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m sh funksiyalarning ko`phadlar yordamidagi kriteriyasi.

5A130101 - Matematika (yo‘nalishlar bo‘yicha)

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M U N D A R I J A

KIRISH

1. Ishning umumiy tavsifi

1.1. Mavzuning dolzarbligi. Plyuripotensiallar nazariyasi yoki kompleks potensiallar nazariyasi $(dd^c u)^n -$ Monje-Ampere operatoriga asoslangan va zamonaviy matematikaning asosiy yo'nalishlaridan biri hisoblanadi. $m-sh$ funksiyalar sinfida potensiallar nazariyasi A.Sadullayev va B.Abdullayevlar tomonidan o'rganilgan va hozirda matematikaning bir qator yo`nalishlarida muvaffaqiyat bilan qo`llanilmoqda. $m-sh$ funksiyalar sinfi subgarmonik va plyurisubgarmonik funksiyalar sinflari orasidagi sinf bo`lgani bois bu soha kompleks potensiallar nazariyasida alohida obyekt sifatida mutaxassislar e'tiborida bo'lib kelmoqda.

1.2. Ishning maqsadi. $m-sh$ funksiyalarning ta'riflari va xossalariga asoslanib qat'iy $m-sh$ funksiyalarni o`rganish.

1.3. Ishning vazifalari.

— $\forall u \in C^2(D) \quad D \subset \mathbb{C}^n$ funksiya uchun $(dd^c u)^k \wedge (dd^c |z|^2)^{n-k} \geq 0 \quad \forall k = \overline{1,m}$ tengsizlikning $k = m$ uchun bajarilishidan qolgan barcha $\forall k = \overline{1, m-1}$ lar uchun bajarilishi yoki bajarilmasligini aniqlash

— $m-sh$ funksiyalar sinfi $m = n$ bo`lgan holda plyurisubgarmonik funksiyalar sinfi bilan ustma-ust tushishini o`rganish.

— $m-sh$ funksiyalarning qat'iy $m-sh$ bo`lish shartlarini o`rganish

1.4.Tadqiqot obyekti. $m-sh$ funksiyalar, qat'iy $m-sh$ funksiyalar.

1.5.Tadqiqot predmeti. $(dd^c u)^n -$ Monje –Ampere operatori hamda gessianlar, oqimlar to'g'risidagi masalalar.

1.6.Tadqiqot usullari. Ko'p kompleks o'zgaruvchining funksiyalar nazariyasi, kompleks potensiallar nazariyasi va chiziqli algebra usullari.

1.7.Tadqiqot natijalarining ilmiy jihatdan yangilik darajasi. Mazkur ishning asosiy natijalari yangi.

1.8.Tadqiqot natijalarining amaliy ahamiyati va tadbiqi. Olingan natijalar va usullar funksiyalar nazariyasining keyingi rivojlanishida va kompleks analizning tadbiqlarida qo'llanilishi mumkin.

1.9.Ish tuzilishi va tarkibi. Dissertatsiya ishi kirish, uchta bob hamda adabiyotlar ro'yhatidan iborat. Har bir bob bo`limlardan tuzilgan.

2. Ishning mazmuni

Dissertatsiya ishi kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yhatidan iborat. Birinchi bob uchta paragrafdan iborat bo`lib, unda garmonik, subgarmonik va plyurisubgarmonik funksiyalar haqida zarur ma'lumotlar hamda ularning xossalari keltirilgan.

Ikkinchi bob ikki paragrafdan iborat. Bu bobda musbat aniqlangan differensial forma va oqimlar hamda musbat gessianlar bilan differensial formalar orasidagi bog`lanishlar o'rganilgan.

Ikki marta silliq ixtiyoriy $u \in C^2(D)$ funksiya uchun uning 2-tartibli differensiali

$$dd^c u = \frac{i}{2}\sum_{j,k} u_{j\bar{k}}\, dz_j \wedge d\bar{z}_k \tag{1}$$

(tayinlangan $z^0 \in D$ nuqtada) ermit kvadratik formasidan iborat bo`ladi. Bu yerda odatdagidek, $d = \partial + \bar{\partial}$, $d^c = \frac{\partial - \bar{\partial}}{4i}$ bo`lib,

$$du = \partial u + \bar{\partial} u = \sum_{k=1}^{n} \frac{\partial u}{\partial z_k} dz_k + \sum_{k=1}^{n} \frac{\partial u}{\partial \bar{z}_k} d\bar{z}_k$$

$$d^c u = \frac{1}{4i}\left(\sum_{k=1}^{n} \frac{\partial u}{\partial z_k} dz_k - \sum_{k=1}^{n} \frac{\partial u}{\partial \bar{z}_k} d\bar{z}_k\right).$$

(1) tenglik unitar ([4] ga qarang) almashtirishlar yordamida quyidagi diagonal formaga keltiriladi:

$$dd^c u = \frac{i}{2}\left(\lambda_1 dz_1 \wedge d\bar{z}_1 + \ldots + \lambda_n dz_n \wedge d\bar{z}_n\right) \tag{2}$$

Bunda $\lambda_1, \ldots, \lambda_n$ lar ermit $\left(u_{j\bar{k}}\right)$ matrisasining xos qiymatlari, bo`lib,

$l = (l_1, l_2, ..., l_n) \in \mathbb{R}^n$ haqiqiy qiymatlardan iborat bo`ladi.

1-ta'rif. Ikki marta silliq $u \in C^2(D)$, $(D \subset \mathbb{C}^n)$ funksiya uchun (tayinlangan $z^0 \in D$ nuqtada)

$$(dd^c u)^k \wedge (dd^c |z|^2)^{n-k} \geq 0, \quad \forall k = 1,2..., m \qquad (3)$$

tengsizlik bajarilsa, u holda bu funksiya D sohada $m-sh$ $(1 \leq m \leq n)$ funksiya deyiladi.

1-teorema. *$D \subset \mathbb{C}^n$ sohada n - sh bo`lgan ixtiyoriy funksiya psh dir.*

2-ta'rif. Agar $u \in m-sh(D) \cap C^2(D), (D \subset \mathbb{C}^n)$ funksiya uchun $\varepsilon > 0$ son topilib, $u - \varepsilon \|z\|^2$ funksiya ham D sohada $m-sh$ funksiya bo`lsa, u holda $u(z)$ funksiya D sohada qat'iy $m-sh$ funksiya deyiladi,

Biz ikki marta silliq $u \in C^2(D)$ funksiya uchun qat'iy $m-sh$ funksiya ta'rifini boshqacha ko`rinishda ham ifodalashimiz mumkin:

3-ta'rif. Agar $u \in C^2(D)$, $(D \subset \mathbb{C}^n)$ funksiya uchun ushbu

$$(dd^c u)^k \wedge (dd^c |z|^2)^{n-k} > 0, \quad \forall k = 1,2..., m \qquad (4)$$

qat'iy tengsizliklar o`rinli bo`lsa, u holda bu funksiya qat'iy $m-sh$ funksiya deyiladi. Bu ikki ta'rif bir-biriga ekvivalent.

1-lemma. Agar

$$\begin{cases} H_1(\lambda) = \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n \geq 0 \\ H_2(\lambda) = \lambda_1\lambda_2 + \lambda_1\lambda_3 + ... + \lambda_{n-1}\lambda_n \geq 0 \\ H_3(\lambda) = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + ... + \lambda_{n-2}\lambda_{n-1}\lambda_n \geq 0 \\ \quad . \quad . \quad . \\ H_{m-1}(\lambda) = \lambda_1...\lambda_{m-1} + ... + \lambda_{n-m+2}...\lambda_n \geq 0 \\ H_m(\lambda) = \lambda_1...\lambda_m + ... + \lambda_{n-m+1}...\lambda_n > 0. \end{cases} \qquad (5)$$

bo`lsa , u holda $H_{m-1} > 0$ bo`ladi.

Yuqoridagi lemma asosida quyidagi teoremani ham isbotlash mumkin:

2-teorema. Agar $u \in m-sh(D)\bigcap C^2(D)$ bo`lib, (3) tengsizliklarda $k = m$ uchun $(dd^c u)^m \wedge (dd^c|z|^2)^{n-m} > 0$ bo`lsa, u holda qolgan barcha $k=\overline{1,m-1}$ lar uchun ham $(dd^c u)^k \wedge (dd^c|z|^2)^{n-k} > 0$ bo`ladi, ya'ni $u \in m-sh(D)\bigcap C^2(D)$ funksiyaning qat'iy $m-sh$ bo`lishligi uchun $(dd^c u)^m \wedge (dd^c|z|^2)^{n-m} > 0$ ning bajarilishi zarur va yetarlidir.

Muallif o'z ilmiy rahbari Fizika-matematika fanlari doktori B.I.Abdullayevga masalani qo'yishda, qo'yilgan masalalarni hal etishda ko`rsatgan doimiy e'tibori uchun chuqur minnatdorchiligini izhor etadi. Shuningdek, A.A.Atamuratovlarga hamda kafedraning barcha o'qituvchilariga dissertatsiya ishini tayyorlashda bergan qimmatli maslahatlari hamda dissertatsiya ishi masalalarini muhokama qilishda ko`rsatgan yordamlari uchun o'zining minnatdorchiligini bildiradi.

I BOB. FUNKSIYALARNING BA'ZI MUHIM SINFLARI

Subgarmonik va plyurisubgarmonik funksiyalar sinflari potensiallar nazariyasining asosiy obyektlaridan bo'lib, bu nazariyaning barcha metodlari asosida ushbu tushunchalar yotadi. Masalan, Laplas tenglamasi uchun Dirixle masalasi, kompleks Monje - Ampere tenglamasi uchun Dirixle masalasi, ko'p argumentli golomorf funksiyalarni analitik davom ettirish masalalari, ko'phad va ratsional funksiyalar bilan yaqinlashtirishlar masalalarida ekstremal plyurisubgarmonik funksiyalar muhim ro'l o'ynaydi.([2],[6],[7],[8],[13],[16] ga qarang)

1.1.Garmonik funksiyalar.

$D \subset \mathbf{R}^n$ sohada ushbu

$$\Delta u \equiv \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \ldots + \frac{\partial^2 u}{\partial x_n^2} = 0$$

tenglamani qanoatlantiruvchi $u \in C^2(D)$ funksiya *garmonik funksiya* deyiladi, bunda

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \ldots + \frac{\partial^2}{\partial x_n^2}$$

laplas operatori.

Barcha garmonik funksiyalar to`plami $H(D)$ kabi belgilanadi.

1.1.1 – teorema. Agar $u(x)$ garmonik funksiya bo`lsa, u holda ixtiyoriy $x^o \in D$ nuqta va $S(x^o, r) \subset D$ sfera uchun

$$u(x^o) = \frac{1}{\sigma_n r^{n-1}} \int\limits_{S(x^o, r)} u(x)\, d\sigma \qquad (1.1.1)$$

bo`ladi, bunda σ_n - birlik sferaning yuzi.

Aksincha, agar D dan olingan har bir $x^o \in D$ nuqta va yetarlicha kichik r $(0 < r \le r_0(x^o))$ lar uchun (1.1.1) formula o`rinli bo`lsa, u holda $u(x) \in H(D)$ bo`ladi.

Endi garmonik funksiyalarning ba'zi bir xossalarini keltiramiz.

a) Agar $u, v \in H(D)$ bo`lib, $\lambda, \mu \in R$ bo`lsa, u holda

$$\lambda u + \mu v \in H(D)$$

bo`ladi;

b) agar umumlashgan funksiya f uchun $\Delta f = 0,$ ya'ni ixtiyoriy $\varphi \in F(D)$ da

$f(\Delta\varphi)=0$ tenglik bajarilsa, u holda f garmonik funksiya bo`ladi, aniqroq qilib aytganda, shunday $v(x)\in C^2(D)$ , $\Delta v=0$ bo`lgan funksiya mavjudki,

$$f(\varphi)=v(\varphi)=\int v(x)\varphi(x)dV$$

bo`ladi;

v) agar $u(x)$ funksiya $B(x_o,r)$ sharda garmonik, uning yopig`ida esa uzluksiz, ya'ni

$$u(x)\in H(B(x_o,r))\cap C(\overline{B(x_o,r)})$$

bo`lsa, u holda

$$u(x)=\int_{S(x^o,r)} u(y)P(x,y)d\sigma(y)$$

formula o`rinli bo`ladi, bunda

$$P(x,y)=\frac{r^2-|x-x_o|^2}{\sigma_n r|x-y|^n}, \qquad n\geq 2 \quad ,$$

Puasson yadrosi.

Ikkinchi tomondan, agar $\varphi(y)$ funksiya $S(x^o,r)$ sferada uzluksiz bo`lsa, u holda

$$u(x)=\int_{S(x^o,r)} \varphi(y)P(x,y)d\sigma(y)$$

funksiya $B(x_o,r)$ sharda Dirixle masalasi

$$\Delta u=0 \ , \qquad u\Big|_{S(x^o,\,r)}=\varphi(y)$$

ning yechimi bo`ladi.

Dirixle masalasini chegarasi silliq bo`lgan ixtiyoriy $D\subset\mathbf{R}^n$ sohada ham qarash mumkin: $\varphi(y)\in C(\partial D)$ uchun yagona $u(x)\in C(\overline{D})$ mavjudki, $\Delta u=0$, $u\,\Big|_{\partial D}=\varphi\,(y)$ bajariladi;

g) kompleks tekislik $\mathbf{C}\approx\mathbf{R}^2$ da garmonik funksiyalar golomorf funksiyalarga bevosita bog`langandir.

Agar $z=x+iy$ deyilsa, unda

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=4\frac{\partial^2}{\partial z\partial \bar{z}}$$

bo`ladi va har bir golomorf $f\in O(D)$ funksiya uchun

$$\operatorname{Re} f=\frac{f+\bar{f}}{2}, \quad \operatorname{Im}\frac{f-\bar{f}}{2i}$$

tengliklardan

$$\Delta \operatorname{Re} f = 0 \quad , \quad \Delta \operatorname{Im} f = 0 \ ,$$

ya’ni $\operatorname{Re} f$ va $\operatorname{Im} f$ larning garmonik funksiyalar ekanligi kelib chiqadi.

Shuningdek, agar $u(z) \in H(D)$ bo`lsa, u holda har bir $z^o \in D$ nuqta va uning atrofi $B(z^o, r) \subset D$ uchun shunday $f \in O(D)$ funksiya topiladiki, $\operatorname{Re} f(z) = u(z), \quad z \in B$ bo`ladi.

