“YANGI O‘ZBEKISTON” UNIVERSITETI HUZURIDAGI ILG‘OR TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSC.03/07.07.2025.FM/T.192.01 RAQAMLI ILMIY KENGASH

FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI ASTRONOMIYA INSTITUTI

TURIMOV BOBUR VALENTINOVICH

SKALYAR-TENZOR-VEKTOR GRAVITATSIYASIDA QORA O’RALARNING ENERGETIK XOSSALARI

01.03.01 – Astronomiya
01.04.02 – Nazariy fizika

FIZIKA-MATEMATIKA FANLARI DOKTORI (DSc) DISSERTATSIYASI AVTOREFARATI

TASHKENT-2026

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Dissertatsiyaning dolzarbligi va talab yuqoriligi. Gravitatsiyaning modifikatsiyalangan nazariyalari (Modified Theories of Gravity) umumiy nisbiylik nazariyasining (GR) kengaytirilgan shakllari bo‘lib, hozirda kosmologiya va astrofizika sohalarida katta e’tiborni qozonmoqda. Ushbu nazariyalar bir necha sabablarga ko‘ra ishlab chiqilmoqda, masalan, Koinotning tezlashib borayotgan kengayishini tushuntirish, umumiy nisbiylikning asosiy tamoyillarini qayta ko‘rib chiqish, hamda bu sohalardagi qadimiy muammolar — singulyarlik va kvant maydon nazariyasi bilan mos kelmaslik muammolarini hal etish maqsadida. Hozirgi vaqtda modifikatsiyalangan gravitatsiya nazariyalari ilmiy tadqiqotlarning markazida turibdi, chunki ular gravitatsiyaning fundamental jihatlari va Koinot tuzilmasi haqida yangi qarashlarni taqdim etishi mumkin. Biroq klassik gravitatsiya kvant maydonlarini kvantlashgan gravitatsiya bilan birlashtirishda jiddiy to‘siqlarga duch keladi. Shu nuqtai nazardan, skalyar-tenzor-vektor gravitatsiya (STVG) jozibali alternativ yondashuv sifatida paydo bo‘ladi. Bu yondashuvda gravitatsiya asosiy kuch sifatida emas, balki turli maydon turlari o‘zaro ta’siri natijasida vujudga keluvchi hodisa sifatida qaraladi. GR gravitatsiyani tasvirlashidagi farqlar va boshqa fundamental kuchlarning xatti-harakatidan ilhomlangan holda, STVG kontseptsiyasi faol tadqiqot va muhokama sohasiga aylangan bo‘lib, u Koinotning chuqur tuzilmasi va gravitatsiya tabiatiga yangi nazariy qarashlarni olib kelish salohiyatiga ega.

STVG nazariyasida gravitatsiya qo‘shimcha maydonlarni kiritish orqali fazo-vaqt egri chizig‘ini (curvature) hosil qiladi. Bu nazariya, shuningdek, modifikatsiyalangan gravitatsiya (MOG) deb ham ataladi, umumiy nisbiylikning eng sodda kengaytirilgan shakllaridan biri bo‘lib, u ayrim kosmologik va astrofizik muammolarni, xususan, qorong‘i modda ehtiyojini izohlashga qaratilgan. STVG umumiy nisbiylikka skalyar, tenzor va vektor maydonlarini qo‘shadi, bu esa gravitatsiyaning katta masshtablardagi xatti-harakatini o‘zgartiradi.

Nurlanish reaksiyasi (radiation reaction) — bu fizika hodisasi bo‘lib, unda zaryadlangan zarracha fazoda harakatlanayotganda elektromagnit nurlanish (fotonlar) chiqarishi natijasida tezligi kamayadi yoki ortadi. Ushbu effekt klassik elektromagnetizmga ko‘ra, zarrachaning o‘z elektromagnit maydoni bilan o‘zaro ta’siri natijasida yuzaga keladi. Zaryadlangan zarracha tezlashganda yoki sekinlashganda, u o‘zgaruvchan elektr va magnit maydonlarni hosil qiladi, ular esa elektromagnit nurlanishni keltirib chiqaradi. Bu nurlanish zarrachadan energiya va impul’sni olib chiqib ketadi, natijada zarracha kinetik energiyasini yo‘qotadi va tezligi kamayadi. Gravitatsiyaning asosiy tamoyillarini o‘zgartirish orqali, modifikatsiyalangan gravitatsiya nazariyalari galaktika aylanish egri chiziqlarini, gravitatsion linzalashni va Koinotdagi katta masshtabli tuzilmalarning dinamikasini tushuntirishga intiladi. Koinot miqyosida gravitatsiyani chuqurroq tushunish astrofizik hodisalarni to‘g‘ri talqin etish va Koinotni shakllantiruvchi fundamental kuchlar haqidagi bilimlarimizni chuqurlashtirish uchun nihoyatda muhimdir. Ushbu ish STVG doirasida Shvartsshild–MOG qora tuynugi atrofida sinov zarrachasining dinamik harakatidagi nurlanish reaksiyasi ta’sirini tahlil qilishga bag‘ishlangan.

Mazkur tadqiqot ishi quyidagi davlat me’yoriy hujjatlarida belgilangan vazifalarga muvofiq keladi: O‘zbekiston Respublikasi Prezidenti 2017-yil 7-fevraldagi PF–4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi Farmoni, shuningdek, 2017-yil 18-fevraldagi PQ–2789-son “Fanlar akademiyasi faoliyatini yanada takomillashtirish, ilmiy tadqiqot faoliyatini tashkil etish, boshqarish va moliyalashtirish chora-tadbirlari to‘g‘risida”gi Prezident qarori va boshqa me’yoriy hujjatlar.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Dissertatsiya tadqiqoti O‘zbekiston Respublikasidagi fan va texnologiyalarning ustuvor yo‘nalishlariga muvofiq amalga oshirilgan, xususan: II. "Energetika, energiya va resurslarni tejash" yo‘nalishi bo‘yicha.

Muammoning o‘rganilganlik darajasi. Kompakt gravitatsion obyektlar atrofidagi zarrachalar harakati dunyo bo‘yicha turli tadqiqotchilar (A. Ovgun, R. Pantig, G. Mustafa, F. Javed, A. Zakharov, J. Schee, M. Kolos, B. Ahmedov, N. Dadhich, S. Ghosh, P. Joshi va boshqalar) tomonidan o‘rganilgan. Biroq, skalyar–tenzor–vektor gravitatsiya (STVG) doirasida qora tuynuklar atrofidagi zarracha harakati tizimli ravishda o‘rganilmagan. Kompakt obyektlarning nol geodezikalari (ya’ni, yorug‘lik nurlari harakat yo‘llari) turli mualliflar (F. Atamurotov, A. de Vries, L. Rezzolla, V. Bozza, Z. Stuchlik, A. Abdujabbarov va boshqalar) tomonidan chuqur o‘rganilgan. Nol geodezikalarni va ular bilan bog‘liq optik xususiyatlarni, masalan, qora tuynuk soyalari kabi hodisalarni tadqiq etish, aylanayotgan qora tuynuk fazo-vaqtlarida hali ham faol ilmiy izlanish mavzusi bo‘lib qolmoqda va bu sohada ko‘plab tadqiqotchilar (A. Belhaj, M. Benali, J. Luminet va boshqalar) tomonidan ishlar olib borilgan. Shunga qaramay, skalyar–tenzor–vektor gravitatsiyaning MOG parametrlarining qora tuynuk soyasi va unga bog‘liq hodisalarga ta’siri masalasi hozirgacha yetarlicha o‘rganilmagan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya Innovatsion rivojlanish agentligi tomonidan moliyalashtirilgan F-FA-2021-510 "Modifikatsiyalangan gravitatsiyada neytron yulduzlar va ularning yadroviy materiyasini tadqiq qilish" ilmiy loyihasi doirasida bajarilgan (2021-2026).

Tadqiqotning maqsadi: skalyar–tenzor–vektor gravitatsiyaga asoslangan nazariy modellarning rivojlantirilishi va takomillashtirilishi hamda kompakt qora tuynuklar uchun mos yechimlarning ishlab chiqilishi.

Tadqiqotning vazifalari:

STVG doirasida Shvartsshild–MOG qora tuynug‘ining termodinamik xususiyatlarini o‘rganish va qora tuynukning bir qancha termodinamik kuzatiladigan miqdorlarini tahlil qilish;

Shvartsshild–MOG fazo-vaqtida geodezik harakatni tahlil qilish va bu jarayonda MOG parametrining ta’sirini o‘rganish;

Qora tuynuk atrofidagi fotonlarning harakatini tahlil qilish va foton sferasi hamda ta’sir parametri (impact parameter) uchun aniq analitik ifodalarni topish;

Shvartsshild–MOG fazo-vaqtida massiv zarrachalarning aylana harakatini o‘rganish, xususan, eng ichki barqaror aylana orbitani (ISCO) va chekka bog‘langan orbitani tahlil qilish;

Shvartsshild–MOG qora tuynugi atrofida aylanayotgan massiv zarrachalarning burchak tezligini tahlil qilish va uni umumiy nisbiylik nazariyasidagi Kepler burchak tezligi bilan solishtirish;

Yulduz qora tuynugi supermassiv qora tuynuk atrofida aylanayotganda yuzaga keladigan nurlanish reaksiyasining gravitatsion analogini o‘rganish;

Penrouz jarayoni orqali STVG nazariyasini sinovdan o‘tkazish va uning Kerr–MOG qora tuynug‘idan energiya ajralishiga ta’sirini tahlil qilish;

Kerr–MOG qora tuynugi kontekstida zarracha to‘qnashuv mexanizmini o‘rganish.

Tadqiqot ob’ekti: Qora o’ralar, muqobil gravitatsiya; skalyar-vektor-tenzor gravitatsiyasi.

Tadqiqot predmeti: foton va massive zarra harakati; elektromagnit maydon uchun Maxwell tenglamalarinign analitik yechimlari. Qora o’ra atrofida yuqori energitik jarayonlar.

Tadqiqot usullari —Skalyar bog‘langan fazo-vaqtlarda (JNW, Ellis) zarrachalar uchun harakatning aniq tenglamalari va effektiv potensiallarini keltirib chiqarish maqsadida analitik modellashtirishdan foydalanildi. Nurlanish reaksiyasi ta’siri ostidagi geodezik tenglamalarni yechish va zarrachalar trayektoriyalarini tahlil qilish uchun sonli simulyatsiyalar (Runge-Kutta integrallash usuli) qo‘llanildi. Kvazinormal modalar va g‘alayonlanish spektrlarini hisoblash uchun yarim analitik WKB yaqinlashuvi hamda vaqt sohasi bo‘yicha integrallash usullari tatbiq etildi. Nazariy kvazi-davriy tebranishlar (KDT) chastotalarini rentgen qo‘shaloq yulduzlaridan olingan kuzatuv ma’lumotlariga moslashtirish uchun Bayesian MCMC tahlili o‘tkazildi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Tadqiqot, skalyar bog‘lanishga ega JNW yalang‘och singulyarlik fazo-vaqtida zarrachalar dinamikasining birinchi to‘liq nazariy modelini yaratadi, bu esa uni qora o‘ralardan farqlovchi noyob IBAO xususiyatini ochib beradi.

Yangi topilmalar skalyar maydonlarning Ellis yumronqoziq ini fizikasini qanday o‘zgartirishini, shu jumladan nurlanish ta’sirida zarrachalarning qochishi va ekzotik ixcham ob’ektlarni qora o‘ralardan ajratib turuvchi xarakterli KDT chastotalarini namoyish etadi.

Ushbu ish Rentgen qo‘shaloq yulduzlari ma’lumotlaridan foydalangan holda skalyar maydon parametrlariga MCMC (Markov zanjiri Monte-Carlo usuli) asosidagi cheklovlarni kashf etadi va kuchli maydon rejimlarida modifikatsiyalangan gravitatsiya nazariyalarini sinash uchun yangi spektral imzolarni aniqlaydi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Skalyar bog‘lanishga ega fazo-vaqtlarda zarrachalar dinamikasi uchun keltirib chiqarilgan analitik ifodalar ekzotik kompakt ob’ektlar atrofidagi akkretsion disklar tuzilmalari va nurlanish profillarini aniq modellashtirish imkonini beradi.

Hisoblangan KDT chastotalari va IBAO radiuslari rentgen qo‘shaloq yulduzlari ma’lumotlaridan foydalanib, JNW yalang‘och singulyarliklari va Ellis yumronqoziq inlarini qora o‘ralardan farqlash uchun kuzatiladigan belgilarni taqdim etadi.

MCMC usuli yordamida moslashtirilgan skalyar maydon parametrlari ($n \sim 0.75$, $g_s \sim 0.2$) kelajakdagi gravitatsion to‘lqin va ko‘p tarmoqli kompaniyalar uchun sinovdan o‘tkazilishi mumkin bo‘lgan tahlillarni taklif qiladi.

Aniqlangan g‘alayonlanish spektrlari (masalan, birlashgan Heun yechimlari, ossillyatsion dumlar) yangi avlod rasadxonalarida skalyar-tenzor gravitatsiya effektlarini aniqlash uchun etalon (benchmark) bo‘lib xizmat qiladi.

Tadqiqot natijalarining ishonchliligi quyidagilardan iborat:

Analitik asos fundamental prinsiplardan qat’iy ravishda keltirib chiqarildi va barcha hisob-kitoblarda matematik izchillikni ta’minlash uchun umumiy nisbiylik nazariyasidagi tasdiqlangan natijalar bilan solishtirib tekshirildi.

Sonli simulyatsiyalarda bir nechta mustaqil usullar (Runge-Kutta integrallash usuli, WKB yaqinlashuvi) qo‘llanildi va ular bir-biriga mos keluvchi natijalarni berdi, bu esa zarrachalar dinamikasi va g‘alayonlanish (perturbatsiya) tahlillarining ishonchliligini tasdiqladi.

MCMC usulidagi parametrlar baholanishi yaqinlashish testlari va mavjud astrofizik cheklovlar bilan taqqoslash orqali tasdiqlandi, bu skalyar maydon bog‘lanish qiymatlarining statistik ishonchliligini namoyish etdi.

Barcha nazariy bashoratlar rentgen qo‘shaloq yulduzlaridan olingan kuzatuv ma’lumotlari va ma’lum bo‘lgan qora o‘ra fenomenologiyasi bilan mos keladi, shu bilan birga kelajakdagi tekshiruvlar uchun sinovdan o‘tkazilishi mumkin bo‘lgan og‘ishlarni aniq belgilaydi.

Tadqiqotning ilmiy va amaliy ahamiyati:

Tadqiqot ekzotik kompakt ob’ektlardagi skalyar maydon ta’sirlarini tahlil qilish uchun keng qamrovli asosni yaratadi va klassik qora o‘ra paradigmalaridan tashqarida gravitatsiya va fundamental maydonlar o‘rtasidagi o‘zaro ta’sir haqida yangi tushunchalarni taqdim etadi.

Keltirib chiqarilgan KDT chastotalari va IBAO modifikatsiyalari qora o‘ralar, yalang‘och singulyarliklar va yumronqoziq inlari o‘rtasidagi muhim kuzatuv diskriminatorlari (ajratuvchilari) bo‘lib xizmat qiladi va ular hozirgi rentgen teleskoplari ma’lumotlarini tahlil qilishda bevosita qo‘llanilishi mumkin.

Ishlab chiqilgan MCMC-KDT tahlil usuli va g‘alayonlanish metodlari astrofizik ma’lumotlardan foydalangan holda modifikatsiyalangan gravitatsiya nazariyalarini cheklash uchun kuchli yangi vositalarni taklif qiladi, bu esa gravitatsiyani yuqori aniqlikdagi sinovlardan o‘tkazish imkoniyatini sezilarli darajada oshiradi.

Skalyar-tenzor o‘zaro ta’sirlarining noyob spektral belgilarini aniqlash orqali, natijalar yangi avlod gravitatsion to‘lqin detektorlari va ko‘p tarmoqli astronomiya kampaniyalarini loyihalash talablarini shakllantirish uchun asos bo‘ladi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Modifikatsiyalangan gravitatsiya nazariyalarida qora o‘ralar atrofidagi zarralar dinamikasini o‘rganish natijalari quyidagi yo‘nalishlarda qo‘llanilgan:

Nazariy tadqiqot natijalari va usullari “Turimov, B., Alibekov, H., Tadjimuratov, P., Abdujabbarov, A., “Gravitational synchrotron radiation and Penrose process in STVG theory”, Phys. Lett. B 843, 138040, (2023), doi: 10.1016/j.physletb.2023.138040.” va “Turimov, B. V., “Comment on ‘Orbital precession of the S2 star in scalar–tensor–vector gravity’”, MNRAS 516, 434–436, (2022), doi:10.1093/mnras/stac2113" nomli ilmiy maqolalarda e'lon qilingan va B.Turimovning fan doktori (DSc) dissertatsiyasida taqdim etilgan. Mazkur ilmiy natijalar Fudan universiteti tomonidan qo'llab-quvvatlangan dasturlar doirasida qo'llanilgan (Prof. Cosimo Bambi tomonidan taqdim etilgan rasmiy xat asosida).

Tadqiqot natijalarining aprobatsiyasi. Dissertatsiya natijalari 5 ta xalqaro va 3 ta mahalliy konferensiyalarda muhokama qilingan.

Tadqiqot natijalarining e'lon qilinganligi. Tadqiqot natijalariga asosan 30 ta ilmiy nashr chop etilgan bo'lib, ularning 20 tasi xorijiy jurnallarda e'lon qilingan maqolalardir.

Dissertatsiya hajmi va tuzilishi. Dissertatsiya kirish qismi, to'rt bob, xulosa va adabiyotlar ro'yxatidan iborat bo'lib, jami 134 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning kirish qismida mavzuning dolzarbligi va zaruriyati, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi, muammoning o'rganilganlik darajasi, dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi, shuningdek, maqsad, vazifalar, tadqiqot ob'ekti, predmeti, metodlari, ilmiy yangiligi, amaliy natijasi, natijalarning ishonchliligi, ilmiy va amaliy ahamiyati, natijalarning amaliyotga joriy etilishi, aprobatsiyasi, e'lon qilinganligi hamda dissertatsiyaning tuzilishi va hajmi to'g'risida qisqacha ma'lumotlar keltirilgan. Dissertatsiyaning "Skalyar-tenzor-vektor gravitatsiyasida qora o'ralarning energetik xossalari" deb nomlangan birinchi bobida Schwarzschild-MOG qora o'ra atrofidagi zarrachalar dinamikasi o'rganiladi. Ikkinchi bobda Kerr-MOG qora o'ra yaqinda Penrose prosesi o'rganilgan. Shuningdek, Schwarzschild-MOG fazo-vaqtida vektor maydon ishtirokida massiv zarrachalarning aylanma harakatini ko'rib chiqamiz. Bundan tashqari, nurlanish reaksiyasi hadini hisobga olgan holda massiv zarrachalarning aylanma harakatini o'rganib, zarrachalar trayektoriyalarini taqdim etamiz. Ohirgi to'rtinchi bobda, qora o'ralarning g'alayonlanishi o'rganilgan. Sferik koordinatalarda $x^\alpha = (t, r, \theta, \phi)$, STVG nazariyasidagi end sodda qora tuynuk yechimi Schwarzschild-MOG fazo-vaqt metrikasi bilan ifodalanadi

$$ds^2 = -fdt^2 + f^{-1}dr^2 + r^2 d\Omega, \qquad f(r) = \left(1 - \frac{2GM}{r} + \frac{G\tilde{Q}^2}{r^2}\right) \quad (1)$$

bu yerda M qora o'ra massasi, G modifikatsiyalangan gravitatsion doimiysi Nyuton gravitatsion doimiysi G_N bilan quyidagicha bog'langan $G = G_N(1+\alpha)$. E'tibor etish kerakki, (1)-ifodadagi fazo-vaqt metrikasi Reissner–Nordström fazo-vaqtiga o'xshash, biroq $\tilde{Q} = \sqrt{\alpha G_N} M$ kattalik STVG nazariyasida tashqi potensialga tegishli bog'lanish doimiysi sifatida qaraladi vaquyidagicha beriladi

$$\Phi_\mu = \Phi_t(1,0,0,0), \qquad \Phi_t = -\frac{\tilde{Q}}{r} = -\frac{\sqrt{\alpha G_N} M}{r} \qquad (2)$$

va α musbat MOG parametri hisoblanadi. Schwarzschild–MOG qora tuynugining tashqi va ichki gorizontlari radiuslari quyidagicha: $r_\pm = G_N M(1 + \alpha \pm \sqrt{1+\alpha})$. Biz STVG doirasida qora tuynuk yaqinida massiv zarrachaning harakatini ko'rib chiqamiz. Umumiy nisbiylik nazariyasi va ko'plab boshqa gravitatsion nazariyalardan farqli ravishda, STVG'da massiv zarralar geodezik chiziqlar bo'ylab harakatlanmaydi. STVG nazariyasida sinov zarrachaning harakati quyidagi nogeodezik tenglama bilan ifodalanadi:

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = \frac{\tilde{q}}{m}B^\alpha_\beta\frac{dx^\beta}{ds}, \qquad B_{\alpha\beta} = \partial_\alpha\Phi_\beta - \partial_\beta\Phi_\alpha \qquad (3)$$

bu yerda m zarra massasi va $\tilde{q} = \sqrt{\alpha G_N} m$ zarracha bilan vektor maydon Φ_μ o'rtasidagi bog'lanish konstatasini ifodalaydi ,$\Gamma^\alpha_{\mu\nu}$ Kristoffel simvollari hisoblanadi and $\dot{x}^\alpha = dx^\alpha/ds$ zarraning 4-tezligi hisoblanib quyidagicha normallashadi $g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = -1$. Maydon kuchlanish tenzori $B^\mu{}_\nu$ ni o'z ichiga olgan qo'shimcha kuch hadi mavjudligi shuni ko'rsatadiki, STVG'da massiv zarralar vektor maydon bilan o'zaro ta'siri natijasida yuzaga keladigan, elektrodinamikadagi Lorens kuchiga o'xshash nogeodezik kuch ta'sirida bo'ladi. Schwarzschild–MOG qora tuynugi yaqinida foton harakatini ko'rib chiqqanimizda, $\tilde{q} = 0$ deb olish mumkin. Fotonning harakat tenglamasi quyidagicha ifodalanadi:

$$\frac{dr}{d\lambda} = \sqrt{\frac{1}{b^2} - \frac{f(r)}{r^2}}, \qquad \frac{d\phi}{d\lambda} = \frac{1}{r^2}, \qquad \frac{dt}{d\lambda} = \frac{1}{bf(r)} \tag{4}$$

bu yerda b fotonning zarba parametri (impact parameter), λ esa fotonning burchak momentiga nisbatan normallashtirilgan affinn parametrdir. Foton massiv obyekt yaqinidan o‘tganda, uning bir qismi qora tuynuk tomonidan tutib olinadi va linzalovchi obyekt soyasining chegarasini belgilovchi sferik orbitalar bo‘ylab harakatlanadi. Agar foton sferik geodeziklar bo‘ylab harakat qilsa, radial tenglama quyidagi shartlarni qanoatlantiradi.

