ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI HUZURIDAGI ILMIY DARAJA BERUVCHI PhD.03/2025.27.12.FM.06.02 RAQAMLI ILMIY KENGASH

ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI

ATANAZAROVA SHOIRA ERKINOVNA

MOSLANGAN MANBALI MANFIY TARTIBLI MODIFITSIRLANGAN KORTEVEG-DE FRIZ TENGLAMASINI INTEGRALLASH

01.01.02 – Differensial tenglamalar va matematik fizika

fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI

Urganch – 2026

UDK: 517.957

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI HUZURIDAGI ILMIY DARAJA BERUVCHI PhD.03/2025.27.12.FM.06.02 RAQAMLI ILMIY KENGASH

ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI

ATANAZAROVA SHOIRA ERKINOVNA

MOSLANGAN MANBALI MANFIY TARTIBLI MODIFITSIRLANGAN KORTEVEG-DE FRIZ TENGLAMASINI INTEGRALLASH

01.01.02 – Differensial tenglamalar va matematik fizika

fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI

Urganch – 2026

Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2024.2.PhD/FM1055 raqam bilan ro'yxatga olingan.

Dissertatsiya O'zbekiston Respublikasi Fanlar akademiyasi V.I. Romanovskiy nomidagi Matematika instituti Xorazm hududiy bo'linmasi va Abu Rayhon Beruniy nomidagi Urganch davlat universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, rus, ingliz (rezyume)) Ilmiy kengash veb-sahifasida (www.ik-mat.urdu.uz) va "ZiyoNet" Axborot ta'lim portalida (www.ziyonet.uz) joylashtirilgan.

Ilmiy rahbar:	**O'razboyev Gayrat O'razaliyevich** fizika-matematika fanlari doktori, dotsent
Rasmiy opponentlar:	**Yaxshumuratov Alisher Bekchanovich** fizika-matematika fanlari doktori
	Baltayeva Umida Ismailovna fizika-matematika fanlari doktori
Yetakchi tashkilot:	**Sharof Rashidov nomidagi Samarqand davlat universiteti**

Dissertatsiya himoyasi Abu Rayhon Beruniy nomidagi Urganch davlat universiteti huzuridagi PhD.03/2025.27.12.FM.06.02 raqamli Ilmiy kengashning 2026 yil "14" fevral soat 15:00 dagi majlisida bo'lib o'tadi. (Manzil: 220100, Urganch sh., H.Olimjon ko'chasi, 14-uy. Tel.: (+99862) 224-66-11, faks: (+99862) 224-67-00, e-mail: ik-mat.urdu@umail.uz

Dissertatsiya bilan Abu Rayhon Beruniy nomidagi Urganch davlat universiteti Axborot-resurs markazida tanishish mumkin (D-939 raqami bilan ro'yxatga olingan). (Manzil: 220100, Urganch sh., H.Olimjon ko'chasi, 14-uy. Tel.: (+99868) 224-67-00, faks: (+99862) 22467-00, e-mail: arm@urdu.uz).

Dissertatsiya avtoreferati 2026 yil "02" fevral kuni tarqatildi.

(2026 yil 02(oy)dagi 1 raqamli reestr bayonnomasi).

O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI

B. I. Abdullayev
Ilmiy daraja beruvchi Ilmiy kengash raisi, f.-m.f.d, professor.

A. A. Reyimberganov
Ilmiy daraja beruvchi Ilmiy kengash ilmiy kotibi, f.-m.f.n.

B. A. Babajanov
Ilmiy daraja beruvchi Ilmiy kengash qoshidagi ilmiy seminar raisi, f.-m.f.d, professor.

Kirish (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlarda chiziqli differensial tenglamalar sistemasiga qo‘yilgan to‘g‘ri va teskari spektral masalalarni tadqiq qilishga alohida ahamiyat berilmoqda. Hozirgi kunda spektral analizning to‘g‘ri va teskari masalalari zamonaviy matematik fizika sohasida murakkab fizik jarayonlarni tavsiflovchi nochiziqli evolyutsion tenglamalarning aniq yechimlarini qurishda muhim tatbiqqa ega bo‘lib, differensial tenglamalar nazariyasi sohasida olib borilayotgan asosiy tadqiqot yo‘nalishlaridan biri hisoblanadi. Bundan tashqari, bu masalalar geofizika, gidrodinamika, nochiziqli optika, akustika va kvant mexanikasining ayrim masalalarini hal qilishda hamda ilmiy-eksperimental tadqiqot usullarini joriy qilishda muhim o‘rin tutmoqda. Shuningdek, optik tolalar va nochiziqli kristallarda impulslarning tarqalish jarayonlarini modellashtiruvchi manfiy tartibli modifitsirlangan Korteveg-de Friz tenglamasiga qo‘yilgan Koshi masalasinining tez kamayuvchi yechimlarini qurishda Zaxarov-Shabat sistemasi uchun sochilish nazariyasining teskari spektral masalalasi usulini tadbiq qilishga doir tadqiqotlarni olib borish bugungi kunda zamonaviy matematik fizikaning muhim vazifalaridan biri bo‘lib qolmoqda.

Hozirgi kunda jahonda matematik fizika sohasida tez kamayuvchi funksiyalar sinfida moslangan manbali manfiy tartibli nochiziqli evolyutsion tenglamalar hamda ularning matritsaviy analoglariga qo‘yilgan Koshi masalasining yechimini topish bo‘yicha olib borilayotgan tadqiqotlarga ustuvor ahamiyat berilmoqda. Bu turdagi masalalar zamonaviy matematik fizikaning istiqbolli yo‘nalishlari qatoriga kirib, elektrodinamika, nochiziqli optika va plazma fizikasida to‘lqinlar evolyutsiyasi hamda ularning muhit bilan o‘zaro ta’sir jarayonlarini o‘rganishda keng qo‘llanilmoqda. Shu sababli, tez kamayuvchi funksiyalar sinfida moslangan manbali manfiy tartibli nochiziqli evolyutsion tenglamalar bilan bir qatorda moslangan manbali matritsaviy manfiy tartibli nochiziqli evolyutsion tenglamalarning yechimlari mavjudligini ko‘rsatishga imkon beruvchi tadqiqotlarni ishlab chiqish maqsadli ilmiy tadqiqotlardan hisoblanadi.

Mamlakatimizda fundamental fanlarning ilmiy va amaliy tadbiqiga ega bo‘lgan chiziqli differensial tenglamalar sistemasi spektral nazariyasining dolzarb masalalarini yechish usullarini ishlab chiqish kabi yo‘nalishlarga alohida e’tibor qaratilmoqda. Xususan, birinchi tartibli oddiy differensial tenglamalar sistemasi spektral nazariyasi masalalarini o‘rganishga e’tibor kuchaydi hamda sochilish nazariyasining to‘g‘ri va teskari masalalari usullaridan foydalanib zamonaviy matematik fizikaning manfiy tartibli nochiziqli evolyutsion tenglamalarining soliton yechimlarini qurish bo‘yicha muhim natijalarga erishildi. Matematika sohasidagi ilmiy tadqiqot ishlari ko‘lamini kengaytirish, ularning natijadorligi va amaliy ahamiyatini oshirish matematika va uning tatbiqlari sohasidagi tadqiqotlarni rivojlantirishning ustuvor yo‘nalishlari etib belgilangan[1]. Ushbu qaror ijrosini ta’minlashda matematik fizikaning moslangan manbali manfiy tartibli nochiziqli

[1] O‘zbekiston Respublikasi Prezidentining 2020 yil 7 maydagi “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi PQ-4708-son qarori.

evolyutsion tenglamalari hamda ularning matritsaviy analoglarini integrallash imkonini beruvchi sochilish nazariyasining to‘g‘ri va teskari masalalarini o‘rganish va tadqiq etish muhim ahamiyat kasb etadi.

O‘zbekiston Respublikasi Prezidentining 2020-yil 29-oktabrdagi PF-6097-son “Ilm-fanni 2030-yilgacha rivojlantirish Konsepsiyasini tasdiqlash to‘g‘risida”gi va 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida”gi Farmonlari, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi va 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi Qarorlari hamda ushbu faoliyat sohasiga oid boshqa me’yoriy-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishi ustuvor yo‘nalishlariga bog‘liqligi. Ushbu dissertatsiyadagi izlanish va tadqiqotlar O‘zbekiston Respublikasida fan va texnika taraqqiyotining IV. «Matematika, mexanika va informatika» ustuvor yo‘nalishiga muvofiq olib borildi.

Muammoni o‘rganilganlik darajasi.

Jahonda zamonaviy matematik fizika sohasida olib borilayotgan tadqiqotlarda solitonlar nazariyasi va nochiziqli evolyutsion tenglamalarni sochilish nazariyasining teskari masalasi usuli yordamida integrallash masalalari ko‘plab tadqiqotchilarda katta qiziqish uyg‘otgan. Sochilish nazariyasining teskari masalasi usuli dastlab K.S. Gardner, J.M. Grin, M.D. Kruskal and R.M. Miuralar tomonidan klassik Korteveg-de Friz tenglamasi uchun qo‘yilgan boshlag‘ich qiymatli masalani yechishda qo‘llanilgan. Keyinchalik, P. Laks nochiziqli evolyutsion tenglamalar va chiziqli operatorlar orasidagi bog‘liqlikni ifodalovchi ushbu $L_t = [L, B]$ formalizmni taklif etgan. Ushbu natijalar asosida V.E Zaxarov va A.B Shabat nochiziqli Shredinger tenglamasini, M. Vadati esa klassik modifitsirlangan Korteweg-de Friz tenglamasini tez kamayuvchi funksiyalar sinfida integrallashga muvaffaq bo‘ldilar. M. J. Ablowits, D.J. Kaup, A.C. Nyuell va H. Segurlar V.E. Zaxarov va A.B. Shabat taklif etgan sxemaga nisbatan umumiyroq bo‘lgan AKNS sxemasini ishlab chiqdilar.

Modifitsirlangan Korteveg-de Friz (mKdF) tenglamasini integrallash Zaxarov-Shabat sistemasi uchun sochilish nazariyasiga asoslanadi. Zaxarov-Shabat sistemasi uchun to‘g‘ri va teskari masalalar G. Gasimov, B.M. Levitan, V.E. Zaxarov, A.B. Shabat , I. S. Frolov, L.A. Takhtajan, L.D. Faddeev , R. K. Dodd, J. S. Eylbek, J.D. Gibbon, H.C. Morris, G.L. Lem, M. J. Ablowits, D.J. Kaup, A.C. Nyuell, H. Segur hamda A.B. Xasanov tomonidan o‘rganilgan. Jahon miqyosida Zaxarov-Shabat sistemasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalarini fizik jarayonlarni modellashtiruvchi nochiziqli evolyutsion tenglamalarga qo‘llash bo‘yicha T.Aktosun, C. van der Mee va F.Demontislar tomonidan salmoqli natijalarga erishilgan. Xususan, ular birinchilardan bo‘lib nochiziqli evolyutsion tenglamalarning oshkormas yechimlarini qurishga imkon beruvchi (A, B, C) uchlik matritsa usulini ishlab chiqishgan. Moslangan manbali nochiziqli evolyutsion tenglamalar sochilish nazariyasining to‘g‘ri va teskari masalalari yordamida dastlab V.K. Melnikovning ishlarida, ularning matritsaviy analoglari esa N. Bondarenko tomonidan o‘rganilgan.

Mamlakatimizda ham sochilish nazariyasining to‘g‘ri va teskari masalalarini xususiy hosilali differensial tenglamalarga tadbiq etishda muhim natijalarga erishilgan. Xususan, moslangan manbali nochiziqli evolyutsion tenglamalar hamda ularning matritsaviy analoglari tez kamayuvchi hamda davriy funksiyalar sinfida A.B. Xasanov, G.O‘. O‘razboyev, A.B. Yaxshimuratov, B.A. Babajanov, U.A. Xoitmetov, A.A. Reyimberganov, I.I. Baltayeva, M.M. Xasanov va A.K. Babadjanovalar tomonidan integrallangan.

Xususiy hosilali differensial tenglamarni integrallash bo‘yicha keyingi tadqiqotlar manfiy tartibli nochiziqli evolyutsion tenglamalar bilan bog‘liq. Manfiy tartibli nochiziqli evolyutsion tenglamalar dastlab J.M. Veroski tomonidan keltirib chiqarilgan. Manfiy tartibli KdF tenglamasi va manfiy tartibli mKdF tenglamasi Z.J. Qiao, E.G. Fan, Sh. Yuan, J.Xu va A.M. Vazvazlar tomonidan turli xil analitik usullar yordamida integrallangan va ushbu tenglamalarning ko‘p solitonli yechimlari topilgan.

Mamlakatimizda manfiy tartibli xususiy hosilali differensial tenglamalar differensial operatorlar uchun to‘g‘ri va teskari masalalar usuli yordamida 2022 yildan buyon o‘rganila boshlagan. Shunga qaramasdan bugungi kunga qadar nafaqat mamlakatimiz, balki jahon miqyosida manfiy tartibli nochiziqli evolyutsion tenglamalarni tadqiq etishda hamyurtlarimiz tomonidan samarali natijalarga erishilgan. Xususan, G.O‘. O‘razboyev, I.I. Baltayeva va A.K. Babadjanovalar tomonidan manfiy tartibli nochiziqli Shredinger tenglamasi keltirib chiqarildi. Manfiy tartibli nochiziqli evolyutsion tenglamalar moslangan manbali va moslangan manbasiz hollarda turli xil funksiyalar sinfida A.B. Xasanov, G.O‘. O‘razboyev, A.B. Yaxshimuratov, A.A. Reyimberganov, I.I. Baltayeva, M.M. Xasanov va A.K. Babadjanovalarning ishlarida o‘rganilgan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya Abu Rayhon Beruniy nomidagi Urganch davlat universiteti “Amaliy matematika va matematik fizika” kafedrasining ilmiy-tadqiqot ishlari rejasiga muvofiq “Differensial operatorlar spektral nazariyasining nochiziqli evolyutsion tenglamalarga tadbiqlari” nomli ilmiy-tadqiqot ishlari rejasi (2021-2025 yillar) asosida amalga oshirilgan.

Tadqiqotning maqsadi. Manfiy tartibli modifitsirlangan Korteveg-de Friz tenglamasi va uning matritsaviy analoglarini moslangan manbali va moslangan manbasiz hollarda sochilish nazariyasining teskari masalasi usuli yordamida tez kamayuvchi funksiyalar sinfida integrallashdan iborat.

Tadqiqotning vazifalari:

manfiy tartibli mKdF va moslangan manbali manfiy tartibli mKdF tenglamalarining to‘la integrallanuvchi bo‘lishini tez kamayuvchi potensialga ega Zaxarov-Shabat sistemasi uchun teskari masala usulidan foydalanib isbotlash;

moslangan manbali manfiy tartibli mKdF tenglamasining tez kamayuvchi funksiyalar sinfida ko‘p solitonli yechimini qurishga (A, B, C) matritsalar metodini tadbiq etish;

matritsaviy manfiy tartibli mKdF va matritsaviy moslangan manbali manfiy tartibli mKdF tenglamalariga qo‘yilgan Koshi masalasini Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida tez kamayuvchi funksiyalar sinfida yechish;

Tadqiqot obyekti manfiy tartibli mKdF tenglamasi, moslangan manbali manfiy tartibli mKdF tenglamasi, harakatlanuvchi xos qiymatlarga mos moslangan manbali manfiy tartibli mKdF tenglamasi, matritsaviy manfiy tartibli mKdF tenglamasi, moslangan manbali matritsaviy manfiy tartibli mKdF tenglamasi.

Tadqiqot predmeti tez kamayuvchi potensialga ega Zaxarov-Shabat sistemasi uchun sochilish nazariyasining teskari masalasi usulini manfiy tartibli nochiziqli evolyutsion tenglamalar va ularning matritsaviy analoglarini integrallash jarayoniga tatbiq etishdan iborat.