Haqiqatan ham, $z^0 = 0$ deb $B = B(0, r)$ doirada Puasson formulasini ushbu

$$u(z) = \frac{1}{2\pi}\int_0^{2\pi} u(\xi)\frac{r^2 - |z|^2}{|\xi - z|^2}\,dt = \frac{1}{2\pi}\int_0^{2\pi} u(\xi)\ \operatorname{Re}\frac{\xi + z}{\xi - z}\,dt\ ,$$

bunda $\xi = re^{it}$, ko`rinishda ifodalaymiz. Bu formuladan $B(o, r)$ da golomorf

$$f(z) = \frac{1}{2\pi}\int_0^{2\pi} u(\xi)\frac{\xi + z}{\xi - z}\,dt \qquad (1.1.2)$$

funksiyasi uchun $\operatorname{Re} f(z) = u(z)$ ekanligi kelib chiqadi.

(1.1.2) formula golomorf funksiyani uning haqiqiy qismi orqali ifodalashda ham ishlatiladi: ixtiyoriy $f(z) \in O(B) \cap C(\overline{B})$ uchun ushbu

$$f(z) = \frac{1}{2\pi}\int_0^{2\pi} u(\xi)\frac{\xi + z}{\xi - z}\,dt + i \operatorname{Im} f(0)$$

formula o’rinlidir.

1.2.Subgarmonik funksiyalar

Faraz qilaylik, $D \subset \mathbf{R}^n$ sohada aniqlangan lokal integrallanuvchi

$$u: \ D \to [-\infty, +\infty)$$

funksiya berilgan bo`lsin, $u \in L^1_{loc}(D)$.

Agar bu funksiya quyidagi ikki shartni bajarsa:

1) $u(x)$ yuqoridan yarim uzluksiz:

ya’ni $\forall x^o \in D$ uchun $\overline{\lim\limits_{x \to x^o}}\, u(x) \le u(x^o)$

(bundan $u(x)$ funksiyaning D ga tegishli ixtiyoriy kompaktda yuqoridan chegaralanganligi kelib chiqadi);

2) har bir $x^o \in D$ nuqta uchun shunday $r(x^o) > 0$ topiladiki, $r \le r(x^o)$ uchun

$$u(x^o) \le \frac{1}{\sigma_n r^{n-1}} \int\limits_{S(x^o,r)} u(x)d\sigma\,; \qquad (1.2.1)$$

u holda $u(x)$ funksiya D sohada *subgarmonik funksiya* deyiladi.

Subgarmonik funksiyalar sinfi $Sh(D)$ kabi belgilanadi. Kelgusida qulaylik uchun biz trivial $u(x) \equiv -\infty$ funksiyasi ham D da $Sh(D)$ ga tegishli deb qaraymiz. Endi subgarmonik funksiyalarning xossalarini keltiramiz.

a) subgarmonik funksiyalarning musbat koeffisentli chiziqli kombinatiyasi subgarmonik funksiya bo`ladi:

$$u_k(x) \in Sh(D), \quad a_k \in R_+ \quad (k = 1,2,...,m) \Rightarrow$$
$$\Rightarrow a_1 u_1(x) + a_2 u_2(x) + ... + a_m u_m(x) \in Sh(D)\,;$$

b) chekli sondagi subgarmonik funksiyalarning maksimumi subgarmonik funksiya bo`ladi:

$$u_1(x), u_2(x),...,u_m(x) \in Sh(D) \Rightarrow$$
$$\Rightarrow \max\{u_1(x), u_2(x),...,u_m(x)\} \in Sh(D)\,;$$

v) monoton kamayuvchi subgarmonik funksiyalar ketma – ketligining limiti subgarmonik funksiya bo`ladi:

$$u_j(x) \in Sh(D)\;, \quad u_j(x) \ge u_{j+1}(x) \quad (j = 1,2,...) \Rightarrow$$
$$\Rightarrow \lim_{j\to\infty} u_j(x) \in Sh(D)\,;$$

g) tekis yaqinlashuvchi subgarmonik funksiyalar ketma – ketligi subgarmonik funksiyaga yaqinlashadi:

$$u_j(x) \in Sh(D) \quad (k = 1,2,3,...)\;, \; u_j(x) \rightrightarrows u(x) \Rightarrow u(x) \in Sh(D)\,.$$

a) – *g*) xossalarning isbot bevosita yuqorida keltirilgan ta'rifdan kelib chiqadi.

d) *Maksimumlar prinsipi.* Agar $u(x) \in Sh(D)$ funksiya biror $x^o \in D$ nuqtada o'zining maksimumiga erishsa, ya'ni

$$u(x^o) = \sup_{x\in D} u(x) \qquad (1.2.2)$$

bo`lsa, u holda $u(x) \equiv const$ bo`ladi.

Ushbu

$$M = \{x \in D:\; u(x) = u(x^o)\}$$

to'plamni qaraylik. $u(x)$ funksiyaning yuqoridan yarim uzluksiz bo`lganligidan va (1.2.2) shartdan M to'plamning D da yopiqligi kelib chiqadi.

Ikkinchi tomondan, ixtiyoriy $p \in M$ uchun (1.2.1) formulaga ko`ra

$$u(x^o) = u(p) \le \frac{1}{\sigma_n r^{n-1}} \int_{S(p,r)} u(x) d\sigma \le u(x^o), \qquad r \le r(p),$$

bo`ladi. Bu munosabatdan $u\big|_{S(p,r)} \equiv u(x^o)$, ya'ni $p$ ning $B(p, r(p))$ atrofida $u(x) = u(x^o)$ bo`lishini ko`ramiz. Bu esa $M$ to'plamning $D$ da ochiq ekanligini ko`rsatadi. Demak, $M = D$.

e) Agar $\vartheta(x) \in Sh(D), \quad u(x) \in H(D)$ funksiyalar uchun $S(x^o, r) \subset\subset D$ sferada

$$\vartheta\big|_S \le u\big|_S$$

bo`lsa, u holda $B(x^o, r)$ sharda $\vartheta(x) \le u(x)$ tengsizlik o'rinli bo`ladi.

Bu xossaning isboti yuqoridagi *d*) xossadan kelib chiqadi;

e) – xossadan quyidagi muxim xulosaga kelamiz. Faraz qilaylik,

$$\vartheta(x) \in Sh(D) \cap C(D)$$

berilgan bo`lib, $B(x^o, r) \subset\subset D$ bo`lsin. Ushbu

$$u(x) = \int_{S(x^o,r)} \vartheta(y) P(x,y) d\sigma(y), \qquad x \in B(x^o, r),$$

Puasson integralini qaraylik. $u(x) \in h(B)$, $u\big|_S = \vartheta\big|_S$ bo`lib, $e$) – xossaga ko`ra $B = B(x^o, r)$ da $\vartheta(x) \le u(x)$ bo`ladi. Uzluksiz funksiyalar uchun isbotlangan bu xossa ixtiyoriy $\vartheta \in Sh(D)$ uchun ham o'rinlidir: $\vartheta(x) \le u(x)$ va deyarli barcha $x \in S$ lar uchun $\vartheta(x) = u(x)$ o'rinlidir;

j) aytaylik, $u(x) \in Sh(D)$ funksiya va $x^o \in D$ nuqta berilgan bo`lsin. Ushbu

$$m(x^o, r) = \frac{1}{\sigma_n r^{n-1}} \int_{S(x^0,r)} u(x) d\sigma$$

va

$$n(x^o, r) = \frac{1}{V_n r^n} \int_{B(x^o,r)} u(x) dV,$$

bunda $V_n r^n$ miqdor $B(x^o, r)$ ning hajmi, integrallarni (o'rta qiymatlarni) qaraylik.

1.2.1 – teorema. Faraz qilaylik, $u(x) \in Sh(D)$ va $u(x) \not\equiv -\infty$ bo`lsin. U holda ixtiyoriy $x^o \in D$ uchun

$$u(x^0) \le n(x^o, r) \le m(x^o, r)\ ,\ n(x^o, r) > -\infty\ ,\ 0 < r < \rho(x^o, \partial D)\ ,$$

tengsizliklar o'rinli bo`ladi. Bundan tashqari, agar r monoton kamayib nolga intilsa, unda $n(x^o, r)$ va $m(x^o, r)$ o'rta qiymatlar monoton kamayib, $u(x^o)$ ga intiladi.

1.2.1 – teoremadan ushbu muhim natijaga ega bo`lamiz: $D \subset \mathbf{R}^n$ sohada subgarmonik $u(x) \ne -\infty$ funksiya D da lokal integrallanuvchidir, ya'ni ixtiyoriy $B \subset\subset D$ uchun $\int_B u(x)dV$ integral mavjuddir;

z) subgarmonik funksiya D sohaning har bir nuqtasida uzilishga ega bo`lishi mumkin. Ayni paytda, quyidagi teorema o'rinli bo`ladi.

1.2.2 – t e o r e m a. Agar $u(x) \in Sh(D)$ bo`lsa, u holda shunday monoton to'plamlar

$$D_j \subset D_{j+1} \subset D\ ,\quad \bigcup_{j=1}^{\infty} D_j = D$$

ketma – ketligi va shunday monoton kamayuvchi funksiyalar

$$u_j(x) \in Sh(D_j) \cap C^\infty(D_j)\ , \qquad j = 1,2,3,...$$

ketma – ketligi mavjudki,

$$u(x) = \lim_{j \to \infty} u_j(x) \qquad (x \in D)$$

bo`ladi.

Isboti. Ushbu

$$K\ \ x\ \ = \begin{cases} ce^{-\frac{1}{1-|x|}}, & agar\ \ |x| \le 1\ \ bo'lsa, \\ 0, & agar\ \ |x| > 1\ bo'lsa. \end{cases}$$

funksiyani qaraylik. Bu funksiya $C^\infty(\mathbf{R}^n)$ sinfga tegishli bo`lib, uning salmog'i $\operatorname{supp} K(x) = B(0,1)$ dir. Endi $K(x)$ ning ifodasidagi o'zgarmas c ni shunday tanlab olamizki,

$$\int_{\mathbf{R}^n} K(x)dV = 1$$

bo`lsin. $u(x) \not\equiv -\infty$ deb, bu yadro yordamida ushbu

$$u_\delta(x) = \frac{1}{\delta^n} \int_{|y-x|\le\delta} u(y)K\left(\frac{y-x}{\delta}\right)dV \ , \quad \delta > 0 \ , \tag{1.2.3}$$

integralni qaraylik. $u_\delta(x)$ funksiyasi

$$D_\delta = \{x \in D:\ \rho(x,\partial D) < \delta\}$$

ochiq to'plamda aniqlangan bo`lib, $x \in D_\delta$ da

$$u_\delta(x) = \frac{1}{\delta^n} \int_{\mathbf{R}^n} u(y)\,K\left(\frac{y-x}{\delta}\right)dV = \int_{\mathbf{R}^n} u(x+\delta y)\,K(y)dV$$

bo`ladi. Keyingi tenglikdagi birinchi integral $C^\infty(D_\delta)$ sinfga, ikkinchi integral esa $Sh(D_\delta)$ sinfga tegishli bo`ladi. Binobarin,

$$u_\delta(x) \in Sh(D_\delta) \bigcap C^\infty(D_\delta)\,.$$

$u(x)$ ning subgarmonik funksiya ekanligidan, u_δ ning $\delta \downarrow 0$ da, monoton kamayuvchi bo`lishligi va

$$\int_{\mathbf{R}^n} K(x)dV = 1$$

tenglikdan, $u_\delta(x)$ ning $u(x)$ ga intilishini topamiz. Teorema isbotlandi.

i) Subgarmonik funksiyalar umumiy holda C^2 sinfga tegishli bo`lmasligidan, $\Delta u$ - Laplas operatorini faqat umumlashgan funksiya sifatida qarash mumkin bo`ladi:

$$\Delta u(\varphi) = \int u\Delta\varphi, \qquad \varphi \in F(D).$$

1.2.3-teorema. Har qanday subgarmonik funksiya $u(x) \in Sh(D)$, $u \not\equiv -\infty$, uchun umumlashgan ma'noda $\Delta u \ge 0$ bo`ladi, ya'ni ixtiyoriy $\varphi \in F(D)$, $\varphi \ge 0$ uchun $\int u\,\Delta\varphi \ge 0$ munosabat o'rinlidir.

Jumladan, $u(x)$ funksiya C^2 sinfga tegishli bo`lib, u subgarmonik bo`lsa, $u \in Sh(D) \bigcap C^2(D)$, u holda $\Delta u = \frac{\partial^2 u}{\partial x_1^2} + ... + \frac{\partial^2 u}{\partial x_n^2} \geq 0$ dir.

Yuqorida keltirilgan teoremaning aksi ham o'rinli: agar umumlashgan funksiya u uchun

$$\Delta u(\varphi) = \int u \Delta \varphi \geq 0 \ , \quad \varphi \in F(D) \ , \quad \varphi \geq 0 \ ,$$

munosabat bajarilsa, u holda u subgarmonik funksiya bo`ladi;

1.3.Plyurisubgarmonik funksiyalar

1.3.1.Plyurigarmonik funksiyalar.

$\mathbf{C}^n$ kompleks fazodagi $D \subset \mathbf{C}^n$ sohada $u(z)$ funksiya berilgan bo`lib, $u(z) \in C^2(D)$ bo`lsin.

Agar ixtiyoriy $z^0 \in D$ va bu nuqtadan o'tuvchi ixtiyoriy kompleks to'g'ri chiziq $l: z = z^0 + w\xi, \quad w \in \mathbf{C}^n, \ \xi \in \mathbf{C}$ uchun ushbu $\varphi(\xi) = u / l = u(z^0 + w\xi)$ kesim – funksiya $\xi = 0$ nuqtada garmonik, ya'ni $\xi = 0$ da $\frac{\partial^2 \varphi}{\partial \xi \cdot \partial \overline{\xi}} = 0$ bo`lsa, $u(z)$ funksiya D da *plyurigarmonik funksiya* deyiladi.

