V.I. ROMANSOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

ESHBEKOV RAYXONBEK XUBAYDULLO O‘G‘LI

CHEKLI ZICHLIK VA MOSLANGAN MANBALI HIROTA TIPIDAGI TENGLAMA UCHUN KOSHI MASALASI

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Toshkent – 2025

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Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiya mavzusi O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2025.2.PhD/FM1294 raqam bilan ro‘yxatga olingan.

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KIRISH (falsafa dokto ri (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va ahamiyati. Jahonda olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlarda Dirak va Shturm-Liuvill operatori uchun qo‘yilgan to‘g‘ri va teskari spektral masalalarni tadqiq qilishga alohida ahamiyat berilmoqda. Hozirgi vaqtda rivojlangan mamlakatlarda spektral analizning to‘g‘ri va teskari masalalari zamonaviy matematik fizikaning evolyutsion tenglamalarining yechimlarini topish va yechimlar sinfini aniqlashda katta ahamiyatga ega. Bundan tashqari, bu masalalar radiotexnika, chiziqli bo‘lmagan optika, kvant mexanikasi va amorf jismlarning kristallik xususiyatlarini modellashtirishda muhim rol o‘ynaydi. Davriy Dirak operatori uchun teskari spektral masalalar usulidan foydalangan holda chekli zichlik va moslangan manbali nochiziqli Hirota tipidagi tenglama uchun boshlang‘ich qiymat masalasini yechish algoritmi ba’zi fizik muhitlarda chiziqli bo‘lmagan to‘lqinlarning harakatini o‘rganishda va optik tolali tizimlarni tahlil qilishda, shu jumladan yuqorida aytib o‘tilgan sohalarda katta ahamiyatga ega.

Jahonda nochiziqli muhitda tarqaladigan turli xil ultraqisqa yorug‘lik impulslari, gidrodinamika, chiziqli bo‘lmagan muhitda lazer nurlarining tarqalishi, chiziqli bo‘lmagan optika, kvant mexanikasi sohalarida olib borilayotgan ilmiy tadqiqotlarda davriy cheksiz zonali funksiyalar sinfida chekli zichlikka ega Hirota tipidagi tenglamasini integrallash katta ahamiyatga ega. Shu bilan birga, davriy cheksiz zonali funksiyalar sinfida qator yoki integral ko‘rinishdagi moslangan manbalarga ega bo‘lgan nochiziqli Hirota tenglamasi uchun Koshi masalasining yechimini topish muammosi yuqorida qayd etilgan fizik jarayonlarning muhim xususiyatlarini o‘rganish va ilmiy tadqiqotlarni rivojlantirishda dolzarb vazifa hisoblanadi.

Respublikamizda matematik fizikaning nochiziqli evolyutsion tenglamalarining yechimlarini Dirak operatoriga qo‘yilgan to‘g‘ri va teskari spektral masalalar usulidan foydalanib aniqlash hamda topilgan yechimlarni amaliyotda qo‘llash bo‘yicha keng ko‘lamli chora-tadbirlar amalga oshirilmoqda. “Algebra va uning tatbiqlari, differensial tenglamalar va uning tatbiqlari, chiziqsiz tizimlar, dinamik tizimlar va ularning tatbiqlarini matematik modellashtirish, stoxastik tahlil, tibbiy-biologik informatika, hisoblash matematikasi[1]” fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo‘nalishlari etib belgilangan. Ushbu vazifani amalga oshirishda, xususan zamonaviy matematik fizikaning nochiziqli evolyutsion tenglamalarni integrallashda teskari spektral masalalar usulini qo‘llab, davriy cheksiz zonali funksiyalar sinfida chekli zichlikka yoki moslangan manbaga ega Hirota tipidagi tenglamaga qo‘yilgan boshlang‘ich qiymat masalasining yechimi mavjudligini ko‘rsatish muhim ilmiy ahamiyatga ega.

[1] O‘zbekiston Respublikasi Prezidentining 2019 yil 9 iyuldagi PQ-4387-son “Matematika ta’limi va fanlarni yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risidagi” qarori

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “O‘zbekston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi Farmoni, 2019 yil 9 iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivoylantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akmademiyaning V.I. Romanoviskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora tadbirlari to‘g‘risida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi

Tadqiqotning respublika fan va texnologiyalari rivojlantirishning ustuvor yo‘nalishlariga muvofiqligi. Ushbu tadqiqot O‘zbekiston Respublikasida fan va texnikani rivojlantirishning IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishiga muvofiq bajarildi.

Muammoning o‘rganilganlik darajasi. 1946-yilda Borg $[0,\pi]$ oraliqdagi Shturm-Liuvill operatori Dirixle chegaraviy shartlari bilan berilgan holda, agar ikki xil ya'ni Dirixle–Dirixle va Dirixle–Neyman chegaraviy shartlari uchun spektr ma’lum bo‘lsa, u holda $q(x)$ potensial funksiyani yagona aniqlanishi mumkinligini ko‘rsatdi.

Ko‘p o‘tmay, 1951-yilda Gelfand va Levitan spektral berilganlardan(xos qiymatlar va normallovchi o‘zgarmaslar) foydalangan holda teskari masalani yechishning analitik usulini taqdim etdilar. Shundan so‘ng, Marchenko teskari sochilish nazariyasi uchun asos bo‘lgan nazariyani ishlab chiqdi va bu nazariya sochilish nazariyasi berilganlaridan foydalanib potensiallarni qayta tiklash imkonini beradi.

1967-yilda Gardner, Grin, Kruskal va Miura quyidagi Shturm–Liuvill operatori

$$Ly \equiv -y'' + q(x,t)y = \mu y,\ \ x \in \mathbb{R}, t > 0.$$

uchun teskari sochilish masala usulidan foydalangan holda ushbu Korteveg–de Friz (KdF) tenglamasi

$$q_t = 6qq_x - q_{xxx},\ \ q(x,t)|_{t=0} = q_0(x),\ \ x \in \mathbb{R}, t > 0,$$

uchun Koshi masalasini yechishni taklif qildilar.

So‘ngra Piter D. Laks nazariyaga Laks juftligi tushunchasini kiritib, KdV kabi ma’lum nochiziqli evolyutsion tenglamalarini izospektral deformatsiyalar, ya’ni $\frac{dL}{dt} = [P, L]$ ko‘rinishda yozish mumkinligini ko'rsatdi va shu bilan nochiziqli xususiy hosilali differensial tenglamalarni spektral nazariya bilan bog‘ladi va integrallanuvchi sistemalarning zamonaviy nazariyasiga asos soldi. Zaxarov va Shabat esa nochiziqli fokuslanuvchi va defokuslanuvchi Shredinger tenglamalari

$$i\psi_t + \psi_{xx} \pm 2|\psi|^2\psi = 0$$

uchun teskari sochilish masalalar usulini ishlab chiqdilar:

Oradan ko‘p vaqt o‘tmasdan Vadati modifitsirlangan Korteveg–de Friz (mKdF) tenglamasi

$$u_t + u^2u_x + \mu u_{xxx} = 0,\ \ 0 < \mu < \frac{1}{6}$$

uchun Zaxarov va Shabat g‘oyalarini rivojlantirdi.

1973-yilda Hirota kompleks mKdF va nochiziqli Shredinger tenglamalarining kombinatsiyasidan tashkil topgan va keyinchalik Hirota tenglamasi deb nomlangan ushbu

$$i\frac{\partial\psi}{\partial t}+i3\alpha|\psi|^2\,\frac{\partial\psi}{\partial x}+\beta\frac{\partial^2\psi}{\partial x^2}+i\gamma\frac{\partial^3\psi}{\partial x^3}+\delta|\psi|^2\psi=0,$$

tenglamaning aniq N –"konvert" soliton yechimlarini topdi. Bu yerda $\alpha\beta = \gamma\delta$, $\alpha,\beta,\gamma,\delta\in\mathbb{R}_+,\ x\in\mathbb{R}, t>0$.

1974-yilda esa Ablovitz, Kaup, Nevell va Sigur odatiy birinchi tartibli matritsaning xos qiymat masalasidan

$$\Psi_{\mathrm{x}}=U(x,t,\lambda)\Psi,\ \ \Psi_t=V(x,t,\lambda)\Psi$$

foydalangan holda keng miqyosidagi integrallanuvchi sistemalar sinfini aniqlash uchun yagona tizimni ishlab chiqdilar.

Shu yili Novikov teskari spektral nazariyani integrallanuvchi sistemalar, xususan, KdF tenglamasi bilan bog'lab, chekli zonali potensiallar tushunchasini kiritdi. Bundan tashqari, Dubrovin, Its, Matveevlar chekli zonali holda Shturm–Liuvill operatori uchun teskari spektral masalalar usulidan foydalanib, KdF tenglamasiga qo'yilgan Koshi masalasi ixtiyoriy boshlang'ich shartlar bilan berilganda yechimga ega ekanligini isbotladilar va Riman teta funksiyalaridan foydalangan holda yechimning aniq formulasini topdilar.

Bundan tashqari, A.B. Xasanov o'zining ilmiy maktabida davriy cheksiz zonali funksiyalar sinfida Dirak operatori uchun teksari spektral masalalar usuli yordamida nochiziqli evolyutsion tenglamalarni integrallash nazariyasiga asos solgan. 2022-yilda Xasanov va Mo'minov defokuslanuvchi Shredinger tenglamasi davriy Dirak operatori uchun teskari spektral masalalar usuli orqali integrallash mumkinligini isbotladilar. Shuningdek, A.B. Xasanov va uning shogirdlari KdF, mKdF, kosinus–Gordon(coshG), mKdF–coshG, Liuvill(L), Hirota, mKdF–L, coshG–L, manfiy tartibli mKdF–L, manfiy tartibli mKdF–coshG kabi turli xil nochiziqli xususiy hosilali differensial tenglamalar teskari spektral masalalar usuli yordamida integrallanishini ko'rsatdilar.

Dissertatsiya tadqiqotining dissertatsiya tugallangan oliy o'quv yurti yoki ilmiy-tadqiqot muassasasining ilmiy tadqiqot rejalari bilan bog'liqligi. Sharof Rashidov nomidagi Samarqand davlat universiteti ilmiy-tadqiqot ishlari rejasiga muvofiq "Differensial operatorlar spektral nazariyasining nochiziqli evolyutsion tenglamalarga tadbiqlari" nomli ilmiy-tadqiqot ishlari rejasi doirasida bajarilgan.

Tadqiqotning maqsadi. Nochiziqli chekli zichlikka ega Hirota tipidagi tenglama va nochiziqli moslangan qator va integral manbali Hirota tipidagi tenglamalarni cheksiz zonali davriy funksiyalar sinfida integrallashdan iborat.

Tadqiqotning vazifalari:

– davriy cheksiz zonali funksiyalar sinfida chekli zichlikka ega Hirota tipidagi tenglamaning integrallanuvchi ekanligini isbotlash;

– davriy Dirak operatori uchun teskari spektral masalalar usulidan foydalanib, qo'shimcha hadlarga ega Hirota tipidagi tenglama uchun boshlang'ich qiymat masalasining yagona yechimga ega ekanligini isbotlash;

– moslangan qator manbali Hirota tipidagi tenglama uchun Koshi masalasining davriy cheksiz zonali funksiyalar sinfida yagona yechimi mavjud ekanligini isbotlash;
– oʻz-oʻziga qoʻshma Dirak operatori uchun teskari spektral masalalar usulidan foydalanib, integral manbali Hirota tipidagi tenglamaning integrallanuvchi ekanligini isbotlash.

Tadqiqot obyektlari chekli zichlikka ega Hirota tipidagi tenglama, qoʻshimcha hadlarga ega Hirota tipidagi tenglama, qator manbali Hirota tipidagi tenglama, integral manbali Hirota tipidagi tenglamalardan iborat.

Tadqiqot predmeti chekli zichlikka va moslangan manbalarga ega Hirota tipidagi tenglamalarni integrallashda davriy Dirak operatori uchun toʻgʻri va teskari spektral masalalardan iborat.

Tadqiqot usullari. Dissertatsiya ishida differensial tenglamalar, matematik fizika, differensial operatorning spektral nazariyasi, kompleks oʻzgaruvchili funksiyalar nazariyasi va funksional analizni masalalarini yechish usullari qoʻllanildi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

–davriy Dirak operatori uchun teskari spektral masalalar usulidan foydalanib, chekli zichlikka ega Hirota tipidagi tenglama olti marta uzluksiz differensiallanuvchi davriy cheksiz zonali funksiyalar sinfida integrallanuvchi ekanligi isbotlangan;

– olti marta uzluksiz differensiallnuvchi funksiyalar sinfida qoʻshimcha hadlarga ega Hirota tipidagi tenglama uchun Koshi masalasining yagona yechimi mavjud ekanligi isbotlangan;

– davriy Dirak operatori uchun teskari spektral masalalar usulidan foydlanib, qator manbali Hirota tipidagi tenglama davriy cheksiz zonali funksiyalar sinfida integrallanuvchi ekanligi isbotlangan;

– olti marta uzluksiz davriy cheksiz zonali funksiyalar sinfida integral manbali Hirota tipidagi tenglama uchun boshlangʻich qiymat masalasining yechimi mavjud va yagona ekanligi isbotlangan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

–davriy cheksiz zonali funksiyalar sinfida chekli zichlikka ega nochiziqli defokuslangan Hirota tipidagi tenglama uchun Koshi masalasini yechish algoritmi yordamida cheksiz oʻlchovli dinamik sistemalarning inersial manifoldlari oʻrganildi;

–davriy cheksiz zonali funksiyalar sinfida moslangan qator manbali Hirota tipidagi tenglamani integrallash algoritmi, kechikkan reaksiya-diffuziya tenglamalari uchun dinamik chegara shartlari ostida attraktorlarning xossalarini oʻrganish imkonini berdi.

Tadqiqot natijalarining ishonchliligi davriy koeffitsiyentli differensial operatorlar uchun teskari spektral masalalarni yechishda va ularni nochiziqli evolyutsion tenglamalarni yechish uchun qoʻllashda matematik fizika, spektral analiz, funksional analiz usullaridan foydalanilgan, shuningdek, matematik mulohazalar qatʼiy isbotlar yordamida asoslangan.

Tadqiqot natijalarining ilmiy ahamiyati zamonaviy matematik fizikaning evolyutsion tenglamalarini davriy cheksiz zonali funksiyalar sinfida integrallash mumkinligi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati nochiziqli muhitlarda optika, kvant mexanikasi, elektrodinamika, gidrodinamika, zamonaviy matematik fizikada qo‘llanilishi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Nochiziqli Hirota tenglamasini integrallash bo‘yicha olingan ilmiy natijalar asosida:

chekli zichlikli nochiziqli Hirota tipidagi tenglama va qo‘shimcha hadlar bilan berilgan nochiziqli Hirota tipidagi tenglamalarni davriy cheksiz zonali funksiyalar sinfida integrallash algoritmidan NRF-2020R1A2C1003119 “Global Attractors and Inertical Manifolds of Infinite Dimensional Dynamical Systems” fundamental loyihada foydalanilgan (Janubiy Koreya Respublikasi, Chonnam milliy universitetining 2025 yil 21 apreldagi ma’lumotnomasi). Ilmiy natijalarning qo‘llanilishi dinamik chegaraviy shartlarga ega kechikilgan reaksiya diffuziya tenglamalari sinfi uchun attraktorlarning xususiyatlarini o‘rganish imkonini bergan.

chekli zichlikli nochiziqli Hirota tipidagi tenglama va moslangan qator va integral manbalar bilan berilgan nochiziqli Hirota tipidagi tenglamalarni davriy cheksiz zonali funksiyalar sinfida integrallash algoritmidan GP/2023/9752700-“Theoretical and numerical studies for some classes of fractional integro-differential equations in Banach space” mavzusidagi ilmiy-tadqiqot ishida foydalanilgan (Malayziya Putra Universitetining 2025-yil 18-sentyabrdagi ma’lumotnomasi). Ilmiy natijalarning qo‘llanilishi Banax fazosida kasr tartibli integro-differensial tenglamalarning ba’zi sinflarini nazariy jihatdan tadqiq qilish imkonini bergan;

Tadqiqot natijalarini aprobatsiyasi. Ushbu tadqiqot natijalari 7 ta ilmiy-amaliy konferensiyalarda, jumladan 3 ta xalqaro va 4 ta Respublika konferensiyalarida muhokama qilingan.

Tadqiqot natijalarini e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha 14 ta ilmiy ish chop etilgan, shundan, O‘zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktori dissertatsiyalarini himoya qilishda tavsiya etilgan ilmiy nashrlarda 7 ta maqola, jumladan, 3 tasi xorijiy va 4 tasi respublika jurnallarida nashr etilgan

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat. Dissertatsiya hajmi 122 betdan iborat.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va ahamiyati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, masalaning o‘rganilganlik darajasi, tadqiqotning maqsad va vazifalari, tadqiqot obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari tavsiflangan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, chop etilgan ishlar va dissertatsiya tuzilishi haqida ma’lumot keltirilgan.

Dissertatsiyaning **“Davriy Dirak operatori haqida”** deb nomlangan birinchi bobida davriy Dirak operatori va Dirak operatori uchun teskari spektral masalalar usuli haqida fundamental va zarur ma’lumotlar keltirilgan. Shuningdek, u fazoviy o‘zgaruvchiga nisbatan Dirak operatorining spektral berilganlarining evolyutsiyasini ifodalovchi birinchi Dubrovin differensial tenglamalari sistemasi keltirib chiqarilgan va ushbu tenglamalar sistemasi uchun Koshi masalasining yagona yechimi mavjudligi isbotlangan. Keyingi boblarda birinchi bobda keltirilgan ma’lumotlardan foydalanilgan.

