OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

ANDIJON MASHINASOZLIK INSTITUTI
ANDIJAN MACHINE-BUILDING INSTITUTE
АНДИЖАНСКИЙ МАШИНОСТРОИТЕЛЬНЫЙ ИНСТИТУТ

“OLIY MATEMATIKA” KAFEDRASI

OLIY MATEMATIKADAN MISOL VA MASALALAR TO’PLAMI
(1-qism)

Andijon – 2019

«TASDIQLAYMAN»

Andijon mashinasozlik instituti

Ilmiy - uslubiy kengashida ko'rib chiqilgan va ma'qullangan

Kengash raisi ________________________________Q.Ermatov

«_______» bayonnoma «_______»________ 2019 yil.

«MA'QULLANGAN»

«Transport va logistika» fakulteti

Kengashida muhokama qilingan va ma'qullangan

Kengash raisi ______________________________A.Ermatov

«_______» bayonnoma «_______»_________2019 yil.

«TAVSIYA ETILGAN»

«Oliy matematika» kafedrasi

majlisida muhokama qilingan va tavsiya etilgan

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KIRISH

Hozirgi kunda ilmiy-texnika taraqqiyoti sharoitida oliy o'quv yurtlarida yuqori malakali mutaxassislar tayyorlash borasida tabiiy fanlar, jumladan matematika faniga katta ahamiyat berilmoqda.

Umumiy muhandislik maxsus fanlari ana shu fanlar asosida qurilib, bu kurslar mutaxassis o'zining amaliy faoliyatida zarur bo'lgan bilimlarni egallashga imkon beradi.

E'tiboringizga havola qilinayotgan mazkur o'quv qo'llanma oliy o'quv yurtlarining muhandis-texnik mutaxassisligi bo'yicha tasdiqlangan oliy matematika kursi o'quv dasturining CHiziqli algebra, CHiziqli tenglamalar sistemasi, Vektorlar, Analitik geometriya elementlari va differensial hisob elementlari asosida tuzilgan.

O'quv qo'llama 6 ta bobdan iborat bo'lib, har qaysi bob uchun qisqacha nazariy ma'lumot bayon qilinib, bob so'ngida misol va masalalar berilgan. SHu bilan birgalikda talabalar mustaqil ishlashlari uchun har biriga alohida variantlar berilgan. (Bunda talabaning guruh jurnalidagi nomeri uning uchun variant nomeri bo'lishi mumkin.)

Bu uslubiy qo'llanma texnika oliy o'quv yurtlari talabalari uchun mo'ljallangan.

MUNDARIJA

CHIZIQLI ALGEBRA

1-§. Matritsalar va ular ustida amallar.

1-ta'rif. Sonlarning m ta satr va n ta ustundan iborat to'g'ri to'rtburchak shaklida, jadval ko'rinishida yozilishi $m\,x\,n$ o'lchovli ***matritsa*** deyiladi va quyidagicha belgilanadi:

$$A = \begin{pmatrix} a_{11} & a_{12} \ \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{pmatrix} \qquad (1)$$

yoki $A = (a_{ij}), i = 1, m; j = 1, n.$

Bu yerda m satrlar soni, n ustunlar soni.

Bu matritsadagi a_{ij}, ($i = 1, m; j = 1, n$) sonlar uning ***elementlari*** deyiladi.

2-ta'rif. Agar $m = 1$ bo`lsa, ***satr matritsa***; $A = (a_{11} \quad a_{12} \quad \cdots \quad a_{1n})$, agar $n = 1$ bo`lsa, ***ustun matritsa***; $A = \begin{pmatrix} a_{11} \\ a_{21} \\ \cdots \\ a_{m1} \end{pmatrix}$ deyiladi.

3-ta'rif. Agar m = n bo`lsa, ***kvadrat matritsa***, n uning tartibi deyiladi.

$$A = \begin{pmatrix} a_{11} & a_{12} \ \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} \cdots & a_{nn} \end{pmatrix} \qquad (2)$$

Kvadrat matritsaning $a_{11}, a_{22}, \ \ldots \ , a_{nn}$ elementlari joylashgan diagonal ***bosh diagonal***, $a_{1n}, a_{2n-1}, \ldots, a_{n1}$ elementlari joylashgan diagonal ***yordamchi diagonal*** deyiladi.

4-ta'rif. Bosh diagonal elementlari noldan farqli, qolgan elementlari nolga teng bo'lgan matritsa, ***diagonal matritsa*** deyiladi.

$$A = \begin{pmatrix} a_{11} & 0 & \ldots & 0 \\ 0 & a_{22} & \ldots & 0 \\ \cdots & & \cdots & \cdots \\ 0 & 0 & \ldots & a_{nn} \end{pmatrix} \qquad (3)$$

5-ta'rif. Bosh diagonal elementlari 1 ga teng bo'lgan diagonal matritsa, ***birlik matritsa*** deyiladi va E harfi bilan belgilanadi.

$$E=\begin{pmatrix}1 & 0 & \dots & 0\\ 0 & 1 & \dots & 0\\ \dots & \dots & \dots & \dots\\ 0 & 0 & \dots & 1\end{pmatrix} \qquad (4)$$

Agar $a_{ij}=a_{ji}$ bo`lsa, matritsa ***simmetrik*** deyiladi.

$A=\begin{pmatrix}a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \dots & \dots & & \dots\\ a_{m1} & a_{m2} & \dots & a_{mn}\end{pmatrix}$ $va\ B=\begin{pmatrix}b_{11} & b_{12} & \dots & b_{1n}\\ b_{21} & b_{22} & \dots & b_{2n}\\ \dots & \dots & & \dots\\ b_{m1} & b_{m2} & \dots & b_{mn}\end{pmatrix}$ matritsalar berilgan bo'lsin.

6-ta'rif. Agar ***A*** va ***B*** matritsalarning o'lchovlari o'zaro teng bo'lsa, ular ***nomdosh*** matritsalar deyiladi.

7-ta'rif. A matritsaning har bir a_{ij} elementi V matritsaning unga mos b_{ij} elementiga teng bo`lsa, bu ikki nomdosh matritsalar ***teng*** deyiladi va $A=B$ kabi yoziladi.

Faqat nomdosh matritsalargina teng bo`lishi mumkin.

Nomdosh bo`lmagan matritsalar umuman ***tengmas*** deb hisoblanadi.

Matritsalar ustida quyidagi amallarni bajarish mumkin;

a) matritsani songa ko'paytirish;

b)matritsani matritsaga qo`shish (ayirish);

c)matritsani matritsaga ko`paytirish;

a) matritsani songa ko'paytirish;

Biror matritsani songa ko`paytirish uchun, uning har bir elementi shu songa ko`paytiriladi, ya'ni;

$A=\begin{pmatrix}a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \dots & \dots & & \dots\\ a_{m1} & a_{m2} & \dots & a_{mn}\end{pmatrix}$ matritsa va $\lambda -$ixtiyoriy son berilgan.

$$\lambda A=\begin{pmatrix}a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \dots & \dots & & \dots\\ a_{m1} & a_{m2} & \dots & a_{mn}\end{pmatrix}=\begin{pmatrix}\lambda a_{11} & \lambda a_{12} & \dots & \lambda a_{1n}\\ \lambda a_{21} & \lambda a_{22} & \dots & \lambda a_{2n}\\ \dots & \dots & & \dots\\ \lambda a_{m1} & \lambda a_{m2} & \dots & \lambda a_{mn}\end{pmatrix} \qquad (5)$$

b) matritsani matritsaga qo`shish (ayirish);

Ikkita nomdosh A va B matritsalarning ***yig`indisi*** deb, elementlari quyidagicha $c_{ij} = a_{ij} + b_{ij}, (i = 1,m; j = 1,n)$ aniqlanadigan o`sha o`lchamli C matritsaga aytiladi, ya'ni; $C = A + B$

Ikki matritsalarning ***ayirmasi*** ham ularning yig`indisi kabi aniqlanadi va $C = A - B$ kabi yoziladi.

Matritsani matritsaga qo`shish, ayrish va songa ko`paytirish amallari chiziqli amallardir.

8-ta'rif. ***A*** matritsani satr elementlarini ustun, ustun elementlarini satr ko'rinishda yozilishi uni ***transpornirlash*** deyiladi va A^T bilan belgilanadi.

s) Matritsani matritsaga ko`paytirish.

9-ta'rif. $m \times k$ o`lchamli $A$ matritsaning $k \times n$ o`lchamli B matritsaga ***ko`paytmasi*** deb, $m \times n$ o`lchamli shunday $C = (c_{ij})$ $(i = 1,m; j = 1,n)$ matritsaga aytiladiki, uning c_{ij} $(i = 1,m; j = 1,n)$ elementi A matritsa i-satri elementlarini B matritsa j - ustunining mos elementlariga ko`paytmalari yig`indisiga teng, ya'ni

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ik}b_{kj} \qquad (6)$$

Umumiy holda, $AB \neq BA$. Agar $AB = BA$ bo`lsa, u holda $A$ va $B$ matritsalar kommutativlanadigan yoki o`rin almashinadigan deb ataladi.

Misollar.

1. Agar $A = \begin{pmatrix} 1 & 2 & -3 \\ 3 & -1 & 0 \end{pmatrix}$ va $B = \begin{pmatrix} 2 & -3 & 6 \\ 0 & 3 & 5 \end{pmatrix}$ bo'lsa, $2A + B$ ni toping.

2. Agar $A = \begin{pmatrix} 5 & -2 & -3 \\ 0 & -1 & 0 \end{pmatrix}$ va $B = \begin{pmatrix} 12 & -3 & 6 \\ 0 & 3 & 0 \end{pmatrix}$ bo'lsa, $A + 3B$ ni toping.

3. Agar $A = \begin{pmatrix} 0 & 2 & -3 \\ 3 & -11 & 10 \end{pmatrix}$ va $B = \begin{pmatrix} 2 & -3 & 0 \\ 0 & -1 & 5 \end{pmatrix}$ bo'lsa, $A - B$ ni toping.

4. Agar $A = \begin{pmatrix} 2 & 4 \\ 2 & -1 \\ 0 & 1 \end{pmatrix}$ va $B = \begin{pmatrix} 3 & -1 \\ 0 & 5 \\ 6 & 7 \end{pmatrix}$ bo'lsa, $A + 3B$ ni toping.

5. Agar $A = \begin{pmatrix} 0 & 1 \\ 2 & -1 \\ 0 & 4 \end{pmatrix}$ va $B = \begin{pmatrix} 6 & -2 \\ 0 & 5 \\ 0 & 7 \end{pmatrix}$ bo'lsa, $5A - B$ ni toping.

6. Agar $A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ 3 & 6 & -6 \end{pmatrix}$ va $B = \begin{pmatrix} 0 & 4 & 2 \\ -4 & -3 & 1 \\ 3 & 6 & 7 \end{pmatrix}$ bo'lsa, $A + 2B$ ni toping.

7. Agar $A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 7 & -1 \\ 3 & 0 & -1 \end{pmatrix}$ va $B = \begin{pmatrix} 0 & 0 & 2 \\ -2 & -3 & 1 \\ 3 & -1 & 10 \end{pmatrix}$ bo'lsa, $3A - B$ ni toping.

8. Agar $A = \begin{pmatrix} 1 & 3 \\ 7 & -1 \\ 3 & 6 \end{pmatrix}$ va $B = \begin{pmatrix} 0 & 4 & 2 \\ -4 & -3 & 1 \\ 3 & 6 & 7 \end{pmatrix}$ bo'lsa, $A + B$ ni toping.

9. Agar $A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ 3 & 6 & -6 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 4 \end{pmatrix}$ bo'lsa, $5A - 2B$ ni toping.

10. Agar $A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 1 & -1 \\ 0 & 6 & -6 \end{pmatrix}$ va $B = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ bo'lsa, $-A + 2B$ ni toping.

11. $A = \begin{pmatrix} 1 & 4 \\ -3 & 0 \end{pmatrix}$ va $B = \begin{pmatrix} -6 & 1 \\ 2 & 0 \end{pmatrix}$ ga teng. $A \cdot B$ ni toping.

12. $A = \begin{pmatrix} 0 & 4 \\ -5 & 1 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & -2 \end{pmatrix}$ ga teng. $A \cdot B$ ni toping.

13. $A = \begin{pmatrix} 3 & 1 \\ -3 & 0 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 2 \\ 3 & -4 \\ 0 & -2 \end{pmatrix}$ ga teng. $A \cdot B$ ni toping.

14. $A = \begin{pmatrix} 5 & 4 \\ 3 & -6 \\ 0 & 1 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & 1 \\ 4 & 0 \end{pmatrix}$ ga teng. $2A \cdot B$ ni toping.

15. $A = \begin{pmatrix} 3 & 0 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 4 & 8 \\ 7 & 0 & 0 \end{pmatrix}$ ga teng. -$3A \cdot B$ ni toping.

16. $A = \begin{pmatrix} 1 & 3 & 8 \\ -5 & -2 & -1 \\ 0 & 1 & 3 \end{pmatrix}$ va $B = \begin{pmatrix} 0 & 2 & 1 \\ 3 & 2 & 0 \\ -1 & 2 & 0 \end{pmatrix}$ ga teng. $A \cdot B$ ni toping.

17. $A = \begin{pmatrix} 1 & 1 & 4 \\ -5 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 2 \\ 3 & 5 \\ 0 & -6 \end{pmatrix}$ ga teng. $A \cdot 2B$ ni toping.

18. $A = \begin{pmatrix} 1 & 3 & 7 \\ 1 & -2 & -1 \\ 0 & 1 & 4 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 0 & 6 \end{pmatrix}$ ga teng. $3A \cdot B$ ni toping.

19. $A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & -2 & 3 \\ 0 & 1 & 3 \end{pmatrix}$ va $B = \begin{pmatrix} 6 \\ 3 \\ 0 \end{pmatrix}$ ga teng. $7A \cdot B$ ni toping.

20. $A = (2 \quad 3 \quad 1)$ va $B = \begin{pmatrix} 0 & 2 & 1 \\ 3 & 2 & 1 \\ -1 & 2 & 0 \end{pmatrix}$ ga teng. $A \cdot B$ ni toping.

Mustaqil yechish uchun misollar.

C matritsani toping:

1. $C = A^T B - 2B^T$, $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

2. $C = AB^T - A^T$, $A = \begin{pmatrix} 3 & 5 \\ 6 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}$.

3. $C = AB + 4A$, $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$, $B = \begin{pmatrix} -1 & -2 & -4 \\ -1 & -2 & -4 \\ 1 & 2 & 4 \end{pmatrix}$.

4. $C = A \cdot B^T - 3B$, $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$.

5. $C = A^T B - BA^T$, $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 3 \\ 6 & 7 \end{pmatrix}$.

6. $C = A^T B - 4B$, $A = \begin{pmatrix} 3 & 1 & 8 \\ 2 & 4 & 3 \\ 7 & 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$.

7. $C = 2A^T B - BA^T$, $A = \begin{pmatrix} 2 & 3 \\ 4 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 1 \\ 2 & 11 \end{pmatrix}$.

8. $C = (A + B)(2B - A)$, $A = \begin{pmatrix} 2 & 3 & -1 \\ 4 & 5 & 2 \\ -1 & 0 & 7 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 5 \\ 0 & 1 & 3 \\ 2 & -2 & 4 \end{pmatrix}$.

9. $C = (B + AB)^T$, $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

10. $C = (A - BA)^T$, $A = \begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 6 & -1 \end{pmatrix}$.

11. $C = B - A \cdot A^T$, $A = \begin{pmatrix} 3 & 2 & 1 & 2 \\ 4 & 1 & 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 20 & 19 \\ 18 & 17 \end{pmatrix}$.

12. $C = (AB + BA)^T$, $A = \begin{pmatrix} 5 & 3 \\ 6 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$.

13. $C = A^T B - 2B^T, \quad A = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}, \quad B = \begin{pmatrix} 11 & 16 \\ 15 & 20 \end{pmatrix}.$

14. $C = 2A(A - B)^T, \quad A = \begin{pmatrix} 3 & 2 \\ 0 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -6 \\ -7 & -5 \end{pmatrix}.$

15. $C = 3B - B^T A^T, \quad A = \begin{pmatrix} 5 & 2 \\ 1 & 11 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 3 \\ 4 & 7 \end{pmatrix}.$

16. $C = AB^T + A, \quad A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 1 & 3 \\ 1 & 3 & 0 \end{pmatrix}.$

17. $C = A^T(B + A), \quad A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & -4 \\ 0 & -1 \end{pmatrix}.$

18. $C = (A - B)B^T, \quad A = \begin{pmatrix} 5 & 2 \\ 7 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}.$

19. $C = (B^T + A)^3, \quad A = \begin{pmatrix} 1 & -2 \\ -1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$

20. $C = (A + 3B)^T B, \quad A = \begin{pmatrix} 2 & 1 \\ -3 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$

21. $C = 3A - 2B^T A^T, \quad A = \begin{pmatrix} 5 & 6 \\ 3 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}.$

22. $C = (A + 3B^T)B, \quad A = \begin{pmatrix} -3 & -22 \\ -21 & -23 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 7 \\ 8 & 9 \end{pmatrix}.$

23. $C = 2A(B - A^T), \quad A = \begin{pmatrix} 5 & 1 \\ 2 & 11 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & 6 \\ 4 & 18 \end{pmatrix}.$

24. $C = A^T \cdot B - 3B, \quad A = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$

25. $C = (AB - BA)^T, \quad A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}.$

26. $C = (A - 2B)B^T, \quad A = \begin{pmatrix} 2 & 4 & 1 \\ 5 & 1 & -3 \\ 7 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & -1 \\ -4 & 0 & 0 \\ 7 & -3 & -1 \end{pmatrix}.$

27. $C = (A + 3B)(A + B), \quad A = \begin{pmatrix} 2 & 4 & 1 \\ 5 & 1 & -3 \\ -5 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 & -1 \\ -5 & 2 & 1 \\ 1 & -3 & -1 \end{pmatrix}.$

28. $C = AB - 4B, \quad A = \begin{pmatrix} 2 & 4 & 1 \\ 4 & -4 & -3 \\ 0 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & -4 \\ 5 & 0 \\ 3 & -3 \end{pmatrix}.$

29. $C = (A - 2B)(A + B), \quad A = \begin{pmatrix} 2 & -3 & 1 \\ 5 & 4 & -3 \\ 0 & 1 & -6 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 10 & -1 \\ -4 & 4 & 5 \\ -2 & 3 & 4 \end{pmatrix}.$

30. $C = (A + 3B)B^T$, $A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 4 & 2 \\ 5 & 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 2 & 3 \\ -2 & 1 & 1 \\ -3 & 8 & 2 \end{pmatrix}$.

2-§. Determinantlar.

1-ta'rif. Ikkinchi tartibli kvadrat matritsaga mos keluvchi ***ikkinchi tartibli determinant*** deb, quyidagi belgi va tenglik bilan aniqlanuvchi songa aytiladi:

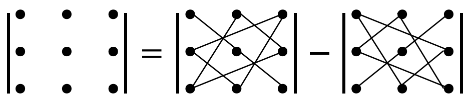
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \qquad (1)$$

Xuddi shunga o'xshash, uchinchi tartibli determinant deb, quyidagi songa aytiladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} -$$

$$-a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{12}a_{21}a_{33}. \qquad (2)$$

Bu "uchburchak qoidasi" deyiladi. Osonroq eslab qolish uchun hisoblash shaklini keltiramiz;



2-ta'rif. Determinantning ixtiyoriy a_{ij} $(i = 1,n; j = 1,n)$ elementining ***minori*** deb, shu element turgan satr va ustunni o'chirish natijasida hosil bo'lgan, tartibi bittaga kamaygan determinantga aytiladi va M_{ij} $(i = 1,n; j = 1,n)$ bilan belgilanadi. Determinant a_{ij} $(i = 1,n; j = 1,n)$ elementining ***algebraik to'ldiruvchisi*** $A_{ij} = (-1)^{i+j}M_{ij}$ formula bilan aniqlanadi, ya'ni u minorning ishorasini aniqlaydi. Bu formula $n \geq 3$ bo'lgandagi determinantlarni tartibini pasaytirib hisoblashda qo'llaniladi.

Xosalari.

1^0. Agar determinant transpornirlansa, uning qiymati o'zgarmaydi.

2^0. Agar determinantning ixtiyoriy satr (ustun) elementlari nollardan iborat bo'lsa, uning qiymati nolga teng bo'ladi.

3^0. Agar determinantning ixtiyoriy ikkita satr (ustun) elementlari o'rinlari almashtirilsa, uning qiymati qarama-qarshisiga o'zgaradi.

4^0. Agar determinantning ixtiyoriy ikkita satr (ustun)i bir xil elementlardan tashkil topgan bo'lsa, uning qiymati nolga teng bo'ladi.

5^0. Determinantning ixtiyoriy satr (ustun) elementlaridan umumiy ko'paytuvchini determinant belgisidan tashqariga chiqarish mumkin.

6^0 .Agar determinantning ixtiyoriy ikkita satr (ustun) elementlari proportsional bo`lsa, uning qiymati nolga teng bo`ladi.

7^0. Agar determinantning biror satr (ustun) elementlari ikkita qo`shiluvchining yig`indisidan iborat bo`lsa, u holda bu determinant qiymati quyidagi ikkita determinantlarning yig`indisiga teng bo'ladi, ya'ni

$$\Delta = \begin{vmatrix} a_{11} & a_{12} + b_1 \\ a_{12} & a_{22} + b_2 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & b_1 \\ a_{12} & b_2 \end{vmatrix}$$

8^0. Agar determinantning ixtiyoriy satr (ustun) elementlari biror songa ko`paytirilib boshqa satr (ustun) elementlariga qo`shilsa, uning qiymati o`zgarmaydi.

9^0. Determinantning qiymati ixtiyoriy satr (ustun) elementlarini ularning mos algebraik to`ldiruvchilariga ko`paytmalari yig`indisiga teng bo`ladi.

$$|A| = \begin{vmatrix} a_{11} & a_{12} \dots & a_{1n} \\ a_{21} & a_{22} \dots & a_{2n} \\ \dots & \dots & \dots \\ a_{i1} & a_{i2} \dots & a_{in} \\ \dots & \dots & \dots \\ a_{n1} & a_{n2} \dots & a_{nn} \end{vmatrix} = a_{i1} \cdot A_{i1} + a_{i2} \cdot A_{i2} + \dots + a_{in} \cdot A_{in}$$

yoki

$$|A| = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nn} \end{vmatrix} = a_{1j} \cdot A_{1j} + a_{2j} + A_{2j} + \dots + a_{2n} \cdot A_{2n}$$

Bular determinantning tartibini pasaytirib hisoblash formulasi deyiladi.

10^0. Determinantning ixtiyoriy satr (ustun) elementlarini boshqa satr (ustun) elementlarining algebraik to`ldiruvchilariga ko`paytmalari yig'indisi nolga teng bo`ladi.

Misollar.

Determinantlarni hisoblang.

1. $\begin{vmatrix} 1 & -4 \\ 3 & 0 \end{vmatrix}=?$

2. $\begin{vmatrix} 3 & -4 \\ 0 & -8 \end{vmatrix}=?$

3. $\begin{vmatrix} a & b \\ d & 2c \end{vmatrix}=?$

4. $\begin{vmatrix} \sin x & cosx \\ cosx & sinx \end{vmatrix}=?$

5. $\begin{vmatrix} \sin x & -cosx \\ -tgx & ctgx \end{vmatrix}=?$

6. $\begin{vmatrix} 1 & 0 & 3 \\ -4 & 0 & 1 \\ -2 & 2 & 5 \end{vmatrix}=?$

7. $\begin{vmatrix} 1 & 3 & 3 \\ -1 & 0 & 1 \\ 2 & 2 & 0 \end{vmatrix}=?$

8. $\begin{vmatrix} 2 & 0 & 3 \\ 0 & 3 & 1 \\ -2 & 1 & -2 \end{vmatrix}=?$

9. $\begin{vmatrix} 3 & 1 & 6 \\ -3 & 1 & 1 \\ -2 & 0 & 5 \end{vmatrix}=?$

10. $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 7 & 1 \\ 1 & 0 & 5 \end{vmatrix}=?$

11. $\begin{vmatrix} 3 & 0 & 0 \\ -4 & 10 & 0 \\ 0 & 2 & 5 \end{vmatrix}=?$

12. $\begin{vmatrix} sinx & 0 & -cosx \\ 0 & 1 & 0 \\ cosx & 0 & sinx \end{vmatrix}=?$

13. $\begin{vmatrix} 2 & 2 & 0 \\ -3 & 1 & 1 \\ -20 & -3 & -1 \end{vmatrix}=?$

14. $\begin{vmatrix} 5 & 2 & 0 \\ -3 & 1 & 11 \\ -2 & 0 & 1 \end{vmatrix}=?$

15. $\begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & 1 \\ -2 & 0 & 5 \end{vmatrix}=?$

16. $\begin{vmatrix} 13 & 2 & 0 \\ 0 & 1 & 1 \\ -2 & 0 & 5 \end{vmatrix}=?$

17. $\begin{vmatrix} a & a & a \\ b & b & b \\ 2 & 0 & 1 \end{vmatrix}=?$

18. $\begin{vmatrix} sinx & 0 & 0 \\ -3 & tgx & 0 \\ -1 & 0 & 2 \end{vmatrix}=?$

19. $\begin{vmatrix} e^{x} & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & e^{-x} \end{vmatrix}=?$

20. $\begin{vmatrix} 1 & 2 & 3 \\ -3 & -1 & -2 \\ -2 & 0 & -2 \end{vmatrix}=?$

Determinantlarni tartibini pasaytirish usuli bilan hisoblang.

1. $\begin{vmatrix} 1 & 2 & 4 & 1 \\ 5 & -7 & 0 & 2 \\ 3 & -2 & 1 & 0 \\ 2 & -5 & 0 & 0 \end{vmatrix}=?$

2. $\begin{vmatrix} 0 & 2 & 1 & 5 \\ 5 & -1 & 0 & 2 \\ 3 & -1 & 1 & 0 \\ 4 & -5 & 0 & 0 \end{vmatrix}=?$

3. $\begin{vmatrix} 1 & 2 & 4 & 5 \\ 2 & 0 & 0 & 2 \\ 5 & -2 & 1 & 0 \\ 2 & -5 & 0 & 1 \end{vmatrix} = ?$

4. $\begin{vmatrix} 2 & 2 & 4 & 0 \\ 0 & -7 & 0 & 2 \\ 3 & 0 & 1 & 0 \\ 0 & -5 & 1 & 2 \end{vmatrix} = ?$

5. $\begin{vmatrix} 3 & 2 & 1 & 2 \\ -1 & -7 & 0 & 2 \\ 2 & 0 & 1 & 4 \\ 2 & 0 & 0 & 1 \end{vmatrix} = ?$

6. $\begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & -7 & 0 & 0 \\ 5 & -2 & 1 & 0 \\ 2 & -5 & 0 & 3 \end{vmatrix} = ?$

7. $\begin{vmatrix} 3 & 2 & 4 & 5 \\ 5 & -7 & 0 & 2 \\ 3 & -2 & -1 & 1 \\ 2 & -5 & 0 & -3 \end{vmatrix} = ?$

8. $\begin{vmatrix} 5 & 2 & 4 & -1 \\ 0 & -7 & 0 & 2 \\ 3 & -2 & 1 & 0 \\ 2 & 0 & 0 & 0 \end{vmatrix} = ?$

9. $\begin{vmatrix} 1 & 1 & 4 & 5 \\ 1 & -7 & 0 & 1 \\ 0 & -2 & 1 & 0 \\ 2 & -5 & 0 & 0 \end{vmatrix} = ?$

10. $\begin{vmatrix} 3 & 2 & 0 & 5 \\ -1 & -7 & 0 & 2 \\ 3 & -2 & 2 & 0 \\ 2 & -5 & 2 & 0 \end{vmatrix} = ?$

11. $\begin{vmatrix} 2 & 3 & -4 & 0 & 5 \\ 0 & 3 & -1 & 7 & 3 \\ 1 & 2 & 3 & 0 & -2 \\ 2 & 0 & 0 & 3 & 0 \\ 2 & 5 & 0 & -4 & 6 \end{vmatrix} = ?$

12. $\begin{vmatrix} 1 & 2 & 3 & 0 & -1 \\ 5 & 0 & 5 & 7 & 8 \\ 0 & -1 & 2 & 6 & -1 \\ -4 & 1 & 0 & 0 & 7 \\ -3 & 0 & 0 & 8 & 0 \end{vmatrix} = ?$

13. $\begin{vmatrix} 3 & -4 & 0 & 1 & -4 \\ 2 & 0 & -5 & 0 & -2 \\ -5 & 4 & 5 & -1 & 6 \\ 0 & 2 & 3 & 0 & -1 \\ -7 & 0 & 0 & 1 & 0 \end{vmatrix} = ?$

14. $\begin{vmatrix} 4 & 0 & 0 & 8 & 3 \\ 0 & 1 & 2 & 3 & 7 \\ 7 & 0 & 3 & 0 & 0 \\ 7 & -1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & -1 \end{vmatrix} = ?$

Mustaqil yechish uchun misollar.

Determinantlarni hisoblang.

a). Ikkinchi tartibli determinantni hisoblang.

1. $\begin{vmatrix} 2 & -4 \\ 3 & 0 \end{vmatrix} = ?$

4. $\begin{vmatrix} \sin x & cosx \\ -cosx & sinx \end{vmatrix} = ?$

2. $\begin{vmatrix} 6 & -4 \\ 0 & -8 \end{vmatrix} = ?$

5. $\begin{vmatrix} 1 & -2 \\ 6 & 6 \end{vmatrix} = ?$

3. $\begin{vmatrix} a & b \\ d & c \end{vmatrix} = ?$

6. $\begin{vmatrix} 2 & -4 \\ 0 & -18 \end{vmatrix} = ?$

7. $\begin{vmatrix} 1 & -2 \\ -5 & 3 \end{vmatrix} = ?$

8. $\begin{vmatrix} 12 & 2 \\ -3 & 0 \end{vmatrix} = ?$

9. $\begin{vmatrix} 21 & -2 \\ 23 & -1 \end{vmatrix} = ?$

10. $\begin{vmatrix} 4 & 7 \\ -6 & -9 \end{vmatrix} = ?$

11. $\begin{vmatrix} 8 & 9 \\ 10 & 11 \end{vmatrix} = ?$

12. $\begin{vmatrix} -3 & -4 \\ -5 & -6 \end{vmatrix} = ?$

13. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = ?$

14. $\begin{vmatrix} 9 & 2 \\ 3 & 4 \end{vmatrix} = ?$

15. $\begin{vmatrix} 1 & 2 \\ 3 & -4 \end{vmatrix} = ?$

16. $\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix} = ?$

17. $\begin{vmatrix} 1 & -12 \\ 6 & 4 \end{vmatrix} = ?$

18. $\begin{vmatrix} 1 & 2 \\ -13 & 4 \end{vmatrix} = ?$

19. $\begin{vmatrix} 1 & -2 \\ 3 & 14 \end{vmatrix} = ?$

20. $\begin{vmatrix} 11 & -2 \\ 3 & 4 \end{vmatrix} = ?$

21. $\begin{vmatrix} 1 & -12 \\ 0 & 4 \end{vmatrix} = ?$

22. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = ?$

23. $\begin{vmatrix} -11 & 2 \\ 3 & 0 \end{vmatrix} = ?$

24. $\begin{vmatrix} 1 & 2 \\ 3 & -7 \end{vmatrix} = ?$

25. $\begin{vmatrix} 11 & 2 \\ 13 & 4 \end{vmatrix} = ?$

26. $\begin{vmatrix} -1 & 12 \\ -3 & 4 \end{vmatrix} = ?$

27. $\begin{vmatrix} 1 & 41 \\ 3 & 4 \end{vmatrix} = ?$

28. $\begin{vmatrix} 7 & 2 \\ 3 & -4 \end{vmatrix} = ?$

29. $\begin{vmatrix} 1 & -12 \\ 5 & 4 \end{vmatrix} = ?$

30. $\begin{vmatrix} 1 & -2 \\ 3 & 14 \end{vmatrix} = ?$

b). Uchinchi tartibli determinantni hisoblang.

1. $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 0 & 1 \\ -2 & 2 & 5 \end{vmatrix} = ?$

2. $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & 2 & 0 \end{vmatrix} = ?$

3. $\begin{vmatrix} 1 & 8 & 3 \\ -4 & 1 & 1 \\ -2 & 2 & 5 \end{vmatrix} = ?$

4. $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 0 & 1 \\ -2 & 2 & 0 \end{vmatrix} = ?$

5. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ -2 & 2 & 5 \end{vmatrix} = ?$

6. $\begin{vmatrix} 1 & 0 & 3 \\ -4 & -1 & 1 \\ -2 & 2 & 5 \end{vmatrix} = ?$

7. $\begin{vmatrix} 0 & 2 & 3 \\ -4 & 0 & 1 \\ -2 & 2 & 5 \end{vmatrix} = ?$

8. $\begin{vmatrix} 2 & 2 & 3 \\ 0 & 3 & 1 \\ -2 & 1 & -2 \end{vmatrix} = ?$

9. $\begin{vmatrix} 3 & 2 & 6 \\ -3 & 1 & 1 \\ -2 & 0 & 5 \end{vmatrix} = ?$

10. $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 7 & 1 \\ 0 & 0 & 5 \end{vmatrix} = ?$

11. $\begin{vmatrix} 3 & 0 & 0 \\ -4 & 10 & 0 \\ 0 & 2 & 5 \end{vmatrix} = ?$

12. $\begin{vmatrix} sinx & 0 & cosx \\ 0 & 1 & 0 \\ -cosx & 0 & sinx \end{vmatrix} = ?$

13. $\begin{vmatrix} 23 & 2 & 0 \\ -3 & 1 & 1 \\ -20 & -3 & -1 \end{vmatrix} = ?$

14. $\begin{vmatrix} 5 & 2 & 0 \\ -3 & 1 & 11 \\ -2 & 0 & 0 \end{vmatrix} = ?$

15. $\begin{vmatrix} 1 & 12 & 6 \\ 0 & 1 & 1 \\ -2 & 0 & 5 \end{vmatrix} = ?$

16. $\begin{vmatrix} 13 & 2 & 0 \\ 0 & 1 & 21 \\ -2 & 0 & 5 \end{vmatrix} = ?$

17. $\begin{vmatrix} a & a & a \\ b & b & b \\ 2 & 0 & 5 \end{vmatrix} = ?$

18. $\begin{vmatrix} sinx & 0 & 0 \\ -3 & tgx & 0 \\ -1 & 0 & 1 \end{vmatrix} = ?$

19. $\begin{vmatrix} e^x & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & e^{-x} \end{vmatrix} = ?$

20. $\begin{vmatrix} 1 & 2 & 3 \\ -3 & 1 & -2 \\ -2 & 0 & -2 \end{vmatrix} = ?$

21. $\begin{vmatrix} -1 & 2 & 5 \\ -3 & 1 & -2 \\ -2 & 0 & -2 \end{vmatrix} = ?$

22. $\begin{vmatrix} 11 & 0 & 3 \\ -3 & 1 & -2 \\ -2 & 0 & -2 \end{vmatrix} = ?$

23. $\begin{vmatrix} 1 & 2 & 0 \\ -3 & 1 & -2 \\ -2 & 0 & -2 \end{vmatrix} = ?$

24. $\begin{vmatrix} 1 & -2 & 3 \\ -3 & 5 & -2 \\ -2 & 0 & -2 \end{vmatrix} = ?$

25. $\begin{vmatrix} 1 & -2 & 3 \\ -3 & 1 & -2 \\ -2 & 0 & -4 \end{vmatrix} = ?$

26. $\begin{vmatrix} 11 & 2 & 3 \\ -3 & 0 & -2 \\ -2 & 0 & -2 \end{vmatrix} = ?$

27. $\begin{vmatrix} 10 & 2 & 3 \\ -5 & 1 & -2 \\ -2 & 0 & -2 \end{vmatrix} = ?$

28. $\begin{vmatrix} 1 & 2 & 0 \\ -3 & 1 & -12 \\ -2 & 0 & -2 \end{vmatrix} = ?$

29. $\begin{vmatrix} 10 & 2 & 3 \\ -3 & 11 & -2 \\ -2 & 0 & -2 \end{vmatrix} = ?$

30. $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 7 & -2 \\ -2 & 0 & -2 \end{vmatrix} = ?$

c). Determinantlarni tartibini pasaytirish usuli bilan hisoblang.