Plyurigarmonik funksiyalar sinfi $Ph(D)$ kabi belgilanadi. Aytaylik, $u(z) \in Ph(D)$ bo`lsin. Unda

$$\frac{\partial^2 u(z^0 + w\xi)}{\partial \xi \partial \overline{\xi}} = 0$$

bo`lib, murakkab funksiyaning differensiallash qoidasidan $\sum_{j,k=1}^{n} \frac{\partial^2 u}{\partial z_j \partial \overline{z_k}} w_j \overline{w_k} = 0$, $w \in \mathbf{C}^n$, ekanligini ko`ramiz. Bundan va $w \in \mathbf{C}^n$ vektorning ixtiyoriyligidan

$$\frac{\partial^2 u}{\partial z_j \partial \overline{z}_k} = 0 \qquad (k, j = 1,2,...,n) \qquad (1.3.1)$$

bo`lishi kelib chiqadi.

Demak, plyurigarmonik funksiyalar sinfi (1.3.1) tenglamalar sistemasi bilan aniqlanadi va ba'zan bu sistema plyurigarmonik funksiyaning ta'rifi sifatida ham qaraladi.

1.3.1 –teorema. Agar $f(z)=u(z)+iv(z)$ funksiya $D\subset\mathbf{C}^n$ sohada golomorf bo`lsa, u holda

$$\operatorname{Re} f(z)=u(z),\quad \operatorname{Im} f(z)=\vartheta(z)$$

funksiyalar D sohada plyurigarmonik bo`ladi va, aksincha, agar $u(z)$ funksiya $D$ sohada plyurigarmonik bo`lsa, u holda ixtiyoriy $B(z^0,r)\subset D$ atrofda $u(z)$ funksiya biror $f(z)\in O(B(z^0,r))$ golomorf funksiyaning haqiqiy qismi bo`ladi: $u(z)\equiv \operatorname{Re} f(z)$.

1.3.2.Plyurisubgarmonik funksiyalar.

Agar $D\subset\mathbf{C}^n$ sohada aniqlangan $u(z):\ D\to[-\infty,\infty)$ funksiya quyidagi ikki shartni bajarsa:

1) $u(z)$ yuqoridagi yarim uzluksiz;

2) ixtiyoriy l kompleks to'g'ri chiziq uchun u/l funksiya $l\cap D$ da subgarmonik bo`lsa, $u(z)$ funksiya D sohada *plyurisubgarmonik* funksiya deyiladi.

Plyurisubgarmonik funksiyalar to'plami $Psh(D)$ kabi belgilanadi. $D\subset\mathbf{C}^n$ sohada plyurisubgarmonik bo`lgan funksiya $D\subset\mathbf{R}^{2n}$ da aniqlangan funksiya sifatida subgarmonik funksiya bo`ladi. Uni subgarmonik funksiyalarning 2 – shartini bajarishini ko`rish qiyin emas. Binobarin, subgarmonik funksiyalarning xossalari plyurisubgarmonik funksiyalar uchun ham saqlanadi.

Endi plyurisubgarmonik funksiyalarning xossalarini keltiramiz:

a) agar $u(z)\in Psh(D)$ bo`lsa, u holda $u_\delta(z)$ funksiya $Psh(D_\delta)\cap C^\infty(D_\delta)$ sinfga tegishli bo`lib, $\delta\downarrow 0$ da $u_\delta(z)\downarrow u(z)$ bo`ladi;

b) agar $u(z)\in Psh(D)$ funksiya $z^0\in D$ nuqtada maksimumga erishsa, ya'ni $u(z^0)\geq u(z)\quad (z\in D)$ bo`lsa, u holda $u(z)\equiv const$ bo`ladi;

v) plyurisubgarmonik funksiyalarning musbat koeffitientli chiziqli kombinatiyasi plyurisubgarmonik funksiya bo`ladi;

g) monoton kamayuvchi yoki tekis yaqinlashuvchi plyurisubgarmonik funksiyalar ketma – ketligining limiti plyurisubgarmonik funksiya bo`ladi;

d) D sohada plyurisubgarmonik funksiyalar sinfi $\{u_\alpha\}_{\alpha\in\wedge}$ berilgan bo`lib, $u=\sup\limits_{\alpha\in\wedge}u_\alpha$ yuqoridan yarim uzluksiz funksiya bo`lsin. U holda $u(z)$ plyurisubgarmonik funksiya bo`ladi, $u \in Psh(D)$. Jumladan, $u_j(z) \in Psh(D) \quad (j = 1,2,...,k)$ bo`lsa,

$$\sup\{u_1(z), u_2(z),...,u_k(z)\} \in Psh(D)$$

bo`ladi;

e) agar $f(z) \in O(D)$ bo`lsa, $\alpha \cdot \ln|f(z)| \in Psh(D)$ bo`ladi, bunda $\alpha \geq 0$;

j) agar $u(z) \in Psh(D)$ bo`lsa, u holda $e^{u(z)} \in Psh(D)$ dir, agar $u(z) \in Psh(D),\ u|_D < 0$ bo`lsa, u holda $-\ln[-u(z)] \in Psh(D)$ dir.

Bu xossaning isboti subgarmonik funksiyalarning shu xossasidan bevosita kelib chiqadi;

z) D sohada aniqlangan $u(z) \in C^2(D)$ funksiyaning D da plyurisubgarmonik bo`lishi uchun ushbu

$$L(u,w) = \sum_{j,k=1}^{n} \frac{\partial^2 u}{\partial z_j \partial \overline{z}_k} w_j \overline{w}_k \qquad (1.3.2)$$

kvadratik formaning (Levi formasining) ixtiyoriy $z \in D$ da musbat aniqlangan bo`lishi zarur va yetarlidir. (1.3.2) munosabatda $w = (w_1, w_2,..., w_n) \in \mathbf{C}^n$ bo`lib, kvadratik formaning musbat aniqlanganligi $\forall w \in \mathbf{C}^n$ da $L(u,w) \geq 0$ bo`lishini anglatadi.

Agar $\forall w \in \mathbf{C}^n,\ w \neq 0$ da $L(u,w) > 0$ bo`lsa, $u(z)$ funksiya $z$ nuqtada ***qat'iy plyurisubgarmonik*** funksiya deyiladi; agar $u(z)$ funksiya $D$ sohaning har bir nuqtasida qat'iy plyurisubgarmonik bo`lsa, $u(z)$ ***sohada qat'iy plyurisubgarmonik*** funksiya deyiladi.

II BOB.MUSBAT GESSIANLAR BILAN DIFFERENSIAL FORMALAR ORASIDAGI BOG`LANISH

2.1.Musbat aniqlangan differensial forma va oqimlar.

2.1.1.Differensial formalar. $f \in C^1(G)$, $G \subset \mathbb{R}^n$ funksiyaning differensiali

$$df = \sum_{j=1}^{n} \frac{\partial f}{\partial x_j} dx_j$$

1-darajali differensial formaga misol bo`ladi. Umuman aytganda, $\omega = \sum_{j=1}^{n} a_j(x)\, dx_j$ ko`rinishdagi ifodaga **1-darajali differensial forma** deyiladi va $\deg \omega = 1$ kabi belgilanadi.Yuqori darajali differensial formalarni ta'riflash uchun biz quyidagi shartlarni qanoatlantiruvchi $\wedge$ —tashqi ko`paytma belgisini kiritamiz:

a) $a(x) \wedge dx_j = dx_j \wedge a(x) = a(x)\, dx_j$

b) $dx_k \wedge dx_j = -dx_j \wedge dx_k, \Rightarrow dx_j \wedge dx_j = 0$

2.1.1-ta'rif. Ushbu

$$\omega = \sum_{0 \le j_1 < j_2 < \ldots < j_p \le n} a_{j_1 j_2 \ldots j_p} dx_{j_1} \wedge dx_{j_2} \wedge \ldots \wedge dx_{j_p}$$

ko`rinishdagi ifodaga p-darajali differensial forma(bunda $a_{j_1 j_2 \ldots j_p}$ - $G$ sohada aniqlangan funksiya) deyiladi va $\deg \omega = p$ kabi belgilanadi. p-darajali differensial formalar to`plami $F^p(G)$ orqali belgilanadi. $\omega \in F^p(G)$ differensial formani quyidagi qulay ko`rinishda ham yozishimiz mumkin:

$$\omega = \sum_{0 \le j_1 < j_2 < \ldots < j_p \le n} a_{j_1 j_2 \ldots j_p} dx_{j_1} \wedge dx_{j_2} \wedge \ldots \wedge dx_{j_p} = \sum_J a_J dx_J$$

bunda $dx_J = dx_{j_1} \wedge dx_{j_2} \wedge \ldots \wedge dx_{j_p}$, $J = (j_1, j_2, \ldots, j_p)$, $1 \le j_1, j_2, \ldots, j_p \le n$— multiindekslar. Ravshanki, 0-darajali differensial forma bu- $a(x)$ skalyar funksiya.

$\mathbb{C}^n \cong \mathbb{R}^{2n}$ kompleks fazoda differensial formani dz_j *va* $d\overline{z}_j$ lardan foydalanib yozish mumkin. $z = (z_1, z_2, \ldots, z_n)$ koordinatalar $\mathbb{C}^n \cong \mathbb{R}^{2n}$, $z_j = x_j + i x_{n+j}$, $j = 1, 2, \ldots$ dagi kompleks koordinatalar bo`lsin. Unda

$$dx_j = \frac{dz_j + d\overline{z_j}}{2}, \quad dx_{n+j} = \frac{dz - d\overline{z_j}}{2i}$$

bo`lganligidan $\omega = \sum_J a_J dx_J$ differensial formani $dz_j$ *va* $d\overline{z_k}$ larning chiziqli kombinatsiyasi ko`rinishida yozish mumkin. Har bir hadda dz_j *va* $d\overline{z_k}$ lar turlicha bo`lishi mumkin. Masalan, $\mathbb{C}^2 \cong \mathbb{R}^4$ da

$$dx_1 \wedge dx_2 \wedge dx_3 = \frac{i}{4} \ -dz_1 \wedge dz_2 \wedge d\overline{z_1} + dz_1 \wedge d\overline{z_1} \wedge d\overline{z_2}$$

Ammo ba'zi hollarda ω dagi barcha qo`shiluvchilarda $dz_j$ larning darajasi $p$ ga, $d\overline{z_k}$ larniki esa $q$ ga teng bo`lib qolishi mumkin. Bunday hollarda biz ω differensial formani p,q bidarajali differensial forma deb aytamiz va $\omega \in F^{p,q}$ deb belgilaymiz.

2.1.2.Musbat aniqlangan differensial formalar.

2.1.2-ta'rif. Ushbu

$$\omega = \mathop{\wedge}_{j=1}^{p} \frac{i}{2} dl_j \wedge d\overline{l_j}$$

ko`rinishdagi differensial forma p,p bidarajali bosh kuchli musbat differensial forma deyiladi , bu yerda $l_j = a_{j_1} dz_1 + ... + a_{j_n} dz_n -$ chiziqli kombinatsiya $dz_1,...,dz_n$ bazis bo'yicha $a_{jk} \in \mathbb{C}$, $j = 1,2,..., p, k = 1,2,...,n$

Bunday formalarning ixtiyoriy chiziqli kombinatsiyasi kuchli musbat differensial forma deb ataladi, ya'ni kuchli musbat differensial forma bu-

$$\omega = \sum_s \lambda_s \ z \ \mathop{\wedge}_{j=1}^{p} \frac{i}{2} dl_{j_s} \wedge d\bar{l}_{j_k}$$

ko`rinishdagi formadir, bu yerda $\lambda_s \ z$ - qaralayotgan $G \subset \mathbb{C}^n$ sohadagi manfiy bo`lmagan funksiya.

Agar $z_j = x_j + ix_{n+j}$ kompleks koordinatalardan $x_j, x_{n+j},$ $j = 1,2,...,n$ haqiqiy koordinatalarga o`tsak, unda ushbu

$$\frac{i}{2} dz_1 \wedge d\overline{z_1} \wedge ... \wedge dz_p \wedge d\overline{z_p} = dx_1 \wedge dx_{n+1} \wedge ... \wedge dx_p \wedge dx_{n+p}$$

ifoda $\mathbb{C}^{2n}$ fazoda $2p$ darajali musbat forma bo`ladi.

n,n bidarajali differensial forma (kuchli) musbat bo`ladi, faqat va faqat shu holdaki, qachonki u

$$\omega = \lambda(z) \bigwedge_{j=1}^{n} \frac{i}{2} dz_j \wedge d\overline{z_j} = \lambda(z) dx_1 \wedge dx_{n+1} \wedge ... \wedge dx_n \wedge dx_{2n}$$

ko`rinishga ega bo`lsa,bu yerda $\lambda(z)$ – musbat funksiya.

n,n bidarajali $\Omega = \bigwedge_{j=1}^{n} \frac{i}{2} dz_j \wedge d\overline{z_j} = dV$ differensial forma $\mathbb{C}^n$ da hajm formasi rolini o'ynaydi.

$p=1$ da $\omega = \sum \lambda_{jk} dz_j \wedge d\overline{z_k}$ – differensial forma musbat bo`ladi, faqat va faqat shu holdaki, qachonki $\sum_{jk} \lambda_{jk} \xi_j \xi_k$ – ermit kvadratik formasi musbat bo`lsa. Shuningdek, ixtiyoriy $z^o \in G$ tayinlangan nuqtada unitar almashtirish yordamida uni

$$\omega = \frac{i}{2} \sum_{j=1}^{n} \mu_j dz_j \wedge d\overline{z_j},\ \mu_j \geq 0$$

ko`rinishda yozish mumkin.

2.1.3-ta'rif. $\omega \in F^{p,p}$ differensial forma kuchsiz musbat yoki musbat deyiladi, agar ixtiyoriy kuchli musbat $\nu \in F^{n-p,n-p}$ forma uchun $\omega \wedge \nu \in F^{n,n}$ forma musbat bo`lsa. $p = 0,1,n-1,n$ da kuchli musbatlik kuchsiz musbatlikka ekvivalent. Biroq boshqa $p$ lar uchun bu sinflar ustma-ust tushmaydi. Buni aniq tushunish uchun kuchli musbat formani quyidagi ko`rinishda yozamiz:

$$\begin{aligned}\omega &= \sum_s \lambda_s(z) \bigwedge_{j=1}^{p} \frac{i}{2} dl_{j_s} \wedge d\bar{l}_{j_s} = \frac{i^{p^2}}{2^p} \sum_s \lambda_s(z) \bigwedge_{j=1}^{p} \frac{i}{2} dl_{j_s} \bigwedge_{j=1}^{p} d\bar{l}_{j_s} = \\ &= \frac{i^{p^2}}{2^p} \sum_s \lambda_s(z)\, dl_{j_s} \wedge d\bar{l}_{j_s}\end{aligned} \qquad (*)$$

Agar $\theta = \sum_s \mu_s(z)\, dl_s$ formani qarasak , bunda μ_s – kompleks qiymatli funksiya, unda $\frac{i^{p^2}}{2^p} \theta \wedge \bar{\theta}$ ko`rinishdagi forma (kuchsiz) musbat bo`ladi, biroq $2 \leq p \leq n-2$ da u har doim ham $*$ ko`rinishga ega bo`lavermaydi.

