V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

TURSUNALIYEV TO‘YCHIVOY G‘ANIJON O‘G‘LI

BOSHQARUV FUNKSIYALARIGA GEOMETRIK VA EKSPONENSIAL CHEGARALI DIFFERENSIAL O‘YINLARDA KO‘P QUVLOVCHIDAN QOCHISH MASALASI

01.01.02 - Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI
bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI

Toshkent - 2025

UDK: 517.977.5/.978.4

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2024.1.PhD/FM1006 raqam bilan ro‘yxatga olingan.

Dissertatsiya Matematika institutida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o‘zbek, ingliz, rus (резюме)) Ilmiy kengash veb-saytida (http://kengash.mathinst.uz) va “Ziyonet” ta’lim axborat tarmog‘ida (www.ziyonet.uz) joylashtirilgan.

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Dissertatsiya himoyasi V.I.Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2025 yil «9» dekabr kuni soat 16:00 dagi majlisida bo‘lib o‘tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99 871) 207 91 40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

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Dissertatsiya avtoreferati 2025 yil «25» noyabr kuni tarqatildi.
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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida ilmiy-texnik taraqqiyotning tez sur'atlar bilan rivojlanishi ko'plab jarayonlarni matematik boshqaruv nazariyasi asosida tahlil qilishni talab etmoqda. Shu jumladan, differensial o'yinlar nazariyasi ham murakkab dinamik tizimlarni modellashtirish va ularning amaliy sohalarda qo'llanilishini ta'minlashda muhim o'rin egallaydi. Konfliktli dinamik jarayonlar differensial tenglamalar orqali ifodalanganda, ular differensial o'yinlar muammosi sifatida o'rganiladi. Ularning umumiy xususiyatlari o'zaro bog'liq bo'lgani sababli, bunday jarayonlar ko'pincha dinamik o'yinlar nazariyasining tarkibiy qismi sifatida qaraladi. Mazkur yondashuv texnika, iqtisodiyot, biologiya, tibbiyot va boshqa ko'plab sohalarda optimal boshqaruv hamda samarali strategiyalarni ishlab chiqishda keng imkoniyatlar yaratadi. Shu bois, differensial o'yinlar nazariyasining elementlarini to'g'ri aniqlash va ulardan samarali foydalanish zamonaviy ilmiy tadqiqotlarda katta ahamiyatga ega hisoblanadi.

Jahonda bugungi kunda konfliktli dinamik jarayonlarni boshqarish va optimallashtirish uchun differensial o'yinlar sohasidagi turli masalalardagi olingan natijalar keng qo'llanilmoqda. Oddiy va chiziqli differensial tenglamalar bilan tavsiflanuvchi ko'p quvlovchidan qochish differensial o'yinlar va uning tatbiqlari bo'yicha birinchi ustuvor yo'nalishlarda qator ilmiy-tadqiqot ishlari olib borilmoqda. O'yinchilarning boshqaruvlariga geometrik chegaralar qo'yilgan holda ko'p quvlovchidan qochish masalalari, amaliyotda ko'p uchraydigan ziddiyatli sharoitlardan kelib chiquvchi fizik jarayonlarga mos keladi. Shu bilan birga, amaliy masalalarni yechishda yanada murakkab cheklovlar paydo bo'lishi mumkin. Jumladan, eksponensial cheklovlar bilan ifodalangan differensial o'yinlarda ko'p quvlovchidan qochish muammolarini o'rganish katta amaliy ahamiyatga ega va bunday hollar dissertatsiyada har tomonlama o'rganiladi. Bu borada texnikada, iqtisodiyotda va boshqa turli sohalarida ularning qo'llanilishiga alohida e'tibor berilmoqda.

Respublikamizda innovatsion texnologiyalarni rivojlatirish uchun fundamental fanlarning asosiy rolini hisobga olgan holda ularning ilmiy va amaliy ahamiyatini oshirish doirasida ishlab chiqilgan strategiyalar doirasida keng qamrovli chora-tadbirlar amalga oshirilib, muayyan natijalarga erishilmoqda. Xususan, differensial tenglamalar, matematik-fizika tenglamalari, algebra va funksional analiz, shuningdek, matematik analiz sohalarida olib borilayotgan ilmiy tadqiqotlarni xalqaro standartlar darajasida rivojlantirish bo'yicha muhim vazifalar belgilangan.[1] Ushbu vazifalarni amalga oshirishda, jumladan, dinamik sistemalar, differensial tenglamalar, optimal boshqaruv va diifferensial o'yinlar nazariyalarini takomillashtirishda geometrik va eksponensial cheklovlarga ega ko'p

[1] O'zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi №PQ-4387 "Matematika ta'limi va fanlarini yanada rivojlantirishni davlat tomonidan qo'llab-quvvatlash, shuningdek, O'zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to'g'risida"gi qarori.

quvlovchidan qochish differensial o‘yinlari uchun optimal yechimlarni ta’minlaydigan boshqaruv modellarini qurish muhim ilmiy-amaliy ahamiyat kasb etmoqda.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PQ-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi, 2017-yil 17-fevraldagi PQ-2789-son “Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2017-yil 20-apreldagi PQ-2909-son “Oliy ta’lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida”gi, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi, 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida”gi qarorlari, hamda mazkur faoliyatga tegishli boshqa me’yoriy-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishga ushbu dissertatsiya ishi muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalarni rivojlantirishning ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Differensial o‘yinlar tushunchasini amerikalik matematik R. Aysaks tomonidan asos solingan. Differensial o‘yinlarning fundamental nazariyasi asoschilari sifatida ko‘riladigan olimlarning ilmiy ishlari ushbu sohaning nazariy asoslarini shakllantirdi, yangi metodlarni ishlab chiqish va ularni turli amaliy jarayonlarga tatbiq etishda muhim o‘rin tutdi. Jumladan, L.S. Pontryagin, N.N. Krasovskiy, L.D. Berkovits, W.H. Fleming, A. Fridman, A. Bryson, E.F. Mishchenko, A.I. Subbotin, Yu.S. Osipov, B.N. Pshenichniy, N.Yu. Satimov, L.A. Petrosyan va A.A. Azamov. A.A. Chikriy, N.L. Grigorenko, N.Yu. Lukoyanov, A.G. Pashkov, N. Nikandr Petrov, B.N. Pshenichniy, I.S. Rappoport, kabi olimlar e’tirof etiladi. Ularning ishlari differensial o‘yinlarning nazariy asoslarini yaratib, boshqaruv nazariyasi, variatsion hisoblash va optimal boshqaruv masalalarida keng qo‘llaniladigan metodlarni ishlab chiqishga zamin yaratdi. Mazkur olimlar tomonidan ishlab chiqilgan konsepsiyalar bugungi kunda ko‘p quvlovchi va qochuvchi ishtirokidagi differensial o‘yinlar, boshqaruv funksiyalariga cheklovlar qo‘yilgan dinamik sistemalar hamda amaliyotda uchraydigan ko‘plab optimallik masalalarini hal etishda asosiy nazariy manba bo‘lib xizmat qilmoqda.

Cheksiz vaqt oraliqida qochish masalasi ilk bor L.S. Pontryagin va E.F. Mishchenko tomonidan kiritilib o‘rganilgan. E.F. Mishchenko esa ko‘p quvlovchilar ishtirokidagi masalalarda qochish uchun yangi manevrni taklif qilgan. Tadqiqotlarning muhim qismi oddiy harakatli ko‘p o‘yinchidan iborat qochish differensial o‘yinlarini tahlil qilishga bag‘ishlangan. Keyinchalik F.L. Chernous’ko tezroq qochuvchi va bir nechta quvlovchilar ishtirokidagi qochish o‘yinini o‘rganib, tezroq qochuvchi quvlovchilardan qutulishi mumkinligini isbotladi. Ushbu natijalar keyinchalik F.L. Chernous’ko va V.L. Zak tomonidan yanada umumiy differensial o‘yin masalalariga kengaytirildi. Shuningdek, quvlovchilar guruhidan qochish bilan bog‘liq masalalar ham o‘rganilgan.

Mamlakatimizda differensial o‘yinlar bo‘yicha A.A. Azamov, G‘.I. Ibragimov, N.A. Mamadaliyev, M.Sh. Mamatov, O.Sh. Qo‘chqarov, B.B. Rixsiyev, N.Yu. Satimov, B.T. Samatov, Tuxtasinov va boshqalar geometrik va integral chegaralanishlar sharoitida ko‘p ishtirokchili diskret va differensial o‘yinlarda quvish-qochish masalalarini tadqiq qilib, bunday masalalarni yechish usullarini yanada kengaytirganlar. Ularning ishlari quvuvchi va qochuvchi strategiyalarining optimal qurilishi evaziga differensial o‘yinlarning nazariy asoslari va amaliy qo‘llanmalarini rivojlantirishga muhim hissa qo‘shgan. Xususan, ba’zi tadqiqotlarda quvuvchining yetib olish vaqti aniq hisoblangan, qochuvchining qochish imkoniyatlarini kafolatlovchi shartlar berilgan.

F.L. Chernous’ko bitta tezroq qochuvchi va bir nechta quvlovchidan iborat differensial o‘yinni tahlil qilib, qochuvchi tezlikda ustunlikka ega bo‘lganda samarali strategiya mavjudligini ko‘rsatdi. V.L. Zak esa bu masalani rivojlantirib, ko‘p quvlovchilar ishtirokidagi qochish strategiyalarini umumiylashtirdi. Keyinchalik, F.L. Chernous’ko va V.L. Zak birgalikda ushbu natijalarni chuqurlashtirib, optimal trayektoriyalar mavjudligi va ularning barqarorligini asoslab berdilar. Natijada, ularning ishlari differensial o‘yinlarda ko‘p quvlovchilardan qochish masalalarining nazariy asoslarini yaratdi.

Dissertatsiya mavzusining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti O‘zbekiston Respublikasi Fanlar akademiyasi V.I. Romanovskiy nomidagi Matematika institutining ilmiy-tadqiqot ishlari rejasi bo‘yicha olib borilayotgan “Dinamik va boshqariluvchi sistemalar” dasturi doirasida tayyorlangan.

Tadqiqotning maqsadi o‘yinchilarning boshqaruv funksiyalari geometrik va eksponensial cheklovlarni qanoatlantiruvchi differensial o‘yinlarda ko‘p quvlovchilardan qochish masalalarini hal qilishdan iborat.

Tadqiqotning vazifalari:

boshqaruv funksiyalari geometrik chegaralanishga ega chiziqli differensial o‘yinlarda ko‘p quvlovchilardan qochish masalalari yechilishini kafolatlovchi yetarli shartlarini aniqlash;

boshqaruv funksiyalari geometrik chegaralanishga ega chiziqli differensial o‘yinlarda quvlovchilar soni ikkiga teng bo‘lgan hol uchun yaqinlashish vaqtlarini aniq qiymatini aniqlash;

boshqaruv funksiyalari geometrik va eksponensial chegaralanishga ega differensial o‘yinlarda ko‘p quvlovchilardan qochish masalalarida qochuvchi uchun strategiya qurish va uning joizligini ko‘rsatish;

boshqaruv funksiyalari eksponensial chegaralanishga ega oddiy differensial o‘yinlarda ko‘p quvlovchilardan qochish masalalari yechilishini kafolatlovchi qochuvchi uchun strategiya qurish va yetarli shartlarni aniqlash.

Tadqiqotning obyekti. Boshqaruvlariga geometrik va eksponensial integral cheklovlar qo‘yilgan holda ko‘p quvlovchidan qochish differensial o‘yinlari.

Tadqiqotning predmeti. Qochuvchi uchun qochishni kafolatlaydigan strategiya qurish hamda geometrik va eksponensial integral cheklovlarda qochish differensial o‘yinini hal qilish.

