ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI PhD.03/2025.27.12.FM.06.02 RAQAMLI ILMIY KENGASH

XORAZM MA’MUN AKADEMIYASI

ABDIKARIMOV FAXRIDDIN BAXROM O‘G‘LI

MOSLANGAN MANBALI BUTUN VA KASR TARTIBLI XUSUSIY HOSILALI NOCHIZIQLI EVOLYUTSION TENGLAMALARNI INTEGRALLASH

01.01.02 – Differensial tenglamalar va matematik fizika

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI

Urganch – 2026

UDK 517.957

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

Content of dissertation abstract of the doctor of philosophy (PhD) on physical and mathematical sciences

ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI HUZURIDAGI ILMIY DARAJALAR BERUVCHI PhD.03/2025.27.12.FM.06.02 RAQAMLI ILMIY KENGASH

XORAZM MA’MUN AKADEMIYASI

ABDIKARIMOV FAXRIDDIN BAXROM O‘G‘LI

MOSLANGAN MANBALI BUTUN VA KASR TARTIBLI XUSUSIY HOSILALI NOCHIZIQLI EVOLYUTSION TENGLAMALARNI INTEGRALLASH

01.01.02 – Differensial tenglamalar va matematik fizika

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI

Urganch – 2026

Fizika-matematika fanlari bo'yicha falsafa doktori (Doctor of Philosophy) dissertasiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestasiya komissiyasida B2024.2.PhD/FM1052 raqam bilan ro'yxatga olingan.

Dissertatsiya Xorazm Ma'mun akademiyasida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, rus, ingliz (rezyume)) Ilmiy kengash veb-sahifasida (www.ik-mat.urdu.uz) va "Ziyonet" axborot ta'lim portalida (www.ziyonet.uz) joylashtirilgan.

Ilmiy rahbar:	**Babajanov Bazar Atajanovich** fizika-matematika fanlari doktori, professor
Rasmiy opponentlar	**Hasanov Aknazar Bekdurdiyevich** fizika-matematika fanlari doktori, professor
	Babadjanova Aygul Komiljanovna fizika-matematika fanlari bo'yicha falsafa doktori(PhD)
Yetakchi tashkilot:	**Buxoro davlat universiteti**

Dissertatsiya himoyasi Abu Rayhon Beruniy nomidagi Urganch Davlat universiteti huzuridagi ilmiy darajalar beruvchi PhD.03/2025.27.12.FM.06.02 raqamli Ilmiy kengashning 2026 yil "16" fevral soat 14:00 dagi majlisida bo'lib o'tadi. (Manzil: 220100, Urganch shahri, H. Alimjon ko'chasi, 14-uy. Tel.: (+99862) 224-66-11, faks: (+99862) 224-67-00, e-mail: info@urdu.uz).

Dissertatsiya bilan Abu Rayhon Beruniy nomidagi Urganch Davlat universitetining Axborot-resurs markazida tanishish mumkin (D-941 -raqami bilan ro'yhatga olingan). (Manzil: 220100, Urganch shahri, H. Alimjon ko'chasi, 14-uy. Tel.: (+99862) 224-66-11, faks: (+99862) 224-67-00, e-mail: arm@urdu.uz).

Dissertatsiya avtoreferati 2026 yil "02" fevral kuni tarqatildi.

(2026 yil "02" fevral dagi 1 raqamli reyestr bayonnomasi).

B. I. Abdullayev
Ilmiy darajalar beruvchi Ilmiy kengash raisi, f.-m.f.d., professor

A. A. Reyimberganov
Ilmiy darajalar beruvchi Ilmiy kengash ilmiy kotibi, f.-m.f.n

B. A. Babajanov
Ilmiy darajalar beruvchi Ilmiy kengash huzuridagi ilmiy seminar raisi, f.-m.f.d., professor

KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahonda olib borilayotgan ko‘plab ilmiy-amaliy tadqiqotlar aksariyat hollarda zamonaviy matematikaning muhim va dolzarb yo‘nalishlaridan biri bo‘lgan nochiziqli evolyutsion tenglamalarni tadqiq qilishga keltiriladi. Hozirgi kunda anomal diffuziya va fraksional dispersiya hodisalari ko‘plab fizik va texnologik jarayonlarda kuzatilayotgani sababli, moslangan manbali butun va kasr tartibli xususiy hosilali nochiziqli evolyutsion tenglamalarni o‘rganish nafaqat nazariy, balki amaliy jihatdan ham nihoyatda muhim tadqiqot yo‘nalishlaridan biri hisoblanadi. Bu yo‘nalish fundamental matematika, kompyuter simulyatsiyasi va eksperimental fizika o‘rtasidagi o‘zaro bog‘liqlikni mustahkamlab, yangi turdagi solitonlar, dispersiv effektlar va integrallanuvchi tenglamalar sinfini aniqlash imkonini beradi. Bundan tashqari bu masalalardan plazma fizikasi, optika, gidrodinamika, biotibbiyot, geofizika, anomal issiqlik tarqalishi, bozor modellaridagi inertsiya effektlari kabi ko‘plab sohalarni tadqiq qilishda foydalanish muhim ahamiyatga ega hisoblanadi.

Hozirgi vaqtda manbali nochiziqli evolyutsion tenglamalarni integrallash bo‘yicha dunyo miqyosida keng ko‘lamli tadqiqotlar olib borilmoqda. Odatda, manbasiz tenglamalar ideal sharoitlarda chiqarilgan model tenglamalar bo‘lib, tabiiy jarayonlarda yuzaga keladigan qo‘shimcha ta’sirlarni hisobga olish zarurati mavjud hisoblanadi. Bunday hollarda nochiziqli evolyutsion tenglamalarni o‘rganishda o‘zgaruvchan koeffitsiyentli va moslangan manbali tenglamalarni kiritish talab etiladi. Shu sababli, ushbu jarayonlarning matematik modellari sifatida o‘zgaruvchan koeffitsiyentli va moslangan manbali nochiziqli evolyutsion tenglamalar qaraladi. Xususan, o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasi va qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz va modifitsirlangan Korteveg-de Friz tenglamalarini funksional o‘zgaruvchilar usuli yordamida tatqiq etish, Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash, o‘zgaruvchan koeffitsiyentli va moslangan manbali umumiy Kaup-Boussinesq tenglamasini tez kamayuvchi funksiyalar sinfida integrallash masalalariga jahon miqiyosida alohida e’tibor berilmoqda.

Respublikamizda kasr tartibli va moslangan mabali nochiziqli evolyutsion tenglamalarning yechimlarini Zaxarov-Shabat sistemasi va Shturm-Liuvill operatorlari kvadratik dastasi uchun qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalar usullari yordamida aniqlash va ularni amaliyotga tatbiq etish bo‘yicha keng ko‘lamli ishlar amalga oshirilmoqda. Xususan, sochilish nazariyasining to‘g‘ri va teskari masalalari usullaridan foydalanib Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz va umumiy Kaup-Boussinesq tenglamalarining soliton yechimlarini qurish bo‘yicha muhim natijalarga erishildi. Differensial tenglamalar, matematik fizika va funksional analiz fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yoʻnalishlari qilib belgilangan[1]. Ushbu vazifalarni amalga oshirishda, xususan zamonaviy matematik fizikaning nochiziqli evolyutsion tenglamalarni integrallashda sochilish nazariyasining toʻgʻri va teskari masalalari usulini qoʻllab, tez kamayuvchi funksiyalar sinfida Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini Zaxarov-Shabat sistemasi uchun qoʻyilgan sochilish nazariyasining toʻgʻri va teskari masalalar usuli yordamida, shuningdek oʻzgaruvchan koeffitsiyentli va moslangan manbali umumiy Kaup-Boussinesq tenglamasini Shturm-Liuvill operatorlari kvadratik dastasi uchun uchun qoʻyilgan sochilish nazariyasining toʻgʻri va teskari masalalar usuli yordamida integrallash muhim ahamiyat kasb etmoqda.

Oʻzbekiston Respublikasi Prezidentining 2020-yil 29-oktabrdagi PF-6097-sonli "Ilm-fanni 2030-yilgacha rivojlantirish Konsepsiyasini tasdiqlash toʻgʻrisida"gi va 2022-yil 28-yanvardagi PF-60-sonli "2022-2026-yillarga moʻljallangan Yangi Oʻzbekistonning taraqqiyot strategiyasi toʻgʻrisida"gi Farmonlari, 2019-yil 9-iyuldagi PQ-4387-sonli "Matematika ta'limi va fanlarini yanada rivojlantirishni davlat tomonidan qoʻllab-quvvatlash, shuningdek, Oʻzbekiston Respublikasi Fanlar akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari toʻgʻrisida"gi va 2020-yil 7-maydagi PQ-4708-sonli "Matematika sohasidagi ta'lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari toʻgʻrisida"gi Qarorlari hamda ushbu faoliyat sohasiga oid boshqa me'yoriy-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishi ustuvor yoʻnalishlariga bogʻliqligi. Ushbu dissertatsiya ishidagi izlanish va tadqiqotlar Oʻzbekiston Respublikasida fan va texnika taraqqiyotining IV. "Matematika, mexanika va informatika" ustuvor yoʻnalishiga muvofiq olib borildi.

Muammoni oʻrganilganlik darajasi. Oʻzgarmas koeffitsiyentli nochiziqli evolyutsion tenglamalarning funksional oʻzgaruvchilar usuli yordamida davriy va soliton yechimlarini topish ilk bor 2010-yilda V. Djoudi va A. Zerarka tomonidan kiritilgan boʻlib, keyinchalik Li va He tomonidan bu usul kasr tartibli differensial tenglamalarning davriy va soliton yechimlarini topishda qoʻllanilgan. 2016-yilda V. Djoudi va A. Zerarka ushbu usulni rivojlantirib, oʻzgaruvchan koeffitsiyentli Korteveg-de Friz tenglamasi va modifitsirlangan Korteveg-de Friz tenglamalarining davriy va soliton yechimlarini funksional oʻzgaruvchilar usuli yordamida qurishga muvaffaq boʻldilar. Funksional oʻzgaruvchilar usulining boshqa usullardan asosiy ustunligi shundaki, bu usul yordamida parametrlarga aniq qiymat berish orqali koʻproq yangi yechimlarni olish mumkin.

Soʻnggi yillarda butun va kasr tartibli nochiziqli evolyutsion tenglamalarni sochilish nazariyasining toʻgʻri va teskari masalalar usuli yordamida integrallash boʻyicha keng koʻlamli tadqiqotlar olib borilmoqda. Xususan, butun tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasi ilk bor 1974-yilda K.

---

[1]Oʻzbekiston Respublikasi Vazirlar Mahkamasining 2017-yil, 18-maydagi 292-sonli "Oʻzbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy-tadqiqot muassasalari faoliyatini tashkil etish chora-tadbirlari toʻgʻrisida"gi qarori.

Konno tomonidan atom panjarasidagi nochiziqli tebranish hodisalarini ifodalovchi matematik model sifatida kiritilgan va sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida to‘la integrallanuvchanligi ko‘rsatilgan. Keyinchalik, bu tenglama H. Leblond va D. Mihalache tomonidan shaffof muhitlarda qisqa optik impulslarning tarqalishini tasvirlashda qo‘llanilgan. Shuningdek, bu tenglamaning davriy cheksiz zonali yechimlari A. Hasanov tomonidan, tez kamayuvchi funksiyalar sinfidagi yechimlari esa U. Hoitmetov tomonidan o‘rganilgan.

2022-yilda Amerikalik olimlar Ablovitz, Been va Carrlar Riss kasr tartibli nochiziqli Korteveg-de Friz tenglamasini sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash mumkinligini ko‘rsatdilar va bu usulni Riss kasr tartibli nochiziqli Shrodinger tenglamasi, Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamasi, Riss kasr tartibli hosilali sinus-Gordon tenglamasi integrallashga qo‘lladilar. Ular tomonidan taklif etilgan bu usul hozirgi kunda ko‘plab kasr tartibli hosilali nochiziqli evolyutsion tenglamalarni integrallashga tadbiq etilmoqda. Jumladan, xitoylik olimlar V. Weng, M. Zhang va Z. Yanlar tomonidan bu usul yordamida Riss kasr tartibli hosilali nochiziqli Shrodinger tenglamasining umumlashmalari integrallangan va N-soliton yechimlarining dinamikasi o‘rganilgan. L. An, L. Ling va H. Zhang esa Riss kasr tartibli hosilali nochiziqli hosilali Shrodinger tenglamasi va Riss kasr tartibli Hirota tenglamasining soliton yechimlarini olganlar. SH. Zhang, H. Li va B. Xu esa Riss kasr tartibli hosilali o‘zgaruvchan koeffitsiyentli Korteveg-de Friz va nochiziqli Shrodinger tenglamasini sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash mumkinligini ko‘rsatganlar.

Shturm-Liuvill operatorining kvadratik dastasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida D. J. Kaup sayoz suvda to‘lqin tarqalishini tasvirlovchi Kaup-Boussinesq tenglamasini tez kamayuvchi funksiyalar sinfida to‘liq integrallanishini ko‘rsatgan. Keyinchalik, M. Jaulent va I. Miodek Kaup-Boussinesq tenglamasi va uning yuqori tartibli analoglari uchun Koshi masalasini yechish algoritmini ishlab chiqdilar. V. B. Matveev va M. I. Yavor Kaup-Boussinesq tenglamasini chegaralangan zona tipidagi boshlang‘ich shartlar bilan o‘rgangan bo‘lib, murakkab ko‘p zonali yechimlarini topganlar va ularning assimptotik xossalarini tahlil qildilar. Kaup-Boussinesq tenglamasining haqiqiy ko‘p zona yechimlari A. O. Smirnov ishlarida tadqiq qilingan.

Bundan tashqari, A. Cabada va A. Yakshimuratov davriy funksiyalar sinfida moslangan manbali Kaup-Boussinesq tenglamasini integrallab, yechimlarning davriylik xususiyatlari va o‘zgaruvchilarga nisbatan analitikligini aniqlash bo‘yicha muhim natijalarni oldilar. Shuningdek, B.A. Babajanov va A.Sh. Azamatovlar moslangan manbali Kaup-Boussinesq tenglamasining integrallanuvchanligini Shturm-Liuvill tenglamalarining kvadratik dastasi uchun sochilish nazariyasining teskari masalalari usuli yordamida asoslab berganlar.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Ushbu dissertatsiya ishi Xorazm Ma’mun akademiyasi “Aniq fanlar” bo‘limining ilmiy-tadqiqot ishlari rejasiga muvofiq “Differensial operatorlar spektral nazariyasining nochiziqli evolyutsion tenglamalarga tadbiqlari” nomli ilmiy-tadqiqot ishlari rejasi (2022-2025 yillar) asosida amalga oshirilgan.

Tadqiqotning maqsadi funksional o‘zgaruvchilar usuli yordamida qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasini, qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasini va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamasining soliton va davriy yechimlarini topish, Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida tadqiq etish va o‘zgaruvchan koeffitsiyentli va moslangan manbali umumiy Kaup-Boussinesq tenglamasini tez kamayuvchi funksiyalar sinfida integrallashdan iborat.

Tadqiqotning vazifalari: qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasini, qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasi va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamasini funksional o‘zgaruvchilar usuli yordamida soliton yechimlarini topish;

Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini Zaxarov-Shabat sistemasi uchun qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash;

o‘zgaruvchan koeffitsiyentli va moslangan manbali umumiy Kaup-Boussinesq tenglamasini Shturm-Liuvill operatorlari kvadratik dastasi uchun uchun qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash.

Tadqiqot obyekti: qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasi, qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasi va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamasi, Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasi, o‘zgaruvchan koeffitsiyentli va moslangan manbali umumiy Kaup-Boussinesq tenglamasi.

Tadqiqot predmeti Zaxarov-Shabat sistemasi va Shturm-Liuvill operatorlari kvadratik dastasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalar usulini butun va kasr tartibli hosilali nochiqli evolyutsion tenglamalarni integrallash jarayoniga tatbiq etishdan iborat. Bundan tashqari, qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli bir qator nochiziqli evolyutsion tenglamalar uchun aniq analitik yechimlarni olish maqsadida funksional o‘zgaruvchi usulining qo‘llanilishi ham o‘rganilgan.

Tadqiqotning usullari. Dissertatsiya ishida matematik analiz, oddiy va xususiy hosilali differensial tenglamalar nazariyasi, matematik fizika tenglamalari, funksional analiz, kompleks o‘zgaruvchili funksiyalar nazariyasi hamda differensial operatorlarning spektral nazariyasi kabi zamonaviy matematik metod va yondashuvlar qo‘llanildi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

funksional o‘zgaruvchilar usuli yordamida qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasi, qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasi va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamasining soliton va davriy yechimlari topilgan;

Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasi Zaxarov-Shabat sistemasi uchun qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalar usulini yordamida tez kamayuvchi funksiylar sinfida integrallanuvchanligi isbotlangan;

o‘zgaruvchan koeffitsiyentli va moslangan manbali umumiy Kaup-Boussinesq tenglamasi Shturm-Liuvill operatorlari kvadratik dastasi uchun qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida tez kamayuvchi funksiylar sinfida integrallanuvchanligi isbotlangan.

Tadqiqotning amaliy natijalari: funksional o‘zgaruvchilar usuli yordamida qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasi, qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasi va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamasining soliton va davriy yechimlari topilgan. Ushbu yechimlar muhitlardagi dispersiv o‘zaro ta’sirlarni chuqur yoritib beradi va ular turbulentlik, gaz dinamikasi hamda nochiziqli diffuziya modellarida to‘g‘ridan-to‘g‘ri qo‘llanishi mumkin.

Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasi Zaxarov-Shabat sistemasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalar usulini yordamida integrallash algoritmi ishlab chiqilgan. Ushbu natijalar kasr tartibli hosilali nochiziqli to‘lqin jarayonlarini tahlil qilish uchun samarali hisoblanadi.

O‘zgaruvchan koeffitsiyentli va moslangan manbali umumiy Kaup-Boussinesq tenglamasi Shturm-Liuvill operatorlari kvadratik dastasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash algoritmi qurilgan va vaqt bo‘yicha evolyutsiyasini ifodalovchi aniq formulalar olingan.

Tadqiqot natijalarining ishonchliligi: qo‘yilgan vazifalarni o‘rganishda matematik fizika tenglamalari, funksional tahlil va kompleks o‘zgaruvchili funksiyalar nazariyasi, shuningdek Zaxarov-Shabat sistemasining spektral xususiyatlari va Shturm-Liuvill operatorlari kvadratik dastasi uchun sochilish nazariyasining teskari masalalar usuli kabi zamonaviy analitik usullardan foydalanilganligi bilan ta’minlanadi. Barcha teoremalar va natijalar qat’iy matematik asoslash, aniq isbotlar, spektral tahlilning fundamental prinsiplariga tayanilgan holda chiqarilgan bo‘lib, ularning to‘g‘riligi tekshirilib, klassik natijalar bilan solishtirish asosida tasdiqlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Ushbu dissertatsiyada olingan asosiy natijalar chiziqli operatorlarning spektral nazariyasi doirasida, shuningdek qattiq jismlar fizikasi, ion akustikasi, plazma fizikasi, radiofizika va kvant fizikasidagi turli masalalarni tahlil qilishda qo‘llanishi mumkin va ishlab chiqilgan nazariy yondashuvlar va analitik usullarni matematik fizikaning moslangan manbali nochiziqli evolyutsion tenglamalarini integrallash jarayonida qo‘llash imkoniyati bilan belgilanadi.

Tadqiqot natijalarining joriy qilinishi.