2.1.3.Oqimlar. Qo`pol qilib aytganda, M ko'pxillikdagi oqim bu- koeffitsiyentlari umumlashgan funksiyalardan iborat differensial formadir. Tayinlangan $G \subset \mathbb{R}^n$ soha uchun asosiy diffferensial formalar fazosi qaraladi:

$$F^p = F^p(G) = \{\omega \in F^p(G) \bigcap C^\infty(G) : \operatorname{supp}\omega \subset\subset G\}$$

F^p da topologiya $F(G)$ fazodagi topologiyadan aniqlanadi.

2.1.4-ta'rif. F^p fazodagi uzluksiz chiziqli (kompleks qiymatli) T funksional $n-p=q$ darajali oqim deyiladi.

Barcha $q-$ darajali oqimlar fazosini F^q orqali belgilaymiz. F^q oqimlar fazosi umumlashgan funksiyalarning barcha xossalarini qanoatlantiradi.Xususan,ular uchun kuchsiz yaqinlashish,kuchsiz chegaralanganlik tushunchalarini kiritish mumkin va $T_\delta \in F^q \cap C^\infty$

o'ramani aniqlash mumkin, ya'ni $\delta \to 0$ da oqim sifatida $T_\delta \circ \omega \to T \circ \omega, \quad \forall \omega \in F^p$ undan tashqari T oqim differensial formadek differensiallanadi: $dT \circ \omega = (-1)^p T \circ d\omega$

2.1.5-ta'rif.Agar $F^p = \{\omega \in F^p(G) \bigcap C^\infty(G) : \operatorname{supp}\omega \subset\subset G\}$ fazodan olingan T oqim (funksional) $\{\omega \in F^p(G) \bigcap C^m(G) : \operatorname{supp}\omega \subset\subset G\}$ fazoga uzluksiz davom qilsa, unda T oqim m dan ko'p bo`lmagan singulyarlikka ega deyiladi, bunda $0 \le m \le \infty$. Agar undan tashqari T oqim $\{\omega \in F^p(G) \bigcap C^{m-1}(G) : \operatorname{supp}\omega \subset\subset G\}$ fazoga davom qilmasa, unda T oqim aniq m singulyarlikka ega deyiladi.

$T = \sum_{|k|=q} \beta_k dx_k$ koeffitsiyentlari umumlashgan funksiya bo`lgan differensial forma oqim deyiladi.

2.1.1-teorema.Ixtiyoriy $q-$ darajali oqim koeffitsiyentlari umumlashgan funksiya bo`lgan differensial forma bo'ladi, shu bilan birga singulyarligi nolga teng bo'lgan oqim o'lchov tipidagi oqim bo'ladi va uni barcha koeffitsiyentlari kompleks qiymatli o'lchov bo`ladi.

O‘lchov tipidagi oqimlar fazosini F_0^q orqali belgilaymiz. F_0^q nafaqat $G \subset \mathbb{R}^n$ soha uchun, balki ixtiyoriy $E \subset \mathbb{R}^n$ borel to‘plami uchun ham aniqlangan. Xususan, $K \subset \mathbb{R}^n$ kompakt uchun u Banax fazosi bo`ladi,xuddi vektorlarni tekis normasi bilan differensial forma koeffitsiyentlaridan tashkil topgan $F_0^q\ K\ = F_0^q\ K\ \bigcap C\ K$ fazoga qo’shma fazo kabi.

$T \circ \omega = \mu\ z\ \Omega$ O‘lchov tipidagi $T \in F_0^q$ oqim uchun

$$T \circ \omega = \int_G d\mu \quad \text{o’rinli.}$$

Bu yerda $\mu -$ kompleks qiymatli o‘lchov bo ‘lib, u $\omega \in F_0^p$ ga bog`liq va $\Omega = dx_1 \wedge ... \wedge dx_n$

Xuddi o‘lchovlar fazosi kabi quyidagi xossalar o ‘rinli:

a) $T_j\ \subset F_0^q$ oqimlar ketma-ketligi T ga kuchsiz yaqinlashadi, ya’ni $\lim\limits_{j\to\infty} T_j \circ \omega = T \circ \omega,\ \forall \omega \in F_0^p$ faqat va faqat shu holdaki,qachon mos o‘lchovlar ketma-ketligini yaqinlashishi T_j ni koeffitsiyentlaridan bo`lsa.

b) $T_j\ \subset F_0^q$ oqimlar ketma-ketligi T ga kuchsiz yaqinlashadi, faqat va faqat shu holdaki,qachon $T_j\ \ \|T_j\|_k \le c,\ \forall K \subset\subset G,\ j = 1, 2, ...$ norma bo`yicha chegaralangan bo`lsa va $j \to \infty$ da $T_j \circ \omega = T \circ \omega$ limit biror hamma yerda zich $E \subset F_0^p\ K$ to’plamdan olingan barcha ω lar uchun o‘rinli bo`lsa, masalan barcha cheksiz differensiallanuvchi $\omega \in F^p \bigcap C^\infty\ K$ lar uchun

d)ixtiyoriy kuchsiz chegaralangan $E \subset F_0^q\ K$ to’plam osti kuchsiz kompakt ya’ni uni ixtiyoriy ketma-ketligi kuchsiz yaqinlashuvchi qismiy ketma-ketligini o`zida saqlaydi.

2.1.6-ta’rif. $q -$ darajali differensial forma (oqim) yopiq deyiladi, agar uni differensiali nolga teng bo`lsa;

U aniq deyiladi, agar u biror $q - 1$ darajali formani(oqimni)differensiali bo`lsa.Aniq formalar(oqimlar) yopiq bo`ladi, teskari tasdiq uchun G sohaga qo'shimcha geometrik shartlarni qo'yish zarur.

Endi $\mathbb{C}^n$ da asosiy differensial formalar fazosini qaraymiz:

$F^{p,p} = F^{p,p}(G) = \{\omega \in F^{p,p}(G) \bigcap C^{\infty}(G) : \operatorname{supp}\omega \subset\subset G\}$

2.1.7-ta'rif. $F^{p,p}$ fazoda uzluksiz chiziqli(kompleks qiymatli) T funksionalga $(n-p, n-p) = (q,q)$ bidarajali oqim deyiladi.

2.1.2-teorema.Musbat oqimlar o'lchov tipidagi oqimlar bo`ladi.

2.1.1-misol. Koeffitsiyentlari L'_{loc} sinfdan olingan ixtiyoriy $T \in F^{q,q}$ differensial forma (q,q) bidarajali oqim bo`ladi.

$T \circ \omega$ funksional $T \circ \omega = \int_G T \wedge \omega, \quad \forall \omega \in F^{p,p}$ kabi aniqlanadi.Bunday oqimlar regulyar oqim deyiladi.

2.1.2-misol. $A \subset \mathbb{C}^n$ $(n-p)$ o'lchamli analitik to'plam bo`lsin.U holda

$[A] \circ \omega = \int_A \omega$ integral (q,q) bidarajali oqimni aniqlaydi.Ko`rish qiyin emaski, $[A]$ musbat oqim bo`ladi.

2.2.Musbat gessianalar bilan differensial formalar orasidagi bog`lanish

2.2.1.Gessianlar. 2-marta silliq $u \in C^2(D)$ funksiya uchun uning ikkinchi tartibli differensiali

$$dd^c u = \frac{i}{2}\sum_{j,k} u_{j\bar{k}} dz_j \wedge d\overline{z_k} \qquad \left(bunda\ u_{j\bar{k}} = \frac{\partial^2 u}{\partial z_j \partial \overline{z_k}} \right) \qquad (2.2.1)$$

(tayinlangan $z^0 \in D$ nuqtada) Ermit kvadratik formasidan iborat bo`ladi.

$(dd^c u)^n$ operator Monje-Ampere operatori deyiladi. Bunda

$$d = \partial + \bar{\partial} \qquad d^c = \frac{1}{4i}(\partial - \bar{\partial})$$

$$du=\partial u+\bar{\partial}u=\sum_{k=1}^{n}\frac{\partial u}{\partial z_k}dz_k+\sum_{k=1}^{n}\frac{\partial u}{\partial \overline{z_k}}d\overline{z_k}$$

$$d^c u=\frac{1}{4i}\left(\sum_{k=1}^{n}\frac{\partial u}{\partial z_k}dz_k-\sum_{k=1}^{n}\frac{\partial u}{\partial \overline{z_k}}d\overline{z_k}\right).$$

(2.2.1) tenglikni $n=1$ uchun ko`rib chiqamiz:

$$du=\frac{\partial u}{\partial z}dz+\frac{\partial u}{\partial \bar{z}}d\bar{z}$$

$$d^c u=\frac{1}{4i}\left(\frac{\partial u}{\partial z}dz-\frac{\partial u}{\partial \bar{z}}d\bar{z}\right)$$

$$dd^c u=\frac{1}{4i}\left(\partial\left(\frac{\partial u}{\partial z}dz-\frac{\partial u}{\partial \bar{z}}d\bar{z}\right)+\bar{\partial}\left(\frac{\partial u}{\partial z}dz-\frac{\partial u}{\partial \bar{z}}d\bar{z}\right)=\right.$$

$$\frac{1}{4i}\left(\frac{\partial^2 u}{\partial z^2}dz\wedge dz-\frac{\partial^2 u}{\partial z\partial \bar{z}}d\bar{z}\wedge dz+\frac{\partial^2 u}{\partial \bar{z}\partial z}dz\wedge d\bar{z}-\frac{\partial^2 u}{\partial^2 \bar{z}}d\bar{z}\wedge d\bar{z}\right)$$

Tashqi ko`paytmada

$dz\wedge dz=0,\quad d\bar{z}\wedge d\bar{z}=0,\quad d\bar{z}\wedge dz=-dz\wedge d\bar{z}$ bo`lishidan foydalansak,

$$dd^c u=\frac{1}{2i}\frac{\partial^2 u}{\partial \bar{z}\partial z}dz\wedge d\bar{z}$$

ifodaga ega bo`lamiz.

Demak, $u\in C^2(D), D\subset\mathbb{C}$ uchun $dd^c u=\frac{i}{2}\partial\bar{\partial}u$ bo`ladi.

(2.2.1) formulani $n=3$ uchun ham ko`rib chiqamiz.

$$du=\partial u+\bar{\partial}u=\frac{\partial u}{\partial z_1}dz_1+\frac{\partial u}{\partial z_2}\partial z_2+\frac{\partial u}{\partial z_3}\partial z_3+\frac{\partial u}{\partial \overline{z_1}}\partial\overline{z_1}+\frac{\partial u}{\partial \overline{z_2}}\partial\overline{z_2}+\frac{\partial u}{\partial \overline{z_3}}\partial\overline{z_3}$$

$$d^c u=\frac{1}{4i}\left(\frac{\partial u}{\partial z_1}dz_1+\frac{\partial u}{\partial z_2}dz_2+\frac{\partial u}{\partial z_3}dz_3-\frac{\partial u}{\partial \bar{z}_1}d\bar{z}_1-\frac{\partial u}{\partial z_2}d\overline{z_2}-\frac{\partial u}{\partial z_3}d\overline{z_3}\right)$$

$$dd^c u=\frac{1}{4i}\left[\partial\left(\frac{\partial u}{\partial z_1}dz_1+\frac{\partial u}{\partial z_2}\partial z_2+\frac{\partial u}{\partial z_3}\partial z_3-\frac{\partial u}{\partial \overline{z_1}}\partial\overline{z_1}-\frac{\partial u}{\partial \overline{z_2}}\partial\overline{z_2}-\frac{\partial u}{\partial \overline{z_3}}\partial\overline{z_3}\right)+\right.$$

$$+\bar{\partial}\left(\frac{\partial u}{\partial z_1}dz_1+\frac{\partial u}{\partial z_2}\partial z_2+\frac{\partial u}{\partial z_3}\partial z_3-\frac{\partial u}{\partial \overline{z_1}}\partial\overline{z_1}-\frac{\partial u}{\partial \overline{z_2}}\partial\overline{z_2}-\frac{\partial u}{\partial \overline{z_3}}\partial\overline{z_3}\right)=$$

$$=\frac{1}{4i}\left[\left(\frac{\partial^2 u}{\partial z_2\partial z_1}dz_1\wedge dz_2+\frac{\partial^2 u}{\partial z_3\partial z_1}dz_1\wedge dz_3+\frac{\partial^2 u}{\partial z_1\partial z_2}dz_2\wedge dz_1+\frac{\partial^2 u}{\partial z_3\partial z_2}dz_2\wedge dz_3+\right.\right.$$

$$+\frac{\partial^2 u}{\partial z_1 \partial z_3} dz_3 \wedge dz_1 + \frac{\partial^2 u}{\partial z_2 \partial z_3} dz_3 \wedge dz_2 - \frac{\partial^2 u}{\partial z_1 \partial \overline{z_1}} d\overline{z_1} \wedge dz_1 - \frac{\partial^2 u}{\partial z_2 \partial \overline{z_1}} d\overline{z_1} \wedge dz_2 -$$

$$-\frac{\partial^2 u}{\partial z_3 \partial \overline{z_1}} d\overline{z_1} \wedge dz_3 - \frac{\partial^2 u}{\partial z_1 \partial \overline{z_2}} d\overline{z_2} \wedge dz_2 - \frac{\partial^2 u}{\partial z_2 \partial \overline{z_2}} d\overline{z_2} \wedge dz_2 - \frac{\partial^2 u}{\partial z_3 \partial \overline{z_2}} d\overline{z_2} \wedge dz_3 -$$