1. $\begin{vmatrix} 1 & 2 & 4 & 5 \\ 5 & -7 & 0 & 2 \\ 3 & -2 & 1 & 0 \\ 2 & -5 & 0 & 0 \end{vmatrix} = ?$

2. $\begin{vmatrix} 0 & 2 & 1 & 5 \\ 5 & -7 & 0 & 2 \\ 3 & -1 & 1 & 0 \\ 4 & -5 & 0 & 0 \end{vmatrix} = ?$

3. $\begin{vmatrix} 1 & 2 & 4 & 5 \\ 2 & -7 & 0 & 2 \\ 5 & -2 & 1 & 0 \\ 2 & -5 & 0 & 1 \end{vmatrix} = ?$

4. $\begin{vmatrix} 2 & 2 & 4 & 0 \\ 0 & -7 & 0 & 2 \\ 3 & 0 & 1 & 0 \\ 0 & -5 & 0 & 2 \end{vmatrix} = ?$

5. $\begin{vmatrix} 3 & 2 & 1 & 2 \\ -1 & -7 & 0 & 2 \\ 2 & -5 & 1 & 4 \\ 2 & 0 & 0 & 1 \end{vmatrix} = ?$

6. $\begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & -7 & 0 & 0 \\ 3 & -2 & 1 & 0 \\ 2 & -5 & 0 & 3 \end{vmatrix} = ?$

7. $\begin{vmatrix} 3 & 2 & 4 & 5 \\ 5 & -7 & 0 & 2 \\ 3 & -2 & -1 & 1 \\ 2 & -5 & 0 & -3 \end{vmatrix} = ?$

8. $\begin{vmatrix} 5 & 2 & 4 & -1 \\ 0 & -7 & 0 & 2 \\ 3 & -2 & 1 & 0 \\ 2 & 0 & 0 & 0 \end{vmatrix} = ?$

9. $\begin{vmatrix} 1 & 1 & 4 & 5 \\ 1 & -7 & 0 & 1 \\ 0 & -2 & 1 & 0 \\ 2 & -5 & 0 & 0 \end{vmatrix} = ?$

10. $\begin{vmatrix} 3 & 2 & 0 & 2 \\ -1 & -7 & 0 & 2 \\ 3 & -2 & 2 & 0 \\ 2 & -5 & 2 & 0 \end{vmatrix} = ?$

11. $\begin{vmatrix} 1 & 1 & 4 & 5 \\ 5 & -7 & 0 & 2 \\ 3 & -2 & 1 & 0 \\ 2 & -5 & 0 & 0 \end{vmatrix} = ?$

12. $\begin{vmatrix} 0 & 2 & 1 & 1 \\ 5 & -7 & 0 & 2 \\ 1 & -1 & 1 & 0 \\ 4 & -5 & 0 & 0 \end{vmatrix} = ?$

13. $\begin{vmatrix} 1 & 2 & 4 & 5 \\ 2 & 7 & 0 & 2 \\ 5 & -2 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{vmatrix} = ?$

14. $\begin{vmatrix} 2 & 2 & 4 & 0 \\ 0 & 1 & 0 & 2 \\ 3 & 0 & 1 & 0 \\ 0 & -5 & 0 & 2 \end{vmatrix} = ?$

15. $\begin{vmatrix} 3 & 2 & 1 & 2 \\ -1 & 8 & 0 & 2 \\ 2 & -5 & 1 & 1 \\ 2 & 0 & 0 & 1 \end{vmatrix} = ?$

16. $\begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 \\ 2 & -5 & 0 & 3 \end{vmatrix} = ?$

17. $\begin{vmatrix} 3 & 2 & 4 & 0 \\ 0 & -7 & 0 & 2 \\ 3 & -2 & -1 & 1 \\ 1 & -5 & 0 & -3 \end{vmatrix} = ?$

18. $\begin{vmatrix} 5 & 2 & 4 & -1 \\ 0 & -7 & 0 & 2 \\ 2 & -2 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{vmatrix} = ?$

19. $\begin{vmatrix} 1 & 1 & 4 & 1 \\ 1 & 4 & 0 & 1 \\ 0 & -2 & 1 & 0 \\ 2 & -5 & 0 & 0 \end{vmatrix} = ?$

20. $\begin{vmatrix} 3 & 2 & 0 & 5 \\ -1 & -7 & 0 & 2 \\ 3 & -2 & 3 & 0 \\ 2 & -5 & 2 & 0 \end{vmatrix} = ?$

21. $\begin{vmatrix} 1 & 2 & 4 & -5 \\ 1 & -7 & 0 & 2 \\ 3 & -2 & 1 & 0 \\ 2 & -5 & 0 & 0 \end{vmatrix} = ?$

22. $\begin{vmatrix} 0 & 2 & -2 & 5 \\ 5 & 8 & 0 & 2 \\ 3 & -1 & 1 & 0 \\ 4 & -5 & 0 & 0 \end{vmatrix} = ?$

23. $\begin{vmatrix} 1 & 2 & 2 & 5 \\ 2 & -7 & 0 & 2 \\ 2 & -2 & -3 & 0 \\ 2 & -5 & 0 & 1 \end{vmatrix} = ?$

24. $\begin{vmatrix} 2 & 3 & 4 & 0 \\ 0 & -7 & 0 & 2 \\ 3 & 0 & 1 & 0 \\ 0 & -5 & 0 & 2 \end{vmatrix} = ?$

25. $\begin{vmatrix} 3 & 2 & 3 & 2 \\ -1 & -7 & 0 & 2 \\ 2 & -5 & 1 & 4 \\ -2 & 0 & 0 & 1 \end{vmatrix} = ?$

26. $\begin{vmatrix} 2 & 0 & -1 & 0 \\ 0 & -7 & -3 & -1 \\ 3 & -2 & -2 & 0 \\ 2 & -5 & 0 & 3 \end{vmatrix} = ?$

27. $\begin{vmatrix} 3 & 2 & 4 & 5 \\ -1 & -7 & 0 & 2 \\ 3 & -2 & -1 & 1 \\ 2 & 0 & 0 & -3 \end{vmatrix} = ?$

28. $\begin{vmatrix} 1 & 2 & 4 & -1 \\ 0 & -7 & 0 & 2 \\ 3 & -2 & 1 & 0 \\ 2 & 0 & 0 & 7 \end{vmatrix} = ?$

29. $\begin{vmatrix} 2 & 1 & 4 & 5 \\ 1 & -7 & 0 & 1 \\ 0 & -2 & 1 & 0 \\ 2 & -5 & 0 & -2 \end{vmatrix} = ?$

30. $\begin{vmatrix} 3 & 2 & 0 & -5 \\ -1 & -7 & 0 & -2 \\ 3 & -2 & -2 & 0 \\ 2 & -5 & 2 & 0 \end{vmatrix} = ?$

3-§. Teskari matritsa.

1-ta'rif. Agar kvadrat A matritsa uchun $AB = BA = E$ tenglik o`rinli bo`lsa, u holda B matritsa A matritsa uchun ***teskari matritsa*** deyiladi.

A matritsaga teskari matritsa A^{-1} kabi belgilanadi.

2-ta'rif. Agar A matritsaning determinanti nolga teng bo'lsa, ***xos*** matritsa, noldan farqli bo'lsa, ***xosmas*** matritsa deyiladi.

Teorema 1. A kvadrat matritsa teskari matritsaga ega bo'lishi uchun A matritsa xosmas matritsa bo'lishi zarur va yetarli.

Teskari matritsa

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix} \qquad (1)$$

formula bilan topiladi.

Misollar

A matritsaga teskari matritsa A^{-1} ni toping va $\mathrm{AA}^{-1} = \mathrm{A}^{-1}\mathrm{A} = \mathrm{E}$ ekanini tekshiring:

1. $A = \begin{pmatrix} 2 & -1 \\ -7 & 2 \end{pmatrix}$ 2. $A = \begin{pmatrix} 1 & 10 \\ -7 & 2 \end{pmatrix}$

3. $A = \begin{pmatrix} -3 & -1 \\ -5 & 2 \end{pmatrix}$ 4. $A = \begin{pmatrix} 3 & -3 \\ 2 & 4 \end{pmatrix}$

5. $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & -2 & 3 \\ 4 & 1 & -1 \end{pmatrix}$ 6. $A = \begin{pmatrix} 3 & 2 & -1 \\ 0 & 3 & 0 \\ 1 & 2 & -2 \end{pmatrix}$

7. $A = \begin{pmatrix} 1 & -1 & 5 \\ 2 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$ 8. $A = \begin{pmatrix} 1 & 1 & 3 \\ 0 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix}$

9. $A = \begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \\ 3 & 1 & 2 \end{pmatrix}$ 10. $A = \begin{pmatrix} -1 & 0 & 1 \\ 2 & -2 & 0 \\ 1 & 3 & 4 \end{pmatrix}$

Mustaqil yechish uchun misollar.

A matritsaga teskari matritsa $\mathbf{A}^{-1}$ ni toping va $\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{E}$ ekanini tekshiring:

1. $A = \begin{pmatrix} 2 & 1 \\ -7 & 2 \end{pmatrix}$

2. $A = \begin{pmatrix} 1 & 0 \\ -7 & 2 \end{pmatrix}$

3. $A = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

4. $A = \begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix}$

5. $A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 3 \\ 4 & 1 & -1 \end{pmatrix}$

6. $A = \begin{pmatrix} 3 & 2 & -1 \\ 7 & 3 & 0 \\ 1 & 2 & -2 \end{pmatrix}$

7. $A = \begin{pmatrix} 1 & -3 & 5 \\ 2 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$

8. $A = \begin{pmatrix} 1 & 1 & 3 \\ 2 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix}$

9. $A = \begin{pmatrix} 1 & -3 & 2 \\ 2 & 4 & 0 \\ 3 & 1 & 2 \end{pmatrix}$

10. $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -2 & 0 \\ 1 & 3 & 4 \end{pmatrix}$

11. $A = \begin{pmatrix} 0 & 1 \\ -7 & 2 \end{pmatrix}$

12. $A = \begin{pmatrix} -5 & 0 \\ -7 & 2 \end{pmatrix}$

13. $A = \begin{pmatrix} 7 & -1 \\ -5 & -2 \end{pmatrix}$

14. $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

15. $A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & -2 & 3 \\ 4 & 1 & -1 \end{pmatrix}$

16. $A = \begin{pmatrix} 0 & 2 & -1 \\ 7 & 3 & 0 \\ 1 & 2 & -2 \end{pmatrix}$

17. $A = \begin{pmatrix} 1 & -3 & 5 \\ 2 & 1 & 0 \\ 3 & -3 & -1 \end{pmatrix}$

18. $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix}$

19. $A = \begin{pmatrix} 1 & -3 & 2 \\ -2 & 4 & 0 \\ 3 & 1 & 2 \end{pmatrix}$

20. $A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & -2 & 0 \\ 1 & 3 & 4 \end{pmatrix}$

21. $A = \begin{pmatrix} -6 & 1 \\ -7 & 2 \end{pmatrix}$

22. $A = \begin{pmatrix} 9 & 0 \\ -7 & 2 \end{pmatrix}$

23. $A = \begin{pmatrix} 8 & -1 \\ -5 & 2 \end{pmatrix}$

24. $A = \begin{pmatrix} 3 & -3 \\ 2 & 4 \end{pmatrix}$

25. $A = \begin{pmatrix} -1 & 0 & -1 \\ 1 & -2 & 3 \\ 4 & 1 & -1 \end{pmatrix}$

26. $A = \begin{pmatrix} 3 & -2 & -1 \\ 1 & 3 & 0 \\ 1 & 2 & -2 \end{pmatrix}$

27. $A = \begin{pmatrix} 1 & -3 & 5 \\ 2 & -4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$

28. $A = \begin{pmatrix} 0 & 1 & 3 \\ -2 & -2 & -5 \\ 1 & 4 & 3 \end{pmatrix}$

29. $A = \begin{pmatrix} 0 & -3 & 2 \\ 2 & 4 & 0 \\ -3 & 1 & 2 \end{pmatrix}$

30. $A = \begin{pmatrix} -2 & 0 & 1 \\ 2 & -2 & 0 \\ 1 & 3 & -4 \end{pmatrix}$

4-§. Matritsaning rangi.

1-ta’rif. A matritsaning k-tartibli ***minori*** deb, bu matritsadan ixtiyoriy k ta satr va k ta ustunni ajratishdan hosil bo’lgan kvadrat matritsaning determinantiga aytiladi.

2-ta’rif. A Matritsaning ***rangi*** deb, uning noldan farqli minorlarining eng katta tartibiga yoki matritsaning chiziqli bog`lanmagan ustun yoki satrlarining eng katta soniga aytiladi va **rangA** kabi belgilanadi.

3-ta’rif. Quyidagi almashtirishlar, chiziqli almashtirish deb ataladi:

a) ixtiyoriy ikkita satr (ustun) elementlarini o’rnini almashtirish;

b) ixtiyoriy satr (ustun) elementlarini biror songa ko’paytirib, boshqa satr (ustun) ning mos elementlariga qo’shish;

c) faqat nollardan iborat satr (ustun) ni o’chirish;

CHiziqli almashtirishlar matritsa rangini o’zgartirmaydi. SHu sababli, elementar almashtirishlardan foydalanib, matritsaning bosh diagonal elementlaridan (kvadrat matritsa bo’lishi shart emas) pastdagi barcha elementlar nolga keltiriladi. Bu holda matritsaning rangi bosh diagonaldagi noldan farqli elementlar soniga teng bo’ladi.

Misollar.Berilgan A matritsaning rangini toping:

1. $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

2. $A = \begin{pmatrix} 1 & -3 \\ -1 & 3 \end{pmatrix}$

3. $A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$

4. $A = \begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix}$

5. $A = \begin{pmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \\ 3 & 1 & 0 \end{pmatrix}$

6. $A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 0 & 3 \\ 1 & -2 & -1 \end{pmatrix}$

7. $A = \begin{pmatrix} 0 & 2 & -1 \\ -2 & 0 & 3 \\ 5 & 1 & 0 \end{pmatrix}$

8. $A = \begin{pmatrix} 3 & 4 & 1 \\ 0 & 1 & 2 \\ 1 & 3 & 4 \end{pmatrix}$

9. $A = \begin{pmatrix} 2 & -2 & -4 \\ -2 & 0 & 2 \\ 1 & -2 & 1 \end{pmatrix}$

10. $A = \begin{pmatrix} 0 & 4 & 0 \\ -2 & 0 & 3 \\ 0 & -2 & -6 \end{pmatrix}$

11. $A=\begin{pmatrix}1 & 4 & -3 & 6\\ 2 & 5 & 1 & -2\\ 17 & -10 & 20 & 0\end{pmatrix}$ 12. $A=\begin{pmatrix}2 & 1 & -2 & 3\\ -3 & 0 & -1 & 1\\ 5 & 1 & -3 & 2;\end{pmatrix}$

13. $A=\begin{pmatrix}1 & -3 & -4 & 1 & 1\\ 5 & 0 & -2 & 8 & 3\\ -2 & -1 & 0 & -5 & 0\end{pmatrix}$ 14. $A=\begin{pmatrix}1 & 3 & -1 & 2 & 0\\ 2 & 0 & -8 & -5 & 6\\ 1 & 4 & 5 & -1 & 0\end{pmatrix}$

15. $A=\begin{pmatrix}0 & 1 & 4 & -2 & 3\\ 1 & 2 & 0 & -1 & 0\\ -1 & -3 & 0 & 3 & -3\\ 1 & 2 & 3 & 4 & 0\end{pmatrix}$ 16. $A=\begin{pmatrix}1 & 0 & 1 & -2 & 3\\ 1 & 2 & 3 & -1 & 0\\ -1 & 0 & -7 & 0 & -3\\ 0 & 1 & 4 & 1 & 3\end{pmatrix}$

17. $A=\begin{pmatrix}-1 & 1 & 2 & 0\\ 1 & 0 & -1 & 2\\ 0 & 1 & 1 & 2\\ 2 & 2 & 0 & 1\end{pmatrix}$ 18. $A=\begin{pmatrix}0 & -1 & 0 & 3\\ 2 & 1 & 3 & 1\\ 0 & 1 & 1 & 0\\ 2 & -4 & -2 & 1\end{pmatrix}$

19. $A=\begin{pmatrix}0 & -1 & 0\\ -3 & 1 & 3\\ 1 & 0 & 3\\ -5 & 1 & -3\end{pmatrix}$ 20. $A=\begin{pmatrix}1 & 2 & 0\\ -2 & -3 & -4\\ 0 & 1 & 3\\ 0 & 2 & 6\end{pmatrix}$

Mustaqil yechish uchun misollar.

Berilgan A matritsaning rangini toping:

1. $A=\begin{pmatrix}1 & 2 & 3\\ -2 & 0 & 3\\ 3 & 1 & 0\end{pmatrix}$ 6. $A=\begin{pmatrix}1 & 2 & 3\\ -2 & 0 & 3\\ 1 & -2 & -6\end{pmatrix}$

2. $A=\begin{pmatrix}1 & 2 & 3\\ -2 & 0 & 3\\ 1 & -2 & -6\end{pmatrix}$ 7. $A=\begin{pmatrix}0 & 2 & 1\\ -2 & 0 & 3\\ 5 & 1 & 0\end{pmatrix}$

3. $A=\begin{pmatrix}0 & 2 & 1\\ -2 & 0 & 3\\ 5 & 1 & 0\end{pmatrix}$ 8. $A=\begin{pmatrix}3 & 4 & 1\\ 0 & -1 & 2\\ 1 & 3 & 4\end{pmatrix}$

4. $A=\begin{pmatrix}3 & 4 & 1\\ 0 & -1 & 2\\ 1 & 3 & 4\end{pmatrix}$ 9. $A=\begin{pmatrix}2 & 2 & -4\\ -2 & 0 & 2\\ 1 & -2 & 1\end{pmatrix}$

5. $A=\begin{pmatrix}1 & 2 & 3\\ -2 & 0 & 3\\ 3 & 1 & 0\end{pmatrix}$ 10. $A=\begin{pmatrix}3 & 4 & 0\\ -2 & 0 & 3\\ 0 & -2 & -6\end{pmatrix}$

11. $A = \begin{pmatrix} 1 & 4 & -3 & 61 \\ 2 & 5 & 1 & -23 \\ 17 & -1 & 20 & 0 \end{pmatrix}$

12. $A = \begin{pmatrix} 2 & 1 & -2 & 3 \\ -3 & 0 & 1 & 1 \\ 5 & 1 & -3 & 2; \end{pmatrix}$

13. $A = \begin{pmatrix} 1 & -3 & -4 & 1 & 1 \\ 5 & 0 & -2 & 0 & 3 \\ 0 & -1 & -10 & -5 & 0 \end{pmatrix}$

14. $A = \begin{pmatrix} 1 & 0 & -1 & 2 & 2 \\ 2 & 5 & 0 & -5 & 0 \\ 1 & 0 & 5 & -1 & 0 \end{pmatrix}$

15. $A = \begin{pmatrix} 0 & 1 & 0 & -2 & 3 \\ 1 & 2 & 3 & -1 & 0 \\ 0 & -3 & 0 & 3 & -3 \\ 1 & 2 & 3 & 4 & 0 \end{pmatrix}$

16. $A = \begin{pmatrix} 1 & 0 & 4 & -2 & 3 \\ 1 & 2 & 0 & -1 & 0 \\ -1 & -3 & -7 & 0 & -3 \\ 0 & 1 & 4 & 1 & 3 \end{pmatrix}$

17. $A = \begin{pmatrix} -1 & 1 & 2 & 0 \\ 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & 2 \\ 2 & 2 & 0 & 8 \end{pmatrix}$

18. $A = \begin{pmatrix} 1 & -1 & 0 & 3 \\ 2 & 1 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & -4 & -2 & 1 \end{pmatrix}$

19. $A = \begin{pmatrix} 4 & -1 & 0 \\ -3 & 1 & 3 \\ 1 & 0 & 3 \\ -5 & 1 & -3 \end{pmatrix}$

20. $A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 0 & 1 & 3 \\ 0 & 2 & 6 \end{pmatrix}$

21. $A = \begin{pmatrix} 1 & 2 & 1 \\ -2 & 0 & 3 \\ 3 & 1 & 0 \end{pmatrix}$

22. $A = \begin{pmatrix} -1 & 2 & -3 \\ -2 & 0 & 3 \\ 1 & -2 & -6 \end{pmatrix}$

23. $A = \begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & -3 \\ 5 & 1 & 0 \end{pmatrix}$

24. $A = \begin{pmatrix} 3 & 7 & 1 \\ 0 & -1 & 2 \\ 1 & 3 & 4 \end{pmatrix}$

25. $A = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 0 & 3 \\ 3 & 1 & 0 \end{pmatrix}$

26. $A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 0 & 11 \\ 1 & -2 & -6 \end{pmatrix}$

27. $A = \begin{pmatrix} 0 & 2 & 1 \\ -2 & 10 & 3 \\ 5 & 1 & 0 \end{pmatrix}$

28. $A = \begin{pmatrix} 3 & 4 & 1 \\ 6 & -1 & 2 \\ 1 & 3 & 1 \end{pmatrix}$

29. $A = \begin{pmatrix} 1 & 2 & -4 \\ -2 & 0 & 2 \\ 5 & -2 & 1 \end{pmatrix}$

30. $A = \begin{pmatrix} 1 & 4 & 0 \\ -2 & 0 & 3 \\ 2 & -2 & -6 \end{pmatrix}$

CHIZIQLI TENGLAMALAR SISTEMASI

1-§.Chiziqli tenglamalar sistemasi.

1-ta'rif. n noma'lumli m ta ***chiziqli tenglamalar sistemasi*** deb,

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases} \qquad (1)$$

ga aytiladi. Bu yerda a_{ij} (i - satr, j - ustun, $i = 1,m; j = 1,n$), b_i ($i = 1,m$) lar berilgan sonlar bo`lib, $a_{ij}$ lar sistemaning koeffitsiyentlari, $b_i$ ($i = 1,m$ ) - lar esa ozod hadlar, $x_i$ ($i = 1,m$ ) - lar ***o`zgaruvchilar yoki noma'lumlar*** deyiladi va ular ixtiyoriy qiymatlar qabul qiladilar.

2-**ta'rif**. Agar $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ sonlarni $(x_1, x_2, \ldots, x_n)$ lar o`rniga mos ravishda qo`yganimizda (1) sistemaning har bir tenglamasi to`g`ri sonli tenglikka aylansa, u holda $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ vektor berilgan sistemaning ***yechimi*** deyiladi.

3-**ta'rif**. Agar (1) sistemaning yechimi bo`lsa, u ***birgalikda;*** yechimi bo`lmasa, ***birgalikda emas;*** faqat bitta yechimi bo`lsa, u ***aniq sistema;*** cheksiz ko`p yechimi bo`lsa, u ***aniqmas sistema*** deyiladi.

4-ta'rif. Agar b_i ($i = 1,m$) ozod hadlarning kamida bittasi noldan farqli bo'lsa***, bir jinsli bo'lmagan tenglamalar sistema*** deyiladi.

5-**ta'rif**. Agar b_i ($i = 1,m$) ozod hadlarning barchasi nolga teng, ya'ni

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0 \end{cases} \qquad (2)$$

bo'lsa***, bir jinsli tenglamalar sistemasi*** deyiladi.

Teorema (Kroneker – Kapelli teoremasi)

(1) sistema birgalikda bulishi uchun uning asosiy A va kengaytirilgan B matritsalarining ranglari teng bo`lishi zarur va yetarlidir.

Bu yerda, $A = \begin{pmatrix} a_{11} & a_{12} \ldots & a_{1n} \\ a_{21} & a_{22} \ldots & a_{2n} \\ \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} \ldots & a_{mn} \end{pmatrix}$, $B = \begin{pmatrix} a_{11} & a_{12} \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} \ldots & a_{2n} & b_2 \\ \ldots & \ldots & \ldots & \\ a_{m1} & a_{m2} \ldots & a_{mn} & b_m \end{pmatrix}$.

(2) sistema $x_1 = x_2 = \cdots = x_n = 0$ yechimga ega. Demak, har qachon birgalikda bo`ladi. Yuqoridagi yechim - trivial yechim bo`lib, amaliyot uchun notrival yechimlarning mavjud bo`lishi muhim ahamiyatga ega.

Teorema. Agar (2) sistemaning rangi r uchun $0 < r < m$ tengsizlik o`rinli bo`lsa, u holda sistema notrival yechimga ega bo`ladi.

2-§.Chiziqli tenglamalar sistemasini yechish.

1.*Kramer usuli.*

Ikki noma'lumli ikkita chiziqli tenglamalar sistemasi:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

Uning asosiy determinant $\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ bo`lganda yagona yechimga ega va Kramer qoidasi bo`yicha quyidagi formulalar bilan hisiblanadi:

$x_1 = \frac{\Delta_{x_1}}{\Delta}$, $x_2 = \frac{\Delta_{x_2}}{\Delta}$ bu yerda $\Delta_{x_1} = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}$, $\Delta_{x_2} = \begin{bmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{bmatrix}$ lar yordamchi determinantlar deyiladi.

Agar $\Delta = 0$ va shu bilan birga Δ_{x_1} , Δ_{x_2} lardan hech bo`lmasa bittasi noldan farqli bo'lsa, sistema yechimga ega bo'lmaydi.

Agar $\Delta = \Delta_{x_1} = \Delta_{x_2} = 0$ bo`lsa, u holda sistema cheksiz ko`p yechimga ega bo`ladi.

Uch noma'lumli uchta chiziqli tenglamalar sistemasi berilgan;

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

Uning asosiy determinanti

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

bo'lganda yagona yechimga ega bo'lib, u Kramer formulalari orqali quyidagicha

$$x_1 = \frac{\Delta_{x_1}}{\Delta}, x_2 = \frac{\Delta_{x_2}}{\Delta}, x_3 = \frac{\Delta_{x_3}}{\Delta}.$$

hisoblanadi.

Bu yerda

$$\Delta_{x_1}=\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix};\quad \Delta_{x_2}=\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix};\quad \Delta_{x_3}=\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

Agar $\Delta = 0$ va shu bilan birga $\Delta_{x_1}, \Delta_{x_2}, \Delta_{x_3}$ lardan hech bo`lmaganda bittasi noldan farqli bo`lsa, sistema yechimga ega bo'lmaydi.

Agar $\Delta = \Delta_{x_1} = \Delta_{x_2} = \Delta_{x_3} = 0$ bo`lsa, u holda sistema cheksiz ko`p yechimga ega bo`ladi.

2.Gauss usuli.

n ta noma'lumli n ta chiziqli tenglamalar sistemasini n ning yetarlicha katta qiymatlarida Kramer qoidasi bilan yechish bir nechta yuqori tartibli determinantlarni hisoblashni talab etadi. Shuning uchun ularni Gauss usulidan foydalanib yechish maqsadga muvofiq. Bu usulda noma'lumlar ketma-ket yo`qotilib, sistema uchburchak shaklga keltiriladi. Agar sistema uchburchaksimon shaklga kelsa, u yagona yechimga ega bo`ladi va uning noma'lumlari oxirgi tenglamadan boshlab topib boriladi. Sistema cheksiz ko`p yechimga ega bo`lsa, noma'lumlar ketma-ket yo`qotilgach, u trapetsiyasimon shaklga keladi.

CHiziqli almashtirishlar bajarilayotganda;

a) Ayrim tenglamalar $0 = 0$ ko`rinishga kelib qolsa, ular tashlab yuboriladi. Bu hol sistemaning rangi *m* dan kichik ekanligini bildiradi;

b) Biror tenglama $0 = a\ (a \neq 0)$ ko`rinishga kelib qolsa, bu hol tenglama birgalikda emasligini bildiradi. U vaqtda barcha hisoblar to`xtatilib "sistema birgalikda emas" deb javob yoziladi.

3.Matritsalar usuli.

n ta noma'lumli n ta chiziqli tenlamalar sistemasi berilgan:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

Bu yerda $A = \begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{12} & a_{22} \cdots & a_{2n} \\ a_{1n} & a_{n2} \cdots & a_{nn} \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$.

Berilgan chiziqli tenglamalar sistemasini matritsa ko`rinishida

$$A \cdot X = B$$

kabi yozish mumkin.

Agar A maxsusmas matritsa, ya`ni $detA \neq 0$ bo`lsa, u holda bu sistemaning matritsa yechimi ushbu ko`rinishga ega bo`ladi: $X = A^{-1} \cdot B$.

Agar $rangA = n$ bo'lsa, sistemaning determinant noldan farqli bo'lib , u yagona yechimga ega bo'ladi; agar $rangA < n$ bo'lsa , u cheksiz ko'p yechimga ega bo'ladi .

Agar barcha b ozod hadlar nolga teng bo'lsa , tenglamalar sistemasi ***bir jinsli*** deyiladi. Bunday tenglamalar sistemasida har doim $rangA = rangB$, shuning uchun bir jinsli sistema birgalikda bo'ladi. Bir jinsli tenglamalar sistemasini $x_1 = 0,\ x_2 = 0, \dots, x_n = 0$ qiymatlar qanoatlantiradi, lekin A matritsaning rangi noma'lumlar soni n dan kichik bo'lganda uning determinanti nolga teng bo'lib , sistema notrivial yechimga ega bo'ladi.

Misollar.

Quyidagi chiziqli tenglamalar sistemasini Kramer formulalaridan foydalanib yeching:

1. $\begin{cases} 3x_1 - 5x_2 + 3x_3 = -3 \\ x_1 + 2x_2 + x_3 = -1 \\ x_1 - 7x_2 - 2x_3 = 5 \end{cases}$ j:(1;0;-2)

2. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 8 \\ 4x_2 + 11x_3 = 8 \\ 7x_1 - 5x_2 = -3 \end{cases}$ j:(1;2;0)

3. $\begin{cases} 2x_1 - x_2 + 5x_3 = 10 \\ 5x_1 + 2x_2 - x_3 = 2{,}5 \\ 4x_1 - x_3 = 0 \end{cases}$ $j: (\frac{1}{2}; 1; 2)$

4. $\begin{cases} x_1 - 4x_2 - 2x_3 = 4 \\ 3x_1 - 5x_2 - 6x_3 = 19 \\ 3x_1 + x_2 + x_3 = 11 \end{cases}$ j:(4;1;-2)

5. $\begin{cases} 2x_1 - 3x_2 + x_3 = 7 \\ x_1 + 2x_2 - x_3 = 2 \\ x_1 + 4x_2 - 2x_3 = 5 \end{cases}$ j:(-1;2;1)

6. $\begin{cases} x_1 + 4x_2 - 3x_3 = 10 \\ 2x_1 + x_2 - x_3 = 3 \\ x_1 - 3x_2 + 2x_3 = -8 \end{cases}$ j:∅

7. $\begin{cases} x_1 + x_2 - 3x_3 = 10 \\ 2x_1 + x_2 - x_3 = 3 \\ x_1 + 2x_3 = -7 \end{cases}$ j:yechim cheksiz ko'p.

8. $\begin{cases} x_1 + 4x_2 - 3x_3 = 10 \\ x_1 + x_2 - x_3 = 3 \\ 3 - 2x_3 = 4 \end{cases}$ j:∅

9. $\begin{cases} 2x_1 + 4x_2 - x_3 = 1 \\ x_1 + x_2 - 2x_3 = 3 \\ x_1 + 3x_2 + x_3 = -2 \end{cases}$ j:yechim cheksiz ko'p.