Quyidagi siljigan argumentli Dirak operatorini qaraymiz:

$$\mathcal{L}(\tau,t)y \equiv J\frac{dy}{dx} + \Lambda(x+\tau,t)y = \mu y,\quad x,\tau \in \mathbb{R}, t>0, \qquad (1)$$

bu yerda $J=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, $\Lambda(x+\tau,t)=\begin{pmatrix}p(x+\tau,t) & q(x+\tau,t)\\ q(x+\tau,t) & -p(x+\tau,t)\end{pmatrix}$, $y(x,t)=\begin{pmatrix}y_1(x,t)\\ y_2(x,t)\end{pmatrix}$, $\mu\in\mathbb{C}$, $p(x+\pi,t)=p(x,t)\in C^2(\mathbb{R})$, $q(x+\pi,t)=q(x,t)\in C^2(\mathbb{R})$ haqiqiy qiymatli funksiyalar.

Ushbu $u(x,\mu,\tau,t)=\big(u_1(x,\mu,\tau,t),u_2(x,\mu,\tau,t)\big)^T$ and $v(x,\mu,\tau,t)=\big(v_1(x,\mu,\tau,t),v_2(x,\mu,\tau,t)\big)^T$ funksiyalar orqali (1) tenglamalar sistemasining mos ravishda $u(0,\mu,\tau,t)=(1,0)^T$ va $v(0,\mu,\tau,t)=(0,1)^T$ shartlarni qanoatlantiruvchi yechimlarini belgilaymiz. $\Delta(\mu,\tau,t)=u_1(\pi,\mu,\tau,t)+v_2(\pi,\mu,\tau,t)$ funksiya (1) sistema uchun Lyapunov funksiyasi deyiladi. Shuningdek, $\mu_n(\tau,t), n\in\mathbb{Z}$ orqali (1) Dirak sistemasiga qo‘yilgan davriy $y(0,\tau,t)=y(\pi,\tau,t)$ va yarimdavriy $y(0,\tau,t)=-y(\pi,\tau,t)$ chegaraviy masalalarning xos qiymatlarini belgilaymiz. Bu xos qiymatlar $\Delta(\mu,\tau,t)\mp 2=0$ tenglamaning ildizlari bilan ustma-ust tushadi. (1) sistema uchun $y_1(0,\tau,t)=y_1(\pi,\tau,t)=0$ shart bilan aniqlangan Dirixle chegaraviy masalasining xos qiymatlarini yordamida belgilaymiz. Osongina tekshirish mumkinki, $\xi_n(\tau,t), n\in\mathbb{Z}$ xos qiymatlari $v_1(\pi,\mu,\tau,t)=0$ tenglamaning ildizlari bilan ustma-ust tushadi.

1-teorema. Ushbu $\Delta(\mu,\tau,t)$ Lyapunov funksiyasi τ parametrga bog‘liq bo‘lmaydi, ya’ni $\Delta(\mu,\tau,t)=\Delta(\mu,t)$. Agar τ parametr $[0;\pi]$ oraliqda o‘zgarsa, u holda Dirixle chegaraviy masalasining $\xi_n(\tau,t)$xos qiymatlari $[\mu_{2n-1},\mu_{2n}], n\in\mathbb{Z}$ oraliqni to‘ldiradi, ya’ni

$$\xi_n(\tau,t)\in[\mu_{2n-1},\mu_{2n}], n\in\mathbb{Z}.$$

1-ta’rif. Ushbu (μ_{2n-1},μ_{2n}) intervallarga lakunalar deyiladi.

2-ta’rif. Agar lakunalar soni cheksiz bo‘lsa, u holda Dirak operatorining $p(x,t)$ va $q(x,t)$ potensiallari cheksiz zonali davriy potensiallar deyiladi. Shuningdek, $\mathcal{L}(\tau,t)$ Dirak operatori davriy cheksiz zonali operator deyiladi.

Biz qaragan hol uchun barcha lakunalar ochiq, ya’ni $\mu_{2n-1} \neq \mu_{2n}, n \in \mathbb{Z}$.

Shuningdek, quyidagi ko‘rinishda lakunlar uzunliklarini aniqlaymiz: $\gamma_n = \mu_{2n} - \mu_{2n-1},\ \ n \in \mathbb{Z}$.

3-ta’rif. (1) Dirak sistemasiga qo‘yilgan Dirixle chegaraviy masalasining $\xi_n(\tau,t), n \in \mathbb{Z}$ xos qiymatlari va ushbu $\sigma_n(\tau,t) = sgn\{v_2(\pi,\xi_n,\tau,t) - u_1(\pi,\xi_n\tau,t)\}, n \in \mathbb{Z}$ ishoralar ketma-ketligi $\mathcal{L}(\tau,t)$ Dirak operatorining spektral parametrlari deyiladi. Spektral parametrlar va lakunalarning chetki nuqtalari $\mu_n, n \in \mathbb{Z}$ lar birgalikda, ya’ni ushbu $\{\mu_n, \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1,\ n \in \mathbb{Z}\}$ to‘plam $\mathcal{L}(\tau,t)$ Dirak operatorining spektral berilganlari deyiladi.

4-ta’rif. Ushbu $p(\tau,t)$ va $q(\tau,t)$ potensiallar yordamida $\mathcal{L}(\tau,t)$ Dirak operatorining spektral berilganlarini topish masalasiga to‘g‘ri spektral masala, spektral berilganlar yordamida $\mathcal{L}(\tau,t)$ Dirak operatorining $\Lambda(\tau,t)$ koeffitsiyentini tiklash masalasiga esa teskari spektral masala deyiladi.

2-teorema[2]. $\mathcal{L}(\tau,t)$ operatorning $p(\tau,t)$ va $q(\tau,t)$ koeffitsiyentlari $\{\mu_n, \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1,\ n \in \mathbb{Z}\}$ spektral berilganlar yordamida bir qiymatli aniqlanadi.

Ushbu

$$\psi^{\pm}(x,\mu,\tau,t) = u(x,\mu,\tau,t) + \frac{v_2(\pi,\mu,\tau,t) - u_1(\pi,\mu,\tau,t) \mp \sqrt{\Delta^2(\mu) - 4}}{2v_1(\pi,\mu,\tau,t)} v(x,\mu,\tau,t) \quad (2)$$

ko‘rinishda aniqlangan $\psi^{\pm}(x,\mu,t) = \left(\psi_1^{\pm}(x,\mu,t), \psi_2^{\pm}(x,\mu,t)\right)$ funksiyalar (1) differensial tenglamalar sistemasining Floke yechimlari deyiladi.

Agar $\Delta^2(\mu) - 4 \neq 0$ bo‘lsa, u holda $\psi^{+}(x,\mu,\tau,t)$ va $\psi^{-}(x,\mu,\tau,t)$ funksiyalar (1) sistemaning fundamental yechimlaridan iborat bo‘ladi.

1-lemma. $\mathcal{L}$ Dirak operatori faqat uzluksiz spektrga ega, ya’ni uning xos qiymatlari yo‘q. $\mathcal{L}$ Dirak operatorining spektri quyidagi to‘plamdan iborat:

$$E = \mathbb{R} \backslash \left(\bigcup_{n=-\infty}^{+\infty} (\mu_{2n-1}, \mu_{2n}) \right).$$

2-lemma. $\mathcal{L}(\tau,t)$ Dirak operatorining potensiallari uchun quyidagi “izlar formulalari” deb ataluvchi formulalar o‘rinli:

$$p(\tau,t) = \sum_{n=-\infty}^{+\infty} \left(\frac{\mu_{2n-1} + \mu_{2n}}{2} - \xi_n(\tau,t) \right), \quad (3)$$

$$q(\tau,t) = \sum_{n=-\infty}^{+\infty} (-1)^{n-1} \sigma_n(\tau,t) h_n\big(\xi(\tau,t)\big), \quad (4)$$

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[2] H.N. Normurodov. *Nochiziqli modifitsirlangan Korteveg-de Friz–sinus–Gordon (mKdF–sG) tenglamalarini davriy cheksiz zonali funksiyalar sinfida integrallash.*. PhD Dissertatsiyasi, 2024, Samarqand, O‘zbekiston.

bu yerda

$$h_n(\xi)=\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})(\mu_{2n}-\xi_n(\tau,t))}\times$$
$$\times\sqrt{\prod_{k=-\infty,k\neq n}^{+\infty}\frac{(\mu_{2k-1}-\xi_n(\tau,t))(\mu_{2k}-\xi_n(\tau,t))}{(\xi_k(\tau,t)-\xi_n(\tau,t))^2}}.$$

Dissertatsiyaning **“Chekli zichlikli yoki qo‘shimcha hadlar bilan berilgan Hirota tipidagi tenglama uchun boshlang‘ich qiymat masalasi”** deb nomlanuvchi ikkinchi bobida dastlab teskari spektral masalalar usuli haqida tarixiy ma’lumotlar keltirilgan, so‘ngra chekli zichlikli Hirota tipidagi tenglama va qo‘shimcha hadlar bilan berilgan Hirota tipidagi tenglama uchun boshlang‘ich qiymat masalalari olti marta uzluksiz differensiallanuvchi davriy cheksiz zonali funksiyalar sinfida yagona global yechimga ega ekanligi isbotlangan.

Ushbu bobning birinchi paragrafida ushbu

$$\begin{cases}p_t=[p_{xxx}+6(\rho^2-\{p^2+q^2\})p_x]-[q_{xx}+2q(\rho^2-\{p^2+q^2\})]\\ q_t=[q_{xxx}+6(\rho^2-\{p^2+q^2\})q_x]+[p_{xx}+2p(\rho^2-\{p^2+q^2\})]\end{cases}. \quad (5)$$

chekli zichlikli Hirota tipidagi tenglamani quyidagi

$$\begin{cases}p(x,t)|_{t=0}=p_0(x), \quad p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}),\\ q(x,t)|_{t=0}=q_0(x), \quad q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}),\end{cases} \quad (6)$$

boshlang‘ich shartlarni qanoatlantiruvchi va ushbu

$$\begin{cases}p(x+\pi,t)=p(x,t), q(x+\pi,t)=q(x,t), x\in\mathbb{R}, t>0\\ p(x,t), q(x,t)\in C_{x,t}^{3,1}(t>0)\cap C(t\geq 0)\end{cases}. \quad (7)$$

silliqlik sinfiga tegishli bo‘lgan x o‘zgaruvchi bo‘yicha π davrli haqiqiy yechimini davriy cheksiz zonali funksiyalar sinfida topish masalasi qaralgan. Bu yerda $\rho>0$ chekli son.

Biz (5)-(7) boshlang‘ich qiymat masalasi(BQM)ning yagona global yechimi mavjud ekanligini (1) Dirak operatori uchun teskari spektral masalalar usuli yordamida isbotlaymiz.

Dastlab, boshlang‘ich shartdagi $p_0(x)$ va $q_0(x)$ funksiyalar yordamida quyidagi $\mathcal{L}(\tau,0)$ Dirak operatorini quramiz:

$$\mathcal{L}(\tau,0)y\equiv\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}y_1'\\y_2'\end{pmatrix}+\begin{pmatrix}p_0(x+\tau)&q_0(x+\tau)\\q_0(x+\tau)&-p_0(x+\tau)\end{pmatrix}\begin{pmatrix}y_1\\y_2\end{pmatrix}=\mu\begin{pmatrix}y_1\\y_2\end{pmatrix}, \quad x,\tau\in\mathbb{R}.$$

To‘g‘ri spektral masalani yechish orqali $\mathcal{L}(\tau,0)$ Dirak operatorining $\{\mu_n,\xi_n^0(\tau),\sigma_n^0(\tau)=\pm1, n\in\mathbb{Z}\}$ spektral berilganlarini topamiz.

Ushbu paragrafning asosiy maqsadi boshlang‘ich shartdagi $p_0(x)$ va $q_0(x)$ funksiyalar qanday mimimal silliqlik shartlarini qanoatlantirganda (5)-(7) BQM global yechimga ega bo‘lishini aniqlashdan iborat.

Asosiy natija quyidagi teorema yordamida ifodalaniladi.

3-teorema. Agar davriy cheksiz zonali $p_0(x), q_0(x)$ funksiyalar $p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R})$, $q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R})$ shartlarni qanoatlantirsa, u holda (5)-(7) BQMning yagona aniqlangan global davriy cheksiz zonali

$p(x,t), q(x,t),\ x \in \mathbb{R}, t > 0$ yechimlari mavjud bo‘ladi va ular mos holda (3) va (4) izlar formulalari yordamida aniqlanadi.

3-teoremani isbotlash uchun $\mathcal{L}(\tau, t)$ Dirak operatorining $\{\mu_n, \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1,\ n \in \mathbb{Z}\}$ spektral berilganlari mavjud va yagona ekanligini isbotlash zarur.

Endi $\mathcal{L}(\tau, t)$ Dirak operatori spektral berilganlarining evolyutsiyasini ifodalovchi cheksiz Dubrovin differensial tenglamalar sistemasini keltiramiz.

3-lemma. Aytaylik, $p(x,t),\ q(x,t), x \in \mathbb{R}, t > 0$ funksiyalar (5)-(7) BQMning yechimlari bo‘lsin. U holda $\mathcal{L}(\tau, t)$ Dirak operatorining spektri chetki nuqtalari τ va t parametrlarga bog‘liq bo‘lmaydi, ya’ni $\mu_n(\tau,t) = \mu_n, n \in \mathbb{Z}$. Shuningdek, $\xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}$ spektral parametrlari quyidagi birinchi va ikkinchi Dubrovin differensial tenglamalar sistemasini qanotlantiradi:

$$1.\ \frac{\partial \xi_n(\tau,t)}{\partial \tau} = 2(-1)^{n-1}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t) - \mu_{2n-1})\big(\mu_{2n} - \xi_n(\tau,t)\big)f_n(\xi) \times \\ \times \{p(\tau,t) + \xi_n(\tau,t)\}}, n \in \mathbb{Z}; \qquad (8)$$

$$2.\ \frac{\partial \xi_n(\tau,t)}{\partial t} = 2(-1)^{n}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t) - \mu_{2n-1})\big(\mu_{2n} - \xi_n(\tau,t)\big)f_n(\xi)g_{n,1}(\xi)}. \quad (9)$$

Quyidagi boshlang‘ich shartlar o‘rinli:

$$\xi_n(\tau,t)|_{t=0} = \xi_n^0(\tau),\ \ \sigma_n(\tau,t)|_{t=0} = \sigma_n^0(\tau), n \in \mathbb{Z}, \qquad (10)$$

bu yerda $\xi_n^0(\tau), \sigma_n^0(\tau) = \pm 1, n \in \mathbb{Z}$ lar $\mathcal{L}(\tau, 0)$ Dirak operatorining spektral parametrlari va

$$f_n(\xi) = \sqrt{\prod_{\substack{k=-\infty \\ k\neq n}}^{+\infty} \frac{(\mu_{2k-1} - \xi_n)(\mu_{2k} - \xi_n)}{(\xi_k - \xi_n)^2}}, \qquad (11)$$

$g_{n,1}(\xi) = [4\xi_n^3 + 4p\xi_n^2 + 2\xi_n(p^2 + q^2 + q_\tau) - p_{\tau\tau} + 2(pq_\tau - p_\tau q) +$
$+2p(p^2 + q^2) - 6\rho^2(p + \xi_n)] + [(p + \xi_n)^2 + \xi_n^2 + q^2 + q_\tau - \rho^2],\ \ n \in \mathbb{Z},$
$p = p(\tau,t), q = q(\tau,t),\ \ \xi \equiv \xi(\tau,t) = (\dots, \xi_{-1}(\tau,t), \xi_0(\tau,t), \xi_1(\tau,t), \dots),$
$\sigma \equiv \sigma(\tau,t) = (\dots, \sigma_{-1}(\tau,t), \sigma_0(\tau,t), \sigma_1(\tau,t), \dots).$

Endi (5)-(7) BQM yagona global yechimga ega ekanligini isbotlaymiz.

Ushbu

$$\xi_n(\tau,t) = \mu_{2n-1} + (\mu_{2n} - \mu_{2n-1})\sin^2 x_n(\tau,t),\ \ n \in \mathbb{Z},$$

almashtirish yordamida (9)–(10) BQM $\mathcal{K}$ Banax fazosida yagona tenglama ko‘rinishida ifodalanadi:

$$\begin{cases} \dfrac{\partial x(\tau,t)}{\partial t} = H\big(x(\tau,t)\big), \\ x(\tau,t)|_{t=0} = x^0(\tau),\ \ x^0(\tau) \in \mathcal{K}, \end{cases} \qquad (12)$$

bu yerda

$$\mathcal{K} = \left\{x = (\dots, x_{-1}, x_0, x_1, \dots) \colon \|x\| = \sum_{n=-\infty}^{+\infty} (1 + |n|^2)(\mu_{2n} - \mu_{2n-1})|x_n| < \infty\right\},$$

$H\big(x(\tau,t)\big) = \big(\dots, H_{-1}\big(x(\tau,t)\big), H_0\big(x(\tau,t)\big), H_1\big(x(\tau,t)\big), \dots\big),$
$x^0(\tau) = (\dots, x_{-1}^0(\tau), x_0^0(\tau), x_1^0(\tau) \dots).$

Demak, (9), (10) BQMning o‘rniga unga ekvivalent bo‘lgan (12) BQMning yagona global yechimi mavjud ekanligini isbotlash yetarli.

1978-yilda Misyura[3] agar $p_0(x) \in C^6(\mathbb{R})$ va $q_0(x) \in C^6(\mathbb{R})$ shartlar o‘rinli bo‘lganda quyidagi asimptotikalar bajarilishini isbotladi:

$$\begin{cases} \mu_{2k}, \mu_{2k-1} = k + \sum_{j=1}^{7} c_j k^{-j} \pm 2^{-6}|k|^{-6}\left|q_{2k}^6\right| + |k|^{-7}\varepsilon_k^{\pm}, \\ \gamma_k \equiv \mu_{2k} - \mu_{2k-1} = \dfrac{\left|q_{2k}^6\right|}{2^5|k|^6} + \dfrac{\delta_k}{|k|^7}, \delta_k = \varepsilon_k^+ - \varepsilon_k^-, \end{cases} \tag{13}$$

bu yerda

$$q_{2k}^2 \equiv \frac{1}{\pi}\int_0^{\pi} Q_0^2(t)e^{-2ikt}dt, \;\; Q_0(t) = q_0(t) - ip_0(t), \;\; \sum_{k=-\infty}^{+\infty} \left(\varepsilon_k^{\pm}\right)^2 < \infty.$$

Shuningdek, ushbu $\xi_n(\tau) \in [\mu_{2n-1}, \mu_{2n}], n \in \mathbb{Z}$ munosabatdan quyidagi

$$\inf_{k \neq n}|\xi_n(\tau) - \xi_k(\tau)| \geq a > 0. \tag{14}$$

tengsizlik kelib chiqadi.

