ox}\begin{bmatrix} a & b \\ c & g(x) \end{bmatrix}\varphi(x) =$$

$$= \begin{cases} \dfrac{1}{\Gamma(-c)}\displaystyle\int_0^x [g(x)-g(t)]^{-c-1}\cdot F\left(a,b,-c;\dfrac{g(x)-g(t)}{g(x)}\right)\varphi(t)\,g'(t)\,dt, & c<0, \\ [g(x)]^b \dfrac{d}{dg(x)} x^{-b} F_{ox}\begin{bmatrix} a+1 & b \\ c-1 & g(x) \end{bmatrix}\varphi(x), & 0<c<1, \end{cases}$$

(2.2.2)

bu yerda a,b va c-haqiqiy sonlar; $g(x)$ – uzluksiz hosilaga ega, monoton funksiya; $\varphi(x)\in L(AB)$; $\Gamma(\cdot)$ – Eyler gamma funksiyasi; $F(a,b,c;z)$ – Gauss gipergeometrik funksiyasi.

2.2.1- masala. Ω sohada (2.2.1) tenglamaning regulyar yechimi va quyidagi shartlarni qanoatlantiruvchi $U(x,y)$ funksiya topilsin.

1. $U(x,y)\in C(\overline{\Omega})\cap C'(\Omega\cup AB)\cap C^2(\Omega)$;

$$\int_0^1 \left|U_y\left(x^{1/q},0\right)\right|[x\cdot(1-x)]^{-\frac{m}{m+2}}dx < \infty;$$

2. $U(x,0)=\tau(x)$, $\qquad (x,0)\in\overline{AB}$; $\qquad$ (2.2.3)

$$(x^{4q})^{-b} F_{ox}\begin{bmatrix} a & b \\ c & (x^{2q})^2 \end{bmatrix} U[\theta(x)] = c(x)\,U_y(x,0) + d(x), \qquad (x,0)\in AB$$

(2.2.4)

bu yerda a,b va c-haqiqiy sonlar; $c(x), d(x)$ -berilgan funksiyalar; $\theta(x)$ -(2.1.1)

Quyidagi teorema o’rinli bo’ladi.

2.2.1. Teorema. Quyidagilar bajarilsin:

$$c+\beta<1, c-a>0,\ c-b\geq 0$$

$$c(x)\neq 0, \forall(x,0)\in\overline{AB};\ c(x), d(x)\in C'(\overline{AB});\ \tau(x)\in C'(\overline{AB})\bigcap C^2(AB).$$

U holda (2.2.1) tenglama yagona yechimga ega.

2.2.2.Masala. Ω sohada (2.2.1) tenglamaning regulyar yechimi va quyidagi shartlarni qanoatlantiruvchi $U(x,y)$ funksiya topilsin.

1. $U(x,y)\in C(\overline{\Omega})\bigcap C'(\Omega\bigcup AB)\bigcap C^2(\Omega)$;

$$\int_0^2\left|U_y\left(x^{1/q},0\right)\right|\left[x\cdot(1-x)\right]^{-\frac{m}{m+2}}dx<\infty;$$

2.

$$U_y(x,0)=\nu(x), \qquad (x,0)\in AB, \qquad (2.2.3)$$

$$x^{2q-a}F_{ox}\begin{bmatrix} a & b \\ c & (x^{2q})^2 \end{bmatrix}U\left[\theta(x)\right]=c(x)U_y(x,0)+g(x)U(x,0), \quad (x,c)\in AB. \qquad (2.2.4)$$

Dissertatsiyaning uchinchi bobi sohaning ichida buziladigan giperbolik tipdagi tenglamalar uchun chegaraviy masalalarni o'rganishga bag'ishlangan.

Uchinchi bobning birinchi paragrafida sohaning ichida buziladigan giperbolik tipdagi tenglamalar uchun chegaraviy masalalar o'rganilgan.

$$|-y|^m U_{xx}-x^n U_{yy}=0, \quad m,n=const>0. \qquad (3.1.1)$$

tenglamani Ω sohada ko'ramiz.

Bu yerdan Ω (3.1.1) tenglamaning $y>0$ dagi xarakteristikalari

$$OC_1:\frac{1}{q}x^q-\frac{1}{p}y^p=0,\ AC_1:\frac{1}{q}x^q+\frac{1}{p}y^p=1$$

xarakteristikalari, $y<0$ bo'lganda

$$OC_2:\frac{1}{q}x^q-\frac{1}{p}(-y)^p=0,\ AC_2:\frac{1}{q}x^q+\frac{1}{p}(-y)^p=1$$

xarakteristikalari bilan chegaralangan bir bog'lamli soha.

bu yerda $2q=n+2,\ 2p=m+2,\ m>n.$

Quyidagi belgilashni kiritamiz:

$$\Omega^+ = \Omega \cap (y>0),\ \Omega^- = \Omega \cap (y<0),$$

$$\theta_1(x) = \left[\frac{x^q}{2}\right]^{1/q} + i\left[\frac{p}{q}\cdot\frac{x^q}{2}\right]^{1/p},\ \theta_2(x) = \left[\frac{x^q}{2}\right]^{1/q} - i\left[\frac{p}{q}\cdot\frac{x^q}{2}\right]^{1/p}. \qquad (3.1.2)$$

bu yerda $\theta_j(x)$, $j=1,2$ - $(x,0) \in J$ nuqtadan chiquvchi, xarakteristikalari bilan OC_j $j=1,2$ xarakteristikalarning kesishgan nuqtalari koordinatalari.

Quyida (3.1.1) tenglamaning regulyar yechimi deganda $C(\bar\Omega) \cap C^1(\Omega) \cap C^2(\Omega^+ \cup \Omega^-)$ sinfga tegishli, bu tenglamani $\Omega^+ \cup \Omega^-$ sohalarda qanoatlantiruvchi va $x=0$, $x = q^{1/q}$ nuqtalarda $U_y(x,0)$ funksiya $(1-2\beta)/(1-2\alpha)$ dan yuqori maxsuslikka ega bo'lmagan $U(x,y)$ funksiyani tushuniladi.

3.1.1.Masala. (3.1.1) tenglamaning Ω sohada regulyar bo'lib quyidagi chegaraviy shartni qanoatlantiruvchi yechimini toping.

$$D_{ox}^{l_1} 2q(x^{2q})^{l_2} u\left[\theta_j(x)\right] = a_j(x) u_y(x,0) + b_j(x), \quad x \in J, \qquad (3.1.3\,j\text{, j=1,2})$$

bu yerda l_1 va l_2-haqiqiy sonlar , $a_j(x), b_j(x)$, $j=1,2$ -berilgan funksiyalar.

Shuni ta'kidlash lozimki, $l_1 = 1-\beta,\ l_2 = 0$ va $l_1 = \beta,\ l_2 = 2\beta - 1$ hollarda soha ichida buziladigan giperbolik turdagi tenglama uchun chegaraviy masala bir qator ishlarda o'rganilgan.

3.1.1.Teorema. Quyidagi shartlar bajarilsin:

1) $l_1 < 1-\beta,\ l_2 > \max\left\{ (\alpha+\beta-2)/2,\ l_1 + \dfrac{2\beta-1}{2} \right\};$

2) $a_2(x) - a_1(x) \neq 0,\ \forall x \in \bar J,\ a_j(x) \in C^{K_8}(\bar J) \cap C^{K_8+2}(J)$,

va $b_j(x) \in C^{K_8}(J)$ va $b_j(x)$ funksiya J interval chetki nuqtalarda $(1-2\beta)/(1-2\alpha)$ dan yuqori bo'lmagan tartibda cheksizlikka aylanishi mumkin.

U holda 3.1.1.masalaning yagona yechimi mavjud bo'ladi.

3.1.2.masala.(3.1.1) tenglamaning Ω sohada regulyar bo'lib quyidagi chegaraviy shartni qanoatlantiruvchi yechimi topilsin:

$$D_{0x^{2q}}^{l_1}(x^{2q})^{l_2}u[\theta(x)] = c_j(x)u(x,0) + d_j(x), \quad x \in J, \qquad (3.1.15\text{ j, j=1,2})$$

bu yerda l_1 va l_2 -haqiqiy sonlar ; $c_j(x)$ va $d_j(x)$, $(j=1,2)$ -berilgan funksiya.

Quyidagi teorema o'rinli bo'ladi.

3.1.2.Teorema. Quyidagi shartlar o'rinli bo'lsin:

1. $l_1 < 1-\beta, \ \ l_2 > \max\left\{ \alpha+\beta-4\big/2, l_1 + 2\beta-1\big/2 \right\}$;

2. $c_1(x) + c_2(x) \neq 0, \ \forall x \in \overline{J}, \quad c_j(x) \in C^1(\overline{J})$;

$d_j(x) \in C^2(J)$ va $d_j(x), \ (j=1,2)$ funksiya $(1-2\beta)\big/1-2\alpha$ dan yuqori bo'lmagan tartib bilan cheksizlikka aylanishi mumkin.

U holda 3.1.2.masala bir qiymatli yechimga ega bo'ladi.

Agar $c_1(x) = -c_2(x), \ \ \forall x \in \overline{J}$ bo'lsa, u holda $\tilde{v}(x)$ oshkor ko'rinishda topiladi.

Uchinchi bobning ikkinchi paragrafida chegaraviy shartida umumlashgan kasr tartibli integro-differensial operatorlar qatnashgan ba'zi siljishli chegaraviy masalalar o'rganilgan.

$$|y|^m U_{xx} - x^n U_{yy} = 0, \quad m,n = const > 0, \qquad (3.2.2)$$

tenglamani Ω sohada ko'ramiz.

3.2.1.Masala. (3.2.1) tenglamaning Ω sohada regulyar bo'lib quyidagi chegaraviy shartni qanoatlantiruvchi yechimini toping.

$$x^{4q}\,{}^{-b}F_{ox}\begin{bmatrix} a & b \\ c & x^{2q} \end{bmatrix}^{2} U\left[\theta_j\ x\ \right] = c_j\ x\ U_y\ \ x,0\ + d_j\ \ x\ , \qquad x,0\ \in J$$

bu yerda a,b va c – haqiqiy sonlar; $c_j\ \ x\ \ ,d_j\ \ x\ \ ,\ \ (j=1,2)$ berilgan funksiyalar.

3.2.1.Teorema. Quyidagi shartlar bajarilsin:

1. $-1<-c-\beta<0$;

2. $a_1(x)-a_2(x)\neq 0,\ \forall x\in \overline{J},\ a_j(x)\in C(\overline{J})\cap C^2(J), b_j(x)C(J)$

va $b_j(x)\in C^{K_8}\ \ J$ va $b_j(x)$ funksiya J interval chetki nuqtalarda $(1-2\beta)\big/1-2\alpha$ dan yuqori bo'lmagan tartibda cheksizlikka aylanishi mumkin.

U holda 3.2.1 masalaning yechimi mavjud va yagona bo'ladi.

I BOB. KASR TARTIBLI INTEGRO-DIFFERENSIAL OPERATORLAR VA ULARNING XOSSALARI

1.1-§. Umumlashgan kasr tartibli integro-differensial operatorlarning va ularning xossalari.

Faraz qilaylik, $a,b,c,m,n \in R$, $0 < m < n$, $c \neq 0,-1,-2,...,$;
$k \in (m,n)$; $f(x), f'(x) \in C(m,n) \cap L_1(m,n)$ bo'lsin.

Quyidagi operatorlarni qaraymiz:

$$F_{kx}\begin{bmatrix} a & ,b \\ c & ;x \end{bmatrix} f(x) \equiv \begin{cases} \dfrac{sign(x-k)}{\Gamma(c)}\displaystyle\int_k^x |x-t|^{c-1} F\left(a,b,c;\dfrac{x-t}{x}\right) f(t)dt, \ c>0; \\ f(x), \qquad c=0; \\ sign(x-k)x^a \dfrac{d}{dx} x^{-a} F_{kx}\begin{bmatrix} a & ,b+1 \\ c+1 & ;x \end{bmatrix} f(x), \ -1<c<0. \end{cases} \qquad (1.1.1)$$

(1.1.1) operatorlar *kasr tartibli umumlashgan integro-differensial operatorlar* deb atalib, ulardan hususiy holda Riman-Luivillning integro-diffrensial operatorlari kelib chiqadi, ya'ni

$$F_{kx}\begin{bmatrix} 0 & ,b \\ c & ;x \end{bmatrix} \equiv F_{kx}\begin{bmatrix} a & ,0 \\ c & ;x \end{bmatrix} \equiv D_{kx}^{-c}, \ c>0;$$

$$F_{kx}\begin{bmatrix} 0 & ,b \\ c & ;x \end{bmatrix} \equiv F_{kx}\begin{bmatrix} a & ,0 \\ c & ;x \end{bmatrix} \equiv \frac{d}{dx} D_{kx}^{-(c+1)} \equiv D_{kx}^{-c}, \ -1<c<0.$$

Demak, (1.1.1) Riman-Luivill ma'nosidagi D_{kx}^{α} integro-differensial operatorlarni $F(a,b,c;x)$ -Gauss gipergeometrik funksiyasi yordamidagi umumlashmasi ekan.

Keyingi satrlarda $k<x$ da $F_{kx} = F_{kx}$, $k>x$ da esa $F_{kx} = F_{xk}$ kabi yozishga kelishib olamiz.

1.1.1-teorema. *Agar* $f(x) \in C(m,n) \cap L_1(m,n)$ *bo'lsa, ixtiyoriy* $a,b \in R, c \in (0,1)$ *va* $x,k \in (m,n)$ *uchun*

$$x^{a}F_{kx}\begin{bmatrix}-a\ ,b-c\\-c\ ;x\end{bmatrix}x^{-a}F_{kx}\begin{bmatrix}a\ ,b\\c\ ;x\end{bmatrix}f(x)=f(x) \tag{1.1.2}$$

tenglik o'rinli bo'ladi.

Isbot. Aniqlik uchun $0<m<x<k<n$ deb olaylik. (1.1.1) ga asosan (1.1.1) ning chap tomonini yopib yozamiz:

$$M=x^{a}F_{kx}\begin{bmatrix}-a\ ,b-c\\-c\ ;x\end{bmatrix}x^{-a}F_{kx}\begin{bmatrix}a\ ,b\\c\ ;x\end{bmatrix}f(x)=$$

$$=-x^{a}\cdot x^{-a}\frac{d}{dx}\left\{x^{a}F_{xk}\begin{bmatrix}-a\ ,b-c+1\\1-c\ ;x\end{bmatrix}x^{-a}F_{kx}\begin{bmatrix}a\ ,b\\c\ ;x\end{bmatrix}f(x)\right\}=$$

$$=-\frac{d}{dx}\left\{x^{a}\frac{1}{\Gamma(1-c)}\int_{x}^{k}(t-x)^{-c}F\left(-a,b-c,1-c;\frac{x-t}{x}\right)t^{-a}\times\right.$$

$$\left.\times\left[\frac{1}{\Gamma(c)}\int_{t}^{k}(z-t)^{c-1}F\left(a,b,c;\frac{t-z}{t}\right)f(z)dz\right]dt\right\}.$$

Takroriy integralga Dirixle formulasini qo'llaymiz:

$$M=-\frac{\sin(c\pi)}{\pi}\frac{d}{dx}\left\{x^{a}\int_{x}^{k}f(z)\left[\int_{x}^{z}t^{-a}(t-x)^{-c}\times\right.\right.$$

$$\left.\left.\times F\left(-a,b-c+1,1-c;\frac{x-t}{x}\right)(z-t)^{c-1}F\left(a,b,c;\frac{t-z}{t}\right)dt\right]dz\right\}.$$

Ikkinchi Gauss funksiyasiga

$$F\ \ \alpha,\beta,\gamma;z\ \ =(1-z)^{-\beta}F\left(\gamma-\alpha,\beta,\gamma;\frac{z}{z-1}\right)$$

formulani qo'llab, quyidagiga ega bo'lamiz:

$$M=-\frac{\sin(c\pi)}{\pi}\frac{d}{dx}\left\{x^{a}\int_{x}^{k}f(z)z^{-b}\times\right.$$

$$\times\left[\int_{x}^{z}t^{b-a}(t-x)^{-c}(z-t)^{c-1}F\left(-a,b-c+1,1-c;1-\frac{t}{x}\right)\times\right.$$

$$\left.\left.\times F\left(c-a,b,c;1-\frac{t}{z}\right)dt\right]dz\right\}.$$

Kvadrat qavs ichidagi integralga

$$\int_{\sigma}^{\omega} x^{\alpha}(x-\sigma)^{c_1-1}(\omega-x)^{c'-1}F\left(a_1,b_1,c_1;1-\frac{x}{\sigma}\right)F\left(a,'b',c';1-\frac{x}{\omega}\right)=$$

$$=B(c_1,c')(\omega-\sigma)^{c_1+c'-1}\sigma^{A}\omega^{B}F\left(C,D,c_1+c';1-\frac{\sigma}{\omega}\right),$$

bu yerda $0<\sigma<\omega, \mathrm{Re}\,c_1, \mathrm{Re}\,c'>0; A=-a, B=b, C=c-a-b, D=0,$ formulani qo'llab

$$M=-\frac{\pi}{\sin(c\pi)}\frac{d}{dx}\left\{x^{a}\int_{x}^{k} z^{-b}f(z)\left[B\ 1-c,c\ \ x^{-a}z^{b}\right]dz\right\}$$

ifodaga ega bo'lamiz.

$B\ 1-c,c\ =\Gamma\ 1-c\ \Gamma\ c\ \big/\ \Gamma\ 1\ =\pi\big/\sin(c\pi)$ tenglikni e'tiborga olsak, oxirgi tenglikdan 1.1.1-teoremaning tasdig'i kelib chiqadi.

Teorema qolgan hollarda ham shunday isbotlanadi.

1.1.2- teorema. *Agar* $f(x)\in C^{0,\delta}(0,1)\cap L_1(0,1)$ *bo'lsa, ixtiyoriy* $a,b\in R, c\in(0,1)\ \delta\in\ 0,1\ , x\in\ 0,1$ *uchun*

$$x^{a}F_{0x}\begin{bmatrix}-a\ ,b-c\\ -c\ ;x\end{bmatrix}x^{-a}F_{x1}\begin{bmatrix}a\ ,b\\ c\ ;x\end{bmatrix}f(x)=$$

$$=\cos(c\pi)f(x)+\frac{\lambda_1}{\pi}\int_0^1\left(\frac{t}{x}\right)^{c-a}\frac{f(t)dt}{t-x}+\frac{\lambda_2}{\pi}\int_0^1\left(\frac{t}{x}\right)^{c-b}\frac{f(t)dt}{t-x}$$

tenglik o'rinli bo'ladi, bu yerda

$$\lambda_1=\frac{\sin(b\pi)\sin\ (c-a)\pi}{\sin\ (b-a)\pi},\qquad \lambda_2=\frac{\sin(a\pi)\sin\ (c-b)\pi}{\sin\ (a-b)\pi}.$$

1.1.3-teorema. *Faraz qilaylik,* $(-1)<b<c<a<0$ *yoki* $(-1)<a<c<b<0,$ $l\geq 0;\omega(x),\ \tau(x)\in C\ 0,1\ ;\ \omega(x)$*-kamaymaydigan musbat funksiya va* $x=x_0\,(0<x_0<1)$ *nuqtaning qisqa atrofida* $\omega(x)\tau(x)$ *ko'paytma* $\gamma>(-c)$ *tartibli Gyolder shartini qanoatlantirsin. Agar* $0,x_0$ *oraliqda* $\tau(x)$ *funksiya eng katta musbat (eng kichik manfiy) qiymatga* $x=x_0$ *nuqtada erishsa, u holda*

$$F_{0x}\begin{bmatrix} a & ,b \\ c & ;x \end{bmatrix} x^l \omega(x)\tau(x)\Big|_{x=x_0} > 0 \qquad (<0) \tag{1.1.3}$$

tengsizlik o'rinli bo'ladi.