Tadqiqotning usullari. Tadqiqot ishida ko‘p quvlovchidan qochish masalasini yechish uchun qochuvchiga strategiya ishlab chiqildi va quvlovchi hamda qochuvchi orasidagi masofani quyidan baholash usuli qo‘llanildi. Yechimlarni asoslashda oddiy differensial tenglamalar nazariyasi, funksional analiz, differensial o‘yinlar nazariyasi hamda optimal boshqaruv nazariyasi usullaridan foydalanildi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

boshqaruv funksiyalari geometrik cheklovli chiziqli differensial o‘yinlarda bir quvlovchi va bir qochuvchidan iborat masalada qochishni kafolatlovchi qochuvchi uchun strategiya qurilgan hamda quvlovchi va qochuvchi orasidagi masofa quyidan baholangan;

boshqaruv funksiyalari geometrik cheklovli chiziqli differensial o‘yinlarda ikki quvlovchiga ega qochish masalasida qochuvchi uchun strategiya qurilib, yaqinlashish vaqtlari soni 3 dan oshib ketmasligi isbotlangan;

boshqaruv funksiyalari geometrik cheklovli chiziqli qochish differensial o‘yinlarida m quvlovchiga ega qochish masalasida qochuvchi uchun strategiya qurilib, har bir guruh hujumidan so‘ng eng kamida bitta quvlovchi keyingi guruh hujumida ishtirok etmasligi hamda quvlovchilarning yaqinlashish vaqtlari soni $m(m+1)/2$ dan oshib ketmasligi isbotlangan;

o‘yinchilarning boshqaruv funksiyalari eksponensial integral cheklovli sodda harakatli differensial o‘yinlarda qochuvchi uchun strategiya qurilib uning joizligi va quvlovchilarning yaqinlashish vaqtlari soni m dan oshmasligi isbotlangan, shuningdek ko‘p quvlovchilardan qochish masalalari yechilishining yetarli shartlari va o‘yinchilar orasidagi masofaning quyi chegarasi topilgan.

Tadqiqotning amaliy natijasi. Dissertatsiya doirasida erishilgan natijalar va ularni isbotlashda qo‘llanilgan usullar ziddiyatli boshqaruv jarayonlarini ifodalovchi matematik modellarni bevosita qochish masalalariga tatbiq etish imkonini beradi. Tadqiqotda olingan natijalar sonli misollar bilan tahlil qilingan va ularning geometrik ko‘rinishlari ham taqdim etilgan. Bu esa tadqiqot ishidagi masalalarning real jarayonlarni matematik modellashtirishda samarali ishlatilishiga muhim hissa qo‘shadi.

Tadqiqot natijalarining ishonchliligi. Oddiy differensial tenglamalar nazariyasi, optimal boshqaruv, funksional analiz, dinamik sistemalar nazariyasi va differensial o‘yinlar nazariyasi metodlaridan foydalangan holda matematikada qabul qilingan xulosalar, jumladan, teorema va xossalarga asoslangan holda kafolatlanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati boshqaruvlari geometrik va eksponensial integral chegaralanish bilan ifodalangan ziddiyatli boshqaruv masalalarining yechimini ta’minlovchi strategiyalarni qurish va ularni amaliyotda qo‘llash bilan belgilanadi. Olingan ilmiy natijalar optimal boshqaruv va differensial o‘yinlar nazariyalaridagi mavjud metodlarni yanada takomillashtirish hamda ularning amaliy tatbiqi bilan bog‘liq murakkab muammolarning samarali yechimlarini aniqlash imkonini beradi.

Tadqiqot natijalarining amaliy ahamiyati oddiy va chiziqli differensial tenglamalar orqali ifodalangan qochish differensial o‘yinlari asosida yaratilgan matematik modellarni turli murakkab real jarayonlarga tatbiq etish imkonini beradi. Ushbu natijalar harbiy sohada havo hujumidan strategik mudofaa, robototexnika va sun’iy intellektda avtonom vositalar uchun xavfsiz boshqaruv strategiyalarini ishlab chiqish, shuningdek, muhandislik va ekologiya sohalarida ziddiyatli boshqaruv masalalari uchun strategiyalarni aniqlash va hisoblash dasturlarini yaratishda qo‘llanilishi mumkin.

Tadqiqot natijalarining joriy qilinishi. Boshqaruvlari geometrik va eksponensial cheklovli ko‘p quvlovchilardan qochish differensial o‘yinlari bo‘yicha olingan natijalar asosida:

boshqaruv funksiyalari geometrik cheklovlar qo‘yilgan chiziqli differensial o‘yinlarda ko‘p quvlovchidan qochish masalalari yechilishini kafolotlovchi shartlaridan 374874-2022 raqamli “Fazaviy o‘tishlar va tahliliy hodisalar masalalari. Ularning tez o‘tish tenglamalari va asimptotikalarining matematik xususiyatlari” mavzusidagi xorijiy grant loyihasida chiziqli differensial tenglamalar uchun analogik masalalarni yechishda foydalanilgan (O‘sh davlat universitetining 2025-yil 17-sentabrdagi 1254-sonli ma’lumotnomasi, Qirg‘iziston Respublikasi). Ilmiy natijaning qo‘llanishi chiziqli differensial tenglamalar bilan ifodalangan fazaviy o‘tishlar va tahliliy hodisalar masalalarining to‘liq yechimlarini ko‘rsatish imkonini bergan;

boshqaruv funksiyalari eksponensial integral cheklovlarga ega oddiy differensial o‘yinlarda ko‘p quvlovchidan qochish masalalari bo‘yicha olingan natijalardan Reggio Kalabriya O‘rta yer dengizi universitetining “Qarorlar laboratoriyasi tadqiqot dasturi” mavzusidagi loyihasida differensial o‘yinlar masalalarini yechishda foydalanilgan (Reggio Kalabriya O‘rta yer dengizi universitetining 2025-yil 4-sentyabrdagi ma’lumotnomasi, Italiya). Ilmiy natijaning qo‘llanishi Evklid fazosida berilgan differensial o‘yinlarda chekli sondagi quvlovchiga ega qochish masalasida agar qochuvchi qochish strategiyasini qo‘llasa quvlovchilarning ixtiyoriy joiz boshqaruvlariga qarshi muvaffaqiyatli qochish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Tadqiqotning asosiy natijalari 9 ta ilmiy-amaliy anjumanlarda muhokama qilingan bo‘lib, shulardan 7 tasi xalqaro va 2 tasi respublika miqyosidagi ilmiy anjumanlardir.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi doirasida jami 14 ta ilmiy maqola chop etilgan bo‘lib, ularning 5 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasi tomonidan falsafa doktori (PhD) dissertatsiyasi himoyasi uchun tavsiya etilgan ilmiy nashrlar ro‘yxatiga kiritilgan. Shundan 4 tasi respublika ilmiy jurnallarida, 1 tasi Scopus bazasiga kiruvchi xalqaro jurnalda, qolgan 9 tasi esa tezislardir.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat bo‘lib, umumiy hajmi 87 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida tadqiqotning dolzarbligi va zaruriyligi asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, dissertatsiya mavzusi bo‘yicha xorijiy ilmiy tadqiqotlarning tahlili hamda muammoning o‘rganilganlik darajasi yoritilgan, tadqiqotning obyekti va predmeti ko‘rsatilgan, maqsad va vazifalari aniqlangan hamda ilmiy yangiligi bayon etilgan. Shuningdek, olingan natijalarning nazariy va amaliy ahamiyati, tadqiqot natijalarining joriy etilishi, nashr etilgan ishlar hamda dissertatsiyaning tuzilishi haqida ma’lumotlar berilgan.

Birinchi bob **“Geometrik cheklovlarga ega ikki quvlovchi va bir qochuvchidan iborat differensial o‘yinda qochish masalasi”** deb nomlanib, boshqaruv funksiyalari geometrik cheklovlarga bo‘ysunuvchi chiziqli differensial o‘yinlardagi ikki quvlovchidan qochish masalalari o‘rganiladi va qochuvchi uchun strategiya quriladi.

Faqar qilaylik, $\mathbb{R}^n$, $n \geq 2$ fazoda harakatlanuvchi x_1 va x_2 quvlovchilar hamda y qochuvchi berilgan bo‘lsin. Quvlovchilar guruhini $x=(x_1,x_2)$ bilan, ularning boshqaruv parametrlarini $u=(u_1,u_2)$ bilan ifodalaymiz va ularning dinamikasi quyidagi tenglamalar bilan ifodalanadi

$$\begin{aligned} &\dot{x}_i = -\lambda x_i + u_i, \quad x_i(0) = x_{i0}, \qquad |u_i| \leq 1, \\ &\dot{y} = -\lambda y + v, \qquad y(0) = y_0, \qquad |v| \leq \sigma, \end{aligned} \tag{1}$$

bu yerda $x_i, x_{i0}, y, y_0, u_i, v \in \mathbb{R}^n$, $\lambda > 0$; $x_i(0) = x_{i0}$ va $y(0) = y_0$ o‘yinchilarning boshlang‘ich holatlari bo‘lib, ular boshlang‘ich vaqtda qochuvchining boshlang‘ich holati bilan ustma-ust tushmaydi ya’ni $x_{i0} \neq y_0$, $i = 1,2$. σ, $\sigma > 1$ berilgan son, u_i va v mos ravishda x_i quvlovchining hamda y qochuvchining boshqaruv parametrlari.

1-ta’rif. O‘lchovli $u_i(t)$, $|u_i(t)| \leq 1$ va $v(t)$, $|v(t)| \leq \sigma$, $t \geq 0$ funksiyalar x_i, $i \in \{1,2\}$ quvlovchining va y qochuvchining joiz boshqaruvlari deyiladi.

2-ta’rif. $V: \mathbb{R}_+ \times \mathbb{R}^{5n} \to \mathbb{R}^n$, $(t, y, x_1, x_2, u_1, u_2) \to V(t, y, x_1, x_2, u_1, u_2)$ funksiya qochuvchining strategiyasi deyiladi agar quvlovchilarning ixtiyoriy joiz $u_1 = u_1(t)$, $u_2 = u_2(t)$ boshqaruvlari uchun quyidagi Koshi masalasi

$$\begin{aligned} &\dot{x}_i = -\lambda x_i + u_i, \quad x_i(0) = x_{i0} \\ &\dot{y} = -\lambda y + V(t, y, x_1, x_2, u_1, u_2), \quad y(0) = y_0 \end{aligned}$$

yagona $(y(t), x_1(t), x_2(t))$ yechimga ega bo‘lsa.

3-ta’rif. Quvlovchilarning ixtiyoriy $u_i = u_i(t),\ i=1,2$ boshqaruvlari uchun qochuvchining shunday V strategiyasi mavjud bo‘lib, barcha $t \ge 0$ va $i=1,2$ uchun $x_i(t) \ne y(t)$ shart bajarilsa, (1) qochish differensial o‘yinida qochish mumkin deyiladi.

1-masala. Masala qochuvchi uchun V strategiya qurish va (1) qochish differensial o‘yini yechishdan iborat.

Dastlab $n=2$ holda bir quvlovchi va bir qochuvchidan iborat qochish differensial o‘yinini ko‘rib chiqamiz. x quvlovchi va y qochuvchining dinamikasi quyidagi tenglamalar orqali berilgan

$$\begin{aligned} &\dot{x} = -\lambda x + u, \quad x(0) = x_0, \quad |u(t)| \le 1, \\ &\dot{y} = -\lambda y + v, \quad y(0) = y_0, \quad |v(t)| \le \sigma, \end{aligned} \tag{2}$$

bu yerda $x, x_0, y, y_0, u, v \in \mathbb{R}^2$, $\lambda > 0$, $x_0 \ne y_0$.

(2) tenglamaning yechimlari

$$x(t) = x_0 e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)} u(s)ds, \quad y(t) = y_0 e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)} v(s)ds.$$

hisoblanadi va bu harakat trayektoriyalarida $\xi(t) = e^{\lambda t} x(t),\ \eta(t) = e^{\lambda t} y(t)$ kabi almashtirish bajaramiz, u holda quyidagiga ega bo‘lamiz

$$\xi(t) = x_0 + \int_0^t e^{\lambda s} u(s)ds, \quad \eta(t) = y_0 + \int_0^t e^{\lambda s} v(s)ds.$$

Buning natijasida, x quvlovchini ξ va y qochuvchini η deb ataymiz.