$$\left.-\frac{\partial^2 u}{\partial z_1 \partial \overline{z_3}} d\overline{z_3} \wedge dz_1 - \frac{\partial^2 u}{\partial z_2 \partial \overline{z_3}} d\overline{z_3} \wedge dz_2 - \frac{\partial^2 u}{\partial z_3 \partial \overline{z_3}} d\overline{z_3} \wedge dz_3 \right) + \frac{\partial^2 u}{\partial \overline{z_1} \partial z_1} dz_1 \wedge d\overline{z_1} +$$

$$+\frac{\partial^2 u}{\partial \overline{z_2} \partial z_1} dz_1 \wedge d\overline{z_2} + \frac{\partial^2 u}{\partial \overline{z_3} \partial z_1} dz_1 \wedge d\overline{z_3} + \frac{\partial^2 u}{\partial \overline{z_1} \partial z_2} dz_2 \wedge d\overline{z_1} + \frac{\partial^2 u}{\partial \overline{z_2} \partial z_2} dz_2 \wedge d\overline{z_2} +$$

$$+\frac{\partial^2 u}{\partial \overline{z_3} \partial z_2} dz_2 \wedge d\overline{z_3} + \frac{\partial^2 u}{\partial \overline{z_1} \partial z_3} dz_3 \wedge d\overline{z_1} + \frac{\partial^2 u}{\partial \overline{z_2} \partial z_3} dz_3 \wedge d\overline{z_2} + \frac{\partial^2 u}{\partial \overline{z_3} \partial z_3} dz_3 \wedge d\overline{z_3} -$$

$$-\frac{\partial^2 u}{\partial \overline{z_2} \partial \overline{z_1}} d\overline{z_1} \wedge d\overline{z_2} - \frac{\partial^2 u}{\partial \overline{z_3} \partial \overline{z_1}} d\overline{z_1} \wedge d\overline{z_3} - \frac{\partial^2 u}{\partial \overline{z_1} \partial \overline{z_1}} d\overline{z_2} \wedge d\overline{z_1} - \frac{\partial^2 u}{\partial \overline{z_3} \partial \overline{z_2}} d\overline{z_2} \wedge d\overline{z_3} -$$

$$\left.\left.-\frac{\partial^2 u}{\partial \overline{z_1} \partial \overline{z_3}} d\overline{z_3} \wedge d\overline{z_1} - \frac{\partial^2 u}{\partial \overline{z_2} \partial \overline{z_3}} d\overline{z_3} \wedge d\overline{z_2} \right)\right] =$$

$$= \frac{1}{4i}\left(2\frac{\partial^2 u}{\partial \overline{z_1} \partial z_1} dz_1 \wedge d\overline{z_1} + 2\frac{\partial^2 u}{\partial \overline{z_2} \partial z_1} dz_1 \wedge d\overline{z_2} + 2\frac{\partial^2 u}{\partial \overline{z_3} \partial z_1} dz_1 \wedge d\overline{z_3} + \right.$$

$$+2\frac{\partial^2 u}{\partial \overline{z_2} \partial z_1} dz_1 \wedge d\overline{z_2} + 2\frac{\partial^2 u}{\partial \overline{z_2} \partial z_3} dz_3 \wedge d\overline{z_2} + 2\frac{\partial^2 u}{\partial \overline{z_3} \partial z_3} dz_3 \wedge d\overline{z_3} +$$

$$\left. 2\frac{\partial^2 u}{\partial z_2 \partial \overline{z_1}} d\overline{z_1} \wedge dz_2 + 2\frac{\partial^2 u}{\partial \overline{z_3} \partial z_1} dz_2 \wedge d\overline{z_3} + 2\frac{\partial^2 u}{\partial \overline{z_1} \partial z_3} dz_3 \wedge d\overline{z_1} \right) =$$

$$= \frac{1}{2i}\; u_{1\bar{1}} dz_1 \wedge d\bar{z}_1 + u_{1\bar{2}} dz_1 \wedge d\bar{z}_2 + u_{1\bar{3}} dz_1 \wedge d\bar{z}_3 + u_{2\bar{1}} dz_2 \wedge d\bar{z}_1 +$$

$$+u_{2\bar{2}} dz_2 \wedge d\bar{z}_2 + u_{2\bar{3}} dz_2 \wedge d\bar{z}_3 + u_{3\bar{1}} dz_3 \wedge d\bar{z}_1 + u_{3\bar{2}} dz_3 \wedge d\bar{z}_2 +$$

$$+u_{3\bar{3}} dz_3 \wedge d\bar{z}_3 = \frac{1}{2i}\sum_{j,k=1}^{3} u_{j\bar{k}} dz_j \wedge d\bar{z}_k .$$

Demak, $n=3$ uchun ham $dd^c u = \frac{i}{2}\partial\bar{\partial}u$ tenglik o`rinli ekan.

Ermit kvadratik formasi bu — $q(x) = \overline{\varphi(x,x)} = \varphi(x,x),$ ya'ni qo`shmasi o`ziga teng bo`lgan haqiqiy kvadratik forma.

2.2.1-ta'rif. Agar elementlari □ maydondan olingan $A=(a_{ik})$ matritsa uchun

$A^T = \overline{A}$ (ya'ni $\overline{a_{ik}} = a_{ki}$) tenglik bajarilsa, u ermit matritsasi deyiladi:

Unitar ([4] ga qarang) almashtirishlardan keyin u quyidagi diagonal formaga keltiriladi:

$$dd^c u = \frac{i}{2}\left[\lambda_1 dz_1 \wedge d\overline{z_1} + ... + \lambda_n dz_n \wedge d\overline{z_n}\right]$$

bunda $\lambda_1(u),..., \lambda_n(u) - \quad u_{j,\bar{k}}$ ermit matritsasini xos qiymatlari, ya'ni

$\lambda = (\lambda_1, \lambda_2, ..., \lambda_n) \in R^n$, haqiqiy qiymatlardan iborat.

Endi xos vektorlar va xos qiymat ta'riflarini ham eslatib o`tamiz:

2.2.2-ta'rif: Agar noldan farqli $x \in u$ vektor va λ son uchun $Ax = \lambda x$ tenglik o`rinli bo`lsa, u holda x vektorga A chiziqli almashtirishni xos vektori deyiladi,unga mos bo`lgan λ songa esa xos qiymat deyiladi.

$dd^c u^{\ n} = 0$ ko`rinishdagi tenglamaga Monje-Ampere tenglamasi deyiladi. Bunda

$$dd^c u^{\ n} = dd^c u \wedge dd^c u \wedge ... \wedge dd^c u$$

$n = 1$ bo`lganda $dd^c u = \Delta u$ Laplas operatori bilan ustma-ust tushadi.

Endi bu tasdiqni isbotlashga harakat qilamiz

$$dd^c u = \frac{i}{2}\partial\bar{\partial}u.$$

Faraz qilaylik, $f(z)$ funksiya $f(z) = u(x, y) + iv(x, y)$ $z_0 = x_0 + iy_0 \in D(D \in \mathbb{C})$ nuqtada $\mathbb{R}^2$ ma'noda differensiallanuvchi bo`lsin. Ushbu $du(x_0, y_0) + idv(x_0, y_0)$ ifoda $f(z)$ funksiyaning z_0 nuqtadagi differensiali deyiladi va $df(z_0)$ kabi belgilanadi:

$$df(z_0) = du(x_0, y_0) + idv(x_0, y_0)$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy \quad dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$$

Shularni e'tiborga olib,

$$df = \left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy\right) + i\left(\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy\right) = \left(\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right)dx +$$
$$+\left(\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}\right)dy = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy \qquad (2.2.2)$$

$$z = x + iy \quad \bar{z} = x - iy \quad dx = \frac{1}{2}\left(dz + d\bar{z}\right), dy = \frac{1}{2i}\left(dz - d\bar{z}\right) \tag{2.2.3}$$

(2.2.2) va (2.2.3) dan

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{\partial f}{\partial x}\cdot\frac{1}{2}\left(dz + d\bar{z}\right) + \frac{\partial f}{\partial y}\cdot\frac{1}{2i}\left(dz - d\bar{z}\right) =$$

$$\frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right)dz + \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right)d\bar{z}$$

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right) = \frac{1}{2}\left(\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial y}\right) + \frac{i}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{2.2.4}$$

$$\frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right) = \frac{1}{2}\left(\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}\right) + \frac{i}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{2.2.5}$$

$$df = \frac{\partial f}{\partial z}dz + \frac{\partial f}{\partial \bar{z}}d\bar{z}$$

$$dd^c u = \frac{i}{2}\partial\bar{\partial}u = \frac{i}{2}\frac{1}{4}\left(\frac{\partial^2 f}{\partial x^2} + i\frac{\partial^2 f}{\partial x \partial y} - i\frac{\partial^2 f}{\partial y \partial x} + \frac{\partial^2 f}{\partial y^2}\right) = 0 \;\Rightarrow$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

Ko`rish qiyin emaski, $\left(dd^c u\right)^m \wedge \beta^{n-m} = m!\left(n-m\right)!H_m\left(u\right)\beta^n$ (2.2.6)

Bunda $H_m\left(u\right) = \sum_{1\le j_1<\ldots<j_m\le n} \lambda_{j_1}, \lambda_{j_2}, \ldots \lambda_{j_m}$ $-$ gessian vektori

Shunday qilib, 2 marta silliq u funksiya uchun $\left(dd^c u\right)^m \wedge \beta^{n-m}$ operator gessian vektori bilan bog‘liq ekan.

2.2.2.Musbat gessianlar.

Berilgan $\lambda = \left(\lambda_1, \lambda_2, \ldots, \lambda_n\right) \in \mathbb{R}^n$ uchun

$$H_m\left(\lambda\right) = \sum_{1\le j_1<\ldots<j_m\le n} \lambda_{j_1}, \lambda_{j_2}, \ldots \lambda_{j_m} \tag{2.2.7}$$

Gessianni tushunish uchun quyidagi yordamchi ifodadan foydalanamiz:

$$\left(t+\lambda_1\right)\left(t+\lambda_2\right)\ldots\left(t+\lambda_n\right) = t^n + H_1\left(\lambda\right)t^{n-1} + \ldots + H_n\left(\lambda\right), \quad t \in \mathbb{R} \tag{2.2.8}$$

(2.2.8) ni $n=2$ uchun yozib ko`ramiz:

$$(t+\lambda_1)(t+\lambda_2)=t^2+(\lambda_1+\lambda_2)t+\lambda_1\lambda_2 \Rightarrow H_1(\lambda)=(\lambda_1+\lambda_2), \quad H_2(\lambda)=\lambda_1\lambda_2$$

(2.2.7) formula bo`yicha yozadigan bo`lsak,

$$H_1(\lambda)=\sum_{1\le j_1\le 2}\lambda_{j_1}=\lambda_1+\lambda_2,\ H_2(\lambda)=\sum_{1\le j_1<j_2\le 2}\lambda_{j_1}\lambda_{j_2}=\lambda_1\lambda_2$$

n=3 uchun :

$$(t+\lambda_1)(t+\lambda_2)(t+\lambda_3)=t^3+(\lambda_1+\lambda_2+\lambda_3)t^2+(\lambda_1\lambda_2+\lambda_3\lambda_2+\lambda_1\lambda_3)t+\lambda_1\lambda_2\lambda_3$$

(2.2.7) formula bo`yicha yozadigan bo`lsak,

$$H_1(\lambda)=\sum_{1\le j_1\le 3}\lambda_{j_1}=\lambda_1+\lambda_2+\lambda_3,$$

$$H_2(\lambda)=\sum_{1\le j_1<j_2\le 3}\lambda_{j_1}\lambda_{j_2}=\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3$$

$$H_3(\lambda)=\sum_{1\le j_1<j_2<j_3\le 3}\lambda_{j_1}\lambda_{j_2}\lambda_{j_3}$$

Endi biz $H_m(\lambda)\ge 0$ bo`ladigan $\lambda\in\mathbb{R}^n$ vektorlar bilan qiziqamiz va shunday vektorlarni birlashtiramiz, ya'ni $S_m=\{\lambda\in\mathbb{R}^n, H_m(\lambda)\ge 0\}$

$m>1$ da bu to`plam qavariq konusni hosil qilmaydi, ya'ni $\lambda',\lambda''\in S_m$ ekanligidan $\lambda'+\lambda''\in S_m$ ekanligi kelib chiqmaydi.Shuning uchun bu to`plamni toraytiramiz, ya'ni $\lambda\in S_m$ vektorlar uchun shunday $\Gamma_m\subset S_m$ to`plamni olamizki, $\{\lambda+tI=(\lambda_1+t_1,...,\lambda_n+t_n), t\in\mathbb{R}^n_+\}\subset S_m$ bo`ladi. Unda Γ_m ning ko`rinishi $\Gamma_m=\{H_1(\lambda)\ge 0,...,H_m(\lambda)\ge 0\}$ bo`ladi.Undan tashqari Γ_m $(1,1,...1)$ vektorni o`z ichiga oluvchi $S_m$ to`plamni $S_m^0=\{\lambda\in\mathbb{R}^n : H_m(\lambda)>0\}$ ochiq yadrosining bog'lamli komponentasini yopig'i bilan ustma-ust tushadi.

Γ_m **quyidagi xossalarga ega:**

1) $\Gamma_n\subset\Gamma_{n-1}\subset...\subset\Gamma_1$ $(\Gamma_n=\{\lambda_1\ge 0,...\lambda_n\ge 0\}=\mathbb{R}^n_+)$

Biz bu xossani $n=3$ bo`lgan holda qaraymiz va $\Gamma_3=\{H_1(\lambda)\ge 0, H_2(\lambda)\ge 0, H_3(\lambda)\ge 0\}$ ekanligidan

$$H_1\;\lambda\; = \sum_{1\le j_1\le 3}\lambda_{j_1} = \lambda_1+\lambda_2+\lambda_3 \ge 0$$

$$H_2\;\lambda\; = \sum_{1\le j_1<j_2\le 3}\lambda_{j_1}\lambda_{j_2} = \lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3 \ge 0 \quad \Rightarrow \lambda_1\ge 0, \lambda_2\ge 0, \lambda_3\ge 0$$

$$H_3\;\lambda\; = \sum_{1\le j_1<j_2<j_3\le 3}\lambda_{j_1}\lambda_{j_2}\lambda_{j_3} = \lambda_1\lambda_2\lambda_3 \ge 0$$

bo`lishi kelib chiqadi.