$$\frac{dr}{d\lambda} = 0, \qquad \frac{d^2r}{d\lambda^2} = 0\,. \tag{5}$$

Yuqoridagi (5)-tenglamasining birinchi qismi yorug’lik nurining burilish nuqtasiga javob beradi, ikkinchi qismi esa sferik orbitaning radiusini ifodalaydi. Foton sferasining radiusi va fotonning kritik nishon parametri quyidagicha hisoblanadi:

$$r_{\rm ph} = \frac{1}{2}\left[3(1+\alpha) + \sqrt{(1+\alpha)(9+\alpha)}\right] G_N M, \qquad b = \frac{r_{\rm ph}}{\sqrt{f(r_{\rm ph})}}, \tag{6}$$

Yuqoridagi ifodalarda α parametri yo‘qligini osongina tekshirish mumkin: bu holda foton sferasi radiusi ($r_{\rm ph} = 3M$) va kritik zarba parametri ($b_0 = 3\sqrt{3}M$) ga teng bo‘ladi, bu Schwarzschild fazo-vaqtiga mos keladi. Qora tuynuk tomonidan fotonning tutib olinadigan maydoni (capture cross-section) quyidagicha aniqlanadi: ($\sigma = \pi b_0^2$), va Schwarzschild–MOG metrikasida u quyidagi ko‘rinishga ega bo‘ladi:

$$\sigma = \frac{2\pi\alpha M^2[3+3\alpha+\sqrt{(1+\alpha)(9+\alpha)}]^2}{\alpha+3+\sqrt{(1+\alpha)(9+\alpha)}}\ . \tag{7}$$

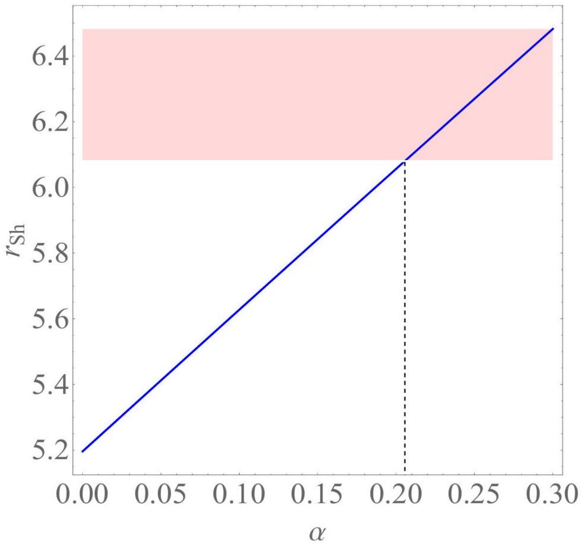


Figure 1.2: Capture cross section of photon by the black hole (shadow of the black hole) in the (x-z) plane for the different values of α parameter.

Qora tuynuk soyasining kattaligi haqida tasavvur hosil qilish uchun, million quyosh massasiga teng supermassiv qora tuynuk (SMBH) uchun tutib olinadigan maydonni quyidagi shaklda baholash mumkin:

$$\sigma \simeq 1.576 \times 10^{16} \left(\frac{M}{10^6 M_\odot}\right)^2 \ \text{km}^2. \tag{8}$$

E’tibor bering, soyadagi bu katta qiymat masofaning uzoqligi sababli kuzatuvchi nuqtadan juda kichik burchak o‘lchamiga ega bo‘ladi. Aniqroq tahlillarni grafik shaklda ham ko‘rsatish mumkin. 1.3-rasmda qora tuynuk tomonidan fotonning turli α qiymatlari uchun tutib olinadigan maydoni tasvirlangan. Bizning tahlillarimiz shuni ko‘rsatadiki, Schwarzschild–MOG fazo-vaqtida fotonning tutib olinadigan maydoni umumiy nisbiylikdagi qiymatdan kattaroq bo‘ladi.

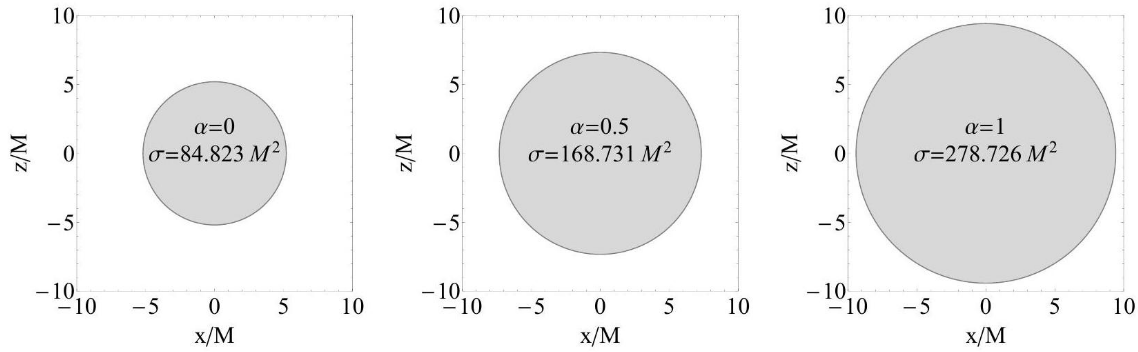


Figure 1.3: Capture cross section of photon by the black hole (shadow of the black hole) in the (x-z) plane for the different values of α parameter.

Nihoyat, kuchsiz maydon yaqinlashuvi holatida nurning og'ish burchagi quyidagicha aniqlanadi:

$$\hat{\alpha}_b = \frac{4(1+\alpha)M}{b} + \frac{(1+\alpha)[(15\pi-16)(\alpha+1)+8]M^2}{4b^2}. \tag{9}$$

Umumiy nisbiylik (GR) nazariyasiga ko'ra, statik, sferik simmetrik gravitatsion manba atrofidagi yorug'lik nurining burilishi $\hat{\alpha} = 4M/b$ bilan ifodalanadi. Bu yerda biz Schwarzschild–MOG fazo-vaqtida zaif gravitatsion linzalash effektini o'rganamiz. Kuchsiz gravitatsion linzalashni o'rganishning oddiy yondashuvlaridan biri metrik tenzorini quyidagicha $g_{\alpha\beta} \simeq \eta_{\alpha\beta} + h_{\alpha\beta}$ bo'ladi, bu yerda $\eta_{\alpha\beta}$ Minkovskiy fazo-vaqtining metrik tenzori, $h_{\alpha\beta}$ esa metrik tenzorning perturbatsiyasidir. Shu orqali fotonning og'ish burchagi aniqlanadi

$$\hat{\alpha}_b = \frac{4M(1+\alpha)}{b} + \frac{\pi M^2}{b^2}(1+\alpha)\left(1+\frac{\alpha}{4}\right). \tag{10}$$

Bu ifoda umumiy nisbiylikda olingan ifoda bilan bir xil, biroq belgisi teskari bo'lishi kerak. Chunki burilish burchagi vektor kattalik hisoblanadi, shuning uchun (10) ifodasidagi belgini osongina almashtirish mumkin.

Endi biz STVG doirasida Schwarzschild–MOG qora tuynugi atrofidagi massiv zarralar harakatini o'rganishga e'tibor qaratamiz. Avval aytib o'tganimizdek, massiv zarralar (3) tenglamasida berilgan Lorenssga o'xshash tenglamani bajaradi. Soddalashtirish maqsadida harakatni ekvatorial tekislikda ($\theta = \pi/2$) ko'rib chiqamiz. Shundan so'ng (3)-tenglamani integrallash orqali massiv zarrachaning harakat tenglamasi quyidagicha ifodalanadi:

$$\frac{dt}{ds} = \frac{1}{f}\left(\mathcal{E} - \frac{\alpha G_N M}{r}\right) \tag{11}$$

$$\frac{d\phi}{ds} = \frac{\mathcal{L}}{r^2} \tag{12}$$

$$\left(\frac{dr}{ds}\right)^2 = \left(\mathcal{E} - \frac{\alpha G_N M}{r}\right)^2 - f\left(1 + \frac{\mathcal{L}^2}{r^2}\right) \tag{13}$$

bu yerda $\mathcal{E}$ va $\mathcal{L}$ sinov zarrachaning specific energiyasi va specific burchak momenti. Soddalashtirish maqsadida, biz Schwarzschild–MOG qora tuynugi ekvatorial tekisligida ($\theta = \pi/2$ va $\dot{\theta} = 0$) sinov zarrachaning aylana harakatini ko'rib chiqamiz. Shundan so'ng radial harakat tenglamasi quyidagicha hosil qilinadi:

$$\dot{r}^2 = \left(\mathcal{E} - \frac{\alpha G_N M}{r}\right)^2 - f\left(1 + \frac{\mathcal{L}^2}{r^2}\right) = [\mathcal{E} - V_+(r)][\mathcal{E} - V_-(r)] \quad (1.50)$$

Bu yerda potensial $V_{\pm}(r)$quyidagicha berilgan:

$$V_{\pm}(r) = \frac{\alpha G_N M}{r} \pm \sqrt{f\left(1 + \frac{\mathcal{L}^2}{r^2}\right)} \tag{14}$$

Effektiv potensial bu sinov zarralarining qora tuynuk atrofida qanday harakat qilishini tushunishda muhim vositadir. 1.4-rasmda turli MOG parametr qiymatlari uchun massiv zarraning effektiv potensiali ko‘rsatilgan. Effektiv potensialning minimum qiymatini aniqlash orqali biz eng ichki barqaror aylana orbitani (ISCO) topishimiz mumkin va massiv zarralar qora tuynukka tushmasdan yoki fazoga uloqtirilmasdan qanday aylanayotganini tushunishimiz mumkin. ISCO astrofizik kuzatishlar uchun muhim nuqta bo‘lib, u qora tuynukka tushayotgan materiya va chiqarilgan nurlanishga ta’sir qiladi. Shuningdek, ISCO’ni o‘rganish qora tuynuklarning xususiyatlari va umumiy nisbiylik nazariyasini, shu jumladan alternativ gravitatsiya nazariyalarini sinashda foydali ma’lumot beradi. ISCO joylashuvi qora tuynukning massasi, aylanish tezligi va hozirgi tadqiqotda STVG parametri kabi turli omillarga bog‘liq bo‘ladi.

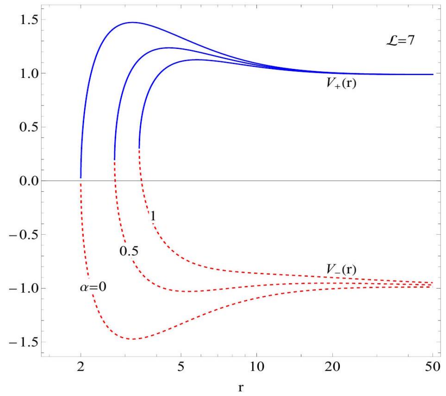


Figure 1.4: Dependence of the ISCO position for massive particle orbiting the SchwarzschildMOG black hole from the MOG parameter α.

Ushbu tadqiqotda biz STVG doirasida qora tuynuk atrofidagi massiv zarralarning ISCO radiusini aniqlashga va uning (\alpha) parametriga qanday bog‘liqligini sinashga e’tibor qaratamiz. Tahlilni soddalashtirish maqsadida, biz massiv zarralarning harakatini ekvatorial tekislikda ko‘rib chiqamiz, ya’ni effektiv potensial faqat radial koordinataga bog‘liq bo‘ladi. ISCO pozitsiyasini aniqlash uchun standart usuldan foydalanish mumkin, bunda quyidagi shartlar qo‘yiladi: $V_+(r) = \mathcal{E}, V_+'(r) = 0$, and $V_+''(r) \leq 0$. ISCO pozitsiyasi uchun analitik ifodalarni olish mumkin, ammo uning aniq joylashuvini aniqlash uchun diqqat bilan sonli tahlillar o‘tkazish zarur.

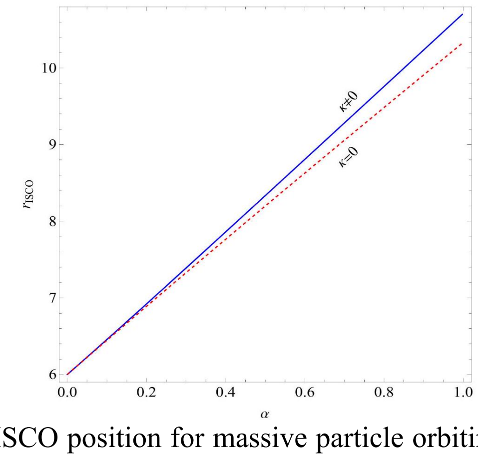


Figure 1.5: Dependence of the ISCO position for massive particle orbiting the SchwarzschildMOG black hole from the MOG parameter α.

Qiziqarli jihati shundaki, bizning natijalarimiz STVG ta’siri sababli ISCO pozitsiyasi kattalashishini ko‘rsatadi. 1.5-rasmda Schwarzschild–MOG atrofida aylanayotgan massiv zarralarning ISCO pozitsiyasining MOG parametriga bog‘liqligi tasvirlangan. Ushbu natijalar STVG ta’sirini hisobga olishning qora tuynuk atrofidagi massiv zarralar harakatini o‘rganishda qanchalik muhimligini ko‘rsatadi.

Shuningdek, Somon yo'li (Milky Way) markazida joylashgan supermassiv qora tuynuk (SMBH) atrofida S2 yulduzlarining harakati juda qiziqarli va muhim vazifa bo'lib, u modifikatsiyalangan yoki alternativ gravitatsiya nazariyalarini nazariy va astrofizik nuqtai nazardan sinash imkonini beradi. Ma'lumki, markaziy gravitatsion obyekt atrofida aylanayotgan osmon jismi uchun periastron ifodasi umumiy nisbiylik doirasida quyidagicha aniqlanadi:

$$\delta\phi_{\rm GR} = \frac{6\pi G_N M}{ac^2(1-e^2)} \tag{15}$$

bu yerda a va e mos ravishda orbitaning yirik yarim o'qi (semi-major axis) va eksentrikligi (eccentricity) ni bildiradi. Bu yerda bizning maqsadimiz S2 yulduzining STVG doirasida SMBH atrofidagi orbitasining periastron pretsessiyasini aniq tarzda chiqarishni ko'rsatishdir. (12) va (13) tenglamalarni hisobga olib, shuningdek yangi o'zgaruvchi $u = \mathcal{L}^2/G_N M r$ ni kiritgandan so'ng, quyidagilarni osongina olish mumkin:

$$u'' + u(1+\alpha\epsilon) = 1 + \alpha\delta + 3\epsilon(1+\alpha)u^2 - 2\epsilon^2\alpha(1+\alpha)u^3 \tag{16}$$

bu yerda $\delta = 1 - \mathcal{E}$, $\epsilon = (G_N M/\mathcal{L})^2 \ll 1$ kichik parameter. Aniq yechimni yuqoridagi tenglama uchun topish qiyin ekanligi aniq. Shuning uchun periastron burilmasining taxminiy qiymatini topish uchun yarim-analitik yondashuv qo'llanishi mumkin. Yechimni kichik parametrning quvvatlari shaklida kengaytiraylik: $u(\phi) = u_0(\phi) + \epsilon u_1(\phi) + \mathcal{O}(\epsilon^2)$. Bu yechimni (16) tenglamaga qo'yganimizda, nolchi va birinchi tartibdagi taxminiy tenglamalar quyidagicha yozilishi mumkin:

$$u_0'' + u_0 = 1 + \alpha\delta \tag{17}$$

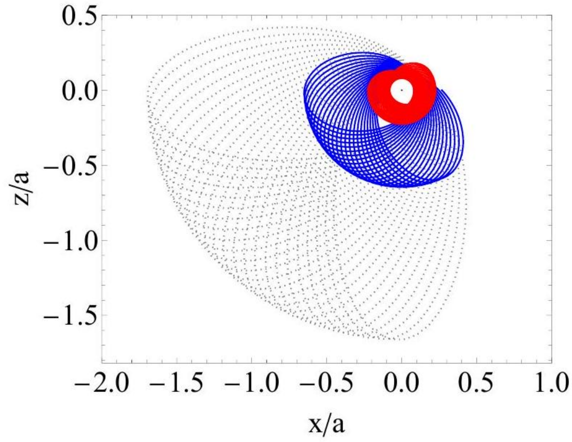
$$u_1'' + u_1 = -\alpha u_0 + 3(1+\alpha)u_0^2 \tag{18}$$

Avval aytib o'tganimizdek, Nyuton nazariyasida osmon jismining trayektoriyasi yopiq bo'lib, sinov zarrachaning pozitsiyasi quyidagi shartni qanoatlantiradi: $u(\phi) = u(\phi + 2\pi)$. Biroq, umumiy nisbiylik nazariyasi ta'siri sababli pozitsiyani quyidagicha topish mumkin: $u(\phi) \simeq u(\phi + 2\pi + \delta\phi)$ va STVG nazariyasida periastron pretsessiyasi quyidagicha ifodalanadi:

$$\delta\phi = \delta\phi_{\rm GR}\left(1 + \frac{5}{6}\alpha\right). \tag{19}$$

Bu ifoda kuchsiz maydon yaqinlashuvi holatida osongina olinishi mumkin. Shu bilan birga, ayniqsa, α parametrini cheklash va parametrlarning o'zaro bog'liqligini Monte Carlo Markov Chain algoritmi yordamida statistik tahlillar bilan aniqlash yaxshi ko'rsatib berilgan. Tadqiqotlarda STVG parametri uchun yuqori chegaraning ($\alpha \leq 0.58$) ekanligi 99,7% ishonchlilik darajasida ko'rsatilgan.

In order to better understand the effects of the STVG we produce the trajectory of the S2 star around the SMBH and probe how the STVG parameter affects on the trajectory of the star. Figure 1.7 draws the trajectory of the S2 star around SMBH in the STVG theory for different values of the parameter α which is produced by a contour plot of $r/a = (\cos\ \phi, \sin\ \phi)(1-e^2)/u$, here u is obtained from equation (1.65) when $\delta = 2$. As one see that the periastron of the S2 star is very sensitive to the parameter α and it might decreases approximately 7.5 times due to the STVG. It should be also noted that the initial position of the S2 star is different from that is predicted in GR.



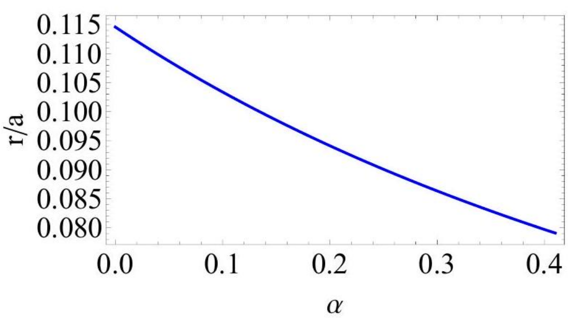


Figure 1.7: (Left) The trajectory of the S2 star around the SMBH in the STVG for different values of α parameter. The dashed gray line represents the trajectory of star in GR, while solid blue and red lines are responsible for $\alpha = 0.1$ and $\alpha = 0.4$. (Right) The initial position of star is a function the STVG parameter α.

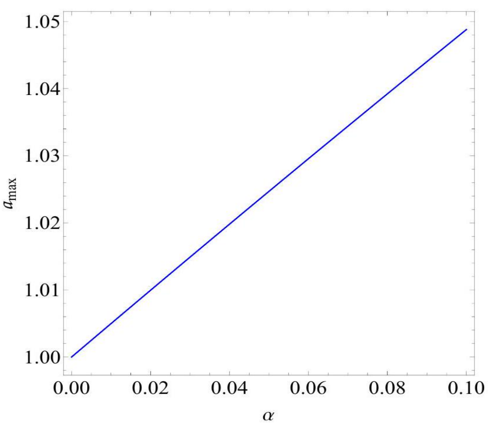
Avval aytilganidek, MOG doirasidagi sferik simmetrik qora tuynuk yechimi Schwarzschild–MOG fazo-vaqti bilan ifodalanadi, bu esa Reissner–Nordström yechimiga yaqin o‘xshashlikka ega. Qora tuynukning aylanma harakati hisobga olinganda, tegishli yechim Kerr–MOG fazo-vaqti bo‘lib, u Kerr–Newman geometriyasiga mos keladi. Boyer–Lindquist koordinatalarida STVG doirasida aylanayotgan qora tuynuk atrofidagi fazo-vaqt Kerr–MOG metrikasi bilan ifodalanadi:

$$ds^2 = -\frac{\Delta}{\Sigma}(dt - a\sin^2\theta d\phi)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\sin^2\theta}{\Sigma}[(r^2 + a^2)d\phi - adt]^2\,, \tag{20}$$

bu yerda $\Delta = r^2 - 2(1+\alpha)Mr + \alpha(1+\alpha)M^2 + a^2$ va $\Sigma = r^2 + a^2\cos^2\theta$. Bu yerda a qora tuynukning specific burchak momenti (spin) bo‘lib. E’tibor bering, (20) fazo-vaqtiga mos keluvchi vektor potensiali Φ_μ sinov jismi bilan tashqi vektor maydon o‘rtasidagi beshinchi o‘zaro ta’sirni tavsiflaydi va statik fazo-vaqtida bu o‘zaro ta’sir Coulombga o‘xshash potensial orqali ifodalanadi. Aylanayotgan fazo-vaqtida esa vektor potensiali quyidagicha kengaytirilishi mumkin:

$$\Phi_\mu = \frac{\sqrt{\alpha}Mr}{\Sigma}(-1,0,0,a\sin^2\theta). \tag{21}$$

STVG doirasida aylanayotgan qora tuynukning gorizonti va ergosferasi quyidagi joylarda joylashgan: $x_+ = 1 + \alpha + \sqrt{1 + \alpha - a_*^2}$, $\quad x_{\rm erg} = 1 + \alpha + \sqrt{1 + \alpha - a_*^2\cos^2\theta}$ bu yerda $x = r/M$ o’lchamsiz radial koordinata va $a_* = a/M$. Shuni takidlash kerakki, (21)-ifodasiga ko‘ra, STVG parametri har doim musbat bo‘lib, ya’ni $\alpha \geq 0$. Qora tuynuk gorizontining mavjudligidan kelib chiqqan holda, qora tuynukning spin parametri uchun chegara aniqlanishi mumkin: $|a_*| \leq \sqrt{1+\alpha}$, ya’ni STVG nazariyasida qora tuynuk Kerr qora tuynugidan tezroq aylanishi mumkin. Qora tuynukning maksimal spini va STVG parametri α o‘rtasidagi bog‘liqlik 2.1-rasmda ko‘rsatilgan. Ma’lumki, Kerr qora tuynukning maksimal spini: $a_{*\,\rm max} = 1$, STVG nazariyasida esa: $a_{*\,\rm max} = \sqrt{1+\alpha}$. 2.2-rasmda turli (\alpha) qiymatlari uchun Kerr–MOG qora tuynuk ergosferasi tasvirlangan. Natijalardan ko‘rinib turibdiki, α parametri kattalashganda, gravitatsion kuch aylanish kuchi ustun bo‘ladi va shu sababli ergosfera hududi kichrayadi.



