V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.05/2025.27.12.FM.11.01.M RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

DO‘SANOVA UMIDA XOMID QIZI

KAPUTO MA’NOSIDAGI KASR TARTIBLI HOSILALI ARALASH TURDAGI TENGLAMALAR UCHUN TO‘G‘RI VA TESKARI MASALALAR

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

TOSHKENT – 2026

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestatsiya komissiyasida № B2025.2.PhD/FM1287 raqam bilan ro‘yxatga olingan.

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Dissertatsiya bilan V.I.Romanovskiy nomidagi Matematika Institutining Axborot-resurs markazida tanishish mumkin (221-raqami bilan ro‘yxatga olingan). (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871)-207-91-40.

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va unga bo‘lgan talab. Butun dunyo miqyosida olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlar xususiy hosilali yuqori tartibli differensial tenglamalar uchun chegaraviy va teskari masalalarni o‘rganish muhimligini ko‘rsatmoqda. So‘nggi yillarda mamlakatimizda matematika, fizika, geologiya va biologiya fanlariga, ya’ni fundamental fanlarda ilmiy va amaliy qo‘llanilish imkoniyatiga ega yo‘nalishlarga katta e’tibor qaratilmoqda. Xususan, differensial tenglamalar va matematik fizika sohalarini amaliy ahamiyatga ega bo‘lgan matematik fanlarning asosiy yo‘nalishlaridan biri sifatida rivojlantirishga alohida ahamiyat berilmoqda.

So‘nggi o‘n yilliklarda kasr tartibli hosilalarga ega differensial tenglamalar nazariyasi sezilarli darajada ommalashdi. Avvalo, bu nazariyaning fan va texnikaning turli, hatto bir-biridan uzoq sohalarida keng qo‘llanilayotgani bilan bog‘liq. Ushbu ilmiy tadqiqotning asosiy maqsadi N o‘lchamli sohada aniqlangan elliptik qismi ixtiyoriy differensial operator bo‘lgan bo‘lgan parabola va giperbola tipidagi tenglamalar uchun chegaraviy va teskari masalalarni hal qilishdan iboratdir.

Yuqori tartibli xususiy hosilali differensial tenglamalarni yechishni o‘rganishdan tashqari, ushbu tenglamalarning koeffitsientlarini, o‘ng tomonini yoki kasr tartibli hosilaning tartibini aniqlash bilan bog‘liq bo‘lgan teskari masalalarni yechishni ham o‘rganish nihoyatda muhimdir. Teskari masalalarni o‘rganish bizga yuqori tartibli tenglamalar orqali ifodalangan jarayonlarni o‘rganish, tahlil qilish va nazorat qilish imkonini beradi. Masalan, agar issiqlik diffuziyasi jarayonlarini olaylik, u holda chegaradagi funksiyani bilgan holda issiqlikning tarqalish jarayonini nazorat qilishimiz mumkin bo‘ladi. Shunga o‘xshash misollar ko‘plab keltirilishi mumkin. Vaqt bo‘yicha kasr tartibli tenglamani ko‘rib chiqaylik. Agar kasr tartibi 1 bilan 2 orasida bo‘lsa, u holda tenglama bir vaqtning o‘zida diffuziya va to‘lqin tarqalishini ifodalaydi; agar kasr tartib 0 bilan 1 orasida bo‘lsa, u holda tenglama sekin tarqalayotgan issiqlik jarayonini ifodalaydi. Xuddi shuningdek, virusologiya sohasidan ham misollar keltirish mumkin: kasr tartibli tenglamalar viruslarning ko‘payish jarayonini tabiiy tarzda ifodalashi mumkin. Yuqoridagilarning barchasini inobatga olgan holda aytish mumkinki, kasr tartibli hosilali tenglamalarni o‘rganish matematikaning dolzarb muammolaridan biridir.

Mamlakatimizda matematika, fizika va optimal boshqaruv kabi fundamental tadqiqotlarda ilmiy va amaliy ahamiyatga ega sohalarga katta e’tibor qaratilmoqda. Xususan, xususiy hosilali differensial tenglamalarning kasr va ixtiyoriy tartibdagilarini o‘rganishga alohida e’tibor berilmoqda. Sababi, bu nazariya mexanika, elektronika, boshqaruv nazariyasi, fiziologiya va biologiyadagi ko‘plab hodisalarni tushunishga yordam beradi. Yuqorida aytib o‘tilganidek, matematikaning muhim yo‘nalishlarida xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish asosiy vazifa va ustuvor yo‘nalish sifatida belgilangan.

Mazkur dissertatsiya O‘zbekiston Respublikasi Prezidentining qarorlarida belgilangan vazifalarni hayotga tatbiq etishga ma’lum darajada xizmat qilmoqda. Mazkur dissertatsiya ishining predmeti va ob’ekti O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi “Fuqarolar akademiyasi faoliyatini yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi PF-4947-sonli qarorida ko‘rsatilgan dolzarb yo‘nalishlarga muvofiq tanlangan. Fanlar, ilmiy-tadqiqot faoliyatini tashkil etish, boshqarish va moliyalashtirish”, 2018-yil 27-apreldagi PQ-3682-sonli “Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliyotga tatbiq etish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi va 2019-yil 9-iyuldagi PQ-4387-sonli qarorlari matematika ta’limi va fanini yanada rivojlantirish, O‘zbekiston Fanlar akademiyasi V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan yaxshilash chora-tadbirlari” hamda 2020-yil 7-maydagi “Matematika bo‘yicha ta’lim va ilmiy tadqiqotlar sifatini oshirish chora-tadbirlari to‘g‘risida”gi PD-4708-sonli qarori.” Bundan tashqari, ushbu dissertatsiya ishi ushbu faoliyat bilan bog‘liq barcha normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda keng qo‘llanilishi mumkin.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga bog‘liqligi. Mazkur dissertatsiya Respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Kasr tartibli xususiy hosilali differensial tenglamalar va ular bilan bog‘liq teskari masalalar zamonaviy matematikada faol tadqiqot olib borilayotgan sohalardan biriga aylangan. Mazkur yo‘nalishda Sh.O. Alimov, M. Ruzhansky, M. Yamamoto, M. Kirane, S.R. Umarov, Z. Li, A.V. Pskhu, Y. Zhang, H.T. Nguyen, A.S. Malik, R.R. Ashurov, D.K. Durdiev, B.X. Turmetov, E.T. Karimov, Yu.E. Fayziyev, Z.A. Sobirov kabi olimlar va boshqa ko‘plab tadqiqotchilar muhim ilmiy hissa qo‘shganlar.

Bugungi kunda aralash turdagi tenglamalar uchun chegaraviy masalalar nazariyasi xususiy hosilali tenglamalar nazariyasining rivojlanayotgan yo‘nalishlaridan biri hisoblanadi. Bu sohaga oid ko‘plab nazariy tadqiqotlar va monografiyalar chop etilgan. Aralash turdagi tenglamalarga oid klassik monografiyalar qatoriga A.V. Bitsadze, M.M. Smirnov va V.N. Vragovlarning asarlarini kiritish mumkin. Tadqiqotchilarning katta qismi turli kasr tartibli aralash turdagi tenglamalarining o‘ng tomonini aniqlash bilan bog‘liq teskari masalalarga ham e’tibor qaratmoqda, chunki bunday masalalar amaliyot uchun muhimdir. Biroq, manba funksiyasining umumiy holati $F(x,t)$ uchun hozircha umumiy nazariya mavjud emas. Ma’lum ishlarda $F(x,t) \equiv f(x)g(t)$ ajraladigan manba funksiyasi ko‘rib chiqilgan va tadqiqot usullari uning qaysi qismi noma’lumligiga bog‘liq holda tanlangan.

Manba funksiyasining fazoviy qismi $f(x)$ ni aniqlashga oid teskari masala ikki xil holatda ko‘rib chiqilgan: $g(t) \equiv 1$ va $g(t) \not\equiv 1$. Aralash turdagi tenglamalar uchun $g(t) \equiv 1$ va $f(x)$ noma’lum bo‘lgan holat R.R. Ashurov, E.T. Karimov, D.K. Durdiev, T. K. Yuldashev, B.Kh. Kadirkulov, B.I. Islomov, M.B. Murzambetova kabi olimlarning ishlarida o‘rganilgan. $g(t) \not\equiv 1$ holati murakkabroq bo‘lib, bunday masalalarning yechiluvchanligi $g(t)$ funksiyaning xatti-harakatiga bog‘liq. Manba funksiyasining vaqtga bog‘liq qismi $g(t)$ ni aniqlashga oid teskari masalalar nisbatan murakkabroq bo‘lib, ularni kamchilik tadqiqotchilar o‘rganishgan. Aralash turdagi tenglamalar uchun bu yo‘nalishda R.R. Ashurov, M.D. Shakarova, S. Umarov, K.B.

Sabitov , S.N. Sidorov kabi olimlar tomonidan natijalar olingan.

Kasr tartibli tenglamalarni turli jarayonlarni tahlil qilish uchun model sifatida qaraganda, kasr hosilaning tartibi koʻpincha nomaʼlum boʻladi va uni bevosita oʻlchash qiyin. Bu nomaʼlum parametrni aniqlash boʻyicha teskari masalalar nazariy jihatdan ham, amaliy jihatdan ham muhim boʻlib, ular boshlangʻich-chegaraviy masalalarni yechishda va yechimlarning xossalarini oʻrganishda zarurdir. Shu sababli soʻnggi yillarda olimlar kasr hosilalar tartibini aniqlashga oid teskari masalalarni faol oʻrganmoqdalar. Ushbu yoʻnalishda chop etilgan ishlarning tahlili shuni koʻrsatadiki, deyarli barcha maʼlum ishlarda yoki subdiffuziya tenglamasi yoki kasr toʻlqin tenglamasi koʻrib chiqilgan. Bizning bilishimizcha, aralash turdagi tenglamalar uchun kasr tartibini aniqlash boʻyicha teskari masalalar hozirgi kunda faqat R.R. Ashurov, R.T. Zunnunov va M.B. Murzambetova kabi olimlar tomonidan oʻrganilgan. Shuni taʼkidlash kerakki, R.R. Ashurov va R.T. Zunnunovlarning ishda sohaning yuqori va quyi qismlaridagi hosilalarning tartibi nomaʼlum deb qaraladi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan Oliy taʼlim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bogʻliqligi.

Dissertatsiya tadqiqoti V.I. Romanovskiy nomidagi Matematika institutida Oʻzbekiston Respublikasi Oliy taʼlim, fan va innovatsiyalar vazirligining F-FA-2021-424-sonli ilmiy tadqiqot grantining rejalashtirilgan mavzusiga muvofiq amalga oshirildi.

Tadqiqot ishining maqsadi parabolik, giperbolik va subdiffuziya turidagi kasr tartibli differensial tenglamalar uchun toʻgʻri va teskari masalalarning klassik maʼnodagi yagona yechimga ega ekanligini isbotlash.

Tadqiqotning vazifalari

Sohaning yuqori qismida subdiffuziya, pastki qismida diffuziya tenglamasi uchun N oʻlchamli ixtiyoriy sohada aniqlangan Laplas operatorli tenglama uchun Dezin maʼnosidagi nolokal shartli toʻgʻri va teskari masalalarning klassik maʼnoda yagona yechimga ega ekanligini isbotlash;

Toʻrtburchak sohada Kaputo operatoriga ega aralash turdagi tenglama uchun toʻgʻri va teskari masalalarning klassik maʼnodagi yagona yechimga egaligini isbotlash;

Har qanday oʻlchamdagi sohada Laplas operatorlarini oʻz ichiga olgan Kaputo kasr hosilasi ishtirok etuvchi aralash turdagi tenglama uchun chegaraviy masalaning yagona yechimga ega ekanligini isbotlash;

Kaputo maʼnosidagi kasr tartibli hosilaning tartibini aniqlashga oid toʻgʻri va teskari masalalarning klassik maʼnodagi yagona yechimga egaligini isbotlash, bunda elliptik operator formal oʻz-oʻziga qoʻshma, N oʻlchamli ixtiyoriy sohada aniqlangan Bessel operatori boʻladi.

Tadqiqotning obyekti. Kaputo kasr hosilalari ishtirokidagi aralash turdagi tenglamalar.

Tadqiqotning predmeti. Aralash turdagi differensial tenglamalar uchun chegaraviy masalalar va oʻng tomon yoki hosila tartibini aniqlashga doir teskari masalalar.

Tadqiqotning usullari. Dissertatsiyada quyidagi usullar qoʻllangan: xususiy hosilali differensial tenglamalar nazariyasi, funksional analiz metodlari, chiziqli operatorlar spektral nazariyasi, Furye usuli.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

sohaning yuqori qismida subdiffuziya, pastki qismida diffuziya tenglamasi uchun N o‘lchamli ixtiyoriy sohada aniqlangan Laplas operatorli tenglama uchun Dezin ma’nosidagi nolokal shartli to‘g‘ri va teskari masalalarning klassik ma’noda yagona yechimga ega ekanligini isbotlangan;

to‘rtburchak sohada Kaputo kasr tartibli aralash turdagi tenglamalar uchun to‘g‘ri va teskari masalalarning yagona yechimga egaligi isbotlangan;

N o‘lchamli ixtiyoriy sohada aniqlangan, o‘z-o‘ziga qo‘shma musbat Laplas operatori ishtirok etuvchi Kaputo kasr tartibli aralash turdagi tenglama uchun chegaraviy masala yechimining mavjudligi va yagonaligi isbotlangan;

Kaputo ma’nosidagi kasr hosila ishtirok etuvchi va musbat o‘z-o‘ziga qo‘shma Bessel operatorli aralash turdagi tenglama uchun to‘g‘ri va kasr tartibini aniqlash teskari masalalarning yagona yechimga egaligi isbotlangan.

Tadqiqot ishining amaliy natijalari. Dissertatsiyada keltirilgan natijalar va usullar oliy ta’lim muassasalarining magistratura va doktorantura bosqichlari uchun mo‘ljallangan o‘quv kurslariga kiritilishi mumkin.

Tadqiqot natijalarining ishonchliligi. Natijalar funksional analiz, spektral nazariya va Furye usuli yordamida olingan. Olingan barcha natijalar matematik jihatdan to‘g‘ri.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati shundaki, olingan natijalar yuqori tartibli xususiy hosilali differensial tenglamalar nazariyasida to‘g‘ri va teskari masalalarni yanada chuqurroq o‘rganishda ishlatiladi. Dissertatsiyaning amaliy ahamiyati shundan iboratki, olingan natijalar texnik, fizik va biologik jarayonlarni matematik modellashtirishda qo‘llanilishi mumkin.