Moslangan manbali butun va kasr tartibli hosilali xususiy hosilali nochiziqli evolyutsion tenglamalarni integrallash bo‘yicha olingan natijalar asosida:

Qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasining soliton yechimlari va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini Zaxarov-Shabat sistemasiga qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash orqali olingan bir solitonli yechimlarning xossalari Muhammad al-Xorazmiy nomidagi Toshkent axborot texnologiyalari universiteti Urganch filialida fizika-matematika fanlari doktori A.B. Yaxshimuratov rahbarligida 2022-2023 yillar davomida bajarilgan AL-42101210 - “Aqlli shahar sensori infratuzilmasining monitoring tizimi” mavzusidagi amaliy tadqiqot loyihasida foydalanilgan(Abu Rayhon Beruniy nomidagi Urganch davlat universitetining ma‘lumotnomasi, O‘zbekiston, 2025-yil 5-noyabr). Kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini Zaxarov-Shabat sistemasiga qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash orqali olingan bir solitonli yechimlarning xossalari signallarning xususiyatlarini o‘zgartirmasdan uzatishda qo‘llanilgan. Shuningdek, o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasining soliton yechimlari sensor tarmoqlarida shovqin kuchayishi va signal buzilishini kamaytirish hamda signal amplitudasi va uzatish tezligini samarali boshqarishda qo‘llanilgan. Ilmiy natijalarning qo‘llanilishi binolarda yong‘in mavjudligini baholashda zarur bo‘lgan dasturiy ta’minotni ishlab chiqish hamda ma’lumotlarni uzatish tarmog‘idagi yukni kamaytirish imkonini bergan.

Umumiy Kaup-Boussinesq tenglamasini sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash orqali olingan ilmiy natijalari Abu Rayhon Beruniy nomidagi Urganch davlat universitetida A.E. Atamuratov rahbarligida 2021-2023 yillar davomida bajarilgan Uzb-Ind-2021-80 raqamli “Zatvor bilan o‘ralgan tartiblangan nanoplastinkalar asosidagi MOYA tranzistorda o‘z-o‘zidan qizish effektini o‘rganish” mavzusidagi fundamental loyihasida foydalanilgan(Abu Rayhon Beruniy nomidagi Urganch davlat universitetining ma‘lumotnomasi, O‘zbekiston, 2025-yil 5-noyabr). Kaup-Boussinesq sistemasini sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash orqali olingan ilmiy natijalari nanoplastinkali MOYA tranzistor kanali markazida yuzaga keladigan maksimal haroratni nazariy asoslash, tranzistordagi lokal issiqlik to‘planish mexanizmini matematik jihatdan tavsiflash, shuningdek, issiqlik oqimi, oqim zichligi, I_{on}/I_{off} nisbati va konstruktiv-geometrik parametrlar o‘rtasidagi modellashtirish algoritmlarini ishlab chiqishda qo‘llanilgan. Ilmiy natijalarning qo‘llanilishi tranzistorning ichki faol sohalarida o‘z-o‘zidan qizish jarayonining boshlanishi va rivojlanishini matematik asoslangan holda tushuntirish, lokal issiqlik to‘planishini oldindan baholash, konstruktiv-geometrik parametrlar bilan issiqlik tarqalish o‘rtasidagi bog‘liqlikni aniqlash, shuningdek, tranzistorlarning barqaror ishlashi uchun optimal struktura parametrlarini tanlashni ta’minlovchi hisoblash va simulyatsiya usullarini ishlab chiqish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Dissertatsiyaning asosiy mazmuni 7 ta ilmiy-amaliy konferensiyalarda, shu jumladan 6 ta xalqaro va 1 ta respublika ilmiy-amaliy konferensiyalarda muhokama qilingan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusida 14 ta ilmiy ish chop qilingan bo‘lib, dissertatsiyalarining asosiy ilmiy natijalarini nashr etish uchun O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasi tomonidan tavsiya etilgan ilmiy nashrlarda 7 ta maqola, shu jumladan 6 tasi xorijiy va 1 tasi respublika jurnallarida chop qilingan.

Dissertatsiyaning hajmi va tuzilishi. Dissertatsiya kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat. Dissertatsiyaning hajmi 102 betni tashkil qiladi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning Respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi keltirilgan, dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi, tadqiqot maqsadi, vazifalari, ob’yekti, predmeti va usullari tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Butun va kasr tartibli hosilali nochiziqli evolyutsion tenglamalarning soliton va davriy to‘lqin yechimlarini funksional o‘zgaruvchilar usuli yordamida topish**” deb nomlangan birinchi bobida, qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasi, qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasi va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamasining soliton va davriy yechimlari topilgan.

Bu bobning birinchi paragrafida funksional o‘zgaruvchilar usulining matematik mohiyati, uning qo‘llanish shartlari, bazaviy tushunchalari va yechimlarni qurishdagi afzalliklari batafsil tushuntirilgan.

Bu bobning ikkinchi paragrafida qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasini funksional o‘zgaruvchilar usuli yordamida yechimlari topilgan.

Bu bobning uchinchi paragrafida qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasi va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamalarining soliton va davriy to‘lqin yechimlari qurilgan. Shuningdek, funksional o‘zgaruvchilar usulining kasr tartibli hosilali modellar uchun qanchalik samarali ishlashi nazariy jihatdan asoslab berilgan.

Quyidagi nochiziqli differensial tenglamani qaraymiz.

$$P(u,u_t,u_x,u_y,u_z,u_{xy},u_{yz},u_{xz},\ldots)=0, \tag{1}$$

1-qadam. Nochiziqli xususiy hosilali tenglamani oddiy differensial tenglamaga keltirish maqsadida quyidagi chiziqli almashtirishni kiritamiz:

$$\xi=\sum_{i=0}^{p}\alpha_i\chi_i+\delta, \tag{2}$$

bu yerda χ_i erkli o‘zgaruvchilar. Agar $p=1$ bo‘lsa, u holda $\xi=\alpha_0\chi_0+\alpha_1\chi_1+\delta$ ko‘rinishga ega bo‘ladi. α_0 va α_1 o‘zgarmas sonlar bo‘lib, ular to‘lqin pulsatsiyasi sifatida talqin qilinadi, χ_0 va χ_1 esa mos ravishda vaqt t va fazoviy koordinata x o‘zgaruvchilarini bildiradi.

(1) tenglamaning yechimlarini izlashda quyidagi almashtirishni kiritamiz:

$$u(\chi_0,\chi_1,\ldots)=u(\xi),$$

va zanjir qoidasiga binoan quyidagi tengliklar o‘rinli bo‘ladi:

$$\frac{\partial u}{\partial \chi_i}=\alpha_i\frac{du}{d\xi},\qquad \frac{\partial^2 u}{\partial \chi_i\partial \chi_j}=\alpha_i\alpha_j\frac{d^2u}{d\xi^2},\quad \ldots, \tag{3}$$

(2) va (3) almashtirishlardan foydalanib (1) nochiziqli xususiy hosilali differensial tenglama oddiy differensial tenglamaga keltiriladi. Natijada quyidagi ko‘rinishdagi tenglama hosil bo‘ladi:

$$Q(u,u',u'',u''',\ldots)=0, \tag{4}$$

bu yerda Q funksional u noma’lum funksiya va uning ξ bo‘yicha hosilalariga bog‘liq va $u'=\frac{du}{d\xi}$.

2-qadam. Keyingi soddalashtirish bosqichida maxsus funksional almashtirish kiritiladi:

$$u'=F(u), \tag{5}$$

Kiritilgan funksional almashtirish natijasida yechim quyidagi integral munosabat orqali aniqlanadi:

$$\int\frac{du}{F(u)}=\xi+C,$$

bu yerda C integrallash doimiysi bo‘lib, qulaylik uchun odatda $C=0$ deb olinadi. Mazkur bosqich funksional o‘zgaruvchilar usulining asosiy g‘oyasini tashkil etadi. u noma’lum funksiyaning ξ bo‘yicha yuqori tartibli hosilalarini $F(u)$ yordamida ifodalash uchun (5) funksional almashtirishdan foydalanib, u funksiyaning yuqori tartibli hosilalari hisoblanadi:

$$\begin{aligned}
u''&=\frac{dF(u)}{du}\frac{du}{d\xi}=\frac{dF(u)}{du}F(u)=\frac{1}{2}\frac{d\left(F^2(u)\right)}{du},\\
u'''&=\frac{1}{2}\frac{d^2\left(F^2(u)\right)}{du^2}\sqrt{F^2(u)},\\
u^{(IV)}&=\frac{1}{2}\left[\frac{d^3\left(F^2(u)\right)}{du^3}F^2(u)+\frac{d^2\left(F^2(u)\right)}{du^2}\frac{d\left(F^2(u)\right)}{du}\right],\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned} \tag{6}$$

3-qadam. (4) oddiy differensial tenglamani u, $F(u)$ va uning hosilalari orqali ifodalash uchun (5) va (6) tengliklardan foydalanamiz. Ushbu ifodalarni (4) ga qo‘yish natijasida tenglama quyidagi ko‘rinishga keltiriladi:

$$R\left(u,\frac{dF(u)}{du},\frac{d^2F(u)}{du^2},\frac{d^3F(u)}{du^3},\ldots\right)=0. \tag{7}$$

(7) tenglamani integrallash natijasida $F(u)$ funksiyasining aniq ifodasi olinadi va (5) tenglama bilan birgalikda qarab, qaralayotgan masalaning yechimlari hosil qilinadi.

Bu bobning ikkinchi paragrafida qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasini funksional o‘zgaruvchilar usuli yordamida aniq yechimlari topilgan. Taklif qilinayotgan usulning boshqa usullarga nisbatan asosiy ustunligi shundaki, bu usul yordamida tenglamaning yanada ko‘proq yangi yechimlarni olish mumkin.

Quyidagi qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasini qaraymiz

$$u_t + h_1(t)u^2u_x - h_2(t)u_{xx} + \omega(t)u_x = 0, \tag{8}$$

bu yerda $u(x,t)$ noma’lum funksiya, $x \in \mathbb{R}$, $t \geq 0$, $h_1(t) \neq 0$, $h_2(t) \neq 0$ va $\omega(t) \neq 0$ berilgan uzluksiz differensiallanuvchi funksiyalar va $h_2(t) > 0$ suyuqlikning kinematik qovushqoqlik koeffitsiyentini ifodalaydi.

1-teorema. Faraz qilaylik $h_1(t) \neq 0$, $h_2(t) \neq 0$ va $\omega(t) \neq 0$ funksiyalar uzluksiz differensiallanuvchi funksiyalar bo‘lib, ular quyidagi tenglikni qanoatlantirsin:

$$h_2(t) = \frac{C_4^2}{3\mu C_2} \cdot \frac{h_1(t)}{\left(\int\limits_0^t h_1(\tau)d\tau + C_3\right)^{4C_2}}.$$

U holda, (8) qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasining yechimi quyidagi ko‘rinishga ega bo‘ladi:

$$u_1(x,t) = \frac{1}{\int\limits_0^t h_1(\tau)d\tau + C_3} + \frac{C_4}{\left(\int\limits_0^t h_1(\tau)d\tau + C_3\right)^{2C_2}} \operatorname{sh}\left(\operatorname{arcth}\left(e^{H_1(x,t)}\right)\right),$$

bu yerda $H_1(x,t) = \int\limits_0^t \left(\lambda C_2^2 h_2(\tau) - C_2\omega(\tau)\right)d\tau + C_1 + C_2x$, λ, μ, C_1, C_2, va C_3 noldan farqli o‘zgarmas sonlar.

2-teorema. Faraz qilaylik $h_1(t) \neq 0$, $h_2(t) \neq 0$ va $\omega(t) \neq 0$ funksiyalar uzluksiz differensiallanuvchi funksiyalar bo‘lib, ular quyidagi tenglikni qanoatlantirsin:

$$h_2(t) = k\,h_1(t),\ k = \text{const}.$$

U holda, (8) qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasining yechimi quyidagi ko‘rinishga ega bo‘ladi:

$$u_2(x,t) = S_3 \operatorname{sh}\left(\operatorname{arcth}(e^{H_2(x,t)})\right).$$

bu yerda $H_2(x,t) = \int\limits_0^t \left(\lambda S_2^2 h_2(\tau) - S_2\omega(\tau)\right)d\tau + S_1 + S_2x$, λ, S_1, S_2, va S_3 noldan farqli o‘zgarmas sonlar.

1-misol. 1-teoremaning qo‘llanilishini quyidagi misol yordamida ko‘rib chiqamiz. Agar $h_1(t) = 2t$, $h_2(t) = \dfrac{t}{t^2+1}$, $\omega(t) = -8t$, $\lambda = 32$ va $\mu = \dfrac{8}{3}$ bo‘lsa, u holda (8) qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasining yechimi quyidagi ko‘rinishga ega bo‘ladi:

$$u_1(x,t) = \frac{1}{t^2+1} + \frac{1}{\sqrt{t^2+1}} \operatorname{sh}\left(\operatorname{arcth}\left((t^2+1)e^{t^2+\frac{1}{4}x}\right)\right).$$

2-misol. 2-teoremaning qo‘llanilishini quyidagi misol yordamida ko‘rib chiqamiz. Agar $h_1(t) = t$, $h_2(t) = t$, $\omega(t) = t$, $\lambda = 1$ va $\mu = 1$ bo‘lsa, u holda (8) qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasining yechimi quyidagi ko‘rinishga ega bo‘ladi:

$$u_2(x,t) = 3\operatorname{sh}\left(\operatorname{arcth}\left(e^{3(t^2+x)}\right)\right).$$

Bu bobning uchinchi paragrafida qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasi va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamalarining soliton va davriy to‘lqin yechimlari funksional o‘zgaruvchilar usuli yordamida topilgan.

Qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasi va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamasini qaraymiz:

$$D_t^{\alpha}u - 6puD_x^{\beta}u + D_x^{3\beta}u + \gamma_1 D_x^{\beta}u = 0, \qquad (9)$$

$$D_t^{\alpha}u - 12pu^2D_x^{\beta}u + D_x^{3\beta}u + \gamma_2 D_x^{\beta}u = 0, \qquad (10)$$

bu yerda $u(x,t)$ noma’lum funksiya, $x \in \mathbb{R}$, $t \geq 0$, $p \neq 0$, $\gamma_1 \neq 0$ va $\gamma_2 \neq 0$ o‘zgarmas sonlar. $0 < \alpha < 1$ va $0 < \beta < 1$ esa Riman-Liuvill kasr tartibli hosila.

1-ta’rif. Agar $u(x)$ funksiya $(a,b) \subseteq \mathbb{R}$ oraliqda aniqlangan silliq funksiya bo‘lsa, u holda $u(x)$ funksiyaning ϵ tartibli chap va o‘ng Riman-Liuvill kasr tartibli hosilalari quyidagi tengliklar bilan aniqlanadi:

$${}_{RL}D_{a,x}^{\epsilon}u(x) = \frac{1}{\Gamma(1-\epsilon)}\frac{d}{dx}\int_a^x \frac{u(\xi)}{(x-\xi)^{\epsilon}}d\xi, \; 0 < \epsilon < 1,$$

$${}_{RL}D_{x,b}^{\epsilon}u(x) = -\frac{1}{\Gamma(1-\epsilon)}\frac{d}{dx}\int_x^b \frac{u(\xi)}{(x-\xi)^{\epsilon}}d\xi, \; 0 < \epsilon < 1.$$

Quyidagi nochiziqli kasr tartibli hosilali differensial tenglamani qaraymiz

$$F\left(u, D_t^{\alpha}u, D_x^{\beta}u, D_t^{\alpha}D_t^{\alpha}u, D_x^{\beta}D_x^{\beta}u, D_t^{\alpha}D_x^{\beta}u, \ldots\right) = 0, \qquad (11)$$

bu yerda $0 < \alpha < 1$, $0 < \beta < 1$, F funksional esa $u(x,t)$ funksiya va uning kasr tartibli hosilalariga bog‘liq.

1-qadam. (11) nochiziqli kasr tartibli hosilali differensial tenglamani oddiy differensial tenglamaga keltirish maqsadida quyidagi almashtirishni kiritamiz:

$$u(x,t) = u(\xi), \; \xi = \frac{cx^{\beta}}{\Gamma(1+\beta)} - \frac{kt^{\alpha}}{\Gamma(1+\alpha)}, \qquad (12)$$

bu yerda c va k noldan farqli ixtiyoriy o‘zgarmas sonlar, k esa to‘lqinning tarqalish tezligi.

(12) dan foydalanib, (11) nochiziqli kasr tartibli hosilali tenglama quyidagi oddiy differensial tenglamaga keltiriladi:

$$P(u, u', u'', u''', \ldots) = 0, \qquad (13)$$

bu yerda $u' = \dfrac{du}{d\xi}$

2-qadam. Noma’lum funksiyaning hosilasini ifodalovchi maxsus funksional almashtirish kiritamiz.

$$u' = F(u), \qquad (14)$$

Mazkur qadam funksional o‘zgaruvchilar usulining asosiy g‘oyasini tashkil etadi. u noma’lum funksiyaning ξ bo‘yicha yuqori tartibli hosilalarini $F(u)$ yordamida ifodalash uchun (14) funksional almashtirishdan foydalanib, u funksiyaning yuqori tartibli hosilalari hisoblanadi:

$$u''=\frac{dF(u)}{du}\frac{du}{d\xi}=\frac{dF(u)}{du}F(u)=\frac{1}{2}\frac{d\left(F^2(u)\right)}{du},$$
$$u'''=\frac{1}{2}\frac{d^2\left(F^2(u)\right)}{du^2}\sqrt{F^2(u)},\qquad (15)$$
$$u^{(IV)}=\frac{1}{2}\left[\frac{d^3\left(F^2(u)\right)}{du^3}F^2(u)+\frac{d^2\left(F^2(u)\right)}{du^2}\frac{d\left(F^2(u)\right)}{du}\right],$$

………………………………………………………………

3-qadam. (13) oddiy differensial tenglamani u, $F(u)$ va uning hosilalari orqali ifodalash uchun (14) va (15) tengliklardan foydalanamiz. Ushbu ifodalarni (13) ga qo‘yish natijasida tenglama quyidagi ko‘rinishga keltiriladi:

$$H\left(u,\frac{dF(u)}{du},\frac{d^2F(u)}{du^2},\frac{d^3F(u)}{du^3},\ldots\right)=0. \qquad (16)$$

(16) tenglamani integrallash natijasida $F(u)$ funksiyasining aniq ifodasi olinadi. Olingan bu funksiya (14) tenglama bilan birgalikda qarab, qaralayotgan masalaning yechimlarini hosil qilish imkonini beradi.