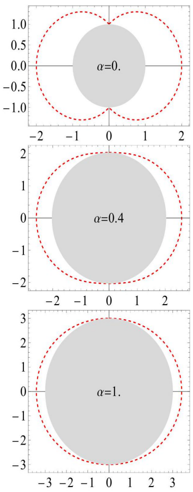


Figure 2.1: Dependence of the maximal spin of the black hole from the STVG parameter α which is given by the equation $a_{\rm max} = \sqrt{1+\alpha}$. Figure 2.2: The ergosphere of the black hole for the different values of the STVG parameter α for $a_* = 1$.

Endi STVG nazariyasi doirasida qora tuynuk atrofidagi massiv zarrachaning harakatini ko‘rib chiqamiz. Ushbu nazariyada, boshqa gravitatsiya nazariyalaridan farqli o‘laroq, massiv zarralar geodezik chiziq bo‘ylab harakat qilmaydi. Sinov zarrachaning saqlanadigan kattaliklari, ya’ni energiyasi E va impuls momenti L, quyidagi munosabatlarni qanoatlantiradi:

$$g_{tt}\dot{t} + g_{t\phi}\dot{\phi} = -\frac{E + q\Phi_t}{m}, \qquad g_{\phi\phi}\dot{\phi} + g_{t\phi}\dot{t} = \frac{L - q\Phi_\phi}{m}, \tag{22}$$

Bu esa $\dot{t}$ va $\dot{\phi}$ ifodalarini chiqarishga imkon beradi. To‘rt tezlikni normallashtirishni hisobga olib va yuqoridagi ifodalarni ishlatgan holda, quyidagi tenglama olinadi. Massiv zarrachaning aylana orbitadagi harakati $\dot{r} = 0$ va $\dot{\theta} = 0$ hisobga olinganda, STVG doirasida massiv zarrachaning harakati quyidagi effektiv potensial bilan cheklangan bo‘ladi:

$$V = -q\Phi_t + \omega(L - q\Phi_\phi) + \sqrt{-\psi\left[m^2 + \frac{(L - q\Phi_\phi)^2}{g_{\phi\phi}}\right]}, \tag{23}$$

bu yerda quyidagi shart $E = V$ bajarilishi kerak. Standart usul yordamida ISCO pozitsiyasi quyidagi shartlar asosida aniqlanishi mumkin: $V(r) = E, V'(r) = 0$ and $V''(r) \leq 0$. ISCO pozitsiyasining analitik ifodasi olinishi mumkin bo‘lsa-da, diqqat bilan olib borilgan sonli tahlillar shuni ko‘rsatdiki, STVG ta’siri sababli ISCO pozitsiyasi kattalashadi. 2.3-rasmda Schwarzschild–MOG va ekstremal Kerr–MOG qora tuynuklar atrofida aylanayotgan massiv zarralarning ISCO pozitsiyasining α parametriga bog‘liqligi ko‘rsatilgan. E’tibor bering, 2.3-rasmda ISCO pozitsiyasi istalgan spin parametri qiymatlari uchun ikki egri chiziq orasidagi hududda joylashgan, bunda spin parametri oralig‘i: $0 < a_* < \sqrt{1+\alpha}$ olinadi.

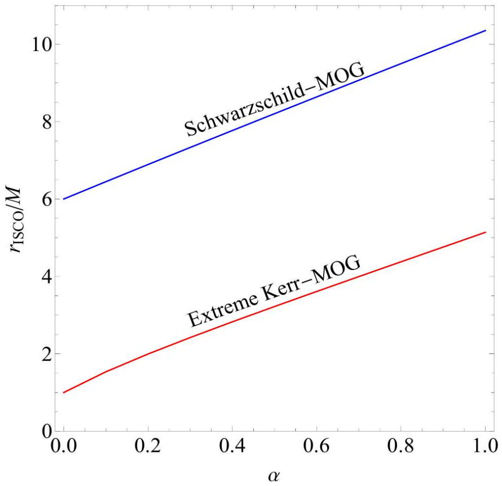


Figure 2.3: Dependence of the ISCO position for massive particle from the STVG parameter α for static and extreme black holes.

Penrose jarayoni qora tuynukning kuchli gravitatsion maydoni va aylanishidan foydalangan holda undan energiya olishni o‘z ichiga oladi. Penrose jarayoniga ko‘ra, qora tuynukka tushayotgan zarracha ergosferada ikki qismga ajraladi: bir qismi qora tuynukka tushadi, ikkinchisi esa tashqariga chiqadi. Tashqariga chiqqan qism qora tuynukning aylanish energiyasining bir qismini olib ketadi, natijada qora tuynuk energiyasining sof yo‘qolishi yuz beradi. Ushbu olinadigan energiya turli astrofizik jarayonlarni quvvatlantirish uchun ishlatilishi mumkin. Zarrachaning energiya samaradorligi uning massasi, spin parametri va STVG parametriga bog‘liq. Faraz qilaylik, massiv zarracha (1) quyidagi parametrlar bilan: $(m_1, E_1, L_1, \dot{r}_1, \dot{\theta}_1, \dot{\phi}_1)$ cheksizlikdan qora tuynukka yaqinlashadi va ergosferada ikki bo‘lakka ajraladi: birinchi zarra quyidagi parametrlar bilan $(m_2, E_2, L_2, \dot{r}_2, \dot{\theta}_2, \dot{\phi}_2)$ qora o’raga qulaydi va ikkinchi zarra quyidagi parametrlar bilan $(m_3, E_3, L_3, \dot{r}_3, \dot{\theta}_3, \dot{\phi}_3)$ aylanoyotgan qora o’ra ergosferasidan chiqib ketadi, bu yerda m_i, E_i, $L_i, \dot{r}_i$ va $\dot{\theta}_i$ mos ravishda massani, energiyani, burchak momentini, radial, vertikal va azimutal tezliklarni bildiradi. Ushbu jarayon uchun saqlanish qonunlari quyidagicha yozilishi mumkin:

$$m_1 \geq m_2 + m_3 \quad (24)$$
$$E_1 = E_2 + E_3 \quad (25)$$
$$L_1 = L_2 + L_3 \quad (25)$$
$$m_1 \dot{r}_1 = m_2 \dot{r}_2 + m_3 \dot{r}_3 \quad (26)$$
$$0 = m_2 \dot{\theta}_2 + m_3 \dot{\theta}_3 \quad (27)$$
$$m_1 \dot{\phi}_1 = m_2 \dot{\phi}_2 + m_3 \dot{\phi}_3 \quad (28)$$

Biz to‘rt-tezligi quyidagicha $\dot{x}^\mu = \dot{t}(1, v, 0, \Omega)$ berilgan massiv zarrachaning aylana harakatini ko‘rib chiqamiz. To‘rt-tezlikni normallashtirish shartidan foydalanib, zarrachaning burchak tezligi

$$\Omega_1 = \frac{-(u^2 + g_{tt})g_{t\phi} + u\sqrt{(-\psi)(u^2 + g_{tt})g_{\phi\phi}}}{u^2 g_{\phi\phi} + g_{t\phi}^2}, \quad (29)$$

bu yerda $u = (E_1 + q_1\Phi_t)/m_1$, ajralgan fragmentlarning burchak tezliklari esa $\Omega_2 = \Omega_+$ va $\Omega_3 = \Omega_-$ deb belgilanadi, bunda Ω_+ va Ω_- quyidagicha aniqlanadi:

$$\Omega_\pm = -\frac{g_{t\phi}}{g_{\phi\phi}} \pm \sqrt{\left(\frac{g_{t\phi}}{g_{\phi\phi}}\right)^2 - \frac{g_{tt}}{g_{\phi\phi}}}. \quad (30)$$

(24) dagi zarracha massasi uchun berilgan tengsizlikdan bog‘lanish konstantalari orasidagi munosabatni $q_1 \geq q_2 + q_3$ ko‘rinishida olish mumkin. (28) tenglamadan foydalanib hamda oddiy algebraik o‘zgartirishlarni bajarib, ichkariga qulayotgan va qochib ketayotgan zarrachalar energiyalari orasidagi munosabatni quyidagicha toppishmumkin:

$$(E_3 + q_3\Phi_t)\frac{\Omega_3 - \Omega_2}{g_{tt} + \Omega_3 g_{t\phi}} \geq (E_1 + q_1\Phi_t)\frac{\Omega_1 - \Omega_2}{g_{tt} + \Omega_1 g_{t\phi}}. \quad (31)$$

Energiya samaradorligi quyidagi ko‘rinishda hisoblanadi $\eta = E_3/E_1 - 1$, Kerr-MOG fazo-vaqtda quyidagicha ifodalanadi

$$\eta = \frac{1}{2}\left(\sqrt{1 + \frac{g_{tt}}{u^2}} - 1\right)\left(1 + \frac{q_1}{E_1}\Phi_t\right) - \frac{q_1 + q_3}{E_1}\Phi_t. \quad (32)$$

STVG nazariyasida energiya samaradorligini sifat jihatdan tahlil qilish uchun qulayotgan zarrachaning energiyasini $E_1 \simeq m_1$ (deb baholash mumkin va ekstremal aylanuvchi qora tuynuk (ya’ni $x_+ = 1 + \alpha$) uchun (32)-ifoda quyidagicha yoziladi:

$$\eta_{\max} \geq \frac{1}{2(1+\alpha)}\left[\sqrt{2+\alpha} - 1 + 2\alpha\left(1 + \frac{m_3}{m_1}\right)\right]. \quad (33)$$

STVG parametrining katta qiymatlarida, ya’ni $\alpha \to \infty$ holatda, energiya samaradorligi $\eta_{\max} \geq 1 + m_3/m_1$, bo‘ladi. Bu qiymat chiqib ketayotgan va qulayotgan zarrachalar massalari nisbatiga bog‘liq ravishda 100% dan ham yuqori bo‘lishi mumkin. Ekstremal Kerr qora tuynugi holatida esa $\eta_{\max} = (\sqrt{2} - 1)/2 \simeq 0.207$, ya’ni taxminan ~21% ga teng bo’ladoi. 2.4-rasm shuni ko‘rsatadiki, Kerr–MOG qora tuynugidan energiya ajratib olishning maksimal samaradorligi chiqib ketayotgan va qulayotgan zarrachalar massalari nisbatining ma’lum qiymatlarida $\sim 200\%$ gacha yetishi mumkin, bu esa juda katta ko‘rsatkich hisoblanadi. Biroq, magnitlangan Kerr qora tuynugi yuqori energiyali zarrachalarni tezlashtirish uchun eng istiqbolli nomzodlardan biri sifatida qaraladi. Tadqiqotlar shuni ko‘rsatadiki, aylanish va magnit maydon ta’siri natijasida hosil bo‘ladigan induksiyalangan elektrostatik potensial sababli magnitlangan Kerr qora tuynugidan energiya ajratib olish samaradorligi 100% dan ham oshishi mumkin.

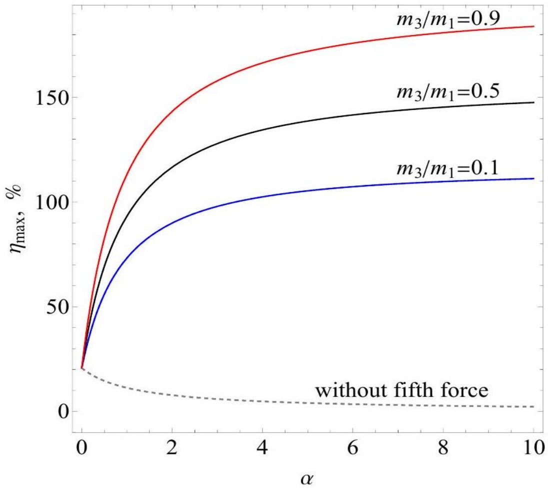


Figure 2.4: Dependence of the maximal energy efficiency of the black hole from the STVG parameter α.

Tezlanayotgan zaryadlangan zarrachalar sinxrotron nurlanishini chiqarishi yaxshi ma'lum. Sinxrotron nurlanish — bu zaryadlangan zarracha magnit maydon mavjud bo'lgan sharoitda egri trayektoriya bo'ylab tezlanganda hosil bo'ladigan elektromagnit nurlanish turidir. Zaryadlangan zarracha chiqaradigan sinxrotron nurlanish intensivligi tezlanishning kvadratiga proporsional, ya'ni $I \sim w_\mu w^\mu$, nurlanish chastotasi esa magnit maydon kuchiga bog'liq bo'ladi. Shuni ta'kidlash joizki, magnitlangan va zaryadlangan qora tuynuklar atrofida relativistik zaryadlangan zarrachalar chiqaradigan sinxrotron nurlanish so'nggi tadqiqotlarning muhim mavzusiga aylangan. Buning sababi shundaki, qora tuynuklar zaryadlangan zarrachalar harakatiga ta'sir ko'rsatishi hamda chiqarilayotgan nurlanish xususiyatlarini o'zgartirishi mumkin bo'lgan kuchli gravitatsion va elektromagnit maydonlarga ega. Ushbu tadqiqotlar qora tuynuklar atrofidagi relativistik zaryadlangan zarrachalarning sinxrotron nurlanish spektri haqida muhim tushunchalar beradi va ularning natijalari qora tuynuklar bilan bog'liq astrofizik hodisalarni yanada chuqurroq anglashimizda katta ahamiyatga ega.

Bu yerda biz STVG nazariyasida qora tuynuk atrofida aylanayotgan massiv zarracha tomonidan chiqariladigan sinxrotron nurlanishning gravitatsion analogini muhokama qilamiz. Avval ta'kidlaganimizdek, ushbu nazariyada beshinchi kuch mavjudligi sababli massiv zarrachalar geodezik chiziqlar bo'ylab harakatlanmaydi. (3)-ifodadagi harakat tenglamasiga ko'ra, massiv zarrachaning to'rt-tezlanishi quyidagicha ifodalanadi:

$$w^\mu = \frac{q}{m} B^\mu_\nu \dot{x}^\nu, \qquad w_\mu \dot{x}^\nu = 0 \tag{34}$$

u har doim zarrachaning to'rt-tezligiga perpendikulyar bo'ladi. E'tibor bering, (34)-tenglama Lorens tenglamasining gravitatsion analogi hisoblanadi: bu yerda zarrachaning zaryadi o'rniga bog'lanish doimiysi q qo'yilgan, hamda Lorens tenglamasidagi Faradey tensori o'rnida anti-simmetrik $B_{\mu\nu}$ tensori keladi.

Bizning asosiy farazimiz shundan iboratki, agar massiv zarracha tashqi kuch mavjud bo'lgan sharoitda to'rt-tezlanishning nolga teng bo'lmagan qiymati bilan orbita bo'ylab harakat qilsa, u holda u nurlanish chiqarishi kerak. Biz bu nurlanishni gravitatsion sinxrotron nurlanishi deb ataymiz. Shunda STVG nazariyasida tezlanayotgan massiv zarrachaning nurlanish intensivligi quyidagicha aniqlanadi:

$$I = -\frac{2q^2}{3} w_\mu w^\mu = \frac{2q^4}{3m^2} B_{\mu\lambda} B^{\mu\nu} \dot{x}^\lambda \dot{x}_\nu \tag{35}$$

Massiv zarrachaning doiraviy harakati $\dot{x}^\mu = \dot{t}(1, v, 0, \Omega)$ ko'rinishidagi to'rt-tezlik bilan qaraladi, bu yerda $v = dr/dt$ va $\Omega = d\phi/dt$ mos ravishda zarrachaning radial va burchak tezliklaridir. Shundan so'ng, oddiy algebraik o'zgartirishlarni bajarish orqali tezlanish ifodasini quyidagi ko'rinishda olish mumkin:

$$w_\mu = \frac{q}{m}\dot{t}\big(vB_{tr}, B_{rt} + \Omega B_{r\phi}, B_{\theta t} + \Omega B_{\theta\phi}, vB_{\phi r}\big), \tag{36}$$

bu yerda anti-simmetrik tensorning nolga teng bo‘lmagan komponentlari quyidagicha aniqlanishi mumkin:

$$B_{rt} = \frac{\sqrt{\alpha}M}{\Sigma^2}(r^2 - a^2\cos^2\ \theta), B_{r\phi} = -B_{rt}a\sin^2\ \theta, \tag{37}$$

$$B_{\theta t} = -\frac{\sqrt{\alpha}Mra^2\sin\ 2\theta}{\Sigma^2}, B_{\theta\phi} = -B_{\theta t}\frac{r^2 + a^2}{a}, \tag{38}$$

va $\dot{t}$ uchun ifoda quyichagicha topiladi $\dot{t}^{-1} = \sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi} - v^2 g_{rr}}$. Ekvatorial tekislikda Kerr–MOG qora tuynuk atrofida aylanayotgan nurlanayotgan massiv zarrachaning nurlanish intensivligi ifodasi quyidagicha yoziladi:

$$I = \frac{2\alpha^3 G^3 m^2 M^2}{3c^3 r^4}\dot{t}^2\left[\frac{\Delta}{r^2}(1 - \Omega a)^2 - \frac{r^2}{\Delta}\frac{v^2}{c^2}\right] \tag{39}$$

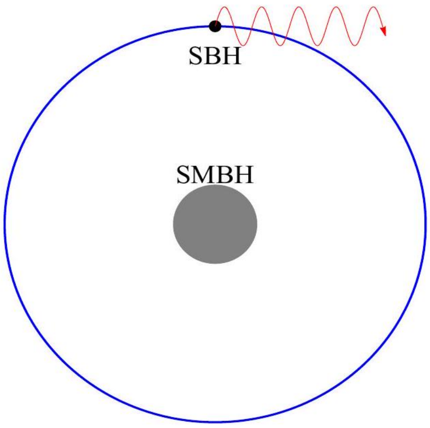


Figure 3.1: The schematical picture of radiated SBH around SMBH in STVG.

Yuqoridagi tenglamadan ko‘rinib turibdiki, massiv zarracha hatto istalgan barqaror orbitada (ya’ni $v = 0$) ham gravitatsion nurlanish chiqaradi, va aylanmaydigan qora tuynuk holatida $\dot{t}^2$ ifodadagi qavs ichidagi ifoda bilan bekor qilinishi mumkin. 3.1-rasmda supermassiv qora tuynuk (SMBH) atrofidagi yulduzli qora tuynuk (SBH) tomonidan chiqarilayotgan gravitatsion nurlanishning sxematik tasviri keltirilgan. Natijada, supermassiv qora tuynuk atrofida aylanayotgan yulduzli qora tuynukdan chiqarilayotgan gravitatsion nurlanish intensivligi quyidagicha taxmin qilinishi mumkin:

$$I \sim 2.43 \times 10^{39}\alpha^3\left(\frac{m}{10M_\odot}\right)^2\left(\frac{M}{10^9 M_\odot}\right)^{-2}\left(\frac{10GM}{c^2 r}\right)^4 \text{erg/s} \tag{40}$$

Bu qiymat radial koordinataga bog‘liq. 3.2-rasmda intensivlikning α parametriga bog‘liqligi ko‘rsatilgan. Rasmdan ko‘rinib turibdiki, STVG parametri mavjud bo‘lmaganda intensivlik nolga teng bo‘ladi, chunki bu holda massiv zarracha qora tuynuk atrofida geodezik chiziq bo‘ylab harakat qiladi va gravitatsion nurlanish chiqarmaydi. Biroq, STVG mavjud bo‘lganda (ya’ni $\alpha \neq 0$) nurlanayotgan zarrachaning intensivligi nolga teng bo‘lmaydi va α parametri oshgani sayin ortib boradi.

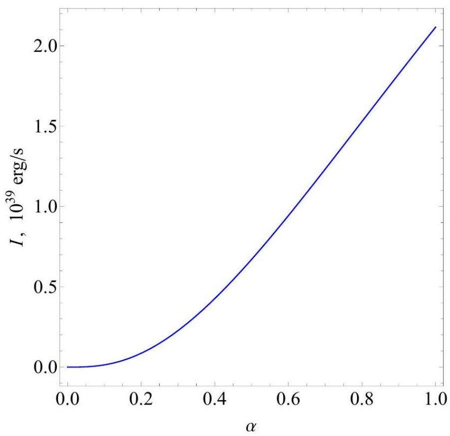


Figure 3.2: Dependence of the intensity of a radiating SBH at the ISCO position from the α parameter.