Tadqiqot ishlarining joriy qilinishi. Xususiy hosilali yuqori tartibli differensial tenglamalar uchun to‘g‘ri va teskari masalalarga oid olingan natijalar amaliyotga quyidagi loyihalarda joriy etilgan:

Kaputo kasr tartibli aralash turdagi tenglamalar uchun Dezin nolokal shartli to‘g‘ri va teskari masalalar uchun topilgan yechimlaridan No. 22-11-00064 raqamli «Geosferadagi dinamik jarayonlarni irsiyatni hisobga olgan holda modellashtirish» mavzusidagi xorijiy grant loyihasida vaqt bo‘yicha o‘zgarib boruvchi fizik tizimni tasniflashda foydalanilgan (Kosmofizik tadqiqotlar va radioto‘lqinlarni tarqatish instituti, Uzoq Sharq bo‘limi, 2025 yil 25-sentyabrdagi №378-sonli ma’lumotnoma, Rossiya Federatsiyasi). Mazkur ilmiy natija subdiffuziya va diffuziya tenglamasida radon chiqishining manba funksiyasini aniqlash imkonini bergan bo‘lib, u geofizika va seysmologiya sohalaridagi amaliy masalalarda qo‘llanishi jihatidan muhim ahamiyatga ega;

To‘rtburchak sohada Kaputo kasr tartibli aralash turdagi tenglamalar uchun to‘g‘ri va teskari masalalarning yechimidan “Izotrop va anizotrop jismlarning statik va dinamik yuklamalar ostidagi deformatsiyasini, shuningdek, chiziqli bo‘lmagan elastik va plastik muhit modellarini ishlab chiqish” mavzusidagi ilmiy-tadqiqot ishlarida amaliy masalalarni nochiziqli modellarini ishlab chiqish hamda izotrop va anizotrop jismlarning deformatsiyalanishini tadqiq etishda qo‘llanilgan (M.T. O‘rozboyev nomidagi Mexanika va inshootlar seysmik mustahkamligi instituti 2025 yil 30 oktabrdagi №1348-3-sonli ma’lumotnomasi). Ilmiy natijalarni qo‘llanilishi elastik va plastik muhitlarning chiziqli bo‘lmagan modellarini yaratish, shuningdek, izotrop va anizotrop jismlarning turli yuklamalar ostida deformatsiyalanishini tahlil qilish imkonini bergan.

Mazkur natijalar elastik va plastik muhitlarning chiziqli bo‘lmagan modellarini yaratish, shuningdek, izotrop va anizotrop jismlarning turli yuklamalar ostida deformatsiyalanishini tahlil qilishda samarali natijalar bergan.

Tadqiqot natijalarining aprobatsiyasi. Tadqiqotning asosiy natijalari 7 ta xalqaro va 2 ta respublika ilmiy anjumanlarida muhokama qilindi.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha 15 ta ilmiy maqola chop etilgan bo‘lib, ulardan 6 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasi tavsiya etgan jurnallarda, 2 tasi xalqaro, 4 tasi esa respublika matematik jurnallarida chop etilgan. 3 ta maqola SCOPUS ma’lumotlar bazasiga kiritilgan. Bundan tashqari, 9 ta tezis e’lon qilingan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat. Dissertatsiya hajmi 120 bet.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertasiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, mavzu bo‘yicha xorijiy ilmiy-tadqiqotlar sharhi, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, ob’ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertasiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning **“Kaputo kasr hosilali va Dezin ma’nosidagi nolokal shartli aralash turdagi tenglamalar uchun to‘g‘ri va teskari masalalar”** deb nomlangan birinchi bobi ikkita bo‘limdan iborat bo‘lib, **“Asosiy ta’rif va tushunchalar”** deb nomlangan birinchi bo‘lim yordamchi xususiyatga ega bo‘lib, dissertatsiyani o‘qish qulay bo‘lishi uchun yaratilgan. Bu yerda yangi natijalar yo‘q va faqat kerakli ta’riflar va tasdiqlar to‘plangan.

Keling, birinchi bo‘limdagi ba’zi kerakli ma’lumotlarni keltiraylik.

Qaralayotgan masalalarni yechishda Furye usulidan foydalanamiz va bu bizni quyidagi spektral masalani ko‘rib chiqishga olib keladi

$$\begin{cases} -\Delta v(x) = \lambda v(x), \quad x \in \Omega, \\ v(x)|_{\partial\Omega} = 0. \end{cases}$$

$\partial\Omega$ chegara yetarlicha silliq bo‘lgani uchun, spektral masala $L_2(\Omega)$ da to‘la ortonormal $\{v_k(x)\}, k \geq 1$, xos funksiyalar to‘plami va λ_k qiymatlar k indeksning ortishi bilan kamaymaydigan nomanfiy sonlar ketma-ketligi $0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \ldots \to \infty$ dan iborat.

Aytaylik, σ ixtiyoriy haqiyqiy son bo‘lsin. $L_2(\Omega)$ fazoda A operatorning ixtiyoriy σ darajasini quyidagicha aniqlaymiz:

$$A^{\sigma} g(x)=\sum_{k=1}^{\infty} \lambda_k^{\sigma} g_k v_k(x), \quad g_k=(g, v_k),$$

bu yerda $g_k=(g, v_k)$ lar $g \in L_2(\Omega)$ funksiyaning Furye koeffitsiyentlari. Shubhasiz, ushbu operatorning aniqlanish sohasi quyidagi ko‘rinishga ega bo‘ladi:

$$D(A^{\sigma})=\{g \in L_2(\Omega): \sum_{k=1}^{\infty} \lambda_k^{2\sigma} |g_k|^2<\infty\}.$$

$D(\hat{A}^{\sigma})$ to‘plam elementlari uchun norma quyidagicha hisoblanadi:

$$\|g\|_{\sigma}^2=\sum_{k=1}^{\infty} \lambda_k^{2\sigma} |g_k|^2=\|A^{\sigma} g\|^2,$$

va bu norma bilan birgalikda $D(A^{\sigma})$ Hilbert fazosiga aylanadi.

Aytaylik, A operator $L_2(\Omega)$ da $Ag(x)=-\Delta g(x)$ kabi aniqlangan, $D(A)=\{g \in C^2(\overline{\Omega}): g(x)=0,\ x \in \partial\Omega\}$ aniqlanish sohasiga ega operator bo‘lsin. U holda $\hat{A} \equiv \hat{A}^1$ operator A operatorning $L_2(\Omega)$ dagi o‘z-o‘ziga qo‘shma kengaytmasi bo‘ladi.

V.A. Il’in teoremasi. Furye usuli yordamida elliptik qismi Laplas operatori Δ bo‘lgan tenglama uchun boshlang‘ich-chegaraviy masalaning yechimi mavjudligini isbotlashda quyidagi qatorning yaqinlashuvchanligini o‘rganish zarur:

$$\sum_{k=1}^{\infty} \lambda_k^{\tau} |h_k|^2, \quad \tau>\frac{N}{2}, \tag{1}$$

bu yerda h_k lar $h(x)$ funksiyalarning Furye koeffitsientlari. τ sonining butun qiymatlarida (1) qatorning yaqinlashishi uchun $h(x)$ funksiyaning qaysi $W_2^k(\Omega)$, $\Omega \subset R^n$ klassik Sobolev fazolariga tegishli bo‘lishlik shartlari A. Il’inning fundamental ishida ko‘rsatilgan. Ushbu shartlarni keltirish uchun biz $\overset{\circ}{W}{}_2^1(\Omega)$ sinfni kiritamiz, ya’ni $\overset{\circ}{W}{}_2^1(\Omega)$ orqali $W_2^1(\Omega)$ fazo normasi bo‘yicha $C_0^{\infty}(\Omega)$ to‘plamining yopilmasini belgilaymiz. Quyida V.A. Il’inning ishidan olingan ba’zi natijalarini keltiramiz.

Agar $h(x)$ funksiya quyidagi

$$h(x) \in W_2^{\left[\frac{N}{2}\right]+1}(\Omega) \text{ va } h(x),\ \Delta h(x), \ldots, \Delta^{\left[\frac{N}{4}\right]} h(x) \in \overset{\circ}{W}{}_2^1(\Omega), \tag{2}$$

shartlarni qanoatlantirsa, u holda (1) qator yaqinlashuvchi bo‘ladi. (agar N juft bo‘lsa, $\tau=\frac{N}{2}+1$, agar N toq bo‘lsa $\tau=\frac{N+1}{2}$ deb olish mumkin).

Xuddi shunday, agar (1) da τ ni $\tau+1$ ga almashtirsak, yaqinlashish shartlari quyidagi ko‘rinishga ega bo‘ladi:

$$h(x) \in W_2^{\left[\frac{N}{2}\right]+2}(\Omega) \text{ va } h(x),\ \Delta h(x), \ldots, \Delta^{\left[\frac{N}{4}\right]} h(x) \in \overset{\circ}{W}{}_2^1(\Omega), \tag{3}$$

Quyida M.M. Dzherbashyanning ishida keltirilgan Mittag-Leffler funksiyasining ta’rifi va keyingi o‘rinlarda bizga kerak bo‘ladigan asosiy xossalaridan birini keltirib o‘tamiz.

1-ta’rif. *Ikkita parametrli Mittag-Leffler funksiyasi quyidagicha aniqlanadi*:

$$E_{\rho,\mu}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\rho k+\mu)},\quad z,\mu\in\mathbb{C},\quad \rho>0.$$

1-lemma. *Agar* $0<\rho<1$ *bo'lsa, u holda ixtiyoriy* $\mu, z\in\mathbb{C}$ *uchun quyidagiga egamiz:*

$$|E_{\rho,\mu}(-z)|\le\frac{C_0}{1+|z|},$$

bu yerda C_0 *o'zgarmas son* z *va* μ *ga bog'liq emas.*

Ikkinchi bo'lim **"Kaputo kasr hosilali va Dezin ma'nosidagi nolokal shartli aralash turdagi tenglama uchun to'g'ri va teskari masalalar"** deb nomlanadi.

Ushbu bo'limda vaqt bo'yicha $t>0$ sohada Kaputo kasr hosilasini o'z ichiga olgan aralash turdagi xususiy hosilali subdiffuziya tenglamasi hamda $t<0$ sohada diffuziya tenglamasi bilan bog'liq to'g'ri va teskari masalalar o'rganildi.

Faraz qilaylik, Ω - yetarlicha silliq chegaraga ega bo'lgan ixtiyoriy N o'lchamli soha bo'lsin. Quyidagi aralash turdagi tenglamani ko'rib chiqamiz:

$$\begin{cases} D_t^{\rho}u-\Delta u=F(x,t), & x\in\Omega,\quad 0<t\le\beta,\\ u_t+\Delta u=F(x,t), & x\in\Omega,\quad -\alpha<t<0,\end{cases}\tag{4}$$

bu yerda $F(x,t)$ - uzluksiz funksiya bo'lib, $\alpha>0$, $\beta>0$- berilgan haqiqiy sonlar, Δ esa Laplas operatori.

1-masala (Dezin masalasi). (4) tenglamani va quyidagi chegaraviy

$$u(x,t)|_{\partial\Omega}=0,\quad t\in[-\alpha,\beta],$$

va ulash sharti:

$$\lim_{t\to+0}u(x,t)=\lim_{t\to-0}u(x,t),\ \ x\in\Omega,$$

hamda quyidagi nolokal shartni:

$$u(x,-\alpha)=\lambda u(x,0),\quad x\in\Omega,\tag{5}$$

qanoatlantiruvchi $u(x,t)$ funksiyani toping, bu yerda $\lambda=const$, $\lambda\ne0$.

(5) shart sababli bu masala Dezin masalasi deb nomlanadi. Agar $\lambda=0$ bo'lsa, bu holda biz subdiffuziya tenglamasi uchun ortga qaytish (**backward**) masalaga ega bo'lamiz.

1- to'g'ri masalaning formal yechimi quyidagi qator ko'rinishiga ega:

$$u(x,t)=\begin{cases}\sum\limits_{k=1}^{\infty}\left(\dfrac{I_\alpha(\lambda_k)}{\delta_k}E_{\rho,1}(-\lambda_k t^{\rho})+I_\rho(\lambda_k)\right)v_k(x), & 0\le t\le\beta,\\ \sum\limits_{k=1}^{\infty}\left(\dfrac{I_\alpha(\lambda_k)}{\delta_k}e^{\lambda_k t}-\displaystyle\int_t^0 F_k(s)e^{\lambda_k(t-s)}ds\right)v_k(x), & -\alpha\le t\le 0.\end{cases}\tag{6}$$

bu yerda

$$I_\alpha(\lambda_k)=\int_{-\alpha}^{0}F_k(s)e^{\lambda_k(-\alpha-s)}ds,\quad I_\rho(\lambda_k)=\int_0^t s^{\rho-1}E_{\rho,\rho}\left(-\lambda_k s^{\rho}\right)F_k(t-s)ds,$$

$$\delta_k=e^{-\lambda_k\alpha}-\lambda,\quad k\ge1.$$

1-teorema. *Aytaylik,* $\lambda\notin[0,1)$ *bo'lsin va* $F(x,t)$ *funksiyasi barcha* $t\in[-\alpha,\beta]$

lar uchun (3) shartni qanoatlantirsin. U holda 1- to'g'ri masalaning yagona yechimi mavjud va u (6) qator orqali aniqlanadi.