Endi (13) asimptotikalar va (14) tengsizlik yordamida quyidagi lemmani isbotlaymiz.

4-lemma. Quyidagi tengsizliklar o‘rinli:

$$C_1 \leq \left|\bar{f}_n(x)\right| \leq C_2, \;\; \left|\frac{\partial \bar{f}_n(x)}{\partial x_m}\right| \leq C_3\gamma_m,$$

$$\left|\bar{g}_{n,1}(x)\right| \leq C_4(|n|^3 + 1), \;\; \left|\frac{\partial \bar{g}_{n,1}(x)}{\partial x_m}\right| \leq C_5\gamma_m(|n|^2 + 1)(|m|^2 + 1), \;\; n, m \in \mathbb{Z},$$

bu yerda $x = (\dots, x_{-1}, x_0, x_1, \dots), \; C_j = const, \; j = \overline{1,5}$.

5-lemma. Agar boslang‘ich shartdagi $p_0(x)$ va $q_0(x)$ funksiyalar ushbu

$$p_0(x + \pi) = p_0(x) \in C^6(\mathbb{R}), \;\; q_0(x + \pi) = q_0(x) \in C^6(\mathbb{R}),$$

shartlarni qanoatlantirsa, u holda (12) BQM yagona global yechimga ega bo‘ladi.

Demak, (9), (10) BQMning yagona global yechimi mavjud.

Endi (5)-(7) BQMning yechimini topish algoritmini keltiramiz:

1) Dastlab, boshlang‘ich shartdagi $p_0(x)$ va $q_0(x)$ funksiyalardan foydalanib, $\mathcal{L}(\tau, 0)$ Dirak operatorining $\{\mu_n, \xi_n^0(\tau), \sigma_n^0(\tau), n \in \mathbb{Z}\}$ spektral berilganlarini topamiz;

2) So‘ngra $\mathcal{L}(\tau, t)$ Dirak operatorining spektral berilganlarini $\{\mu_n, \xi_n(\tau, t), \sigma_n(\tau, t), n \in \mathbb{Z}\}$ orqali belgilaymiz va (9), (10) BQMni ixtiyoriy τ qiymatlarda yechib, $\{\xi_n(\tau, t), \sigma_n(\tau, t), n \in \mathbb{Z}\}$ spektral parametrlarni topamiz;

3) (3) va (4) izlar formulalaridan foydalanib, $p(\tau, t)$ va $q(\tau, t)$ funksiyalarni, ya’ni (5)-(7) BQMning yechimini topamiz.

Endi $\mathcal{L}(\tau, 0)$ Dirak operatorining spektrida faqatgina bitta lakuna ochiq bo‘lgan holda (5)-(7) BQMning yechimini topamiz. Boshqacha aytganda $\mathcal{L}(0)$

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[3] T.V. Misura, Characterization of the spectra of the periodic and antiperiodic boundary value problems that are generated by the Dirac operator, I. Function theory, functional analysis and their applications. 1978, Vol. 30, pp. 90-101 [in Russian].

Dirak operatorining yagona lakunali potensiali $u_0(x)=q_0(x)-ip_0(x)$ ushbu $\{\mu_{-1},\mu_0,\xi_0(0),\sigma_0(0)=\pm1\}$ spektral berilganlar bilan keltirilgan holni qaraymiz. Ushbu holda chekli zichlikli Hirota tipidagi tenglama uchun BQM quyidagi koʻrinishda boʻladi:

$$\begin{cases} p_t=[p_{xxx}+6(\rho^2-\{p^2+q^2\})p_x]-[q_{xx}+2q(\rho^2-\{p^2+q^2\})], \\ q_t=[q_{xxx}+6(\rho^2-\{p^2+q^2\})q_x]+[p_{xx}+2p(\rho^2-\{p^2+q^2\})], \end{cases} \qquad (15)$$

$$\begin{cases} p(x,t)|_{t=0}=\dfrac{\mu_0-\mu_{-1}}{2}\cos\{-(\mu_0+\mu_{-1})x+2\sigma_0(0)\phi(0)\}, \\ q(x,t)|_{t=0}=-\sigma_0(\tau)\dfrac{\mu_0-\mu_{-1}}{2}\sin\{-(\mu_0+\mu_{-1})x+2\sigma_0(0)\phi(0)\}, \end{cases} \qquad (16)$$

bu yerda

$$\phi(0)=\arcsin\sqrt{\frac{\xi_0(0)-\mu_{-1}}{\mu_0-\mu_{-1}}},\quad \sigma_0(\tau)=\operatorname{sgn}\{\sin\{-(\mu_{-1}+\mu_0)x+2\sigma_0(0)\phi(0)\}\}.$$

$\mathcal{L}(\tau,t)$ Dirak operatori uchun teskari spketral masalalar usulini qoʻllagan holda (15), (16) BQMning yechimi topamiz:

$$p(x,t)=\frac{\mu_0-\mu_{-1}}{2}\cos\big(2K(\rho)t-(\mu_{-1}+\mu_0)x+2\sigma_0(0)\phi(0)\big),$$

$$q(x,t)=-\frac{\mu_0-\mu_{-1}}{2}\sin\big(2K(\rho)t-(\mu_{-1}+\mu_0)x+2\sigma_0(0)\phi(0)\big),$$

bu yerda

$$K(\rho)=\frac{\mu_{-1}+\mu_0}{4}[5\mu_{-1}^2+5\mu_0^2-2\mu_{-1}\mu_0+3\mu_{-1}+3\mu_0-12\rho^2]-\rho^2-\mu_{-1}\mu_0.$$

“Qoʻshimcha hadlar bilan berilgan Hirota tipidagi tenglama uchun boshlangʻich qiymat masalasi” deb nomlanuvchi ikkinchi bobning ikkinchi paragrafida

$$\begin{cases} p_t=[p_{xxx}-6p_x(p^2+q^2)]+[-q_{xx}+2q(p^2+q^2)]+p_x+q, \\ q_t=[q_{xxx}-6q_x(p^2+q^2)]+[p_{xx}-2p(p^2+q^2)]+q_x-p, \end{cases} \qquad (17)$$

ushbu qoʻshimcha hadlar bilan berilgan Hirota tipidagi tenglamaning quyidagi

$$\begin{cases} p(x,t)|_{t=0}=p_0(x),\;\; p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}), \\ q(x,t)|_{t=0}=q_0(x),\;\; q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}), \end{cases} \qquad (18)$$

boshlangʻich va

$$\begin{cases} p(x+\pi,t)=p(x,t),\;\; q(x+\pi,t)=q(x,t),\quad x\in\mathbb{R},t>0, \\ p(x,t),q(x,t)\in C_x^3(t>0)\cap C_t^1(t>0)\cap C(t\geqslant 0). \end{cases} \qquad (19)$$

silliqlik shartlarini qanoatlantiruvchi x oʻzgaruvchi boʻyicha π davrli cheksiz zonali yagona global yechimi mavjud ekanligini (1) Dirak operatori uchun teskari spektral masalalar usuli yordamida isbotlangan.

Dastlab $\mathcal{L}(\tau,t)$ Dirak operatorining spektral berilganlari evolyutsiyasini ifodalovchi Dubrovin differensial tenglamalar sistemasini keltiramiz.

6-lemma. Aytaylik, $\{p(x,t),q(x,t),x\in\mathbb{R},t>0\}$ funksiyalar (17)-(19) BQM yechimidan iborat boʻlsin. U holda $\mathcal{L}(\tau,t)$ Dirak operatorining

$\{\mu_n(t), \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}\}$ spektral berilganlari Quyidagi Dubrovin differensial tenglamalar sistemlarini qanoatlantiradi:

$$1. \frac{\partial \mu_n(t)}{\partial t} = 0, \quad n \in \mathbb{Z}, \tag{20}$$

$$2. \frac{\partial \xi_n(\tau,t)}{\partial \tau} = 2(-1)^{n-1}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})(\mu_{2n}-\xi_n(\tau,t))f_n(\xi)} \times$$
$$\times \{p(\tau,t)+\xi_n(\tau,t)\}, n \in \mathbb{Z}, \tag{21}$$

$$3. \frac{\partial \xi_n(\tau,t)}{\partial t} = 2(-1)^{n}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})(\mu_{2n}-\xi_n(\tau,t))f_n(\xi)g_{n,2}(\xi)}. \tag{22}$$

Shuningdek, quyidagi boshlang‘ich shartlar o‘rinli:

$$\xi_n(\tau,t)|_{t=0} = \xi_n^0(\tau),\ \sigma_n(\tau,t)|_{t=0} = \sigma_n^0(\tau),\ \ n \in \mathbb{Z}, \tag{23}$$

bu yerda $\xi_n^0(\tau), \sigma_n^0(\tau) = \pm 1, n \in \mathbb{Z}$ lar $\mathcal{L}(\tau,0)$ Dirak operatorining spektral parametrlari. $f_n(\xi), n \in \mathbb{Z}$ ketma-ketlik esa (11) formula yordamida aniqlanadi va

$$g_{n,2}(\xi) = [4\xi_n^3 + 4p\xi_n^2 + 2\xi_n(p^2+q^2+q_\tau) - p_{\tau\tau} + 2(pq_\tau - p_\tau q) +$$
$$+2p(p^2+q^2)] + [2\xi_n^2 + 2p\xi_n + p^2 + q^2 + q_\tau] + \left[\frac{1}{2} - \xi_n - p\right], n \in \mathbb{Z},$$

$p = p(\tau,t), q = q(\tau,t),\ \xi \equiv \xi(\tau,t) = (\dots, \xi_{-1}(\tau,t), \xi_0(\tau,t), \xi_1(\tau,t), \dots),$
$\sigma \equiv \sigma(\tau,t) = (\dots, \sigma_{-1}(\tau,t), \sigma_0(\tau,t), \sigma_1(\tau,t), \dots).$

Ushbu

$$\xi_n(\tau,t) = \mu_{2n-1} + (\mu_{2n} - \mu_{2n-1})\sin^2\big(x_n(\tau,t)\big), n \in \mathbb{Z},$$

almashtirish yordamida (22), (23) BQM $\mathcal{K}$ Banax fazosida yagona masala sifatida ifodalaniladi:

$$\frac{dx(\tau,t)}{dt} = \mathcal{H}\big(x(\tau,t)\big),\ \ x(\tau,t)|_{t=0} = x^0(\tau),\ \ x^0(\tau) \in \mathcal{K}, \tag{24}$$

bu yerda

$$\mathcal{K} = \Big\{x = (\dots, x_{-1}(\tau,t), x_0(\tau,t), x_1(\tau,t), \dots):$$
$$\|x\| = \sum_{n=-\infty}^{+\infty}(1+|n|^2)(\mu_{2n}-\mu_{2n-1})|x_n| < \infty\Big\},$$

$\mathcal{H}(x) = (\dots, \mathcal{H}_{-1}(x), \mathcal{H}_1(x), \dots),$

$$\mathcal{H}_n(x(\tau,t)) = (-1)^n\sigma_n^0(\tau)\bar{f}_n\big(\dots, \lambda_1 + (\lambda_2-\lambda_1)\sin^2\big(x_1(\tau,t)\big), \dots\big) \times$$
$$\times\, \overline{g}_{n,2}\big(\dots, \lambda_1 + (\lambda_2-\lambda_1)\sin^2\big(x_1(\tau,t)\big), \dots\big) = (-1)^n\sigma_n^0(\tau)\overline{f}_n\big(x(\tau,t)\big)\overline{g}_{n,2}\big(x(\tau,t)\big).$$

7-lemma. Agar boshlang‘ich shartdagi $p_0(x)$ va $q_0(x)$ funksiyalar

$$p_0(x+\pi) = p_0(x) \in C^6(\mathbb{R}),\ \ q_0(x+\pi) = q_0(x) \in C^6(\mathbb{R}),$$

shartlarni qanoatlantirsa, u holda $\mathcal{H}(x)$ vektor funksiya $\mathcal{K}$ Banax fazosida Lipshits shartini qanoatlantiradi, ya’ni, shunday o‘zgarmas $N = \text{const} > 0$ mavjudki, barcha $x, y \in \mathcal{K}$ elementlar uchun ushbu

$$\|\mathcal{H}(x) - \mathcal{H}(y)\| \le N\|x-y\|,$$

tengsizlik o‘rinli bo‘ladi. Bu yerda $N = A\sum_{n=-\infty}^{+\infty}\gamma_n(1+|n|^3)(1+|n|^2) < \infty.$

7-lemmadan (24) BQMning, boshqa so‘z bilan aytganda (22), (23) BQMning barcha $t > 0$ va $\tau \in \mathbb{R}$ qiymatlarda yagona yechimi mavjud ekanligi kelib chiqadi.

Endi 6-lemma va 7-lemmalardan foydalanib, (17)-(19) BQMning yechimini topish algoritmini keltiramiz:

1) Dastlab, $\mathcal{L}(\tau,0)$ Dirak operatorining $\{\mu_n,\xi_n^0(\tau),\sigma_n^0(\tau),n\in\mathbb{Z}\}$ spektral berilganlarini topamiz;

2) So‘ngra $\mathcal{L}(\tau,t)$ Dirak operatorining spektral berilganlarini $\{\mu_n,\xi_n(\tau,t),\sigma_n(\tau,t),n\in\mathbb{Z}\}$ orqali belgilaymiz va (22), (23) BQMni ixtiyoriy τ qiymatlarda yechib, $\{\xi_n(\tau,t),\sigma_n(\tau,t),n\in\mathbb{Z}\}$ spektral parametrlarni topamiz;

3) (3) va (4) izlar formulalaridan foydalanib, $p(\tau,t)$ va $q(\tau,t)$ funksiyalarni, ya’ni (22)-(23) BQMning yechimini topamiz.

Shunday qilib biz quyidagi teoremani isbotladik.

4-teorema. Agar boshlang‘ich shartdagi $p_0(x),q_0(x)$ funksiyalar ushbu

$$p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}),q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}),$$

shartlarni qanoatlantirsa, u holda (17), (19) BQM yagona $C_x^3(t>0)\cap C_t^1(t>0)\cap C(t\geqslant 0)$ sinfga tegishli bo‘lgan $p(x,t)$ va $q(x,t)$ yechimlarga ega. Ushbu yechimlar mos ravishda (3) va (4) formulalar yordamida aniqlanadi.

Dissertatsiyaning uchinchi bobi **“Moslangan manbalar bilan berilgan Hirota tipidagi tenglama uchun Koshi masalasi”** deb nomlanadi va ushbu bobning birinchi paragrafida ushbu

$$\begin{cases} p_t=[p_{xxx}-6p_x(p^2+q^2)]+[-q_{xx}+2q(p^2+q^2)]- \\ -\sum\limits_{k=-\infty}^{+\infty}\zeta_k(t)v_1(\pi,\mu_k,t)\big(\psi_1^+(x,\mu_k,t)\psi_2^-(x,\mu_k,t)+\psi_1^-(x,\mu_k,t)\psi_2^+(x,\mu_k,t)\big), \\ q_t=[q_{xxx}-6q_x(p^2+q^2)]+[p_{xx}-2p(p^2+q^2)]+ \\ +\sum\limits_{k=-\infty}^{+\infty}\zeta_k(t)v_1(\pi,\mu_k,t)\big(\psi_1^+(x,\mu_k,t)\psi_1^-(x,\mu_k,t)-\psi_2^+(x,\mu_k,t)\psi_2^-(x,\mu_k,t)\big), \end{cases} \tag{25}$$

qator moslangan manbali Hirota tipidagi tenglamaning quyidagi

$$\begin{cases} p(x,t)|_{t=0}=p_0(x)\in C^6(\mathbb{R}),\ \ p_0(x)=p_0(x+\pi), \\ q(x,t)|_{t=0}=q_0(x)\in C^6(\mathbb{R}),\ \ q_0(x)=q_0(x+\pi), \end{cases} \tag{26}$$

$$\begin{cases} p(x+\pi,t)=p(x,t),\ \ q(x+\pi,t)=q(x,t), \\ p(x,t),q(x,t)\in C_{x,t}^{3,1}(t>0)\cap C(t\geq 0),\ \ x\in\mathbb{R},t>0. \end{cases} \tag{27}$$

boshlang‘ich va silliqlik shartlarini qanoatlantiruvchi yechimi topish masalasi qaralgan. Bu yerda $\zeta_n(t)$ –berilgan haqiqiy uzluksiz funksiyalardan iborat ketma-ketlik bo‘lib, u quyidagi asimptotikani qanoatlantiradi:

$$\zeta_n(t)=O\left(\frac{1}{1+n^2}\right),\ \ |n|\to\infty. \tag{28}$$

Shuningdek, $\psi^\pm(x,\mu,t)=\left(\psi_1^\pm(x,\mu,t),\psi_2^\pm(x,\mu,t)\right)^T$ funksiyalar Dirak sistemasining Floke yechimlari, ya’ni

$$\mathcal{L}(t)\psi^\pm\equiv\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}(\psi_1^\pm)'\\ (\psi_2^\pm)'\end{pmatrix}+\begin{pmatrix}p(x,t) & q(x,t)\\ q(x,t) & -p(x,t)\end{pmatrix}\begin{pmatrix}\psi_1^\pm\\ \psi_2^\pm\end{pmatrix}=\mu\begin{pmatrix}\psi_1^\pm\\ \psi_2^\pm\end{pmatrix},x\in\mathbb{R},t>0.$$

Ushbu

$$v_1(\pi,\mu,t)[\psi_1^+(\mu,\tau,t)\psi_2^-(\mu,\tau,t)+\psi_1^-(\mu,\tau,t)\psi_2^+(\mu,\tau,t)]=$$
$$=v_2(\pi,\mu,\tau,t)-u_1(\pi,\mu,\tau,t), \tag{29}$$
$$v_1(\pi,\mu,t)[\psi_1^+(\mu,\tau,t)\psi_1^-(\mu,\tau,t)-\psi_2^+(\mu,\tau,t)\psi_2^-(\mu,\tau,t)]=$$
$$=v_1(\pi,\mu,\tau,t)+u_2(\pi,\mu,\tau,t),$$
ayniyatlar va quyidagi asimptotikalar
$$\begin{aligned} v_2(\pi,\mu,\tau,t)-u_1(\pi,\tau,\mu,\tau,t)&=O\left(\frac{1}{1+\mu}\right), \quad |\mu|\to\infty,\\ v_1(\pi,\tau,\mu,\tau,t)+u_2(\pi,\mu,\tau,t)&=O\left(\frac{1}{1+\mu}\right), \quad |\mu|\to\infty, \end{aligned} \tag{30}$$
shuningdek, (28) munosbatdan foydalanib, (25) sistema o‘ng tomonidagi funksional qatorlarning tekis yaqinlashishini va (25) sistema o‘ng tomoni x o‘zgaruvchi bo‘yicha π davrli ekanligini isbotlash mumkin.