Isbot. (1.1.1) ta'rifga asosan

$$M_1 F_{0x}\begin{bmatrix} a & ,b \\ c & ;x \end{bmatrix} x^l \omega(x)\tau(x) = x^a \frac{d}{dx} x^{-a} F_{kx}\begin{bmatrix} a & ,b+1 \\ c+1 & ;x \end{bmatrix} =$$

$$= \frac{x^a}{\Gamma(c+1)} \frac{d}{dx} x^{-a} \int_0^x z^l (x-z)^c \omega(z)\tau(z) F\left(a, b+1, c+1; \frac{x-z}{x}\right) dz.$$

Quyidagi ifodani qaraymiz:

$$M_{1\varepsilon} = \frac{x^a}{\Gamma(c+1)} \frac{d}{dx} x^{-a} \int_0^{x-\varepsilon} z^l (x-z)^c \omega(z)\tau(z) F\left(a, b+1, c+1; \frac{x-z}{x}\right) dz,$$

bu yerda ε -yetarlicha kichik musbat son. Aniqki, $\lim\limits_{\varepsilon\to 0} M_{1\varepsilon} = M_1$.

Differensiallash amalini bajarib,

$$M_{1\varepsilon} \frac{1}{\Gamma(c+1)} \ (x-\varepsilon)^l \omega(x-\varepsilon)\tau(x-\varepsilon)\varepsilon^c F\left(a, b+1, c+1; \frac{\varepsilon}{x}\right) +$$

$$\left. + c\int_0^{x-\varepsilon} z^l \omega(z)\tau(z)(x-z)^{c-1} F\left(a, b, c; \frac{x-z}{x}\right) dz \right\}.$$

tenglikka ega bo'lamiz. Bu tenglikni

$$M_{1\varepsilon} \frac{1}{\Gamma(c+1)} \ (x-\varepsilon)^l \omega(x-\varepsilon)\tau(x-\varepsilon)\varepsilon^c F\left(a, b+1, c+1; \frac{\varepsilon}{x}\right) +$$

$$- c\int_0^{x-\varepsilon} z^{a+b-c}\left[x^{l+c-a-b}\omega(x)\tau(x) - z^{l+c-a-b}\omega(z)\tau(z)\right] \times$$

$$\times (x-z)^{c-1} F\left(a, b, c; \frac{x-z}{x}\right) dz + \tag{1.1.4}$$

$$\left. + c\omega(x)\tau(x) \int_0^{x-\varepsilon} z^{a+b-c} x^{l+c-a-b} (x-z)^{c-1} F\left(a, b, c; \frac{x-z}{x}\right) dz \right\}$$

ko'rinishda yozib olamiz.(1.1.4) dagi oxirgi integralni M_2 orqali belgilab olib, so'ngra Gauss funksiyasi uchun avtotransformatsiya formulasini qo'llaymiz:

$$M_2=-x^{l+c}\int_0^{x-\varepsilon} c\left(-\frac{1}{x}\right)\left(\frac{x-z}{x}\right)^{c-1}F\left(c-a,c-b,c;\frac{x-z}{x}\right)dz.$$

Ushbu

$$\frac{d}{d\theta}\left[\theta^{c_1-1}F(a_1,b_1,c_1;\theta)\right]=(c_1-1)\,\theta^{c_1-2}F(a_1,b_1,c_1-1;\theta)$$

formula yordamida oxirgi integralni hisoblaymiz:

$$M_2=x^{l+c}F(c-a,c-b,c+1;1)-x^l\varepsilon^c F\left(c-a,c-b,c+1;\frac{\varepsilon}{x}\right)$$

Buni (1.3.4) ga qo'yib, topamiz:

$$M_{1\varepsilon}=\frac{x^{l+c}}{\Gamma(c+1)}\omega(x)\tau(x)F(c-a,c-b,c+1;1)+\frac{M_{3\varepsilon}}{\Gamma(c+1)}-$$
$$-\frac{1}{\Gamma(c)}\int_0^{x-\varepsilon} z^{a+b-c}\left[x^{l+c-a-b}\omega(x)\tau(x)-z^{l+c-a-b}\omega(z)\tau(z)\right]\times \tag{1.1.5}$$
$$\times(x-z)^{c-1}F\left(a,b,c;\frac{x-z}{x}\right)dz,$$

bu yerda

$$M_{3\varepsilon}=(x-\varepsilon)^l\omega(x-\varepsilon)\tau(x-\varepsilon)\varepsilon^c F\left(a,b+1,c+1;\frac{\varepsilon}{x}\right)-$$
$$-x^l\omega(x)\tau(x)\varepsilon^c F\left(c-a,c-b,c+1;\frac{\varepsilon}{x}\right).$$

$M_{3\varepsilon}$ ifodani quyidagicha yozish mumkin:

$$M_{3\varepsilon}=\left[(x-\varepsilon)^l-x^l\right]\varepsilon^c\omega(x-\varepsilon)\tau(x-\varepsilon)F\left(a,b+1,c+1;\frac{\varepsilon}{x}\right)+$$
$$+\left[\omega(x-\varepsilon)\tau(x-\varepsilon)-\omega(x)\tau(x)\right]\varepsilon^c x^l F\left(a,b+1,c+1;\frac{\varepsilon}{x}\right)+$$
$$+x^l\omega(x)\tau(x)\varepsilon^c\left[F\left(a,b+1,c+1;\frac{\varepsilon}{x}\right)-F\left(c-a,c-b,c+1;\frac{\varepsilon}{x}\right)\right].$$

$0<\delta_1<x-\varepsilon<x$ bo'lsin, bu yerda δ_1 yetarlicha kichik musbat tayinlangan son.U holda

$$\omega(x-\varepsilon)\tau(x-\varepsilon)-\omega(x)\tau(x)=\varepsilon^\gamma\cdot O(1),\quad \gamma>-c,$$

$$\omega(x-\varepsilon)-x^{l}=\varepsilon\cdot O(1).$$

$$F\left(a,b+1,c+1;\frac{\varepsilon}{x}\right)-F\left(c-a,c-b,c+1;\frac{\varepsilon}{x}\right)=\varepsilon\cdot O(1).$$

Bularni e'tiborga olsak,

$$\lim_{\varepsilon\to 0}M_{3\varepsilon}=0,\qquad \forall x\in(\delta_1,1). \tag{1.1.6}$$

(1.1.5) dan $\varepsilon\to 0$da limitga o'tib va (1.3.6) ni inobatga olib, har bir $x\in(\delta_1,1)$ uchun

$$M_{1\varepsilon}=\frac{1}{\Gamma(c+1)}\Big\{x^{l+c}\omega(x)\tau(x)F(c-a,c-b,c+1;1)-$$
$$-c\int_0^x z^{a+b-c}\left[x^{l+c-a-b}\omega(x)\tau(x)-z^{l+c-a-b}\omega(z)\tau(z)\right]\times$$
$$\times(x-z)^{c-1}F\left(a,b,c;\frac{x-z}{x}\right)dz\Big\}$$

tenglikka ega bo'lamiz.

Bu yerda $x=x_0,\ x_0\in(0,1)$ desak,

$$M_1\Big|_{x=x_0}=\frac{1}{\Gamma(c+1)}\Big\{x_0^{\,l+c}\omega(x_0)\tau(x_0)F(c-a,c-b,c+1;1)-$$
$$-c\int_\delta^{x_0} z^{a+b-c}\left[x_0^{\,l+c-a-b}\omega(x_0)\tau(x_0)-z^{l+c-a-b}\omega(z)\tau(z)\right]\times$$
$$\times(x_0-z)^{c-1}F\left(a,b,c;\frac{x_0-z}{x_0}\right)dz- \tag{1.1.7}$$
$$-c\int_0^{\delta} z^{a+b-c}\left[x_0^{\,l+c-a-b}\omega(x_0)\tau(x_0)-z^{l+c-a-b}\omega(z)\tau(z)\right]\times$$
$$\times(x_0-z)^{c-1}F\left(a,b,c;\frac{x_0-z}{x_0}\right)dz\Big\}$$

tenglik kelib chiqadi, bu yerda $\delta-x_0$ga chapdan yetarlicha yaqin son.

$(-1)<b<c<a<0$ [yoki $(-1)<a<b<c<0$] shartga asosan $a+1>0,b+1>0,c+1>0,0<c-a-b<1$ tengsizliklar o'rinli. Bularni va gamma-funksiyaning xossalarini e'tiborga olsak, $F(c-a,c-b,c+1;1)>0$ tengsizlik o'rinli ekanligi kelib chiqadi. Agar $(-1)<a<0,(-1)<b<0,(-1)<c<0,0<\left[{x-z}\big/{x}\right]<1$ ekanligi hamda gipergeometrik funksiyaning qator ko'rinishidagi ifodasini e'tiborga olsak,

$$\frac{d}{dz}F\left(a,b,c;\frac{x-z}{x}\right)=-\frac{ab}{cx}F\left(a+1,b+1,c+1;\frac{x-z}{x}\right)>0$$

tengsizlikka ega bo'lamiz. Demak, $F\left(a,b,c;{x-z}\big/{x}\right)$ funksiya z bo'yicha o'suvchi ekan. Shu sababli, $\forall x\in(0,1)$ uchun

$$F\left(a,b,c;\frac{x-z}{x}\right)>F(a,b,c;1)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}>0.$$

$(-1)<b<c<a<0$ $(-1)<a<b<c<0$ tengsizlikka asosan $c-1>0,c-a<0$ $c-1<0,c-a>0$. Bularni va $l\geq c-a-b>0$ tengsizlikni hamda $\omega(x)$ va $\tau(x)$ funksiyalarga qo'yilgan shartlarni hisobga olsak, (1.1.7) tenglikdan darhol (1.1.3) tengsizlik kelib chiqadi. 1.1.3- teorema isbotlandi.

1.1.4-teorema. *Faraz qilaylik,* $(-1)<c<a<0<b<1+c$ *yoki* $(-1)<a<c<b<0;$ $\omega(x),\tau(x)\in C(0,1)$; $\omega(z)$ *-o'smaydigan musbat funksiya va* $x=x_0$ $(0<x_0<1)$ *nuqtaning qisqa atrofida* $\omega(x)\tau(x)$ *ko'paytma* $\gamma[>(-c)]$ *ko'rsatkichli Gyolder shartini qanoatlantirsin. Agar* $(x_0,1)$ *oraliqda* $\tau(x)$ *funksiya eng katta musbat (eng kichik manfiy) qiymatga* $x=x_0$ *nuqtada erishsa, u holda*

$$F_{x1}\begin{bmatrix}a, & b\\ c; & x\end{bmatrix}\omega(x)\tau(x)\Big|_{x=x_0}>0\quad(<0)\tag{1.1.8}$$

tengsizlik o'rinli bo'ladi.

Isbot. (1.1.1) ta'rifiga asosan

$$M_2=F_{x1}\begin{bmatrix} a,\ b \\ c;\ x\end{bmatrix}\omega(x)\tau(x)=-x^a\frac{d}{dx}x^{-a}F_{x1}\begin{bmatrix} a,\ b+1 \\ c+1;\ x\end{bmatrix}\omega(x)\tau(x)=$$

$$=-x^a\frac{d}{dx}\frac{x^{-a}}{\Gamma(c+1)}\int\limits_x^1 (z-x)^c\omega(z)\tau(z)F\left(a,b+1,c+1;\frac{x-z}{x}\right)dz.$$

Gipergeometrik funksiyaga

$$F(\alpha,\beta,\gamma;y)=(1-y)^{-\alpha}F\left[\alpha,\gamma-\beta,\gamma;{}^{y}\!/\!_{y-1}\right]$$

formulani qo'llab, M_2ni quyidagicha yozib olamiz:

$$M_2=-\frac{x^{-a}}{\Gamma(c+1)}\frac{d}{dx}\int\limits_x^1 z^{c-a}\omega(z)\tau(z)\left(\frac{z-x}{z}\right)^c F\left(a,c-b,c+1;\frac{z-x}{z}\right)dz.$$

Xuddi 1.3.3- teoremadagi kabi, ushbu funksiyani kiritamiz:

$$M_{2\varepsilon}=-\frac{x^{a}}{\Gamma(c+1)}\frac{d}{dx}\int\limits_{x+\varepsilon}^1 z^{c-a}\omega(z)\tau(z)\left(\frac{z-x}{z}\right)^c F\left(a,c-b,c+1;\frac{z-x}{z}\right)dz,$$

bu yerda ε -yetarlicha kichik musbat son. Aniqqi, $\lim\limits_{\varepsilon\to 0}M_{2\varepsilon}=M_2$.

Differensiallash amalini bajaramiz:

$$M_{2\varepsilon}=\frac{x^a}{\Gamma(c+1)}\left\{(x+\varepsilon)^{-a}\varepsilon^c\omega(x+\varepsilon)\tau(x+\varepsilon)F\left(a,c-b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)+\right.$$

$$\left.+c\int\limits_{x+\varepsilon}^1 z^{-a}\omega(z)\tau(z)(z-x)^{c-1}F\left(a,c-b,c;\frac{z-x}{z}\right)dz\right\}.$$

Bu ifodani quyidagicha yozib olamiz:

$$M_{2\varepsilon}=\frac{x^a}{\Gamma(c+1)}\left\{(x+\varepsilon)^{-a}\varepsilon^c\omega(x+\varepsilon)\tau(x+\varepsilon)F\left(a,c-b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)-\right.$$

$$-c\int\limits_{x+\varepsilon}^1 z^{-a}\left[\omega(x)\tau(x)-\omega(z)\tau(z)\right](z-x)^{c-1}F\left(a,c-b,c;\frac{z-x}{z}\right)dz+ \qquad (1.1.9)$$

$$\left.+\omega(x)\tau(x)\int\limits_{x+\varepsilon}^1 cz^{-a}(z-x)^{c-1}F\left(a,c-b,c;\frac{z-x}{z}\right)dz\right\}.$$

Gipergeometrik funksiya uchun o'rinli bo'lgan ushbu:

$$F(\alpha,\beta,\gamma;z)=(1-z)^{\gamma-\alpha-\beta}F(\alpha-\gamma,\beta-\gamma,\gamma;z),$$

$$\frac{d}{dz}\left[z^{\gamma-1}(1-z)^{\beta-\gamma+1}F(\alpha,\beta,\gamma;z)\right]=(\gamma-1)z^{\gamma-2}(1-z)^{\beta-\gamma}F(\alpha-1,\beta,\gamma-1;z)$$

formulalarni qo'llab, oxirgi integralni

$$(1-x)^{c}x^{b-a}F(c-a+1,b,c+1;1-x)-$$
$$-\varepsilon^{c}x^{b-a}(x+\varepsilon)^{-b}F\left(c-a+1,b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)$$

ga tengligini ko'rsatish qiyin emas. Buni e'tiborga olib, (1.1.9) ni quyidagicha yozish mumkin:

$$M_{2\varepsilon}=\frac{1}{\Gamma(c+1)}\varepsilon^{c}\left[\left(\frac{x+\varepsilon}{x}\right)^{-a}\omega(x+\varepsilon)\tau(x+\varepsilon)F\left(a,c-b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)-\right.$$
$$\left.-\left(\frac{x+\varepsilon}{x}\right)^{-b}\omega(x)\tau(x)F\left(c-a+1,b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)\right]+$$
$$+\frac{(1-x)^{c}x^{b}}{\Gamma(c+1)}F(c-a+1,b,c+1;1-x)- \tag{1.1.10}$$
$$-\frac{x^{a}}{\Gamma(c)}\int_{x+\varepsilon}^{1}z^{-a}\left[\omega(x)\tau(x)-\omega(z)\tau(z)\right](z-x)^{c-1}F\left(a,c-b,c;\frac{z-x}{z}\right)dz.$$

Birinchi kvadrat qavs ichidagi ifodani

$$\omega(x+\varepsilon)\tau(x+\varepsilon)\left[\left(1+\frac{\varepsilon}{x}\right)^{-a}-1\right]F\left(a,c-b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)+$$
$$+\left[\omega(x+\varepsilon)\tau(x+\varepsilon)-\omega(x)\tau(x)\right]F\left(a,c-b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)+$$
$$+\omega(x)\tau(x)\left[1-\left(1+\frac{\varepsilon}{x}\right)^{-b}\right]F\left(c-a+1,b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)+$$
$$+\omega(x)\tau(x)\left[F\left(a,c-b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)-F\left(c-a+1,b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)\right]$$

ko'rinishda yozish mumkin. Bu yerda

$$\left(1+\frac{\varepsilon}{x}\right)^{-a}-1=\varepsilon\cdot O(1),\qquad 1-\left(1+\frac{\varepsilon}{x}\right)^{-b}-1=\varepsilon\cdot O(1),$$

$$F\left(a,c-b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)-F\left(c-a+1,b,c+1;\frac{\varepsilon}{x+\varepsilon}\right)=\varepsilon\cdot O(1),$$

$$\omega(x-\varepsilon)\tau(x-\varepsilon)-\omega(x)\tau(x)=\varepsilon^{\gamma}\cdot O(1)$$

ekanligini e'tiborga olsak, (1.1.10) tenglikdan $\varepsilon\to 0$da limitga o'tib,

$$M_2=\Gamma^{-1}(1+c)(1+x)^{c}x^{b}F(c-a+1,b,c+1;1-x)-$$

$$-\frac{x^{a}}{\Gamma(c)}\int_x^1 z^{-a}\left[\omega(x)\tau(x)-\omega(z)\tau(z)\right](z-x)^{c-1}F\left(a,c-b,c;\frac{z-x}{z}\right)dz$$

tenglikka ega bo'lamiz.

Bu yerda dastlab birinchi gipergeometrik funksiyaga avtotransformatsiya formulasini qo'llab, so'ngra $x=x_0$desak, $x_0$ga o'ngdan yetarlicha yaqin bo'lgan δ son uchun

$$M_2\Big|_{x=x_0}=\Gamma^{-1}(1+c)x_0^{\,a}(1-x_0)^{c}F(c-a+1,b,c+1;1-x_0)-$$

$$-\frac{x_0^{\,a}}{\Gamma(c)}\int_\delta^1 z^{-a}\left[\omega(x_0)\tau(x_0)-\omega(z)\tau(z)\right](z-x_0)^{c-1}F\left(a,c-b,c;\frac{z-x_0}{z}\right)dz-$$

$$-\frac{x_0^{\,a}}{\Gamma(c)}\int_{x_0}^1 z^{-a}\left[\omega(x_0)\tau(x_0)-\omega(z)\tau(z)\right](z-x_0)^{c-1}F\left(a,c-b,c;\frac{z-x_0}{z}\right)dz$$

tenglikning o'rinli ekanligi kelib chiqadi.