2-teorema. *Aytaylik,* $0<\lambda<1$ *bo'lsin va* $F(x,t)$ *funksiyasi barcha* $t\in[-\alpha,\beta]$ *uchun (3) shartlarni qanoatlantirsin. U holda:*

1) Agar barcha $k\geq 1$ *uchun* $\lambda_k\neq\lambda_0$ *bajarilsa, unda* ***1- to'g'ri masala****ning yagona yechimi mavjud bo'lib, u (6) ko'rinishda ifodalanadi.*

2) Agar ba'zi k *uchun* $\lambda_k=\lambda_0$ *bajarilsa u holda va* ***1- to'g'ri masala****ning yechimining mavjudligi uchun quyidagi ortogonallik shartlari bajarilishi zarur va yetarlidir:*

$$F_k^*=(F^*,v_k)=0,\ k\in K_0,\ K_0=\{k_0,k_0+1,\ldots,k_0+p_0-1\},\ F_k^*=\int_{-\alpha}^{0}F_k(s)e^{\lambda_k(-\alpha-s)}ds.$$

Bu holda 1- to'g'ri masalaning yechimi yagona emas va ixtiyoriy $a_k,\ k\in K_0$ *koeffitsientlar bilan quyidagi ko'rinishga ega bo'ladi:*

$$u(x,t)=\begin{cases}\sum\limits_{k\notin K_0}\left(\dfrac{I_\alpha(\lambda_k)}{\delta_k}E_{\rho,1}(-\lambda_k t^\rho)+I_\rho(\lambda_k)\right)v_k(x)+\sum\limits_{k\in K_0}a_kE_{\rho,1}(-\lambda_k t^\rho)v_k(x),\ t>0,\\ \sum\limits_{k\notin K_0}\left(\dfrac{F_k^*}{\delta_k}e^{\lambda_k t}-\int_t^0F_k(s)e^{\lambda_k(t-s)}ds\right)v_k(x)+\sum\limits_{k\in K_0}a_ke^{\lambda_k t}v_k(x),\quad t<0.\end{cases}$$

2-masala (teskari masala). Faraz qilaylik, $F(x,t)=f(x)g(t)$ va $g(t)$ berilgan funksiya bo'lsin. 1- to'g'ri masalaning yechimi bo'lib, quyidagi qo'shimcha shartni

$$u(x,t_0)=\varphi_0(x),\quad x\in\Omega,\tag{7}$$

qanoatlantiruvchi $u(x,t)\in AC([0,\beta];C(\overline{\Omega}))\cup C(\overline{\Omega}\times[-\alpha,0])$ va $f(x)\in C(\overline{\Omega})$ funksiyalar juftligini toping, bu yerda $\varphi_0(x)$ - yetarlicha silliq berilgan funksiya, $t_0\in(0,\beta)$ berilgan nuqta.

f_k koeffitsiyentlarni topish uchun, biz quyidagi tenglikka ega bo'lamiz:

$$f_k\Delta_k(t_0)=\delta_k\varphi_{0k}=(e^{-\lambda_k\alpha}-\lambda)\varphi_{0k},$$

bu yerda

$$\Delta_k(t_0)=E_{\rho,1}(-\lambda_k t_0^\rho)\int_{-\alpha}^{0}g(s)e^{\lambda_k(-\alpha-s)}ds+$$

$$+(e^{-\lambda_k\alpha}-\lambda)\int_0^{t_0}s^{\rho-1}E_{\rho,\rho}(-\lambda_k s^\rho)g(t_0-s)ds.\tag{8}$$

$\Delta_k(t_0)$ uchun quyi baho keltiramiz. Buning uchun $g(t)$ funksiyasi nolga teng emasligi talab qilinadi. $g(t)\neq 0$ bo'lib, u uzluksiz bo'lganligi sababli, tahlil $g(t)>0$ yoki $g(t)<0$ bo'lishidan qat'i nazar, bir xil bo'ladi. Shuning uchun quyida keltiriladigan to'rtta lemma davomida umumiylikni yo'qotmasdan $g(t)>0$ deb faraz qilamiz.

Aytaylik, $g\in C[-\alpha,\beta]$ va $g(t)\neq 0$ bo'lsin. Quyidagicha belgilaymiz:

$$m=\min_{t\in[-\alpha,t_0]}|g(t)|>0,\quad M=\max_{t\in[-\alpha,t_0]}|g(t)|>0.$$

2-lemma. *Aytaylik,* $\lambda < 0$, $g(t) \in C[-\alpha, \beta]$, $g(t) \neq 0$, $t \in [-\alpha, \beta]$ *bo'lsin. U holda* t_0 *va* α *ga bog'liq bo'lgan* $C > 0$ *o'zgarmas son mavjud bo'lib, barcha* k *lar uchun quyidagicha quyidan baho o'rinli:*

$$\Delta_k(t_0) \geq \frac{C}{\lambda_k}.$$

3-lemma. *Aytaylik,* $\lambda \geq 1$, $g(t) \in C[-\alpha, \beta]$, $g(t) \neq 0$, $t \in [-\alpha, \beta]$ *bo'lsin. Agar* t_0 *soni quyidagi shartni qanoatlantirsa:*

$$t_0^\rho > \frac{C_0}{\lambda_1}\left(1 + \frac{M}{m}\right),$$

bu yerda C_0 *- 1-lemmada keltirilgan son, u holda* t_0, ρ *va* α *ga bog'liq* $C > 0$ *doimiy mavjud bo'lib, barcha* k *lar uchun*

$$|\Delta_k(t_0)| \geq \frac{C}{\lambda_k}. \qquad (9)$$

tengsizlik bajariladi.

Agar t_0 *soni quyidagi shartni bajarsa,*

$$t_0^\rho \leq \frac{C_0}{\lambda_1}\left(1 + \frac{M}{m}\right),$$

u holda shunday $k_1 \in \mathbb{N}$ *soni topiladiki, (9) baho barcha* $k > k_1$ *lar uchun bajariladi.*

4-lemma. *Aytaylik,* $0 < \lambda < 1$, $g(t) \in C[-\alpha, \beta]$ *va* $g(t) \neq 0$, $t \in [-\alpha, \beta]$ *bo'lsin. U holda shunday* $k_2 \in \mathbb{N}$ *soni topiladiki, barcha* $k > k_2$, *uchun quyidagi baho o'rinli:*

$$|\Delta_{k,\rho}(t_0)| \geq \frac{C}{\lambda_k},$$

bu yerda $C > 0$ *-* ρ, t_0 *va* α *ga bog'liq o'zgarmas son.*

Quyidagi to'plamni kiritamiz:

$$\mathbb{K}_0 = \{k \in \mathbb{N} : \Delta_k(t_0) = 0\}.$$

5-lemma. $\mathbb{K}_0$ *to'plam yoki bo'sh bo'ladi, yoki chekli sondagi elementlarni o'z ichiga oladi.*

3-teorema. *Aytaylik,* $\lambda < 0$, $g(t) \in C[-\alpha, \beta]$ *va* $g(t) \neq 0$, $t \in [-\alpha, \beta]$, *hamda* $\varphi_0(x)$ *funksiyasi (2) shartlarni qanoatlantirsin. U holda* ***2- masala****ning yagona yechimi mavjud va u quyidagicha ifodalanadi:*

$$u(x,t) = \sum_{k=1}^{\infty}\left(\frac{\varphi_{0k}}{\Delta_k(t_0)} E_{\rho,1}(\lambda_k t^\rho) \int_{-\alpha}^{0} g(s) e^{\lambda_k(-\alpha - s)} ds\right) v_k(x) +$$

$$+ \sum_{k=1}^{\infty}\left(\frac{\varphi_{0k}(e^{-\lambda_k \alpha} - \lambda)}{\Delta_k(t_0)} \int_0^t s^{\rho-1} E_{\rho,\rho}\left(-\lambda_k s^\rho\right) g(t-s) ds\right) v_k(x), \quad t \geq 0,$$

$$u(x,t) = \sum_{k=1}^{\infty}\left(\frac{\varphi_{0k}}{\Delta_k(t_0)} e^{\lambda_k t} \int_{-\alpha}^{0} g(s) e^{\lambda_k(-\alpha - s)} ds - \frac{\varphi_{0k}(e^{-\lambda_k \alpha} - \lambda)}{\Delta_k(t_0)} \int_t^0 g(s) e^{\lambda_k(t-s)} ds\right) v_k(x), \quad t \leq 0.$$

$$f(x)=\sum_{k=1}^{\infty}\frac{\varphi_{0k}(e^{-\lambda_k\alpha}-\lambda)}{\Delta_k(t_0)}v_k(x).$$

4-teorema. *Aytaylik,* $g(t)\in C[-\alpha,\beta]$ *va* $g(t)\neq 0$, $t\in[-\alpha,\beta]$, $\varphi_0(x)$ *funksiyasi (2) shartlarni qanoatlantirsin, hamda barcha* k *lar uchun* $\delta_k\neq 0$ *bo'lsin. Shuningdek, 3-lemma yoki 4-lemma shartlari bajarilsin.*

1) Agar $\mathbb{K}_0$ *to'plam bo'sh bo'lsa, u holda* ***2- masala****ning yechimi mavjud va yagona;*

2) Agar $\mathbb{K}_0$ *to'plam bo'sh bo'lmasa, unda* ***2- masala*** *yechimi mavjud bo'lishi uchun quyidagi ortogonallik sharti bajarilishi zarur va yetarlidir:*

$$\varphi_{0k}=(\varphi_0,v_k)=0,\ k\in\mathbb{K}_0.$$

Ikkinchi bob **"Elliptik qismi x bo'yicha ikkinchi tartibli hosilaga ega bo'lgan aralash turdagi tenglamalar uchun to'rtburchak sohada to'g'ri va teskari masalalar"** deb nomlanadi.

Ushbu bobning birinchi bo'limida to'rtburchak sohada Kaputo operatori bilan berilgan bir jinsli parabolo-giperbolik turdagi tenglama uchun quyidagicha

$$au(x,-\alpha)-bu(x,\beta)=\varphi(x),\quad 0\leq x\leq 1, \tag{10}$$

nolokal shartli masala yechimining turg'unligi hamda yechimning mavjudligi va yagonaligi o'rganilgan, bu yerda a va b - ixtiyoriy o'zgarmas sonlar, $\varphi(x)$ - berilgan yetarlicha silliq funksiya bo'lib, $\varphi(0)=\varphi(1)=0$ shartni qanoatlantiradi.

Ushbu bobning ikkinchi bo'limida birinchi bo'limda qaralgan tenglama bir jinsli bo'lmagan holi uchun o'rganilgan va asosan teskari masalani o'rganishga e'tibor qaratilgan. Lekin, (10) nolokal shart soddaroq holda (ya'ni $a=1,\ b=0$) olingan.

3-masala. Aytaylik, $0<\rho\leq 1$ bo'lsin. To'rtburchak $D=\{(x,t)|\ 0<x<1,\ -\alpha<t<\beta\}$ sohada quyidagi aralash turdagi tenglamani

$$Lu=F(x,t),$$

bu yerda

$$Lu=\begin{cases}D_{0t}^{\rho}u-u_{xx}, & 0<t<\beta,\\ u_{tt}-u_{xx}, & -\alpha<t<0,\end{cases}\qquad F(x,t)=\begin{cases}f_1(x,t), & 0<t<\beta,\\ f_2(x,t), & -\alpha<t<0.\end{cases}$$

Quyidagi chegaraviy shartni:

$$u(0,t)=u(1,t)=0,\quad -\alpha\leq t\leq\beta,$$

quyidagicha ulash shartlarini:

$$\lim_{t\to+0}u(x,t)=\lim_{t\to-0}u(x,t),\ \ \lim_{t\to+0}D_t^{\rho}u(x,t)=\lim_{t\to-0}u_t(x,t),$$

hamda quyidagi boshlang'ich shartni:

$$u(x,-\alpha)=\varphi(x),\quad 0<x<1,$$

qanoatlantiruvchi $u(x,t)\in C(\overline{D}_-)$, $u(x,t)\in AC([0,\beta];C[0,1])$ funksiyani toping, bu yerda $\varphi(x)$ va $F(x,t)$ berilgan yetarlicha silliq funksiyalar, $\alpha>0$, $\beta>0$ -berilgan haqiqiy sonlar.

3-to'g'ri masalaning yechimini qurish uchun Furye usulidan foydalanamiz.

Yechimning maxrajida paydo bo'ladigan ifodani quyidagicha belgilaymiz: $\Delta_\alpha(k)=\cos(\sqrt{\lambda_k}\alpha)+\sqrt{\lambda_k}\sin(\sqrt{\lambda_k}\alpha)=\sqrt{1+\lambda_k}\sin(\sqrt{\lambda_k}\alpha+\theta_k)\neq 0,$

bu yerda $k\to\infty$ da $\theta_k=\arcsin\left(\dfrac{1}{\sqrt{1+\lambda_k}}\right)\to 0$.

Aytaylik, $\mathbb{N}=K\cup K_0$ deb olaylik. Bu yerda $\mathbb{N}$ barcha natural sonlar to‘plami, K va K_0 esa quyidagi shartlarni qanoatlantiruvchi to‘plamlardir:

1) Agar $\Delta_\alpha(k)\neq 0$ bo‘lsa, u holda $k\in K$,

2) Aks holda, agar $\Delta_\alpha(k)=0$ bo‘lsa, u holda $k\in K_0$.

5-teorema. *Aytaylik, α ixtiyoriy natural son yoki musbat darajasi ikki bo‘lgan irratsional algebraik son bo‘lib, $f_1(x,t)$, $f_2(x,t)$ va $\varphi(x)$ funksiyalar uchun quyidagi shartlar bajarilsin:*

1) $f_1(x,t)\in C([0,\beta],W_2^3(0,1)),\quad f_1(0,t)=f_1(1,t)=0,\ f_1''(0,t)=f_1''(1,t)=0,$

2) $f_2(x,t)\in C([-\alpha,0],W_2^3(0,1)),\quad f_2(0,t)=f_2(1,t)=0,\ f_2''(0,t)=f_2''(1,t)=0,$

3) $\varphi(x)\in W_2^4(0,1),\ \varphi(0)=\varphi(1)=\varphi''(0)=\varphi''(1)=0.$

*U holda **3- to‘g‘ri masala**ning yagona yechimi mavjud va u quyidagi qator ko‘rinishida ifodalanadi:*

$$u(x,t)=\sum_{k=1}^{\infty}\frac{\omega_k E_{\rho,1}(-(\pi k)^2t^\rho)}{\Delta_\alpha(k)}\sin\pi kx+$$

$$+\sum_{k=1}^{\infty}\left(\int_0^t\tau^{\rho-1}E_{\rho,\rho}(-\pi k\tau^\rho)\,f_{1k}(t-\tau)d\tau\right)\sin\pi kx,\quad t>0, \tag{11}$$

$$u(x,t)=\sum_{k=1}^{\infty}\left(\frac{\omega_k}{\Delta_\alpha(k)}(\cos\pi kt-\pi k\sin\pi kt)+\frac{f_{1k}(0)}{\pi k}\sin\pi kt\right)\sin\pi kx$$

$$+\sum_{k=1}^{\infty}\left(\frac{1}{\pi k}\int_0^t f_{2k}(\tau)\sin(\pi k(t-\tau))d\tau\right)\sin\pi kx,\quad t<0, \tag{12}$$

bu yerda

$$\omega_k=\varphi_k+\frac{f_{1k}(0)\sin(\pi k\alpha)}{\pi k}-\frac{1}{\pi k}\int_{-\alpha}^0 f_{2k}(\tau)\sin(\pi k(\alpha+\tau))d\tau.$$

6-teorema. *Agar $\varphi(x)$ va $f_1(x,t)$, $f_2(x,t)$ funksiyalar 5-teoremaning shartlarini qanoatlantirib, α musbat ratsional son bo‘lsa, u holda*

*1) agar K_0 to‘plam bo‘sh bo‘lsa, **3- to‘g‘ri masala**ning yechimi mavjud, yagona va (11), (12) qator ko‘rinishida ifodalanadi.*

*2) agar K_0 bo‘sh bo‘lmasa, unda **3-to‘g‘ri masala**ning yechimi mavjud bo‘lishi uchun quyidagi ortogonallik shartlari bajarilishi zarur va yetarli:*

$$\varphi_k=\frac{1}{\pi k}\int_{-\alpha}^{0}f_{2k}(\tau)\sin\pi k(\alpha+\tau)d\tau-\frac{f_{1k}(0+0)\sin\pi k\alpha}{\pi k},\quad k\in K_0.$$

4-masala. Aytaylik,

$$F(x,t)=\begin{cases} f_1(x)\phi_1(t), & 0<t<\beta,\\ f_2(x,t), & -\alpha<t<0.\end{cases}$$

bo‘lsin. To‘g‘ri masalaning yechimi bo‘lib, quyidagi

$$u(x_0,t) = h_1(t), \quad 0 \le t \le \beta, \tag{13}$$

qo‘shimcha shartni qanoatlantiruvchi $u(x,t)$ va $\phi_1(t) \in C[0,\beta]$ funksiyalar juftligini toping, bu yerda $f_1(x),\ f_2(x,t), h_1(t)$ berilgan yetarlicha silliq funksiyalar, $x_0 \in (0,1)$ berilgan nuqta.