Tadqiqotning usullari. Dissertatsiya ishida xususiy hosilali tenglamalar, funksional tahlil va kompleks o‘zgaruvchili funksiyalar nazariyasi usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

manfiy tartibli mKdF tenglamasi bilan bog‘liq tez kamayuvchi potensialga ega Zaxarov-Shabat sistemasining sochilish nazariyasi berilganlari uchun evolyutsion tenglamalar sochilish nazariyasining teskari masalasi usuli yordamida keltirib chiqarilgan;

Zaxarov-Shabat sistemasining sochilish nazariyasi berilganlari uchun keltirib chiqarilgan evolyutsion tenglamalar yordamida qurilgan potensial manfiy tartibli mKdF tenglamasiga qo‘yilgan Koshi masalasining yechimidan iborat bo‘lishi tez kamayuvchi funksiyalar sinfida isbotlangan;

moslangan manbali manfiy tartibli mKdF tenglamasining to‘la integrallanuvchi bo‘lishi tez kamayuvchi funksiyalar sinfida Zaxarov-Shabat sistemasi uchun sochilish nazariyasining teskari masalasi usulidan foydalanib isbotlangan;

moslangan manbali manfiy tartibli mKdF tenglamasining ko‘p solitonli yechimini tez kamayuvchi funksiyalar sinfida qurishga (A, B, C) uchlik matritsa usuli tadbiq etilgan;

Zaxarov-Shabat sistemasi harakatlanuvchi xos qiymatlarga ega bo‘lgan holda moslangan manbali manfiy tartibli mKdF tenglamasi tez kamayuvchi funksiyalar sinfida sochilish nazariyasining teskari masalasi usuli yordamida integrallangan;

matritsaviy manfiy tartibli mKdF tenglamasi tez kamayuvchi potensialga ega matritsaviy Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida integrallangan;

matritsaviy Zaxarov-Shabat sistemasi uchun teskari masala usulidan foydalanib matritsaviy moslangan manbali mKdF tenglamasi uchun Koshi masalasi tez kamayuvchi funksiyalar sinfida yechilgan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

- manfiy tartibli mKdF tenglamasi va uning matritsaviy analoglarini moslangan manbali va moslangan manbasiz hollarda tez kamayuvchi yechimlarini topish algoritmlari ishlab chiqilgan;

- natijalarni MATLAB dasturiy ta’minoti orqali amalga oshirish imkonini beruvchi moslangan manbali manfiy tartibli mKdF tenglamasining yopiq shakldagi yechimini olish uchun (A,B,C) uchlik matritsa usulidan foydalanilgan;

Tadqiqot natijalarining ishonchliligi qo‘yilgan vazifalarni o‘rganishda matematik fizika, funksional tahlil va kompleks o‘zgaruvchili funksiyalar nazariyasi, chekli ayirmali operatorlar uchun sochilish nazariyasining teskari masalasi usullaridan foydalanilgan va barcha isbotlar qat’iy matematik mulohazalarga tayanganligi bilan asoslanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati olingan ilmiy natijalarning chiziqli operatorlarning spektral nazariyasi va turli fizik hodisalarni, masalan, transport oqimi modellari, doimiy manfiy egrilikka ega sirtlar geometriyasi, nochiziqli optika, lokal bo‘lmagan effektlarga ega ion-akustik solitonlarni tavsiflovchi matematik modellarga qo‘llanilishiga asoslangan.

Dissertatsiyaning amaliy ahamiyati shundaki, taklif qilingan algoritmlar va aniq yechim usullari manfiy tartibli nochiziqli evolyutsion tenglamalarni yechishda foydalanilishi va matematik fizika va muhandislik qo‘llanmalarida muhim ahamiyatga ega bo‘lgan sonli muhitlarda bevosita amalga oshirilishi mumkin.

Tadqiqot natijalarining joriy qilinishi. Moslangan manbali manfiy tartibli mKdF tenglamasini tez kamayuvchi funksiyalar sinfida integrallash bo‘yicha:

Zaxarov-Shabat sistemasi uchun sochilish nazariyasining teskari masalasi usulidan foydalanib, manfiy tartibli mKdF tenglamasi uchun qo‘yilgan Koshi masalasini tez kamayuvchi funksiyalar sinfida yechish bo‘yicha olingan natijalardan Bayroyt universitetida DAAD dasturi tomonidan moliyalashtirilgan “Universitet professor-o‘qituvchilari va olimlari uchun ilmiy tadqiqotlar, 2024 (57693448)” ilmiy loyihasi doirasida matritsaviy manfiy tartibli Korteveg-de Friz tenglamasi uchun evolyutsion tenglamalarni keltirib chiqarishda foydalanilgan (Bayroyt universitetining ma‘lumotnomasi, Germaniya, 2025-yil 14-oktabr). Ilmiy natijalarning qo‘llanilishi sochilish nazariyasining teskari masalasi usulidan foydalanib matritsaviy manfiy tartibli Korteveg-de Friz tenglamasiga qo‘yilgan Koshi masalasi yechimini qurish algoritmini ishlab chiqish hamda olingan yechimlarning to‘lqinsimon harakatlarini MATLAB dasturiy ta’minoti yordamida vizuallashtirish imkonini bergan.;

Matritsaviy manfiy tartibli mKdF tenglamasini sochilish nazariyasining teskari masalasi usulida integrallash bo‘yicha olingan natijalardan Slovakiya Respublikasining “Talabalar, PhD talabalari, universitet o‘qituvchilari, tadqiqotchilar va rassomlarning harakatchanligini qo‘llab-quvvatlash, 2024” Milliy stipendiya dasturi tomonidan moliyalashtirilgan ilmiy loyihada tez kamayuvchi funksiyalar sinfida moslangan manbali matritsaviy sinus-Gordon tenglamasi bilan bog‘liq bo‘lgan operatorning sochilish nazariyasi berilganlari uchun evolyutsion tenglamalarni keltirib chiqarishda foydalanilgan (Slovakiya Fanlar akademiyasining Matematika institutining № 288\D1 sonli ma’lumotnomasi, Bratislava, 2025-yil 14-

oktabr). Ilmiy natijalarning qo‘llanilishi moslangan manbali matritsaviy sinus-Gordon tenglamasining oshkormas yechimlarini (A,B,C) uchlik matritsa usuli yordamida qurish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Dissertatsiyaning asosiy natijalari 8 ta ilmiy-amaliy konferensiyalarda, shu jumladan 4 ta xalqaro ilmiy-amaliy konferensiyalarda muhokama qilingan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi doirasida 12 ta ilmiy ish chop qilingan. Doktorlik dissertatsiyalarining asosiy ilmiy natijalarini nashr etish uchun O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasi tomonidan tavsiya etilgan ilmiy nashrlarda 4 ta maqola, shu jumladan 2 tasi xorijiy jurnallarda chop qilingan.

Dissertatsiyaning hajmi va tuzilishi. Dissertatsiya kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat. Dissertatsiyaning hajmi 89 betni tashkil qiladi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning Respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi keltirilgan, dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi, tadqiqot maqsadi, vazifalari, obyekti, predmeti va usullari tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “Manfiy tartibli modifitsirlangan Korteveg-de Friz tenglamasi yechimini sochilish nazariyasining teskari masalasi usuli yordamida qurish” deb nomlangan birinchi bobida manfiy tartibli mKdF tenglamasiga qo‘yilgan Koshi masalasi sochilish nazariyasining teskari masalasi usuli yordamida tez kamayuvchi funksiyalar sinfida yechilgan.

Bu bobning birinchi paragrafida manfiy tartibli nochiziqli evolyutsion tenglamalarni keltirib chiqarish masalalari qaralgan. Rekursiya operatori nochiziqli xususiy hosilali differensial tenglamalar va ularning integrallanuvchanlik xossalarini o‘rganishda muhim matematik instrument hisoblanadi. P.J. Olver nochiziqli evolyutsion tenglamalarni ushbu

$$u_t = R^n \Delta_i(x,t), \quad i \ge 1, \quad n = 1,2,3,\dots \tag{1}$$

ko‘rinishda ifodalash mumkin ekanligini isbotlagan, bu yerda $\Delta_i(x,t)$ - qaralayotgan nochiziqli tenglama simmetriyalari, R esa ushbu tenglama uchun rekursiya operatori.

J.M. Veroski esa P.J. Olver go‘yasidan foydalanib manfiy tartibli nochiziqli evolyutsion tenglamalarni keltirib chiqarish metodini taklif etgan. Xususan, (1) tenglikda R rekursiya operatorini manfiy darajalarini qo‘llab, ya’ni ushbu

$$R^n u_t = \Delta_i(x,t), \quad i \ge 1, \; n = 1,2,3,\dots$$

tenglikdan foydalanib, manfiy tartibli nochiziqli evolyutsion tenglamalarni keltirib chiqarish mumkin ekanligini isbotlagan. Keyinchalik, A.M. Vazvaz quyidagi

$$R = -\partial_x^2 - 4u - 4u_x \partial_x^{-1}(\cdot u), \quad (Rf = -f_{xx} - 4uf - 4u_x \partial_x^{-1}(fu))$$

rekursiya operatorini ushbu $Ru_t = u_x$ tenglikka qo‘llash natijasida ushbu

$$-u_{xxt} - 4u^2 u_t - 4u_x \partial_x^{-1}(u \cdot u_t) = u_x \tag{3}$$

manfiy tartibli mKdF tenglamasini keltirib chiqargan. Agar $\partial_x^{-1} = \int\limits_{-\infty}^{x} dx$ deb olsak hamda ushbu $\int\limits_{-\infty}^{x} 2u \cdot u_t dx = \eta$ belgilashni kiritsak, u holda (3) tenglama quyidagi

$$-u_{xxt} - 2u(\eta_x + 2uu_t\big|_{-\infty}) - 2u_x\eta - u_x = 0$$

ko‘rinishni oladi. Aytaylik, $u = u(x,t)$ funksiya tez kamayuvchi funksiyalar sinfiga tegishli bo‘lsin. U holda manfiy tartibli mKdF tenglamasi ushbu

$$\begin{cases} \eta_x = 2uu_t \\ u_{xt} + 2u\eta + u = 0 \end{cases} \tag{4}$$

tenglamalar sistemasi ko‘rinishida ifodalanadi. Keyingi paragraflarda (4) manfiy tartibli mKdF tenglamasi sochilish nazariyasining teskari masalasi usuli yordamida tez kamayuvchi funksiyalar sinfida integrallash masalalari qaralgan.

Ushbu bobning ikkinchi paragrafida quyidagi

$$\begin{cases} y_{1x} + i\xi y_1 = u(x) y_2 \\ y_{2x} - i\xi y_2 = -u(x) y_1, \end{cases} \quad -\infty < x < \infty, \tag{5}$$

Zaxarov-Shabat sistemasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalariga oid zaruriy ma’lumotlarni keltirilgan. (5) sistemani $Ly = \xi y$ ko‘rinishda yozish mumkin, bunda $y = (y_1, y_2)^T$ va L operator quyidagi ko‘rinishga ega

$$L = i\begin{pmatrix} \dfrac{d}{dx} & -u(x) \\ -u(x) & -\dfrac{d}{dx} \end{pmatrix}.$$

(5) sistemadagi $u(x)$ potensial haqiqiy funksiyadan iborat bo‘lib, ushbu

$$\int_{-\infty}^{\infty} (1+|x|)|u(x)|dx < \infty$$

shartni qanoatlantirsin. U holda (5) sistema $\phi(x,\xi)$ va $\psi(x,\xi)$ Yost yechimlarga ega bo‘lib, bu yechimlar $\mathrm{Im}\,\xi = 0$ da quyidagi asimptotikalar yordamida yagona aniqlanadi

$$\left.\begin{aligned} \phi(x,\xi) &\sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x} \\ \bar{\phi}(x,\xi) &\sim \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{i\xi x} \end{aligned}\right\}, \quad x \to -\infty, \qquad \left.\begin{aligned} \psi(x,\xi) &\sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi x} \\ \bar{\psi}(x,\xi) &\sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x} \end{aligned}\right\}, \quad x \to \infty.$$

$\{\psi, \bar{\psi}\}$ funksiyalar juftligi (5) sistemaninng chiziqli erkli yechimlarini tashkil qilganligi uchun, quyidagi yoyilma o‘rinli

$$\phi = a(\xi)\bar{\psi} + b(\xi)\psi, \quad \bar{\phi} = -\bar{a}(\xi)\psi + \bar{b}(\xi)\bar{\psi}, \quad \mathrm{Im}\,\xi = 0.$$

Bunda

$$a(\xi) = W\{\phi, \psi\}, \quad b = -W\{\phi, \bar{\psi}\},$$

bo‘lib, ushbu ayniyatlar o‘rinli

$$a(\xi)\bar{a}(\xi) + b(\xi)\bar{b}(\xi) = 1, \; \bar{a}(\xi) = a(-\xi), \; \bar{b}(\xi) = b(-\xi).$$

$a(\xi)$ funksiya yuqori yarim tekislikka ($\mathrm{Im}\,\xi > 0$) analitik davom qilib, u yerda ushbu

$$a(\xi) = 1 + \underline{\underline{O}}\left(|\xi|^{-1}\right), \qquad |\xi| \to \infty$$

asimptotiklarga ega bo‘ladi.

Bundan tashqari, $a(\xi)$ funksiya yuqori yarim tekislikda cheklita oddiy nollarga ega bo‘ladi va bu nollar L operatorning ξ_k, $k = 1, \ldots, N$, xos qiymatlari bilan ustma-ust tushadi. $a(\xi)$ funksiyaning haqiqiy o‘qda yotuvchi nollari L operatorning spektral maxsusliklari deb ataladi. Umuman olganda (5) o‘z-o‘ziga qo‘shma bo‘lmagan Zaxarov-Shabat sistemasi karrali xos qiymatlarga hamda spektral maxsusliklarga ega bo‘lishi mumkin. Ammo mazkur dissertatsiya ishida o‘z-o‘ziga qo‘shma bo‘lmagan Zaxarov-Shabat sistemasi oddiy xos qiymatlarga ega bo‘lib, spektral maxsusliklarga ega bo‘lmasin deb faraz qilinadi. Shuning uchun ushbu tenglik

$$\phi(x,\xi_k)=b_k\psi(x,\xi_k),\ \ k=1,\dots,N.$$

o‘rinli.

Ushbu

$$\left\{ r^{+}\left(\xi\right)=\frac{b(\xi)}{a(\xi)},\ \xi\in R;\ \xi_k,\operatorname{Im}\xi_k>0;\ \ b_k,\ \ k=1,\dots,N \right\} \tag{6}$$

jamlanma L operatorning sochilish nazariyasi berilganlari deb ataladi. (5) sistemaning $\psi(x,\xi)$ Yost yechimi uchun ushbu

$$\psi(x,\xi)=\begin{pmatrix}0\\1\end{pmatrix}e^{i\xi x}+\int_x^{\infty}K\left(x,s\right)e^{i\xi s}ds$$

integral tasvir o‘rinli bo‘lib, bunda $K\left(x,s\right)=\begin{pmatrix}K_1\left(x,s\right)\\K_2\left(x,s\right)\end{pmatrix}$ yadro ushbu

$$\begin{cases} K_2(x,y,t)+\int_x^{\infty}K_1(x,s,t)F(s+y,t)ds=0 \\ -K_1(x,y,t)+F(x+y,t)+\int_x^{\infty}K_2(x,s,t)F(s+y,t)ds=0 \end{cases}$$

Gelfand-Levitan-Marchenko integral tenglamalar sistemasini yechimlaridan iborat bo‘ladi. Bu yerda $F(x)=\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}\frac{b(\xi)}{a(\xi)}e^{i\xi x}d\xi-i\sum_{j=1}^{N}c_je^{i\xi_j x},$ bo‘lib $c_k=\dfrac{b_k}{\dot a(\xi_k)}$, $\dot a(\xi_k)=\dfrac{\partial a(\xi)}{\partial\xi}\Big|_{\xi=\xi_k}$.

Gelfand-Levitan-Marchenko integral tenglamalar sistemasini yechimlari (5) sistemaning potensiali bilan quyidagicha

$$u(x)=-2K_1(x,x),\qquad -2K_2(x,x)=\int_x^{\infty}\left|u(s)\right|^2ds$$

bog‘langan.

Zaxarov-Shabat sistemasi uchun sochilish nazariyasining to‘g‘ri masalasi deb $u(x)$, $x\in R$ potensial berilgan holda sochilish nazariyasining berilganlari (6) ni topish masalasiga aytiladi. Va aksincha, sochilish nazariyasining berilganlari (6) ma’lum bo‘lgan holda $u(x)$, $x\in R$ potensialni aniqlash masalasiga Zaxarov-Shabat sistemasi uchun sochilish nazariyasining teskari masalasi deyiladi.

Ushbu bobning uchinchi paragrafida manfiy tartibli mKdF tenglamasi uchun qo‘yilgan Koshi masalasi tez kamayuvchi funksiyalar sinfida Zaxarov-Shabat sistemasi uchun teskari masala usulida yechilgan. Xususan, ushbu

$$\begin{cases} \eta_x = 2uu_t \\ u_{xt} + 2\eta u + u = 0, \;\; x\in\mathbb{R}, \;\; t\geq 0 \end{cases} \tag{7}$$

sistema quyidagi

$$u(x,0) = u_0(x)\,, \; x\in\mathbb{R} \tag{8}$$

boshlang‘ich shartda qaralgan. Bu yerda $u_0(x)$ boshlang‘ich funksiya haqiqiy funksiya bo‘lib, quyidagi shartlarni qanoatlantiradi

1. $\int_{\mathbb{R}} \left(1+|x|\right)^5 |u_0(x)| dx < \infty$,
2. $L(0) = i\begin{pmatrix} \frac{d}{dx} & -u_0 \\ -u_0 & -\frac{d}{dx} \end{pmatrix}$ operator spektral maxsusliklarga ega emas va $2N$ ta $\xi_1(0), \xi_2(0), \ldots \xi_{2N}(0)$ oddiy xos qiymatlarga ega bo‘lib, bunda $\operatorname{Im}\xi_k(0) > 0$, $\xi_{N+k}(0) = -\xi_k(0)$, $k = 1,\ldots,N$.