Ushbu paragrafda (25)-(27) Koshi masalasining yagona global cheksiz zonali $\{p(x,t),q(x,t),x\in\mathbb{R},t>0\}$ yechimi mavjud ekanligini $\mathcal{L}(\tau,t)$ Dirak operatori uchun teskari spektral masalalar usuli yordamida isbotlangan.

Quyidagi teorema asosiy natijani ifodalaydi.

5-teorema. Agar davriy cheksiz zonali $p_0(x)$ va $q_0(x)$ funksiyalar ushbu
$$p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}),\ \ q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}),$$
shartlarni qanoatlantirsa, u holda (25)-(27) Koshi masalasining yagona aniqlangan davriy cheksiz zonali $p(x,t),q(x,t),\ x\in\mathbb{R},t>0$ yechimi mavud bo‘ladi va ubu yechimlar mos ravishda (3), (4) formulalar yordamida topiladi.

Endi $\mathcal{L}(\tau,t)$ Dirak operatorining spektral berilganlari evolyutsiyasini ifodalovchi Dubrovin differensial tenglamalar sistemasini keltiramiz.

8-lemma. Aytaylik, $\{p(x,t),q(x,t),x\in\mathbb{R},t>0\}$ funksiyalar (25)-(27) Koshi masalasining yechimidan iborat bo‘lsin. U holda $\mu_n(t),n\in\mathbb{Z}$ xos qiymatlar t parametrga bog‘liq bo‘lmaydi, ya’ni, $\mu_n(t)=\mu_n,n\in\mathbb{Z}$ va $\xi_n(\tau,t),\sigma_n(\tau,t)=\pm1,n\in\mathbb{Z}$ spektral parametrlar mos ravishda birinchi va ikkinchi Dubrovin differensial tenglamalar sistemasini qanotlantiradi:

1. $$\frac{\partial\xi_n(\tau,t)}{\partial\tau}=2(-1)^{n-1}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})\big(\mu_{2n}-\xi_n(\tau,t)\big)}\times$$
$$\times f_n(\xi)\{p(\tau,t)+\xi_n(\tau,t)\},\ n\in\mathbb{Z}; \tag{31}$$

2. $$\frac{\partial\xi_n(\tau,t)}{\partial t}=2(-1)^{n}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})\big(\mu_{2n}-\xi_n(\tau,t)\big)}f_n(\xi)g_{n,3}(\xi). \tag{32}$$

Shuningdek, quyidagi boshlang‘ich shartlar o‘rinli:
$$\xi_n(\tau,t)|_{t=0}=\xi_n^0(\tau),\ \ \sigma_n(\tau,t)|_{t=0}=\sigma_n^0(\tau),n\in\mathbb{Z}, \tag{33}$$
bu yerda $\xi_n^0(\tau),\sigma_n^0(\tau)=\pm1,n\in\mathbb{Z}$ lar $\mathcal{L}(\tau,0)$ Dirak operatorining spektral parametrlari. $f_n(\xi),\ n\in\mathbb{Z}$ ketma-ketlik (11) formula yordamida aniqlanadi va
$$g_{n,3}(\xi)=[4\xi_n^3+4p\xi_n^2+2\xi_n(p^2+q^2+q_\tau)-p_{\tau\tau}+2(pq_\tau-p_\tau q)+2p(p^2+q^2)]$$
$$+[2\xi_n^2+2\xi_n p+p^2+q^2+q_\tau]+\sum_{k=-\infty}^{+\infty}\frac{\zeta_k(t)v_1(\pi,\mu_k,\tau,t)}{\xi_n-\mu_k},\ \ n\in\mathbb{Z},$$
bunda $p=p(\tau,t),q=q(\tau,t),\ \xi\equiv\xi(\tau,t)=(\dots,\xi_{-1}(\tau,t),\xi_0(\tau,t),\xi_1(\tau,t),\dots)$, $\sigma\equiv\sigma(\tau,t)=(\dots,\sigma_{-1}(\tau,t),\sigma_0(\tau,t),\sigma_1(\tau,t),\dots)$,

$$v_1(\pi,\mu_k,\tau,t)=\pi\prod_{n=-\infty}^{+\infty}\frac{\xi_n(\tau,t)-\mu_k}{\beta_n},\quad \beta_n=\begin{cases}n, & n\neq 0,\\ 1, & n=0,\end{cases}$$

Cheksiz Dubrovin differensial tenglamalar sistemasi uchun Koshi masalasining yechimi mavjud va yagona ekanligini isbotlash uchun quyidagi almashtirishdan foydalanamiz:

$$\xi_n(\tau,t)=\mu_{2n-1}+(\mu_{2n}-\mu_{2n-1})\sin^2 x_n(\tau,t)\,,\quad n\in\mathbb{Z}.$$

U holda (32), (33) Koshi masalasi $\mathcal{K}$ Banax fazosida yagona masala ko‘rinishida ifodalanadi:

$$\begin{cases}\dfrac{\partial x(\tau,t)}{\partial t}=\mathbb{H}\big(x(\tau,t)\big),\\ x(\tau,t)|_{t=0}=x^0(\tau)\in\mathcal{K},\end{cases}\tag{34}$$

bu yerda $\mathbb{H}\big(x(\tau,t)\big)=\big(\dots,\mathbb{H}_{-1}\big(x(\tau,t)\big),\mathbb{H}_0\big(x(\tau,t)\big),\mathbb{H}_1\big(x(\tau,t)\big),\dots\big)$,

$$\mathcal{K}=\Big\{x=(\dots,x_{-1}(\tau,t),x_0(\tau,t),x_1(\tau,t),\dots):\ \|x\|=\sum_{n=-\infty}^{+\infty}(1+|n|^2)(\mu_{2n}-\mu_{2n-1})|x_n|<\infty\Big\}.$$

9-lemma. Agar boshlang‘ich shartdagi $p_0(x)$ va $q_0(x)$ funksiyalar ushbu

$$p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}),\ \ q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}),$$

shartlarni qanoatlantirsa, u holda $\mathbb{H}(x)$ vector funksiya $\mathcal{K}$ Banax fazosida Lipshits shartini qanoatlantiradi.

Shuning uchun (34) Koshi masalasibarcha $t>0$ va $\tau\in\mathbb{R}$ qiymatlarda yagona yechimga ega bo‘ladi. Bundan esa (32), (33) Koshi masalasining barcha $t>0$ va $\tau\in\mathbb{R}$ qiymatlarda yagona yechimi mavjud ekanligi kelib chiqadi.

5-teorema va 9-lemmalardan foydalanib, (25)-(27) Koshi masalasining yechimini topish algoritmini keltiramiz:

1) Dastlab, $\mathcal{L}(\tau,0)$ Dirak operatorining $\{\mu_n,\xi_n^0(\tau),\sigma_n^0(\tau),n\in\mathbb{Z}\}$ spektral berilganlarini topamiz;

2) So‘ngra (32), (33) Koshi masalasini ixtiyoriy τ qiymatlarda yechib, $\{\xi_n(\tau,t),\sigma_n(\tau,t),n\in\mathbb{Z}\}$ spektral parametrlarni topamiz;

3) (3) va (4) izlar formulalaridan foydalanib, $p(\tau,t)$ va $q(\tau,t)$ funksiyalarni, ya’ni (37)-(39) Koshi masalasining yechimini topamiz;

4) Topilgan $p(\tau,t)$ va $q(\tau,t)$ funksiyalar yordamida $\psi^{\pm}(x,\mu,t)=\Big(\psi_1^{\pm}(x,\mu,t),\psi_2^{\pm}(x,\mu,t)\Big)^T$ Floke yechimlarini aniqlaymiz.

Endi yagona lakuna $u_0(x)=q_0(x)-ip_0(x)$ potensialli, $E=\mathbb{R}\backslash(\mu_{-1},\mu_0)$ spektr va $\xi_0(0)\in[\mu_{-1},\mu_0]$, $\sigma_0(0)=\pm1$ spektral parametrlar yordamida aniqlangan $\mathcal{L}(\tau,0)$ Dirak operatorini qaraymiz. Bu holda qaralgan Koshi masalasi quyidagi ko‘rinishda bo‘ladi:

$$
\begin{cases}
p_t = [p_{xxx} - 6p_x(p^2+q^2)] + [-q_{xx} + 2q(p^2+q^2)] - \\
- \sum\limits_{k=-1}^{0} \zeta_k(t) v_1(\pi, \mu_k, t)\big(\psi_1^+(x,\mu_k,t)\psi_2^-(x,\mu_k,t) + \psi_1^-(x,\mu_k,t)\psi_2^+(x,\mu_k,t)\big), \\
q_t = [q_{xxx} - 6q_x(p^2+q^2)] + [p_{xx} - 2p(p^2+q^2)] + \\
+ \sum\limits_{k=-1}^{0} \zeta_k(t) v_1(\pi, \mu_k, t)\big(\psi_1^+(x,\mu_k,t)\psi_1^-(x,\mu_k,t) - \psi_2^+(x,\mu_k,t)\psi_2^-(x,\mu_k,t)\big),
\end{cases} \tag{35}
$$

$$
\begin{cases}
p(x,t)|_{t=0} = \dfrac{\mu_0 - \mu_{-1}}{2}\cos\{-(\mu_0 + \mu_{-1})x + 2\sigma_0(0)\phi(0)\}, \\
q(x,t)|_{t=0} = -\dfrac{\mu_0 - \mu_{-1}}{2}\sin\{-(\mu_0 + \mu_{-1})x + 2\sigma_0(0)\phi(0)\}.
\end{cases} \tag{36}
$$

Yuqorida keltirilgan algoritmdan foydalanib, (35), (36) Koshi masalasining yechimi topamiz:

$$
p(\tau,t) = \frac{\mu_0 - \mu_{-1}}{2}\cos\{(2K+2M)t + 2\pi\int_0^t \left(\zeta_{-1}(s) + \zeta_0(s)\right)ds -
$$

$$
-(\mu_0 + \mu_{-1})\tau + 2\sigma_0(0)\phi(0)\},
$$

$$
q(\tau,t) = -\frac{\mu_0 - \mu_{-1}}{2}\sin\{(2K+2M)t + 2\pi\int_0^t \left(\zeta_{-1}(s) + \zeta_0(s)\right)ds -
$$

$$
-(\mu_0 + \mu_{-1})\tau + 2\sigma_0(0)\phi(0)\},
$$

bu yerda

$$
K = \frac{5}{4}(\mu_{-1} + \mu_0)^3 - 3\mu_{-1}\mu_0(\mu_{-1} + \mu_0), \;\; M = \frac{3}{4}(\mu_{-1} + \mu_0)^2 - \mu_{-1}\mu_0,
$$

$$
\phi(0) = \arcsin\sqrt{\frac{\xi_0(0) - \mu_{-1}}{\mu_0 - \mu_{-1}}}.
$$

Uchinchi bobning **“Integral moslangan manbali Hirota tipidagi tenglama uchun Koshi masalasi”** deb nomalanuvchi ikkinchi paragrafida quyidagi integral manbali Hirota tipidagi tenglama uchun Koshi masalasi qaralgan:

$$
\begin{cases}
p_t = [p_{xxx} - 6p_x(p^2+q^2)] + [-q_{xx} + 2q(p^2+q^2)] - \\
- \int_{-\infty}^{+\infty} \zeta(\mu,t) v_1(\pi,\mu,t)\big(\psi_1^+(x,\mu,t)\psi_2^-(x,\mu,t) + \psi_2^+(x,\mu,t)\psi_1^-(x,\mu,t)\big)d\mu, \\
q_t = [q_{xxx} - 6q_x(p^2+q^2)] + [p_{xx} - 2p(p^2+q^2)] + \\
+ \int_{-\infty}^{+\infty} \zeta(\mu,t) v_1(\pi,\mu,t)\big(\psi_1^+(x,\mu,t)\psi_1^-(x,\mu,t) - \psi_2^+(x,\mu,t)\psi_2^-(x,\mu,t)\big)d\mu,
\end{cases} \tag{37}
$$

$$
\begin{cases}
p(x,t)|_{t=0} = p_0(x), \;\; p_0(x+\pi) = p_0(x) \in C^6(\mathbb{R}), \\
q(x,t)|_{t=0} = q_0(x), \;\; q_0(x+\pi) = q_0(x) \in C^6(\mathbb{R})
\end{cases} \tag{38}
$$

$$
\begin{cases}
p(x+\pi,t) = p(x,t), \;\; q(x+\pi,t) = q(x,t), \;\; x \in \mathbb{R}, \;\; t > 0, \\
p(x,t), q(x,t) \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0).
\end{cases} \tag{39}
$$

Bu yerda $\zeta(\mu,t)$ –berilgan uzluksiz funksiya bo‘lib, u quyidagi asimptotikani qanoatlantiradi:

$$\zeta(\mu,t)=O\left(\frac{1}{1+\mu^2}\right),\quad |\mu|\to\infty. \tag{40}$$

(29) ayniyatlar, (28) va (30) asimptotikalardan foydalanib, (37) sistemaning o‘ng tomonidagi xosmas integrallarning tekis yaqinlashuvchi ekanligini va (37) sistemaning o‘ng tomoni x o‘zgaruvchi bo‘yicha π davrli ekanligini isbotlash mumkin.

Ushbu paragrafning asosiy maqsadi boshlang‘ich shartdagi $p_0(x)$ va $q_0(x)$ funksiyalar qanday mimimal silliqlik shartlarini qanoatlantirganda (37)-(39) Koshi masalasi yagona global yechimga ega bo‘lishini aniqlashdan iborat.

Quyidagi lemma $\mathcal{L}(\tau,t)$ Dirak operatori spektral berilganlarining evolyutsiyasini ifodalaydi.

10-lemma. Aytaylik, $p(x,t),\ q(x,t),\psi^{\pm}(x,\mu,t),x\in\mathbb{R},t>0$ funksiyalar (37)–(39) Koshi masalasining yechimidan iborat bo‘lsin. U holda $\mu_n(t)=\mu_n$va $\{\xi_n(\tau,t),\sigma_n(\tau,t)=\pm1,n\in\mathbb{Z}\}$ spektral parametrlar Dubrovin differensial tenglamalar sistemalarini qanoatlantiradi:

$$1.\frac{\partial\xi_n(\tau,t)}{\partial\tau}=2(-1)^{n-1}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})(\mu_{2n}-\xi_n(\tau,t))f_n(\xi)}\times$$
$$\times\left(p(\tau,t)+\xi_n(\tau,t)\right),n\in\mathbb{Z}. \tag{41}$$

$$2.\frac{\partial\xi_n(\tau,t)}{\partial t}=2(-1)^{n}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})(\mu_{2n}-\xi_n(\tau,t))f_n(\xi)}g_{n,4}(\xi). \tag{42}$$

$f_n(\xi)$ ketma-ketlik (11) formula yordamida topiladi va

$$g_{n,4}\big(\xi(\tau,t)\big)=\{4\xi_n^3+4p\xi_n^2+2\xi_n(p^2+q^2+q_\tau)-p_{\tau\tau}+2(pq_\tau-p_\tau q)+$$
$$+2p(p^2+q^2)\}+\{2\xi_n^2+2\xi_np+p^2+q^2+q_\tau\}+\int_{-\infty}^{+\infty}\frac{\zeta(\mu,t)v_1(\pi,\mu,\tau,t)}{\xi_n-\mu}d\mu,\ \ n\in\mathbb{Z}.$$

Bu yerda $\xi_n=\xi_n(\tau,t),p=p(\tau,t),q=q(\tau,t)$ va

$$v_1(\pi,\mu,\tau,t)=\pi\prod_{k=-\infty}^{+\infty}\frac{\xi_k(\tau,t)-\mu}{\beta_k},\quad \beta_k=\begin{cases}1, & k=0\\ k, & k\neq0\end{cases}.$$

Shuningdek, quyidagi boshlang‘ich shartlar o‘rinli:

$$\xi_n(\tau,t)|_{t=0}=\xi_n^0(\tau),\ \ \sigma_n(\tau,t)|_{t=0}=\sigma_n^0(\tau),\ n\in\mathbb{Z}. \tag{43}$$

11-lemma. Agar boshlang‘ich shartdagi $p_0(x),q_0(x)$ funksiyalar ushbu

$$p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}),q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}),$$

shartlarni qanoatlantirsa, u holda (42), (43) Koshi masalasining barcha $t>0$ va $\tau\in\mathbb{R}$ qiymatlarda yechimi mavjud va yagona bo‘ladi.