3-teorema. Aytaylik $c\neq 0$, $k\neq 0$, $p\neq 0$ va γ_1 haqiqiy o‘zgarmas sonlar bo‘lsin. Agar $\frac{\gamma_1 c-k}{c}<0$ bo‘lsa, u holda (9) qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Vries tenglamasi quyidagi soliton yechimga ega bo‘ladi:

$$u(x,t)=\frac{\gamma_1 c-k}{2cp}\cdot\frac{1}{\mathrm{ch}^2\left(\frac{1}{2c}\sqrt{\frac{k-\gamma_1 c}{c}}\left(\frac{cx^\beta}{\Gamma(1+\beta)}-\frac{kt^\alpha}{\Gamma(1+\alpha)}\right)\right)}.$$

Agar $\frac{\gamma_1 c-k}{c}>0$ bo‘lsa, u holda (9) qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Vries tenglamasi quyidagi davriy yechimga ega bo‘ladi:

$$u(x,t)=\frac{\gamma_1 c-k}{2cp}\cdot\frac{1}{\cos^2\left(\frac{1}{2c}\sqrt{\frac{\gamma_1 c-k}{c}}\left(\frac{cx^\beta}{\Gamma(1+\beta)}-\frac{kt^\alpha}{\Gamma(1+\alpha)}\right)\right)}.$$

4-teorema. Aytaylik $c\neq 0$, $k\neq 0$, $p>0$ va γ_2 haqiqiy o‘zgarmas sonlar bo‘lsin. Agar $\frac{\gamma_2 c-k}{c}<0$ bo‘lsa, u holda (10) qo‘shimcha hadli kasr tartibli hosilali modifitsirlangan Korteveg-de Vries tenglamasi quyidagi soliton yechimga ega bo‘ladi:

$$u(x,t)=\sqrt{\frac{k-\gamma_2 c}{2pc}}\cdot\frac{1}{\mathrm{ch}\left(\frac{1}{c}\sqrt{\frac{k-\gamma_2 c}{c}}\left(\frac{cx^\beta}{\Gamma(1+\beta)}-\frac{kt^\alpha}{\Gamma(1+\alpha)}\right)\right)}.$$

Agar $\frac{\gamma_2 c - k}{c} > 0$ bo‘lsa, u holda (10) qo‘shimcha hadli kasr tartibli hosilali modifitsirlangan Korteveg-de Vries tenglamasi quyidagi davriy yechimga ega bo‘ladi:

$$u(x,t) = \sqrt{\frac{\gamma_2 c - k}{2pc}} \cdot \frac{1}{\cos\left(\frac{1}{c}\sqrt{\frac{\gamma_2 c - k}{c}}\left(\frac{cx^\beta}{\Gamma(1+\beta)} - \frac{kt^\alpha}{\Gamma(1+\alpha)}\right)\right)}.$$

3-misol. 3-teoremaning qo‘llanilishini quyidagi misol yordamida ko‘rib chiqamiz. Agar $k = -2$, $\alpha = 0.5$, $\beta = 0.5$, $\gamma_1 = 1$, $p = 0.5$ va $c = -1$ bo‘lsa, u holda (9) qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Vries tenglamasi quyidagi soliton yechimga ega bo‘ladi:

$$u(x,t) = -\frac{1}{\mathrm{ch}^2\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(\sqrt{x} - 2\sqrt{t}\right)\right)}.$$

Agar $k = -1$, $\alpha = 0.5$, $\beta = 0.5$, $\gamma_1 = 1$, $p = 1$ va $c = 1$ bo‘lsa, u holda (9) qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Vries tenglamasi quyidagi davriy yechimga ega bo‘ladi:

$$u(x,t) = \frac{1}{\cos^2\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(\sqrt{x} + \sqrt{t}\right)\right)}.$$

4-misol. 4-teoremaning qo‘llanilishini quyidagi misol yordamida ko‘rib chiqamiz. Agar $k = -2$, $\alpha = 0.5$, $\beta = 0.5$, $\gamma_2 = 1$, $p = 0.5$ va $c = -1$ bo‘lsa, u holda (9) qo‘shimcha hadli kasr tartibli hosilali modifitsirlangan Korteveg-de Vries tenglamasi quyidagi soliton yechimga ega bo‘ladi:

$$u(x,t) = \frac{1}{\mathrm{ch}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(\sqrt{x} - 2\sqrt{t}\right)\right)}.$$

Agar $k = -1$, $\alpha = 0.5$, $\beta = 0.5$, $\gamma_2 = 1$, $p = 1$ va $c = 1$ bo‘lsa, u holda (9) qo‘shimcha hadli kasr tartibli hosilali modifitsirlangan Korteveg-de Vries tenglamasi quyidagi davriy yechimga ega bo‘ladi:

$$u(x,t) = \frac{1}{\cos\left(\frac{2\sqrt{2}}{\sqrt{\pi}}\left(\sqrt{x} + \sqrt{t}\right)\right)}.$$

Dissertatsiyaning “**Kasr tartibli hosilali hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallash**” deb nomlangan ikkinchi bobida, kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini Zaxarov-Shabat sistemasi uchun qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida tez kamayuvchi funksiyalar sinfida integrallash algoritmlari ishlab chiqilgan.

Bu bobning birinchi paragrafi Zaxarov-Shabat sistemasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalar usuli haqida zaruriy tushunchalarni taqdim etishdan boshlangan. Ushbu nazariy asoslar bobning keyingi bo‘limlarida bajariladigan integrallash jarayoni uchun tayanch vazifasini bajaradi.

Ushbu

$$\begin{cases} v_x^{(1)} = -ikv^{(1)} + q(x,t)v^{(2)}, \\ v_x^{(2)} = ikv^{(2)} + r(x,t)v^{(1)}, \end{cases} \tag{16}$$

Zaxarov-Shabat sistemasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalari haqidagi zaruriy ma’lumotlarni keltiramiz

2-ta’rif. (16) Zaxarov-Shabat sistemasining ushbu

$$\phi(x,k,t)e^{ikx} \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ \overline{\phi}(x,k,t)e^{-ikx} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ x \to -\infty,$$

$$\psi(x,k,t)e^{-ikx} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ \overline{\psi}(x,k,t)e^{ikx} \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ x \to +\infty,$$

asimtotikalarni qanoatlantiruvchi yechimlarini $\phi(x,k,t)$, $\overline{\phi}(x,k,t)$, $\psi(x,k,t)$ va $\overline{\psi}(x,k,t)$ orqali belgilaymiz va bu yechimlar (16) Zaxarov-Shabat sistemasining Yost yechimlari deb ataladi.

Yost yechimlari uchun quyidagi yoyilmalar o‘rinli:

$$\phi(x,k,t) = b(k,t)\psi(x,k,t) + a(k,t)\overline{\psi}(x,k,t),$$

$$\overline{\phi}(x,k,t) = \overline{a}(k,t)\psi(x,k,t) + \overline{b}(k,t)\overline{\psi}(x,k,t),$$

bu yerda $a(k,t) = W(\phi,\psi)$, $\overline{a}(k,t) = W(\overline{\psi},\overline{\phi})$, $b(k,t) = W(\overline{\psi},\phi)$, $\overline{b}(k,t) = W(\overline{\phi},\psi)$. Oxirgi tenglikda $W(u,v)$ Vronskiy determinanti va u quyidagicha aniqlanadi:

$$W(u,v) = u^{(1)}v^{(2)} - u^{(2)}v^{(1)}.$$

3-ta’rif. Quyidagi

$$\tau(k,t) = \frac{1}{a(k,t)},\ \rho(k,t) = \frac{b(k,t)}{a(k,t)},$$

tengliklar bilan aniqlangan $\tau(k,t)$ funksiyaga o‘tish koeffitsiyenti va $\rho(k,t)$ funksiyaga esa qaytish koeffitsiyenti deyiladi.

Agar $r(x,t) = -q(x,t)$ bo‘lsa, u holda $a(k,t)(\overline{a}(k,t))$ funksiya yuqori(quyi) yarim tekislikda cheklita $k_j = \xi_j + i\eta_j\,(\overline{k}_j = \overline{\xi}_j + i\overline{\eta}_j)$, $j = 1,2,\dots,N$ nollarga ega bo‘lib, ular (16) Zaxarov-Shabat sistemasining xos qiymatlaridan iborat bo‘ladi va quyidagi tengliklar o‘rinli bo‘ladi:

$$\phi_j(x,t) = \chi_j(t)\psi_j(x,t),\ \overline{\phi}_j(x,t) = \overline{\chi}_j(t)\overline{\psi}_j(x,t),$$

$$C_j(t) = \frac{\chi_j(t)}{a_j'(t)},\ \overline{C}_j(t) = \frac{\overline{\chi}_j(t)}{\overline{a}_j'(t)},$$

bu yerda $\chi_j(t) = \chi\left(k_j,t\right)$, $\dot{a}_j(t) = \left.\frac{da(k,t)}{dk}\right|_{k=k_j}$.

4-ta’rif. Ushbu jamlanmaga

$$\left\{\rho(k,t),\, k\in\mathbb{R},\, k_j(t),\, C_j(t),\, j=1,2,...,N\right\},$$

(16) Zaxarov-Shabat sistemasi uchun sochilish nazariyasining berilganlari deyiladi.

(16) Zaxarov-Shabat sistemasi uchun qo'yilgan sochilish nazariyasi teskari masalasining asosiy integral tenglamalar sistemasi quyidagicha bo'ladi:

$$\mathbf{N}(x,k,t)=\begin{pmatrix}0\\1\end{pmatrix}-\sum_{j=1}^{N}\frac{C_j(t)e^{2ik_jx}}{k+k_j}\sigma^{-1}\mathbf{N}_j(x,t)+\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\rho(\xi,t)e^{2i\xi x}}{\xi+k+i0}\sigma^{-1}\mathbf{N}(x,\xi,t)d\xi,\quad(17)$$

$$\mathbf{N}_\ell(x,t)=\begin{pmatrix}0\\1\end{pmatrix}-\sum_{j=1}^{N}\frac{C_j(t)e^{2ik_jx}}{k_\ell+k_j}\sigma^{-1}\mathbf{N}_j(x,t)+\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\rho(\xi,t)e^{2i\xi x}}{\xi+k_\ell}\sigma^{-1}\mathbf{N}(x,\xi,t)d\xi,\quad(18)$$

bu yerda $\mathbf{N}(x,k,t)=\left(N^{(1)}(x,k,t),N^{(2)}(x,k,t)\right)^T$, $\mathbf{N}\left(x,k_l,t\right)=\mathbf{N}_\ell(x,t)$, $\ell=1,2,\ldots,N$,

$$\sigma^{-1}=\begin{pmatrix}0&-1\\1&0\end{pmatrix}.$$

(16) Zaxarov-Shabat sistemasining koeffitsiyenti $q(x,t)$ sochilish nazariyasining berilganlari orqali quyidagicha aniqlanadi:

$$q(x,t)=2i\sum_{j=1}^{N}e^{2ik_jx}C_j(t)N_j^{(2)}(x,t)-\frac{1}{\pi}\int_{-\infty}^{\infty}\rho(\xi,t)e^{2i\xi x}N^{(2)}(x,\xi,t)d\xi.\quad(19)$$

Bu bobning ikkinchi paragrafida esa quyidagi

$$q_t+\Omega_f\left(L^A\right)q_x=0,\ x\in\mathbb{R},\ t>0,\quad(20)$$

Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini mazkur

$$q(x,0)=q_0(x),\ x\in\mathbb{R},\quad(21)$$

boshlang'ich shart bilan qaraymiz. Bu yerda $p_1,p_2\neq 0$ o'zgarmas sonlar va

$$\Omega_f\left(L^A\right)=\left(p_2\left(4L^A\right)^{-1}-4p_1L^A\right)\cdot\left|4L^A\right|^{\epsilon},\quad(22)$$

$$L^A=-\frac{1}{4}\frac{\partial^2}{\partial x^2}-q^2-q_xI_-q,\ I_-=\int_{-\infty}^{x}dy.$$

5-ta'rif. Agar $q(x,t)$ funksiya $(a,b)\subseteq\mathbb{R}$ oraliqda aniqlangan silliq funksiya bo'lsa, u holda $q(x,t)$ funksiyaning ϵ tartibli Riss kasr tartibli hosilasi quyidagi tenglik bilan aniqlanadi:

$$\left(-\partial_x^2\right)^{\epsilon}q(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{q}(k,t)\,|k|^{2\epsilon}\,e^{ikx}\,dk,$$

bu yerda $\hat{q}(k,t)=\int_{-\infty}^{\infty}q(x,t)e^{-ikx}dx$, $\hat{q}(k,t)$ funksiya $q(x,t)$ funksiyaning Furye almashtirishi.

Boshlang'ich shartdagi $q_0(x)$ funksiya quyidagi xossalarga ega:

1. $\int_{-\infty}^{\infty}(1+|x|)|q_0(x)|dx<\infty$,

2. $H(0)=i\begin{pmatrix}\frac{d}{dx} & -q_0(x)\\ -q_0(x) & -\frac{d}{dx}\end{pmatrix}$ operator yuqori yarim tekislikda N ta $k_1(0)$, $k_2(0)$, …, $k_N(0)$ xos qiymatlarga ega va spektral maxsusliklarga ega emas.

(22) tenglamada keltirilgan $\Omega_f\left(L^A\right)$ ifodani aniqlashda Ablovitz, Bin va Carrlar ishida keltirilgan algoritmni qo'llab, ya'ni Riss kasr tartibli hosilali chiziqli modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasining

$$u_{xt}+\left(-\partial_x^2\right)^{\epsilon}\left(p_1u_{xxxx}-p_2u\right)=0, \qquad (23)$$

dispersiya munosabatidan foydalanamiz. Buning uchun $u(x,t)=e^{i(kx-\omega(k)t)}$ ifodani (23) tenglamaga qo'yamiz va natijada quyidagi dispersion munosabatga ega bo'lamiz:

$$\omega(k)=\left(\frac{p_2}{k}-p_1k^3\right)|k|^{2\epsilon}.$$

Ushbu

$$\Omega_f(k^2)=\frac{\omega(2k)}{2k},$$

tenglikni e'tiborga olsak, $\Omega_f\left(L^A\right)$ifoda (22) ko'rinishda bo'lishi kelib chiqadi.

5-teorema. Agar $q(x,t)$ funksiya Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz sinus-Gordon tenglamasiga qo'yilgan Koshi masalasi, ya'ni (20)-(21) masalaning yechimi bo'lsa, u holda (16) Zaxarov-Shabat sistemasi uchun sochilish nazariyasi berilganlarining vaqt bo'yicha evolyutsiyalari quyidagi differensial tenglamalarni qanoatlantiradi:

$$\frac{\partial\rho(k,t)}{\partial t}=-2ik\Omega_f(k^2)\cdot\rho(k,t),\ k\in\mathbb{R}\setminus\{0\},$$

$$\frac{dk_j(t)}{dt}=0,\ j=1,2,\ldots,N,$$

$$\frac{dC_j(t)}{dt}=-2ik_j\Omega_f(k_j^2)\cdot C_j(t),\ j=1,2,\ldots,N,$$

bu yerda $\Omega_f(k^2)=\left(\frac{p_2}{4k^2}-4p_1k^2\right)|2k|^{2\epsilon}$.

Olingan natijalar sochilish nazariyasining vaqt bo'yicha evolyutsiyalarini to'liq aniqlab, (17) tenglamani quyidagi algoritm asosida yechish imkonini beradi.

Bizga $q(x,0)$ berilgan bo'lsin.

Dastlab berilgan $q(x,0)$ boshlang'ich shart orqali (16) sistemaning $t=0$ dagi sochilish nazariyasining berilganlarini(SNB) topamiz:

$$\left\{\rho(k,0),\ k\in\mathbb{R},\ k_j(0),\ C_j(0),\ j=1,2,...,N\right\},$$

$t=0$ paytidagi SNB ni boshlang'ich shart sifatida olib, 5-teoremadagi differensial tenglamalarni yechib, $t>0$ paytidagi SNB ning vaqt bo'yicha evolyutsiyalarini keltirib chiqaramiz:

$$\left\{\rho(k,t),\, k\in\mathbb{R},\, k_j(t),\, C_j(t),\, j=1,2,...,N\right\}.$$

3. Topilgan SNB yordamida (17) va (18) integral tenglamalar sistemasini yechish orqali $N_j^{(2)}(x,t)$, $j=1,2,...,N$ funksiyalarni aniqlaymiz;

4. (19) tenglikdan foydalanib $q(x,t)$ funksiya topiladi;

5. Nihoyat, $q(x,t)=\frac{1}{2}u_x(x,t)$ tenglikga ko‘ra, (20) tenglamaning $u(x,t)$ yechimi aniqlanadi.

5-misol. (20) tenglamani quyidagi

$$q(x,0)=\frac{2}{ch\left(2(x-x_0)\right)},$$

boshlang‘ich shart bilan qaraymiz. Yuqoridagi keltirilgan algoritm yordamida $q(x,t)$ funksiyani topamiz:

$$q(x,t)=\frac{2}{ch\left(2(x-x_0)-2\left(4p_1-\frac{p_2}{4}\right)|2|^{2\epsilon}\, t\right)},$$

$q(x,t)=\frac{1}{2}u_x(x,t)$ tenglikga ko‘ra, (20) tenglamaning $u(x,t)$ yechimi quyidagicha bo‘ladi:

$$u(x,t)=2\arctan\left(sh\left((x-x_0)-2\left(4p_1-\frac{p_2}{4}\right)|2|^{2\epsilon}\, t\right)\right).$$

Bu bobning uchinchi paragrafida esa o‘zgaruvchan koeffitsiyentli Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasi Zaxarov-Shabat sistemasi uchun qo‘yilgan sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida integrallangan.

Quyidagi

$$q_t+\Omega_f\left(L^A\right)q_x=0,\; x\in\mathbb{R},\; t>0, \tag{24}$$

o‘zgaruvchan koeffitsiyentli Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasini mazkur

$$q(x,0)=q_0(x),\; x\in\mathbb{R}, \tag{25}$$

boshlang‘ich shart bilan qaraymiz. Bu yerda $p_1(t), p_2(t)\neq 0$ vaqtga bog‘liq funksiyalar va

$$\Omega_f\left(L^A\right)=\left(p_2(t)\left(4L^A\right)^{-1}-4p_1(t)L^A\right)\cdot\left|4L^A\right|^{\epsilon}, \tag{26}$$

$$L^A=-\frac{1}{4}\frac{\partial^2}{\partial x^2}-q^2-q_x I_- q,\; I_-=\int_{-\infty}^{x}dy.$$

(24) tenglamada keltirilgan $\Omega_f\left(L^A\right)$ ifodani aniqlashda Ablovitz, Bin va Carrlar ishida keltirilgan algoritmni qo‘llab, ya’ni o‘zgaruvchan koeffitsiyentli Riss kasr tartibli hosilali chiziqli modifitsirlangan Korteveg-de Friz-sinus-Gordon

$$u_{xt}+\left(-\partial_x^2\right)^{\epsilon}\left(p_1(t)u_{xxxx}-p_2(t)u\right)=0, \tag{27}$$

tenglamasining dispersiya munosabatidan foydalanamiz. Buning uchun $u(x,t)=e^{i(kx-\omega(k)t)}$ ifodani (27) tenglamaga qo'yamiz va natijada quyidagi dispersion munosabatga ega bo'lamiz:

$$\omega(k)=\left(\frac{p_2(t)}{k}-p_1(t)k^3\right)|k|^{2\epsilon}.$$

Ushbu

$$\Omega_f(k^2)=\frac{\omega(2k)}{2k},$$

tenglikni e'tiborga olsak, $\Omega_f\left(L^A\right)$ifoda (26) ko'rinishda bo'lishi kelib chiqadi.

6-teorema. Agar $q(x,t)$ funksiya o'zgaruvchan koeffitsiyentli Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz sinus-Gordon tenglamasiga qo'yilgan Koshi masalasi, ya'ni (24)-(25) masalaning yechimi bo'lsa, u holda (16) Zaxarov-Shabat sistemasi uchun sochilish nazariyasi berilganlarining vaqt bo'yicha evolyutsiyalari quyidagi differensial tenglamalarni qanoatlantiradi:

$$\frac{\partial\rho(k,t)}{\partial t}=-2ik\Omega_f(k^2)\cdot\rho(k,t),\ k\in\mathbb{R}\setminus\{0\},$$

$$\frac{dk_j(t)}{dt}=0,\ j=1,2,\ldots,N,$$

$$\frac{dC_j(t)}{dt}=-2ik_j\Omega_f(k_j^2)\cdot C_j(t),\ j=1,2,\ldots,N,$$

bu yerda $\Omega_f(k^2)=\left(\frac{p_2(t)}{4k^2}-4p_1(t)k^2\right)|2k|^{2\epsilon}$.