10. $\begin{cases} -x_1 + x_2 - 2x_3 = -2 \\ x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + x_2 - x_3 = 5 \end{cases}$ j:∅

11. $\begin{cases} x_1 + 4x_2 - 3x_3 = 10 \\ 2x_1 + x_2 - x_3 = 3 \\ 4x_1 + 2x_2 + 5x_3 = -8 \end{cases}$ j:(0;1;-2)

12. $\begin{cases} x_1 - 3x_2 = 5 \\ 4x_1 + 3x_3 = 8 \\ 5x_1 + 2x_2 - 4x_3 = 8 \end{cases}$ j:(2;-1;0)

13. $\begin{cases} 2x_1 - x_2 + 4x_3 = -3 \\ x_1 + x_2 - 4x_3 = 12 \\ x_1 + 4x_2 - x_3 = 9 \end{cases}$ j:(3;1;-2)

14. $\begin{cases} x_1 - 3x_2 + x_3 = 2 \\ 2x_1 + x_2 - 3x_3 = 9 \\ 4x_1 - x_2 = 12 \end{cases}$ j:(3;0;-1)

15. $\begin{cases} x_1 - 3x_2 - x_3 = 2 \\ 2x_1 + x_2 + x_3 = 0 \\ 3x_1 - 2x_2 = 12 \end{cases}$ j: ∅

Quyidagi tenglamalar sistemasini Gauss usuli bilan yeching:

16. $\begin{cases} x_1 - 2x_2 + 3x_3 = 6 \\ 2x_1 + 3x_2 - 4x_3 = 20 \\ 3x_1 - 2x_2 - 5x_3 = 6 \end{cases}$ J:(8;4;2)

17. $\begin{cases} x_1 - 9x_2 + 2x_3 = 14 \\ x_1 + 3x_2 - 8x_3 = -18 \\ x_1 - 3x_2 - 2x_3 = 0 \end{cases}$ J:(1;-1;2)

18. $\begin{cases} x_1 + x_2 + 3x_3 = -1 \\ 2x_1 - x_2 + 2x_3 = -1 \\ 4x_1 + x_2 + 4x_3 = 1 \end{cases}$ J:(1;1;-1)

19. $\begin{cases} 3x_1 - 3x_2 - x_3 = -3 \\ 3x_1 + 5x_2 + 4x_3 = -2 \\ 4x_1 - 2x_2 + 3x_3 = -13 \end{cases}$ J:(0;2;-3)

20. $\begin{cases} 3x_1 + 4x_2 + 2x_3 = 0 \\ 2x_1 - x_2 - 3x_3 = -6{,}5 \\ x_1 + 5x_2 + x_3 = 3{,}5 \end{cases}$ J:(-2;1;0,5)

21. $\begin{cases} 3x_1 + x_2 - x_3 = -2 \\ x_1 - 2x_2 + 2x_3 = 11 \\ 4x_1 - x_2 + x_3 = 9 \end{cases}$ J:(1;-3;2)

22. $\begin{cases} x_1 - 4x_2 - 2x_3 = 7 \\ 3x_1 + x_2 - x_3 = 16 \\ -3x_1 + 5x_2 - 6x_3 = -9 \end{cases}$ J:(5;0;-1)

23. $\begin{cases} 3x_1 + 2x_2 - x_3 = 4 \\ x_1 - 3x_2 + 2x_3 = 1 \\ 4x_1 - x_2 + x_3 = 6 \end{cases}$ J:$\emptyset$

24. $\begin{cases} x_1 + 2x_2 + x_3 = 4 \\ 2x_1 - 3x_2 - x_3 = 1 \\ 3x_1 - x_2 = 0 \end{cases}$ J:$\emptyset$

25. $\begin{cases} x_1 + 5x_2 + x_3 = -2 \\ 2x_1 - 4x_2 - 3x_3 = 0 \\ 3x_1 + x_2 - 2x_3 = 3 \end{cases}$ J:$\emptyset$

26. $\begin{cases} 3x_1 + x_2 - x_3 = -2 \\ 4x_1 - x_2 + x_3 = 9 \end{cases}$ J:U.Ye $\begin{cases} x_1 = 1 \\ x_2 = x_3 - 5 \end{cases}$;

27. $\begin{cases} -x_1 + x_2 - x_3 = -2 \\ x_1 - 2x_2 + 2x_3 = 9 \\ 2x_1 - 3x_2 + 3x_3 = 11 \end{cases}$ J:U.Ye $\begin{cases} x_1 = -5 \\ x_2 = x_3 - 7 \end{cases}$;

28. $\begin{cases} 3x_1 + 2x_2 - x_3 = 0 \\ x_1 - 2x_2 + x_3 = 4 \\ 2x_1 + 4x_2 - 2x_3 = -4 \end{cases}$ J:U.Ye $\begin{cases} x_1 = 1 \\ x_3 = 2x_2 + 3 \end{cases}$;

29. $\begin{cases} x_1 - x_2 + 2x_3 = 9 \\ 2x_1 - 3x_2 - 3x_3 = -2 \end{cases}$ J:U.Ye $\begin{cases} x_1 = -9x_3 + 29 \\ x_2 = -7x_3 + 20 \end{cases}$;

30. $\begin{cases} x_1 + x_2 - x_3 = -2 \\ x_1 - 2x_2 + 2x_3 = 9 \\ 5x_1 - 2x_2 - 2x_3 = 0 \end{cases}$ J:∅

Berilgan chiziqli tenglamalar sistemasini matritsalar usuli bilan yeching;

31. $\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 3 \\ x_1 + x_2 + 3x_3 = -1 \end{cases}$ j:(2;0;-1)

32. $\begin{cases} x_1 + 2x_2 - x_3 = 8 \\ 2x_1 + x_2 + 5x_3 = 8 \\ 3x_1 + 2x_2 + 4x_3 = 11 \end{cases}$ j:(-1;5;1)

33. $\begin{cases} 4x_1 - x_2 + 2x_3 = 6 \\ 2x_1 + 5x_2 - 3x_3 = 7 \\ x_1 + x_2 + 2x_3 = 0 \end{cases}$ j:(2;0;-1)

34. $\begin{cases} x_1 + x_2 - x_3 = -1 \\ 2x_1 + 8x_2 + 5x_3 = -3 \\ 3x_1 + 9x_2 = 0 \end{cases}$ j:(-3;1;-1)

35. $\begin{cases} 2x_1 - x_2 - x_3 = 2 \\ 3x_1 + 4x_2 - 2x_3 = 6 \\ x_1 - 2x_2 + 4x_3 = -2 \end{cases}$ j:(1;0,5;-0,5)

36. $\begin{cases} x_1 + x_2 + 3x_3 = 7 \\ 2x_1 + 6x_2 - x_3 = 2 \\ 3x_1 + 7x_2 + 2x_3 = 9 \end{cases}$ j:yechim cheksiz ko’p

37. $\begin{cases} x_1 + x_2 - x_3 = 1 \\ 2x_1 + 3x_2 - 6x_3 = 2 \\ -x_1 - 2x_2 + 5x_3 = -3 \end{cases}$ J:∅

38. $\begin{cases} 2x_1 - x_2 + 3x_3 = 3 \\ x_1 + 3x_2 - 2x_3 = 2 \\ -3x_1 + 5x_2 - 8x_3 = 8 \end{cases}$ J:∅

39. $\begin{cases} 2x_1 - 5x_2 = 3 \\ x_1 + 11x_3 = -4 \\ 3x_1 - 5x_2 + 11x_3 = -1 \end{cases}$ j:yechim cheksiz ko’p

40. $\begin{cases} 5x_1 - x_2 + 3x_3 = 2 \\ -x_1 + 3x_2 - 2x_3 = 1 \\ 3x_1 + 5x_2 - x_3 = 4 \end{cases}$ j:yechim cheksiz ko’p

Bir jinsli tenglamalar sistemasini yeching.

41. $\begin{cases} x_1 - 2x_2 + 3x_3 = 0 \\ 2x_1 + 3x_2 - x_3 = 0 \\ x_1 + 4x_2 - 2x_3 = 0 \end{cases}$ J:(0;0;0)

42. $\begin{cases} 4x_1 - x_2 + 3x_3 = 0 \\ 2x_1 + 3x_2 - 5x_3 = 0 \\ x_1 - 2x_2 + 4x_3 = 0 \end{cases}$ J: umumiy yechim ($x_1 = -\frac{2}{7}x_3;\ \ x_2 = 1\frac{6}{7}x_3$).

43. $\begin{cases} x_1 - 4x_2 + 3x_3 = 0 \\ x_1 - x_2 - x_3 = 0 \\ 3x_1 + 3x_2 - 4x_3 = 0 \end{cases}$ J:(0;0;0)

44. $\begin{cases} 3x_1 + 2x_2 - x_3 = 0 \\ 4x_1 + x_2 - 2x_3 = 0 \\ x_1 - x_2 - x_3 = 0 \end{cases}$ J:(0;0;0)

45. $\begin{cases} x_1 - x_2 + 5x_3 = 0 \\ 2x_1 + 3x_2 - x_3 = 0 \\ 3x_1 + 2x_2 + 4x_3 = 0 \end{cases}$ J: umumiy yechim ($x_1 = -2\frac{4}{5}x_3; x_2 = 2\frac{1}{5}x_3$).

46. $\begin{cases} 2x_1 - x_2 - 2x_3 = 0 \\ x_1 - 4x_2 + 3x_3 = 0 \end{cases}$ J: umumiy yechim ($x_1 = 1\frac{4}{7}x_3; x_2 = 1\frac{1}{7}x_3$).

47. $\begin{cases} x_1 + 2x_2 + x_3 = 0 \\ 4x_1 - x_2 + 3x_3 = 0 \end{cases}$ J: umumiy yechim ($x_1 = 7x_2; x_3 = -9x_2$).

48. $\begin{cases} 2x_1 + 7x_2 - 11x_3 = 0 \\ 4x_1 + x_2 - 3x_3 = 0 \\ x_1 - 3x_2 + 4x_3 = 0 \end{cases}$ J: umumiy yechim ($x_1 = \frac{5}{13}x_3; x_2 = 1\frac{6}{13}x_3$).

49. $\begin{cases} x_1 - 4x_2 + 3x_3 = 0 \\ 3x_1 - 2x_2 - x_3 = 0 \end{cases}$ J: umumiy yechim ($x_1 = x_3; x_2 = x_3$).

50. $\begin{cases} 3x_1 + 5x_2 - x_3 = 0 \\ 2x_1 - 3x_2 + 3x_3 = 0 \\ x_1 + 8x_2 - 4x_3 = 0 \end{cases}$ J: umumiy yechim ($x_1 = -\frac{12}{19}x_3; x_2 = \frac{11}{19}x_3$).

Mustaqil yechish uchun misollar.

CHiziqli tenglamalar sistemasini Kramer usuli, Gauss usuli va matritsalar usuli bilan yeching. Yechimni tekshiring:

1. $\begin{cases} 2x + 6y + 5z = 1 \\ 5x + 3y - 2z = 0 \\ 7x + 4y - 3z = 2. \end{cases}$

2. $\begin{cases} 3x + 2y + 3z = -2 \\ -4x - 3y - 5z = 1 \\ 5x + y - z = 3. \end{cases}$

3. $\begin{cases} 2x + 3y + z = 0 \\ 7x + 9y + 5z = -3 \\ 3x + 4y + 3z = 5. \end{cases}$

4. $\begin{cases} x + 2y + 3z = 3 \\ 3x + z = 9 \\ 2x + 4y + 5z = 6. \end{cases}$

5. $\begin{cases} 5x + 2y + 3z = 1 \\ x + 2y = 1 \\ 3x + 4y + 7z = 1. \end{cases}$

6. $\begin{cases} x + 2y + 2z = 10 \\ 2x + y - 2z = 1 \\ 2x - 2y + z = 2. \end{cases}$

7. $\begin{cases} x + 2y + 3z = 2 \\ 4x + z = 1 \\ 6x + 2y + 5z = 2. \end{cases}$

8. $\begin{cases} x + 2y - z = 2 \\ x + 3y - 2z = 3 \\ x + 5y + z = 4. \end{cases}$

9. $\begin{cases} 6x + 5y + 2z = 5 \\ 3x - 2y + 5z = 1 \\ 4x - 3y + 7z = 2. \end{cases}$

10. $\begin{cases} 2y - z = 12 \\ 2x + y - 2z = 15 \\ -3x + 2y + z = 1. \end{cases}$

11. $\begin{cases} 2x + y - z = 5 \\ 3x + y - 2z = 10 \\ 5x + y + z = 5. \end{cases}$

12. $\begin{cases} x + 3y + 2z = -3 \\ 4x + y = 5 \\ 6x + 5y + 2z = 3. \end{cases}$

13. $\begin{cases} 3x + 3y + 2z = 0, \\ -5x - 3y - 3z = 7 \\ -x + 5y + z = 1. \end{cases}$

14. $\begin{cases} 3x + 2y + z = -1 \\ x + 4z = 1 \\ 5x + 2y + 6z = 0. \end{cases}$

15. $\begin{cases} -2x + y + 8z = 2 \\ 5x + 3y + 2z = 3 \\ 6x + y + z = 1. \end{cases}$

16. $\begin{cases} 6x + 2y + 5z = 2 \\ 3x + 5y - 2z = 1 \\ 4x + 7y - 3z = 1. \end{cases}$

17. $\begin{cases} 2x + y + 3z = 6 \\ 7x + 5y + 9z = 3 \\ 3x + 3y + 4z = 10. \end{cases}$

18. $\begin{cases} 2x + y + 3z = -6 \\ 3y + z = 12 \\ 4x + 2y + 5z = 3. \end{cases}$

19. $\begin{cases} 2x + 3y + 3z = -2 \\ -3x - 4y - 5z = 3 \\ x + 5y - z = 1. \end{cases}$

20. $\begin{cases} 3x + 2y + 5z = 7 \\ 2y + z = -1 \\ 7x + 4y + 3z = 1. \end{cases}$

21. $\begin{cases} -x + 2y = 6 \\ -2x + y + 2z = 7 \\ x + 2y - 3z = 2. \end{cases}$

22. $\begin{cases} -x + y + 2z = 3, \\ -2x + y + 3z = 3 \\ x + y + 5z = 8. \end{cases}$

23. $\begin{cases} 2x + 6y + 5z = 1 \\ 5x + 3y - 2z = 0 \\ 7x + 4y - 3z = 2. \end{cases}$

24. $\begin{cases} 2x + 2y + z = 27 \\ -2x + y + 2z = 9 \\ x - 2y + 2z = 18. \end{cases}$

25. $\begin{cases} -x + 2z = 1 \\ -2x + 2y + z = 4 \\ x - 3y + 2z = -6. \end{cases}$

26. $\begin{cases} x + 4y - 3z = 2 \\ 3y - 3z = 1 \\ 4x + 5y - 2z = 0. \end{cases}$

27. $\begin{cases} 4x - 2y + 4z = 7 \\ 3x + z = 2 \\ 4x - 2y + z = 2. \end{cases}$

28. $\begin{cases} 2x + 4y + 3z = -4 \\ x + 4y = 3 \\ x - y + 4z = 2. \end{cases}$

29. $\begin{cases} 6x + 2y + 3z = -3 \\ 4x + 3z = 1 \\ 4x - 3y + z = 4. \end{cases}$

30. $\begin{cases} 2x - 3y - 4z = 1 \\ 5x + 2z = 3 \\ 3x - 4y - 2z = 7. \end{cases}$

Matritsali tenglamani yeching:

1. $\begin{pmatrix} 1 & 1 & 1 \\ 4 & -1 & 2 \\ -4 & 0 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ -1 & 6 \end{pmatrix}$

2. $\begin{pmatrix} 5 & 2 & -1 \\ 1 & -1 & -2 \\ 1 & 0 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} -1 & 2 \\ -2 & -3 \\ 4 & -1 \end{pmatrix}$

3. $\begin{pmatrix} -1 & 3 & 3 \\ 1 & -2 & 2 \\ -1 & 4 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$

4. $X \cdot \begin{pmatrix} -1 & 5 & 3 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 5 & 7 \end{pmatrix}$

5. $X \cdot \begin{pmatrix} 1 & -1 & 2 \\ -1 & 2 & -3 \\ 2 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 1 & 5 \end{pmatrix}$

6. $X \cdot \begin{pmatrix} 1 & 3 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 3 & 1 \\ -3 & 2 & -1 \end{pmatrix}$

7. $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$

8. $X \cdot \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}.$

9. $\begin{pmatrix} 3 & -1 & 2 \\ 4 & -1 & 1 \\ 1 & 3 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 9 & 7 \\ 1 & 11 & 7 \\ 7 & 5 & 7 \end{pmatrix}.$

10. $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ -1 & -2 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}.$

11. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} \cdot X = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$

12. $\begin{pmatrix} 4 & 2 & -1 \\ 5 & 3 & -2 \\ 3 & 2 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & 2 & 3 & 2 \\ 5 & 3 & 3 & 3 \\ 3 & 2 & 2 & 2 \end{pmatrix}$.

13. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & 1 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 10 \\ 8 \\ -1 \end{pmatrix}$.

14. $X \cdot \begin{pmatrix} 3 & 2 & 3 \\ -1 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 0 & 2 \end{pmatrix}$.

15. $X \cdot \begin{pmatrix} 5 & 6 & 3 \\ 0 & 1 & 0 \\ 7 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 26 & 20 & 18 \end{pmatrix}$.

16. $\begin{pmatrix} 2 & 0 & 3 \\ 7 & 1 & 6 \\ 6 & 0 & 5 \end{pmatrix} \cdot X = \begin{pmatrix} 5 & 2 \\ 12 & 7 \\ 11 & 6 \end{pmatrix}$.

17. $\begin{pmatrix} 3 & 2 & -5 \\ 2 & 6 & -10 \\ 1 & 2 & -3 \end{pmatrix} \cdot X = \begin{pmatrix} 12 & 5 \\ 24 & 8 \\ 8 & 3 \end{pmatrix}$.

18. $X \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ -1 & 0 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & 3 & 13 \end{pmatrix}$.

19. $\begin{pmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} 9 & 7 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$.

20. $X \cdot \begin{pmatrix} 4 & 3 & 8 \\ 5 & 0 & 2 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 29 & 16 & 49 \end{pmatrix}$.

21. $\begin{pmatrix} 2 & 3 & -4 \\ 1 & -1 & 1 \\ 5 & -4 & -2 \end{pmatrix} \cdot X = \begin{pmatrix} 12 \\ -2 \\ -1 \end{pmatrix}$.

22. $X \cdot \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ 2 & 3 & 4 \end{pmatrix}$.

23. $X \cdot \begin{pmatrix} 3 & 3 & 2 \\ 3 & 8 & 3 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$.

24. $\begin{pmatrix} 3 & 2 & -5 \\ 2 & 6 & -10 \\ 1 & 5 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} -4 & 2 \\ 3 & 5 \\ -1 & 2 \end{pmatrix}$.

25. $\begin{pmatrix} 6 & -1 & 8 \\ -1 & 4 & -3 \\ 5 & -1 & 7 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & -2 & 1 \\ 0 & 2 & 3 \\ -2 & 0 & -1 \end{pmatrix}.$

26. $\begin{pmatrix} -2 & 3 & -2 \\ -3 & -1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & 4 \\ -1 & 4 \\ 1 & 0 \end{pmatrix}.$

27. $\begin{pmatrix} 1 & 0 & 3 \\ 3 & -4 & -1 \\ -4 & 3 & -2 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 7 & 1 \\ 0 & -3 \end{pmatrix}.$

28. $X \cdot \begin{pmatrix} -2 & 3 & -2 \\ 1 & -1 & 1 \\ 3 & 5 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 3 \\ -1 & 1 & 0 \end{pmatrix}.$

29. $X \cdot \begin{pmatrix} 3 & 4 & 1 \\ -1 & -1 & -1 \\ 3 & 7 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 3 \\ -1 & 1 & 0 \end{pmatrix}.$

30. $X \cdot \begin{pmatrix} 6 & 3 & 1 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 4 & 8 \end{pmatrix}.$

Chiziqli tenglamalar sistemasini yeching yoki birgalikda emasligini isbotlang:

1. $\begin{cases} x_1 - 3x_2 + 4x_3 - x_4 = 1 \\ 7x_1 + 3x_2 - 5x_3 + 5x_4 = 10 \\ 2x_1 + 2x_2 - 3x_3 + 2x_4 = 3. \end{cases}$

2. $\begin{cases} 4x_1 + 2x_2 + x_3 = 7 \\ x_1 - x_2 + x_3 = -2 \\ 2x_1 + 3x_2 - 3x_3 = 11 \\ 4x_1 + x_2 - x_3 = 7. \end{cases}$

3. $\begin{cases} x_1 - x_3 + x_4 = 3 \\ 2x_1 + 3x_2 - x_3 - x_4 = 2 \\ 5x_1 - 3x_4 = -6 \\ x_1 + x_2 + x_3 + x_4 = 2. \end{cases}$

4. $\begin{cases} 2x_1 + 3x_2 + 4x_3 + 6x_4 = 0 \\ 3x_2 + x_3 - 3x_4 = 7 \\ 2x_1 - x_2 + x_3 + x_4 = 2. \end{cases}$

5. $\begin{cases} x_1 + 2x_2 + 3x_3 = 6 \\ 2x_1 + 3x_2 + 4x_3 = 9 \\ 3x_1 + 4x_2 + 5x_3 = 12 \\ x_1 - x_2 - x_3 = -1. \end{cases}$

6. $\begin{cases} 3x_1 + 2x_2 - x_3 = 1 \\ x_1 + 3x_2 + 2x_3 = 5 \\ 5x_1 + 8x_2 + 3x_3 = 11 \\ x_1 + x_2 = 1. \end{cases}$

7. $\begin{cases} x_1 + 6x_2 + 3x_3 + x_4 = 7 \\ 2x_1 - 5x_2 + 4x_4 = 3 \\ x_1 + 2x_2 - 2x_3 = 6. \end{cases}$

8. $\begin{cases} 3x_1 + 2x_2 - 3x_3 + 4x_4 = 1 \\ 2x_1 + 3x_2 - 2x_3 + 3x_4 = 2 \\ 4x_1 + 2x_2 - 3x_3 + 2x_4 = 3. \end{cases}$

9. $\begin{cases} 3x_1 - 2x_2 + x_3 - x_4 = 0 \\ 3x_1 - 2x_2 - x_3 + x_4 = 1 \\ x_1 - x_2 + 2x_3 + 5x_4 = 3. \end{cases}$

10. $\begin{cases} 2x_1 + x_2 - x_3 - x_4 = 1 \\ x_1 - x_2 + x_3 + x_4 = 0 \\ 3x_1 + 3x_2 - 3x_3 - 3x_4 = 2 \\ 4x_1 + 5x_2 - 5x_3 - 5x_4 = 3. \end{cases}$

11. $\begin{cases} x_1 - 2x_2 - 3x_3 = -3 \\ x_1 + 3x_2 - 5x_3 = 0 \\ -x_1 + 4x_2 + x_3 = 3 \\ 3x_1 + x_2 - 13x_3 = 7. \end{cases}$

12. $\begin{cases} 4x_1 - x_2 + 7x_4 = 2 \\ 4x_1 - 3x_2 + x_3 - x_4 = 2 \\ x_1 - 3x_2 + x_3 = 4. \end{cases}$

13. $\begin{cases} x_1 - 7x_2 + x_3 + 6x_4 = -2 \\ 4x_2 - 3x_3 + 4x_4 = 7 \\ 2x_1 - x_2 + x_3 + x_4 = 2. \end{cases}$

14. $\begin{cases} 2x_1 + x_2 - x_3 = 3 \\ x_1 + x_2 + x_3 = 2 \\ 3x_1 + x_2 + 5x_3 = 4 \\ x_1 - x_2 + x_3 = 0 \\ 5x_1 + 2x_2 - x_3 = 7. \end{cases}$

15. $\begin{cases} 2x_1 - x_2 + x_4 = 4 \\ x_1 + 3x_2 + 5x_3 + 3x_4 = 8 \\ -3x_1 + x_2 + 3x_3 = 1 \\ 3x_2 + 8x_3 + 5x_4 = 13. \end{cases}$

16. $\begin{cases} 2x_1 + 4x_2 - 5x_3 = 7 \\ x_1 + 3x_3 = 4 \\ x_1 + x_2 + 6x_3 = 7 \\ 2x_1 + 5x_2 - 2x_3 = 10. \end{cases}$

17. $\begin{cases} 2x_1 - 3x_2 - x_3 + 2x_4 = 3 \\ x_1 + x_2 + x_3 - x_4 = -8 \\ x_1 - 3x_2 + 5x_3 + 7x_4 = 4. \end{cases}$

18. $\begin{cases} x_1 + 2x_2 - 3x_3 + 4x_4 = 7 \\ 2x_1 + 4x_2 + 5x_3 - x_4 = 2 \\ x_1 + x_2 + 7x_3 + 2x_4 = 11. \end{cases}$

19. $\begin{cases} 3x_1 - x_2 + x_3 = 6 \\ x_1 - 5x_2 + x_3 = 12 \\ 2x_1 + 4x_2 = -6 \\ 2x_1 + x_2 + 3x_3 = 3 \\ 5x_1 + 4x_3 = 9. \end{cases}$

20. $\begin{cases} x_1 + 2x_2 - 4x_3 = 1 \\ 2x_1 + x_2 - 5x_3 = -1 \\ x_1 - x_2 - x_3 = -2 \\ 4x_1 + 5x_2 - 13x_3 = 1. \end{cases}$

21. $\begin{cases} 2x_1 + x_2 - 5x_3 = 7 \\ x_1 - 8x_3 = 4 \\ 3x_1 + 5x_2 + x_3 = 7 \\ x_1 + x_2 - 2x_3 = 3. \end{cases}$

22. $\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = 1 \\ 2x_1 - x_2 + x_3 + 2x_4 = 2 \\ 4x_1 + 3x_2 - 5x_3 + 2x_4 = 4 \\ 7x_1 + 4x_2 - 7x_3 + 5x_4 = 7. \end{cases}$

23. $\begin{cases} x_1 + 6x_3 = -5 \\ 4x_1 + 3x_2 - x_3 = 5 \\ x_1 + x_2 + 4x_3 = -4 \\ 2x_1 + x_2 + 10x_3 = -9 \\ 3x_1 + 2x_2 + 14x_3 = -13. \end{cases}$

24. $\begin{cases} x_1 + x_2 - 3x_3 = 0 \\ x_1 + x_2 - x_3 + 2x_4 = 1 \\ 2x_1 + 2x_2 + x_3 - x_4 = 0. \end{cases}$

25. $\begin{cases} x_1 + 2x_2 + 3x_3 = 6 \\ 2x_1 - 3x_2 + x_3 = 0 \\ 3x_1 - 2x_2 + 4x_3 = 5 \\ x_1 - x_2 + 3x_3 = 3. \end{cases}$

26. $\begin{cases} 2x_1 - x_2 + 4x_4 = 9 \\ x_1 + 4x_3 - x_4 = 3 \\ x_1 + 2x_2 - 4x_3 + 5x_4 = 4. \end{cases}$

27. $\begin{cases} 2x_1 - x_2 + 4x_4 = 2 \\ 3x_1 - x_3 - x_4 = -5 \\ x_1 + 2x_2 - 4x_3 + 6x_4 = 4. \end{cases}$

28. $\begin{cases} x_1 + 3x_3 = 2 \\ -2x_1 + 4x_2 + 3x_3 = 1 \\ 3x_1 - 4x_2 = 1 \\ 8x_1 + 6x_2 + x_3 = 0. \end{cases}$

29. $\begin{cases} 4x_1 + 10x_2 - 3x_3 = 2 \\ x_2 + x_3 = 4 \\ 3x_1 + 3x_2 - 4x_3 = 2 \\ x_1 + 7x_2 + x_3 = 0. \end{cases}$

30. $\begin{cases} x_1 + 4x_2 - x_3 = 6 \\ x_1 + 2x_2 + 3x_3 + x_4 = -2 \\ 3x_1 + 4x_2 - 5x_3 + x_4 = 2. \end{cases}$

Bir jinsli chiziqli tenglamalar sistemasini umumiy yechimini toping:

1. $$\begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 = 0; \\ 2x_1 - 4x_2 + 5x_3 + 7x_4 = 0; \\ 6x_1 - 12x_2 + 17x_3 - 9x_4 = 0; \\ 7x_1 - 14x_2 + 18x_3 + 17x_4 = 0. \end{cases}$$

2. $$\begin{cases} 2x_1 + 3x_2 - x_3 - 5x_4 = 0; \\ 4x_1 + 6x_2 + 2x_3 - x_4 = 0; \\ 2x_1 + 3x_2 - 5x_3 - 14x_4 = 0; \\ 10x_1 + 15x_2 + 3x_3 - 7x_4 = 0. \end{cases}$$

3. $$\begin{cases} 14x_1 + 35x_2 - 7x_3 - 63x_4 = 0; \\ -10x_1 - 25x_2 + 5x_3 + 45x_4 = 0; \\ 26x_1 + 65x_2 - 13x_3 - 117x_4 = 0. \end{cases}$$

4. $$\begin{cases} 9x_1 + 21x_2 - 15x_3 + 5x_4 = 0; \\ 12x_1 + 28x_2 - 20x_3 + 7x_4 = 0. \end{cases}$$

5. $$\begin{cases} x_1 + 4x_2 + 2x_3 - 3x_5 = 0; \\ 2x_1 + 9x_2 + 5x_3 + 2x_4 + x_5 = 0; \\ x_1 + 3x_2 + x_3 - 2x_4 - 9x_5 = 0. \end{cases}$$

6. $$\begin{cases} 3x_1 + 4x_2 + 3x_3 + 2x_4 = 0; \\ 5x_1 + 7x_2 + 4x_3 + 3x_4 = 0; \\ 4x_1 + 5x_2 + 5x_3 + 3x_4 = 0; \\ 5x_1 + 6x_2 + 7x_3 + 4x_4 = 0. \end{cases}$$

7. $$\begin{cases} x_1 + 3x_2 + 2x_3 = 0; \\ 2x_1 - x_2 + 3x_3 = 0; \\ 3x_1 - 5x_2 + 4x_3 = 0; \\ x_1 + 17x_2 + 4x_3 = 0. \end{cases}$$

8. $$\begin{cases} x_1 - 2x_2 + x_3 + x_4 - x_5 = 0; \\ 2x_1 + x_2 - x_3 - x_4 + x_5 = 0; \\ x_1 + 7x_2 - 5x_3 - 5x_4 + 5x_5 = 0; \\ 3x_1 - x_2 - 2x_3 + x_4 - x_5 = 0. \end{cases}$$

9. $$\begin{cases} x_1 + x_2 - 3x_4 - x_5 = 0; \\ x_1 - x_2 + 2x_3 - x_4 = 0; \\ 4x_1 - 2x_2 + 6x_3 + 3x_4 - 4x_5 = 0; \\ 2x_1 + 4x_2 - 2x_3 + 4x_4 - 7x_5 = 0. \end{cases}$$

10. $$\begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0; \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0; \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0; \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0. \end{cases}$$

11. $\begin{cases} 2x_1 - 4x_2 + 5x_3 + 3x_4 = 0; \\ 3x_1 - 6x_2 + 4x_3 + 2x_4 = 0; \\ 4x_1 - 8x_2 + 17x_3 + 11x_4 = 0. \end{cases}$

12. $\begin{cases} 3x_1 + 5x_2 + 2x_3 = 0; \\ 4x_1 + 7x_2 + 5x_3 = 0; \\ x_1 + x_2 - 4x_3 = 0; \\ 2x_1 + 9x_2 + 6x_3 = 0. \end{cases}$

13. $\begin{cases} 2x_1 - x_2 + 5x_3 + 7x_4 = 0; \\ 4x_1 - 2x_2 + 7x_3 + 5x_4 = 0; \\ 2x_1 - x_2 + x_3 - 5x_4 = 0. \end{cases}$

14. $\begin{cases} 3x_1 + 2x_2 + x_3 + 3x_4 + 5x_5 = 0; \\ 6x_1 + 4x_2 + 3x_3 + 5x_4 + 7x_5 = 0; \\ 9x_1 + 6x_2 + 5x_3 + 7x_4 + 9x_5 = 0. \end{cases}$

15. $\begin{cases} 5x_1 + 6x_2 - 2x_3 + 7x_4 + 4x_5 = 0; \\ 2x_1 + 3x_2 - x_3 + 4x_4 + 2x_5 = 0; \\ 5x_1 + 9x_2 - 3x_3 + x_4 + 6x_5 = 0. \end{cases}$

16. $\begin{cases} 3x_1 + 4x_2 + x_3 + 2x_4 + 3x_5 = 0; \\ 5x_1 + 7x_2 + x_3 + 3x_4 + 4x_5 = 0; \\ 4x_1 + 5x_2 + 2x_3 + x_4 + 5x_5 = 0. \end{cases}$

17. $\begin{cases} 3x_1 + 4x_2 - 5x_3 + 7x_4 = 0; \\ 2x_1 - 3x_2 + 3x_3 - 2x_4 = 0; \\ 4x_1 + 11x_2 - 13x_3 + 16x_4 = 0; \\ 7x_1 - 2x_2 + x_3 + 3x_4 = 0. \end{cases}$

18. $\begin{cases} x_1 - 2x_2 + x_3 - x_4 + x_5 = 0; \\ 2x_1 + x_2 - x_3 + 2x_4 - 3x_5 = 0; \\ 3x_1 - 2x_2 - x_3 + x_4 - 2x_5 = 0; \\ 2x_1 - 5x_2 + x_3 - 2x_4 + 2x_5 = 0. \end{cases}$

19. $\begin{cases} 2x_1 + 3x_2 - x_3 + 5x_4 = 0; \\ 3x_1 - x_2 + 2x_3 - 7x_4 = 0; \\ 4x_1 + x_2 - 3x_3 + 6x_4 = 0; \\ x_1 - 2x_2 + 4x_3 - 7x_4 = 0. \end{cases}$

20. $\begin{cases} 3x_1 + 5x_2 + 2x_3 + 4x_4 = 0; \\ 5x_1 + 4x_2 + 3x_3 + 5x_4 = 0; \\ 9x_1 + 2x_2 + 5x_3 + 7x_4 = 0; \\ 5x_1 - 9x_2 + 2x_3 = 0. \end{cases}$

21. $\begin{cases} 3x_1 + 3x_2 + 5x_3 + 7x_4 + 4x_5 = 0; \\ 2x_1 + 2x_2 + 3x_3 + 5x_4 + 3x_5 = 0; \\ 4x_1 + 4x_2 + 7x_3 + 9x_4 + 5x_5 = 0; \\ 5x_1 + 5x_2 + 9x_3 + 11x_4 + 6x_5 = 0. \end{cases}$

22. $\begin{cases} 2x_1 + x_2 + 4x_3 + x_4 = 0; \\ 3x_1 + 2x_2 - x_3 - 6x_4 = 0; \\ 7x_1 + 4x_2 + 6x_3 - 5x_4 = 0; \\ x_1 + 8x_3 + 7x_4 = 0. \end{cases}$

23. $\begin{cases} 2x_1 - 2x_2 + 3x_3 + 6x_4 + 5x_5 = 0; \\ -4x_1 + 5x_2 - 7x_3 - 3x_4 + 8x_5 = 0; \\ -6x_1 + 7x_2 - 10x_3 - 9x_4 + 3x_5 = 0; \\ 8x_1 - 9x_2 + 13x_3 + 15x_4 + 2x_5 = 0. \end{cases}$

24. $\begin{cases} 2x_1 - x_2 - x_3 - x_4 - x_5 = 0; \\ -x_1 + 2x_2 - x_3 - x_4 - x_5 = 0; \\ 4x_1 + x_2 - 5x_3 - 5x_4 - 5x_5 = 0; \\ x_1 + x_2 + 2x_3 + x_4 + x_5 = 0; \\ x_1 + x_2 + x_3 + 2x_4 + x_5 = 0. \end{cases}$

25. $\begin{cases} 3x_1 + 6x_2 + 10x_3 + 4x_4 - 2x_5 = 0; \\ 6x_1 + 10x_2 + 17x_3 + 7x_4 - 3x_5 = 0; \\ 9x_1 + 3x_3 + 2x_4 + 3x_5 = 0; \\ 12x_1 - 2x_2 + x_3 + 8x_4 + 5x_5 = 0. \end{cases}$

26. $\begin{cases} x_1 + x_2 + x_3 + 2x_4 + x_5 = 0; \\ x_1 - 2x_2 - 3x_3 + x_4 - x_5 = 0; \\ 2x_1 - x_2 - 2x_3 + 3x_4 = 0. \end{cases}$

27. $\begin{cases} 3x_1 + 3x_2 + 5x_3 + 7x_4 + 4x_5 = 0; \\ 2x_1 + 2x_2 + 3x_3 + 5x_4 + 3x_5 = 0; \\ 4x_1 + 4x_2 + 7x_3 + 9x_4 + 5x_5 = 0; \\ 5x_1 + 5x_2 + 9x_3 + 11x_4 + 6x_5 = 0. \end{cases}$

28. $\begin{cases} 2x_1 - 5x_2 + 4x_3 + 3x_4 = 0; \\ 3x_1 - 4x_2 + 7x_3 + 5x_4 = 0; \\ 4x_1 - 9x_2 + 8x_3 + 5x_4 = 0; \\ -3x_1 + 2x_2 - 5x_3 + 3x_4 = 0. \end{cases}$

29. $\begin{cases} x_1 + x_3 + x_5 = 0; \\ x_2 - x_4 + x_6 = 0; \\ x_1 - x_2 + x_5 - x_6 = 0; \\ x_2 + x_3 + x_6 = 0. \end{cases}$

30. $\begin{cases} x_1 + x_2 - 2x_3 + 3x_4 - 3x_5 = 0; \\ 2x_1 + 2x_2 + 3x_3 - x_4 + 4x_5 = 0; \\ 4x_2 + x_3 - x_4 + 2x_5 = 0; \\ x_1 + 2x_2 - 4x_3 + 5x_4 + 2x_5 = 0. \end{cases}$

VEKTORLAR.