(13) shartni 3-to‘g‘ri masalaning yechimiga qo‘llab, quyidagiga ega bo‘lamiz:

$$f_1(x_0)\phi_1(t) - \int_0^t \phi_1(s)\bar{K}_1(s,t)ds = D_t^\rho \bar{h}_1(t), \tag{14}$$

(14) tenglamada agar $f_1(x_0) \neq 0$ bo‘lsa, u holda $[0,\beta]$ oraliqda uzluksiz yadro $\bar{K}_1(s,t)$ va uzluksiz o‘ng tomonli $D_t^\rho \bar{h}_1(t)$ ikkinchi tur Volterra integral tenglamasiga ega bo‘lamiz. Shuningdek, bu tenglama o‘ng tarafining maxrajida quyidagi ifodaning nolga teng emaslik sharti muhimdir

$$f_1(x_0) + D_t^\rho S_1(t)|_{t=0} = \sum_{k=1}^\infty f_{1k} v_k(x_0) - \sum_{k=1}^\infty \frac{\lambda_k f_{1k} \sin\sqrt{\lambda_k}\alpha}{\sqrt{\lambda_k}\Delta_\alpha(k)} v_k(x_0)$$

$$= \sum_{k=1}^\infty f_{1k}\left[1 - \frac{\sqrt{\lambda_k}\sin\sqrt{\lambda_k}\alpha}{\Delta_\alpha(k)}\right] v_k(x_0) = \sum_{k=1}^\infty \frac{f_{1k}\cos\sqrt{\lambda_k}\alpha}{\Delta_\alpha(k)} v_k(x_0) \neq 0. \tag{15}$$

O‘z-o‘zidan ko‘rinib turibdiki, (15) shart masalan, $\alpha \in \mathbb{N}$ bo‘lsa, aniq bajariladi. Chunki oxirgi qator bu ikkita qatorning ayirmasidan iborat. λ_k ning ta’rifiga ko‘ra $\alpha \in \mathbb{N}$ bo‘lsa, ikkinchi qator nolga teng. Birinchi qator esa $f(x)$ funksiyaning x_0 nuqtadagi Furye qatori. Farazimizga ko‘ra u noldan farqli.

7-teorema. *Aytaylik, α ixtiyoriy natural son yoki darajasi ikki bo‘lgan musbat irratsional algebraik son, $D_t^\rho h_1(t) \in C[0,\beta]$ hamda $f_1(x)$, $f_2(x,t)$, $\varphi(x)$ funksiyalar uchun quyidagi shartlar bajarilsin:*

1) $f_1(x) \in W_2^3(0,1),\ f_1(0) = f_1(1) = 0,\ f_1''(0) = f_1''(1) = 0,$

2) $f_2(x,t) \in C([-\alpha,0], W_2^3(0,1)),\quad f_2(0,t) = f_2(1,t) = 0,\ f_{2xx}''(0,t) = f_{2xx}''(1,t) = 0,$

3) $\varphi(x) \in W_2^4(0,1),\ \varphi(0) = \varphi(1) = \varphi''(0) = \varphi''(1) = 0.$

*Agar $f_1(x_0) \neq 0$ va (15) shart bajarilsa, u holda **4-masala** yechimi mavjud va yagona.*

5-masala. Aytaylik,

$$F(x,t) = \begin{cases} f_1(x,t), & 0 < t < \beta, \\ f_2(x)\phi_2(t), & -\alpha < t < 0. \end{cases}$$

bo‘lsin. 3-to‘g‘ri masalaning yechimi bo‘lib, quyidagi

$$u(x_0,t) = h_2(t), \quad -\alpha \le t \le 0. \tag{16}$$

qo‘shimcha shartni qanoatlantiruvchi $u(x,t)$ va $\phi_2(t) \in C[-\alpha,0]$ funksiyalar juftligini toping, bu yerda $f_1(x,t),\ f_2(x), h_2(t)$ - berilgan yetarlicha silliq funksiyalar.

Bu yerda ham, yuqorida bajarilgan amallarni takrorlab, hamda $f_2(x_0) \neq 0$ shartini talab qilib, quyidagi ikkinchi tur Fredgolm integral tenglamasiga ega bo‘lamiz:

$$\phi_2(t)-\lambda\int_{-\alpha}^{0}\phi_2(\tau)\Phi(\tau,t)dt=\mu(t),\qquad \lambda=\frac{1}{f_2(x_0)}. \tag{17}$$

(17) integral tenglama yadrosi $\Phi(\tau,t)$, $-\alpha\le\tau,t\le 0$ oraliqda uzluksizdir. Agar $h_2(t)\in C^2[-\alpha,0]$ bo‘lsa, u holda o‘ng tomondagi $\mu(t)$ ham $[-\alpha,0]$ da uzluksiz bo‘ladi. Shuning uchun (17) tenglama uzluksiz yadroli va uzluksiz o‘ng tomonga ega ikkinchi tur Fredgolm integral tenglamasidir.

8-teorema. *Aytaylik, α ixtiyoriy natural son yoki darajasi ikki bo‘lgan musbat irratsional algebraik son, $h_2(t)\in C^2[-\alpha,0]$, $f_2(x_0)\ne 0$ hamda $f_1(x,t)$, $f_2(x)$ va $\varphi(x)$ funksiyalar uchun quyidagi shartlar bajarilsin:*

1) $f_1(x,t)\in C([0,\beta],W_2^3(0,1))$, $f_1(0,t)=f_1(1,t)=0$, $f_{1xx}''(0,t)=f_{1xx}''(1,t)=0$,

2) $f_2(x)\in W_2^3(0,1)$, $f_2(0)=f_2(1)=0$, $f_2''(0)=f_2''(1)=0$,

3) $\varphi(x)\in W_2^4(0,1)$, $\varphi(0)=\varphi(1)=\varphi''(0)=\varphi''(1)=0$.

Agar λ parametr uchun quyidagi

a) λ soni yadrosi $\Phi(\tau,t)$ bo‘lgan integral operatorning xarakteristik soni bo‘lmasin;

b) $|\lambda|<\dfrac{1}{M\alpha}$;

*shartlardan biri bajarilsa, u holda, **5-masala** yechimi mavjud va yagona.*

6-masala. Aytaylik,

$$F(x,t)=\begin{cases} f_1(x)\phi_1(t), & 0<t<\beta,\\ f_2(x)\phi_2(t), & -\alpha<t<0.\end{cases}$$

bo‘lsin. To‘g‘ri masalaning yechimi bo‘lib, (13), (16) qo‘shimcha shartlarni qanoatlantiruvchi, $\{u(x,t),\phi_1(t),\phi_2(t)\}$ funksiyalar juftligini toping, bu yerda $f_1(x)$, $f_2(x)$ - berilgan yetarlicha silliq funksiyalar.

(13) va (16) shartlarni 3-to‘g‘ri masalaning yechimiga qo‘llab, quyidagi tenglamalarga ega bo‘lamiz:

$$\phi_1(t)\,f_1(x_0)-\int_0^t\phi_1(s)\bar{K}_1(s,t)ds-\int_{-\alpha}^{0}\phi_2(s)D_t^{\rho}\bar{\bar{K}}_1(s,t)ds=D_t^{\rho}\bar{\bar{h}}_1(t), \tag{18}$$

$$\phi_2(t)\,f_2(x_0)-\int_t^0\phi_2(\tau)K_{2tt}(\tau,t)d\tau-\int_{-\alpha}^{0}\phi_2(\tau)K_{3tt}(\tau,t)d\tau=\bar{h}_2''(t). \tag{19}$$

$f_1(x_0)\ne 0$ shartni hisobga olgan holda, topilgan $\phi_1(0)$ ni qiymatini (19) tenglamaning o‘ng tomoniga qo‘yib, berilgan oraliqlarda uzluksiz $K_{2tt}(\tau,t)$, $K_{3tt}(\tau,t)$ yadroli va uzluksiz o‘ng tomonili $\bar{h}_2''(t)$ ikkinchi tur Fredgolm integral tenglamasini olamiz. Funksiya $\phi_2(t)$ topilgandan so‘ng, $\phi_1(t)$ ham $f_1(x_0)\ne 0$ va (15) shart bajarilganda (18) tenglama berilgan oraliqlarda uzluksiz $\bar{K}_1(s,t)$, $D_t^{\rho}\bar{\bar{K}}_1(s,t)$ yadroli va uzluksiz o‘ng tomonli $D_t^{\rho}\bar{\bar{h}}_1(t)$ ikkinchi tur Volterra integral tenglamasi sifatida topiladi.

9-teorema. *Aytaylik,* $D_t^{\rho} h_1(t) \in C[0,\beta],\ h_2(t) \in C^2[-\alpha,0],\ \ f_1(x_0) \neq 0,\ \ f_2(x_0) \neq 0$ *bo'lsin va* α *ixtiyoriy natural son yoki darajasi ikki bo'lgan musbat irratsional algebraik son bo'lsin, hamda* $f_1(x)$, $f_2(x)$ *va* $\varphi(x)$ *funksiyalar quyidagi shartlarni qanoatlantirsin:*

1) $f_1(x) \in W_2^3(0,1),\ \ f_1(0) = f_1(1) = 0,\ \ f_1''(0) = f_1''(1) = 0,$

2) $f_2(x) \in W_2^3(0,1),\ \ \ f_2(0) = f_2(1) = 0,\ \ f_2''(0) = f_2''(1) = 0,$

3) $\varphi(x) \in W_2^4(0,1),\ \varphi(0) = \varphi(1) = \varphi''(0) = \varphi''(1) = 0.$

Agar λ *parametr uchun 8-teoremada keltirilgan shartlardan biri* $a)$ *yoki* $b)$ *bajarilsa va* $\phi_1(0) = 0$ *yoki* $\phi_2(0) = 0$ *bo'lsa, u holda* ***6-masala*** *yechimi mavjud va yagona.*

Dissertatsiyaning **uchinchi bobi "Elliptik qismi Laplas va Bessel operatorlari bo'lgan aralash turdagi tenglamalar uchun to'g'ri va teskari masalalar"** deb nomlanadi.

Ushbu bobning birinchi bo'limida ixtiyoriy N o'lchovli fazoda elliptik qismi Laplas operatoridan iborat Kaputo kasr hosilali bir jinsli bo'lmagan parabolo-giperbolik turdagi tenglamalar uchun uchta ulash shartli masala yechimining mavjudligi va yagonaligi o'rganilgan. Shuningdek, uchta ulash sharti berilgan bu masalani o'rganish jarayonida agar tenglamaning o'ng tomoni t ga bog'liq bo'lmasa, u holda bunday masala yechimi ham t ga bog'liq bo'lmasligi, ya'ni tenglamaning t ga bog'liqligi yo'qolishi va u oddiy elliptik tenglamaga aylananishi ham ko'rsatilgan.

Ushbu bobning ikkinchi bo'limida quyidagi to'g'ri va daraja aniqlash teskari masalasi o'rganildi:

Aytaylik, $\rho \in (0,1)$ bo'lsin. $Q := \{(x,t) \mid 0 < x < 1,\ -\alpha < t < \infty\}$ sohada quyidagi aralash turdagi tenglamani ko'rib chiqamiz:

$$f(x,t) = \begin{cases} D_t^{\rho} u - B_0 u, & 0 < x < 1, \quad 0 < t < \infty, \\ u_{tt} - B_0 u, & 0 < x < 1, \quad -\alpha < t < 0, \end{cases} \tag{20}$$

bu yerda $f(x,t)$ - yetarlicha silliq berilgan funksiya, $\alpha > 0$ - haqiqiy son va

$$B_0 u = u_{xx}(x,t) + \frac{1}{x} u_x(x,t)$$

nolinchi tartibli birinchi tur Bessel operatori bo'lib, tenglamaning Bessel qismi hisoblanadi.

7-masala. (20) tenglamani va quyidagi chegaraviy shartlarni

$$\lim_{x \to 0} x u_x(x,t) = 0,\ \ u(1,t) = 0,$$

hamda quyidagi ulash shartlarini:

$$\lim_{t \to +0} u(x,t) = \lim_{t \to -0} u(x,t),\ \ \lim_{t \to +0} D_t^{\rho} u(x,t) = \lim_{t \to -0} u_t(x,t),$$

shuningdek, boshlang'ich shartni:

$$u(x,-\alpha) = \varphi(x), \quad 0 < x < 1,$$

qanoatlantiruvchi $u(x,t)$ funksiyani toping. Bu yerda $\varphi(x)$ - berilgan funksiya.

Masala yechimini qurishda Furye usuli qo'llaniladi va yechimning maxrajida uchraydigan ifoda quyidagicha aniqlanadi:

$$\Delta_\alpha(k)=\sqrt{1+\lambda_k^2}\,\sin\left(\lambda_k\alpha+\theta_k\right),\quad k\geq 1.$$

6- lemma. *Agar* α *musbat ratsional son bo'lsa, u holda shunday* $C_0>0$ *va* $k_0\in\mathbb{N}$ *mavjudki, barcha* $k>k_0$ *uchun quyidagi baho o'rinli:*

$$|\Delta_\alpha(k)|>C_0.$$

Quyidagi to'plamni kiritamiz:

$$\mathbb{K}_0=\{k:\Delta_\alpha(k)=0,\ k\in\mathbb{N}\}.$$

$\mathbb{K}_0$ to'plam 6-lemmaga ko'ra chekli sondagi elementlarga ega.