Endi biz STVG nazariyasi doirasida, radiatsiya reaksiyasi (radiation reaction) terminini o‘z ichiga olgan holda, massiv zarralarning dinamikasini ko‘rib chiqamiz. Lorentz–Abraham–Dirak (LAD) tenglamasi zaryadlangan zarrachalar uchun radiatsiya reaksiyasini tavsiflovchi tenglama sifatida keng tanilgan bo‘lsa-da, bizning tadqiqotimizda ushbu hodisaning gravitatsion analogiga e’tibor qaratiladi. Shu kontekstda, harakat tenglamasi quyidagicha ifodalanadi

$$\frac{Du^{\mu}}{ds} = \frac{\tilde{q}}{m} B^{\mu}_{\nu} u^{\nu} + \frac{1}{2}\tau_0 \left(R^{\mu}_{\nu} + u^{\mu} u_{\lambda} R^{\lambda}_{\nu} \right) u^{\nu} + \tau_0 \left(\frac{D^2 u^{\mu}}{ds^2} + u^{\mu} u_{\lambda} \frac{D^2 u^{\lambda}}{ds^2} \right), \tag{41}$$

bu yerda τ_0 gravitatsion nurlanishning so’nish vaqti $\tau_0 = 2\tilde{q}^2/(3m) \ll 1s$ va ushbu parametr kichik parametri sifatida ishlatilishi mumkin. Tasavvur qilish uchun, elektronning demping vaqtini taxminan hisoblasak, u quyidagi qiymatga ega bo‘ladi

$$\tau_0 = \frac{2\alpha G_N m_e}{3c^3} \left(\frac{m}{m_e} \right) \sim 10^{-66} \alpha \left(\frac{m}{m_e} \right) \mathrm{s}\,,$$

ammo while the stellar black hole orbiting around supermassiv qora o’ra atrifida aylanayotgan yulduzsimon qora o’ra uchun esa

$$\tau_0 \simeq 3.3 \times 10^{-4} \alpha \left(\frac{m}{10 M_{\odot}} \right) \mathrm{s}\,.$$

Shuni hisobga olsakki, (41)-tenglamaning oxirgi termini boshqa terminlarga nisbatan sezilarli darajada kichik, shuning uchun tenglamani τ_0 so‘nish vaqti bo‘yicha kengaytirish uchun **Landau usuli** qo‘llanishi mumkin. Oddiy algebraik manipulyatsiyalarni bajargach, harakat tenglamasi quyidagicha qayta yozilishi mumkin:

$$\frac{Du^{\mu}}{ds} = \frac{\tilde{q}}{m} B^{\mu}_{\nu} u^{\nu} + \frac{1}{2}\tau_0 h^{\mu\lambda} R_{\lambda\nu} u^{\nu} + \tau_0 \frac{\tilde{q}}{m} \left(u^{\alpha} \nabla_{\alpha} B^{\mu}_{\nu} + \frac{\tilde{q}}{m} h^{\mu\lambda} B_{\lambda\alpha} B^{\alpha}_{\nu} \right) u^{\nu}\,, \tag{42}$$

bu yerda $h^{\mu\nu} = g^{\mu\nu} + u^{\mu} u^{\nu}$ va $h_{\mu\nu} u^{\nu} = 0$. Ko‘rinib turibdiki, egri fazo-vaqt ichida zarrachaning harakati bo‘yicha o‘n ikki ta harakat konstantasini topish talab qilinadi, bu esa zarracha harakati bilan bog‘liq asosiy muammolardan biridir. Biroq, ushbu muammo Landau usuli yordamida chetlab o‘tish mumkin va harakat tenglamasi to‘rt koordinata x^{μ} uchun ikkinchi tartibli tenglamalar sistemasiga kamayadi. (3.12) dagi har bir koordinata uchun analitik tenglamalar juda uzun bo‘lgani sababli ularni maqolada keltirmaymiz. Biroq, berilgan boshlang‘ich shartlar $(0, r_0, \theta_0, 0)$ va $\{-\mathcal{E}/f(r_0), 0,0, \mathcal{L}/(r_0 \sin\,\theta_0)^2\}$, uchun sonli tahlillar orqali har bir koordinata affina parametrga bog‘liq funksiyasi sifatida topilishi mumkin. Keyinchalik, koordinata transformatsiyasini bajarish orqali: $x = r\cos\phi\sin\theta, y = r\sin\phi\sin\theta$, va $z = r\cos\theta$ zarrachaning trayektoriyalari Dekart koordinatalarida $2D$ va $3D$ parametrik grafiklar yordamida tasvirlanishi mumkin.

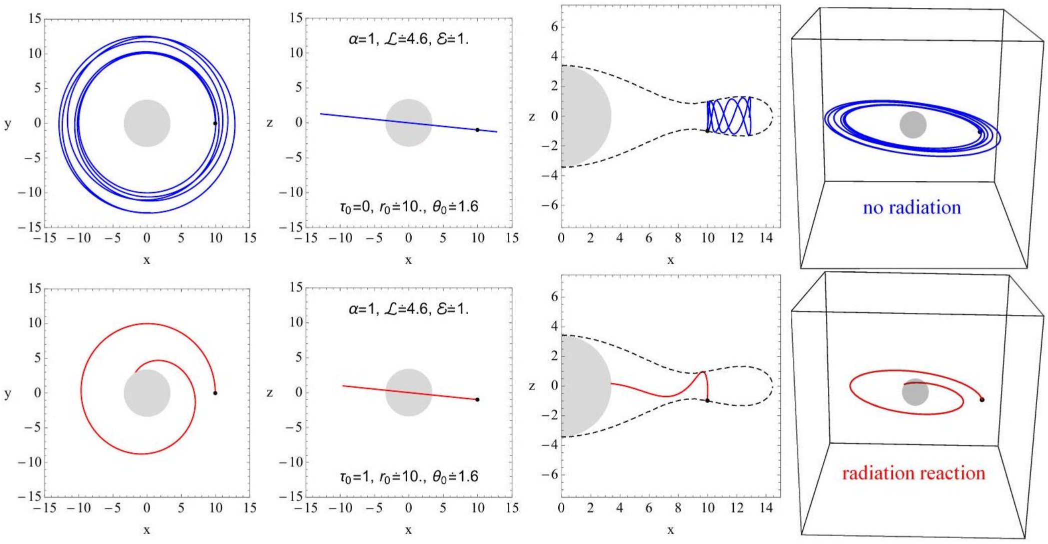


Figure 3.3: The trajectories of massive particle orbiting the Schwarschild-MOG black hole with (bottom panel) and without (top panel) including radiation reaction force, respectively. Initial conditions are chosen to be the same for each cases and shown in the second row. The initial position of particle is depicted by black dot in each plots. The first and second columns represent the trajectory of massive particle in ($x-y$) and ($x-z$) planes, respectively. The third column represents finite motion of test particle in the region given by dashed line (contour plot of the effective potential), while in the last column particle's trajectory is shown in 3D.

3.3-rasmda STVG doirasida radiatsiya reaksiyasi terminini hisobga olgan va olmagan holda massiv zarrachaning trayektoriyasi ko‘rsatilgan. Aylana harakatda radiatsiya reaksiyasi ta’sirini ko‘rish uchun demping vaqtini $\tau_0 = 1$ s deb belgiladik. Biroq, demping vaqtining haqiqiy qiymatini elektron uchun (3.9) va yulduzli qora tuynuk uchun (3.10) formulalar yordamida hisoblash mumkin. Bu shuni anglatadiki, demping vaqti τ_0 1 s dan ancha kichik va zarracha qora tuynuk atrofida 3.3-rasmda ko‘rsatilganidan ko‘proq aylanadi va oxir-oqibat radiatsiya reaksiyasi kuchi ta’sirida qora tuynukka tushadi. 3.4-rasmdan ko‘rinib turibdiki, MOG parametri α va demping vaqtiτ_0 kattalashgan sari, zarracha qora tuynukka tezroq tushadi.

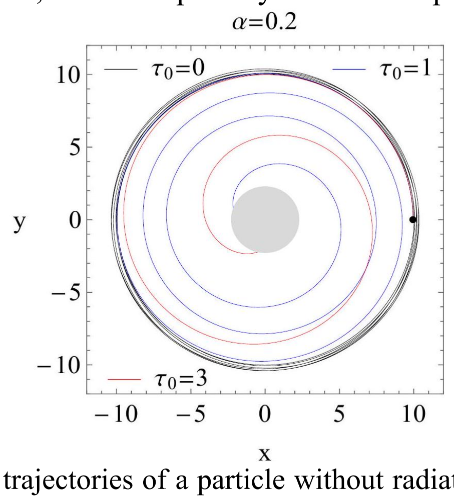


Figure 3.4: Plot is comparing the trajectories of a particle without radiation reaction force (black circular line) and with radiation reaction force for different τ parameter (blue and red). In each cases, initial positions are chosen to be the same and depicted by black dot.

Albatta, STVG doirasida qora tuynuklarning termodinamik xossalarini o‘rganish va MOG parametrining ushbu kattaliklarga ta’sirini tahlil qilish juda qiziqarli vazifa hisoblanadi. Biz STVG doirasida qora tuynuklar termodinamikasining birinchi qonunini ko‘rib chiqishdan boshlaymiz, u quyidagicha ifodalanadi:

$$dM = TdS + \Phi_t dQ \quad (43)$$

bu yerda T va S bu yerda qora o‘raning mos ravishda Hawking harorati va entropiyasi hisoblanadi. E’tiborli jihati shundaki, (43)-tenglamaning ikkinchi hadini STVG mavjudligi tufayli paydo bo‘ladi va u qora tuynukning termodinamik xossalariga ta’sir ko‘rsatishi mumkin. Schwarzschild–MOG qora tuynugi sirtidagi Hawking harorati va entropiyasi quyidagicha ifodalanadi:

$$T = \frac{1}{4\pi}\left(\frac{dg_{tt}}{dr}\right)\Bigg|_{r=r_+} = \frac{1}{2\pi G_N M\sqrt{1+\alpha}(1+\sqrt{1+\alpha})^2}, \quad (44)$$

$$S_A = \frac{A}{4G} = \frac{\pi r_+^2}{4G_N(1+\alpha)} = \pi G_N M^2(1+\sqrt{1+\alpha})^2. \quad (45)$$

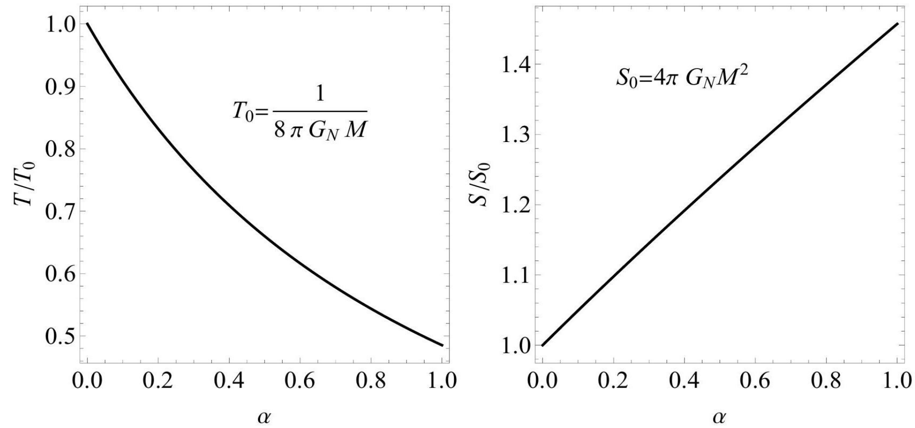


Figure 4.1: Dependence of the Hawking temperature (left panel) and entropy (right panel) of the Schwarzschild-MOG black hole form MOG parameter α.

4.1-rasmda berilga natijalardan xulosamin shundan iboratki, MOG ta’siri tufayli qora tuynukning Hokking harorati kamayadi, shu bilan birga uning entropiyasi qora tuynuk fazo-vaqtida oshadi.

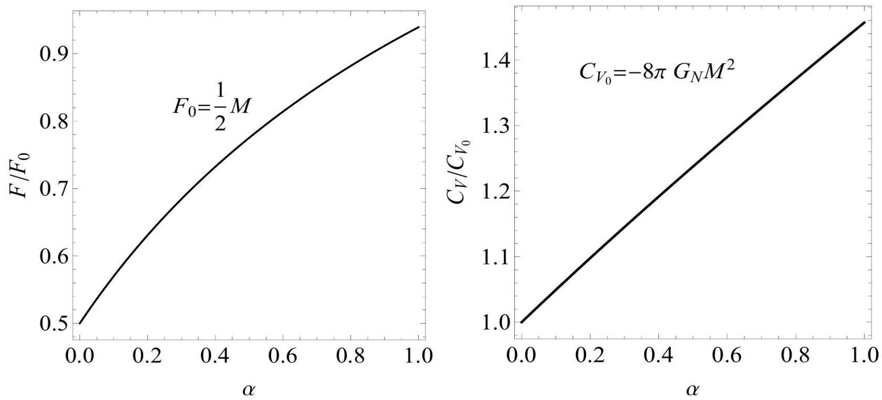


Figure 4.2: Dependence of the free energy (left panel) and heat capacity (right panel) of the Schwarzschild-MOG black hole from MOG parameter α.

Endi, qora tuynukning harorati va entropiya ifodalaridan foydalanib, Schwarzschild–MOG qora tuynugining erkin energiyasini aniqlashimiz mumkin:

$$F = M - ST - Q\Phi_t = \frac{M}{2\sqrt{1+\alpha}}. \quad (46)$$

Endi biz Schwarzschild–MOG qora tuynugining issilik sig‘imini hisoblashga e’tibor qaratamiz. Termodinamika kursidan ma’lumki, izoxorik issilik sig‘imi $C_V = T(\partial S/\partial T)$ qora tuynukning entropiyasiga bog‘liq bo‘ladi. Ushbu ifodadan foydalanib, Schwarzschild–MOG qora tuynugining issilik sig‘imi quyidagicha aniqlanishi mumkin:

$$C_V = T\frac{\partial S}{\partial M}\left(\frac{\partial T}{\partial M}\right)^{-1} = -2\pi G_N M^2(1+\sqrt{1+\alpha})^2. \quad (47)$$

4.2-rasmda qora tuynukning erkin energiya issiqlik sig’imi qiymatlari MOG parametriga bog‘liq holda taqqoslangan. Mavjud ma’lumotlarga asoslanib, natija shuni ko‘rsatadiki, Schwarzschild-MOG qora o‘rasining erkin energiya va issiqlik sig’imi o‘ziga xos ravishda STVG nazariyasida ortib boradi. Qora tuynukning yana bir muhim xususiyati uning yashash vaqti hisoblanadi. Schwarzschild–MOG qora tuynugidan energiya yo‘qotish tezligi Stefan–Boltzmann nurlanish qonuni yordamida taxminan quyidagicha baholanishi mumkin:

$$\frac{dM}{dt} = -\sigma A T^4 + \frac{Q}{r_+}\frac{dQ}{dt} = -\sigma A T^4 + \frac{\alpha G_N M}{r_+}\frac{dM}{dt} \qquad (48)$$

bu yerda σ Stefan–Boltzmann konstantasi, va ikkinchi termin STVGdagi tashqi potensial ta’siridan kelib chiqadi. Keyinchalik, (4.7)-tenglamadan dt va dMni ajratib olib, M dan 0 gacha integrallash orqali Schwarzschild–MOG qora tuynugining umrini quyidagicha ifodalash mumkin:

$$\tau = \frac{4\pi}{3\sigma} G_N^2 M^3 \sqrt{1+\alpha}(1+\sqrt{1+\alpha})^6 . \qquad (49)$$

4.3-rasmda qora tuynukning umrining MOG parametriga bog‘liqligi ko‘rsatilgan. Rasmdan ko‘rinib turibdiki, MOG parametri tufayli Schwarzschild–MOG qora tuynugining umri ortadi.

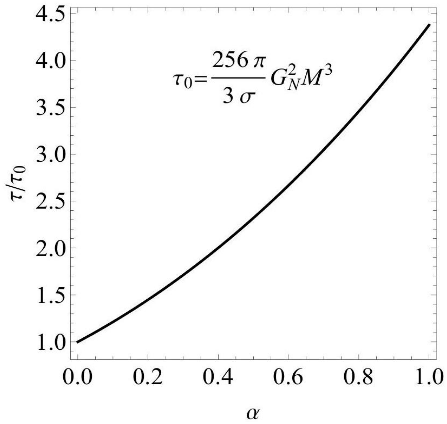


Figure 4.3: Dependence of the heat capacity (left panel) and lifetime (right panel) of the Schwarzschild-MOG black hole from MOG parameter α.

Qora tuynuk fazo-vaqtidagi metrikaning perturbaatsiyasi $h_{\mu\nu}$ bilan ifodalanadi, bunda asosiy fon metrikaning $g_{\mu\nu}$ bo‘lib, u Einshteyn maydon tenglamalarining yechimi sifatida olinadi: $G_{\mu\nu}(g) = 8\pi T_{\mu\nu}(q)$ bu yerda $G_{\mu\nu}$ metrikaning funksiyasi sifatida Einshteyn tensori, $T_{\mu\nu}$ esa moddiy maydon q ga bog‘liq energiya-moment tensori hisoblanadi. E’tibor bering, energiya-moment tensori modda turiga bog‘liq holda aniq ko‘rinishga ega bo‘ladi. Einshteyn maydon tenglamalari metrikaning perturbaatsiyasi orqali quyidagicha qayta yozilishi mumkin: $G_{\mu\nu}(g+h) = 8\pi T_{\mu\nu}(q+\delta q)$ va metrikaning perturbaatsiyasi kuchiga nisbatan kengaytirilishi mumkin:

$$G_{\mu\nu}(g) - \frac{1}{2}H_{\mu\nu} + \cdots = 8\pi\left[T_{\mu\nu}(q) + \delta T_{\mu\nu}(\delta q) + \cdots\right], \qquad (50)$$

$$H_{\mu\nu} = -16\pi\delta T_{\mu\nu}(\delta q), \qquad (51)$$

where $H_{\mu\nu}$ is Einstein tensor for perturbed field. We must emphasize that according to Regge and Wheeler the gravitational perturbations can be expseessed as $h_{\mu\nu}$. The gravitational perturbation is

$$h_{\mu\nu}^{-} = \begin{bmatrix} 0 & 0 & -h_0 \csc\theta\partial_\phi & h_0 \sin\theta\partial_\theta \\ 0 & 0 & -h_1 \csc\theta\partial_\phi & h_1 \sin\theta\partial_\theta \\ * & * & h_2 \csc\theta(\partial_{\theta\phi}^2 - \cot\theta\partial_\phi) & \frac{1}{2}h_2(\csc\theta\partial_\phi^2 + \cos\theta\partial_\theta - \sin\theta\partial_\theta^2) \\ * & * & * & -h_2 \sin\theta(\partial_{\theta\phi}^2 - \cot\theta\partial_\phi) \end{bmatrix} Y_\ell^m, \qquad (52)$$

where $h_0(t,r), h_2(t,r)$ and $h_2(t,r)$ are profile functions in axial perturbation, $Y_\ell^m(\theta,\phi)$ is the spherical harmonics satisfied the following equation: $\nabla_\Omega^2 Y_\ell^m(\theta,\phi) = -\ell(\ell+1)Y_\ell^m(\theta,\phi)$, where ∇_Ω^2 is the angular Laplacian operator defined as

$$\nabla_{\Omega}^{2}=\frac{1}{\sin\,\theta}\frac{\partial}{\partial\theta}\left(\sin\,\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^{2}\,\theta}\frac{\partial^{2}}{\partial\phi^{2}} \tag{4.17}$$

and it also satisfies the following normalisation condition:

$$\int d\Omega Y_{l}^{m*}(\theta,\phi)Y_{l'}^{m'}(\theta,\phi)=\delta_{ll'}\delta_{mm'} \tag{4.18}$$

Apply a gauge transformation to remove the second angular derivatives. We use the following gauge $h_2 = 0$, the gravitational perturbation to metric function with $\ell \neq 0$ and $m = 0$. After simpe algebraic manupilations this equation can be reduced to the Regge-Wheeler equation

$$\left[\frac{d^{2}}{dr_{*}^{2}}+\omega^{2}-V(r)\right]\psi=0 \tag{4.26}$$

where $\psi = h_1 f/r$ is new radial function, $r_* = \int dr/f$ is a tortoise coordinate, and $V(r)$ is the potential defined as
$V(r) = f\left(\frac{\ell(\ell+1)}{r^2} + f'' - \frac{f'}{r}\right)$. From equation (4.26) equation, it is evident that radial function in the axial perturbation can be governed by the standard Schrodinger-like equation.
There exist simpler types of black hole perturbations in a fixed background spacetime, known as scalar and electromagnetic perturbations. In order to describe these two particular perturbations, we use the Klein-Gordon and Maxwell equations in curved spacetime:

$$\nabla_{\mu}\nabla^{\mu}\Phi=0, \tag{4.43}$$

$$\nabla_{\mu}F^{\mu\nu}=0, \tag{4.44}$$

where ∇_μ is the covariant derivative, Φ is a scalar field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Faraday tensor and A_μ is a vector potential of the electromagnetic field. Equations (4.43) and (4.44) can be rewritten as

$$\frac{1}{\sqrt{-g}}\partial_{\mu}\left(g^{\mu\nu}\sqrt{-g}\partial_{\nu}\Phi\right)=0, \tag{4.45}$$

$$\frac{1}{\sqrt{-g}}\partial_{\mu}\left(\sqrt{-g}F^{\mu\nu}\right)=0, \tag{4.46}$$

One can see that equations (4.45) and (4.46) can be separated into the radial and angular parts, if the wave function is chosen as harmonically time dependent form as follows

$$\Phi(x)=e^{-i\omega t}\frac{R_{\ell}(r)}{r}Y_{\ell}^{m}(\theta,\phi) \tag{4.47}$$

$$A_{\mu}(x)=e^{-i\omega t}\left[0,0,\frac{R_{\ell}^{B}(r)}{\sin\,\theta}\partial_{\phi},-R_{\ell}^{B}(r)\sin\,\theta\partial_{\theta}\right]Y_{\ell}^{m}(\theta,\phi) \tag{4.48}$$

where ω is the frequency of quasinormal modes for scalar and vector fields, $R_\ell(r)$ is a radial wave function for scalar field and $R_\ell^E(r), R_\ell^B(r)$ are profile functions for (electric and magnetic cases) the electromagnetic field. Substituting expression (4.47) and (4.48) into (4.45) and (4.46), one can write the following stationary wave equations:

$$\left[\frac{d^{2}}{dr_{*}^{2}}+\omega^{2}-V_{s}(r)\right]\psi_{s}=0 \tag{4.49}$$

where $V_s(r)$ is the effective potential for a spin weighted field described as

$$V_{s}(r)=f\left[\frac{\ell(\ell+1)}{r^{2}}+\frac{1-s^{2}}{r}\frac{df}{dr}\right], \tag{4.50}$$

Ushbu tadqiqotda biz Schwarzschild–MOG (Modified Gravity) qora tuynugining kvazi-normal modlarini (QNMs)o‘rgandik, bunda e’tibor skalyar va elektromagnit perturbatsiyalarga qaratildi. QNMs chastotasi qora tuynuklar perturbatsiyaga uchraganda ularning barqarorligi va tebranish xatti-harakatini tushunishda muhim ahamiyatga ega. Biz turli MOG parametri α qiymatlari uchun QNM chastotalarini aniqladik, bu tafsilotlar 4.1 va 4.3-jadvallarda keltirilgan.