1-§. Vektorlar. Vektorlar ustida chiziqli amallar.

1-ta'rif. Boshi A nuqtada, oxiri B nuqtada bo'lgan yo'naltirilgan kesma ***vektor*** deyiladi va u $\overrightarrow{AB}$ yoki $\vec{a}$ kabi belgilanadi.

Vektorning o`lchami uning koordinatalari (komponentalari) orqali aniqlanadi.

2-ta'rif. Koordinatasi (komponentasi) n ta bo'lgan vektor, n o'lchovli vektor deyiladi.

$$\vec{a} = (a_1, a_2, \dots, a_n) \qquad (1)$$

$n = 1; 2; 3$ bo`lganda geometrik vektorlar hosil bo`ladi, ya'ni ularni chizmada tasvirlash mumkin. $n > 3$ bo`lganda vektorni geometrik tasvirlab bo`lmaydi.

3-ta'rif. Ikkita vektorning o`lchamlari bir xil va mos koordinatalari teng bo`lsa, ular ***o`zaro teng vektorlar*** deyiladi.

4-ta'rif. Vektorning ***moduli yoki uzunligi*** deb, uning koordinatalari kvadratlari yig`indisidan chiqarilgan kvadrat ildizga aytiladi va quyidagicha belgilanadi:

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2} \qquad (2)$$

5-ta'rif. Barcha koordinatalari nollardan iborat bo`lgan vektor ***nol vektor*** deyiladi va u quyidagicha yoziladi:

$$\vec{0} = (0,0, \dots ,0) \qquad (3)$$

Bunday vektor tayin yo'nalishga ega emas, uning moduli nolga teng.

Uzunligi birga teng vektor birlik vektor deyiladi.

6-ta'rif. Bir to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda yotuvchi vektorlar ***kollinear vektorlar*** deyiladi.

Agar ikki vektor o'zaro kollinear, bir xil yo'nalgan va modullari teng bo'lsa, bu vektorlar ***teng*** vektorlar deyiladi.

7-ta'rif. Bir tekislikda yoki parallel tekisliklarda yotuvchi vektorlar ***komplanar vektorlar*** deyiladi.

Vektorlar ustida songa ko`paytirish, qo`shish va ayirish kabi chiziqli amallarni bajarish mumkin.

8-ta'rif. $\vec{a} = (a_1, a_2, \dots, a_n)$ vektorni λ haqiqiy songa ***ko`paytmasi*** deb,

$$\lambda\vec{a} = (\lambda a_1, \lambda a_2, \dots, \lambda a_n) \qquad (4)$$

vektorga aytiladi, ya'ni vektorni songa ko`paytirish uchun uning barcha koordinatalari shu songa ko`paytiriladi.

Agar $\lambda > 0$ bo'lsa, yo'nalish o'zgarmaydi, $\lambda < 0$ bo'lsa, yo'nalish qarama-qarshisiga o'zgaradi. Vektor uzunligi $|\lambda|$ marta ortadi.

9-ta'rif. $\vec{a} = (a_1, a_2, \dots, a_n)$ *va* $\vec{b} = (b_1, b_2, \dots, b_n)$ vektorlarning ***yig`indisi*** (ayirmasi) deb,

$$\vec{a} \pm \vec{b} = (a_1, a_2, \dots, a_n) \pm (b_1, b_2, \dots, b_n)$$
$$= (a_1 \pm b_1, a_2 \pm b_2, \dots, a_n \pm b_n) \qquad (5)$$

formula bilan aniqlanuvchi vektorga aytiladi.

$\vec{a}$ vektor OX o'q bilan φ burchak hosil qilsin. U holda vektorning bu o'qdagi proektsiyasi;

$$pr_x\vec{a} = |\vec{a}| \cdot cos\varphi \qquad (6)$$

formula bilan topiladi.

Bir nechta vektorlar yig'indisini o'qdagi proektsiyasi qo'shiluvchi vektorlar proektsiyalarining yig'indisiga teng:

$$pr_x(\vec{a} + \vec{b} + \dots + \vec{c}) = pr_x\vec{a} + pr_x\vec{b} + \dots + pr_x\vec{c} \qquad (7)$$

O`zaro perpendikulyar kesishuvchi uchta o`qlar, ularning kesishish nuqtasi bo`lgan koordinata boshi va birlik masshtabga ega bo`lgan tartiblangan sistema, fazoda to`g`ri burchakli ***dekart koordinatalar sistemasi*** deyiladi.

10-ta'rif. Nuqtaning radius-vektori $\overrightarrow{OM} = \vec{r}$ ning o'qlardagi $x = OM_1$, $y = OM_2$ va $z = OM_3$ proektsiyalari $\vec{r} = \overrightarrow{OM}$ vektorning ***to'g'ri burchakli koordinatalari*** deyiladi.

OX **−*abtsissa*,** OY **−*ordinata*** va OZ **−*applikata o`qlari*** deyiladi.

$\vec{r} = \overrightarrow{OM}$ radius-vektorning moduli yoki uzunligi:

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} \qquad (8)$$

formula bilan topiladi. Koordinata o'qlaridagi $i,\ j,\ k$ birlik vektorlar ***ortlar*** deyiladi. Radius-vektorlar ortlar orqali quyidagicha ifodalanadi.

$$\vec{r} = xi + yj + zk \qquad (9)$$

U holda $\vec{a} = xi + yj + zk$ vektorni $\boldsymbol{\lambda}$ **songa ko'paytmasi** deb;

$$\lambda\vec{a} = \lambda xi + \lambda yj + \lambda zk \qquad (10)$$

ga aytiladi.

$\vec{a} = x_1 i + y_1 j + z_1 k\ va\ \vec{b} = x_2 i + y_2 j + z_2 k$ vektorlarni yig'indisi (ayirmasi) deb,

$$\vec{a} \pm \vec{b} = (x_1 \pm x_2)i + (y_1 \pm y_2)j + (z_1 \pm z_2)k \qquad (11)$$

ga atiladi.

$A(x_1, y_1, z_1)$ va $B(x_2, y_2, z_2)$ nuqtalar berilgan. $\vec{a} = \overrightarrow{AB}$ vektorning koordinata o'qlaridagi proektsiyalari:

$$\begin{cases} a_x = pr_x\overrightarrow{AB} = X = x_2 - x_1 \\ a_y = pr_y\overrightarrow{AB} = Y = y_2 - y_1 \\ a_z = pr_z\overrightarrow{AB} = Z = z_2 - z_1 \end{cases} \qquad (12)$$

formula bilan topiladi.

Agar $\vec{a} = \overrightarrow{AB}$ vektor koordinata o'qlari bilan α, β va γ burchaklar tashkil etsa, u holda bu vektorning yo'naltiruvchi kosinuslari:

$$cos\alpha = \frac{x}{|\vec{a}|}; \qquad cos\beta = \frac{y}{|\vec{a}|}; \qquad cos\gamma = \frac{z}{|\vec{a}|} \qquad (13)$$

formula bilan topiladi.

Har qanday vektorning yo'naltiruvchi kosinuslari kvadratlarining yig'indisi 1 ga teng:

$$cos^2\alpha + cos^2\beta + cos^2\gamma = 1 \qquad (14)$$

2-§. Ikki vektorning skalyar ko'paytmasi.

1-ta'rif. Ikki vektorning ***skalyar ko`paytmasi*** deb, shu vektorlar modullarining ular orasidagi burchak kosinusi ko`paytmasiga aytiladi.

$\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko'paytmasi $\vec{a} \cdot \vec{b}$ ko'rinishda belgilanadi. Demak,

$$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot cos\varphi \qquad (1)$$

Endi *n* o`lchovli vektorlarning skalyar ko`paytmasiga ta'rif beramiz.

Agar vektorlar $\vec{a}=(a_1,a_2,\dots,a_n)$ va $\vec{b}=(b_1,b_2,\dots,b_n)$ koordinatalar ko'rinishida berilsa, skalyar ko'paytma;

$$\vec{a}\cdot\vec{b}=a_1\cdot b_1+a_2\cdot b_2+\cdots+a_n\cdot b_n \qquad (2)$$

formula bilan topiladi, ya'ni ikki vektorning skalyar ko`paytmasi shu vektorlar mos koordinatalari ko`paytmalarining yig`indisiga teng.

Skalyar ko`paytmaning xossalari.

1^0. $\vec{a}\cdot\vec{a}\geq 0$, agar $\vec{a}=0$ bo'lsa, $\vec{a}\cdot\vec{a}=0$ bo'ladi;

2^0. $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$-o'rin almashtirish qonuni;

3^0. $\left(\vec{a}+\vec{b}\right)\cdot\vec{c}=\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}$-taqsimot qonuni;

4^0. $(\lambda\cdot\vec{a})\cdot\vec{b}=\lambda\cdot(\vec{a}\cdot\vec{b})$-bu yerda $\lambda=const.$

5^0. Ortlarning skalyar ko'paytmasi:

$$i\cdot j=0,\quad j\cdot k=0,\quad i\cdot k=0,\quad i\cdot i=1,\quad j\cdot j=1,\quad k\cdot k=1$$

Ikki vektor orasidagi burchak:

$$cos\varphi=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{a_1\cdot b_1+a_2\cdot b_2+\cdots+a_n\cdot b_n}{\sqrt{a_1^2+a_2^2+\cdots+a_n^2}\cdot\sqrt{b_1^2+b_2^2+\cdots+b_n^2}} \qquad (3)$$

Parallellik sharti:

$$\frac{a_1}{b_1}=\frac{a_2}{b_2}=\cdots=\frac{a_n}{b_n} \qquad (4)$$

Perpendikulyarlik sharti:

$$a_1\cdot b_1+a_2\cdot b_2+\cdots+a_n\cdot b_n=0 \qquad (5)$$

3-§. Ikki vektorning vektor ko'paytmasi.

1-ta'rif. $\vec{a}$ va $\vec{\mathrm{b}}$ vektorlarning ***vektor ko'paytmasi*** deb, $\vec{\mathrm{c}}=\vec{a}\times\vec{\mathrm{b}}$ ko'rinishda belgilanuvchi va quyidagi shartlarni qanoatlantiruvchi $\vec{\mathrm{c}}$ vektorga aytiladi:

1. $\vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarga perpendikulyar;

2. $\vec{c}$ vektor uchidan qaraganda $\vec{a}$ vektordan $\vec{b}$ vektorga eng qisqa burilish soat strelkasi yo'nalishiga qarama-qarshi yo'nalishda ($\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlarning bunday joylashuvi o'ng uchlik deyiladi) bo'ladi;

3. $\vec{c}$ vektorning moduli $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogrammning yuziga teng, ya'ni $|\vec{c}| = S = |\vec{a}| \cdot |\vec{b}| \cdot sin\varphi$ ($\varphi - \vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak)

Vektor ko'paytmaning xossalari.

1^0. $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$.

2^0. $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ –taqsimot qonuni.

3^0. Ortlarning vektor ko'paytmasi:

$$i \times j = k, \quad j \times k = i, \quad k \times i = j, \quad j \times i = -k, \quad k \times j = -i,$$
$$i \times k = -j, \quad i \times i = 0, \quad j \times j = 0, \quad k \times k = 0 \qquad (6)$$

4^0. Agar vektorlar $\vec{a}(a_x, a_y, a_z)$ va $\vec{b}(b_x, b_y, b_z)$ ko'rinishda berilsa, u holda vektor ko'payma;

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \qquad (7)$$

ga teng bo'ladi.

5^0. $\vec{a}$ va $\vec{b}$ vektorlarga yasalgan parallelogrammning yuzi:

$$S_{par} = |\vec{a} \times \vec{b}| \qquad (8)$$

shu vektorlarga yasalgan uchburchak yuzi:

$$S_{\Delta} = \frac{1}{2}|\vec{a} \times \vec{b}| \qquad (9)$$

4-§. Uch vektorning aralash ko'paytmasi.

1-ta'rif. $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarning ***aralash ko'paytmasi*** deb $(\vec{a} \times \vec{b}) \cdot \vec{c}$ ko'rinishdagi ifodaga aytiladi.

Agar $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar o'zlarining koordinatalari bilan berilgan bo'lsa, u holda

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \qquad (1)$$

1^0. Aralash ko'paytmaning istalgan ikkita ko'paytuvchisining o'rinlari o'zaro almashtirilsa, ko'paymaning ishorasi qarama-qarshisiga o'zgaradi:

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = -(\vec{a} \times \vec{c}) \cdot \vec{b} = -(\vec{c} \times \vec{b}) \cdot \vec{a} \qquad (2)$$

2^0. Agar berilgan uchta vektorda ikkitasi o'zaro teng yoki parallel bo'lsa, aralash ko'paytma 0 ga teng bo'ladi.

3^0. Skalyar ko'paytma- $(\cdot)$ va vektor ko'paytma- $(\times)$ amallarining o'rinlarini almashtirish mumkin:

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c});$$

shuning uchun ham aralash ko'paymani $\vec{a}\vec{b}\vec{c}$ ko'rinishda, ya'ni qavslarni va ko'paytirish amallarini ko'rsatmasdan yozish qabul qilingan.

$\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarga yasalgan parallelepipedning hajmi:

$$V = |\vec{a}\vec{b}\vec{c}| \qquad (3)$$

$\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarga yasalgan piramidaning hajmi:

$$V_{pir} = \frac{1}{6}|\vec{a}\vec{b}\vec{c}| \qquad (4)$$

Agar $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar o'zaro komplanar bo'lsa, $\vec{a}\vec{b}\vec{c} = 0$, va aksincha $\vec{a}\vec{b}\vec{c} = 0$ bo'lsa, berilgan uch vektor komplanar bo'ladi.

5 §.Vektorlar sistemasi.

1-ta'rif. $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ vektorlarning ***chiziqli kombinatsiyasi*** deb,

$$\lambda_1\overrightarrow{a_1} + \lambda_2\overrightarrow{a_2} + \dots + \lambda_n\overrightarrow{a_n}$$

yig`indiga aytiladi. Bu yerda $\lambda_1; \lambda_2; \dots \lambda_n$ haqiqiy sonlar bo'lib, bu ***chiziqli kombinatsiyaning koeffitsiyentlari*** deyiladi.

2-ta'rif. $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ chekli sondagi vektorlar uchun kamida bittasi noldan farqli shunday $\lambda_1; \lambda_2; \dots \lambda_n$ sonlar topilsaki, ular uchun

$$\lambda_1\overrightarrow{a_1} + \lambda_2\overrightarrow{a_2} + \dots + \lambda_n\overrightarrow{a_n} = 0 \qquad (1)$$

tenglik bajarilsa, u holda berilgan $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ sistema ***chiziqli bog`langan sistema*** deyiladi.

3-ta'rif. Agar (3) tenglik faqat $\lambda_1 = \lambda_2 = \dots = \lambda_n = 0$ bo`lgandagina bajarilsa, u holda $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ sistema, ***chiziqli erkli yoki chiziqli bog`lanmagan sistema*** deyiladi.

4-ta'rif. Agar $\overrightarrow{\lambda_\iota}(i = 1, n)$ sonlar uchun

$$\vec{a} = \lambda_1\overrightarrow{a_1} + \lambda_2\overrightarrow{a_2} + \dots + \lambda_n\overrightarrow{a_n} \qquad (2)$$

tenglik bajarilsa, u holda $\vec{a}$ vektor $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ vektorlar orqali chiziqli ifodalanadi yoki $\vec{a}$ vektor $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ vektorlarning ***chiziqli kombinatsiyasidan iborat*** deyiladi.

Fazodagi chekli vektorlar sistemasining chiziqli bog`lanishi quyidagi xossalarga ega:

1^0. $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ vektorlar sistemasining:

a) kamida bitta vektori nol vektordan iborat bo`lsa;

b) qandaydir 2 ta vektori proportsional bo`lsa, bu sistema chiziqli bog`langan bo`ladi.

2^0. Agar $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ sistema chiziqli bog`langan bo`lsa, istalgan $\overrightarrow{b_1}; \overrightarrow{b_2}; \dots; \overrightarrow{b_k}$ sistema uchun

$$\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}; \overrightarrow{b_1}; \overrightarrow{b_2}; \dots; \overrightarrow{b_k} \qquad (3)$$

sistema ham chiziqli bog`langan bo`ladi.

3^0. Berilgan V fazoda $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ sistema chiziqli bog`lanmagan bo`lsa, uning har qanday qism sistemasi ham chiziqli bog`lanmagan bo`ladi.

4^0. $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ vektorlar sistemasining istalgan vektori bu sistema orqali chiziqli ifodalanadi, ya'ni

$$\overrightarrow{a_\iota} = 0 \cdot \overrightarrow{a_1} + 0 \cdot \overrightarrow{a_2} + \dots + 1 \cdot \overrightarrow{a_\iota} + 0 \cdot \overrightarrow{a_{\iota+1}} + \dots + 0 \cdot \overrightarrow{a_n}$$

5^0. $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$ vektorlar sistemasi chiziqli bog`langan bo`lish uchun, ulardan kamida bittasi qolganlari orqali chiziqli ifodalanishi zarur va yetarlidir.

5-ta'rif. Agar V vektorlar fazosining o`zaro chiziqli bog`lanmagan shunday

$$\overrightarrow{a_1}; \overrightarrow{a_2}; \dots; \overrightarrow{a_n}$$

vektorlar sistemasi mavjud bo`lsaki, bu vektorlar fazosining qolgan barcha vektorlari shu sistema orqali chiziqli ifodalansa, u holda $\overrightarrow{a_1}; \overrightarrow{a_2}; \dots ; \overrightarrow{a_n}$ vektorlar sistemasi V ***vektor fazoning bazisi*** deyiladi.

6-ta'rif. Chekli vektorlar sistemasining ***rangi*** deb undagi chiziqli bog`lanmagan vektorlarning maksimal soniga aytiladi.

7-ta'rif. Agar V vektor fazoning biror

$$\overrightarrow{a_1}; \overrightarrow{a_2}; \dots ; \overrightarrow{a_n} \qquad (4)$$

vektorlari sistemasining istalgan ikki vektorlari o`zaro ortogonal bo`lsa, u holda (4) sistema ***ortogonal vektorlar sistemasi*** deyiladi.

8-ta'rif. Agar ortogonal sistema qaralayotgan fazoning bazisi bo`lsa, bunday sistemaga ***ortogonal bazis*** deyiladi.

Misollar.

1. $\vec{a}(3;1;2)$ va $\vec{b}(0;-2;-3)$ vektorlar berilgan. $\vec{a}\ va\ \vec{b}$ vektorlarning yig'indisini toping.

2. $\vec{a}(2;1)$ va $\vec{b}(4;-3)$ vektorlar berilgan. $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlarni toping va geometrik tasvirlang.

3. $\vec{a}(1;-3)$ va $\vec{b}(-4;-1)$ vektorlar berilgan. $\vec{a}+\vec{b}$ vektorning uzunligini toping va geometrik tasvirlang.

4. $A(1;4)\ va\ B(-3;0)$ nuqtalar berilgan. $\overrightarrow{AB}$ vektorning uzunligini toping va geometrik tasvirlang.

5. $\vec{a}(-3;1;0)$ va $\vec{b}(2;5;-1)$ vektorlar berilgan. $\left|2\vec{a}-\vec{b}\right|$ ni topin.

6. $M(0;3;-4)$ nuqta yasalsin va uning radius-vektori uzunligi hamda yo'nalishini aniqlang.

7. $r = 2i + 3j - 6k$ vektor yasalsin va uning radius-vektorining uzunligi, yo'nalishi va yo'naltiruvchi kosinuslarini toping.

8. M nuqtaning radius-vektori OX o'qi bilan 45^0 va OY o'qi bilan 60^0 burchak tashkil etadi. Bu $\overrightarrow{OM}$ vektorning uzunligi $r = 6$ ga teng. Agar M nuqtaning

applikatasi manfiy bo'lsa, uning koordinatalarini aniqlang va $r = \overrightarrow{OM}$ vektorni $i;\ j;\ k$ ortlar orqali ifodalang.

9. $A(1;2;3)\ va\ B(3;-4;6)$ nuqtalar berilgan. $\vec{a} = \overrightarrow{AB}$ vektor va uning koordinata o'qlaridagi proyeksiyalarini toping. Uning uzunligi va yo'nalishini aniqlang. $\vec{a}$ vektorni koordinata o'qlari bilan tashkil etgan burchaklarini aniqlang.

10. $\overrightarrow{OA} = i + j\ va\ \overrightarrow{OB} = k - 3j$ vektorlarga yasalgan parallelogramning dioganal-larini toping.

11. $\vec{a}(-1;1;0)\ va\ \vec{b}(1;-2;2)$ vektorlar orasidagi burchakni toping.

12. Uchlari $A(2;-1;3),\ B(1;1;1)\ va\ C(0;0;5)$ nuqtalarda bo'lgan ABC uchburchakning burchaklarini toping.

13. Tekislikda uchlari $O(0;0), A(2;0)\ va\ B(1;-1)$ nuqtalarida bo'lgan uchburchak berilgan. Shu uchburchakning OB tomoni bilan OM medianasi orasidagi burchakni toping.

14. $\vec{a}(2;1;0)\ va\ \vec{b}(0;-2;1)$ vektorlarga yasalgan parallelogramm dioganallari orasidagi burchakni toping.

15. $\vec{a} = i + j + 2k\ \ va\ \vec{b} = i - j + 4k$ vektorlar berilgan. $pr_{\vec{b}}\vec{a}$ ni toping.

16. $(2i - j) \cdot j + (j - 2k) \cdot k + (i - 2k)^2$ ifodani soddalashtiring.

17. $A(-2;3;-4), B(3;2;5), C(1;-1;2)\ va\ D(3;2;-4)$ nuqtalar berilgan. $\vec{a} = \overrightarrow{AB}$ vektorning $\vec{c} = \overrightarrow{CD}$ vektordagi proyeksiyani toping.

18.Uchlari $A(-1;-2;4), B(-4;-2;0)\ va\ C(3;-2;1)$ nuqtalarda bo'lgan uchburchak berilgan. Uchburchakning B uchidagi tashqi burchagini toping.

19. Parallelogramning ketma-ket uchta $A(-3;-2;0), B(3;-3;1)\ va\ C(5;0;2)$ uchlari berilgan. Uning to'rtinchi uchi D hamda $\overrightarrow{AC}\ \ va\ \overrightarrow{BD}$ vektorlar orasidagi burchakni toping.

20. $m\ \ va\ n$ lar o'zaro 120^0 burchak tashkil etuvchi birlik vektorlar bo'lsa, $\vec{a} = 2m + 4n\ va\ \vec{b} = m - n$ vektorlar orasidagi burchakni toping.

21. $\vec{a} = 3i, \vec{b} = 2k$ bo'lsa, $\vec{c} = \vec{a} \times \vec{b}$ ni toping va shaklini chizing.

22. Uchlari $A(7;3;4), B(1;0;6)\ va\ C(4;5;-2)$ nuqtalarda bo'lgan uchburchakning yuzini toping

23. $\vec{a}(2;0;1)\ va\ \vec{b}(1;2;2)$ vektorlarga parallelogramm yasalgan. Parallelogrammning yuzi va balandligini toping.

24. Ushbu $i \times (j + k) - j \times (i + k) + k \times (i + j + k)$ ifodani soddalashtiring.

25. $\vec{a}\ va\ \vec{b}$ vektorlar o'zaro 45^0 burchak tashkil etadi. Agar $|\vec{a}| = |\vec{b}| = 5$ bo'lsa, $\vec{a} - 2\vec{b}\ va\ 3\vec{a} + 2\vec{b}$ vektorlarga yasalgan parallelogrammning yuzini toping.

26. $\vec{a} = -j + k\ \ va\ \ \vec{b} = i + j + k$ vektorlarga yasalgan uchburchakning yuzini toping.

27. $m\ va\ n$ o'zaro 30^0 burchak tashkil etuvchi birlik vektorlar bo'lsa, $\vec{a} = m + 2n$

$va\ \ \vec{b} = 2m + n$ vektorlarga yasalgan uchburchakning yuzini toping.

28. $\vec{a} = 2i + 3j\ va\ \ \vec{b} = i - j + 2k$ vektorlarning vektor ko'paytmasi $\vec{c} = \vec{a} \times \vec{b}$ ni toping.

29. $\vec{a} = 3i + 4j, \vec{b} = -3j + k\ va\ \vec{c} = 2j + 5k$ vektorlarga yasalgan parallelepipedning hajmini toping.

30. Uchlari $O(0;0;0), A(5;2;0), B(2;5;0)\ va\ C(1;2;4)$ nuqtalarda bo'lgan piramida hajmini toping. ABC yog'ining yuzi va shu yoqqa tushirilgan balandligini hisoblang.

31. $A(2;-1;-2), B(1;2;1), C(2;3;0)\ va\ D(5;0;-6)$ nuqtalarning bir tekislikda yotishini ko'rsating.

32. $\vec{a}(-1;3;2),\ \vec{b}(2;-3;-4)\ \ va\ \ \vec{c}(-3;12;6)$ vektorlarning o'zaro komplanar ekanligini ko'rsating. $\vec{c}$ vektorni $\vec{a}\ \ va\ \vec{b}$ vektorlar orqali ifodalang.

33. $\vec{a} = 2i - j, \vec{b} = j + 3k\ \ va\ \vec{c} = 3i + k$ vektorlarning aralash ko'paytmasini toping.

Mustaqil yechish uchun misollar.

Variant 1

1.$\vec{a} = \{3; -2; 6\}$, $\vec{b} = \{-2; 2; 0\}$, $\vec{c} = \{4; -2, ; -3\ \}$ vektorlar berilgan.

$\vec{d} = 3\vec{a} - 2\vec{b} + \vec{c}$ vektorni koordinatalarini toping

2. $\vec{a} = 4\vec{\imath} - 2\vec{\imath}$, $\vec{b} = 3\vec{\imath} + 5\vec{\jmath}$, $\vec{c} = \vec{\imath} - 7\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = \{4; -3;\ 2\}$ vektorni koordinata o'qlari bilan bir xil o'tkir burchak hosil qiluvchi o'qdagi proyeksiyasini toping.

4. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{\pi}{6}$ va $|\vec{a}| = \frac{1}{5}$, $|\vec{b}| = 2$ bo'lsa, $\left|\left[\vec{a} + 3\vec{b}, 3\vec{a} - \vec{b}\right]\right|$ ni toping.

5. $\overrightarrow{AB} = 2\vec{\jmath} + \vec{k}$, $\overrightarrow{BC} = 2\vec{\imath} - 3\vec{\jmath} - 2\vec{k}$, $\overrightarrow{CA} = -2\vec{\imath} + 4\vec{\jmath} + 3\vec{k}$ vektorlarga parallelepiped qurilgan. Uning A uchidan tushirilgan balandlik uzunligi va hajmini toping.

6. Soddalashtiring: $\left(\vec{a}; \vec{b} - 2\vec{a} + \vec{c}; \vec{b}\right) + \left(\vec{a} - 2\vec{b}; \vec{c}; \vec{a} - \vec{c}\right)$.

Javoblar: **1**. *J:* $\vec{a} = \{17; -10;\ 15\}$. **2**. *J:* $\vec{c} = \vec{a} - \vec{b}$. **3**. *J:* $\sqrt{3}$. **4**. *J :* 2.
5. $J:\ V = 2\ ,\ H = \frac{2}{3}$. **6**. *J :* $-3\left(\vec{a}; \vec{b}; \vec{c}\right)$.

Variant 2

1. $\vec{a} = \{3; -2; 6\}$, $\vec{b} = \{-2; 1; 0\}, \vec{c} = \{4; -2;\ -3\ \}$ vektorlar berilgan. $\vec{d} = 5\vec{a} + 6\vec{b} - 4\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = -6\vec{\imath} + 2j$, $\vec{b} = 4\vec{\imath} + 7\vec{\jmath}$, $\vec{c} = 9\vec{\imath} - 3\vec{\jmath}$ vektorlar berilgan. $\vec{c}$ vertorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $A(1; 2; 3)$, $B(-1; 2; 0)$, $C(3; 4; 2\ )$, $D(-7; 0; 5)$ nuqtalar berilgan. $\text{пр}_{(\overrightarrow{AD} + \overrightarrow{BC})}\overrightarrow{BA}$ ni hisoblang.

4. $\overrightarrow{AB} = 2\vec{\imath} - 6j$, $\overrightarrow{BC} = \vec{\imath} + 7\vec{\jmath}$, $\overrightarrow{CA} = -3\vec{\imath} - \vec{\jmath}$ vektorlar uchburchak tomonlari bo'lsa, A uchidagi ichki burchakni toping.

5. $\overrightarrow{a}$ va $\overrightarrow{b}$ vektorlar orasidagi burchak $\varphi = \frac{\pi}{3}$ va $|\vec{a}\,| = 4$, $|\vec{b}\,| = \sqrt{3}$ bo'lsa, $\left|[5\vec{a} + 2\vec{b}, \vec{a} - 3\vec{b}]\right|$ ni hisoblang.

6. Soddalashtiring: $(\vec{a}; \vec{b} + 2\vec{c};\ \ 3\vec{a} - \vec{c}) + (\vec{a} + \vec{b} + 2;\ \vec{a}; \vec{b})$.

Javoblar: **1**. *J:* $\{-13; 4; 42\}$. **2**. *J:* $\vec{c} = -1{,}5\vec{a}$. **3**. *J:* $\frac{\sqrt{2}}{2}$ **4**. *J:* 90^0. **5.** *J:* 102.
6. *J :* $(\vec{a}; \vec{b}; \vec{c})$.

Variant 3

1. $\vec{a} = \{3; -2; 6\}$, $\vec{b} = [-2; 1; 0]$, $\vec{c} = \{4; -2;\ -3\ \}$ vektorlar berilgan. $\vec{d} = 3\vec{a} + \vec{b} - 5\vec{c}$ vektorning koordinatasini toping.

2. $\vec{a} = 3\vec{\imath} - 2j$, $\vec{b} = -2\vec{\imath} + \vec{\jmath}$, $\vec{c} = 7\vec{\imath} - 4\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar orqali ifodalang.

3. $\vec{a} = \{2; -3; 2\}$ vektorning OX va OZ o'qlari bilan mos ravishda $\alpha = 120°$ va $\gamma = 45°$, OY o'qi bilan esa β o'tkir burchak hosil qiluvchi o'qdagi proyeksiyasini toping.

4. $\vec{a} = 5\vec{\imath} - j + \overrightarrow{2k}$, $\vec{b} = 2\vec{\imath} + \vec{\jmath} + 2k$ vektorlarga qurilgan parallelogramm diagonallari orasidagi burchakni toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\frac{\pi}{4}$ ga teng. Agar $|\vec{a}| = 2\sqrt{2}$, $|\vec{b}| = 3$ bo'lsa, $\left|[4\vec{a} - 5\vec{b};\ \vec{a} + 2\vec{b}]\right|$ ni hisoblang.

6. Soddalashtiring: $(2\vec{a} + \vec{b} + \vec{c};\ \ \vec{a} + \vec{b}; \vec{c} - \vec{a}) + (\vec{a}; \vec{a} + \vec{b} - \vec{c}; \vec{c})$.

Javoblar: **1**. *J :* $\vec{a} = \{-13; 5; 33\}$. **2.** *J :* $\vec{c} = \vec{a} - 2\vec{b}$. **3**. *J :* $\sqrt{2} - \frac{5}{2}$. **4.** *J :* $cos\varphi = \frac{21}{13\sqrt{5}}$. **5**. *J:* 78. **6**. J: $3\,(\vec{a}; \vec{b}; \vec{c})$.

Variant 4

1. $\vec{a} = \{3; -2; 6\}$, $\vec{b} = \{-2; 1; 0\}$, $\vec{c} = \{4; -2; -3\ \}$ vektorlar berilgan. $\vec{d} = \vec{a} + 24\vec{b} + 4\vec{c}$ vektorni koordinatasini toping.

2. $\vec{a} = 3\vec{\imath} + 5j,\ \ \vec{b} = \vec{\imath} - 7\vec{\jmath},\ \vec{c} = 7\vec{\imath} + 3\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vertorlar orqali ifodalang.

3. Agar $\vec{a} = 3\vec{m} - \vec{n}\,, \vec{b} = 2\vec{m} + 5\vec{n},$ va $|\vec{m}| = \frac{1}{\sqrt{2}},\ |\vec{n}| = 2,\ (\vec{m}\hat{}\vec{n},) = \frac{\pi}{4}$ bo'lsa, $\text{пр}_{\vec{a}}\vec{b}$ ni toping.