10-teorema. *Aytaylik,* α *musbat ratsional son bo'lib,* $\varphi(x)$ *va* $f(x,t)$ *funksiyalar* x *bo'yicha to'rt marta differensiallanuvchi bo'lsin va* x *bo'yicha hosilalar uchun quyidagi tengliklar bajarilsin:*

1. $f(0,t)=f'(0,t)=f''(0,t)=f'''(0,t)=0$*;* $f(1,t)=f'(1,t)=f''(1,t)=0$, $\forall t\in(-\alpha,\beta)$*;*

2. $\varphi(0)=\varphi'(0)=\varphi''(0)=\varphi'''(0)=0$*;* $\varphi(1)=\varphi'(1)=\varphi''(1)=0$*;*

3. $\partial^4 f(x,t)/\partial x^4$ *va* $\partial^4\varphi(x)/\partial x^4$ *funksiyalar chegaralangan bo'lsin,* $\forall t\in(-\alpha,\beta),\ x\in(0,1)$*;*

4. $f(x,t)$ *funksiyasi* t *bo'yicha uzluksiz va uzluksiz diferensiallanuvchi bo'lsin.*

U holda, ***7-masala****ning yechimi mavjudligi uchun quyidagi ortogonallik shartlarining bajarilishi zarur va yetarli*

$$(\varphi(x),J_0(\lambda_k x)))=0,\quad (f(x),J_0(\lambda_k x)))=0,\quad k\in\mathbb{K}_0.$$

2-ta'rif. P_k deb birinchi k ta xos funksiyalarning chiziqli qobig'ining ortogonal qo'shimchasiga ortogonal proyektorni belgilaymiz, ya'ni

$$P_k=I-\sum_{j=1}^{k}(\cdot,J_0(\lambda_j x))J_0(\lambda_j x),$$

bu yerda I - $L_2(\Omega)$ fazodagi birlik operator.

Yuqoridagi ta'rifga ko'ra, operator P_k ni funksiya $f(x)$ ga ta'siri quyidagicha yoziladi:

$$P_k f(x)=\sum_{j=1}^{\infty}(f,J_0(\lambda_j x))J_0(\lambda_j x)-\sum_{j=1}^{k}(f,J_0(\lambda_j x))J_0(\lambda_j x)=$$

$$=\sum_{j=k+1}^{\infty}(f,J_0(\lambda_j x))J_0(\lambda_j x).$$

Endi faraz qilaylikki, $u(x,t)$ funksiyasi bilan birgalikda kasr tartibli hosila tartibi ρ ham noma'lum bo'lsin. Biz ushbu noma'lum parametr ρ ni aniqlash masalasini, ya'ni teskari masalani ham ko'rib chiqamiz.

8-masala. Shunday $\{u(x,t),\rho\}$ funksiyalar juftligini topingki, bunda $\rho\in[\rho_0,1],\ 0<\rho_0<1$ va $u(x,t)$ quyidagi qo'shimcha shartni qanoatlantirsin:

$$U(t_0,\rho)=\|P_{k_0}(u(t_0)-A^{-2}f)\|^2=u_0,\quad 0<t_0<\infty, \tag{21}$$

bu yerda u_0 berilgan son.

Qo'shimcha shart (21) da aniqlangan $U(t_0,\rho)$ funksiyasi quyidagi ko'rinishga ega:

$$U(t_0,\rho)=\sum_{k=k_0+1}^{\infty}\left|E_{\rho,1}(-\lambda_k t_0^{\rho})\right|^2\left|\frac{\varphi_k\lambda_k^2}{\Delta_\alpha(k)}-\frac{f_k}{\lambda_k^2\Delta_\alpha(k)}\right|^2,\quad 0<t_0<\infty.$$

11-teorema. *Aytaylik, α musbat ratsional son bo'lib, $\varphi(x)$ va $f(x,t)$ funksiyalar uchun 10-teoremaning shartlari bajarilsin. U holda shunday $T_0=T_0(\rho_0)>0$ soni mavjud bo'lib, barcha $t_0\geq T_0$ larda teskari masalaning yagona yechimi $\{u(t),\rho\}$, $\rho\in[\rho_0,1]$ mavjud bo'lishi uchun quyidagi shartning bajarilishi zarur va yetarli*

$$U(t_0,1)\leq u_0\leq U(t_0,\rho_0).$$

XULOSA

Ushbu dissertatsiya ishi Kaputo kasr hosilali aralash turdagi tenglamalar uchun to'g'ri va teskari masalalarni yechishga bag'ishlangan bo'lib, kasr tartibli differensial tenglamalar sohasiga yangi hissa qo'shadi.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. Vaqt bo'yicha $t>0$ sohada Kaputo kasr hosilasini o'z ichiga olgan aralash turdagi xususiy hosilali subdiffuziya tenglamasi, hamda $t<0$ sohada klassik parabolik tenglama qatnashgan Dezin nolokal shartli to'g'ri va teskari masalalar o'rganilgan. To'g'ri masala uchun boshlang'ich ma'lumotlar va o'ng tomondagi funksiyaga qo'yilgan mos shartlar asosida Furye usuli yordamida yechimning mavjudligi va yagonaligi isbotlandi. Shuningdek, yechimning mavjudligi parametr λ (Dezin ma'nosidagi shartdan kelib chiqadigan parametr) ga qanday bog'liq ekani ham tahlil qilingan.

Teskari masala uchun esa, o'ng tomoni ajraladigan ko'rinishda $F(x,t)=f(x)g(t)$ deb olinib, bunda noma'lum funksiya $f(x)$ hisoblanadi. Ushbu holatda $g(t)$ funksiyaga qo'yilgan ayrim shartlar (masalan, doimiy ishoraga ega bo'lishi) bajarilganda yechimning mavjudligi va yagonaligi isbotlangan.

2. Kaputo kasr tartibli bir jinsli aralash turdagi tenglamalar uchun nolokal shartli masala o'rganildi. Bunda tenglamaning elliptik qismi oddiy ikkinchi tartibli hosiladan iborat. Furye usulidan foydalanib, nolokal masalaning klassik yechimlari mavjudligi, yagonaligi va turg'unligi isbotlandi.

3. Kaputo kasr hosilali aralash turdagi, elliptik qismi ikkinchi tartibli hosiladan iborat tenglamalarning o'ng tomoni tarkibidagi fazoga bog'liq qismni aniqlashga oid teskari masalalar o'rganildi. Bu masalalarda tenglamaning o'ng tomoni $f(x)\phi(t)$, ko'rinishda bo'lib, bu yerda $\phi(t)$ noma'lum, $f(x)$ esa berilgan funksiya. Bunday teskari masalalar tegishli integral tenglamani yechishga keltirilgan.

4. Kaputo kasr tartibli aralash turdagi tenglama uchun uchta ulash shartli masala tahlil qilingan, bunda elliptik qism Laplas operatoridan iborat. Furye usulidan foydalanib, boshlang'ich-chegaraviy masalaning klassik yechimlari mavjudligi va yagonaligi isbotlangan.

5. Kaputo kasr tartibli aralash turdagi tenglama elliptik qismi Bessel operatori bo'lgan holda o'rganilgan. Shuningdek, teskari masala sifatida yangicha qo'shimcha shart ostida kasr tartibi ρ ni aniqlash masalasi qaralgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.05/2025.27.12.FM.11.01.M AT V.I. ROMANOVSKIY INSTITUTE OF MATHEMATICS

INSTITUTE OF MATHEMATICS

DUSANOVA UMIDA KHOMID QIZI

FORWARD AND INVERSE PROBLEMS FOR MIXED-TYPE EQUATIONS WITH FRACTIONAL DERIVATIVES IN THE SENSE OF CAPUTO

01.01.02 – Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

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INTRODUCTION

Actuality and demand of the theme of the dissertation. Numerous scientific and applied studies conducted worldwide have demonstrated the significance of investigating boundary value and inverse problems for higher-order partial differential equations. In recent years, considerable attention has been paid in our country to mathematics, physics, geology, and biology that is, to fundamental sciences that possess both theoretical depth and practical applicability. In particular, the development of differential equations and mathematical physics as key branches of mathematics with broad practical relevance has been given special emphasis.

In recent decades, the theory of differential equations involving fractional derivatives has gained considerable popularity. This is primarily due to its wide application in various, even vastly different, fields of science and technology. The main objective of this research is to solve boundary value and inverse problems for equations of parabolic and hyperbolic type in an N dimensional domain, where the elliptic part is represented by an arbitrary differential operator.

In addition to investigating the solvability of higher-order partial differential equations, it is of great importance to study inverse problems related to identifying the coefficients, the right-hand side, or the order of the fractional derivative appearing in such equations. Research on inverse problems provides valuable tools for analyzing, understanding, and controlling processes described by higher-order equations. For instance, in the study of heat diffusion processes, knowing the boundary function allows us to control the distribution of heat. Many similar examples can be given. Consider a time-fractional differential equation: if the fractional order lies between 1 and 2, the equation simultaneously characterizes diffusion and wave propagation; if the fractional order lies between 0 and 1, it describes a slow diffusion process. Likewise, in virology, fractional-order equations can naturally represent the dynamics of virus replication. Taking all these aspects into account, it can be concluded that the study of fractional-order differential equations constitutes one of the most relevant and challenging problems in modern mathematics.

In Uzbekistan, substantial attention is being forwarded toward scientific and applied research in fundamental disciplines such as mathematics, physics, and optimal control. In particular, special focus is placed on the study of fractional and arbitrary-order partial differential equations, as this theory provides a powerful framework for understanding a wide variety of phenomena in mechanics, electronics, control theory, physiology, and biology. As noted above, conducting scientific research in these key areas of mathematics at a level consistent with international standards has been defined as a major objective and priority forwardion.

This dissertation contributes, to some extent, to the implementation of the tasks and objectives set forth in the decrees of the President of the Republic of Uzbekistan. The subject and object of research of this thesis are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On the strategy of action for the further development Of the Republic of Uzbekistan”, PP-2789 dated February 17, 2017 “On measures to further improve of the activities of the Academy of Sciences, organization, management and financing of research activities", PP-3682 from April 27, 2018 "On measures to further improve the system of practical implementation of innovative ideas, technologies and projects" and PP-4387 from July 9, 2019 "On measures to further development of mathematical education and science, total improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovskiy Institute of Mathematics" andalso PP-4708 from May 7, 2020 "On measures to improve the quality of education and research in mathematics" as well as in other regulations related to basic science.

Connection of research to priority forwardions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, «Mathematics, Mechanics and Computer Science».

The degree of scrutiny of the problem. Fractional order partial differential equations and the related inverse problems have become one of the most actively studied areas in modern mathematics. In this field, significant scientific contributions have been made by scholars such as Sh.O. Alimov, R.R. Ashurov, S.R. Umarov, M. Yamamoto, Z. Li, A.V. Pskhu, M. Kirane, M. Ruzhansky, D.K. Durdiev, Yu.E. Fayziyev, Z.A. Sobirov, E.T. Karimov, B.Kh. Turmetov, Y. Zhang, H.T. Nguyen, A.S. Malik, and many other researchers.

At present, the theory of boundary value problems for mixed-type equations is considered one of the developing branches of the theory of partial differential equations. Numerous theoretical studies and monographs have been published in this area. The classical monographs by A.V. Bitsadze, M.M. Smirnov, and V.N. Vragov can be regarded as fundamental works devoted to mixed-type equations. Many researchers have also focused on inverse problems related to determining the right-hand side of mixed-type equations of various fractional orders, since such problems are of great practical importance. However, a general theory for the most general form of the source function $F(x,t)$ has not yet been developed. In several studies, a separable source function of the form $F(x,t) \equiv f(x)g(t)$ has been considered, and the research methods have been chosen depending on which part of the function is unknown.

The inverse problem of determining the spatial part $f(x)$ of the source function has been investigated in two different cases: $g(t) \equiv 1$ and $g(t) \not\equiv 1$. For mixed-type equations, the case $g(t) \equiv 1$ with unknown $f(x)$, has been studied in the works of R.R. Ashurov, E.T. Karimov, D.K. Durdiev, T.K. Yuldashev, B.Kh. Kadirkulov, B.I. Islomov, and M.B. Murzambetova. The case $g(t) \not\equiv 1$ is more complicated, and the solvability of such problems essentially depends on the behavior of the function $g(t)$.

Inverse problems concerning the determination of the time-dependent part $g(t)$ of the source function are relatively more complex and have been studied by a smaller number of researchers. For mixed-type equations, results in this forwardion have been obtained by R.R. Ashurov, M.D. Shakarova, S. Umarov, K.B. Sabitov, and S.N. Sidorov.

When fractional-order equations are considered as models for describing various processes, the order of the fractional derivative is often unknown and difficult to measure forwardly. Inverse problems aimed at determining this unknown parameter are of both theoretical and practical significance, as they are essential for solving initial–boundary value problems and for studying the qualitative properties of solutions. Therefore, in recent years, researchers have been actively investigating inverse problems concerned with identifying the order of the fractional derivative. An analysis of the works published in this field shows that almost all existing studies deal either with the subdiffusion equation or with the fractional wave equation. To the best of our knowledge, inverse problems concerning the determination of the fractional order for mixed-type equations have so far been studied only by R.R. Ashurov and R.T. Zunnunov, and M.B. Murzambetova. It should be noted that in the work of R.R. Ashurov and R.T. Zunnunov, the orders of the derivatives in the upper and lower parts of the domain are assumed to be unknown.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation research is done in accordance with the planned theme of scientific research grant no. F-FA-2021-424 Ministry of higher education, science and innovations of the Republic of Uzbekistan, at the Institute of Mathematics after named V.I. Romanovskiy.

The aim of research work is to prove the existence and uniqueness, in the classical sense, of solutions to forward and inverse problems for fractional-order differential equations of parabolic, hyperbolic, and subdiffusion types.

Research problems:

To prove, in the classical sense, the existence and uniqueness of solutions to forward and inverse nonlocal problems in the sense of Dezin for an equation with the Laplace operator, where the subdiffusion equation is considered in the upper part of the domain and the diffusion equation in the lower part of the domain of arbitrary dimension;

To prove, in the classical sense, the existence and uniqueness of solutions to forward and inverse problems for a mixed-type equation with the Caputo fractional derivative in a rectangular domain;

To prove the uniqueness of the solution to the boundary value problem for a mixed-type equation involving Caputo fractional derivatives and Laplace operators in a domain of arbitrary dimension;

To prove, in the classical sense, the existence and uniqueness of solutions to forward and inverse problems of determining the order of the fractional derivative in the sense of Caputo, where the elliptic operator is formally self-adjoint and represented by the Bessel operator defined in a domain of arbitrary dimension.

The research object. Mixed-type equations involving Caputo fractional derivatives.

The research subject. Boundary value problems and inverse problems for mixed-type differential equations related to determining the right-hand side or the order of the derivative.

Research methods. In the research the methods of functional analysis, spectral theory and the Fourier methods are used.