$(-1)<c<a<0<b<c+1$ yoki $(-1)<a<c<b<0$ shartlar bajarilganda, differensiallash yordamida $F(a,c-b,c+1;1-x)$ va $F(a,c-b,c;{}^{z-x}\!/_{z})$ funksiyalar $x$bo'yicha o'suvchi ekanligini ko'rsatish qiyin emas. Shu sababli, $\forall x\in(0,1)$ uchun

$$F(a,c-b,c+1;1-x_0)>F(a,c-b+1,c+1;1)>0,$$

$$F\left(a,c-b,c;{}^{z-x_0}\!/_{z}\right)>F(a,c-b,c;1)>0.$$

Bularni, $\Gamma(c)<0$tengsizlikni va teoremaning qolgan shartlarini e'tiborga olsak, oxirgi tenglikdan (1.1.8) tengsizliklar darxol kelib chiqadi.1.1.4- teorema isbotlandi.

Odatda 1.1.3-va 1.1.4- teoremalar F_{0x}va F_{1x} operatorlar uchun *ekstremum prinsipi* deb atalib, ularga o'xshash teoremalar [17] da $a$va b parametrlarning boshqa qiymatlari uchun isbotlangan.

1.2-§. Funksiyadan funksiya bo'yicha olingan kasr tartibli hosila va integrallar

Umumlashgan Abel integral tenglamasi. Ushbu

$$\frac{1}{\Gamma(\alpha)}\int_a^x \frac{\varphi(t)dg(t)}{(g(x)-g(t))^{1-\alpha}} = f(x), \quad 0<\alpha<1 \tag{1.2.1}$$

ko'rinishdagi integral tenglama *umumlashgan Abel integral tenglamasi* deyiladi. Bu yerda $g(x)$ uzluksiz differensiallanuvchi, o'suvchi va $g'(x)\neq 0$ funksiya.

Bu tenglama quyidagi usulda yechiladi. Tenglamada [1] ishdagi kabi x ni t bilan, t ni s bilan almashtirib, so'ngra tenglamaning har ikki tomonini $(g(x)-g(t))^{-\alpha}$ ifodaga ko'paytiramiz va $g(t)$ bo'yicha a dan x gacha integrallaymiz:

$$\int_a^x \frac{dg(t)}{(g(x)-g(t))^{\alpha}}\int_a^t \frac{\varphi(s)dg(s)}{(g(t)-g(s))^{1-\alpha}} = \Gamma(\alpha)\int_a^x \frac{f(t)dg(t)}{(g(x)-g(t))^{\alpha}}.$$

Dirixle formulasiga ko'ra integrallash tartibini almashtirib,

$$\int_a^x \varphi(s)dg(s)\int_s^x \frac{dg(t)}{(g(x)-g(t))^{\alpha}(g(t)-g(s))^{1-\alpha}} = \Gamma(\alpha)\int_a^x \frac{f(t)dg(t)}{(g(x)-g(t))^{\alpha}}. \tag{1.2.2}$$

tenglikni hosil qilamiz. Tenglikning chap tomonidagi ichki integralda $g(t)=g(s)+z(g(x)-g(s))$ almashtirish bajarsak va

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx,$$

bu yerda $a>0, b>0$

formuladan foydalansak,

$$\int_s^x (g(x)-g(t))^{-\alpha}(g(t)-g(s))^{\alpha-1}dg(t) = \Gamma(\alpha)\Gamma(1-\alpha)$$

tenglik kelib chiqadi. U holda, (1.2.2) ga asosan

$$\int_a^x \varphi(s)dg(s) = \frac{1}{\Gamma(1-\alpha)}\int_a^x \frac{f(t)dg(t)}{(g(x)-g(t))^{\alpha}}. \tag{1.2.3}$$

Bu tenglikning har ikki tomonini $g(x)$ funksiya bo'yicha differensiallab, umumlashgan Abel integral tenglamasining yechimini hosil qilamiz:

$$\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dg(x)}\int_a^x\frac{f(t)dg(t)}{\left(g(x)-g(t)\right)^{\alpha}}. \tag{1.2.4}$$

Shunday qilib, agar (1.2.1) tenglamaning yechimi mavjud bo'lsa, u (1.2.4) ko'rinishda bo'lar ekan. Bu formulani hosil qilish jarayonidan kelib chiqadiki, agar yechim mavjud bo'lsa, u yagona.

Shu usulda ko'rsatish mumkinki, ushbu

$$\frac{1}{\Gamma(\alpha)}\int_x^b\frac{\varphi(t)dg(t)}{\left(g(x)-g(t)\right)^{1-\alpha}}=f(x),\ \ 0<\alpha<1 \tag{1.2.5}$$

umumlashgan integral tenglamaning yechimi

$$\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dg(x)}\int_x^b\frac{f(t)dg(t)}{\left(g(x)-g(t)\right)^{\alpha}} \tag{1.2.6}$$

formula bilan aniqlanadi.Bu yerda $g(x)$ kamayuvchi va $g'(x)\neq 0$ funksiya.

Ta'rif. $\varphi(x)\in L_1(a,b)$ $(a<b<+\infty)$ *va* $g(x)$ *uzluksiz differensiallanuvchi, o'suvchi (kamayuvchi) funksiya bo'lsin. Ushbu*

$$D^{-\alpha}_{a,x,g(x)}\varphi(x)=\frac{1}{\Gamma(\alpha)}\int_a^x\left(g(x)-g(t)\right)^{\alpha-1}\varphi(t)dg(t),\qquad \alpha>0, \tag{1.2.7}$$

$$\left[D^{-\alpha}_{a,b,g(x)}\varphi(x)=\frac{1}{\Gamma(\alpha)}\int_x^b\left(g(t)-g(x)\right)^{\alpha-1}\varphi(t)dg(t),\qquad \alpha>0\right] \tag{1.2.8}$$

ko'rinishdagi ifodalar $\varphi(x)$ *funksiyaning* $g(x)$ *funksiya bo'yicha olingan* α *(kasr) tartbli (Riman- Luivill ma'nosida) integrallari deyiladi.*

$D^{-\alpha}_{a,x,g(x)}\varphi(x)$ va $D^{-\alpha}_{a,b,g(x)}\varphi(x)$ funksiyalar (a,b) oraliqning deyarli barcha nuqtalarida aniqlangan bo'lib, $L_1(a,b)$ sinfga tegishli bo'ladi.

Agar $0<\alpha,\beta<+\infty$ bo'lsa, deyarli hamma $x\in(a,b)$ uchun

$$D^{-\beta}_{a,x;g(x)}D^{-\alpha}_{a,x;g(x)}\varphi(x)=D^{-\alpha}_{a,x;g(x)}D^{-\beta}_{a,x;g(x)}\varphi(x)=D^{-(\alpha+\beta)}_{a,x;g(x)}\varphi(x) \tag{1.2.9}$$

tenglik o'rinli bo'ladi. Xaqiqatan ham,

$$D^{-\beta}_{a,x;g(x)}D^{-\alpha}_{a,x;g(x)}\varphi(x)=$$

$$=\frac{1}{\Gamma(\alpha)}D^{-\beta}_{a,x;g(x)}\int_a^x[g(x)-g(s)]^{\alpha-1}\varphi(s)dg(s)=$$

$$=\frac{1}{\Gamma(\alpha)\Gamma(\beta)}\int_a^x\left[\int_a^t[g(t)-g(s)]^{\alpha-1}\varphi(s)dg(s)\right][g(x)-g(t)]^{\beta-1}dg(t)=$$

$$=\frac{1}{\Gamma(\alpha)\Gamma(\beta)}\int_a^x\varphi(s)dg(s)\int_s^x[g(x)-g(t)]^{\beta-1}[g(t)-g(s)]^{\alpha-1}dg(t).$$

Oxirgi ichki integralda $g(t)=g(s)+z[g(x)-g(s)]$ almashtirish bajarish natijasida quyidagi tenglikni hosil qilamiz:

$$\int_s^x[g(x)-g(t)]^{\beta-1}[g(t)-g(s)]^{\alpha-1}dg(t)=[g(x)-g(t)]^{\alpha+\beta-1}\int_0^1 z^{\alpha-1}(1-z)^{\beta-1}dz=$$

$$=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}[g(x)-g(t)]^{\alpha+\beta-1}$$

Bu esa (9) tenglikning to'g'riligini ko'rsatadi.

Ta'rif. *$\varphi(x)$ funksiya $[a,b]$ kesmada aniqlangan va $g(x)$ uzluksiz differensiallanuvchi, o'suvchi (kamayuvchi) bo'lib, $g'(x)\neq 0$ bo'lsin.*

$$D^{\alpha}_{a,x;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dg(x)}\int_a^x\frac{\varphi(t)dg(t)}{[g(x)-g(t)]^{\alpha}},\quad 0<\alpha<1, \tag{1.2.10}$$

$$\left[D^{\alpha}_{x,b;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dg(x)}\int_x^b\frac{\varphi(t)dg(t)}{[g(x)-g(t)]^{\alpha}},\quad 0<\alpha<1\right] \tag{1.2.11}$$

ko'rinishdagi ifodalar $\varphi(x)$ funksiyadan $g(x)$ funksiya bo'yicha olingan α (kasr) tartibli (Luivill ma'nosidagi) hosilalari deyiladi.

Lemma. *Agar $\varphi(x)$ funksiya kesmada absolyut uzluksiz bo'lsa, $[a,b]$ kesmaning deyarli barcha nuqtalarida $\varphi(x)$ funksiyaning kasr tartibli hosilalari mavjud bo'lib, quyidagi formula o'rinli bo'ladi:*

$$D^{\alpha}_{a,x;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\left[\frac{\varphi(a)}{[g(x)-g(a)]^{\alpha}}+\int_a^x\frac{\varphi'(t)dg(t)}{[g(x)-g(t)]^{\alpha}}\right],\quad 0<\alpha<1$$

$$D^{\alpha}_{x,b;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\left[\frac{\varphi(b)}{[g(b)-g(x)]^{\alpha}}+\int_x^b\frac{\varphi'(t)dg(t)}{[g(x)-g(t)]^{\alpha}}\right],\quad 0<\alpha<1$$

Misol. $\varphi(x)=[g(x)-g(a)]^{\alpha-1}$, $0<\alpha<1$ bo'lsin.U holda, (1.2.10) tenglikka asosan,

$$D^{\alpha}_{a,x;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dg(x)}\int_a^x[g(x)-g(t)]^{-\alpha}[g(t)-g(s)]^{\alpha-1}dg(t)\,.$$

Integral o'zgaruvchisini $g(t)=g(a)+z[g(x)-g(a)]$ formula bilan almashtirsak,

$$D^{\alpha}_{a,x;g(x)}\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dg(x)}\int_a^x z^{\alpha-1}(1-z)^{-\alpha}dz=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dg(x)}B(\alpha,1-\alpha)=0$$

tenglik kelib chiqadi. Demak, $\varphi(x)[g(x)-g(a)]^{\alpha-1}$funksiya $\alpha\in(0,1)$ tartibli hosila uchun o'zgarmas son vazifasini bajaradi.

Teorema. $\alpha>0$ *bo'lsin. U holda*

$$D^{\alpha}_{a,x;g(x)}D^{-\alpha}_{a,x;g(x)}\varphi(x)=\varphi(x)\ ,\qquad D^{\alpha}_{x,b;g(x)}D^{-\alpha}_{x,b;g(x)}\varphi(x)=\varphi(x)\qquad(1.2.12)$$

tengliklar barcha $\varphi(x)\in L_1(a,b)$ *funksiyalar uchun,*

$$D^{-\alpha}_{a,x;g(x)}D^{\alpha}_{a,x;g(x)}\varphi(x)=\varphi(x),\qquad D^{-\alpha}_{x,b;g(x)}D^{\alpha}_{x,b;g(x)}\varphi(x)=\varphi(x)\qquad(1.2.13)$$

tengliklar mos ravishda barcha

$$\varphi(x)\in D^{-\alpha}_{a,x;g(x)}(L_1),\qquad \varphi(x)\in D^{-\alpha}_{x,b;g(x)}(L_1)$$

funksiyalar uchun bajariladi.

Yuqoridagi $\varphi(x)$funksiyaning $g(x)$funksiya bo'yicha olingan kasr tartibli hosilasi va integrali yordamida [3] ishdagi kabi quyidagi integro-differensial operatorlarni kiritamiz.

$$D^{\alpha}_{a,x;g(x)}\varphi(x)=\begin{cases}\dfrac{1}{\Gamma(-\alpha)}\displaystyle\int_a^x[g(x)-g(t)]^{-\alpha-1}\varphi(t)dg(t), \; agar \;\; \alpha<0 \;\;\; bo'lsa,\\ \dfrac{d^n}{d[g(x)]^n}D^{\alpha-n}_{a,x;g(x)}\varphi(x), \;\;\; agar \;\; \alpha>0 \;\;\; bo'lsa,\end{cases}$$

$$D^{\alpha}_{x,b;g(x)}\varphi(x)=\begin{cases}\dfrac{1}{\Gamma(-\alpha)}\displaystyle\int_x^b[g(x)-g(t)]^{-\alpha-1}\varphi(t)dg(t), \; agar \;\; \alpha<0 \;\;\; bo'lsa,\\ (-1)^n\dfrac{d^n}{d[g(x)]^n}D^{\alpha-n}_{x,b;g(x)}\varphi(x), \;\;\; agar \;\; \alpha>0 \;\;\; bo'lsa,\end{cases}$$

bu yerda $n=[\alpha]+1.$

Bundan tashqari bu operatorlar quyidagi hossalarga ega:

1) Agar $0<\alpha,\beta<1$ va $[g(x)-g(a)]^{-\alpha}f(x),[g(b)-g(x)]^{-\beta}f(x)\in L_1(a,b)$ bo'lsa, u holda deyarli hamma $x\in(a,b)$ uchun

$$D^{-\beta}_{a,x;g(x)}\; g(x)-g(a)\;^{-\beta}D^{-\alpha}_{a,x;g(x)}\; g(x)-g(a)\;^{-\alpha}f(x)=$$
$$=D^{-\alpha}_{a,x;g(x)}\; g(x)-g(a)\;^{-\alpha}D^{-\beta}_{a,x;g(x)}\; g(x)-g(a)\;^{-\beta}f(x)$$

munosabat o'rinli bo'ladi.

2) Agar $0<2\alpha<1$ va $[g(x)-g(a)]^{-\alpha}f(x),[g(b)-g(x)]^{-\alpha}f(x)\in L_1(a,b)$ bo'lsa, u holda hamma $x\in(a,b)$ uchun quyidagi ayniyatlar o'rinli bo'ladi:

$$D^{\alpha}_{a,x;g(x)}[g(x)-g(a)]^{2\alpha-1}D^{\alpha-1}_{a,x;g(x)}[g(x)-g(a)]^{-\alpha}f(x)=$$
$$=[g(x)-g(a)]^{\alpha-1}D^{2\alpha-1}_{a,x;g(x)}f(x),$$

$$D^{\alpha}_{x,b;g(x)}[g(b)-g(x)]^{2\alpha-1}D^{\alpha-1}_{x,b;g(x)}[g(b)-g(x)]^{-\alpha}f(x)=$$
$$=[g(b)-g(x)]^{\alpha-1}D^{2\alpha-1}_{x,b;g(x)}f(x).$$

3) Agar $0<2\beta<1$va

$g(x)-g(a)\ ^{\beta-1} f(x),\ [g(b)-g(x)]^{\beta-1} f(x)\in L_1(a,b)$ bo’lsa, u holda deyarli hamma $x\in(a,b)$ uchun quyidagi ayniyatlar o’rinli bo’ladi:

$$D_{a,x;g(x)}^{1-\beta}\ g(x)-g(a)\ ^{1-2\beta} D_{a,x;g(x)}^{-\beta}\ g(x)-g(a)\ ^{\beta-1} f(x)=$$
$$=\ g(x)-g(a)\ ^{-\beta} D_{a,x;g(x)}^{1-2\beta} f(x),$$

$$D_{x,b;g(x)}^{1-\beta}[g(b)-g(x)]^{1-2\beta} D_{x,b;g(x)}^{-\beta}[g(b)-g(x)]^{\beta-1} f(x)=$$
$$=[g(b)-g(x)]^{-\beta} D_{x,b;g(x)}^{1-2\beta} f(x).$$

4) $f(x)\in C^{(0,\gamma)}(a,b),\ 0<\gamma\le 1$ va $0<\alpha<1$ bo’lsin. U holda ushbu

$$D_{a,x;g(x)}^{\alpha} D_{a,x;g(x)}^{-\alpha} f(x)=\cos(\alpha\pi) f(x)+\frac{\sin(\alpha\pi)}{\pi}\int_a^b\left(\frac{g(t)-g(a)}{g(x)-g(a)}\right)^{\alpha}\frac{f(t)dg(t)}{g(t)-g(x)},$$

$$D_{x,b;g(x)}^{\alpha} D_{x,b;g(x)}^{-\alpha} f(x)=\cos(\alpha\pi) f(x)-\frac{\sin(\alpha\pi)}{\pi}\int_a^b\left(\frac{g(b)-g(t)}{g(b)-g(x)}\right)^{\alpha}\frac{f(t)dg(t)}{g(t)-g(x)}.$$

5) Agar $\upsilon(x)\in C^{(0,\gamma)}(-1,-1),\ 0<\gamma\le 1$ va $0<2\beta<1$bo’lsa, u holda quyidagi ayniyatlar o’rinli bo’ladi:

$$\frac{d}{dg(x)}\int_{-1}^{x}\ g(x)-g(\xi)\ ^{2\beta-1}\ dg(\xi)\times$$
$$\times\int_{-1}^{1}\left[\left|g(\xi)-g(t)\right|^{-2\beta}-\ 1-g(\xi)g(t)\ ^{-2\beta}\right]\upsilon(t)dg(t)=$$
$$=\pi tg(\beta\pi)\upsilon(x)+\int_{-1}^{1}\left(\frac{1+g(t)}{1+g(x)}\right)^{1-2\beta}\left(\frac{1}{g(t)-g(x)}-\frac{1}{1-g(x)g(t)}\right)\upsilon(t)dg(t),$$

$$\frac{d}{dg(x)}\int_{x}^{1}\ g(\xi)-g(x)\ ^{2\beta-1}\ dg(\xi)\int_{-1}^{1}\left[\left|g(\xi)-g(t)\right|^{-2\beta}-\ 1-g(\xi)g(t)\ ^{-2\beta}\right]\upsilon(t)dg(t)=$$
$$=\pi tg(\beta\pi)\upsilon(x)+\int_{-1}^{1}\left(\frac{1-g(t)}{1-g(x)}\right)^{1-2\beta}\left(\frac{1}{g(t)-g(x)}+\frac{1}{1-g(x)g(t)}\right)\upsilon(t)dg(t).$$

1.3-§. Turli kasr tartibli integro-differensial operatorlar kompozitsiyasi xossalari haqida

Ushbu paragrafda quyidagi ko'rinishdagi kasr tartibli integro-differensial operatorlar kompozitsiyasining ba'zi xossalari haqida so'z yuritilgan:

$$A\left[\varphi(x)\right] \equiv D_{ax}^{l_1}(x-a)^{l_2} D_{ax}^{l_2}(x-a)^{l_4} D_{ax}^{l_5}\varphi(x). \qquad (1.3.1)$$

1.3.1-Lemma. Quyidagi shartlar bajarilsa

1) $\varphi(x) \in L(a,b), \quad -\infty < a < b < \infty;$

2) $l_1 < -l_3 - l_5,\ l_2 > l_3 - l_4 + l_5,\ l_3 < -l_5,\ l_4 > l_5, l_5 < 0,$

(a,b) ning hamma yerida quyidagi formula o'rinli bo'ladi:

$$A\left[\varphi(x)\right] \equiv (x-a)^{l_2-l_1-l_3+l_4-l_5-1}\times$$
$$\times\int_a^x G_{33}^{03}\left(\frac{z-a}{x-a}\middle| \begin{matrix} l_4-l_3-l_5, -l_5, l_2-l_1-l_3+l_4-l_5 \\ l_4-l_5,\ 0\ , l5-l_3+l_4-l_5 \end{matrix}\right)\varphi(z)dz, \qquad (1.3.2)$$

bu yerda $G_{33}^{03}(y|...)$ - Meyer funksiyasi

Isbot. (1.3.1) ifodadan

$$A\left[\varphi(x)\right] = \frac{1}{\Gamma(k_1-l_1)}\cdot\frac{1}{\Gamma(k_3-l_1)}\cdot\frac{1}{\Gamma(-l_5)}\times$$
$$\times\frac{d^{k_1}}{dx^{k_1}}\int_a^x (x-t)^{k_1-l_1-1}(t-a)^{l_2}dt\frac{d^{k_3}}{dx^{k_3}}\times$$
$$\times\int_a^t (t-y)^{k_3-l_3-1}(y-a)^{l_4}dy\int_a^y (y-z)^{-l_5-1}\varphi(z)dz.$$

Integrallash tartibini almashtirsak ,

$$A\left[\varphi(x)\right] = \frac{1}{\Gamma(k_1-l_1)}\cdot\frac{1}{\Gamma(k_3-l_1)}\cdot\frac{1}{\Gamma(-l_5)}\times$$
$$\times\frac{d^{k_1}}{dx^{k_1}}\int_a^x (x-t)^{k_1-l_1-1}(t-a)^{l_2}dt\frac{d^{k_3}}{dx^{k_3}}\times \qquad (1.3.3)$$
$$\times\int_a^t \varphi(z)dz\int_z^t (t-y)^{k_3-l_3-1}(y-z)^{-l_5-1}(y-a)^{l_4}dy,$$

ifodaga ega bo'lamiz. Bu yerda $k_1 = \max\{0; k_2+1\}$, $k_3 = \max\{0; k_4+1\}$, $k_2(k_4)$-$l_1(l_3)$ ning butun qismi.