4. ΔABC uchlari berilgan: $A(1;2;-1),\ B(3;4;0),\ C(2;5;-2)$. B uchidagi ichki burchagi kosinusini toping.

5. $|\vec{a}| = 2,\ \ |\vec{b}| = 7, \left|[\vec{a},\vec{b}]\right| = 7\sqrt{3}$ lar berilgan. $(\vec{a},\vec{b})$ ni hisoblang.

6. Soddalashtiring: $(2\vec{a}; 3\vec{b} - \vec{a} + 2\vec{c}; \vec{b} - \vec{a}) +\ (\vec{a}; \vec{a} + \vec{b} - \vec{c}; \vec{c})$

Javoblar: **1**. *J :* $\{-29; 14; -6\}$. **2**. *J :* $\vec{c} = 2\vec{a} + \vec{b}$. **3**. *J :* $-\frac{4\sqrt{2}}{\sqrt{5}}$. **4**. *J :* $cosB = \frac{2}{3\sqrt{6}}$. **5**. *J :* 7. **6**. *J :* $-2\ (\vec{a}; \vec{b}; \vec{c})$

Variant 5

1. $\vec{a} = 2\vec{\imath} - 3j + \vec{k},\ \ \vec{b} = -\vec{\imath} + 2\vec{\jmath}, -3k, c = 3i + \vec{k}$ vektorlar berilgan. $\vec{d} = \vec{a} + 2\vec{b} + 4\vec{c}$ vektorning koordinatasini toping.

2. $\vec{a} = 3\vec{\imath} + j = 2\vec{\imath} - \vec{\jmath},\ \vec{c} = 8i + \vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar orqali ifodalang.

3. $\vec{a} = 3\vec{\imath} - 5j + \vec{k},\ \vec{b} = -\vec{\imath} + \vec{\jmath} + 4k,\ \vec{c} = 4i + 4j - 2\vec{k}$ vektorlar berilgan. $\text{пр}_{\vec{c}}(3\vec{a} - 2\vec{b}\,)$ ni hisoblang.

4. $|\vec{a}| = 5,\ \ |\vec{b}| = 4\ (\vec{a},\vec{b}) = 10$ berilgan. $\left|[\vec{a},\vec{b}]\right|$ ni hisoblang.

5. $\vec{a} = \vec{\imath} - 3\vec{\jmath} + \vec{k},\ \vec{b} = 2\vec{\imath} + \vec{\jmath} - 3k,\ \vec{c} = \vec{\imath} + \vec{\jmath} + \vec{k}$ vektorlarga qurilgan parallelopipedning asosi $\vec{b}$ va $\vec{c}$ vektorlarga qurilgan parallelogram bo'lsa , parallelepiped hajmi va balandligini toping.

6. Soddalashtiring:$(\vec{c} + \vec{a}; 2\vec{a} - 3\vec{b} + \vec{c};\ \vec{a} - 2\vec{b} + 2\vec{c}) + (\vec{a} - \vec{b}; \vec{a}; 2\,\vec{a} - 5\vec{b} + \vec{c})$

Javoblar: **1**. *J:* $\{12;\ 1; -9\}$. **2**. *J:*$\vec{c} = 2\vec{a} + \vec{b}$. **3**. *J:*$-\frac{7}{3}$. **4.** *J:* $10\sqrt{3}$. **5**. *J:* $V = 25,\ H = 25/\sqrt{83}$. **6**. *J:* $-4\ (\vec{a}; \vec{b}; \vec{c})$.

6- variant

1. $\vec{a} = 2\vec{\imath} - 3j + \vec{k}$, $\vec{b} = -\vec{\imath} + 2\vec{\jmath}, -3k$, $\vec{c} = 3i - \vec{k}$ vektorlar berilgan. $\vec{d} = -2\vec{a} + 3\vec{b} + 5\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 3\vec{\imath} + j$, $\vec{b} = 2\vec{\imath} - \vec{\jmath}$, $\vec{c} = \vec{\imath} - 3\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = \sqrt{8}i + 5\vec{\jmath} - 7\vec{k}$ vektorning OX va OY o'qlari bilan mos ravishda α= $45°$ va $\beta = 60°$ va OZ o'qi bilan o'tmas burchak hosil qiluvchi o'qdagi proyeksiyasini toping.

4. ABC uchburchak uchlari berilgan: $A(1;2;-1)$, $B(3;4;0)$, $C(2;5;-2)$. C uchidagi ichki burchagi kosinusini toping.

5. $|\vec{a}| = 4$, $|\vec{b}| = 5$, $|[\vec{a},\vec{b}]| = 10$ lar berilgan bo'lsa, $(\vec{a},\vec{b})$ ni hisoblang.

6. Soddalashtiring: $(3\vec{a} + 2b - 5\vec{c},\ \vec{a} - \vec{b} + 4\vec{c}, \vec{a} - 3\vec{b} + \vec{c})$.

Javoblar: **1.** *J:* $\{8,12,-16\}$. **2**. *J:* $\vec{c} = -\vec{a} + \vec{b}$. **3**. *J: 8.* 4. *J:* $cos\varphi = \frac{4}{\sqrt{66}}$. **5**. *J:* $5\sqrt{7}$.

6. J: $49\ (\vec{a},\vec{b},\vec{c})$.

Variant 7

1. $\vec{a} = 2\vec{\imath} - 3j + \vec{k}$, $\vec{b} = -\vec{\imath} + 2\vec{\jmath}, -3k$, $\vec{c} = 3\vec{\imath} - \vec{k}$ vektorlar berilgan. $\vec{d} = 3\vec{a} - 2\vec{b} + \vec{c}$. vektorning koordinatalarini toping.

2. $\vec{a} = 3\vec{\imath} + j$, $\vec{b} = 2\vec{\imath} - \vec{\jmath}$, $\vec{c} = 5\vec{\jmath} + 5\vec{\imath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = 2\vec{m} + 2n$, va $\vec{b} = 2\vec{m} - \vec{n}$, vektorlar berilgan. Agar $|\vec{m}| = 2$, $|\vec{n}| = 1$, $(\vec{m}\hat{}\vec{n}) = \frac{\pi}{3}$ bo'lsa $\text{пр}_{\vec{b}}\vec{a}$ ni toping.

4. $\vec{a} = 2\vec{m} - \vec{n}$ vektor berilgan va $\vec{m}$ va $\vec{n}$ birlik vektorlar orasidagi burchak $120°$ ga teng bo'lsa, $\cos(\vec{a}\hat{}\vec{m})$ va $cos(\vec{a}\hat{}\vec{n})$ ni toping.

5. $\vec{a} = \vec{m} + 3\vec{n}$, $\quad \vec{b} = 4\vec{m} - \vec{n}$, $\quad \vec{c} = 3\vec{m} + \vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = 4$, $|\vec{n}| = \sqrt{3}$, $(\vec{m}\hat{}\vec{n}) = \frac{\pi}{3}$ bo'lsa, $\left|[\vec{a} + \vec{b}, \vec{c}]\right|$ ni toping.

6. Soddalashtiring: $\left(\vec{a} - 3\vec{b}, +\vec{c}, 2\,\vec{a} + \vec{b} - 3\vec{c}, \vec{a} + 2\vec{b} + \vec{c}\right)$.

Javoblar: 1. *J:*$\{11, -13{,}8\}$. 2. *J:* $\vec{c} = 3\vec{a} - 2\vec{b}$. 3. *J:* $16/\sqrt{13}$. 4. *J:* $\frac{2.5}{\sqrt{7}}$; $-2/\sqrt{7}$. **5.** *J: 6*.**6**. *J:* $25\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 8

1. $\vec{a} = 2\vec{\imath} - 3j + \vec{k}$, $\vec{b} = -\vec{\imath} + 2\vec{\jmath}, -3k, c = 3i - \vec{k}$ vektorlar berilgan. $\vec{d} = 2\vec{a} - 3\vec{b} - \vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 4\vec{\imath} - 2j$, $\vec{b} = 3\vec{\imath} + 5\vec{\jmath}$, $\vec{c} = 5\vec{\imath} - 9\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3.$\vec{a} = 2\vec{m} + 2n$, $\vec{b} = 2\vec{m} - \vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = \frac{1}{\sqrt{8}}$, $|\vec{n}| = \frac{1}{4}$, $(\vec{m}\hat{}\vec{n}) = \frac{\pi}{4}$ bo'lsa $\text{пр}_{\vec{b}}\vec{a}$ ni toping.

4. $\vec{a} = 2\vec{\imath} + j$ va $\vec{b} = -2\vec{\jmath} + \vec{k}$ vektorlarga qurilgan parallelogrammning diagonallari orasidagi burchakni toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{3}{4}\pi$ va $|\vec{a}| = 2$, $\left|\vec{b}\right| = \sqrt{2}$ bo'lsa, $|[\vec{a} - 3\vec{b}, 2\vec{a} + 2\vec{b}]|$ ni toping.

6. $\vec{a} = 2\vec{\imath} + 3j - \vec{k}$, $\vec{b} = 5\vec{\imath} - 2\vec{\jmath}$, $\vec{c} = 3\vec{\jmath} + 5\vec{k}$ vektorlarga qurilgan parallelepiped asosi $\vec{a}$ va $\vec{c}$ vektorlarga qurilgan parallelogram bo'lsa, parallelepipedning balandligi va hajmini toping.

7. Soddalashtiring: $\left(\vec{a} + 2\vec{b} - \vec{c};\ \vec{a} - \vec{b}; \vec{a} - \vec{b} - \vec{c}\right)$.

Javoblar: **1**. *J:*$\{4; -12; 12\}$. **2**. *J:* $\vec{c} = 2\vec{a} - \vec{b}$. **3**. *J:* $1/\sqrt{5}$. **4**. *J:* $\varphi = 90°$. **5**. *J:16*. **6.** *J:* $V = 78, H = \frac{39}{\sqrt{115}}$. **7**. *J:* $3\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 9

1. $\vec{a} = 2\vec{\imath} - 3j + \vec{k}$, $\vec{b} = -\vec{\imath} + 2\vec{\jmath}, -3k$, $\vec{c} = 3\vec{\imath} - \vec{k}$ vektorlar berilgan.

$\vec{d} = 4\vec{a} - \vec{b} + 2\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 2\vec{i} + \vec{j}$, $\vec{b} = -\vec{i} + 2\vec{j}$, $\vec{c} = \vec{i} + 8\vec{j}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = 2\vec{m} + 2n$, va $\vec{b} = 2\vec{m} - \vec{n}$ vektorlar hamda $|\vec{m}| = \frac{1}{\sqrt{8}}$, $|\vec{n}| = \frac{1}{4}$, $(\vec{m}\hat{\ }\vec{n},) = \frac{\pi}{4}$ lar berilgan bo'lsa, $\text{пр}_{\vec{a}}\vec{b}$ ni toping.

4. ABC uchburchak uchlari berilgan: $A(1;2;-1)$, $B(3;4;0)$, $C(2;5;-2)$. B uchidagi ichki burchagini toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{\pi}{6}$ ga teng. Agar $|\vec{a}| = 5$, $|\vec{b}| = 4$ bo'lsa, $|[2\vec{a} + 5\vec{b}, \vec{a} - 2\vec{b}]|$ ni hisoblang.

6. Soddalashtiring: $(\vec{a} - 2b; 2\vec{a} - 3\,\vec{c}; \vec{a} + \vec{b} - 2\vec{c})$.

Javoblar: 1. *J:* $\{15; -14; 5\}$. 2. *J:* $\vec{c} = 2\vec{a} + 3\vec{b}$. 3. *J:* $1/\sqrt{5}$. 4. *J:* $cosB = \frac{2}{2\sqrt{6}}$. **5.** *J:* 90^0. 6. *J:* $(\vec{a}, \vec{b}, \vec{c})$.

Variant 10

1. $\vec{a} = \{5,7,2\}$, $\vec{b} = \{3,0,4\}$, $\vec{c} = \{-6,1,-1\}$ vektorlar berilgan.
$\vec{d} = 2\vec{a} - 3\vec{b} - \vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 2\vec{i} + \vec{j}$, $\vec{b} = -\vec{i} + 2\vec{j}$, $\vec{c} = 12\vec{i} + \vec{j}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $A(1,2,3,)$, $B(-1,2,0)$, $C(3,4,2\,)$, $D(-7,0,5)$ nuqtalar berilgan. $\text{пр}_{(\overrightarrow{AD}+\overrightarrow{BC})}\overrightarrow{BA}$ ni hisoblang.

4. $\vec{a} = 6\vec{i} - 2\vec{j} + 3\vec{k}$ vektor bazislari $\vec{i}, \vec{j}, \vec{k}$ bo'lsa, har bir vektor ortlarini ifodalovchi burchaklarini toping.

5. $\vec{a} = 4\vec{m} + 3\vec{n}$, $\vec{b} = 3\vec{m} - \vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = 2, |\vec{n}| = 3$, $(\vec{m}\hat{\ }\vec{n},) = \frac{\pi}{6}$ bo'lsa, $|[\vec{a}\vec{b}, -\vec{a}]|$ ni hisoblang.

6. Soddalashtiring: $(\vec{a}, -3b, +c, 2\vec{a} + b - 3\,\vec{c}, \vec{a} + 2\vec{b} + \vec{c})$.

Javoblar: **1**. *J:*$\{7,13,-7\}$. **2**. *J:*$\vec{c}=5\vec{a}-2\vec{b}$. **3**. *J:*$-\sqrt{16}$. **4**. *J:* $cos(\vec{a}\wedge\vec{n})=\frac{6}{7}$; $cos(\vec{a}\wedge\vec{n})=-\frac{2}{7}$; $cos(\vec{a}\wedge\vec{n})=\frac{3}{7}$. **5**. *J: 21*. **6**. *J:* $25(\vec{a},\vec{b},\vec{c})$.

Variant 11

1. $\vec{a}=\{5,7,2\}$, $\vec{b}=\{3,0,4\}$, $\vec{c}=\{-6,1,-1\}$ vektorlar berilgan. $\vec{d}=3\vec{a}-2\vec{b}+\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a}=\vec{\imath}-7\vec{\jmath}$, $\vec{b}=4\vec{\imath}-2\vec{\jmath}$, $\vec{c}=3\vec{\imath}+5\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $A(2,-5,4), B(1,0,-1)$ nuqtalar berilgan. $\overrightarrow{AB}$ vekrorning OY va OZ o'qlari bilan $\beta=\gamma=60°$ burchak hosil qiluvchi va OX bilan esa o'tkir burchak hosil qiluvchi o'qdagi proyeksiyasini toping.

4. $A(a,0,0)$, $B(0,0,a)$, $C(a,0,a)$ nuqtalar berilgan. $\overrightarrow{OC}$ va $\overrightarrow{AB}$ vektorlarni yasang va ular orasidagi burchakni toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi=\frac{\pi}{3}$ ga teng. Agar $|\vec{a}|=3$, $|\vec{b}|=1$ bo'lsa, $|[3\vec{a}+2\vec{b},2\vec{a}+2\vec{b}]|$ ni hisoblang.

6. Soddalashtiring: $(\vec{a}-3\vec{b}+2\vec{c},2\vec{a}-3\vec{b}+\vec{c},\ \vec{a}+\vec{c})$.

Javoblar: **1**. *J:* $\{3,22,-3\}$. **2**. *J:*$\vec{c}=-\vec{a}+\vec{b}$. **3**. *J:* $-\frac{1}{\sqrt{2}}$. **4**. *J:* $cos\varphi=\frac{1}{\sqrt{10}}$. **5**. *J:*$3\sqrt{3}$ **6**. *J:* $6(\vec{a},\vec{b},\vec{c})$.

Variant 12

1. $\vec{a}=\{5,7,2\}$, $\vec{b}=\{3,0,4\}$, $\vec{c}=\{-6,1,-1\}$ vektorlar berilgan. $\vec{d}=-2\vec{a}+3\vec{b}+5\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a}=5\vec{\imath}+2\vec{\jmath}$, $\vec{b}=3\vec{\imath}-4\vec{\jmath}$, $\vec{c}=16\vec{\imath}-4\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a}=3\vec{m}+2n$, va $\vec{b}=\vec{m}-\vec{n}$, vektorlar berilgan. Agar $|\vec{m}|=1$, $|\vec{n}|=2$, $(\vec{m}\wedge\vec{n},)=\frac{2\pi}{3}$ bo'lsa, $\text{пр}_{\vec{a}}\vec{b}$ ni hisoblang.

4. Uchburchak tomonlari $\overrightarrow{AB} = 2\vec{\imath} - 6\vec{\jmath},\ \overrightarrow{BC} = \vec{\imath} + 7\vec{\jmath},\ \overrightarrow{CA} = -3\vec{\imath} - \vec{\jmath}$, vektorlardan iborat bo'lsa, uning B ichki burchagini toping.

5. Tomonlari $\vec{a} = 2\vec{\jmath} + \vec{k},\ \vec{b} = \vec{\imath} + 2\vec{k}$ vektorlardan iborat parallelogram yasang va uning balandligi va yuzini toping.

6. $|\vec{a}| = 3,\ \ |\vec{b}| = 7$ va $(\vec{a}, \vec{b}) = 9$ lar berilgan. $|[\vec{a}, \vec{b}]|$ ni hisoblang.

7. Soddalashtiring: $(3\vec{a}, +b - 2\vec{c}, \vec{b} + 5\vec{c}, -\vec{a} + \vec{b} - \vec{c})$.

Javoblar: 1. *J:*$\{-31, -9{,}3\}$. 2. *J:*$\vec{c} = -2\vec{a} + 2\vec{b}$. 3. *J:*$2\sqrt{3}$. 4. *J:* $cos\varphi = \frac{2}{\sqrt{5}}$. 5. *J:* $S = \sqrt{21}, H = \sqrt{21}/5$. 6. *J:* $6\sqrt{10}$. **7**. *J:*$-13\ (\vec{a}, \vec{b}, \vec{c})$.

Variant 13

1. $\vec{a} = \{5.7.2\},\ \vec{b} = \{3.0.4\}, \vec{c} = \{-.6.1 - 1\}$ vektorlar berilgan.
$\vec{d} = \vec{a} + 4\vec{b} - \vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 5\vec{\imath} + 2\vec{\jmath},\ \vec{b} = 3\vec{\imath} - 4\vec{\jmath},\ \vec{c} = 4\vec{\imath} - 12\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = 3\vec{\imath} - 5j + \vec{k},\ \vec{b} = -\vec{\imath} + \vec{\jmath}, +4k$ va $c = 4\vec{\imath} + 4\vec{\jmath} - 2\vec{k}$ vektorlar berilgan. $\text{пр}_{\vec{a}+2\vec{b}}\vec{c}$ ni hisoblang.

4. $\overrightarrow{OA} = \vec{a}$ va $\overrightarrow{OB} = \vec{b}$, vektorlar berilgan. Agar $\overline{|a|} = 2,\ |\vec{b}| = 4,\ (\vec{a}, \vec{b}) = 60$ bo'lsa, AOB uchburchakning OM medianasi va OA tomoni orasidagi burchakni toping.

5.$\vec{a} = 2\vec{m} + \vec{n}$va $\vec{b} = \vec{m} + 2\vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = 2,\ |\vec{n}| = \sqrt{2}$, $(\vec{m}\wedge\vec{n}) = \frac{\pi}{4}$ bo'lsa, $|[\vec{a}, \vec{b}] + 5[\vec{m}, \vec{n}] - 3[\vec{n}, \vec{m}]|$ ni toping.

6.Soddalashtiring:$(2\vec{a}, -b, +2c, \vec{a}, -2b - \vec{c}, \vec{a} + \vec{b})$

Javoblar: 1. *J:* $\{29.5.20\}$. 2. *J:* $\vec{c} = 2\vec{a} - 2\vec{b}$. 3. *J:*$-26/\sqrt{91}$. 4. *J:* $cos\varphi = 2/\sqrt{7}$. **5**. *J: 22*. 6. *J:* $9(\vec{a}, \vec{b}, \vec{c})$

Variant 14

1.$\vec{a} = \{2; 7; 5\}$, $\vec{b} = \{3; 0; 4\}$, $\vec{c} = \{-6; -1; 3\}$ vektorlar berilgan.

$\vec{d} = \vec{a} - 2\vec{b} + 2\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 5\vec{\imath} + 2\vec{\jmath}$, $\vec{b} = -\vec{\imath} + 10\vec{\jmath}$, $\vec{c} = 3\vec{\imath} - 4\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $A(3; -2; 1), B(2; 1; -1)$ nuqtalar berilgan. $\overrightarrow{AB}$ vekrorning OY va OZ o'qlari bilan $\beta = \gamma = 60°$ burchak hosil qiluvchi va OX bilan esa o'tmas burchak hosil qiluvchi o'qdagi proyeksiyasini toping.

4. ABC uchburchak uchlari berilgan: $A(2, -2,1)$, $B(3,0,4)$, $C(1,1, -3)$. C uchidagi ichki burchagi kosinusini toping.

5. ABCD parallelogrammda $\overrightarrow{AB} = \vec{N} - 0.5\vec{M}$ ė $\overrightarrow{AD} = 5.5\vec{m}$ vektorlar berlgan. Agar $|\vec{m}| = \sqrt{2}$, $|\vec{n}| = 2$, $(\vec{m}\hat{}\vec{n}) = \frac{\pi}{4}$ bo'lsa, $\left|\left[\overrightarrow{AC}, \overrightarrow{BD}\right]\right|$ ni toping.

6. Soddalashtiring: $\left(\vec{a} - \vec{b} + 3\vec{c},\ -2\vec{a} + 2\vec{b} + \vec{c},\ 3\vec{a} - 2\vec{b} + 5\vec{c}\right)$

Javoblar: 1. *J:* $\{-16,5,3\}$. 2. *J:* $\vec{c} = \frac{1}{2}\vec{a} - \frac{1}{2}\vec{b}$. 3. *J:* $(+\sqrt{2})/2$. 4. *J:* $cosC = -5.5/\sqrt{39}$. 5. *J: 22.* **6.** *J*: $-7\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 15

1. $\vec{a} = \{2,7,5\}$, $\vec{b} = \{3,0,4\}, \vec{c} = \{-6, -1,3\}$ vektorlar berilgan.

$\vec{d} = -2\vec{a} + 3\vec{b} + 2\vec{c}$ vektorning koordinatalarini toping.

2.$\vec{a} = 2\vec{\imath} - 3\vec{\jmath}$, $\vec{b} = 5\vec{\imath} + 4\vec{\jmath}$, $\vec{c} = \vec{\imath} + 10\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $3\vec{\imath} - 5\vec{\jmath} + \vec{k}$, $\vec{b} = -\vec{\imath} + \vec{\jmath}, +4\vec{k}$, $\vec{c} = 4\vec{\imath} + 4\vec{\jmath} - 2\vec{k}$ vektorlar berilgan. $\text{пр}_{\vec{a}+2\vec{b}}\vec{c}$ ni toping.

4. $\overrightarrow{AB} = 2\vec{\imath} - 6\vec{\jmath}$, $\overrightarrow{BC} = \vec{\imath} + 7\vec{\jmath}$, $\overrightarrow{CA} = -3\vec{\imath} - \vec{\jmath}$ vektorlar uchburchak tomonlari bo'lsa, uchburchakning C uchidagi burchagini toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{\pi}{4}$. Agar $|\vec{a}| = 4$, $\left|\vec{b}\right| = 1$ bo'lsa, $\left|\left[3\vec{a} - \vec{b}, \vec{a} - \overline{2b}\right]\right|$ ni hisoblang.

6. Soddalashtiring: $\left(5\vec{a}, +3b - \vec{c}, \vec{a} + 2\vec{b}, \vec{b} - \vec{c}\right)$.

***Javoblar**:* 1. *J:* $\{-7, -16{,}8\}$. 2.*J:* $\vec{c} = -2\vec{a} + \vec{b}$. 3. *J:* -2. 4. *J:* $cos\alpha = 1/\sqrt{5}$. 5. *J:* $10\sqrt{3}$. 6. $Javob$: $-8\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 16

1. $\vec{a} = \{1{,}2{,}3\}$, $\vec{b} = \{4{,}5{,}0\}, \vec{c} = \{8{,}3{,}10\}$ vektorlar berilgan. $\vec{d} = -3\vec{a} + 5\vec{b} - 4\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 4\vec{\imath} - 2\vec{\jmath}$, $\vec{b} = 3\vec{\imath} + 5\vec{\jmath}$, $\vec{c} = \vec{\imath} - 7\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = \{4, -3{,}2\}$ vertorning koordinata o'qlari bilan bir xil o'tkir burchak hosil qiluvchi o'qdagi proyeksiyasini toping.

4. ABC uchburchak uchlari berilgan: $A(1{,}2, -1)$, $B(3{,}4{,}0)$, $C(2{,}5, -2)$.uchburchakning A uchidagi burchagini toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{\pi}{6}$. Agar $|\vec{a}| = \frac{1}{5}$, $\left|\vec{b}\right| = 2$ bo'lsa, $|[\vec{a} + 3\vec{b}, 3\vec{a} - \vec{b}]|$ ni hisoblang.

6. $\overrightarrow{AB} = 2\vec{\jmath} + \vec{k}$,$\overrightarrow{BC} = 2\vec{\imath} - 3\vec{\jmath} - 2\vec{k}$, $\overrightarrow{CD} = -2\vec{\imath} + 4\vec{\jmath} + 3\vec{k}$ vektorlarga parallelepiped qurilgan. Uning hajmini va A uchidan tushirilgan balandligini toping.

7. Soddalashtiring:$\left(\vec{a}, \vec{b} - 2\vec{a} + \vec{c}, \vec{b}\right) + \left(\vec{a}, -2\vec{b}, \vec{c}, \vec{a} - \vec{c}\right)$.

Javoblar: 1. *J:* $\{-15{,}7, -49\}$. 2. *J:* $\vec{c} = \vec{a} - \vec{b}$ 3. *J:*$\sqrt{3}$. 4. *J:* $cosA = 7/3\sqrt{11}$ .5. *J:* 2. 6. *J:* $V = 2$, $H = \frac{2}{3}$. 7. *J:* $-3\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 17

1. $\vec{a} = \{3, -2{,}6\}$, $\vec{b} = \{-2{,}1{,}0\}, \vec{c} = \{4, -2, -3\}$ vektorlar berilgan.
$\vec{d} = 5\vec{a} + 6\vec{b} - 4\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = -6\vec{\imath} + 2\vec{\jmath}$, $\vec{b} = 4\vec{\imath} + 7\vec{\jmath}$, $\vec{c} = 9\vec{\imath} - 3\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $A(1,2,3,),\ \ B(-1,2,0),\ C(3,4,2\ ),\ D(-7,0,5)$ nuqtalar berilgan. $\text{пр}_{\overrightarrow{AD}+\overrightarrow{BC}}\overrightarrow{BA}$.

4. $\overrightarrow{AB}=2\vec{\imath}-6\vec{\jmath},\ \ \overrightarrow{BC}=\vec{\imath}+7\vec{\jmath},\ \overrightarrow{CA}=-3\vec{\imath}-\vec{\jmath}$ vektorlar uchburchak tomonlari bo’lsa, uchburchakning A uchidagi burchagini toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi=\frac{\pi}{3}$ ga teng. Agar $|\vec{a}|=4,\ \ |\vec{b}|=\sqrt{3}$ bo’lsa, $|[5\vec{a}+2\vec{b},\vec{a}-3\vec{b}]|$ ni hisoblang.

6. Soddalashtiring: $(\vec{a},\vec{b}+2\vec{c},\ 3\vec{a}-\vec{c})+(\vec{a}+\vec{b}+2\vec{c},\vec{a},\vec{b})$

Javoblar: 1. *J:* $\{-13,4,42\}$. 2. *J:* $\vec{c}=-1{,}5\vec{a}$. 3. *J:* $\frac{\sqrt{2}}{2}$. 4. *J:* 90^0 5. *J:* 102. 6. *J:* $(\vec{a},\vec{b},\vec{c})$

Variant 18

1. $\vec{a}=\{3,-2,6\},\ \vec{b}=[-2,1,0],\vec{c}=\{4,-2,-3\ \}$ vektorlar berilgan.
$\vec{d}=3\vec{a}+\vec{b}-5\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a}=3\vec{\imath}-2\vec{\jmath},\ \ \vec{b}=-2\vec{\imath}+\vec{\jmath},\ \vec{c}=7\vec{\imath}-4\vec{\jmath}$ vektorlarni yasang. c⃗ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo’yicha yoying.

3. $\vec{a}=\{2,-3,2\}$ vektorning OX va OZ o’qlaridan mos ravishda $\alpha=120^\circ$ va $\gamma=45^\circ$ OY o’qidan esa o’tkir burchak hosil qiluvchi o’qdagi proyeksiyasini toping.

4. Tomonlari $\vec{a}=5\vec{\imath}-\vec{\jmath}+2\vec{k},\ \ \vec{b}=2\vec{\imath}+\vec{\jmath}+2\vec{k}$ vektorlarga qurilgan parallelogrammning diagonallari orasidagi burchakni toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi=\frac{\pi}{4}$ ga teng. Agar $|\vec{a}|=2\sqrt{2},\ \ |\vec{b}|=3$ bo’lsa, $|[4\vec{a}-5\vec{b},\vec{a}+2\vec{b}]|$ ni hisoblang.

6. Soddalashtiring:$(2\vec{a}+\vec{b}+\vec{c},\ \vec{a}+\vec{b},\vec{c}-\vec{a})+(\vec{a},\vec{a}+\vec{b}-\vec{c},\vec{c})$.

Javoblar: 1. *J:* $\vec{a}=\{-13,5,33\}$. 2. *J:* $\vec{c}=\vec{a}-2\vec{b}$. 3. *J:* $\sqrt{2}-\frac{5}{2}$. 4. *J:* $arccosx=\frac{21}{13\sqrt{5}}$. 5. *J:* 78.. 6. *J:* $3\ (\vec{a},\vec{b},\vec{c})$.

Variant 19

1. $\vec{a}=\{3,-2,6\},\ \vec{b}=\{-2,1,0\},\ \ \vec{c}=\{4,-2,-3\ \}$ vektorlar berilgan.

$\vec{d} = \vec{a} + 24\vec{b} + 4\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 3\vec{\imath} + 5\vec{\jmath}$, $\vec{b} = \vec{\imath} - 7\vec{\jmath}$, $\vec{c} = 7\vec{\imath} + 3\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. Agar $\vec{a} = 3\vec{m} - \vec{n}$ va $\vec{b} = 2\vec{m} + 5\vec{n}$ vektorlar va $|\vec{m}| = \frac{1}{\sqrt{2}}$, $|\vec{n}| = 2$, $(\vec{m} \wedge \vec{n}) = \frac{\pi}{4}$ berilgan bo'lsa, $\text{пр}_{\vec{a}}\vec{b}$ ni toping.

4. ABC uchburchak uchlari berilgan: $A(1,2-1), B(3,4,0), C(2,5-2)$. Uning B uchidagi nburchagini toping.

5. $|\vec{a}| = 2$, $|\vec{b}| = 7$ va $|[\vec{a}, \vec{b}]| = 7\sqrt{3}$ lar berilgan bo'lsa, $(\vec{a}, \vec{b})$ ni hisoblang.

6. Soddalashtiring: $(2\vec{a}, 3\vec{b} - \vec{a} + 2\vec{c}, \vec{b} - \vec{a}) + (3\vec{a} - \vec{b}, \vec{b} + \vec{c}, \vec{a} + \vec{c})$.

Javoblar: 1. *J:* $\{-29,14,-6\}$. 2. *J:* $\vec{c} = 2\vec{a} + \vec{b}$. 3. *J:* $-\frac{4\sqrt{2}}{\sqrt{5}}$. 4. *J:* $cosa = \frac{2}{3\sqrt{6}}$. 5. *J:* 7. 6. *J:*$-2\ (\vec{a}, \vec{b}, \vec{c})$.

Variant 20

1. $\vec{a} = 2\vec{\imath} - 3\vec{\jmath} + \vec{k}$, $\vec{b} = -\vec{\imath} + 2\vec{\jmath}, -3\vec{k}$, $\vec{c} = 3\vec{\imath} + \vec{k}$ vektorlar berilgan.

$\vec{d} = \vec{a} + 2\vec{b} + 4\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 3\vec{\imath} + \vec{\jmath}$, $\vec{b} = 2\vec{\imath} - \vec{\jmath}$, $\vec{c} = 8\vec{\imath} + \vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = 3\vec{\imath} - 5\vec{\jmath} + \vec{k}$, $\vec{b} = -\vec{\imath} + \vec{\jmath} + 4\vec{k}$, $\vec{c} = 4\vec{\imath} + 4\vec{\jmath} - 2\vec{k}$ vektorlar berilgan. $\text{пр}_{\vec{c}}(3\vec{a} - 2\vec{b})$ ni toping.

4. ABC uchburchak uchlari berilgan: $A(2; -2; 1)$, $B(3; 4; 0)$, $C(1; 1; -3)$. Uning A uchidagi burchagini toping.

5. $|\vec{a}| = 5$, $|\vec{b}| = 4$, $(\vec{a}, \vec{b}) = 10$ lar berilgan. $|[\vec{a}, \vec{b}]|$ ni toping.

6. $\vec{a} = \vec{\imath} - 3\vec{\jmath} + \vec{k}$, $\vec{b} = 2\vec{\imath} + \vec{\jmath} - 3\vec{k}$, $\vec{c} = \vec{\imath} + 2\vec{\jmath} + \vec{k}$ vektorlarga qurilgan parallelepiped asosi $\vec{b}$ va $\vec{c}$ vektorlarga qurilgan parallelogram bo'lsa, parallelopipedning hajmini va balandligini toping.

7.Soddalashtiring: $\left(\vec{c}+\vec{a}, 2a-3\overrightarrow{b,}+\vec{c}, \vec{a}-2\vec{b}+2\vec{c}\right)+\left(\vec{a}-\vec{b}, \vec{a}, 2\,\vec{a}-5\vec{b}+\vec{c}\right)$.

Javoblar: 1. *J:* $\{12,1,-9\}$. 2. *J:* $\vec{c}=2\vec{a}+\vec{b}$. 3. *J:* $-7/3$. 4. *J:* $cos\alpha=\frac{\sqrt{7}}{2\sqrt{13}}$. 5. *J:* $10\sqrt{3}$. 6. *J:* $V=25,\ H=\frac{25}{\sqrt{83}}$. 7. *J:* $-4\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 21

1.$\vec{a}=2\vec{\imath}-3\vec{\jmath}+\vec{k},\ \ \vec{b}=-\vec{\imath}+2\vec{\jmath},-3\vec{k},\ \ \vec{c}=3\vec{\imath}-\vec{k}$ vektorlar berilgan.