Biz α va a sonlarini quyidagicha kiritib olamiz

$$0 < \alpha < \frac{1}{2}(\sigma - 1), \qquad 0 < a < |x_0 - y_0|.$$

Biz endi qochuvchi uchun strategiya qurib olamiz. Dastlab, qochuvchi boshlang‘ich $t=0$ vaqtdan boshlab Oy-o‘qiga parallel quyidagi o‘zgarmas boshqaruv bilan harakatlansin

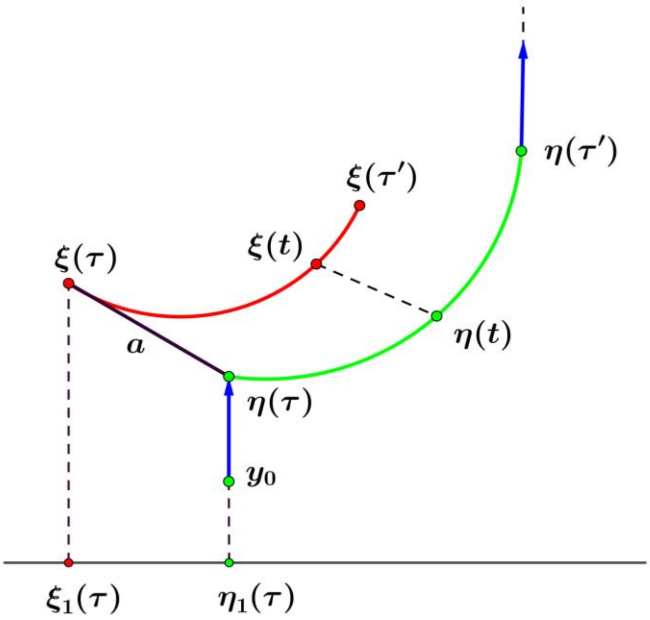
$$V(t) = V_0 = (0, \sigma),\ t \in [0, \tau), \tag{3}$$

bu yerda $V_1(t) = 0$, $V_2(t) = \sigma$ hamda $t = \tau$ vaqt $|\xi(t) - \eta(t)| = a$ shart bajariladigan birinchi vaqt hisoblanadi. Biz τ vaqtni quvlovchini qochuvchiga a-yaqinlashish vaqti deb ataymiz. 1-chizmadagi y_0 va $\eta(\tau)$ ga to‘g‘ri keluvchi kesma qochuvchining (3) boshqaruviga mos trayektoriyasidir.

Shuni ta’kidlash kerakki τ vaqt ro‘y bermasligi mumkin, bu holda barcha $t \ge 0$ uchun $|\xi(t) - \eta(t)| > a$ tengsizlikga ega bo‘lamiz va ravshanki barcha $t \ge 0$ uchun $\xi(t) \ne \eta(t)$ shart bajariladi. Shuning uchun, biz τ vaqt ro‘y bersin deb tasavvur qilamiz.

Shuningdek, τ' vaqtni quyidagicha aniqlaymiz

$$\tau' = \frac{1}{\lambda} \ln\left(e^{\lambda\tau} + \frac{2\lambda a}{\sigma - 1 - \alpha} \right).$$



Chizma 1. Qochuvchi trayektoriyasi, bunda $\xi_1(\tau) \le \eta_1(\tau)$.

Qochuvchi $[\tau, \tau')$ oraliqda quyidagi strategiyani qo‘llaydi

$$V(t) = (V_1(t), V_2(t)), \quad t \in [\tau, \tau'), \tag{4}$$

bu yerda

$$V_1(t) = \begin{cases} |u_1(t)| + \alpha, & \xi_1(\tau) \le \eta_1(\tau) \\ -(|u_1(t)| + \alpha), & \xi_1(\tau) > \eta_1(\tau) \end{cases}, \quad V_2(t) = \sqrt{\sigma^2 - V_1^2(t)}. \tag{5}$$

Ravshanki, $|V_1(t)| \le |u_1(t)| + \frac{1}{2}(\sigma - 1) \le \sigma$, shuning uchun $V_2(t)$ aniqlangan. Biz (4) orqali aniqlangan $V(t)$ ni ξ quvlovchining η qochuvchiga qarshi manyovri deb ataymiz. Qochuvchi strategiyasining yakuniy qismi sifatida, quyidagini olamiz

$$V(t) = V_0 = (0, \sigma), \ t \ge \tau'. \tag{6}$$

1-lemma. Qochuvchi (3), (4) va (6) strategiyalardan foydalansin, bu yerda τ vaqt ξ quvlovchining η qochuvchiga a-yaqinlashish vaqti. U holda

$$|\eta(t) - \xi(t)| \ge a, \qquad 0 \le t \le \tau,$$

$$|\eta(t) - \xi(t)| > \frac{\alpha a}{2\sigma}, \ \tau \le t \le \tau',$$

$$\eta_2(t) - \xi_2(t) > a, \qquad t \ge \tau',$$

tengsizliklar bajariladi.

1-teorema. Ikki quvlovchi va bitta qochuvchining har qanday boshlang‘ich holatlari uchun, (1) o‘yinda qochish mumkin bo‘ladi.

Agar $\tau_2 < \tau_1'$ bo‘lsa, u holda $\tau_2' - \tau_1' \le \dfrac{2a_2}{\sigma - 1 - \alpha_1}$ tengsizlik bajariladi.

2-lemma. Qochuvchi $v(t) = \begin{cases} V_1(t), & t \in [\tau_1, \tau_2), \\ V_2(t), & t \in [\tau_2, \tau_2'), \end{cases}$ strategiyadan foydalansin va $\tau_1' \le \tau_2'$ bo‘lsin, u holda

$$|\eta(t)-\zeta(t)|\le \frac{8\sigma e^{\lambda T}}{\sigma-1-\alpha_1}a_2, \qquad \tau_1\le t\le \tau_1'$$

tengsizlik bajariladi.

3-lemma. Qochuvchi $v(t)=\begin{cases}V_1(t), & t\in[\tau_1,\tau_2)\\ V_2(t), & t\in[\tau_2,\tau_2')\end{cases}$ strategiyadan foydalansin va $\tau_1'\le\tau_2'$ bo‘lsin, u holda ξ_1 quvlovchi uchun ushbu

$$|\eta(t)-\xi_1(t)|>a_2, \quad \tau_1\le t\le\tau_2'$$
$$\eta_2(t)-\xi_{12}(t)>a_2, \quad \tau_1'\le t\le\tau_2'$$

tengsizliklar bajariladi.

4-lemma. Qochuvchi $v(t)=\begin{cases}V_1(t), & t\in[\tau_1,\tau_2)\\ V_2(t), & t\in[\tau_2,\tau_2')\end{cases}$, strategiyadan foydalansin va $\tau_1'\le\tau_2'$ bo‘lsin, u holda har ikki quvlovchi uchun

$$\eta_2(t)-\xi_{p2}(t)>a_2, \quad t\ge\tau_2'$$

tengsizlik barcha $p=1,2$ larda bajariladi.

5-lemma. Qochuvchi $v(t)=\begin{cases}V_1(t), & t\in[\tau_1,\tau_2)\\ V_2(t), & t\in[\tau_2,\tau_2')\end{cases}$, strategiyadan foydalansin va $\tau_2'\le\tau_1'$ bo‘lsin, u holda ikkinchi quvlovchi uchun

$$\eta_2(t)-\xi_{22}(t)>a_2, \quad t\ge\tau_2'$$

tengsizlik bajariladi.

Ikkinchi bob **“Ko‘p quvlovchilardan qochish chiziqli differensial o‘yini”** deb nomlanadi va bunda quvlovchilar soni umumlashtirilgan hol uchun qochish masalasi hal qilinadi, ya’ni m ta quvlovchili qochish differensial o‘yini ko‘rilgan. Guruh hujumi tushunchasi kiritilib, har bir guruh hujumdan so‘ng kamida bitta quvlovchi qochuvchidan ortda qolishi isbotlanadi. Bu esa ushbu bobning asosiy natijasi quvlovchilarning τ_k yaqinlashish vaqtlari soni ko‘pi bilan

$$m+(m-1)+\ldots+1=m(m+1)/2$$

dan oshmasligini ko‘rsatadi.

$\mathbb{R}^n$, $n\ge 2$ fazoda m ta $x_1,\ldots,x_m$ quvlovchilar va bitta y qochuvchi ishirokidagi qochish differensial o‘yini quyidagi chiziqli tenglamalar bilan berilgan. Quvlovchilar guruhini $x=(x_1,x_2,\ldots,x_m)$ bilan, ularning boshqaruv parametrlarini $u=(u_1,u_2,\ldots,u_m)$ bilan belgilaymiz

$$\begin{aligned}\dot{x}_i&=-\lambda x_i+u_i, & x_i(0)&=x_{i0}, & |u_i|&\le 1\\ \dot{y}&=-\lambda y+v, & y(0)&=y_0, & |v|&\le\sigma\end{aligned} \tag{7}$$

bu yerda $x_i,x_{i0},y,y_0,u_i,v\in\mathbb{R}^n$, $\lambda>0$; $x_i(0)=x_{i0}$ va $y(0)=y_0$ o‘yinchilarning boshlang‘ich holatlari, boshlang‘ich vaqtda hech bir quvlovchining boshlang‘ich holati qochuvchining boshlang‘ich holati bilan ustma-ust tushmaydi ya’ni $x_{i0}\ne y_0$, $i=1,2,\ldots,m$, va σ, $\sigma>1$ berilgan son, bu yerda u_i va v mos ravishda x_i quvlovchining hamda y qochuvchining boshqaruv parametrlari.

4-ta'rif. O'lchovli $u_i(t)$, $|u_i(t)|\le 1$, va $v(t)$, $|v(t)|\le\sigma$, $t\ge 0$ funksiyalar mos ravishda x_i, $i\in\{1,2,\ldots,m\}$ quvlovchilarning va y qochuvchining boshqaruvlari deyiladi.

5-ta'rif. $V:\mathbb{R}_+\times\mathbb{R}_+\times\mathbb{R}_+\times\mathbb{R}^{(2m+1)n}\to\mathbb{R}^n$ funksiya qochuvchining strategiyasi deyiladi

$$(t,\varphi_1,\varphi_2,y,x_1,\ldots,x_m,u_1,\ldots u_m)\to V(t,\varphi_1,\varphi_2,y,x_1,\ldots,x_m,u_1,\ldots u_m)$$

agar quvlovchilarning ixtiyoriy joiz $u_1=u_1(t),\ldots,u_m=u_m(t)$ boshqaruvlari va $\varphi_1=\varphi_1(t)$, $\varphi_2=\varphi_2(t)$ funksiyalar uchun quyidagi Koshi masalasi

$$\dot{x}_i=-\lambda x_i+u_i,\quad x_i(0)=x_{i0}$$

$$\dot{y}=-\lambda y+V(t,\varphi_1,\varphi_2,y,x_1,\ldots,x_m,u_1,\ldots,u_m),\quad y(0)=y_0$$

yagona $(y(t),x_1(t),\ldots,x_m(t))$ yechimga ega bo'lsa, bu yerda $\varphi_1(t)$ va $\varphi_2(t)$, $t\ge 0$, berilgan funksiyalar.

6-ta'rif. Quvlovchilarning ixtiyoriy joiz $u_i=u_i(t)$, $i=1,\ldots,m$ boshqaruvlari uchun qochuvchining shunday V strategiyasi mavjud bo'lib, barcha $t\ge 0$ va $i=1,\ldots,m$ uchun $x_i(t)\ne y(t)$ shart bajarilsa, (7) qochish differensial o'yinida qochish mumkin deyiladi.

2-masala. Masala qochuvchi uchun V strategiya qurish va (7) qochish differensial o'yini yechishdan iborat.

2-teorema. $x_1,\ldots,x_m$ quvlovchilarning va y qochuvchining ixtiyoriy boshlang'ich holatlari uchun, (7) differensial o'yinda qochish mumkin bo'ladi.

1-tasdiq. Ixtiyoroy butun $p\ge 0$ son uchun $\sum_{k=p+1}^{\infty}a_k=\dfrac{a_{p+1}}{1-\beta}\le 2a_{p+1}$ bajariladi.