Scientific novelty of the research work consists of the following:

It has been proved that the forward and inverse problems with nonlocal conditions in the sense of Dezin for the equation with the Laplace operator defined in an arbitrary multidimensional domain where the subdiffusion equation is considered in the upper part of the domain and the diffusion equation in the lower part possess a unique classical solution.

The uniqueness of the solution to the forward and inverse problems for mixed-type equations with the Caputo fractional derivative has been established in a rectangular domain.

The existence and uniqueness of the solution to the boundary value problem for a mixed-type equation with the Caputo fractional derivative involving a self-adjoint positive Laplace operator defined in an arbitrary multidimensional domain have been proved.

The uniqueness of the solution to the forward and inverse problems of determining the fractional order for a mixed-type equation with the Caputo fractional derivative involving a positive self-adjoint Bessel operator has been established.

Practical results of the research. The results and methodologies presented in this dissertation can be incorporated into graduate-level courses designed for master's and doctoral students at higher education institutions.

The reliability of the results of the study. The results were derived using the techniques from functional analysis, spectral theory, and the Fourier method. All obtained results are mathematically correct.

Scientific and practical significance of the research results. The scientific significance of the research lies in the fact that the obtained results can be used for a deeper study of forward and inverse problems in the theory of higher-order partial differential equations.

The practical significance of the dissertation is that the obtained results can be applied to the mathematical modeling of technical, physical, and biological processes.

Implementation of the research results. The results obtained for forward and inverse problems of higher-order partial differential equations have been implemented in practice within the following projects:

the solutions obtained for the forward and inverse problems with Dezin-type nonlocal condition for mixed-type equations involving the Caputo fractional derivative were used in the framework of the international grant project No. 22-11-00064 titled “Modeling of dynamic processes in the geosphere taking into account heredity” to classify time-varying physical systems (Institute of cosmophysical research and radio wave propagation, Far Eastern branch, Certificate No. 378 dated September 25, 2025, Russian Federation). This scientific result made it possible to determine the source function of radon emission in the subdiffusion and diffusion equations, which is of significant importance for applications in geophysics and seismology.

the scientific solutions obtained in the study of forward and inverse problems for mixed-type equations involving fractional derivatives were also practically applied in the state-funded research projects conducted at the Institute of Seismically Resistant Construction and Mechanics named after M. T. O‘rozboyev of the Academy of Sciences of the Republic of Uzbekistan, within the research area “Development of models for the deformation of isotropic and anisotropic bodies under static and dynamic loads, as well as nonlinear elastic and plastic media.” (Certificate No. 1348-3 dated October 30, 2025, Institute of Seismic Stability of Structures, Academy of Sciences of the Republic of Uzbekistan).

These results have proven effective in developing nonlinear models of elastic and plastic media and in analyzing the deformation of isotropic and anisotropic bodies under various loading conditions.

Approbation of the research results. The main results of the research have been discussed at 7 international and 2 national scientific conferences.

Publications of the research results. On the topic of the dissertation 15 research papers have been published in the scientific journals, 6 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis and 2 and 4 of them were published in international and national mathematical journals, respectively. 3 of them are included the SCOPUS information database, and 9 theses.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The total volume of the dissertation is 120 pages.

MAIN CONTENT OF THE DISSERTATION

The introduction substantiates the actuality and demand of the theme of the dissertation, determines the correspondence of the study to priority areas of development of science and technology, provides an overview of foreign scientific research on the dissertation topic and the degree of study of the problem, formulates goals and objectives, identifies the object and subject of the study, outlines the scientific novelty and practical results of the study, the theoretical and practical significance of the results obtained is disclosed, information is given on the implementation of the research results, on published works and information on the structure of the dissertation.

The first chapter of the dissertation, entitled **"Forward and inverse problems for a mixed-type equation with the Caputo fractional derivative and Dezin-type non-local condition"** consists of two sections. The first section, called **"Basic definitions and concepts"** is auxiliary in nature and it was created for the convenience of reading the dissertation. There are no new results here, and only the necessary definitions and assertions are collected.

When solving the considered problems, we use the Fourier method, which leads us to the following spectral problem:

$$\begin{cases} -\Delta v(x) = \lambda v(x), \quad x \in \Omega, \\ v(x)|_{\partial\Omega} = 0. \end{cases}$$

Since the boundary $\partial\Omega$ is sufficiently smooth, the spectral problem in $L_2(\Omega)$ has a complete orthonormal set of eigenfunctions $\{v_k(x)\}$, $k \geq 1$ and the corresponding eigenvalues λ_k form a nondecreasing sequence of positive numbers $0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \ldots \to \infty.$

Let σ be an arbitrary real number. We introduce the power of operator A, acting in $L_2(\Omega)$ according to the rule

$$\hat{A}^{\sigma} g(x) = \sum_{k=1}^{\infty} \lambda_k^{\sigma} g_k v_k(x),$$

where $g_k = (g, v_k)$ is the Fourier coefficient of the function $g \in L_2(\Omega)$. Obviously, the domain of definition of this operator has the form

$$D(\hat{A}^{\sigma}) = \{g \in L_2(\Omega) : \sum_{k=1}^{\infty} \lambda_k^{2\sigma} | g_k |^2 < \infty\}.$$

The norm for the elements of the set $D(\hat{A}^{\sigma})$ is calculated as follows:

$$\| g \|_{\sigma}^2 = \sum_{k=1}^{\infty} \lambda_k^{2\sigma} | g_k |^2 = \| A^{\sigma} g \|^2 .$$

Self-adjoint extension of the Laplace operator. If A stands for the operator acting in $L_2(\Omega)$ as $Ag(x) = -\Delta g(x)$ with the domain of definition $D(A) = \{g \in C^2(\Omega) : g(x) = 0,\ x \in \partial\Omega\}$, then $\hat{A} \equiv \hat{A}^1$ is a self-adjoint extension of A in $L_2(\Omega)$.

Theorem of V.A. Il'in. In order to prove the existence of solutions of initial-boundary value problems by the Fourier method when the elliptic part of the equation is Laplace operator Δ, it is necessary to study the convergence of the following series:

$$\sum_{k=1}^{\infty} \lambda_k^{\tau} | h_k |^2, \quad \tau > \frac{N}{2}, \tag{1}$$

where h_k are the Fourier coefficients of function $h(x)$. In the case of integers τ, in the fundamental paper by V.A. Il'in, conditions are obtained for the convergence of such series in terms of the membership of function $h(x)$ in the classical Sobolev spaces $W_2^k(\Omega)$, $\Omega \subset R^n$. To formulate these conditions, we introduce the class $\overset{\circ}{W}{}_2^1(\Omega)$ as the closure in the $W_2^1(\Omega)$ norm of the set of all functions that are continuously differentiable in Ω and vanish near the boundary of Ω.

The theorem of V.A. Il'in states that, if function $h(x)$ satisfies the conditions

$$h(x) \in W_2^{\left[\frac{N}{2}\right]+1}(\Omega) \text{ and } h(x),\ \Delta h(x), \ldots ,\ \Delta^{\left[\frac{N}{4}\right]} h(x) \in \overset{\circ}{W}{}_2^1(\Omega), \tag{2}$$

then the number series (1) converges. Here $[a]$ denotes the integer part of the number a. Similarly, if in (1) we replace τ by $\tau + 2$, then the convergence conditions will have the form:

$$h(x) \in W_2^{\left[\frac{N}{2}\right]+2}(\Omega) \text{ and } h(x),\ \Delta h(x), \ldots ,\ \Delta^{\left[\frac{N}{4}\right]} h(x) \in \overset{\circ}{W}{}_2^1(\Omega). \tag{3}$$

Below, we present the definition of the Mittag-Leffler function given in the work of M. M. Dzherbashyan, as well as one of its main properties that will be needed later.

Definition 1. *The function defined by the following equation*

$$E_{\rho,\mu}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\rho k + \mu)}, \quad z, \mu \in \mathbb{C}, \quad \rho > 0.$$

is called Mittag-Leffler function with two-parameters.

Lemma 1. *If* $0 < \rho < 1$ *then for any* $\mu, z \in \mathbb{C}$, *we have*

$$|E_{\rho,\mu}(-z)|\le \frac{C_0}{1+|z|},$$

where C_0 is a constant independent of z and μ.

The second paragraph of this chapter studies forward and inverse problems associated with a mixed-type partial differential equation involving the Caputo fractional derivative with respect to time in the domain $t>0$ and the diffusion equation in the domain $t<0$, whose elliptic part is the Laplace operator, in an arbitrary N dimensional domain.

Let Ω be an arbitrary N dimensional domain with a sufficiently smooth boundary $\partial\Omega$.

Consider the following mixed type equation:

$$\begin{cases} D_t^{\rho}u-\Delta u=F(x,t), & x\in\Omega, \quad 0<t\le\beta, \\ u_t+\Delta u=F(x,t), & x\in\Omega, \quad -\alpha<t<0, \end{cases} \tag{4}$$

where $F(x,t)$ is a continuous function and $\alpha>0$, $\beta>0$ are given real numbers and Δ is the Laplace operator.

Problem 1. Find a function $u(x,t)$ satisfying equation (4) and the boundary condition

$$u(x,t)|_{\partial\Omega}=0, \quad t\in[-\alpha,\beta],$$

and gluing condition

$$\lim_{t\to+0}u(x,t)=\lim_{t\to-0}u(x,t), \quad x\in\Omega,$$

and also a non-local condition

$$u(x,-\alpha)=\lambda u(x,0), \quad x\in\Omega, \tag{5}$$

where $\lambda=const$, $\lambda\ne 0$.

This problem is called the Dezin problem due to condition (5). Note, that if $\lambda=0$ then we arrive at the backward problem for subdiffusion equation.

Formal solution to Problem 1 which is represented in the form

$$u(x,t)=\begin{cases} \sum_{k=1}^{\infty}\left(\frac{I_\alpha(\lambda_k)}{\delta_k}E_{\rho,1}(-\lambda_k t^{\rho})+I_\rho(\lambda_k)\right)v_k(x), & 0\le t\le\beta, \\ \sum_{k=1}^{\infty}\left(\frac{I_\alpha(\lambda_k)}{\delta_k}e^{\lambda_k t}-\int_t^0 F_k(s)e^{\lambda_k(t-s)}ds\right)v_k(x), & -\alpha\le t\le 0. \end{cases} \tag{6}$$

where

$$I_\alpha(\lambda_k)=\int_{-\alpha}^{0}F_k(s)e^{\lambda_k(-\alpha-s)}ds, \quad I_\rho(\lambda_k)=\int_0^t s^{\rho-1}E_{\rho,\rho}\left(-\lambda_k s^{\rho}\right)F_k(t-s)ds,$$

$$\delta_k=e^{-\lambda_k\alpha}-\lambda, \quad k\ge 1.$$

Theorem 1. *Let $\lambda\notin[0,1)$. Let the function $F(x,t)$ be continuous for all $t\in[-\alpha,\beta]$ and satisfy condition (3) uniformly with respect to t. Then there exists a unique solution of the forward* ***problem 1****, which is determined by the series (6).*

Theorem 2. *Let $0<\lambda<1$ and let the function $F(x,t)$ be continuous for all*

$t\in[-\alpha,\beta]$ and satisfy condition and satisfy condition (3) uniformly with respect to t.

1) If $\lambda_k \neq \lambda_0$, for all $k\geq 1$, then there exists a unique solution of the ***problem 1*** *and it can be represented as the form (6).*

2) If $\lambda_k = \lambda_0$, for some k and the orthogonality conditions

$$F_k^* = (F^*, v_k) = 0,\ k\in K_0,\ K_0=\{k_0, k_0+1,\ldots,k_0+p_0-1\},\ F_k^* = \int_{-\alpha}^{0} F_k(s)e^{\lambda_k(-\alpha-s)}ds.$$

holds for indices $k\in K_0$, then the ***problem 1*** *has a solution, which is given by*

$$u(x,t)=\begin{cases}\sum\limits_{k\notin K_0}\left(\dfrac{I_\alpha(\lambda_k)}{\delta_k}E_{\rho,1}(-\lambda_k t^\rho)+I_\rho(\lambda_k)\right)v_k(x)+\sum\limits_{k\in K_0}a_k E_{\rho,1}(-\lambda_k t^\rho)v_k(x),\ t>0,\\ \sum\limits_{k\notin K_0}\left(\dfrac{F_k^*}{\delta_k}e^{\lambda_k t}-\int_t^0 F_k(s)e^{\lambda_k(t-s)}ds\right)v_k(x)+\sum\limits_{k\in K_0}a_k e^{\lambda_k t}v_k(x),\quad t<0,\end{cases}$$

with arbitrary coefficients a_k.

Problem 2. Let $F(x,t)=f(x)g(t)$ be a given function. Let $u(x,t)$ be the solution of the problem 1 satisfying the additional condition

$$u(x,t_0)=\varphi_0(x),\quad x\in\Omega, \tag{7}$$

Find the pair of functions $u(x,t)\in AC([0,\beta];C(\overline{\Omega}))\cup C(\overline{\Omega}\times[-\alpha,0])$ and $f(x)\in C(\overline{\Omega})$ that satisfy these conditions, where $\varphi_0(x)$ is a sufficiently smooth given function, and $t_0\in(0,\beta)$ is a given point.

To determine the coefficients f_k, we have the following equality:

$$f_k\Delta_k(t_0)=\delta_k\varphi_{0k}=(e^{-\lambda_k\alpha}-\lambda)\varphi_{0k},$$

where

$$\Delta_k(t_0)=E_{\rho,1}(-\lambda_k t_0^\rho)\int_{-\alpha}^{0} g(s)e^{\lambda_k(-\alpha-s)}ds+$$

$$+(e^{-\lambda_k\alpha}-\lambda)\int_0^{t_0} s^{\rho-1}E_{\rho,\rho}(-\lambda_k s^\rho)g(t_0-s)ds. \tag{8}$$

We now provide a lower estimate for $\Delta_k(t_0)$. To do this, it is required that the function $g(t)$ does not vanish. Since $g(t)\neq 0$, and it is continuous, the analysis remains the same whether $g(t)>0$ or $g(t)<0$. Therefore, in the four lemmas to be proved below, we will assume without loss of generality that $g(t)>0$.