$\vec{d}=-2\vec{a}+3\vec{b}+5\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a}=3\vec{\imath}+\vec{\jmath},\ \vec{b}=2\vec{\imath}-\vec{\jmath},\ \vec{c}=\vec{\imath}-3\vec{\jmath}$ vektorlarni yasang. c⃗ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo’yicha yoying.

3. $\vec{a}=\sqrt{8\vec{\imath}}+5\vec{\jmath}-7\vec{k}$ vektorning OX va OY o’qlardan mos ravishda $\alpha=45°$ va $\beta=60°$ li burchaklar OZ o’qidan esa o’tmas burchak hosil qiluvchi o’qdagi proyeksiyasini toping.

4. ABC uchburchak uchlari berilgan: $A(1,2,-1),\ B(3,4,0),\ C(2,5,-2)$. Uning C burchagini toping.

5. $|\vec{a}|=4,\ \ |\vec{b}|=5|,\left[\vec{a}, \vec{b}\right]|=15$ lar berilgan . $\left(\vec{a}, \vec{b}\right)$

6. Soddalashtiring: $\left(3\vec{a}+2\vec{b}-5\vec{c},\ \vec{a}-\vec{b}+4\vec{c}, \vec{a}-3\vec{b}+\vec{c}\right)$.

Javoblar: 1. *J:* $\{8,12,-16\}$. 2. *J:* $\vec{c}=-\vec{a}+2\vec{b}$. 3. *J:* *8.* 4. *J:* $cos\varphi=\frac{4}{\sqrt{66}}$. 5. *J:* $5\sqrt{5}$. 6. *J:* $49\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 22

1. $\vec{a}=2\vec{\imath}-3j+\vec{k},\ \ \vec{b}=-\vec{\imath}+2\vec{\jmath}-3\vec{k},\ \ \ \vec{c}=3\vec{\imath}-\vec{k}$ vektorlar berilgan.

$\vec{d}=3\vec{a}-2\vec{b}+\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a}=3\vec{\imath}+j,\ \ \vec{b}=2\vec{\imath}-\vec{\jmath},\ \vec{c}=5\vec{\jmath}+5\vec{\imath}$ vektorlarni yasang. c⃗ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo’yicha yoying.

3. $\vec{a} = 2\vec{m} + 2n$ va $\vec{b} = 2\vec{m} - \vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = 2$, $|\vec{n}\,| = 1$ va $(\vec{m}\hat{}\vec{n}) = \frac{\pi}{3}$ bo'lsa, $\overline{\text{пр}}_{\vec{b}}\vec{a}$ ni toping.

4. $\vec{a} = 2\vec{m} - \vec{n}$ vektor va $\vec{m}$ va $\vec{n}$ –birlik vektorlar orasidagi burchak $120°$ bo'lsa, $\cos(\vec{a}\hat{}\vec{m})$ va $cos(\vec{a}\hat{}\vec{n})$ ni toping.

5. $\vec{a} = \vec{m} + 3\vec{n}$, $\vec{b} = 4\vec{m} - \vec{n}$, $\vec{c} = 3\vec{m} + \vec{n}$ vektorlar brilgan. Agar $|\vec{m}| = 4$, $|\vec{n}| = \sqrt{3}$, $(\vec{m}\hat{}\vec{n}) = \frac{\pi}{3}$ bo'lsa $\left|\left[\vec{a} + \vec{b}, \vec{c}\right]\right|$ ni hisoblang.

6. Soddalashtiring: $\left(\vec{a} - 3\vec{b}, +\vec{c}, 2\,\vec{a} + \vec{b} - 3\vec{c}, \vec{a} + 2\vec{b} + \vec{c}\right)$.

Javoblar: 1. *J:* $\{11, -13{,}8\}$. 2. *J:* $\vec{c} = 3\vec{a} - 2\vec{b}$. 3. *J:* $16/\sqrt{13}$. 4. *J:* $\frac{2.5}{\sqrt{7}}$; $-2/\sqrt{7}$. 5. *J: 6*. 6. *J:* $25\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 23

1. $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = -\vec{i} + 2\vec{j}, -3\vec{k}$, $\vec{c} = 3\vec{i} - \vec{k}$ vektorlar berilgan. $\vec{d} = 2\vec{a} - 3\vec{b} - \vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 4\vec{i} - 2\vec{j}$, $\vec{b} = 3\vec{i} + 5\vec{j}$, $\vec{c} = 5\vec{i} - 9\vec{j}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = 2\vec{m} + 2\vec{n}$, и $\vec{b} = 2\vec{m} - \vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = \frac{1}{\sqrt{8}}$, $|\vec{n}\,| = \frac{1}{4}$ va $(\vec{m}\hat{}\vec{n}) = \frac{\pi}{4}$ bo'lsa, $\overline{\text{пр}}_{\vec{a}}\vec{b}$ ni toping.

4. $\vec{a} = 2\vec{i} + \vec{j}$ va $\vec{b} = -2\vec{j} + \vec{k}$ vektorlarga qurilgan parallelogrammning diagonallari orasidagi burchakni toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{3}{4}\pi$ ga teng. Agar $|\vec{a}| = 2$, $\left|\vec{b}\right| = \sqrt{2}$ bo'lsa, $|[\vec{a} - 3\vec{b}, 2\vec{a} + 2\vec{b}]|$ ni hisoblang.

6. $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = 5\vec{i} - 2\vec{j}$, $\vec{c} = 3\vec{j} + 5\vec{k}$ vektorlarga qurilgan parallelepiped asosi $\vec{a}$ va $\vec{c}$ vektorlarga qurilgan parallelogram bo'lsa, uning hajmi va balandligini hisoblang.

7. Soddalashtiring: $\left(\vec{a} + 2\vec{b}, -\vec{c}, \vec{a} - \vec{b}, \vec{a} - \vec{b} - \vec{c}\right)$

Javoblar: 1. *J:* $\{4, -12{,}12\}$. 2. *J:* $\vec{c} = 2\vec{a} - \vec{b}$. 3. *J:* $1/\sqrt{5}$. 4. *J:* $\varphi = 90°$. 5. *J: 16.*

6. $J: V = 78,\ \ H = \frac{39}{\sqrt{115}}$. 7. J: $3(\vec{a},\vec{b},\vec{c})$.

Variant 24

1.$\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k},\ \ \vec{b} = -\vec{i} + 2\vec{j}, -3\vec{k},\ \ \vec{c} = 3\vec{i} - \vec{k}$ vektorlar berilgan.

$\vec{d} = 4\vec{a} - \vec{b} + 2\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 2\vec{i} + \vec{j},\ \ \vec{b} = -\vec{i} + 2\vec{j},\ \ \vec{c} = \vec{i} + 8\vec{j}$ vektorlarni yasang. c⃗ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = 2\vec{m} + 2\vec{n}$ va $\vec{b} = 2\vec{m} - \vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = \frac{1}{\sqrt{8}},\ |\vec{n}| = \frac{1}{4}$, $(\vec{m}\hat{}\vec{n}) = \frac{\pi}{4}$ bo'lsa, $\text{пр}_{\vec{a}}\vec{b}$ ni toping.

4. ABC uchburchak uchlari berilgan: $A(1,2,-1),\ B(3,4,0),\ C(2,5,-2)$. Uchburchakning B uchidagi ichki burchagini toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{\pi}{6}$ ga teng. Agar $|\vec{a}| = 5,\ \ |\vec{b}| = 4$ bo'lsa, $\left|\left[2\vec{a} + 5\vec{b}, \vec{a} - 2\vec{b}\right]\right|$ ni hisoblang.

6. Soddalashtiring: $(\vec{a} - 2b, 2\vec{a} - 3\,\vec{c}, \vec{a} + \vec{b} - 2\vec{c})$.

Javoblar: 1. *J:* $\{15,-14,5\}$. 2. *J:* $\vec{c} = 2\vec{a} + 3\vec{b}$. 3. *J:* $1/\sqrt{5}$. 4. *J:* $cosa = \frac{2}{2\sqrt{6}}$. 5. *J:* 90^0. 6. $Javob$: $(\vec{a},\vec{b},\vec{c})$.

Variant 25

1.$\vec{a} = \{5,7,2\},\ \vec{b} = \{3,0,4\},\ \vec{c} = \{-6,1,-1\}$ vektorlar berilgan.

$\vec{d} = 2\vec{a} - 3\vec{b} - \vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 2\vec{i} + \vec{j},\ \ \vec{b} = -\vec{i} + 2\vec{j},\ \ \vec{c} = 12\vec{i} + \vec{j}$ vektorlarni yasang. c⃗ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $A(1,2,3,),\ \ B(-1,2,0),\ \ C(3,4,2\ ),\ \ D(-7,0,5)$ nuqtalar berilgan. $\text{пр}_{\overline{BC}}\overrightarrow{AD}$ ni hisoblang.

4. $\vec{a} = 6\vec{i} - 2\vec{j} + 3\vec{k}$ vektorni $\vec{i},\vec{j},\vec{k}$ bazis bo'yicha yoyilmasi ma'lum bo'lsa, bu vektorni ortlar bilan hosil qilgan burchaklarini toping.

5. $\vec{a} = 4\vec{m} + 3\vec{n}$, $\vec{b} = 3\vec{m} - \vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = 2$, $|\vec{n}| = 3$, $(\vec{m}\wedge\vec{n}) = \frac{\pi}{6}$ lar berilgan bo'lsa, $\left|\left[\vec{a}, \vec{b} - \vec{a}\right]\right|$ ni toping.

6. Soddalashtiring: $\left(\vec{a} - 3\vec{b} + \vec{c}, 2\vec{a} + b - 3\,\vec{c}, \vec{a} + 2\vec{b} + \vec{c}\right)$

Javoblar: 1. *J:*$\{7,13,-7\}$. 2. *J:*$\vec{c} = 5\vec{a} - 2\vec{b}$. 3. *J:*$-\sqrt{16}$. 4. *J:* $cos(\vec{a}\wedge\vec{\imath}) = \frac{6}{7}$; $cos(\vec{a}\wedge\vec{\jmath}) = -\frac{2}{7}$; $cos\left(\vec{a}\wedge\vec{k}\right) = \frac{3}{7}$. 5. *J: 21.* 6. *J*: $25\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 26

1. $\vec{a} = \{5,7,2\}$, $\vec{b} = \{3,0,4\}$, $\vec{c} = \{-6,1,-1\}$ vektorlar berilgan.
$\vec{d} = 3\vec{a} - 2\vec{b} - \vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = \vec{\imath} - 7\vec{\jmath}$, $\vec{b} = 4\vec{\imath} - 2\vec{\jmath}$, $\vec{c} = 3\vec{\imath} + 5\vec{\jmath}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $A(2,-5,4)$, $B(1,0,-1)$ nuqtalar berilgan. $\overrightarrow{AB}$ vektorning OY va OZ o'qlari bilan teng $\beta = \gamma = 60°$, OX o'qi bilan esa o'tkir burchak hosil qiluvchi o'qdagi proyeksiyalarini toping.

4. $A(a, 0,0)$, $B(0,0,2a)$, $C(a, 0, a)$ vektorlar berilgan. $\overrightarrow{OC}$ va $\overrightarrow{AB}$ vektorlarni yasang va bu vektorlar orasidagi burchagini toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{\pi}{3}$ ga teng . Agar $|\vec{a}| = 3$, $\left|\vec{b}\right| = 1$ bo'lsa, $\left|\left[3\vec{a} + 2\vec{b}, 2\vec{a} + 2\vec{b}\right]\right|$ ni hisoblang.

6. Soddalashtiring: $\left(\vec{a} - 3\vec{b} + 2\vec{c},\ 2\vec{a} - 3\vec{b} + \ \vec{c}, \vec{a} + \vec{c}\right)$.

Javoblar: 1. *J:*$\{3,22,-3\}$. 2. *J:* $\vec{c} = -\vec{a} + \vec{b}$. 3. *J:*$-\frac{1}{\sqrt{2}}$. 4. *J:* $cos\varphi = \frac{1}{\sqrt{10}}$. 5. *J:* $3\sqrt{3}$. 6. $Javob$: $6\left(\vec{a}, \vec{b}, \vec{c}\right)$.

Variant 27

1. $\vec{a} = \{5,7,2\}$, $\vec{b} = \{3,0,4\}$, $\vec{c} = \{-6,1,-1\}$ vektorlar berilgan.
$\vec{d} = -2\vec{a} + 3\vec{b} + 5\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 5\vec{i} + 2\vec{j}$, $\vec{b} = 3\vec{i} - 4\vec{j}$, $\vec{c} = 16\vec{i} - 4\vec{j}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = -3\vec{m} + 2\vec{n}$ va $\vec{b} = \vec{m} + \vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = 1$, $|\vec{n}| = 2$, $(\vec{m}\hat{}\vec{n}) = \frac{2\pi}{3}$ bo'lsa, $\text{пр}_{\vec{b}}\vec{a}$ ni hisoblang.

4. Uchburchak tomonlari $\overrightarrow{AB} = 2\vec{i} - 6\vec{j}$, $\overrightarrow{BC} = \vec{i} + 7\vec{j}$, $\overrightarrow{CA} = -3\vec{i} - \vec{j}$ vektorlardan iborat bo'lsa, uning B uchidagi burchagini toping.

5. Tomonlari $\vec{a} = 2\vec{j} + \vec{k}$ va $\vec{b} = \vec{i} + 2\vec{k}$ vektorlarga qurilgan parallelogrammning balandligi va yuzini toping.

6. $|\vec{a}| = 3$, $|\vec{b}| = 7, (\vec{a}, \vec{b}) = 9$ lar berilgan. $|[\vec{a}, \vec{b}]|$ ni hisoblang.

7. Soddalashtiring: $(\vec{a} + \vec{b} - 2\vec{c},\ \vec{b} + 5\vec{c}, -\vec{a} + \vec{b} - \vec{c})$.

Javoblar: 1. *J:*$\{-31, -9{,}3\}$. 2. *J:*$\vec{c} = 2\vec{a} + 2\vec{b}$. 3. *J:* $2\sqrt{3}$. 4. *J:* $cos\varphi = 2/\sqrt{5}$. 5. *J:* $S = \sqrt{21}, H = \sqrt{21}/5$. 6. *J:* $6\sqrt{10}$. 7. *Javob*: $-13(\vec{a}, \vec{b}, \vec{c})$.

Variant 28

1. $\vec{a} = \{5,7,2\}$, $\vec{b} = \{3,0,4\}$, $\vec{c} = \{-6, 1, -1\}$ vektorlar berilgan.
$\vec{d} = \vec{a} + 4\vec{b} - 2\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 5\vec{i} + 2\vec{j}$, $\vec{b} = 3\vec{i} - 4\vec{j}$, $\vec{c} = 4\vec{i} + 12\vec{j}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = 3\vec{i} - 5\vec{j} + \vec{k}$, $\vec{b} = -\vec{i} + \vec{j} + 4\vec{k}$, $\vec{c} = 4\vec{i} + 4\vec{j} - 2\vec{k}$ vektorlar berilgan. $\text{пр}_{(\vec{a}+2\vec{b})}\vec{c}$ ni toping.

4. $\overrightarrow{OA} = \vec{A}$ va $\overrightarrow{OB} = \vec{b}$ vektorlar berilgan. Agar $|\vec{a}| = 2$, $|\vec{b}| = 4$, $(\vec{a} \hat{} \vec{b}) = 60°$ bo'lsa, AOB uchburchakning OM medianasi va OA tomoni orasidagi burchagini toping.

5. $\vec{a} = 2\vec{m} + \vec{n}$, $\vec{b} = \vec{m} + 2\vec{n}$ vektorlar berilgan. Agar $|\vec{m}| = 2$, $|\vec{n}| = \sqrt{2}$, $(\vec{m}\hat{}\vec{n}) = \frac{\pi}{4}$ bo'lsa, $|[\vec{a}, \vec{b}] + 5[\vec{m}, \vec{n}] - 3[\vec{n}, \vec{m}]|$ ni toping.

6. Soddalashtiring: $(2\vec{a} - \vec{b} + 2\vec{c},\ \vec{a} - 2b - \vec{c},\ \vec{a} + \vec{b})$.

Javoblar: 1. *J:* $\{29{,}5{,}20\}$. 2. *J:* $\vec{c} = 2\vec{a} - 2\vec{b}$. 3. *J:* $-26/\sqrt{91}$. 4. *J:* $cos\varphi = 2/\sqrt{7}$. 5. *J: 22*. 6. $Javob$: $9(\vec{a}, \vec{b}, \vec{c})$.

Variant 29

1. $\vec{a} = \{2{,}7{,}5\}$, $\vec{b} = \{3{,}0{,}4\}$, $\vec{c} = \{-6, -1{,}3\}$ vektorlar berilgan. $\vec{d} = \vec{a} - 2\vec{b} + 2\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 5\vec{i} + 2\vec{j}$, $\vec{b} = -\vec{i} + 10\vec{j}$, $\vec{c} = 3\vec{i} - 4\vec{j}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $A(3, -2{,}1)$, $B(2{,}1, -1)$ nuqtalar berilgan. $\overline{AB}$ vektorning OY va OZ o'qlari bilan bir hil $\beta = \gamma = 60°$ burchak ,OX o'qi bilan esa o'tmas burchak hosil qiluvchi o'qdagi proyeksiyasini toping.

4. ABC uchburchak uchlari berilgan. $A(2, -2{,}1)$, $B(3{,}0{,}4)$, $C(1{,}1, -3)$. Uning C burchagini toping.

5. ABCD parallelogrammda $\overrightarrow{AB} = \vec{n} - 0{,}5\vec{m}$, $\overrightarrow{AD} = 5{,}5\vec{m}$ vektorlar berilgan. Agar $|\overline{m}| = \sqrt{2}, |\overline{n}| = 2, (\vec{m}\hat{}\vec{n},) = \frac{\pi}{4}$ lar berilgan bo'lsa, $|[\overline{AC}, \overline{BD}]|$ ni hisoblang.

6. Soddalashtiring: $(\vec{a} - \vec{b} + 3\vec{c}, -2\vec{a} + 2\vec{b} + \vec{c},\ 3\vec{a} - 2\vec{b} + 5\vec{c})$.

Javoblar: 1. *J:* $\{-16{,}5{,}3\}$. 2. *J:* $\vec{c} = \frac{1}{2}\vec{a} - \frac{1}{2}\vec{b}$. 3. *J:* $(1 + \sqrt{2})/2$. 4. *J:* $cos\varphi = -5{,}5/\sqrt{39}$. **5**. *J:* 22. **6**. *J:* $-7(\vec{a}, \vec{b}, \vec{c})$.

Variant 30

1. $\vec{a} = \{2{,}7{,}5\}$, $\vec{b} = \{3{,}0{,}4\}$, $\vec{c} = \{-6, -1{,}3\}$ vektorlar berilgan. $\vec{d} = -2\vec{a} + 3\vec{b} + 2\vec{c}$ vektorning koordinatalarini toping.

2. $\vec{a} = 2\vec{i} - 3\vec{j}$, $\vec{b} = 5\vec{i} + 4\vec{j}$, $\vec{c} = \vec{i} + 10\vec{j}$ vektorlarni yasang. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar bo'yicha yoying.

3. $\vec{a} = 3\vec{i} - 5\vec{j} + \vec{k}$, $\vec{b} = -\vec{i} + \vec{j} + 4\vec{k}$, $\vec{c} = 4\vec{i} + 4\vec{j} - 2\vec{k}$ vektorlar berilgan. $\text{пр}_{\vec{c}}(2\vec{a} - \vec{b})$ ni hisoblang.

4. $\overrightarrow{AB} = 2\vec{\imath} - 6\vec{\jmath},\ \overrightarrow{BC} = \vec{\imath} + 7\vec{\jmath},\ \overrightarrow{CA} = -3\vec{\imath} - \vec{\jmath}$ vektorlar uchburchak tomonlari bo'lsa, uchburchakning C uchidagi ichki burchagini toping.

5. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{2\pi}{3}$ ga teng. Agar $|\vec{a}| = 4, |\vec{b}| = 1$ bo'lsa, $\left|\left[3\vec{a} - \vec{b}, \vec{a} - 2\vec{b}\right]\right|$ ni hisoblang.

6. Soddalashtiring: $\left(5\vec{a} + 3\vec{b} - \vec{c},\ \vec{a} +\ 2\vec{b},\ \vec{b} - \vec{c}\right)$.

Javoblar: 1. *J:* $\{-7, -16, 8\}$. **2**. *J:* $\vec{c} = -2\vec{a} + \vec{b}$. **3**. *J:* -2. **4**. *J:* $cos\alpha = \frac{1}{\sqrt{5}}$. *J:* $10\sqrt{3}$. **6**. *J:*$-8\left(\vec{a}, \vec{b}, \vec{c}\right)$.

ANALITIK GEOMETRIYA ELEMENTLARI.

1§.Ikki nuqta orasidagi masofa.

O'qdagi $A(x_1)$ va $B(x_2)$ nuqtalar orasidagi masofa:

$$d = |x_2 - x_1| = \sqrt{(x_2 - x_1)^2} \qquad (1)$$

AB kesmaning (algebraik) kattaligi:

$$AB = x_2 - x_1 \qquad (2)$$

Tekislikdagi $A(x_1; y_1)$ va $B(x_2; y_2)$ nuqtalar orasidagi masofa:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (3)$$

R^3 fazodagi $A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ nuqtalar orasidagi masofa:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \qquad (4)$$

2§.Kesmani berilgan nisbatda bo'lish.

$A(x_1; y_1)$ va $B(x_2; y_2)$ nuqtalar berilgan. AB kesmani $AN{:}NB = \lambda$ nisbatda bo'luvchi $N(x; y)$ nuqtaning koordinatalari;

$$x = \frac{x_1 + \lambda x_2}{1 + \lambda}; \qquad y = \frac{y_1 + \lambda y_2}{1 + \lambda} \qquad (1)$$

Xususiy holda kesmani teng ikkiga, ya'ni λ=1:1=1 nisbatda bo'lish:

$$x = \frac{x_1 + x_2}{2}; \qquad y = \frac{y_1 + y_2}{2} \qquad (2)$$

Uchlari $A(x_1; y_1)$, $B(x_2; y_2)$, $C(x_3; y_3)$,…,M$(x_n; y_n)$ nuqtalarda bo'lgan ko'pburchak yuzi:

$$S=\frac{1}{2}\left|\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\end{vmatrix}+\begin{vmatrix}x_2 & y_2\\ x_3 & y_3\end{vmatrix}+\cdots+\begin{vmatrix}x_n & y_n\\ x_1 & y_1\end{vmatrix}\right| \qquad (3)$$

formula bilan topiladi.

Uchlari $A_1(x_1, y_1), A_2(x_2, y_2), \dots, A_n(x_n, y_n)$, nuqtalarda bo'lgan qavariq ko'p burchak uchlariga mos ravishda $m_1, m_2, \dots, m_n$ og'irliklar qo'yilgan bo'lsa, u holda og'irlik markazi koordinatasi

$$x=\frac{m_1x_1+m_2x_2+\cdots+m_nx_n}{m_1+m_2+\cdots+m_n}; \qquad y=\frac{m_1y_1+m_2y_2+\cdots+m_ny_n}{m_1+m_2+\cdots+m_n} \qquad (4)$$

formula bilan topiladi.

3§.Tekislikda to'g'ri chiziq tenglamasi.

1. To'g'ri chiziqning umumiy tenglamasi.

$$Ax+By+C=0 \qquad (1)$$

Bu yerda A, B, C lar o'zgarmas sonlar.

a) $C=0$ bo'lsa, $y=-\frac{A}{B}x$ –to'g'ri chiziq koordinatalar boshidan o'tadi.

b) $B=0$ bo\`lsa, $x=-\frac{C}{A}$ –to\`g\`ri chiziq $OY$ o\`qqa parallel bo\`ladi.

c) $A=0$ bo\`lsa, $y=-\frac{C}{B}$ –to\`g\`ri chiziq $OX$ o\`qqa parallel bo\`ladi.

d) $A=C=0$ bo\`lsa, $y=0$ –to\`g\`ri chiziq $OX$ o\`qdan iborat bo\`ladi.

e) $B=C=0$ bo\`lsa, $x=0$ to\`g\`ri chiziq $OY$ o\`qdan iborat bo\`ladi.

2. To\`g\`ri chiziqning burchak koeffitsiyentli tenglamasi.

$$y=kx+b \qquad (2)$$

Bu yerda k parametr-to'g'ri chiziqning OX o'qini musbat yo'nalishi bilan hosil qilgan burchakning tangensiga teng, ya'ni $k=tg\alpha$, b –ozod son.

3 .To\`g\`ri chiziqning kesmalar bo\`yicha tenglamasi.

$$\frac{x}{a}+\frac{y}{b}=1 \qquad (3)$$

Bu yerda a va b to\`g\`ri chiziqni OX va OY o\`qlardan mos ravishda ajratgan kesmalari.

4 .Ikki nuqtadan o\`tuvchi to\`g\`ri chiziq tenglamasi.

$A(x_1; y_1)$ va $B(x_2; y_2)$ nutalardan o'tuvchi to'g'ri chiziq tenglamasi;

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \qquad (4)$$

5. Berilgan nuqtadan berilgan yo`nalish bo`yicha o`tuvchi to`g`ri chiziq tenglamasi;

$A(x_0; y_0)$ nuqtadan o'tuvchi va OX o'qini musbat yo'nalishi bilan α burchak hosil qiluvchi ($k = tg\alpha$) to'g'ri chiziq tenglamasi;

$$y - y_0 = k(x - x_0) \qquad (5)$$

6. Ikki to`g`ri chiziq orasidagi burchak.

$y_1 = k_1 x + b_1$ to'g'ri chiziqdan $y_2 = k_2 x + b_2$ to'g'ri chiziqqacha soat strelkasiga qarama-qarshi yo'nalishda hisoblanuvchi φ burchak

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 \cdot k_2} \qquad (6)$$

formula bilan topiladi.

To'g'ri chiziqlar umumiy ko'rinishda

$$A_1 x + B_1 y + C_1 = 0 \;\; va \;\; A_2 x + B_2 y + C_2 = 0$$

berilsa, ikki to`g`ri chiziq orasidagi burchak ularning normal vektorlari $\overrightarrow{n_1}(A_1; B_1)$ va $\overrightarrow{n_2}(A_2; B_2)$ orasidagi burchak

$$cos\varphi = \frac{\overrightarrow{n_1} \cdot \overrightarrow{n_2}}{|\overrightarrow{n_1}| \cdot |\overrightarrow{n_2}|} = \frac{A_1 \cdot A_2 + B_1 \cdot B_2}{\sqrt{A_1^2 + B_1^2} \cdot \sqrt{A_2^2 + B_2^2}} \qquad (7)$$

ga teng bo`ladi.

Ikki to'g'ri chiziqning parallellik sharti:

$$k_1 = k_2 \;\; yoki \quad \frac{A_1}{A_2} = \frac{B_1}{B_2} \qquad (8)$$

Ikki to'g'ri chiziqning perpendikulyarlik sharti:

$$k_1 \cdot k_2 = -1 \qquad yoki \qquad A_1 \cdot A_2 + B_1 \cdot B_2 = 0 \qquad (9)$$

7. Parallel bo'lmagan ikki $A_1 x + B_1 y + C_1 = 0$ va $A_2 x + B_2 y + C_2 = 0$ to'g'ri chiziqlarning kesishish nuqtasini topish uchun ularning tenglamalarini birgalikda yechish kerak.

$$x=\frac{\begin{vmatrix}-C_1 & B_1\\ -C_2 & B_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}};\qquad y=\frac{\begin{vmatrix}A_1 & -C_1\\ A_2 & -C_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}} \qquad (10)$$

$A(x_0;y_0)$ nuqtadan $Ax+By+C=0$ to'g'ri chiziqqacha bo'lgan masofa;

$$d=\left|\frac{Ax_0+By_0+C}{\sqrt{A^2+B^2}}\right| \qquad (11)$$

$Ax+By+C=0\ va\ Ax_1+By_1+C_1=0$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamalari;

$$\frac{Ax+By+C}{\sqrt{A^2+B^2}}=\pm\frac{Ax_1+By_1+C_1}{\sqrt{A_1^2+B_1^2}} \qquad (12)$$

FAZODA TO`G`RI CHIZIQ VA TEKISLIK TENGLAMALARI.

1-§.Tekislik tenglamasi.

1-ta'rif. Quyidagi

$$Ax+By+Cz+D=0 \qquad (1)$$

tenglama ***tekislikning umumiy tenglamasi*** deyiladi.

Bu yerda $A,B,C\ va\ D$ berilgan sonlar bo'lib, A,B,C lar noma'lumlar oldidagi koeffitsient, D ozod son deyiladi.

$\vec{n}(A;B;C)$ vektor tekislikka perpendikulyar bo'lib, uning normal (yo'naltiruvchi) vektori deyiladi.

1^0.1. Agar $D=0$ bo`lsa, $Ax+By+Cz=0$ tekislik koordinatalar boshidan o`tadi.

2.a) Agar $A=0$ bo`lsa, $By+Cz+D=0$ tekislik $OX$ o`qiga parallel bo`ladi.

b) Agar $B=0$ bo`lsa, $Ax+Cz+D=0$ tekislik $OY$ o`qiga parallel bo`ladi.

c) Agar $C=0$ bo`lsa, $Ax+By+D=0$ tekislik $OZ$ o`qiga parallel bo`ladi.

3.a) Agar $A=D=0$ bo`lsa, $By+Cz=0$ tekislik $OX$ o`qidan o`tadi.

b) Agar $B=D=0$ bo`lsa, $Ax+Cz=0$ tekislik $OY$ o`qidan o`tadi.

c) Agar $C=D=0$ bo`lsa, $Ax+By=0$ tekislik $OZ$ o`qidan o`tadi.

4.a) Agar $A=B=0$ bo`lsa, $Cz+D=0$ tekislik $XOY$ tekislikka parallel bo`ladi.

b) Agar $A = C = 0$ bo`lsa, $By + D = 0$ tekislik $XOZ$ tekislikka parallel bo`ladi.

c) Agar $B = C = 0$ bo`lsa, $Ax + D = 0$ tekislik $YOZ$ tekislikka parallel bo`ladi.

2^0. Koordinata tekisliklarining tenglamalari: $x = 0,\ y = 0\ va\ z = 0$.

3^0.Tekislikning koordinata o'qlaridan ajratgan kesmalar bo'yicha tenglamasi:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad (2)$$

4^0. Berilgan uchta $M_1(x_1, y_1, z_1),\ M_2(x_2, y_2, z_2)\ va\ M_3(x_3, y_3, z_3)$ nuqtalardan o'tuvchi tekislik tenglamasi:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \qquad (3)$$

5^0. $M_0(x_0, y_0, z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tekislikkacha bo'lgan masofa:

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} \qquad (4)$$

6^0. $A_1x + B_1y + C_1z + D_1 = 0\ va\ A_2x + B_2y + C_2z + D_2 = 0$ tekisliklar orasidagi burchak, ularning normal $\vec{m}(A_1; B_1; C_1)\ va\ \vec{n}(A_2; B_2; C_2)$ vektorlari orasidagi burchakka teng:

$$cos\varphi = \frac{\vec{m} \cdot \vec{n}}{|\vec{m}| \cdot |\vec{n}|} = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}} \qquad (5)$$

a) tekisliklarning parallellik sharti:

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \qquad (6)$$

b) tekisliklarning perpendikulyarlik sharti:

$$A_1A_2 + B_1B_2 + C_1C_2 = 0 \qquad (7)$$

2§.Fazoda to'g'ri chiziq.

1^0. $M(x_0, y_0, z_0)$ nuqtadan o'tuvchi va $\vec{s}(m; n; p)$ yo'naltiruvchi vektorga ega bo'lgan to'g'ri chiziqning kanonik tenglamasi:

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} = \frac{z - z_0}{p} \qquad (1)$$

2^0. (1) tenglamadagi har bir nisbatni t parametrga tenglab, to'g'ri chiziqning

$$\begin{cases} x = mt + x_0 \\ y = nt + y_0 \\ z = pt + z_0 \end{cases} \qquad (2)$$

parametrik tenglamasini hosil qilamiz.

3^0. Berilgan $M(x_1, y_1, z_1)\ va\ N(x_2, y_2, z_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \qquad (3)$$

4^0. Fazodagi to'g'ri chiziqning umumiy tenglamasi:

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0 \\ A_2x + B_2y + C_2z + D_2 = 0 \end{cases} \qquad (4)$$

bu yerda

$$\frac{A_1}{A_2} \neq \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$$

Bu to'g'ri chiziqning yo'naltiruvchi vektori

$$\vec{s} = \vec{m} \times \vec{n} = \begin{vmatrix} i & j & k \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} \qquad (5)$$

5^0. (4) tenglamadan bir marta y ni, ikkinchi marta x yo'qotib, to'g'ri chiziqning parametrlari bo'yicha yozilgan tenglamasiga ega bo'lamiz:

$$\begin{cases} x = mz + x_0 \\ y = nz + y_0 \end{cases} \qquad (6)$$

(6) tenglamani ushbu

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} = \frac{z - z_0}{1}$$

kanonik ko'rinishda yozish mumkin.

