**QARSHI DAVLAT UNIVERSITETI HUZURIDAGI
ILMIY DARAJALAR BERUVCHI
PhD.03/30.06.2020.FM.70.04 RAQAMLI ILMIY KENGASH**

QARSHI DAVLAT UNIVERSITETI

BARATOV BAHODIR SOYIB O‘G‘LI

SEPARABEL KUBIK STOXASTIK OPERATORLARNING DINAMIKASI

01.01.01 – Matematik analiz

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Qarshi – 2025

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Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahonda olib borilayotgan ko‘plab amaliy masalalar nochiziqli operatorlar bilan hosil qilingan dinamik sistemalarni ochib berishni taqozo qilmoqda. Kubik operatorlar matematik biologiya va genetika masalalarini hal qilishda muhim amaliy ahamiyatga egadir. Xususan, bunday evolyutsion operatorlar yirtqich va o‘ljalar (masalan, faglar va bakteriyalar)ning o‘zaro ta’sirini matematik modellashtirish jarayonida namoyon bo‘ladi. Populyatsiyalarning evolyutsiyasi (yoki dinamikasi) ko‘payish yoki kamayish va yashab qolish natijasida keyingi avlodlardagi holatning aniq o‘zgarishini o‘z ichiga oladi. Nochiziqli operatorlarning dinamik sistemasi orqali populyatsiya evolyutsiyasini aniqlash, simpleksni invariant saqlaydigan kubik stoxastik operatorlarning dinamikasini ochib berish muhim ahamiyat kasb etadi.

Hozirgi vaqtda dunyo bo‘ylab matematik biologiyaning ko‘plab amaliy masalalarini yechishda nochiziqli dinamik sistemalar nazariyasi asosiy vosita sifatida qo‘llanilmoqda. Jumladan, separabel kubik stoxastik operatorlarning dinamikasini aniqlash masalasi, mazkur operatorlarni nochiziqli bo‘lganligi sababli ko‘plab nazariy va amaliy masalalarni keltirib chiqaradi. Bu borada maqsadli ilmiy izlanishlar bunday operatorlarning invariant to‘plamlarini tavsiflash, davriy nuqtalarini topish va ularning turlarini aniqlash hamda orbitalarning limit nuqtalari to‘plamini tavsiflashga katta e’tibor qaratilmoqda.

Mamlakatimizda so‘nggi yillarda fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo‘lgan statistik fizika, biologiyaning dolzarb yo‘nalishlariga e’tibor qaratilmoqda. Jumladan, matematik biologiyada uchraydigan asosiy obyektlardan bo‘lgan simpleksni invariant saqlovchi Volterra kubik stoxastik operatorlar dinamikasini topishga oid salmoqli natijalarga erishildi. “Funksional analiz, matematik fizika, ehtimollar nazariyasi va dinamik sistemalar” fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo‘nalishlari etib belgilandi[1]. Ushbu vazifalardan kelib chiqqan holda ilmiy natijalardan, ilm-fanning turdosh sohalaridan foydalanish maqsadida nochiziqli operatorlar bilan hosil qilingan diskret vaqtli dinamik sistemalar nazariyasini tadqiqi muhim ilmiy-amaliy ahamiyat kasb etadi.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF–4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi farmoni, 2019-yil 9-iyuldagi PQ–4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi farmoni, 2020-yil 7-maydagi PQ–4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan

[1] O‘zbekiston Respublikasi Vazirlar Mahkamasining 2017-yil 18-maydagi “O‘zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish to‘g‘risida”gi 292-sonli qarori.

vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishi ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishiga muvofiq amalga oshirildi.

Muammoning o‘rganilganlik darajasi. So‘nggi yillarda fizika yoki biologiyadagi matematik modellarni o‘rganish uzluksiz yoki diskret vaqtli nochiziqli operatorlarning dinamikasini o‘rganishga keltiriladi. Kvadratik stoxastik operatorlar nochiziqli operatorlarning sodda sinfi bo‘lib sanaladi. Kvadratik stoxastik operatorlarni birinchi marta S. Bernshteyn tomonidan 1924-yilda kiritilgan. S. Ulam tomonidan kvadratik stoxastik operatorlar orbitalarining xarakterini tekshirish muammosi qo‘yildi.

Jumladan, o‘zbek matematiklari N.N. G‘anixo‘jayev, U.A. Roziqov, F.M. Muhamedov, U.U. Jamilov, A.Y. Hamrayev, R.T. Muhitdinovlarning ishlarida ba’zi novolterra kvadratik stoxastik operatorlari sinflariga tegishli operatorlari tadqiq qilindi. Ta’kidlab o‘tish joizki, novolterra kvadratik stoxastik operatorlar dinamikalari uchun umumiy nazariya kichik o‘lchamli simplekslarda ham mavjud emas. R.N. G‘anixo‘jayev tomonidan turnirlar va Lyapunov funksiyalari nazariyalaridan foydalanib Volterra kvadratik stoxastik operatorlari uchun orbitalarining asimptotik xarakteri tadqiq qilindi. Nochiziqli stoxastik operatorlarning yana bir sinfi kubik stoxastik operatordan tashkil topgan. U.A. Roziqov va Y.A. Hamrayevlar tomonidan ilk bor 2004-yilda kubik stoxastik operatorlarning ta’rifi berildi. Mualliflar tomonidan chekli o‘lchamli simpleksda aniqlangan kubik stoxastik operatorlar yagona qo‘zg‘almas nuqtaga ega bo‘lishi uchun operator koeffisiyentlariga yetarli shart topilgan. Ular tomonidan, bir o‘lchamli simpleksda aniqlangan Volterra kubik operatorlari dinamikasi o‘rganilgan. Chekli o‘lchamli simpleksda aniqlangan simmetrik Volterra kubik stoxastik operatori uchun orbitalarning limit nuqtalar to‘plami to‘liq tavsiflandi. Hoffman va boshqalar tomonidan klassik Lotka-Volterra tenglamalarini umumlashtirish g‘oyasi ilgari surildi. Shuningdek, dengiz faglari va bakteriyalarining o‘zaro ta’sirini aniqroq tasvirlash uchun tajribalar natijalari asosida Lotka-Volterra tenglamalariga darajani qo‘llab yangicha talqin qilish taklif qilindi. Yangi hosil bo‘lgan umumlashgan Lotka-Volterra tenglamalari C. Gavin, A. Pokrovskii, M. Prentice va V. Sobolevlar tomonidan atroflicha tadqiq qilingan. E’tiborli joi shundaki, yuqorida keltirib o‘tilgan Volterra kubik stoxastik operatorlari umumlashtirilgan Lotka-Volterra tenglamalarining diskret analogi bo‘ladi. U.A. Roziqov va A.Y. Hamrayev tomonidan kubik stoxastik operatorning konstruktsiyasi berildi. Ushbu konstruksiya fiksirlangan chekli grafda berilgan ehtimollik o‘lchoviga bog‘liqdir. R.R. Davronov va U.U. Jamilovlar va M. Ladralar tomonidan shartli kubik stoxastik operatorlar deb atalgan novolterra kubik stoxastik operatorlari o‘rganilgan. Shuningdek, barcha shartli kubik stoxastik operatorlar yagona qo‘zg‘almas nuqtasi mavjudligi va bunday operatorlar regularlik xususiyatiga ega ekanligi ko‘rsatilgan. U.U. Jamilov, A.Y. Hamrayev va

M. Ladralar tomonidan, chekli o‘lchamli simpleksda aniqlangan bitta parametrga bog‘liq Volterra kubik stoxastik operatorlarining ixtiyoriy orbitasi yaqinlashuvchiligi isbotlangan.

Keyinchalik, Volterra kubik stoxastik operatorlar nazariyasi U.A. Roziqov, F.M. Muhamedov, U.U. Jamilov, A.Y. Hamrayevlarning ishlarida rivojlantirilgan. U.U. Jamilov va A. Reinfeldslar tomonidan qo‘shimcha shart bilan aniqlangan Volterra kubik stoxastik operatorlari oilasining ta’rifi berildi, hamda mazkur kubik stoxastik operatorlari uchun orbitaning limit nuqtalar to‘plami tavsiflandi.

F. Muhamedov, Ch.H. Pah va A. Roslilar tomonidan, noergodik Volterra kubik stoxastik operatorlari o‘rganildi. F. Muhamedov, A.F. Embong va A. Roslilar tomonidan kubik stoxastik operatorlar uchun operatorlarning suryektivligi ularning ortogonallikni saqlash xossasiga ekvivalent ekanligi isbotlangan. Shuningdek, ikki o‘lchamli simpleksda aniqlangan ortogonallikni saqlovchi (yoki suryektiv) kubik stoxastik operatorlari to‘liq tavsiflangan. U.U. Jamilov va M. Ladralar tomonidan chekli o‘lchamli simpleksda aniqlangan o‘rin almashtirish va bitta parametrga bog‘liq novolterra kubik stoxastik operatorning dinamikasi o‘rganilgan. Mazkur kubik stoxastik operatorlar uchun ixtiyoriy orbitaning yoki qo‘zg‘almas nuqtaga yoki davriy orbitaga yaqinlashuvchiligi ko‘rsatilgan.

U.A. Roziqov, S. Nazir va A. Zada va boshqalarning ilmiy ishlarida separabel kvadratik stoxastik operatorlar dinamikasini o‘rganish masalalariga bag‘ishlangan. Separabel kvadratik stoxastik operatorlarning analogi bo‘lgan separabel kubik stoxastik operatorlaning dinamikasini o‘rganish masalasi ancha murakkabdir. Shu bilan birga bunday operatorlarni o‘rganish uchun umumiy nazariya mavjud emasligi dissertatsiya ishning salohiyatini yanada oshiradi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya Qarshi davlat universiteti ilmiy-tadqiqot ishlar rejasidagi OT-F-4-03 “Uzluksiz hamda diskret vaqtli aniq dinamik sistemalar, qismiy integral operatorlar spektrlari” (2017-2020-yillar) nomli fundamental loyihalari doirasida bajarilgan.

Tadqiqotning maqsadi. Separabel kubik stoxastik operatorlar bilan hosil qilingan diskret vaqtli dinamik sistemalar uchun ixtiyoriy boshlang‘ich nuqta orbitasining limit nuqtalari to‘plamini tavsiflashdan iborat.

Tadqiqotning vazifalari:

separabel kubik stoxastik operatorlarning invariant to‘plamlarini tavsiflash;

separabel kubik stoxastik operatorlarning davriy nuqtalarini topish va ularning tiplarini aniqlash;

separabel kubik stoxastik operatorlari uchun Lyapunov funksiyalarini qurish;

separabel kubik stoxastik operatorlari ixtiyoriy orbitasining asimptotik xarakterini tavsiflash.

Tadqiqotning obyekti: kubik stoxastik operatorlar bilan hosil qilingan diskret vaqtli dinamik sistemalardir.

Tadqiqotning predmeti. Chekli o‘lchamli simpleksda aniqlangan separabel kubik stoxastik operatorlar.

Tadqiqotning usullari. Dissertatsiyada matematik analizning metodlari, funksional analiz, ehtimollar va dinamik sistemalar nazariyalari usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

separabel kubik stoxastik operatorlar uchun qurilgan chiziqli funksional shaklidagi Lyapunov funksiyalari koeffisiyentlari bilan dastlabki uchta matritsalar elementlari orasida bog‘liqliklar ifodalangan;

separabel kubik stoxastik operatorning Volterra kubik stoxastik operatori bo‘lishi uchun, chekli o‘lchamli simpleksning ichki sohasidan olingan ixtiyoriy nuqtada mos koordinatasining o‘zi qatnashmaydigan kubik formalarning nolga teng bo‘lishi zarur va yetarli ekanligi ko‘rsatilgan;

parametrlarga ayrim shartlar ostida bir o‘lchamli diskret dinamik sistemalar nazariyasi va monoton ketma-ketliklar usulidan foydalanib ikki o‘lchamli simpleksda aniqlangan separabel kubik stoxastik operatorning regulyar bo‘lishi, ya’ni ixtiyoriy orbitaning qo‘zg‘almas nuqtalardan biriga yaqinlashishi isbotlangan;

chekli o‘lchamli simpleksda aniqlangan o‘rin almashtirishga bog‘liq separabel kubik stoxastik operatorning ixtiyoriy orbitasi yoki qo‘zg‘almas nuqtaga yoki davri o‘rin almashtirish sikllarining tartiblarini eng kichik umumiy karralisiga teng davriy orbitaga yaqinlashishi ko‘rsatilgan.

Tadqiqotning amaliy natijalari. Tadqiqot yangi qiziqarli natijalardan iborat va ular kubik operatorlar bilan hosil qilingan dinamik sistemalarni tadqiq qilishda qo‘llanilishi mumkin. Separabel kubik stoxastik operatorlarning ixtiyoriy orbitasini limit nuqtalari to‘plami chekli to‘plam bo‘lishidan matematik biologiya masalalarida biologik sistemaning kelajagini prognozlashda qo‘lanilgan.

Tadqiqot natijalarining ishonchliligi natijalar matematik analiz, funksional analiz, diskret vaqtli dinamik sistemalar nazariyalarining fundamental natijalaridan foydalanganligi, hamda matematik mulohazalar va isbotlarning qat’iyligi bilan izohlanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati kubik stoxastik operatorlar nazariyasida va nochiziqli diskret vaqtli dinamik sistemalar nazariyasida qo‘llanilishi mumkinligi bilan izohlanadi.

Tadqiqotning amaliy ahamiyati separabel operator bilan qurilgan dinamik sistema orbitasining limit nuqtalari to‘plamini tavsiflash orqali matematik biologiyadagi populyatsiya evolyutsiyasi xususiyatlarini prognozlashda qo‘llanilishi bilan izohlanadi. Shuningdek, olingan natijalar va dissertatsiyada qo‘llanilgan usullardan oliy o‘quv yurtlarida soha bo‘yicha magistratura talabalari va tayanch doktorantlar uchun maxsus kurs sifatida o‘qitishda foydalanish mumkin.

Tadqiqot natijalarining joriy qilinishi. Dissertatsiya tadqiqoti jarayonida olingan ilmiy natijalar quyidagi yo‘nalishlarda amaliyotga joriy qilingan:

separabel kubik stoxastik operatorlar bilan hosil qilingan diskret vaqtli dinamik sistemalarning davriy nuqtalari to‘plamidan va orbitalarning tavsifidan G00003447 raqamli “Kvant genetik algebralar va ularning tatbiqlari” mavzusidagi xorijiy loyihada novolterra kubik stoxastik operatorlarning dinamikasini tadqiq qilishda foydalanilgan (Birlashgan Arab Amirliklari universitetining 2024-yil

17-oktabrdagi ma’lumotnomasi, BAA). Ilmiy natijaning qo‘llanilishi biologik populyatsiya evolyutsiyasini ifodalaydigan kubik stoxastik operatorlarning davriy orbitalarini tavsiflash imkonini bergan;

separabel kubik stoxastik operatorlar bilan hosil qilingan diskret vaqtli dinamik sistemalar uchun orbitalarning limit nuqtalari to‘plami tavsifidan OT–F-4-03 raqamli “Uzluksiz hamda diskret vaqtli aniq dinamik sistemalar, qismiy integral operatorlar spektrlari” mavzusidagi loyihada nochiziqli stoxastik operatorlarning orbitalarini tadqiq qilishda foydalanilgan (Qarshi davlat universitetining 2024-yil 26-sentabrdagi 04/3052-son ma’lumotnomasi). Ilmiy natijaning qo‘llanilishi nochiziqli diskret vaqtli dinamik sistema orbitasining limit nuqtalari to‘plamini tavsiflash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Dissertatsiyaning asosiy natijalari 8 ta ilmiy amaliy anjumanlarda, jumladan 3 ta xalqaro va 5 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha 14 ta ilmiy ish chop etilgan, shundan 6 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasi tomonidan falsafa doktorlik dissertatsiyalari himoyasi uchun tavsiya etilgan ilmiy nashrlarda, shu jumladan, 1 tasi xorijiy jurnalda va 5 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning hajmi va tuzilishi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning hajmi 86 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiyada tanlangan mavzuning dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi yoritilgan, mavzu bo‘yicha xorijiy va mahalliy ilmiy-tadqiqot ishlari sharhi, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan maqolalar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning **“Kvadratik va kubik stoxastik operatorlar”** deb nomlanuvchi birinchi bobida dissertatsiya mavzusini to‘la yoritish uchun zarur bo‘lgan asosiy ta’riflar va muhim tushunchalar keltirilgan. Shuningdek Lotka-Volterra operatorning kiritilishi va dinamikasi haqidagi natijalar bayon qilingan.

Birinchi bobning birinchi paragrafida Lotka-Volterra operatorlarining uchun ta’rifi keltirilgan. Mashhur Lotka-Volterra tenglamalari turli xil ekologik va kimyoviy sistemalarni matematik modellashtirishda muhim rol o‘ynaydi. Hoffmann va boshqalar tomonidan dengiz faglari va bakteriyalar populyatsiyalari o‘zaro ta’sirini modellashtirish uchun klassik Lotka-Volterra tenglamalari umumlashtirildi. Ya’ni Lotka-Volterra tenglamalarini eksperimental ma’lumotlar asosida fag-bakteriyalarning o‘zaro ta’sirini kengroq tasvirlash uchun ushbu tenglamalarga darajani qo‘llab modifikatsiya qilishni taklif qilindi. Hosil bo‘lgan yangi umumlashgan Lotka-Volterra tenglamalari atroflicha tadqiq qilindi. Umumlashgan Lotka-Volterra tenglamalari quyidagicha ifodalanadi:

$$\dot{x} = \alpha x - \beta x y^p, \qquad \dot{y} = -\gamma y^p + \delta x y^p. \tag{1}$$

Bu tenglamalarda, x va y bakteriyalar va fag populyatsiyalarining miqdoriga mos keladigan vaqt bo‘yicha o‘zgaruvchilar va α, β, γ va δ sonlar esa ikki tur o‘rtasidagi va ularning tegishli populyatsiyalari ichidagi o‘zaro ta’sirlarning intensivligini boshqaradigan nomanfiy o‘zgarmas qiymatlardir. Shuni ta’kidlash joizki, har doim $p > 1$ deb olinadi, chunki $p = 1$ bo‘lganda (1) tenglamalar, klassik Lotka-Volterra tenglamalariga to‘g‘ri keladi. Klassik Lotka-Volterra tenglamalarini umumlashtirish bilan dengiz faglari bakteriyalar populyatsiyalarini modellashtirish imkoni paydo bo‘ladi.