Dissertatsiyaning "**Moslangan manbali umumiy Kaup-Boussinesq tenglamasini integrallash**" deb nomlangan uchinchi bobida, umumiy Kaup-Boussinesq tenglamasini sochilish nazariyasining to'g'ri va teskari masalalar usuli yordamida integrallash masalasi o'rganilgan. Bobning asosiy maqsadi Shturm-Liuvill operatorlarining kvadratik dastasi asosida qurilgan spektral nazariya va uning yordamida umumiy Kaup-Boussinesq tenglamasining o'zgaruvchan koeffitsiyentli hamda moslangan manbali ko'rinishlari uchun aniq integrallash algoritmini ishlab chiqishdan iborat.

Bu bobning birinchi paragrafida Shturm-Liuvill operatorining kvadratik dastasi uchun sochilish nazariyasining to'g'ri va teskari masalalar yechishning Maqsudov-Guseynov usuli haqida zaruriy ma'lumotlar keltirilgan.

Ushbu Shturm-Liuvill operatorlari kvadratik dastasini qaraymiz

$$L(k)y\equiv -y''+v(x)y+2ku(x)y-k^2y=0,\ x\in R, \tag{28}$$

bunda $v(x)$ va $u(x)$ haqiqiy funksiyalar, $u(x)$ absolyut uzluksiz va ular quyidagi

$$\int_{-\infty}^{\infty}|u(x)|dx<\infty,\ \int_{-\infty}^{\infty}(1+|x|)[|v(x)|+|u'(x)|]dx<\infty. \tag{29}$$

shartlarni qanoatlantiradi.

(29) shart bajarilganda (28) tenglama $Imk \geq 0$ yuqori yarim tekislikka tegishli barcha k lar uchun quyidagi

$$f_+(x,k) = e^{ikx}[1+o(1)],\ x \to +\infty,$$
$$f_-(x,k) = e^{-ikx}[1+o(1)],\ x \to -\infty,$$

asimptotikalarni qanoatlantiruvchi $f_+(x,k),\ f_-(x,k)$ yechimlarga ega. Noldan farqli haqiqiy $k \neq 0$ larda, (28) tenglama $f_+(x,k),\ \overline{f}_+(x,k)$ va $f_-(x,k),\ \overline{f}_-(x,k)$ kabi yechimlar fundamental sistemasiga ega bo‘ladi, va bu fundamental sistemalar o‘zaro quyidagi ko‘rinishda bog‘langan:

$$f_+(x,k) = b(k)f_-(x,k) + a(k)\overline{f}_-(x,k),$$
$$f_-(x,k) = -\overline{b}(k)f_+(x,k) + a(k)\overline{f}_+(x,k).$$

bu yerda

$$a(k) = -\frac{1}{2ik}W\left\{f_+(x,k), f_-(x,k)\right\},\ b(k) = \frac{1}{2ik}W\left\{f_+(x,k), \overline{f}_-(x,k)\right\}.$$

Bunda $a(k)$ funksiya $Imk > 0$ yuqori yarim tekislikka analitik davom qiladi va cheklita $k_1, k_2, ..., k_N$ nollarga ega bo‘ladi hamda quyidagi tengliklar o‘rinli bo‘ladi

$$f_\mp(x,k_n) = B_n^\pm f_\pm(x,k_n),$$

bu yerdagi $B_n^\pm$ kattaliklar x o‘zgaruvchiga bog‘liq bo‘lmaydi.

6-ta’rif. Ushbu

$$\left\{r_-(k) = \frac{b(k)}{a(k)},\ k \in R \setminus \{0\},\ k_1, k_2, ..., k_N,\ \gamma_1^-, \gamma_2^-, ..., \gamma_N^-\right\}$$

va

$$\left\{r_+(k) = -\frac{\overline{b}(k)}{a(k)},\ k \in R \setminus \{0\}, k_1, k_2, ..., k_N, \gamma_1^+, \gamma_2^+, ..., \gamma_N^+\right\} \tag{30}$$

jamlanmalarga (28) tenglamaning mos ravishda chap va o‘ng sochilish nazariyasining berilganlari deyiladi, bunda $\gamma_n^\pm$ quyidagicha aniqlanadi:

$$\gamma_n^\pm = B_n^\pm \left(\left.\frac{da(k)}{dk}\right|_{k=k_n} \right)^{-1},\ n = 1, 2, ..., N.$$

Chap yoki o‘ng sochilish nazariyasining berilganlari orqali $u(x)$ va $v(x)$ koeffitsiyentlarni tiklash masalasiga (28) tenglama uchun qo‘yilgan teskari masala deyiladi.

O‘ng sochilish nazariyasi berilganlari (30) yordamida $u(x)$ va $v(x)$ koeffitsiyentlarni tiklash masalasini ko‘rib chiqamiz.

O‘ng sochilish nazariyasining berilganlari (30) yordamida $F_+(x)$ funksiyani qurib olamiz

$$F_+(x) = -i\sum_{n=1}^{N}\gamma_n^+ e^{ik_n x} + \frac{1}{2\pi}\int_{-\infty}^{\infty} r_+(k)e^{ikx}dk.$$

Topilgan $F_+(x)$ funksiyani quyidagi integral tenglamalarga qo‘yamiz

$$F_+(x+y) + \overline{K_+^{(0)}(x,y)} + \int_x^{\infty} K_+^{(0)}(x,\tau)F_+(\tau+y)d\tau = 0, x \le y < \infty,$$

$$iF_+(x+y) + \overline{K_+^{(1)}(x,y)} + \int_x^{\infty} K_+^{(1)}(x,\tau)F_+(\tau+y)d\tau = 0, x \le y < \infty.$$

Bu integral tenglamalarni yechib $K_+^{(0)}(x,y)$ va $K_+^{(1)}(x,y)$ larni topamiz. Bular yordamida quyidagi

$$K_+(x,y) = K_+^{(0)}(x,y)\cos\alpha_+(x) + K_+^{(1)}(x,y)\sin\alpha_+(x)$$

funksiyani qurib olamiz. Bu tenglikdagi $\alpha_+(x)$ funksiya quyidagi

$$\alpha_+(x) = \int_x^{\infty}\Phi(s,\alpha_+(s))ds, \ -\infty < x < \infty$$

Volterra integral tenglamasining yechimidan iborat bo‘ladi. Bu yerda

$$\Phi(s,z) = \left[\mathrm{Re}\, K_+^{(0)}(s,s) - \mathrm{Im}\, K_+^{(1)}(s,s)\right]\sin 2z +$$
$$+2[\mathrm{Re}\, K_+^{(1)}(s,s)]\sin^2 z - 2[\mathrm{Im}\, K_+^{(0)}(s,s)]\cos^2 z.$$

Natijada, $u(x)$ va $v(x)$ koeffitsiyentlar ushbu tengliklar orqali topiladi

$$u(x) = -\alpha'_+(x),$$

$$v(x) = -u^2(x) - 2\frac{d}{dx}\left\{\left[\mathrm{Re}\, K_+(x,x)\right]\cos\alpha_+(x) + \left[\mathrm{Im}\, K_+(x,x)\right]\sin\alpha_+(x)\right\}.$$

Bu bobning ikkinchi paragrafida quyidagi o‘zgaruvchan koeffitsiyentli umumiy Kaup-Boussinesq tenglamasini

$$U_t + \Omega(L^*)U_x = G, \tag{31}$$

ushbu

$$v(x,t)\big|_{t=0} = v_0(x), \quad u(x,t)\big|_{t=0} = u_0(x), \quad x \in R, \tag{32}$$

boshlang‘ich shartlar bilan qaraymiz. Bu yerda

$$U = \begin{pmatrix} v(x,t) \\ u(x,t) \end{pmatrix}, \ G = \begin{pmatrix} G_1(x,t) \\ G_2(x,t) \end{pmatrix}, \ G_1(x,t) = \mu(t)v_x, \ G_2(x,t) = \mu(t)u_x$$

$$L^* = \begin{pmatrix} 0 & -\frac{\partial^2}{\partial x^2} + 4v - 2v_x\int_x^{\infty}d\tau \\ 1 & 4u - 2u_x\int_x^{\infty}d\tau \end{pmatrix}, \tag{33}$$

$\Omega(s)$- s bo‘yicha istalgan darajali ko‘phad bo‘lib(uning koeffitsiyentlari vaqtga bog‘liq bo‘lishi mumkin), $\mu(t)$ esa ixtiyoriy uzluksiz funksiya hisoblanadi va $v_0(x)$, $u_0(x)$ funksiyalar haqiqiy va quyidagi shartlarni qanoatlantiradi:

i) $u_0(x)$ absolyut uzluksiz funksiya va ushbu tengsizliklar o‘rinli:

$$\int_{-\infty}^{+\infty} |u_0(x)|\,dx < \infty,\ \int_{-\infty}^{+\infty} (1+|x|)[|v_0(x)|+|u_0'(x)|]dx < \infty \tag{34}$$

ii) Shturm-Liuvill operatorlarining kvadratik dastasi

$$L(0,k)y \equiv -y'' + v_0(x)y + 2ku_0(x)y - k^2 y = 0,\ x \in R$$

$2N$ ta $k_1(0), k_2(0), ..., k_{2N}(0)$ oddiy xos qiymatlarga ega.

7-teorema. Agar $v = v(x,t)$ va $u = u(x,t)$ funksiyalari (31)-(34) masalasining yechimlari bo‘lsa, u holda Shturm-Liuvill operatorlarining kvadratik dastasi $T(t,k)$ uchun sochilish nazariyasi berilganlarining vaqt bo‘yicha evolyutsiyalari quyidagi differensial tenglamalarni qanoatlantiradi:

$$\frac{dr_+(t,k)}{dt} = -2ik[\Omega(2k) - \mu(t)]r_+(t,k),\ k \in \mathbb{R},$$

$$\frac{dk_n(t)}{dt} = 0,\ n = 1,2,\dots,N,$$

$$\frac{d\gamma_n^+(t)}{dt} = 2ik_n[\Omega(2k_n) + \mu(t)]\gamma_n^+(t),\ n = 1,2,\dots,N.$$

Olingan natijalar sochilish nazariyasi berilganlarining vaqt bo‘yicha evolyutsiyasini to‘la aniqlaydi va (31)-(34) masalani teskari masala usulida yechish imkonini beradi.

Bu bobning uchinchi paragrafida moslangan manbali umumiy Kaup-Boussinesq tenglamasi tez kamayuvchi funksiyalar sinfida integrallangan.

Ushbu

$$L(k)y = -y'' + (V - k^2)y = 0, x \in R, \tag{35}$$

Shturm-Liuvill operatorlari kvadratik dastasini qaraymiz. Bu yerda $V(x,k) = v(x) + 2ku(x)$ hamda $v(x)$ va $u(x)$ funksiyalar kompleks qiymatli funksiyalar bo‘lib quyidagi shartlarni qanoatlantiradi:

$$\int_{-\infty}^{+\infty} x^2\left[|v(x)| + |u'(x)|\right]dx < \infty,\ \int_{-\infty}^{+\infty} |x|\left[|v'(x)| + |u''(x)|\right]dx < \infty, \tag{36}$$

(36) shart bajarilganda, (35) tenglama ixtiyoriy $k \in R$ larda ushbu asimptotikalarni

$$\left[f_1(x,k), g_1(x,k)\right] \sim \left[e^{-ikx}, e^{ikx}\right],\ x \to \infty,$$

$$\left[f_2(x,k), g_2(x,k)\right] \sim \left[e^{ikx}, e^{-ikx}\right],\ x \to -\infty$$

qanoatlantiruvchi $\{f_1(x,k),\ g_1(x,k)\}$ va $\{f_2(x,k),\ g_2(x,k)\}$ Yost yechimlariga ega. Noldan farqli haqiqiy $k \neq 0$ da $\{f_1(x,k), g_1(x,k)\}$ va $\{f_2(x,k), g_2(x,k)\}$ funksiyalar (35) tenglamaning yechimlar fundamental sistemasini tashkil qiladi. Bu yechimlar uchun quyidagi tengliklar o‘rinli

$$f_2 = c_{11}f_1 + c_{12}g_1,\ g_2 = d_{12}f_1 + d_{11}g_1,$$

$$f_1 = c_{22}f_2 + c_{21}g_2,\ g_1 = d_{21}f_2 + d_{22}g_2,$$

$$c_{12} = c_{21} = (2ik)^{-1}W[f_1, f_2],\quad c_{11} = -d_{22} = (2ik)^{-1}W[f_2, g_1],$$

$$d_{12} = d_{21} = (2ik)^{-1}W[g_2, g_1],\quad d_{11} = -c_{22} = (2ik)^{-1}W[f_1, g_2],$$

bunda $c_{11}, c_{12}, c_{21}, c_{22}, d_{11}, d_{12}, d_{21}, d_{22}$ funksiyalar x o‘zgaruvchiga bog‘liq emas hamda $c_{21}(k),\ (Imk < 0)$ funksiya quyi yarim tekislikka analitik davom qiladi. (35) tenglamada $V(x,k)$ o‘rniga $V(x,\pm k)$ ni qarasak, u holda (35) ni ikkita tenglama deb qarash mumkin. Shu sababli yuqorida keltirilgan barcha tengliklarda "$\pm$" indeks bor deb tushunamiz. $c_{21}^{\pm}(k)(Imk < 0)$ funksiya cheklita $N^{\pm}$ nollarga ega va bu nollarni $k_n^{\pm},\ n = 1,2,...,N^{\pm}$ orqali belgilaymiz.

7-ta’rif. Ushbu

$$\left\{ R^{\pm}(k) = \frac{c_{11}^{\pm}(-k)}{c_{21}^{\mp}(-k)},\ k \in R \setminus \{0\},\ k_n^{\pm},\ C_n^{\pm},\ n = 1,2,...,N^{\pm} \right\}$$

jamlanmaga (35) tenglamaning sochilish nazariyasining berilganlari deyiladi.
Bu yerda

$$C_n^{\pm} = \left[c_{11}^{\pm}(k_n^{\pm}) \right]^{-1} \left[i\frac{d}{dk} c_{21}^{\pm}(k) \right]_{k=k_n^{\pm}}.$$

$u(x)$ va $v(x)$ koeffitsiyentlar sochilish nazariyasi berilganlari orqali yagona aniqlanadi.

Bu bobning uchinchi paragrafida moslangan manbali umumiy Kaup-Boussinesq tenglamasi uchun sochilish nazaryasi berilganlarining vaqt bo‘yicha o‘zgarish evolyutsiyalari keltirib chiqarilgan va moslangan manbali umumiy Kaup-Boussinesq tenglamasi uchun qo‘yilgan Koshi masalasini teskari masala usuli yordamida yechish algoritmi keltirilgan.

Quyidagi moslangan manbali umumiy Kaup-Boussinesq tenglamasini

$$\begin{cases} U_t + \Omega(L^*)U_x = G, \\ (\phi_m)_{xx} + \left[k_m^2 - v - 2k_m u \right]\phi_m = 0,\ m = 1,2,\dots,N, \end{cases} \tag{37}$$

ushbu

$$v(x,t)\big|_{t=0} = v_0(x),\quad u(x,t)\big|_{t=0} = u_0(x),\quad x \in R, \tag{38}$$

boshlang‘ich va quyidagi

$$\int_{-\infty}^{+\infty}(2k_m-2u)\varphi_m^2dx=A_m(t),\ m=1,2,...,N, \tag{39}$$

normallovchi shartlar bilan qaraymiz. Bu yerda

$$U=\begin{pmatrix} v(x,t)\\ u(x,t)\end{pmatrix},\ G=\begin{pmatrix} G_1(x,t)\\ G_2(x,t)\end{pmatrix},$$

$$G_1(x,t)=2\sum_{m=1}^{N}\left[-u_x\phi_m^2+(k_m-2u)\frac{\partial}{\partial x}\phi_m^2\right],\ G_2(x,t)=\sum_{m=1}^{N}\frac{\partial}{\partial x}\phi_m^2,$$

$$L^*=\begin{pmatrix} 0 & -\dfrac{\partial^2}{\partial x^2}+4v-2v_x\displaystyle\int_x^{\infty}d\tau \\ 1 & 4u-2u_x\displaystyle\int_x^{\infty}d\tau \end{pmatrix}, \tag{40}$$

bu yerda $\Omega(s)$ - s bo‘yicha istalgan darajali ko‘phad bo‘lib(uning koeffitsiyentlari vaqtga bog‘liq bo‘lishi mumkin), $\varphi_1=\varphi_1(x,t),\ \varphi_2=\varphi_2(x,t),...,\varphi_N=\varphi_N(x,t)$ funksiyalar (35) tenglamaning $k_1=k_1(t), k_2=k_2(t),...,k_N=k_N(t)$, $\operatorname{Im}k_m<0,\ m=1,2,...,N$ xos qiymatlariga mos keluvchi xos funksiyalari, hamda $A_1(t), A_2(t),..., A_N(t)$ lar oldindan berilgan ixtiyoriy uzluksiz funksiyalar. $v_0(x), u_0(x)$ funksiyalar quyidagi shartlarni qanoatlantiradi:

i) $v_0(x)$, $u_0(x)$ funksiyalar kompleks qiymatli funksiyalar bo‘lib quyidagi shartlarni qanoatlantiradi:

$$\int_{-\infty}^{+\infty}x^2\left[|v_0(x)|+|u_0'(x)|\right]dx<\infty,\ \int_{-\infty}^{+\infty}|x|\left[|v_0'(x)|+|u_0''(x)|\right]dx<\infty \tag{41}$$

ii) Shturm-Liuvill operatorlari kvadratik dastasi

$$L(0,k)y\equiv -y''+v_0(x)y+2ku_0(x)y-k^2y=0,\ x\in R$$

chekli N ta oddiy xos qiymatlarga ega.

8-teorema. Agar $v=v(x,t)$ va $u=u(x,t)$ funksiyalari (37)-(40) masalasining yechimlari bo‘lsa, u holda Shturm-Liuvill operatorlarining kvadratik dastasi $T(t,\pm k)$ uchun sochilish nazariyasi berilganlarining vaqt bo‘yicha evolyutsiyalari quyidagi differensial tenglamalarni qanoatlantiradi:

$$\frac{dR^{\pm}(t,k)}{dt}=-2ik\Omega(\pm k)R^{\pm}(t,k),\ k\in\mathbb{R},$$

$$\frac{dk_n^{\pm}(t)}{dt}=0,\ n=1,2,...,N^{\pm},$$

$$\frac{dC_n^{+}(t)}{dt}=-[2ik_n^{+}\Omega(k_n^{+})+2ik_n^{+}A_n(t)]C_n^{+}(t),\ n=1,2,...,N^{+},$$

$$\frac{dC_n^-(t)}{dt} = 2ik_n^-\Omega(-k_n^-)C_n^-(t), \; n = 1, 2, ..., N^- .$$

Olingan natijalar sochilish nazariyasi berilganlarining vaqt bo‘yicha evolyutsiyasini to‘la aniqlaydi va (37)-(40) masalani teskari masala usulida yechish imkonini beradi.