Let $g\in C[-\alpha,\beta]$ and $g(t)\neq 0$, we define

$$m=\min_{t\in[-\alpha,t_0]}|g(t)|>0,\quad M=\max_{t\in[-\alpha,t_0]}|g(t)|>0.$$

Lemma 2. *Let $\lambda<0$, $g(t)\in C[-\alpha,\beta]$, and $g(t)\neq 0$, $t\in[-\alpha,\beta]$. Then, there is a constant $C>0$, depending on t_0 and α, such that for all k:*

$$\Delta_k(t_0) \ge \frac{C}{\lambda_k}.$$

Lemma 3. *Let , $\lambda \ge 1$, $g(t) \in C[-\alpha,\beta]$, and $g(t) \ne 0$, $t \in [-\alpha,\beta]$. If number t_0 satisfies following condition*

$$t_0^\rho > \frac{C_0}{\lambda_1}\left(1+\frac{M}{m}\right),$$

where C_0 is the number in the Lemma 1 then, there is constant $C > 0$ depending on t_0, ρ and α, such that for all k:

$$|\Delta_k(t_0)| \ge \frac{C}{\lambda_k}. \tag{9}$$

If the number t_0 satisfies condition

$$t_0^\rho \le \frac{C_0}{\lambda_1}\left(1+\frac{M}{m}\right),$$

then there exist a number $k_1 \in \mathbb{N}$ such that the estimate (9) holds for all $k > k_1$.

Lemma 4. *Let $0 < \lambda < 1$, $g(t) \in C[-\alpha,\beta]$ and $g(t) \ne 0$, $t \in [-\alpha,\beta]$. Then there exist a number $k_2 \in \mathbb{N}$ such that the for all $k > k_2$ the following estimate*

$$|\Delta_{k,\rho}(t_0)| \ge \frac{C}{\lambda_k},$$

is valid, where constant $C > 0$ depend on ρ, t_0 and α.

Hence, we introduce the set

$$\mathbb{K}_0 = \{k \in \mathbb{N} : \Delta_k(t_0) = 0\}.$$

Lemma 5. *The set $\mathbb{K}_0$ is either empty or contains only finitely many elements.*

Theorem 3. Let $g(t) \in C[-\alpha,\beta]$, $g(t) \ne 0$, $t \in [-\alpha,\beta]$. Let $\lambda < 0$, and function $\varphi_0(x)$ satisfy the conditions (2). Then there exists a unique solution of the inverse problem 2 and it can be represented as:

$$u(x,t) = \sum_{k=1}^{\infty}\left(\frac{\varphi_{0k}}{\Delta_k(t_0)} E_{\rho,1}(\lambda_k t^\rho)\int_{-\alpha}^{0} g(s)e^{\lambda_k(-\alpha-s)}ds\right)v_k(x) +$$

$$+\sum_{k=1}^{\infty}\left(\frac{\varphi_{0k}(e^{-\lambda_k\alpha}-\lambda)}{\Delta_k(t_0)}\int_0^t s^{\rho-1}E_{\rho,\rho}\left(-\lambda_k s^\rho\right)g(t-s)ds\right)v_k(x), \quad t \ge 0,$$

$$u(x,t) = \sum_{k=1}^{\infty}\left(\frac{\varphi_{0k}}{\Delta_k(t_0)} e^{\lambda_k t}\int_{-\alpha}^{0} g(s)e^{\lambda_k(-\alpha-s)}ds - \frac{\varphi_{0k}(e^{-\lambda_k\alpha}-\lambda)}{\Delta_k(t_0)}\int_t^0 g(s)e^{\lambda_k(t-s)}ds\right)v_k(x), \quad t \le 0.$$

$$f(x) = \sum_{k=1}^{\infty}\frac{\varphi_{0k}(e^{-\lambda_k\alpha}-\lambda)}{\Delta_k(t_0)}v_k(x).$$

Theorem 4. Let $\varphi_0(x)$ satisfy the conditions (2) and $g(t) \in C[-\alpha,\beta]$, $g(t) \ne 0$,

$t \in [-\alpha, \beta]$ and let $\delta_k \neq 0$ for all k. Moreover, let the assumptions of Lemma 3 or Lemma 4 hold.

1) If set $\mathbb{K}_0$ is empty, then there exists a unique solution of the inverse **problem 2** and it can be represented as the series in Theorem 3.

2) If set $\mathbb{K}_0$ is not empty, then for the existence of a solution to the inverse **problem 2**, it is necessary and sufficient that the following conditions

$$\varphi_{0k} = (\varphi_0, v_k) = 0, \; k \in \mathbb{K}_0,$$

be satisfied.

The second chapter of the dissertation is titled **"Forward and inverse problems for mixed-type equations with second-order derivatives with respect to the elliptic part in a rectangular domain."**

In the first section of this chapter the stability, existence, and uniqueness of the solution to a nonlocal problem for a homogeneous parabolic-hyperbolic type equation with the Caputo operator in a rectangular domain are studied. The nonlocal condition is given as follows:

$$au(x,-\alpha) - bu(x,\beta) = \varphi(x), \quad 0 \le x \le 1, \tag{10}$$

where a and b are arbitrary constants, and $\varphi(x)$ is a sufficiently smooth function satisfying the condition $\varphi(0) = \varphi(1) = 0$.

In the second section of this chapter, the inhomogeneous case of the equation considered in the first section is studied, with the main focus on the investigation of the inverse problem. However, the nonlocal condition (10) is considered in a simplified form (that is, $a = 1, \; b = 0$).

Problem 3. Let $0 < \rho \le 1$. In the domain $D = \{(x,t) | \; 0 < x < 1, \; -\alpha < t < \beta\}$ find the function $u(x,t) \in C(\overline{D}_-)$, $u(x,t) \in AC([0,\beta]; C[0,1])$ satisfying following equation

$$Lu = F(x,t),$$

here

$$Lu = \begin{cases} D_{0t}^{\rho} u - u_{xx}, & 0 < t < \beta, \\ u_{tt} - u_{xx}, & -\alpha < t < 0, \end{cases} \qquad F(x,t) = \begin{cases} f_1(x,t), & 0 < t < \beta, \\ f_2(x,t), & -\alpha < t < 0, \end{cases}$$

and also boundary condition

$$u(0,t) = u(1,t) = 0, \quad -\alpha \le t \le \beta,$$

and initial condition

$$u(x,-\alpha) = \varphi(x), \quad 0 < x < 1,$$

also gluing conditions

$$u(x,+0) = u(x,-0), \quad \lim_{t \to +0} D_{0t}^{\rho} u(x,t) = u_t(x,-0), \quad 0 < x < 1,$$

where $\varphi(x)$ and $F(x,t)$ are given sufficiently smooth functions, $\alpha > 0$, $\beta > 0$ are given real numbers.

To construct the solution of the problem 3, we use the Fourier method.

We denote the expression that appears in the denominator of the solution as follows:

$$\Delta_\alpha(k) = \cos(\sqrt{\lambda_k}\alpha) + \sqrt{\lambda_k}\sin(\sqrt{\lambda_k}\alpha) = \sqrt{1+\lambda_k}\sin(\sqrt{\lambda_k}\alpha + \theta_k) \neq 0,$$

where in $k \to \infty$ da $\theta_k = \arcsin\left(\frac{1}{\sqrt{1+\lambda_k}}\right) \to 0$.

Let $\mathbb{N} = K \cup K_0$. Here $\mathbb{N}$ is the set of all natural numbers and K and K_0 are sets such that

1) If $\Delta_\alpha(k) \neq 0$, then $k \in K$,

2) Otherwise, if $\Delta_\alpha(k) = 0$, then $k \in K_0$.

Theorem 5. *Let α be an arbitrary natural number or an irrational algebraic number of degree two, and let the following conditions be satisfied for the functions $f_1(x,t)$, $f_2(x,t)$ and $\varphi(x)$:*

1) $f_1(x,t) \in C([0,\beta], W_2^3(0,1))$, $f_1(0,t) = f_1(1,t) = 0$, $f_1''(0,t) = f_1''(1,t) = 0$,

2) $f_2(x,t) \in C([-\alpha,0], W_2^3(0,1))$, $f_2(0,t) = f_2(1,t) = 0$, $f_2''(0,t) = f_2''(1,t) = 0$,

3) $\varphi(x) \in W_2^4(0,1)$, $\varphi(0) = \varphi(1) = \varphi''(0) = \varphi''(1) = 0$.

*In this case, **problem 3** has a unique solution, which can be represented in the following series form:*

$$u(x,t) = \sum_{k=1}^{\infty} \frac{\omega_k E_{\rho,1}(-(\pi k)^2 t^\rho)}{\Delta_\alpha(k)} \sin \pi k x +$$

$$+\sum_{k=1}^{\infty}\left(\int_0^t \tau^{\rho-1} E_{\rho,\rho}(-\pi k \tau^\rho) f_{1k}(t-\tau)d\tau\right)\sin \pi k x, \quad t > 0, \tag{11}$$

$$u(x,t) = \sum_{k=1}^{\infty}\left(\frac{\omega_k}{\Delta_\alpha(k)}(\cos \pi k t - \pi k \sin \pi k t) + \frac{f_{1k}(0)}{\pi k}\sin \pi k t\right)\sin \pi k x$$

$$+\sum_{k=1}^{\infty}\left(\frac{1}{\pi k}\int_0^t f_{2k}(\tau)\sin(\pi k(t-\tau))d\tau\right)\sin \pi k x, \quad t < 0, \tag{12}$$

where

$$\omega_k = \varphi_k + \frac{f_{1k}(0)\sin(\pi k \alpha)}{\pi k} - \frac{1}{\pi k}\int_{-\alpha}^0 f_{2k}(\tau)\sin(\pi k(\alpha+\tau))d\tau.$$

Theorem 6. *Let the functions $\varphi(x)$ and $f_1(x,t)$, $f_2(x,t)$ satisfy conditions of Theorem 5. Further, let α is a positive rational number.*

*1) If set K_0 is empty, then there exists a unique solution of the **problem 3** and it can be represented as the series (11), (12).*

*2) If set K_0 is not empty, then for the existence of a solution to the **problem 3**, it is necessary and sufficient that the following conditions*

$$\varphi_k = \frac{1}{\pi k}\int_{-\alpha}^{0} f_{2k}(\tau)\sin \pi k(\alpha+\tau)d\tau - \frac{f_{1k}(0+0)\sin \pi k \alpha}{\pi k}, \quad k \in K_0.$$

be satisfied.

Problem 4. Let

$$F(x,t) = \begin{cases} f_1(x)\phi_1(t), & 0 < t < \beta, \\ f_2(x,t), & -\alpha < t < 0. \end{cases}$$

Let $u(x,t)$ be the solution of the problem 3 satisfying the following additional condition:

$$u(x_0,t) = h_1(t), \quad 0 \le t \le \beta, \tag{13}$$

Find the pair of functions $u(x,t)$ and $\phi_1(t) \in C[0,\beta]$ satisfying this condition, where $f_1(x),\ f_2(x,t), h_1(t)$ are given sufficiently smooth functions, and $x_0 \in (0,1)$ is a given point.

Applying condition (13) to the solution of the problem 3, we obtain the following:

$$f_1(x_0)\phi_1(t) - \int_0^t \phi_1(s)\bar{K}_1(s,t)ds = D_t^\rho \bar{h}_1(t), \tag{14}$$

If $f_1(x_0) \ne 0$, then equation (14) is a Volterra integral equation of the second kind with a continuous kernel $\bar{K}_1(s,t)$ and continuous right-hand side $D_t^\rho \bar{h}_1(t)$. Moreover, it is important that the expression in the denominator of the right-hand side of this equation does not vanish.

$$f_1(x_0) + D_t^\rho S_1(t)|_{t=0} = \sum_{k=1}^{\infty} f_{1k} v_k(x_0) - \sum_{k=1}^{\infty} \frac{\lambda_k f_{1k} \sin\sqrt{\lambda_k}\alpha}{\sqrt{\lambda_k}\Delta_\alpha(k)} v_k(x_0)$$

$$= \sum_{k=1}^{\infty} f_{1k}\left[1 - \frac{\sqrt{\lambda_k}\sin\sqrt{\lambda_k}\alpha}{\Delta_\alpha(k)}\right] v_k(x_0) = \sum_{k=1}^{\infty} \frac{f_{1k}\cos\sqrt{\lambda_k}\alpha}{\Delta_\alpha(k)} v_k(x_0) \ne 0. \tag{15}$$

It is obvious that condition (15) is satisfied exactly when, for example, $\alpha \in \mathbb{N}$. Therefore, knowing that the first term on the left is nonzero, it suffices to show that the second term is zero.

Theorem 7. *Let α be an arbitrary natural number or an irrational algebraic number of degree two, $D_t^\rho h_1(t) \in C[0,\beta]$ and let the following conditions be satisfied for the functions $f_1(x),\ f_2(x,t)$ and $\varphi(x)$:*

1) $f_1(x) \in W_2^3(0,1),\ f_1(0) = f_1(1) = 0,\ f_1''(0) = f_1''(1) = 0,$

2) $f_2(x,t) \in C([-\alpha,0], W_2^3(0,1)), \quad f_2(0,t) = f_2(1,t) = 0,\ f_{2xx}''(0,t) = f_{2xx}''(1,t) = 0,$

3) $\varphi(x) \in W_2^4(0,1),\ \varphi(0) = \varphi(1) = \varphi''(0) = \varphi''(1) = 0.$

If $f_1(x_0) \ne 0$ and condition (15) is satisfied, then the solution of ***problem 4*** *exists and is unique.*

Problem 5. Let

$$F(x,t) = \begin{cases} f_1(x,t), & 0 < t < \beta, \\ f_2(x)\phi_2(t), & -\alpha < t < 0. \end{cases}$$

Let $u(x,t)$ be the solution of the problem 3 satisfying the following additional condition:

$$u(x_0,t) = h_2(t), \quad -\alpha \le t \le 0. \tag{16}$$

Find the pair of functions $u(x,t)$ and $\phi_2(t) \in C[-\alpha,0]$ satisfying this condition, where $f_1(x,t),\ f_2(x), h_2(t)$ are given sufficiently smooth functions.

Here, by repeating the operations performed above and assuming that $f_2(x_0) \ne 0$, we obtain the following second-kind Fredholm integral equation:

$$\phi_2(t) - \lambda \int_{-\alpha}^{0} \phi_2(\tau)\Phi(\tau,t)dt = \mu(t), \qquad \lambda = \frac{1}{f_2(x_0)}. \tag{17}$$

The kernel $\Phi(\tau,t)$ of integral equation (17) is continuous in the domain $-\alpha \le \tau, t \le 0$. If $h_2(t) \in C^2[-\alpha,0]$, then the right-hand side $\mu(t)$ is also continuous on $[-\alpha,0]$. Therefore, equation (17) is a second-kind Fredholm integral equation with a continuous kernel and a continuous right-hand side.

Theorem 8. *Let α be an arbitrary natural number or an irrational algebraic number of degree two, $h_2(t) \in C^2[-\alpha,0]$, $f_2(x_0) \neq 0$ and let the following conditions be satisfied for the functions $f_1(x,t)$ $f_2(x)$ and $\varphi(x)$:*

1) $f_1(x,t) \in C([0,\beta], W_2^3(0,1))$, $f_1(0,t) = f_1(1,t) = 0$, $f_{1xx}''(0,t) = f_{1xx}''(1,t) = 0$,

2) $f_2(x) \in W_2^3(0,1)$, $f_2(0) = f_2(1) = 0$, $f_2''(0) = f_2''(1) = 0$,

3) $\varphi(x) \in W_2^4(0,1)$, $\varphi(0) = \varphi(1) = \varphi''(0) = \varphi''(1) = 0$.