Aytaylik $u = u(x,t)$ va $\eta = \eta(x,t)$ funksiyalar yetarlicha silliq haqiqiy funksiyalar bo‘lib, ushbu sinfga tegishli bo‘lsin

$$\mathcal{L} = \left\{ \begin{array}{c} u\in C^2(\mathbb{R}\times(0,\infty))\cap(\mathbb{R}\times[0,\infty)), \;\; \eta\in C^{1,0}(\mathbb{R}\times(0,\infty)); \\ \lim\limits_{x\to\pm\infty}\eta(x,t) = 0, \;\; t\in\mathbb{R}; \\ \int\limits_{-\infty}^{+\infty}\left(1+|x|\right)\left(|u(x,t)| + |u_x(x,t)| + |u_t(x,t)| + |u_{xt}(x,t)|\right)dx < \infty, \;\; t\in\mathbb{R} \end{array} \right\}. \tag{9}$$

Ushbu paragrafda olingan asosiy natija quyidagi teoremada keltirilgan

1-Teorema. Aytaylik $\{u(x,t), \eta(x,t)\}$ funksiyalar juftligi (7)-(9) masalaning yechimi bo‘lsin. U holda ushbu

$$L(t) = i\begin{pmatrix} \frac{d}{dx} & -u(x,t) \\ -u(x,t) & -\frac{d}{dx} \end{pmatrix}$$

operatorning sochilish nazariyasi berilganlari vaqt bo‘yicha quyidagicha o‘zgaradi

$$\frac{\partial r^+(\xi,t)}{\partial t} = \frac{i}{2\xi} r^+(\xi,t)\,, \; \xi\in\mathbb{R}\setminus\{0\},$$

$$\frac{d\xi_k}{dt} = 0\,, \qquad \frac{db_k(t)}{dt} = \frac{i}{2\xi_k} b_k(t), \quad k = 1,2,...,N.$$

Aytaylik (5) sistemaning sochilish nazariyasi berilganlari uchun quyidagi shartlar bajarilsin:

a) $\{\xi_k\}_{k=1}^N \subset \mathbb{C}$- diskret xos qiymatlar, $\operatorname{Im}\xi_k \geq 0$ va $\{c_k\}_{k=1}^N \subset \mathbb{C}$ - normallovchi o‘zgarmaslar bo‘lib, $c_k = \overline{c}_k$;

b) $r_0^+ : \mathbb{R}\to\mathbb{C}$ qaytish koeffisienti uchun ushbu

$$\left|r_0^+(\xi)\right| < 1, \; r_0^+(-\xi) = \overline{r}_0^+(\xi)\,, \; r_0^+(\xi) = o(\xi^{-1}), \;\; \xi\to\pm\infty,$$

$$\int_{\mathbb{R}}\left(\left|r_0^+(\xi)\right|+\left|\frac{d}{d\xi}r_0^+(\xi)\right|+\left|\frac{d^2}{d\xi^2}r_0^+(\xi)\right|\right)\left(1+|\xi|\right)^3 d\xi<\infty$$

munosabatlar o‘rinli.

1-Teoremaning natijalaridan foydalanib, (7)-(9) masalaning yechimini topish uchun quyidagi algoritmdan foydalanilgan

Algoritm 1.

1. $u(0)$ berilgan holda $\left\{r^+{}_0(\xi);\ \xi_k(0);\ b_k(0),\quad k=1,\ldots,N\right\}$ jamlanma topiladi.

2. 1– Teorema natijalaridan foydalanib, ushbu funksiya aniqlanadi

$$F(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}r_0^+(0,\xi)e^{\frac{i}{2\xi}t+i\xi x}d\xi-i\sum_{k=1}^{N}c_k(0)e^{\frac{i}{2\xi_k}t+i\xi_k x}.$$

3. Ushbu $\begin{cases} K_2(x,y,t)+\int_x^{\infty}K_1(x,s,t)F(s+y,t)ds=0 \\ -K_1(x,y,t)+F(x+y,t)+\int_x^{\infty}K_2(x,s,t)F(s+y,t)ds=0\end{cases}$ sistemaning

$K(x,y,t)=\begin{pmatrix}K_1(x,y,t)\\K_2(x,y,t)\end{pmatrix}$ yechimi topiladi.

4. Ushbu

$$u(x,t)=-2K_1(x,x),\quad \eta_x(x,t)=2u(x,t)u_t(x,t)$$

formulalar yordamida $u(x,t)$ va $\eta(x,t)$ funksiyalar quriladi.

Mazkur pargrafning keyingi asosiy natijasi quyidagi teoremadan iborat

Teorema 2. Aytaylik a) va b) shartlar bajarilsin. U holda *Algoritm 1* yordamida qurilgan $\{u(x,t),\eta(x,t)\}$ funksiyalar (7)-(9) masalaning yechimidan iborat bo‘ladi.

Dissertatsiyaning ikkinchi bobi “Moslangan manbali manfiy tartibli modifitsirlangan Korteveg-de Friz tenglamasini tez kamayuvchi funsiyalar sinfida integrallash” deb nomlanib, moslangan manbali manfiy tartibli mKdF tenglamasiga qo‘yilgan Koshi masalasi Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida yechilgan.

Mazkur bobning birinchi paragrafida quyidagi

$$\begin{cases}\eta_x-2uu_t=G_1\\ u_{xt}+2\eta u+u=G_2\end{cases}\tag{10}$$

sistema qaralgan bo‘lib, bunda $G_1=G_1(x,t)$ va $G_1=G_2(x,t),\quad t\in\mathbb{R}$ funksiyalar yetralicha silliq bo‘lib, $G_1(x,t)=o(1)$ va $G_1(x,t)=o(1)$ on $x\to\pm\infty$ xossalarga ega. (10) sistema ushbu

$$u(x,0)=u_0(x),\ x\in\mathbb{R},\tag{11}$$

boshlang‘ich shart bilan qaralgan bo‘lib, $u_0(x)$ haqiqiy funksiya quyidagi xossalarga ega

1. $\int_{\mathbb{R}}\left(1+|x|\right)^5|u_0(x)|dx<\infty$,

2. $L(0)=i\begin{pmatrix} \frac{d}{dx} & -u_0 \\ -u_0 & -\frac{d}{dx} \end{pmatrix}$ operator spektral maxsusliklarga ega emas va $2N$ ta $\xi_1(0),\xi_2(0),\ldots\xi_{2N}(0)$ oddiy xos qiymatlarga ega bo‘lib, bunda $\operatorname{Im}\xi_k(0)>0$, $\xi_{N+k}(0)=-\xi_k(0)$, $k=1,\ldots,N$.

Aytaylik $u=u(x,t)$ va $\eta=\eta(x,t)$ funksiyalar yetarlicha silliq haqiqiy funksiyalar bo‘lib, ushbu sinfga tegishli bo‘lsin

$$\mathcal{L}=\left\{\begin{array}{c} u\in C^2(\mathbb{R}\times(0,\infty))\cap(\mathbb{R}\times[0,\infty)),\quad \eta\in C^{1,0}(\mathbb{R}\times(0,\infty)); \\ \lim\limits_{x\to\pm\infty}\eta(x,t)=0,\quad t\in\mathbb{R}; \\ \int\limits_{-\infty}^{+\infty}\left(1+|x|\right)\left(|u(x,t)|+|u_x(x,t)|+|u_t(x,t)|+|u_{xt}(x,t)|\right)dx<\infty,\quad t\in\mathbb{R} \end{array}\right\}. \quad (12)$$

Quyidagi lemma o‘rinli

1-Lemma. Agar $\{u(x,t),\eta(x,t)\}$ juftlik (10)-(12) masalaning yechimidan iborat bo‘lsa, u holda $L(t)$ operatorning sochilish nazariyasi berilganlari t vaqt bo‘yicha quyidagicha o‘zgaradi

$$\frac{\partial r^+(\xi,t)}{\partial t}=\frac{i}{2\xi}r^+(\xi,t)-\frac{i}{a^2(\xi)}\int\limits_{-\infty}^{+\infty}\hat{\phi}^T G\phi dx,\qquad \xi\in\mathbb{R}\setminus\{0\},$$

$$\frac{d\xi_n(t)}{dt}=-\frac{i}{b_n\dot{a}\left(\xi_n\right)}\int\limits_{-\infty}^{\infty}\hat{\phi}_n^T G\phi_n dx,\quad n=1,\ldots,N,$$

$$\frac{db_n(t)}{dt}=\left(\frac{i}{2\xi_n}-\frac{i}{2b_n\dot{a}(\xi_n)}\int\limits_{-\infty}^{\infty}\hat{\chi}_n^T G\phi_n\,dx\right)b_n(t).$$

Ushbu bobning ikkinchi paragrafida quyidagi

$$\begin{cases} \eta_x=2uu_t \\ u_{xt}+2\eta u+u=\sum\limits_{k=1}^{2N}\left(f_{k1}^2+f_{k2}^2\right),\quad x\in\mathbb{R}, \\ Lf_k=\xi_k f_k,\quad k=1,2,\ldots,2N, \end{cases} \quad (13)$$

moslangan manbali manfiy tartibli mKdF tenglamasi ushbu

$$u(x,0)=u_0(x),\ x\in\mathbb{R}, \quad (14)$$

boshlang‘ich shart bilan qaralgan bo‘lib, $u_0(x)$ haqiqiy funksiya quyidagi xossalarga ega

1. $\int_{\mathbb{R}}\left(1+|x|\right)^5|u_0(x)|dx<\infty$,

2. $L(0)=i\begin{pmatrix} \frac{d}{dx} & -u_0 \\ -u_0 & -\frac{d}{dx} \end{pmatrix}$ operator spektral maxsusliklarga ega emas va $2N$ ta $\xi_1(0),\xi_2(0),\ldots\xi_{2N}(0)$ oddiy xos qiymatlarga ega bo‘lib, bunda $\operatorname{Im}\xi_k(0)>0$, $\xi_{N+k}(0)=-\xi_k(0)$, $k=1,\ldots,N$.

Qaralayotgan masalada, $f_k=(f_{k1},f_{k2})^T$ funksiya $L(t)$ operatorninig ξ_k, $k=1,2,\ldots,N$ xos qiymatlariga mos keluvchi xos funksiyalaridan iborat va ushbu

$$\int_{-\infty}^{+\infty} f_{k1}f_{k2}dx=A_k(t),\ k=1,2,\ldots,2N \tag{15}$$

shartni qanoatlantiradi, bu yerda $A_k(t)$ oldindan berilga haqiqiy uzluksiz funksiyalar bo‘lib, ushbu $A_k(t)=A_{N+k}(t)$, $\xi_k=-\xi_{N+k}$, $k=1,2,\ldots,N$ xossaga ega.

Shuningdek, $u=u(x,t)$ va $\eta=\eta(x,t)$ funksiyalar yetarlicha silliq haqiqiy funksiyalar bo‘lib, ushbu sinfga tegishli bo‘lsin

$$\mathcal{L}=\left\{\begin{array}{c} u\in C^2(\mathbb{R}\times(0,\infty))\cap(\mathbb{R}\times[0,\infty)),\quad \eta\in C^{1,0}(\mathbb{R}\times(0,\infty)); \\ \lim\limits_{x\to\pm\infty}\eta(x,t)=0,\quad t\in\mathbb{R}; \\ \int_{-\infty}^{+\infty}\left(1+|x|\right)\left(|u(x,t)|+|u_x(x,t)|+|u_t(x,t)|+|u_{xt}(x,t)|\right)dx<\infty,\quad t\in\mathbb{R} \end{array}\right\}. \tag{16}$$

Ushbu paragrafda olingan asosiy natija quyidagicha

3-Teorema. Agar ushbu $\{u(x,t),\eta(x,t),f_k(x,t),k=1,2,\ldots,N\}$ jamlanma (13) – (16) masalaning yechimi bo‘lsa, u holda potensiali $u(x,t)$ bo‘lgan $L(t)$ operatorning sochilish nazariyasi berilganlari quyidagi evolyutsion tenglamalarni qanoatlantiradi

$$\frac{\partial r^+(\xi,t)}{\partial t}=\frac{i}{2\xi_n}r^+(\xi,t),\qquad \xi=\mathbb{R}\setminus\{0\},$$

$$\frac{d\xi_n}{dt}=0,\quad \frac{db_n(t)}{dt}=\left(\frac{i}{2\xi_n}+\frac{i}{\xi_n}A_n(t)\right)b_n(t),\ \ n=1,2,\ldots,N.$$

3- Teorema natijalaridan foydalanib, (13)-(16) masalaning ushbu

$$u(x,t)=-\frac{2}{\cosh(2x-\gamma(t))},\qquad \eta(x,t)=-\frac{2}{\cosh^2(2x-\gamma(t))},$$

$$f_1(x,t)=\frac{\sqrt{A_1(t)}}{\cosh(2x-\gamma(t))}\begin{pmatrix} e^{-x+\frac{\gamma(t)}{2}} \\ e^{x-\frac{\gamma(t)}{2}} \end{pmatrix}$$

bir solitonli yechimi topilgan, bunda $u(x,0)=-\dfrac{2}{\cosh(2x)}$, $\gamma(t)=\dfrac{1}{2}t+\displaystyle\int_0^t A_1(\rho)d\rho.$

Mazkur bobning uchinchi paragrafida harakatlanuvchi xos qiymatlarga mos moslangan manbali manfiy tartibli mKdF tenglamasi tez kamayuvchi funksiyalar sinfida integrallangan. Ushbu

$$\begin{cases} \eta_x - 2uu_t = -2\sum_{k=1}^{2N} u(f_{k1}g_{k1} - f_{k2}g_{k2}) \\ u_{xt} + u + 2\eta u = \sum_{k=1}^{2N} \frac{d}{dx}(f_{k1}g_{k1} - f_{k2}g_{k2}), \\ (L-\xi_k)f_k = 0,\ (L-\xi_k)g_k = 0, \quad k = 1, 2, \ldots, 2N, \end{cases} \tag{17}$$

sistema

$$u(x,0) = u_0(x), \quad x \in R, \tag{18}$$

boshlang‘ich shart bilan qaralgan bo‘lib, $u_0(x)$ haqiqiy funksiya quyidagi xossaga ega

$$\int_{\mathbb{R}} (1+|x|)^5 |u_0(x)| dx < \infty .$$

Aytaylik, $L(0) = i\begin{pmatrix} \frac{d}{dx} & -u_0 \\ -u_0 & -\frac{d}{dx} \end{pmatrix}$ operator spektral maxsusliklarga ega emas va $2N$ ta $\xi_1(0), \xi_2(0), \ldots \xi_{2N}(0)$ oddiy xos qiymatlarga ega bo‘lib, $\operatorname{Im}\xi_k(0) > 0$, $\xi_{N+k}(0) = -\xi_k(0)$, $k = 1, \ldots, N$ bo‘lsin.

Bundan tashqari, $u = u(x,t)$ va $\eta = \eta(x,t)$ funksiyalar yetarlicha silliq haqiqiy funksiyalar bo‘lib, ushbu sinfga tegishli bo‘lsin

$$\mathcal{L} = \left\{ \begin{array}{c} u \in C^2(\mathbb{R}\times(0,\infty)) \cap (\mathbb{R}\times[0,\infty)), \quad \eta \in C^{1,0}(\mathbb{R}\times(0,\infty)); \\ \lim_{x\to\pm\infty} \eta(x,t) = 0, \quad t \in \mathbb{R}; \\ \int_{-\infty}^{+\infty} (1+|x|)\left(|u(x,t)| + |u_x(x,t)| + |u_t(x,t)| + |u_{xt}(x,t)|\right) dx < \infty, \quad t \in \mathbb{R} \end{array} \right\}. \tag{19}$$

Qaralayotgan masalada $f_k = (f_{k1}(x,t), f_{k2}(x,t))^T$ funksiya $L(t)$ operatorning ξ_k xos qiymatiga mos xos vektor-funksiyasi, $g_k = (g_{k1}(x,t), g_{k2}(x,t))^T$ esa $L(t)g_k = \xi_k g_k$ tenglamaning f_k bilan chiziqli erkli yechimi bo‘lib, ushbu shartlar

$$W\{f_k, g_k\} \equiv f_{k1}g_{k2} - f_{k2}g_{k1} = \omega_k(t), \quad k = 1, 2, \ldots 2N, \tag{20}$$

$$\lim_{\delta\to\infty} \int_{-\delta}^{\delta} (f_{k1}g_{k2} + f_{k2}g_{k1}) dx = A_k(t), \ k = 1, 2, \ldots 2N \tag{21}$$

bajarilsin. Bunda $A_k(t)$ va $\omega_k(t)$ oldindan berilgan uzluksiz funksiyalar bo‘lib, ushbu

$$\omega_k(t) = -\omega_{N+k}(t), A_k(t) = -A_{N+k}(t), \quad \xi_n = -\xi_k, \ k = 1, 2, \ldots N$$

xossalarga ega.

Ushbu paragrafning asosiy natijasi quyidagi teoremada aks etgan

4-Teorema. Aytaylik ushbu $\{u(x,t), \eta(x,t), f_k(x,t), g_k(x,t), k = 1, 2, \ldots N\}$, jamlanma (17)-(21) masalaning yechimi bo‘lsin. U holda potensiali $u(x,t)$ bo‘lgan $L(t)$ operatorning sochilish nazariyasi berilganlari uchun quyidagi

$$\frac{\partial r^{+}(\xi,t)}{\partial t}=\left(\frac{i}{2\xi}+i\sum_{k=1}^{N}\omega_k(t)\left(\frac{1}{\xi-\xi_k}+\frac{1}{\xi_k+\xi}\right)\right)r^{+}(\xi,t),$$
$$\frac{d\xi_n(t)}{dt}=i\omega_n(t),\quad n=1,2,\ldots,N,$$
$$\frac{db_n(t)}{dt}=\left(\frac{i}{2\xi_n}+i\beta_n(t)\omega_n(t)\right)b_n(t)$$
evolyutsion tenglamalar o‘rinli.