$\Gamma_2=\{H_1\;\lambda\;\ge 0, H_2\;\lambda\;\ge 0\}$ esa

$$H_1\;\lambda\; = \sum_{1\le j_1\le 3}\lambda_{j_1} = \lambda_1+\lambda_2+\lambda_3 \ge 0$$

$$H_2\;\lambda\; = \sum_{1\le j_1<j_2\le 3}\lambda_{j_1}\lambda_{j_2} = \lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3 \ge 0$$

shartlarni qanoatlantiradi.

$\Gamma_1=\{H_1\;\lambda\;\ge 0\}$ ekanligidan esa $H_1\;\lambda\; = \sum_{1\le j_1\le 3}\lambda_{j_1} = \lambda_1+\lambda_2+\lambda_3 \ge 0$ tengsizlikning bajarilishi yetarli ekan.

2)Γ_m qavariq konusdan iborat, ya'ni agar $\lambda,\mu\in\Gamma_m$ bo`lsa, u holda $\forall a,b\in\mathbb{R}^+$ uchun $a\lambda+b\mu\in\Gamma_m$ o`rinli.

3) $H_m^{1/m}\;\lambda\;,\lambda\in\Gamma_m$ o`suvchi va botiq konusdan iborat, ya'ni

$$H_m^{1/m}\;\mu\;\le H_m^{1/m}\;\lambda\;+\sum_j\;\mu_j-\lambda_j\;\frac{\partial}{\partial\lambda_j}H_m^{1/m}\;\lambda\;\;\forall\lambda,\mu\in\Gamma_m$$

Soddalik uchun m=1, n=1 bo`lgan holni qarab chiqamiz:

$$H_1\;\mu\;\le H_1\;\lambda\;+\;\mu-\lambda\;\frac{\partial}{\partial\lambda}H_1\;\lambda\quad\forall\lambda,\mu\in\Gamma_m\Rightarrow$$

$$\mu\le\lambda+\;\mu-\lambda\;\Rightarrow\mu=\mu$$

Endi $m=1, n=2$ bo`lgan holni qaraymiz:

$\mu_1+\mu_2\le\lambda_1+\lambda_2+\;\mu_1-\lambda_1\;+\;\mu_2-\lambda_2$ demak, $n=k, m=1$ uchun hamma vaqt tenglik bajarilar ekan. $m=2, n=2$ bo`lgan holda esa quyidagicha ko`rinishga o`tadi:

$$\sqrt{\mu_1\mu_2} \le \sqrt{\lambda_1\lambda_2} + (\mu_1-\lambda_1)\frac{\lambda_2}{2\sqrt{\lambda_1\lambda_2}} + (\mu_2-\lambda_2)\frac{\lambda_1}{2\sqrt{\lambda_1\lambda_2}}$$

$$\sqrt{\mu_1\mu_2} \le \frac{2\lambda_1\lambda_2+\mu_1\lambda_2-2\lambda_1\lambda_2+\mu_2\lambda_1}{2\sqrt{\lambda_1\lambda_2}} \Rightarrow$$

$$2\sqrt{\lambda_1\lambda_2\mu_1\mu_2} \le \mu_1\lambda_2+\mu_2\lambda_1,\ a=\mu_1\lambda_2, b=\mu_2\lambda_1$$

$$\Rightarrow a+b \ge 2\sqrt{ab}$$

4) $\sum_j \mu_j \frac{\partial}{\partial\lambda_j} H_m(\lambda) \ge m H_m^{1/m}(\mu)\, H_m^{1-1/m}(\lambda)\quad \forall\lambda,\mu\in\Gamma_m$

$m=1, n=1$ bo`lsa, $\mu\cdot 1\ge\mu, \Rightarrow \mu=\mu$

$m=1, n=2 \Rightarrow \mu_1+\mu_2\ge\mu_1+\mu_2$

$m=2,\ n=2 \Rightarrow \mu_1\lambda_2+\mu_2\lambda_1 \ge 2\sqrt{\mu_1\mu_2}\sqrt{\lambda_1\lambda_2}$

$$m=2, n=3 \Rightarrow \mu_1\frac{\partial}{\partial\lambda_1}(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_3\lambda_2)+$$

$$+\mu_2\frac{\partial}{\partial\lambda_2}(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_3\lambda_2)+\mu_3\frac{\partial}{\partial\lambda_3}(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_3\lambda_2)\ge$$

$$\ge 2\sqrt{\mu_1\mu_2+\mu_1\mu_3+\mu_3\mu_2}\cdot\sqrt{\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_3\lambda_2}$$

$$\mu_1(\lambda_2+\lambda_3)+\mu_2(\lambda_1+\lambda_3)+\mu_3(\lambda_1+\lambda_2)\ge 2\sqrt{\mu_1\mu_2+\mu_1\mu_3+\mu_3\mu_2}\cdot\sqrt{\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_3\lambda_2}$$

5) $\left[\frac{H_m}{\binom{n}{m}}\right]^{1/m} \le \left[\frac{H_m}{\binom{n}{k}}\right]^{1/k}\quad \forall\ 1\le k\le m\le n$ Bu tengsizlik Makloren tengsizligi deb yuritiladi.

2.2.3. Musbat gessianlar bilan differensial formalar

Bizga $(1,1)$ - bidarajali $\alpha=\frac{i}{2}\sum_{j,k} a_{j\bar{k}}\, dz_j\wedge d\bar{z}_k$ haqiqiy differensial forma berilgan bo`lsin. Bunda $(a_{j\bar{k}})$ – ermit matrisa. Agar $\lambda(\alpha)=(\lambda_1(\alpha),...,\lambda_n(\alpha))$ – bu matrisaning xos qiymati bo`lsa, u holda $H_m(\alpha)=H_m(\lambda(\alpha))$ gessian aniqlash mumkin. Bu esa (1,1) – ermit formasi va $\lambda\in\mathbb{R}^n$ vektor orasidagi bog‘liklikni aniqlashga yordam beradi:

$$\hat{\Gamma}_m=\{\alpha:\lambda(\alpha)\in\Gamma_m\}=\left\{\begin{matrix} H_m(\alpha+t_1dd^c|z_1|^2+...+t_ndd^c|z_n|^2)\ge 0, \\ \forall t=(t_1,...,t_n)\in\mathbb{R}_+^n \end{matrix}\right\}= \qquad (2.2.9)$$

$$=\{\alpha\wedge\beta^{n-1}\ge 0,...,\alpha^m\wedge\beta^{n-m}\ge 0\}.$$

Gessianning 2-xossasiga ko`ra agar $\lambda',\lambda''\in\Gamma_m$ bo`lsa, u holda $a,b\in\mathbb{R}_+$ uchun

$a\lambda' + b\lambda'' \in \Gamma_m$ bo`ladi. Biz λ' sifatida α differensial formaning xos qiymatlarini, λ'' sifatida $\lambda'' = (t_1, t_2, ..., t_n) \in \mathbb{R}^n_+$ ni belgilasak va $a, b = 1$ xususiy holni qarasak

$$H_m\left(\alpha + t_1 dd^c |z_1|^2 + ... + t_n dd^c |z_n|^2\right) \geq 0 \ \forall t = (t_1, ..., t_n) \in \mathbb{R}^n_+$$

ekanligiga ishonch hosil qilishimiz mumkin. Chunki Γ_m ni

$\Gamma_m = \{H_1(\lambda) \geq 0, ..., H_m(\lambda) \geq 0\}$ ko`rinishda aniqlagan edik.

Endi boshqa holatlarni ko`rib chiqamiz, ya'ni $\lambda' \notin \Gamma_m$ bo`lsin. Buni misollar yordamida tushunib olishimiz osonroq bo`ladi:

2.2.1-misol $\alpha = \frac{i}{2}\left(-7dz_1 \wedge d\overline{z_1} - 11dz_2 \wedge d\overline{z_2} + 3dz_3 \wedge d\overline{z_3}\right)$ bo`lsin.

$\lambda(\alpha) = (\lambda_1, \lambda_2, \lambda_3) = (-7, -11, 3) \notin \Gamma_2 = \{H_1(\lambda) \geq 0, H_2(\lambda) \geq 0\}$ Chunki

$$H_1(\lambda(\alpha)) = \lambda_1 + \lambda_2 + \lambda_3 = -7 - 11 + 3 = -15 < 0,$$
$$H_2(\lambda(\alpha)) = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = 77 - 21 - 33 = 23 > 0$$
$$H_3(\lambda(\alpha)) = \lambda_1\lambda_2\lambda_3 = 231 > 0$$

Biz $t = (t_1, t_2, t_3) \in \mathbb{R}^3_+$ ning ixtiyoriyligidan foydalanib $t = (6, 1, 1) \in \mathbb{R}^3_+$ deb tanlab olamiz va

$$H_2\left(\frac{i}{2}\left(-7dz_1 \wedge d\overline{z_1} - 11dz_2 \wedge d\overline{z_2} + 3dz_3 \wedge d\overline{z_3}\right) + 6dd^c |z_1|^2 + dd^c |z_2|^2 + dd^c |z_3|^2\right) =$$
$$= H_2(-1, -10, 4) = -44 + 10 = -34 < 0,$$

bo`lishini ko`ramiz. Xulosa qilishimiz uchun yana bir misolni ko`rib o`taylik:

2.2.2-misol. $\alpha = \frac{i}{2}\left(-2dz_1 \wedge d\overline{z_1} + dz_2 \wedge d\overline{z_2} + 3dz_3 \wedge d\overline{z_3}\right)$ bo`lsin.

$\lambda(\alpha) = (\lambda_1, \lambda_2, \lambda_3) = (-2, 1, 3) \notin \Gamma_2, \Gamma_3$ Chunki

$$H_1(\lambda(\alpha)) = \lambda_1 + \lambda_2 + \lambda_3 = -2 + 1 + 3 = 2 > 0$$
$$H_2(\lambda(\alpha)) = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = -2 - 6 + 3 = -5 < 0$$
$$H_3(\lambda(\alpha)) = \lambda_1\lambda_2\lambda_3 = -6 < 0$$

Biz $t = (t_1, t_2, t_3) \in \mathbb{R}^3_+$ ning ixtiyoriyligidan foydalanib $t = (0, 1, 1) \in \mathbb{R}^3_+$ deb tanlab olamiz va

$$H_2\left(\frac{i}{2}\left(-2dz_1\wedge d\overline{z_1}+1dz_2\wedge d\overline{z_2}+3dz_3\wedge d\overline{z_3}\right)+dd^c|z_2|^2+dd^c|z_3|^2\right)=$$

$$=H_2(-2,2,4)=-4-8+8=-4<0$$

tengsizlik o`rinli bo`ladi. Agar $t=(1,3,5)\in\mathbb{R}^3_+$ deb olsak, unda ushbu

$$H_3\left(\frac{i}{2}\left(-2dz_1\wedge d\overline{z_1}+1dz_2\wedge d\overline{z_2}+3dz_3\wedge d\overline{z_3}\right)+dd^c|z_1|^2+3dd^c|z_2|^2+5dd^c|z_3|^2\right)=$$

$$=H_3(-1,4,8)=-32<0$$

ifodaga ega bo`lamiz. Shunday qilib, biz quyidagi xulosaga keldik:

$\lambda(\alpha)\in\Gamma_m$ bo`lgandagina $H_m\left(\alpha+t_1dd^c|z_1|^2+...+t_ndd^c|z_n|^2\right)\geq 0\ \forall t=(t_1,...,t_n)\in\mathbb{R}^n_+$ tengsizlik o`rinli bo`lar ekan. Boshqa hollarda bu tengsizlik bajarilmas ekan.

2.2.1-teorema.[18] Agar $\alpha_1,...,\alpha_k\in\hat{\Gamma}_m$, $1\leq k\leq m$ bo`lsa, u holda $\alpha_1\wedge...\wedge\alpha_k\wedge\beta^{n-m}\geq 0$, bo`ladi.

II BOB bo`yicha xulosa

Ushbu bobda differensial forma va oqimlar, musbat aniqlangan differensial formalar o`rganildi. Xususan, musbat gessianlar bilan differensial formalar orasidagi bog`lanish o`rganildi. Musbat gessianlardan biz ikki marta silliq $m-sh$ funksiyalarga ta’rif berishda va ularning xossalarini isbotlashda hamda qat’iy $m-sh$ (ikki marta silliq funksiyalar uchun) funksiyalarni ta’riflashda foydalanamiz.