2-tasdiq. ***A*** holdagi ixtiyoriy $p\in\{1,...,k_1-1\}$ lar uchun va ***B*** holda ixtiyoriy musbat butun p lar uchun

$$t-\tau_{p+1}\le\frac{4}{\sigma-1-\alpha_1}a_{p+1},\qquad t\in[\tau_{p+1},\overline{\tau}]$$

munosabat bajariladi.

6-lemma. Qochuvchi ***A*** holda

$$v(t)=\begin{cases}V_k(t), & t\in[\tau_k,\tau_{k+1}),\ k=p,p+1,\ldots,k_1-1\\ V_{k1}(t), & t\in[\tau_{k1},\tau'_{k1})\end{cases}$$

strategiyadan va ***B*** holda

$$v(t)=V_k(t),\ t\in[\tau_k,\tau_{k+1}),\ \tau_k<\tau'_{k-1},\ k=p,p+1,\ldots.$$

strategiyadan foydalansin, u holda

$$|\eta(t)-\zeta_p(t)|\le\frac{16\sigma e^{\lambda T}}{\sigma-1-\alpha_1}a_{p+1},\qquad t\in[\tau_p,\tau'_p]\cap[\tau_p,\overline{\tau}]$$

tengsizlik bajariladi.

7-lemma. Qochuvchi ***A*** holda

$$v(t)=\begin{cases} V_k(t), & t\in[\tau_k,\tau_{k+1}),\ k=p,p+1,\dots,k_1-1, \\ V_{k1}(t), & t\in[\tau_{k1},\tau'_{k1}), \end{cases}$$

strategiyadan foydalansin va ***B*** holda

$$v(t)=V_k(t),\ t\in[\tau_k,\tau_{k+1}),\ \tau_k<\tau'_{k-1},\ k=p,p+1,\dots .$$

strategiyadan foydalansin, u holda

$$|\eta(t)-\xi_p(t)|>a_{p+1},\qquad \tau_p\le t\le\overline{\tau}$$

munosabat bajariladi.

8-lemma. ξ_p quvlovchi uchun $\tau'_p\le\tau'_{k_1}$ bo‘lsin, u holda

$$\eta_2(t)-\xi_{p2}(t)>a_{p+1},\quad t\ge\tau'_{k_1}$$

munosabat bajariladi.

Uchinchi bob **“Eksponensial integral cheklovli ko‘p quvlovchi va bitta qochuvchidan iborat qochish differensial o‘yini”** deb nomlanadi. Boshqaruv funksiyalariga eksponensial cheklovlar qo‘yilgan holda ko‘p quvlovchilardan qochish differensial o‘yini sodda harakatli tenglamalar bilan berilgan holda o‘rganiladi. Ushbu bobda har bir vaqt oralig‘ida qochuvchi uchun manyovrini tasdiqlovchi $r(t)$ funksiyasi quriladi va ko‘p quvlovchilardan qochishni kafolatlovchi qochuvchi strategiyasi quriladi. Shuningdek, har bir quvlovchi uchun yaqinlashish vaqti ko‘pi bilan bir marta sodir bo‘lishini va o‘yindagi quvlovchilarning umumiy yaqinlashishlar soni m dan oshmasligi isbotlanadi.

Bizga $\mathbb{R}^n$, $n\ge 2$ fazoda harakatlanayotgan m ta x_i, $i=1,...,m$ quvlovchilar va bitta y qochuvchi berilgan bo‘lsin. Ularning dinamikasi quyidagi tenglamalar orqali berilgan:

$$\begin{aligned} &\dot{x}_i=u_i,\quad x_i(0)=x_i^0\quad i=1,\dots,m,\\ &\dot{y}=v,\quad\ \ y(0)=y^0, \end{aligned}\tag{8}$$

bu yerda $x_i,x_i^0,y,y^0,u_i,v\in\mathbb{R}^n$, $n\ge 2$; $x_i^0\ne y^0$, $i=1,...,m$. o‘yinchilarning boshlang‘ich holatlari va ular boshlang‘ich vaqtda bir biriga teng emas ya’ni $x_{i0}\ne y_0$. $u_1,\dots,u_m$ va v mos ravishda quvlovchilar va qochuvchining boshqaruv parametrlari.

7-ta’rif. O‘lchovli $u_i(t)$, $t\ge 0$ funksiya x_i quvlovchining joiz boshqaruvi deyiladi, agar u quyidagi eksponensial chegaralanishni qanoatlantirsa:

$$\int_0^t|u_i(s)|^2\,ds\le\rho_i^2e^{2lt},\quad i=1,\dots,m,\tag{9}$$

bu yerda $\rho_1,\rho_2,...,\rho_m$ va l berilgan musbat sonlar.

8-ta’rif. O‘lchovli $v(t)$, $t\ge 0$ funksiya y qochuvchining joiz boshqaruvi deyiladi, agar u quyidagi eksponensial chegaralanishni qanoatlantirsa:

$$\int_0^t|v(s)|^2\,ds\le\sigma^2e^{2lt},\tag{10}$$

bu yerda σ berilgan musbat son.

9-ta’rif. $V:[0,\infty)\times\mathbb{R}^{(2m+1)n}\to\mathbb{R}^n$

$$(t, y, x_1, ..., x_m, u_1, ..., u_m) \mapsto V(t, y, x_1, ..., x_m, u_1, ..., u_m)$$

funksiya qochuvchining strategiyasi deyiladi agar quvlovchilarning ixtiyoriy joiz $u_i = u_i(t)$, $i = 1, ..., m$ boshqaruvlari uchun quyidagi Koshi masalasi

$$\dot{x}_i = u_i, \quad x_i(0) = x_i^0, \quad i = 1, 2, \ldots, m,$$
$$\dot{y} = V(t, y, x_1, ..., x_m, u_1, ..., u_m), \quad y(0) = y^0,$$

yagona $(x_1(t), \ldots, x_m(t), y(t))$, $t \geq 0$ yechimga ega bo‘lsa va u quyidagi tengsizlikni qanoatlantirsa

$$\int_0^t |V(s, y(s), x_1(s), \ldots x_m(s), u_1(s), \ldots, u_m(s))|^2 \, ds \leq \sigma^2 e^{2lt}.$$

10-ta’rif. Quvlovchilarning ixtiyoriy joiz $u_i = u_i(t)$, $i = 1, ..., m$ boshqaruvlari uchun qochuvchining shunday V strategiyasi mavjud bo‘lib, barcha $t \geq 0$ va $i = 1, \ldots, m$ lar uchun $x_i(t) \neq y(t)$ shart bajarilsa, u holda (8)-(10) o‘yinda qochish mumkin deyiladi.

3-masala. (8)-(10) ko‘p quvlovchilardan qochish differensial o‘yinini kafolatlovchi y qochuvchi uchun V strategiya qurish hamda ρ_i, $i = 1, ..., m$ va σ parametrlar uchun yetarli shartlarni topishdan iborat.

3-teorema. Agar quyidagi tengsizlik bajarilsa

$$\rho_1^2 + \ldots + \rho_m^2 < \sigma^2$$

u holda (8)-(10) differensial o‘yinda ko‘p quvlovchilardan qochish ketish mumkin bo‘ladi.

$\{a_k\}_{k=1}^{\infty}$ ketma ketlik $a_{k+1} = \beta \cdot a_k^4$, $k = 1, 2, ..$ formula bilan aniqlangan bo‘lsin. Bu ketma ketlik quyidagi xossaga ega:

2-xossa. Ixtiyoriy $p \geq 1$ uchun $\sum\limits_{k=p+1}^{\infty} a_k \leq 2a_{p+1}$ tengsizlik bajariladi.

I_k quyidagi $I_k = \cup_{j=k}^{m_0} [\tau_j, \tau_j')$, $I_{m_0+1} = \varnothing$ ko‘rinishdagi to‘plam bo‘lsin. Biz har bir vaqtda qochuvchi uchun manyovrni tayinlashda asosiy rol o‘ynaydigan $r : [0, T] \to \{0, 1, \ldots, m_0\}$ funksiyani quyidagicha aniqlaymiz

$$r(t) = \begin{cases} 0, & t \in [0, T] \setminus I_1, \\ k, & t \in [\tau_k, \tau_k'] \setminus I_{k+1}, \; k = 1, ..., m_0, \end{cases}$$

hamda $r(t)$ funksiya quyidagi xossaga ega.

3-xossa. $m_0 > 1$ bo‘lsin. U holda, $k = 1, 2, \ldots, (m_0 - 1)$ uchun quyidagi o‘rinli Agar $\tau_k' \leq \tau_{k+1}$ bajarilsa, u holda $\tau_k \leq t < \tau_k'$ uchun $r(t) = k$ bo‘ladi.

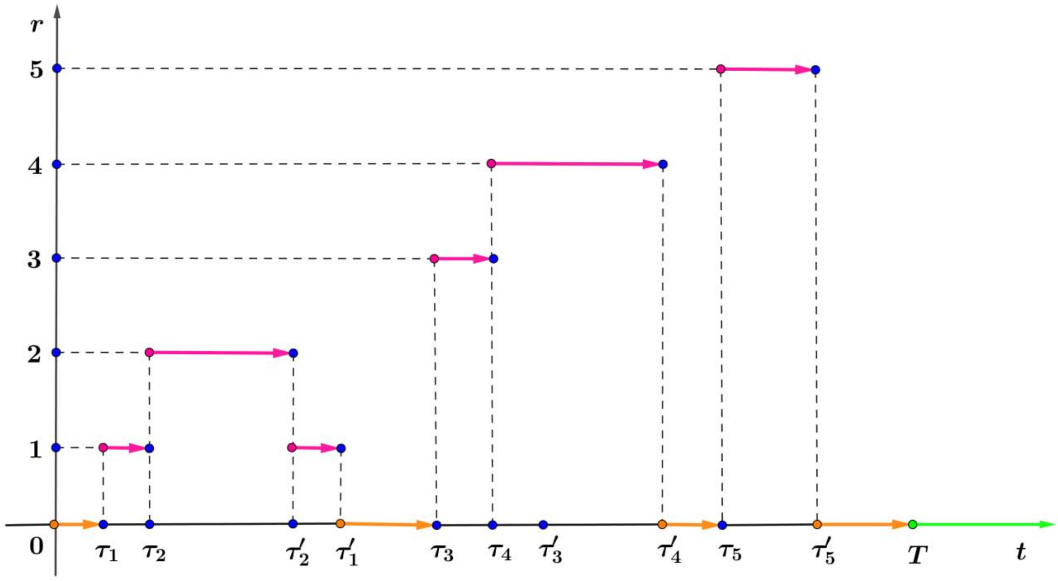
(i) Agar $\tau_k' \leq \tau_{k+1}$ bo'lsa, u holda $\tau_k \leq t < \tau_k'$ uchun $r(t) = k$ bo'ladi,

(ii) Agar $\tau_{k+1} \leq \tau_k'$ bo'lsa, u holda $\tau_k \leq t < \tau_{k+1}$ uchun $r(t) = k$ bo'ladi.

Misol. Agar quyidagi shart bajarilsa

$$0 = \tau_0 < \tau_1 < \tau_2 < \tau_2' < \tau_1' < \tau_3 < \tau_4 < \tau_3' < \tau_4' < \tau_5 < \tau_5',$$

u holda $r(t)$ funksiya 2-chizmada ko’rsatilganidek grafikka ega bo‘ladi.



Chizma 2. $r(t)$ funksiya grafigi.