α	$\omega(\ell=2)$	$\omega(\ell=3)$	$\omega(\ell=4)$
0.	0.506317-0.0961232i	0.691728-0.0961481i	0.880197-0.0961713i
0.2	0.433660-0.0807996i	0.592954-0.0808258i	$0.754779-0.0808460i$
0.4	$0.379726-0.0696219i$	0.519562-0.0696467i	$0.661552-0.0696639i$
0.6	0.338011-0.0611098i	$0.462752-0.0611321i$	$0.589363-0.0611467i$
0.8	0.304735-0.0544140i	0.417402-0.0544334i	0.531717-0.0544456i
1.	$0.277542-0.0490113i$	$0.380319-0.0490279i$	0.484568-0.049038i

Table 4.1: Quasi-normal modes of the massless scalar perturbation in the Schwarzschild-MOG spacetime for $n=0$.

α	$\omega(\ell=2)$	$\omega(\ell=3)$	$\omega(\ell=4)$
0.	0.480754-0.0943536i	0.673438-0.0952575i	0.865960-0.0956364i
0.2	$0.412178-0.0793758i$	$0.577579-0.0801080i$	0.742810-0.0804145i
0.4	0.361213-0.0684375i	0.506310-0.0690487i	$0.651235-0.0693041i$
0.6	0.321755-0.0600997i	0.451114-0.0606215i	$0.580301-0.0608392i$
0.8	0.290252-0.0535355i	0.407032-0.0539888i	$0.523644-0.0541777i$
1.	0.264488-0.0482354i	$0.370972-0.0486347i$	$0.477291-0.048801i$

Table 4.2: Quasi-normal modes of the electromagnetic perturbation in the Schwarzschild-MOG spacetime for $n=0$.

α	$\omega(\ell=2)$	$\omega(\ell=3)$	$\omega(\ell=4)$
0.	0.398850-0.0882854i	0.616561-0.0923181i	0.822303-0.0939294i
0.2	$0.351606-0.0750712i$	0.535440-0.0780357i	$0.710420-0.0792072i$
0.4	0.314453-0.0652472i	$0.473723-0.0675159i$	$0.626160-0.0684089i$
0.6	0.284483-0.0576534i	0.425099-0.0594465i	0.560266-0.0601515i
0.8	0.259796-0.0516084i	0.385746-0.0530628i	$0.507239-0.0536349i$
1.	0.239104-0.0466838i	0.353210-0.0478886i	$0.463594-0.048363i$

Table 4.3: Quasi-normal modes of the axial gravitational perturbation in the SchwarzschildMOG spacetime for $n=0$.

4.1-jadvalda massasiz skalyar perturbatsiya uchun kvazi-normal chastotalar keltirilgan, 4.3-jadval esa elektromagnit perturbatsiya uchun mos natijalarni ko‘rsatadi. Ikkala perturbatsiya turida ham bir xil tendensiya kuzatiladi: MOG parametri (\alpha) oshgan sari chastotalarning haqiqiy qismi kamayadi, shuningdek, mavhum qismi ham kamayadi, bu esa tebranishlarning sekinlashishi va kuchliroq dampinglanishini bildiradi. MOG parametri mavjud bo‘lmaganda ((\alpha=0)), natijalar Schwarzschild qora tuynugiga mos keladi va taqqoslash uchun asos bo‘lib xizmat qiladi. (\alpha) 0 dan 1 gacha oshganda, skalyar va elektromagnit perturbatsiyalarning chastotalari monoton ravishda kamayadi. Bu shuni anglatadiki, MOG parametri mavjudligi gravitatsion ta’sirlarni kuchaytiradi, natijada damping vaqtlari uzoqroq bo‘ladi va tebranish chastotalari sekinlashadi, Schwarzschild qora tuynugiga nisbatan. Ushbu jadvallarda keltirilgan natijalar MOG parametrining qora tuynukning tashqi perturbatsiyalarga javobini qanday o‘zgartirishini, uning barqarorligi va muvozanatga qaytish vaqt o‘lchovlariga ta’sirini ko‘rsatadi. Qayd etish joizki, gravitatsion g‘alayonlanishlarni boshqaruvchi tenglamalar ancha murakkab bo‘lishiga qaramay, ularni Regge–Wheeler–Zerilli tenglamasi shakliga keltirish orqali soddalashtirish mumkin. Nihoyat, biz bu tenglama uchun radial funksiyalar bo‘yicha sonli yechimlarni ham taqdim etdik. Biz tashqi asimptotik jihatdan bir jinsli magnit maydonga joylashtirilgan Shvartsshildga o‘xshash qora o‘ra uchun kvazinormal rejimlar (QNM) bo‘yicha olib borilgan raqamli hisoblash natijalarini qisqacha taqdim etamiz. Chastota sohasidagi tahlil uchun biz yarim-analitik WKB usulidan foydalanamiz. Ushbu usulda yechim har ikki cheksizlikda WKB qatoriga kengaytiriladi va bu asimptotik kengaytmalar effektive potensialning maksimumi yaqinidagi Teylor qatoriga yoyiladi.

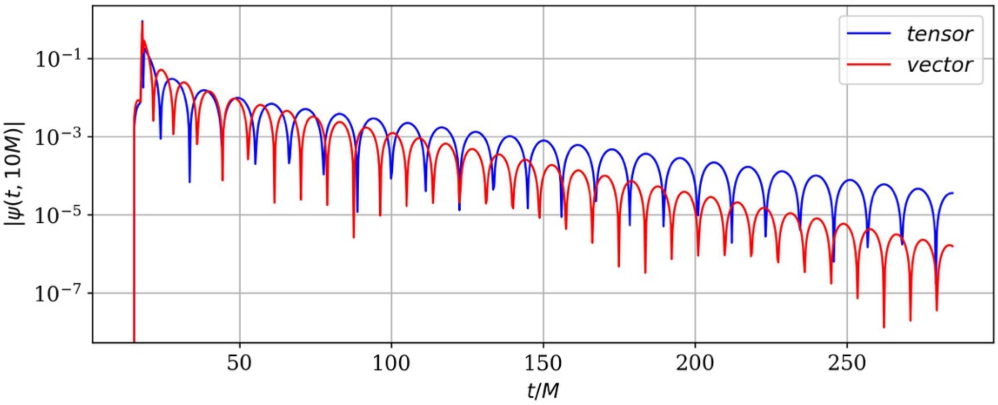
Yuqori tartibli WKB formulasi quyidagicha beriladi:

$$\begin{aligned}\omega^2 &= V_0 + A_2(K^2) + A_4(K^2) + A_6(K^2) + \dots \\ &- iK\left(-2V_1 + A_3(K^2) + A_5(K^2) + A_7(K^2) + \dots\right)\end{aligned} \tag{27}$$

bu yerda $K = n + 1/2$, va $n = 0, 1, 2, 3, \dots$.

12-rasm tashqi magnit maydonda joylashgan massasiz skalyar maydon uchun WKB usuli yordamida olingan ma’lumotlarga mos keluvchi eng yaxshi polinom funksiyani ko‘rsatadi. Rasmda ikki xil egri chiziq tasvirlangan: kulrang va qora – ular fazo-vaqt parametrlarining turli qiymatlariga mos keladi, ya’ni: a/M=0.3 (kulrang chiziq) va a/M=0.5 (qora chiziq). Bu parametr qiymatlari uchun kvazinormal rejim chastotasi ω ning mavhum qismlari rasmning o‘ng tomonida tasvirlangan.

Bu yerda WKB natijalari vaqt sohasidagi integrallash orqali tekshirildi. Biroq, bunday taqqoslashni to‘g‘ri amalga oshirish uchun ikki muhim jihatni hisobga olish zarur: $\ell = 0$ perturbatsiyalar uchun kvazinormal tebranish davri juda qisqa bo‘ladi, chunki u tezda tebranma xarakterdagi kuch qonuni (power-law) quyruqlari bilan almashtiriladi (14-rasmga qarang). Shu sababli, bu holatda vaqt sohasidagi profildan chastotani aniqlik bilan ajratib olish qiyin bo‘ladi. Ikkinchi muhim jihat — bu kechki vaqt oralig‘ida perturbatsiyaning o‘sishi (13-rasmga qarang). Bunday o‘sish, ehtimol, m > 0 bo‘lgan holatda katta r qiymatlarida effektiv potensialning manfiy bo‘lishi tufayli yuzaga keladigan barqarorlik buzilishi (noaniqlik) deb talqin qilinishi mumkin.



13-Rasm: Yarim-logarifmik shkalada tenzor va vektor g’alayonlanishlarining vaqt bo’yicha evalyutsiyasi ko’rsatilgan.

Garchi regular qora o‘ralarning kvazinormal modalari ko‘plab ishlarda keng qamrovli o‘rganilgan bo‘lsa-da, tashqi magnit maydon ishtirokidagi regular qora o‘ralar uchun hech qanday keng qamrovli tadqiqotlar olib borilmagan. Ushbu ishda biz bu bo‘shliqni to‘ldirishga harakat qildik va magnit maydon zaryadlangan skalyar maydon spektrini sezilarli darajada o‘zgartirishini namoyish etdik, bu esa o‘z navbatida kvazi-rezonanslar deb ataluvchi uzoq yashovchi kvazinormal modalarning paydo bo‘lishiga olib keladi.

Magnit maydon ishtirokida g‘alayonlanishlar (perturbatsiyalar) evolyutsiyasining yana bir o‘ziga xos jihati asimptotik dumlarning g‘ayrioddiy harakatidir. Parametrlarning muayyan qiymatlarida bu dumlar darajali qobiqni namoyon etmaydi, aksincha, ossillyatsion (tebranuvchi) qobiqni namoyish etadi.

XULOSA

“Skalyar-tenzor-vektor gravitatsiyasida qora o’ralarning energetik xossalari” mavzusida olib borilgan ilmiy tadqiqot asosida quyidagi ilmiy xulosalarga kelindi:

Schwarzschild–MOG qora tuynuk atrofidagi foton harakati o‘rganildi va nazariy natijalar fenomenologik kuzatuvlar bilan solishtirildi. EHT (Event Horizon Telescope) hamkorligi orqali MOG parametri uchun quyidagi cheklov $\alpha < 0.21$ topildi.

Shuningdek, STVG doirasida neytral massiv zarracha qora tuynuk bilan vektor maydon orqali o‘zaro ta’sir natijasida tezlanishi mumkinligi ko‘rsatildi. S2 yulduzining Sgr A* atrofidagi orbitasi bo‘yicha STVG nazariyasi va kuzatuv ma’lumotlarini solishtirish orqali MOG parametri uchun yuqori chegara quyidagicha $\alpha < 0.58$ aniqlangan.

Shuningdek, MOG parametri mavjud bo‘lganda ISCO orbitasining xarakteristik radiuslari oddiy Schwarzschild fazo-vaqtiga nisbatan kattaroq bo‘lishi ko‘rsatildi. Bundan tashqari, Schwarzschild–MOG qora tuynuk atrofida sinov zarralarning maksimal energiya samaradorligi taxminan ~8.14% ga yetishi mumkinligi aniqlangan, bu esa Schwarzschild holatiga nisbatan nisbatan yuqori samaradorlikni bildiradi.

STVG doirasida aylanayotgan qora tuynuklar kvazarlar va blazarlar kabi yuqori energiyali manbalar uchun yaxshi nomzod bo‘lishi mumkinligi ko‘rsatildi. Penrose jarayoniga ko‘ra, energiya samaradorligi 100% dan oshishi mumkin.

Birinchi marta, STVG doirasida tashqi vektor maydon bilan o‘zaro ta’sir qiluvchi massiv zarracha tomonidan chiqariladigan gravitatsion sinkrotron radiatsiyasining analoqlari muhokama qilindi. Tadqiqotlar shuni ko‘rsatdiki, tipik yulduzli qora tuynuk 10 $M_{\odot}$ va uning atrofida aylanayotgan supermassiv qora tuynuk 10^9 $M_{\odot}$ uchun chiqariladigan radiatsiya intensivligi taxminan:~2.43×10^{39}erg/s ekanligi aniqlandi

Schwarzschild–MOG qora tuynukning termodinamik xususiyatlari o‘rganildi. Tadqiqotlar shuni ko‘rsatdiki, MOG parametri STVG doirasida qora tuynukning termodinamik kattaliklariga sezilarli ta’sir qiladi. Kuchli gravitatsiya ostida Schwarzschild–MOG qora tuynukning barcha termodinamik kattaliklari oshadi.

Shuningdek, Schwarzschild–MOG qora tuynuk perturbatsiyalari o‘rganildi. Tadqiqotlar shuni ko‘rsatdiki, kuchli gravitatsiya sababli scalar, elektromagnit va tensor perturbatsiyalarning kvazi-normal rejimlari (QNM) sezilarli darajada o‘zgaradi. STVG nazariyasida tensor g’alayonlanish vector g’alayonlashdan qaraganda sekonroq so’nishi ko’rsatildi

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ENERGETIC PRPERTIES OF BLACK HOLES IN SCALAR-TENSOR-VECTOR GRAVITY

01.03.01 – Astronomy
01.04.02 – Theoretical physics

ABSTRACT ON AWARDING THE SCIENTIFIC DEGREE OF DOCTOR OF SCIENCE ON DISSERTATION

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INTRODUCTION (presentation abstract)

Relevance and necessity of the topic. Modified theories of gravity represent extensions of general relativity (GR) that are gaining substantial attention in cosmology and astrophysics. These theories are being developed for several reasons, such as explaining the Universe's accelerating expansion, examining the foundational principles of GR, and addressing longstanding issues within these fields including singularity and non-compatibility with quantum field theory. Currently, modified gravity theories are a focal point of research, as they may offer new insights into both the fundamental aspects of gravity and the universe's structure. However, classical gravity presents considerable obstacles when attempting to unify quantum fields with quantized gravity. In this context, the scalar-tensor-vector gravity (STVG) emerges as a compelling alternative. This approach considers gravity not as a fundamental force, but as an interaction that arises from different types of fields. Inspired by the differences between GR's description of gravity and the behavior of other fundamental forces, the concept of STVG is an area of active research and discussion, with the potential to reveal new perspectives on the universe's underlying structure and the nature of gravity.

In STVG, gravity is generated by introducing additional field generating spacetime curvature. This theory, also known as Modified Gravity (MOG), is one of the simplest extensions of general relativity that seeks to address certain cosmological and astrophysical challenges, such as the need for dark matter. STVG introduces additional fields to general relativity, specifically scalar, tensor, and vector fields, to modify the behavior of gravity on large scales.

Radiation reaction is a phenomenon in physics where a charged particle slows down or speeds up due to the emission of electromagnetic radiation (photons) as it moves through space. This effect arises from the particle's interaction with its own electromagnetic fields, as described by classical electrodynamics. When a charged particle accelerates or decelerates, it generates changing electric and magnetic fields, which in turn produce electromagnetic radiation. This radiation carries away energy and momentum from the particle, causing it to lose kinetic energy and reduce its speed.

By modifying the fundamental gravitational principles, modified gravity theories seek to elucidate galactic rotation curves, gravitational lensing, and the dynamics of large-scale structures in the universe. A deeper comprehension of gravity on cosmic scales is vital for accurately interpreting astrophysical phenomena and advancing our understanding of the fundamental forces shaping the Universe. This work is devoted to analysis of the effect of radiation reaction in dynamical motion of test particle around the Schwarzschild-MOG black hole in STVG.

This research work corresponds to the tasks by the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. PD-4947 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan" dated February 07, 2017, Resolution of the President of the Republic of Uzbekistan No. PR-2789 "On measures for further improvement of the activities of the Academy of Sciences, organization, management and financing of research activities" dated February 18, 2017 and others.

Conformity of the research to the main priorities of science and technology development of the Republic. The dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. “Power, energy and resource-saving”.

The degree of knowledge of the problem. The particle motion around compact gravitating object have been investigated by the different researcher worldwide (A. Ovgun, R. Pantig, G. Mustafa, F. Javed, A. zakharov, J.Schee, M. Kolos, B Ahmedov, N. Dadhich, S. Ghosh, P. Joshi, etc.). However, particle motion around black holes in scalar-tensor-vector gravity have not been systematically studied.

The null geodesic of the compact objects are properly studied by different authors (F. Atamurotov, A. de-Vries, L. Rezzolla, V. Bozza, Z. Stuchlik, A. Abdujabbarov, etc). The investigation of null geodesics and the associated optical properties of black holes, such as black hole shadows, have been remain an active topic of research in rotating black hole spacetimes and have been studied by various researchers (A. Belhaj, M. Benali, J. Luminet, etc.). However, the question of effect of MOG parameters of scalar-tensor-vector gravity on shadow and related phenomenon have not been studied yet.

Connection of the topic of the dissertation topic to the scientific works of higher education and research institutions, where the dissertation is carried out. The dissertation was done in the framework of the scientific projects funded by the Ministry of Innovative Development. F-FA-2021-510 "Investigations of nuclear matter of neutron stars in modified gravity".

The aim of the research is the development and improvement of theoretical models of astrophysical processes based on scalar-tensor-vector gravity and corresponding black holes solutions.

The tasks of the research:

to analyse photon motion around the Schwarzschild-MOG black hole and find analytical expressions for the photon sphere and critical impact parameter and to obtain constraints for MOG parameter using EHT result for a black hole shadow;

to investigate the circular motion of massive particles in the Schwarzschild-MOG spacetime, specifically examining the innermost stable circular orbit and obtain constraint for MOG parameter from the observation of S2 Star orbiting around Sagittarius A*;

to test the Penrose process and analyse the energy extraction from a Kerr-MOG black hole in STVG;

to study the gravitational radiation emitted by a massive particle orbiting the Schwarzschild-MOG black hole;

to study thermodynamic properties of the Schwarzschild-MOG black hole and to investigate several thermodynamic observational quantities;

to analyse photon motion around the black hole and find analytical expressions for the photon sphere and impact parameter;

to investigate the circular motion of massive particles in the Schwarzschild-MOG spacetime, specifically examining the innermost stable circular orbit (ISCO) and marginally bound orbit;

to find an exact analytical solution for the components of the vector potential of the electromagnetic field in the Schwarzschild-MOG background;

to investigate the black hole perturbation in the Schwarzschild-MOG spacetime, particularly examining quasi-normal modes of the black hole;

The object of the research are astrophysical compact objects, modified gravity, scalar-tensor-vector gravity.

The subject of the research are massive and massless particles motion, analytic solutions for the electromagnetic field, energetic processes around black holes, thermodynamics of the black holes.

The methods of the research are methods of computational mathematics, methods of theoretical astrophysics, modern methods of mathematical physics, analytical and numerical methods of calculating differential equations for field and particle motion.

The scientific novelty of the research is the following:

For the first time it has been found the exact formulation of the first law of black hole thermodynamics in STVG including the additional term arising from the presence of scalar-vector-tensor field. It has been shown that the Hawking temperature decreases due to the influence of the MOG parameter. It has been observed that the positive nature of the MOG parameter ensures that the temperature of the Schwarzschild-MOG black hole is consistently smaller than that predicted in the Schwarzschild spacetime.

It is shown that entropy of the Schwarzschild-MOG black hole is always greater than the entropy of the Schwarzschild black hole.

The consistency between the area law and the first law of black hole thermodynamics enhances our understanding of the entropy of Schwarzschild-MOG black holes and contributes valuable insights to the broader field of gravitational physics.

It has been shown that the heat capacity for the Schwarzschild-MOG black hole is negative. It has been concluded that within the context of the STVG theory, the fractional heat capacity of the black hole increases with the augmentation of the MOG parameter.

The results prompt a reevaluation of the thermodynamic properties of black holes within the Schwarzschild-MOG framework, challenging the conventional expectations derived from general relativity.

It has been stated that the black hole's lifetime is longer in the context of STVG. It has been shown that both the photon sphere and the size of the black hole shadow increase due to the presence of MOG.

The practical results of the research are the following:

It has been also shown that in the presence of MOG parameter the characteristic radii of ISCO orbits are larger compared to those in the pure Schwarzschild spacetime. Furthermore, it was obtained that the maximum energy efficiency of test particles around the Schwarzschild-MOG black hole can reach up to approximately ~8.14 % corresponding a relatively large efficiency compared to the Schwarzschild case.

It has been shown that the magnetic field exhibits a non-uniform behavior in the vicinity of the Schwarzschild-MOG black hole, with denser field lines compared to the pure Schwarzschild case.

It was estimated that for a typical SMBH with a mass of $10^9\ M_\odot$ and black hole with mass 10 $M_\odot$ orbiting around it, the intensity of the radiation takes the value ~2.43×10^{39} erg.

For the first time the gravitational analogue of radiation reaction experienced by a massive particle orbiting a Schwarzschild-MOG black hole has been developed. Within this model the equation of motion has been simplified by discarding high-order terms concerning damping time.

It has been demonstrated that the maximum center-of-mass energy of a colliding pair of particles depends on the disparity in the specific energy values of the individual particles. Moreover, it remains independent of the vector field, and the effect of STVG emerges solely from the spacetime geometry.