XULOSA

Mazkur dissertatsiya ishi moslangan manbali butun va kasr tartibli hosilali xususiy hosilali nochiziqli evolyutsion tenglamalarni tez kamayuvchi funksiyalar sinfida sochilish nazariyasining to‘g‘ri va teskari masalalari usulida integrallashga bag‘ishlangan.

Dissertatsiya ishining asosiy natijalari quyidagilardan iborat:

1) funksional o‘zgaruvchilar usuli yordamida qo‘shimcha hadli va o‘zgaruvchan koeffitsiyentli modifitsirlangan Burgers tenglamasi, qo‘shimcha hadli kasr tartibli hosilali Korteveg-de Friz tenglamasi va kasr tartibli hosilali modifitsirlangan Korteveg-de Friz tenglamasining aniq soliton va davriy yechimlari topilgan;

2) Riss kasr tartibli hosilali modifitsirlangan Korteveg-de Friz-sinus-Gordon tenglamasi Zaxarov-Shabat sistemasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalar usulini yordamida tez kamayuvchi funksiylar sinfida integrallanuvchanligi isbotlangan;

3) o‘zgaruvchan koeffitsiyentli va moslangan manbali umumiy Kaup-Boussinesq tenglamasi Shturm-Liuvill operatorlari kvadratik dastasi uchun sochilish nazariyasining to‘g‘ri va teskari masalalar usuli yordamida tez kamayuvchi funksiylar sinfida integrallanuvchanligi isbotlangan;

Olingan natijalar dissertatsiya tadqiqotining maqsadga erishilganligini tasdiqlaydi. Barcha asosiy natijalar nochiziqli evolyutsion tenglamalarni integrallash nazariyasiga ma’lum darajada hissa qo‘shadi.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES PhD.03/2025.27.12.FM.06.02 URGENCH STATE UNIVERSITY NAMED AFTER ABU RAYHAN BIRUNI

KHOREZM MAMUN ACADEMY

ABDIKARIMOV FAXRIDDIN BAXROM UGLI

INTEGRATION OF INTEGER AND FRACTIONAL ORDER NONLINEAR EVOLUTION EQUATIONS WITH SELF-CONSISTENT SOURCES

01.01.02 – Differential Equations and Mathematical Physics

ABSTRACT
of dissertation of the doctor of philosofhy (PhD) on physical and matematical sciences

Urgench – 2026

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Comission at the Ministry of Higher education, science and innovations of the Republic of Uzbekistan under number B2024.2.PhD/FM1052.

The dissertation was prepared at Khorezm Mamun Academy.

The abstract of the dissertation is posted in three languages (uzbek, russian, english (resume)) on the website (www.ik-mat.urdu.uz) and the "ZiyoNet" Information and educational portal (www.ziyonet.uz).

Scientific supervisor:	**Babajanov Bazar Atajanovich** Doctor of Physical and Matematical Sciences, Professor
Official opponents:	**Khasanov Aknazar Bekdurdiyevich** Doctor of Physical and Matematical Sciences, Professor
	Babajanova Aygul Komiljanovna Doctor of Philosophy in Physical and Mathematical Sciences
Leading organization:	**Bukhara State University**

Defense will take place "16" february 2026 at 14 : 00 the meeting of Scientific Council number PhD.03/2025.27.12.FM.06.02 at Urgench State University named after Abu Rayhan Biruni. (Address: Kh.Alimdjan str.14, Urgench, 220100, Uzbekistan, Ph.: (+99862) 224-66-11, fax: (+99862) 224-67-00, e-mail: info@urdu.uz).

Dissertation in possible to review in Information-resource centre at Urgench State University named after Abu Rayhan Biruni (is registred №D-941). (Address: Kh.Alimdjan str.14, Urgench, 220100, Uzbekistan, Ph.: (+99862) 224-66-11, fax: (+99862) 224-67-00, e-mail: arm@urdu.uz).

Abstract of dissertation sent out on "02" february 2026 year

(Mailing report № 1 on "02" february 2026 year)

B. I. Abdullayev
Chairman of scientific council on award of scientific degrees, Doctor of Physical and Mathematical Sciences, Professor

A. A. Reyimberganov
Scientific secretary of scientific council on award of scientific degrees, Candidate of Physical and Mathematical Sciences

B. A. Babajanov
Chairman of Scientific seminar under scientific council on award of scientific degrees, Doctor of Physical and Mathematical Sciences, Professor

INTRODUCTION (Annotation of the PhD Dissertation in Philosophy)
The actuality (relevance) and demand of the theme of the dissertation.

Today, many scientific and practical studies around the world are connected to one of the important and active areas of modern mathematics the study of nonlinear evolution equations. These equations describe many real processes in nature. Since anomalous diffusion and fractional dispersion effects appear in many physical and technological systems, studying nonlinear evolution equations with integer and fractional derivatives and with self-consistent sources have become very important. This is not only a theoretical topic but also a practical one. Research in this area strengthens the connection between pure mathematics, computer simulations, and experimental physics. It helps scientists discover new types of solitons, new kinds of wave behaviors, and new classes of integrable equations. In addition, these mathematical models are very useful in many fields such as plasma physics, optics, fluid dynamics, biomedicine, geophysics, heat transfer, and even economic models where inertia effects appear.

At present, large-scale research is being carried out around the world on the integration of nonlinear evolution equations that include self-consistent sources. Usually, equations without such sources are model equations derived under ideal conditions, but real natural processes always involve additional effects. Therefore, when studying nonlinear evolution equations, it becomes necessary to introduce equations with variable coefficient and self-consistent sources. For this reason, nonlinear evolution equations with variable coefficients and self-consistent sources are considered more realistic mathematical models of these processes. In particular, special attention worldwide is being given to the solution of the modified Burgers equation with variable-coefficient and additional term, as well as the fractional Korteweg-de Vries and modified Kortewe-de Vries equations with additional terms using the functional variable method, integration of the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation using the inverse scattering method, the integration of the hierarchy Kaup-Boussinesq equation with variable-coefficient and self-consistent sources in the class of rapidly decaying functions.

In the Republic of Uzbekistan, extensive research is being carried out to determine solutions of fractional-order and integrable nonlinear evolution equations using the methods of the direct and inverse scattering problems formulated for the Zakharov-Shabat system and the quadratic pencil of Sturm-Liouville operators, as well as to apply the obtained solutions in practice. In particular, significant results have been achieved in constructing soliton solutions of the modified Korteweg-de Vries equation with Riesz fractional derivatives and the generalized Kaup-Boussinesq equation by employing the direct and inverse scattering methods.

In accordance with the Decision of the Cabinet of Ministers of the Republic of Uzbekistan adopted in 2019 on further development of mathematical education and sciences, conducting scientific research at the level of international standards in priority areas of mathematics namely algebra and functional analysis, differential equations and mathematical physics, dynamical systems theory, geometry and topology, probability theory and mathematical statistics, applied mathematics and mathematical modelinghas been defined as the main objectives and areas of activity of mathematical sciences[2].

In implementing these tasks, special importance is attached to the application of the direct and inverse scattering methods in the integration of nonlinear evolution equations of modern mathematical physics. In particular, the integration of the Riesz fractional derivative modified Korteweg-de Vries-sine-Gordon equation in the class of rapidly decaying functions using the direct and inverse scattering methods formulated for the Zakharov-Shabat system, as well as the integration of the generalized Kaup-Boussinesq equation with variable coefficients and self-consistent sources using the direct and inverse scattering methods formulated for the quadratic pencil of Sturm-Liouville operators, is of great significance.

The present dissertation research contributes, to a certain extent, to the implementation of the tasks set forth in the Decrees of the President of the Republic of Uzbekistan No. PF-6097 dated October 29, 2020, “On Approval of the Concept for the Development of Science until 2030,” and No. PF-60 dated January 28, 2022, “On the Development Strategy of New Uzbekistan for 2022-2026,” as well as in the Decisions of the President of the Republic of Uzbekistan No. PQ-4387 dated July 9, 2019, “On State Support for Further Development of Mathematical Education and Sciences and Measures for Radical Improvement of the Activities of the V. I. Romanovsky Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan,” and No. PQ-4708 dated May 7, 2020, “On Measures to Improve the Quality of Education in the Field of Mathematics and to Develop Scientific Research,” along with other regulatory and legal documents related to this field of activity.

Connection of the research to the priority areas of the development of science and technology of the Republic. The research and studies in this dissertation work were conducted in accordance with the development of science and technology in the Republic of Uzbekistan IV. “Mathematics, mechanics and computer science”.

The degree of scrutiny of the problem. The functional variable method for obtaining periodic and soliton solutions of nonlinear evolution equations with constant coefficients was first introduced by W. Djoudi and A. Zerarka in 2010. Subsequently, this method was extended by Li and He, who successfully applied it to derive periodic and soliton solutions of fractional differential equations.

In 2016, W. Djoudi and A. Zerarka further developed this method and constructed periodic and soliton solutions of the Korteweg-de Vries equation and the modified Korteweg-de Vries equation variable coefficient using the functional variable method. The main advantage of the proposed functional variable method over other methods is that it provides more accurate traveling wave solutions with additional free parameters.

In recent years, extensive research has been conducted on the integration of nonlinear evolution equations of integer and fractional order using the method of direct and inverse problems of scattering theory. In particular, the modified Korteweg-de Vries-sine-Gordon equation with integral derivatives was first introduced in 1974 by

[2]Decision No. 292 of the Cabinet of Ministers of the Republic of Uzbekistan dated May 18, 2017 “On Measures to Organize the Activities of Newly Established Research Institutions of the Academy of Sciences of the Republic of Uzbekistan.”

K. Konno as a mathematical model describing nonlinear vibrational phenomena in an atomic lattice, and its complete integrability using the method of direct and inverse problems of scattering theory was shown. Later, this equation was used by H. Leblond and D. Mihalache to describe the propagation of short optical pulses in transparent media. Also, periodic infinite zone solutions of this equation were studied by A. Khasanov, and solutions in the class of rapidly decreasing functions were studied by U. Hoitmetov.

In 2022, American scientists Ablowitz, Been, and Carr showed that the Riesz fractional nonlinear Korteweg-de Vries equation can be integrated using the method of direct and inverse problems of scattering theory, and applied this method to the integration of the Riesz fractional nonlinear Schrodinger equation, the Riesz fractional derivative modified Korteweg-de Vries equation, and the Riesz fractional derivative sine-Gordon equation. This method proposed by them is currently being applied to the integration of many fractional nonlinear evolutionary equations. In particular, Chinese scientists W. Weng, M. Zhang, and Z. Yan used this method to integrate generalizations of the Riesz fractional nonlinear Schrödinger equation and study the dynamics of N-soliton solutions. L. An, L. Ling, and H. Zhang obtained soliton solutions to the Riesz fractional derivative nonlinear Schrodinger equation and the Riesz fractional Hirota equation. SH. Zhang, H. Li, and B. Xu showed that the Riesz fractional derivative Korteweg-de Vries and nonlinear Schrodinger equations with variable coefficients can be integrated using the direct and inverse problem method of scattering theory.

Using the direct and inverse scattering theory for the quadratic pencil of Sturm-Liouville operators, D. J. Kaup demonstrated the complete integrability of the Kaup-Boussinesq equation, which models the propagation of waves in shallow water in the class of rapidly decaying functions. Subsequently, M. Jaulent and I. Miodek developed an algorithm for solving the Cauchy problem for the Kaup-Boussinesq equation and its higher-order analogues.

V. B. Matveev and M. I. Yavor studied the Kaup-Boussinesq equation and obtaining multi solutions and analyzing their asymptotic properties. The real multi solutions of the Kaup-Boussinesq system were further investigated in the works of A. O. Smirnov.

In addition, A. Cabada and A. Yakshimuratov integrated the Kaup-Boussinesq equation with self-consistent sources in the class of periodic functions, obtaining important results concerning the periodicity properties of the solutions and their analyticity with respect to the independent variables. Furthermore, B. A. Babajanov and A. Sh. Azamatov established the integrability of the Kaup-Boussinesq equation with self-consistent sources by applying the inverse scattering method for the quadratic pencil of Sturm-Liouville equations.

Relevance of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation was carried out in accordance with the research plan of the Department of “Exact Sciences” of the Khorezm Ma’mun Academy, within the framework of the scientific project “Applications of the Spectral Theory of Differential Operators to Nonlinear Evolution Equations” for the period 2022-2025.

The aims of research work are:

to obtain soliton and periodic solutions of the modified Burgers equation with variable-coefficient and an additional term, the fractional Korteweg-de Vries equation with an additional term, and the fractional modified Korteweg-de Vries equation with an additional term using the functional variable method;

to study the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation by the direct and inverse scattering problems of scattering theory;

to integrate the hierarchy Kaup-Boussinesq with variable-coefficient and self-consistent sources in the class of rapidly decaying functions.

Problems of the research:

to obtain soliton solutions of the modified Burgers equation with variable-coefficient and an additional term using the functional variable method and to derive soliton and periodic solutions of the fractional Korteweg-de Vries equation with an additional term and the fractional modified Korteweg-de Vries equation with an additional term by functional variable method;

to integrate the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation using the direct and inverse scattering problems of scattering theory for the Zakharov-Shabat system;

to integrate the hierarchy Kaup-Boussinesq with variable-coefficient and self-consistent sources by applying the direct and inverse scattering problems associated with the quadratic pencil of Sturm-Liouville operators.

The object of the research consists of the modified Burgers equation equation with variable-coefficient and an additional term, the fractional Korteweg-de Vries equation with an additional term, the fractional modified Korteweg-de Vries equation with an additional term, the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation, and the hierarchy Kaup-Boussinesq with variable coefficient and self-consistent sources.

The subject of the research is the application of the direct and inverse scattering problems of scattering theory for the Zakharov-Shabat system and for the quadratic pencil of Sturm-Liouville operators to the integration of nonlinear evolution equations with integer and fractional derivatives. In addition, the study investigate the use of the functional variable method to obtain exact analytical solutions for several nonlinear evolution equations with variable coefficients.

Research methods. The dissertation uses a range of modern mathematical methods and approaches, including mathematical analysis, the theory of ordinary and partial differential equations, equations of mathematical physics, functional analysis, the theory of functions of a complex variable, and the spectral theory of differential operators.

Scientific novelty of research work consists of the followings:

soliton and periodic solutions of the modified Burgers equation with variable-coefficient and an additional term, the fractional Korteweg-de Vries equation with an additional term, and the fractional modified Korteweg-de Vries an additional term equation have been obtained using the functional variable method;

the integrability of the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation in the class of rapidly decaying functions has been established by applying the direct and inverse scattering problems of scattering theory associated with the Zakharov-Shabat system;

the integrability of the hierarchy Kaup-Boussinesq with variable-coefficient and self-consistent sources in the class of rapidly decaying functions has been proved using the direct and inverse scattering problems of scattering theory formulated for the quadratic pencil of Sturm-Liouville operators.

The practical results of the research include the derivation of soliton and periodic solutions of the modified Burgers equation with variable coefficient and an additional term, the fractional Korteweg-de Vries equation with an additional term, and the fractional modified Korteweg-de Vries equation with an additional term using the functional variable method. These solutions provide deeper insight into dispersive interactions in various media and can be directly applied in models of turbulence, gas dynamics, and nonlinear diffusion. An integration algorithm for the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation has been constructed by applying the direct and inverse scattering problems of scattering theory formulated for the Zakharov-Shabat system. These results are effective for analyzing nonlinear wave processes involving fractional derivatives. Furthermore, an integration algorithm for the hierarchy Kaup-Boussinesq with variable-coefficient and self-consistent sources has been developed using the direct and inverse scattering problems associated with the quadratic pencil of Sturm-Liouville operators, and explicit formulas describing its time evolution have been obtained.

The reliability of the research results is ensured by the use of modern analytical methods, including equations of mathematical physics, functional analysis, the theory of functions of a complex variable, the spectral properties of the Zakharov-Shabat system, and the inverse scattering problems associated with the quadratic pencil of Sturm-Liouville operators. All theorems and results have been derived on the basis of rigorous mathematical argumentation, precise proofs, and the fundamental principles of spectral analysis. Their correctness has been verified through detailed examination and confirmed by comparison with known classical results.

Scientific and practical significance of research results. The main results obtained in the dissertation can be applied within the spectral theory of linear operators, as well as in the analysis of various problems in solid-state physics, ion acoustics, plasma physics, radiophysics, and quantum physics. The significance of the work is further determined by the possibility of applying the developed theoretical approaches and analytical methods to the integration of nonlinear evolution equations with self-consistent sources in mathematical physics.

Implementation of research results. Based on the results obtained for the integration of nonlinear evolution equations with integer and fractional derivatives and self-consistent sources:

The soliton solutions of the modified Burgers equation with additional terms and variable coefficients, as well as the properties of one-soliton solutions obtained by integrating the fractional modified Korteweg-de Vries-sine-Gordon equation using the direct and inverse scattering transform associated with the Zakharov-Shabat system, were applied in the applied research project AL-42101210 “Monitoring System of Smart City Sensor Infrastructure”, carried out during 2022-2023 under the supervision of Doctor of Physical and Mathematical Sciences A. B. Yakhshimuratov at the Urgench Branch of Tashkent University of Information Technologies named after Muhammad al-Khwarizmi(Reference issued by Urgench State University named after Abu Rayhon Beruni, Uzbekistan, November 5, 2025). The properties of the one-soliton solutions obtained from the fractional modified Korteweg-de Vries-sine-Gordon equation were used for signal transmission without altering the essential characteristics of the signals. In addition, the soliton solutions of the modified Burgers equation with variable coefficients were applied in sensor networks to reduce noise amplification and signal distortion, as well as to effectively control signal amplitude and transmission speed. The implementation of these scientific results enabled the development of software tools for assessing fire presence in buildings and contributed to reducing the data load in communication and transmission networks.

The scientific results obtained by integrating the hierarchy Kaup-Boussinesq equation using the direct and inverse scattering transform were applied in the fundamental research project Uzb-Ind-2021-80 “Investigation of the Self-Heating Effect in a MOYA Transistor Based on Ordered Nanoplates Surrounded by a Gate”, carried out during 2021-2023 under the supervision of A. E. Atamuratov at Urgench State University named after Abu Rayhon Beruni(Reference issued by Urgench State University named after Abu Rayhon Beruni, Uzbekistan, November 5, 2025). The results derived from the integration of the hierarchy Kaup-Boussinesq system using the direct and inverse scattering transform were used to provide a theoretical justification for the maximum temperature arising at the center of the nanoplate-based MOYA transistor channel, to mathematically describe the mechanisms of local heat accumulation within the transistor, and to develop modeling algorithms describing the relationships between heat flux, current density, the I_{on}/I_{off} ratio, and constructive-geometric parameters. The implementation of these scientific results made it possible to give a mathematically grounded explanation of the initiation and development of self-heating processes in the active regions of the transistor, to perform early assessment of local heat accumulation, to identify the relationship between structural-geometric parameters and heat dissipation, and to develop computational and simulation methods for selecting optimal structural parameters to ensure stable transistor operation.

Approbation of the research results. The research results were discussed at 7 scientific and practical conferences, including 6 international and 1 national scientific-practical conferences.

Publication of the research results. A total of 14 scientific works were published on the subject of the dissertation, 6 of them were published in international scientific journals included, and 1 was published in national journals recommended by the Supreme Attestation Comission at the Ministry of Higher education, science and innovations of the Republic of Uzbekistan for the defense of doctoral dissertations.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, a conclusion and a list of the used references. The volume of the dissertation is 102 pages.