Endi 10-lemma va 11-lemmalardan foydalanib, (37)-(39) Koshi masalasining yechimini topish algoritmini keltiramiz:

1) Dastlab, $\mathcal{L}(\tau,0)$ Dirak operatorining $\{\mu_n,\xi_n^0(\tau),\sigma_n^0(\tau),n\in\mathbb{Z}\}$ spektral berilganlarini topamiz;

2) So‘ngra (22), (23) Koshi masalasini ixtiyoriy τ qiymatlarda yechib, $\{\xi_n(\tau,t),\sigma_n(\tau,t),n\in\mathbb{Z}\}$ spektral parametrlarni topamiz;

3) (3) va (4) izlar formulalaridan foydalanib, $p(\tau,t)$ va $q(\tau,t)$ funksiyalarni, ya’ni (37)-(39) Koshi masalasining yechimini topamiz;

4) Topilgan $p(\tau,t)$ va $q(\tau,t)$ funksiyalar yordamida $\psi^{\pm}(x,\mu,t)=\left(\psi_1^{\pm}(x,\mu,t),\psi_2^{\pm}(x,\mu,t)\right)^T$ Floke yechimlarini aniqlaymiz.

Shunday qilib biz quyidagi teoremani isbotladik.

6-teorema. Agar boshlang‘ich shartdagi $p_0(x), q_0(x)$ funksiyalar ushbu

$$p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}), q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}),$$

shartlarni qanoatlantirsa, u holda (37), (39) BQM yagona $C_x^3(t>0)\cap C_t^1(t>0)\cap C(t\geqslant 0)$ sinfga tegishli bo‘lgan $p(x,t)$ va $q(x,t)$ yechimlarga ega. Ushbu yechimlar mos ravishda (3) va (4) formulalar yordamida aniqlanadi.

XULOSA

Dissertatsiya ishi davriy cheksiz zonali funksiyalar sinfidagi chekli zichlikli Hirota tipidagi tenglamani, qo‘shimcha hadlar bilan berilgan Hirota tipidagi tenglamani va moslangan qator va integral manbalarga ega Hirota tipidagi tenglamalarni integrallashga bag‘ishlangan.

Dissertatsiyaning asosiy natijalari quyidagilardan iborat:

– davriy Dirak operatori uchun teskari spektral masalalar usulidan foydalanib, olti marta uzluksiz differensiallanuvchi davriy cheksiz zonali funksiyalar sinfida chekli zichlikli Hirota tipidagi tenglama integrallanuvchi ekanligi isbotlangan;

– olti marta uzluksiz differensiallanuvchi funksiyalar sinfidagi qo‘shimcha hadlar bilan berilgan Hirota tipidagi tenglama uchun Koshi masalasining yagona yechimi mavjudligi isbotlangan;

– davriy Dirak operatori uchun teskari spektral masalalar usulidan foydalanib, moslangan qator manbaga ega bo‘lgan Hirota tipidagi tenglamaning integrallanuvchi ekanligi isbotlangan;

– olti marta uzluksiz differentsiallanuvchi davriy cheksiz zonali funksiyalar sinfida integral ko‘rinishdagi moslangan manbaga ega Hirota tipidagi tenglama uchun Koshi masalasining yechimi mavjud va yagona ekanligi isbotlangan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 AT V.I. ROMANOVSKY INSTITUTE OF MATHEMATICS, UZBEKISTAN ACADEMY OF SCIENCES

SAMARKAND STATE UNIVERSITY NAMED AFTER SHAROF RASHIDOV

ESHBEKOV RAYKHONBEK KHUBAYDULLO UGLI

THE CAUCHY PROBLEM FOR THE HIROTA TYPE EQUATION WITH A FINITE DENSITY AND SELF-CONSISTENT SOURCE

01.01.02 - Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

Tashkent – 2025

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Abstract of dissertation sent out on «____» _______________2025 year
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INTRODUCTION (Abstract of the PhD dissertation)

Relevance and importance of the dissertation topic. In many scientific and applied researches conducted in the world, special importance is given to the study of the direct and inverse spectral problems for the Dirac and Sturm-Liouville operators. Currently, in developed countries, the direct and inverse problems of spectral analysis are of great importance in finding solutions of the evolutionary equations of modern mathematical physics and determining the class of solutions. In addition, these issues play an important role in radio engineering, nonlinear optics, quantum mechanics, and modeling the crystalline properties of amorphous bodies. The algorithm for solving the initial problem for the nonlinear Hirota equation with finite density and self-consistent sources using the method of inverse spectral problems for the periodic Dirac operator is of great importance in the study of the behavior of nonlinear waves in some physical media and in the analysis of fiber-optic systems, including in the areas mentioned above.

In scientific research conducted in the world in the fields of various ultrashort light pulses propagating in nonlinear media, hydrodynamics, laser beam propagation in nonlinear media, nonlinear optics, quantum mechanics, the integration of the Hirota equation with finite density in the class of periodic infinite-gap functions is of great importance. At the same time, the problem of finding a solution of the Cauchy problem for the nonlinear Hirota equation with a self-consistent integral or series sources in the class of periodic infinite-gap functions is considered an urgent task in studying the important properties of the above-mentioned physical processes and in developing scientific research.

In our republic, large-scale measures are being taken to determine solutions of nonlinear evolutionary equations of mathematical physics using the method of direct and inverse spectral problems for the Dirac operator and to apply the solutions found in practice. The main tasks and areas of activity of the mathematical science are to conduct scientific research at the level of international standards in the priority areas of the disciplines “Algebra and its applications, differential equations and their applications, nonlinear systems, dynamic systems and their applications, stochastic analysis, medical and biological informatics, computational mathematics[4]”. In implementing this task, in particular, in the integration of nonlinear evolutionary equations of modern mathematical physics, it is of great scientific importance to show the existence of a solution of the Initial Value Problem for nonlinear Hirota equation with finite density or self-consistent source in the class of periodic infinite-gap functions using the method of inverse spectral problems.

[4] Resolution of the President of the Republic of Uzbekistan No. PR-4387 “On measures of state support for the further development of mathematical education and science, as well as the radical improvement of the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan” dated July 9, 2019.

This dissertation research will serve to a certain extent in the implementation of the tasks set out in the Decree of the President of the Republic of Uzbekistan No. PD-4947 “On the Strategy of Actions for the Further Development of the Republic of Uzbekistan” dated February 7, 2017, the Resolution No. PR-4387 “On measures of state support for the further development of mathematical education and science, as well as the radical improvement of the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan” dated July 9, 2019, and other regulatory legal acts related to this activity.

Compliance of the research with the priority areas of development of science and technology of the republic. This research was carried out in accordance with the priority area of development of science and technology in the Republic of Uzbekistan IV. “Mathematics, mechanics and computer science”.

Level of problem study. In 1946, Borg demonstrated that for a Sturm–Liouville operator on the interval $[0,\pi]$ with Dirichlet boundary conditions at both ends, the potential function $q(x)$ can be uniquely determined if two different spectra are known–for instance, the spectrum for Dirichlet-Dirichlet and Dirichlet-Neumann boundary conditions.

Shortly thereafter, in 1951, Gelfand and Levitan provided an analytical method for solving the inverse problem using spectral data (eigenvalues and norming constants). After that, Marchenko developed the theory, central to inverse scattering theory, enabling the reconstruction of potentials from scattering data.

In 1967, Gardner, Greene, Kruskal, and Miura introduced the solving the Cauchy problem for Korteweg–de Vries(KdV) equation

$$q_t = 6qq_x - q_{xxx}, \ \ q(x,t)|_{t=0} = q_0(x), \ \ x \in \mathbb{R}, t > 0,$$

using Inverse Scattering Problem Method (ISPM) for the Sturm-Liouville operator

$$Ly \equiv -y'' + q(x,t)y = \mu y, \ \ x \in \mathbb{R}, t > 0.$$

Then Peter D. Lax introduced the concept of a Lax pair, showing that certain nonlinear evolution equations like the KdV can be written as isospectral deformations $\frac{dL}{dt} = [P,L]$, thereby linking nonlinear PDEs to spectral theory and laying the foundation for the modern theory of integrable systems. Zakharov and Shabat developed the Inverse Scattering Transfer(IST) method for the focusing and defocusing Nonlinear Schrodinger (NLS) equations:

$$i\psi_t + \psi_{xx} \pm 2|\psi|^2\psi = 0.$$

Soon Wadati developed the ideas of Zakharov and Shabat for the modified Korteweg–de Vries(mKdV) equation:

$$u_t + u^2u_x + \mu u_{xxx} = 0, \ \ 0 < \mu < \frac{1}{6}.$$

In 1973, Hirota obtained the exact $N-$envelope-soliton solutions of the following equation, which is a combination of the complex mKdV and NLS equations and was later called the Hirota equation:

$$i\frac{\partial\psi}{\partial t} + i3\alpha|\psi|^2\ \frac{\partial\psi}{\partial x} + \beta\frac{\partial^2\psi}{\partial x^2} + i\gamma\frac{\partial^3\psi}{\partial x^3} + \delta|\psi|^2\psi = 0,$$

where $\alpha\beta\ =\ \gamma\delta,\ \alpha,\beta,\gamma,\delta \in \mathbb{R}_+,\ x \in \mathbb{R}, t > 0$.

In 1974, Ablowitz, Kaup, Newell, and Segur developed a unified framework to handle a broad class of integrable systems using a common first-order matrix eigenvalue problem:

$$\Psi_x = U(x,t,\lambda)\Psi,\ \ \Psi_t = V(x,t,\lambda)\Psi.$$

In 1974, Novikov related inverse spectral theory to integrable systems, in particular to the Korteweg-de Vries (KdV) equation, introducing the concept of finite-gap potentials. In addition, Dubrovin, Its, Matveev using the method of inverse spectral problems for the Sturm-Liouville operator, they proved that the Cauchy problem Korteweg-de Vries equation for the finite-gap case, has a solution with arbitrary initial conditions, and found an exact formula for the solution using Riemann theta functions.

Moreover, A.B. Khasanov founded the theory of integration of nonlinear evolutionary equations using the ISPM for the Dirac operator in the class of periodic infinite-gap functions in his scientific school. In 2022, Khasanov and Muminov proved that the defocusing NLS integrated via ISPM for the periodic Dirac operator. In addition, he and his students proved various nonlinear PDEs, such as KdV, mKdV, cosine–Gordon(coshG), mKdV–coshG, Liouville(L), Hirota, mKdV–L, coshG–L, negative order mKdV–L can be integrated ISPM method.

The connection of the dissertation research with the scientific research plans of the higher educational institution or research institution where the dissertation was completed. The dissertation thesis was carried out within the framework of the scientific research plan entitled “Applications of the spectral theory of differential operators to nonlinear evolutionary equations” in accordance with the scientific research plan of Samarkand State University named after Sharof Rashidov.

The purpose of the research is to integrate the nonlinear Hirota-type equation with finite density and the nonlinear Hirota-type equations with self-consistent integral and series sources in the class of periodic infinite-gap functions.

Objectives of the study:

– prove that a Hirota-type equation with finite density is integrable in the class of periodic infinite-gap functions;

– using the inverse spectral problem method for a periodic Dirac operator, prove that the initial value problem for Hirota-type equation with additional terms has a unique solution;

– prove that the Cauchy problem for a Hirota-type equation with a self-consistent series source has a unique solution in the class of periodic infinite-gap functions;

– using the inverse spectral problem method for a self-adjoint Dirac operator, prove that a Hirota-type equation with a self-consistent source in integral form is integrable.

T**he research objects** consist of Hirota-type equation with finite density, Hirota-type equation with additional terms, Hirota-type equation with a self-consistent series source and Hirota-type equation with a self-consistent source in integral form.

The subject of the research is the direct and inverse spectral problems for the periodic Dirac operator in the integration of nonlinear defocused Hirota-type equation with finite density or self-consistent sources.

Research methods. The dissertation used methods in differential equations, mathematical physics, spectral theory of differential operators, theory of functions of complex variables, and functional analysis.

The scientific novelty of the research is as follows:

– using the method of inverse spectral problems for the periodic Dirac operator, it is proved that a Hirota-type equation with a finite density is integrable in the class of six-times continuously differentiable periodic infinite-gap functions;

– it is proved that a unique solution of the Cauchy problem exists for a Hirota-type equation with additional terms in the class of six-times continuously differentiable functions;

– using the method of inverse spectral problems for the periodic Dirac operator, it is proved that a Hirota-type equation with a self-consistent series source is integrable in the class of periodic infinite-gap functions;

– it is proved that a solution of the initial value problem exists and is unique for a Hirota-type equation with a self-consistent source in integral form in the class of six-times continuously differentiable infinite-gap periodic functions.

The practical results of the research are as follows:

– the inertial manifolds of infinite-dimensional dynamical systems were studied using the Cauchy problem solving algorithm for the nonlinear defocused Hirota-type equation with finite density in the class of periodic infinite-gap functions;

– the algorithm for integrating the Hirota-type equation with a self-consistent series source in the class of periodic infinite-gap functions made it possible to study the properties of attractors for the class of delayed reaction diffusion equations with dynamic boundary conditions.

The reliability of the research results is based on the use of methods from mathematical physics, spectral analysis, and functional analysis in solving inverse spectral problems for differential operators with periodic coefficients and their application to solving nonlinear evolutionary equations, as well as the use of mathematical reasoning based on rigorous proofs.

The scientific significance of the research results is explained by the fact that the evolutionary equations of modern mathematical physics can be integrated in the class of periodic infinite-gap functions.

The practical significance of the research results is explained by their application in optics, quantum mechanics, electrodynamics, hydrodynamics, and modern mathematical physics in nonlinear environments.

Implementation of research results. Based on the scientific results obtained on the integration of the nonlinear Hirota type equation with finite density and self-consistent sources:

the algorithm for integrating nonlinear Hirota-type equation with a finite density and nonlinear Hirota-type equation with additional terms in the class of periodic infinite gap functions were applied in the fundamental scholarship NRF-2020R1A2C1003119 “Global Attractors and Inertial Manifolds of Infinite Dimensional Dynamical Systems” (reference of Chonnam National University, South Korea, dated April 21, 2025). The application of scientific results made it possible to study the properties of attractors for a class of delayed reaction diffusion equations with dynamic boundary conditions.

the algorithm for finding a solution of the nonlinear Hirota-type equation with a finite density and nonlinear Hirota-type equation with self-consistent series and integral source in the class of periodic infinite gap functions were applied in the fundamental scholarship “GP/2023/9752700 Theoretical and numerical studies for some classes of fractional integro-differential equations in Banach space” (reference of University Putra Malaysia, Malaysia, dated September 18, 2025). The application of scientific results made it possible to study the theoretical side of classes of fractional integro-differential equations in Banach space.

Approbation of research results. The results of this research were discussed at 7 scientific and practical conferences, including 3 international and 4 republican conferences.

Publication of research results. 14 scientific works have been published on the topic of the dissertation, of which 7 papers were published in scientific publications recommended for the defense of dissertations of the Higher Attestation Commission of the Republic of Uzbekistan, including 3 in foreign and 4 in republican journals.

Structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, a conclusion and a list of references. The volume of the dissertation is 122 pages.

MAIN CONTENT OF THE DISSERTATION

The Introduction of the dissertation establishes the relevance and importance of the dissertation topic, indicates the relevance of the research to the priority areas of development of science and technology in the republic, provides reviews of foreign scientific research on the topic of the dissertation, the level of study of the issue, describes the goals and objectives, object and subject of the research, describes the scientific novelty and practical results of the research, reveals the theoretical and practical significance of the results obtained, provides information on the implementation of the research results, published works, and the structure of the dissertation.

The first chapter of the dissertation, entitled **"On the Periodic Dirac Operator"** presents fundamental and necessary information about the periodic Dirac operator and inverse spectral problem for the Dirac operator. It also presents a system of the first Dubrovin differential equations that represents the evolution of the spectral data of the Dirac operator with respect to the spatial variable, and proves that there is a unique solution of the Cauchy problem for it. The next chapters use the information presented in the first chapter.

We consider the following Dirac operator with a shifted argument:

$$\mathcal{L}(\tau,t)y \equiv J\frac{dy}{dx} + \Lambda(x+\tau,t)y = \mu y,\ \ x,\tau \in \mathbb{R}, t > 0, \qquad (1)$$

where $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\Lambda(x+\tau,t) = \begin{pmatrix} p(x+\tau,t) & q(x+\tau,t) \\ q(x+\tau,t) & -p(x+\tau,t) \end{pmatrix}$, $y(x,t) = \begin{pmatrix} y_1(x,t) \\ y_2(x,t) \end{pmatrix}$, $\mu \in \mathbb{C}$, $p(x+\pi,t) = p(x,t) \in C^2(\mathbb{R})$, $q(x+\pi,t) = q(x,t) \in C^2(\mathbb{R})$ are real valued functions.

Let us denoted the by $u(x,\mu,\tau,t) = \left(u_1(x,\mu,\tau,t), u_2(x,\mu,\tau,t)\right)^T$ and $v(x,\mu,\tau,t) = \left(v_1(x,\mu,\tau,t), v_2(x,\mu,\tau,t)\right)^T$ solutions of the system of equations (1) with conditions $u(0,\mu,\tau,t) = (1,0)^T$ and $v(0,\mu,\tau,t) = (0,1)^T$, respectively. The function $\Delta(\mu,\tau,t) = u_1(\pi,\mu,\tau,t) + v_2(\pi,\mu,\tau,t)$ is called the Lyapunov function for the system of equations (1). Also we denote by $\mu_n(\tau,t), n \in \mathbb{Z}$, the eigenvalues of the periodic and antiperiodic boundary value problems for the Dirac system (1) with conditions $y(0,\tau,t) = y(\pi,\tau,t)$ and $y(0,\tau,t) = -y(\pi,\tau,t)$, respectively. They coincide with the roots of the equation $\Delta(\mu,\tau,t) \mp 2 = 0$. In addition, we denote the eigenvalues of the Dirichlet boundary value problem for the equation (1) with conditions $y_1(0,\tau,t) = y_1(\pi,\tau,t) = 0$ by $\xi_n(\tau,t), n \in \mathbb{Z}$. It is easy to check that, the eigenvalues $\xi_n(\tau,t), n \in \mathbb{Z}$ coincide with the roots of the equation $v_1(\pi,\mu,\tau,t) = 0$.