Ushbu
$$u(x,0)=\frac{-2}{\cosh(2x)}.$$
boshlang‘ich shart bilan berilgan (17)-(21) masalaning quyidagi bir solitonli yechimi qurilgan
$$u(x,t)=\frac{-2\mu_1}{\cosh(2\mu_1 x-\tilde{\sigma})},\ \eta(x,t)=\frac{-1}{\cosh^2(2\mu_1 x-\tilde{\sigma})},$$
$$f_{11}=\frac{e^{-\mu_1 x}}{2\cosh(2\mu_1 x-\tilde{\sigma})},\ f_{12}=\frac{e^{\mu_1 x-\tilde{\sigma}}}{2\cosh(2\mu_1 x-\tilde{\sigma})},$$
$$g_{11}=\frac{\left((2i\mu_1\beta_1-4\mu_1 x-1)e^{-\mu_1 x+\tilde{\sigma}}-e^{3\mu_1 x-\tilde{\sigma}}\right)\omega_1}{2\cosh(2\mu_1 x-\tilde{\sigma})},$$
$$g_{12}=\frac{\left((2i\mu_1\beta_1-4\mu_1 x+1)e^{\mu_1 x}+e^{-3\mu_1 x+2\tilde{\sigma}}\right)\omega_1}{2\cosh(2\mu_1 x-\tilde{\sigma})}.$$
Bu yerda $\xi_1=i\mu_1(t)$, $\mu_1(t)=\left(1+\frac{1}{2}\int_0^{\tau}\omega_1(\tau)d\tau\right)$, $\frac{d\tilde{\sigma}(t)}{dt}=\frac{1}{2\mu_1}+i\beta_1\omega_1$, bo‘lib, μ_1, β_1, ω_1, $\tilde{\sigma}$ - t parametrga bog‘liq haqiqiy funksiyalar. β_1 kattalikni (21) shartdan foydalanib topish mumkin.

Dissertatsiya ishining uchinchi bobi “Moslangan manbali matritsaviy manfiy tartibli modifitsirlangan Korteveg-de Friz tenglamasini tez kamayuvchi funsiyalar sinfida integrallash” deb nomlanib, moslangan manbali matritsaviy manfiy tartibli mKdF tenglamasiga qo‘yilgan Koshi masalasi matritsaviy Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida yechilgan.

Mazkur bobning birinchi paragrafida ushbu
$$-iJ\frac{d}{dx}Y(x,\xi)-V(x)Y(x,\xi)=\xi Y(x,\xi),\tag{22}$$
matritsaviy Zaxarov-Shabat sistemasi uchun sochilish nazariyasiga oid zaruriy ma’lumotlar keltirilgan. Bunda $J=\begin{bmatrix} I_m & 0\\ 0 & -I_m\end{bmatrix}$, I_m- m o‘lchamli birlik matritsa, $V(x)=\begin{bmatrix} 0 & i\upsilon(x)\\ i\upsilon(x) & 0\end{bmatrix}$ bo‘lib, uning komponentalari $m\times m$ o‘lchamli haqiqiy $\upsilon(x)$ matritsalardan iborat. $V(x)$ potensial quyidagi

$$\int_{-\infty}^{\infty}\left(1+|x|\right)\|V(x)\|dx<\infty$$

shartni qanoatlantirsin, bu yerda matritsa normasi quyidagicha $\|V(x)\|=\max_{j}\sum_{k=1}^{m}\left|V_{jk}(x)\right|$ kiritiladi. (22) sistemada $Y(x,\xi)$ - ustun $Y=(y_j)_{j=1}^{2m}$ vektor funksiyadan iborat. Haqiqiy ξ larda (22) sistema $2m\times 2m$ o‘lchamli $\Phi_l(x,\xi)$ va $\Phi_r(x,\xi)$ Yost matritsalariga ega bo‘lib, bu matritsalar ushbu

$$\Phi_l(x,\xi)=[\bar{\psi}(x,\xi)\;\;\psi(x,\xi)]\to e^{i\xi Jx}I_{2m},\quad x\to\infty,$$
$$\Phi_r(x,\xi)=[\phi(x,\xi)\;\;\bar{\phi}(x,\xi)]\to e^{i\xi Jx}I_{2m},\quad x\to-\infty$$

asimptotikalar yordamida yagona aniqlanadi. Haqiqiy ξ larda Yost matritsalar uchun ushbu

$$\Phi_l(x,\xi)=\Phi_r(x,\xi)A_l(\xi)$$

yoyilma o‘rinli bo‘lib, bunda $A_l(\xi)=\begin{bmatrix}a_{l1}(\xi) & a_{l2}(\xi)\\ a_{l3}(\xi) & a_{l4}(\xi)\end{bmatrix}$, $a_{li}(\xi)$, $i=1,2,3,4$, - $m\times m$ o‘lchamli matritsalar. Ushbu $\det a_{l1}(\xi)=0$ tenglama yuqori yarim tekislikda ξ_j $j=1,2,\ldots,N$ cheklita oddiy nollarga ega bo‘lib, (22) sistemaning xos qiymatlari bilan ustma-ust tushadi. $\det a_{l1}(\xi)=0$ tenglamaning haqiqiy nollari (22) sistemaning spektral maxsusliklari deb ataladi. (22) matritsaviy Zaxarov-Shabat sistemasi karrali xos qiymatlarga va spektral maxsusliklari ega bo‘lishi mumkin. Ammo ushbu bobda (22) sistema oddiy xos qiymatlarga ega bo‘lib, spektral maxsusliklari ega bo‘lmasin deb faraz qilinadi.

Aytaylik, $\tau_j=\operatorname*{Res}_{\xi=\xi_j}(a_1(\xi))^{-1}$, $j=1,2,\ldots,N$ bo‘lsin. U holda shunday Γ_j matritsalar mavjud bo‘lib, ushbu

$$\bar{\phi}(\xi_j,x)\tau_j=i\bar{\psi}(\xi_j,x)\Gamma_j,\quad j=1,2,\ldots,N.$$

munosabat bajariladi.

Ushbu $\left\{R(\xi)=-a_{l1}^{-1}(\xi)a_{l2}(\xi),\xi\in\mathbb{R};\xi_1,\xi_2,...,\xi_N;\Gamma_1,\Gamma_2,...,\Gamma_N\right\}$ jamlanmaga (22) sistemaning sochilish nazariyasi berilganlari deyiladi.

$\bar{\psi}(\xi,x)$ va $\psi(\xi,x)$, $\xi\in\mathbb{R}$, Yost matritsalar uchun ushbu

$$\begin{aligned}\bar{\psi}(\xi,x)&=e^{i\xi x}\begin{bmatrix}I_m\\ 0_m\end{bmatrix}+\int_x^{\infty}e^{i\xi y}\bar{K}(x,y)dy,\\ \psi(\xi,x)&=e^{-i\xi x}\begin{bmatrix}0_m\\ I_m\end{bmatrix}+\int_x^{\infty}e^{-i\xi y}K(x,y)dy,\end{aligned}\tag{23}$$

integral tasvirlar o‘rinli. Bunda $\left[\bar{K}(x,y)\;K(x,y)\right]=\begin{bmatrix}K_1(x,y) & K_2(x,y)\\ K_3(x,y) & K_4(x,y)\end{bmatrix}$,

$K_s(x,y),\ s=\overline{1,4}$ - $m\times m$ matritsalar ushbu

$$K(x,y)+\begin{bmatrix} I_m \\ 0_m \end{bmatrix}\Omega(x+y)+\int_x^{\infty}\bar{K}(x,z)\Omega(z+y)dz=0,\quad x>y \tag{24}$$

Gelfand-Levitan-Marchenko integral tenglamalar sistemasini qanoatlantiradi. Bu yerda

$$\Omega(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}R(\xi)e^{i\xi x}d\xi+i\sum_{j=1}^{N}\Gamma_j e^{i\xi_j x}\ .$$

(24) sistemaning yechimlari $\upsilon(x)$ potensial bilan quyidagicha bog‘langan

$$\upsilon(x)=2K_2(x,x)=2K_3(x,x)\,.$$

Ushbu bobning ikkinchi paragrafida matritsaviy manfiy tartibli mKdF tenglamasi uchun qo‘yilgan Koshi masalasi tez kamayuvchi funksiyalar sinfida yechilgan. Xususan, ushbu

$$\begin{cases} w_x(x,t)=\upsilon(x,t)\upsilon_t(x,t)+\upsilon_t(x,t)\upsilon(x,t) \\ \upsilon_{xt}(x,t)+\upsilon(x,t)w(x,t)+w(x,t)\upsilon(x,t)+\upsilon(x,t)=0 \end{cases} \tag{25}$$

sistema

$$\upsilon(x,t)\,|_{t=0}=\upsilon_0(x)\,, \tag{26}$$

boshlang‘ich shart bilan qaralgan bo‘lib, $\upsilon_0(x)$ haqiqiy funksiya quyidagi xossalarga ega

1. $\int_{-\infty}^{\infty}(1+|x|)\|\upsilon_0(x)\|dx<\infty$;

2. Ushbu $L(0)=-iJ\dfrac{d}{dx}-V_0(x)$ sistema N ta $\xi_1(0),\xi_2(0),...,\xi_{2N}(0)$ oddiy xos qiymatlarga ega bo‘lsin, bu yerda $V_0(x)=\begin{pmatrix} 0 & i\upsilon_0(x) \\ i\upsilon_0(x) & 0 \end{pmatrix}$.

Aytaylik $\upsilon(x,t)$ va $w(x,t)$ - $m\times m$ haqiqiy simmetrik matritsalar bo‘lib, ushbu sinfga tegishli bo‘lsin

$$\mathcal{L}=\left\{\begin{array}{c} \upsilon\in C^2(\mathbb{R}\times(0,\infty))\cap(\mathbb{R}\times[0,\infty)),\quad w\in C^{1,0}(\mathbb{R}\times(0,\infty)); \\ \lim\limits_{x\to\pm\infty}w(x,t)=0,\quad t\in\mathbb{R}; \\ \int_{-\infty}^{+\infty}\left(1+|x|\right)\left(\|\upsilon(x,t)\|+\|\upsilon_x(x,t)\|+\|\upsilon_t(x,t)\|+\|\upsilon_{xt}(x,t)\|\right)dx<\infty,\quad t\in\mathbb{R} \end{array}\right\}. \tag{27}$$

Ushbu paragrafning asosiy natijasi quyidagicha

5-Teorema. Agar ushbu $\{\upsilon(x,t),w(x,t)\}$ juflik (25)-(27) masalaning yechimi bo‘lsa, u holda $L(t)=-iJ\dfrac{d}{dx}-V(x,t)$ operatorning sochilish nazariyasi berilganlari quyidagi evolyutsion tenglamalarni qanoatlantiradi

$$\frac{\partial R(\xi,t)}{\partial t}=\frac{i}{2\xi}R(\xi,t)\,,\ \xi\in\mathbb{R}\setminus\{0\},$$

$$\frac{d\xi_j}{dt}=0\,,\qquad \frac{d\Gamma_j(t)}{dt}=\frac{i}{2\xi_j}\Gamma_j(t)\,,\qquad j=1,2,\ldots,N\,.$$

Mazkur bobning uchinchi paragrafi moslangan manbali matritsaviy manfiy tartibli mKdF tenglamasini tez kamayuvchi funksiyalar sinfida integrallashga bag‘ishlangan. Quyidagi

$$\begin{cases} w_x(x,t)=\upsilon(x,t)\upsilon_t(x,t)+\upsilon_t(x,t)\upsilon(x,t) \\ \upsilon_{xt}(x,t)+\upsilon(x,t)w(x,t)+w(x,t)\upsilon(x,t)+\upsilon(x,t)=2\sum_{n=1}^{N}(\varphi_{1n}\hat{\varphi}_{2n}^T+\hat{\varphi}_{1n}\varphi_{2n}^T) \\ -iJ\varphi_n'-V\varphi_n=\xi_n\varphi_n\,, \qquad n=1,2,\ldots,N, \end{cases} \quad (28)$$

sistema ushbu

$$\upsilon\,|_{t=0}=\upsilon_0(x) \qquad (29)$$

boshlang‘ich shart bilan qaralgan. Bu yerda $\varphi_n=\begin{pmatrix}\varphi_{1n}\\ \varphi_{2n}\end{pmatrix}$, $\varphi_n=\varphi_n(x,\xi,t,)$ - (22) sistemaning ξ_n xos qiymatiga mos $m\times m$ o‘lchamli matritsaviy xos vektor funksiyalari, $\hat{\varphi}_n=\varphi_n(x,-\xi,t,)$, $\hat{\varphi}_n=\begin{pmatrix}\hat{\varphi}_{1n}\\ \hat{\varphi}_{2n}\end{pmatrix}$ esa shu sistemaning $-\xi_n$ xos qiymatiga mos $m\times m$ o‘lchamli matritsaviy xos vektor funksiyalari bo‘lib, quyidagi

$$\begin{gathered}\int_{-\infty}^{\infty}\varphi_n^T(x,t)\sigma J\varphi_n dx=\alpha_n(t)I_m\,,\\ \int_{-\infty}^{\infty}\hat{\varphi}_n^T(x,t)J\hat{\varphi}_n dx=\beta_n(t)\,I_m,\qquad n=1,2,...,N\end{gathered} \qquad (30)$$

shartlar bajarilsin. Bu yerda $\upsilon=\upsilon(x,t)$, $\upsilon=\upsilon^T$- $m\times m$ o‘lchamli haqiqiy matritsa, $J=\begin{bmatrix}I_m & 0_m\\ 0_m & -I_m\end{bmatrix}$, I_m -birlik matritsa, $V(x,t)=\begin{bmatrix}0_m & i\upsilon(x,t)\\ i\upsilon(x,t) & 0_m\end{bmatrix}$ - $2m\times 2m$ matritsa, $\sigma=\begin{pmatrix}0 & -I_m\\ I_m & 0\end{pmatrix}$ va $\alpha_n(t)\,,\beta_n(t)$ $\quad n=1,2,\ldots,N$ - oldindan berilgan uzluksiz skalyar funsiyalar.

$\upsilon_0(x)$ haqiqiy matritsa quyidagi xossalarga ega bo‘lsin:

1. $\int_{-\infty}^{\infty}(1+|x|)\|\upsilon_0(x)\|dx<\infty$;

2. Ushbu $L(0)=-iJ\dfrac{d}{dx}-V_0(x)$ sistema N ta $\xi_1(0),\xi_2(0),...,\xi_{2N}(0)$ oddiy xos qiymatlarga ega bo‘lsin.

Ushbu paragrafda (28)-(30) masalaning ushbu

$$\mathcal{L} = \left\{ \begin{array}{c} \upsilon \in C^2(\mathbb{R} \times (0,\infty)) \cap (\mathbb{R} \times [0,\infty)), \quad w \in C^{1,0}(\mathbb{R} \times [0,\infty)); \\ \lim\limits_{x \to \pm\infty} w(x,t) = 0, \quad t \in \mathbb{R}; \\ \int\limits_{-\infty}^{+\infty} \left(1+|x|\right)\left(\|\upsilon(x,t)\| + \|\upsilon_x(x,t)\| + \|\upsilon_t(x,t)\| + \|\upsilon_{xt}(x,t)\|\right)dx < \infty, \quad t \in \mathbb{R} \end{array} \right\} \qquad (31)$$

sinfga tegishli $\left\{\upsilon(x,t), w(x,t), \varphi_n(x,\xi,t), \ n = 1,2,\ldots,N\right\}$ yechimi matritsaviy Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida topilgan bo‘lib, quyidagi teorema isbotlangan

6-Teorema. Aytaylik ushbu $\left\{\upsilon(x,t), w(x,t), \varphi_j(x,\xi,t), \quad j = 1,2,\ldots,N\right\}$ jamlanma (28)-(31) masalaning yechimi bo‘lsin. U holda $L(t) = -iJ\dfrac{d}{dx} - V(x,t)$ operatorning sochilish nazariyasi berilganlari vaqt bo‘yicha quyidagicha o‘zgaradi

$$\frac{\partial R(\xi,t)}{\partial t} = \frac{i}{2\xi} R(\xi,t)\,, \ \xi \in \mathbb{R} \setminus \left\{0\right\},$$

$$\frac{d\xi_j}{dt} = 0\,, \quad \frac{d\Gamma_j(t)}{dt} = \left(\frac{i}{2\xi_j} + \left(\alpha(t) + \beta(t)\right)\right)\Gamma_j(t)\,, \quad \mathrm{Im}\,\xi_j > 0\,, \quad n = 1,2,\ldots,N\,.$$

XULOSA

Dissertatsiya ishining birinchi bobi manfiy tartibli mKdF tenglamasiga qo'yilgan Koshi masalasini sochilish nazariyasining teskari masalasi usuli yordamida tez kamayuvchi funksiyalar sinfida yechishga bag'ishlangan.

Ikkinchi bobida esa moslangan manbali mKdF tenglamasi tez kamayuvchi potensilga ega Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida integrallangan.