BASIC CONTENT OF THE DISSERTATION

In the Introduction, the relevance and necessity of the dissertation topic are justified, the correspondence of the research to the priority directions of scientific and technological development in the Republic is indicated, and the degree of study of the problem is presented. The connection of the dissertation with the research plans of the higher educational institution where it was carried out is described. The aim, objectives, object, subject, and methods of the research are outlined, the scientific novelty and practical results are stated, the theoretical and practical significance of the obtained findings is explained, and information is provided on the implementation of the results, the published works, and the structure of the dissertation.

The first chapter of the dissertation, titled "**Soliton and periodic wave solutions of nonlinear evolution equations with integer and fractional derivatives via the functional variable method**" is devoted to obtaining soliton and periodic solutions of the modified Burgers equation with an additional term and variable coefficients, the fractional-order Korteweg-de Vries equation with an additional term, and the fractional-order modified Korteweg-de Vries equation. In the first paragraph of this chapter, the mathematical essence of the functional variable method, its conditions of application, basic concepts, and its advantages in constructing solutions are explained in detail. In the second paragraph, the solutions of the modified Burgers equation with an additional term and variable coefficients are obtained by applying the functional variable method. In the third paragraph, the soliton and periodic wave solutions of the fractional-order Korteweg-de Vries equation with an additional term and the fractional-order modified Korteweg-de Vries equation are constructed. In addition, the effectiveness of the functional variable method for fractional-order models is theoretically substantiated.

Let us consider the following nonlinear differential equation:

$$P(u,u_t,u_x,u_y,u_z,u_{xy},u_{yz},u_{xz},\ldots)=0, \qquad (1)$$

Step 1. To reduce the nonlinear partial differential equation to an ordinary differential equation, we introduce the following linear transformation:

$$\xi=\sum_{i=0}^{p}\alpha_i\chi_i+\delta, \qquad (2)$$

where χ_i are independent variables. If $p=1$ then the expression becomes $\xi=\alpha_0\chi_0+\alpha_1\chi_1+\delta$. α_0 and α_1 are constants interpreted as the wave frequency and wave number, respectively, while χ_0 and χ_1 represent the time variable t and the spatial coordinate x.

We can introduce the following transformation for a travelling wave solution of equation (2)

$$u(\chi_0,\chi_1,\ldots)=u(\xi),$$

and by the chain rule we have the following relations:

$$\frac{\partial u}{\partial\chi_i}=\alpha_i\frac{du}{d\xi}, \qquad \frac{\partial^2 u}{\partial\chi_i\partial\chi_j}=\alpha_i\alpha_j\frac{d^2u}{d\xi^2}, \quad \ldots, \qquad (3)$$

Using substitutions (2) and (3), the nonlinear partial differential equation (1) is reduced to an ordinary differential equation. As a result, we obtain an equation of the form

$$Q(u,u',u'',u''',\ldots)=0, \tag{4}$$

where Q is a functional of the unknown function u, depending on u and its derivatives with respect to ξ. Here $u'=\dfrac{du}{d\xi}$.

Step 2. We make a transformation in which the unknown function u is considered as a functional variable in the form

$$u'=F(u), \tag{5}$$

Applying this functional substitution, the solution is obtained from the integral relation

$$\int\frac{du}{F(u)}=\xi+C,$$

where C is a constant of integration, which is usually taken as $C=0$ for convenience. This step forms the main idea of the functional variable method, since it allows the reduction of the ordinary differential equation to a form that can be solved explicitly. To express the higher-order derivatives of u with respect to ξ in terms of we use substitution (5). For this purpose, the higher-order derivatives of the function u

$$\begin{aligned} u''&=\frac{dF(u)}{du}\frac{du}{d\xi}=\frac{dF(u)}{du}F(u)=\frac{1}{2}\frac{d\left(F^2(u)\right)}{du},\\ u'''&=\frac{1}{2}\frac{d^2\left(F^2(u)\right)}{du^2}\sqrt{F^2(u)},\\ u^{(IV)}&=\frac{1}{2}\left[\frac{d^3\left(F^2(u)\right)}{du^3}F^2(u)+\frac{d^2\left(F^2(u)\right)}{du^2}\frac{d\left(F^2(u)\right)}{du}\right],\end{aligned} \tag{6}$$

……………..

3-qadam. To express the ordinary differential equation (4) in terms of u, $F(u)$ and their derivatives, we use the functional substitutions given in (5) and (6). Substituting these expressions into equation (4), the equation is transformed into the following form:

$$R\left(u,\frac{dF(u)}{du},\frac{d^2F(u)}{du^2},\frac{d^3F(u)}{du^3},\ldots\right)=0. \tag{7}$$

By integrating equation (7), the explicit form of the function $F(u)$ is obtained and together with equation (5), this result allows us to construct the solutions of the problem under consideration.

In the second paragraph of this chapter, the exact solutions of the modified Burgers equation with an additional term and variable coefficients are obtained using the functional variable method. The main advantage of the proposed method over other methods is that it provides more new exact traveling wave solutions.

We consider the following modified Burgers equation with an additional term and variable coefficients:

$$u_t + h_1(t)u^2u_x - h_2(t)u_{xx} + \omega(t)u_x = 0, \qquad (8)$$

where $u(x,t)$ is the unknown function, $x \in \mathbb{R}$, $t \geq 0$, $h_1(t) \neq 0$, $h_2(t) \neq 0$ and $\omega(t) \neq 0$ are given continuously differentiable functions. The condition $h_2(t) > 0$ represents the kinematic viscosity coefficient of the fluid.

Theorem 1. Let $h_1(t) \neq 0$, $h_2(t) \neq 0$ va $\omega(t) \neq 0$ be continuously differentiable functions, and suppose that they satisfy the relation

$$h_2(t) = \frac{C_4^2}{3\mu C_2} \cdot \frac{h_1(t)}{\left(\int_0^t h_1(\tau)d\tau + C_3\right)^{4C_2}}.$$

Then the solution of the modified Burgers equation with an additional term and variable coefficients given in (8) has the form

$$u_1(x,t) = \frac{1}{\int_0^t h_1(\tau)d\tau + C_3} + \frac{C_4}{\left(\int_0^t h_1(\tau)d\tau + C_3\right)^{2C_2}} \operatorname{sh}\left(\operatorname{arcth}\left(e^{H_1(x,t)}\right)\right),$$

where $H_1(x,t) = \int_0^t \left(\lambda C_2^2 h_2(\tau) - C_2\omega(\tau)\right)d\tau + C_1 + C_2 x$, λ, μ, C_1, C_2, and C_3 are nonzero constants.

Theorem 2. Let $h_1(t) \neq 0$, $h_2(t) \neq 0$ va $\omega(t) \neq 0$ continuously differentiable functions, and suppose that they satisfy the relation

$$h_2(t) = k\,h_1(t),\ k = \text{const}.$$

Then the solution of the modified Burgers equation with an additional term and variable coefficients given in (8) has the form

$$u_2(x,t) = S_3 \operatorname{sh}\left(\operatorname{arcth}(e^{H_2(x,t)})\right).$$

where $H_2(x,t) = \int_0^t \left(\lambda S_2^2 h_2(\tau) - S_2\omega(\tau)\right)d\tau + S_1 + S_2 x$, λ, S_1, S_2, and S_3 are nonzero constants.

Example 1. We show the application of Theorem 1 with the following example. Let $h_1(t) = 2t$, $h_2(t) = \frac{t}{t^2+1}$, $\omega(t) = -8t$, $\lambda = 32$ and $\mu = \frac{8}{3}$. Then the solution of the modified Burgers equation with an additional term and variable coefficients given in (8) takes the form

$$u_1(x,t) = \frac{1}{t^2+1} + \frac{1}{\sqrt{t^2+1}} \operatorname{sh}\left(\operatorname{arcth}\left((t^2+1)e^{t^2+\frac{1}{4}x}\right)\right).$$

Example 2. We show the application of Theorem 2 with the following example. Let $h_1(t) = t$, $h_2(t) = t$, $\omega(t) = t$, $\lambda = 1$ and $\mu = 1$. Then the solution of the modified Burgers equation with an additional term and variable coefficients given in (8) takes the form

$$u_2(x,t) = 3\operatorname{sh}\left(\operatorname{arcth}\left(e^{3(t^2+x)}\right)\right).$$

In the third paragraph of this chapter, the soliton and periodic wave solutions of the fractional Korteweg-de Vries equation with an additional term and the fractional modified Korteweg-de Vries equation are constructed using the functional variable method.

We consider the fractional Korteweg-de Vries equation and the fractional modified Korteweg-de Vries equation with an additional term:

$$D_t^{\alpha}u - 6puD_x^{\beta}u + D_x^{3\beta}u + \gamma_1 D_x^{\beta}u = 0, \tag{9}$$

$$D_t^{\alpha}u - 12pu^2 D_x^{\beta}u + D_x^{3\beta}u + \gamma_2 D_x^{\beta}u = 0, \tag{10}$$

where $u(x,t)$ is the unknown function, $x \in \mathbb{R}$, $t \geq 0$, $p \neq 0$, $\gamma_1 \neq 0$ va $\gamma_2 \neq 0$ are constants. $0 < \alpha < 1$ va $0 < \beta < 1$ are the orders of the Riemann-Liouville fractional derivatives.

Definition 1. Let $u(x)$ be a smooth function defined on the interval $(a,b) \subseteq \mathbb{R}$. Then the left and right Riemann-Liouville fractional derivatives of order ϵ of the function $u(x)$ are defined by the following formulas:

$$_{RL}D_{a,x}^{\epsilon}u(x) = \frac{1}{\Gamma(1-\epsilon)} \frac{d}{dx} \int_a^x \frac{u(\xi)}{(x-\xi)^{\epsilon}} d\xi, \; 0 < \epsilon < 1,$$

$$_{RL}D_{x,b}^{\epsilon}u(x) = -\frac{1}{\Gamma(1-\epsilon)} \frac{d}{dx} \int_x^b \frac{u(\xi)}{(x-\xi)^{\epsilon}} d\xi, \; 0 < \epsilon < 1.$$

Let us consider the following nonlinear fractional differential equation

$$F\left(u, D_t^{\alpha}u, D_x^{\beta}u, D_t^{\alpha}D_t^{\alpha}u, D_x^{\beta}D_x^{\beta}u, D_t^{\alpha}D_x^{\beta}u, \ldots\right) = 0, \tag{11}$$

where $0 < \alpha < 1$, $0 < \beta < 1$, The functional F depends on the function u and its fractional derivatives.

Step 1. To reduce the nonlinear fractional differential equation (11) to an ordinary differential equation, we introduce the following transformation:

$$u(x,t) = u(\xi), \; \xi = \frac{cx^{\beta}}{\Gamma(1+\beta)} - \frac{kt^{\alpha}}{\Gamma(1+\alpha)}, \tag{12}$$

where c and k are arbitrary nonzero constants, and k represents the propagation speed of the wave.

Using transformation (12), the nonlinear fractional differential equation (11) is reduced to the following ordinary differential equation

$$P(u, u', u'', u''', \ldots) = 0, \tag{13}$$

here $u' = \dfrac{du}{d\xi}$.

Step 2. We now introduce a special functional substitution that expresses the derivative of the unknown function

$$u' = F(u). \tag{14}$$

This step represents the main idea of the functional variable method and makes it possible to determine the solution of the ordinary differential equation. To express the higher order derivatives of the unknown function u with respect to ξ in terms of $F(u)$, we use the functional substitution given in (14). For this purpose, the higher order derivatives of u are computed as follows:

$$u'' = \frac{dF(u)}{du}\frac{du}{d\xi} = \frac{dF(u)}{du}F(u) = \frac{1}{2}\frac{d\left(F^2(u)\right)}{du},$$

$$u''' = \frac{1}{2}\frac{d^2\left(F^2(u)\right)}{du^2}\sqrt{F^2(u)}, \tag{15}$$

$$u^{(IV)} = \frac{1}{2}\left[\frac{d^3\left(F^2(u)\right)}{du^3}F^2(u) + \frac{d^2\left(F^2(u)\right)}{du^2}\frac{d\left(F^2(u)\right)}{du}\right],$$

……………………………………………………………………

Step 3. To rewrite the ordinary differential equation (13) in terms of u, $F(u)$, and their derivatives, we apply the functional substitutions given in (14) and (15). Substituting these expressions into equation (13), the equation is transformed into the following form:

$$H\left(u, \frac{dF(u)}{du}, \frac{d^2F(u)}{du^2}, \frac{d^3F(u)}{du^3}, \ldots\right) = 0. \tag{16}$$

By integrating equation (16), the explicit form of the function $F(u)$ is obtained and together with equation (14), this function allows us to construct the solutions of the problem under consideration.

Theorem 3. Let $c \neq 0$, $k \neq 0$, $p \neq 0$ va γ_1 be real constants. If $\frac{\gamma_1 c - k}{c} < 0$ then the fractional Korteweg-de Vries equation with an additional term given in (9) has the following soliton solution:

$$u(x,t) = \frac{\gamma_1 c - k}{2cp} \cdot \frac{1}{\mathrm{ch}^2\left(\frac{1}{2c}\sqrt{\frac{k - \gamma_1 c}{c}}\left(\frac{cx^\beta}{\Gamma(1+\beta)} - \frac{kt^\alpha}{\Gamma(1+\alpha)}\right)\right)}.$$

If $\frac{\gamma_1 c - k}{c} > 0$ then the fractional Korteweg-de Vries equation with an additional term given in (9) has the following periodic solution:

$$u(x,t) = \frac{\gamma_1 c - k}{2cp} \cdot \frac{1}{\cos^2\left(\frac{1}{2c}\sqrt{\frac{\gamma_1 c - k}{c}}\left(\frac{cx^\beta}{\Gamma(1+\beta)} - \frac{kt^\alpha}{\Gamma(1+\alpha)}\right)\right)}.$$

Theorem 4. Let $c \neq 0$, $k \neq 0$, $p > 0$ va γ_2 be real constants. If $\frac{\gamma_2 c - k}{c} < 0$ then the fractional modified Korteweg-de Vries equation with an additional term given in (10) has the following soliton solution:

$$u(x,t) = \sqrt{\frac{k - \gamma_2 c}{2pc}} \cdot \frac{1}{\mathrm{ch}\left(\frac{1}{c}\sqrt{\frac{k - \gamma_2 c}{c}}\left(\frac{cx^\beta}{\Gamma(1+\beta)} - \frac{kt^\alpha}{\Gamma(1+\alpha)}\right)\right)}.$$

Agar $\frac{\gamma_2 c-k}{c}>0$ then the fractional modified Korteweg-de Vries equation with an additional term given in (10) has the following periodic solution:

$$u(x,t)=\sqrt{\frac{\gamma_2 c-k}{2pc}}\cdot\frac{1}{\cos\left(\frac{1}{c}\sqrt{\frac{\gamma_2 c-k}{c}}\left(\frac{cx^{\beta}}{\Gamma(1+\beta)}-\frac{kt^{\alpha}}{\Gamma(1+\alpha)}\right)\right)}.$$

Example 3. We show the application of Theorem 3 by means of the following example. If $k=-2$, $\alpha=0.5$, $\beta=0.5$, $\gamma_1=1$, $p=0.5$ and $c=-1$ then the fractional Korteweg-de Vries equation with an additional term given in (9) possesses the following soliton solution:

$$u(x,t)=-\frac{1}{\mathrm{ch}^2\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(\sqrt{x}-2\sqrt{t}\right)\right)}.$$

If $k=-1$, $\alpha=0.5$, $\beta=0.5$, $\gamma_1=1$, $p=1$ and $c=1$ then the fractional Korteweg-de Vries equation with an additional term given in (9) possesses the following periodic solution:

$$u(x,t)=\frac{1}{\cos^2\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(\sqrt{x}+\sqrt{t}\right)\right)}.$$

Example 4. We show the application of Theorem 4 by means of the following example. If $k=-2$, $\alpha=0.5$, $\beta=0.5$, $\gamma_2=1$, $p=0.5$ and $c=-1$ then the fractional modified Korteweg-de Vries equation with an additional term given in (10) possesses the following soliton solution:

$$u(x,t)=\frac{1}{\mathrm{ch}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(\sqrt{x}-2\sqrt{t}\right)\right)}.$$

If $k=-1$, $\alpha=0.5$, $\beta=0.5$, $\gamma_2=1$, $p=1$ va $c=1$ then the fractional modified Korteweg-de Vries equation with an additional term given in (10) possesses the following periodic solution:

$$u(x,t)=\frac{1}{\cos\left(\frac{2\sqrt{2}}{\sqrt{\pi}}\left(\sqrt{x}+\sqrt{t}\right)\right)}.$$

In the second chapter of the dissertation, entitled "**Integration of the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation via the inverse scattering methods**", the integration algorithms for the fractional modified Korteweg-de Vries-sine-Gordon equation are developed within the class of rapidly decaying functions by applying the inverse scattering methods associated with the Zakharov-Shabat system.

The first paragraph of this chapter begins by presenting the essential concepts of the direct and inverse scattering methods for the Zakharov-Shabat system. These theoretical foundations serve as the basis for the integration procedures carried out in the subsequent sections of the chapter.

We now present the necessary information regarding the inverse scattering problems for the Zakharov-Shabat system

$$\begin{cases} v_x^{(1)} = -ikv^{(1)} + q(x,t)v^{(2)}, \\ v_x^{(2)} = ikv^{(2)} + r(x,t)v^{(1)}, \end{cases} \tag{16}$$

Definition 2. The functions $\phi(x,k,t)$, $\overline{\phi}(x,k,t)$, $\psi(x,k,t)$ and $\overline{\psi}(x,k,t)$ are defined as the solutions of the Zakharov-Shabat system (16) that satisfy the following asymptotic conditions:

$$\phi(x,k,t)e^{ikx} \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ \overline{\phi}(x,k,t)e^{-ikx} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ x \to -\infty,$$

$$\psi(x,k,t)e^{-ikx} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ \overline{\psi}(x,k,t)e^{ikx} \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ x \to +\infty,$$

These solutions are called the Jost solutions of the Zakharov-Shabat system (16).

The Jost solutions admit the following expansions:

$$\phi(x,k,t) = b(k,t)\psi(x,k,t) + a(k,t)\overline{\psi}(x,k,t),$$

$$\overline{\phi}(x,k,t) = \overline{a}(k,t)\psi(x,k,t) + \overline{b}(k,t)\overline{\psi}(x,k,t),$$

where $a(k,t) = W(\phi,\psi)$, $\overline{a}(k,t) = W(\overline{\psi},\overline{\phi})$, $b(k,t) = W(\overline{\psi},\phi)$, $\overline{b}(k,t) = W(\overline{\phi},\psi)$. $W(u,v)$ denotes the Wronskian determinant, which is defined by:

$$W(u,v) = u^{(1)}v^{(2)} - u^{(2)}v^{(1)}.$$

Definition 3. The functions

$$\tau(k,t) = \frac{1}{a(k,t)},\ \rho(k,t) = \frac{b(k,t)}{a(k,t)},$$

are called the transmission coefficient and the reflection coefficient, respectively.

If $r(x,t) = -q(x,t)$, then the function $a(k,t)\,(\overline{a}(k,t))$ has finitely many zeros $k_j = \xi_j + i\eta_j\ (\overline{k}_j = \overline{\xi}_j + i\overline{\eta}_j),\ j = 1,2,\ldots,N$ in the upper (respectively, lower) half-plane. These zeros correspond to the eigenvalues of the Zakharov-Shabat system (16), and the following relations hold:

$$\phi_j(x,t) = \chi_j(t)\psi_j(x,t),\ \overline{\phi}_j(x,t) = \overline{\chi}_j(t)\overline{\psi}_j(x,t),$$

$$C_j(t) = \frac{\chi_j(t)}{a_j'(t)},\ \overline{C}_j(t) = \frac{\overline{\chi}_j(t)}{\overline{a}_j'(t)},$$

where $\chi_j(t) = \chi\left(k_j,t\right)$, $\dot{a}_j(t) = \left.\frac{da(k,t)}{dk}\right|_{k=k_j}$.