Theorem 1. The Lyapunov function $\Delta(\mu,\tau,t)$ does not depend on the parameter τ, that is, $\Delta(\mu,\tau,t) = \Delta(\mu,t)$. If the parameter τ varies in the interval $[0;\pi]$, then the eigenvalues of the Dirichlet boundary value problem $\xi_n(\tau,t)$ fill the intervals $[\mu_{2n-1},\mu_{2n}], n \in \mathbb{Z}$, that is,

$$\xi_n(\tau,t) \in [\mu_{2n-1},\mu_{2n}], n \in \mathbb{Z}.$$

Definition 1. The intervals (μ_{2n-1},μ_{2n}) are called gaps.

Definition 2. If the number of gaps is infinite, then the periodic potentials of the Dirac operator $p(x,t)$ and $q(x,t)$ are called infinite-gap periodic potentials. Also Dirac operator $\mathcal{L}(\tau,t)$ is called periodic infinite-gap operator.

In our case all gaps are open, in other words, $\mu_{2n-1} \neq \mu_{2n}, n \in \mathbb{Z}$.

Also we define the lengths of the gaps: $\gamma_n = \mu_{2n} - \mu_{2n-1},\ \ n \in \mathbb{Z}$.

Definition 3. The eigenvalues of the Dirichlet boundary value problem for the Dirac system (1) $\xi_n(\tau,t), n \in \mathbb{Z}$ and signs $\sigma_n(\tau,t) = sgn\{v_2(\pi,\xi_n,\tau,t) - u_1(\pi,\xi_n\tau,t)\}, n \in \mathbb{Z}$ are called the spectral parameters of the operator $\mathcal{L}(\tau,t)$. Also spectral parameters and boundary of the gaps $\mu_n, n \in \mathbb{Z}$, that is, the set $\{\mu_n, \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1,\ n \in \mathbb{Z}\}$ is called the spectral data of the Dirac operator $\mathcal{L}(\tau,t)$.

Definition 4. The problem of finding the spectral data of the operator $\mathcal{L}(\tau,t)$ using the potentials $p(\tau,t)$ and $q(\tau,t)$ is called the direct problem, and the problem of recovering the coefficient $\Lambda(\tau,t)$ of the operator $\mathcal{L}(\tau,t)$ from spectral data is called the inverse problem.

Theorem[5] 2. The coefficients $p(\tau,t)$ and $q(\tau,t)$ of the operator $\mathcal{L}(\tau,t)$ are determined uniquely from the spectral data $\{\mu_n, \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1,\ n \in \mathbb{Z}\}$.

The functions $\psi^{\pm}(x,\mu,t) = \left(\psi_1^{\pm}(x,\mu,t), \psi_2^{\pm}(x,\mu,t)\right)$ defined the form

$$\psi^{\pm}(x,\mu,\tau,t) = u(x,\mu,\tau,t) + \frac{v_2(\pi,\mu,\tau,t) - u_1(\pi,\mu,\tau,t) \mp \sqrt{\Delta^2(\mu) - 4}}{2v_1(\pi,\mu,\tau,t)} v(x,\mu,\tau,t) \quad (2)$$

are called Floquet solutions of the system of differential equations (1):

If $\Delta^2(\mu) - 4 \neq 0$, then the functions $\psi^+(x,\mu,\tau,t)$ and $\psi^-(x,\mu,\tau,t)$ are the fundamental solutions of the system of differential equations (1).

Lemma 1. The Dirac operator $\mathcal{L}$ has only a continuous spectrum, that is, it has no eigenvalues. The spectrum of the Dirac operator $\mathcal{L}$ is consists of the set:

$$E = \mathbb{R} \backslash \left(\bigcup_{n=-\infty}^{+\infty} (\mu_{2n-1}, \mu_{2n}) \right).$$

Lemma 2. The following traces formulas hold for the potentials of the Dirac operator $\mathcal{L}(\tau,t)$:

$$p(\tau,t) = \sum_{n=-\infty}^{+\infty} \left(\frac{\mu_{2n-1} + \mu_{2n}}{2} - \xi_n(\tau,t) \right), \quad (3)$$

$$q(\tau,t) = \sum_{n=-\infty}^{+\infty} (-1)^{n-1} \sigma_n(\tau,t) h_n\big(\xi(\tau,t)\big), \quad (4)$$

where

[5] H.N. Normurodov. *Integration of the nonlinear modified Korteveg-de Vries–sine–Gordon (mKdV–sG) equations in the class of periodic infinite-gap functions.* PhD Dissertation, 2024, Samarkand, Uzbekistan [in Uzbek].

$$h_n(\xi) = \sqrt{(\xi_n(\tau,t) - \mu_{2n-1})\big(\mu_{2n} - \xi_n(\tau,t)\big)} \times$$
$$\times \sqrt{\prod_{k=-\infty,k\neq n}^{+\infty} \frac{\big(\mu_{2k-1} - \xi_n(\tau,t)\big)\big(\mu_{2k} - \xi_n(\tau,t)\big)}{\big(\xi_k(\tau,t) - \xi_n(\tau,t)\big)^2}}.$$

The second chapter of the dissertation, entitled "**The Initial Value Problem for the Hirota-type Equation with A Finite Density or Additional Terms**," first presents historical information about the method of inverse spectral problems, and then proves that there is a unique global solution of the initial value problem for a Hirota-type equation with a finite density and a Hirota-type equation with additional terms in the class of six-times continuously differentiable periodic infinite-gap functions.

In the first paragraph of this section we consider the Hirota-type equation with a finite density in the form

$$\begin{cases} p_t = [p_{xxx} + 6(\rho^2 - \{p^2 + q^2\})p_x] - [q_{xx} + 2q(\rho^2 - \{p^2 + q^2\})] \\ q_t = [q_{xxx} + 6(\rho^2 - \{p^2 + q^2\})q_x] + [p_{xx} + 2p(\rho^2 - \{p^2 + q^2\})] \end{cases}. \quad (5)$$

with initial conditions

$$\begin{cases} p(x,t)|_{t=0} = p_0(x), \quad p_0(x+\pi) = p_0(x) \in C^6(\mathbb{R}), \\ q(x,t)|_{t=0} = q_0(x), \quad q_0(x+\pi) = q_0(x) \in C^6(\mathbb{R}), \end{cases} \quad (6)$$

in the class of real infinite-gap π $-$periodic functions with respect to x :

$$\begin{cases} p(x+\pi,t) = p(x,t), q(x+\pi,t) = q(x,t), x \in \mathbb{R}, t > 0 \\ p(x,t), q(x,t) \in C^{3,1}_{x,t}(t>0) \cap C(t \geq 0) \end{cases}. \quad (7)$$

Here $\rho > 0$ is a finite number.

We prove the Initial Value Problem(IVP) (5)–(7) has a unique solution using the Invere Spectral Problem Method(ISPM) for the periodic Dirac operator (1).

First using the initial functions $p_0(x)$ and $q_0(x)$, we construct the Dirac operators $\mathcal{L}(\tau,0)$:

$$\mathcal{L}(\tau,0)y \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} p_0(x+\tau) & q_0(x+\tau) \\ q_0(x+\tau) & -p_0(x+\tau) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \mu\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \ \ x,\tau \in \mathbb{R}.$$

Solving the direct problem, we find the spectral data $\{\mu_n, \xi_n^0(\tau), \sigma_n^0(\tau) = \pm 1, n \in \mathbb{Z}\}$ of the operator $\mathcal{L}(\tau,0)$.

The main object of study in this section is the solvability of the initial value problem (5)–(7) that is, establishing the minimum smoothness requirements for the initial data $p_0(x)$ and $q_0(x)$, under which there is a unique global solution of this problem.

The main result of this paragpraph contains the following theorem.

Theorem 3. If periodic infinite-gap functions $p_0(x), q_0(x)$ satisfy the conditions $p_0(x+\pi) = p_0(x) \in C^6(\mathbb{R})$, $q_0(x+\pi) = q_0(x) \in C^6(\mathbb{R})$, then there exist uniquely determined global periodic infinite-gap solutions $p(x,t), q(x,t)$, $x \in \mathbb{R}, t > 0$ of the IVP (5)-(7) and they are determined by the formulas (3), (4), respectively.

To prove Theorem 3, it is necessary to prove that the spectral data $\{\mu_n, \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1,\ n \in \mathbb{Z}\}$ of the operator $\mathcal{L}(\tau,t)$ exist and are unique.

Now we derive an infinite Dubrovin system of differential equations that represents the evolution of the spectral data of the operator $\mathcal{L}(\tau,t)$.

Lemma 3. Let $p(x,t),\ q(x,t), x \in \mathbb{R}, t > 0$ be a solution to the IVP (5)-(7). Then the boundaries of the spectrum $\mu_n(\tau,t), n \in \mathbb{Z}$ of the operator $\mathcal{L}(\tau,t)$ do not depend on τ and t, that is, $\mu_n(\tau,t) = \mu_n, n \in \mathbb{Z}$, and the spectral parameters $\xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}$ satisfy the first and second Dubrovin systems of differential equations, respectively:

$$1. \frac{\partial \xi_n(\tau,t)}{\partial \tau} = 2(-1)^{n-1}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})\big(\mu_{2n}-\xi_n(\tau,t)\big)} f_n(\xi) \times \times \{p(\tau,t)+\xi_n(\tau,t)\}, n \in \mathbb{Z}; \quad (8)$$

$$2. \frac{\partial \xi_n(\tau,t)}{\partial t} = 2(-1)^{n}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})\big(\mu_{2n}-\xi_n(\tau,t)\big)} f_n(\xi) g_{n,1}(\xi). \quad (9)$$

In addition, the following initial conditions are satisfied:

$$\xi_n(\tau,t)|_{t=0} = \xi_n^0(\tau),\ \ \sigma_n(\tau,t)|_{t=0} = \sigma_n^0(\tau), n \in \mathbb{Z}, \quad (10)$$

where $\xi_n^0(\tau), \sigma_n^0(\tau) = \pm 1, n \in \mathbb{Z}$ are the spectral parameters of the Dirac operator $\mathcal{L}(\tau,0)$ and

$$f_n(\xi) = \sqrt{\prod_{\substack{k=-\infty \\ k\neq n}}^{+\infty} \frac{(\mu_{2k-1}-\xi_n)(\mu_{2k}-\xi_n)}{(\xi_k-\xi_n)^2}}, \quad (11)$$

$g_{n,1}(\xi) = [4\xi_n^3 + 4p\xi_n^2 + 2\xi_n(p^2+q^2+q_\tau) - p_{\tau\tau} + 2(pq_\tau - p_\tau q) +$
$+2p(p^2+q^2) - 6\rho^2(p+\xi_n)] + [(p+\xi_n)^2 + \xi_n^2 + q^2 + q_\tau - \rho^2],\ \ n \in \mathbb{Z},$
$p = p(\tau,t), q = q(\tau,t),\ \ \xi \equiv \xi(\tau,t) = (\dots, \xi_{-1}(\tau,t), \xi_0(\tau,t), \xi_1(\tau,t), \dots),$
$\sigma \equiv \sigma(\tau,t) = (\dots, \sigma_{-1}(\tau,t), \sigma_0(\tau,t), \sigma_1(\tau,t), \dots).$

Now we prove that the IVP (5)–(7) has a unique global solution.

Using the change of variables

$$\xi_n(\tau,t) = \mu_{2n-1} + (\mu_{2n}-\mu_{2n-1})\sin^2 x_n(\tau,t),\ \ n \in \mathbb{Z},$$

the IVP (9)–(10) can be written as a single equation in the Banach space $\mathcal{K}$:

$$\begin{cases} \dfrac{\partial x(\tau,t)}{\partial t} = H\big(x(\tau,t)\big), \\ x(\tau,t)|_{t=0} = x^0(\tau),\ \ x^0(\tau) \in \mathcal{K}, \end{cases} \quad (12)$$

where

$$\mathcal{K} = \left\{ x = (\dots, x_{-1}, x_0, x_1, \dots) \colon \|x\| = \sum_{n=-\infty}^{+\infty} (1+|n|^2)(\mu_{2n}-\mu_{2n-1})|x_n| < \infty \right\},$$

$H\big(x(\tau,t)\big) = \big(\dots, H_{-1}\big(x(\tau,t)\big), H_0\big(x(\tau,t)\big), H_1\big(x(\tau,t)\big), \dots\big),$
$x^0(\tau) = (\dots, x_{-1}^0(\tau), x_0^0(\tau), x_1^0(\tau) \dots).$

Thus, we study the solvability of the problem (12) instead of solvability of the problem of (9), (10).

In 1978, Misura[6] proved that if $p_0(x) \in C^6(\mathbb{R})$ and $q_0(x) \in C^6(\mathbb{R})$, then the following asymptotes hold:

$$\begin{cases} \mu_{2k}, \mu_{2k-1} = k + \sum_{j=1}^{7} c_j k^{-j} \pm 2^{-6}|k|^{-6}\left|q_{2k}^6\right| + |k|^{-7}\varepsilon_k^{\pm}, \\ \gamma_k \equiv \mu_{2k} - \mu_{2k-1} = \dfrac{\left|q_{2k}^6\right|}{2^5|k|^6} + \dfrac{\delta_k}{|k|^7}, \delta_k = \varepsilon_k^+ - \varepsilon_k^-, \end{cases} \tag{13}$$

where

$$q_{2k}^2 \equiv \frac{1}{\pi}\int_0^{\pi} Q_0^2(t)e^{-2ikt}dt, \;\; Q_0(t) = q_0(t) - ip_0(t), \;\; \sum_{k=-\infty}^{+\infty} \left(\varepsilon_k^{\pm}\right)^2 < \infty.$$

In addition, from $\xi_n(\tau) \in [\mu_{2n-1}, \mu_{2n}], n \in \mathbb{Z}$, we obtain,

$$\inf_{k \neq n} |\xi_n(\tau) - \xi_k(\tau)| \geq a > 0. \tag{14}$$

Now using asymptotes (13) and inequality (14) we can prove the lemma.

Lemma 4. The following inequalities hold:

$$C_1 \leq \left|\bar{f}_n(x)\right| \leq C_2, \;\; \left|\frac{\partial \bar{f}_n(x)}{\partial x_m}\right| \leq C_3\gamma_m,$$

$$\left|\bar{g}_{n,1}(x)\right| \leq C_4(|n|^3 + 1), \;\; \left|\frac{\partial \bar{g}_{n,1}(x)}{\partial x_m}\right| \leq C_5\gamma_m(|n|^2 + 1)(|m|^2 + 1), \;\; n, m \in \mathbb{Z},$$

where $x = (\dots, x_{-1}, x_0, x_1, \dots)$, $C_j = const$, $j = \overline{1,5}$.

Next we prove IVP (9), (10) has a unique global solution.

Lemma 5. If the initial functions $p_0(x)$ and $q_0(x)$ satisfy the conditions

$$p_0(x + \pi) = p_0(x) \in C^6(\mathbb{R}), \;\; q_0(x + \pi) = q_0(x) \in C^6(\mathbb{R}),$$

then IVP (11) has a unique global solution.

The solvability of IVP (9), (10) is equivalent to the solvability of IVP (12). Thus, IVP (9), (10) has a unique global solution.

Now we give the algorithm for finding a solution of the problem (5)–(7):

1) First, using the initial functions $p_0(x)$ and $q_0(x)$ we find the spectral data $\{\mu_n, \xi_n^0(\tau), \sigma_n^0(\tau), n \in \mathbb{Z}\}$ of the Dirac operator $\mathcal{L}(\tau, 0)$;

2) Then denote the spectral data of the Dirac operator $\mathcal{L}(\tau, t)$ by $\{\mu_n, \xi_n(\tau, t), \sigma_n(\tau, t), n \in \mathbb{Z}\}$ and solving IVP (9), (10) for arbitrary value of τ, find spectral parameters $\{\xi_n(\tau, t), \sigma_n(\tau, t), n \in \mathbb{Z}\}$;

3) Using traces formulas (3) and (4) we find the functions $p(\tau, t)$ and $q(\tau, t)$, that is, the solution of IVP (5)–(7).