$y = z + (t-z)s$ almashtirish bajarsak, quyidagi ifoda hosil bo'ladi:

$$A[\varphi(x)] = \frac{1}{\Gamma(k_1-l_1)}\cdot\frac{1}{\Gamma(k_3-l_1)}\cdot\frac{1}{\Gamma(-l_5)}\times$$
$$\times\frac{d^{k_1}}{dx^{k_1}}\int_a^x (x-t)^{k_1-l_1-1}(t-a)^{l_2}dt\times\frac{d^{k_3}}{dx^{k_3}}\int_a^t (z-a)^{l_4}(t-z)^{k_3-l_3-l_5-1}\varphi(z)dz\times \tag{1.3.4}$$
$$\times\int_0^1 s^{-l_5-1}(1-s)^{k_3-l_3-1}\left(1-\frac{z-t}{z-a}s\right)^{l_4}ds.$$

Gaussning gipergeometrik funksiyasining integral tasviriga ko'ra:

$$F(a',b',c';z) =$$
$$=\frac{\Gamma(c')}{\Gamma(a')\Gamma(c'-a')}\int_0^1 t^{a'-1}(1-t)^{c'-a'-1}(1-zt)^{-b'}dt \tag{1.3.5}$$

$$\operatorname{Re}c' > \operatorname{Re}a' > 0.$$

(1.4.4) ifodani quyidagi ko'rinishda yozish mumkin.

$$A[\varphi(x)] = \frac{1}{\Gamma(k_1-l_1)}\cdot\frac{1}{\Gamma(k_3-l_3-l_5)}\cdot\times$$
$$\times\frac{d^{k_1}}{dx^{k_1}}\int_a^x (x-t)^{k_1-l_1-1}(t-a)^{l_2}dt\int_a^x (z-a)^{l_4}(t-z)^{k_3-l_3-l_5-1}\times \tag{1.3.6}$$
$$\times F\left(-l_5,-l_4,k_3-l_3-l_5;\frac{z-t}{z-a}\right)\varphi(z)dz.$$

[3] formuladan foydalansak,

$$F(a',b',c';z) = (1-z)^{-b'}F\left(c'-a',b',c';\frac{z}{1-z}\right) \tag{1.3.7}$$

$$\frac{d^{n'}}{dz^{n'}}\left[z^{c'-1}(1-z)^{b'-c'+n'}F(a',b',c';z)\right]=$$
$$=(-1)^{n'}(1-c')_{n'}z^{c'-n'-1}(1-z)^{b'-c'}\cdot F(a'-n',b',c'-n';z) \tag{1.3.8}$$

(1.3.6) dan integrallash tartibini almashtirsak,

$$A\left[\varphi(x)\right]=\frac{1}{\Gamma(k_1-l_1)}\cdot\frac{1}{\Gamma(-l_3-l_5)}\cdot\frac{d^{k_1}}{dx^{k_1}}\int\limits_a^x (z-a)^{l_4}\varphi(z)dz\times$$

$$\times\int\limits_z^x (x-t)^{k_1-l_1-1}(t-z)^{-l_3-l_5-1}(t-a)^{l_2}\cdot F\left(-l_5,-l_4,-l_3-l_5;\frac{z-t}{z-a}\right)dt. \tag{1.3.9}$$

(1.3.9) ifodada $t=a+(x-a)r$ almashtirishni bajarsak,

$$\begin{gathered} A\left[\varphi(x)\right]=\frac{1}{\Gamma(k_1-l_1)}\cdot\frac{1}{\Gamma(-l_3-l_5)}\times \\ \times\frac{d^{k_1}}{dx^{k_1}}\int\limits_a^x (z-a)^{l_2-l_1-l_3+l_4-l_5+k_1-1}Q_1(\sigma)\varphi(z)dz \end{gathered} \tag{1.3.10}$$

bu yerda

$$Q_1(\sigma)=\sigma^{l_2-l_1+k_1}\int\limits_0^\infty r^{l_2} f_1(\sigma r) f_2(r)dr\ , \tag{1.3.11}$$

$$f_1(r)=(r-1)^{-l_3-l_5-1}F(-l_5,-l_4,-l_3-l_5;1-r)\ , \tag{1.3.12}$$

$$f_2(r)=(1-r)_+^{k_1-l_1-1}, \tag{1.3.13}$$

$$\sigma=\frac{x-a}{z-a};\qquad r_{1+}^l=\begin{cases}0\ ,\ r_1\le 0,\\ r_1^l\ ,\ r_1>0\,.\end{cases}$$

(1.3.11) integralni hisoblash uchun Mellin integral almashtirishidan foydalanamiz [21]

$$f^*(s)=M\left\{f(x);s\right\}=\int\limits_0^\infty f(x)x^{s-1}dx \tag{1.3.14}$$

U holda quyidagi

$$\begin{gathered} M\left\{x^{\delta_1}\int\limits_0^\infty \xi^{\delta_2}g_1(x\xi)g_2(\xi)d\xi;s\right\}= \\ =g_1^*(s+\delta_1)g_2^*(1-\delta_1+\delta_2-s) \end{gathered} \tag{1.3.15}$$

formulani e'tiborga olsak,

$$M\left\{Q_1(\sigma);s\right\}=f_1^*(l_2-l_1+k_1+s)\,f_2^*(1+l_1-k_1-s) \tag{1.3.16}$$

olamiz.

Agar quyidagi formulalarni qo'llasak,

$$M\ \ (x-1)_+^{c'-1}F(a',b',c';1-x);s\ \ =$$

$$=\Gamma(c')\Gamma\begin{bmatrix}1+a'-c'-s,\ 1+b'-c'-s & \\ 1-s, & 1+a'+b'-c'-s\end{bmatrix} \tag{1.3.17}$$

$$\operatorname{Re}c'>0,\operatorname{Re}s<1+\operatorname{Re}(a'-c'),1+\operatorname{Re}(b'-c'),$$

$$M\ \ (1-x)_+^{c'-1};s\ \ =\Gamma(c')\Gamma\begin{bmatrix}s\\ s+c'\end{bmatrix},$$

$$\operatorname{Re}c'<0,\operatorname{Re}s>0 \tag{1.3.18}$$

bu yerda

$$\Gamma\begin{bmatrix}a_1,\ a_2,...,a_i\\ b_1,\ b_2,...,b_i\end{bmatrix}=\frac{\Gamma\ \ a_1\ \ \Gamma\ \ a_2\ \ ...\Gamma\ \ a_i}{\Gamma\ \ b_1\ \ \Gamma\ \ b_2\ \ ...\Gamma\ \ b_i} \tag{1.3.19}$$

quyidagini topamiz:

$$M\ \ f_1(\sigma r);s\ \ =$$

$$=\Gamma(-l_3-l_5)\Gamma\begin{bmatrix}1+l_3-s,\ 1+l_3-l_4+l_5-s\\ 1+l_3-l_4-s,\ 1-s\end{bmatrix} \tag{1.3.20}$$

$$s<\min\ \ 1+l_3,1+l_3-l_4+l_5$$

va

$$M\ \ f_2(r);s\ \ =\Gamma(k_1-l_1)\Gamma\begin{bmatrix}s\\ s-l_1+k_1\end{bmatrix},\ \operatorname{Re}s>0. \tag{1.3.21}$$

(1.3.20), (1.3.21)ni (1.3.16) ga qo'ysak,

$$M\ \ Q_1(\sigma);s\ \ =\Gamma(k_1-l_1)\Gamma(-l_3-l_5)\times$$

$$\times\Gamma\begin{bmatrix}1+l_1-l_2+l_3-k_1-s, & 1+l_1-l_2+l_3-l_4+l_5-k_1-s, & 1+l_1-k_1-s\\ 1+l_1-l_2-k_1-s, & 1+l_1-l_2+l_3-l_4-k_1-s, & 1-s\end{bmatrix} \tag{1.3.22}$$

$$s<\min\ \ 1+l_1-l_2+l_3-k_1,1+l_1-l_2+l_3-l_4+l_5-k_1,1+l_1-k_1\ \ .$$

ifodani olamiz [21]

$$M\left\{G_{p'q'}^{m'n'}\left(z\left|\begin{matrix}a_1,\ a_2,...,a_{p'}\\ b_1,\ b_2,...,b_{q'}\end{matrix}\right.\right);s\right\}=$$

$$=\Gamma\left[\begin{matrix} s+b_1, s+b_2, ..., s+b_m, 1-a_1-s, ..., 1-a_{n'}-s \\ s+a_{n'+1}, s+a_{n'+2}, ..., s+a_{p'}, 1-b_{m'+1}-s, 1-b_{m'+2}-s, ..., 1-b_{q'}-s \end{matrix}\right]$$

(1.3.23)

$$-\min_{1\le k'\le m'} \operatorname{Re} b_{k'} < \operatorname{Re} s < 1-\max_{1\le k'\le n'} \operatorname{Re} a_j,$$

$$p'=q'\ge 1, m'+n'=p', \quad \sum_{j=1}^{p'} a_j - b_j \quad >0.$$

(1.3.22) dan quyidagiga ega bo'lamiz:

$$Q_1(\sigma)=\Gamma(k_1-l_1)\Gamma(-l_3-l_5)\times$$
$$\times G_{33}^{03}\left(\sigma\left|\begin{matrix} l_2-l_1+l_3+k_1, l_2-l_1-l_3+l_4-l_5, k_1-l_1 \\ l_2-l_1+k_1, \quad l_2-l_1+l_3+l_4+k_1,\ 0 \end{matrix}\right.\right). \tag{1.3.24}$$

(1.3.24) ifodaga (1.3.12) ni qo'ysak,

$$A[\varphi\ x\ ]=\frac{d^{k_1}}{dx^{k_1}}\int_a^x z-a^{\ l_2-l_1-l_3+l_4-l_5+k_1-1}\varphi(z)\times$$
$$\times G_{33}^{03}\left(\frac{x-a}{z-a}\left|\begin{matrix} l_2-l_1+l_3+k_1, l_2-l_1-l_3+l_4-l_5+k_1, k_1-l_1 \\ l_2-l_1+k_1, \quad l_2-l_1+l_3+l_4+k_1,\ 0 \end{matrix}\right.\right)dz \tag{1.3.25}$$

ifodani olamiz.

Keyin [20] formulani qo'llaymiz.

$$x^{\delta}G_{p'q'}^{m'n'}\left(x\left|\begin{matrix} a_1, a_2, ..., a_{p'} \\ b_1, b_2, ..., b_{q'} \end{matrix}\right.\right)=G_{p'q'}^{m'n'}\left(x\left|\begin{matrix} a_1+\delta, ..., a_{p'}+\delta \\ b_1+\delta, ..., b_{q'}+\delta \end{matrix}\right.\right), \tag{1.3.26}$$

$$G_{p'q'}^{m'n'}\left(x\left|\begin{matrix} a_1, a_2, ..., a_{p'} \\ b_1, b_2, ..., b_{q'} \end{matrix}\right.\right)=G_{q'p'}^{n'm'}\left(\frac{1}{x}\left|\begin{matrix} 1-b_1, ..., 1-b_{q'} \\ 1-a_1, ..., 1-a_{p'} \end{matrix}\right.\right), \tag{1.3.27}$$

(1.3.25) dan

$$A[\varphi\ x\ ]=\frac{d^{k_1}}{dx^{k_1}}\ x-a^{\ l_2-l_1-l_3+l_4-l_5+k-1}\times$$
$$\times\int_a^x G_{33}^{30}\left(\frac{z-a}{x-a}\left|\begin{matrix} l_4-l_3-l_5,\ l_2-l_1-l_3+l_4-l_5+k \\ l_4-l_5, \quad 0\ ,\ l_2-l_3+l_4-l_5 \end{matrix}\right.\right)\varphi(z)dz,$$

Bu yerdan [20] formuladan foydalanib,

$$\frac{d^{n'}}{dx^{n'}}\left\{x^{a_{p'}-1}G^{n'm'}_{q'p'}\left(\frac{1}{x}\middle|\begin{matrix}a_1, \ldots, a_{p'}\\ b_1, \ldots, b_{q'}\end{matrix}\right)\right\}=$$

$$= x^{a_{p'}-n'-1}G^{n'm'}_{q'p'}\left(\frac{1}{x}\middle|\begin{matrix}a_1, \ldots, a_{p'}-n'\\ b_1, \ldots, b_{q'}\end{matrix}\right), \quad n' < p'-1, \tag{1.3.28}$$

ega bo'lamizki,

$$A[\varphi(x)] = (x-a)^{l_2-l_1-l_3+l_4-l_5-1}\times$$

$$\times\int_a^x G^{30}_{33}\left(\frac{z-a}{x-a}\middle|\begin{matrix}l_4-l_3-l_5,\ -l_5,\ l_2-l_1-l_3+l_4-l_5\\ l_4-l_5,\ \ 0,\ l_2-l_3+l_4-l_5\end{matrix}\right)\varphi(z)dz.$$

1.3.1-lemma isbotlandi.

1.3.2-Lemma. $\varphi^{(k+1)}(x)\in L(a,b)$,

$l_1 < -l_3 - l_5,\ l_2 > l_3 - l_4 + l_5,\ l_3 < -l_5,\ l_4 > l_5, l_5 < 0$.

U holda deyarli barcha (a,b) uchun quyidagi munosabat bajariladi:

$$A[\varphi(x)] = \frac{d^{k+1}}{dx^{k+1}}(x-a)^{l_2-l_1-l_3+l_4-l_5+k}\times$$

$$\times\int_a^x G^{30}_{33}\left(\frac{z-a}{x-a}\middle|\begin{matrix}l_4-l_3-l_5,\ \ l_2-l_1-l_3+l_4-l_5 k+1\\ l_4-l_5,\ \ 0,\ l_2-l_3+l_4-l_5\end{matrix}\right)\varphi(z)dz, \tag{1.3.29}$$

bu yerda k - $l_1+l_3+l_5$ sonining butun qismi.

Isbot. Ba'zi hisoblashlardan so'ng quyidagiga ega bo'lamiz:

$$A[\varphi(x)] = \frac{1}{\Gamma(k_1-l_1)}\cdot\frac{1}{\Gamma(-l_3-l_5)}\cdot\frac{d^{k+1}}{dx^{k+1}}\times$$

$$\times\int_a^x (z-a)^{l_2-l_1-l_3+l_4-l_5+k_1-1}Q_1(z)\varphi(z)dz, \tag{1.3.30}$$

bu yerda

$$Q_1(\sigma) = \sigma^{l_2-l_1+k_1}\int_0^\infty r^{l_2} f_1(\sigma r) f_2(r)dr\ , \tag{1.3.31}$$

$$f_1(r) = (r-1)_+^{-l_3-l_5-1}F(-l_5,-l_4,-l_3-l_5;1-r)\ , \tag{1.3.32}$$

$$f_2(r) = (1-r)_+^{k_1-l_1-1}, \tag{1.3.33}$$

Quyidagi tenglikni ko'rib chiqamiz:

$$
A\left[\varphi\ \ x\ \ \right]=\lim A^{\varepsilon}\left[\varphi\ \ x\ \ \right]\frac{1}{\Gamma(k_1-l_1)}\cdot\frac{1}{\Gamma(-l_3-l_5)}\times
$$

$$
\times\lim_{\varepsilon\to 0}\frac{d^{k_1}}{dx^{k_1}}\int\limits_a^{x-\varepsilon} z-a\ \ ^{l_2-l_1-l_3+l_4-l_5+k_1-1}Q_1(\sigma)\varphi(z)dz=
$$

$$
=\frac{1}{\Gamma(k_1-l_1)}\cdot\frac{1}{\Gamma(-l_3-l_5)}\lim_{\varepsilon\to 0}\left\{\sum_{j=0}^{k_1-1}B_j^{\varepsilon}\left[\varphi\ \ x\ \ \right]+\int\limits_a^{x-\varepsilon} z-a\ \ ^{l_2-l_1-l_3+l_4-l_5+k_1-1}\times\right.
$$

$$
\left.\times\frac{d^{k_1}}{d\sigma^{k_1}}Q_1(\sigma)\left(\frac{\partial\sigma}{\partial x}\right)^{k_1}\varphi(z)dz,\right.
$$

(1.3.34)

bu yerda

$$
B_j^{\varepsilon}\left[\varphi\ \ x\ \ \right]=\frac{d^{k_1-1}}{dx^{k_1-1}}\ x-\varepsilon-a\ \ ^{l_2-l_1-l_3+l_4-l_5+k_1-1}\times
$$

$$
\times\varphi(x-\varepsilon)\left\{\frac{d^{j}}{d\sigma^{j}}Q_1(\sigma)\left(\frac{\partial\sigma}{\partial x}\right)^{j}\right\}\Bigg|_{\sigma=\frac{x-a}{x-\varepsilon-a}}
$$

$\dfrac{d^{j}}{d\sigma^{j}}Q_1(\sigma),\ \ j=\overline{0,k_1}$ ni hisoblaymiz.