III BOB. QAT'IY $m-sh$ FUNKSIYALAR

3.1. $m-sh$ funksiylar va ularning xossalari

3.1.1-ta'rif. Ikki marta silliq $u\in C^2(D)$ funksiya ($D\subset\mathbb{C}^n$) $z^0\in D$ nuqtada $m-sh$ $(1\le m\le n)$ deyiladi, agar $(u_{j\bar k})|_{z=z^0}$ matrisani $\lambda(u)=(\lambda_1(u),...,\lambda_n(u))$ xos qiymatlari Γ_m ga tegishli bo`lsa yoki $dd^c u\in\hat{\Gamma}_m$ bo`lsa. $u\in C^2(D)$ funksiya $D\subset\mathbb{C}^n$ sohada $m-sh,$ $(1\le m\le n)$ deyiladi, agar u har bir $z^0\in D$ nuqtada $m-sh$ bo`lsa. Boshqacha aytganda, $u\in C^2(D)$ funksiya $m-sh$ $(1\le m\le n)$ deyiladi, agar $dd^c u\in\hat{\Gamma}_m$ bo`lsa yoki unga quyidagi tengsizlik teng kuchli:

$$(dd^c u)^k\wedge\beta^{n-k}\ge 0,\ \ \forall\ k=1,2,...,m \qquad (3.1.1)$$

(3.1.1) ga quyidagi tengsizlik ham ekvivalent:

$$\left[\left(dd^c u+t_1 dd^c|z_1|^2+...+t_n dd^c|z_n|^2\right)\right]^m\wedge\beta^{n-m}\ge 0\ \forall\ t=(t_1,...,t_n)\in\mathbb{R}^n_+]$$

3.1.1-misol. $u(z)=-|z_1|^2+2|z_2|^2+3|z_3|^2$ funksiya **(**$m=1,2$ bo'lgan hol uchun**)** $m-sh$ bo'ladi.

$$dd^c u=\frac{i}{2}\left(-dz_1\wedge d\overline{z_1}+2dz_2\wedge d\overline{z_2}+3dz_3\wedge d\overline{z_3}\right)$$

$k=1$ bo'lganda

$$dd^c u\wedge\left(dd^c|z|^2\right)^2=\frac{i}{2}\left(-dz_1\wedge d\overline{z_1}+2dz_2\wedge d\overline{z_2}+3dz_3\wedge d\overline{z_3}\right)\wedge$$

$$\wedge\frac{i}{2}\left(dz_1\wedge d\overline{z_1}+dz_2\wedge d\overline{z_2}+dz_3\wedge d\overline{z_3}\right)\wedge\frac{i}{2}\left(dz_1\wedge d\overline{z_1}+dz_2\wedge d\overline{z_2}+dz_3\wedge d\overline{z_3}\right)=$$

$$=\left(\frac{i}{2}\right)^3\left(-dz_1\wedge d\overline{z_1}+2dz_2\wedge d\overline{z_2}+3dz_3\wedge d\overline{z_3}\right)\wedge(dz_1\wedge d\overline{z_1}\wedge dz_2\wedge d\overline{z_2}+$$

$$+dz_1\wedge d\overline{z_1}\wedge dz_3\wedge d\overline{z_3}+dz_2\wedge d\overline{z_2}\wedge dz_1\wedge d\overline{z_1}+dz_2\wedge d\overline{z_2}\wedge dz_3\wedge d\overline{z_3}+$$
$$+dz_3\wedge d\overline{z_3}\wedge dz_1\wedge d\overline{z_1}+dz_3\wedge d\overline{z_3}\wedge dz_2\wedge d\overline{z_2})$$

Tashqi ko'paytmada

$dz_k\wedge dz_j=-dz_j\wedge dz_k$ bo'lishidan foydalansak,

$$dd^c u \;\wedge\; dd^c|z|^2 \;{}^2 = \left(\frac{i}{2}\right)^3 \;-dz_1\wedge d\overline{z_1} + 2dz_2\wedge d\overline{z_2} + 3dz_3\wedge d\overline{z_3}$$

$$\wedge 2(dz_1\wedge d\overline{z_1}\wedge dz_2\wedge d\overline{z_2} +$$

$$+dz_1\wedge d\overline{z_1}\wedge dz_3\wedge d\overline{z_3} + dz_2\wedge d\overline{z_2}\wedge dz_3\wedge d\overline{z_3}) =$$

$$= \left(\frac{i}{2}\right)^3 4dz_1\wedge d\overline{z_1}\wedge dz_2\wedge d\overline{z_2}\wedge dz_3\wedge d\overline{z_3}$$

$$dz_1 = dx_1 + idy_1, \quad d\overline{z_1} = dx_1 - idy_1$$

$$dz_1\wedge d\overline{z_1} = \; dx_1 + idy_1 \quad dx_1 - idy_1 \; = -idx_1\wedge dy_1 + idy_1\wedge dx_1 =$$

$$= -2idx_1\wedge dy_1$$

ekanidan foydalanib

$$dz_1\wedge d\overline{z_1}\wedge dz_2\wedge d\overline{z_2}\wedge dz_3\wedge d\overline{z_3} = \; -2i \;{}^3 dx_1\wedge dy_1\wedge dx_2\wedge dy_2\wedge dx_3\wedge dy_3$$

SHunday qilib,

$$2\left(\frac{i}{2}\right)^3 \;8i\; dx_1\wedge dy_1\wedge dx_2\wedge dy_2\wedge dx_3\wedge dy_3 =$$

$$= 2dx_1\wedge dy_1\wedge dx_2\wedge dy_2\wedge dx_3\wedge dy_3 \geq 0$$

Demak, $dd^c u \;\wedge\; dd^c|z|^2 \;{}^2 \geq 0$ bo‘lishi uchun $\lambda_1+\lambda_2+\lambda_3 \;\geq 0$ bo‘lishi kerak

ekan.

Endi $k=2$ uchun (3.1.1) tengsizlikni tekshiramiz:

$$dd^c u \;{}^2 \wedge\; dd^c|z|^2 \; = dd^c u\wedge dd^c u\wedge dd^c|z|^2 =$$

$$= \left(\frac{i}{2}\right)^2 \;-dz_1\wedge d\overline{z_1} + 2dz_2\wedge d\overline{z_2} + 3dz_3\wedge d\overline{z_3} \;\wedge$$

$$\wedge\; -dz_1\wedge d\overline{z_1} + 2dz_2\wedge d\overline{z_2} + 3dz_3\wedge d\overline{z_3} \;\wedge$$

$$\wedge\frac{i}{2}\; dz_1\wedge d\overline{z_1} + dz_2\wedge d\overline{z_2} + dz_3\wedge d\overline{z_3} \; =$$

$$= 2\left(\frac{i}{2}\right)^3 dz_1\wedge d\overline{z_1}\wedge dz_2\wedge d\overline{z_2}\wedge dz_3\wedge d\overline{z_3} \geq 0$$

$$\Rightarrow \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \geq 0$$

$k=3$ da esa:

$$(dd^c u)^3 = -6\left(\frac{i}{2}\right)^3 dz_1 \wedge d\overline{z_1} \wedge dz_2 \wedge d\overline{z_2} \wedge dz_3 \wedge d\overline{z_3}$$

Demak , $\lambda_1\lambda_2\lambda_3 < 0$ ekan.

3.1.2-misol. $u(z) = -7|z_1|^2 - 11|z_2|^2 + 3|z_3|^2$ .Bu funksiya uchun ham yuqoridagi mulohazalarni yuritish mumkin.

$\lambda_1 = -7,\ \lambda_2 = -11,\ \lambda_3 = 3$ bo'lganda $k=2$ uchun bajarilgan tengsizlik $k=1$ uchun bajarilmaydi, ya'ni $(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_3\lambda_2) = 77 - 21 - 33 = 23 > 0$

$(\lambda_1 + \lambda_2 + \lambda_3) = -15 < 0$, $k=3$ uchun $\lambda_1\lambda_2\lambda_3 = 231 > 0$ o'rinli. Ammo bu funksiya plyurisubgarmonik ham , $m-sh$ ham emas. Demak, (3.1.1) tengsizlik $k=m$ uchun bajarilishidan qolgan barcha $k = \overline{1, m-1}$ lar uchun ham bajarilishi kelib chiqmas ekan.

Undan tashqari quyidagi tasdiq ham o`rinli:

$$dd^c u \wedge \alpha_1 \wedge ... \wedge \alpha_{k-1} \wedge \beta^{n-m} \geq 0 \quad \forall\ \alpha_1, ..., \alpha_{k-1} \in \hat{\Gamma}_m\ , 1 \leq k \leq m \qquad (3.1.2)$$

Bu tasdiq ikki xil xarakterga ega: agar u ikki marta silliq bo`lib, $\alpha_1, ..., \alpha_{m-1} \in \hat{\Gamma}_m$ uchun (3.1.2) shartni qanoatlantirsa, u holda u funksiya $m-sh$ bo`ladi.

u funksiyani $m-sh$ ligini tekshirish uchun (3.1.2) da faqat $\alpha_j = dd^c u_j\ , u_j \in C^2(D)\ ,\ j = 1, 2, ..., m-1$ differensial formani chegaralangan bo`lishi yoki $m-sh$ ko`phad bo`lishi yetarli.

Bu esa bizga L^1_{loc} funksiyalar sinfida $m-sh$ funksiyalarni aniqlashga yordam beradi.

3.1.2-ta'rif. $u \in L^1_{loc}(D)$ funksiya $D \subset \mathbb{C}^n$ sohada $m-sh$ deyiladi, agar u yuqoridan yarim uzluksiz va ixtiyoriy ikki marta silliq $v_1, ..., v_{m-1}$ $m-sh$ funksiyalar uchun $dd^c u \wedge dd^c v_1 \wedge ... \wedge dd^c v_{m-1} \wedge \beta^{n-m}$ oqim

$$\begin{aligned} &\left[dd^c u \wedge dd^c v_1 \wedge ... \wedge dd^c v_{m-1} \wedge \beta^{n-m}\right](\omega) = \\ &\int u\, dd^c v_1 \wedge ... \wedge dd^c v_{m-1} \wedge \beta^{n-m} \wedge dd^c \omega,\ \omega \in F^{0,0} \end{aligned} \qquad (3.1.3)$$

musbat aniqlangan bo`lsa.

Biz bilamizki, p,p – finit differensial formalar fazosidagi chiziqli, uzluksiz T funksional (kompleks qiymatli)

$$F^{p,p}=F^{p,p}\ D\ =\ \omega\in^{p,p}\ D\ \bigcap C^{\infty}\ D\ :\operatorname{supp}\omega\subset\subset D\quad,\ n-p,n-p\ =\ q,q$$

bidarajali oqim deyilar edi.

$m-sh$ funksiyalarning xossalari:

1) $u\in C^2\ D$ funksiya $m-sh$ $(1\le m\le n)$ bo`ladi, faqat va faqat shu holdaki, qachonki $dd^c u\in\hat{\Gamma}\ \forall z^0\in D$ bo`lsa.

2)agar $u\in m-sh\ D$ bo`lsa, u holda $u_j\ z\ =u*K_j\ z-w$ standart approksimatsiya (o`rama) ham $m-sh$ bo`ladi, ( $j\to\infty$ da $u_j\downarrow u$ bo`ladi)

3) agar $u,v\in m-sh$ bo`lsa, u holda ixtiyoriy $a,b\ge 0$ uchun $au+bv\in m-sh\ D$.

$au+bv\in m-sh\ D\quad\Rightarrow\lambda\ u\ ,\lambda\ v\ \in\Gamma_m$ $\qquad a,b\ge 0$ $\qquad$ ekanligidan $a\lambda\ u\ +b\lambda\ v\ \in\Gamma_m\Rightarrow au+bv\in m-sh\ D$ bo`lishi kelib chiqadi.

4) $psh=n-sh\subset...\subset 1-sh=sh$

Bu xossani $n=3$ hol uchun isbotlaymiz:

$$dd^c u^{\ k}\wedge\ dd^c|z|^{2\ \ 3-k}\ge 0,\quad k=1,2,3$$

$k=1$ bo`lganda $dd^c u\ \wedge\ dd^c|z|^{2\ \ 2}\ge 0$ bo`lishi uchun $\lambda_1+\lambda_2+\lambda_3\ \ge 0$ bo`lishi kerak ekan.

Endi $k=2$ uchun tengsizlikni tekshiramiz:

$$dd^c u^{\ 2}\wedge\ dd^c|z|^2\ \ge 0\Rightarrow\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3\ge 0$$

$k=3$ uchun:

$$dd^c u^{\ 3}=dd^c u\wedge dd^c u\wedge dd^c u=6\left(\frac{i}{2}\right)^3\lambda_1\lambda_2\lambda_3 dz_1\wedge d\overline{z_1}\wedge dz_2\wedge d\overline{z_2}\wedge dz_3\wedge d\overline{z_3}$$

$$\Rightarrow\qquad\qquad\lambda_1\lambda_2\lambda_3\ge 0$$

Yuqoridagi uchala tengsizlikni birgalikda yechamiz:

$$\begin{cases} \lambda_1 + \lambda_2 + \lambda_3 \ \ \geq 0 \\ \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \geq 0 \\ \lambda_1\lambda_2\lambda_3 \geq 0 \end{cases} \tag{3.1.4}$$

(3.1.4) tengsizlikning bajarilishi uchun λ_1 , λ_2 , λ_3 larning 2 tasi manfiy, bittasi musbat yoki uchalasi ham musbat bo\`lishi kerak. Ikkala holni ham tekshirib ko\`ramiz:

1) $\lambda_1 \geq 0,\ \lambda_2 \leq 0,\ \lambda_3 \leq 0$ bo\`lsin.

$$\begin{cases} \lambda_1 + \lambda_2 + \lambda_3 \ \ \geq 0 \\ \lambda_1\lambda_2 + \lambda_3\lambda_2 + \lambda_1\lambda_3 \geq 0 \end{cases} \quad \Rightarrow \quad \begin{cases} \lambda_1 \geq -(\lambda_2 + \lambda_3) \\ \lambda_1(\lambda_2 + \lambda_3) + \lambda_2\lambda_3 \geq 0 \end{cases}$$

$$\begin{cases} \lambda_1 \geq -(\lambda_2 + \lambda_3) \\ \lambda_2\lambda_3 \geq -\lambda_1(\lambda_2 + \lambda_3) \geq (\lambda_2 + \lambda_3)^2 \end{cases} \Rightarrow \begin{cases} \lambda_1 \geq -(\lambda_2 + \lambda_3) \\ (\lambda_2 + \lambda_3)^2 \leq \lambda_2\lambda_3 \end{cases}$$

$\lambda_2^{\ 2} + 2\lambda_2\lambda_3 + \lambda_3^{\ 2} \leq \lambda_2\lambda_3 \ \Rightarrow \lambda_2^{\ 2} + \lambda_3^{\ 2} \leq -\lambda_2\lambda_3$ Bu esa ziddiyat. Bu holda tengsizliklar sistemasi yechimga ega emas ekan. 2-holda tengsizliklar sistemasi doimo yechimga ega. Ravshanki, $m=1$ bo\`lganda m-subgarmonik funksiya subgarmonik funksiyaga aylanadi va $\lambda_1 + \lambda_2 + \lambda_3 \ \geq 0$ tengsizlikning bajarilishi yetarli.Yuqoridagilardan ko\`rinadiki, pyurisubgarmonik bo\`lgan funksiya albatta $m-sh$ va $sh$ funksiya bo\`la oladi. O\`z navbatida $m-sh$ bo\`lgan funksiya subgarmonik funksiya bo\`la oladi.