Now we find the solution of IVP (5)–(7) when the spectrum of the Dirac operator $\mathcal{L}(\tau, 0)$ has only one gap. In other words, we consider single-gap potential $u_0(x) = q_0(x) - ip_0(x)$ of the Dirac operator $\mathcal{L}(0)$ with spectral data $\{\mu_{-1}, \mu_0, \xi_0(0), \sigma_0(0) = \pm 1\}$. In this case we obtain the following IVP for the Hirota-type equation with a finite density:

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[6] T.V. Misura, Characterization of the spectra of the periodic and antiperiodic boundary value problems that are generated by the Dirac operator, I. Function theory, functional analysis and their applications. 1978, Vol. 30, pp. 90-101 [in Russian].

$$\begin{cases} p_t = [p_{xxx} + 6(\rho^2 - \{p^2 + q^2\})p_x] - [q_{xx} + 2q(\rho^2 - \{p^2 + q^2\})], \\ q_t = [q_{xxx} + 6(\rho^2 - \{p^2 + q^2\})q_x] + [p_{xx} + 2p(\rho^2 - \{p^2 + q^2\})], \end{cases} \quad (15)$$

$$\begin{cases} p(x,t)|_{t=0} = \dfrac{\mu_0 - \mu_{-1}}{2}\cos\{-(\mu_0 + \mu_{-1})x + 2\sigma_0(0)\phi(0)\}, \\ q(x,t)|_{t=0} = -\sigma_0(\tau)\dfrac{\mu_0 - \mu_{-1}}{2}\sin\{-(\mu_0 + \mu_{-1})x + 2\sigma_0(0)\phi(0)\}, \end{cases} \quad (16)$$

where

$$\phi(0) = \arcsin\sqrt{\frac{\xi_0(0) - \mu_{-1}}{\mu_0 - \mu_{-1}}}, \quad \sigma_0(\tau) = \operatorname{sgn}\{\sin\{-(\mu_{-1} + \mu_0)x + 2\sigma_0(0)\phi(0)\}\}.$$

Using the inverse spectral problem method for the Dirac operator $\mathcal{L}(\tau,t)$ we find the solutions of IVP (15), (16) as follows:

$$p(x,t) = \frac{\mu_0 - \mu_{-1}}{2}\cos\big(2K(\rho)t - (\mu_{-1} + \mu_0)x + 2\sigma_0(0)\phi(0)\big),$$

$$q(x,t) = -\frac{\mu_0 - \mu_{-1}}{2}\sin\big(2K(\rho)t - (\mu_{-1} + \mu_0)x + 2\sigma_0(0)\phi(0)\big),$$

where

$$K(\rho) = \frac{\mu_{-1} + \mu_0}{4}[5\mu_{-1}^2 + 5\mu_0^2 - 2\mu_{-1}\mu_0 + 3\mu_{-1} + 3\mu_0 - 12\rho^2] - \rho^2 - \mu_{-1}\mu_0.$$

In the second paragraph of the second chapter titled **“The initial value problem for the Hirota-type equation with additional terms”**, proved that the following IVP for the Hirota-type equation with additional terms:

$$\begin{cases} p_t = [p_{xxx} - 6p_x(p^2 + q^2)] + [-q_{xx} + 2q(p^2 + q^2)] + p_x + q, \\ q_t = [q_{xxx} - 6q_x(p^2 + q^2)] + [p_{xx} - 2p(p^2 + q^2)] + q_x - p, \end{cases} \quad (17)$$

with initial conditions

$$\begin{cases} p(x,t)|_{t=0} = p_0(x), \;\; p_0(x + \pi) = p_0(x) \in C^6(\mathbb{R}), \\ q(x,t)|_{t=0} = q_0(x), \;\; q_0(x + \pi) = q_0(x) \in C^6(\mathbb{R}), \end{cases} \quad (18)$$

in the class of real π periodic in x infinite-gap functions:

$$\begin{cases} p(x + \pi, t) = p(x,t), \;\; q(x + \pi, t) = q(x,t), \quad x \in \mathbb{R}, t > 0, \\ p(x,t), q(x,t) \in C_x^3(t > 0) \cap C_t^1(t > 0) \cap C(t \geqslant 0). \end{cases} \quad (19)$$

has a unique global infinite-gap solution using the inverse spectral problem method for the Dirac operator (1).

First we derive Dubrovin system of differential equations, that is, evolution of the spectral data of Dirac operator $\mathcal{L}(\tau,t)$.

Lemma 6. Let $\{p(x,t), q(x,t), x \in \mathbb{R}, t > 0\}$ be solution of IVP (17)–(19). Then spectral data $\{\mu_n(t), \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}\}$ of the Dirac operator $\mathcal{L}(\tau,t)$ satisfy an analogue of the system infinite Dubrovin system of equations:

$$1. \frac{\partial \mu_n(t)}{\partial t} = 0, \quad n \in \mathbb{Z}, \quad (20)$$

$$2. \frac{\partial \xi_n(\tau,t)}{\partial \tau} = 2(-1)^{n-1}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t) - \mu_{2n-1})\big(\mu_{2n} - \xi_n(\tau,t)\big)}f_n(\xi) \times$$

$$\times \{p(\tau,t)+\xi_n(\tau,t)\}, n\in\mathbb{Z}, \quad (21)$$

3. $$\frac{\partial \xi_n(\tau,t)}{\partial t}=2(-1)^n\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})\big(\mu_{2n}-\xi_n(\tau,t)\big)f_n(\xi)g_{n,2}(\xi)}. \quad (22)$$

In addition, the following initial conditions are satisfied:

$$\xi_n(\tau,t)|_{t=0}=\xi_n^0(\tau),\ \sigma_n(\tau,t)|_{t=0}=\sigma_n^0(\tau),\ \ n\in\mathbb{Z}, \qquad (23)$$

where $\xi_n^0(\tau),\sigma_n^0(\tau)=\pm 1, n\in\mathbb{Z}$ are the spectral parameters of the Dirac operator $\mathcal{L}(\tau,0)$. Also sequence $f_n(\xi), n\in\mathbb{Z}$ is defined by the formula (11) and

$$g_{n,2}(\xi)=[4\xi_n^3+4p\xi_n^2+2\xi_n(p^2+q^2+q_\tau)-p_{\tau\tau}+2(pq_\tau-p_\tau q)+$$
$$+2p(p^2+q^2)]+[2\xi_n^2+2p\xi_n+p^2+q^2+q_\tau]+\left[\frac{1}{2}-\xi_n-p\right], n\in\mathbb{Z},$$

where $p=p(\tau,t), q=q(\tau,t),\ \xi\equiv\xi(\tau,t)=(\dots,\xi_{-1}(\tau,t),\xi_0(\tau,t),\xi_1(\tau,t),\dots)$, and $\sigma\equiv\sigma(\tau,t)=(\dots,\sigma_{-1}(\tau,t),\sigma_0(\tau,t),\sigma_1(\tau,t),\dots)$.

Using the change of variables

$$\xi_n(\tau,t)=\mu_{2n-1}+(\mu_{2n}-\mu_{2n-1})\sin^2\big(x_n(\tau,t)\big), n\in\mathbb{Z},$$

the IVP for the system of Dubrovin equations (22)-(23) can be written as a single equation

$$\frac{dx(\tau,t)}{dt}=\mathcal{H}\big(x(\tau,t)\big),\ \ x(\tau,t)|_{t=0}=x^0(\tau),\ \ x^0(\tau)\in\mathcal{K}, \quad (24)$$

in the Banach space $\mathcal{K}$, where

$$\mathcal{K}=\Big\{x=(\dots,x_{-1}(\tau,t),x_0(\tau,t),x_1(\tau,t),\dots):$$
$$\|x\|=\sum_{n=-\infty}^{+\infty}(1+|n|^2)(\mu_{2n}-\mu_{2n-1})|x_n|<\infty\Big\},$$

$$\mathcal{H}(x)=(\dots,\mathcal{H}_{-1}(x),\mathcal{H}_1(x),\dots),$$
$$\mathcal{H}_n(x(\tau,t))=(-1)^n\sigma_n^0(\tau)\bar{f}_n\big(\dots,\lambda_1+(\lambda_2-\lambda_1)\sin^2\big(x_1(\tau,t)\big),\dots\big)\times$$
$$\times\,\overline{g}_{n,2}\big(\dots,\lambda_1+(\lambda_2-\lambda_1)\sin^2\big(x_1(\tau,t)\big),\dots\big)=(-1)^n\sigma_n^0(\tau)\overline{f}_n\big(x(\tau,t)\big)\overline{g}_{n,2}\big(x(\tau,t)\big).$$

Lemma 7. If the initial functions $p_0(x)$ and $q_0(x)$ satisfy

$$p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}),\ \ q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}),$$

then the vector function $\mathcal{H}(x)$ is Lipschitz continuous in the Banach space $\mathcal{K}$, that is, there is a constant $N=\text{const}>0$ such that, for all $x,y\in\mathcal{K}$,

$$\|\mathcal{H}(x)-\mathcal{H}(y)\|\le N\|x-y\|,$$

where $N=A\sum_{n=-\infty}^{+\infty}\gamma_n(1+|n|^3)(1+|n|^2)<\infty$.

It follows from Lemma 7 that for all $t>0$ and $\tau\in\mathbb{R}$, the solution of IVP (24), in other words, IVP (22), (23) exists and is unique.

Now we give the algorithm for finding a solution of the problem (17), (19). Lemma 6 and Lemma 7 give a method for finding a solution to IVP (17), (19).

1. We first find the spectral data $\mu_n,\xi_n^0(\tau),\sigma_n^0(\tau)=\pm1, n\in\mathbb{Z}$, of the Dirac operator $\mathcal{L}(\tau,0)$;

2. Denote the spectral data of the operator $\mathcal{L}(\tau,t)$ by $\mu_n,\xi_n(\tau,t),\sigma_n(\tau,t)=\pm1,\ n\in\mathbb{Z}$. Solving IVP (22), (23), we find $\xi_n(\tau,t),\ \sigma_n(\tau,t), n\in\mathbb{Z}$;

3. The functions $p(\tau,t)$ and $q(\tau,t)$, that is, the solutions to IVP (17), (19), are found from the traces formulas (3) and (4), respectively.

Thus, we have proved the following theorem.

Theorem 4. If the initial functions $p_0(x), q_0(x)$ satisfy the conditions

$$p_0(x+\pi) = p_0(x) \in C^6(\mathbb{R}), q_0(x+\pi) = q_0(x) \in C^6(\mathbb{R}),$$

then IVP (17), (19) has unique solutions $p(x,t)$ and $q(x,t)$ from the class $C_x^3(t > 0) \cap C_t^1(t > 0) \cap C(t \geqslant 0)$. These solutions are defined, by the sum of series (6) and (7), respectively.

The third chapter of the dissertation is called "**The Cauchy problem for the Hirota-type equation with self-consistent sources**" and in the first paragraph of this chapter consider the Hirota-type equation with self-consistent series source

$$\begin{cases} p_t = [p_{xxx} - 6p_x(p^2+q^2)] + [-q_{xx} + 2q(p^2+q^2)] - \\ -\sum\limits_{k=-\infty}^{+\infty} \zeta_k(t) v_1(\pi,\mu_k,t)\big(\psi_1^+(x,\mu_k,t)\psi_2^-(x,\mu_k,t) + \psi_1^-(x,\mu_k,t)\psi_2^+(x,\mu_k,t)\big), \\ q_t = [q_{xxx} - 6q_x(p^2+q^2)] + [p_{xx} - 2p(p^2+q^2)] + \\ +\sum\limits_{k=-\infty}^{+\infty} \zeta_k(t) v_1(\pi,\mu_k,t)\big(\psi_1^+(x,\mu_k,t)\psi_1^-(x,\mu_k,t) - \psi_2^+(x,\mu_k,t)\psi_2^-(x,\mu_k,t)\big), \end{cases} \tag{25}$$

with the following initial and smoothness conditions, respectively

$$\begin{cases} p(x,t)|_{t=0} = p_0(x) \in C^6(\mathbb{R}), \;\; p_0(x) = p_0(x+\pi), \\ q(x,t)|_{t=0} = q_0(x) \in C^6(\mathbb{R}), \;\; q_0(x) = q_0(x+\pi), \end{cases} \tag{26}$$

$$\begin{cases} p(x+\pi,t) = p(x,t), \;\; q(x+\pi,t) = q(x,t), \\ p(x,t), q(x,t) \in C_{x,t}^{3,1}(t>0) \cap C(t \geq 0), \;\; x \in \mathbb{R}, t > 0. \end{cases} \tag{27}$$

Here $\zeta_n(t)$ is given real sequence of continuous functions that has asymptotes

$$\zeta_n(t) = O\left(\frac{1}{1+n^2}\right), \;\; |n| \to \infty. \tag{28}$$

In addition, the functions $\psi^\pm(x,\mu,t) = \left(\psi_1^\pm(x,\mu,t), \psi_2^\pm(x,\mu,t)\right)^T$ are Floquet solutions of the Dirac system of equations, that is,

$$\mathcal{L}(t)\psi^\pm \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} (\psi_1^\pm)' \\ (\psi_2^\pm)' \end{pmatrix} + \begin{pmatrix} p(x,t) & q(x,t) \\ q(x,t) & -p(x,t) \end{pmatrix}\begin{pmatrix} \psi_1^\pm \\ \psi_2^\pm \end{pmatrix} = \mu \begin{pmatrix} \psi_1^\pm \\ \psi_2^\pm \end{pmatrix}, x \in \mathbb{R}, t > 0.$$

Using identities

$$\begin{aligned} & v_1(\pi,\mu,t)[\psi_1^+(\mu,\tau,t)\psi_2^-(\mu,\tau,t) + \psi_1^-(\mu,\tau,t)\psi_2^+(\mu,\tau,t)] = \\ & = v_2(\pi,\mu,\tau,t) - u_1(\pi,\mu,\tau,t), \\ & v_1(\pi,\mu,t)[\psi_1^+(\mu,\tau,t)\psi_1^-(\mu,\tau,t) - \psi_2^+(\mu,\tau,t)\psi_2^-(\mu,\tau,t)] = \\ & = v_1(\pi,\mu,\tau,t) + u_2(\pi,\mu,\tau,t), \end{aligned} \tag{29}$$

and asymptotes

$$\begin{aligned} & v_2(\pi,\mu,\tau,t) - u_1(\pi,\tau,\mu,\tau,t) = O\left(\frac{1}{1+\mu}\right), \quad |\mu| \to \infty, \\ & v_1(\pi,\tau,\mu,\tau,t) + u_2(\pi,\mu,\tau,t) = O\left(\frac{1}{1+\mu}\right), \quad |\mu| \to \infty, \end{aligned} \tag{30}$$

also relation (28), we can prove the right-hand side series of system of equations (25) are uniformly convergent and the right-hand side of (25) are π periodic functions with respect to x.

In this paragraph, we prove the Cauchy problem (25)–(27) has a unique infinite-gap solution $\{p(x,t), q(x,t), x \in \mathbb{R}, t > 0\}$, using the inverse spectral problem method for the periodic Dirac operator $\mathcal{L}(\tau, t)$.

The following theorem expressing the main result.

Theorem 5. If periodic infinite-gap functions $p_0(x)$ and $q_0(x)$ satisfy the conditions

$$p_0(x+\pi) = p_0(x) \in C^6(\mathbb{R}),\ \ q_0(x+\pi) = q_0(x) \in C^6(\mathbb{R}),$$

then there exist uniquely determined global infinite-gap solutions $p(x,t), q(x,t)$, $x \in \mathbb{R}, t > 0$ of the Cauchy problem (25)–(27), which are determined, by the sum of formulas (3) and (4), respectively.

Now we derive an infinite Dubrovin system of differential equations that represent the evolution of the spectral data of the operator $\mathcal{L}(\tau, t)$.

Lemma 8. Let $p(x,t),\ q(x,t), x \in \mathbb{R}, t > 0$ be solutions of the Cauchy problem (25)–(27). Then the eigenvalues $\mu_n(t), n \in \mathbb{Z}$ do not depend on t, that is, $\mu_n(t) = \mu_n, n \in \mathbb{Z}$, and the spectral parameters $\xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}$, satisfy the first and second Dubrovin system of differential equations, respectively:

$$1. \frac{\partial \xi_n(\tau,t)}{\partial \tau} = 2(-1)^{n-1}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})\big(\mu_{2n}-\xi_n(\tau,t)\big)} \times$$
$$\times f_n(\xi)\{p(\tau,t)+\xi_n(\tau,t)\},\ n \in \mathbb{Z}; \qquad (31)$$

$$2. \frac{\partial \xi_n(\tau,t)}{\partial t} = 2(-1)^{n}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t)-\mu_{2n-1})\big(\mu_{2n}-\xi_n(\tau,t)\big)} f_n(\xi) g_{n,3}(\xi). \qquad (32)$$

In addition, the following initial conditions are satisfied

$$\xi_n(\tau,t)|_{t=0} = \xi_n^0(\tau),\ \ \sigma_n(\tau,t)|_{t=0} = \sigma_n^0(\tau), n \in \mathbb{Z}, \qquad (33)$$

where $\xi_n^0(\tau), \sigma_n^0(\tau) = \pm 1, n \in \mathbb{Z}$ are the spectral parameters of the Dirac operator $\mathcal{L}(\tau, 0)$. The sequence $f_n(\xi),\ n \in \mathbb{Z}$ is defined by formula (11) and

$$g_{n,3}(\xi) = [4\xi_n^3 + 4p\xi_n^2 + 2\xi_n(p^2+q^2+q_\tau) - p_{\tau\tau} + 2(pq_\tau - p_\tau q) + 2p(p^2+q^2)]$$
$$+[2\xi_n^2 + 2\xi_n p + p^2 + q^2 + q_\tau] + \sum_{k=-\infty}^{+\infty} \frac{\zeta_k(t) v_1(\pi, \mu_k, \tau, t)}{\xi_n - \mu_k},\ \ n \in \mathbb{Z}.$$

where $p = p(\tau,t), q = q(\tau,t),\ \ \xi \equiv \xi(\tau,t) = (\dots, \xi_{-1}(\tau,t), \xi_0(\tau,t), \xi_1(\tau,t), \dots)$, $\sigma \equiv \sigma(\tau,t) = (\dots, \sigma_{-1}(\tau,t), \sigma_0(\tau,t), \sigma_1(\tau,t), \dots)$,

$$v_1(\pi, \mu_k, \tau, t) = \pi \prod_{n=-\infty}^{+\infty} \frac{\xi_n(\tau,t) - \mu_k}{\beta_n},\ \ \beta_n = \begin{cases} n, & n \neq 0, \\ 1, & n = 0, \end{cases}$$

To prove the existence and uniqueness of the solution of the Cauchy problem for the infinite Dubrovin system of differential equations (32), (33), we use the following change of variables:

$$\xi_n(\tau,t) = \mu_{2n-1} + (\mu_{2n} - \mu_{2n-1})\sin^2 x_n(\tau,t)\,,\ \ n \in \mathbb{Z}.$$

Then the Cauchy problem (32), (33) can be written as a single equation in a Banach space $\mathcal{K}$:

$$\begin{cases}\dfrac{\partial x(\tau,t)}{\partial t}=\mathbb{H}\big(x(\tau,t)\big),\\ x(\tau,t)|_{t=0}=x^0(\tau)\in\mathcal{K},\end{cases}\tag{34}$$

where $\mathbb{H}\big(x(\tau,t)\big)=\big(\dots,\mathbb{H}_{-1}\big(x(\tau,t)\big),\mathbb{H}_0\big(x(\tau,t)\big),\mathbb{H}_1\big(x(\tau,t)\big),\dots\big)$,

$$\mathcal{K}=\{x=(\dots,x_{-1}(\tau,t),x_0(\tau,t),x_1(\tau,t),\dots):\\ \|x\|=\sum_{n=-\infty}^{+\infty}(1+|n|^2)(\mu_{2n}-\mu_{2n-1})|x_n|<\infty\Bigg\}.$$

Lemma 9. If the initial functions $p_0(x)$ and $q_0(x)$ satisfy the conditions

$$p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}),\ \ q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}),$$

then the vector function $\mathbb{H}(x)$ satisfies the Lipschitz condition in the Banach space $\mathcal{K}$.