[20] formuladan

$$
M\left\{\frac{d^{n'}}{dx^{n'}}f(x);s\right\}=\Gamma\begin{bmatrix}1+n'-s\\ 1-s\end{bmatrix}f^{*}(s-n') \tag{1.3.35}
$$

(1.3.22) dan :

$$
M\left\{\frac{d^{j}}{d\sigma^{j}}Q_1(\sigma);s\right\}=\Gamma\ \ k_1-l_1\ \ \Gamma\ \ -l_3-l_5\ \ \times
$$

$$
\times\begin{bmatrix}1+l_1-l_2+l_3+j-k_1-s,1+l_1-l_2+l_3-l_4+l_5+j-k_1-s,1+l_1+j-k_1-s\\ 1+l_1-l_2+j-k_1-s,\quad 1+l_1-l_2+l_3-l_4+j-k_1-s,\quad 1-s\end{bmatrix}
$$

(1.3.36)

topamiz.

(1.3.23) va (1.3.27) ga asosan (1.3.36) dan

$$\frac{d^j}{d\sigma^j}Q_1(\sigma)=\Gamma\left(k_1-l_1\right)\Gamma\left(-l_3-l_5\right)\times$$

$$\times G_{33}^{30}\left(\frac{1}{\sigma}\left|\begin{matrix}1+l_1-l_2+j-k_1, & 1+l_1-l_2+l_3-l_4+j-k_1, & 1\\ 1+l_1-l_2+l_3+j-k_1, & 1+l_1-l_2+l_3-l_4+l_5+j-k_1, & 1+l_1+j-k_1\end{matrix}\right.\right)$$

(1.3.37)

(1.3.37) ifodani (1.3.34) ga qo'ysak,

$$A\left(\varphi(x)\right)=\frac{1}{\Gamma\left(k_1-l_1\right)}\cdot\frac{1}{\Gamma\left(-l_3-l_5\right)}\times$$

$$\times\lim_{\varepsilon\to 0}\left\{\sum_{j=0}^{k_1-1}B_j^{\varepsilon}\left(\varphi(x)\right)+\Gamma\left(k_1-l_1\right)\cdot\Gamma\left(-l_3-l_5\right)\int_a^{x-\varepsilon}\left(z-a\right)^{l_2-l_1-l_3+l_4-l_5-1}\times\right. \tag{1.3.38}$$

$$\left.\times G_{33}^{30}\left(\frac{1}{\sigma}\left|\begin{matrix}1+l_1-l_2, & 1+l_1-l_2+l_3-l_4, & 1\\ 1+l_1-l_2+l_3, & 1+l_1-l_2+l_3-l_4+l_5, & 1+l_1\end{matrix}\right.\right)\varphi(z)dz\right\}$$

bu yerda

$$B_j^{\varepsilon}\left(\varphi(x)\right)=\frac{d^{k_1-j-1}}{d\sigma^{k_1-j-1}}\left(x-\varepsilon-a\right)^{l_2-l_1-l_3+l_4-l_5+k_1-j-1}\times$$

$$\times G_{33}^{30}\left(\frac{x-\varepsilon-a}{x-a}\left|\begin{matrix}1+l_1-l_2+k_1-j, 1+l_1-l_2+l_3-l_4+l_5+k_1-j, 1+k_1-j\\ 1+l_1-l_2+l_3+k_1-j,\ 1+l_1-l_2+l_3-l_4+l_5+k_1-j,\ 1+l_1+k_1-j\end{matrix}\right.\right)\times$$

$$\times\varphi\left(x-\varepsilon\right)$$

ko'rsatish qiyin emaski,

$$\frac{d^{k+1}}{dx^{k+1}}\left(x-a\right)^{l_2-l_1-l_3+l_4-l_5+k}\times$$

$$\times\int_a^{x-\varepsilon}G_{33}^{30}\left(\frac{z-a}{x-a}\left|\begin{matrix}l_4-l_3-l_5, & -l_5, & l_2-l_1-l_3+l_4-l_5+k+1\\ l_4-l_5, & 0, & l_2-l_3+l_4-l_5\end{matrix}\right.\right)\times$$

$$\times\varphi\left(z\right)dz=\sum_{j=0}^{k-1}\tilde{B}_j^{\varepsilon}\left[\varphi\left(x\right)\right]+\left(x-a\right)^{l_2-l_1-l_3+l_4-l_5-1}\times$$

$$\times\int_a^{x-\varepsilon}G_{33}^{30}\left(\frac{z-a}{x-a}\left|\begin{matrix}l_4-l_3-l_5, & -l_5, & l_2-l_1-l_3+l_4-l_5\\ l_4-l_5, & 0, & l_2-l_3+l_4-l_5\end{matrix}\right.\right)\varphi\left(z\right)dz.$$

(1.3.39)

bu yerda

$$\tilde{B}_j^{\varepsilon}\ \varphi(x)\ =\frac{d^{k-j}}{dx^{k-j}}\ x-\varepsilon-a\ ^{l_2-l_1-l_3+l_4-l_5+k-j}\times$$

$$\times G_{33}^{30}\left(\frac{x-\varepsilon-a}{x-a}\middle|\begin{matrix}2+l_1-l_2+k-j,\ 2+l_1-l_2+l_3-l_4+l_5+k-j, 2+k-j\\ 2+l_1-l_2+l_3+k-j,\ 2+l_1-l_2+l_3-l_4+l_5+k-j,\ 2+l_1+k-j\end{matrix}\right)\times$$

$$\times\varphi\ \ x-\varepsilon$$

(1.3.39) ni e’tiborga olsak, (1.3.38) dan

$$A\ \varphi(x)\ =\lim_{\varepsilon\to 0}\left\{\sum_{j=0}^{k_1-1}B_j^{\varepsilon}\ \varphi(x)\ -\sum_{j=0}^{k_1-1}\tilde{B}_j^{\varepsilon}\ \varphi(x)\ +\right.$$

$$+\frac{d^{k+1}}{dx^{k+1}}\ x-a\ ^{l_2-l_1-l_3+l_4-l_5+k}\times$$

$$\times\int_a^{x-\varepsilon}G_{33}^{30}\left(\frac{z-a}{x-a}\middle|\begin{matrix}l_4-l_3-l_5,\ -l_5,\ \ l_2-l_1-l_3+l_4-l_5+k+1\\ l_4-l_5,\ \ 0,\ \ l_2-l_3+l_4-l_5\end{matrix}\right)\varphi\ \ z\ \ dz.$$

olamiz.

Keyin (1.3.29) tenglikni hosil qilamiz.

Lemma isbotlandi.

I BOB YUZASIDAN XULOSA

Dissertatsiyaning ushbu bobida birinchi paragrafida umumlashgan kasr tartibli integro-differensial operatorlarga ta’rif berilgan.

Ikkinchi paragrafida funksiyadan funksiya bo’yicha olingan kasr tartibli hosila va integrallarni o’rganilgan.

Uchinchi paragrafida kasr tartibli integro-differensial operatorlar kompozitsiyasining ba’zi xossalari haqida so’z yuritilgan.

II BOB. IKKITA BUZILISH CHIZIG'IGA EGA BO'LGAN GIPERBOLIK TIPDAGI TENGLAMALAR UCHUN CHEGARAVIY MASALALAR

2.1-§.Soha chegarasida buziluvchi giperbolik tipdagi tenglamalar uchun nolokal chegaraviy masalalar

Ushbu paragraf (2.1.1) tenglama uchun chegaraviy masalalar yechishga kasr tartibli integro-differensial operatorlar qo'llanilishiga bag'ishlangan.

$$-y^{m}U_{xx}-x^{n}U_{yy}=0,\qquad m,n=const>0. \tag{2.1.1}$$

tenglamani Ω sohada ko'ramiz.

Bu yerda Ω- (2.1.1) tenglamaning $A(0,0)$, $B(h_1,0)$ nuqtadan chiquvchi xarakteristikalari

$$AC:\frac{1}{q}x^{q}-\frac{1}{p}(-y)^{p}=0,\ \ BC:\frac{1}{q}x^{q}+\frac{1}{p}(-y)^{p}=1$$

va $y=0$ to'g'ri chiziqning AB kesmasi bilan chegaralangan bir bog'lamli soha.

bu yerda $2p=m+2,\ 2q=n+2,\ h_1=q^{\frac{1}{q}}$

Quyidagi belgilashni kiritamiz:

$$J=\ \ x,y\ \ :0<x<h_1, y=0\ \ ,$$

$$\theta(x)=\left[\frac{x^{q}}{2}\right]^{1/q}-i\left[\frac{p}{q}\cdot\frac{x^{q}}{2}\right]^{1/p}, \tag{2.1.2}$$

$$2\alpha=n/(n+2),\quad 2\beta=m/(m+2)$$

bu yerda $\theta(x)$ -(2.1.1)

2.1.1. Masala. Quyidagi hossalarga ega bo'lgan $U\ \ x,y$ funksiya topilsin.

1) $U(x,y)\in C(\overline{\Omega})\cap C^{1}(\Omega\cup J),$

$$\int_0^1\left|U_y(x^{1/q},0)\right|\ x(1-x)^{\ -\beta}dx<\infty\,;$$

2) $U(x,y)$ (2.1.1) tenglamaning Ω sohadagi regulyar yechimi;

3) $U(x,y)$ quyidagi shartlarni qanoatlantiradi:

$$U(x,0)=\tau(x),\quad x\in\overline{J}, \tag{2.1.3}$$

$$D_{0x^{2x}}^{l_1}(x^{2q})^{l_2}u[\theta(x)]=a_1(x)u_y(x,0)+b_1(x),\quad x\in J, \tag{2.1.4}$$

bu yerda l_1 va l_2-xaqiqiy sonlar; $\tau(x)$, $a_1(x)$ va $b_1(x)$-berilgan funksiyalar.

2.1.1.Teorema. Quyidagi shartlar bajarilganda 2.1. masala yagona yechimga ega bo’ladi:

4. $l_1<1-\beta,\quad l_2>\max\left\{\dfrac{\alpha+\beta-2}{2},l_1+\dfrac{2\beta-1}{2}\right\}$;

5. $a_1(x)\neq 0,\ \forall x\in\overline{J}$, $a_1(x),b_1(x)\in C^1(\overline{J})$;

6. $\tau(x)\in C^{K_8}(\overline{J})\cap C^{K_8+2}(J)$, $K_8=\max\ 0,K_7+1$,(K_7-$l_1-\beta$ ning butun qismi).

Isbot. (2.1.1) tenglama uchun Koshi masalasining boshlang’ich $u(x,0)=\tau(x)$, $u_y(x,0)=\nu(x)$ shartli yechimi Ω sohada [II],[I9] ko’rinishga ega.

$$u(x,y)=\frac{\Gamma\left(\frac{1}{2}+\beta\right)}{\sqrt{\pi}\,\Gamma\ \beta}2^{2\beta-1}\left(\frac{1}{q}x^q\right)^{-\alpha}\times$$

$$\times\int_0^1\left[\frac{1}{q}x^q+\frac{1}{p}(-y)^p(2z-1)\right]\ z(1-z)\ ^{\beta-1}\times$$

$$\times\tau\left\{\left[x^q+\frac{q}{p}(-y)^p(2z-1)\right]^{1/q}\right\}F\ \ \alpha,1-\alpha;\beta;\rho\ \ dz-$$

$$-\frac{\Gamma\left(\frac{3}{2}-\beta\right)}{\sqrt{\pi}\,\Gamma(1-\beta)}2^{2\beta-1}\left(\frac{1}{q}x^{q}\right)^{-\alpha}\left[\frac{1}{p}(-y)^{p}\right]^{-2\beta}\times$$

$$\times\int_{0}^{1}\left[\frac{1}{q}x^{q}+\frac{1}{p}(-y)^{p}(2z-1)\right]^{\alpha}\left(z(1-z)\right)^{-\beta}\times$$

$$\times\nu\left\{\left[x^{q}+\frac{q}{p}(-y)^{p}(2z-1)\right]^{1/q}\right\}F\left(\alpha,1-\alpha;1-\beta;\rho\right)dz, \tag{2.1.5}$$

bu yerda

$$\rho=\frac{q(-y)^{2p}z(1-z)}{p^{2}x^{q}\left[\frac{1}{q}x^{q}+\frac{1}{p}(-y)^{p}\right]}.$$

(2.1.5) formuladan foydalanib quyidagini topamiz:

$$\begin{gathered}u\left(\theta(x)\right)=u\left(\operatorname{Re}\theta(x),\operatorname{Im}\theta(x)\right)=\frac{\Gamma(2\beta)}{\Gamma(\beta)}2^{\alpha}(x^{q})^{1-\alpha-2\beta}\times\\ \times\int_{0}^{x}(t^{q})^{1+\beta-1}\tau(t)(x^{q}-t^{q})^{\beta-1}F\left(\alpha,1-\alpha;\beta;\frac{x^{q}-t^{q}}{2x^{q}}\right)qt^{q-1}dt-\\ -\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\left(\frac{p}{q}\right)^{1-2\beta}2^{1+2\beta-1}(x^{q})^{-\alpha}\int_{0}^{x}(t^{q})^{\alpha-\beta}\nu(t)\times\\ \times(x^{q}-t^{q})^{-\beta}F\left(\alpha,1-\alpha;1-\beta;\frac{x^{q}-t^{q}}{2x^{q}}\right)qt^{q-1}dt.\end{gathered} \tag{2.1.6}$$

[3] formulaga asosan

$$F\left(a',b',c';z\right)=\left(1-z\right)^{c'-1}F\left(\frac{c'-a'}{2},\frac{c'+a'-1}{2};c';4z\left(1-z\right)\right). \tag{2.1.7}$$

(2.1.6) dan

$$u\ \theta(x)\ =$$

$$=\gamma_1 (x^{2q})^{\frac{2-\alpha-3\beta}{2}} F_{ox}\begin{bmatrix} \dfrac{\beta-\alpha}{2} & \dfrac{\alpha+\beta-1}{2} \\ \beta & x^{2q} \end{bmatrix} (x^{2q})^{\frac{\alpha+\beta-2}{2}} \tau(x) - \tag{2.1.8}$$

$$-\gamma_2 (x^{2q})^{\frac{\beta-\alpha}{2}} F_{ox}\begin{bmatrix} \dfrac{1-\alpha-\beta}{2} & \dfrac{\alpha-\beta}{2} \\ 1-\beta & x^{2q} \end{bmatrix} (x^{2q})^{\frac{\alpha-\beta-1}{2}} \nu(x).$$

bu yerda

$$\gamma_1 = \frac{\Gamma\ 2\beta}{\Gamma\ \beta}, \qquad \gamma_2 = \frac{\Gamma\ 2-2\beta}{\Gamma\ 1-\beta}\left(\frac{p}{q}\right)^{1-2\beta} 2^{1+3\beta-2}.$$

(2.1.4) chegaraviy masalaga (2.1.8) ni olib borib qo'yib, $x = x^{1/2q}$, $t = t^{1/2q}$ almashtirish bajarsak, quyidagi ifodaga ega bo'lamiz:

$$\gamma_1 J_1(x) - \gamma_2 J_2(x) = \tilde{a}_1(x)\tilde{\nu}(x) + \tilde{b}_1(x), \quad x \in J, \tag{2.1.9}$$

$$J_1(x) = D_{0x}^{l_1} x^{l_2+\frac{2-\alpha-3\beta}{2}} F_{ox}\begin{bmatrix} \dfrac{\beta-\alpha}{2} & \dfrac{\alpha+\beta-1}{2} \\ \beta & x \end{bmatrix} x^{\frac{\alpha+\beta-2}{2}} \tilde{\tau}(x), \tag{2.1.10}$$

$$J_2(x) = D_{0x}^{l_1} x^{l_2+\frac{\beta-\alpha}{2}} F_{ox}\begin{bmatrix} \dfrac{1-\alpha-\beta}{2} & \dfrac{\alpha-\beta}{2} \\ 1-\beta & x \end{bmatrix} x^{\frac{\alpha-\beta-1}{2}} \tilde{\nu}(x) \tag{2.1.11}$$

$$\tilde{a}_1(x) = a_1(x^{1/2q}), \quad \tilde{b}_1(x) = b_1(x^{1/2q}), \ \tilde{\tau}(x) = \tau(x^{1/2q}), \ \tilde{\nu}(x) = \nu(x^{1/2q}).$$

Ba'zi almashtirishlardan keyin

$$J_1(x) = \frac{d^{K_8}}{dx^{K_8}} x^{l_2-l_1-\frac{\alpha+\beta}{2}+K_8} \times$$

$$\times \int_a^x G_{33}^{30}\left(\frac{y}{x} \left| \begin{matrix} \dfrac{1-\alpha-\beta}{2}, & \dfrac{\alpha+\beta}{2}, & l_2 - l_1 - \dfrac{\alpha+\beta}{2} + K_8 \\ \dfrac{1}{2}, & 0, & l_2 + \dfrac{2-\alpha-\beta}{2} \end{matrix} \right. \right) \times$$

$$\times y^{\frac{\alpha+\beta-2}{2}} \tilde{\tau}(y)dy, \tag{2.1.12}$$

$$J_2(x)=x^{l_2-l_1-\frac{\alpha+\beta}{2}}\int_a^x y^{\frac{\alpha+\beta-2}{2}}\tilde{\nu}(y)\times$$

$$\times\int_a^x G_{33}^{30}\left(\frac{y}{x}\left|\begin{matrix}\frac{2-\alpha+\beta}{2}, & \frac{1+\alpha-\beta}{2}, & l_2-l_1+\frac{2-\alpha-\beta}{2}\\ \frac{1}{2} & , \quad 0 \quad , & l_2+\frac{2-\alpha-\beta}{2}\end{matrix}\right.\right)dy. \tag{2.1.13}$$

(2.1.12), (2.1.13) larni (2.1.9) ga olib borsak, Volterraning II tur integral tenglamasiga ega bo'lamiz

$$\tilde{\nu}(x)+\int_0^x R(x,y)\tilde{\nu}(y)dy=\Phi(x),\ \ x\in J, \tag{2.1.14}$$

bu yerda

$$R(x,y)=\frac{\gamma_2}{\tilde{a}_1(x)}x^{l_2-l_1-\frac{1+2\beta}{2}}\times$$

$$\times\int_a^x G_{33}^{30}\left(\frac{y}{x}\left|\begin{matrix}\frac{1-2\beta}{2}, & \alpha-\beta, & l_2-l_1+\frac{1-2\beta}{2}\\ \frac{\alpha-\beta}{2}, & \frac{\alpha-\beta-1}{2}, & l_2+\frac{1-2\beta}{2}\end{matrix}\right.\right), \tag{2.1.15}$$

$$\Phi(x)=\frac{\gamma_1}{\tilde{a}_1(x)}\frac{d^{K_8}}{dx^{K_8}}x^{l_2-l_1-\frac{1+\beta}{2}+K_8}\times$$

$$\times\int_a^x y^{\frac{\alpha+\beta-2}{2}}\tilde{\tau}(y)\times$$

$$\times G_{33}^{30}\left(\frac{y}{x}\left|\begin{matrix}\frac{1-\alpha+\beta}{2}, & \frac{\alpha+\beta}{2}, & l_2-l_1+\frac{2-\alpha-\beta}{2}+K_8\\ \frac{1}{2} & , \quad 0 \quad , & l_2+\frac{2-\alpha-\beta}{2}\end{matrix}\right.\right)dy- \tag{2.1.16}$$

$$-\frac{\tilde{b}_1(x)}{\tilde{a}_1(x)}.$$

2.1.1. teoremaning shartidan quyidagilarni ko'rsatish qiyin emas:

$$\left|R(x,y)\right|\le\gamma_3 x^{l_2-\frac{1}{2}}\left|x-y\right|^{-l_1-\beta},\ \gamma_3=const>0, \tag{2.1.17}$$

$$\left|\Phi(x)\right|\le\gamma_4 x^{l_2-l_1},\gamma_4=const>0. \tag{2.1.18}$$

Volterra integral tenglamalar nazariyasidan (2.1.14) integral tenglama yagona yechimga ega. [14]

$u(x,y)$ yechimi (2.1.5) formuladan aniqlanadi,bu yerda $\nu(x)$ (2.1.14)dan topiladi. Bundan 2.1. masala yagona yechimga ega.