Aslini olganda, (1) tenglamalar fag populyatsiyasining “samarali hajmi” uning “jismoniy hajmidan” farq qilishini anglatadi, bu samarali kattalik jismoniy hajmning p ko‘rsatkichiga to‘g‘ri proporsionaldir. Klassik Lotka-Volterra tenglamalarida $p = 1$ bo‘ladi va bu holda o‘zaro ta’sirlarda bitta yirtqich bilan bitta o‘lja to‘qnashadi deb faraz qilinadi. Umumlashgan Lotka-Volterra tenglamalarida $p > 1$ bo‘lgan hol uchun bir nechta yirtqich bitta o‘lja bilan to‘qnashadi deb faraz qilinadi. Umumlashgan tenglamalardagi $p = 2$ ko‘rsatkichi o‘zaro ta’sirni ifodalashda muhim bo‘lgan parametrning qiymatidir, ya’ni bu qiymat ikkita fagning bitta bakteriya bilan to‘qnashishini ta’minlaydi, deb faraz qilishimiz mumkin. Hoffmann buni bakteriya dengizda bir nechta fagni jalb qilishi bilan bog‘laydi, bu kontseptsiya laboratoriyada o‘tkazilgan tajribalar natijalari bilan tasdiqlanadi, haqiqatan ham natijalar har bir bakteriya 2-3 faglarni yuqtirishini ko‘rsatadi. Jumladan, (1) tenglamalardagi y^p belgilash p ta faglardan tashkil topgan “ov jamoalari” mavjudligini bildiradi va ularning o‘zaro birgalikdagi ta’sirida samarali bo‘ladi. Lotka-Volterra tenglamalarining darajadan foydalanib p ko‘rsatkichni kiritilgan modifikatsiyasi fag-bakteriyalarning o‘zaro ta’sirini yaxshiroq tushunish imkoni va an’anaviy yirtqich va o‘lja munosabatlaridan farqli dinamik sistemalarni tadqiq qilishni beradi. Yuqorida aytib o‘tilgan tenglamalar yirtqichlar va o‘ljalarning o‘zaro ta’sirini boshqaradigan diskret vaqtli Kolmogorov sistemasining o‘ziga xos misoli bo‘ladi.

Diskret vaqtli Kolmogorov sistemasi deb nomlanuvchi $K : \mathbb{R}^m_+ \to \mathbb{R}^m_+$ akslantirish quyidagicha aniqlanadi:

$$K(\mathbf{x}) = \left(x_1 g_1(\mathbf{x}), x_2 g_2(\mathbf{x}), \ldots, x_m g_m(\mathbf{x})\right), \qquad \mathbf{x} \in \mathbb{R}^m_+.$$

Bu sistemada, har bir x_i, kattalik i turlarning populyatsiyasi zichligini ifodalaydi va g_i funksiya i turlarning o‘zgarish sur’atini tavsiflaydi. Bu o‘zgarish sur’atlari populyatsiya zichligi $\mathbf{x} = (x_1, x_2, \ldots, x_m)$ vektorga bog‘liq. Diskret vaqtli Kolmogorov sistemasi g_i funksiyalarining o‘ziga xos xususiyatlariga qarab, turlarning o‘zaro ta’sirining har xil turlarini qamrab olishi mumkin.

Aytaylik, $E=\{1,2,\ldots,m\}$ chekli to‘plam bo‘lsin va E dagi barcha ehtimollik taqsimotlari to‘plami

$$S^{m-1}=\left\{\mathbf{x}\in\mathbb{R}_{+}^{m}:\ \sum_{i=1}^{m}x_i=1\right\},$$

kabi aniqlanadi va bu to‘plam $(m-1)$-o‘lchamli simpleks deb ataladi.

1-ta’rif. $\Phi:\mathbb{R}_{+}^{m}\to\mathbb{R}_{+}^{m}$ uzluksiz akslantirish uchun $\Phi\left(S^{m-1}\right)\subset S^{m-1}$ munosabat o‘rinli bo‘lsa, bunday akslantirishga stoxastik operator (SO) deyiladi.

2-ta’rif. $K:\mathbb{R}_{+}^{m}\to\mathbb{R}_{+}^{m}$ stoxastik Kolmogorov sistemasiga Lotka-Volterra operatori deyiladi.

Aytaylik, $\mathbf{x}^{(0)}\in S^{m-1}$ nuqta berilgan bo‘lsin va Φ SOni $\mathbf{x}^{(0)}\in S^{m-1}$ nuqtaga n marta ta’sirini $\Phi^{n}\left(\mathbf{x}^{(0)}\right)$ bilan belgilaymiz. $\mathbf{x}^{(0)}\in S^{m-1}$ boshlang‘ich nuqtaning Φ SO ta’siri natijasidagi $\left\{\mathbf{x}^{(n)}\right\}_{n\geq 0}$ orbitasi (trayektoriyasi) deb

$$\mathbf{x}^{(n+1)}=\Phi\left(\mathbf{x}^{(n)}\right)=\Phi^{n+1}\left(\mathbf{x}^{(0)}\right),\qquad n=0,1,2\ldots$$

qonuniyat bilan aniqlangan ketma-ketlikka aytiladi.

Dinamik sistemalar nazariyasining asosiy masalasi berilgan operator uchun orbitalarning asimptotik xarakterini o‘rganishdan iborat.

$\left\{\mathbf{x}^{(n)}\right\}_{n\geq 0}$ orbitaning ω-limit nuqtalari to‘plamini $\omega_{\Phi}\left(\mathbf{x}^{(0)}\right)$ bilan belgilaylik. S^{m-1} simpleks kompakt to‘plam va $\left\{\mathbf{x}^{(n)}\right\}_{n\geq 0}\subset S^{m-1}$ bo‘lgani uchun $\omega_{\Phi}\left(\mathbf{x}^{(0)}\right)\neq\varnothing$ bo‘ladi.

Simpleksni ichki sohasi va chegarasini quyidagicha belgilab olamiz

$$\mathrm{int}S^{m-1}=\left\{x\in S^{m-1}:\prod_{i=1}^{m}x_i>0\right\},\qquad \partial S^{m-1}=S^{m-1}\setminus\mathrm{int}S^{m-1}.$$

3-ta’rif. Agar $\varphi:S^{m-1}\to\mathbb{R}$ uzluksiz funksiya va barcha $\mathbf{x}^{(0)}\in S^{m-1}$ nuqtalar uchun $\lim\limits_{n\to\infty}\varphi\left(\mathbf{x}^{(n)}\right)$ limit mavjud bo‘lsa, u holda φ funksiyaga Φ operator uchun Lyapunov funksiyasi deyiladi.

Agar $\mathbf{x}\in S^{m-1}$ nuqta $\Phi(\mathbf{x})=\mathbf{x}$ tenglikni qanoatlantirsa, u holda $\mathbf{x}$ nuqta Φ SO ning qo‘zg‘almas nuqtasi deyiladi. Φ SOning barcha qo‘zg‘almas nuqtalari to‘plamini Fix(Φ) bilan belgilaymiz, ya’ni Fix(Φ) = $\left\{\mathbf{x}\in S^{m-1}:\Phi(\mathbf{x})=\mathbf{x}\right\}$.

Agar $\mathbf{x}\in S^{m-1}$ nuqta $\Phi^{n}(\mathbf{x})=\mathbf{x}$ tenglikni qanoatlantirsa, u holda $\mathbf{x}$ nuqta Φ SOning davri n ga teng davriy nuqtasi deyiladi. $\Phi^{n}(\mathbf{x})=\mathbf{x}$ tenglikni qanoatlantiruvchi n sonlarning eng kichigi, $\mathbf{x}$ nuqtaning asosiy davri yoki eng kichik davri deb ataladi.

Ma’lumki, qo‘zg‘almas nuqtalar $\Phi(\mathbf{x})=\mathbf{x}$ tenglikni qanoatlantiradi. Asosiy davri n ga teng bo‘lgan barcha davriy nuqtalari to‘plamini (qo‘zg‘almas nuqtalardan tashqari) Per$_n$(Φ) bilan belgilaymiz. Ma’lumki, davriy nuqtaning barcha iteratsiyalari to‘plami davriy orbita hosil qiladi.

Aytaylik $D\Phi(\mathbf{x}^*) = (\partial\Phi_i / \partial x_j(\mathbf{x}^*))_{i,j=1}^m$ matritsa SOning $\mathbf{x}^*$ nuqtadagi Yakobi matritsasi bo‘lsin. Agar $\mathbf{x}^*$ qo‘zg‘almas nuqtaning $D\Phi(\mathbf{x}^*)$ Yakobi matritsasi birlik aylanada xos songa ega bo‘lmasa, u holda $\mathbf{x}^*$ giperbolik qo‘zg‘almas nuqta deb ataladi.

Agar $D\Phi(\mathbf{x}^*)$ Yakobi matritsasining barcha xos sonlari moduli birdan kichik bo‘lsa $\mathbf{x}^*$ giperbolik nuqta o‘ziga tortuvchi nuqta deb ataladi.

Agar $D\Phi(\mathbf{x}^*)$ Yakobi matritsasining barcha xos sonlari moduli birdan katta bo‘lsa $\mathbf{x}^*$ giperbolik nuqta o‘zidan itaruvchi nuqta deb ataladi.

Qolgan hollarda $\mathbf{x}^*$ giperbolik nuqta egar nuqta deb ataladi.

Nochiziqli SO larning eng sodda vakili kvadratik stoxastik operatorlardir.

4-ta’rif. S^{m-1} simpleksni o‘zini o‘ziga o‘tkazuvchi V akslantirish

$$V : x'_k = \sum_{i,j\in E} P_{ij,k} x_i x_j , \quad \forall k \in E, \tag{2}$$

ko‘rinishga ega bo‘lsa va $P_{ij,k}$ koeffitsiyentlar quyidagi shartlarni qanoatlantirsa

$$P_{ij,k} \geq 0, \quad \forall i,j,k \in E \quad \text{va} \quad \sum_{k\in E} P_{ij,k} = 1, \quad \forall i,j \in E, \tag{3}$$

V ga kvadratik stoxastik operator deyiladi.

Matematik biologiyada V kvadratik stoxastik operatorga populyatsiyaning evolyutsion operatori deb ataladi. Biologik populyatsiya deganda ko‘payishga nisbatan yopiq bo‘lgan organizmlar jamoasi tushuniladi. Har bir V kvadratik stoxastik operatorni (3) shartlarni qanoatlantiruvchi $\mathbf{P} = (P_{ij,k})_{i,j,k=1}^m$ kubik matritsasi yordamida yagona usulda aniqlash mumkin.

Masalaning murakkabligi berilgan $\mathbf{P}$ matritsaga bog‘liq. Shuni ta’kidlash joizki, asosiy masala umumiy holda kvadratik stoxastik operatorlar uchun hatto ikki o‘lchamli holda ham ochiq masala bo‘lib hisoblanadi. Biroq, ushbu masala Volterra kvadratik stoxastik operatorlari uchun kengroq o‘rganilgan.

Volterra kvadratik stoxastik operator (2) va (3) shartlarga qo‘shimcha

$$P_{ij,k} = 0 \qquad \text{agar} \qquad k \notin \{i, j\}, \qquad \forall i,j,k \in E, \tag{4}$$

munosabatlar bilan aniqlanadi.

(4) shartga tug‘ilgan farzand faqat ota-onasidan birining genotipini takrorlaydi deb biologik talqin berish mumkin.

Shuni ta’kidlash kerakki, Volterra kvadratik stoxastik operator an’anaviy yirtqich-o‘lja modelining diskret vaqtli ko‘rinishini ifodalaydi.

R.N. G‘anixo‘jayev tomonidan turnirlar va Lyapunov funksiyalari nazariyalaridan foydalanib Volterra kvadratik stoxastik operatorlari uchun orbitalarining asimptotik xarakteri o‘rganildi. Biroq, novolterra kvadratik stoxastik operatorlar dinamikalari uchun umumiy nazariya kichik o‘lchamli simplekslarda ham mavjud emas.

§ 1.2 paragrafda chekli o‘lchamli simpleksda aniqlangan, ikkita matritsaga bog‘liq bo‘lgan, separabel kvadratik stoxastik operatorlarning dinamikasini o‘rganish haqidagi masala qaralgan.

Aytaylik, kvadrat stoxastik operatorlarni (2) va (3) shartlariga qo‘shimcha quyidagi shart bilan ko‘rib chiqaylik:

$$P_{ij,k} = a_{ik} b_{jk}, \quad \forall i, j, k \in E. \tag{5}$$

Bunda, $a_{ik}, b_{jk} \in \mathbb{R}$ sonlar va mos ravishda $A = (a_{ik})$ va $B = (b_{jk})$ matritsalarning elementlari, shuningdek (5) koeffitsiyentlar uchun (3) shartlar o‘rinli. U holda, (5) koeffitsiyentlarga mos kvadrat stoxastik operator V quyidagi shaklda bo‘ladi:

$$x'_k = (V(\mathbf{x}))_k = (A(\mathbf{x}))_k (B(\mathbf{x}))_k, \qquad \forall k \in E, \tag{6}$$

bu yerda $(A(\mathbf{x}))_k = \sum_{i=1}^{m} a_{ik} x_i$, $(B(\mathbf{x}))_k = \sum_{j=1}^{m} b_{jk} x_j$.

5-ta’rif. (6) shakldagi operatorga separabel kvadrat stoxastik operator deyiladi.

§ 1.3 paragrafda kubik stoxastik operatorlar haqida ma’lum tasdiq va teoremalar keltirilgan. Nochiziqli operatorlarning yana bir sinfi kubik stoxastik operatordan iborat.

6-ta’rif. S^{m-1} simpleksni o‘zini o‘ziga o‘tkazuvchi W akslantirish

$$W : x'_l = \sum_{i,j,k \in E} P_{ijk,l} x_i x_j x_k, \qquad \forall l \in E, \tag{7}$$

ko‘rinishda aniqlangan bo‘lib, uning $P_{ijk,l}$ koeffitsiyentlari

$$P_{ijk,l} \geq 0, \quad \forall i, j, k, l \in E \quad \text{va} \quad \sum_{l \in E} P_{ijk,l} = 1, \quad \forall i, j, k \in E, \tag{8}$$

shartlarni qanoatlantirsa W ga kubik stoxastik operator (KSO) deyiladi.

Shuni ta’kidlash kerakki, KSOlar nafaqat matematik biologiyada qo‘llaniladi, balki fizikaning spin sistemalarida uchlik o‘zaro ta’sirga ega bo‘lgan sistemalarni modellashtirish uchun ham qo‘llaniladi. Matematik biologiyada x_i o‘zgaruvchining vaqt bo‘yicha o‘zgaruvchi xarakteri haqidagi asosiy masalani hal qilish uchun orbitalarning asimptotik xarakterini o‘rganish kerak. Bundan tashqari, bu asosiy muammo, Volterra KSOlari uchun ko‘proq o‘rganilgan. Volterra KSOlari (7) va (8) shartlar bilan birga

$$P_{ijk,l} = 0 \qquad \text{agar} \qquad l \notin \{i, j, k\}, \quad \forall i, j, k, l \in E. \tag{9}$$

(9) shartlar quyidagicha biologik talqinga ega: keyingi avlod faqat ota-onasidan birining genotipini meros qilib olishini ko‘rsatadi.

Dissertatsiya ishining ikkinchi bobi **“Separabel kubik stoxastik operatorlar”** deb nomlanadi. Separabel kubik stoxastik operatorlarning dinamikasini tadqiq qilishga bag‘ishlangan.

§ 2.1 paragrafda uchta kvadratik matritsaga bog‘liq bo‘lgan, separabel kubik stoxastik operatorlari qurilgan. Separabel kubik stoxastik operatorlar uchun qurilgan chiziqli funksional shaklidagi Lyapunov funksiyalari koeffisiyentlari bilan dastlabki uchta matritsalar elementlari orasida bog‘liqliklar topilgan. Shuningdek, separabel kubik stoxastik operatorlarni Volterra operatorlari bilan ustma-ust tushishi uchun zaruriy va yetarli shartlar topilgan.

Aytaylik, elementlari haqiqiy sonlardan iborat $A=(a_{il})_{i,l=1}^{m}$, $B=(b_{jl})_{j,l=1}^{m}$ va $C=(c_{kl})_{k,l=1}^{m}$, kvadratik matritsalar bo‘lsin. (7) va (8) shartlar bilan berilgan KSOning koeffitsiyentlarini qo‘shimcha quyidagi shart bilan qaraymiz:

$$P_{ijk,l}=a_{il}b_{jl}c_{kl} \qquad \forall i,j,k,l\in E, \tag{10}$$

bu yerda $a_{il},b_{jl},c_{kl}\in\mathbb{R}$ haqiqiy sonlar va mos ravishda A, B va C matritsalarning elementlari. U holda, (10) shartda berilgan koeffitsiyentlarga mos keladigan W KSO quyidagicha:

$$x'_l=\big(W(\mathbf{x})\big)_l=\big(A(\mathbf{x})\big)_l\cdot\big(B(\mathbf{x})\big)_l\cdot\big(C(\mathbf{x})\big)_l \qquad \forall l\in E, \tag{11}$$

bu yerda $\big(A(\mathbf{x})\big)_l=\sum_{i=1}^{m}a_{il}x_i$, $\big(B(\mathbf{x})\big)_l=\sum_{j=1}^{m}b_{jl}x_j$, $\big(C(\mathbf{x})\big)_l=\sum_{k=1}^{m}c_{kl}x_k$.

7-ta’rif. (11) shakldagi operatorga separabel kubik stoxastik operator (SKSO) deyiladi.

1-izoh. (10) qo‘shimcha shart bilan aniqlangan KSOni uchta chiziqli operatorlarning ko‘paytmasi ko‘rinishida ifodalanishi uchun yetarli shart biroq shuni ta’kidlash kerakki, ushbu shart har doim ham zaruriy shart emas.

Izoh 1 ga ko‘ra $P_{ijk,l}$ koeffitsiyentli KSO uchun quyidagi

$$\begin{aligned}&P_{ijk,l}+P_{kij,l}+P_{ikj,l}+P_{kji,l}+P_{jik,l}+P_{jki,l}=\\&a_{il}b_{jl}c_{kl}+a_{kl}b_{il}c_{jl}+a_{il}b_{kl}c_{jl}+a_{kl}b_{jl}c_{il}+a_{jl}b_{il}c_{kl}+a_{jl}b_{kl}c_{il},\end{aligned} \tag{12}$$

shart o‘rinli bo‘lsa, KSOni uchta

$$\big(A(\mathbf{x})\big)_l=\sum_{i=1}^{m}a_{il}x_i,\ \big(B(\mathbf{x})\big)_l=\sum_{j=1}^{m}b_{jl}x_j,\ \big(C(\mathbf{x})\big)_l=\sum_{k=1}^{m}c_{kl}x_k,\ \forall l\in E,$$

chiziqli operatorlarning ko‘paytmasi sifatida ifodalash mumkinligini oson tekshirish mumkin. Demak, (10) shart (12) shartning xususiy holini ifodalar ekan.