6^0. $\frac{x - x_1}{m_1} = \frac{y - y_1}{n_1} = \frac{z - z_1}{p_1}\ \ va\ \ \frac{x - x_2}{m_2} = \frac{y - y_2}{n_2} = \frac{z - z_2}{p_2}$.

to'g'ri chiziqlar orasidagi burchak, ularning yo'naltiruvchi vektorlari $\vec{s_1}(m_1; n_1; p_1)\ \ va\ \ \vec{s_2}(m_2; n_2; p_2)$ orasidagi burchakka teng:

$$cos\varphi = \frac{\vec{s_1} \cdot \vec{s_2}}{|\vec{s_1}| \cdot |\vec{s_2}|} = \frac{m_1m_2 + n_1n_2 + p_1p_2}{\sqrt{m_1^2 + n_1^2 + p_1^2}\sqrt{m_2^2 + n_2^2 + p_2^2}} \qquad (7)$$

a) Ikki to'g'ri chiziqning perpendikulyarlik sharti:

$$\overrightarrow{s_1}\cdot\overrightarrow{s_2}=0 \ \ yoki \quad m_1m_2+n_1n_2+p_1p_2=0 \tag{8}$$

b) Ikki to'g'ri chiziqning parallellik sharti:

$$\frac{m_1}{m_2}=\frac{n_1}{n_2}=\frac{p_1}{p_2} \tag{9}$$

c) Ikki to'g'ri chiziqning ustma-ust tushish sharti:

$$\frac{m_1}{m_2}=\frac{n_1}{n_2}=\frac{p_1}{p_2} \quad va \quad \frac{x_2-x_1}{m_1}=\frac{y_2-y_1}{n_1}=\frac{z_2-z_1}{p_1} \tag{10}$$

d) Ikki to'g'ri chiziqning kesishish sharti:

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix}=0 \tag{11}$$

e) Ikki to'g'ri chiziqning ayqash bo'lishlik sharti:

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix}\neq 0 \tag{12}$$

7^0. $a)$. $M_1(x_1,y_1,z_1)$ nuqtadan $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$

to'g'ri chiziqqacha bo'lgan masofa:

$$d=\frac{|\vec{s}\times\overrightarrow{M_1M_0}|}{|\vec{s}|}=\frac{\sqrt{\begin{vmatrix} m & n \\ x_0-x_1 & y_0-y_1 \end{vmatrix}^2+\begin{vmatrix} n & p \\ y_0-y_1 & z_0-z_1 \end{vmatrix}^2\begin{vmatrix} p & m \\ z_0-z_1 & x_0-x_1 \end{vmatrix}^2}}{\sqrt{m^2+n^2+p^2}} \tag{13}$$

bu yerda $M_0(x_0,y_0,z_0)$ to'g'ri chiziqqa tegishli nuqta va $\vec{s}(m;n;p)$ vektor to'g'ri chiziqning yo'naltiruvchi vektori.

$$b). \ \frac{x-x_1}{m_1}=\frac{y-y_1}{n_1}=\frac{z-z_1}{p_1} \ \ va \ \frac{x-x_2}{m_2}=\frac{y-y_2}{n_2}=\frac{z-z_2}{p_2}$$

ikki ayqash to'g'ri chiziqlar orasidagi eng qisqa masofa:

$$d=\frac{|\overrightarrow{M_1M_2}\cdot\overrightarrow{s_1}\cdot\overrightarrow{s_2}|}{|\overrightarrow{s_1}\times\overrightarrow{s_2}|}=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix}}{\sqrt{\begin{vmatrix} m_1 & n_1 \\ m_2 & n_2 \end{vmatrix}^2+\begin{vmatrix} n_1 & p_1 \\ n_2 & p_2 \end{vmatrix}^2+\begin{vmatrix} p_1 & m_1 \\ p_2 & m_2 \end{vmatrix}^2}} \tag{14}$$

bu yerda $M_1(x_1,y_1,z_1)$ va $M_2(x_2,y_2,z_2)$ nuqtalar mos ravishda to'g'ri chiziqlarga tegishli, $\overrightarrow{s_1}(m_1;n_1;p_1)$ va $\overrightarrow{s_2}(m_2;n_2;p_2)$ lar esa ularning yo'naltiruvchi vektorlari.

3§.To'g'ri chiziq va tekislik.

1^0. $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ to'g'ri chiziq va $Ax+By+Cz+D=0$ tekislik orasidagi burchak:

$$sin\varphi=\frac{Am+Bn+Cp}{\sqrt{A^2+B^2+C^2}\sqrt{m^2+n^2+p^2}} \qquad (1)$$

Bu yerda $\vec{s}(m;n;p)$ –to'g'ri chiziqning yo'naltiruvchi vektori, $\vec{k}(A;B;C)$-tekislikning normal vektori.

a) To'g'ri chiziq va tekislikning parallellik sharti:

$$Am+Bn+Cp=0 \qquad (2)$$

b) Ularning perpendikulyarlik sharti:

$$\frac{A}{m}=\frac{B}{n}=\frac{C}{p} \qquad (3)$$

c) To'g'ri chiziqning tekislikda yotish sharti:

$$Am+Bn+Cp=0 \quad va \quad Ax_0+By_0+Cz_0+D=0 \quad (4)$$

d) To'g'ri chiziq bilan tekislikning kesishgan nuqtasi. To'g'ri chiziq tenglamalarini parametrik $x=mt+x_0$, $y=nt+y_0$, $z=pt+z_0$ ko'rinishda yozib, tekislikning $Ax+By+Cz+D=0$ tenglamasidagi x; y; z larni o'rniga qo'yamiz. Natijada t ga nisbatan tenglama hosil bo'ladi. Hosil bo'lgan tenglamadan t_0 ni topib, so'ngra kesishgan nuqta koordinatalari $(x_1;y_1;z_1)$ topiladi.

e) Ikki to'g'ri chiziqning bir tekislikda yotish sharti:

$$\begin{vmatrix} x_1-x_0 & y_1-y_0 & z_1-z_0 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix}=0 \qquad (5)$$

Misollar.

1. Koordinatalar boshidan $A(-5;12)$ nuqtagacha bo'lgan masofani toping.

$j: d=13.$

2. $A(3;1)$ va $B(5;4)$ nuqtalar orasidagi masofani toping.

$j: d=4\sqrt{2}.$

3. Uchlari $A(1;0)$, $B(1;3)$ va $C(6;3)$ nuqtalarda bo'lgan uchburchak tomonlari uzunliklarini toping va grafigini yasang.

$j: AB = 3, BC = 5, AC = \sqrt{34}.$

4. Koordonatalar boshidan va $A(6;0)$ nuqtadan 5 birlik masofada yotuvchi nuqtalar topilsin.

j: $A(3;4), B(3;-4)$.

5. Abstsissalar o'qida $A(0;3)$ nuqtadan 5 birlik masofada yotuvchi nuqtalar topilsin.

j: $A(-4;0), B(4;0)$.

6. $A(1;1)$ va $B(5;4)$ nuqtalar hamda OX o'qqa nisbatan simmetrik bo'lgan A_1, B_1 nuqtalar yasalsin. Bu nuqtalarni tutashtirish natijasida hosil bo'lgan ABA_1B_1 trapetsiyaning tomonlar uzunliklarini toping.

j: $AA_1 = 2, AB = 5, BB_1 = 8, A_1B_1 = 5.$

7. Uchlari $A(1;1)$ va $B(10;4)$ nuqtalarda bo'lgan AB kesmani $2:1$ nisbatda bo'luvchi nuqtani toping.

j: $(3;3)$

Ko'rsatma. Uchburchakning og'irlik markazi medianalar kesishish nuqtasida yotadi.

8. Uchlari $A(-3;2)$ va $B(5;-4)$ nuqtalarda bo'lgan kesmani teng ikkiga bo'luvchi $N(x;y)$ nuqtaning koordinatasini toping.

j: $N(1;-1)$

9. Uchlari $A(-1;-1)$ va $B(4;-6)$ nuqtalarda bo'lgan kesmani 3:2 nisbatda bo'luvchi nuqtaning koordinatalari yig'indisini toping.

j: -2

10. Uchlari $A(-2;4)$, $B(3;-1)$ va $C(2;3)$ nuqtalarda bo'lgan uchburchak uchlariga mos ravishda 60, 40 va 100 kg toshlar (og'irliklar) osilgan. Og'irlik markazi $M(x;y)$ ni toping.

j: $M(1;2{,}5)$

11. Uchlari $A(5;-3)$, $B(-1;-1)$ va $C(7;1)$ nuqtalarda bo'lgan uchburchak tomonlarining o'rtalarini toping.

j: $(2;-2),(3;0),(6;-1)$

12. Uchlari $O(0;0)$, $A(8;0)$ va $B(0;6)$ nuqtalarda bo'lgan uchburchakning OC medianasi uzunligini toping.

j; 5

13. Uchlari $A(1;-1)$, $B(6;4)$ va $C(2;6)$ nuqtalarda bo'lgan uchburchakning og'irlik markazini toping.

j: $(3;3)$

Ko'rsatma. Uchburchakning og'irlik markazi medianalar kesisishish nuqtasida yotadi.

14. Uchlari $A(2;0)$, $B(5;3)$ va $C(2;6)$ nuqtalarda bo'lgan uchburchakning yuzini hisoblang.

j: $9\ kv\ birlik$

15. $A(-1;2)$, $B(1;3)$ va $C(5;5)$ nuqtalarning bir to'g'ri chiziqda yotishini ko'rsating.

16. Uchlari $A(3;1)$, $B(4;6)$ va $C(6;3)$ nuqtalarda bo'lgan to'rtburchakning yuzini hisoblang.

j: $13\ kv\ birlik$

17. $F(2;2)$ nuqtadan va OX o'qdan teng uzoqlashgan nuqtalar geometrik o'rnining tenglamasini tuzingva grafigini chizing.

18. OY o'qdan $b=5$ kesma ajratib, OX o'q bilan 1) 30^0 ;2) 45^0; 3) 60^0 burchak tashkil qiluvchi to'g'ri chiziqlarning tenglamasini tuzing va grafigini yasang.

$$j:\ y_1 = \frac{1}{\sqrt{3}}x + 5;\ y_2 = x + 5;\ y_3 = \sqrt{3}x + 5$$

19. Koordinatalar boshidan o'tib, OX o'qi bilan; 1) 45^0; 2) 90^0; 3) 120^0 burchak tashkil qiluvchi to'g'ri chiziqlarning tenglamasini tuzing va grafigini yasang.

j: 1) $y = x$; 2) $x = 0$; 3) $y = -\sqrt{3}x$.

20. OX o'qidan 5 birlik va OY o'qidan 4 birlik ajratuvchi to'g'ri chiziqlar tenglamalarini tuzing va grafigini chizing.

$$j:\quad \frac{x}{5}+\frac{y}{4}=1;\ \frac{x}{5}+\frac{y}{4}=-1$$

21. 1) $2x+3y=6$; 2) $x-3y=4$ to'g'ri chiziq tenglamalarini o'qlardan ajratgan kesmalar bo'yicha yozing.

$$J:\ \frac{x}{3}+\frac{y}{2}=1;\ \frac{x}{4}+\frac{y}{\left(-\frac{4}{3}\right)}=1$$

22. Koordinatalar boshidan va $A(3;-4)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzing va grafigini yasang.

$$J: 4x+3y=0$$

23. Uchlari $A(1;2)$, $B(4;4)$, $C(7;0)$ nuqtalarda bo'lgan uchburchak tomonlarining tenglamasini tuzing va grafigini yasang.

$$j: AB: 2x-3y+4=0\,; BC: 4x+3y-28=0\,; AC: x+3y-7=0$$

24. $A(-3;1)$ nuqtadan o'tuvchi va OX o'qining musbat yo'nalishi bilan 1) 30^0; 2) 45^0; 3) 60^0 tashkil etuvchi to'g'ri chiziq tenglamasini tuzing va grafigini yasang.

$$j: 1).\ y=\frac{1}{\sqrt{3}}x+\sqrt{3+1};\ \ 2).\ y=x+4;\ \ 3).\, y=\sqrt{3}x+3\sqrt{3}+1$$

25. Quyidagi to'g'ri chiziqlar orasidagi burchakni toping:

1) $y=2x+3\ \ va\ \ y=-\frac{1}{2}x+4$; 2) $5x-y=-7\ \ va\ \ 2x-3y=-1$;

3) $2x+y=0\ \ va\ \ 3x-y-4=0$; 4) $x+2y=0\ \ va\ \ 2x+4y=7$;

$$j;\ 1)\ 90^0, 2)\ 45^0;\ 3)\ 45^0; 4)\ 0$$

26. 1) $3x-2y-5=0$, 2). $6x-4y+1=0$, 3). $6x+4y-3=0$, 4). $2x+3y-6=0$ to'g'ri chiziqlardan parallel va perpendikulyar bo'lganlarini ko'rsating.

J: 1 va 2-to'g'ri chiziqlar parallel, 1 va 4-to'g'ri chiziqlar hamda 2 va 4-to'g'ri chiziqlar perpendikulyar.

27. Uchlari $A(1;2)$, $B(4;5)$ va $C(7;2)$ nuqtalarda bo'lgan uchburchak tomonlari tenglamalarini tuzing va uning ichki burchaklarini toping.

j: $AB: x - y + 1 = 0$; $BC: x + y - 9 = 0$; $AC: y = 2$

$< A = < C = 45^0; < B = 90^0$

28. Uchlari $A(-1; 1)$, $B(3; 3)$ va $C(5; 0)$ nuqtalarda bo'lgan uchburchakning medianalari tenglamalarini yozing.

j: $AN: x - 10y + 16 = 0$; $BM: 5x - 2y - 9 = 0$; $CK: x + 2y - 5 = 0$

29. Uchlari $A(-2; 0)$, $B(2; 4)$ va $C(3; -1)$ nuqtalarda bo'lgan uchburchakning balandliklari tenglamalarini tuzing.

j: $AN: y = \frac{1}{5}x + \frac{2}{5}$; $BM: y = 5x - 6$; $CK: y = -x + 2$.

30. $x + y - 4 = 0$ va $2x - 2y + 5 = 0$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamasini tuzing.

j: $y = \frac{3}{4}; y = 3\frac{1}{4}$

Mustaqil yechish uchun misollar.

Variant 1

1. Uchburchak uchlari berilgan: $A(1; 2; -1,)$, $B(3; 4; 0)$, $C(2;\ 5; -2\ )$. A uchidagi ichki burchagi kosinusini toping.

2. $A(3; 0; -3)$, $B(1; 2; 3)$, $C(2; -2; 1\ )$ nuqtalar berilgan. ABC uchburchakning yuzini toping.

3. $M(1; 1)$ nuqtadan o'tib, $a)\ x + 2y + 2 = 0$ to'g'ri chiziqqa parallel; $b)$ $x + 2y + 2 = 0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(-8; -2)$, $B(2; 1; 0)$, $C(4; 4\ )$. a) A uchidan tushirilgan balandlik tenglamasini, b) C uchidan tushirilgan mediana tenglamasini, c) B burchakni toping.

5. M(5;-2) nuqtani l: $2x - 3y - 3 = 0$ to'g'ri chiziqqa proyeksiyasini toping.

6. Parallelogrammnining ikkita tomoni tenglamasi berilgan: $3x - 2y + 12 = 0$ va $x - 3y + 11 = 0$ va diagonallari kesishish nuqtasi $(\ 2; 2)$. Uning qolgan ikki tomoni va diagonallari tenglamasini tuzing.

7. Oz o'qiga parallel va $M(\,2,-1,3)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(\,3;-6;2)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$x-2y+z-1=0,\ 2\,x-4y+2z-1=0, 3\,x+4y-z+2=0.$

10. $\begin{cases} 2x-3y-3z-9=0 \\ x-2y+z+3=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(-1;3;9)$ nuqtadan o'tib, $\vec{q}\{2;-4;7\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(3;-1;4)$ nuqtani $2x+y-z+5=0$ tekislikka proyeksiyasini toping.

13. $\frac{x-1}{2}=\frac{y+2}{1}=\frac{z}{1}$ to'g'ri chiziq va $x+2y-z+5=0$ tekislik orasidagi burchakni toping.

14. $\frac{x+1}{1}=\frac{y}{1}=\frac{z-1}{2}$, $\frac{x}{1}=\frac{y+1}{3}=\frac{z-2}{4}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: **1**. J : $\cos a=\frac{7}{3\sqrt{11}}$ **2** J :$\sqrt{10}$. **3**. J : $a)\ x+2y-3=0,\ \ b)\ 2x-y-1=0$. **4**. J : $a)\ \ x-3y+2=0,\ \ b)\ y-4=0\ ,\ \ c)\ arccos\left(\frac{13}{\sqrt{610}}\right)$. **5**. J : $(\,3;1)$. **6**. J: $3x-2y-16=0, x-3y-3=0,\ x+4y-10=0,\ 5x-8y+6=0.$ **7**. J: $x-2=0,\ y+1=0$. **8**. J: $3x-6y+2z-49=0$. **9**. J:$d=\frac{1}{2\sqrt{6}}$ **10**. J: $\frac{x}{y}=\frac{y}{5}=\frac{z+3}{1}$. **11**. J: $\frac{x+1}{2}=\frac{y-3}{-4}=\frac{z+9}{7}$. **12**. J: $(1;-2;5)$. **13** J: $\frac{\pi}{6}$. **14** J: $d=\frac{1}{\sqrt{5}}$..

Variant 2

1. $A(2;\ -2;1),\ \ B(1;1;1),\ \ C(7;3;2\ )$ nuqtalar berilgan. ABC uchburchak yuzini toping.

2. Piramida uchlarining koordinatalari berilgan : $A(1;2;-1),\ B(0;1;5),\ C(0;4;-1\,), D(3;2;1)$. Piramidaning hajmini ABD qirraga tushirilgan balandlik uzunligini toping..

3. $M_0(2;1)$ nuqtadan o'tib, a) $2x+3y+4=0$ to'g'ri chiziqqa parallel; b) $2x+3y+4=0$ ga perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(-5;-2)$, $B(7;6)$, $C(5;-4\,)$.a) C uchidan tushirilgan balandlik tenglamasini, b) A uchidan tushirilgan mediana tenglamasini, c) B va C burchaklarini toping.

5. M(-1;-5) nuqtani l: $4x + 7y - 26 = 0$ to'g'ri chiziqqa proyeksiyasini toping.

6. ABCD parallelogrammning $A(2;5)$ va $B(5;3)$ uchlari va diagonallarini kesishish nuqtasi $M(-2;0)$ berilgan. Parallelogramm tomonlari tenglamasini tuzing.

7. $M(-1;2;3)$ nuqtadan o'tib, $M_1(1;0;-2)$, $M_2(3;4;5)$, $M_3(-1;2;0)$ nuqtalardan o'tuvchi tekislikka parallel tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(\,4;-3;12)$ bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$2\,x - 3y + 6z - 21 = 0;\ \ x + y + z = 0;\ 4\,x - 6y + 12z + 35 = 0;$

10. $\begin{cases} x - 2y + 3z - 4 = 0 \\ 3x + 2y - 5z - 4 = 0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(3;0;-2)$ nuqtadan o'tib, $\vec{q} = \{2;1;1\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(1;-3;2)$ nuqtani $2x + 5y - 3z - 19 = 0$ tekislikka proyeksiyasini toping.

13. $\frac{x}{2} = \frac{y-5}{2} = \frac{z+1}{-1}$ to'g'ri chiziq va $4x + y + z - 3 = 0$ tekislik orasidagi burchakni toping.

14. $\frac{x-2}{1} = \frac{y+2}{-3} = \frac{z+1}{-2}\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{1}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: **1**. *J:* 0,5 $\sqrt{410}$. **2.** *J* $:V = 5,\ \ H = \frac{15}{\sqrt{51}}$ **.3**. *J* : $a)\ 2x + 3y - 7 = 0$; $b)\ 3x - 2y - 4 = 0$. **4**. *J* : $a)\ 3x + 2y - 7 = 0$ $b)\ 3x - 11y - 7 = 0$ $c)\ 45^0, 90^0$. **5**. *J :* $(\,3;2)$. **6**. *J :* AB: $2x + 3y - 19 = 0$, BC: $8x - 11y - 7 = 0$,CD: $2x + 3y + 27 = 0$, AD: $8x - 11y + 39 = 0$. **7.** *J:* $x + 3y - 2z =$

0. **8.** *J:* $4x-3y+12z-169=0$. **9**. *J:* $d=5{,}5$. **10.** *J:* $\frac{x}{2}=\frac{y+8}{7}=\frac{z+4}{4}$. **11.** *J:* $\frac{x-3}{2}=\frac{y}{1}=\frac{z+2}{1}$. **12.** *J:* $(3;2;-1)$. **13.** *J:* $\frac{\pi}{4}$. **14**. *J:* $d=\frac{4}{\sqrt{26}}$.

Variant 3

1. $A(5;-4;1)$, $B(3;2;3)$, $C(1;-1;-2\,)$ nuqtalar berilgan. Uchlari berilgan nuqtalardan iborat uchburchak yuzini toping..

2. Uchlari $A(7;12;3)$, $B(3;0;1)$, $C(2;1;1\,)$ nuqtalarda bo'lgan tetraedrning hajmi 10 ga teng. Agar tetraedrning D uchi OY o'qida yotsa, D nuqtaning koordinatasini toping.

3. $M_0(2;-1)$ nuqtadan o'tib, a) $2x+3y=0$ to'g'ri chiziqqa parallel , b) $2x+3y=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlari berilgan: $A(-4;-5)$, $B(4;1)$, $C\left(-\frac{1}{2};7\right)$.a) C uchidan tushirilgan balandligi tenglamasini; b) A uchidan o'tkazilgan medianasi tenglamasini ; c) A ichki burchagi bissektrisasi tenglamasini tuzing.

5. $M(-8;12)$ nuqtani $A(2;-3)$, $B(-5;1)$ nuqtalardan o'tuvchi to'g'ri chiziqqa proyeksiyasini toping

6. ABCD parallelogrammning qo'shni uchlari berilgan: $A(1;-2)$, $B(3;2)$ va $M(1;1)$ nuqta diagonallari kesishish nuqtasi bo'lsa, parallelogramm tomonlari tenglamasini tuzing.

7. $M(-2;7;3)$ nuqtadan o'tib, $x-4y+5z-1=0$ tekislikka parallel tekislik tenglamasini tuzing,

8. Koordinata boshidan tushirilgan perpendikulyar asosi berilgan: $P(3;-2;4)$. Shu nuqtadan o'tuvchi tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$x-2y+z-1=0, \qquad x+2y+3z+1=0, \qquad 2\,x-4y+2z-1=0$

10. $\begin{cases} 3x-5y+z-8=0 \\ 2x+y-z+2=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(1;0;-2)$ nuqtadan o'tib, $\vec{q}\{2;1;0\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(1;-3;2)$ nuqtaning $6x+3y-z-41=0$ tekislikdagi proyeksiyasini toping.

13. $\frac{x+3}{1}=\frac{y-2}{-2}=\frac{z+1}{2}$ to'g'ri chiziq va $4x+2y+2z-5=0$ tekislik orasidagi burchakni toping.

14. $\frac{x-2}{4}=\frac{y+1}{1}=\frac{z-1}{-1}$va $\frac{x+4}{2}=\frac{y-2}{-2}=\frac{z+2}{-3}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. $J:\sqrt{274}$. 2. $J:D_1(0;25;0)D_2(0;-35;0)$. 3. *J:* a) $2x+3y-1=0$ $b)$ $3x-2y-8=0$. 4. J $:a)$ $4x+3y-19=0$, $b)$ $36x-23y+29=0$, $c)$ $13x-9y+7=0$. 5. $J:(1;2;5)$. 6. $J:$ $AB: 2x-y-4=0$, $BC: x+y+5=0$, $CD: 2x-y+2=0$, $AD: x+y+1=0$. 7. $J: x-4y+5z+15=0$. 8. $J: 3x-2y+4z-29=0$. 9. $J: d=\frac{1}{2\sqrt{6}}$. 10. $J:\frac{x-2}{4}=\frac{y-1}{5}=\frac{z-7}{13}$.
11. $J:\begin{cases} x-2y-z=0 \\ z+2=0 \end{cases}$. 12. $J:(7;0;1)$. 13. $J: arcsin\left(\frac{2}{3\sqrt{6}}\right)$. 14. $J: d=6$.

Variant 4

1. Uchburchak uchlari berilgan: $A(5;-6;2)$, $B(1;3;-1)$, $C(1;-1;2)$. A uchidan BC tomoniga tushirilgan balandlikni toping.

2. Piramida uchlari berilgan: $A(2;0;0)$, $B(0;0;3)$, $C(0;0;6)$, $D(2;3;8)$. Piramida hajmini va D uchidan tushirilgan balandligini toping.

3. $M_0(-1;3)$ nuqtadan o'tib, a) $\frac{x-1}{2}=\frac{y+1}{3}$ to'g'ri chiziqqa parallel , b) $\frac{x-1}{2}=\frac{y+1}{3}$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlari berilgan: $A(4;8)$, $B(2;-10)$, $C(-6;-2)$. a) B uchidan tushirilgan balandligi tenglamasini; b) A uchidan o'tkazilgan medianasi tenglamasini; c) C ichki burchagi bissektrisasi tenglamasini tuzing.

5. $M(-4;6)$ nuqtaning $l: 4x-5y+3=0$ to'g'ri chiziqqa proyeksiyasini toping.

6. Uchlari $A(-2;-2)$, $B(-3;1)$, $\left(\frac{5}{2};\frac{5}{2}\right)$, $D(3;1)$ nuqtalarda bo'lgan to'rtburchak trapetsiya ekanligini ko'rsating va shu trapetsiyaning o'rta chizig'i va diagonallari tenglamasini tuzing.

7. $M_0(-3;-2;4)$ nuqtadan o'tib, $x-2y-3z+5=0$ tekislikka parallel tekislik tenglamasini tuzing.

8. Koordinata boshidan tushirilgan perpendikulyar asosi berilgan: $P(1;2;3)$. Shu nuqtadan o'tuvchi tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:
$2x-3y+6z-21=0, 4x-6y+12z+35=0, 7x-3y+z-15=0$

10. $\begin{cases} 2x-y+3z-5=0 \\ 4x+3y-2z+8=0 \end{cases}$ to'g'ri chiziqni kanonik ko'rinishga keltiring.

11. $M(1;0;-2)$ nuqtadan o'tib, $\vec{q}\{5;-1;8\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(5;2;-1)$ nuqtaning $2x-y+3z+23=0$ tekislikka proyeksiyasini toping.

13. $\frac{x}{2}=\frac{y+12}{3}=\frac{z-4}{6}$ to'g'ri chiziq va $6x+15y-10z=0$ tekislik orasidagi burchakni toping.

14. $\frac{x}{1}=\frac{y+3}{2}=\frac{z-2}{1}$va $\frac{x-3}{1}=\frac{y+2}{2}=\frac{z-2}{1}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J* : 5. 2. *J* : $V=14$, $H=\sqrt{14}$. 3. *J* : $a)\ 3x-2y+9=0$, $b)\ 2x+3y-7=0$. 4. *J* : $a) x+y+8=0$, $b)\ 7x-3y-4=0$, $c)\ y+2=0$. 5. *J* : $(-2;-1)$. 6. *J* : $AB \parallel CD$, $3x+y-1=0, x-y=0, y-1=70$. 7. *J* : $x-2y-3z+11=0$. 8. *J* : $x+2y+3z-14=0$. 9. *J*: $d=5{,}5$. 10. *J* : $\frac{x}{-7}=\frac{y+1}{16}=\frac{z-1}{10}$. 11. *J* : $\frac{x-1}{5}=\frac{y+1}{-1}=\frac{z}{8}$. 12. *J* : $(1;4;7)$.
13. *J*: $arcsin(3/133)$. 14. $d=5\sqrt{5}/\sqrt{6}$.

Variant 5

1. Uchburchak uchlari $A(2;-2;1)$, $B(3;4;0)$, $C(1;1;-3)$ nuqtalarda yotsa, A uchidagi ichki burchagini toping.

2. $A(5;-6;2)$, $B(1;3;-1)$, $C(1;-1;2)$ nuqtalar uchburchak uchlari bo'lsa, B uchidan AC tomonga tushirilgan balandlik uzunligini toping.

3. $M(2;3)$ nuqtadan o'tib, a) $5x-y+3=0$ to'g'ri chiziqqa parallel; c) $0{,}5x-y+3=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(4;8)$, $B(2;-10)$, $C(-6;-2)$. a) B uchidan tushirilgan balandlik tenglamasini, b) A uchidan tushirilgan mediana tenglamasini, c) B burchakni toping.

5. $M(1;2)$ nuqtaning $l: 2x-y-5=0$ to'g'ri chiziqdagi proyeksiyasini toping.

6. Parallelogrammning bir uchidan chiqqan ikkita tomoni tenglamasi berilgan: $5x-3y+28=0$,$x-3y-4=0$va diagonallari kesishish nuqtasi $(10;6)$. Uning qolgan ikki tomoni va diagonallari tenglamasini tuzing.

7. $M(2;-1;1)$ nuqtadan o'tib, $a)$ $3x-y-Z+1=0$, $b)$ $x-y+2z=0$ tekisliklar kesishish chizig'iga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-1;2;3)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$3x-6y-2z+35=0, 3x-6y-2z-7=0, 4x-5y+3z-1=0.$

10. $\begin{cases} 2x+3y-16z-7=0 \\ 3x+y-17z=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(2;-5;3)$ nuqtadan o'tib, $\vec{q}\{4;-6;9\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(2;1;1)$ nuqtaning $x+y+z+5=0$ tekislikdagi proyeksiyasini toping.

13. $\frac{x-1}{0}=\frac{y}{2}=\frac{z+1}{1}$ to'g'ri chiziq va $x+y-z+1=0$ tekislik orasidagi burchakni toping.

14. $\frac{x+7}{1}=\frac{y+4}{14}=\frac{z+3}{2},\frac{x-21}{6}=\frac{y+5}{-4}=\frac{-2}{-1}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $cos\alpha=\frac{\sqrt{7}}{2\sqrt{13}}$. 2. *J:* $25/\sqrt{41}$. 3. *J:* $a)$ $0{,}5x-y+2=0$, $b)$ $2x+y-7=0$. 4. *J:* $a) x+y+8=0$, $b)$ $7x-3y-4=0$, $c)$ $arccos(4/\sqrt{41})$. 5. *J:* $(3;1)$. 6. *J:* $x-3y+8=0; 5x-3y-32=0; 5x-3y+4=0; y-1=0$. 7. *J:* $3x+7y+2z-1=0$. 8. *J:*$-x+2y+3z-14=0$. 9. *J:*$d=6$. 10. *J:* $\frac{x+1}{5}=\frac{y-3}{2}=\frac{z}{1}$. 11. *J:* $\frac{x-2}{4}=\frac{y+5}{-6}=\frac{z-3}{9}$. 12. *J:* $(1{,}0,-2)$. 13. *J:* $arcsin\left(\frac{1}{\sqrt{15}}\right)$.
14. *J: d=13.*

Variant 6

1. ABC uchburchak berilgan: $A(7;3;4)$, $B(1;0;6)$, $C(4;5;-2)$. A uchidan BC tomonga tushirilgan balandligini toping.

2. $|\vec{a}|=4$, $|\vec{b}|=5$, $|[\vec{a},\vec{b}]|=10$ lar berilgan bo'lsa, $(\vec{a},\vec{b})$ ni hisoblang.

3. Piramida uchlari berilgan: $A(2;0;0)$, $B(2;3;8)$, $C(0;0;6)$, $D(0;3;0)$. Uning hajmini va ACD yoqiga tushirilgan balandligini toping.

4. $M(2;3)$ nuqtadan o'tib, a) $3x-2y+2=0$ to'g'ri chiziqqa parallel, c) $3x-2y+2=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

5. Uchburchak uchlarining koordinatalari berilgan: $A(4;8)$, $B(2;-10)$, $C(-6;-2)$. a) A uchidan tushirilgan balandlik tenglamasini, b) C uchidan tushirilgan mediana tenglamasini, c) B burchakni toping.

6. $M(0;0)$ nuqtani l: $2x+3y=0$ to'g'ri chiziqqa proyeksiyasini toping.

7. Parallelogrammning ikki tomoni tenglamasi $y=2$ va $2x-3y+12=0$ hamda diagonallari kesishish nuqtasi $M(2;4)$ berilgan. Parallelogrammning qolgan ikki tomoni tenglamasini va diagonallari tenglamasini tuzing.

8. Koordinatalar boshidan o'tib, $2x-y+5z+3=0$, $x+3y-z-7=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

9. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(1;0;2)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

10. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$2x-3y+6z+28=0, 2x-3y+6z-14=0, x-4y-z+9=0.$

11. $\begin{cases} x+y-z=0 \\ 2x-y+2=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

12. $M(2;-5;3)$ nuqtadan o'tib, $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z+2}{1}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

13. $A(3;-1;4)$ nuqtaning $\frac{x-5}{13}=\frac{y-6}{1}=\frac{z+3}{-4}$, $\frac{x-2}{13}=\frac{y-3}{1}=\frac{z+3}{-4}$ parallel to'g'ri chiziqlar orqali o'tuvchi tekislikdagi proyeksiyasini toping.

14. $\frac{x-3}{2}=\frac{y+4}{2}=\frac{z-4}{3}$ to'g'ri chiziq va $x+2y+3z-5=0$ tekislik orasidagi burchakni toping.

15. $\frac{x-1}{2}=\frac{y}{-1}=\frac{z+1}{3}$, $\frac{x}{2}=\frac{y+1}{-1}=\frac{z-2}{3}$ to'g'ri chiziqlar orasidagi masofani toping.

***Javoblar:* 1**. *J:* $\frac{7}{\sqrt{2}}$. **2**. *J:* $V=14,\ H=\sqrt{14}$. 3. *J:* $3x-2y=0, c)2x+3y-13=0$. 4. *J:* $a)x+8y+22=0$ $\quad b)\ 11x+3y+2=0$ $c)\ arccos(3/\sqrt{15})$. 5. *J: 0.* 6. *J:* $y=6, 2x-3y+4=0, 2x-5y+16=0, 2x-y=0$. 7. *J:* $-13x+24y+10z=0$. 8. *J:* $x+2y-5=0$. 9. *J:*$d=6$. 10. *J:* $\frac{x}{1}=\frac{y-2}{2}=\frac{z-2}{3}$. 11. *J:* $\frac{x-2}{2}=\frac{y+5}{3}=\frac{z-3}{1}$. 12. *J:*$(2;-3;-5)$. 13. *J:* $arcsin(19/\sqrt{406})$. 14. *J:* $3\sqrt{5}/\sqrt{7}$.

Variant 7

1. ABC uchburchak uchlari berilgan: $A(4;-2;3)$, $B(0;-1;3), C(3;-4;5)$. A uchidan VS tomonga tushirilgan balandligini toping.

2. Uchlari $A(-1;1;1)$, $B(1;2;1)$, $C(7;12;3)$ nuqtalarda bo'lgan tetraedrning hajmi 2 ga teng. Agar uning D uchi OY o'qida yotsa, shu uch koordinatasini toping.

3. $M(2;-3)$ nuqtadan o'tib, a) $3x-7y+3=0$ to'g'ri chiziqqa parallel; c) $3x-7y+3=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(2;6)$, $B(4;-2)$, $C(-2;-6)$. a) A uchidan tushirilgan balandlik tenglamasini, b) C uchidan tushirilgan mediana tenglamasini, c) B burchakni toping.

5. $M(4,3)$ nuqtaning $l: 3x-4y+10=0$ to'g'ri chiziqdagi proyeksiyasini toping.