2.1.2. Masala. Ω sohada quyidagi shartlarni qanoatlantiruvchi $u(x,y)$ funksiyasi topilsin.

4. $u(x,y)\in C\left(\overline{\Omega}\right)$;

5. $u(x,y)$ -(2.1.1) tenglamaning regulyar yechimi;

6. $u_y(x,0)=\nu(x),\ \ x\in J,$ (2.1.19)

$$D_{0x}^{l_1}(x^{2q})^{l_2}u[\theta(x)]=a_2(x)u(x,0)+b_2(x),\quad x\in J, \tag{2.1.20}$$

bu yerda $\nu(x)$, $a_2(x)$ va $b_2(x)$ berilgan funksiyalar; l_1 va l_2 -xaqiqiy sonlar.

2.1.2. Teorema. Agar quyidagi shartlar bajarilsa

4) $l_1<\beta,\ \ l_2>\max\left\{\dfrac{\alpha+\beta-2}{2},l_1\right\}$;

5) $a_2(x)\neq 0\ \ \forall x\in\bar{J}$, $a_2(x),b_2(x)\in C\left(\bar{J}\right)\cap C^2\left(J\right)$;

6) agar $\nu(x)\in C^2\left(J\right)$ va $\nu(x)$ funksiya J ning chetki nuqtalarida $(1-2\beta)\big/_{1-2\alpha}$ tartibli maxsuslikka ega bo'lsa, 2.2.masalaning yagona yechimi mavjud.

Teorema isboti 2.1. teorema isbotiga o'xshash bo'ladi.

Ta'kidlaymizki, 1.1. va 1.2. masalalar $l_1=1-\beta,\ \ l_2=0; l_1=\beta,\ \ l_2=2\beta-1$ va $l_1<1-\beta,\ \ l_2=0$ bo'lganda bir chiziqda buziluvchi giperbolik tipdagi tenglamalar uchun bir qator ishlarda o'rganilgan.

2.1.Ilova. 2.1 va 2.2 masalalar $a_i(x)\equiv 0$, $i=1,2$ bo'lganda Darbu masalasiga keladi va bu masala yagona yechimga ega.

2.2-§. Chegaraviy shartlarda umumlashgan kasr tartibli integro-differensial operator qatnashgan siljishli chegaraviy masalalar

Ushbu paragraf (2.2.1) tenglama uchun chegaraviy masalalar yechishga umumlashgan kasr tartibli integro-differensial operatorlar qo'llanilishiga bag'ishlangan.

$$-y^{m}U_{xx}-x^{n}U_{yy}=0\,, \tag{2.2.1}$$

Umumlashgan kasr tartibli integro-differensial operatorni quyidagi ko'rinishda olamiz [3] :

$$F_{ox}\begin{bmatrix} a & b \\ c & g(x)\end{bmatrix}\varphi(x)=$$

$$=\begin{cases} \dfrac{1}{\Gamma(-c)}\displaystyle\int_0^x\left[g(x)-g(t)\right]^{-c-1}\cdot F\left(a,b,-c;\dfrac{g(x)-g(t)}{g(x)}\right)\varphi(t)\,g'(t)\,dt, & c<0,\\ \varphi(x), & c=0,\\ \left[g(x)\right]^{b}\dfrac{d}{dg(x)}x^{-b}F_{ox}\begin{bmatrix} a+1 & b \\ c-1 & g(x)\end{bmatrix}\varphi(x), & 0<c<1,\end{cases}$$

(2.2.2)

bu yerda a,b va c-haqiqiqy sonlar; $g(x)-$uzluksiz hosilaga ega, monoton funksiya; $\varphi(x)\in L(AB)$; $\Gamma(\cdot)-$Eyler gamma funksiyasi [2]; $F(a,b,c;z)-$Gauss gipergeometrik funksiyasi [4].

2.2.1- masala. Ω sohada (2.2.1) tenglamaning regulyar yechimi va quyidagi shartlarni qanoatlantiruvchi $U(x,y)$ funksiya topilsin.

1. $U(x,y)\in C(\overline{\Omega})\cap C'(\Omega\cup AB)\cap C^2(\Omega)$;

$$\int_0^1\left|U_y\left(x^{1/q},0\right)\right|\left[x\cdot(1-x)\right]^{-\frac{m}{m+2}}dx<\infty;$$

2. $U(x,0)=\tau(x)$, $\quad (x,0)\in\overline{AB}$; (2.2.3)

$$x^{4q(\ldots)-b}F_{ox}\begin{bmatrix} a & b \\ c & (x^{2q})^{2}\end{bmatrix}U\left[\theta(x)\right]=c(x)U_y(x,0)+d(x), \qquad (x,0)\in AB$$

(2.2.4)

bu yerda a,b va c-haqiqiy sonlar; $c(x),d(x)$-berilgan funksiyalar; $\theta(x)$-(2.1.1) tenglamaning xarakteristikalari bilan kesishgan nuqtasi affiksi.

Quyidagi teorema o'rinli bo'ladi.

2.2.1. Teorema. Quyidagilar bajarilsin:

$$c+\beta<1, c-a>0,\ c-b\geq 0$$

$$C(x)\neq 0,\forall(x,0)\in\overline{AB};\ C(x),d(x)\in C'(\overline{AB});\ \tau(x)\in C'(\overline{AB})\cap C^2(AB).$$

U holda (2.2.1) tenglama yagona yechimga ega.

Isbot. (2.2.1) tenglama uchun $U(x,0)=\tau(x),\ \ (x,0)\in\overline{AB}, U_y(x,0)=\nu(x),\ \ (x,0)\in AB$ Koshi masalasining yechimi Ω sohada quyidagi formula bilan beriladi [5]:

$$
\begin{aligned}
u(x,y) = {} & \frac{\Gamma\left(\frac{1}{2}+\beta\right)}{\sqrt{\pi}\,\Gamma(\beta)} 2^{2\beta-1}\left(\frac{1}{q}x^q\right)^{-\alpha} \times \int_0^1 \left[\frac{1}{q}x^q + \frac{1}{p}(-y)^p(2z-1)\right] [z(1-z)]^{\beta-1} \times \\
& \times \tau\left\{\left[x^q + \frac{q}{p}(-y)^p(2z-1)\right]^{1/q}\right\} F(\alpha, 1-\alpha; \beta; \rho)\, dz - \\
& - \frac{\Gamma\left(\frac{3}{2}-\beta\right)}{\sqrt{\pi}\,\Gamma(1-\beta)} 2^{2\beta-1}\left(\frac{1}{q}x^q\right)^{-\alpha}\left[\frac{1}{p}(-y)^p\right]^{-2\beta} \times \\
& \times \int_0^1 \left[\frac{1}{q}x^q + \frac{1}{p}(-y)^p(2z-1)\right]^{\alpha} [z(1-z)]^{-\beta} \times \\
& \times \nu\left\{\left[x^q + \frac{q}{p}(-y)^p(2z-1)\right]^{1/q}\right\} F(\alpha, 1-\alpha; 1-\beta; \rho)\, dz,
\end{aligned}
$$

(2.2.5)

bu yerda

$$
\rho = \frac{q(-y)^{2p} z(1-z)}{p^2 x^q \left[\frac{1}{q}x^q + \frac{1}{p}(-y)^p\right]}.
$$

$2\alpha = n/(n+2)$, $2\beta = m/(m+2)$.

(2.2.5) formuladan foydalanib,

$$
\begin{aligned}
U[\theta(x)] = {} & \gamma_1 \cdot \left(x^{2q}\right)^{\frac{2-\alpha-3\beta}{q}} F_{ox}\begin{bmatrix} \frac{\beta-\alpha}{2} & \frac{4\beta-1}{2} \\ \beta & x^{2q} \end{bmatrix} \left(x^{2q}\right)^{\frac{\alpha+\beta-2}{2}} \tau(x) - \\
& - \lambda_2 \left(x^{2q}\right)^{\frac{\beta-\alpha}{2}} F_{ox}\begin{bmatrix} \frac{1-\alpha-\beta}{2} & \frac{\alpha-\beta}{2} \\ 1-\beta & x^{2q} \end{bmatrix} \left(x^{2q}\right)^{\frac{\alpha-p-1}{2}} \nu(x),
\end{aligned}
$$

(2.2.6)

ifodani topamiz.

bu yerda

$$\gamma_1=\frac{\Gamma(q\beta)}{\Gamma(\beta)},\qquad \gamma_2=\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\cdot\left(\frac{p}{q}\right)^{1-2\beta}\cdot 2^{\alpha+3\beta-2}, \tag{2.2.7}$$

(2.2.6) ni (2.2.4) chegaraviy shartga qo'yib, x ni $x^{1/2q}$ bilan, t ni $t^{1/2q}$ bilan almashtirsak, quyidagi tenglik hosil bo'ladi:

$$\gamma_1\cdot J_1(x)-\gamma_2 J_2(x)=\tilde{c}(x)\tilde{\nu}(x)+\tilde{d}(x),\qquad (x,\cdot)\in AB$$

bu yerda

$$J_1(x)=x^{-b}F_{ox}\begin{bmatrix} a & b\\ c & x\end{bmatrix}\left(\sqrt{x}\right)^{\frac{2-\alpha-3\beta}{2}}F_{ox}\begin{bmatrix}\frac{\beta-\alpha}{2} & \frac{\alpha+\beta-1}{2}\\ \beta & \sqrt{x}\end{bmatrix}\left(\sqrt{x}\right)^{\frac{\alpha-\beta-1}{2}}\tilde{\tau}(x) \tag{2.2.8}$$

$$J_2(x)=x^{-b}F_{ox}\begin{bmatrix} a & b\\ c & x\end{bmatrix}\left(\sqrt{x}\right)^{\frac{\beta-\alpha}{2}}F_{ox}\begin{bmatrix}\frac{1-\alpha-\beta}{2} & \frac{\alpha-\beta}{2}\\ 1-\beta & \sqrt{x}\end{bmatrix}\left(\sqrt{x}\right)^{\frac{\alpha-\beta-1}{2}}\tilde{\nu}(x) \tag{2.2.9}$$

$$\tilde{c}(x)=c\left(x^{1/2q}\right),\ \tilde{d}(x)=d\left(x^{1/2q}\right),\ \tilde{\tau}(x)=\tau\left(x^{1/2q}\right),\ \tilde{\nu}(x)=\nu\left(x^{1/2q}\right)$$

(2.2.8) va (2.2.9) tengliklarni hisoblash uchun Mellin almashtirishidan foydalanamiz :

$$f(x)\to\int_0^\infty x^{s-1}f(x)\,dx, \tag{2.2.10}$$

Ko'rsatish mumkinki,

$$x^{-b}F_{ox}\begin{bmatrix} a & b\\ c & x\end{bmatrix}\varphi(x)\to\Gamma\begin{bmatrix}1+c+b-s & 1-c-s\\ 1-a+b-s & 1-s\end{bmatrix}\varphi^{\alpha}(s-c-b), \tag{2.2.11}$$

$$\operatorname{Re}s<1+c+b,\ 1-a.$$

va

$$\left(\sqrt{x}\right)^{-b} F_{ox}\begin{bmatrix} a & b \\ c & \sqrt{x} \end{bmatrix}\varphi\left(x\right) \to \Gamma\begin{bmatrix} 1+c+b-2s & 1-c-2s \\ 1-a+b-2s & 1-2s \end{bmatrix}\varphi^{\alpha}\left(ks-c-b\right), \qquad (2.2.12)$$

$$\operatorname{Re} s < 1+c+b,\ 1-a.$$

(2.2.11) va (2.2.12) larni hisobga olib, (2.2.9) dan quyidagi tenglik kelib chiqadi:

$$J_2\left(x\right) = 2^{\beta}\Gamma\begin{bmatrix} 1+b-c-b, & -1-s, & \frac{3\beta-\alpha}{4}+b-c-s, & \frac{2+3\beta-\alpha}{4}+b-c-s \\ 1-c+b-s, & \frac{1+\beta}{4}+b-c-s, & \frac{2+\beta}{4}+1-c-s & \end{bmatrix}\times$$

$$\times\begin{bmatrix} \frac{2+\alpha+\beta}{4}+b-c-s, & \frac{2+\alpha+\beta}{4}+b-c-s \\ \frac{1}{2}+b-c-s, & 1+b-c-s \end{bmatrix}\varphi\left(\frac{2-\alpha-\beta}{2}+2c+2b+2s\right),$$

(2.2.13)

Quyidagi formulani inobatga olamiz [2] .

$$C_{p,q}^{m,n}\left(y\middle|\begin{matrix} a_1, a_2, ..., a_n \\ b_1,\ b_2, ..., b_n \end{matrix}\right) \to \Gamma\begin{bmatrix} s+b_1, s+b_{n-1}, s+b_m, 1-c_n-s, ..., 1-a_n-s \\ s+a_{n+1}, s+n_{m-c_2}, ..., s+a_{p1}, 1-b, ..., 1-b_q-s \end{bmatrix}, \qquad (2.2.14)$$

$$\int_0^{\infty} k_1\left(\frac{x}{t}\right)k_2\left(t\right)\frac{dt}{t} \to k_1^{*}\left(t\right)\left(s\right) k_2^{*}\left(t\right)\left(s\right), \qquad (2.2.15)$$

Quyidagilarga ega bo'lamiz:

$$J_2\left(x\right) = 2^{\beta}\cdot\frac{x^{c-b}}{y}\times$$

$$\times\int_0^x C_{6,6}^{6,0}\left(\frac{y}{x}\middle|\begin{matrix} c-b,\ a, & \frac{4-\beta-\alpha}{4}+c-b, & \frac{2-\beta-\alpha}{4}+c-b, & \frac{3-\beta-\alpha}{4}+c-b, & \frac{1-\beta-\alpha}{4}+c-b \\ a-b,\ 0, & \frac{3-2\alpha}{4}+c-b, & \frac{1-2\alpha}{4}+c-b, & \frac{1}{2}+c-b,\ c-b & \end{matrix}\right)\times$$

$$\times y^{\frac{1-2\beta}{2}} \tilde{\nu}\left(y\right) dy \tag{2.2.16}$$

Keyin [4] formulani qo'llaymiz:

$$C_{p,q}^{m,n}\left(z\left|\begin{matrix}a_1,a_2,...,a_n\\ b_1,b_2,...,b_n\end{matrix}\right.\right)=C_{p,q}^{m,n}\left(\frac{1}{z}\left|\begin{matrix}1-b_1,1-b_2,...,1-b_n\\ 1-a_1,1-a_2,...,1-a_n\end{matrix}\right.\right) \tag{2.2.17}$$

va

$$z^{\alpha}C_{p,q}^{m,n}\left(z\left|\begin{matrix}a_1,a_2,...,a_n\\ b_1,b_2,...,b_n\end{matrix}\right.\right)=C_{p,q}^{m,n}\left(z\left|\begin{matrix}a_1+\alpha,a_2+\alpha,...,a_n+\alpha\\ b_1+\alpha,b_2+\alpha,...,b_n+\alpha\end{matrix}\right.\right) \tag{2.2.18}$$

(2.2.16) dan

$$J_2\left(x\right)=2^{\beta}\cdot x^{c-b}\times$$

$$\times\int_0^x C_{5,5}^{5,0}\left(\frac{y}{x}\left|\begin{matrix}1+c-a_1,\ 1+c-b,\ \frac{1+2\alpha}{4},\ \frac{2+2\alpha}{4},\ \frac{1}{2}\\ 1+c-a-b,\ \frac{\alpha+\beta}{4},\ \frac{2+\alpha+\beta}{4},\ \frac{1+\alpha+\beta}{4},\frac{3+\alpha+\beta}{4}\end{matrix}\right.\right)\times$$

$$\times y^{-\frac{1-2\beta}{4}} \tilde{\nu}\left(y\right) dy$$

(2.2.19)

Bundan kelib chiqdiki,

$$J_1\left(x\right)=2^{1-\beta}x^{c-b}\int_0^x C_{6,6}^{6,0}\left(\frac{y}{x}\left|\begin{matrix}1+c+a,\ 2+c+b,\ \frac{\alpha+\beta}{2},\ \frac{1+\alpha+\beta}{2},\ \frac{1+2\beta}{4},\ \frac{3+2\beta}{4}\\ 1,\ 1-c+b-a,\ \frac{\alpha+\beta}{4},\ \frac{\alpha+\beta+2}{4},\ \frac{\alpha+\beta+1}{2},\ \frac{3+\alpha+\beta}{4}\end{matrix}\right.\right)\times$$

$$\times\tau_1(x)dx$$

(2.2.20)

(2.2.19) va (2.2.20) larni (2.2.7) ga qo'ysak, II tur Volterra integral tenglamasiga ega bo'lamiz.

$$\tilde{\nu}\left(x\right)+\int_0^x R(x,y)\tilde{\nu}\left(y\right)dy=\Phi(x), \qquad (x,0)\in GJ \tag{2.2.21}$$

bu yerda

$$R(x,y)=\frac{\gamma_2\cdot 2^{\beta}}{\bar{c}(x)}x^{c-b}\times$$

$$\times y^{\frac{1-2\beta}{4}}C_{5,5}^{5,0}\left(\frac{y}{x}\left|\begin{matrix}1+c-a_1,1+c-b,\frac{1+2\alpha}{4},\frac{2+2\alpha}{4},\frac{1}{2}\\ 1+c-a-b,\frac{\alpha+\beta}{4},\frac{2+\alpha+\beta}{4},\frac{1+\alpha+\beta}{4},\frac{3+\alpha+\beta}{4}\end{matrix}\right.\right),$$

(2.2.22)

$$\Phi(x)=\frac{\gamma_1}{\tau(x)}x^{c-b}\int_0^x C_{6,6}^{6,0}\left(\frac{y}{x}\left|\begin{matrix}1+c-a_1,1+c-b,\frac{1+2\alpha}{4},\frac{2+2\alpha}{4},\frac{1}{2}\\ 1+c-a-b,\frac{\alpha+\beta}{4},\frac{2+\alpha+\beta}{4},\frac{1+\alpha+\beta}{4},\frac{3+\alpha+\beta}{4}\end{matrix}\right.\right)\tau_1(x)dx$$

(2.2.23)

Teorema shartlarini e'tiborga olib, quyidagi ifodani ko'rsatish qiyin emas:

$$\left|R(x,y)\right|\le\gamma_3 x^{-b+\frac{1+2\beta}{4}-\frac{1}{2}}(x-y)^{c-\beta}, \quad \gamma_3=const>0 \tag{2.2.24}$$

$$\left|\Phi(x,y)\right|\le\gamma_4 x^{c-b}, \quad \gamma_4=const>0 \tag{2.2.25}$$

Volterra integral tenglamasi nazariyasidan [6] ko'rinadiki, (2.2.21) tenglama bir qiymatli yechimga ega. Bundan qo'yilgan masalaning mavjudligi ham kelib chiqadi. Uning yechimi (2.2.25) formuladan, $\nu\left(x\right)$ esa (2.2.21) formuladan topiladi.