Dissertatsiya ishining uchinchi bobida manfiy tartibli mKdF tenglamasining moslangan manbali va moslangan manbasiz hollarda matritsaviy Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida to'la integrallanuvchi bo'lishi isbotlangan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1) manfiy tartibli mKdF tenglamasining Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida tez kamayuvchi funksiyalar sinfida to'la integrallanuvchi bo'lishi isbotlangan;

2) moslangan manbali manfiy tartibli mKdF tenglamasiga qo'yilgan Koshi masalasi Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida yechilgan;

3) matritsaviy manfiy tartibli mKdF tenglamasi matritsaviy Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida tez kamayuvchi funksiyalar sinfida integrallangan;

4) moslangan manbali matritsaviy manfiy tartibli mKdF tenglamasining to'la integrallanuvchi bo'lishi tez kamayuvchi potensialga ega matritsaviy Zaxarov-Shabat sistemasi uchun teskari masala usuli yordamida isbotlangan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREE PhD.03/2025.27.12.FM.06.02 UNDER URGENCH STATE UNIVERSITY NAMED AFTER ABU RAYHAN BIRUNI

URGENCH STATE UNIVERSITY NAMED AFTER ABU RAYHAN BIRUNI

ATANAZAROVA SHOIRA ERKINOVNA

INTEGRATION OF THE NEGATIVE ORDER MODIFIED KORTEWEG-DE VRIES EQUATION WITH A SELF-CONSISTENT SOURCE

01.01.02 – Differential equations and mathematical physics

ABSTRACT
of dissertation of the Doctor of philosofhy (PhD) on physical and matematical sciences

Urgench – 2026

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Comission at the Ministry of Higher education, science and innovations of the Republic of Uzbekistan under number B2024.2.PhD/FM1055.

Dissertation was prepared at the Khorezm regional branch of V.I. Romanovskiy Institute of Mathematics of Uzbekistan Academy of Sciences and Urgench State University named after Abu Rayhan Biruni.

The abstract of the dissertation is posted in three languages (uzbek, russian, english (resume)) on the website (www.ik-mat.urdu.uz) and the "ZiyoNet" Information and educational portal (www.ziyonet.uz).

Scientific supervisor:	**Urazboev Gayrat Urazaliyevich** Doctor of Physical and Matematical Sciences
Official opponents:	**Yaxshimuratov Alisher Bekchanovich** Doctor of Physical and Matematical Sciences
	Baltaeva Umida Ismailovna Doctor of Physical and Matematical Sciences
Leading organization:	**Samarkand State University named after Sharof Rashidov**

Defense will take place "14" February 2026 at 15 : 00 the meeting of Scientific Council number PhD.03/2025.27.12.FM.06.02 at Urgench State University named after Abu Rayhan Biruni. (Address: Kh.Alimdjan str.14, Urgench, 220100, Uzbekistan, Ph.: (+99862) 224-66-11, fax: (+99862) 224-67-00, e-mail: ik-mat.urdu@umail.uz

Dissertation in possible to review in Information-resource centre at Urgench State University named after Abu Rayhan Biruni (is registred № D-939). (Address: Kh.Alimdjan str.14, Urgench, 220100, Uzbekistan, Ph.: (+99862) 224-66-11, fax: (+99862) 224-67-00, e-mail: arm@urdu.uz).

Abstract of dissertation sent out on "02" February 2026 year

(Mailing report № 1 on "02" February 2026 year)

O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI

B. I. Abdullaev
Chairman of Scientific council on award of scientific degree, DSc.

A. A. Reyimberganov
Scientific secretary of Scientific council on award of scientific degree, PhD

B. A. Babajanov
Chairman of Scientific seminar under scientific council on award of scientific degree, DSc.

INTRODUCTION (abstract of the PhD dissertation)

The actuality and demand of the theme of the dissertation. In numerous scientific and applied studies conducted globally, special attention is paid to the study of direct and inverse spectral problems for the system of linear differential equations. Currently, direct and inverse problems of spectral analysis have important applications in constructing exact solutions of nonlinear evolution equations describing complex physical processes and form one of the main research areas in the theory of differential equations. In addition, these problems play a crucial role in solving some problems in geophysics, hydrodynamics, nonlinear optics, acoustics, and quantum mechanics, as well as in the implementation of scientific and experimental research methods. Moreover, the application of the inverse scattering transform method for the Zakharov-Shabat system to construct rapidly decreasing solutions of the Cauchy problem for the negative order modified Korteweg-de Vries equation modeling the propagation of pulses in optical fibers and nonlinear crystals remains one of the important problems of modern mathematical physics today.

Currently, special attention in mathematical physics is focused on finding solutions to the Cauchy problem for negative order nonlinear evolution equations with a self-consistent source and their matrix analogues in the class of rapidly decreasing functions. Such problems represent the perspective direction in modern mathematical physics and are widely used in analyzing wave evolution and their interaction with the environment in electrodynamics, nonlinear optics, and plasma physics. Therefore, it is a priority scientific research to develop the methods that establish the existence of solutions to negative order nonlinear evolution equations with a self-consistent source and their matrix analogues in the class of rapidly decreasing functions.

In our country, special attention is paid to the development of methods for solving actual problems in the spectral theory for the system of linear differential equations, which have scientific and practical applications in fundamental sciences. In particular, growing attention has been directed toward the study of spectral theory for the system of first order ordinary differential operators, and significant results have been achieved in constructing soliton solutions of negative order nonlinear evolution equations of modern mathematical physics using the methods of direct and inverse scattering theory. Expanding the scope of mathematical research and enhancing its effectivness and practical significance have been identified as priority areas for the development of mathematics and its applications[2]. In order to ensure the implementation of this decision, it is essential to study the direct and inverse scattering problems, which enable the integration of negative order nonlinear evolution equations with a self-consistent sources and their matrix analogues.

This dissertation, to a certain extent, contributes to the fulfillment of the objectives defined in Decrees of the President of the Republic of Uzbekistan No. DP-6097 dated October 29, 2020 “On the approval of the concept of science development

---

[2] Resolution of the President of the Republic of Uzbekistan “On measures for improving the quality of education and developing scientific research in the field of mathematics”, May 7, 2020, No.
RP-4708.

until 2030” and No. DP-60 dated January 28, 2022 “On the development strategy of New Uzbekistan for 2022-2026”, Resolutions No. RP-4387 dated July 9, 2019 “On measures for state support for the further development of mathematics education and sciences, and for the radical improvement of the activities of the V.I. Romanovsky Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan” and Resolutions No.RP-4708 dated May 7, 2020 “On measures for improving the quality of education and developing scientific research in the field of mathematics” as well as in other regulatory legal acts related to the basic sciences.

Relevance of the research to the priority areas of the development of science and technology of the Republic. This research was conducted within the framework of the priority area for the development of science and technology in the Republic of Uzbekistan IV. "Mathematics, mechanics and computer science".

The degree of scrutiny of the problem.

In modern mathematical physics, soliton theory and integrating of nonlinear evolution equations (NEEs) through the inverse scattering method (IST) has attracted great interest among researcher. The fundamental study of C.S. Gardner, J.M. Greene, M.D. Kruskal and R.M. Miura (GGKM) is the first fundamental source in the world, where it was solved the initial value problem for the classical Korteweg-de Vries (KdV) equation by employing the IST method to the associated Sturm-Liouville operator. Further significant advancement was made by V.E. Zakharov and A.B Shabat, who developed an analogue of GGKM theory for another physically important NEE known as the nonlinear Schrödinger equation (NLS). Subsequently, M. Wadati, Sh. Tanaka applied the IST technique to modified Korteweg-de Vries (mKdV) equation marking another contribution to the theory of integrable systems. M. J. Ablowitz, D.J. Kaup, A.C. Newell and H. Segur (AKNS) introduced a generalized framework, referred to as AKNS system, by extending approach originally proposed by V. E. Zakharov and A.B. Shabat.

The integration of the NLS and mKdV equations relies on the scattering theory for the Dirac operator, the inverse spectral problem on the real line for which was systematically investigated at the international level in the classical works of M. G. Gasymov, B.M. Levitan, V.E. Zakharov, A.B. Shabat , I. S. Frolov, L.A. Takhtajan, L.D. Faddeev , R. K. Dodd, J. S. Eilbeck, J.D. Gibbon, H.C. Morris, G.L. Lamb, M. J. Ablowitz, D.J. Kaup, A.C. Newell, H. Segur and further developed in our country by A.B. Khasanov and his collaborators. Significant results have been achieved worldwide in the application of the direct and inverse problems of scattering theory for the Dirac operator to NEEs modeling physical processes. In particular, the integration of the NLS equation, the sine-Gordon equation, the mKdV equation, and their matrix analogues using direct and inverse problems of scattering theory was studied in detail in the studies of T. Aktosun, C. van der Mee, and F. Demontis. In particular, they firstly developed the matrix triplet (A,B,C) method to construct the closed form solutions of NEEs. V.K. Melnikov made a great contribution to the solution of NEEs with a self-consistent source using direct and inverse problems of scattering theory. The results obtained by N. Bondarenko in the integration of the matrix KdV equation with a self-consistent source using direct and inverse problems of scattering theory deserve special attention.

In our country, significant results have been achieved in the application of the direct and inverse problems of scattering theory to partial differential equations. In particular, the NLS equation with a self-consistent, the sine-Gordon equation with self-consistent source and the mKdF equation with a self-consistent and their matrix analogues in the class of rapidly decreasing and periodic functions were integrated by A.B. Khasanov, G.U. Urazboev, A.B. Yakshimuratov, B.A. Babajanov, U.A. Khoitmetov, A.A. Reyimberganov, I.I. Baltaeva, M.M. Khasanov and A.K. Babadjanova.

Further studies on the integration of partial differential equations in the world are related to negative order NEEs, which firstly proposed by J.M. Veroski. The negative order KdV equation and the negative order mKdV equation were integrated by Z.J. Qiao, E.G. Fan, Sh. Yuan, J.Xu and A.M. Wazwaz using various analytical methods, and multi-soliton solutions of these equations were found.

In our country, negative order partial differential equations have been studied using the method of direct and inverse problems for differential operators since 2022. Nevertheless, to date, significant progress has also been made by our researchers in the study of negative order NEEs not only in our country, but also worldwide. In particular, G.U. Urazbaev, I.I. Baltaeva and A.K. Babadjanova firstly derived the negative order NLS equation. The negative order KdV equation, the negative order mKdV equation, and the negative order NLS equation in the presense of self-consistent source were studied in the works of A.B. Khasanov, G.O. Urazboyev, A.B. Yakshimuratov, A.A. Reyimberganov, I.I. Baltayeva, M.M. Khasanov, and A.K. Babadjanova in various classes of functions.

Relevance of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation has been executed in a basis of the research plan entitled "Applications of the spectral theory of differential operators to nonlinear evolution equations" in accordance with the research plan of the Department of "Applied mathematics and mathematical Physics" of the Urgench State University named after Abu Rayhan Biruni.

The aim of research work is integration of the negative order mKdV equation with and without a self-consistent source along with its matrix analogs via IST method in the class of rapidly decreasing functions.

Research problems are:

showing the complete integrability of the negative order mKdV equation and negative order mKdV equation with a self-consistent source using IST method for the Zakharov-Shabat system with rapidly decaying potential;

applying the matrix triplet (A, B, C) method for constructing the multisoliton solution of the negative order mKdV equation with a self-consistent source in the class of rapidly decreasing functions;

solving the Cauchy problem for the matrix negative order mKdV equation and matrix negative order mKdV equation with a self-consistent source in the class of rapidly decreasing functions using IST method for the matrix Zakharov-Shabat system.

The research objects are: negative order mKdV equation, negative order mKdV equation with a self-consistent source, negative order mKdV equation with a self-consistent source corresponding to moving eigenvalues, matrix negative order mKdV equation, matrix negative order mKdV equation with a self-consistent source.

The research subject is application of IST method for the non-selfadjoint Zakharov-Shabat system with a rapidly decaying potential to integrating the nonlinear evolution equations and its matrix analogs.

Research methods. Within the framework of the dissertation, methods of mathematical physics, spectral theory of differential operators, functional analysis, theory of functions of complex variables and methods of solving partial differential equations are employed.

Scientific novelty of research work includes the following:

derived the evolution equations for the scattering data of the Zakharov-Shabat system with a rapidly decaying potential associated with the negative order mKdV equation using IST method;

proved that the potential constructed by the derived evolution equations for the scattering data of the Zakharov-Shabat system by IST method is the rapidly decreasing solution of the Cauchy problem for the negative order mKdV equation;

proved the complete integrability of the negative order mKdV equation with a self consistent source via IST method for the Zakharov-Shabat system in the class of rapidly decreasing functions;

applied the matrix triplet (A, B, C) method for constructing the multisoliton solution of the negative order mKdV equation with a self consistent source in the class of rapidly decreasing functions;

integrated the negative order mKdV equation with a self-consistent source corresponding to moving eigenvalues of the Zakharov-Shabat system using IST method in the class of rapidly decreasing functions;

integrated the matrix negative order mKdV equation using IST method for the matrix Zakharov-Shabat system with rapidly decaying potential;

solved the Cauchy problem for the matrix negative order mKdV equation with a self-consistent source via IST method in the class of rapidly decreasing functions.

The practical results of the study include:

developing and applying algorithms to find the solution of the negative order mKdV equation within the class of rapidly decreasing functions with and without a self-consistent source, and for their matrix analogs;

using the matrix triplet (A, B, C) method to obtain closed form solution of the negative order mKdV equation with the self-consistent source, which enable the results to be implemented by MATLAB software package.

Reliability of the research results is substantiated by the rigor of the mathematical reasoning, as well as by the application of methods of mathematical physics, spectral analysis, functional analysis and the integrability theory to solve inverse spectral and scattering problems related to differential operators with rapidly decreasing coefficients. In addition, the publication of the research results in reputable scientific journals with impact-factors and their discussion at scientific seminars also confirm reliability of the dissertation results.

Scientific and practical significance of the research results. The scientific significance of the research results based on applicability of the obtained scientific results to the spectral theory of linear operators and mathematical models describing various physical phenomena such as the traffic flow models, geometry of surfaces with constant negative curvature, nonlinear optics, ion-acoustic solitons with nonlocal effects.

The practical significance of the dissertation is that the proposed algorithms and explicit solution methods can be used for solving nonlinear evolution equations with negative powers and directly implemented in numerical environments, which have significance in mathematical physics and engineering applications.

Implementation of research results. The results on integration of the negative order mKdV equation with a self-consistent source were applied in the following fields:

the results on solving the Cauchy problem for the negative order mKdV equation in the class of rapidly decreasing functions by the IST method for the Zakharov-Shabat system were used for deriving the evolution equations for the matrix negative order KdV equation within the framework of the scientific project "Research Stays for University Academics and Scientists, 2024 (57693448)" in University of Bayreuth, founded by the DAAD. The application of scientific result allowed developing an algorithm for constructing the solution to the initial value problem for the matrix negative order KdV equation via inverse scattering transform method and visualize the wave behavior of obtained solutions of the considered problem using MATLAB software package (Reference from the University of Bayreuth, Germany, October 14, 2025);

the results on integration of matrix negative order mKdV by IST method were used for deriving the evolution equations for scattering data of the operator associated to the matrix sine-Gordon equation with a source in the class of decreasing functions under the scientific project founded by the National Scholarship Programme of the Slovak Republic "Support of Mobility of Students, PhD Students, University Teachers, Researchers and Artists, 2024". The application of scientific result allowed constructing the closed form solutions of the matrix sine-Gordon equation with a source in the class of rapidly decreasing functions by matrix triplet (A,B,C) method. (Reference № 288\D1 from the Institute of Mathematics Slovak Academy of Sciences, Bratislava, October 14, 2025).

Approbation of the research results.

The main results of the dissertation were discussed at 8 scientific and practical conferences, including 4 international ones.

Publication of the research results. According to the theme of the dissertation, 12 scientific works were published, 2 of them were published in the international journals, included in Scopus list, and 2 were published in the national scientific journals included in the list of scientific journals proposed by the Supreme Attestation Commission of the Republic of Uzbekistan for the defense of doctoral dissertations.

The structure and volume of the dissertation. The dissertation consists of three chapters, conclusion and list of the used literatures. The volume of the dissertation is 89 pages.

MAIN CONTENT OF THE DISSERTATION

The first chapter of the dissertation is titled as "**Construction of solution to the negative order modified Korteweg-de Vries equation via inverse scattering transform method**" and it concerns on construction of solution to Cauchy problem for negative order mKdV equation using IST approach. In the first paragraph it is formulated the negative order mKdV equation.

The recursion operator is an important instrument in investigating of nonlinear partial differential equations (PDEs), especially their integrability properties. P.J. Olver showed that, nonlinear evolution equations can be expressed in the operator form

$$u_t = R^n \Delta_i(x,t), \quad i \geq 1, \quad n = 1,2,3,\dots \tag{1}$$

where $\Delta_i(x,t)$ are symmetries and R is recursion operator corresponding to considered evolution equation.