$u_i(t)$, $i=1,...,m$ quvlovchilarning joiz boshqaruvlari bo‘lsin. Biz qochuvchi uchun strategiyani quyidagi (11)-(14) ko‘rinishida qurib olamiz.

$$v(t)=V_0(t)=\left(0,\alpha+\left(\sum_{i=1}^{m}|u_i(t)|^2\right)^{1/2}\right),\quad t\in[0,T]\setminus I_1, \tag{11}$$

$$v(t)=V_r(t)=(V_{r1}(t),U(t)),\quad t\in[0,T]\cap I_1, \tag{12}$$

bu yerda $r=r(t)$, $V_k(t)=(V_{k1}(t),U(t))$, $\tau_k\le t<\tau_k'$, $k=1,\dots,m_0$ bo‘lib, ular quyidagicha aniqlangan

$$V_{k1}(t)=\begin{cases}\alpha+|u_{k1}(t)|, & y_1(\tau_k)\ge x_{k1}(\tau_k),\\ -(\alpha+|u_{k1}(t)|), & y_1(\tau_k)<x_{k1}(\tau_k),\end{cases} \tag{13}$$

$$U(t)=\alpha+\left(\sum_{i=1}^{m}u_{i2}^2(t)\right)^{1/2}.$$

Eslatib o‘tamizki $U(t)$ k ga bog‘liq emas. Nihoyat, quyidagicha bo‘lsin

$$v(t)=\left(0,\left(\sum_{i=1}^{m}|u_i(t)|^2\right)^{1/2}\right),\quad t>T. \tag{14}$$

9-lemma. Agar qochuvchi (11)-(14) strategiyadan foydalansa, u holda
(a) Barcha $k\in J_0=\{1,...,m_0\}$ lar uchun, (i) $\tau_k\le T_0$ va (ii) $\tau_k'\le T$ larga ega bo'lamiz
(b) Agar biror $i\in\{1,...,m\}$ lar uchun $y_2^0\ge x_{i2}^0$ bo'lsa, u holda barcha $t>0$ uchun $y_2(t)>x_{i2}(t)$ bo‘ladi.

10-lemma. x_p quvlovchi $\tau_p\le t\le\tau_p'$ oraliqda ixtiyoriy joiz $u_p(t)$ boshqaruv qo‘llasin, u holda quyidagi tengsizliklar bajariladi.

$$|z_p(t)-x_p(t)|>\frac{\alpha a_p^2}{6\sigma^2e^{2lT}},\ \ \tau_p\le t\le\tau_p',\quad \text{va}\quad y_2(t)-x_{p2}(t)\ge a_p,\ \ t\ge\tau_p'.$$

11-lemma. Quyidagi baho o‘rinli bo‘ladi

$$| y(t) - z_p(t) | \le 6\sigma e^{lT} \sqrt{\frac{a_{p+1}}{\alpha}}, \quad \tau_p \le t \le \tau'_p.$$

XULOSA

Dissertatsiyada boshqaruv funksiyalari geometrik va eksponensial cheklovlarga bo‘ysunuvchi ko‘p quvlovchidan iborat qochish masalalari o‘rganildi, bunda o‘yinchilar dinamikasi sodda va chiziqli differensial tenglamalar bilan ifodalangan.

Tadqiqotda asosiy natijalar quyidagilardan iborat:

1. Geometrik cheklovlarga ega ikki quvlovchi va bitta qochuvchili chiziqli differensial tenglamalar bilan berilgan qochish differensial o‘yini xal qilingan, qochuvchi uchun strategiya qurilgan hamda quvlovchilarning yaqinlashish soni ko‘pi bilan 3 dan oshmasligi isbotlangan.
2. Ko‘p quvlovchili qochish differensial o‘yinida qochuvchi uchun strategiya qurilgan. O‘yinchilar dinamikasi chiziqli differensial tenglamalar bilan berilgan. Quvlovchilarning yaqinlashish soni ko‘pi bilan $m(m+1)/2$ dan oshmasligi ko‘rsatilgan va ko‘p quvlovchidan qochishni ta’minlovchi yetarli shartlar olingan.
3. Boshqaruv funksiyalariga geometrik cheklovlar qo‘yilgan chiziqli differensial tenglamalar bilan berilgan qochish differensial o‘yinida har bir quvlovchi guruh hujumida faqat bir marta yaqinlashish sodir etishi isbotlanib, guruh hujumidan so‘ng kamida bitta quvlovchi keyingi guruh hujumida ishtirok etmasligi ko‘rsatilgan. Buning natijasida yaqinlashish vaqtlari soni chekli ekanligi ko‘rsatilgan.
4. Boshqaruv funksiyalariga eksponensial integral cheklovlar qo‘yilgan sodda harakatli qochish differensial o‘yinida qochuvchi uchun strategiya qurilgan va uning joizligi isbotlangan. Shuningdek, quvlovchilarning yaqinlashish soni m dan oshmasligi ko‘rsatilgan. Bundan tashqari, ko‘p quvlovchili qochish masalalarini yechish uchun yetarli shartlar olingan hamda o‘yinchilar orasidagi masofa uchun quyi chegara aniqlangan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I.ROMANOVSKIY

INSTITUTE OF MATHEMATICS

TURSUNALIEV TUYCHIVOY GANIJON UGLI

DIFFERENTIAL GAMES OF EVASION FROM MANY PURSUERS WITH GEOMETRIC AND EXPONENTIAL CONSTRAINTS ON CONTROL FUNCTIONS

01.01.02 - Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT - 2025

The theme of dissertation of doctor of philosophy (PhD) on physical and mather sciences was registered at the Supreme Attestation Commission at the Ministry of] education, Science and Innovations of the Republic of Uzbekistan under n B2024.1.PhD/FM1006.

Dissertation has been prepared at Institute of Mathematics.

The abstract of the dissertation is posted in three languages (Uzbek, English, Russian (resume)) on the website (http://kengash.mathinst.uz) and the "ZiyoNet" information and educational portal (www.ziyonet.uz).

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Defense will take place on «9» December 2025 at 16:00 at the meeting of Scientific Council DSc.02/30.12.2019.FM.86.01 at Institute of Mathematics named after V.I.Romanovsky. (Address: University str. 9, Almazar district, Tashkent, 100174, Uzbekistan, Ph.: (+998 78) 207 91 40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

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Abstract of dissertation sent out on «25» November 2025 year.
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INTRODUCTION (abstract of PhD thesis)

Actuality and demand of the theme of dissertation. At the global level, the rapid advancement of scientific and technological progress increasingly demands the analysis of complex processes within the framework of mathematical control theory. In this regard, the theory of differential games occupies a central position, as it enables the modeling of conflict-driven dynamic systems and ensures their practical applications. When such processes are described by differential equations, they are treated as problems of differential games, while models based on discrete equations are studied within the framework of discrete game theory. Since these approaches share common characteristics, they are often regarded as components of the broader theory of dynamic games. This methodology plays a crucial role in developing optimal control strategies and effective solutions across various fields such as engineering, economics, biology, and medicine. Consequently, the correct identification and application of the elements of differential game theory remain of great importance in modern scientific research.

At present, the results obtained in various problems of differential game theory are extensively applied to the control and optimization of conflicting dynamic processes worldwide. Priority research directions include the study of multi-pursuer evasion differential games governed by ordinary and linear differential equations and their applications. Evasion problems with geometric constraints imposed on the players' controls correspond to physical processes that arise in practice under conflict-driven conditions. Moreover, in applied settings, more sophisticated types of constraints may emerge. In particular, the investigation of multi-pursuer evasion problems in differential games with exponential constraints is of significant practical importance, and such cases are analyzed comprehensively in the dissertation. Special emphasis is placed on their applications in engineering, economics, and other domains.

In our republic, large-scale measures are being undertaken within the framework of strategies aimed at strengthening the scientific and practical significance of fundamental sciences, recognizing their pivotal role in the development of innovative technologies. In particular, priority tasks[1] have been set to advance scientific research in the fields of differential equations, mathematical physics equations, algebra and functional analysis, as well as mathematical analysis, in line with international standards. Within the implementation of these objectives, special attention is devoted to the enhancement of dynamical systems, differential equations, optimal control, and differential game theories. Of particular importance is the construction of control models that ensure optimal solutions for the evasion problem with geometric and exponential constraints involving multiple pursuers, which carries both high scientific value and practical relevance.

The subject and object of this dissertation align with the tasks outlined in the Decrees of the President of the Republic of Uzbekistan UP-4947 dated February 7, 2017 "On the strategy of action for the further development of the Republic of Uzbekistan", UP-2789 dated February 17, 2017 "On measures to further improvement of the activities of the Academy of Sciences, organization, management, and financing of research activities, PP-4387 dated July 9, 2019

“On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics”, UP-4708 dated May 7, 2020 “Quality of education in the field of mathematics” , PP-60 dated January 28, 2022, “On the Development Strategy of New Uzbekistan for 2022-2026”, and furthermore, regulations that encompass this research.

Connection of research to priority directions of development of science and technologies of the Republic. This research was carried out in connection with the priority areas of science and technology of the Republic of Uzbekistan IV, “Mathematics, Mechanics, and Computer Science”.

The degree of scrutiny of the problem. The concept of differential games was introduced by the American mathematician R. Isaacs. The works of the scholars regarded as the founders of the fundamental theory of differential games played a crucial role in shaping its theoretical foundations, developing new methods, and applying them to various practical processes. Among those recognized are L.S. Pontryagin, N.N. Krasovskii, L.D. Berkovitz, W.H. Fleming, A. Friedman, A. Bryson, E.F. Mishchenko, A.I. Subbotin, Yu.S. Osipov, B.N. Pshenichnyi, N.Yu. Satimov, L.A. Petrosyan, A.A. Azamov, A.A. Chikrii, N.L. Grigorenko, N.Yu. Lukoyanov, A.G. Pashkov, N. Nikandr Petrov, and I.S. Rappoport. Their contributions laid the groundwork for the theoretical structure of differential games and provided methodologies that are widely used in control theory, variational calculus, and optimal control problems. The concepts they developed continue to serve as a theoretical basis for addressing problems involving differential games with multiple pursuers and an evader, dynamic systems with constrained control functions, and numerous optimization problems encountered in practice.

The pursuit-evasion problem over an infinite time horizon was first introduced and studied by L.S. Pontryagin and E.F. Mishchenko. Notably, Mishchenko proposed a new evasion maneuver in the setting of multiple pursuers. A significant body of research has been devoted to the analysis of evasion games with several players moving under simple dynamics. Later, F.L. Chernous’ko investigated the case of a faster evader against multiple pursuers and proved that evasion is possible if the evader enjoys a speed advantage. These findings were subsequently extended by F.L. Chernous’ko and V.L. Zak to more general classes of differential game problems, including those involving groups of pursuers.

In our country, in the field of differential games, scholars such as A.A. Azamov, G.I. Ibragimov, N.A. Mamadaliyev, Sh.M. Mamatov, O.Sh. Qo‘chqarov, B.B. Rixsiyev, N.Yu. Satimov, B.T. Samatov, and M. Tuxtasinov have conducted extensive research on pursuit–evasion problems in multi-player discrete and differential games under geometric and integral constraints. Their works have broadened the range of solution techniques and made significant contributions to both the theoretical foundations and practical applications of differential games. In particular, several studies provided exact calculations of capture times for pursuers and identified sufficient conditions guaranteeing the evader's ability to escape.

F.L. Chernous'ko demonstrated that in the case of one faster evader against multiple pursuers, an effective evasion strategy exists when the evader has a speed advantage. V.L. Zak further generalized these results to broader classes of pursuit-evasion games involving multiple pursuers. Later, Chernous'ko and Zak jointly deepened these investigations, establishing the existence and stability of optimal trajectories. Collectively, their contributions laid the theoretical foundation for modern studies of pursuit-evasion problems in differential games with multiple pursuers.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation research was carried out within the framework of the program "Dynamic and Control Systems" conducted according to the research plan of the V.I. Romanovskiy Institute of Mathematics of the Academy of Sciences of Uzbekistan.

The aim of the research is to solve evasion problems with multiple pursuers in differential games under geometric and exponential constraints on the players' control functions.

Problems of the research:

identifying sufficient conditions that guarantee the solvability of multiple-pursuer evasion problems in linear differential games with geometrically constrained control functions;

determining the exact approach times in the case of two pursuers in linear differential games with geometrically constrained control functions;

demonstrating the admissibility of the strategy constructed for the evader in multiple-pursuer evasion problems of differential games with control functions subject to geometric and exponential constraints;

constructing an evader's strategy that guarantees the solvability of multiple-pursuer evasion problems in ordinary differential games with exponentially constrained control functions and identifying the corresponding sufficient conditions.

The object of the research. Differential games of evasion from multiple pursuers under geometric and exponential integral constraints on controls.

The subject of the research. Constructing a strategy that guarantees evasion for the evader and solving the differential game of evasion under geometric and exponential integral constraints.