Therefore, the solution of the Cauchy problem (34) for all $t>0$ and $\tau\in\mathbb{R}$ exists and is unique. It follows that the global solution of the Cauchy problem (32), (33) exists and is unique for all $t>0$ and $\tau\in\mathbb{R}$.

Theorem 5 and Lemma 9 provide a method for finding a solution of the initial value problem (25)–(27):

1) First, we find the spectral data $\{\mu_n,\xi_n^0(\tau),\sigma_n^0(\tau)=\pm1,n\in\mathbb{Z}\}$, of the Dirac operator $\mathcal{L}(\tau,0)$;

2) Now, solving the Cauchy problem (32), (33) for an arbitrary value of τ, we find $\{\mu_n,\xi_n(\tau,t),\sigma_n(\tau,t)=\pm1,n\in\mathbb{Z}\}$;

3) From the traces formulas (3), (4) we determine the functions $p(\tau,t)$ and $q(\tau,t)$, that is, the solutions to problem (25)–(27);

4) Then, using the found functions $p(\tau,t)$ and $q(\tau,t)$, we find the Floquet solutions $\psi^{\pm}(x,\mu,t)=\Big(\psi_1^{\pm}(x,\mu,t),\psi_2^{\pm}(x,\mu,t)\Big)^T$.

Now let us consider the single-gap potential $u_0(x)=q_0(x)-ip_0(x)$ of the Dirac operator $\mathcal{L}(\tau,0)$ defined by the spectrum $E=\mathbb{R}\backslash(\mu_{-1},\mu_0)$ and spectral parameters $\xi_0(0)\in[\mu_{-1},\mu_0]$, $\sigma_0(0)=\pm1$. In this case consider Cauchy problem takes the following form:

$$\begin{cases}p_t=[p_{xxx}-6p_x(p^2+q^2)]+[-q_{xx}+2q(p^2+q^2)]-\\ -\displaystyle\sum_{k=-1}^{0}\zeta_k(t)v_1(\pi,\mu_k,t)\big(\psi_1^+(x,\mu_k,t)\psi_2^-(x,\mu_k,t)+\psi_1^-(x,\mu_k,t)\psi_2^+(x,\mu_k,t)\big),\\ q_t=[q_{xxx}-6q_x(p^2+q^2)]+[p_{xx}-2p(p^2+q^2)]+\\ +\displaystyle\sum_{k=-1}^{0}\zeta_k(t)v_1(\pi,\mu_k,t)\big(\psi_1^+(x,\mu_k,t)\psi_1^-(x,\mu_k,t)-\psi_2^+(x,\mu_k,t)\psi_2^-(x,\mu_k,t)\big),\end{cases}\tag{35}$$

$$\begin{cases}p(x,t)|_{t=0}=\dfrac{\mu_0-\mu_{-1}}{2}\cos\{-(\mu_0+\mu_{-1})x+2\sigma_0(0)\phi(0)\},\\ q(x,t)|_{t=0}=-\dfrac{\mu_0-\mu_{-1}}{2}\sin\{-\big(\mu_0+\mu_{-1}\big)x+2\sigma_0(0)\phi(0)\}.\end{cases}\tag{36}$$

Using above algorithm we find the solution of the Cauchy problem (35), (36):

$$p(\tau,t)=\frac{\mu_0-\mu_{-1}}{2}\cos\{(2K+2M)t+2\pi\int\limits_0^t\Big(\zeta_{-1}(s)+\zeta_0(s)\Big)ds-$$

$$-(\mu_0+\mu_{-1})\tau+2\sigma_0(0)\phi(0)\},$$

$$q(\tau,t)=-\frac{\mu_0-\mu_{-1}}{2}\sin\{(2K+2M)t+2\pi\int\limits_0^t\Big(\zeta_{-1}(s)+\zeta_0(s)\Big)ds-$$

$$-(\mu_0+\mu_{-1})\tau+2\sigma_0(0)\phi(0)\},$$

where

$$K=\frac{5}{4}(\mu_{-1}+\mu_0)^3-3\mu_{-1}\mu_0(\mu_{-1}+\mu_0),\ \ M=\frac{3}{4}(\mu_{-1}+\mu_0)^2-\mu_{-1}\mu_0,$$

$$\phi(0)=\arcsin\sqrt{\frac{\xi_0(0)-\mu_{-1}}{\mu_0-\mu_{-1}}}.$$

In the second paragraph of the third chapter, titled "**The Cauchy problem for the Hirota-type equation with a self-consistent integral source in the class of periodic infinite-gap functions**", we consider the following Cauchy problem for the Hirota-type equation with a self-consistent source in integral form

$$\begin{cases} p_t=[p_{xxx}-6p_x(p^2+q^2)]+[-q_{xx}+2q(p^2+q^2)]- \\ -\int_{-\infty}^{+\infty}\zeta(\mu,t)v_1(\pi,\mu,t)\big(\psi_1^+(x,\mu,t)\psi_2^-(x,\mu,t)+\psi_2^+(x,\mu,t)\psi_1^-(x,\mu,t)\big)d\mu, \\ q_t=[q_{xxx}-6q_x(p^2+q^2)]+[p_{xx}-2p(p^2+q^2)]+ \\ +\int_{-\infty}^{+\infty}\zeta(\mu,t)v_1(\pi,\mu,t)\big(\psi_1^+(x,\mu,t)\psi_1^-(x,\mu,t)-\psi_2^+(x,\mu,t)\psi_2^-(x,\mu,t)\big)d\mu, \end{cases} \quad (37)$$

$$\begin{cases} p(x,t)|_{t=0}=p_0(x),\ \ p_0(x+\pi)=p_0(x)\in C^6(\mathbb{R}), \\ q(x,t)|_{t=0}=q_0(x),\ \ q_0(x+\pi)=q_0(x)\in C^6(\mathbb{R}) \end{cases} \quad (38)$$

$$\begin{cases} p(x+\pi,t)=p(x,t),\ \ q(x+\pi,t)=q(x,t),\ \ x\in\mathbb{R},\ \ t>0, \\ p(x,t),q(x,t)\in C_x^3(t>0)\cap C_t^1(t>0)\cap C(t\geq 0). \end{cases} \quad (39)$$

Here $\zeta(\mu,t)$ is a given real continuous function with the uniform asymptote

$$\zeta(\mu,t)=O\left(\frac{1}{1+\mu^2}\right),\ \ |\mu|\to\infty. \quad (40)$$

Using identities (29), asymptotes (28) and (30) we can prove the right-hand side improper integrals of system of equations (37) are uniformly convergent and the right-hand side of (37) are π periodic functions with respect to x.

The main object of study in this paragraph is the solvability of the Cauchy problem (37)–(39), that is, establishing the minimum smoothness requirements for the initial data $p_0(x)$ and $q_0(x)$, under which there is a unique global solution of this problem.

The following lemma represents the evolution of spectral data of the operator $\mathcal{L}(\tau,t)$.

Lemma 10. Let $p(x,t),\ q(x,t), \psi^{\pm}(x,\mu,t), x \in \mathbb{R}, t > 0$ be a solution of the Cauchy problem (37)–(39). Then $\mu_n(t) = \mu_n$ and $\{\xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}\}$ satisfy the systems of Dubrovin differential equations:

$$1.\ \frac{\partial \xi_n(\tau,t)}{\partial \tau} = 2(-1)^{n-1}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t) - \mu_{2n-1})\big(\mu_{2n} - \xi_n(\tau,t)\big) f_n(\xi) \times} \\ \times \big(p(\tau,t) + \xi_n(\tau,t)\big), n \in \mathbb{Z}. \quad (41)$$

$$2.\ \frac{\partial \xi_n(\tau,t)}{\partial t} = 2(-1)^{n}\sigma_n(\tau,t)\sqrt{(\xi_n(\tau,t) - \mu_{2n-1})\big(\mu_{2n} - \xi_n(\tau,t)\big) f_n(\xi)}\, g_{n,4}(\xi). \quad (42)$$

The sequence $f_n\big(\xi(\tau,t)\big)$ is defined by the formula (11) and

$$g_{n,4}\big(\xi(\tau,t)\big) = \{4\xi_n^3 + 4p\xi_n^2 + 2\xi_n(p^2 + q^2 + q_\tau) - p_{\tau\tau} + 2(pq_\tau - p_\tau q) + \\ +2p(p^2 + q^2)\} + \{2\xi_n^2 + 2\xi_n p + p^2 + q^2 + q_\tau\} + \int_{-\infty}^{+\infty} \frac{\zeta(\mu,t) v_1(\pi,\mu,\tau,t)}{\xi_n - \mu} d\mu,\ \ n \in \mathbb{Z}.$$

Here $\xi_n = \xi_n(\tau,t), p = p(\tau,t), q = q(\tau,t)$ and

$$v_1(\pi,\mu,\tau,t) = \pi \prod_{k=-\infty}^{+\infty} \frac{\xi_k(\tau,t) - \mu}{\beta_k}, \quad \beta_k = \begin{cases} 1, & k = 0 \\ k, & k \neq 0 \end{cases}.$$

In addition, the following initial conditions are satisfied:

$$\xi_n(\tau,t)|_{t=0} = \xi_n^0(\tau),\ \ \sigma_n(\tau,t)|_{t=0} = \sigma_n^0(\tau),\ n \in \mathbb{Z}. \quad (43)$$

Lemma 11. If the initial functions $p_0(x)$ and $q_0(x)$ satisfy the conditions

$$p_0(x+\pi) = p_0(x) \in C^6(\mathbb{R}),\ \ q_0(x+\pi) = q_0(x) \in C^6(\mathbb{R}),$$

then the solution of the Cauchy problem (42), (43) for all $t > 0$ and $\tau \in \mathbb{R}$ exists and is unique.

Lemma 10 and Lemma 11 provide a method for finding a solution to problem (37)–(39).

1) First, we find the spectral data $\mu_n, \xi_n^0(\tau), \sigma_n^0(\tau) = \pm 1, n \in \mathbb{Z}$, of the Dirac operator $\mathcal{L}(\tau, 0)$;

2) Now, solving the Cauchy problem (42), (43) for an arbitrary value of τ, we find $\mu_n, \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}$;

3) From the trace formula (3) and (4) we determine the functions $p(\tau,t)$ and $q(\tau,t)$, that is, the solutions of the Cauchy problem (37)–(39);

4) Then, using the found functions $p(\tau,t)$ and $q(\tau,t)$, we find the Floquet solutions $\psi^{\pm}(x,\mu,t) = \Big(\psi_1^{\pm}(x,\mu,t), \psi_2^{\pm}(x,\mu,t)\Big)^T$.

Thus, we have proved the following theorem.

Theorem 6. If the initial functions $p_0(x)$ and $q_0(x)$ satisfy the conditions

$$p_0(x+\pi) = p_0(x) \in C^6(\mathbb{R}),\ \ q_0(x+\pi) = q_0(x) \in C^6(\mathbb{R}),$$

then there exists a uniquely determined global solution $p(\tau,t),\ q(\tau,t)$ of problem (37)–(39) belonging to the class $C_x^3(t > 0\ ) \cap C_t^1(t > 0) \cap C(t \geq 0)$, which are determined by the sum of series (3) and (4), respectively.

CONCLUSION

The dissertation is devoted to the integration of nonlinear Hirota-type equation with a finite density, Hirota-type equation with additional terms and Hirota-type equation with self-consistent series and integral sources in the class of periodic infinite-gap functions.

The main results of the dissertation thesis are as follows:

– using the method of inverse spectral problems for the periodic Dirac operator, it is proved that a Hirota-type equation with a finite density is integrable in the class of six-times continuously differentiable periodic infinite-gap functions;

– it is proved that a unique solution of the Cauchy problem exists for a Hirota-type equation with additional terms in the class of six-times continuously differentiable functions;

– using the method of inverse spectral problems for the periodic Dirac operator, it is proved that a Hirota-type equation with a self-consistent series source is integrable;

– it is proved that a solution of the Cauchy problem exists and is unique for a Hirota-type equation with a self-consistent source in integral form in the class of six-times continuously differentiable infinite-gap periodic functions.

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ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО АКАДЕМИИ НАУК РЕСПУБЛИКИ УЗБЕКИСТАН

САМАРКАНДСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМЕНИ ШАРОФА РАШИДОВА

ЭШБЕКОВ РАЙХОНБЕК ХУБАЙДУЛЛО УГЛИ

ЗАДАЧА КОШИ ДЛЯ УРАВНЕНИЯ ТИПА ХИРОТЫ С КОНЕЧНОЙ ПЛОТНОСТЬЮ И САМОСОГЛАСОВАННЫМ ИСТОЧНИКОМ

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

Ташкент–2025

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является интегрирование нелинейного уравнения типа Хироты с конечной плотностью и нелинейных уравнений типа Хироты с самосогласованными интегральными и рядовыми источниками в классе периодических бесконечнозонных функций.

Научная новизна исследования заключается в следующем:

– методом обратных спектральных задач для периодического оператора Дирака доказано, что уравнение типа Хироты с конечной плотностью интегрируемо в классе шестикратно непрерывно дифференцируемых периодических бесконечнозонных функций;

– доказано, что для уравнения типа Хироты с дополнительными членами в классе шестикратно непрерывно дифференцируемых функций существует единственное решение задачи Коши;

– методом обратных спектральных задач для периодического оператора Дирака доказано, что уравнение типа Хироты с самосогласованным рядовым источником интегрируемо в классе периодических бесконечнозонных функций;

– доказано, что для уравнения типа Хироты с самосогласованным источником в интегральной форме в классе шестикратно непрерывно дифференцируемых бесконечнозонных периодических функций существует и единственно решение начальной задачи.

Внедрение результатов исследования. На основе научных результатов, полученных при интегрировании нелинейного уравнения типа Хироты с конечной плотностью и самосогласованными источниками:

алгоритм интегрирования нелинейного уравнения типа Хироты с конечной плотностью и нелинейного уравнения типа Хироты с дополнительными членами в классе периодических бесконечнозонных функций были применены в фундаментальном гранте NRF-2020R1A2C1003119 «Global Attractors and Inertial Manifolds of Infinite Dimensional Dynamical Systems» (справка Национального университета Чоннам, Южная Корея, от 21 апреля 2025 г.). Применение научных результатов позволило исследовать свойства аттракторов для класса уравнений реакции-диффузии с запаздывающей задержкой и динамическими граничными условиями.

алгоритм нахождения решения нелинейного уравнения типа Хироты с конечной плотностью и нелинейного уравнения типа Хироты с самосогласованным рядом и интегральным источником в классе периодических бесконечнозонных функций были применены в фундаментальном гранте «GP/2023/9752700 Theoretical and numerical studies for some classes of fractional integro-differential equations in Banach space» (справка Университета Путра Малайзия, Малайзия, от 18 сентября 2025 г.). Применение научных результатов позволило изучить теоретическую сторону классов дробных интегро-дифференциальных уравнений в банаховом пространстве.

Апробация результатов исследования. Результаты исследования обсуждались на 7 научно-практических конференциях, в том числе на 3 международных и 4 республиканских.

Публикация результатов исследования. По теме диссертации опубликовано 14 научных работ, из них 7 статей в научных изданиях, рекомендованных к защите диссертаций ВАК Республики Узбекистан, в том числе 3 в зарубежных и 4 в республиканских журналах.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, заключения и списка литературы. Объем диссертации составляет 122 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI

LIST OF PUBLISHED WORKS

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I bo‘lim (I part; I часть)

1. A.B Khasanov, R.Kh. Eshbekov, T. G. Hasanov. Integration of a non-linear Hirota type equation with additional terms. Izvestiya: Mathematics. 2025, Vol. 89, No. 1, pp. 208–232 (Scopus IF 0.978). https://doi.org/10.4213/im9559

2. A.B. Khasanov, R.Kh Eshbekov. The Cauchy Problem for a Nonlinear Hirota Type Equation with a Self-Consistent Series Source. Differential Equations. 2025, Vol. 61, no. 10, pp. 1369–1381. (Scopus IF 0.8). https://doi.org/10.7868/S3034503025100063

3. A.B. Khasanov, R.Kh Eshbekov, K.N. Normurodov. Integration of a Nonlinear Hirota Type Equation with Finite Density in the Class of Periodic Functions. Lobachevskii Journal of Mathematics. 2023, Vol. 44, No. 10, pp. 4329–4347 (Scopus IF 0.8). https://doi.org/10.1134/S1995080223100220

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II bo‘lim (II part; II часть)

1. Хасанов А.Б., Эшбеков Р.Х., Жонузаков Ш. Интегрирование нелинейного уравнения типа Хироты конечной плотности в классе периодических функций. "Системы управления, сложные системы: моделирование, устойчивость, стабилизация, интеллектуальные технологии". IX Международной научно-практической конференции, 2023, Елец, Российская Федерация.

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3. A.B. Khasanov, R.Kh. Eshbekov. About a Nonlinear Hirota Equation with Finite Density. "Mathematical methods and models in high-tech production". III International forum. 2023, Saint Petersburg, Russian Federation.

4. R.Kh. Eshbekov. On The Hirota-Type Equation With A Self-Consistent Source In Integral Form. "Modern methods of mathematical physics and their applications". Republican Conference, 2024, Tashkent, Uzbekistan.

5. A.B. Khasanov, R.Kh. Eshbekov. On the Hirota-type equation with a finite density and integral self–consistent source. "Mathematical analysis and its applications to modern mathematical physics". 2025, Samarkand, Uzbekistan.

6. Р.Х.Эшбеков. Задача Коши для уравнения Хироты с самосогласованным источником в виде ряда. "Актуальные проблемы современной математики и ее приложения". 2025, Ташкент, Узбекистан.

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