Definition 4. The set

$$\left\{\rho(k,t),\ k \in \mathbb{R},\ k_j(t),\ C_j(t),\ j = 1,2,...,N\right\},$$

is called the scattering data for the Zakharov-Shabat system (16).

The main system of integral equations corresponding to the inverse scattering problem for the Zakharov-Shabat system (16) is given by

$$\mathbf{N}(x,k,t)=\begin{pmatrix}0\\1\end{pmatrix}-\sum_{j=1}^{N}\frac{C_j(t)e^{2ik_jx}}{k+k_j}\sigma^{-1}\mathbf{N}_j(x,t)+\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\rho(\xi,t)e^{2i\xi x}}{\xi+k+i0}\sigma^{-1}\mathbf{N}(x,\xi,t)d\xi, \quad (17)$$

$$\mathbf{N}_\ell(x,t)=\begin{pmatrix}0\\1\end{pmatrix}-\sum_{j=1}^{N}\frac{C_j(t)e^{2ik_jx}}{k_\ell+k_j}\sigma^{-1}\mathbf{N}_j(x,t)+\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\rho(\xi,t)e^{2i\xi x}}{\xi+k_\ell}\sigma^{-1}\mathbf{N}(x,\xi,t)d\xi, \quad (18)$$

where $\mathbf{N}(x,k,t)=\left(N^{(1)}(x,k,t),N^{(2)}(x,k,t)\right)^T$, $\mathbf{N}\left(x,k_l,t\right)=\mathbf{N}_\ell(x,t)$, $\ell=1,2,\dots,N$, $\sigma^{-1}=\begin{pmatrix}0&-1\\1&0\end{pmatrix}$.

The potential \$q(x,t)\$ in the Zakharov-Shabat system (16) is recovered from the scattering data via the following formula:

$$q(x,t)=2i\sum_{j=1}^{N}e^{2ik_jx}C_j(t)N_j^{(2)}(x,t)-\frac{1}{\pi}\int_{-\infty}^{\infty}\rho(\xi,t)e^{2i\xi x}N^{(2)}(x,\xi,t)d\xi. \quad (19)$$

In the second paragraph of this chapter, we consider the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation

$$q_t+\Omega_f\left(L^A\right)q_x=0,\ x\in\mathbb{R},\ t>0, \quad (20)$$

with the initial condition

$$q(x,0)=q_0(x),\ x\in\mathbb{R}, \quad (21)$$

where $p_1,p_2\neq 0$ are nonzero constants, and

$$\Omega_f\left(L^A\right)=\left(p_2\left(4L^A\right)^{-1}-4p_1L^A\right)\cdot\left|4L^A\right|^{\epsilon}, \quad (22)$$

$$L^A=-\frac{1}{4}\frac{\partial^2}{\partial x^2}-q^2-q_xI_-q,\ I_-=\int_{-\infty}^{x}dy.$$

Definition 5. The Riesz fractional derivative of the function $q(x,t)$ is defined by

$$\left(-\partial_x^2\right)^{\epsilon}q(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{q}(k,t)|k|^{2\epsilon}e^{ikx}dk,$$

where $\hat{q}(k,t)=\int_{-\infty}^{\infty}q(x,t)e^{-ikx}dx$ is the Fourier transform of $q(x,t)$.

The initial function $q_0(x)$ is assumed to satisfy the following conditions:

1. $\int_{-\infty}^{\infty}(1+|x|)|q_0(x)|dx<\infty$,

2. The operator

$$H(0)=i\begin{pmatrix}\frac{d}{dx}&-q_0(x)\\-q_0(x)&-\frac{d}{dx}\end{pmatrix}$$

has N simple eigenvalues $k_1(0)$, $k_2(0)$, … , $k_N(0)$ in the upper half-plane and possesses no spectral singularities.

To determine the operator $\Omega_f\left(L^A\right)$ in (20), we follow the algorithm proposed by Ablowitz, Been, and Carr. We use the dispersion relation associated with the linear Riesz fractional modified Korteweg-de Vries-sine-Gordon equation

$$u_{xt} + \left(-\partial_x^2\right)^{\epsilon}\left(p_1 u_{xxxx} - p_2 u\right) = 0, \tag{23}$$

Substituting $u(x,t) = e^{i(kx-\omega(k)t)}$ into (23), we obtain the dispersion relation

$$\omega(k) = \left(\frac{p_2}{k} - p_1 k^3\right)|k|^{2\epsilon}.$$

Taking into account the identity

$$\Omega_f(k^2) = \frac{\omega(2k)}{2k},$$

we find that the operator $\Omega_f\left(L^A\right)$ has the form given in (22).

Theorem 5. Let $q(x,t)$ be a solution of the Cauchy problem for the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation, i.e., the problem (20)-(21). Then, the time evolution of the scattering data associated with the Zakharov-Shabat system (16) is satisfied by the differential equations:

$$\frac{\partial \rho(k,t)}{\partial t} = -2ik\Omega_f(k^2)\cdot\rho(k,t),\ k \in \mathbb{R}\setminus\{0\},$$

$$\frac{dk_j(t)}{dt} = 0,\ j = 1,2,\ldots,N,$$

$$\frac{dC_j(t)}{dt} = -2ik_j\Omega_f(k_j^2)\cdot C_j(t),\ j = 1,2,\ldots,N,$$

where $\Omega_f(k^2) = \left(\frac{p_2}{4k^2} - 4p_1k^2\right)|2k|^{2\epsilon}$.

The obtained results completely define the time evolution of the scattering data, which allows us to solve the problem (20)-(21).

Let the initial function $q(x,0)$ be given.

First, using the initial condition $q(x,0)$, we compute the scattering data (SD) of system (16) at $t = 0$:

$$\left\{\rho(k,0),\ k \in \mathbb{R},\ k_j(0),\ C_j(0),\ j = 1,2,...,N\right\},$$

Taking this set of scattering data at $t = 0$ as the initial data, we solve the differential equations provided in Theorem 5 and obtain the time evolution of the scattering data for $t > 0$:

$$\left\{\rho(k,t),\ k \in \mathbb{R},\ k_j(t),\ C_j(t),\ j = 1,2,...,N\right\}$$

Using the obtained scattering data, we solve the system of integral equations (17) and (18) to determine the functions $N_j^{(2)}(x,t)$, $j = 1,2,...,N$.

Then, by applying relation (19), the function $q(x,t)$ is reconstructed;

Finally, using the identity $q(x,t) = \frac{1}{2}u_x(x,t)$ we obtain the solution $u(x,t)$ of equation (20).

Example 5. Consider equation (20) with the initial condition

$$q(x,0)=\frac{2}{ch\left(2(x-x_0)\right)},$$

Using the algorithm described above, we obtain the solution $q(x,t)$ in the form

$$q(x,t)=\frac{2}{ch\left(2(x-x_0)-2\left(4p_1-\frac{p_2}{4}\right)|2|^{2\epsilon}\, t\right)},$$

Given that $q(x,t)=\frac{1}{2}u_x(x,t)$, one can reconstruct the solution $u(x,t)$ of equation (20) as follows

$$u(x,t)=2\arctan\left(sh\left((x-x_0)-2\left(4p_1-\frac{p_2}{4}\right)|2|^{2\epsilon}\, t\right)\right).$$

In the third paragraph of this chapter, the Riesz fractional-derivative variable-coefficient modified Korteweg-de Vries-sine-Gordon equation is integrated by using the inverse scattering methods associated with the Zakharov-Shabat system.

We consider the Riesz fractional-derivative variable-coefficient modified Korteweg-de Vries-sine-Gordon equation

$$q_t+\Omega_f\left(L^A\right)q_x=0,\ x\in\mathbb{R},\ t>0, \tag{24}$$

with the initial condition

$$q(x,0)=q_0(x),\ x\in\mathbb{R}, \tag{25}$$

here $p_1(t),p_2(t)\neq 0$ are nonzero time-dependent functions, and

$$\Omega_f\left(L^A\right)=\left(p_2(t)\left(4L^A\right)^{-1}-4p_1(t)L^A\right)\cdot\left|4L^A\right|^{\epsilon}, \tag{26}$$

$$L^A=-\frac{1}{4}\frac{\partial^2}{\partial x^2}-q^2-q_xI_-q,\ I_-=\int_{-\infty}^{x}dy.$$

To determine the operator $\Omega_f\left(L^A\right)$ in (24), we follow the algorithm proposed by Ablowitz, Been, and Carr. We use the dispersion relation associated with the linear Riesz fractional modified Korteweg-de Vries-sine-Gordon equation

$$u_{xt}+\left(-\partial_x^2\right)^{\epsilon}\left(p_1(t)u_{xxxx}-p_2(t)u\right)=0, \tag{27}$$

Substituting $u(x,t)=e^{i(kx-\omega(k)t)}$ into (27), we obtain the dispersion relation

$$\omega(k)=\left(\frac{p_2(t)}{k}-p_1(t)k^3\right)|k|^{2\epsilon}.$$

Taking into account the identity

$$\Omega_f(k^2)=\frac{\omega(2k)}{2k},$$

we find that the operator $\Omega_f\left(L^A\right)$ has the form given in (26).

Theorem 6. Let $q(x,t)$ be a solution of the Cauchy problem for the Riesz fractional modified Korteweg-de Vries-sine-Gordon equation, i.e., the problem (24)-(25). Then, the time evolution of the scattering data associated with the Zakharov-Shabat system (16) is satisfied by the differential equations:

$$\frac{\partial \rho(k,t)}{\partial t} = -2ik\Omega_f(k^2)\cdot \rho(k,t)\,,\ k\in \mathbb{R}\setminus\{0\}\,,$$
$$\frac{dk_j(t)}{dt} = 0\,,\ j = 1,2,\ldots,N\,,$$
$$\frac{dC_j(t)}{dt} = -2ik_j\Omega_f(k_j^2)\cdot C_j(t)\,,\ j = 1,2,\ldots,N\,,$$

where $\Omega_f(k^2) = \left(\frac{p_2(t)}{4k^2} - 4p_1(t)k^2\right)|2k|^{2\epsilon}$.

In the third chapter of the dissertation, titled **"Integration of the hierarchy Kaup-Boussinesq equation with self-consistent sources"** the integration of the hierarchy Kaup-Boussinesq equation is studied using the inverse scattering method. The main purpose of this chapter is to develop a integration algorithm for the variable-coefficient and self-consistent-source forms of the hierarchy Kaup-Boussinesq equation for the spectral theory that comes from the quadratic pencil of Sturm-Liouville operators. The analysis is carried out by identifying the scattering data, deriving their time-evolution equations, and reconstructing the solutions through the inverse scattering approach.

In the first paragraph of the first chapter, the well-known information about the Maksudov-Guseynov method of solving the inverse scattering problem for the quadratic pencil of Sturm-Liouville equations is presented.

We consider

$$L(k)y \equiv -y'' + v(x)y + 2ku(x)y - k^2y = 0,\ x\in R, \tag{28}$$

where the functions $v(x)$ and $u(x)$ are real, moreover, $u(x)$ is absolutely continuous and the inequalities hold:

$$\int_{-\infty}^{\infty}|u(x)|\,dx < \infty,\ \int_{-\infty}^{\infty}(1+|x|)[|v(x)|+|u'(x)|]dx < \infty. \tag{29}$$

Under condition (29), Eq. (28) for all k from the half-plane $Imk \ge 0$ has solutions $f_+(x,k),\ f_-(x,k)$ satisfying asymptotics

$$f_+(x,k) = e^{ikx}[1+o(1)], x\to+\infty,\ \ f_-(x,k) = e^{-ikx}[1+o(1)], x\to-\infty.$$

For real $k\neq 0$, the pairs $f_+(x,k), \overline{f}_+(x,k)$ and $f_-(x,k), \overline{f}_-(x,k)$ form two fundamental systems of solutions to equation (1). The following relations hold

$$f_+(x,k) = b(k)f_-(x,k) + a(k)\overline{f}_-(x,k),$$
$$f_-(x,k) = -\overline{b}(k)f_+(x,k) + a(k)\overline{f}_+(x,k),$$
$$a(k) = -\frac{1}{2ik}W\{f_+(x,k), f_-(x,k)\},\qquad b(k) = \frac{1}{2ik}W\{f_+(x,k), \overline{f}_-(x,k)\}.$$

The function $a(k)$ admits an analytic continuation to the half-plane $Imk > 0$ and can have at most a finite number of zeros $k_1,k_2,...,k_N$, besides, at $k = k_n,\ n = 1,2,...,N$ the following equality holds

$$f_{\mp}(x,k_n)=B_n^{\pm}f_{\pm}(x,k_n),$$

where the quantities $B_n^{\pm}$ are independent of x.

Definition 6. The set of the quantities

$$\left\{r_-(k)=\frac{b(k)}{a(k)},\ k\in R\setminus\{0\},\ k_1,k_2,...,k_N,\ \gamma_1^-,\gamma_2^-,...,\gamma_N^-\right\},$$

and

$$\left\{r_+(k)=-\frac{\overline{b}(k)}{a(k)},\ k\in R\setminus\{0\},\ k_1,k_2,...,k_N,\ \gamma_1^+,\gamma_2^+,...,\gamma_N^+\right\},\tag{30}$$

are called the left and right scattering data of Eq. (1), respectively, here

$$\gamma_n^{\pm}=B_n^{\pm}\left(\frac{da(k)}{dk}|_{k=k_n}\right)^{-1},\ n=1,2,...,N.$$

The problem of finding the coefficients $u(x)$ and $v(x)$ through the left or right scattering data is called the inverse problem for equation (28).

We now turn to the question of constructing $u(x)$ and $v(x)$ from scattering data (30).

We constructing the function $F_+(x)$ using the given right scattering data as follows

$$F_+(x)=-i\sum_{n=1}^{N}\gamma_n^+e^{ik_nx}+\frac{1}{2\pi}\int_{-\infty}^{\infty}r_+(k)e^{ikx}dk.$$

Puting $F_+(x)$ into the following integral equations

$$F_+(x+y)+\overline{K_+^{(0)}(x,y)}+\int_x^{\infty}K_+^{(0)}(x,\tau)F_+(\tau+y)d\tau=0,x\le y<\infty,$$

$$iF_+(x+y)+\overline{K_+^{(1)}(x,y)}+\int_x^{\infty}K_+^{(1)}(x,\tau)F_+(\tau+y)d\tau=0,x\le y<\infty$$

and solving them we find $K_+^{(0)}(x,y)$ and $K_+^{(1)}(x,y)$. Using these, we construct the following function

$$K_+(x,y)=K_+^{(0)}(x,y)\cos\alpha_+(x)+K_+^{(1)}(x,y)\sin\alpha_+(x).$$

Here the function $\alpha_+(x)$ is the solution of the following Volterra integral equation

$$\alpha_+(x)=\int_x^{\infty}\Phi(s,\alpha_+(s))ds,\ -\infty<x<\infty,$$

where

$$\Phi(s,z)=\left[\mathrm{Re}\,K_+^{(0)}(s,s)-\mathrm{Im}\,K_+^{(1)}(s,s)\right]\sin 2z+$$
$$+2\left[\mathrm{Re}\,K_+^{(1)}(s,s)\right]\sin^2 z-2\left[\mathrm{Im}\,K_+^{(0)}(s,s)\right]\cos^2 z.$$

Finally, the coefficients $u(x)$ and $v(x)$ are determined as follows

$$u(x) = -\alpha'_+(x),$$

$$v(x) = -u^2(x) - 2\frac{d}{dx}\left\{\left[\mathrm{Re}\, K_+(x,x)\right]cos\alpha_+(x) + \left[\mathrm{Im}\, K_+(x,x)\right]sin\alpha_+(x)\right\}.$$

In the second section of the first chapter**,** we consider the following hierarchy Kaup-Boussinesq equation with a time-dependent coefficients

$$U_t + \Omega(L^*)U_x = G \tag{31}$$

under initial condition

$$v(x,t)\big|_{t=0} = v_0(x),\ u(x,t)\big|_{t=0} = u_0(x),\ x \in R, \tag{32}$$

where

$$U = \begin{pmatrix} v(x,t) \\ u(x,t) \end{pmatrix},\ G = \begin{pmatrix} G_1(x,t) \\ G_2(x,t) \end{pmatrix},\ G_1(x,t) = \mu(t)v_x,\ G_2(x,t) = \mu(t)u_x$$

$$L^* = \begin{pmatrix} 0 & -\dfrac{\partial^2}{\partial x^2} + 4v - 2v_x\displaystyle\int_x^\infty d\tau \\ 1 & 4u - 2u_x\displaystyle\int_x^\infty d\tau \end{pmatrix}, \tag{33}$$

$\Omega(s)$ is any polynomial function of s (whose coefficients may depend on time) and $\mu(t)$ is an arbitrary prescribed continuous function. The functions $v_0(x)$, $u_0(x)$ are real and satisfy the following conditions:

i) $u_0(x)$ is absolutely continuous and the following inequalities hold:

$$\int_{-\infty}^{+\infty} |u_0(x)|\, dx < \infty,\ \int_{-\infty}^{+\infty} (1+|x|)[|v_0(x)| + |u'_0(x)|]dx < \infty \tag{34}$$

ii) the quadratic pencil of Sturm-Liouville operators

$$L(0,k)y \equiv -y'' + v_0(x)y + 2ku_0(x)y - k^2 y = 0,\ x \in R$$

has exactly $2N$ simple eigenvalues $k_1(0), k_2(0), ..., k_{2N}(0)$.

Theorem 7. Let $v = v(x,t)$ and $u = u(x,t)$ be a solution of the problem for (31)-(34). Then, the time evolution of the scattering data associated with the quadratic pencil of Sturm-Liouville equations $T(t,k)$ is satisfied by the differential equations:

$$\frac{dr_+(t,k)}{dt} = -2ik[\Omega(2k) - \mu(t)]r_+(t,k),\ k \in \mathbb{R},$$

$$\frac{dk_n(t)}{dt} = 0,\ n = 1,2,\ldots,N,$$

$$\frac{d\gamma_n^+(t)}{dt} = 2ik_n[\Omega(2k_n) + \mu(t)]\gamma_n^+(t),\ n = 1,2,\ldots,N.$$

The obtained results completely define the time evolution of the scattering data, which allows us to solve the problem (20)-(22).

In the third paragraph of this chapter, the hierarchy Kaup-Boussinesq equation with a self-consistent source is integrated within the class of rapidly decaying functions.

We consider equation

$$L(k)y = -y'' + (V - k^2)y = 0, x \in R, \tag{35}$$

where $V(x,k) = v(x) + 2ku(x)$, and $v(x)$, $u(x)$ are continuously differentiable complex valued functions and the following inequalities hold:

$$\int_{-\infty}^{\infty} x^2 \left[|v(x)| + |u'(x)|\right] dx < \infty, \quad \int_{-\infty}^{\infty} |x| \left[|v'(x)| + |u''(x)|\right] dx < \infty. \tag{36}$$

Under condition (36), Eq. (35) for all $k \in R$ has Jost solutions $\{f_1(x,k), g_1(x,k)\}$ and $\{f_2(x,k), g_2(x,k)\}$ which satisfy the conditions

$$\left[f_1(x,k), g_1(x,k)\right] \sim \left[e^{-ikx}, e^{ikx}\right], x \to \infty, \quad \left[f_2(x,k), g_2(x,k)\right] \sim \left[e^{ikx}, e^{-ikx}\right], x \to -\infty.$$

For real $k \neq 0$, the pairs $\{f_1(x,k), g_1(x,k)\}$ and $\{f_2(x,k), g_2(x,k)\}$ form two fundamental systems of solutions to equation (35).