The reliability of the research results provided by applying modern proven methods of mathematical physics, computational mathematics, and relativistic astrophysics. The results were obtained strictly within the mathematical apparatus of general relativity and theoretical physics. Modern numerical and analytical methods of calculation are also used, and the results are compared with available observational data and the results of other authors. The structured conclusions of the thesis correspond to the basic rules of astrophysics of compact objects.

The scientific and practical significance of the research results. The ongoing exploration of the research results not only contributes to our understanding of black hole physics but also underscores the intricate interplay between gravitational theories and thermodynamics, paving the way for further advancements in our comprehension of the fundamental nature of the cosmos.

The practical significance of the research results is that the positive nature of the MOG parameter ensures that the temperature of the Schwarzschild-MOG black hole is consistently smaller than one predicted in the Schwarzschild spacetime, providing valuable insights into the thermal behavior of black holes in the context of Modified Gravity.

Application of the research results. The developed theoretical models of the photon and particle motion in the MOG theory have been applied to the followings:

scientific results obtained on the motion of particles and photons have been used by scientists from Fudan University (FU) in Shanghai (FU, China, November 4, 2024 reference);

For the first time the thermodynamics of the MOG black hole, including enthalpy, Hawking temperature, and Gibbs free energy have been explored in detail. Results on the dynamics of particles have been used in the works of foreign researchers, in foreign journals with a high impact factor (Physics Letters B, Volume 854, id.138758, Web-Sc, IF-4.4; Communications in Theoretical Physics, Volume 76, Issue 8, id.085402, Web-Sc, IF-2.4; Pramana, Volume 98, Issue 3, id.92, Web-Sc, IF-2.219 and others) are used in more than 30 published scientific papers to describe the effects of particles around a black hole.

Approbation of the research results. The research results were reported in the form of reports and tested at 8 international and local scientific conferences. The main results of the study were tested at the scientific seminars of the Institute of Fundamental and Applied Research (2023), Astronomical Institute (2020-2022), of the Department of Theoretical Physics of Samarkand State University of Uzbekistan (2019), Department of Chemistry-Physics in Basque Country University (Spain, 2017-2018), Department of Theoretical Physics in University of Zurich (Switzerland, 2016) etc.

Publication of research results. The results of DSc research have been presented in 30 peer-reviewed articles published in prestigious Q1/Q2 quartile scientific journals recommended by Supreme Attestation Commission at the Ministry of higher education, science and innovations of the Republic of Uzbekistan.

The main content of the dissertation: The dissertation presents brief information on the relevance and necessity of the research topic, its correspondence to the priority directions of the development of science and technology in the Republic, the degree to which the problem has been studied, and its connection with the research plans of the higher education institution where the dissertation was carried out. In addition, the objectives and tasks of the research, the object and subject of the study, the applied methods, scientific novelty, practical results, reliability of the obtained results, their scientific and practical significance, implementation of the results into practice, approbation, publication record, as well as the structure and volume of the dissertation are described. In the first chapter of the dissertation, entitled "Energetic Properties of Black Holes in Scalar–Tensor–Vector Gravity," the dynamics of particles around a Schwarzschild–MOG black hole are investigated. The second chapter is devoted to the study of the Penrose process in the vicinity of a Kerr–MOG black hole. Furthermore, the circular motion of massive particles in the Schwarzschild–MOG spacetime in the presence of a vector field is analyzed. In addition, taking into account the radiation reaction term, the circular motion of massive particles is studied and their trajectories are presented. In the final, fourth chapter, black hole perturbations are investigated.

In the spherical coordinates $x^\alpha = (t, r, \theta, \phi)$, the simple black hole solution in STVG is given by the Schwarzschild-MOG spacetime

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega, \qquad f(r) = \left(1 - \frac{2GM}{r} + \frac{G\tilde{Q}^2}{r^2}\right) \quad (1)$$

where M is a mass of a black hole, G is a modified gravitational constant related to Newtonian gravitational constant G_N as fallows $G = G_N(1+\alpha)$. Notice that the spacetime metric (1) is similar to the Reissner-Nordström spacetime, however, the quantity $\tilde{Q} = \sqrt{\alpha G_N} M$ is the coupling constant regarded to the external potential in STVG which is given as

$$\Phi_\mu = \Phi_t(1,0,0,0), \qquad \Phi_t = -\frac{\tilde{Q}}{r} = -\frac{\sqrt{\alpha G_N} M}{r} \qquad (2)$$

and α is positively defined MOG parameter. The radius of the outer and inner horizons of Schwarzschild-MOG black hole are $r_\pm = G_N M(1 + \alpha \pm \sqrt{1+\alpha})$. We now examine the motion of a massive particle in the vicinity of a black hole within the framework of STVG. Unlike in general relativity and many other gravitational theories, in STVG massive particles do not follow geodesic line. The motion of test particle within STVG theory is described by the following non-geodesic equation:

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = \frac{\tilde{q}}{m} B^\alpha_\beta \frac{dx^\beta}{ds}, \qquad B_{\alpha\beta} = \partial_\alpha \Phi_\beta - \partial_\beta \Phi_\alpha \qquad (3)$$

where m denotes the mass of the test particle, while $\tilde{q} = \sqrt{\alpha G_N} m$ represents the coupling strength between the particle and the vector field Φ_μ, $\Gamma^\alpha_{\mu\nu}$ are the Christoffel symbols and $\dot{x}^\alpha = dx^\alpha/ds$ is the 4-velocity of test particle normalized as $g_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta = -1$. The presence of the additional force term involving the field strength tensor $B^\mu{}_\nu$ indicates that massive particles in STVG are subject to a non-geodesic force analogous to a Lorentz force in electrodynamics, arising from their interaction with the vector field.

When we consider the photon motion near a Schwarzschild-MOG black hole, one can set $\tilde{q} = 0$. Equation of motion for photon can be expressed as

$$\frac{dr}{d\lambda} = \sqrt{\frac{1}{b^2} - \frac{f(r)}{r^2}}, \qquad \frac{d\phi}{d\lambda} = \frac{1}{r^2}, \qquad \frac{dt}{d\lambda} = \frac{1}{bf(r)} \tag{4}$$

where b is the impact parameter of photon and λ is an affine parameter normalized by the angular momentum of the photon. When the photon passes near the massive object, some part of the photon will be captured by the black hole and then will move along spherical orbits defining the boundary of the shadow of the lensing object. If photon moves along the spherical geodesics the radial equation satisfies the following conditions

$$\frac{dr}{d\lambda} = 0, \frac{d^2r}{d\lambda^2} = 0 \tag{1.28}$$

The first equation of (1.28) is responsible for the turning point of the light ray while second one is represents the radius of the spherical orbit.

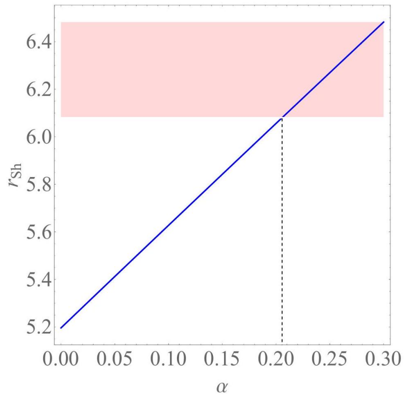


Figure 1.2: Capture cross section of photon by the black hole (shadow of the black hole) in the (x-z) plane for the different values of α parameter.

the radius of the photonsphere and the critical impact factor of photon is determined as

$$r_{\rm ph} = \frac{1}{2}\left[3(1+\alpha) + \sqrt{(1+\alpha)(9+\alpha)}\right] G_N M, \qquad b = \frac{r_{\rm ph}}{\sqrt{f\left(r_{\rm ph}\right)}}, \tag{}$$

One can easily check that absence of α parameter above expressions will be $r_{\rm ph} = 3M$ and $b_0 = 3\sqrt{3}M$ corresponding to the photon-sphere and the critical impact parameter in the Schwarzschild space.

The capture cross section of photon by the black hole can be determined as $\sigma = \pi b_0^2$ and in the Schwarzschild-MOG metric it takes a form:

$$\sigma = \frac{2\pi\alpha M^2[3 + 3\alpha + \sqrt{(1+\alpha)(9+\alpha)}]^2}{\alpha + 3 + \sqrt{(1+\alpha)(9+\alpha)}}, \tag{1.31}$$

while in the Schwarzschild one it reduces to $\sigma = 27\pi M^2 \simeq 84.823 M^2$.

To have

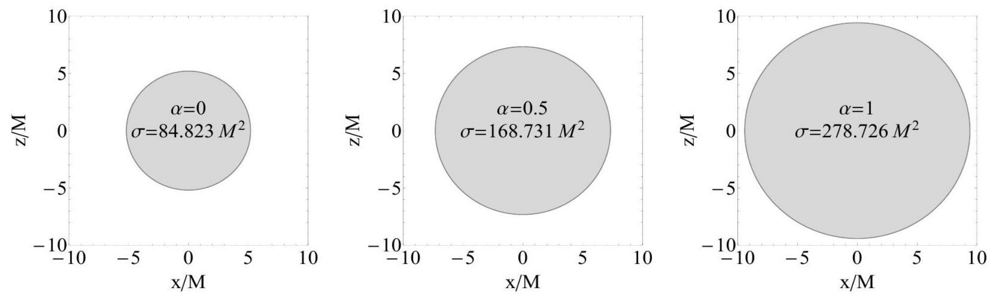


Figure 1.3: Capture cross section of photon by the black hole (shadow of the black hole) in the (x-z) plane for the different values of α parameter.

an idea about size of the black hole's shadow one can estimate cross section for the supermassive black hole (SMBH) with a million solar mass in the form:

$$\sigma \simeq 1.576 \times 10^{16} \left(\frac{M}{10^6 M_\odot}\right)^2 \text{ km}^2. \tag{1.32}$$

Note, that this large valued size of the shadow will have very small angular size due to the far distance to the super massive black hole from observer. More precise analyses can be presented in graphical way. Figure 1.3 illustrates capture cross section of photon by the black hole for different values of α parameter. Our analyses show that capture cross section in Schwarzschild-MOG space gets larger than that obtained in general relativity.

Finally, in the case of weak field approximation, the deflection angle is determined as

$$\hat{\alpha}_b = \frac{4(1+\alpha)M}{b} + \frac{(1+\alpha)[(15\pi - 16)(\alpha+1)+8]M^2}{4b^2}, \tag{1.37}$$

Because the leading term already contains $1+\alpha$, observational constraints on the deflection angle from precise lensing measurements (e.g. light deflection near the Sun or strong-lens systems) primarily constrain α through that linear scaling. The second-order difference is numerically tiny for typical astrophysical lensing situations it scale $O(M^2/b^2)$, but could be relevant in precision tests (e.g. very close grazing by compact objects or future high-precision astrometric missions). Therefore, it is important to check which definition of b was used in the derivation and to verify the expansion keeping consistent coordinate choices when comparing second-order coefficients to the canonical Schwarzschild result. It is well-known that gravitational lensing is one of the powerful tools to test GR versus alternate theories of gravity. According to GR theory the deflection of the light-ray around static spherically symmetric gravitational source with the total mass M is $\hat{\alpha} = 4M/b$, where b is the impact parameter of the light-ray. Here we study the weak gravitational lensing effect in the Schwarzschild-MOG spacetime. The one of simple approaches for studying weak gravitational lensing has been developed by expanding metric tensor as follows:$g_{\alpha\beta} \simeq \eta_{\alpha\beta} + h_{\alpha\beta}$, where $\eta_{\alpha\beta}$ is the metric tensor of a flat Minkowski spacetime and $h_{\alpha\beta}$ is a perturbation of the metric tensor. The deflection angle of the light-ray can be determined as

$$\hat{\alpha}_b = \frac{4M}{b}(1+\alpha) + \frac{\pi M^2}{b^2}(1+\alpha)\left(1+\frac{\alpha}{4}\right), \tag{1.42}$$

which is the exactly same expression as obtained in general relativity, however, the sign should be opposite. Since the deflection angle is vector quantity one can easily replace sign of expression (1.42).

Now we focus on investigating massive particle motion aroud the SchwarzschildMOG black hole in STVG. As we mentioned before that massive particle follow Lorentz-like equation given in equation (1.22). For simplicity, we consider motion in the equatorial plane (i.e. $\theta = \pi/2$). Hereafter integrating (1.22), equation of
motion of massive particle can be written as

$$\frac{dt}{ds} = \frac{1}{f}\left(\mathcal{E} - \frac{\alpha G_N M}{r}\right) \quad (1.47)$$

$$\frac{d\phi}{ds} = \frac{\mathcal{L}}{r^2} \quad (1.48)$$

$$\left(\frac{dr}{ds}\right)^2 = \left(\mathcal{E} - \frac{\alpha G_N M}{r}\right)^2 - f\left(1 + \frac{\mathcal{L}^2}{r^2}\right) \quad (1.49)$$

where $\mathcal{E}$ and $\mathcal{L}$ are the specific energy and specific angular momentum of test particle. For simplicity, we consider circular motion of test particle in the equatorial plane of the Schwarzschild-MOG black hole i.e., $\theta = \pi/2$ and $\dot{\theta} = 0$. The radial equation of motion can be derived as

$$\dot{r}^2 = \left(\mathcal{E} - \frac{\alpha G_N M}{r}\right)^2 - f\left(1 + \frac{\mathcal{L}^2}{r^2}\right) = [\mathcal{E} - V_+(r)][\mathcal{E} - V_-(r)] \quad (1.50)$$

where $V_\pm(r)$ is defined as

$$V_\pm(r) = \frac{\alpha G_N M}{r} \pm \sqrt{f\left(1 + \frac{\mathcal{L}^2}{r^2}\right)} \quad (1.51)$$

The effective potential is a valuable tool for understanding how test particles move around black holes. Figure 1.4 shows the effective potential for massive particle for different values of the MOG parameter. By locating the minimum value of the effective potential, we can determine the position of the innermost stable circular orbit (ISCO) and understand how massive particles orbit around black holes without getting drawn into them or flung out into space. The ISCO is a significant location for astrophysical observations, as it affects the emission of radiation and matter falling onto the black hole. Additionally, studying the

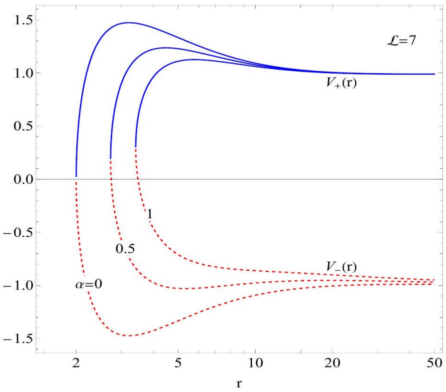


Figure 1.4: Dependence of the ISCO position for massive particle orbiting the SchwarzschildMOG black hole from the MOG parameter α.

ISCO can provide insights into the properties of black holes and tests of general relativity, including alternative theories of gravity. The location of the ISCO depends on various factors such as the mass and spin of the black hole and the STVG parameter in the current research work.

In this study, we focus on finding the radius of the ISCO of massive particles around black holes in STVG and testing how it depends on the α parameter. To simplify the analysis, we consider the motion of massive particles in the equatorial plane, meaning that the effective potential depends on only the radial coordinate. We can find the ISCO position using the standard method with the following conditions: $V_+(r) = \mathcal{E}, V_+'(r) = 0$, and $V_+''(r) \leq 0$. Analytical expressions can be obtained for the ISCO position, but careful numerical analyses are necessary to accurately determine its location.

Interestingly, our results indicate that the ISCO position gets larger due to the

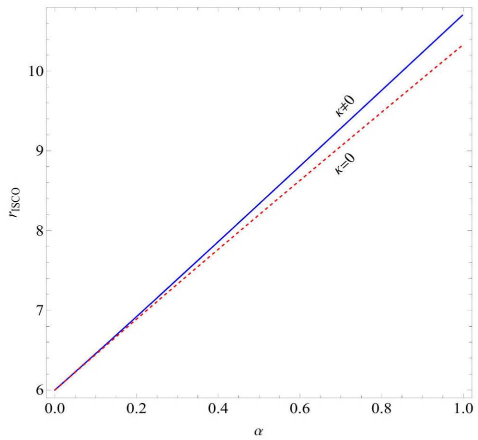


Figure 1.5: Dependence of the ISCO position for massive particle orbiting the SchwarzschildMOG black hole from the MOG parameter α.

effect of STVG. Figure 1.5 illustrates the dependence of the ISCO position of massive particles orbiting around the Schwarzschild-MOG. These findings highlight the importance of considering the effects of STVG when studying the motion of massive particles around black holes.

A motion of S2-stars around the SMBH located in center of the Milky Way is quit interesting and important task to test the modified or/and alternative theories of gravity in both theoretical and astrophysical point view. It is well-known that the periastron expression of celestial object orbiting around central gravitational object in the framework of general relativity is determined as

$$\delta\phi_{\rm GR} = \frac{6\pi G_N M}{ac^2(1-e^2)} \tag{1.54}$$

where a and e being the orbital semi-major axis and eccentricity, respectively. Here we are aimed to shown the derivation of the periastron precession of S2-star orbiting around SMBH in the STVG in the precise way. Taking into account equations (1.48), (1.49), and after introducing new variable $u = \mathcal{L}^2/G_N M r$, one can easily obtain

$$\left(\frac{du}{d\phi}\right)^2 = \frac{1}{\epsilon}(\mathcal{E} - \alpha\epsilon u)^2 - [1 - 2(1+\alpha)\epsilon u + \alpha(1+\alpha)\epsilon^2 u^2]\left(u^2 + \frac{1}{\epsilon}\right), \tag{1.55}$$

where $\epsilon = (G_N M/\mathcal{L})^2 \ll 1$ is a small parameter. Notice that the solution of above equation represents the trajectory of celestial object orbiting around the SMBH. Hereafter differentiating the equation (1.55), one can get

$$u'' + u(1+\alpha\epsilon) = 1 + \alpha\delta + 3\epsilon(1+\alpha)u^2 - 2\epsilon^2\alpha(1+\alpha)u^3 \tag{1.56}$$

where $\delta = 1 - \mathcal{E}$ and double primes denotes second order derivative with respect to the angle ϕ. Obviously, it is difficult to find exact solution to above equation. Therefore the semi-analytical approach can be performed to find approximate value of the periastron shift. Let's expand the solution in power of the small parameter in the form: $u(\phi) = u_0(\phi) + \epsilon u_1(\phi) + \mathcal{O}(\epsilon^2)$. Substituting the equation (1.57) into (1.56), the equations in the zeroth and first order approximations can be written as

$$u_0'' + u_0 = 1 + \alpha\delta \tag{1.58}$$

$$u_1'' + u_1 = -\alpha u_0 + 3(1+\alpha)u_0^2 \tag{1.59}$$

As we mentioned before that the trajectory of celestial body in Newtonian theory is closed and the position of test particle satisfied the following condition $u(\phi) = u(\phi + 2\pi)$. However due to the general relativistic effect the position can be found $u(\phi) \simeq u(\phi + 2\pi + \delta\phi)$ and the periastron precession in STVG theory reads

$$\delta\phi = \delta\phi_{\rm GR}\left(1 + \frac{5}{6}\alpha\right) \tag{1.67}$$

which can be easily obtained in the weak field approximation. It is worth to noting that, in particular, statistical analyses on constraining the α parameter and cross correlations of the parameters by using Monte Carlo Markov Chain algorithm are nicely presented. It has been shown the upper limit for the STVG parameter is $\alpha \leq 0.58$ at 99.7% confidence level.

In order to better understand the effects of the STVG we produce the trajectory of the S2 star around the SMBH and probe how the STVG parameter affects on the trajectory of the star. Figure 1.7 draws the trajectory of the S2 star around SMBH in the STVG theory for different values of the parameter α which is produced by a contour plot of $r/a = (\cos\,\phi, \sin\,\phi)(1-e^2)/u$, here u is obtained from equation (1.65) when $\delta = 2$. As one see that the periastron of the S2 star is very sensitive to the parameter α and it might decreases approximately 7.5 times due to the STVG. It should be also noted that the initial position of the S2 star is different from that is predicted in GR.

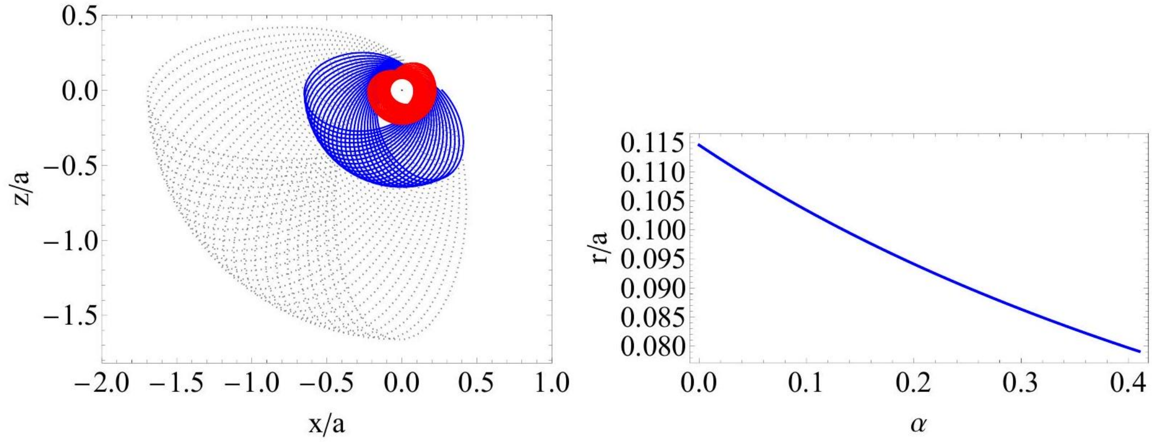


Figure 1.7: (Left) The trajectory of the S2 star around the SMBH in the STVG for different values of α parameter. The dashed gray line represents the trajectory of star in GR, while solid blue and red lines are responsible for $\alpha = 0.1$ and $\alpha = 0.4$. (Right) The initial position of star is a function the STVG parameter α.