(11) SKSOning ba’zi xossalarini keltirib o‘tamiz.

i) Agar A, B va C matritsalarning har birida satrlari bir xil bo‘lsa, u holda bu matritsalarga mos (11) operator o‘zgarmas (konstanta) operator bo‘ladi.

ii) Agar A va B matritsalarning har birida satrlari bir xil va C matritsa satrlari turlicha bo‘lsa, u holda ushbu matritsalarga mos (11) operator chiziqli stoxastik operator bo‘ladi.

iii) Agar A matritsaning satrlari bir xil va B, C matritsalarning satrlari esa turli bo‘lsa, u holda ushbu matritsalarga mos (11) operator separabel kvadrat stoxastik operator bo‘ladi.

iv) Agar A, B va C matritsalar turli satrli bo‘lsa, u holda ushbu matritsalarga mos (11) operator quyidagicha bo‘ladi:

$$x'_l=\left(\sum_{i=1}^{m}a_{il}x_i\right)\cdot\left(\sum_{j=1}^{m}b_{jl}x_j\right)\cdot\left(\sum_{k=1}^{m}c_{kl}x_k\right), \qquad \forall l\in E. \tag{13}$$

Shuni ta’kidlash joizki, yuqorida aytib o‘tilgan faktlarga ko‘ra, biz uchun faqat va faqat (13) SKSOning dinamikasini o‘rganish qiziqarli hol hisoblanadi.

Umuman olganda, chekli o‘lchamli simpleksda (13) SKSOning dinamikasini (fiksirlangan matritsalar uchun) o‘rganish murakkab masala hisoblanadi.

1-lemma. Agar barcha $i,j,k,l\in E$ lar uchun $A=(a_{il})_{i,l=1}^{m}$ yoki $B=(b_{jl})_{j,l=1}^{m}$ yoki $C=(c_{kl})_{k,l=1}^{m}$ matritsalardan biri kososimmetrik matritsa va qolgan ikkita matritsaning elementlari bir xil ishorali bo‘lsa, u holda ushbu matritsalarga mos operator SKSO emas.

Endi, SKSOlarni Volterra operatori bo‘lishi uchun zaruriy va yetarli shartini keltiramiz.

1-teorema. Volterra kubik stoxastik operatorning SKSO bo‘lishi uchun $\forall \mathbf{x}\in \operatorname{int} S^{m-1}$ nuqta va $\forall l\in E$ uchun $\sum\limits_{i,j,k\in E\setminus\{l\}} a_{il}b_{jl}c_{kl}x_ix_jx_k=0$ bo‘lishi zarur va yetarli.

Faraz qilaylik $A=(a_{il})_{i,l=1}^{m}$, $B=(b_{jl})_{j,l=1}^{m}$ va $C=(c_{kl})_{k,l=1}^{m}$ matritsalar $W:S^{m-1}\to S^{m-1}$ SKSOning matritsalari va $\mathbf{I}$ matritsa $m\times m$ tartibli birlik matritsa bo‘lsin. U holda, quyidagi teoremani keltiramiz.

2-teorema. Agar barcha $i\in E$ uchun $\mathbf{d}=(d_1,\dots,d_m)$ vektorning koordinatalari $d_i\geq 0$ munosabatni qanoatlantirsa va barcha $i,j,k\in E$ uchun quyidagi $\beta\gamma A\mathbf{d}^T\leq \mathbf{Id}^T$, $\alpha\gamma B\mathbf{d}^T\leq \mathbf{Id}^T$, $\alpha\beta C\mathbf{d}^T\leq \mathbf{Id}^T$ tengsizliklardan kamida bittasi o‘rinli bo‘lsin, u holda quyidagicha aniqlangan

$$\varphi_d(\mathbf{x})=\sum_{l\in E} d_lx_l,$$

uzluksiz $\varphi:S^{m-1}\to\mathbb{R}$ funksiya $W:S^{m-1}\to S^{m-1}$ SKSO uchun Lyapunov funksiyasi bo‘ladi, bu yerda $\forall i,j,k\in E$ uchun A, B va C matritsalar W operatorning nomanfiy elementli matritsalari va $\alpha=\max\limits_{i,l\in E}\{a_{il}\}$, $\beta=\max\limits_{j,l\in E}\{b_{jl}\}$ va $\gamma=\max\limits_{k,l\in E}\{c_{kl}\}$.

SKSO uchun aniqlangan Lyapunov funksiyasidan foydalanib SKSOlarning limint nuqtalari to‘plamini yuqoridan baholaymiz. Quyidagi

$$D=\left\{\mathbf{d}\in\mathbb{R}^m: d_i\geq 0, d_1+\cdots+d_2>0, \beta\gamma A\mathbf{d}^T\leq I\mathbf{d}^T \text{ yoki } \alpha\gamma B\mathbf{d}^T\leq I\mathbf{d}^T \text{ yoki } \alpha\gamma C\mathbf{d}^T\leq I\mathbf{d}^T\right\}.$$

to‘plam berilgan bo‘lsin. U holda, har qanday $\mathbf{d}\in D$ uchun $\varphi_{\mathbf{d}}$ Lyapunov funksiyasi bo‘ladi. Ya’ni $\forall \mathbf{x}^{(0)}\in S^{m-1}$ uchun $\lim\limits_{n\to\infty}\varphi_{\mathbf{d}}\left(\mathbf{x}^{(n)}\right)=\lambda_{\mathbf{d}}\left(\mathbf{x}^{(0)}\right)$, $\mathbf{d}\in D$ o‘rinli bo‘ladi. Demak, har qanday $\mathbf{d}\in D$ uchun $\omega\left(\mathbf{x}^{(0)}\right)\subset\left\{\mathbf{x}\in S^{m-1}:\varphi_{\mathbf{d}}(\mathbf{x})=\lambda_{\mathbf{d}}\left(\mathbf{x}^{(0)}\right)\right\}$, bu munosabatdan esa $\omega\left(\mathbf{x}^{(0)}\right)\subset\bigcap\limits_{\mathbf{d}\in D}\left\{\mathbf{x}\in S^{m-1}:\varphi_{\mathbf{d}}(\mathbf{x})=\lambda_{\mathbf{d}}\left(\mathbf{x}^{(0)}\right)\right\}$ ekanligi kelib chiqadi.

§ 2.2 paragrafda bir o‘lchamli simpleksda aniqlangan regular va davriy orbitaga yaqinlashuvchi SKSOlarga misollar keltirilgan.

Quyidagi musbat elementli matritsalarni qaraylik:

$$A=\begin{pmatrix} a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix},\quad B=\begin{pmatrix} b_{11} & b_{12}\\ b_{21} & b_{22}\end{pmatrix},\quad C=\begin{pmatrix} c_{11} & c_{12}\\ c_{21} & c_{22}\end{pmatrix}. \tag{14}$$

Faraz qilaylik A, B va C matritsaning elementlari uchun

$$a_{11}b_{11}c_{11}+a_{12}b_{12}c_{12}=1,\ a_{21}b_{21}c_{21}+a_{22}b_{22}c_{22}=1,$$
$$a_{21}b_{11}c_{11}+a_{11}b_{21}c_{11}+a_{11}b_{11}c_{21}+a_{22}b_{12}c_{12}+a_{12}b_{22}c_{12}+a_{12}b_{12}c_{22}=1,$$
$$a_{21}b_{21}c_{11}+a_{11}b_{21}c_{21}+a_{21}b_{11}c_{21}+a_{22}b_{22}c_{12}+a_{12}b_{22}c_{22}+a_{22}b_{12}c_{22}=1 \qquad (15)$$
shartlar o‘rinli bo‘lsin.

U holda (14) matritsalarga mos $W:S^1\to S^1$ separabel kubik stoxastik operator (SKSO) quyidagicha ko‘rinishga ega:
$$W:\begin{cases} x_1'=\left(a_{11}x_1+a_{21}x_2\right)\left(b_{11}x_1+b_{21}x_2\right)\left(c_{11}x_1+c_{21}x_2\right),\\ x_2'=\left(a_{12}x_1+a_{22}x_2\right)\left(b_{12}x_1+b_{22}x_2\right)\left(c_{12}x_1+c_{22}x_2\right).\end{cases} \qquad (16)$$

Endi $x_1+x_2=1$ va $x_1=x \ \Rightarrow\ x_1'=f(x_1)$ tengliklardan foydalanib (16) operatorning 2-koordinatasini quyidagicha bir o‘zgaruvchili kubik funksiya ko‘rinishida qayta yozishimiz mumkin
$$f\left(x\right)=ax^3+bx^2+cx+d, \qquad (17)$$
bu yerda $a=a_{22}b_{22}c_{22}-a_{12}b_{12}c_{12}+a_{12}b_{12}c_{22}+a_{12}b_{22}c_{12}+a_{22}b_{12}c_{12}-$
$a_{12}b_{22}c_{22}-a_{22}b_{12}c_{22}-a_{22}b_{22}c_{12}$,
$b=3a_{12}b_{12}c_{12}-2a_{12}b_{12}c_{22}-2a_{12}b_{22}c_{12}-2a_{22}b_{12}c_{12}+a_{12}b_{22}c_{22}+a_{22}b_{22}c_{12}-a_{22}b_{12}c_{22}$,
$c=a_{12}b_{12}c_{22}+a_{12}b_{22}c_{12}+a_{22}b_{12}c_{12}-3a_{22}b_{12}c_{12}$, $\quad d=a_{12}b_{12}c_{12}$.

Demak, (16) operatorning dinamikasini o‘rganish uchun (17) kubik funksiyaning dinamikasini o‘rganish yetarli ekan. Biroq (17) kubik funksiyaning dinamikasini o‘rganish murakkab hisoblanadi. Shu sababdan, dinamik sistemalarda uchrashi mumkin bo‘lgan hollarga misollar keltiramiz.

1-misol. Aytaylik,
$$A=\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad B=\begin{pmatrix}1&3\\0&1\end{pmatrix},\quad C=\begin{pmatrix}1&0\\3&1\end{pmatrix},$$
matritsalar berilgan bo‘lsin. A, B va C matritsalarning elementlari (15) shartni qanoatlantiradi va ularga mos SKSO quyidagichadir
$$W:\begin{cases} x_1'=x_1^2\left(x_1+3x_2\right),\\ x_2'=x_2^2\left(x_2+3x_1\right).\end{cases} \qquad (18)$$

Ushbu belgilashlarni olamiz $\mathbf{e}_1=\left(1,0\right)$, $\mathbf{e}_2=\left(0,1\right)$ vektorlar bir o‘lchamli simpleksning uchlari va $\mathbf{c}=\left(\frac{1}{2},\frac{1}{2}\right)$ bo‘lsin. Quyidagi tasdiqni (18) operator uchun keltiramiz.

1-tasdiq. (18) SKSO uchun quyidagi tasdiqlar o‘rinli:

a) $\mathrm{Fix}\left(W\right)=\left\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{c}\right\}$, hamda $\mathbf{c}$ nuqta itaruvchi qo‘g‘almas nuqta va $\mathbf{e}_1$, $\mathbf{e}_2$ nuqtalar tortuvchi qo‘zg‘almas nuqtalar;

b) $\mathbf{x}^{(0)}\in S^1\setminus \mathrm{Fix}\left(W\right)$ uchun
$$\lim_{n\to\infty}W^n\left(x\right)=\begin{cases}\mathbf{e}_2, & \text{agar } 0\le x_1^{(0)}<\frac{1}{2},\\ \mathbf{e}_1, & \text{agar } \frac{1}{2}<x_1^{(0)}\le 1.\end{cases}$$

2-misol. Quyidagi matritsalarni qaraylik:

$$A=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\quad B=\begin{pmatrix}3 & 1\\ 1 & 0\end{pmatrix},\quad C=\begin{pmatrix}0 & 1\\ 1 & 3\end{pmatrix}.$$

U holda, A, B va C matritsalarga mos separabel kubik stoxastik operator quyidagicha

$$W:\begin{cases}x_1' = x_2^2\left(x_2+3x_1\right),\\ x_2' = x_1^2\left(x_1+3x_2\right).\end{cases} \tag{19}$$

2-tasdiq. (19) SKSO uchun quyidagi tasdiqlar o'rinli:

i) $\mathrm{Fix}(W)=\{\mathbf{c}\}$ va $\mathbf{c}$ nuqta itaruvchi qo'zg'almas nuqtadir;

ii) $\mathrm{Per}_2(W)=\{\mathbf{e}_1,\mathbf{e}_2\}$;

iii) $\mathbf{x}^{(0)}\in S^1\setminus \mathrm{Fix}(W)$ uchun $\omega_W\left(\mathbf{x}^{(0)}\right)=\{\mathbf{e}_1,\mathbf{e}_2\}$.

M. Akbari va M. Rabiilar tomonidan $f_a(x)=ax^2(x-1)+x$ (bu yerda $x\in\mathbb{R}$, $a\in\mathbb{R}\setminus\{0\}$) funksiya dinamikasi o'rganilgan. Agar $a<0$ bo'lsa, u holda $f_a(x)$ funksiya orbitasi xaotik bo'ladi. Shuningdek, $f_a(x)$ funksiya agar $0<a\le 4$ bo'lsa $f_a:[0,1]\to[0,1]$ bo'ladi. Agar $f_a(c)\in J_1$ bo'lsa, u holda $f_a(x)$ funksiya $\Lambda_a=\{x\in[0,x_1]: f_a^n(x)\in[0,x_1];\forall n\ge 1\}$ to'plamda orbitasi xaotik bo'ladi, bu yerda $a<0,\ c\in\mathbb{R}$ va $x_1>0$, $x_3\in(x_1,\infty)$, $J_1=[x_1,x_3]$ sonlar o'qidagi yopiq kesma. Demak, (17) funksiya $f_a(x)$ kubik funksiya bilan a parametrning qiymatlarida kesishmaydi.

Quyidagi kvadratik matritsalarni qaraylik:

$$A=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix},\ B=\begin{pmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{pmatrix},\ C=\begin{pmatrix}c_{11} & c_{12} & c_{13}\\ c_{21} & c_{22} & c_{23}\\ c_{31} & c_{32} & c_{33}\end{pmatrix}.$$

U holda, ushbu matritsalarga mos $W:S^2\to S^2$ SKSO

$$W:\begin{cases}x_1'=\left(a_{11}x_1+a_{21}x_2+a_{31}x_3\right)\left(b_{11}x_1+b_{21}x_2+b_{31}x_3\right)\left(c_{11}x_1+c_{21}x_2+c_{31}x_3\right),\\ x_2'=\left(a_{12}x_1+a_{22}x_2+a_{32}x_3\right)\left(b_{12}x_1+b_{22}x_2+b_{32}x_3\right)\left(c_{12}x_1+c_{22}x_2+c_{32}x_3\right),\\ x_3'=\left(a_{13}x_1+a_{23}x_2+a_{33}x_3\right)\left(b_{13}x_1+b_{23}x_2+b_{33}x_3\right)\left(c_{13}x_1+c_{23}x_2+c_{33}x_3\right),\end{cases}$$

kabi ko'rinishda bo'ladi va bu yerda parametrlar quyidagi

$$a_{11}b_{11}c_{11}+a_{12}b_{12}c_{12}+a_{13}b_{13}c_{13}=1,\ a_{21}b_{21}c_{21}+a_{22}b_{22}c_{22}+a_{23}b_{23}c_{23}=1,$$
$$a_{31}b_{31}c_{31}+a_{32}b_{32}c_{32}+a_{33}b_{33}c_{33}=1,$$
$$a_{21}b_{11}c_{11}+a_{11}b_{21}c_{11}+a_{11}b_{11}c_{21}+a_{12}b_{12}c_{22}+a_{12}b_{22}c_{12}+a_{22}b_{12}c_{12}+a_{13}b_{13}c_{23}+$$
$$a_{13}b_{23}c_{13}+a_{23}b_{13}c_{13}=3,$$
$$a_{31}b_{11}c_{11}+a_{11}b_{31}c_{11}+a_{11}b_{11}c_{31}+a_{13}b_{13}c_{33}+a_{13}b_{33}c_{13}+a_{33}b_{13}c_{13}+a_{12}b_{12}c_{32}+$$
$$a_{12}b_{32}c_{12}+a_{32}b_{12}c_{12}=3,$$

$$a_{12}b_{22}c_{22}+a_{22}b_{12}c_{22}+a_{22}b_{22}c_{12}+a_{21}b_{21}c_{11}+a_{21}b_{11}c_{21}+a_{11}b_{21}c_{21}+a_{23}b_{23}c_{13}+$$
$$a_{23}b_{13}c_{23}+a_{13}b_{23}c_{23}=3,$$
$$a_{13}b_{33}c_{33}+a_{33}b_{13}c_{33}+a_{33}b_{33}c_{13}+a_{31}b_{31}c_{11}+a_{31}b_{11}c_{31}+a_{11}b_{31}c_{31}+a_{32}b_{32}c_{12}+$$
$$a_{32}b_{12}c_{32}+a_{12}b_{32}c_{32}=3,$$
$$a_{32}b_{22}c_{22}+a_{22}b_{32}c_{22}+a_{22}b_{22}c_{32}+a_{23}b_{23}c_{33}+a_{23}b_{33}c_{23}+a_{33}b_{23}c_{23}+a_{21}b_{21}c_{31}+$$
$$a_{21}b_{31}c_{21}+a_{31}b_{21}c_{21}=3,$$
$$a_{23}b_{33}c_{33}+a_{33}b_{23}c_{33}+a_{33}b_{33}c_{23}+a_{32}b_{32}c_{22}+a_{32}b_{22}c_{32}+a_{22}b_{32}c_{32}+a_{21}b_{31}c_{31}+$$
$$a_{31}b_{21}c_{31}+a_{31}b_{31}c_{21}=3,$$
$$a_{21}b_{11}c_{31}+a_{11}b_{21}c_{31}+a_{11}b_{31}c_{21}+a_{31}b_{11}c_{21}+a_{21}b_{31}c_{11}+a_{31}b_{21}c_{11}+a_{12}b_{22}c_{32}+$$
$$a_{22}b_{12}c_{32}+a_{12}b_{32}c_{22}+a_{32}b_{12}c_{22}+a_{22}b_{32}c_{12}+a_{32}b_{22}c_{12}+a_{13}b_{23}+a_{23}b_{13}c_{33}+$$
$$a_{13}b_{33}c_{23}+a_{33}b_{13}c_{23}+a_{23}b_{33}c_{13}+a_{33}b_{23}c_{13}=6 \qquad (20)$$

tengliklarni qanoatlantiradi. (20) tenglamalar sistemasining dastlabki uchta tenglamasida $a_{ii}=b_{ii}=c_{ii}=1$ $(i=1,2,3)$, $a_{12}=b_{13}=a_{21}=c_{23}=a_{31}=b_{32}=c_{32}=0$, $b_{21}=b_{31}$ va $c_{21}=c_{31}$ bo‘lsin deb faraz qilaylik, u holda mos $W:S^2\to S^2$ SKSO quyidagicha

$$W:\begin{cases}x_1'=x_1\left(x_1+b_{21}x_2+b_{21}x_3\right)\left(x_1+c_{21}x_2+c_{21}x_3\right),\\ x_2'=\left(x_2+a_{32}x_3\right)\left(b_{12}x_1+x_2\right)\left(c_{12}x_1+x_2\right),\\ x_3'=\left(a_{13}x_1+a_{23}x_2+x_3\right)\left(b_{23}x_2+x_3\right)\left(c_{13}x_1+x_3\right),\end{cases} \qquad (21)$$

bu yerda (21) operatorning parametrlari uchun quyidagi

$$b_{21}+c_{21}+b_{12}c_{12}+a_{13}b_{23}c_{13}=3,\ b_{21}+c_{21}+a_{13}c_{13}+a_{32}b_{12}c_{12}=3,$$
$$b_{12}+c_{12}+b_{21}c_{21}+a_{23}b_{23}c_{13}=3,\ a_{13}+c_{13}+b_{21}c_{21}=3,$$
$$a_{32}+a_{23}b_{23}=3,\ a_{23}+b_{23}=3,$$
$$2b_{21}c_{21}+a_{32}b_{12}+a_{32}c_{12}+a_{13}b_{23}+a_{23}c_{13}+b_{23}c_{13}=6, \qquad (22)$$

shartlar o‘rinli.