The following relations hold

$$f_2 = c_{11}f_1 + c_{12}g_1, \;\; g_2 = d_{12}f_1 + d_{11}g_1, \quad f_1 = c_{22}f_2 + c_{21}g_2, \;\; g_1 = d_{21}f_2 + d_{22}g_2,$$

$$c_{12} = c_{21} = (2ik)^{-1}W[f_1, f_2], \quad c_{11} = -d_{22} = (2ik)^{-1}W[f_2, g_1],$$

$$d_{12} = d_{21} = (2ik)^{-1}W[g_2, g_1], \quad d_{11} = -c_{22} = (2ik)^{-1}W[f_1, g_2],$$

where $c_{11}, c_{12}, c_{21}, c_{22}, d_{11}, d_{12}, d_{21}, d_{22}$ are independent on x. Moreover, the function $c_{21}(k)$ admits an analytic continuation to the half-plane $Imk < 0$. We can take the point of view that (35) is a pair of equations $(35)^{\pm}$ having potentials $V^{\pm}(x,k) = V(x, \pm k)$. Now all the above equations can be understood to have superscripts "$\pm$". $c_{21}^{\pm}(k)\ (Imk < 0)$ each have a finite number of zeros $N^{\pm}$, located at the points $k = k_n^{\pm}$, $n = 1, 2, ..., N^{\pm}$.

Definition 7. The set of the quantities

$$\left\{ R^{\pm}(k) = \frac{c_{11}^{\pm}(-k)}{c_{21}^{\mp}(-k)}, k \in R \setminus \{0\}, k_n^{\pm}, \; C_n^{\pm}, n = 1, 2, ..., N^{\pm} \right\}$$

is called the scattering data of Eq. (35), where

$$C_n^{\pm} = [c_{11}^{\pm}(k_n^{\pm})]^{-1} \left[i \frac{d}{dk} c_{21}^{\pm}(k) \right]_{k = k_n^{\pm}}.$$

The coefficients $u(x)$ and $v(x)$ are uniquely recovered by the scattering data.

In the third paragraph of this chapter, the time evolution of the scattering data for the hierarchy Kaup-Boussinesq equation with a self-consistent source is derived, and an algorithm for solving the Cauchy problem for this equation via the inverse scattering method is presented.

We consider the following hierarchy Kaup-Boussinesq system with a self-consistent source

$$\begin{cases} U_t + \Omega(L^*)U_x = G, \\ (\phi_m)_{xx} + \left[k_m^2 - v - 2k_m u\right]\phi_m = 0, \, m = 1,2,\ldots,N, \end{cases} \tag{37}$$

under the initial condition

$$v(x,t)\big|_{t=0} = v_0(x), \; u(x,t)\big|_{t=0} = u_0(x), \; x \in R \tag{38}$$

and the normalizing conditions

$$\int_{-\infty}^{+\infty} (2k_m - 2u)\varphi_m^2 dx = A_m(t), \; m = 1,2,...,N, \tag{39}$$

where

$$U = \begin{pmatrix} v(x,t) \\ u(x,t) \end{pmatrix}, \; G = \begin{pmatrix} G_1(x,t) \\ G_2(x,t) \end{pmatrix},$$

$$G_1(x,t) = 2\sum_{m=1}^{N}\left[-u_x\phi_m^2 + (k_m - 2u)\frac{\partial}{\partial x}\phi_m^2\right], \; G_2(x,t) = \sum_{m=1}^{N}\frac{\partial}{\partial x}\phi_m^2,$$

$$L^* = \begin{pmatrix} 0 & -\dfrac{\partial^2}{\partial x^2} + 4v - 2v_x\displaystyle\int_x^{\infty} d\tau \\ 1 & 4u - 2u_x\displaystyle\int_x^{\infty} d\tau \end{pmatrix}, \tag{40}$$

$\Omega(s)$ is any polynomial function of s (whose coefficients may depend on time), $\varphi_1 = \varphi_1(x,t), \; \varphi_2 = \varphi_2(x,t),...,\varphi_N = \varphi_N(x,t)$ are eigenfunctions corresponding to the eigenvalues $k_1 = k_1(t), k_2 = k_2(t),...,k_N = k_N(t)$, $\mathrm{Im}k_m < 0, \; m = 1,2,...,N$ of the equation (31). Moreover, $A_1(t), A_2(t),..., A_N(t)$ are given arbitrary continuous functions and $v_0(x), u_0(x)$ are complex valued functions satisfying conditions:

1. $$\int_{-\infty}^{+\infty} x^2\left[\left|v_0(x)\right| + \left|u_0'(x)\right|\right]dx < \infty, \; \int_{-\infty}^{+\infty} |x|\left[\left|v_0'(x)\right| + \left|u_0''(x)\right|\right]dx < \infty, \tag{41}$$

2. The quadratic pencil of Sturm-Liouville operators

$$L(0,k)y \equiv -y'' + v_0(x)y + 2ku_0(x)y - k^2y = 0, \; x \in R$$

has exactly N simple eigenvalues.

Theorem 8. Let $v = v(x,t)$ and $u = u(x,t)$ be a solution of the problem for (37)-(40). Then, the time evolution of the scattering data associated with the quadratic pencil of Sturm-Liouville equations $T(t,k)$ is satisfied by the differential equations:

$$\frac{dR^{\pm}(t,k)}{dt} = -2ik\Omega(\pm k)R^{\pm}(t,k),\ k \in \mathbb{R},$$

$$\frac{dk_n^{\pm}(t)}{dt} = 0,\ n = 1,2,...,N^{\pm},$$

$$\frac{dC_n^{+}(t)}{dt} = -[2ik_n^{+}\Omega(k_n^{+}) + 2ik_n^{+}A_n(t)]C_n^{+}(t),\ n = 1,2,...,N^{+},$$

$$\frac{dC_n^{-}(t)}{dt} = 2ik_n^{-}\Omega(-k_n^{-})C_n^{-}(t),\ n = 1,2,...,N^{-}.$$

The obtained results completely define the time evolution of the scattering data, which allows us to solve the problem (37)-(40).

CONCLUSION

This dissertation is devoted to the integration of nonlinear evolutionary partial differential equations with integer and fractional derivatives and with self-consistent sources within the class of rapidly decaying functions, using the direct and inverse scattering methods.

The main results of the dissertation can be summarized as follows:

Using the functional variable method, explicit soliton and periodic solutions of the modified Burgers equation with an additional term and variable coefficients, the fractional Kortewegde Vries equation with an additional term, and the fractional modified Korteweg-de Vries equation were obtained.

The integrability of the fractional modified Korteweg-de Vries-sine-Gordon equation within the class of rapidly decaying functions was shown by applying the direct and inverse scattering methods for the associated Zakharov-Shabat system.

The integrability of the hierarchy Kaup-Boussinesq equation with variable coefficients and a self-consistent source within the class of rapidly decaying functions was proved using the direct and inverse scattering methods for the quadratic pencil of Sturm-Liouville operators.

The obtained results confirm that the objectives of the dissertation research have been fully achieved. All major findings contribute to the theory of integration of nonlinear evolutionary equations and enrich the mathematical framework of modern mathematical physics.

**НАУЧНЫЙ СОВЕТ PhD.03/2025.27.12.FM.06.02
ПО ПРИСУЖДЕНИЮ УЧЕНОЙ СТЕПЕНИ ПРИ УРГЕНЧСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ ИМЕНИ АБУ РАЙХАНА БЕРУНИ**

ХОРЕЗМСКАЯ АКАДЕМИЯ МАЪМУНА

АБДИКАРИМОВ ФАХРИДДИН БАХРОМ УГЛИ

ИНТЕГРИРОВАНИЕ НЕЛИНЕЙНЫХ ЭВОЛЮЦИОННЫХ УРАВНЕНИЙ С ЧАСТНЫМИ ПРОИЗВОДНЫМИ ЦЕЛЫХ И ДРОБНЫМ ПОРЯДКОМ С САМОСОГЛАСОВАННЫМ ИСТОЧНИКОМ

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ
диссертации доктора философии (PhD) по физико-математическим наукам

Ургенч - 2026

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при министерстве Высшего образования, науки и инноваций Республики Узбекистан за № B2024.2.PhD/FM1052.

Диссертация выполнена в Хорезмской академии Маъмуна.

Автореферат диссертации на трех языках (узбекский, русский, английский (резюме) размещен на веб-странице Научного совета (www.ik-mat.urdu.uz) и на Информационно-образовательном портале “ZiyoNet” (www.ziyonet.uz).

Научный руководитель:	**Бабажанов Базар Атаджанович** Доктор физико-математических наук, профессор
Официальные оппоненты:	**Хасанов Акназар Бекдурдиевич** Доктор физико-математических наук, профессор
	Бабажанова Айгул Комилжановна Доктор философии по физико-математическим наукам
Ведущая организация	**Бухарский государственный университет**

Защита диссертации состоится “16” февраля 2026 года в 14 : 00 часов на заседании Научного совета PhD.03/2025.27.12.FM.06.02 при Ургенчском государственном университете имени Абу Райхана Беруни. (Адрес: 220100, г. Ургенч, ул. Х. Алимджана, дом 14. Тел:(+99862) 224-66-11, факс: (+99862) 224-67-00, e-mail: info@urdu.uz).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Ургенчском государственном университете имени Абу Райхана Беруни. (зарегистрирована за № D-941). (Адрес: 220100, г. Ургенч, ул. Х. Алимджана, дом 14. Тел:(+99862) 224-66-11, факс: (+99862) 224-67-00 arm@urdu.uz).

Автореферат диссертации разослан “02” февраля 2026 года.

(протокол рассылки № 1 от “02” февраля 2026 года).

O‘ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI ABU RAYHON BERUNIY NOMIDAGI URGANCH DAVLAT UNIVERSITETI

Б. И. Абдуллаев
Председатель Научного совета по присуждению ученой степени, д.ф.-м.н., профессор

А. А. Рейимберганов
Ученый секретарь Научного совета по присуждению ученой степени, к.ф.-м.н.

Б. А. Бабажанов
Председатель Научного семинара при научном совете по присуждению ученой степени, д.ф.-м.н., профессор

ВВЕДЕНИЕ (аннотация диссертации доктора философии(PhD))

Цель исследования: с помощью метода функциональных переменных получить солитонные и периодические решения модифицированного уравнения Бюргерса с переменным коэффициентом и дополнительным членом, дробного уравнения Кортевега-де Фриза с дополнительным членом и дробного модифицированного уравнения Кортевега-де Фриза, исследовать дробное модифицированное уравнение Рисса Кортевега-де Фриза-синус-Гордона методом прямой и обратной задачи рассеяния, а также интегрировать обобщённое уравнение Каупа-Буссинеска с переменными коэффициентами и согласованным источником в классе быстро убывающих функций.

Задачи исследования: получить солитонные решения модифицированного уравнения Бюргерса с переменными коэффициентами и дополнительным членом с использованием метода функциональных переменных;

найти солитонные и периодические решения дробного уравнения Кортевега-де Фриза с дополнительным членом и дробного модифицированного уравнения Кортевега-де Фриза посредством расширенного метода функциональных переменных;

интегрировать дробное модифицированное уравнение Рисса Кортевега-де Фриза-синус-Гордона с применением метода прямой и обратной задач рассеяния для системы Захарова-Шабата;

интегрировать обобщённое уравнение Каупа-Буссинеска с переменными коэффициентами и согласованным источником с использованием метода прямой и обратной задач рассеяния для квадратического пучка операторов Штурма-Лиувилля.

Научная новизна исследовательской работы состоит в следующем:

с помощью метода функциональных переменных получены солитонные и периодические решения модифицированного уравнения Бюргерса с переменными коэффициентами и дополнительным членом, дробного уравнения Кортевега-де Фриза с дополнительным членом и дробного модифицированного уравнения Кортевега-де Фриза;

доказана интегрируемость дробного модифицированного уравнения Рисса Кортевега-де Фриза-синус-Гордона в классе быстро убывающих функций посредством применения методов прямой и обратной задач рассеяния для системы Захарова-Шабата;

доказана интегрируемость обобщённого уравнения Каупа-Буссинеска с переменными коэффициентами и согласованным источником в классе быстро убывающих функций с использованием методов прямой и обратной задач рассеяния для квадратического пучка операторов Штурма-Лиувилля.

Внедрение результатов исследований. На основе полученных результатов по интегрированию нелинейных эволюционных уравнений частных производных с целыми и дробными производными и согласованным источником:

Солитонные решения модифицированного уравнения Бюргерса с переменными коэффициентами и дополнительным членом и свойства одномерных солитонных решений дробного модифицированного уравнения Рисса Кортевега-де Фриза-синус-Гордона, полученные посредством интегрирования средствами прямой и обратной задач рассеяния для системы Захарова-Шабата, были использованы в 2022-2023 годах в прикладном исследовательском проекте AL-42101210 “Система мониторинга инфраструктуры датчиков «Умного города»”, выполненном в Ургенчском филиале Ташкентского университета информационных технологий имени Мухаммада аль-Хоразмий под руководством доктора физико-математических наук А. Б. Яхшимуратова. Полученные результаты применялись при передаче сигналов без искажения их характеристик, снижении шумов и помех в сенсорных сетях, эффективном управлении амплитудой сигнала и скоростью передачи данных.

Научные результаты, полученные при интегрировании обобщённого уравнения Каупа-Буссинеска с использованием методов прямой и обратной задач рассеяния для квадратического пучка операторов Штурма-Лиувилля, были внедрены в рамках фундаментального проекта Uzb-Ind-2021-80 “Исследование эффекта самонагревания в МОЯ-транзисторе на основе упорядоченных нанопластинок, заключённых в затвор”, выполненного в 2021-2023 годах в Ургенчском государственном университете имени Абу Райхона Беруни под руководством А. Э. Атамуратова.

Эти результаты использовались для теоретического обоснования максимальной температуры, возникающей в центре канала нанопластинчатого МОЯ-транзистора, математического описания механизма локального теплового накопления в транзисторе, разработки алгоритмов моделирования взаимосвязей между тепловым потоком, плотностью тока, отношением I_{on}/I_{off} и конструктивно-геометрическими параметрами.

Кроме того, они позволили математически обосновать начало и развитие процесса самонагревания во внутренних активных областях транзистора, предварительно оценить локальное накопление тепла, установить связь между тепловым распространением и конструктивно-геометрическими параметрами, а также разработать вычислительные и симуляционные методы для выбора оптимальных структурных параметров, обеспечивающих стабильную работу транзистора.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, заключения и списка использованной литературы. Объем диссертации 102 страниц.

E'LON QILINGAN ISHLAR RO'YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo'lim (I часть; I part)

1. Babajanov B.A., Abdikarimov F.B. New exact soliton and periodic wave solutions of nonlinear fractional evolution equations with an additional term. Partial Differential Equations in Applied Mathematics, 2023, Vol. 8, 100567. (3. Scopus, IF 5.9).
2. Babajanov B.A., Abdikarimov F.B. Integration of the fractional modified Korteweg-de Vries-sine-Gordon equation by the inverse scattering method. Results in Applied Mathematics, 2025, Vol. 26, 100586. (3. Scopus, IF 2.9).
3. Babajanov B.A., Abdikarimov F.B. Integration of the hierarchy of the Kaup-Boussinesq system via the inverse scattering method. Azerbaijan Journal of Mathematics, 2025, Vol. 2, Issue 23, pp. 23-26. (3. Scopus, IF 1.6).
4. Babajanov B.A., Abdikarimov F.B., Sulaymonov F.U. On the integration of the hierarchy of the Kaup-Boussinesq system with a self-consistent source. Lobachevskii Journal of Mathematics, 2024, Vol. 45, Issue 7, pp. 3233-3245. (3. Scopus, IF 1.5).
5. Babajanov B.A., Abdikarimov F.B., Bazarbaeva S.K. Exact solutions for the modified Burgers equation with an additional time-dependent variable coefficient. European Journal of Pure and Applied Mathematics, 2024, Vol.17, Issue 1, pp. 582-592. (3. Scopus, IF 1.8).
6. Babajanov B.A., Abdikarimov F.B. New soliton solutions of the Burgers equation with an additional time-dependent variable coefficient. WSEAS Transactions on Fluid Mechanics, 2024, Vol. 19, pp. 59-63. 3. (3. Scopus, IF 1.7).
7. Abdikarimov F.B. Two-soliton solution of the fractional modified Korteweg-de Vries-sine-Gordon equation with variable time-dependent coefficients. Ilm Sarchashmalari, 2025, Vol. 6, pp. 41-47. (01.00.00; №12).

II bo'lim (II часть; II part)

1. Babajanov B.A., Abdikarimov F.B. Traveling wave solutions of the space-time fractional Burgers and modified Burgers equations with variable coefficients. Abstracts of the VII World Congress of Turkic World Mathematicians, Turkestan, September 20-23, 2023, p. 52.
2. Babajanov B.A., Abdikarimov F.B. Integration of the fractional modified Korteweg-de Vries-sine-Gordon equation by the inverse scattering method. Abstracts of the Republican Scientific Conference of Modern Methods of Mathematical Physics and Their Applications, Tashkent, April 22-24, 2025, pp. 256-257.
3. Babajanov B.A., Abdikarimov F.B. New soliton solutions of the Burgers equation with variable coefficients. Abstracts of the International Scientific Conference of Modern Problems of Differential Equations and Their Applications Tashkent, November 23-25, 2023, pp. 162-163.
4. Babajanov B.A., Abdikarimov F.B. Soliton and periodic wave solutions of the loaded nonlinear fractional evolution equations. Abstracts of the Conference of Actual

Problems of Applied Mathematics and Information Technologies, Samarkand, September 25-26, 2023, p. 184.

5. Babajanov B.A., Abdikarimov F.B. Solitary and periodic wave solutions of the loaded nonlinear Klein-Gordon equation. Abstracts of the International Scientific Conference of Actual Problems of Mathematics, Mechanics and Informatics, Karakanda, September 8-9, 2022, pp. 84-85.

6. Babajanov B.A., Abdikarimov F.B. New solitary and periodic wave solutions of the loaded nonlinear three-dimensional modified Zaxarov-Kuznetsov equation. Abstracts of the International Scientific Conference of Mathematical Analysis and Its Applications in Modern Mathematical Physics, Samarkand, September 23-24, 2022, pp. 165-167.

7. Babajanov B.A., Abdikarimov F.B. Solitary and periodic wave solutions of the loaded cubic nonlinear Klein-Gordon equation. Abstracts of the II International Forum on Computational and Mathematical Methods and Modelling in High-Tech Manufacturing, Saint Petersburg, November 9, 2022, pp. 69-72.

Dissertatsiya avtoreferati “Khwarezm publication” nashriyotida tahrir qilindi.

Bosishga ruxsat etildi: 30.01.2026-yil.
Bichimi 60x84 $^{1/16}$, “Times New Roman”
garniturada raqamli bosma usulida bosildi.
Shartli bosma tabog‘i 3,7. Adadi: 100. Buyurtma: № 218
“Khwarezm travel” bosmaxonasida chop etildi
220502, Xorazm, Urganch tumani, Zargarlar mahallasi,
Marvarid ko‘cha 7-yo‘lak 4-uy