J.M. Verosky introduced a method for constructing negative order evolution equations by extending the Olver's idea in negative direction. Namely, for creating the sequence of negative order nonlinear equations he proposed the relation (1) with negative power of R, which is equivalent to

$$R^n u_t = \Delta_i(x,t), \quad i \geq 1, \; n = 1,2,3,\dots$$

Inspired by the work of J.M. Verosky, A.M. Wazwaz applyed the following recursion operator

$$R = -\partial_x^2 - 4u - 4u_x \partial_x^{-1}(\cdot u), \quad (Rf = -f_{xx} - 4uf - 4u_x \partial_x^{-1}(fu))$$

to the relation $Ru_t = u_x$ and obtained the negative order mKdV equation in the following form

$$-u_{xxt} - 4u^2 u_t - 4u_x \partial_x^{-1}(u \cdot u_t) = u_x. \tag{3}$$

If we assume $\partial_x^{-1} = \int\limits_{-\infty}^{x} dx$ and make denotaion $\int\limits_{-\infty}^{x} 2u \cdot u_t dx = \eta$, then the equation (3) reduces to the equation

$$-u_{xxt} - 2u(\eta_x + 2uu_t\big|_{-\infty}) - 2u_x\eta - u_x = 0.$$

Suppose that the function $u = u(x,t)$ belongs to the class of rapidly decreasing functions. Then we arrive at the negative order mKdV equation in the following form

$$\begin{cases} \eta_x = 2uu_t \\ u_{xt} + 2u\eta + u = 0. \end{cases} \tag{4}$$

In the following paragraphs, we will examine the complete integrability of resulting system (4) by the IST method in the class of rapidly decreasing functions.

The second paragraph provides the basic facts concerning on scattering theory for the following well-known Zakharov-Shabat system of equations

$$\begin{cases} y_{1x} + i\xi\, y_1 = u(x) y_2, \\ y_{2x} - i\xi\, y_2 = -u(x) y_1, \end{cases} \tag{5}$$

on a line ($-\infty < x < \infty$), where the potential $u(x)$ satisfies the condition

$$\int_{-\infty}^{\infty}(1+|x|)|u(x)|dx<\infty\,. \tag{6}$$

The system (5) can be rewritten in the form $Ly=\xi y$, where $y=(y_1,y_2)^T$,

$$L=i\begin{pmatrix}\dfrac{d}{dx} & -u(x)\\ -u(x) & -\dfrac{d}{dx}\end{pmatrix}.$$

Due to the condition (6), the system (5) has specific solutions $\phi(x,\xi)$ and $\psi(x,\xi)$, usually known as the Jost solutions, which are uniquely determined by the following asymptotics on real values of ξ

$$\left.\begin{array}{l}\phi(x,\xi)\sim\begin{pmatrix}1\\0\end{pmatrix}e^{-i\xi x}\\ \bar{\phi}(x,\xi)\sim\begin{pmatrix}0\\-1\end{pmatrix}e^{i\xi x}\end{array}\right\},\quad x\to-\infty,\qquad \left.\begin{array}{l}\psi(x,\xi)\sim\begin{pmatrix}0\\1\end{pmatrix}e^{i\xi x}\\ \bar{\psi}(x,\xi)\sim\begin{pmatrix}1\\0\end{pmatrix}e^{-i\xi x}\end{array}\right\},\quad x\to\infty.$$

At any real $\xi\neq 0$, the pair of functions $\psi(x,\xi)$ and $\bar{\psi}(x,\xi)$ forms a complete system of solutions to (5). Consequently, the following equality is fulfilled

$$\phi(x,\xi)=a(\xi)\bar{\psi}(x,\xi)+b(\xi)\psi(x,\xi)\,,$$

where the quantities $a(\xi)$ and $b(\xi)$ are independent of x. An elementary verification shows that

$$a(\xi)=W\{\phi,\psi\}\equiv\phi_1\psi_2-\phi_2\psi_1.$$

The function $a(\xi)$ analytically continues into the upper half-plane $\operatorname{Im}\xi>0$ and may have a finite number of simple zeros there. The discrete eigenvalues of the system (5) coincide with the zeros of the function $a(\xi)$ in the upper half-plane $\operatorname{Im}\xi>0$. Therefore,

$$\phi_k(x)=b_k\psi_k(x)\,,\ k=1,2,\ldots,N.$$

Definition 1. The set of quantities

$$\left\{r^+(\xi)=\frac{a(\xi)}{b(\xi)},\ \xi\in R;\ \xi_k,\operatorname{Im}\xi_k>0;\ \ b_k,\quad k=1,2,\ldots,N\right\}$$

is called the scattering data for the system (5).

It is well-known that for Jost solutions $\phi(x,\xi)$ and $\psi(x,\xi)$ the following integral representations are valid

$$\psi(x,\xi)=\begin{pmatrix}0\\1\end{pmatrix}e^{i\xi x}+\int_x^{\infty}K(x,s)e^{i\xi s}ds,$$

$$\bar{\psi}(x,\xi)=\begin{pmatrix}1\\0\end{pmatrix}e^{-i\xi x}+\int_x^{\infty}\bar{K}(x,s)e^{-i\xi s}ds,$$

where $K\left(x,s\right)=\begin{pmatrix} K_1\left(x,s\right) \\ K_2\left(x,s\right) \end{pmatrix}$ and $\bar{K}\left(x,s\right)=\begin{pmatrix} \bar{K}_1\left(x,s\right) \\ \bar{K}_2\left(x,s\right) \end{pmatrix}$ are solutions to the following Gelfand-Levitan-Marchenko (GLM) system of integral equations

$$\bar{K}(x,y)+\begin{pmatrix} 0 \\ 1 \end{pmatrix}F(x+y)+\int_x^\infty K(x,s)F(s+y)ds=0,$$

$$K(x,y)-\begin{pmatrix} 1 \\ 0 \end{pmatrix}\bar{F}(x+y)-\int_x^\infty \bar{K}(x,s)\bar{F}(s+y)ds=0,$$

where

$$F(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{b(\xi)}{a(\xi)}e^{i\xi x}d\xi-i\sum_{j=1}^{N}c_je^{i\xi_j x},\ \bar{F}(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{b}(\xi)}{\bar{a}(\xi)}e^{i\xi x}d\xi+i\sum_{j=1}^{N}\bar{c}_je^{-i\bar{\xi}_j x},$$

where $c_k=\dfrac{b_k}{\dot{a}(\xi_k)}$, $\bar{c}_k=\dfrac{\bar{b}_k}{\dot{\bar{a}}(\bar{\xi}_k)}$, and $\dot{a}(\xi_k)=\left.\dfrac{\partial a(\xi)}{\partial\xi}\right|_{\xi=\xi_k}$, $\dot{\bar{a}}(\xi_k)=\left.\dfrac{\partial a(\bar{\xi})}{\partial\bar{\xi}}\right|_{\bar{\xi}=\bar{\xi}_k}$, $k=1,\dots,N$. The solution of GLM system of equations has the following relations with the potential of the system (5)

$$u(x)=-2K_1(x,x),\qquad -2K_2(x,x)=\int_x^\infty |u(s)|^2\,ds.$$

The third parapraph is devoted to solve the Cauchy problem for negative order mKdV equation in the class of rapidly decreasing functions. We analyze the following system of equations

$$\begin{cases} \eta_x=2uu_t \\ u_{xt}+2\eta u+u=0,\ \ x\in\mathbb{R},\ \ t\geq 0 \end{cases} \tag{7}$$

under the initial condition

$$u(x,0)=u_0(x),\ x\in\mathbb{R} \tag{8}$$

where the initial function is real-valued function with the following properties:

1. $\int_{\mathbb{R}}\left(1+|x|\right)^5|u_0(x)|dx<\infty$,
2. The operator $L(0)=i\begin{pmatrix} \dfrac{d}{dx} & -u_0(x) \\ -u_0(x) & -\dfrac{d}{dx} \end{pmatrix}$ possesses a finite number of eigenvalues, precisely, $2N$ simple eigenvalues $\xi_1(0),\xi_2(0),\dots\xi_{2N}(0)$ such that $\operatorname{Im}\xi_k(0)>0$, $\xi_{N+k}(0)=-\xi_k(0)$, $k=1,\dots,N$ and has no spectral singularities.

Let the functions $u=u(x,t)$ and $\eta=\eta(x,t)$ are sufficiently smooth real-valued functions belonging to the class

$$\mathcal{L} = \left\{ \begin{array}{c} u \in C^2(\mathbb{R}\times(0,\infty)) \cap (\mathbb{R}\times[0,\infty)), \quad \eta \in C^{1,0}(\mathbb{R}\times(0,\infty)); \\ \lim\limits_{x\to\pm\infty} \eta(x,t) = 0, \quad t\in\mathbb{R}; \\ \int\limits_{-\infty}^{+\infty} \left(1+|x|\right)\left(|u(x,t)| + |u_x(x,t)| + |u_t(x,t)| + |u_{xt}(x,t)|\right) dx < \infty, \quad t \in \mathbb{R} \end{array} \right\} \qquad (9)$$

The primary aim of this section is constructing the solution $\{u(x,t),\eta(x,t)\}$ of the problem (7)-(9) in the class of rapidly decreasing functions by employing the inverse scattering technique for operator $L(t)$

Theorem 1. Suppose that $\{u(x,t),\eta(x,t)\}$ is solution of the problem (7)-(9). Then the scattering data of the operator

$$L(t) = i\begin{pmatrix} \frac{d}{dx} & -u(x,t) \\ -u(x,t) & -\frac{d}{dx} \end{pmatrix}$$

evolves in time according to the following differential equations,

$$\frac{\partial r^+(\xi,t)}{\partial t} = \frac{i}{2\xi} r^+(\xi,t)\,,\ \xi \in \mathbb{R}\setminus\{0\},$$

$$\frac{d\xi_k}{dt} = 0\,, \qquad \frac{dc_k(t)}{dt} = \frac{i}{2\xi_k} c_k(t), \quad k = 1,2,...,N\,.$$

General assumptions 1. Let $\{\xi_k\}_{k=1}^N \subset \mathbb{C}\setminus\{0\}$ are discrete eigenvalues, $\operatorname{Im}\xi_k \ge 0$ and $\{c_k\}_{k=1}^N \subset \mathbb{C}$ are corresponding norming constants with $c_k = \overline{c}_k$. Moreover, $r_0^+ : \mathbb{R} \to \mathbb{C}$ satisfies the following conditions

$$\left|r_0^+(\xi)\right| < 1,\ r_0^+(-\xi) = \overline{r}_0^+(\xi)\,,\ r_0^+(\xi) = o(\xi^{-1}) \text{ for } \xi \to \pm\infty\,,$$

$$\int_{\mathbb{R}} \left(1+|\xi|\right)^3 \left(\left|r_0^+(\xi)\right| + \left|\frac{d}{d\xi} r_0^+(\xi)\right| + \left|\frac{d^2}{d\xi^2} r_0^+(\xi)\right| \right) d\xi < \infty,$$

Based on results of Theorem 1 and the inverse scattering construction presented in the previous paragraph, we propose the following algorithm for obtaining solution of the problem (7)-(9).

Algorithm 1.

5. Find the set of scattering data $\left\{ {r^+}_0(\xi);\ \xi_k(0);\ b_k(0), \quad k = 1,\ldots,N \right\}$ *for a given* $u(0)$.

6. Define the function

$$F(x,t) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} r_0^+(0,\xi) e^{\frac{i}{2\xi}t + i\xi x} d\xi - i\sum_{k=1}^N c_k(0) e^{\frac{i}{2\xi_k}t + i\xi_k x}.$$

7. Find the function $K(x,y,t) = \begin{pmatrix} K_1(x,y,t) \\ K_2(x,y,t) \end{pmatrix}$ *by solving the integral equation*

$$\begin{cases} K_2(x,y,t)+\int\limits_x^\infty K_1(x,s,t)F(s+y,t)ds=0 \\ -K_1(x,y,t)+F(x+y,t)+\int\limits_x^\infty K_2(x,s,t)F(s+y,t)ds=0. \end{cases}$$

8. Recover the functions $u(x,t)$ and $\eta(x,t)$ *by the formulas*

$$u(x,t)=-2K_1(x,x),\quad \eta_x(x,t)=2u(x,t)u_t(x,t).$$

Theorem 2. Suppose that the *General assumptions 1* are satisfied. Then the pair of functions $\{u(x,t),\eta(x,t)\}$, constructed using the *Algorithm 1*, solves the problem (7)-(9).

In the second chapter **"Intergration of the negative order modified Korteweg-de Vries equation with a self-consistent source in the class of rapidly decreasing functions"** it was solved the Cauchy problem for the negative order mKdV equation with a self-consistent sources via IST method for Zakharov-Shabat system.

In the first paragraph, we consider the following system of equations

$$\begin{cases} \eta_x-2uu_t=G_1 \\ u_{xt}+2\eta u+u=G_2 \end{cases} \tag{10}$$

with sufficiently smooth functions $G_1=G_1(x,t)$ and $G_1=G_2(x,t),\ \ t\in\mathbb{R}$ having properties $G_1(x,t)=o(1)$ and $G_1(x,t)=o(1)$ on $x\to\pm\infty$.

The system (10) is considered under the initial condition

$$u(x,0)=u_0(x),\ x\in\mathbb{R}, \tag{11}$$

where the initial real-valued function $u_0(x)$ satisfies the following conditions

1. $\int_{\mathbb{R}}\left(1+|x|\right)^5|u_0(x)|dx<\infty$,
2. The operator $L(0)$ does not have spectral singularities and possesses exactly $2N$ simple eigenvalues $\xi_1(0),\xi_2(0),\ldots\xi_{2N}(0)$ such, that $\operatorname{Im}\xi_k(0)>0$, $\xi_{N+k}(0)=-\xi_k(0)$, $k=1,\ldots,N$.

Let the functions $u=u(x,t)$ and $\eta=\eta(x,t)$, $t\in\mathbb{R}$ are sufficiently smooth real-valued functions belonging to the class

$$\mathcal{L}=\left\{\begin{array}{c} u(x,t)\in C^2(\mathbb{R}\times(0,\infty))\cap(\mathbb{R}\times[0,\infty)),\quad \eta(x,t)\in C^{1,0}(\mathbb{R}\times[0,\infty)), \\ \lim\limits_{x\to\pm\infty}\eta(x,t)=0,\quad t\in\mathbb{R}, \\ \int\limits_{-\infty}^{+\infty}\left(1+|x|\right)\left(|u(x,t)|+|u_x(x,t)|+|u_t(x,t)|+|u_{xt}(x,t)|\right)dx<\infty, \end{array}\right\}. \tag{12}$$

In this paragraph we find the solution to Cauchy problem (10)-(12) for the negative order mKdV equation with a source by employing the IST method to the operator $L(t)$.

Lemma 1. If the pair $\{u(x,t),\eta(x,t)\}$ is a solution of the problem (10)-(12), then the scattering data of the operator $L(t)$ depends on t as follows

$$\frac{\partial r^{+}(\xi,t)}{\partial t}=\frac{i}{2\xi}r^{+}(\xi,t)-\frac{i}{a^{2}(\xi)}\int\limits_{-\infty}^{+\infty}\hat{\phi}^{T}G\phi dx,\qquad \xi\in\mathbb{R}\setminus\{0\},$$

$$\frac{d\xi_n(t)}{dt}=-\frac{i}{b_n\dot{a}\left(\xi_n\right)}\int\limits_{-\infty}^{\infty}\hat{\phi}_n^{T}G\phi_n dx,\quad n=1,\ldots,N,$$

$$\frac{dc_n(t)}{dt}=\left(\frac{i}{2\xi_n}-\frac{i}{2b_n\dot{a}(\xi_n)}\int\limits_{-\infty}^{\infty}\hat{\chi}_n^{T}G\phi_n\,dx-\frac{\partial(\dot{a}(\xi_n))}{\dot{a}(\xi_n)\partial t}\right)c_n(t).$$

In the second paragraph we analyze the following system of equations

$$\begin{cases}\eta_x=2uu_t\\ u_{xt}+2\eta u+u=\sum\limits_{k=1}^{2N}\left(f_{k1}^{2}-f_{k2}^{2}\right),\quad x\in\mathbb{R},\\ Lf_k=\xi_k f_k,\ \ k=1,2,\ldots,2N,\end{cases}\tag{13}$$

under the initial condition

$$u(x,0)=u_0(x),\ x\in\mathbb{R},\tag{14}$$

where the initial real-valued function $u_0(x)$ satisfies the following conditions

3. $\int_{\mathbb{R}}\left(1+|x|\right)^{5}|u_0(x)|dx<\infty$,

4. The operator $L(0)$ does not have spectral singularities and possesses exactly $2N$ simple eigenvalues $\xi_1(0),\xi_2(0),\ldots\xi_{2N}(0)$ such, that $\operatorname{Im}\xi_k(0)>0$, $\xi_{N+k}(0)=-\xi_k(0)$, $k=1,2,\ldots,N,$.

In the considered problem, $f_k=(f_{k1},f_{k2})^{T}$ is eigenfunction of the operator $L(t)$ corresponding to the eigenvalue ξ_k normalized by the condition

$$\int\limits_{-\infty}^{+\infty}f_{k1}f_{k2}dx=A_k(t),\ \ k=1,2,\ldots,2N,\tag{15}$$

where $A_k(t)$ is given continuous nonzero functions satisfying the conditions

$$A_k(t)=A_{N+k}(t)\ \text{ for }\ \xi_k=-\xi_{N+k},\ \ k=1,2,\ldots,N.$$

Additionally, the functions $u=u(x,t)$ and $\eta=\eta(x,t)$ are sufficiently real-valued smooth functions from the class

$$\mathcal{L}=\left\{\begin{array}{c}u(x,t)\in C^{2}(\mathbb{R}\times(0,\infty))\cap(\mathbb{R}\times[0,\infty)),\quad \eta(x,t)\in C^{1,0}(\mathbb{R}\times[0,\infty));\\ \lim\limits_{x\to\pm\infty}\eta(x,t)=0,\ \ t\in\mathbb{R};\\ \int\limits_{-\infty}^{+\infty}\left(1+|x|\right)\left(|u(x,t)|+|u_x(x,t)|+|u_t(x,t)|+|u_{xt}(x,t)|\right)dx<\infty.\end{array}\right\}.\tag{16}$$

In this paragraph, we aim to derive the representation for solution $\{u(x,t),\eta(x,t), f_k(x,t),\ k=1,2,\dots,N\}$ of the problem (13)-(16) in the framework of IST method for the operator $L(t)$.