Shuni ta’kidlash joizki, (22) tenglamalar sistemasining yechimlari to‘plami bo‘sh emas, uning quvvati kontinuum (cheksiz) bo‘lganligi uchun (21) SKSO oila tashkil etadi.

Dastlab, bizga ba’zi yordamchi natijalar kerak bo‘ladi. (21) SKSOning 1-koordinatasini quyidagi $x_1'=f_{bc}\left(x_1\right)$ shaklida yozish mumkin. Shu sababli,

$$f_{bc}\left(x\right)=\left(1-b\right)\left(1-c\right)x^3+\left(b+c-2bc\right)x^2+bcx,$$

yordamchi funksiyaning dinamikasini ko‘rib chiqaylik, bu yerda $x\in\left[0,1\right]$, $b,c\in\left[0,3\right]$.

Agar $b=c=1$ bo‘lsa, u holda $f_{bc}\left(x\right)$ funksiya ayniy (birlik) akslantirish bo‘ladi. $b\neq 1$ va $c\neq 1$ qiymatlar uchun $x^*=\dfrac{bc-1}{\left(1-b\right)\left(1-c\right)}$ va $f_{bc}\left(x\right)$ funksiyaning o‘zini-o‘ziga n marta ta’sirini $f_{bc}^{n} = \underbrace{f_{bc} \circ \ldots \circ f_{bc}}_{n \ \ marta}$ bilan belgilaylik. Quyidagi teoremada $f_{bc}(x)$ funksiyaning dinamik xossalari ifodalanadi.

3-teorema. $f_{bc}(x)$ funksiya uchun quyidagilar o‘rinli:

$$\text{i) Fix}(f_{bc}) = \begin{cases} [0,1], \ \text{agar } b = c = 1, \\ \{0,1\}, \ \text{agar } b = 1, c \neq 1 \text{ yoki } b \neq 1, c = 1 \\ \qquad \text{yoki } b \neq 1, c \neq 1, bc = 1 \\ \qquad \text{yoki } b \neq 1, c \neq 1, b + c = 2 \\ \qquad \text{yoki } b \neq 1, c \neq 1, bc < 1, b + c < 2 \\ \qquad \text{yoki } b \neq 1, c \neq 1, bc > 1, b + c > 2, \\ \{0,1,x^*\}, \text{agar } b \neq 1, c \neq 1, bc < 1, b + c > 2; \end{cases}$$

$$\text{ii) } x = 0 \text{ qo‘zg‘almas nuqta } \begin{cases} \text{tortuvchi}, & \text{agar } bc < 1, \\ \text{nogiperbolik}, & \text{agar } bc = 1, \\ \text{itaruvchi}, & \text{agar } bc > 1, \end{cases}$$

$$x = 1 \text{ qo‘zg‘almas nuqta } \begin{cases} \text{tortuvchi}, & \text{agar } 2 < b + c \leq 3, \\ \text{nogiperbolik}, & \text{agar } b + c = 2, \\ \text{itaruvchi}, & \text{agar } 0 \leq b + c < 2, \end{cases}$$

Agar $b \neq 1, c \neq 1, bc < 1, b + c > 2$ bo‘lsa, $x = x^*$ nuqta itaruvchi tipli bo‘ladi;

i) ixtiyoriy $x \in (0,1)$ nuqta uchun quyidagi limit o‘rinli:

$$\lim_{n\to\infty} f_{bc}^{n}(x) = \begin{cases} 0, & \text{agar } b = 1, \ 0 < c < 1 \text{ yoki } c = 1, 0 < b < 1, \\ 1, & \text{agar } b = 1, \ 1 < c \leq 2 \text{ yoki } c = 1, 1 < b \leq 2; \end{cases}$$

ii) Agar $b \neq 1$, $c \neq 1$ bo‘lsa, u holda

iv1) barcha $x \in (0,1)$ nuqtalar uchun

$$\lim_{n\to\infty} f_{bc}^{n}(x) = \begin{cases} 0, & \text{agar } bc < 1, b + c < 2 \text{ yoki } b + c = 2, \\ 1, & \text{agar } bc > 1, b + c > 2 \text{ yoki } bc = 1; \end{cases}$$

iv2) $bc < 1$, $b + c > 2$ bo‘lganda esa

$$\lim_{n\to\infty} f_{bc}^{n}(x) = \begin{cases} 0, & \text{agar } 0 \leq x < x^*, \\ 1, & \text{agar } x^* < x \leq 1. \end{cases}$$

4-teorema. (21) SKSO uchun quyidagi tasdiqlar o‘rinli:

i) S^2 simpleksning $\Gamma_{\{2,3\}}$ qirrasi invariant to‘plam bo‘ladi;

ii) $\text{Fix}(W) = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} \cup \{\hat{\mathbf{x}}, \text{ agar } 2 < a_{32} \leq 3 \text{ bo‘lsa}\}$,

bu yerda $\hat{\mathbf{x}} = \left(0, \dfrac{1}{a_{32} - 1}, \dfrac{a_{32} - 2}{a_{32} - 1}\right)$.

Agar $b_{21}c_{21} < 1$, $b_{21} + c_{21} > 2$ bo‘lsa, u holda $|\text{Fix}(W)| > 3$ bo‘ladi;

$$\text{iii) } \mathbf{e}_1 \text{ nuqta} \begin{cases} \text{tortuvchi}, & \text{agar } b_{21}+c_{21}\in(2,4) \text{ va } \left|b_{12}c_{12}(1-a_{32})\right|<1, \\ \text{nogiperbolik}, & \text{agar } b_{21}+c_{21}\in\{2,4\} \text{ yoki } \left|b_{12}c_{12}(1-a_{32})\right|=1, \\ \text{itaruvchi}, & \text{agar } b_{21}+c_{21}<2, b_{21}+c_{21}>4 \text{ va } \left|b_{12}c_{12}(1-a_{32})\right|>1, \\ \text{egar}, & \text{agar } b_{21}+c_{21}<2, b_{21}+c_{21}>4 \text{ va } \left|b_{12}c_{12}(1-a_{32})\right|<1 \\ & \quad \text{yoki } b_{21}+c_{21}\in(2,4) \text{ va } \left|b_{12}c_{12}(1-a_{32})\right|>1, \end{cases}$$

$$\mathbf{e}_2 \text{ nuqta} \begin{cases} \text{tortuvchi}, & \text{agar } |3-a_{32}|<1 \text{ va } |b_{21}c_{21}|<1, \\ \text{nogiperbolik}, & \text{agar } |3-a_{32}|=1 \text{ yoki } |b_{21}c_{21}|=1, \\ \text{itaruvchi}, & \text{agar } |3-a_{32}|>1 \text{ va } |b_{21}c_{21}|>1, \\ \text{egar}, & \text{agar } |3-a_{32}|>1 \text{ va } |b_{21}c_{21}|<1 \\ & \quad \text{yoki } |3-a_{32}|<1 \text{ va } |b_{21}c_{21}|>1, \end{cases}$$

$$\mathbf{e}_3 \text{ nuqta} \begin{cases} \text{tortuvchi}, & \text{agar } |b_{21}c_{21}|<1, \\ \text{nogiperbolik}, & \text{agar } |b_{21}c_{21}|=1, \\ \text{egar}, & \text{agar } |b_{21}c_{21}|>1, \end{cases}$$

$$\hat{\mathbf{x}} \text{ nuqta} \begin{cases} \text{egar}, & \text{agar } 1+\dfrac{a_{32}-2}{a_{32}-1}>1 \text{ va } |b_{21}c_{21}|<1, \\ \text{nogiperbolik}, & \text{agar } 1+\dfrac{a_{32}-2}{a_{32}-1}>1 \text{ yoki } |b_{21}c_{21}|=1, \\ \text{itaruvchi}, & \text{agar } 1+\dfrac{a_{32}-2}{a_{32}-1}>1 \text{ va } |b_{21}c_{21}|>1. \end{cases}$$

Agar $b_{21}c_{21}<1$, $b_{21}+c_{21}>2$ holdagi (21) SKSOning boshqa qo‘zg‘almas nuqtalari tortuvchi bo‘la olmaydi.

Dinamik sistemalar nazariyasidan quyidagi faktlar kelib chiqadi. S^2 simpleksning $\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3$ uchlari tortuvchi nuqtalar bo‘lganda ularning mos ravishda shunday O_1, O_2 va O_3 ochiq atroflari topiladiki, ixtiyoriy $\mathbf{x}\in O_1$ ($\mathbf{x}\in O_2$, $\mathbf{x}\in O_3$) boshlang‘ich nuqta uchun $\lim\limits_{n\to\infty}W^n(\mathbf{x})=\mathbf{e}_1$ ($\lim\limits_{n\to\infty}W^n(\mathbf{x})=\mathbf{e}_2$, $\lim\limits_{n\to\infty}W^n(\mathbf{x})=\mathbf{e}_3$) bo‘ladi. $\Omega_1=\left\{\mathbf{x}\in S^2:\lim\limits_{n\to\infty}W^n(\mathbf{x})=\mathbf{e}_2\right\}$ to‘plamni belgilab olamiz. 4-teoremaga ko‘ra parametrlarning $|3-a_{32}|>1$, $|b_{21}c_{21}|<1$ yoki $|3-a_{32}|<1$, $|b_{21}c_{21}|>1$ shartlarni qanoatlantiruvchi qiymatlarida S^2 simpleksning $\mathbf{e}_2$ uchi egar tipli qo‘zg‘almas nuqta bo‘ladi. U holda $\mathbf{e}_2$ uchning shunday atrofi va (atrofida yotuvchi qo‘zg‘almas nuqtadan o‘tuvchi) γ egri chiziq mavjudki, $\forall\mathbf{x}\in\gamma$ uchun $\lim\limits_{n\to\infty}W^n(\mathbf{x})=\mathbf{e}_2$ bo‘ladi. Demak, Ω_1 to‘plam S^2 simpleksning bo‘sh bo‘lmagan qism to‘plami bo‘ladi. Xuddi shunday usulda $\Omega_2 = \left\{ \mathbf{x} \in S^2 : \lim_{n\to\infty} W^n(\mathbf{x}) = \hat{\mathbf{x}} \right\}$ to‘plamni belgilab, uni S^2 simpleksning bo‘sh bo‘lmagan qism to‘plami bo‘lishini ko‘rsatiladi. Endi, o‘zaro kesishmaydigan quyidagi to‘plamlarni belgilab olamiz:

$$P_1 = \left\{ (\alpha,\beta) \in \mathbb{R}_+^2 : \alpha = 1,\ 0 \le \beta < 1 \right\},\ P_2 = \left\{ (\alpha,\beta) \in \mathbb{R}_+^2 : \beta = 1,\ 0 \le \alpha < 1 \right\},$$

$$P_3 = \left\{ (\alpha,\beta) \in \mathbb{R}_+^2 : \alpha = 1,\ 1 < \beta \le 3 \right\},\ P_4 = \left\{ (\alpha,\beta) \in \mathbb{R}_+^2 : \beta = 1,\ 1 < \alpha \le 3 \right\}$$

$$P_5 = \left\{ (\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha\beta > 1, \alpha + \beta > 2 \right\},$$

$$P_6 = \left\{ (\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha\beta = 1 \right\},$$

$$P_7 = \left\{ (\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha\beta < 1,\ \alpha + \beta > 2 \right\},$$

$$P_8 = \left\{ (\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha + \beta = 2 \right\},$$

$$P_9 = \left\{ (\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha\beta < 1, \alpha + \beta < 2 \right\}.$$

5-teorema. (21) SKSO uchun quyidagi tasdiqlar o‘rinli:

i) agar $(b_{21}, c_{21}) \in P_1 \cup P_2 \cup P_3 \cup \cdots \cup P_9$ va

i*a*) $a_{32} \in [0,2]$ bo‘lsa, u holda $\lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \begin{cases} \mathbf{e}_3,\ \forall \mathbf{x}^{(0)} \in \Gamma_{\{2,3\}} \setminus \{\mathbf{e}_2\}, \\ \mathbf{e}_2,\ \forall \mathbf{x}^{(0)} \in \Omega_1, \end{cases}$

i*b*) $a_{32} \in (2,3]$ bo‘lsa, u holda barcha $\mathbf{x}^{(0)} \in \Omega_2$ uchun $\lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \hat{\mathbf{x}}$ bo‘ladi,

i*c*) $a_{32} \in (2,3]$ bo‘lsa, u holda $\forall \mathbf{x}^{(0)} \in \Gamma_{\{2,3\}} \setminus \{\mathbf{e}_2, \mathbf{e}_3\}$ uchun

$$\lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \begin{cases} \mathbf{e}_3, \text{ agar } 0 \le x_2^{(0)} < \hat{x}, \\ \hat{\mathbf{x}}, \text{ agar } x_2^{(0)} = \hat{x}, \\ \mathbf{e}_2, \text{ agar } \hat{x} < x_2^{(0)} \le 1; \end{cases}$$

ii) agar $(b_{21}, c_{21}) \in P_1 \cup P_2 \cup P_8 \cup P_9$ va

ii*a*) $a_{32} \in [0,2]$ bo‘lsa, u holda barcha $\mathbf{x}^{(0)} \in S^2 \setminus \left(\mathrm{Fix}(W) \cup \Gamma_{\{2,3\}} \cup \Omega_1 \right)$ uchun $\lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_3$;

ii*b*) $a_{32} \in (2,3]$ bo‘lsa, u holda $\forall \mathbf{x}^{(0)} \in S^2 \setminus \left(\mathrm{Fix}(W) \cup \Gamma_{\{2,3\}} \cup O_2 \cup \Omega_2 \right)$ uchun $\omega_W\left(\mathbf{x}^{(0)}\right) \subset \Gamma_{\{2,3\}}$ bo‘ladi. Undan tashqari, olingan $\mathbf{x}^{(0)}$ uchun $x_2^{(n_0)} < \hat{x}$ o‘rinli bo‘lsa, u holda $\lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_3$;

ii*c*) $a_{32} \in (2,3]$ bo‘lsa, u holda $\forall \mathbf{x}^{(0)} \in S^2 \setminus \left(\mathrm{Fix}(W) \cup \Gamma_{\{2,3\}} \cup O_3 \cup \Omega_2 \right)$ uchun $\omega_W\left(\mathbf{x}^{(0)}\right) \subset \Gamma_{\{2,3\}}$ bo‘ladi. Undan tashqari, olingan $\mathbf{x}^{(0)}$ uchun $x_2^{(n_0)} > \hat{x}$ o‘rinli bo‘lsa, u holda $\lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_2$;

iii) agar $(b_{21},c_{21})\in P_3\cup P_4\cup P_5\cup P_6$ va $a_{32}\in[0,3]$ bo‘lsa, u holda barcha $\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}})$ uchun $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_1$;

iv) agar $(b_{21},c_{21})\in P_7$ va

iv$_a$) $a_{32}\in[0,2]$ bo‘lsa, u holda $0\le x_1^{(0)}<\dfrac{b_{21}c_{21}-1}{(1-b_{21})(1-c_{21})}$ ni qanoatlantiruvchi $\forall\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}}\cup\Omega_1)$ uchun $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_3$;

iv$_b$) $a_{32}\in(2,3]$ bo‘lsa, u holda $0\le x_1^{(0)}<\dfrac{b_{21}c_{21}-1}{(1-b_{21})(1-c_{21})}$ ni qanoatlantiruvchi $\forall\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}}\cup O_3\cup\Omega_2)$ uchun $\omega_W(\mathbf{x}^{(0)})\subset\Gamma_{\{2,3\}}$ bo‘ladi. Undan tashqari, olingan $\mathbf{x}^{(0)}$ uchun $x_2^{(n_0)}<\hat{x}$ o‘rinli bo‘lsa, u holda $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_3$;

iv$_c$) $a_{32}\in(2,3]$ bo‘lsa, u holda $0\le x_1^{(0)}<\dfrac{b_{21}c_{21}-1}{(1-b_{21})(1-c_{21})}$ ni qanoatlantiruvchi $\forall\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}}\cup O_2\cup\Omega_2)$ uchun $\omega_W(\mathbf{x}^{(0)})\subset\Gamma_{\{2,3\}}$ bo‘ladi. Undan tashqari, olingan $\mathbf{x}^{(0)}$ uchun $x_2^{(n_0)}>\hat{x}$ o‘rinli bo‘lsa, $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_2$;

iv$_d$) $a_{32}\in[0,3]$ bo‘lsa, u holda $\dfrac{b_{21}c_{21}-1}{(1-b_{21})(1-c_{21})}<x_1^{(0)}\le 1$ ni qanoatlantiruvchi $\forall\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}})$ uchun $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_1$.

Endi, $b_{21}=1$, $c_{21}=1$ va $b_{21}\ne 1$, $c_{21}\ne 1$, $x_1^{(0)}=x^*$ bo‘lgan holatlar uchun (21) operatorni qaraymiz.

6-teorema. Agar $b_{21}=c_{21}=a_{32}=1$, $b_{12}=c_{12}$ bo‘lsa, u holda barcha $\mathbf{x}^{(0)}\in S^2\setminus\mathrm{Fix}(W)$ nuqtalar uchun quyidagi limit o‘rinli bo‘ladi

$$\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{x}^*(x_1^{(0)})=(x_1^*,x_2^*,x_3^*),$$

bu yerda $x_1^*=x_1^{(0)}$, $x_2^*=\dfrac{1-x_1^{(0)}}{1+\sqrt{1-4(1-b_{12})(1-x_1^{(0)})x_1^{(0)}}}$, $x_3^*=1-x_1^*-x_2^*$.