If for the parameter λ the following conditions hold:

a) the number λ be not the characteristic number of the kernel $\Phi(\tau,t)$;

b) $|\lambda| < \dfrac{1}{M\alpha}$;

*then, there exists a unique solution to **problem 5**.*

Problem 6. Let

$$F(x,t) = \begin{cases} f_1(x)\phi_1(t), & 0 < t < \beta, \\ f_2(x)\phi_2(t), & -\alpha < t < 0. \end{cases}$$

Find the pair of functions $\{u(x,t), \phi_1(t), \phi_2(t)\}$ that constitute the solution of the problem 3 and satisfy the additional conditions (13) and (16), where $f_1(x), f_2(x)$ are given sufficiently smooth functions.

Taking into account conditions (13) and (16), and applying them to the solution of the forward problem 3, we obtain the following integral equations:

$$\phi_1(t) f_1(x_0) - \int_0^t \phi_1(s)\bar{K}_1(s,t)ds - \int_{-\alpha}^{0} \phi_2(s) D_t^\rho \bar{\bar{K}}_1(s,t)ds = D_t^\rho \bar{\bar{h}}_1(t), \tag{18}$$

$$\phi_2(t) f_2(x_0) - \int_t^0 \phi_2(\tau) K_{2tt}(\tau,t)d\tau - \int_{-\alpha}^{0} \phi_2(\tau) K_{3tt}(\tau,t)d\tau = \bar{h}_2''(t). \tag{19}$$

Taking into account the condition $f_1(x_0) \neq 0$ and substituting the obtained value of $\phi_1(0)$ into equation (19), we get a second-kind Fredholm integral equation with kernel $K_{2tt}(\tau,t)$, $K_{3tt}(\tau,t)$ that is continuous in the given domain and with continuous right-hand side $\bar{h}_2''(t)$. After finding the function $\phi_2(t)$, the function $\phi_1(t)$ is determined as well, since $f_1(x_0) \neq 0$ and condition (15) is satisfied. When this condition holds, equation (18) represents a second-kind Volterra integral equation with continuous kernels $\bar{K}_1(s,t)$, and $D_t^\rho \bar{\bar{K}}_1(s,t)$, and with continuous right-hand side $D_t^\rho \bar{\bar{h}}_1(t)$.

Theorem 9. *Let α be an arbitrary natural number or an irrational algebraic* *number of degree two,* $D_t^\rho h_1(t) \in C[0,\beta]$, $h_2(t) \in C^2[-\alpha,0]$, $f_1(x_0) \neq 0$, $f_2(x_0) \neq 0$ *and let the following conditions be satisfied for the functions* $f_1(x)$ $f_2(x)$ *and* $\varphi(x)$:

1) $f_1(x) \in W_2^3(0,1)$, $f_1(0) = f_1(1) = 0$, $f_1''(0) = f_1''(1) = 0$,

2) $f_2(x) \in W_2^3(0,1)$, $f_2(0) = f_2(1) = 0$, $f_2''(0) = f_2''(1) = 0$,

3) $\varphi(x) \in W_2^4(0,1)$, $\varphi(0) = \varphi(1) = \varphi''(0) = \varphi''(1) = 0$.

and one of assumptions $a)$ *or* $b)$ *of Theorem 8 hold. In that case, if* $\phi_1(0) = 0$ *or* $\phi_2(0) = 0$, *then there exists a unique solution to* ***problem 6****.*

The third chapter of the dissertation is titled **"Forward and inverse problems for mixed-type equations with the elliptic part being the Laplace and Bessel operators".**

In the first section of this chapter, the existence and uniqueness of the solution to a problem with three gluing conditions are investigated for a nonhomogeneous parabolic-hyperbolic type equation with a fractional Caputo derivative, whose elliptic part is represented by the Laplace operator in an arbitrary N-dimensional space. Moreover, in the process of studying this problem with three conjugation conditions, it is shown that if the right-hand side of the equation does not depend on t, then the solution of such a problem is also independent of t; that is, the dependence of the equation on t disappears, and it reduces to an ordinary elliptic equation.

In the second section of this chapter, the following forward and inverse problem of determining the order are studied:

Let $\rho \in (0,1)$. In the domain $Q := \{(x,t) \mid 0 < x < 1, -\alpha < t < \infty\}$ consider the following mixed type equation:

$$f(x,t) = \begin{cases} D_t^\rho u - B_0 u, & 0 < x < 1, \quad 0 < t < \infty, \\ u_{tt} - B_0 u, & 0 < x < 1, \quad -\alpha < t < 0, \end{cases} \tag{20}$$

where $f(x,t)$ is a given sufficiently smooth function and $\alpha > 0$ is given real number and

$$B_0 u = u_{xx}(x,t) + \frac{1}{x} u_x(x,t)$$

is the Bessel part of the equation.

Problem 7. Find a function $u(x,t)$ satisfying equation (20) and the boundary conditions

$$\lim_{x\to 0} x u_x(x,t) = 0, \quad u(1,t) = 0,$$

and gluing conditions

$$\lim_{t\to +0} u(x,t) = \lim_{t\to -0} u(x,t), \ \lim_{t\to +0} D_t^\rho u(x,t) = \lim_{t\to -0} u_t(x,t),$$

and also a initial condition

$$u(x,-\alpha) = \varphi(x), \quad 0 < x < 1,$$

where $\varphi(x)$ is a given function.

In constructing the solution of the problem, the Fourier method is applied, and the expression appearing in the denominator of the solution is defined as follows:

$$\Delta_\alpha(k)=\sqrt{1+\lambda_k^2}\sin\left(\lambda_k\alpha+\theta_k\right),\quad k\geq 1.$$

Let α be a positive rational number, and let $\Delta_\alpha=0$ for some values of $k=k_1,k_2,...,k_m$ $(1\leq k_1<k_2<\ldots<k_m\leq k_0)$.

We introduce the following set:

$$\mathbb{K}_0=\{k:\Delta_\alpha(k)=0,\ k\in\mathbb{N}\}.$$

Theorem 10. *Let the functions $\varphi(x)$ and $f(x,t)$ are differentiable four times with respect to x, and for derivatives with respect to x the following equalities be satisfied:*

1. $f(0,t)=f'(0,t)=f''(0,t)=f'''(0,t)=0$; $f(1,t)=f'(1,t)=f''(1,t)=0$, $\forall t\in(-\alpha,\beta)$;
2. $\varphi(0)=\varphi'(0)=\varphi''(0)=\varphi'''(0)=0$; $\varphi(1)=\varphi'(1)=\varphi''(1)=0$;
3. $\partial^4 f(x,t)/\partial x^4$ *and* $\partial^4\varphi(x)/\partial x^4$, $\forall t\in(-\alpha,\beta)$, $x\in(0,1)$ *are bounded;*
4. $f(x,t)$ *is continuous and continuously differentiable with respect to* t;

and let α be a positive rational number. Then for the existence of a solution to the inverse ***problem 7****, it is necessary and sufficient that the following conditions*

$$(\varphi(x),J_0(\lambda_k x)))=0,\quad (f(x),J_0(\lambda_k x)))=0,\quad k\in\mathbb{K}_0,$$

be satisfied.

Definition 2. By P_k we denote the orthogonal projector onto the orthogonal complement of the linear span of the first k eigenfunctions, that is,

$$P_k=I-\sum_{j=1}^{k}(\cdot,J_0(\lambda_j x))J_0(\lambda_j x),$$

where I is the identity operator in $L_2(\Omega)$.

According to the above definition, the action of the operator P_k on the function $f(x)$ is written as follows:

$$P_k f(x)=\sum_{j=1}^{\infty}(f,J_0(\lambda_j x))J_0(\lambda_j x)-\sum_{j=1}^{k}(f,J_0(\lambda_j x))J_0(\lambda_j x)=$$

$$=\sum_{j=k+1}^{\infty}(f,J_0(\lambda_j x))J_0(\lambda_j x).$$

Let us now assume that together with the function $u(x,t)$ the order of the fractional derivative ρ is also unknown. We will also study the inverse problem of determining this unknown parameter ρ.

Problem 8. Find a pair of fuctions $\{u(x,t),\rho\}$ such that $\rho\in[\rho_0,1]$, $0<\rho_0<1$ and satisfy the additional condition :

$$U(t_0,\rho)=\|P_{k_0}(u(t_0)-A^{-2}f)\|^2=u_0,\quad 0<t_0<\infty. \tag{21}$$

When solving the inverse problem, we will assume that $f(x,t)=f(x)$.

The function $U(t_0,\rho)$ defined in the additional condition (21) has the following form:

$$U(t_0,\rho)=\sum_{k=k_0+1}^{\infty}\left|E_{\rho,1}(-\lambda_k t_0^\rho)\right|^2\left|\frac{\varphi_k\lambda_k^2}{\Delta_\alpha(k)}-\frac{f_k}{\lambda_k^2\Delta_\alpha(k)}\right|^2,\quad 0<t_0<\infty.$$

Theorem 11. *Let* α *be a positive rational number, and let* $\varphi(x)$ *and* $f(x,t)$ *be functions for which the conditions of Theorem 10 are satisfied. Then there exists a number* $T_0 = T_0(\rho_0) > 0$ *such that for all* $t_0 \geq T_0$, *the inverse problem has a unique solution* $\{u(t), \rho\}$, $\rho \in [\rho_0, 1]$. *For the existence of such a solution, it is necessary and sufficient that the following condition holds:*

$$U(t_0, 1) \leq u_0 \leq U(t_0, \rho_0).$$

CONCLUSION

This dissertation is devoted to solving forward and inverse problems for mixed-type equations with the Caputo fractional derivative and contributes new results to the field of fractional differential equations.

The main results of the study are as follows:

1. In the domain , a mixed-type partial differential subdiffusion equation involving the Caputo fractional derivative, and in the domain , a classical parabolic equation are considered under Dezin-type nonlocal conditions for both forward and inverse problems.

For the forward problem, under appropriate conditions imposed on the initial data and the right-hand side function, the existence and uniqueness of the solution are proved using the Fourier method. Moreover, the dependence of the existence of the solution on the parameter (arising from the Dezin-type condition) is also analyzed.

For the inverse problem, the right-hand side is assumed to have the separable form, where is the unknown function.

It is proved that, under certain conditions on (for example, having a constant sign), the existence and uniqueness of the solution hold.

2. A nonlocal problem for homogeneous mixed-type equations with the Caputo fractional derivative is studied, where the elliptic part of the equation consists of a simple second-order derivative.

Using the Fourier method, the existence, uniqueness, and stability of classical solutions to the nonlocal problem are proved.

3. Inverse problems for mixed-type equations with the Caputo fractional derivative, in which the elliptic part is of second order, are investigated for determining the space-dependent part of the right-hand side of the equation. In these problems, the right-hand side has the form where is the unknown function and is a given function.

Such inverse problems are reduced to solving the corresponding integral equation.

4. A problem with three gluing conditions for a mixed-type equation with the Caputo fractional derivative is analyzed, where the elliptic part is represented by the Laplace operator. Using the Fourier method, the existence and uniqueness of classical solutions to the initial-boundary value problem are established.

5. A mixed-type equation with the Caputo fractional derivative, in which the elliptic part is represented by the Bessel operator, is studied. Moreover, as an inverse problem, the determination of the fractional order under a novel additional condition is investigated.

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ИНСТИТУТ МАТЕМАТИКИ

ДУСАНОВА УМИДА ХОМИД КИЗИ

ПРЯМЫЕ И ОБРАТНЫЕ ЗАДАЧИ ДЛЯ УРАВНЕНИЙ СМЕШАННОГО ТИПА С ДРОБНОЙ ПРОИЗВОДНОЙ В СМЫСЛЕ КАПУТО

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования

Объектом исследования являются уравнения смешанного типа с производными в смысле Капуто.

Научная новизна исследования заключается в следующем:

Доказано существование и единственность классического решения прямых и обратных задач с нелокальными условиями в смысле Дезина для уравнения с оператором Лапласа, определённого в произвольной N-мерной области, где в верхней части области рассматривается субдиффузионное уравнение, а в нижней диффузионное;

Доказано существование и единственность решения прямых и обратных задач для уравнений смешанного типа с дробной производной Капуто в прямоугольной области;

Доказано существование и единственность решения краевой задачи для уравнения смешанного типа с дробной производной Капуто, содержащего самосопряжённый положительный оператор Лапласа, определённый в произвольной N-мерной области;

Доказано существование и единственность решения прямых и обратных задач по определению дробного порядка для уравнения смешанного типа с дробной производной в смысле Капуто и положительным самосопряжённым оператором Бесселя.

Внедрение результатов исследования.

Результаты, полученные для прямых и обратных задач по дифференциальным уравнениям в частных производных более высокого порядка, внедрены в практику в рамках следующих проектов:

решения прямых и обратных задач с нелокальными условиями в смысле Дезина для уравнений смешанного типа с дробным порядком производной Капуто были использованы в рамках зарубежного грантового проекта № 22-11-00064 на тему «Моделирование динамических процессов в геосфере с учётом наследственности» при классификации физической системы, изменяющейся во времени. (Институт космофизических исследований и распространения радиоволн, Дальневосточного отделения Российской Академии наук, справка № 378 от 25 сентября 2025 года). Применение научного результата позволило изучить определение неизвестного источника эманации радона в уравнении субдиффузии и диффузии на основе экспериментальных данных, что представляет значительный практический интерес в приложениях геофизики и сейсмологии;

научные результаты, полученные при исследовании прямых и обратных задач для смешанного типа уравнений с дробными производными, были практически применены в рамках бюджетных научно-исследовательских работ, выполненных в Институте механики и сейсмостойкости сооружений имени М. Т. Уразбаева Академии наук Республики Узбекистан по направлению «Разработка моделей деформирования изотропных и анизотропных тел под действием статических и динамических нагрузок, а также моделей нелинейных упругих и пластических сред» (Справка № 1348-3 от 30 октября 2025 г., выданная Институтом механики и сейсмостойкости сооружений Академии наук Республики Узбекистан).

Полученные результаты оказались эффективными при разработке нелинейных моделей упругих и пластических сред, а также при анализе деформирования изотропных и анизотропных тел под различными нагрузками.

Объем и структура диссертации. Диссертация состоит из введения, трёх глав, заключения и списка использованной литературы. Общий объем диссертации составляет 120 стр.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo‘lim (part I; I часть)

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