Endi quyidagi teoremani isbotlaymiz:

3.1.1-teorema.([24 ga qarang) $D \subset \mathbb{C}^n$ *sohada* n - sh *bo\`lgan ixtiyoriy funksiya* psh *dir.*

Isboti. Faraz qilaylik, (3.1.1) tengsizliklar barcha k lar uchun bajarilsin, $k = 1,2,...,n.$ Natijada quyidagi tengsizliklar sistemasi hosil bo\`ladi:

$$\begin{cases} \lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n \geq 0 \\ \lambda_1\lambda_2 + \lambda_1\lambda_3 + ... + \lambda_{n-1}\lambda_n \geq 0 \\ \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + ... + \lambda_{n-2}\lambda_{n-1}\lambda_n \geq 0 \\ \quad \quad \quad \quad . \ \ . \ \ . \\ \quad \lambda_1\lambda_2\lambda_3...\lambda_{n-1}\lambda_n \geq 0 \end{cases} \tag{3.1.5}$$

$\lambda_1, \lambda_2, \ldots \lambda_n$ larning haqiqiy sonlar ekanligidan (3.1.5) tengsizliklar sistemasini o`rganish uchun biz quyidagicha belgilashlarni kiritaylik:

$$\begin{cases} a_1 = \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_n \ \mathrm{i} \ 0 \\ a_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \ldots + \lambda_{n-1}\lambda_n \ \mathrm{i} \ 0 \\ a_3 = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \ldots + \lambda_{n-2}\lambda_{n-1}\lambda_n \ \mathrm{i} \ 0 \\ \quad \quad . \ . \ . \\ a_n = \lambda_1\lambda_2\lambda_3\ldots\lambda_{n-1}\lambda_n \ \mathrm{i} \ 0 \end{cases} \tag{3.1.6}$$

Endi koeffitsiyentlari $1, -a_1, a_2, \ldots, (-1)^n a_n$ bo`lgan

$$f(\lambda) = \lambda^n - a_1\lambda^{n-1} + a_2\lambda^{n-2} - a_3\lambda^{n-3} + \ldots + (-1)^n a_n \tag{3.1.7}$$

ko`rinishdagi ko`phadni qaraylik. Bu ko`phad uchun Viyet teoremasini qo`llasak (3.1.6) sistema hosil bo`ladi.

Bu ko`phadning ildizlari xarakterini aniqlash uchun quyidagi teoremadan foydalanamiz:

3.1.2-teorema (**Dekart) ([5] ga qarang)** $f(\lambda)$ ko`phadning karrasini hisobga olgan holda hisoblangan musbat ildizlarining soni bu ko`phadning koeffitsiyentlaridan tuzilgan sistemadagi ishora o`zgarishlar soniga teng yoki bu sondan juft songa kam bo`ladi. (Bunda nolga teng bo`lgan koeffitsiyentlar hisobga olinmaydi)

SHubhasiz, $f(\lambda)$ ko`phadning manfiy ildizlarining sonini topish uchun

Dekart teoremasini $f(-\lambda)$ ko`phadga qo`llash kifoyadir.

Ko`phadning barcha ildizlari haqiqiy ekanligi avvaldan ma'lum bo`lgan xususiy holda Dekart teoremasiga birmuncha aniqlik kiritish imkoni tug`iladi: *agar $f(\lambda)$ ko`phadning barcha ildizlari haqiqiy bo`lib, ozod had noldan farqli bo`lsa, u holda bu ko`phad musbat ildizlarining soni $k_1$ uning koeffitsiyentlari sistemasidagi ishora o`zgarishlar soni s_1 ga teng, manfiy ildizlarining soni k_2 esa $f(-\lambda)$ ko`phadning koeffitsiyentlari sistemasidagi ishora o`zgarishlar soni s_2 ga teng bo`ladi.*

Endi yuqoridagi tasdiqni biz qurgan (3.1.7) ko`phad uchun qo`llaylik. Oldin biz $a_1,a_2,...,a_n \neq 0$ bo`lgan holni qaraymiz. Agar $a_1,a_2,...,a_n \neq 0$ bo`lsa, u holda (3.1.7) ko`phadning koeffitsiyentlari sistemasi

$$1,-a_1,a_2,...,(-1)^n a_n$$

uchun $a_1,a_2,...,a_n > 0$ ligidan bu sistemadagi ishora o`zgarishlar soni $n$ ga teng bo`ladi. Bundan (3.1.7) ko`phad $n$ ta musbat ildizga ega bo`lishi kelib chiqadi. Bunda $\lambda_1,\lambda_2,...,\lambda_n > 0$ ekanini ko`ramiz. Endi ixtiyoriy $u \in C^2(D) \cap (m-sh(D))$ funksiyani quyidagi yordamchi funksiya bilan almashtiramiz:

$$v_\varepsilon(z) = u(z) + \varepsilon|z|^2 \in C^2(D) \cap (m-sh(D))$$

Bu funksiya uchun ham yuqoridagi mulohazalarni yuritish orqali koeffitsiyentlari $1,-a_1',a_2',...,(-1)^n a_n'$ bo`lgan

$$f(\lambda) = \lambda^n - a_1'\lambda^{n-1} + a_2'\lambda^{n-2} - a_3'\lambda^{n-3} + ... + (-1)^n a_n' \qquad (3.1.8)$$

ko`rinishdagi ko`phadni hosil qilamiz. Bu ko`phad uchun ham Viyet teoremasini qo`llasak ushbu

$$\begin{cases} a_1' = \lambda_1 + \varepsilon + \lambda_2 + \varepsilon + ... + \lambda_n + \varepsilon > 0 \\ a_2' = (\lambda_1+\varepsilon)(\lambda_2+\varepsilon) + (\lambda_1+\varepsilon)(\lambda_3+\varepsilon) + ... + (\lambda_{n-1}+\varepsilon)(\lambda_n+\varepsilon) > 0 \\ \quad . \quad . \quad . \\ a_n' = (\lambda_1+\varepsilon)(\lambda_2+\varepsilon)(\lambda_n+\varepsilon) > 0 \end{cases}$$

(3.1.9)

sistema hosil bo`ladi. $a_1',a_2',...,a_n' > 0$ bo`lgani uchun (3.1.8) ko`phad koeffitsiyentlari sistemasida ishora o`zgarishlar soni n ga teng bo`ldi. Demak, bu ko`phad n ta musbat $\lambda_1(v_\varepsilon),\lambda_2(v_\varepsilon),...,\lambda_n(v_\varepsilon)$ ildizga ega ekan. Bunda $\lambda_j = \lim_{\varepsilon \to 0} \lambda_j(v_\varepsilon) \geq 0,\ j = \overline{1,n}$. SHunday qilib, biz $\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, ..., \lambda_n \geq 0$ ekanligini aniqladik. Demak, agar $u \in C^2(D)$ funksiyasi uchun (1) tengsizliklar barcha k lar uchun bajarilsa, u holda $u(z)$ funksiya psh bo`lar ekan, ya'ni

$$n-sh(D)\cap C^{2}(D)=psh(D)\cap C^{2}(D). \qquad (3.1.10)$$

Endi biz (3.1.10) ni ixtiyoriy $n-sh$ funksiyalar uchun ko`rsatamiz: faraz qilaylik $u-$ ixtiyoriy $n-sh$ funksiya bo`lsin, $u\in n-sh(D)$. U holda $u_j=u*K_j(z-w)$ standart approksimatsiya olamiz,

$$u_j\in n-sh(D_j)\cap C^{\infty}(D_j),\ u_j\downarrow u,\ D_j\Subset D_{j+1}\Subset D,\ D=\bigcup_{j=1}^{\infty}D_j.$$

Unda (3.1.10) ga ko`ra $u_j\in psh(D_j)$ bo`lib, plyurisubgarmonik funksiyalarning xossasiga ko`ra $j\to\infty$ da $u_j\downarrow u$ bo`lganligidan $u\in psh(D)$ bo`ladi. **Teorema isbotlandi**.

5)agar $\gamma(t)-t\in\mathbb{R}_+$ parametrning o`suvchi qavariq funksiyasi va $u\in m-sh$ bo`lsa, u holda $\gamma\circ u\in m-sh$ bo`ladi.

6)tekis yaqinlashuvchi yoki kamayuvchi $m-sh$ funksiyalar ketma-ketligining limiti yana $m-sh$ bo`ladi.

7)chekli sondagi $m-sh$ funksiyalarning maksimumi $m-sh$ funksiyadan iborat bo`ladi; ixtiyoriy lokal tekis chegaralangan $\{u_\theta\}\subset m-sh$ m-subgarmonik funksiyalar oilasining supremumi $u(z)=\sup\limits_{\theta}u_\theta(z)$ regulyarizatsiyasi $u^*(z)$ ham $m-sh$ bo`ladi. $m-sh\subset sh$ ekanligidan $\{u(z)<u^*(z)\}-$toplam $\mathbb{C}^n\approx\mathbb{R}^{2n}$ da polyar toplam bo`ladi. Qisqacha aytganda uning Lebeg o`lchovi nolga teng. Ixtiyoriy lokal tekis chegaralangan $\{u_j\}\subset m-sh$ funksiyalar ketma-ketligining limiti $u(z)=\overline{\lim\limits_{j\to\infty}}u_j(z)$ regulyarizatsiyasi $u^*(z)$ ham $m-sh$ va $\{u(z)<u^*(z)\}-$polyar to`plam bo`ladi.

8)agar $u\in m-sh$ bo`lsa, u holda ixtiyoriy $P\subset\mathbb{C}^n$ kompleks gipertekislik uchun $u|_{P}\in sh_{m-1}$ bo`ladi.

Haqiqatdan, gessianning 4-xossasidan foydalanib, $\lambda,\mu\in\Gamma_m$ vektor uchun

$$\sum_j \mu_j \frac{\partial}{\partial \lambda_j} H_m(\lambda) \geq m H_m^{1/m}(\mu) H_m^{1-1/m}(\lambda)$$

$\mu = (0,...,0,1)$ deb oladigan bo`lsak, $\frac{\partial}{\partial \lambda_n} H_m(\lambda) = H_{m-1}('\lambda) \geq 0$. Bu shuni anglatadiki, agar $\lambda = (\lambda_1,...,\lambda_{n-1},\lambda_n) \in \Gamma_m$ bo`lsa, u holda $'\lambda = (\lambda_1,...,\lambda_{n-1}) \in \Gamma_{m-1}$ bo`ladi. Endi $u \in m-sh(D)$ va $P \subset \mathbb{C}^n$ gipertekislik bo`lsin.Umumiylikka zid ish qilmagan holda biz $u \in C^2(D)$, $\mathrm{P} = \{z_n = 0\}$ deb faraz qilamiz va $u|_{\mathrm{P}} = u('z,0) \in sh_m$ ekanini isbotlaymiz.

$\lambda = (\lambda_1,...,\lambda_n) - U = (u_{j\bar{k}})$, $j,k = 1,2,...,n$ matrisani xos qiymatlari bo`lsin. $\mu = (\mu_1,...,\mu_{n-1})$ – esa $'U = (u_{j\bar{k}})$, $j,k = 1,2,...,n-1$ matrisani xos qiymatlari bo`lsin. Unitar almashtirishlardan keyin (tayinlangan $('z^0,0) \in D$ nuqtada) U quyidagi ko`rinishga keladi:

$$U = \begin{pmatrix} \mu_1 & 0 & ... & 0 & u_{1\bar{n}} \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & \mu_{n-1} & u_{n-1\bar{n}} \\ u_{n\bar{1}} & u_{n\bar{2}} & ... & u_{n\overline{n-1}} & u_{n\bar{n}} \end{pmatrix}$$

$u(z) \in m-sh$ ekan, u holda (ixtiyoriy tayinlangan $t \in \mathbb{R}_+$ parametr uchun) $u_t(z) = u(z) + t|z_n|^2 \in m-sh$ bo`ladi.

Natijada $\lambda(t) = \lambda(u_t) \in \Gamma_m$ vektor yuqorida isbotlanganiga ko`ra $'\lambda(t) = (\lambda_1(t),...,\lambda_{n-1}(t)) \in \Gamma_{m-1}$. Boshqa tomondan $\lambda(t)$ quyidagi

$$\begin{vmatrix} \lambda - \mu_1 & 0 & ... & 0 & u_{1\bar{n}} \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & \lambda - \mu_{n-1} & u_{n-1\bar{n}} \\ u_{n\bar{1}} & u_{n\bar{2}} & ... & u_{n\overline{n-1}} & \lambda - t - u_{n\bar{n}} \end{vmatrix} = 0$$

tenglamaning ildizi.Yana almashtirishlar bajarsak,

$$\begin{vmatrix} \lambda-\mu_1 & 0 & \dots & 0 & \dfrac{u_{1\bar{n}}}{t} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda-\mu_{n-1} & \dfrac{u_{n-1\bar{n}}}{t} \\ u_{n\bar{1}} & u_{n\bar{2}} & \dots & u_{n\overline{n-1}} & \dfrac{\lambda-u_{n\bar{n}}}{t}-1 \end{vmatrix}=0$$

$t\to\infty$ da $\lambda_j(t)\to\mu_j$, $'\lambda(t)\in\Gamma_{m-1}$ ekan, u holda Γ_{m-1} ning yopiqligidan $\mu\in\Gamma_{m-1}$ ya'ni $u|_{P}=u'(z,0)\in m-sh$ ekanligi kelib chiqadi. **8-xossa isbotlandi.**

III BOB bo`yicha xulosa

Ushbu bobda $m-sh$ funksiyalar va ularning xossalari hamda qat'iy $m-sh$ funksiyalar o`rganildi. Ixtiyoriy $n-sh$ funksiyalar plyurisubgarmonik bo`lishi polinomlar nazariyasining asosiy teoremalaridan biri bo`lgan Dekart teoremasi yordamida isbotlandi. $m-sh$ funksiyalarning qat'iy $m-sh$ bo`lish shartlari o`rganildi. Olingan natijalar va usullarni funksiyalar nazariyasining keyingi rivojlanishida va kompleks analizning tadbiqlarida qo'llanilishi mumkin.

XULOSA

Dissertatsiya Qat’iy $m-sh$ funksiyalarni o‘rganishga bag‘ishlangan.

Dissertatsiyada olingan asosiy natijalar yuzasidan quyidagi xulosaga kelindi:

1. $\forall u \in C^2(D) \quad D \subset \mathbb{C}^n$ funksiya uchun $\left(dd^c u\right)^k \wedge \left(dd^c |z|^2\right)^{n-k} \geq 0 \quad \forall k = \overline{1,m}$ tengsizlikning $k=m$ uchun bajarilishidan qolgan barcha $\forall k = \overline{1, m-1}$ lar uchun bajarilmasligini aniqlandi.

2. $m-sh$ funksiyalar sinfi $m=n$ bo`lgan holda plyurisubgarmonik funksiyalar sinfi bilan ustma-ust tushishi isbotlandi.

3. $m-sh$ funksiyalarning qat’iy $m-sh$ bo`lish shartlari o`rganildi.

Umuman, dissertatsiya ishidagi olingan asosiy natijalar yangi bo‘lib, kompleks o‘zgaruvchining funksiyalar nazariyasining keyingi rivojida qo‘llanilishi mumkin.

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