Dissertatsiyaning “**Separabel kubik stoxastik operatorlarning davriy orbitalari”** nomli uchinchi bobida chekli o‘lchamli simpleksda aniqlangan o‘rin almashtirishga va parametrlarga bog‘liq separabel kubik stoxastik operatorlarning dinamik xossalari o‘rganilgan.

§ 3.1 paragrafda separabel kubik stoxastik operatorlar qaraymiz.

Ma’lumki, $E_{m-1}=\{1,\dots,m-1\}$ to‘plamning berilgan π o‘rin almashtirishni $(m-1)\times(m-1)$ o‘lchamli matritsa shaklida ifodalash mumkin va uni $(m\times m)$ o‘lchamli A matritsagacha quyidagicha kengaytiramiz: $a_{mm}=1$, $a_{mk}=a_{km}=0$,

$k=1,\dots,m-1$ shartlarni qanoatlantiruvchi oxirgi satr va oxirgi ustunni qo‘shamiz. B, C matritsalar esa $(m\times m)$ o‘lchamli bo‘lib quyidagi ko‘rinishda tanlab olamiz

$$B=\begin{pmatrix}1 & 1 & \dots & a\\ 1 & 1 & \dots & a\\ \vdots & \vdots & \ddots & \vdots\\ c & c & \dots & 1\end{pmatrix},\qquad C=\begin{pmatrix}1 & 1 & \dots & b\\ 1 & 1 & \dots & b\\ \vdots & \vdots & \ddots & \vdots\\ d & d & \dots & 1\end{pmatrix}. \qquad (23)$$

U holda berilgan π o‘rin almashtirishi va A,B,C matritsalar bilan hosil qilingan SKSO quyidagicha ko‘rinishda aniqlanadi

$$W:\begin{cases}x'_k=x_{\pi(k)}\left(1+(c-1)x_m\right)\cdot\left(1+(d-1)x_m\right),\ k\in E_{m-1},\\ x'_m=x_m\left(a+(1-a)x_m\right)\cdot\left(b+(1-b)x_m\right),\end{cases} \qquad (24)$$

va uning koeffisiyentlari uchun quyidagi shartlar o‘rinli bo‘ladi

$$a+b+cd=3,\ ab+c+d=3,\ 0\le a,c\le 3,\ 0\le b\le 3-a,\ 0\le d\le 3-c. \qquad (25)$$

Agar $a=1$, $b=1$ yoki $c=1$, $d=1$ bo‘lsa, u holda $a=b=c=d=1$ tenglik kelib chiqadi. Xuddi shunday usulda $c=1$ va $d=1$ holini ham ko‘rsatish mumkin. Demak, $a=1$, $b=1$ yoki $c=1$, $d=1$ $\Rightarrow$ $a=b=c=d=1$. Shu usulda agar $a\neq 1$, $b=1$ yoki $b\neq 1$, $a=1$ bo‘lsa, u holda $c=1$ yoki $d=1$ bo‘ladi. Shuningdek, agar $a\neq 1$, $b\neq 1$ yoki $c\neq 1$, $d\neq 1$ bo‘lsa, u holda $a\neq 1$, $b\neq 1$, $c\neq 1$, $d\neq 1$ bo‘ladi. Ravshanki, agar $a=b=c=d=1$ va $\pi=Id$ bo‘lsa, u holda (24) SKSO birlik operator bo‘ladi.

Agar $a=b=c=d=1$ va $\pi\neq Id$ bo‘lsa, u holda (24) SKSO π o‘rin almashtirishiga mos chiziqli bo‘ladi.

§ 3.2 paragrafda (24) separabel kubik stoxastik operatorning davriy nuqtalari topilgan.

Aytaylik $\pi=\tau_1\cdots\tau_q$ bo‘lsin. Barcha $i,j\in\{1,2,\dots,q\}$ va $\eta>0$ uchun quyidagi to‘plamlarni belgilaymiz

$$M_\eta^{(ij)}=\left\{\mathbf{x}\in S^{m-1}:\prod_{k\in\operatorname{supp}(\tau_i)}x_k=\eta\prod_{k\in\operatorname{supp}(\tau_j)}x_k\right\},\qquad \Omega(\alpha)=\left\{\mathbf{x}\in S^{m-1}:x_m=\alpha\right\},$$

$$X(\alpha)=\left\{\mathbf{x}\in S^{m-1}:x_k=x_l,\forall k,l\in\operatorname{supp}(\tau_i),i=1,\dots,q,x_m=\alpha\right\},$$

bu yerda $\forall\alpha\in[0,1]$.

7-teorema. (24) SKSO uchun quyidagi tasdiqlar o‘rinli:

i) Invariant to‘plamlar:

i$_a$) $M_0=\left\{\mathbf{x}\in S^{m-1}:x_1\cdots x_{m-1}=0\right\}$ va $M_{\tau_i}=\left\{\mathbf{x}\in S^{m-1}:\prod_{k\in\operatorname{supp}(\tau_i)}x_k=0\right\}$, $i=\overline{1,q}$ invariant to‘plamlar;

i$_b$) agar $\operatorname{ord}(\tau_i)=\operatorname{ord}(\tau_j)$, $i\neq j$ va $i,j\in\{1,\dots,q\}$ va $\eta>0$ bo‘lsa, u holda $M_\eta^{(ij)}$ to‘plam W operatorga nisbatan invariant to‘plamdir;

i$_c$) π o'rin almashtirishning har qanday τ_i, $i=1,\ldots,q$ sikli uchun $N(\tau_i)=\{\mathbf{x}\in S^{m-1} x_u = x_v,\ \forall u,v\in \operatorname{supp}(\tau_i)\}$ invariant to'plam;

ii) Qo'zg'almas nuqtalar to'plami:

ii$_a$) Agar $a=1$, $b=1$ $\pi\neq Id$ bo'lsa, u holda $\operatorname{Fix}(W)=\bigcup\limits_{\alpha\in[0,1]} X(\alpha)$;

ii$_b$) Agar $a=1$, $b\neq 1$ yoki $b=1$, $a\neq 1$ bo'lsa, u holda

$$\operatorname{Fix}(W)=\begin{cases}\Gamma_{E_{m-1}}\cup\{\mathbf{e}_m\}, & \text{agar } \pi=Id,\\ X(0)\cup\{\mathbf{e}_m\}, & \text{agar } \pi\neq Id;\end{cases}$$

ii$_c$) Agar $a\neq 1$, $b\neq 1$ va $\pi=Id$ bo'lsa, u holda

$$\operatorname{Fix}(W)=\begin{cases}\Gamma_{E_{m-1}}\cup\Omega(x^*)\cup\{\mathbf{e}_m\}, & \text{agar } ab<1, a+b>2,\\ \Gamma_{E_{m-1}}\cup\{\mathbf{e}_m\}, & \text{aks holda;}\end{cases}$$

ii$_d$) Agar $a\neq 1$, $b\neq 1$ va $\pi\neq Id$ bo'lsa, u holda

$$\operatorname{Fix}(W)=\begin{cases}X(0)\cup X(x^*)\cup\{\mathbf{e}_m\}, & \text{agar } ab<1,\ a+b>2,\\ X(0)\cup\{\mathbf{e}_m\}, & \text{aks holda;}\end{cases}$$

iii) Aytaylik $\pi\neq Id$ bo'lsin. U holda davriy nuqtalar to'plami:

iii$_a$) $$\operatorname{Per}_s(W)=\begin{cases}\Omega(0), & \text{agar } a=1, b\neq 1 \text{ yoki } b=1, a\neq 1\\ & \text{yoki } a\neq 1, b\neq 1,\\ \bigcup\limits_{\alpha\in[0,1)}\Omega(\alpha), & \text{agar } a=1, b=1,\end{cases}$$

bu yerda $s=\operatorname{lcm}(\operatorname{ord}(\tau_1),\ldots,\operatorname{ord}(\tau_q))$;

iii$_b$) barcha $n>s$ uchun $\operatorname{Per}_n(W)=\varnothing$.

§ 3.3 paragrafda (24) separabel kubik stoxastik operator orbitasining limit nuqtalari to'plami tavsiflangan.

Dastlab $x_m^{(0)}<1$ uchun

$$\hat{\mathbf{x}}(\mathbf{x}^{(0)})=\left(\frac{x_1^{(0)}}{1-x_m^{(0)}},\frac{x_2^{(0)}}{1-x_m^{(0)}},\ldots,\frac{x_{m-1}^{(0)}}{1-x_m^{(0)}},0\right),$$

nuqtani belgilaymiz va yuqorida keltirilgan P_i $(i=1,2,\ldots,9)$ to'plamlardan quyidagi teoremada foydalanamiz.

8-teorema. (24) SKSO uchun quyidagi tasdiqlar o'rinli:

i) Agar $(a,b)\in P_3\cup P_4\cup P_5\cup P_6$ bo'lsa, u holda $\forall\pi$ va $\forall\mathbf{x}^{(0)}\in S^{m-1}\setminus\operatorname{Fix}(W)$ uchun $\omega_W(\mathbf{x}^{(0)})=\{\mathbf{e}_m\}$;

ii) Aytaylik $\pi=Id$.

ii$_a$) Agar $a=1$, $b=1$ bo'lsa, u holda $\forall\mathbf{x}^{(0)}\in S^{m-1}$ uchun $\omega_W(\mathbf{x}^{(0)})=\{\mathbf{x}^{(0)}\}$;

ii$_b$) Agar $(a,b)\in P_1\cup P_2$ bo'lsa, u holda $\forall\mathbf{x}^{(0)}\in S^{m-1}\setminus\operatorname{Fix}(W)$ uchun $\omega_W(\mathbf{x}^{(0)})=\{\tilde{\mathbf{x}}\}$ bu yerda $\tilde{\mathbf{x}}\in X(0)$;

ii$_c$) Agar $(a,b)\in P_7$ bo‘lsa, u holda $\forall \mathbf{x}^{(0)} \in S^{m-1} \setminus \mathrm{Fix}(W)$ uchun

$$\omega_W\left(\mathbf{x}^{(0)}\right)=\begin{cases}\left\{\hat{\mathbf{x}}(\mathbf{x}^{(0)})\right\}, & \text{agar} \quad 0\le x_m^{(0)}<\dfrac{ab-1}{(1-a)(1-b)},\\ \{\mathbf{e}_m\}, & \text{agar} \quad \dfrac{ab-1}{(1-a)(1-b)}<x_m^{(0)}\le 1;\end{cases}$$

ii$_d$) Agar $(a,b)\in P_8\cup P_9$ bo‘lsa, u holda $\forall \mathbf{x}^{(0)} \in S^{m-1} \setminus \mathrm{Fix}(W)$ uchun $\omega_W(\mathbf{x}^{(0)})=\left\{\hat{\mathbf{x}}(\mathbf{x}^{(0)})\right\}$;

iii) Aytaylik $\pi\neq Id$.

iii$_a$) Agar $(a,b)\in P_1\cup P_2$ bo‘lsa, u holda $\forall \mathbf{x}^{(0)} \in S^{m-1} \setminus \mathrm{Fix}(W)$ uchun $\mathbf{x}_\xi=\lim\limits_{n\to\infty} W^{ns}\left(\mathbf{x}^{(0)}\right)$ mavjud va

$$\omega_W(\mathbf{x}^{(0)})=\left\{\mathbf{x}_\xi, W\left(\mathbf{x}_\xi\right),\ldots,W^{s-1}\left(\mathbf{x}_\xi\right)\right\};$$

iii$_b$) Agar $(a,b)\in P_7$ bo‘lsa, u holda $\forall \mathbf{x}^{(0)} \in S^{m-1} \setminus \mathrm{Fix}(W)$ uchun

$$\omega_W\left(\mathbf{x}^{(0)}\right)=\begin{cases}\left\{\mathbf{x}_\xi, W\left(\mathbf{x}_\xi\right),\ldots,W^{s-1}\left(\mathbf{x}_\xi\right)\right\}, & \text{agar} \quad 0\le x_m^{(0)}<\dfrac{ab-1}{(1-a)(1-b)},\\ \{\mathbf{e}_m\}, & \text{agar} \quad \dfrac{ab-1}{(1-a)(1-b)}<x_m^{(0)}\le 1,\end{cases}$$

bu yerda $\lim\limits_{n\to\infty} W^{sn}\left(\mathbf{x}^{(0)}\right)=\mathbf{x}_\xi$;

iii$_c$) Agar $(a,b)\in P_8\cup P_9$ bo‘lsa, u holda $\forall \mathbf{x}^{(0)} \in S^{m-1} \setminus \{\mathbf{e}_m\}$ uchun $\lim\limits_{n\to\infty} W^{sn}\left(\mathbf{x}^{(0)}\right)=\mathbf{x}_\mu$ mavjud va $\omega_W(\mathbf{x}^{(0)})=\left\{\mathbf{x}_\mu, W\left(\mathbf{x}_\mu\right),\ldots,W^{s-1}\left(\mathbf{x}_\mu\right)\right\}$;

XULOSA

Dissertatsiya ishida asosan separabel kubik stoxastik operatorlarga mos diskret vaqtli dinamik sistemalar o‘rganilgan.

Birinchi bobda dissertatsiya mavzusiga tegishli bo‘lgan ta’riflar, asosiy tushunchalar va ma’lum natijalar qisqacha keltirib o‘tilgan. Jumladan, Lotka-Volterra operatorlarning aniqlanishi va stoxastik operatorlar, hamda ularning dinamikasiga oid asosiy tushunchalar, shu bilan birga kvadratik stoxastik operatorlarning aniqlanishi, Volterra kvadratik stoxastik operatorlari separabel kvadratik stoxastik operatorlarning kubik stoxastik operatorlarning ta’riflari va ularga oid ma’lum natijalar keltirib o‘tilgan.

Ikkinchi bobda chekli o‘lchamli simpleksda aniqlangan, uchta matritsaga bog‘liq bo‘lgan, separabel KSOlarning dinamikasi o‘rganilgan. Separabel KSOlar uchun aniqlangan chiziqli funksionalning Lyapunov funksiyasi bo‘lishi uchun berilgan uchta matritsalarning elementlariga shartlar topilgan. Shuningdek, separabel KSOlarni Volterra operatorlari bilan ustma-ust tushishi uchun yetarli shartlar topilgan. Bir o‘lchamli simpleksda aniqlangan separabel KSOlar uchun hosil bo‘lishiga misollar keltirildi. Ikki o‘lchamli simpleksda aniqlangan fiksirlangan matritsalarga mos separabel KSOlar uchun ularning qo‘zg‘almas nuqtalari va ularning tegishli tiplarining tavsifi keltirilgan. Ushbu operatorlar uchun ixtiyoriy orbitaning yaqinlashuvchiligi, ya’ni bunday operatorlarning regulyarlik xossasiga ega bo‘lishi ko‘rsatildi.

Uchinchi bobda 4 ta parametrlar va bitta o‘rin almashtirishga mos kelgan separabel kubik stoxastik operatorlar aniq ko‘rinishi keltirildi. Shuningdek, chekli o‘lchamli simpleksda aniqlangan separabel kubik stoxastik operatorning qo‘zg‘almas nuqtalari topilgan. Agar o‘rin almashtirishi ayniy bo‘lsa mos separabel kubik stoxastik operator uchun har qanday orbitasi qo‘zg‘almas nuqtalardan biriga yaqinlashishi ko‘rsatildi. Shu bilan birga agar o‘rin almashtirish ayniydan farqli bo‘lsa separabel kubik stoxastik operator uchun ixtiyoriy orbita davriy orbitaga yaqinlashishi ko‘rsatildi.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIK DEGREES PhD.03/30.06.2020.FM.70.04 KARSHI STATE UNIVERSITY

KARSHI STATE UNIVERSITY

BARATOV BAKHODIR SOYIB UGLI

DYNAMICS OF SEPARABLE CUBIC STOCHASTIC OPERATORS

01.01.01 – Mathematical analysis

ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

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O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI QARSHI DAVLAT UNIVERSITETI

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INTRODUCTION (abstract of PhD dissertation)

Actuality and demand of the theme of the dissertation. Nowadays in the world many scientific and practical researches are reduced to the investigation of nonlinear dynamical systems. Evolutionary operators are important in solving problems in mathematical biology and genetics. In particular, such evolutionary operators appear in the process of mathematical modeling of the interaction between predators and prey (for example, phages and bacteria). In biology, a population is a community of individuals closed with respect to reproduction. The evolution (or dynamics) of populations involves the apparent change of state in subsequent generations as a result of growth or decline and survival. It is possible to study population evolution through the dynamical system of nonlinear operators. Therefore, studying the dynamics of simplex-preserving cubic stochastic operators remains one of the important and urgent tasks.

In recent years, the theory of nonlinear dynamical systems is used as the main tool for solving many practical problems of mathematical biology. In particular, the issue of studying the dynamics of separable cubic stochastic operators raises many theoretical and practical issues due to the fact that these operators are nonlinear. Targeted scientific research in this regard includes description of invariant sets of such operators, finding periodic points and determining their types, as well as descriptions of sets of limit points of trajectories.

In last years, in our country, attention has been paid to the current directions of statistical physics and biology, which have scientific and practical applications of fundamental sciences. In particular, significant results have been achieved on finding the dynamics of simplex-preserving Volterra cubic stochastic operators, which are among the main objects encountered in mathematical biology.

As the main tasks and areas of activity of mathematics, the conduct of scientific research at the level of international standards in the priority areas of functional analysis, mathematical physics, statistical physics dynamical systems are determined[1]. It is important to develop the theory of dynamical systems in order to use scientific results in related fields of science to ensure the implementation of the solution.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan of February 7, 2017, PF–4947, “On the strategy of action for the further development of the Republic of Uzbekistan”, PQ–4387 dated July 9, 2019 “On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovskiy of the Academy of Sciences of the Republic of Uzbekistan”, PQ–4708 of May 7, 2020 “On measures to improve the quality of education and research in the field of mathematics” as well as in other regulations related to basic sciences.

[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 may “On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan” № 292 dated May 17, 2017.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of the Republic of Uzbekistan IV, "Mathematics, Mechanics and Computer Science".

The degree of scrutiny of the problem. The study of mathematical models in physics or biology is reduced to the study of continuous or discrete-time dynamical system of nonlinear operators. Quadratic stochastic operators are a simple class of nonlinear operators. Quadratic stochastic operators were first introduced by S. Bernstein in 1924. S. Ulam posed the problem of investigating the behaviour of the orbits of quadratic stochastic operators.