As previously noted, the spherically symmetric black hole solution in MOG is represented by the Schwarzschild-MOG spacetime, which closely resembles the Reissner-Nordström solution. When the black hole's rotation is taken into account, the corresponding solution becomes the Kerr-MOG spacetime, analogous to the Kerr-Newman geometry. In the Boyer-Lindquist coordinates, the spacetime around rotating black hole in the STVG is described by the Kerr-MOG metric:

$$ds^2 = -\frac{\Delta}{\Sigma}(dt - a\sin^2\theta d\phi)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\sin^2\,\theta}{\Sigma}[(r^2+a^2)d\phi - adt]^2 \tag{2.1}$$

where $\Delta = r^2 - 2(1+\alpha)Mr + \alpha(1+\alpha)M^2 + a^2$ and $\Sigma = r^2 + a^2\cos^2\,\theta$. Here a is specific angular momentum (spin) of the black hole along mass of black hole M and parameter α in STVG theory. Notice that the associated vector potential, Φ_μ, to the spacetime (2.1) which characterize the fifth interaction between test body with external vector field and this interaction is described by the Coulomb-like potential in the static spacetime. While in the rotating spacetime the vector potential can be extended as

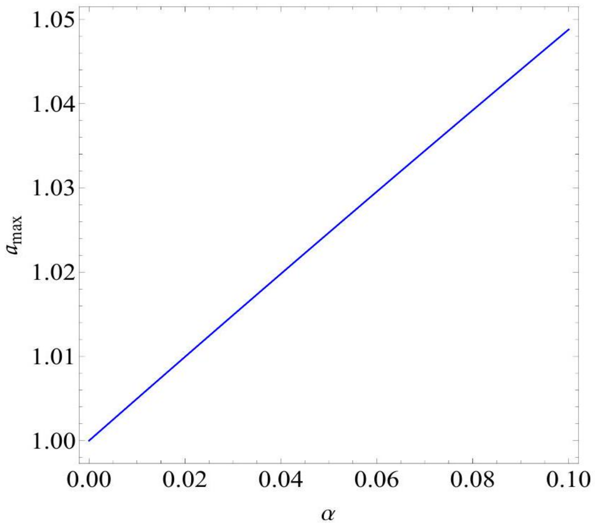
$$\Phi_\mu = \frac{\sqrt{\alpha}Mr}{\Sigma}(-1,0,0,a\sin^2\,\theta). \tag{2.2}$$

The horizon and ergosphere of the rotating black hole in STVG is located at

$$x_+ = 1 + \alpha + \sqrt{1+\alpha-a_*^2}, \quad x_{\rm erg} = 1 + \alpha + \sqrt{1+\alpha-a_*^2\cos^2\,\theta}$$

where $x = r/M$ is a dimensionless radial coordinate and $a_* = a/M$. According to the expression (1.21) the STVG parameter is always positive, i.e. $\alpha \geq 0$. From the existence of the black hole horizon one can find the limit the black hole's spin parameter as $|a_*| \leq \sqrt{1+\alpha}$ which means in the STVG theory the black hole can rotate faster than Kerr black hole. The dependence of the maximal spin of the black hole from STVG parameter α is illustrated in Fig. 2.1. It is known that the maximal spin of Kerr black hole is $a_{*\,\max} = 1$ while in the STVG theory it equal to $a_{*\,\max} = \sqrt{1+\alpha}$. Figure 2.2 draws the ergosphere of the Kerr-MOG black hole for different values of the

STVG parameter α. As one can see that from this result that when the α parameter gets a large the gravitational force dominates rotational force, therefore the region of the ergosphere gets smaller.



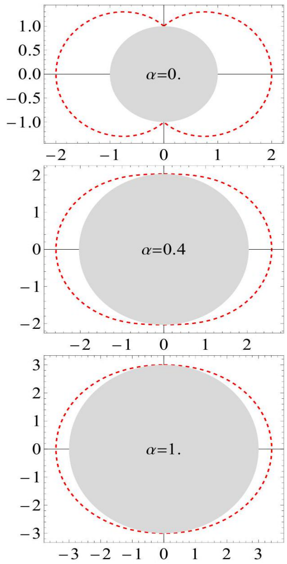


Figure 2.1: Dependence of the maximal spin of the black hole from the STVG parameter α which is given by the equation $a_{\max} = \sqrt{1+\alpha}$. Figure 2.2: The ergosphere of the black hole for the different values of the STVG parameter α for $a_* = 1$.

Now we consider massive particle motion around black hole in the STVG theory. In this theory massive particles do not follow the geodesic line in unlike other gravity theories. The conserved quantities of motion, namely, the energy E, and the angular momentum L, of test particle are satisfied the following relations:

$$g_{tt}\dot{t} + g_{t\phi}\dot{\phi} = -\frac{E + q\Phi_t}{m}, \qquad g_{\phi\phi}\dot{\phi} + g_{t\phi}\dot{t} = \frac{L - q\Phi_\phi}{m}, \tag{2.4}$$

which allows to derive the following expressions for $\dot{t}$ and $\dot{\phi}$. Using the normalization of the four-velocity and taking into account above expressions, one can have the following equation. Considering motion of massive particle in circular orbit with the following conditions $\dot{r} = 0$ and $\dot{\theta} = 0$, one can find that in STVG, massive particle motion is bounded by the following effective potential:

$$V = -q\Phi_t + \omega(L - q\Phi_\phi) + \sqrt{-\psi\left[m^2 + \frac{(L - q\Phi_\phi)^2}{g_{\phi\phi}}\right]}, \tag{2.8}$$

satisfying the following condition $E = V$. It is well-known that the effective potential is a useful tool for understanding the motion of test particles around black holes, and the position of the innermost stable circular orbit (ISCO) can be determined by finding the minimum value of the effective potential. The ISCO position of massive particle around a black hole is the smallest possible orbit where a test particle can maintain a stable circular orbit without being drawn into the black hole or flung away into space. The ISCO is a significant location for astrophysical observations, as it can affect the emission of radiation and matter falling onto the black hole. The study of the ISCO can also provide insight into the properties of black holes and tests of general relativity including alternative theories of gravity. The ISCO position depends on the mass and spin of the black hole, as well as the STVG parameter in the present research work. Next thing what we discuss is finding the radius of the ISCO of massive particle around the black hole in STVG and to test how it depends on the α parameter. For simplicity, we consider motion of massive particle in the equatorial plane that means the effective potential depends only on the radial coordinate.

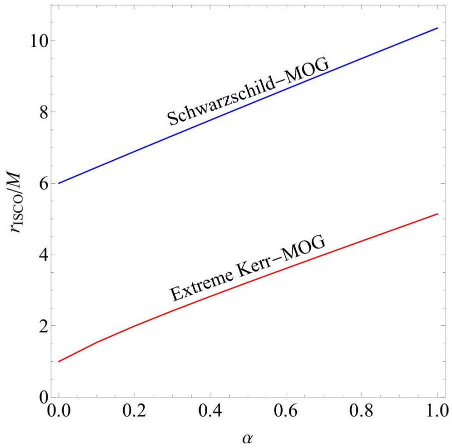


Figure 2.3: Dependence of the ISCO position for massive particle from the STVG parameter α for static and extreme black holes.

Using the standard method, the ISCO position can be found from the following conditions: $V(r) = E, V'(r) = 0$ and $V''(r) \leq 0$. Obtaining the analytical expression for the ISCO position, therefore, careful numerical analyzes showed that the ISCO position gets larger due the effect of STVG. Figure 2.3 shows the dependence of the ISCO position of massive particles orbiting around the Schwarzschild-MOG and extreme Kerr-MOG black holes from the α parameter. Notice that the ISCO position is located in the region between two curves in Fig. 2.3 for arbitrary values of the spin parameter with range of $0 < a_* < \sqrt{1+\alpha}$.

The Penrose process involves taking advantage of the strong gravitational field and rotation of a black hole to extract energy from it. According to the Penrose process the particle falling onto the black hole separates into two parts in the ergosphere, with one part falling into black hole and other part escaping outwards. The part that escapes carries away some of the black hole's rotational energy, resulting in a net loss of energy for the black hole. This extracted energy can then be used to power various astrophysical phenomena. The energy efficiency of the particle depends of the mass, spin parameter and the STVG parameter. Assume that massive particle (1) of parameters ($m_1, E_1, L_1, \dot{r}_1, \dot{\theta}_1, \dot{\phi}_1$) falls onto black hole from infinity and decays into two fragments (2) and (3) with parameters of ($m_2, E_2, L_2, \dot{r}_2, \dot{\theta}_2, \dot{\phi}_2$) and ($m_3, E_3, L_3, \dot{r}_3, \dot{\theta}_3, \dot{\phi}_3$) in the ergosphere of the rotating black hole, where m_i, E_i , $L_i, \dot{r}_i$ and $\dot{\theta}_i$ are, respectively, the mass, energy, angular momentum, radial, vertical and azimuthal velocities. The conservation laws for this process can be written as

$$m_1 \geq m_2 + m_3 \tag{2.9}$$

$$E_1 = E_2 + E_3 \tag{2.10}$$

$$L_1 = L_2 + L_3 \tag{2.11}$$

$$m_1\dot{r}_1 = m_2\dot{r}_2 + m_3\dot{r}_3 \tag{2.12}$$

$$0 = m_2\dot{\theta}_2 + m_3\dot{\theta}_3 \tag{2.13}$$

$$m_1\dot{\phi}_1 = m_2\dot{\phi}_2 + m_3\dot{\phi}_3 \tag{2.14}$$

Again we consider circular motion of a massive particle with four-velocity of $\dot{x}^\mu = \dot{t}(1, v, 0, \Omega)$. Using the normalization of the four-velocity, the angular velocity of particle (1) is determined as

$$\Omega_1 = \frac{-(u^2 + g_{tt})g_{t\phi} + u\sqrt{(-\psi)(u^2 + g_{tt})g_{\phi\phi}}}{u^2 g_{\phi\phi} + g_{t\phi}^2}, \tag{2.15}$$

where $u = (E_1 + q_1\Phi_t)/m_1$, while the angular velocity of the splitted fragments denote $\Omega_2 = \Omega_+$and $\Omega_3 = \Omega_-$, where $\Omega_\pm$are defined as

$$\Omega_\pm = -\frac{g_{t\phi}}{g_{\phi\phi}} \pm \sqrt{\left(\frac{g_{t\phi}}{g_{\phi\phi}}\right)^2 - \frac{g_{tt}}{g_{\phi\phi}}}. \tag{2.16}$$

From the inequality for particles mass in (2.9), relation between coupling constants can be obtained as $q_1 \geq q_2 + q_3$. Using equation (2.14) and after performing simple algebraic manipulations, the relation between energies of in falling and escaping particles can be found as

$$(E_3 + q_3\Phi_t)\frac{\Omega_3 - \Omega_2}{g_{tt} + \Omega_3 g_{t\phi}} \geq (E_1 + q_1\Phi_t)\frac{\Omega_1 - \Omega_2}{g_{tt} + \Omega_1 g_{t\phi}}. \tag{2.17}$$

The energy efficiency is determined as $\eta = E_3/E_1 - 1$, while in the Kerr-MOG spacetime, it reduces to

$$\eta = \frac{1}{2}\left(\sqrt{1 + \frac{g_{tt}}{u^2}} - 1\right)\left(1 + \frac{q_1}{E_1}\Phi_t\right) - \frac{q_1 + q_3}{E_1}\Phi_t. \tag{2.18}$$

To make qualitative analyses of the energy efficiency in STVG, one can estimate the energy of falling particle as $E_1 \simeq m_1$ and the expression (4.11) for the extreme rotating black hole (i.e. $x_+ = 1 + \alpha$) reads

$$\eta_{\max} \geq \frac{1}{2(1+\alpha)}\left[\sqrt{2+\alpha} - 1 + 2\alpha\left(1 + \frac{m_3}{m_1}\right)\right] \tag{2.19}$$

For large value of the STVG parameter, i.e. $\alpha \to \infty$, the energy efficiency is $\eta_{\max} \geq 1 + m_3/m_1$, which is greater than 100% because of mass ratio of escaping and falling particles, while in the case of the extreme Kerr black black hole, $\eta_{\max} = (\sqrt{2} - 1)/2 \simeq 0.207$, which is around $\sim 21\%$. Figure 2.4 shows that maximal efficiency of energy extraction from the Kerr-MOG black hole reaches up to $\sim 200\%$ for particular value of mass ratio of escaping and falling which is quite huge. However, it has been demonstrated that a magnetized Kerr black hole is considered one of the most promising candidates for accelerating high-energy particles. The study reports that the efficiency of energy extraction from a magnetized Kerr black hole can exceed 100% due to the induced electrostatic potential resulting from rotation and magnetic field effects.

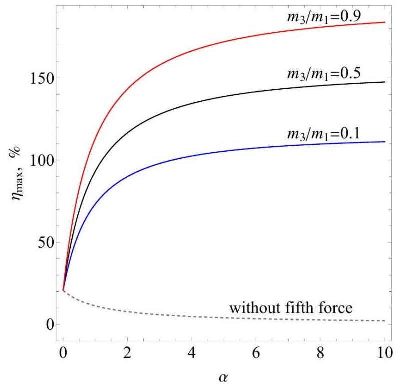


Figure 2.4: Dependence of the maximal energy efficiency of the black hole from the STVG parameter α.

It is well established that accelerating charged particles emit synchrotron radiation, which is a form of electromagnetic radiation that is emitted when a charged particle is accelerated in a curved path in the presence of a magnetic field. The intensity of synchrotron radiation emitted by the charged particle is proportional to the square of the acceleration $I \sim w_\mu w^\mu$ and the frequency of the emitted radiation depends on the strength of the magnetic field. It is interesting to note that the synchrotron radiation emitted by relativistic charged particles around magnetized and charged black holes has been the subject of recent research. This is because black holes are known to have strong gravitational and electromagnetic fields that can affect the motion of charged particles and alter the characteristics of the emitted radiation. It is provided insights into the synchrotron radiation spectrum of relativistic charged particles around black holes, and their results have important implications for our understanding of the astrophysical phenomena associated with black holes.
Here we will discuss gravitational analogue of the synchrotron radiation from massive particle orbiting around black hole in STVG. As we mentioned before that in this theory, massive particle does not follow geodesics line due to the fifth force and from equation of motion in (2.3), the four-acceleration of massive particle is given as

$$w^\mu = \frac{q}{m} B^\mu_\nu \dot{x}^\nu, w_\mu \dot{x}^\nu = 0 \quad (3.1)$$

which is always perpendicular to the four-velocity of particle. Notice that equation (3.1) is gravitational analogue of the Lorentz equation, and charge of particle is replaced by the coupling constant q, while anti-symmetric tensor $B_{\mu\nu}$ stands instead of the Faraday tensor in Lorentz equation.

Our main assumption is that if massive particle orbits with non-zero value of the four-acceleration in the presence of the external force then it should emit radiation. We call this radiation as gravitational synchrotron radiation. Then the intensity of accelerating massive particle in STVG is determined as

$$I = -\frac{2q^2}{3} w_\mu w^\mu = \frac{2q^4}{3m^2} B_{\mu\lambda} B^{\mu\nu} \dot{x}^\lambda \dot{x}_\nu \quad (3.2)$$

Considering circular motion of a massive particle with four-velocity of $\dot{x}^\mu = \dot{t}(1, v, 0, \Omega)$, where $v = dr/dt$ and $\Omega = d\phi/dt$ are radial and angular velocities of massive particle. Hereafter, performing simple algebra manipulations one can obtain the expression for the acceleration the following the form:

$$w_\mu = \frac{q}{m} \dot{t}\left(v B_{tr}, B_{rt} + \Omega B_{r\phi}, B_{\theta t} + \Omega B_{\theta\phi}, v B_{\phi r}\right), \quad (3.3)$$

where non-zero components of the anti-symmetric tensor can be found as

$$B_{rt} = \frac{\sqrt{\alpha} M}{\Sigma^2}(r^2 - a^2 \cos^2 \theta), B_{r\phi} = -B_{rt} a \sin^2 \theta, \quad (3.4)$$

$$B_{\theta t} = -\frac{\sqrt{\alpha} M r a^2 \sin 2\theta}{\Sigma^2}, B_{\theta\phi} = -B_{\theta t} \frac{r^2 + a^2}{a}, \quad (3.5)$$

and $\dot{t}$ can be found using normalization of the four-velocity $\dot{t}^{-1} = \sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi} - v^2 g_{rr}}$.

The expression for the intensity of the radiating massive particle orbiting around Kerr-MOG black hole in equatorial plane reads

$$I = \frac{2\alpha^3 G^3 m^2 M^2}{3c^3 r^4} \dot{t}^2 \left[\frac{\Delta}{r^2}(1 - \Omega a)^2 - \frac{r^2}{\Delta} \frac{v^2}{c^2}\right] \quad (3.6)$$

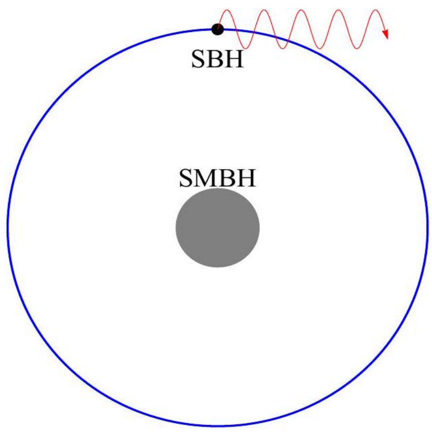


Figure 3.1: The schematical picture of radiated SBH around SMBH in STVG.

As one can see from above equation that massive particle emits gravitational radiation even in arbitrary stable orbit (i.e. $v = 0$) and in the case of a nonrotating black hole $\dot{t}^2$ can be canceled with expression in bracket. In Fig. 3.1 a schematically picture of gravitational radiating SBH in the vicinity of a SMBH is illustrated. As a result one can estimate the intensity of gravitational radiation from the stellar black hole orbiting around supermassive black hole as

$$I \sim 2.43 \times 10^{39} \alpha^3 \left(\frac{m}{10 M_\odot}\right)^2 \left(\frac{M}{10^9 M_\odot}\right)^{-2} \left(\frac{10GM}{c^2 r}\right)^4 \text{erg/s} \quad (3.7)$$

which depends on the radial coordinate. Figure 3.2 shows the dependence of the intensity on the α parameter. As one can see from the figure, in the absence of the STVG parameter, the intensity becomes zero because, in this case, the massive particle follows a geodesic line near the black hole and does not emit gravitational radiation. However, in the presence of the STVG (i.e., $\alpha \neq 0$), the intensity of the radiating particle becomes non-zero and increases with an increase in the α parameter.

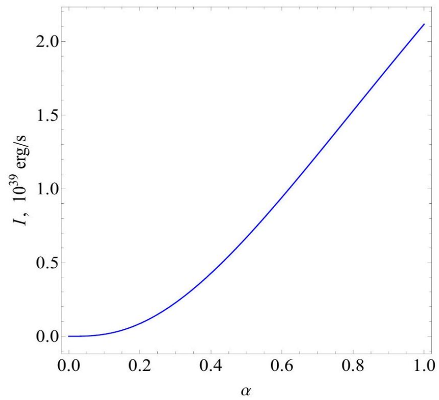


Figure 3.2: Dependence of the intensity of a radiating SBH at the ISCO position from the α parameter.

Now we examine the dynamics of massive particles within the framework of the STVG theory, which incorporates a radiation reaction term. While the Lorentz-Abraham-Dirac (LAD) equation is widely recognized as the equation governing radiation reaction for charged particles, our research focuses on the gravitational counterpart of this phenomenon. In this context, the equation of motion can be modified as

$$\frac{Du^\mu}{ds} = \frac{\tilde{q}}{m} B^\mu_\nu u^\nu + \frac{1}{2}\tau_0\left(R^\mu_\nu + u^\mu u_\lambda R^\lambda_\nu\right)u^\nu + \tau_0\left(\frac{D^2u^\mu}{ds^2} + u^\mu u_\lambda \frac{D^2u^\lambda}{ds^2}\right), \tag{3.8}$$

whereτ_0 is a damping time of gravitational radiation defined as $\tau_0 = 2\tilde{q}^2/(3m) \ll 1s$ and this parameter can play a role of expansion parameter. To have an idea one can estimate of damping time for electron and it yields

$$\tau_0 = \frac{2\alpha G_N m_e}{3c^3}\left(\frac{m}{m_e}\right) \sim 10^{-66}\alpha\left(\frac{m}{m_e}\right)\text{s}, \tag{3.9}$$

while the stellar black hole orbiting around supermassive black hole is

$$\tau_0 \simeq 3.3 \times 10^{-4}\alpha\left(\frac{m}{10M_\odot}\right)\text{s}\,. \tag{3.10}$$

Given that the final term of equation (3.8) is significantly smaller when compared to the other terms, therefore one can employ the Landau trick to expand the equation in terms of the damping time τ_0. After simple algebraic manipulations, equation of motion can be rewritten as

$$\frac{Du^\mu}{ds} = \frac{\tilde{q}}{m} B^\mu_\nu u^\nu + \frac{1}{2}\tau_0 h^{\mu\lambda} R_{\lambda\nu} u^\nu + \tau_0 \frac{\tilde{q}}{m}\left(u^\alpha \nabla_\alpha B^\mu_\nu + \frac{\tilde{q}}{m} h^{\mu\lambda} B_{\lambda\alpha} B^\alpha_\nu\right)u^\nu \tag{3.12}$$

where $h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ and $h_{\mu\nu}u^\nu = 0$. It turns out that one has to find twelve constants of motion which one of main issue in particle motion in curved spacetime. However, this problem can be avoided using the Landau trick and equation of motion reduces to the second order system of equations for four coordinates x^μ. The analytical form equations for each coordinates in (3.12) are too long therefore we will not report them in the Letter. However, careful numerical analyses for given initial conditions $(0, r_0, \theta_0, 0)$ and $\{-\mathcal{E}/f(r_0), 0,0, \mathcal{L}/(r_0 \sin\,\theta_0)^2\}$, allows to find each coordinates as function of affine parameter $x^\mu = x^\mu(s)$. Hereafter performing coordinate transformation $x = r\cos\phi\sin\theta, y = r\sin\phi\sin\theta$, and $z = r\cos\theta$ the trajectories of particle can be produced in Cartesian coordinates using parametric plots in $2D$ or $3D$.