Methods of the research. In the research work, the evasion problem from multiple pursuers is studied, the strategy for the evader is constructed, and a method for estimating from below the distance between the pursuers and the evader is applied. To substantiate the solutions, methods of the theory of ordinary differential equations, functional analysis, differential game theory, and optimal control theory are employed.

Scientific novelty of the research is composed of the following:

in linear differential games with geometrically constrained control functions involving one pursuer and one evader, a strategy guaranteeing successful evasion was constructed for the evader, and a lower bound for the distance between the pursuer and the evader was obtained;

in linear differential games with geometrically constrained control functions involving two pursuers, a strategy for the evader was constructed, and it was proved that the number of approach times does not exceed 3;

in linear evasion differential games with geometrically constrained control functions involving m pursuers, a strategy for the evader was constructed. It was shown that, following each group's attack, at least one pursuer does not participate in the next group's attack, and that the number of pursuers' approach times does not exceed $m(m+1)/2$;

in simple motion differential games with exponential integral constraints on the players' control functions, a strategy for the evader is constructed, and its admissibility is proved. It is also shown that the number of approach times of the pursuers does not exceed m. Furthermore, sufficient conditions for solving the evasion problem against multiple pursuers are established, and a lower bound on the distance between the players is obtained.

ractical results of the research. The results achieved in the dissertation and the methods used to prove them allow the direct application of mathematical models representing conflicting control processes to avoidance problems. The results obtained in the study are analyzed with numerical examples, and their geometric representations are also presented. This makes a significant contribution to the effective use of the issues in the research work in mathematical modeling of real processes.

The reliability of the results of the research. The validity of the results is ensured based on mathematically established conclusions, including theorems and their properties, employing methods from the theory of ordinary differential equations, optimal control, functional analysis, dynamic systems theory, and differential game theory.

The scientific and practical significance of the research results. The scientific significance of the research results focuses on the development of strategies that provide solutions to conflict-control problems described by geometric and exponential integral constraints, along with their practical implementation. These results allow for the refinement of existing methods in optimal control and differential game theories and support the identification of effective solutions to complex problems encountered in real-world applications.

The practical significance of the research is reflected in the development of mathematical models based on evasion differential games, represented by simple and linear differential equations, which can be applied to a wide range of complex real-world processes. In particular, these results can be utilized in the military domain for strategic air defense, in robotics and artificial intelligence for designing safe control strategies for autonomous systems, and in engineering and ecological fields for developing strategies and computational tools to address conflict control challenges.

Implementation of the research result. Based on the results obtained on differential games of multiple pursuers avoidance with geometric and exponential constraints, the following conditions are established:

among the sufficient conditions that guarantee the solvability of multiple-pursuer evasion problems in games described by linear differential equations with geometrically constrained control functions, some were applied in the international grant project number 374874-2022 entitled “Problems of phase transitions and analytical phenomena. Mathematical properties of their fast transition equations and asymptotics” to solve analogous problems for linear differential equations (the reference with number 1254 of Osh State University dated September 17, 2025, Kyrgyz Republic). The application of these scientific results made it possible to present complete solutions to problems of phase transitions and analytical phenomena formulated by linear differential equations;

the results obtained on evasion problems from multiple pursuers in simple differential games with exponentially integral constrained control functions were applied in the scientific project *“Decisions_lab research program”* (https://www.decisionslab.eu/aree-di-ricerca/), at the Department of Law, Economics and Human Sciences, University “Mediterranea” of Reggio Calabria (Italy), for addressing and solving differential game problems. The application of this result to the evasion problem with a finite number of pursuers in Euclidean-space differential games ensures successful escape against all admissible pursuer controls, provided the evader follows the corresponding strategy.

Approbation of the research results. The principal findings of the research were discussed at 9 scientific-practical conferences, including 7 international and 2 national conferences.

Publications of the research results. Within the scope of the dissertation topic, a total of 14 scientific articles were published, 5 of which are included in the list of scientific publications recommended by the Higher Attestation Commission of the Republic of Uzbekistan as qualifying for the Doctor of Philosophy (PhD) thesis defense. Among these, 1 were published in international journals and 4 in national scientific journals, while the remaining 9 are abstracts.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, conclusion, and bibliography, with a total length of 87 pages.

THE MAIN CONTENT OF THE DISSERTATION

In introduction, the motivation of the research and correspondence to the priority research areas of science and technology of the Republic are given, we present a review of international research on the topic of the dissertation and the degree of scrutiny of the problem, formulate the object and subject of the study, identify the goals and objectives, and state the scientific novelty and practical results of the research. Moreover, the theoretical and practical importance of the obtained results is provided, along with information on the implementation of the research findings, the published works, and the structure of the dissertation.

The first Chapter is entitled "**Evasion problem in a differential game with two pursuers under geometric constraints**" the evasion problems involving two pursuers with geometric constraints on controls in linear differential games are investigated.

Suppose that two pursuers x_1, x_2 and one evader y move in the space $\mathbb{R}^n$, $n \geq 2$. The notation $x = (x_1, x_2)$ for the team of pursuers and $u = (u_1, u_2)$ for their control parameters, whose dynamics are described by the following equations

$$\begin{aligned} &\dot{x}_i = -\lambda x_i + u_i, \quad x_i(0) = x_{i0}, \qquad |u_i| \leq 1, \\ &\dot{y} = -\lambda y + v, \qquad y(0) = y_0, \qquad |v| \leq \sigma, \end{aligned} \tag{1}$$

where $x_i, x_{i0}, y, y_0, u_i, v \in \mathbb{R}^n$, $\lambda > 0$; $x_i(0) = x_{i0}$ and $y(0) = y_0$ are the initial positions, it is assumed that $x_{i0} \neq y_0$, $i = 1, 2$, and σ, $\sigma > 1$ is a given number, where u_i is the control parameter of pursuer x_i, and v is that of evader y.

Definition 1. Measurable functions $u_i(t)$, $|u_i(t)| \leq 1$, and $v(t)$, $|v(t)| \leq \sigma$, $t \geq 0$, are called controls of the pursuer x_i, $i \in \{1, 2\}$, and the evader y, respectively.

Definition 2. The strategy of evader is defined as a function

$$V : \mathbb{R}_+ \times \mathbb{R}^{5n} \to \mathbb{R}^n,$$
$$(t, y, x_1, x_2, u_1, u_2) \to V(t, y, x_1, x_2, u_1, u_2)$$

for which initial value problem (1) with $v = V(t, y, x_1, x_2, u_1, u_2)$ has a unique solution $(y(t), x_1(t), x_2(t))$ for arbitrary controls $u_1 = u_1(t), u_2 = u_2(t)$ of pursuers.

Definition 3. We say that evasion is possible in a game (1) if there exists a strategy V of evader such that, for any controls of pursuers, we have $x_i(t) \neq y(t)$ for all $t \geq 0$ and $i = 1, 2$.

Problem 1. Construct a strategy V for the evader, for which evasion is possible in game (1).

First, the evasion problem of one pursuer and one evader is considered, in the case where $n = 2$. The dynamics of the pursuer x and the evader y are governed by the following equations

$$\begin{aligned} &\dot{x} = -\lambda x + u, \quad x(0) = x_0, \quad |u(t)| \leq 1, \\ &\dot{y} = -\lambda y + v, \quad y(0) = y_0, \quad |v(t)| \leq \sigma, \end{aligned} \tag{2}$$

where $x, x_0, y, y_0, u, v \in \mathbb{R}^2$, $\lambda > 0$, $x_0 \neq y_0$.

The solutions of equations (2) are

$$x(t) = x_0 e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)} u(s) ds, \quad y(t) = y_0 e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)} v(s) ds.$$

We introduce the following by noting $\xi(t) = e^{\lambda t} x(t)$, $\eta(t) = e^{\lambda t} y(t)$,

$$\xi(t) = x_0 + \int_0^t e^{\lambda s} u(s) ds, \quad \eta(t) = y_0 + \int_0^t e^{\lambda s} v(s) ds.$$

From now on, we refer to the pursuer and evader as ξ and η, instead of x and y, respectively.

We temporarily fix the numbers α and a

$$0 < \alpha < \frac{1}{2}(\sigma - 1), \qquad 0 < a < |x_0 - y_0|.$$

We construct a strategy for the evader. First the evader starting from the initial time $t = 0$ moves with the constant control

$$V(t) = V_0 = (0, \sigma), \ t \in [0, \tau), \tag{3}$$

i.e., $V_1(t) = 0$, $V_2(t) = \sigma$, parallel to the Oy-axis, where $t = \tau$ is the first time when $|\xi(t) - \eta(t)| = a$. We call τ the a-approach time of pursuer to the evader. The segment between the points y_0 and $\eta(\tau)$ in Fig.1 is the trajectory of the evader corresponding to (3).

Note that time τ may not occur. In this case, we have $|\xi(t) - \eta(t)| > a$ for all $t \geq 0$ and, clearly, $\xi(t) \neq \eta(t)$ for all $t \geq 0$. Therefore, we assume that the time τ occurs.

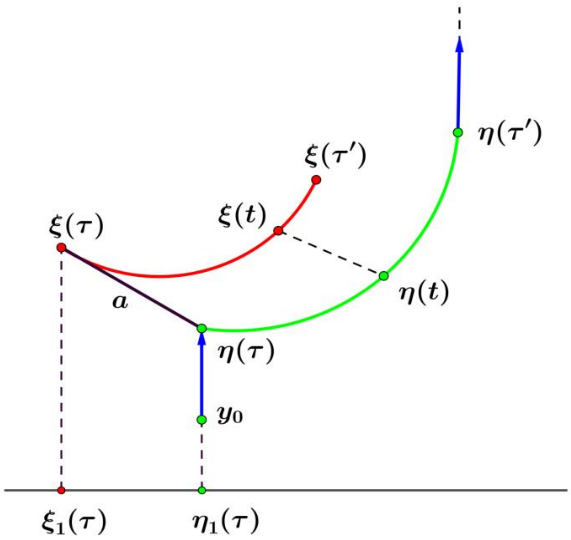

Figure 1. The trajectory of the evader when $\xi_1(\tau) \leq \eta_1(\tau)$.

Also, we define the number $\tau' = \frac{1}{\lambda} \ln\left(e^{\lambda\tau} + \frac{2\lambda a}{\sigma - 1 - \alpha} \right)$.

The evader applies the following strategy on $[\tau, \tau')$

$$V(t)=(V_1(t),V_2(t)),\ t\in[\tau,\tau'). \tag{4}$$

where

$$V_1(t)=\begin{cases}|u_1(t)|+\alpha, & \xi_1(\tau)\le\eta_1(\tau)\\ -(|u_1(t)|+\alpha), & \xi_1(\tau)>\eta_1(\tau)\end{cases},\quad V_2(t)=\sqrt{\sigma^2-V_1^2(t)}. \tag{5}$$

Clearly, $|V_1(t)|\le|u_1(t)|+\frac{1}{2}(\sigma-1)\le\sigma$, and so $V_2(t)$ in (5) is defined. We call $V(t)$ defined by (4) a maneuver of the evader η against the pursuer ξ.

For the final part of evader's strategy, we let

$$V(t)=V_0=(0,\sigma),\ t\ge\tau'. \tag{6}$$

Lemma 1. Let the evader use strategy (3), (4), and (6), where τ is the a-approach time of the pursuer ξ to the evader η. Then

$$|\eta(t)-\xi(t)|\ge a,\qquad 0\le t\le\tau,$$

$$|\eta(t)-\xi(t)|>\frac{\alpha a}{2\sigma},\ \ \tau\le t\le\tau',$$

$$\eta_2(t)-\xi_2(t)>a,\qquad t\ge\tau'.$$

Theorem 1. For any initial positions of two pursuers and one evader, evasion is possible in game (1).

Property 1. If $\tau_2<\tau_1'$ and $t<\tau_2'$, then $t-\tau_1'\le\dfrac{2a_2}{\sigma-1-\alpha_1}$.