Among them, in the works of N.N. Ganikhodjaev, U.A. Rozikov, F.M. Mukhamedov, U.U. Jamilov, A.Y. Khamrayev, R.T. Mukhitdinov, some classes of novolterra quadratic stochastic operators were studied. It should be noted that the general theory for the dynamics of non-Volterra quadratic stochastic operators does not exist even for small simplecies dimensional. By R.N. Ganikhodjaev the problem of researching the asymptotic behavior of trajectories for Volterra quadratic stochastic operators using the theories of tournaments and Lyapunov functions was solved. Another class of nonlinear operators consists of the cubic stochastic operator. By U.A. Rozikov and A.Y. Khamrayev the cubic stochastic operators were first defined in 2004. Also, the authors found a sufficient condition for the of the coefficients of the operator so that the cubic operators defined in the finite-dimensional simplex have a unique fixed point. The set of limit points trajectory for a symmetric Volterra cubic stochastic operator defined in a finite-dimensional simplex is fully described. The idea of generalizing the classical Lotka-Volterra equations was put forward by Hoffman et al. This work also proposes a new interpretation of the Lotka-Volterra equations using a power law based on experimental results to more accurately describe the interaction between marine phages and bacteria. The newly developed generalized Lotka-Volterra equations were extensively studied by C. Gavin, A. Pokrovskii, M. Prentice, and V. Sobolev. Note that the Volterra cubic stochastic operators mentioned above can be considered as the discrete analogues of the generalized Lotka-Volterra equations. U.A. Rozikov and A.Y. Khamrayev gave the construction of the non-Volterra cubic stochastic operator. This construction depends on the probability measure given on a fixed finite graph. R.R. Davronov, U.U. Jamilov, and M. Ladra studied non-Volterra cubic stochastic operators, also called conditional cubic stochastic operators. Also shows that all conditional cubic stochastic operators have a unique fixed point and such operators have the property being regular. By F. Mukhamedov, A.F. Embong, and A. Rosli considered cubic stochastic operators and proved that surjectivity of such operators is equivalent to their orthogonal preserving property. Also, this article provides a complete description of orthogonality-preserving cubic stochastic operators (respectively surjective) defined in a two-dimensional simplex. By U.U. Jamilov, A.Y. Khamrayev, and M. Ladra, the convergence of an arbitrary orbit for Volterra cubic stochastic operators depending on a parameter defined on a finite-dimensional simplex was proved.

Later, the theory of Volterra cubic stochastic operators was developed in the works of U.A. Rozikov, F.M. Mukhamedov, U.U. Jamilov, and A.Y. Khamrayev. U.U. Jamilov and A. Reinfelds defined the family of Volterra cubic stochastic operators defined by an additional condition and described the set of limit points of the orbit for these cubic stochastic operators.

By F. Mukhamedov, Ch.H. Pah, and A. Rosli a family of non-ergodic Volterra cubic stochastic operators was studied. F.M. Mukhamedov, A.F. Embong, and A. Rosli proved that for cubic stochastic operators, the surjectivity of operators is equivalent to their orthogonality-preserving property. Also fully describes orthogonality-preserving (or surjective) cubic stochastic operators defined on the two-dimensional simplex. U.U. Jamilov and M. Ladra studied the dynamics of a non-Volterra cubic stochastic operator defined on a finite-dimensional simplex and depending of a permutation and one-parameter. It has been shown that an arbitrary orbit of such cubic operators either converges to a fixed point or to a periodic orbit.

The problems of studying the dynamics of separable quadratic stochastic operators are mentioned in the works of U.A. Rozikov, S. Nazir and A. Zada and others. The problem of studying the dynamics of separable cubic stochastic operators, which is an analogue of separable quadratic stochastic operators, is quite complicated. At the same time, the lack of a general theory of learning such operators increases the potential of the thesis.

The connection of the dissertation research with the scientific research plans of the higher educational institution where the dissertation was completed. The dissertation was carried out within the framework of fundamental projects OT-F-4-03 “Continuous and discrete-time exact dynamical systems, spectra of partially integral operators” (2017-2020) of the research plan of Karshi State University.

The aim of the research work is description of the set of limit points of the trajectory for an arbitrary initial point in discrete-time dynamical systems generated by separable cubic stochastic operators.

Tasks of research:

Description of invariant sets for the separable cubic stochastic operators;

Finding periodic points of the separable cubic stochastic operators and determining their types;

Construction of Lyapunov functions for separable cubic stochastic operators;

To study of the asymptotic behavior of arbitrary orbit of separable cubic stochastic operators.

The research object. Discrete-time dynamical systems by cubic stochastic operators.

The research subject. Separable cubic stochastic operators defined in a finite-dimensional simplex.

Research methods. The methods of mathematical analysis, functional analysis, probabilities and dynamical systems theories were used in the dissertation.

Scientific novelty of the research work consists of the following:

relationships were found between the coefficients of the Lyapunov functions in the linear functional form constructed for separable cubic stochastic operators and the elements of the first three matrices;

necessary and sufficient conditions have been found for a separable cubic stochastic operator to be a Volterra operator;

it was proved that a separable cubic stochastic operator defined on a two-dimensional simplex under certain conditions is regular, that is, all orbits converge to one of the fixed points;

the periodic orbits of a separable cubic stochastic operator, determined by a displacement on a finite-dimensional simplex, are described.

Practical results of the research. in mathematical biology, the results are used in solving mathematical biology problems as follows:

The fixed points of the separable cubic stochastic operators corresponding the equilibrium states of the biology system;

The regularity property of separable cubic stochastic operators makes it possible to forecast the future of the biology system;

Periodic trajectories of separable cubic stochastic operators represent future iterations of finite states of the biology system;

Also, the obtained results and the methods used in the dissertation can be used in teaching as a special course for master's students and basic doctoral students in higher educational institutions.

The reliability of the results of the study. The results have been obtained by using the methods of functional analysis, theory of dynamical systems. The obtained results are mathematically strongly proved.

Scientific and practical significance of the research results. The scientific significance of the research results is explained by the fact that they can be used in the theory of cubic stochastic operators and in the theory of nonlinear discrete-time dynamic systems, as well as in solving problems in biology.

The practical significance of the research is explained by its use in forecasting the features of population evolution in mathematical biology by describing the set of limit points of the trajectory of the dynamical systems genrated by the separable cubic operator.

Implementation of the research results. The results related to separable cubic stochastic operator were used in the following research projects:

the periodic points and the description of orbits of discrete-time dynamical systems generated by separable cubic stochastic operators have been used in the research project "Quantum genetic algebras and their applications" with the reference number G0003447 for investigation the dynamics of cubic stochastic operators (Reference of United Arab Emirates University dated October 17, 2024, UAE). The application of the scientific results made it possible to describe the periodic orbits of cubic stochastic operators that represent the evolution of biological populations;

for the discrete-time dynamical systems generated by separable cubic stochastic operators, the limit points of orbits have been used in the research project "Certime continuous and discrete-time dynamical systems, spectra of partial integral operators", with reference number OT–F-4-03 for study the asymptotical behaviour of orbits of cubic stochastic operators (Reference No. 04/3052 of Karshi state university dated September 26, 2024, Karshi). The application of the scientific result made it possible to describe the set of limit points of the orbit of a nonlinear discrete-time dynamical system.

Approbation of the research results. The main results of the research have been discussed at 3 international and 5 national scientific conferences.

Publications of the research results. On the topic of the dissertation 14 research papers have been published in the scientific journals, 6 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, in addition 1 of them were published in international journals and 5 papers published in national mathematical journals.

The structure and volume of the thesis. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The general volume of the thesis is 86 pages.

THE MAIN CONTENT OF THE THESIS

The **introduction** of the thesis includes the motivation of the research, the relevance of the research to the priorities of science and technology, the review of foreign research on the topic, the degree of scrutiny of the problem, the aim, research problems, object and subject of research, scientific novelty and practical results, theoretical and practical significance of the results obtained, the statement of research results, published works and information on the structure of the thesis.

In the first chapter of the thesis, entitled **"Quadratic and cubic stochastic operators"**, contains the basic definitions and important concepts necessary for a complete coverage of the topic of the dissertation.

The well-known Lotka-Volterra equations play a pivotal role in mathematically modeling various ecological and chemical systems. Recently, a new adaptation of the classical Lotka-Volterra equations has emerged for modeling marine phage-bacteria populations, as introduced in the work by Hoffmann. Building upon this, we revisit these modified Lotka-Volterra equations, as outlined.

Hoffmann proposed a modification of the Lotka-Volterra equations based on experimental data, suggesting the application of a power law to these equations to better describe phage-bacteria interactions. The modified Lotka-Volterra equations are represented as follows:

$$\dot{x} = \alpha x - \beta x y^p, \qquad \dot{y} = -\gamma y^p + \delta x y^p. \tag{1}$$

In these equations, the variables x and y correspond to the populations of bacteria and phage over time, respectively. The constants α, β, γ and δ are positive values governing the intensity of interactions between the two species and within their respective populations. It is essential to note that we always consider the parameter p to be greater than 1, because when $p = 1$, (1) coincides with the classical Lotka-Volterra equations.

In essence, the equations (1) imply that the "effective size" of the phage population differs from its "physical size", with this effective size being directly proportional to the exponent p of the physical size. A key assumption underlying these equations is that, in the context of interactions, one predator engages with one prey in the classical Lotka-Volterra equations with $p=1$, whereas in the modified Lotka-Volterra equations with $p>1$, there is a departure from this one-to-one interaction.

For $p=2$, which provides a strong fit in the modified equations, we can postulate that the significant interaction involves the encounter of two phages with one bacterium. Hoffmann attributes this to the attraction of multiple hosts to the area of phage lysis, a concept supported by laboratory experiments indicating that 2-3 phages indeed infect each bacterium. Consequently, the term y^p in (1) signifies the presence of "hunting-teams" composed of p phages, which are collectively effective in their interactions. This modification of the Lotka-Volterra equations with a power law exponent p provides a better understanding of phage-bacteria interactions, shedding light on the dynamics of these systems beyond the traditional one-to-one predator-prey relationships.

We would like to highlight that the mentioned equations represent a specific instance of a Kolmogorov system governing predator-prey interactions. A discrete-time Kolmogorov system, denoted as $K:\mathbb{R}_+^m \to \mathbb{R}_+^m$, is defined as follows:

$$K(x):=\left(x_1g_1(\mathbf{x}),x_2g_2(\mathbf{x}),\ldots,x_mg_m(\mathbf{x})\right), \qquad \mathbf{x}\in\mathbb{R}_+^m.$$

In this system, each x_i represents the population density of species i, and the function g_i characterizes the growth rate of species i. These growth rates are dependent on the population density vector $\mathbf{x}=(x_1,x_2,\ldots,x_m)$. Depending on the specific properties of the functions g_i, the discrete-time Kolmogorov system can encapsulate various types of species interactions.

Consider a finite set $E=\{1,2,\ldots,m\}$ and define the set of all probability distributions on E as:

$$S^{m-1}=\left\{\mathbf{x}\in\mathbb{R}_+^m : \sum_{i=1}^{m} x_i=1\right\},$$

which is often referred to as the $(m-1)$-dimensional simplex.

Definition 1. A continuous mapping $\Phi:\mathbb{R}_+^m\to\mathbb{R}_+^m$ is called as a *stochastic operator* (SO) if $\Phi:\left(S^{m-1}\right)\subset S^{m-1}$.

Definition 2. A stochastic Kolmogorov system $K:\mathbb{R}_+^m\to\mathbb{R}_+^m$ is called a *Lotka-Volterra operator.*

Now, let $\Phi:S^{m-1}\to S^{m-1}$ be a SO. A point $\mathbf{x}\in S^{m-1}$ is considered a *periodic point* of Φ if there exists an integer n such that $\Phi^n(\mathbf{x})=\mathbf{x}$. The smallest positive integer n that satisfies this condition is known as the *prime period* or *least period* of the point $\mathbf{x}$. A period-one point is termed a *fixed point* of Φ. We denote the set of all fixed points of the operator Φ as $\mathrm{Fix}(\Phi)$, which is defined as:

$$\mathrm{Fix}(\Phi)=\left\{\mathbf{x}\in S^{m-1}:\Phi(\mathbf{x})=\mathbf{x}\right\}.$$

In the types of fixed points defined as follows: let $D\Phi(\mathbf{x}^*) = (\partial\Phi_i / \partial x_j)(\mathbf{x}^*)$ be the Jacobian of Φ at the point $\mathbf{x}^*$. A fixed point $\mathbf{x}^*$ is called hyperbolic if its Jacobian $D\Phi(\mathbf{x}^*)$ has no eigenvalues on the unit circle in $\mathbb{C}$. A hyperbolic fixed point $\mathbf{x}^*$ is called:

(i) *attracting* if all the eigenvalues of the Jacobian $D\Phi(\mathbf{x}^*)$ are inside the unit ball;

(ii) *repelling* if all the eigenvalues of the Jacobian $D\Phi(\mathbf{x}^*)$ are outside the unit ball;

(iii) a *saddle* otherwise.

These definitions and notations serve as the foundation for our subsequent discussions and analyses.

For a given initial point $\mathbf{x}^{(0)} \in S^{m-1}$, the orbit $\{\mathbf{x}^{(n)}\}_{n\geq 0}$ of Φ is defined as follows:

$$\mathbf{x}^{(n+1)} = \Phi(\mathbf{x}^{(n)}) = \Phi^{n+1}(\mathbf{x}^{(0)}), \quad n = 0,1,2,\ldots .$$

We denote by $\omega_\Phi(\mathbf{x}^{(0)})$ the ω-limit set of the orbit $\{\mathbf{x}^{(n)}\}_{n\geq 0}$. Since S^{m-1} is a compact set, and $\{\mathbf{x}^{(n)}\}_{n\geq 0} \subset S^{m-1}$, it follows that $\omega_\Phi(\mathbf{x}^{(0)}) \neq \varnothing$. The primary problem in mathematical biology is to describe the set $\omega_\Phi(\mathbf{x}^{(0)})$ for any initial point $\mathbf{x}^{(0)} \in S^{m-1}$ for a given SO Φ.

A continuous function $\varphi : S^{m-1} \to \mathbb{R}$ is called a Lyapunov function for a SO Φ if the limit $\lim_{n\to\infty} \varphi(\mathbf{x}^{(n)})$ exists for all $\mathbf{x} \in S^{m-1}$.

Now, let's introduce the concept of a "quadratic stochastic operator" (QSO) using the following definition:

Definition 3. A mapping V is called *quadratic stochastic operator* if it has the form

$$V : x'_k = \sum_{i,j\in E} P_{ij,k} x_i x_j, \quad \forall k \in E, \tag{2}$$

and the coefficients $P_{ij,k}$ satisfy

$$P_{ij,k} \geq 0, \ \forall i,j,k \in E \quad \text{and} \quad \sum_{k\in E} P_{ij,k} = 1, \qquad \forall i,j \in E. \tag{3}$$

For a given initial distribution $\mathbf{x}^{(0)} \in S^{m-1}$, we define the orbit $\{\mathbf{x}^{(n)}\}_{n\geq 0}$ by the QSO (1.2) as follows $\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)})$ where $n = 0,1,2,\ldots$

One of the main problems in mathematical biology is to study the asymptotic behavior of the orbits. Note this problem in general case is an open problem for QSOs even in the two-dimensional case. However, it has been subject to more in-depth study in the context of Volterra QSOs.

Definition 4. A QSO is called Volterra if $P_{ij,k} = 0$ for any $k \notin \{i,j\}$, $\forall i,j,k \in E$.

Another class of nonlinear operators is the class of cubic stochastic operators. Now, moving on to cubic stochastic operators:

Definition 5. A SO is called a cubic stochastic operator (CSO) if it has the form:

$$W: x_l' = \sum_{i,j,k\in E} P_{ijk,l} x_i x_j x_k, \qquad \forall l\in E, \tag{4}$$

where $P_{ijk,l}$ are coefficients of heredity such that:

$$P_{ijk,l} \geq 0, \quad \forall i,j,k,l \in E \quad \text{and} \quad \sum_{l\in E} P_{ijk,l} = 1, \quad \forall i,j,k \in E. \tag{5}$$

Furthermore, this problem was specifically explored for a class of Volterra CSOs. A Volterra CSO is defined by (4), (5) and an additional crucial condition:

$$P_{ijk,l} = 0 \qquad \text{if} \qquad l \notin \{i,j,k\}, \quad \forall i,j,k,l \in E.$$

It's important to note that CSOs are not only used in mathematical biology but also have applications in physics, such as modeling systems with ternary interactions in spin systems.

In the second chapter of the dissertation **"Separable cubic stochastic operators"** dedicated to the study of the dynamics of separable cubic stochastic operators.

Let $A = (a_{il})_{i,l=1}^m$ $B = (b_{jl})_{j,l=1}^m$ and $C = (c_{kl})_{k,l=1}^m$, be some matrices with the real entries. Consider a CSO (2.4), (2.5) with additional conditions

$$P_{ijk,l} = a_{il} b_{jl} c_{kl} \qquad \text{for all} \qquad i,j,k,l \in E. \tag{6}$$

where $a_{il}, b_{jl}, c_{kl} \in \mathbb{R}$ entries of matrices A B and C. Then, the CSO W corresponding to the coefficients defined in (6) takes the following form:

$$x_l' = (W(\mathbf{x}))_l = (A(\mathbf{x}))_l \cdot (B(\mathbf{x}))_l \cdot (C(\mathbf{x}))_l, \qquad l \in E \tag{7}$$

where

$$(A(\mathbf{x}))_l = \sum_{i=1}^m a_{il} x_i, \quad (B(\mathbf{x}))_l = \sum_{j=1}^m b_{jl} x_j, \quad (C(\mathbf{x}))_l = \sum_{k=1}^m c_{kl} x_k.$$

Definition 6. The CSO (7) is called a separable cubic stochastic operator (SCSO).

Let $A = (a_{il})$, $B = (b_{jl})$ and $C = (c_{kl})$ be the matrices of the SCSO $W: S^{m-1} \to S^{m-1}$ and $\mathbf{I}$ be the identity matrix of order *m*. Then, we can state the following theorem.

Theorem 1. The function $\varphi_{\mathbf{d}}: S^{m-1} \to \mathbb{R}$ defined as $\varphi_{\mathbf{d}}(\mathbf{x}) = \sum_{l\in E} d_l x_l$ is a Lyapunov function for the SCSO W if $\mathbf{d} = (d_1, \ldots, d_m)$ satisfies $d_i \geq 0$ for all $i \in E$ and either one of the following inequalities $\beta\gamma A\mathbf{d}^T \leq \mathbf{I}\mathbf{d}^T$, $\alpha\gamma B\mathbf{d}^T \leq \mathbf{I}\mathbf{d}^T$, $\alpha\beta C\mathbf{d}^T \leq \mathbf{I}\mathbf{d}^T$ holds, where A, B and C are the matrices of W with non-negative entries and $\alpha = \max_{i,l\in E}\{a_{il}\}$, $\beta = \max_{j,l\in E}\{b_{jl}\}$ and $\gamma = \max_{k,l\in E}\{c_{kl}\}$.