Theorem 3. If the set of functions $\{u(x,t),\eta(x,t), f_k(x,t), k=1,2,\dots,N\}$ is solution of the problem (13) – (16), then the scattering data of the operator $L(t)$ with rapidly decreasing potential $u(x,t)$, satisfy the following evolution equations

$$\frac{\partial r^+(\xi,t)}{\partial t}=\frac{i}{2\xi_n}r^+(\xi,t),\qquad \xi=\mathbb{R}\setminus\{0\},$$

$$\frac{d\xi_n(t)}{dt}=0,\quad \frac{db_n(t)}{dt}=\left(\frac{i}{2\xi_n}+\frac{i}{\xi_n}A_n(t)\right)b_n(t),\ n=1,2,\dots,N.$$

Based on the results of Theorem 3, we construct the following one-soliton solution of the problem (13)-(16)

$$u(x,t)=-\frac{2}{\cosh(2x-\gamma(t))},\qquad \eta(x,t)=-\frac{2}{\cosh^2(2x-\gamma(t))},$$

$$f_1(x,t)=\frac{\sqrt{A_1(t)}}{\cosh(2x-\gamma(t))}\begin{pmatrix} e^{-x+\frac{\gamma(t)}{2}} \\ e^{x-\frac{\gamma(t)}{2}}\end{pmatrix}$$

under the initial condition

$$u(x,0)=-\frac{2}{\cosh(2x)},$$

where $\gamma(t)=\frac{1}{2}t+\int_0^t A_1(\rho)d\rho.$

In the third paragraph we have integrated the negative order mKdV equation with a self-consistent source corresponding to moving eigenvalues in the class of rapidly decreasing functions. We consider the system of equations

$$\begin{cases} \eta_x-2uu_t=-2\sum_{k=1}^{2N}u(f_{k1}g_{k1}-f_{k2}g_{k2}) \\ u_{xt}+u+2\eta u=\sum_{k=1}^{2N}\frac{d}{dx}\left(f_{k1}g_{k1}-f_{k2}g_{k2}\right), \\ (L-\xi_k)f_k=0,\ (L-\xi_k)g_k=0,\ k=1,2,\dots,2N, \end{cases} \tag{17}$$

under the initial condition

$$u(x,0)=u_0(x),\quad x\in R, \tag{18}$$

where $u_0(x)$ is real-valued function having the property

$$\int_{\mathbb{R}}(1+|x|)^5|u_0(x)|dx<\infty.$$

We assume that the operator $L(0)$ does not have spectral singularities, as well as, it possesses exactly $2N$ simple eigenvalues $\xi_1(0), \xi_2(0), ..., \xi_{2N}(0)$, such that $\operatorname{Im}(\xi_k(0)) > 0, \ \ \xi_{N+k}(0) = -\xi_k(0)$, $k = 1, 2, \ldots, N$.

Let the real-valued functions $u(x,t)$ and $\eta(x,t)$ belong to the class

$$\mathcal{L} = \left\{ \begin{array}{c} u(x,t) \in C^2(\mathbb{R}\times(0,\infty)) \cap (\mathbb{R}\times[0,\infty)), \quad \eta(x,t) \in C^{1,0}(\mathbb{R}\times[0,\infty)) \\ \lim\limits_{x\to\pm\infty} \eta(x,t) = 0, \quad t \in \mathbb{R}; \\ \int\limits_{-\infty}^{+\infty} \left(1+|x|\right)\left(|u(x,t)| + |u_x(x,t)| + |u_t(x,t)| + |u_{xt}(x,t)|\right) dx < \infty, \quad t \in \mathbb{R} \end{array} \right\}. \tag{19}$$

Furthermore, let $f_k = \left(f_{k1}(x,t), \ f_{k2}(x,t)\right)^T$ is an eigenfunction of the operator $L(t)$ corresponding to the eigenvalue ξ_k . $g_k = \left(g_{k1}(x,t), \ g_{k2}(x,t)\right)^T$ is a solution of the equation $L(t) g_k = \xi_k g_k$, which is the linearly independent of f_k, such that

$$W\{f_k, g_k\} \equiv f_{k1} g_{k2} - f_{k2} g_{k1} = \omega_k(t), \quad k = 1, 2, \ldots, 2N, \tag{20}$$

$$\lim_{\delta\to\infty} \int_{-\delta}^{\delta} \left(f_{k1} g_{k2} + f_{k2} g_{k1} \right) dx = A_k(t), \quad k = 1, 2, \ldots, 2N \tag{21}$$

where $A_k(t)$ and quantities $\omega_k(t)$ are real continuous functions of t satisfying the following condition

$$\omega_k(t) = -\,\omega_{N+k}(t)\,, A_k(t) = -\,A_{N+k}(t) \quad \text{for} \quad \xi_n = -\,\xi_k, \ \ k = 1, 2, \ldots, N.$$

We aim to find the solution $\left\{u\left(x,t\right), \eta(x,t), f_k(x,t), g_k(x,t), k = 1, 2, \ldots N\right\}$ to the problem (17)-(21) in the framework of IST method for the operator $L(t)$.

The obtained results were formulated as the following theorem

Theorem 4. If the set of functions $\left\{u\left(x,t\right), \eta(x,t), f_k(x,t), g_k(x,t), k = 1, 2, \ldots N\right\}$ satisfies the problem (17)-(21), then the scattering data associated with operator $L(t)$ evolve over time t in accordance with equations

$$\frac{\partial r^+(\xi,t)}{\partial t} = \left(\frac{i}{2\xi} + i \sum_{k=1}^{N} \omega_k(t) \left(\frac{1}{\xi - \xi_k} + \frac{1}{\xi_k + \xi} \right) \right) r^+(\xi,t),$$

$$\frac{d\xi_n(t)}{dt} = i\omega_n(t), \quad n = 1, 2, \ldots, N,$$

$$\frac{db_n(t)}{dt} = \left(\frac{i}{2\xi_n} + i\beta_n(t)\omega_n(t) \right) b_n(t).$$

We have derived the single soliton solution of the problem (17)-(21) under the initial condition

$$u(x,0) = \frac{-2}{\cosh(2x)}.$$

The corresponding scattering data take the form

$$r(x,0) = 0, \ \xi_1(x,0) = i, \ c_1(x,0) = 2i.$$

Solving the inverse scattering problem for $L(t)$ with resulting initial values of scattering data, we find

$$u(x,t)=\frac{-2\mu_1}{\cosh(2\mu_1 x-\tilde{\sigma})},$$

with $\xi_1=i\mu_1(t)$, $\mu_1(t)=\left(1+\frac{1}{2}\int_0^\tau \omega_1(\tau)d\tau\right)$, $\frac{d\tilde{\sigma}(t)}{dt}=\frac{1}{2\mu_1}+i\beta_1\omega_1$, where $\mu_1,\ \beta_1,\ \omega_1,\ \tilde{\sigma}$, are real functions of t.

Consequently, we obtain

$$f_{11}=\frac{e^{-\mu_1 x}}{2\cosh(2\mu_1 x-\tilde{\sigma})},\quad f_{12}=\frac{e^{\mu_1 x-\tilde{\sigma}}}{2\cosh(2\mu_1 x-\tilde{\sigma})},$$

$$g_{11}=\frac{\left((2i\mu_1\beta_1-4\mu_1 x-1)e^{-\mu_1 x+\tilde{\sigma}}-e^{3\mu_1 x-\tilde{\sigma}}\right)\omega_1}{2\cosh(2\mu_1 x-\tilde{\sigma})},$$

$$g_{12}=\frac{\left((2i\mu_1\beta_1-4\mu_1 x+1)e^{\mu_1 x}+e^{-3\mu_1 x+2\tilde{\sigma}}\right)\omega_1}{2\cosh(2\mu_1 x-\tilde{\sigma})}.$$

From the first equation of the system (17) ,we can find

$$\eta(x,t)=\frac{-1}{\cosh^2(2\mu_1 x-\tilde{\sigma})}.$$

Note that the quantity β_1 can be found by the condition (21).

The third chapter **“Intergration of the matrix negative order modified Korteweg-de Vries equation with a self-consistent source in the class of rapidly decreasing functions”** concerns on the integration of matrix negative order mKdV equation with and without a self-consistent source by applying the IST technique. In the first paragraph we analyze the following matrix Zakharov-Shabat system

$$-iJ\frac{d}{dx}Y(x,\xi)-V(x)Y(x,\xi)=\xi Y(x,\xi), \tag{22}$$

where $J=\begin{bmatrix} I_m & 0 \\ 0 & -I_m \end{bmatrix}$ with I_m the identity matrix of order m, and the potential $V(x)=\begin{bmatrix} 0 & i\upsilon(x) \\ i\upsilon(x) & 0 \end{bmatrix}$ with $m\times m$ real matrix $\upsilon(x)$ satisfies the following condition

$$\int_{-\infty}^{\infty}\left(1+|x|\right)\|V(x)\|dx<\infty,$$

where the matrix norm is defined as $\|V(x)\|=\max_j\sum_{k=1}^{m}\left|V_{jk}(x)\right|$. In (22) $Y(x,\xi)$ is column vector function $Y=(y_j)_{j=1}^{2m}$.

For real ξ the Jost solution $\Phi_l(x,\xi)$ and $\Phi_r(x,\xi)$ defined as the $2m\times 2m$ matrix solutions of (22) satisfying the following boundary conditions:

$$\Phi_l(x,\xi)=[\bar{\psi}(x,\xi)\ \ \psi(x,\xi)]\to e^{i\xi Jx}I_{2m},\quad x\to\infty,$$
$$\Phi_r(x,\xi)=[\phi(x,\xi)\ \ \bar{\phi}(x,\xi)]\to e^{i\xi Jx}I_{2m},\quad x\to-\infty.$$

For $\xi\in\mathbb{R}$ we have

$$\Phi_l(x,\xi)=\Phi_r(x,\xi)A_l(\xi),$$

where $A_l(\xi)=\begin{bmatrix} a_{l1}(\xi) & a_{l2}(\xi) \\ a_{l3}(\xi) & a_{l4}(\xi)\end{bmatrix}$, $a_{li}(\xi)$, $i=1,2,3,4$, are $m\times m$ matrices.

The equation $\det a_{l1}(\xi)=0$ has exactly a finite number of simple zeros ξ_j $j=1,2,\dots,N$ in the half-plane $\operatorname{Im}\xi>0$, which correspond to the eigenvalues of the operator L. The real zeros of the equation $\det a_{l1}(\xi)=0$ is called the spectral singularities of the system (22). In this chapter we assume that the system (22) has no spectral singularities and all eigenvalues are simple.

Let $\tau_j=\operatorname*{Res}_{\xi=\xi_j}s(a_1(\xi))^{-1}$, $j=1,2,\dots,N$, then there exist matrices Γ_j such that

$$\bar{\phi}(\xi_j,x)\tau_j=i\bar{\psi}(\xi_j,x)\Gamma_j,\quad j=1,2,\dots,N.$$

The set $\left\{R(\xi)=-a_{l1}^{-1}(\xi)a_{l2}(\xi),\xi\in\mathbb{R};\xi_1,\xi_2,...,\xi_N;\Gamma_1,\Gamma_2,...,\Gamma_N\right\}$ is called the scattering data associated with the equation (22).

The Jost solutions $\bar{\psi}(\xi,x)$ and $\psi(\xi,x)$ of the equation (22) at any $\xi\in\mathbb{R}$ can be represented in the following form

$$\begin{aligned}\bar{\psi}(\xi,x)&=e^{i\xi x}\begin{bmatrix} I_m \\ 0_m\end{bmatrix}+\int_x^\infty e^{i\xi y}\bar{K}(x,y)dy,\\ \psi(\xi,x)&=e^{-i\xi x}\begin{bmatrix} 0_m \\ I_m\end{bmatrix}+\int_x^\infty e^{-i\xi y}K(x,y)dy,\end{aligned}\tag{23}$$

where

$$\left[\bar{K}(x,y)\ K(x,y)\right]=\begin{bmatrix} K_1(x,y) & K_2(x,y) \\ K_3(x,y) & K_4(x,y)\end{bmatrix},$$

$K_s(x,y)$, $s=\overline{1,4}$ are $m\times m$ matrices.

The kernels of (23) satisfy the following Gelfand-Levitan-Marchenko (GLM) integral equations for $x>y$

$$K(x,y)+\begin{bmatrix} I_m \\ 0_m\end{bmatrix}\Omega(x+y)+\int_x^\infty \bar{K}(x,z)\Omega(z+y)dz=0,\tag{24}$$

where

$$\Omega(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}R(\xi)e^{i\xi x}d\xi+i\sum_{j=1}^{N}\Gamma_j e^{i\xi_j x}.$$

The solution of (24) have relations with the potential

$$\upsilon(x)=2K_2(x,x)=2K_3(x,x).$$

In the second paragraph we have solved the Cauchy problem for matrix negative order mKdV equation in the class of rapidly decreasing functions.

We consider the integration of the following problem

$$\begin{cases} w_x(x,t)=\upsilon(x,t)\upsilon_t(x,t)+\upsilon_t(x,t)\upsilon(x,t) \\ \upsilon_{xt}(x,t)+\upsilon(x,t)w(x,t)+w(x,t)\upsilon(x,t)+\upsilon(x,t)=0 \end{cases} \tag{25}$$

under the initial condition

$$\upsilon(x,t)|_{t=0}=\upsilon_0(x), \tag{26}$$

where $\upsilon_0(x)$ is real-valued matrix function having the following properties

1. $\int_{-\infty}^{\infty}(1+|x|)\|\upsilon_0(x)\|dx<\infty$;

2. Matrix operator $L(0)=-iJ\frac{d}{dx}-V_0(x)$ possess exactly N simple eigenvalues $\xi_1(0),\xi_2(0),...,\xi_N(0)$, where $V_0(x)=\begin{pmatrix} 0 & i\upsilon_0(x) \\ i\upsilon_0(x) & 0 \end{pmatrix}$.

Let $\upsilon(x,t)$ and $w(x,t)$ are real-valued symmetric $m\times m$ matrices belonging to the class

$$\mathcal{L}=\left\{\begin{array}{c} \upsilon\in C^2(\mathbb{R}\times(0,\infty))\cap(\mathbb{R}\times[0,\infty)),\quad w\in C^{1,0}(\mathbb{R}\times(0,\infty)); \\ \lim_{x\to\pm\infty}w(x,t)=0,\quad t\in\mathbb{R}; \\ \int_{-\infty}^{+\infty}\left(1+|x|\right)\left(\|\upsilon(x,t)\|+\|\upsilon_x(x,t)\|+\|\upsilon_t(x,t)\|+\|\upsilon_{xt}(x,t)\|\right)dx<\infty,\quad t\in\mathbb{R} \end{array}\right\}. \tag{27}$$

The main purpose of this paragraph is finding the solution $\{\upsilon(x,t),w(x,t)\}$ of the problem (25)-(27) in the framework of IST method.

Theorem 5. If the pair $\{\upsilon(x,t),w(x,t)\}$ is a solution of the problem (25)-(27), then the scattering data for the operator

$$L(t)=-iJ\frac{d}{dx}-V(x,t)$$

satisfy the relations

$$\frac{\partial R(\xi,t)}{\partial t}=\frac{i}{2\xi}R(\xi,t),\ \xi\in\mathbb{R}\setminus\{0\},$$

$$\frac{d\xi_j}{dt}=0,\quad \frac{d\Gamma_j(t)}{dt}=\frac{i}{2\xi_j}\Gamma_j(t),\quad j=1,2,\ldots,N.$$

In the third paragraph we have integrated the matrix negative order mKdV equation with a self-consistent source in the class rapidly decreasing functions.