Lemma 2. Let the evader use strategy $v(t)=\begin{cases}V_1(t), & t\in[\tau_1,\tau_2),\\ V_2(t), & t\in[\tau_2,\tau_2'),\end{cases}$ and $\tau_1'\le\tau_2'$,

then

$$|\eta(t)-\zeta(t)|\le\frac{8\sigma e^{\lambda T}}{\sigma-1-\alpha_1}a_2,\qquad \tau_1\le t\le\tau_1'.$$

Lemma 3. Let the evader use strategy $v(t)=\begin{cases}V_1(t), & t\in[\tau_1,\tau_2),\\ V_2(t), & t\in[\tau_2,\tau_2'),\end{cases}$ and $\tau_1'\le\tau_2'$,

then

$$|\eta(t)-\xi_1(t)|>a_2,\quad \tau_1\le t\le\tau_2',$$

and for the pursuer ξ_1

$$\eta_2(t)-\xi_{12}(t)>a_2,\quad \tau_1'\le t\le\tau_2'.$$

Lemma 4. Let the evader use strategy $v(t)=\begin{cases}V_1(t), & t\in[\tau_1,\tau_2),\\ V_2(t), & t\in[\tau_2,\tau_2'),\end{cases}$ and $\tau_1'\le\tau_2'$,

then

$$\eta_2(t)-\xi_{p2}(t)>a_2,\quad t\ge\tau_2',$$

Lemma 5. Let the evader use strategy $v(t)=\begin{cases}V_1(t), & t\in[\tau_1,\tau_2),\\ V_2(t), & t\in[\tau_2,\tau_2'),\end{cases}$, then

$$\eta_2(t)-\xi_{22}(t)>a_2,\quad t\ge\tau_2'$$

The second Chapter is entitled **"A linear differential game of evasion from multiple pursuers"**, and it is shown that the number of pursuers is generalized; that is, the evasion differential game with m pursuers is considered. The concept of a group attack is introduced, and it is proved that after each group attack at least one pursuer is left behind. This leads us to one of the main results that the total number of approach times τ_k of m pursuers during the game cannot exceed

$$m+(m-1)+\ldots+1=m(m+1)/2.$$

We consider an evasion linear differential game of m pursuers $x_1,\ldots,x_m$ and one evader y that move in the space $\mathbb{R}^n$, $n\geq 2$. We use the notation $x=(x_1,x_2,\ldots,x_m)$ for the team of pursuers and $u=(u_1,u_2,\ldots,u_m)$ for their control parameters, whose dynamics are described by the following equations

$$\begin{aligned} &\dot{x}_i=-\lambda x_i+u_i, \quad x_i(0)=x_{i0}, \qquad |u_i|\leq 1,\\ &\dot{y}=-\lambda y+v, \qquad y(0)=y_0, \qquad |v|\leq\sigma, \end{aligned} \tag{7}$$

where $x_i,x_{i0},y,y_0,u_i,v\in\mathbb{R}^n$, $\lambda>0$; $x_i(0)=x_{i0}$ and $y(0)=y_0$ are the initial positions, it is assumed that $x_{i0}\neq y_0$, $i=1,2,\ldots,m$, and σ, $\sigma>1$ is a given number, where u_i is the control parameter of pursuer x_i, and v is that of evader y.

Definition 4. Measurable functions $u_i(t)$, $|u_i(t)|\leq 1$, and $v(t)$, $|v(t)|\leq\sigma$, $t\geq 0$, are called controls of the pursuer x_i, $i\in\{1,2,\ldots,m\}$, and the evader y, respectively.

Definition 5. The strategy of evader is defined as a function

$$V:\mathbb{R}_+\times\mathbb{R}_+\times\mathbb{R}_+\times\mathbb{R}^{(2m+1)n}\to\mathbb{R}^n,$$

$$(t,\varphi_1,\varphi_2,y,x_1,\ldots,x_m,u_1,\ldots u_m)\to V(t,\varphi_1,\varphi_2,y,x_1,\ldots,x_m,u_1,\ldots u_m),$$

for which initial value problem (7) with $v=V(t,\varphi_1,\varphi_2,y,x_1,\ldots,x_m,u_1,\ldots,u_m)$ has a unique solution $(y(t),x_1(t),\ldots,x_m(t))$ for $\varphi_1=\varphi_1(t)$, $\varphi_2=\varphi_2(t)$, and arbitrary controls $u_1=u_1(t),\ldots,u_m=u_m(t)$ of pursuers, where $\varphi_1(t)$ and $\varphi_2(t)$, $t\geq 0$, are given functions.

Definition 6. We say that evasion is possible in a game (7) if there exists a strategy V of evader such that, for any controls of pursuers, we have $x_i(t)\neq y(t)$ for all $t\geq 0$ and $i=1,\ldots,m$.

Problem 2. Construct a strategy V for the evader, for which evasion is possible in game (7).

Theorem 2. For any initial positions of m pursuers and one evader, evasion is possible in game (7).

Proposition 1. $\sum\limits_{k=p+1}^{\infty}a_k=\dfrac{a_{p+1}}{1-\beta}\leq 2a_{p+1}$ for any integer $p\geq 0$.

Proposition 2. For any $p\in\{1,...,k_1-1\}$ in Case A, and for any positive integer p in Case B,

$$t-\tau_{p+1}\leq\frac{4}{\sigma-1-\alpha_1}a_{p+1}, \quad t\in[\tau_{p+1},\overline{\tau}].$$

Lemma 6. Let the evader use strategy

$$v(t)=\begin{cases} V_k(t), & t\in[\tau_k,\tau_{k+1}),\ k=p,p+1,\dots,k_1-1, \\ V_{k1}(t), & t\in[\tau_{k1},\tau'_{k1}), \end{cases} \quad \text{in Case A,}$$

and

$$v(t)=V_k(t),\ t\in[\tau_k,\tau_{k+1}),\ \tau_k<\tau'_{k-1},\ k=p,p+1,\dots. \quad \text{in Case B,}$$

then

$$|\eta(t)-\zeta_p(t)|\le\frac{16\sigma e^{\lambda T}}{\sigma-1-\alpha_1}a_{p+1}, \qquad t\in[\tau_p,\tau'_p]\cap[\tau_p,\overline{\tau}].$$

Lemma 7. Let the evader use strategy

$$v(t)=\begin{cases} V_k(t), & t\in[\tau_k,\tau_{k+1}),\ k=p,p+1,\dots,k_1-1, \\ V_{k1}(t), & t\in[\tau_{k1},\tau'_{k1}), \end{cases} \quad \text{in Case A,}$$

and

$$v(t)=V_k(t),\ t\in[\tau_k,\tau_{k+1}),\ \tau_k<\tau'_{k-1},\ k=p,p+1,\dots \quad \text{in Case B,}$$

then

$$|\eta(t)-\xi_p(t)|>a_{p+1}, \qquad \tau_p\le t\le\overline{\tau}\ .$$

Lemma 8. Let $\tau'_p\le\tau'_{k_1}$ for the pursuer ξ_p, then

$$\eta_2(t)-\xi_{p2}(t)>a_{p+1}, \quad t\ge\tau'_{k_1}.$$

The third Chapter is entitled **"Differential game of one evader and multiple pursuers with exponential integral constraints"**, and it is explored differential game of evasion from multiple pursuers under exponential constraints on controls, where the dynamics of the players are described by simple differential equations. In this Chapter we introduced a function $r(t)$ for the evader to confirm maneuvers at each interval and constructed the strategy for the evader under exponential constraints on control functions. We proved that each pursuer's approach time occured at most once and the total number of approach times in evasion game does not exceed m.

We considered a simple motion evasion differential game of one evader y and m pursuers x_i, $i=1,...,m$, in $\mathbb{R}^n$, $n\ge2$. Game is described by the following equations:

$$\begin{aligned} &\dot{x}_i=u_i, \quad x_i(0)=x_i^0 \quad i=1,\dots,m, \\ &\dot{y}=v, \qquad y(0)=y^0, \end{aligned} \tag{8}$$

where $x_i,x_i^0,y,y^0,u_i,v\in\mathbb{R}^n$, $n\ge2$, $x_i^0\ne y^0$, $i=1,...,m$ and $u_1,\dots,u_m$ are the control parameters of pursuers and v is that of evader.

Definition 7. A measurable function $u_i(t)$, $t\ge0$, is called an admissible control of the pursuer x_i if

$$\int_0^t|u_i(s)|^2\,ds\le\rho_i^2e^{2lt}, \quad i=1,\dots,m, \tag{9}$$

where $\rho_1,\rho_2,...,\rho_m$ and l are given positive numbers.

Definition 8. A measurable function $v(t)$, $t \ge 0$, is called an admissible control of the evader y if

$$\int_0^t |v(s)|^2\, ds \le \sigma^2 e^{2lt}, \tag{10}$$

where σ is a given positive number.

Definition 9. A function $V:[0,\infty)\times\mathbb{R}^{(2m+1)n} \to \mathbb{R}^n$,

$$(t,y,x_1,...,x_m,u_1,...,u_m) \mapsto V(t,y,x_1,...,x_m,u_1,...,u_m),$$

is called a strategy of evader if the following initial value problem

$$\dot{x}_i = u_i, \quad x_i(0) = x_i^0, \quad i = 1,2,\ldots,m,$$
$$\dot{y} = V(t,y,x_1,...,x_m,u_1,...,u_m), \quad y(0) = y^0,$$

has a unique solution $(x_1(t),\ldots,x_m(t),y(t))$, $t \ge 0$, for any admissible controls of the pursuers $u_i = u_i(t)$, $i = 1,...,m$, and along this solution the following inequality

$$\int_0^t |V(s,y(s),x_1(s),\ldots x_m(s),u_1(s),\ldots,u_m(s))|^2\, ds \le \sigma^2 e^{2lt}.$$

holds.

Definition 10. If there exists a strategy V of the evader such that for any admissible controls of pursuers $x_i(t) \ne y(t)$ for all $t \ge 0$, and $i = 1,\ldots,m$, then we say that evasion is possible.

Problem 3. Construct a strategy V for the evader y and find a condition for the parameters ρ_i, $i = 1,...,m$, σ, which guarantees the evasion in game (8)-(10).

Theorem 3. If

$$\rho_1^2 + \ldots + \rho_m^2 < \sigma^2,$$

then evasion is possible in game (8)-(10).

Let a sequence $\{a_k\}_{k=1}^{\infty}$ be defined by the formula $a_{k+1} = \beta \cdot a_k^4$, $k = 1,2,\ldots$ It is not difficult to prove that this sequence has the following:

Property 2. $\sum\limits_{k=p+1}^{\infty} a_k \le 2a_{p+1}$ for any $p \ge 1$.

We have defined a function $r:[0,T] \to \{0,1,\ldots,m_0\}$, which plays a key role in assigning a maneuver for the evader. We denoted $I_k = \cup_{j=k}^{m_0}[\tau_j,\tau_j')$, $I_{m_0+1} = \varnothing$. Set

$$r(t) = \begin{cases} 0, & t \in [0,T] \setminus I_1, \\ k, & t \in [\tau_k,\tau_k'] \setminus I_{k+1},\ k = 1,...,m_0. \end{cases}$$

The function $r(t)$ has the following property.

Property 3. Let $m_0 > 1$. Then, for $k = 1,2,\ldots,(m_0 - 1)$,

(i) If $\tau_k' \le \tau_{k+1}$, then $r(t) = k$ for $\tau_k \le t < \tau_k'$,

(ii) If $\tau_{k+1} \le \tau_k'$, then $r(t) = k$ for $\tau_k \le t < \tau_{k+1}$.

Example . If

$$0 = \tau_0 < \tau_1 < \tau_2 < \tau_2' < \tau_1' < \tau_3 < \tau_4 < \tau_3' < \tau_4' < \tau_5 < \tau_5',$$

then $r(t)$ has the graph shown in Fig.2.