Denote
$D=\left\{\mathbf{d}\in\mathbb{R}^m : d_i\geq 0, d_1+\cdots+d_2>0, \beta\gamma A\mathbf{d}^T\leq I\mathbf{d}^T \text{ or } \alpha\gamma B\mathbf{d}^T\leq I\mathbf{d}^T \text{ or } \alpha\gamma C\mathbf{d}^T\leq I\mathbf{d}^T\right\}$.
Then, we have that $\varphi_{\mathbf{d}}$ is a Lyapunov function for any $\mathbf{d}\in D$. That is for any initial point $\mathbf{x}^{(0)}\in S^{m-1}$ we have $\lim_{n\to\infty}\varphi_{\mathbf{d}}\left(\mathbf{x}^{(n)}\right)=\lambda_{\mathbf{d}}\left(\mathbf{x}^{(0)}\right)$, $\mathbf{d}\in D$. Thus $\omega\left(\mathbf{x}^{(0)}\right)\subset\left\{\mathbf{x}\in S^{m-1}:\varphi_{\mathbf{d}}(\mathbf{x})=\lambda_{\mathbf{d}}\left(\mathbf{x}^{(0)}\right)\right\}$ for any $\mathbf{d}\in D$ which implies

$$\omega\left(\mathbf{x}^{(0)}\right)\subset\bigcap_{\mathbf{d}\in D}\left\{\mathbf{x}\in S^{m-1}:\varphi_{\mathbf{d}}(\mathbf{x})=\lambda_{\mathbf{d}}\left(\mathbf{x}^{(0)}\right)\right\}.$$

In general, the problem of study the behavior of any SCSO is also a difficult problem.

We study the dynamics of SCSOs defined on the two-dimensional simplex under certain conditions on the parameters.

Consider the following matrices:

$$A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix},\ B=\begin{pmatrix}b_{11}&b_{12}&b_{13}\\b_{21}&b_{22}&b_{23}\\b_{31}&b_{32}&b_{33}\end{pmatrix},\ C=\begin{pmatrix}c_{11}&c_{12}&c_{13}\\c_{21}&c_{22}&c_{23}\\c_{31}&c_{32}&c_{33}\end{pmatrix}.$$

Then the corresponding SCSO $W:S^2\to S^2$ can be written as

$$W:\begin{cases}x_1'=\left(a_{11}x_1+a_{21}x_2+a_{31}x_3\right)\left(b_{11}x_1+b_{21}x_2+b_{31}x_3\right)\left(c_{11}x_1+c_{21}x_2+c_{31}x_3\right),\\x_2'=\left(a_{12}x_1+a_{22}x_2+a_{32}x_3\right)\left(b_{12}x_1+b_{22}x_2+b_{32}x_3\right)\left(c_{12}x_1+c_{22}x_2+c_{32}x_3\right),\\x_3'=\left(a_{13}x_1+a_{23}x_2+a_{33}x_3\right)\left(b_{13}x_1+b_{23}x_2+b_{33}x_3\right)\left(c_{13}x_1+c_{23}x_2+c_{33}x_3\right),\end{cases}$$

where the parameters satisfy the conditions

$$a_{11}b_{11}c_{11}+a_{12}b_{12}c_{12}+a_{13}b_{13}c_{13}=1,\ a_{21}b_{21}c_{21}+a_{22}b_{22}c_{22}+a_{23}b_{23}c_{23}=1,$$
$$a_{31}b_{31}c_{31}+a_{32}b_{32}c_{32}+a_{33}b_{33}c_{33}=1,$$
$$a_{21}b_{11}c_{11}+a_{11}b_{21}c_{11}+a_{11}b_{11}c_{21}+a_{12}b_{12}c_{22}+a_{12}b_{22}c_{12}+a_{22}b_{12}c_{12}+a_{13}b_{13}c_{23}+$$
$$a_{13}b_{23}c_{13}+a_{23}b_{13}c_{13}=3,$$
$$a_{31}b_{11}c_{11}+a_{11}b_{31}c_{11}+a_{11}b_{11}c_{31}+a_{13}b_{13}c_{33}+a_{13}b_{33}c_{13}+a_{33}b_{13}c_{13}+a_{12}b_{12}c_{32}+$$
$$a_{12}b_{32}c_{12}+a_{32}b_{12}c_{12}=3,$$
$$a_{12}b_{22}c_{22}+a_{22}b_{12}c_{22}+a_{22}b_{22}c_{12}+a_{21}b_{21}c_{11}+a_{21}b_{11}c_{21}+a_{11}b_{21}c_{21}+a_{23}b_{23}c_{13}+$$
$$a_{23}b_{13}c_{23}+a_{13}b_{23}c_{23}=3,$$
$$a_{13}b_{33}c_{33}+a_{33}b_{13}c_{33}+a_{33}b_{33}c_{13}+a_{31}b_{31}c_{11}+a_{31}b_{11}c_{31}+a_{11}b_{31}c_{31}+a_{32}b_{32}c_{12}+$$
$$a_{32}b_{12}c_{32}+a_{12}b_{32}c_{32}=3,$$
$$a_{32}b_{22}c_{22}+a_{22}b_{32}c_{22}+a_{22}b_{22}c_{32}+a_{23}b_{23}c_{33}+a_{23}b_{33}c_{23}+a_{33}b_{23}c_{23}+a_{21}b_{21}c_{31}+$$
$$a_{21}b_{31}c_{21}+a_{31}b_{21}c_{21}=3,$$
$$a_{23}b_{33}c_{33}+a_{33}b_{23}c_{33}+a_{33}b_{33}c_{23}+a_{32}b_{32}c_{22}+a_{32}b_{22}c_{32}+a_{22}b_{32}c_{32}+a_{21}b_{31}c_{31}+$$
$$a_{31}b_{21}c_{31}+a_{31}b_{31}c_{21}=3,$$
$$a_{21}b_{11}c_{31}+a_{11}b_{21}c_{31}+a_{11}b_{31}c_{21}+a_{31}b_{11}c_{21}+a_{21}b_{31}c_{11}+a_{31}b_{21}c_{11}+a_{12}b_{22}c_{32}+$$
$$a_{22}b_{12}c_{32}+a_{12}b_{32}c_{22}+a_{32}b_{12}c_{22}+a_{22}b_{32}c_{12}+a_{32}b_{22}c_{12}+a_{13}b_{23}+a_{23}b_{13}c_{33}+$$

$$a_{13}b_{33}c_{23}+a_{33}b_{13}c_{23}+a_{23}b_{33}c_{13}+a_{33}b_{23}c_{13}=6. \tag{8}$$

Let $a_{ii}=b_{ii}=c_{ii}=1$ (i=1,2,3), $a_{12}=b_{13}=a_{21}=c_{23}=a_{31}=b_{32}=c_{32}=0$, $b_{21}=b_{31}$ and $c_{21}=c_{31}$. Then the corresponding SCSO $W:S^2\to S^2$ has the form

$$W:\begin{cases} x_1'=x_1(x_1+b_{21}x_2+b_{21}x_3)(x_1+c_{21}x_2+c_{21}x_3), \\ x_2'=(x_2+a_{32}x_3)(b_{12}x_1+x_2)(c_{12}x_1+x_2), \\ x_3'=(a_{13}x_1+a_{23}x_2+x_3)(b_{23}x_2+x_3)(c_{13}x_1+x_3), \end{cases} \tag{9}$$

where the parameters satisfy the conditions

$$\begin{aligned} &b_{21}+c_{21}+b_{12}c_{12}+a_{13}b_{23}c_{13}=3,\ b_{21}+c_{21}+a_{13}c_{13}+a_{32}b_{12}c_{12}=3, \\ &b_{12}+c_{12}+b_{21}c_{21}+a_{23}b_{23}c_{13}=3,\ a_{13}+c_{13}+b_{21}c_{21}=3, \\ &a_{32}+a_{23}b_{23}=3,\ a_{23}+b_{23}=3, \\ &2b_{21}c_{21}+a_{32}b_{12}+a_{32}c_{12}+a_{13}b_{23}+a_{23}c_{13}+b_{23}c_{13}=6. \end{aligned} \tag{10}$$

Note that a family of SCSOs (9) with conditions (10) is non-empty and the set of solutions of the system of equations (10) has infinitely many solutions.

Theorem 2. For the SCSO W (9), the following assertions hold:

i) The face $\Gamma_{\{2,3\}}$ of the simplex S^2 is an invariant set;

ii) $\mathrm{Fix}(W)=\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}\cup\{\hat{\mathbf{x}},\text{ if } 2<a_{32}\le 3\}$, where $\hat{\mathbf{x}}=\left(0,\frac{1}{a_{32}-1},\frac{a_{32}-2}{a_{32}-1}\right)$.

If $b_{21}c_{21}<1$, $b_{21}+c_{21}>2$ then $|\mathrm{Fix}(W)|>3$;

iii)

$$\mathbf{e}_1 \text{ is } \begin{cases} \text{attracting}, & \text{if } b_{21}+c_{21}\in(2,4),\ |b_{12}c_{12}(1-a_{32})|<1, \\ \text{non-hyperbolic}, & \text{if } b_{21}+c_{21}\in\{2,4\} \text{ or } |b_{12}c_{12}(1-a_{32})|=1, \\ \text{repelling}, & \text{if } b_{21}+c_{21}<2, b_{21}+c_{21}>4 \text{ and } |b_{12}c_{12}(1-a_{32})|>1, \\ \text{saddle}, & \text{if } b_{21}+c_{21}<2, b_{21}+c_{21}>4 \text{ and } |b_{12}c_{12}(1-a_{32})|<1 \\ & \quad \text{or } b_{21}+c_{21}\in(2,4) \text{ and } |b_{12}c_{12}(1-a_{32})|>1, \end{cases}$$

$$\mathbf{e}_2 \text{ is } \begin{cases} \text{attracting}, & \text{if } a_{32}\in(2,3] \text{ and } |b_{21}c_{21}|<1, \\ \text{non-hyperbolic}, & \text{if } a_{32}=2 \text{ or } |b_{21}c_{21}|=1, \\ \text{repelling}, & \text{if } a_{32}\in[0,2) \text{ and } |b_{21}c_{21}|>1, \\ \text{saddle}, & \text{if } a_{32}\in[0,2) \text{ and } |b_{21}c_{21}|<1 \\ & \quad \text{or } a_{32}\in(2,3] \text{ and } |b_{21}c_{21}|>1, \end{cases}$$

$$\mathbf{e}_3 \text{ is } \begin{cases} \text{attracting}, & \text{if } |b_{21}c_{21}|<1, \\ \text{non-hyperbolic}, & \text{if } |b_{21}c_{21}|=1, \\ \text{saddle}, & \text{if } |b_{21}c_{21}|>1, \end{cases}$$

$$\hat{\mathbf{x}} \text{ is } \begin{cases} \text{saddle}, & \text{if } a_{32} \in (2,3] \text{ and } |b_{21}c_{21}| < 1, \\ \text{non-hyperbolic}, & \text{if } a_{32} \in (2,3] \text{ and } |b_{21}c_{21}| = 1, \\ \text{repelling}, & \text{if } a_{32} \in (2,3] \text{ and } |b_{21}c_{21}| > 1. \end{cases}$$

In the case $b_{21}c_{21} < 1$, $b_{21} + c_{21} > 2$ the other fixed points are non-attractive.

The following facts follow from the theory of dynamical systems: if the vertex $\mathbf{e}_1$ (resp. $\mathbf{e}_2$, $\mathbf{e}_3$) of the simplex is an attracting point, there is an open neighborhood of O_1 (resp. O_2, O_3) such that for an arbitrary point $\mathbf{x}^{(0)} \in O_1$ (resp. $\mathbf{x}^{(0)} \in O_2$, $\mathbf{x}^{(0)} \in O_3$) we have $\lim_{n\to\infty} W^n(\mathbf{x}^{(0)}) = \mathbf{e}_1$ (resp. $\lim_{n\to\infty} W^n(\mathbf{x}^{(0)}) = \mathbf{e}_2$, $\lim_{n\to\infty} W^n(\mathbf{x}^{(0)}) = \mathbf{e}_3$). Denote $\Omega_1 = \left\{\mathbf{x}^{(0)} \in S^2 : \lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \mathbf{e}_2\right\}$. Due to Theorem 2 for the parameters satisfying the conditions $|3 - a_{32}| > 1$, $|b_{21}c_{21}| < 1$ or $|3 - a_{32}| < 1$, $|b_{21}c_{21}| > 1$ the vertex $\mathbf{e}_2$ of the simplex S^2 is a saddle-type fixed point. Then there exists a curve γ (passing through the fixed point $\mathbf{e}_2$) such that for $\forall \mathbf{x}^{(0)} \in \gamma$ we have $\lim_{n\to\infty} W^n(\mathbf{x}^{(0)}) = \mathbf{e}_2$. Hence the set Ω_1 is a non-empty subset of the simplex S^2. Similarly, we can show that $\Omega_2 = \left\{\mathbf{x}^{(0)} \in S^2 : \lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \hat{\mathbf{x}}\right\}$ is a non-empty subset of the simplex S^2.

Now, we define the following disjoint sets:

$$P_1 = \left\{(\alpha,\beta) \in \mathbb{R}_+^2 : \alpha = 1,\ 0 \le \beta < 1\right\},\ P_2 = \left\{(\alpha,\beta) \in \mathbb{R}_+^2 : \beta = 1,\ 0 \le \alpha < 1\right\},$$
$$P_3 = \left\{(\alpha,\beta) \in \mathbb{R}_+^2 : \alpha = 1,\ 1 < \beta \le 3\right\},\ P_4 = \left\{(\alpha,\beta) \in \mathbb{R}_+^2 : \beta = 1,\ 1 < \alpha \le 3\right\},$$
$$P_5 = \left\{(\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha\beta > 1, \alpha + \beta > 2\right\},$$
$$P_6 = \left\{(\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha\beta = 1\right\},$$
$$P_7 = \left\{(\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha\beta < 1,\ \alpha + \beta > 2\right\},$$
$$P_8 = \left\{(\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha + \beta = 2\right\},$$
$$P_9 = \left\{(\alpha,\beta) \in \mathbb{R}_+^2 : \alpha \ne 1, \beta \ne 1,\ \alpha\beta < 1, \alpha + \beta < 2\right\}.$$

Theorem 3. For the SCSO W (9) the following assertions hold:

i) if $(b_{21}, c_{21}) \in P_1 \cup P_2 \cup P_3 \cup \cdots \cup P_9$ and

i$_a$) $a_{32} \in [0,2]$ then $\lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \begin{cases} \mathbf{e}_3, & \forall \mathbf{x}^{(0)} \in \Gamma_{\{2,3\}} \setminus \{\mathbf{e}_2\}, \\ \mathbf{e}_2, & \forall \mathbf{x}^{(0)} \in \Omega_1, \end{cases}$

i$_b$) $a_{32} \in (2,3]$ then $\lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \hat{\mathbf{x}}$ for all $\mathbf{x}^{(0)} \in \Omega_2$,

i$_c$) $a_{32} \in (2,3]$ then $\lim_{n\to\infty} W^n\left(\mathbf{x}^{(0)}\right) = \begin{cases} \mathbf{e}_3, & \text{if } 0 \le x_2^{(0)} < \hat{x}, \\ \hat{\mathbf{x}}, & \text{if } x_2^{(0)} = \hat{x}, \\ \mathbf{e}_2, & \text{if } \hat{x} < x_2^{(0)} \le 1, \end{cases}$

for all $\mathbf{x}^{(0)} \in \Gamma_{\{2,3\}} \setminus \{\mathbf{e}_2, \mathbf{e}_3\}$;

ii) if $(b_{21},c_{21})\in P_1\cup P_2\cup P_8\cup P_9$ and

ii$_a$) $a_{32}\in[0,2]$ then $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_3$ for all $\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}}\cup\Omega_1)$;

ii$_b$) $a_{32}\in(2,3]$ then $\omega_W(\mathbf{x}^{(0)})\subset\Gamma_{\{2,3\}}$ for all $\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}}\cup O_2\cup\Omega_2)$. Also for any $\mathbf{x}^{(0)}$ with $x_2^{(n_0)}<\hat{x}$, we have $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_3$;

ii$_c$) $a_{32}\in(2,3]$ then $\omega_W(\mathbf{x}^{(0)})\subset\Gamma_{\{2,3\}}$ for all $\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}}\cup O_3\cup\Omega_2)$. Also for any $\mathbf{x}^{(0)}$ with $x_2^{(n_0)}>\hat{x}$, we have $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_2$;

iii) if $(b_{21},c_{21})\in P_3\cup P_4\cup P_5\cup P_6$ and $a_{32}\in[0,3]$ then $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_1$ for all $\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}})$;

iv) if $(b_{21},c_{21})\in P_7$ and

iv$_a$) $a_{32}\in[0,2]$ then $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_3$ for all $\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}}\cup\Omega_1)$ with $0\le x_1^{(0)}<\dfrac{b_{21}c_{21}-1}{(1-b_{21})(1-c_{21})}$;

iv$_b$) $a_{32}\in(2,3]$ then $\omega_W(\mathbf{x}^{(0)})\subset\Gamma_{\{2,3\}}$ for all $\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}}\cup O_3\cup\Omega_2)$ with $0\le x_1^{(0)}<\dfrac{b_{21}c_{21}-1}{(1-b_{21})(1-c_{21})}$. Also, for all $\mathbf{x}^{(0)}$ with $x_2^{(n_0)}<\hat{x}$, we have $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_3$;

iv$_c$) $a_{32}\in(2,3]$ then $\omega_W(\mathbf{x}^{(0)})\subset\Gamma_{\{2,3\}}$ for all $\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}}\cup O_2\cup\Omega_2)$ with $0\le x_1^{(0)}<\dfrac{b_{21}c_{21}-1}{(1-b_{21})(1-c_{21})}$. Also, for any $\mathbf{x}^{(0)}$ with $x_2^{(n_0)}>\hat{x}$, we have $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_2$;

iv$_d$) $a_{32}\in[0,3]$ then $\lim_{n\to\infty}W^n(\mathbf{x}^{(0)})=\mathbf{e}_1$ for all $\mathbf{x}^{(0)}\in S^2\setminus(\mathrm{Fix}(W)\cup\Gamma_{\{2,3\}})$ with $\dfrac{b_{21}c_{21}-1}{(1-b_{21})(1-c_{21})}<x_1^{(0)}\le 1$;

In the third chapter of the dissertation **"Periodic orbits of separable cubic stochastic operators"** deals to with dynamical properties of separable cubic stochastic operators depending on permutation and parameters defined in a finite-dimensional simplex.