Teorema isbotlandi.

2.2.2.Masala. Ω sohada (2.2.1) tenglamaning regulyar yechimi va quyidagi shartlarni qanoatlantiruvchi $U\left(x,y\right)$ funksiya topilsin.

1. $U(x,y)\in C(\overline{\Omega})\cap C'(\Omega\cup AB)\cap C^2(\Omega)$;

$$\int_0^2\left|U_y\left(x^{1/q},0\right)\right|\left[x\cdot(1-x)\right]^{-\frac{m}{m+2}}dx<\infty;$$

2. $U(x,y)$ quyidagi shartlarni qanoatlantiradi:

$$U_y(x,0)=\nu(x),\qquad (x,0)\in AB, \tag{2.2.3}$$

$$x^{(2q)-a}F_{ox}\begin{bmatrix} a & b \\ c & (x^{2q})^2 \end{bmatrix}U\left[\theta(x)\right]=c(x)U_y(x,0)+g(x)U(x,0),\quad (x,c)\in AB. \tag{2.2.4}$$

II BOB YUZASIDAN XULOSA

Ikkinchi bobda ikkita buzilish chizig'iga ega bo'lgan giperbolik tipdagi tenglamalar uchun ikkita chegaraviy masala o'rganilgan. Bundan tashqari sohaning chegarasida buziladigan giperbolik tipdagi tenglamalar uchun chegaraviy masalalar yechimining mavjud va yagonaligi isbotlangan.

III BOB.SOHANING ICHIDA BUZILADIGAN GIPERBOLIK TIPDAGI TENGLAMALAR UCHUN CHEGARAVIY MASALALAR

3.1-§ Chegaraviy shartlari xarakterisrikalarda berilgan chegaraviy masalalar.

$$|-y|^m U_{xx} - x^n U_{yy} = 0, \qquad m,n = const > 0. \tag{3.1.1}$$

tenglamani Ω sohada ko'ramiz.

Bu yerda Ω - (3.1.1) tenglamaning $y > 0$ dagi

$$OC_1 : \frac{1}{q} x^q - \frac{1}{p} y^p = 0, \;\; AC_1 : \frac{1}{q} x^q + \frac{1}{p} y^p = 1$$

xarakteristikalar, $y < 0$ da

$$OC_2 : \frac{1}{q} x^q - \frac{1}{p} (-y)^p = 0, \;\; AC_2 : \frac{1}{q} x^q + \frac{1}{p} (-y)^p = 1$$

xarakteristikalar bilan chegaralangan bir bog'lamli soha.

bu yerda $2q = n + 2, \; 2p = m + 2, \; m > n.$

Quyidagi belgilashni kiritamiz:

$$\Omega^+ = \Omega \cap (y > 0), \; \Omega^- = \Omega \cap (y < 0),$$

$$\theta_1(x) = \left[\frac{x^q}{2}\right]^{1/q} + i\left[\frac{p}{q} \cdot \frac{x^q}{2}\right]^{1/p}, \;\; \theta_2(x) = \left[\frac{x^q}{2}\right]^{1/q} - i\left[\frac{p}{q} \cdot \frac{x^q}{2}\right]^{1/p}. \tag{3.1.2}$$

bu yerda $\theta_j(x)$, $j = 1,2$ - $x,0 \in J$ nuqtadan chiquvchi, xarakterisrikalari bilan OC_j $j = 1,2$ xarakteristikalarning kesishgan nuqtalari koordinatalari.

Quyida (3.1.1) tenglamaning regulyar yechimi deganda $C\;\bar{\Omega} \cap C^1\;\Omega \cap C^2\;\Omega^+ \cup \Omega^-$ sinfga tegishli, bu tenglamani $\Omega^+ \cup \Omega^-$ sohalarda qanoatlantiruvchi va $x = 0$, $x = q^{1/q}$ nuqtalarda $U_y\;x,0$ funksiya $(1-2\beta)/_{1-2\alpha}$ dan yuqori maxsuslikka ega bo'lmagan $U\;x,y$ funksiyani tushuniladi.

3.1.1.Masala. (3.1.1) tenglamaning Ω sohada regulyar bo'lib quyidagi chegaraviy shartni qanoatlantiruvchi yechimini toping.

$$D_{ox}^{l_1} 2q(x^{2q})^{l_2} u\left[\theta_j(x)\right] = a_j(x)u_y(x,0) + b_j(x), \quad x \in J, \qquad (3.1.3\,j,\ j=1,2)$$

bu yerda l_1 va l_2-haqiqiy sonlar , $a_j(x), b_j(x), \quad j=1,2$ -berilgan funksiyalar.

Shuni ta'kidlash lozimki, $l_1 = 1-\beta,\, l_2 = 0$ va $l_1 = \beta,\, l_2 = 2\beta - 1$ hollarda soha ichida buziladigan giperbolik turdagi tenglama uchun chegaraviy masala bir qator ishlarda o'rganilgan.

3.1.1.Teorema. Quyidagi shartlar bajarilsin:

1) $l_1 < 1-\beta,\, l_2 > \max\left\{ \alpha+\beta-2 \big/ 2,\, l_1 + \frac{2\beta-1}{2} \right\};$

3) $a_2(x) - a_1(x) \neq 0,\ \forall x \in \bar{J},\ a_j(x) \in C^{K_8}(\bar{J}) \cap C^{K_8+2}(J)$,

va $b_j(x) \in C^{K_8}(J)$ va $b_j(x)$ funksiya J intervalning chetki nuqtalarida $(1-2\beta) \big/ 1-2\alpha$ dan yuqori bo'lmagan tartibda cheksizlikka aylanishi mumkin.

U holda 3.1.1.masalaning yagona yechimi mavjud bo'ladi.

Isbot. (3.1.1) tenglama uchun $u(x,0) = \tau(x)$, $u_y(x,0) = \nu(x)$ Koshi masalasining yechimi Ω^- va Ω^+ sohalarda quyidagi formula bilan beriladi:

$$u(x,y) = \frac{\Gamma(\frac{1}{2}+\beta)}{\sqrt{\pi}\Gamma(\beta)} 2^{2\beta-1} (\frac{1}{q} x^q)^{-\alpha} \times$$

$$\times \int_0^1 \left[\frac{1}{q} x^q + \frac{1}{p} y^p (2z-1) \right] z(1-z)^{\beta-1} \times \tau \left\{ \left[x^q + \frac{p}{q}(-y)^p (2z-1) \right]^{1/q} \right\} \times$$

$$\times F(\alpha, 1-\alpha, \beta; \rho) dz - \frac{\Gamma(\frac{3}{2}-\beta)}{\sqrt{\pi}\Gamma(1-\beta)} \times$$

$$\times(2\beta)^{1-2\beta}(\frac{1}{q}x^q)^{-\alpha}(\frac{1}{p}y^p)^{-2\beta}\int_0^1\left[\frac{1}{q}x^q+\frac{1}{p}y^p(2z-1)\right]^{\alpha}z(1-z)^{-\beta}\times$$

$$\times\nu\left\{\left[x^q+\frac{p}{q}(-y)^p(2z-1)\right]^{1/q}\right\}F(\alpha,1-\alpha,1-\beta;\rho)dz$$

(3.1.4)

bu yerda

$$\rho=\frac{q\,y^{2p}\,z(1-z)}{p^2\,x^q\left[\frac{1}{q}x^q+\frac{1}{p}y^p\right]}$$

(3.1.2) ga ko'ra (2.1.5), (3.1.4) lardan quyidagilarga ega bo'lamiz:

$$u\left[\theta_j(x)\right]=u\left[\mathrm{Re}\,\theta_j(x),\mathrm{Im}\,\theta_j(x)\right]=\frac{\Gamma(2\beta)}{\Gamma(\beta)}2^{\alpha}\times$$

$$\times(x^q)^{1-\alpha-2\beta}\int_0^x(x^q)^{\alpha+\beta-1}\tilde{\tau}(t)(x^q-t^q)^{\beta-1}\times$$

$$\times F(\alpha,1-\alpha,\beta;\frac{x^q-t^q}{2x^q})qt^{q-1}dt+(-1)^{j+1}\times$$

$$\times\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}\left(\frac{p}{q}\right)^{1-2\beta}(x^q)^{-\alpha}\times\int_0^x(t^q)^{\alpha-\beta}(x^q-t^q)^{-\beta}\times$$

$$\times F(\alpha,1-\alpha,1-\beta;\frac{x^q-t^q}{2x^q})q\cdot t^{q-1}dt$$

(3.1.5 j, j=1,2)

(3.1.5 j, j=1,2) ifodani (3.1.3 j, j=1,2) ga qo'yib quyidagiga ega bo'lamiz:

$$\gamma_1J_1(x)+\gamma_2J_2(x)=\tilde{a}_1(x)\tilde{\nu}(x)+b_1(x),\quad x\in J \tag{3.1.6}$$

$$\gamma_1J_1(x)-\gamma_2J_2(x)=\tilde{a}_1(x)\tilde{\nu}(x)+\tilde{b}_2(x),\quad x\in J \tag{3.1.7}$$

bu yerda $\tilde{a}_1(x)=a_j\left(x^{1/2q}\right)$, $\tilde{b}_j(x)=b_j\left(x^{1/2q}\right)$, $(j=1,2)$, $J_1(x)$ va $J_2(x)$ (2.1.12), (2.1.13) formulalardan aniqlanadi.

Agar 3.1.1.teorema 2-shartidan foydalansak, va (2.1.12), (2.1.13) e'tiborga olib, (3.1.6), (3.1.7) lardan quyidagi ifodani hosil qilamiz:

$$\tilde{\nu}(x)-\int_0^x R(x,y)\tilde{\nu}(y)dy=F(x), \tag{3.1.8}$$

bu yerda

$$R(x,y)=\frac{2\gamma_2}{\tilde{a}_1(x)-\tilde{a}_2(x)}x^{l_2-l_1-\frac{1+\beta}{2}}y^{\frac{\alpha-\beta-1}{2}}\times$$

$$\times G_{33}^{30}\left(\frac{y}{x}\left|\begin{matrix}\frac{2-\alpha-\beta}{2}, & \frac{1+\alpha-\beta}{2}, & l_2-l_1+\frac{2-\alpha-\beta}{2}\\ \frac{1}{2}, & 0, & l_2+\frac{2-\alpha-\beta}{2}\end{matrix}\right.\right)$$

$$F(x)=\frac{\tilde{b}_2(x)-\tilde{b}_1(x)}{\tilde{a}_1(x)-\tilde{a}_2(x)}$$

va

$$\frac{d^{K_8}}{dx^{K_8}}x^{l_2-l_1-\frac{1+\beta}{2}}\int_0^x y^{\frac{\alpha-\beta-1}{2}}\tilde{\tau}(y)\times$$

$$\times G_{33}^{30}\left(\frac{y}{x}\left|\begin{matrix}\frac{1-\alpha+\beta}{2}, & \frac{\alpha+\beta}{2}, & l_2-l_1+\frac{2-\alpha-\beta}{2}+K_8\\ \frac{1}{2}, & 0, & l_2+\frac{2-\alpha-\beta}{2}\end{matrix}\right.\right)dt= \tag{3.1.9}$$

$$=\frac{1}{2\gamma_1}\quad \tilde{a}_1(x)+\tilde{a}_2(x)\ \ \tilde{\nu}(x)+\tilde{b}_1(x)+\tilde{b}_2(x)$$

Shunday qilib,

$$\left|R(x,y)\right|\le c_2x^{l_2-\frac{1}{2}}\ \ x-y\ ^{-l_1-\beta},\ c_2=const>0, \tag{3.1.10}$$

$$\left|F(x)\right|\le c_3x^{l_2-l_1},l_3=const>0 \tag{3.1.11}$$

U holda (2.1.12) tenglama kuchsiz maxsuslikka ega bo'lgan II tur Volterra integral tenglama, bu tenglamaning bir qiymatliligi integral tenglamalar nazariyasidan kelib chiqadi.

(3.1.8) dan

$$\tilde{\nu}(x)=F(x)+\int_0^x R(x,y)F(y)dy, \tag{3.1.12}$$

kelib chiqadi. Bu yerda $\Re(x,y)$- $R(x,y)$ ning rezolventasi.

Keyin (3.1.9) operatorni ikki tomoniga

$$\frac{d^{K_{10}}}{dx^{K_{10}}} x^{l_1-l_2+\frac{1-\beta-3}{2}+K_{10}} \times$$

$$\times G_{33}^{30}\left(\frac{y}{x} \left| \begin{matrix} l_1-l_2+\dfrac{\alpha-\beta}{2}, & l_1, & l_1-l_2+\dfrac{\alpha-\beta-1}{2}+K_{10} \\ l_1-l_2 \quad , & l_1-l_2+\dfrac{2\alpha-1}{2} \quad , & 0 \end{matrix} \right. \right) ...dy$$

operatorni qo'llaymiz.

bu yerda $K_{10}=\max\ 0, K_9+1\ ,\ K_9$ - $\beta-l_1$,haqiqiy sonning butun qismi.

U holda quyidagiga ega bo'lamiz:

$$\tilde{\tau}\ x\ =\frac{1}{2\gamma_1}\frac{d^{K_{10}}}{dx^{K_{10}}} x^{l_1-l_2+\frac{1+\beta-3}{2}+K_{10}} \times$$

$$\times G_{33}^{30}\left(\frac{y}{x} \left| \begin{matrix} l_1-l_2+\dfrac{\alpha-\beta}{2}, & l_1, & l_1-l_2+\dfrac{\alpha-\beta-1}{2}+K_{10} \\ l_1-l_2 \quad , & l_1-l_2+\dfrac{2\alpha-1}{2} \quad , & 0 \end{matrix} \right. \right) \times \tag{3.1.13}$$

$$\times\ \ \tilde{a}_1(y)+\tilde{a}_2(y)\ \tilde{\nu}(y)+\tilde{b}_1(y)+\tilde{b}_2(y)$$

(3.1.12) ifodani (3.1.13) ga qo'yib $\tilde{\tau}\ x$ ni topamiz.

3.1.1.teoremaning 1),2) shartlaridan foydalanib,

$\tilde{\tau}\ x\ \in C\ \bar{J}\ \cap C^2\ J\ ,\ \tilde{\nu}(x)\in C^2\ J$ ko'rsatish mumkin.

$\tilde{\nu}(x)$ funksiya $x\to 0$ va $x\to q^{1/q}$ ga intilganda $1-2\beta/2$ dan yuqori bo'lmagan tartibda cheksizlikka aylanishi mumkin.

Agar $a_1(x)\equiv a_2(x)\ ,\forall x\in\bar{J}$ bo'lsa,u holda $\tilde{\nu}(x)$ funksiya

$$\tilde{\nu}(x)=\frac{1}{2\gamma_2}\frac{d^{K_{11}+1}}{dx^{K_{11}+1}} x^{l_1-l_2+\frac{1+\beta-2}{2}+K_{11}} \int_0^x \left[\tilde{b}_1(t)-\tilde{b}_2(t)\right]\times$$

$$\times G_{33}^{30}\left(\frac{t}{x} \left| \begin{matrix} l_1-l_2+\dfrac{\alpha+\beta-1}{2}, & l_1, & l_1-l_2+\dfrac{\alpha+\beta-2}{2}+K_{11}+1 \\ l_1-l_2 \quad , & l_1-l_2+\dfrac{2\alpha-1}{2} \quad , & 0 \end{matrix} \right. \right) dt \tag{3.1.14}$$

formula bilan aniqlanadi.

bu yerda K_{11}- $l_1+\beta-1$ sonning haqiqiy butun qismi.

Shunday qilib, 3.1.1. masala bir qiymatli yechimga ega. $u(x,y)$ (2.1.5),(3.1.4) formula bilan beriladi, bu yerda $\tau(x)$ va $\nu(x)$ mos ravishda (3.1.13) va (3.1.12) tengliklardan topiladi.

3.1.2.masala.(3.1.1) tenglamaning Ω sohada regulyar bo'lib quyidagi chegaraviy shartni qanoatlantiruvchi yechimi topilsin:

$$D^{l_1}_{0x^{2q}}(x^{2q})^{l_2}u[\theta(x)]=c_j(x)u(x,0)+d_j(x),\quad x\in J, \qquad \text{(3.1.15 j, j=1,2)}$$

bu yerda l_1 va l_2-haqiqiy sonlar ; $c_j(x)$ va $d_j(x)$, $(j=1,2)$-berilgan funksiya.

Quyidagi teorema o'rinli bo'ladi.

3.1.2.Teorema. Quyidagi shartlar o'rinli bo'lsin:

3. $l_1<1-\beta,\ \ l_2>\max\ {\alpha+\beta-4}/{2},\ l_1+{2\beta-1}/{2}$;

4. $c_1(x)+c_2(x)\neq 0,\ \forall x\in\overline{J}$, $\quad c_j(x)\in C^1(\overline{J})$;

$d_j(x)\in C^2\ \ J$ va $d_j(x),\ (j=1,2)$ funksiya ${(1-2\beta)}/{1-2\alpha}$ dan yuqori bo'lmagan tartib bilan cheksizlikka teng bo'ladi.

U holda 3.1.2.masala bir qiymatli yechimga ega bo'ladi.

Agar $c_1(x)=-c_2(x),\ \ \forall x\in\overline{J}$ bo'lsa, u holda $\tilde{\nu}(x)$ oshkor ko'rinishda topiladi.

Bu teoremaning isboti 3.1.1. teoremaning isbotiga o'xshash bo'ladi.

Izox. 3.1.1 va 3.1.2 masalalar $a_j(x)\equiv 0$ va $c_j(x)\equiv 0,\ \ (j=1,2)$ bo'lgan hollarda Gursa masalasiga keltiriladi.

3.2-§. Chegaraviy shartida umumlashgan kasr tartibli integro-differensial operator qatnashgan ba'zi siljishli chegaraviy masalalar

Ushbu paragraf (3.2.1) tenglama uchun chegaraviy masalalar yechishga umumlashgan kasr tartibli integro-differensial operatorlar qo'llanilishiga bag'ishlangan.

3.2.1.Masala. (3.2.1) tenglamaning Ω sohada regulyar bo'lib quyidagi chegaraviy shartni qanoatlantiruvchi yechimini toping.

$$x^{4q}\ ^{-b}F_{ox}\begin{bmatrix} a & b \\ c & x^{2q} \end{bmatrix}^{2} U\left[\theta_j\ x\ \right]=c_j\ x\ U_y\ \ x,0\ +d_j\ \ x\ ,\qquad x,0\ \in J$$

bu yerda a,b va c – haqiqiy sonlar; $c_j\ \ x\ , d_j\ \ x\ ,\ (j=1,2)$ berilgan funksiyalar.