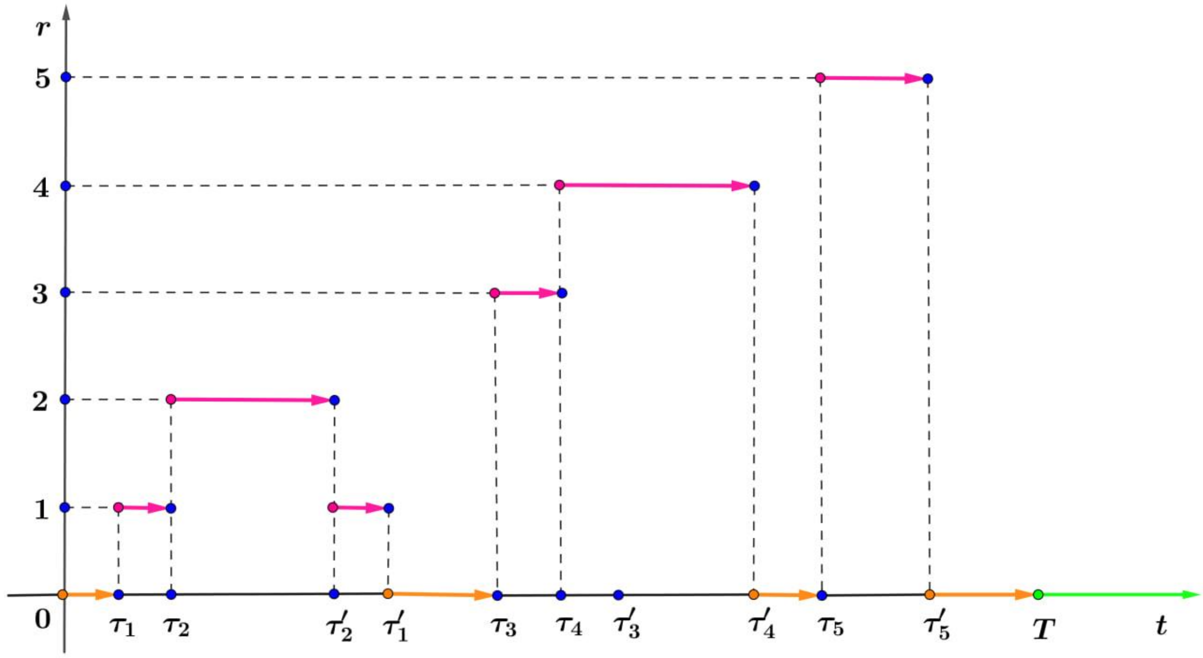

Figure 2. The graph of function $r(t)$.

We now construct a strategy for the evader. Let $u_i(t)$, $i=1,...,m$, be arbitrary controls of pursuers. Set

$$v(t) = V_0(t) = \left(0, \alpha + \left(\sum_{i=1}^{m} |u_i(t)|^2\right)^{1/2}\right), \quad t \in [0,T] \setminus I_1, \tag{11}$$

$$v(t) = V_r(t) = (V_{r1}(t), U(t)), \quad t \in [0,T] \cap I_1, \tag{12}$$

where $r = r(t)$, $V_k(t) = (V_{k1}(t), U(t))$, $\tau_k \le t < \tau_k'$, $k = 1, \ldots, m_0$, is defined as follows

$$V_{k1}(t) = \begin{cases} \alpha + |u_{k1}(t)|, & y_1(\tau_k) \ge x_{k1}(\tau_k), \\ -(\alpha + |u_{k1}(t)|), & y_1(\tau_k) < x_{k1}(\tau_k), \end{cases} \tag{13}$$

$$U(t) = \alpha + \left(\sum_{i=1}^{m} u_{i2}^2(t)\right)^{1/2}.$$

Note that $U(t)$ doesn't depend on k. Finally, let

$$v(t) = \left(0, \left(\sum_{i=1}^{m} |u_i(t)|^2\right)^{1/2}\right), \quad t > T. \tag{14}$$

Lemma 9. If the evader uses strategy (11)-(14), then

(a) For all $k \in J_0 = \{1,...,m_0\}$, we have (i) $\tau_k \le T_0$ and (ii) $\tau_k' \le T$.

(b) If $y_2^0 \ge x_{i2}^0$ for some $i \in \{1,...,m\}$, then $y_2(t) > x_{i2}(t)$ for all $t > 0$.

Lemma 10. Let the pursuer x_p apply an arbitrary admissible control $u_p(t)$ on $\tau_p \le t \le \tau_p'$, then

$$|z_p(t)-x_p(t)|>\frac{\alpha a_p^2}{6\sigma^2 e^{2lT}},\quad \tau_p\le t\le\tau'_p,\quad \text{and}\quad y_2(t)-x_{p2}(t)\ge a_p,\quad t\ge\tau'_p.$$

Lemma 11. The following estimate holds

$$|y(t)-z_p(t)|\le 6\sigma e^{lT}\sqrt{\frac{a_{p+1}}{\alpha}},\quad \tau_p\le t\le\tau'_p.$$

CONCLUSION

In the dissertation, evasion problems involving multiple pursuers whose control functions satisfy geometric and exponential constraints are investigated, where the players' dynamics are described by simple and linear differential equations:

The main results achieved in the research are as follows:

1. In the linear evasion differential game with two pursuers and one evader under geometric constraints, the strategy was constructed for the evader, and established that the number of approach times did not exceed 3, explicit conditions guaranteeing successful evasion were derived.
2. In evasion differential game from multiple pursuers, a strategy for the evader was constructed. The players' dynamics are given by linear differential equations with positive parameter. It was shown that the number of pursuers' approach times did not exceed $m(m+1)/2$, and sufficient conditions guaranteeing the solvability of evasion problems from multiple pursuers were established.
3. In a linear differential game of evasion described by differential equations with geometrically constrained control functions, it was proven that during each group attack, every pursuer can approach the evader only once. Furthermore, after each group attack, at least one pursuer does not participate in the subsequent group attack. As a consequence, it was shown that the total number of approach times is finite.
4. Simple motion evasion differential game subject to exponential integral constraints on control functions, a strategy for the evader was constructed, and its admissibility was proved. It was also shown that the number of pursuers' approach times did not exceed m. Furthermore, sufficient conditions for solving evasion problems with multiple pursuers were established, and a lower bound for the distance between the players was derived.

НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

ИНСТИТУТ МАТЕМАТИКИ

ТУРСУНАЛИЕВ ТЎЙЧИВОЙ ҒАНИЖОН ЎҒЛИ

ДИФФЕРЕНЦИАЛЬНЫЕ ИГРЫ УБЕГАНИЯ ОТ МНОГИХ ПРЕСЛЕДОВАТЕЛЕЙ С ГЕОМЕТРИЧЕСКИМ И ЭКСПОНЕНЦИАЛЬНЫМ ОГРАНИЧЕНИЯМИ НА ФУНКЦИИ УПРАВЛЕНИЯ

01.01.02 - Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

ТАШКЕНТ - 2025

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве Высшего образования, Науки и Инноваций Республики Узбекистан за № B2024.1.PhD/FM1006.

Диссертация выполнена в Институте Математики им. В.И.Романовского.

Автореферат диссертации на трёх языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (http://kengash.mathinst.uz) и на Информационно-образовательном портале «ZiyoNet» (www.ziyonet.uz).

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Защита диссертации состоится «9» декабря 2025 года в 16:00 часов на заседании Научного совета DSc.02/30.12.2019.FM.86.01 при Институте Математики имени В.И.Романовского. (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+998 78) 207 91 40, e-mail: mathinst@umail.uz, Website: www.mathinst.uz).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Института Математики имени В.И.Романовского (зарегистрирована за № 215). (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+998 78) 207 91 40).

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является решение задачи уклонения от нескольких преследователей с геометрическими и экспоненциальными интегральными ограничениями на управления.

Объектом исследования являются дифференциальные игры уклонения от нескольких преследователей при геометрических и экспоненциальных интегральных ограничениях на управления.

Научная новизна исследования заключается в следующем:

в линейных дифференциальных играх с геометрически ограниченными управляющими функциями, включающих одного преследователя и одного убегающего, была разработана стратегия убегающего, обеспечивающая гарантированное уклонение, а также получена нижняя граница расстояния между преследователем и убегающим;

в линейных дифференциальных играх с геометрически ограниченными управляющими функциями для задачи уклонения с двумя преследователями была построена стратегия убегающего, и доказано, что количество моментов сближения не превышает трёх.

в линейных дифференциальных играх уклонения с геометрически ограниченными управляющими функциями для задачи уклонения от m преследователей была построена стратегия убегающего, и доказано, что после атаки каждой группы как минимум один преследователь не участвует в следующей атаке, а число моментов сближения преследователей не превышает $m(m+1)/2$;

в дифференциальных играх с простым движением, где управляющие функции игроков подчинены экспоненциально-интегральным ограничениям, была построена стратегия для убегающего, доказана её допустимость, и показано, что число моментов сближения преследователей не превышает m. Кроме того, получены достаточные условия допустимости задач уклонения от нескольких преследователей и выведена нижняя граница расстояния между игроками.

Внедрение результатов исследования. На основе результатов, полученных в дифференциальных играх уклонения нескольких преследователей с геометрическими и экспоненциальными ограничениями, установлены следующие условия:

среди достаточных условий, гарантирующих допустимость задач уклонения от преследования несколькими преследователями в играх, описываемых линейными дифференциальными уравнениями с геометрически ограниченными управляющими функциями, некоторые были применены в международном грантовом проекте № 374874-2022 под названием «Проблемы фазовых переходов и аналитических явлений. Математические свойства их уравнений быстрых переходов и асимптотики» для решения аналогичных задач для линейных дифференциальных уравнений (справка № 1254 Ошского государственного университета от 17 сентября 2025 года, Кыргызская Республика). Применение этих научных результатов позволило получить полные решения задач фазовых переходов и аналитических явлений, формулируемых в терминах линейных дифференциальных уравнений;

результаты, полученные при изучении задачи уклонения от нескольких преследователей в простых дифференциальных играх с экспоненциально-интегральными ограничениями на управляющие функции, были применены в научном проекте Средиземноморского университета Реджо-ди-Калабрия «Исследовательская программа лаборатории принятия решений» для решения задач в дифференциальных играх (Рекомендация Средиземноморского университета Реджо-Калабрия, Италия, 4 сентября 2025 г.). Применение этого научного результата к задаче уклонения с конечным числом преследователей в дифференциальных играх, определённых в евклидовом пространстве, обеспечивает успешное уклонение от любых допустимых управлений преследователей при условии, что убегающий использует соответствующую стратегию.

Структура и объем диссертации. Диссертация объемом 87 страниц состоит из введения, трех глав, заключения и списка литературы.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

1-bo‘lim (part 1; часть 1)

1. Tursunaliev T.G. Evasion differential game of two pursuers and one evader // *Bulletin of National University of Uzbekistan: Mathematics and Natural Sciences Natural*. Vol. 6, Issue 3, 2023, pp. 132–140. (01.00.00; №8)
2. Ibragimov G.I., Tursunaliev T.G. Evasion problem in a differential game with geometric constraints // *Uzbek Mathematical Journal*. Vol. 68, Issue 2, 2024, pp. 81–91. (01.00.00; №6)
3. Ibragimov G.I., Tursunaliev T.G., Luckraz Sh. Evasion in a linear differential game with many pursuers // *Discrete and Continuous Dynamical Systems – Series B (DCDS-B)*. Vol. 29, Issue 12, 2024, pp. 4929–4945. (3. Scopus, IF=1.3).
4. Ibragimov G.I., Tursunaliev T.G. Evasion game of two pursuers with a single evader // *Bulletin of the Institute of Mathematics*. Vol. 8, Issue 1, 2025, pp.1–7. (01.00.00; №17)
5. Ibragimov G.I., Tursunaliev T.G. Differential game of one evader and multiple pursuers with exponential integral constraints // *Uzbek Mathematical Journal*. Vol. 69, Issue 4, 2025, pp.113–127. (01.00.00; №6)

2-bo‘lim (part 2; часть 2)

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Avtoreferat “Oʻzbekiston matematika jurnali” tahririyatida
2025 yil 17-noyabrda tahrirdan oʻtkazilib, oʻzbek, ingliz rus tillaridagi
matnlar oʻzaro muvofiqlashtirildi.

Bichimi: 84x60 $^{1}/_{16}$. «Times New Roman» garniturasi.
Raqamli bosma usulda bosildi.
Shartli bosma tabogʻi: 2,5. Adadi 50 dona. Buyurtma № ___.

Guvohnoma № ___.
«Tipograff» MChJ bosmaxonasida chop etilgan.
Bosmaxona manzili: 100011, Toshkent sh., Beruniy koʻchasi, 83-uy.