It is known that a permutation π of E_m is a one function from E_m onto E_m. The set of all permutations on the set E_m is a group S_m. A permutation π of the set $E_m=\{1,2,\dots,m\}$ is a k*-cycle* if there exists a positive integer k and an integer $i\in E_m$ such that

(1) k is the smallest positive integer such that $\pi^k(i)=i$ and

(2) π fixes each $j\in E_m\setminus\{i,\pi(i),\dots,\pi^{k-1}(i)\}$.

The $k-cycle$ π is usually denoted $(i,\pi(i),\dots,\pi^{k-1}(i))$. The set $\operatorname{supp}(\pi)=\{i\in E_m:\pi(i)\neq i\}$ denote the support of π and we let $\operatorname{supp}(k)$ denote the support of the k-cycle, $\operatorname{supp}(k)=\{i,\pi(i),\dots,\pi^{k-1}(i)\}$. It is known that any permutation can be represented in the form of a product of disjoint cycles and this representation is unique to within the order of the factors.

Also known that any permutation π of the set $E_{m-1}=\{1,\dots,m-1\}$ is can be expressed in the form of a $(m-1)\times(m-1)$ matrix and we expand it to the A $(m\times m)$ matrix as follows: we add the last row and the last column defined as $a_{mm}=1$, $a_{mk}=a_{km}=0$, $k\in E_{m-1}$. We choose the $(m\times m)$ matrices B and C as follows

$$B=\begin{pmatrix}1 & 1 & \dots & a\\ 1 & 1 & \dots & a\\ \vdots & \vdots & \ddots & \vdots\\ c & c & \dots & 1\end{pmatrix},\qquad C=\begin{pmatrix}1 & 1 & \dots & b\\ 1 & 1 & \dots & b\\ \vdots & \vdots & \ddots & \vdots\\ d & d & \dots & 1\end{pmatrix}.$$

Then the corresponding SCSO $W:S^{m-1}\to S^{m-1}$ can be written as

$$W:\begin{cases}x'_k=x_{\pi(k)}\left(1+(c-1)x_m\right)\cdot\left(1+(d-1)x_m\right),\ k\in E_{m-1},\\ x'_m=x_m\left(a+(1-a)x_m\right)\cdot\left(b+(1-b)x_m\right),\end{cases}\tag{11}$$

where π is a permutation of the set E_{m-1} and

$$a+b+cd=3,\ ab+c+d=3,\ 0\le a,c\le 3,\ 0\le b\le 3-a,\ 0\le d\le 3-c.\tag{12}$$

First, we need some auxiliary results. We claim that if either $a=1$, $b=1$ or $c=1$, $d=1$ then it follows $a=b=c=d=1$. Indeed, if $a=1$ and $b=1$ then from the condition (12) we obtain

$$cd=1,\ c+d=2\ \Rightarrow\ c^2-2c=-1\ \Rightarrow\ c=1\ \Rightarrow\ d=1.$$

Therefore if $a=1$ and $b=1$ then we have $a=b=c=d=1$. The case $d=1$ and $c=1$ can be considered in a similar manner.

Similarly can be shown that if either $a\neq 1$, $b=1$ or $b\neq 1$, $a=1$ then it follows that $c=1$ or $d=1$. If either $a\neq 1$, $b\neq 1$ or $c\neq 1$, $d\neq 1$ then it follows that $a\neq 1$, $b\neq 1$ and $c\neq 1$, $d\neq 1$.

Evidently, if $a=b=c=d=1$ and $\pi=Id$ then the SCSO W (11) is the identity map.

Let $\pi=\tau_1\cdots\tau_q$. For any different $i,j\in\{1,2,\dots,q\}$ and $\eta>0$ we denote

$$M_\eta^{(ij)}=\left\{\mathbf{x}\in S^{m-1}:\prod_{k\in\operatorname{supp}(\tau_i)}x_k=\eta\prod_{k\in\operatorname{supp}(\tau_j)}x_k\right\},\qquad \Omega(\alpha)=\left\{\mathbf{x}\in S^{m-1}:x_m=\alpha\right\},$$

$$X(\alpha)=\left\{\mathbf{x}\in S^{m-1}:x_k=x_l,\forall k,l\in\operatorname{supp}(\tau_i),i=1,\dots,q,x_m=\alpha\right\},$$

where $\forall\alpha\in[0,1]$.

Theorem 4. For the SCSO W (11) the following statements are hold:

i) The invariant sets have form:

i$_a$) The sets $M_0 = \{\mathbf{x} \in S^{m-1} : x_1 \cdots x_{m-1} = 0\}$ and

$M_{\tau_i} = \left\{ \mathbf{x} \in S^{m-1} : \prod_{k \in \operatorname{supp}(\tau_i)} x_k = 0 \right\}$, $i = \overline{1,q}$ are invariant sets;

i$_b$) The set $M_\eta^{(ij)}$ is an invariant set under W for any $\eta > 0$, when $\operatorname{ord}(\tau_i) = \operatorname{ord}(\tau_j)$, $i \neq j$ and $i, j \in \{1, \dots, q\}$;

i$_c$) The set $N(\tau_i) = \{\mathbf{x} \in S^{m-1} x_u = x_v, \ \forall u, v \in \operatorname{supp}(\tau_i)\}$ is an invariant set for any cycle τ_i, $i = 1, \dots, q$ of π;

ii) The set of fixed points has form:

ii$_a$) If $a = 1$, $b = 1$ $\pi \neq Id$ then we have $\operatorname{Fix}(W) = \bigcup_{\alpha \in [0,1]} X(\alpha)$;

ii$_b$) If $a = 1$, $b \neq 1$ or $b = 1$, $a \neq 1$ then we have

$$\operatorname{Fix}(W) = \begin{cases} \Gamma_{E_{m-1}} \cup \{\mathbf{e}_m\}, & \text{if } \pi = Id, \\ X(0) \cup \{\mathbf{e}_m\}, & \text{if } \pi \neq Id; \end{cases}$$

ii$_c$) If $a \neq 1$, $b \neq 1$ and $\pi = Id$ then we have

$$\operatorname{Fix}(W) = \begin{cases} \Gamma_{E_{m-1}} \cup \Omega(x^*) \cup \{\mathbf{e}_m\}, & \text{if } ab < 1, a + b > 2, \\ \Gamma_{E_{m-1}} \cup \{\mathbf{e}_m\}, & \text{otherwise}; \end{cases}$$

ii$_d$) If $a \neq 1$, $b \neq 1$ and $\pi \neq Id$, then

$$\operatorname{Fix}(W) = \begin{cases} X(0) \cup X(x^*) \cup \{\mathbf{e}_m\}, & \text{if } ab < 1, a + b > 2, \\ X(0) \cup \{\mathbf{e}_m\}, & \text{otherwise}; \end{cases}$$

iii) Let $\pi \neq Id$. Then the set of periodic points has form

$$\text{iii}_a)\ \operatorname{Per}_s(W) = \begin{cases} \Omega(0), & \text{if } a = 1, b \neq 1 \text{ or } b = 1, a \neq 1, \\ & \quad \text{or } a \neq 1, b \neq 1, \\ \bigcup_{\alpha \in [0,1)} \Omega(\alpha), & \text{if } a = 1, b = 1, \end{cases}$$

where $s = \operatorname{lcm}(\operatorname{ord}(\tau_1), \dots, \operatorname{ord}(\tau_q))$;

iii$_b$) $\operatorname{Per}_n(W) = \varnothing$ for $n > s$.

For $x_m^{(0)} < 1$ we denote

$$\hat{\mathbf{x}}\left(\mathbf{x}^{(0)}\right) = \left(\frac{x_1^{(0)}}{1 - x_m^{(0)}}, \frac{x_2^{(0)}}{1 - x_m^{(0)}}, \dots, \frac{x_{m-1}^{(0)}}{1 - x_m^{(0)}}, 0 \right).$$

Also, we use the sets P_i ($i = 1, 2, \dots, 9$) given above on page 41.

In the following theorem, we present the dynamical properties of the separable cubic stochastic operator (11).

Theorem 5. For the SCSO W (11) the following statements are hold:

i) If $(a,b)\in P_3\cup P_4\cup P_5\cup P_6$ then for $\forall\pi$ and any $\mathbf{x}^{(0)}\in S^{m-1}\setminus\mathrm{Fix}(W)$ we have $\omega_W(\mathbf{x}^{(0)})=\{\mathbf{e}_m\}$;

ii) Let $\pi = Id$.

ii$_a$) If $a=1$, $b=1$ then for any $\mathbf{x}^{(0)}\in S^{m-1}$ we have $\omega_W(\mathbf{x}^{(0)})=\{\mathbf{x}^{(0)}\}$;

ii$_b$) If $(a,b)\in P_1\cup P_2$ then for any $\mathbf{x}^{(0)}\in S^{m-1}\setminus\mathrm{Fix}(W)$ we have $\omega_W(\mathbf{x}^{(0)})=\{\tilde{\mathbf{x}}\}$ where $\tilde{\mathbf{x}}\in X(0)$;

ii$_c$) If $(a,b)\in P_7$ then for all $\mathbf{x}^{(0)}\in S^{m-1}\setminus\mathrm{Fix}(W)$ we have

$$\omega_W\left(\mathbf{x}^{(0)}\right)=\begin{cases}\left\{\hat{\mathbf{x}}(\mathbf{x}^{(0)})\right\}, & \text{if } \quad 0\le x_m^{(0)}<\dfrac{ab-1}{(1-a)(1-b)},\\ \{\mathbf{e}_m\}, & \text{if } \quad \dfrac{ab-1}{(1-a)(1-b)}<x_m^{(0)}\le 1;\end{cases}$$

ii$_d$) If $(a,b)\in P_8\cup P_9$ then for any $\mathbf{x}^{(0)}\in S^{m-1}\setminus\mathrm{Fix}(W)$ we have $\omega_W(\mathbf{x}^{(0)})=\left\{\hat{\mathbf{x}}(\mathbf{x}^{(0)})\right\}$;

iii) Let $\pi\ne Id$.

iii$_a$) If $(a,b)\in P_1\cup P_2$ then for any $\mathbf{x}^{(0)}\in S^{m-1}\setminus\mathrm{Fix}(W)$ there is $\mathbf{x}_\xi=\lim\limits_{n\to\infty}W^{ns}\left(\mathbf{x}^{(0)}\right)$ and we have

$$\omega_W(\mathbf{x}^{(0)})=\left\{\mathbf{x}_\xi, W\left(\mathbf{x}_\xi\right),\dots,W^{s-1}\left(\mathbf{x}_\xi\right)\right\};$$

iii$_b$) If $(a,b)\in P_7$ then for all $\mathbf{x}^{(0)}\in S^{m-1}\setminus\mathrm{Fix}(W)$ we have

$$\omega_W\left(\mathbf{x}^{(0)}\right)=\begin{cases}\left\{\mathbf{x}_\xi, W\left(\mathbf{x}_\xi\right),\dots,W^{s-1}\left(\mathbf{x}_\xi\right)\right\}, & \text{if } \quad 0\le x_m^{(0)}<\dfrac{ab-1}{(1-a)(1-b)},\\ \{\mathbf{e}_m\}, & \text{if } \quad \dfrac{ab-1}{(1-a)(1-b)}<x_m^{(0)}\le 1,\end{cases}$$

where $\lim\limits_{n\to\infty}W^{sn}\left(\mathbf{x}^{(0)}\right)=\mathbf{x}_\xi$;

iii$_c$) If $(a,b)\in P_8\cup P_9$ then for any $\mathbf{x}^{(0)}\in S^{m-1}\setminus\{\mathbf{e}_m\}$ there is $\lim\limits_{n\to\infty}W^{sn}\left(\mathbf{x}^{(0)}\right)=\mathbf{x}_\mu$ and we have $\omega_W(\mathbf{x}^{(0)})=\left\{\mathbf{x}_\mu, W\left(\mathbf{x}_\mu\right),\dots,W^{s-1}\left(\mathbf{x}_\mu\right)\right\}$;

CONCLUSION

The thesis is devoted to investigation of the dynamical systems generated by separable cubic stochastic operator.

In the first chapter, the definitions, views and certain pictures related to the topic of the dissertation are briefly mentioned. Including the definition of Lotka-Volterra operators and stochastic operators, as well as overviews of file dynamics, including the definition of quadratic stochastic operators, definitions of Volterra quadratic stochastic operators, separable quadratic stochastic operators, cubic stochastic operators, and some reports on them.

In the second chapter, the dynamics of separable CSOs defined in a finite-dimensional simplex, depending on three matrices, is studied. Conditions for the elements of the given three matrices are found for the linear functional defined for separable CSOs to be a Lyapunov function. Also, sufficient conditions were found for overlapping of separable CSOs with Volterra operators. Examples of formation of regular and ergodic trajectories for separable CSOs defined on the one-dimensional simplex are given. For separable CSOs corresponding to fixed matrices defined on the two-dimensional simplex, a description of their fixed points and their corresponding types is given. Convergence of an arbitrary orbit for these operators, that is, such operators have the regularity property, was shown.

In the third chapter, the four parameters and the separable cubic stochastic operators corresponding to permutation were presented. Also, the fixed points of this separable cubic stochastic operator defined in a finite-dimensional simplex are found and their types are determined. It was shown that any orbit of a suitable separable cubic stochastic operator converges to one of the fixed points if permutation is the identity permutation. At the same time, it was shown that any trajectory of a separable cubic stochastic operator approaches a periodic orbit if permutation is a non-identity permutation.

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КАРШИНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

БАРАТОВ БАХОДИР СОЙИБ УГЛИ

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является описание множества предельных точек орбиты для произвольной начальной точки в дискретных динамических системах, порожденных сепарабельными кубическими стохастическими операторами.

Объекты исследования: дискретные динамические системы, порожденные сепарабельными кубическими стохастическим операторами.

Научная новизна исследования заключается в следующем:

Найдена связь между коэффициентами линейных функций Ляпунова для сепарабельных кубических стохастических операторов и элементами исходных трех матриц;

получены необходимые и достаточные условия для того, чтобы сепарабельный кубический стохастический оператор являлся оператором Вольтерра;

доказано, что сепарабельный кубический стохастический оператор, определенный на двумерном симплексе, при определенных условиях на параметры, является регулярным, то есть все орбиты сходятся к одной из неподвижных точек;

описаны периодические орбиты сепарабельного кубического стохастического оператора, зависящего от перестановки и определенного на конечномерном симплексе;

Внедрение результатов исследования. Результаты, связанные с динамикой сепарабельных кубических стохастических операторов, были использованы в следующих исследовательских проектах:

периодические точки и описание орбит дискретных динамических систем, порожденных сепарабельными кубическими стохастическими операторами были использованы в исследовательском проекте «Квантовые генетические алгебры и их приложения» с номером G0003447 для анализа динамики нелинейных стохастических операторов (Справка от Университета Объединенных Арабских Эмиратов, 17 октябрь 2024 г., ОАЭ). Применение научного результата позволило описать периодические орбиты кубических стохастических операторов, представляющих эволюцию биологических популяций;

для дискретной динамической системы, порожденной сепарабельными кубическими стохастическими операторами, предельные точки орбит были использованы в исследовательском проекте «Непрерывные и дискретные точные динамические системы, спектры частичных интегральных операторов» с номером OT–F-4-03 для исследования орбит кубических стохастических операторов (Справка от Каршинского государственного университета от 26 сентября 2024 года № 04/3052). Применение научного результата позволило описать множество предельных точек орбит нелинейной дискретной динамической системы.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав, заключения и списка использованной литературы. Объем диссертации составляет 86 страниц.

E'LON QILINGAN ISHLAR RO'YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo'lim (part I; часть I)

1. Baratov B.S., Jamilov U.U. On separable cubic stochastic operators // *Qual. Theory Dyn. Syst,* 2024, Vol. 23, Issue 2, a.n. 93. – 28 p. (3. Scopus, IF=0.41).
2. Baratov B.S. The dynamics of a separable cubic operator // *Uz. Math. Jour.* 2024, Vol 68, Issue 2. – Pp. 27-41. (01.00.00. № 6)
3. Baratov B.S. On dynamics of a separable cubic stochastic operator // *Bull. Inst. Math.* 2023, Vol. **6**, Issue 1, – Pp. 17-25. (01.00.00. № 17)
4. Baratov B.S., Jamilov U.U. Periodicity of the discrete dynamical systems of separable cubic operators // *Reports of the Academy of Sciences of the Republic of Uzbekistan*, 2024 № 3. – Pp. 3-8. (01.00.00. № 7)
5. Eshkabilov Y.Kh., Baratov B.S. On the dynamics of a separable cubic stochastic operator on the two-dimensional simplex // *Bull. Inst. Math*, 2022, Vol. 5, Issue 2. – Pp. 97-104. (01.00.00. № 17)
6. Baratov B.S. Separable cubic stochastic operators // *Bukhara State University scientific information,* 2019, Vol. 2, Issue 2. – Pp. 29-37. (01.00.00. № 3)

II bo'lim (part II; часть II)

7. Baratov B.S. Some properties of separable cubic stochastic operators // *Republican scientific conference*, "Problems and solutions to modern mathematical problems". – Termiz, 2020 october 21-23. – Pp. 94-96.
8. Baratov B.S. On a class separable cubic stochastic operators // *International scientific conference*, "Theories of functions of one and several complex variables". – Nukus, 2020 november 26-28. – Pp. 14-16.
9. Baratov B.S., Eshkabilov Y.X. Separable cubic stochastic operators // *International scientific conference*, "Algebraic and geometric methods of analysis". – Ukraine, 2021 may 25-28. – Pp. 124-125.
10. Baratov B.S. О матричной факторизации сепарабельных кубических стохастических операторов // *International scientific conference* "Modern problems in science and technology theory and practice". – Воронеж, 2020 december 21-23. – Pp. 33-36.
11. Jamilov U.U., Baratov B.S. On dynamics of separable cubic stochastic operators // *Republican scientific conference* "Operator algebras, non-associative structures and related problems". – Tashkent, 2022 september 14-15. – Pp. 282-284.
12. Jamilov U.U., Baratov B.S. On periodic trajectories of a cubic operator // *II Republican scientific conference of young scientists* "Mathematics, mechanics and intellectual technologies Tashkent-2023". – Tashkent, 2023 march 28-29. – Pp. 35-36.
13. Jamilov U.U., Baratov B.S. Periodicity of the discrete-time dynamical systems of separable cubic operators // *Republican scientific conference* "Modern problems of analysis". – Karshi, 2023 june 2-3. – Pp. 47-49.

14. Baratov B.S. The dynamics of a separable cubic stochastic operator // *Republican scientific conference* “Current and contemporary issues in mathematics and its teaching”, 2024. – Termiz, 2024 october 24-25. – Pp. 93-95.

Avtoreferat Qarshi davlat universitetining “QarDU xabarlari” ilmiy-nazariy uslubiy jurnali tahririyatida tahrirdan o‘tkazildi (27.03. 2025 yil).

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Qarshi davlat universiteti
Kichik bosmaxonasida chop etildi.