We consider the integration of the following problem

$$\begin{cases} w_x(x,t) = \upsilon(x,t)\upsilon_t(x,t) + \upsilon_t(x,t)\upsilon(x,t) \\ \upsilon_{xt}(x,t) + \upsilon(x,t)w(x,t) + w(x,t)\upsilon(x,t) + \upsilon(x,t) = 2\sum_{n=1}^{N}(\varphi_{1n}\hat{\varphi}_{2n}^T + \hat{\varphi}_{1n}\varphi_{2n}^T) \\ -iJ\varphi_n' - V\varphi_n = \xi_n\varphi_n \ , \quad -iJ\hat{\varphi}_n' - V\hat{\varphi}_n = -\xi_n\hat{\varphi}_n \qquad n = 1,2,...,N, \end{cases} \tag{28}$$

under the initial condition

$$\upsilon\,|_{t=0} = \upsilon_0(x). \tag{29}$$

Here $\varphi_n = \begin{pmatrix} \varphi_{1n} \\ \varphi_{2n} \end{pmatrix}$, $\varphi_n = \varphi_n(x,\xi,t)$ are $m \times m$ matrix eigenfunction that corresponds to the eigenvalue ξ_n, and $\hat{\varphi}_n = \varphi_n(x,-\xi,t)$, $\hat{\varphi}_n = \begin{pmatrix} \hat{\varphi}_{1n} \\ \hat{\varphi}_{2n} \end{pmatrix}$, are $m \times m$ matrix eigenfunction that corresponds to the eigenvalue $-\xi_n$ of the system (22), which are normalized by the following conditions

$$\begin{gathered} \int_{-\infty}^{\infty} \varphi_n^T(x,t)\sigma J\varphi_n dx = \alpha_n(t) I_m, \\ \int_{-\infty}^{\infty} \hat{\varphi}_n^T(x,t) J\hat{\varphi}_n dx = \beta_n(t) I_m, \quad n = 1,2,...,N \end{gathered} \tag{30}$$

furthermore $\upsilon = \upsilon(x,t)$ is a real symmetric $m \times m$ matrix $\upsilon = \upsilon^T$, $J = \begin{bmatrix} I_m & 0_m \\ 0_m & -I_m \end{bmatrix}$, I_m is the unit matrix, $V(x,t) = \begin{bmatrix} 0_m & i\upsilon(x,t) \\ i\upsilon(x,t) & 0_m \end{bmatrix}$ is $2m \times 2m$ matrix, $\sigma = \begin{pmatrix} 0 & -I_m \\ I_m & 0 \end{pmatrix}$ and $\alpha_n(t)$, $\beta_n(t)$ $n = 1,2,\ldots,N$ are nonzero continuous scalar functions.

The real-valued matrix function $\upsilon_0(x)$ satisfy the following properties:

1. $\int_{-\infty}^{\infty} (1+|x|)\|\upsilon_0(x)\| dx < \infty$;

2. The operator $L(0) = -iJ\dfrac{d}{dx} - V_0(x)$ possess exactly $2N$ simple eigenvalues $\xi_1(0), \xi_2(0), ..., \xi_{2N}(0)$.

In this paragraph we apply the IST approach to find the solution $\left(\upsilon(x,t), w(x,t), \varphi_n(x,\xi,t),\ n = 1,2,\ldots,N\right)$ of the Cauchy problem (30)-(31) in the following class

$$\mathcal{L} = \left\{ \begin{array}{c} \upsilon \in C^2(\mathbb{R}\times(0,\infty)) \cap (\mathbb{R}\times[0,\infty)), \quad w \in C^{1,0}(\mathbb{R}\times[0,\infty)); \\ \lim\limits_{x\to\pm\infty} w(x,t) = 0, \quad t \in \mathbb{R}; \\ \int\limits_{-\infty}^{+\infty} \left(1+|x|\right)\left(\|\upsilon(x,t)\| + \|\upsilon_x(x,t)\| + \|\upsilon_t(x,t)\| + \|\upsilon_{xt}(x,t)\|\right) dx < \infty, \quad t \in \mathbb{R} \end{array} \right\}. \quad (32)$$

We have proved the following theorem

Theorem 6. If the collection $\left\{\upsilon(x,t), w(x,t), \varphi_j(x,\xi,t), \quad j=1,2,\dots,N\right\}$ is a solution of the problem (30)-(32), then the scattering data for the Zakharov-Shabat system $L(t) = -iJ\dfrac{d}{dx} - V(x,t)$ satisfy the following relations

$$\frac{\partial R(\xi,t)}{\partial t} = \frac{i}{2\xi} R(\xi,t)\,, \; \xi \in \mathbb{R} \setminus \{0\},$$

$$\frac{d\xi_j}{dt} = 0\,, \quad \frac{d\Gamma_j(t)}{dt} = \left(\frac{i}{2\xi_j} + \left(\alpha(t) + \beta(t)\right)\right)\Gamma_j(t)\,, \quad \operatorname{Im}\xi_j > 0\,, \; j = 1,2,\dots,N\,.$$

CONCLUSION

The first chapter of the dissertation is devoted to the solution of the Cauchy problem for the negative order mKdV equation in the class of rapidly decreasing functions using the IST method.

In the second chapter, the negative order mKdV equation with a self-consistent sources is integrated using the IST method for the Zakharov-Shabat system with a rapidly decaying potential.

In the third chapter of the dissertation, it is proved complete integrability of the negative order mKdV equation with and without a self-consistent source using the IST method for the matrix Zakharov-Shabat system.

The main results of the research:

1) it is shown the completely integrablity of negative order mKdV equation in the class of rapidly decreasing functions using the IST method for the Zakharov-Shabat system;

2) solved the Cauchy problem for the negative order mKdV equation with a self-consistent source using the IST method for the Zakharov-Shabat system in the class of rapidly decreasing functions;

3) integrated the matrix negative order mKdV equation within the class of rapidly decreasing functions using IST approach for the matrix Zakharov-Shabat system;

4) proved the complete integrability of the matrix negative order mKdV equation with a self-consistent source using the IST technique for the matrix Zakharov-Shabat system with rapidly decaying potential.

НАУЧНЫЙ СОВЕТ PhD.03/2025.27.12.FM.06.02
ПО ПРИСУЖДЕНИЮ УЧЕНОЙ СТЕПЕНИ ПРИ УРГЕНЧСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ ИМЕНИ АБУ РАЙХАНА БЕРУНИ

УРГЕНЧСКИЙ ГОСУДАРСТВУННЫЙ УНИВЕРСИТЕТ ИМЕНИ АБУ РАЙХАНА БЕРУНИ

АТАНАЗАРОВА ШОИРА ЭРКИНОВНА

ИНТЕГРИРОВАНИЕ МОДИФИЦИРОВАННОГО УРАВНЕНИЯ КОРТЕВЕГА-ДЕ ФРИЗА ОТРИЦАТЕЛЬНОГО ПОРЯДКА С САМОСОГЛАСОВАННЫМ ИСТОЧНИКОМ

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ
диссертации доктора философии (PhD) по физико-математическим наукам

Ургенч – 2026

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при министерстве Высшего образования, науки и инноваций Республики Узбекистан за № B2024.2.PhD/FM1055.

Диссертация выполнена в Хорезмском территориальном подразделении Института математики имени В.И. Романовского Академии наук Республики Узбекистан и Ургенчском государственном университете имени Абу Райхана Беруни.

Автореферат диссертации на трех языках (узбекский, русский, английский (резюме)) размещен на веб-странице Научного совета (www.ik-mat.urdu.uz) и на Информационно-образовательном портале «ZiyoNet» (www.ziyonet.uz).

Научный руководитель:	**Уразбоев Гайрат Уразалиевич** доктор физико-математических наук, доцент
Официальные оппоненты:	**Яхшимуратов Алишер Бекчанович** доктор физико-математических наук **Балтаева Умида Исмаиловна** доктор физико-математических наук
Ведущая организация:	**Самаркандский государственный университет имени Шарофа Рашидова**

Защита диссертации состоится «14» февраля 2026 года в 15:00 часов на заседании Научного совета PhD.03/2025.27.12.FM.06.02 при Ургенчском государственном университете имени Абу Райхана Беруни (Адрес: 220100, г.Ургенч, ул. Х. Алимджана, дом 14. Тел:(+99862) 224-66-11, факс: (+99862) 224-67-00, e-mail: ik-mat.urdu@umail.uz

С диссертацией можно ознакомиться в Информационно-ресурсном центре Ургенчского государственного университета имени Абу Райхана Беруни (зарегистрирована за № D-939). (Адрес: 220100, г.Ургенч, ул. Х. Алимджана, дом 14. Тел:(+99862) 224-66-11, факс: (+99862) 224-67-00, e-mail: arm@urdu.uz).

Автореферат диссертации разослан «02» февраля 2026 года.

(протокол рассылки № 1 от «02» февраля 2026 года).

O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI

Б. И. Абдуллаев
Председатель Научного совета по присуждению ученой степени, д.ф-м.н., профессор.

А. А. Рейимберганов
Ученый секретарь Научного совета по присуждению ученой степени, к.ф-м.н.

Б. А. Бабажанов
Председатель Научного семинара при научном совете по присуждению ученой степени, д.ф-м.н., профессор.

ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является интегрирование модифицированного уравнения Кортевега-де Фриза (мКдФ) отрицательного порядка и уравнения мКдФ отрицательного порядка с самосогласованным источником, а также их матричных аналогов в классе быстроубывающих функций методом обратной задачи рассеяния.

Задачи исследования:

доказательство полной интегрируемости уравнения мКдФ отрицательного порядка и уравнения мКдФ отрицательного порядка с самосогласованным источником методом обратной задачи рассеяния для системы Захарова-Шабата с быстроубывающим потенциалом;

применение матричного метода (A, B, C) для построения многосолитонного решения уравнения мКдФ отрицательного порядка с самосогласованным источником в классе быстроубывающих функци;

решение задачи Коши для матричного уравнения мКдФ отрицательного порядка и матричного уравнения мКдФ отрицательного порядка с самосогласованным источником методом обратной задачи рассеяния для системы Захарова-Шабата в классе быстроубывающих функций.

Объектом исследования: уравнение мКдФ отрицательного порядка, уравнение мКдФ отрицательного порядка с самосогласованным источником, уравнение мКдФ отрицательного порядка с самосогласованным источником соответсвующий движущимся собственным значениям, матричное уравнение мКдФ отрицательного порядка, матричное уравнение мКдФ отрицательного порядка с самосогласованным источником.

Предметом исследования является применение метода обратной задачи рассеяния для системы Захарова-Шабата с быстроубывающим потенциалом к интегрированию нелинейных эволюционных уравнений отрицательного порядка и их матричных аналогов.

Научная новизна исследования состоит в следующем:

выведены эволюционные уравнения для данных рассеяния системы Захарова-Шабата с быстроубывающим потенциалом, связанным с уравнением мКдФ отрицательного порядка методом обратной задачи рассеяния;

доказано, что потенциал, построенный по выведенным эволюционным уравнениям для данных рассеяния системы Захарова-Шабата методом обратной задачи рассеяния, является быстроубывающим решением задачи Коши для уравнения мКдФ отрицательного порядка;

доказана вполне интегрируемость уравнения мКдФ отрицательного порядка с самосогласованным источником в классе быстроубывающих функций методом обратной задачи рассеяния для системы Захарова-Шабата;

применен матричный метод (A, B, C) для построения многосолитонного решения уравнения мКдФ отрицательного порядка с самосогласованным источником в классе быстроубывающих функций;

интегрировано уравнение мКдФ отрицательного порядка с самосогласованным источником соответствующий движущимся собственным значениям системы Захарова-Шабата в классе быстроубывающих функций методом обратной задачи рассеяния;

интегрировано матричное уравнения мКдФ отрицательного порядка методом обратной задачи рассеяния для матричной системы Захарова-Шабата с быстроубывающим потенциалом;

решена задача Коши для уравнения мКдФ отрицательного порядка с самосогласованным источником методом обратной задачи рассеяния для матричной системы Захарова-Шабата в классе быстроубывающих функций.

Внедрение результатов исследования. Результаты интегрирования уравнения мКдФ отрицательного порядка с самосогласованным источником были применены в следующих областях:

результаты, полученные при решении задачи Коши для уравнения мКдФ отрицательного порядка в классе быстроубывающих функций методом обратной задачи рассеяния для системы Захарова-Шабата использованы для вывода эволюционных уравнений для матричного уравнения КдФ отрицательного порядка в рамках научного проекта, финансируемого DAAD, «Научные исследования для ученых и преподавателей университетов, 2024 (57693448)» в Университете Байройта. Применение научных результатов позволило разработать алгоритм для построения решения начальной задачи для матричного уравнения КдФ отрицательного порядка методом обратной задачи рассеяния и визуализировать волновое поведение полученных решений рассматриваемой задачи с использованием программного пакета MATLAB. (Справка Университета Байройта, Германия, от 14 октября 2025 г.);

на основе результатов, полученных при интегрировании матричного уравнения мКдФ отрицательного порядка методом обратной задачи рассеяния, выведены эволюционные уравнения для данных рассеяния оператора связанного с матричным уравнением синус-Гордона с источником в классе убывающих функций в рамках научного проекта «Поддержка мобильности студентов, аспирантов, преподавателей вузов, исследователей и деятелей искусства, 2024» финансиреумого Национальной стипендиальной программой Словацкой Республики. Применение научных результатов позволило построить неявное решение матричного уравнения синус-Гордона с источником методом матриц (A, B, C). (Справка института математики Словацкой академии наук, Братислава, от 14 октября 2025 г.).

Объем и структура диссертации. Диссертация состоит из введения, трех глав, заключения и списка использованной литературы. Объем диссертации составляет 89 страниц.

E'LON QILINGAN ISHLAR RO'YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo'lim (Part I; Часть I)

1. Urazboev G.U., Baltaeva I. I., Atanazarova Sh.E. Soliton solutions of the negative order modified Korteweg-de Vries equation. The Bulletin of Irkutsk State University. Series Mathematics. 2024, vol. 47, pp. 63–77.(IF. 1,7)
2. Urazboev G.U., Baltaeva I. I., Atanazarova Sh.E. Exploring solutions for the negative-order modified Korteweg–de Vries equation with a self-consistent source corresponding to moving eigenvalues. Theoretical and mathematical physics. 2025, vol.225, № 2, pp. 1896-1909.(IF.1,8)
3. Atanazarova Sh.E. On the negative order matrix modified Korteweg-deVries equation with a source. Actual problems of mathematics, physics and mechanics. 2025, vol. 6, pp. 103-107. (01.00.00, OAK Rayosatining 2019 yil 28 martdagi 263/7.1-son qarori)
4. Atanazarova Sh.E. Интегрирование матричного модифицированного уравнения Кортевега-де Фриза отрицательного порядка. Ilm sarchashmalari. 2025, №.8. стр.50-53. (01.00.00, OAK Rayosatining 2019 yil 28 martdagi 263/7.1-son qarori)

II bo'lim (Part II; Часть II)

1. Уразбоев, Г.У., Бабаджанова А.К., Атаназарова Ш.Э. О матричном модифицированном уравнении Кортевега-де Фриза отрицательного порядка. Материалы XIII Международной научно-практической конференции «Наука и образование в современном мире: ВызовыXXI», Астана, 27-31 мая, 2023г., стр.105-108
2. Baltaeva I. I., Babadjanova A.K., Atanazarova Sh.E. Inverse scattering transform for the negative order modified Korteweg-de Vries equation with a source. Abstracts of the IX international Scientific conference "Actual problems of applied mathematics and information technologies Al-Khwarizmi 2024" Tashkent, 23-24 October, 2024, pp. 137-138.
3. O'razboyev G.O', Babadjanova A.K., Atanazarova Sh.E. Manfiy tartibli matritsaviy modifitsirlangan Korteveg-de Friz tenglamasining solitonsimon yechimlarini topish. "Analizning zamonaviy muammolari" Respublika ilmiy anjumani materiallari, Qarshi, 2-3 iyun, 2023-yil., 179-181 betlar.
4. Уразбоев Г.У., Балтаева И.И., Атаназарова Ш.Э. Обратная задача рассеяния для модифицированного уравнения Кортевега–де Фриза отрицательного порядка с самосогласованным источником. Ali Qushchining 620 yillik tavaluddiga bag'ishlangan "Ali Qushchi – Mirzo Ulug'bek ilmiy maktabining buyuk elchisi" mavzusidagi xalqaro ilmiy anjuman materiallari, 21-22 sentabr 2023-yil., 284-286 betlar.
5. Уразбоев Г.У., Балтаева И.И., Атаназарова Ш.Э. Об интегрировании модифицированного уравнения Кортевега-де Фриза отрицательного порядка с самосогласованным источником, соответствующим движущимся собственным

значениям. Тезисы докладов международной научной конференции «Современные проблемы дифференциальных уравнений и их приложения», Ташкент, 23-25 ноября, 2023, стр. 263-264.

6. Urazboev G.U, Babadjanova A.K., Atanazarova Sh.E. Explicit Solution For The Matrix Negative Order Modified Korteweg-De Vries Equation Via Matrix Triplet. “Matematik fizikaning zamonaviy usullari va ularning tadbiqlari” akademik Sh.Alimovning 80 yilligiga bag‘ishlangan respublika ilmiy anjumani, Toshkent, 22-24 aprel, 2025, 32-35 betlar.

7. Уразбоев Г.У., Бабаджанова А.К. Атаназарова Ш.Э. Об интегрировании матричного модифицированного уравенения Кортевега-де Фриза отрицательного порядка. “Zamonaviy matematikaning dolzarb muammolari va ularning tadbiqlari” respublika ilmiy anjumani, Toshkent, 10-11 sentabr, 2025, 236-238 betlar.

8. Babadjanova A.K. Atanazarova Sh.E. Inverse scattering for the negative order matrix modified Korteweg-de Vries equation with a self-consistent source. Abstracts of the conference “New theorems of young mathematicians”, Namangan, 20-21,2025, pp.129-131.

Dissertatsiya avtoreferati “Khwarezm publication” nashriyotida tahrir qilindi.

Bosishga ruxsat etildi: 20.01.2026-yil.
Bichimi 60x84 $^{1/16}$, “Times New Roman”
garniturada raqamli bosma usulida bosildi.
Shartli bosma tabog‘i 3,5. Adadi: 100. Buyurtma: № 218
“Khwarezm travel” bosmaxonasida chop etildi
220502, Xorazm, Urganch tumani, Zargarlar mahallasi,
Marvarid ko‘cha 7-yo‘lak 4-uy