3.2.1.Teorema. Quyidagi shartlar bajarilsin:

1. $-1<-c-\beta<0$;

2. $a_1(x)-a_2(x)\neq 0,\ \forall x\in \overline{J},\ a_j(x)\in C(\overline{J})\cap C^2(J), b_j(x)C(J)$

va $b_j(x)\in C^{K_8}\ J$ va $b_j(x)$ funksiya J interval chetki nuqtalarda $(1-2\beta)\big/1-2\alpha$ dan yuqori bo'lmagan tartibda cheksizlikka aylanishi mumkin.

Isbot. (3.2.1) tenglama uchun $u(x,0)=\tau(x)$, $u_y(x,0)=\nu(x)$ Koshi masalasining yechimi Ω^- va Ω^+ sohalarda quyidagi formula bilan beriladi: [6]

$$U(x,y)=\frac{\Gamma(\frac{1}{2}+\beta)}{\sqrt{\pi}\Gamma(\beta)}2^{2\beta-1}(\frac{1}{q}x^q)^{-\alpha}\times\int_0^1\left[\frac{1}{q}x^q+\frac{1}{p}y^p(2z-1)\right]\ z(1-z)\ ^{\beta-1}\times$$

$$\times\tau\left\{\left[x^q+\frac{p}{q}(-y)^p(2z-1)\right]^{1/q}\right\}\times F(\alpha,1-\alpha,\beta;\rho)dz\pm\frac{\Gamma(\frac{3}{2}-\beta)}{\sqrt{\pi}\Gamma(1-\beta)}\times$$

$$\times(2\beta)^{1-2\beta}(\frac{1}{q}x^q)^{-\alpha}(\frac{1}{p}y^p)^{-2\beta}\int_0^1\left[\frac{1}{q}x^q+\frac{1}{p}y^p(2z-1)\right]^{\alpha}\ z(1-z)\ ^{-\beta}\times$$

$$\times\nu\left\{\left[x^q+\frac{p}{q}(-y)^p(2z-1)\right]^{1/q}\right\}F(\alpha,1-\alpha,\beta;\rho)dz,\tag{3.2.4}$$

bu yerda

$$\rho=\frac{qy^{2p}z(1-z)}{p^2x^q\left[\frac{1}{q}x^q+\frac{1}{p}y^p\right]}$$

$$U\left[\theta_j(x)\right]=U\left[\operatorname{Re}\theta_j(x),\operatorname{Im}\theta_j(x)\right]=$$

$$=\frac{\Gamma(2\beta)}{\Gamma(\beta)}2^{\alpha}\times(x^q)^{1-\alpha-2\beta}\int_0^x(x^q)^{\alpha-\beta-1}\,\tilde{\tau}(t)(x^q-t^q)^{\beta-1}\times$$

$$\times F(\alpha,1-\alpha,\beta;\frac{x^q-t^q}{2x^q})q\cdot t^{q-1}dt+(-1)^{j+1}\times\frac{\Gamma(2-2\beta)}{\Gamma(1-\beta)}(\frac{p}{q})^{1-2\beta}(x^q)^{-\alpha}$$

$$\times\int_0^x(x^q)^{\alpha-\beta}\,(x^q-t^q)^{-\beta}\times F(\alpha,1-\alpha,1-\beta;\frac{x^q-t^q}{2x^q})q\cdot t^{q-1}dt$$

(3.2.5)

$$\gamma_1 J_1(x)+\gamma_2 J_2(x)=\tilde{a}_1(x)\cdot\tilde{\nu}(x)+\tilde{b}_1(x),\ x\in J, \qquad (3.2.6)$$

va

$$\gamma_1 J_1(x)-\gamma_2 J_2(x)=\tilde{a}_2(x)\cdot\tilde{\nu}(x)+\tilde{b}_2(x),\ x\in J, \qquad (3.2.7)$$

bu yerda $\tilde{a}_j(x)=a_j(x^{1/2q}),\ \tilde{b}_j(x)=b_j(x^{1/2q}),\ (j=1,2),$

$$J_1\ x\ =x^{-b}F_{ox}\begin{bmatrix}a & b\\ c & x\end{bmatrix}\ \sqrt{x}\ ^{\frac{2-\alpha-3\beta}{2}}F_{ox}\begin{bmatrix}\frac{\beta-\alpha}{2} & \frac{\alpha+\beta-1}{2}\\ \beta & \sqrt{x}\end{bmatrix}\ \sqrt{x}\ ^{\frac{\alpha-\beta-1}{2}}\ \tilde{\tau}\ \ x \qquad (3.2.8)$$

$$J_2\ x\ =x^{-b}F_{ox}\begin{bmatrix}a & b\\ c & x\end{bmatrix}\ \sqrt{x}\ ^{\frac{\beta-\alpha}{2}}F_{ox}\begin{bmatrix}\frac{1-\alpha-\beta}{2} & \frac{\alpha-\beta}{2}\\ 1-\beta & \sqrt{x}\end{bmatrix}\ \sqrt{x}\ ^{\frac{\alpha-\beta-1}{2}}\ \tilde{\nu}\ \ x \qquad (3.2.9)$$

(3.2.8) va (3.2.9) ifodalarni hisoblash uchun Mellin almashtirishidan foydalanamiz [5].

$$f\ \ x\ \to\int_0^{\infty}x^{s-1}f\ \ x\ \ dx \qquad (3.2.10)$$

Ko'rsatish qiyin emaski,

$$x^{-b}F_{ox}\begin{bmatrix}a & b\\ c & x\end{bmatrix}\varphi\ \ x\ \to\Gamma\begin{bmatrix}1+c+b-s & 1-c-s\\ 1-a+b-s & 1-s\end{bmatrix}\varphi\ \ s-c-b\ \ , \qquad (3.2.11)$$

$$\operatorname{Re}s<\operatorname{Re}\ 1+c+b,\ 1-a$$

va

$$\left(\sqrt{x}\right)^{-b} F_{ox}\begin{bmatrix} a & b \\ c & \sqrt{x} \end{bmatrix}\varphi(x) \to \Gamma\begin{bmatrix} 1+c+b-2s & 1-c-2s \\ 1-a+b-2s & 1-2s \end{bmatrix}\varphi(2s-c-b), \quad (3.2.12)$$

$$\mathrm{Re}\, s < \mathrm{Re}(1+c+b,\ 1-a)$$

(3.2.11),(3.2.12) larni hisobga olib, (3.2.9) dan

$$J_2(x) = 2^{\beta}\Gamma\begin{bmatrix} 1+b-c-s, & -1-s, & \frac{3\beta-\alpha}{4}+b-c-s, & \frac{2+3\beta-\alpha}{4}+b-c-s \\ 1-c+b-s, & \frac{1+\beta}{4}+b-c-s, & \frac{2+\beta}{4}+1-c-s & \end{bmatrix}\times$$

$$\times\begin{bmatrix} \frac{2+\alpha+\beta}{4}+b-c-s, & \frac{3+\alpha+\beta}{4}+b-c-s \\ \frac{1}{2}+b-c-s, & 1+b-c-s \end{bmatrix}\times$$

$$\times\varphi\left(\frac{2-\alpha-\beta}{2}+2c+2b+2s\right)$$

(3.2.13)

ifodani topamiz.

Bu yerda formula teng kuchli [4].

$$C_{p,q}^{m,n}\left(y\middle|\begin{matrix} a_1, a_2, ..., a_n \\ b_1, b_2, ..., b_n \end{matrix}\right) \to \Gamma\begin{bmatrix} s+b_1, s+b_{n-1}, s+b_m, 1-a_1-s, ..., 1-a_n-s \\ s+a_{n+1}, s+n_{m-c_2}, ..., s+a_{p1}, 1-b, ..., 1-b_q-s \end{bmatrix}$$

(3.2.14)

$$\int_0^{\infty} k_1\left(\frac{x}{t}\right)k_2(t)\frac{dt}{t} \to k_1^{*}(t)(s)\, k_2^{*}(t)(s) \quad (3.2.15)$$

quyidagilarga ega bo'lamiz:

$$J_2(x) = 2^{\beta}\cdot\frac{x^{c-b}}{y}\times$$

$$\times\int_0^x C_{6,6}^{6,0}\left(\frac{y}{x}\middle|\begin{matrix} a, & \frac{4-\beta-\alpha}{4}+c-b, & \frac{2-\beta-\alpha}{4}+c-b, & \frac{3-\beta-\alpha}{4}+c-b, & \frac{1-\beta-\alpha}{4}+c-b \\ a-b, & 0, \quad \frac{3-2\alpha}{4}+c-b, & \frac{1-2\alpha}{4}+c-b, & \frac{1}{2}+c-b, & \end{matrix}\right)\times$$

$$\times y^{\frac{1-2\beta}{2}}\tilde{\nu}(y)\,dy \quad (3.2.16)$$

Keyin quyidagi formuladan foydalanib [6].

$$C_{p,q}^{m,n}\left(z \left| \begin{matrix} a_1, a_2, ..., a_n \\ b_1, b_2, ..., b_n \end{matrix} \right. \right) = C_{p,q}^{m,n}\left(\frac{1}{z} \left| \begin{matrix} 1-b_1, 1-b_2, ..., 1-b_n \\ 1-a_1, 1-a_2, ..., 1-a_n \end{matrix} \right. \right) \tag{3.2.17}$$

va

$$z^{\alpha} C_{p,q}^{m,n}\left(z \left| \begin{matrix} a_1, a_2, ..., a_n \\ b_1, b_2, ..., b_n \end{matrix} \right. \right) = C_{p,q}^{m,n}\left(z \left| \begin{matrix} a_1+\alpha, a_2+\alpha, ..., a_n+\alpha \\ b_1+\alpha, b_2+\alpha, ..., b_n+\alpha \end{matrix} \right. \right) \tag{3.2.18}$$

(3.2.16) dan

$$J_2 \;\; x \; = 2^{\beta} \cdot x^{c-b} \times$$

$$\times \int_0^x C_{5,5}^{5,0}\left(\frac{y}{x} \left| \begin{matrix} 1-a+c,\ 1-b+c,\ \dfrac{1+2\alpha}{4},\ \dfrac{3+2\alpha}{4},\ \dfrac{1}{2} \\ 1-a+b+c,\ \dfrac{\beta+\alpha}{4},\ \dfrac{2+\beta+\alpha}{4},\ \dfrac{1+\alpha+\beta}{4}, \dfrac{3+\alpha+\beta}{4} \end{matrix} \right. \right) y^{-\frac{1-2\beta}{4}} \tilde{\nu} \;\; y \;\; dy \tag{3.2.19}$$

ga ega bo'lamiz.

$$J_1 \;\; x \; = 2^{2-\beta} x^{-b+c-\frac{3+\beta+\alpha}{4}} \int_0^x y^{\frac{2-\alpha+3\beta}{2}-\frac{3+\beta+\alpha}{4}+\frac{\alpha-\beta-1}{4}} \tilde{\tau} \;\; y \;\; \times$$

$$\times C_{6,6}^{6,0}\left(\frac{y}{x} \left| \begin{matrix} -a+c+\dfrac{1-\beta-\alpha}{4},\ -b+c-\dfrac{1-\beta-\alpha}{4},\ \dfrac{-3+\beta+\alpha}{4},\ \dfrac{-1+\beta+\alpha}{4},\ \dfrac{-2+\beta-\alpha}{4}, \dfrac{\beta-\alpha}{4} \\ \dfrac{1-\beta-\alpha}{4},\ \dfrac{1-\beta-\alpha}{4}-a-b+c,\ -\dfrac{3}{4},\ -\dfrac{1}{4},\ -\dfrac{2}{4},\ 0 \end{matrix} \right. \right) dy \tag{3.2.20}$$

Teoremaning 2) shartidan foydalanib, (3.2.6) va (3.2.7) dan

$$\tilde{\nu}(x) - \int_0^x R(x,y)\tilde{\nu}(y)dy = F(x), \tag{3.2.21}$$

$$R(x,y) = \frac{\gamma_2 \cdot 2^{\beta}}{\tilde{a}_1(x)-\tilde{a}_2(x)} x^{c-b} \cdot y^{\frac{1-2\beta}{4}} C_{5,5}^{5,0}\left(\frac{y}{x} \left| \begin{matrix} 1-a+c,\ 1-b+c,\ \dfrac{1+2\alpha}{4},\ \dfrac{3+2\alpha}{4},\ \dfrac{1}{2} \\ 1-a+b+c,\ \dfrac{\beta+\alpha}{4},\ \dfrac{2+\beta+\alpha}{4},\ \dfrac{1+\alpha+\beta}{4}, \dfrac{3+\alpha+\beta}{4} \end{matrix} \right. \right) \tag{3.2.22}$$

$$F(x)=\frac{\tilde{b}_2(x)-\tilde{b}_1(x)}{\tilde{a}_1(x)-\tilde{a}_2(x)}$$

va

$$2^{1-\beta}x^{c-b}\times$$
$$\times\int_0^x C_{6,6}^{6,0}\left(\frac{y}{x}\left|\begin{matrix}-a+c+\frac{1-\beta-\alpha}{4},\ -b+c-\frac{1-\beta-\alpha}{4},\ \frac{-3+\beta+\alpha}{4},\ \frac{-1+\beta+\alpha}{4},\ \frac{-2+\beta-\alpha}{4},\frac{\beta-\alpha}{4}\\ \frac{1-\beta-\alpha}{4},\ \frac{1-\beta-\alpha}{4}-a-b+c,\ -\frac{3}{4},\ -\frac{1}{4},\ -\frac{2}{4},\ 0\end{matrix}\right.\right)\times$$
$$\times\tau_1(x)dx=\frac{1}{2\gamma_1}\quad\tilde{a}_1(x)+\tilde{a}_2(x)\ \tilde{\nu}(x)+\tilde{b}_1(x)+\tilde{b}_2(x)\ .$$

(3.2.23)

Shunday qilib,

$$\left|R(x,y)\right|\le c_2x^{l_2-\frac{1}{2}}(x-y)^{-l_1-\beta},\ c_2=const>0, \qquad (3.2.24)$$

U holda tenglama kuchsiz maxsuslikka ega bo'lgan II tur Volterra integral tenglamasidir. Bu tenglamaning bir qiymatliligi integral tenglamalar nazariyasidan kelib chiqadi. [8]

Keyin (3.2.23)operatorni ikki tomoniga

$$\frac{d}{dx}x^{\frac{\alpha-\beta}{4}}\times$$
$$\times\int_0^x C_{6,6}^{6,0}\left(\frac{y}{x}\left|\begin{matrix}\frac{1-2\beta}{4},\ \frac{1-2\beta}{4}-a-b+c,\ \frac{-3-\beta+\alpha}{4},\ \frac{-1-\beta+\alpha}{4},\frac{-2-\beta+\alpha}{4},\frac{4-\beta+\alpha}{4}\\ -a+c+\frac{1-2\beta}{4},-b+c+\frac{1-2\beta}{4},\ \frac{-3+2\alpha}{4},\ \frac{1-2\alpha}{4},\ -\frac{2}{4},0\end{matrix}\right.\right)\times$$
$$\times\ x^{b-c+\frac{2\beta-1}{4}}...dy$$

(3.2.25)

$$\begin{cases}c+\beta-1>0\\ -c-\beta<0\end{cases},-c-\beta>-1,-1<-c-\beta<0$$

operatorni qo'llaymiz.

Quyidagi ifodani topamiz:

$$\tilde{\tau}(x)=\frac{1}{2\gamma_1}x^{c-b}\frac{d}{dx}x^{\frac{\alpha-\beta}{4}}\times$$

$$\times\int_0^x C_{6,6}^{6,0}\left(\frac{y}{x}\left|\begin{matrix}\frac{1-2\beta}{4},\ \frac{1-2\beta}{4}-a-b+c,\ \frac{-3-\beta+\alpha}{4},\ \frac{-1-\beta+\alpha}{4},\frac{-2-\beta+\alpha}{4},\frac{4-\beta+\alpha}{4}\\ -a+c+\frac{1-2\beta}{4},-b+c+\frac{1-2\beta}{4},\ \frac{-3+2\alpha}{4},\ \frac{1-2\alpha}{4},\ -\frac{2}{4},\ 0\end{matrix}\right.\right)\times$$

$$\times x^{b-c+\frac{2\beta-1}{4}}\ldots dy$$

(3.2.26)

(3.2.25) ni (3.2.26)ga qo'yib $\tilde{\tau}(x)$ ni topamiz.

Teoremaning 1), 2) shartlaridan foydalanib,

$$\tilde{\tau}(x)\in C(\bar{J})\cap C^2(J),\ \tilde{\nu}(x)\in C^2(J)$$

ko'rsatish mumkin. $\tilde{\nu}(x)$ funksiya $x\to 0$ va $x\to q^{1/q}$ ga intilganda $1-2\beta/2$ dan yuqori bo'lmagan tartibda cheksizlikka aylanishi mumkin.

III BOB YUZASIDAN XULOSA

Uchinchi bobda sohaning ichida buziladigan giperbolik tipdagi tenglamalar uchun chegaraviy masalalarni o'rganilgan. Bu masalalarning chegaraviy shartida umumlashgan kasr tartibli integro-differensial operatorlarni ba'zi siljishli chegaraviy masalalarda qo'llash mumkinligi isbotlangan.

XULOSA

Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yhatidan iborat.

Dissertatsiyaning kirish qismida mavzuning dolzarbligi, o'rganilganlik darajasi berilgan.Birinchi bob uchta paragrafdan iborat. Birinchi bobning birinchi paragrafida umumlashgan kasr tartibli integro-differensial operatorlarga ta'rif beriladi.

Ikkinchi paragrafida funksiyadan funksiya bo'yicha olingan kasr tartibli hosila va integrallarni o'rganishga bag'ishlangan.

Uchinchi paragrafida kasr tartibli integro-differensial operatorlar kompozitsiyasining ba'zi xossalari haqida so'z yuritilgan.

Dissertatsiyaning ikkinchi bobi ikkita buzilish chizig'iga ega bo'lgan giperbolik tipdagi tenglamalar uchun chegaraviy masalalarni o'rganishga bag'ishlangan.

Ikkinchi bobning birinchi paragrafida sohaning chegarasida buziladigan giperbolik tipdagi tenglamalar uchun chegaraviy masalalar o'rganiladi.

Ikkinchi paragrafi chegaraviy shartlarida umumlashgan kasr tartibli integro-differensial operator qatnashgan siljishli chegaraviy masalalarga bag'ishlangan.

Dissertatsiyaning uchinchi bobi sohaning ichida buziladigan giperbolik tipdagi tenglamalar uchun chegaraviy masalalarni o'rganishga bag'ishlangan.

Uchinchi bobning birinchi paragrafida sohaning ichida buziladigan giperbolik tipdagi tenglamalar uchun chegaraviy masalalar o'rganilgan.

Uchinchi bobning ikkinchi paragrafida chegaraviy shartida umumlashgan kasr tartibli integro-differensial operatorlar qatnashgan ba'zi siljishli chegaraviy masalalar o'rganilgan.

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