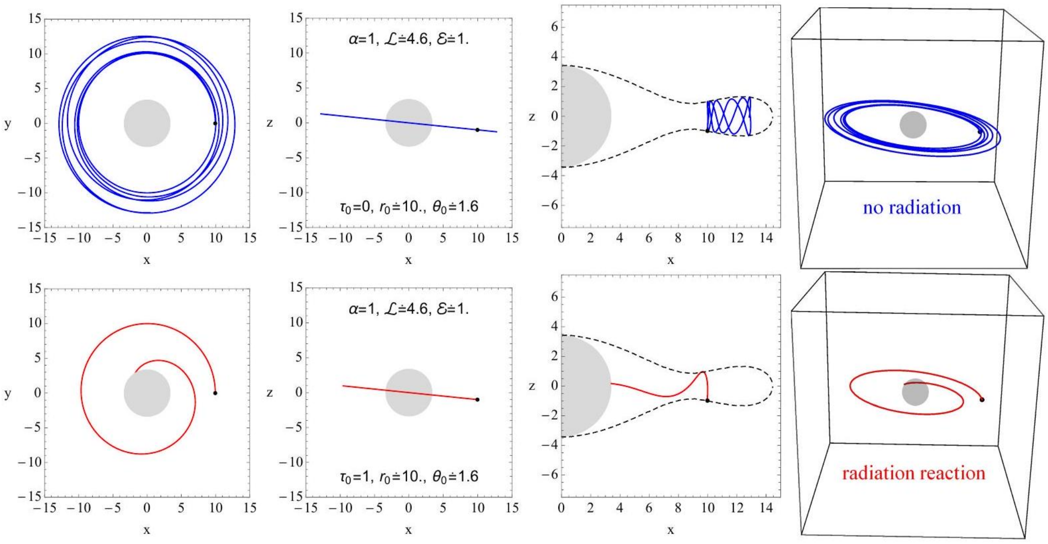


Figure 3.3: The trajectories of massive particle orbiting the Schwarschild-MOG black hole with (bottom panel) and without (top panel) including radiation reaction force, respectively. Initial conditions are chosen to be the same for each cases and shown in the second row. The initial position of particle is depicted by black dot in each plots. The first and second columns represent the trajectory of massive particle in ($x-y$) and ($x-z$) planes, respectively. The third column represents finite motion of test particle in the region given by dashed line (contour plot of the effective potential), while in the last column particle's trajectory is shown in 3D.

In Fig 3.3, we present trajectory of massive particle including and without including radiation reaction term in STVG. In order to see the effect of radiation reaction term in circular motion we set damping time to $\tau_0 = 1$ s. However, real value of the damping time τ_0 can be calculated by following formulae for electron and for stellar black hole. This means that damping time is much lower than 1 s and particle rounds many times than shown in Fig 3.3 around black hole and eventually fall into black hole due to radiation reaction force. It is evident from Fig. 3.4 that for the larger values of MOG parameter α and damping time τ_0, a particle falls into black hole faster.

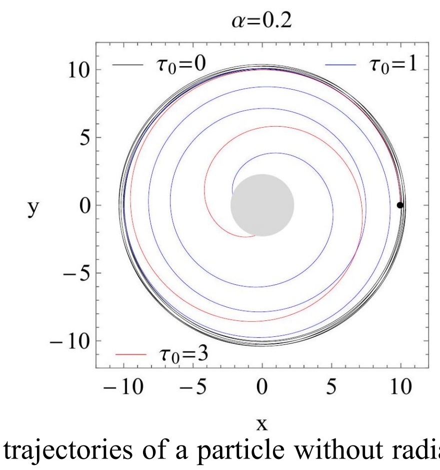


Figure 3.4: Plot is comparing the trajectories of a particle without radiation reaction force (black circular line) and with radiation reaction force for different τ parameter (blue and red). In each cases, initial positions are chosen to be the same and depicted by black dot.

Certainly, studying the thermodynamic properties of black holes in STVG and investigating the influence of the MOG parameter on these quantities is an intriguing endeavor. We begin by considering the first law of black hole thermodynamics in STVG, which can be expressed as:

$$dM = TdS + \Phi_t dQ \tag{4.1}$$

where T and S denote the Hawking temperature and entropy of the black hole, respectively. Notably, the second term of equation (4.1) emerges due to the presence of STVG, and it has the potential to impact the thermodynamic properties of the black hole. The Hawking temperature on the surface of the Schwarzschild-MOG black hole

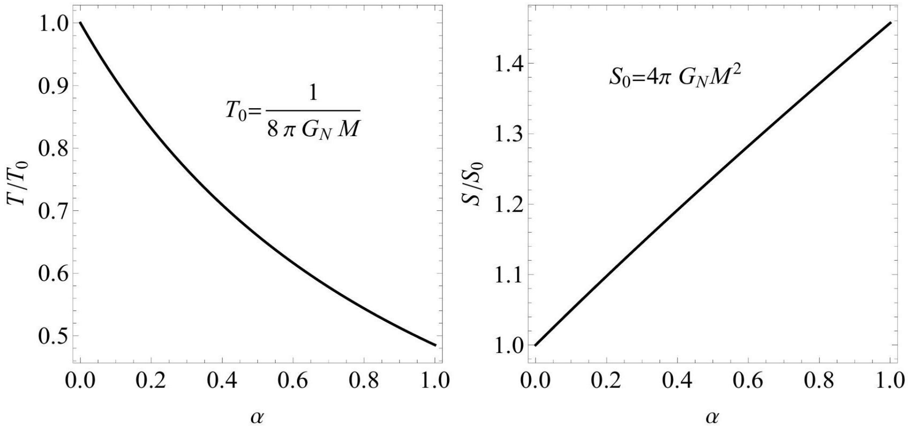


Figure 4.1: Dependence of the Hawking temperature (left panel) and entropy (right panel) of the Schwarzschild-MOG black hole form MOG parameter α.

can be determined as follows:

$$T = \frac{1}{4\pi}\left(\frac{dg_{tt}}{dr}\right)\bigg|_{r=r_+} = \frac{1}{2\pi G_N M\sqrt{1+\alpha}(1+\sqrt{1+\alpha})^2}, \quad (4.2)$$

$$S_A = \frac{A}{4G} = \frac{\pi r_+^2}{4G_N(1+\alpha)} = \pi G_N M^2(1+\sqrt{1+\alpha})^2. \quad (4.3)$$

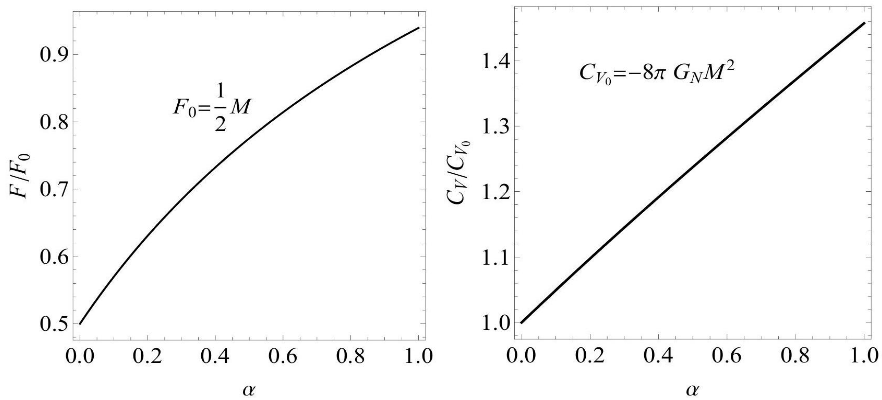


Figure 4.2: Dependence of the free energy (left panel) and heat capacity (right panel) of the Schwarzschild-MOG black hole from MOG parameter α.

Using the area law the entropy of the Schwarzschild-MOG black hole reads of the black hole determined via using area law and the first law of black hole thermodynamics give the same result. It is evident from equation (4.2), the Hawking temperature of the black hole decreases due to the influence of MOG in the black hole's spacetime, as illustrated in Fig. 4.1. Now, utilizing the expressions for the temperature and entropy, we can determine the free energy of the Schwarzschild-MOG black hole:

$$F = M - ST - Q\Phi_t = \frac{M}{2\sqrt{1+\alpha}}. \quad (4.5)$$

Now we focus on calculating the heat capacity of the Schwarzschild-MOG black hole. From the thermodynamics course the isochoric heat capacity is given as $C_V = T(\partial S/\partial T)$, which depends on the entropy of the black hole. Using this expression heat capacity of the Schwarzschild-MOG black hole can be determined as

$$C_V = T\frac{\partial S}{\partial M}\left(\frac{\partial T}{\partial M}\right)^{-1} = -2\pi G_N M^2(1+\sqrt{1+\alpha})^2. \quad (4.6)$$

In Figure 4.2, we provide a comparison of the entropy and free energy values of the black hole based on the MOG parameter. The solid lines represent the actual entropy and free energy of the black hole, calculated using equations (4.3) or (4.4) and (4.5), which presumably correspond to a new approach. Based on the available information, the statement concludes that the entropy and free energy exhibit a unique behavior. This potentially implies that the new formulation provides a consistent and unified description of these quantities for black holes. However, given the limited context and details provided, it is challenging to provide a more comprehensive interpretation without further clarification on the specific content of the paper and the equations involved. Another important feature of the black hole is it's lifetime. The rate of energy loss from the Schwarzschild-MOG black hole can be approximated with the Stephan-Boltzmann radiation law as

$$\frac{dM}{dt} = -\sigma A T^4 + \frac{Q}{r_+}\frac{dQ}{dt} = -\sigma A T^4 + \frac{\alpha G_N M}{r_+}\frac{dM}{dt} \quad (4.7)$$

where σ is the Stephan-Boltzmann constant and the second term arises due to the external potential in STVG. Hereafter isolating dt and dM in equation (4.7) and integrating from M to 0 , the life time of the Schwarzschild-MOG black hole can be expressed as

$$\tau = \frac{4\pi}{3\sigma} G_N^2 M^3 \sqrt{1+\alpha}(1+\sqrt{1+\alpha})^6 \quad (4.8)$$

In Fig. 4.3 dependence of the lifetime of the black hole from MOG parameter is illustrated.

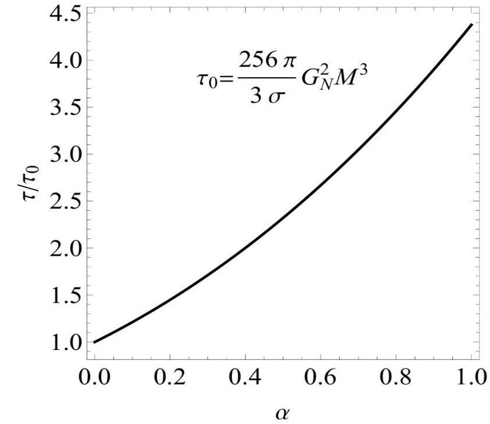


Figure 4.3: Dependence of the heat capacity (left panel) and lifetime (right panel) of the Schwarzschild-MOG black hole from MOG parameter α.

The metric perturbation in the black hole spacetime is indicated with $h_{\mu\nu}$, along background metric metric $g_{\mu\nu}$ which is solution of the Einstein field equations $G_{\mu\nu}(g) = 8\pi T_{\mu\nu}(\Phi)$, where $G_{\mu\nu}$ is the Einstein tensor dependence on the metric tensor and $T_{\mu\nu}$ is the energy momentum tensor dependent of matter field Φ. Notice that the explicit form of the energy-momentum tensor depends on type of the matter field. The Einstein field equation can be rewritten in terms of the metric perturbation: $G_{\mu\nu}(g+h) = 8\pi T_{\mu\nu}(\Phi + \delta\Phi)$ and can be expended in the power of metric perturbation as follows

$$G_{\mu\nu}(g) - \frac{1}{2}H_{\mu\nu} + \cdots = 8\pi\left[T_{\mu\nu}(\Phi) + \delta T_{\mu\nu}(\delta\Phi) + \cdots\right] \quad (4.12)$$

$$H_{\mu\nu} = -16\pi\delta T_{\mu\nu}(\delta\Phi) \quad (4.13)$$

where $H_{\mu\nu}$ is Einstein tensor for perturbed field. We must emphasize that according to Regge and Wheeler the gravitational perturbations $h_{\mu\nu}$, can be expsessed as

$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & -h_0 \csc\theta\partial_\phi & h_0 \sin\theta\partial_\theta \\ 0 & 0 & -h_1 \csc\theta\partial_\phi & h_1 \sin\theta\partial_\theta \\ * & * & h_2 \csc\theta\left(\partial^2_{\theta\phi} - \cot\theta\partial_\phi\right) & \frac{1}{2}h_2\left(\csc\theta\partial^2_\phi + \cos\theta\partial_\theta - \sin\theta\partial^2_\theta\right) \\ * & * & * & -h_2 \sin\theta\left(\partial^2_{\theta\phi} - \cot\theta\partial_\phi\right) \end{bmatrix} Y_\ell^m, \quad (4.15)$$

where $h_0(t,r), h_2(t,r)$ and $h_2(t,r)$ are profile functions in axial perturbation $Y_\ell^m(\theta,\phi)$ is the spherical harmonics satisfied the following equation: $\nabla^2_\Omega Y_\ell^m(\theta,\phi) = -\ell(\ell+1)Y_\ell^m(\theta,\phi)$, where ∇^2_Ω is the angular Laplacian operator defined as

$$\nabla^2_\Omega = \frac{1}{\sin\,\theta}\frac{\partial}{\partial\theta}\left(\sin\,\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\,\theta}\frac{\partial^2}{\partial\phi^2} \tag{4.17}$$

and it also satisfies the following normalisation condition:

$$\int d\Omega Y_l^{m*}(\theta,\phi) Y_{l'}^{m'}(\theta,\phi) = \delta_{ll'}\delta_{mm'} \tag{4.18}$$

Apply a gauge transformation to remove the second angular derivatives. We use the following gauge $h_2 = 0$, the axial gravitational perturbation to metric function with $\ell \neq 0$ and $m = 0$. this equation can be reduced to the Regge-Wheeler equation

$$\left[\frac{d^2}{dr_*^2} + \omega^2 - V_1(r)\right]\psi_1 = 0 \tag{4.26}$$

where $\psi = h_1 f/r$ is new radial function, $r_* = \int dr/f$ is a tortoise coordinate, and $V(r)$ is the potential defined as

$V(r) = f\left(\frac{\ell(\ell+1)}{r^2} + f'' - \frac{f'}{r}\right)$.

From equation (4.26) equation, it is evident that radial function in the axial perturbation can be governed by the standard Schrodinger-like equation.

There exist simpler types of black hole perturbations in a fixed background spacetime, known as scalar and electromagnetic perturbations. In order to describe these two particular perturbations, we use the Klein-Gordon and Maxwell equations in curved spacetime:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu\Phi\right) = 0, \tag{4.45}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = 0, \tag{4.46}$$

where Φ is a scalar field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Faraday tensor and A_μ is a vector potential of the electromagnetic field. One can see that equations (4.45) and (4.46) can be separated into the radial and angular parts, if the wave function is chosen as harmonically time dependent form as follows

$$\Phi(x) = e^{-i\omega t}\frac{R_\ell(r)}{r}Y_\ell^m(\theta,\phi) \tag{4.47}$$

$$A_\mu(x) = e^{-i\omega t}\left[0,0,\frac{R_\ell^B(r)}{\sin\,\theta}\partial_\phi, -R_\ell^B(r)\sin\,\theta\partial_\theta\right]Y_\ell^m(\theta,\phi) \tag{4.48}$$

where ω is the frequency of quasinormal modes for scalar and vector fields, $R_\ell(r)$ is a radial wave function for scalar field and $R_\ell^E(r), R_\ell^B(r)$ are profile functions for (electric and magnetic cases) the electromagnetic field. Substituting expression (4.47) and (4.48) into (4.45) and (4.46), one can write the following stationary wave equations:

$$\left[\frac{d^2}{dr_*^2} + \omega^2 - V_s(r)\right]\psi_s = 0 \tag{4.49}$$

where $V_s(r)$ is the effective potential for a spin weighted field described as

$$V_s(r) = f\left[\frac{\ell(\ell+1)}{r^2} + \frac{1-s^2}{r}\frac{df}{dr}\right], \tag{4.50}$$

where s=0 scalar field and s=1 vector field. In this study, we investigated the quasi-normal modes (QNMs) of the Schwarzschild-MOG (Modified Gravity) black hole, focusing on both scalar and electromagnetic perturbations. The frequency of QNMs is a crucial aspect in understanding the stability and oscillatory behavior of black holes when perturbed. Table 4.1 presents the quasi-normal frequencies for the massless scalar perturbation, while Table 4.3 shows the corresponding results for the electromagnetic perturbation. For both types of perturbations, we observe a consistent trend: as the MOG parameter α increases, the real part of the frequency decreases, while the imaginary part also decreases, indicating that the oscillations become slower and more heavily damped. In the absence of the MOG parameter ($\alpha = 0$), the results reduce to those for the Schwarzschild black hole, providing a baseline for comparison. As α increases from 0 to 1 , the frequencies of both scalar and electromagnetic perturbations decrease monotonically. This suggests that the presence of the MOG parameter enhances the gravitational effects, leading to longer damping times and slower oscillation frequencies, as compared to the Schwarzschild black hole. The results presented in these tables highlight how the inclusion of the MOG parameter modifies the black hole's response to external perturbations, affecting both its stability and the time scales over which it returns to equilibrium. We obtained the QNM frequencies for various values of the MOG parameter α, as detailed in Tables 4.1 and 4.3.

α	$\omega(\ell = 2)$	$\omega(\ell = 3)$	$\omega(\ell = 4)$
0.	0.506317-0.0961232i	0.691728-0.0961481i	0.880197-0.0961713i
0.2	0.433660-0.0807996i	0.592954-0.0808258i	$0.754779 - 0.0808460i$
0.4	$0.379726 - 0.0696219i$	0.519562-0.0696467i	$0.661552 - 0.0696639i$
0.6	0.338011-0.0611098i	$0.462752 - 0.0611321i$	$0.589363 - 0.0611467i$
0.8	0.304735-0.0544140i	0.417402-0.0544334i	0.531717-0.0544456i
1.	$0.277542 - 0.0490113i$	$0.380319 - 0.0490279i$	0.484568-0.049038i

Table 4.1: Quasi-normal modes of the massless scalar perturbation in the Schwarzschild-MOG spacetime for $n = 0$.

α	$\omega(\ell = 2)$	$\omega(\ell = 3)$	$\omega(\ell = 4)$
0.	0.480754-0.0943536i	0.673438-0.0952575i	0.865960-0.0956364i
0.2	$0.412178 - 0.0793758i$	$0.577579 - 0.0801080i$	0.742810-0.0804145i
0.4	0.361213-0.0684375i	0.506310-0.0690487i	$0.651235 - 0.0693041i$
0.6	0.321755-0.0600997i	0.451114-0.0606215i	$0.580301 - 0.0608392i$
0.8	0.290252-0.0535355i	0.407032-0.0539888i	$0.523644 - 0.0541777i$
1.	0.264488-0.0482354i	$0.370972 - 0.0486347i$	$0.477291 - 0.048801i$

Table 4.2: Quasi-normal modes of the electromagnetic perturbation in the Schwarzschild-MOG spacetime for $n = 0$.

α	$\omega(\ell = 2)$	$\omega(\ell = 3)$	$\omega(\ell = 4)$
0.	0.398850-0.0882854i	0.616561-0.0923181i	0.822303-0.0939294i
0.2	$0.351606 - 0.0750712i$	0.535440-0.0780357i	$0.710420 - 0.0792072i$
0.4	0.314453-0.0652472i	$0.473723 - 0.0675159i$	$0.626160 - 0.0684089i$
0.6	0.284483-0.0576534i	0.425099-0.0594465i	0.560266-0.0601515i
0.8	0.259796-0.0516084i	0.385746-0.0530628i	$0.507239 - 0.0536349i$
1.	0.239104-0.0466838i	0.353210-0.0478886i	$0.463594 - 0.048363i$

Table 4.3: Quasi-normal modes of the axial gravitational perturbation in the SchwarzschildMOG spacetime for $n = 0$.

CONCLUSION

Based on the scientific research conducted on the topic "Energetic properties of black holes in scalar-tensor-vector gravity" the following scientific conclusions are drawn:

1. For the first time it has been found the exact formulation of the first law of black hole thermodynamics in STVG including the additional term arising from the presence of scalar-vector-tensor field. It has been shown that the Hawking temperature decreases due to the influence of the MOG parameter. It has been observed that the positive nature of the MOG parameter ensures that the temperature of the Schwarzschild-MOG black hole is consistently smaller than that predicted in the Schwarzschild spacetime.
2. It is shown that entropy of the Schwarzschild-MOG black hole is always greater than the entropy of the Schwarzschild black hole. It has been shown that the heat capacity for the Schwarzschild-MOG black hole is negative. It has been concluded that within the context of the STVG theory, the fractional heat capacity of the black hole increases with the augmentation of the MOG parameter.
3. It has been stated that the black hole's lifetime is longer in the context of STVG. It has been shown that both the photon sphere and the size of the black hole shadow increase due to the presence of MOG.
4. It has been also shown that in the presence of MOG parameter the characteristic radii of ISCO orbits are larger compared to those in the pure Schwarzschild spacetime. Furthermore, it was obtained that the maximum energy efficiency of test particles around the Schwarzschild-MOG black hole can reach up to approximately ~8.14 % corresponding a relatively large efficiency compared to the Schwarzschild case.

5. It was estimated that for a typical SMBH with a mass of $10^9\ M_\odot$ and black hole with mass 10 $M_\odot$ orbiting around it, the intensity of the radiation takes the value ~2.43×10^{39} erg.
6. For the first time the gravitational analogue of radiation reaction experienced by a massive particle orbiting a Schwarzschild-MOG black hole has been developed. Within this model the equation of motion has been simplified by discarding high-order terms concerning damping time.
7. It has been demonstrated that the maximum center-of-mass energy of a colliding pair of particles depends on the disparity in the specific energy values of the individual particles. Moreover, it remains independent of the vector field, and the effect of STVG emerges solely from the spacetime geometry.

E’LON QILINGAN ISHLAR RO‘YXATI
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