6. Parallelogrammning qo'shni uchlari $A(-3;2)$ va $B(3;6)$ hamda diagonallari kesishish nuqtasi $M(2;4)$ berilgan bo'lsa, parallelogrammning diagonallari va qolgan ikki tomoni tenglamasini tuzing.

7. Koordinatalar boshidan o'tib, $2x-y+5z+3=0, x+3y-z-7=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-1;2;0)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping: $x+5y-z+10=0,\ x+2y-4=0,\ x-y+2z+10=0,\ x-y+2z-4=0$.

10. $\begin{cases}4x-y+3z-7=0\\2x+y-4z+10=0\end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(-1;2;1)$ nuqtadan o'tib, $\begin{cases}x+y-2z-1=0\\x+2y-z+1=0\end{cases}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(4;3;-1)$ nuqtaning $x+2y-y-z-3=0$ tekislikdagi proyeksiyasini toping.

13. $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-3}{2}$ to'g'ri chiziq va $x+2y-3z+4=0$ tekislik orasidagi burchakni toping.

14. $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z}{1}$, $\frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\sqrt{\frac{149}{22}}$. 2. *J:* $D_1(0;1;0),\ D_2(0;-5;0)$. 3. *J:* $3x-7y+3=0$, $c)7x+3y-5=0$. 4. *J:* $a)\ 3x+2y-18=0,\quad b)\ 8x-5y-14=0$, $c)\ arccos(-5/\sqrt{211})$. 5. *J:2*. 6. *J:* $2x-3y+12=0,\ y=6, 2x-3y+4=0,$

$y = 2$. 7. *J:* $2x - y - z = 0$. 8. *J:* $-x + 2y - 5 = 0$. 9. *J:*$d = \frac{14}{\sqrt{6}}$. 10. *J:* $\frac{x}{1} = \frac{y-2}{22} = \frac{z-3}{6}$. 11. *J:* $\frac{x+1}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$. 12. *J:*$(5; -1; 0)$. 13. *J:* $arcsin(5/\sqrt{406})$. 14. *J:* $d = 3$.

Variant 8

1. ABC uchburchak uchlari berilgan: $A(4; -2; 3)$, $B(0; -1; 3)$, $C(3; -4; 5)$. B uchidan AS tomonga tushirilgan balandligini toping.

2. $M(2; -3)$ nuqtadan o'tib, a) $x + 9y - 11 = 0$ to'g'ri chiziqqa parallel; c) $x + 9y - 11 = 0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

3. Uchburchak uchlarining koordinatalari berilgan: $A(-7,3)$, $B(2,-1)$, $C(-1,-5)$.a) B uchidan tushirilgan balandlik tenglamasini, c) B uchidan tushirilgan mediana tenglamasini, c) C burchakni toping.

4. $M(2; -5)$ nuqtaning $l: 4x - 3y + 25 = 0$ to'g'ri chiziqdagi proyeksiyasini toping.

5. Parallelogramning ikki tomoni tenglamasi $x - y + 1 = 0$ va $x - 2y + 6 = 0$ hamda diagonallari kesishish nuqtasi $M(4; 4)$ berilgan. Parallelogrammning qolgan ikki tomoni tenglamasini va diagonallari tenglamasini tuzing.

6. Koordinatalar boshidan o'tib, $2x - y + 3z - 1 = 0, x + 2y + z = 0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

7. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(2; 1; 1)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

8. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$a) \; x - 2y + z - 1 = 0, b) 2x - 4y + 2z - 1 = 0, c) 5x - 6y + 7z - 1 = 0.$

9. $\begin{cases} 2x + y + 3z - 4 = 0 \\ x - y + 2z - 1 = 0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

10. $M(2; -3; -1)$ nuqtadan o'tib, $\frac{x-4}{4} = \frac{y+1}{3} = \frac{z+3}{2}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

11. $A(3;1;-1)$ nuqtaning $x+2y+3z-30=0$ tekislikdagi proyeksiyasini toping.

12. $\frac{x+4}{3}=\frac{y-1}{2}=\frac{z-3}{4}$ to'g'ri chiziq va $2x-3y-2z+5=0$ tekislik orasidagi burchakni toping.

13. $\frac{x-9}{4}=\frac{y+2}{-3}=\frac{z}{1}$, $\frac{x}{-2}=\frac{y+7}{9}=\frac{z-2}{2}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $2\sqrt{10}$. 2. *J:* a) $x+9y+25=0, b)9x-y-21=0$. 3. *J:*$a)3x-4y-10=0$ $b)$ $4x+5y+13=0$ $c)$ $x+1=0$. 4. *J:* 9,6. 5. *J:* $x-y-1=0$, $x-2y+2=0, 3x-4y+4=0$, $x-4=0$. 6. *J:* $-7x+y+5z=0$. 7. *J:* $2x+y+z-6=0$. 8. *J:*$d=\frac{\sqrt{6}}{12}$. 9. *J:* $\frac{x}{5}=\frac{y-1}{-1}=\frac{z-1}{-3}$. 10. *J:*$\frac{x-2}{4}=\frac{y+3}{3}=\frac{z+1}{2}$. 11. *J:*$(5,5,5)$. 12. *J:* $arcsin\left(\frac{8}{\sqrt{493}}\right)$. 13. *J:* $d=29/7$.

Variant 9

1. ABC uchburchak uchlari berilgan: $A(2;5;3)$, $B(1;2;3)$, $C(0;2;5)$.C uchidan AB tomoniga tushirilgan balandligini hisoblang.

2. Uchlari $A(3;5;4)$, $B(8;7;4)$, $C(5;10;4)$, $D(4;7;8)$ nuqtalarda bo'lgan piramidaning hajmini va ABC yoqqa tushirilgan balandligini toping.

3. $M(-2;-5)$ nuqtadan o'tib, a) $3x+4y+2=0$ to'g'ri chiziqqa parallel; c) $3x+4y+2=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(2;-1)$, $B(-7;3)$, $C(-1;-5)$. a) B uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) B burchakni toping.

5. $(-1;5)$ nuqtaning $l: 4x+3y-5=0$ to'g'ri chiziqqa proyeksiyani toping.

6. Parallelogrammning qo'shni uchlari $A(0;1)$ va $B(4;5)$ hamda diagonallari kesishish nuqtasi $M(4;4)$ berilgan bo'lsa, Parallelogrammning diagonallari va qolgan ikki tomoni tenglamasini tuzing.

7. $M(-1;-1;2)$ nuqtadan o'tib, $x+2y-2z+4=0,\ x-2y+z=0$ tekisliklar kesishish chizig'iga perpendikulyar tekislik tenglamasini tuzing.

8 .Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-1;2;1)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$6x-18y-9z-28=0, 5x-6y+7z+1=0, 4x-12y-6z-7=0.$

10. $\begin{cases} x+2y-3z-7=0 \\ 2x-y+4z+1=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(2;-5;3)$ nuqtadan o'tib, $\begin{cases} 2\,x-y+3z-1=0 \\ 5x+4y-z-7=0 \end{cases}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(1;0;2)$ nuqtaning $\frac{x-2}{-1}=\frac{y+3}{2}=\frac{z-1}{-1}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x-1}{4}=\frac{y}{12}=\frac{z-1}{-3}$ to'g'ri chiziq va $6x-3y-2z=0$ tekislik orasidagi burchakni toping.

14. $\frac{x+3}{4}=\frac{y-6}{-3}=\frac{z-3}{2}$, $\frac{x-4}{8}=\frac{y+1}{-3}=\frac{z+7}{3}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\frac{3}{\sqrt{2}}$. 2. *J:* $V=14, H=7/\sqrt{14}$. 3. *J:* $a) \ 3x+4y+26=0$, $c) \ 4x-3y-7=0$. 4.*J:*$a)3x+4y+9=0$; $b)\ y+1=0$; $c)\ arccos\left(\frac{43}{5\sqrt{97}}\right)$. 5. *J:* 1,2. 6. *J:* $x-y+1=0, x-2y+6=0,\ x-y-1=0, x-2\,y+2=0$. 7. *J:* $2x+3y+4z-3=0$. 8 .*J:*$-x+2y+z-6=0$. 9. *J:*$d=\frac{5}{6}$. 10. *J:* $\frac{x-1}{11}=\frac{y-3}{-10}=\frac{z}{-5}$. 11. *J:* $\frac{x-2}{-11}=\frac{y+5}{17}=\frac{z-3}{13}$. 12. *J:*$(1;-1;0)$. 13. *J:* $arcsin(6/91)$. 14. *J:* $d=13$.

Variant 10

1. ABC uchburchak uchlari berilgan: $A(-1,2,-3)$, $B(3,4,-6)$, $C(1,1,-1)$. A uchidan BC tomoniga tushirilgan balandligini hisoblang.

2. Piramida uchlari berilgan: $A(4,6,5)$, $B(6,9,4)$, $C(2,10,10)$, D$(7,5,9)$. Shu piramida hajmini toping.

3. $M(2;-3)$ nuqtadan o'tib, a) $2x+3=0$ to'g'ri chiziqqa parallel; c)$2x+3=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(-2,-2)$, $B(7,-6)$, $C(1,2)$.a) B uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) B burchakni toping.

5. $M(0;6;3)$ nuqtaning $l: 5x-12y-6=0$ to'g'ri chiziqdagi proyeksiyasini toping.

6. Uchlari $A(3,-1)$, $B(2,4)$, $C(5,6)$, $D(9,3)$ nuqtalarda bo'lgan to'rtburchak trapetsiya ekanligini ko'rsating. Trapetsiya o'rta chizig'i va diagonallari tenglamasini tuzing.

7. M(1;2;3) nuqtadan o'tib, $2x-y+5z+3=0$, $x+3y-z-7=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(2;3;4)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$3x-6y-2z+35=0, 3x-6y-2z-1=0, x-4y-z+9=0.$

10. $\begin{cases} 2x+y-z+1=0 \\ 3x-4y+z=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(2;0;-3)$ nuqtadan o'tib, $\begin{cases} 3x-y+2z-7=0 \\ x+3y-2z-3=0 \end{cases}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(1;1;1)$ nuqtaning $\frac{x+4}{-2}=\frac{y-3}{4}=\frac{z-4}{1}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x-5}{2}=\frac{y+1}{-3}=\frac{z}{-1}$ to'g'ri chiziq va $2x+y+z=0$ tekislik orasidagi burchakni toping.

14. $\begin{cases} 2x+2y-z-10=0 \\ x-y-z-22=0 \end{cases}$ va $\frac{x+7}{3}=\frac{y-5}{-1}=\frac{z-9}{4}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\frac{3\sqrt{29}}{\sqrt{38}}$. 2. *J:* $V=\frac{121}{6}$, $H=\frac{121}{3\sqrt{69}}$. 3. *J:*$a)x-2=0, c)y+3=0$. 4. *J:* $a)3x+4y+3=0$ $b)$ $y+2=0$ $c)$ $arccos(8.6/\sqrt{97})$. 5. *J:* 3. 6. *J:* $BC \parallel AD$, $4x-6y-1=0, 7x-2y-23=0, x+7y-30=0$. 7. *J:* $2x+3y+z-11=0$. 8. *J:*$2x+3y+4z-29=0$. 9. *J:*$d=6$. 10. *J:* $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-5}{11}$. 11. *J:*$\frac{x-2}{-2}=\frac{y}{4}=\frac{z+3}{5}$. 12. *J:*$(-2,-1,3)$. 13. *J:0*. 14. *J:*$d=25$.

Variant 11

1. ABC uchburchak uchlari berilgan: $A(-1,2,-3)$, $B(3,4,-6)$, $C(1,1,-1)$.C uchidan AB tomoniga tushirilgan balandligini toping.

2. Piramida uchlari berilgan: $A(1,3,6)$, $B(2,2,1)$, $C(-1,0,1)$, $D(-4,6,-3)$. Piramida hajmini va D uchidan tushirilgan balandligini hisoblang.

3. $M(2;-3)$ nuqtadan o'tib, a) $16x-24y-7=0$ to'g'ri chiziqqa parallel; c)$16x-24y-7=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(7,-6)$, $B(-2,-2)$, $C(1,2)$.a) B uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) B burchakni toping.

5. $x+2y+5=0$ va $x+2y+3=0$ parallel to'g'ri chiziqlar orasidagi masofani toping.

6. Uchlari $A(6;-3)$, $B(3;3)$, $C(5,6)$, $D(10,3)$ nuqtalarda bo'lgan to'rtburchak trapetsiya ekanligini ko'rsating. Trapetsiya o'rta chizig'I va diagonallari tenglamasini tuzing.

7. $M_0(3;-1;-5)$ nuqtadan o'tib, $3x-2y+2z+7=0, 5x-4y+3z+1=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-2;1;3)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko’rsating va ular orasidagi masofani toping: $x-2y+3z-1=0, 3x+y-z+5=0, 2x-4y+6z+3=0.$

10. $\begin{cases} x+y+3z-2=0 \\ 3x-2y+7z-1=0 \end{cases}$ to’g’ri chiziq tenglamasini kanonik ko’rinishga keltiring.

11. $M(1;1;1)$ nuqtadan o’tib, OX o’qiga parallel to’g’ri chiziq tenglamasini tuzing.

12. $A(2;3;1)$ nuqtaning $\frac{x+7}{1}=\frac{y+2}{2}=\frac{z+2}{3}$ to’g’ri chiziqdagi proyeksiyasini toping.

13. $\begin{cases} x+y-2z+3=0 \\ x+2y-3z-1=0 \end{cases}$ to’g’ri chiziq va $2x-y-z+7=0.$ tekislik orasidagi burchakni toping.

14. $\frac{x+1}{1}=\frac{y}{1}=\frac{z-1}{2}$, $\frac{x}{1}=\frac{y+1}{3}=\frac{z-2}{4}$ to’g’ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:*3. 2. *J:* $V=\frac{70}{3}, H=2\sqrt{14}$. 3. *J:*$a)$ $2x-3y-13=0$, $c)$ $3x+2y=0$. 4. *J:*$a)3x-4y-2=0$ $b)$ $4x+5y+2=0$ $c)$ $arccos(11\sqrt{97}/485)$. 5. *J:* $8/\sqrt{5}$. 6. *J:* $BC \parallel AD$, $6x-4y-27=0, y=3, 9x+y-51=0$. 7. *J:* $2x+y-2z-15=0$. 8. *J:*$-2x+y+3z-29=0$. 9. *J:*$d=\frac{5}{2\sqrt{14}}$. 10. *J:* $\frac{x-1}{13}=\frac{y-1}{2}=\frac{z}{-5}$. 11. *J:*$\begin{cases} y-1=0 \\ z-1=0. \end{cases}$ 12. *J:*$(-5,2,4)$. 13. *J:0*. 14. *J:*$d=\sqrt{3}/3$.

Variant 12

1. Piramida uchlari berilgan: $A(-4,2,6)$, $B(2,-3,0)$, $C(-10,5,8), D(-5,2,-4)$. Piramida hajmini va D uchidan tushirilgan balandligini toping.

2. $M(-2;4)$ nuqtadan o’tib, a) $2x-3y+6=0$ to’g’ri chiziqqa parallel; c)$2x-3y+6=0$ to’g’ri chiziqqa perpendikulyar bo’lgan to’g’ri chiziq tenglamasini tuzing.

3. Uchburchak uchlarining koordinatalari berilgan: $A(-5,3)$, $B(3,4)$, $C(7,-3)$.a) A uchidan tushirilgan balandlik tenglamasini, c) C uchidan tushirilgan mediana tenglamasini, c) B burchakni toping.

4. $x-3=0$ va $2x+5=0$ to'g'ri chiziqlar orasidagi masofani toping.

5. Parallelogrammning bir uchidan chiquvchi ikki tomoni tenglamasi $2x-3y+3=0$, $2x-y+9=0$ vashu uchga qarama-qarshi uchining koordinatasi $(2,5)$ bo'lsa, parallelogrammning qolgan ikki tomoni va diagonallari tenglamasini tuzing.

6. M(1;1;-2) nuqtadan o'tib, $2x+2z=0$, $x-y+z-1=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

7. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(4;1;5)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

8. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$3x+2y-z-6=0, 9x+6y-3z=0, x+4y-5z+9=0.$

9. $\begin{cases} 3\,x-y+4z-7=0 \\ x+5y-3z+1=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

10. $M(2;4;5)$ nuqtadan o'tib, OX o'qiga parallel to'g'ri chiziq tenglamasini tuzing.

11. $A(1;2;1)$ nuqtaning $\frac{x+2}{3}=\frac{y}{-1}=\frac{z-1}{2}$ to'g'ri chiziqdagi proyeksiyasini toping.

12. $\frac{x-1}{2}=\frac{y+2}{1}=\frac{z}{1}$ to'g'ri chiziq va $x-2y+z-5=0$ tekislik orasidagi burchakni toping.

13. $\frac{x-2}{4}=\frac{y+1}{1}=\frac{z-1}{-1}$, $\frac{x+4}{2}=\frac{y-2}{-2}=\frac{z+2}{-3}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $V=56/3, H=4$. 2. *J:* $a)\ 2x-3y+6=0, c)3x+2y-2=0$. 3. *J:* $a)4x-7y+41=0$ $\quad b)x-16y-55=0$ $c)\ arccos(2.5/\sqrt{221})$. 4. *J: 5,5*. 5. *J:* $2x-3y+11=0; 2x-y+1=0; x-y+3=0; y-1=0$. 6. *J:* $3x+y-2z-8=0$. 7. *J:* $4x+y+5z-42=0$. 8. *J:* $d=\frac{6}{\sqrt{14}}$. 9. *J:* $\frac{x}{-17}=\frac{y-1}{13}=\frac{z-2}{-16}$. 10. *J:* $\begin{cases} y-4=0 \\ z-5=0. \end{cases}$ 11. *J:* $(-0{,}5;-0{,}5;2)$. 12. *J:* $arcsin\,1/\,6$. 13. *J:* $d=6$.

Variant 13

1. ABC uchburchak uchlari berilgan: $A(-2.1.1)$, $B(2.3.-2)$, $C(1.1.-1)$ A uchidan BC tomoniga tushirilgan balandligini hisoblang.

2. Tetraedr uchlari berilgan: $A(1,1,2), B(-1,1,3)$, $C(2,-2,4)$, $D(-1,0,-2)$. Tetraedr hajmini va B uchidan tushirilgan balandligini toping.

3. $M(2;-1)$ nuqtadan o'tib, a) $3x+5y-7=0$ to'g'ri chiziqqa parallel; c)$3x+5y-7=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(-2;0)$, $B(2;6)$, $C(4;2)$. a) AC tomonini tenglamasini ,c) B uchidan tushirilgan balandlik tenglamasini, c) B uchidan tushirilgan mediana tenglamasini tuzing.

5. $3x-4y+5=0$ va $3x-4y+1=0$ parallel to'g'ri chiziqlar orasidagi masofani toping.

6. Parallelogrammning qo'shni uchlari $A(-6;-3)$ va $B(-4;1)$ hamda diagonallari kesishish nuqtasi $M(-2;1)$ berilgan bo'lsa, Parallelogrammning diagonallari va qolgan ikki tomoni tenglamasini tuzing.

7. M(0;2;1) nuqtadan o'tib, $x+5y+9z-13=0, 3x-y-5z+1=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(1;-2;1)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping: $3x+4y-5=0,\ 5x-7y+8z=0,\ 6x+8y-16-7=0.$

10. $\begin{cases} x-3y+z-5=0 \\ 2x+4y-2z+10=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(1;0;-3)$ nuqtadan o'tib, OX o'qiga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(2;0;0)$ nuqtaning $\frac{x-30}{1}=\frac{y-2}{3}=\frac{z+2}{4}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x+2}{3}=\frac{y}{-1}=\frac{z-1}{2}$ to'g'ri chiziq va $4x+2y+2z-5=0$ tekislik orasidagi burchakni toping.

14. $\frac{x+7}{1}=\frac{y+4}{14}=\frac{z+3}{-2}$, $\frac{x-21}{6}=\frac{y+5}{-4}=\frac{z-1}{-1}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\frac{3\sqrt{29}}{\sqrt{38}}$. 2. *J:* $V=35/6,\ H=\sqrt{5}$. 3. *J:* $a)\ 3x-5y-11=0, c)$ $5x+3y-1=0$. 4. *J:* $a) x-3y+2=0;$ $b)$ $5x-y\pm 4=0;$ $c)\ 3x+y-12=0$. 5. *J: 0,8.* 6. *J:* $2x-3y+3=0,\ 2x-y+9=0,\ 2x-3y+11=0, 2x-\ y+1=0$. 7. *J:* $x+y+z-3=0$. 8. *J:*$x-2y+z-6=0$. 9. *J:* $d=0{,}6$. 10. *J:* $\frac{x}{1}=\frac{y}{2}=\frac{z-5}{5}$. 11. *J:* $\begin{cases} x-1=0 \\ z+3=0 \end{cases}$. 12. *J:* $(29.-1.-6)$. 13. *J:* $arcsin\left(\frac{\sqrt{7}}{2\sqrt{3}}\right)$. 14. *J:* $d=13$.

Variant 14

1. ABC uchburchak uchlari berilgan: $A(-2{,}1{,}1)$, $B(2{,}3,-2)$, $C(0{,}0{,}3)$. B uchidan AC tomoniga tushirilgan balandligini toping.

2. Tetraedr uchlari berilgan: $A(2{,}3{,}1)$, $B(4{,}1,-2)$, $C(6{,}3{,}7)$, $D(7{,}5,-3)$. Tetraedr hajmini va ACD yoqqa tushirilgan balandligini toping.

3. $M(-1;-4)$ nuqtadan o'tib, $a)\ \frac{x}{4}-\frac{y}{3}=1$ to'g'ri chiziqqa parallel; c) $\frac{x}{4}-\frac{y}{3}=1$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(1;1)$, $B(2;4)$, $C(-3{,}1)$. a) AC tomonini tenglamasini ,c) B uchidan tushirilgan balandlik tenglamasini, c) B uchidan tushirilgan mediana tenglamasini tuzing.

5. $5x-12y+52=0$ va $10x-24y-39=0$ to'g'ri chiziqlar orasidagi masofani toping.

6. Uchlari $A(4,12),\ B(6,5)$, $C(3,2),\ D(-3,5)$ nuqtalarda bo'lgan to'rtburchak trapetsiya ekanini ko'rsating. Trapetsiyaning o'rta chizig'i va diagonallari tenglamasini tuzing.

7. $4x - y + 3z - 1 = 0$va$x + 5y - z + 2 = 0$ tekisliklar kesishish chizig'idan o'tib, $2x - y + 5z - 3 = 0$ tekislikka perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(2;1;3)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$2x - 2y + z + 3 = 0, 6x - 6y + 3z - 12 = 0, 4x - y + 2z - 4 = 0.$

10. $\begin{cases} 2x + 3y - 4z - 3 = 0 \\ 3x - 5y + z + 5 = 0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(2;-5;3)$ nuqtadan o'tib, OZ o'qiga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(0;0;-1)$ nuqtaning $\frac{x+2}{-3} = \frac{y-1}{-2} = \frac{z-9}{1}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x}{2} = \frac{y}{-2} = \frac{z-1}{1}$ to'g'ri chiziq va $x - 2y + 2z - 5 = 0$ tekislik orasidagi burchakni toping.

14. $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z}{1},\ \frac{x-7}{3} = \frac{y-1}{4} = \frac{z-3}{2}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\sqrt{29}$. 2. *J:* $V = \frac{70}{3},\ H = 70/\sqrt{581}$. 3. *J:* $3x + 4y + 19 = 0, c)4x - 3y - 8 = 0$. 4. *J:* $a) y - 1 = 0$; $b)\ x - y + 2 = 0$; $c)\ x - 2 = 0$. 5. *J: 5,5.* 6. *J:* $BC \parallel AD,\ 2x - 2y - 7 = 0, y = 5, 10x - y - 28 = 0$. 7. *J:* $7x + 14y + 5 = 0$. 8. *J:* $2x + y + 3z - 14 = 0$. 9. *J:*$d = \frac{7}{3}$. 10. *J:* $\frac{x}{17} = \frac{y-1}{14} = \frac{z}{19}$. 11. *J:* $\begin{cases} x - 2 = 0 \\ y + 5 = 0. \end{cases}$ 12. *J:* $(1;3;8)$. 13. *J:* $arcsin(8/9)$. 14. *J:* $d = 3$.

Variant 15

1. $A(2,0,-2),\ B(1,2,3),\ C(3,-3,1)$ nuqtalar berilgan. ABC uchburchak yuzini toping.

2. Tetraedr uchlari berilgan: $A(7,2,4)$, $B(7,-1,-2)$, $C(3,3,1), \ddot{A}(-4,2,1)$. Tetraedr hajmini va ABC yoqqa tushirilgan balandligini toping.

3. $M_0(-3,-1)$ nuqtadan o'tib, a) AB to'g'ri chiziqqa parallel, c)AB to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing. Bu yerda: A(-2;6), B(3;-1).

4. Uchburchak uchlarining koordinatalari berilgan: $A(0,0)$, $B(1,6)$, $C(4,2\ )$. a) AC tomonini tenglamasini ,c) B uchidan tushirilgan balandlik tenglamasini, c) B uchidan tushirilgan mediana tenglamasini tuzing.

5. $3x + y - 3\sqrt{10} = 0$ va $6x + 2y + 5\sqrt{10} = 0$. To'g'ri chiziqlar orasidagi masofani toping.

6. Parallelogrammning qo'shni uchlari $A(0;0)$ va $B(-2;5)$ hamda diagonallari kesishish nuqtasi $M(1;3)$ berilgan bo'lsa, Parallelogrammning diagonallari va qolgan ikki tomoni tenglamasini tuzing.

7. A$(1,-1,3)$ va B$(1,2,4)$ nuqtalardan o'tib, $2x - 3y + z + 1 = 0$ tekislikka parallel tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-5;0;3)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$4x - 3y + 4 = 0, 8x - 6z - 12 = 0, 2x - z + 3 = 0$

10. $\begin{cases} 5x - 2y + 3z + 1 = 0 \\ x + 3y - 5z - 1 = 0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(3;-1;1)$ nuqtadan o'tib, OZ o'qiga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(0;3;0)$ nuqtaning $\frac{x-4}{2} = \frac{y-6}{1} = \frac{z+1}{-3}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x+1}{0} = \frac{y-5}{-1} = \frac{z}{2}$ to'g'ri chiziq va $x + 2y + 2z - 5 = 0$ tekislik orasidagi burchakni toping.

14. $\frac{x+3}{4}=\frac{y-6}{-3}=\frac{z-3}{2}$, $\frac{x-4}{8}=\frac{y+1}{-3}=\frac{z+7}{3}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1 *J:* $0.5\sqrt{506}$. 2. *J:* $V=\frac{43}{2}$, $H=43/\sqrt{105}$. 3. *J:* $a)\ \frac{x+3}{5}=\frac{y+1}{-7}$, $b)\ 5x-7y+8=0$. 4. *J:* $a)\ x-2y=0$, $b)\ 5x+y-11=0$, $c)\ 2x+y-8=0$. 5. *J:* *5,5*. 6. *J:* $5x+2y+3=0$, $x-2y+12=0$, $5x+2y-24=0$, $x-\ 2y=0$. 7. *J:* $3x+y-3z+7=0$. 8. *J:* $-5x+3z-34=0$. 9. *J:*$d=2$. 10. *J:* $\frac{x}{1}=\frac{y}{28}=\frac{z-1}{17}$. 11. *J:* $\begin{cases} x-3=0 \\ y+1=0. \end{cases}$ 12. *J:* (2,5,2). 13. *J:* $arcsin\left(\frac{2}{3\sqrt{5}}\right)$. 14. *J:* $d=13$.

Variant 16

1. $A(3,0,-3)$, $B(1,2,3)$, $C(2,-2,1)$ nuqtalar berilgan. ABC uchburchak yuzini toping.

2. $M(1;1)$ nuqtadan o'tib, $a)\ x+2y+2=0$ to'g'ri chiziqqa parallel; c) $x+2y+2=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

3. Uchburchak uchlarining koordinatalari berilgan: $A(-8,-2)$, $B(2,10)$, $C(4,4)$. a) A uchidan tushirilgan balandlik tenglamasini, b) C uchidan tushirilgan mediana tenglamasini, c) B burchakni toping.

4. $M(5,-2)$ nuqtaning $2x-3y-3=0$ to'g'ri chiziqdagi proyeksiyasini toping.

5. Parallelogrammning ikki tomoni tenglamasi $3x-2y+12=0$, $x-3y+11=0$ va diagonallari kesishish nuqtasi (2;2) bo'lsa , parallelogrammning qolgan ikki yomoni tenglamasi va diagonallari tenglamasini tuzing.

6. $M(2;-1;3)$ nuqtadan o'tib, Oz o'qiga parallel tekislik tenglamasini tuzing.

7. Koordinatalar boshidan tekislikka tushirilgan perpendikulya rasosidagi nuqta $P(3;-6;2)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

8. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$x-2y+z-1=0$; $2\,x-4y+2z-1=0$;.3 $x+4y-z+2=0$

9. $\begin{cases} 2x - 3y - 3z - 9 = 0 \\ x - 2y + z + 3 = 0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

10. $M(-1;3;9)$ nuqtadan o'tib, $\vec{q}\{2;-4;7\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

11. $A(3;-1;4)$ nuqtaning $2x + y - z + 5 = 0$ tekislikdagi proyeksiyasini toping.

12. $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z}{1}$ to'g'ri chiziq va $x + 2y - z + 5 = 0$ tekislik orasidagi burchakni toping.

13. $\frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2}, \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\sqrt{110}$. 2. *J:* $a)\ x + 2y - 3 = 0$; $b)\ 2x - y - 1 = 0$. 3. *J:* $a)\ x - 3y + 2 = 0$; $b)\ y - 4 = 0$; $c)\ arccos\left(\frac{13}{\sqrt{610}}\right)$. 4. *J:* $(3,1)$. 5. *J*: $3x - 2y - 16 = 0, x - 3y - 3 = 0, x + 4y - 10 = 0, 5x - 8y + 6 = 0$. 6. *J:* $x - 2 = 0$; $y + 1 = 0$. 7. *J:* $3x - 6y + 2z - 49 = 0$. 8. *J:*$d = \frac{1}{2\sqrt{6}}$. 9. *J:* $\frac{x}{9} = \frac{y}{5} = \frac{z+3}{1}$. 10. *J:* $\frac{x+1}{2} = \frac{y-3}{-4} = \frac{z-9}{7}$. 11. *J:* $(1,-2,5)$. 12. *J:* $\frac{\pi}{6}$. 13. *J:* $d = \frac{1}{\sqrt{3}}$.

Variant 17

1. $A(2,-2,1)$, $B(1,1,1)$, $C(7,3,2)$ nuqtalar berilgan. ABC uchburchak yuzini toping.

2. Piramida uchlari berilgan. $A(1,2,-,1)$, $B(0,1,5)$, $C(0,4,-1), D(3,2,1)$. Piramida hajmini va ABD yoqqa tushirilgan balandligini toping.

3. $M(2;1)$ nuqtadan o'tib, a) $2x + 3y + 4 = 0$ to'g'ri chiziqqa parallel; b) $2x + 3y + 4 = 0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(-5,-2)$, $B(7,6)$, $C(5,-4)$.a) C uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) B va C burchaklarni toping.

5. $M(-1,-5)$ nuqtani $l: 4x+7y-26=0$ to'g'ri chiziqqa proyeksiyasini toping.

6. Parallelogrammning qo'shni uchlari $A(2;5)$ va $B(5;3)$ hamda diagonallari kesishish nuqtasi $M(-2;0)$ berilgan bo'lsa, Parallelogrammning diagonallari va qolgan ikki tomoni tenglamasini tuzing.

7. $M(-1;2;3)$ nuqtadan o'tib ,$M_1(1;0;-2)$, $M_2(3;4;5)$, $M_3(-1;2;0)$ nuqtalardan o'tuvchi tekislikka parallel tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(3;-6;2)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$2\,x-3y+6z-21=0;\ x+y+z=0;\ 4\,x-6y+12z+35=0$

10. $\begin{cases} x-2y+3z-4=0 \\ 3x+2y-5z-4=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(3;0;-2)$ nuqtadan o'tib, $\vec{q}\{2;1;1\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(1;-3;2)$ nuqtaning $2x+5y-3z-19=0$ tekislikdagi proyeksiyasini toping.

13. $\frac{x}{2}=\frac{y-5}{2}=\frac{z+1}{-1}$ to'g'ri chiziq va $4x+y+z-3=0$ tekislik orasidagi burchakni toping.

14. $\frac{x-2}{1}=\frac{y+2}{-3}=\frac{z+1}{-2}\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-1}{1}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $0{,}5\sqrt{410}$. 2. *J:* $V=5,\ H=\frac{15}{\sqrt{51}}$. 3. *J:* $a)\ 2x+3y-7=0\ b)\ 3x-2y-4=0$. 4. *J:* $a)\ 3x+2y-7=0,\ b)\ 3x-11y-7=0, c)\ 45^0, 90^0$. 5. *J:* (3,2). 6. *J:*$AB$: $2x+3y-19=0,\ BC: 8x-11y-7=0,\ CD: 2x+3y+27=0,\ AD:\ 8x-11y+39=0$. 7. *J:* $x+3y-2z=0$. 8. *J:* $3x-6y+2z-49=0$. 9. *J:*$d=5{,}5$. 10. *J:* $\frac{x}{2}=\frac{y+8}{7}=\frac{z+4}{4}$. 11. *J:* $\frac{x-3}{2}=\frac{y}{1}=\frac{z+2}{1}$. 12. *J:* $(3{,}2-1)$. 13. *J:* $\frac{\pi}{4}$. 14. *J:* $d=\frac{4}{\sqrt{26}}$.

Variant 18

1. $A(5,-4,1)$, $B(3,2,3)$, $C(1,-1,-2\ )$ nuqtalar berilgan. ABC uchburchak yuzini toping.

2. Tetraedr hajmi $V=10$ ga teng. $A(7,12,3)$, $B(3,0,1)$, $C(2,1,1\ )$ nuqtalar uning uchlari bo'lsa, uning OY o'qida yotgan D uchini koordinatasini toping.

3. $M(2;-1)$ nuqtadan o'tib, a) $2x+3y=0$ to'g'ri chiziqqa parallel; c)$2x+3y=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(-4,-5)$, $B(4,1)$, $C\left(-\frac{1}{2},7\right)$.a) C uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) A burchagi bissektrisasi tenglamasini tuzing.

5. $M(-8,1,2)$ nuqtaning $A(2,-3)$, $B(-5,1)$ nuqtalardan o'tuvchi to'g'ri chiziqqa proyeksiyasini toping.

6. Parallelogrammning qo'shniuchlari $A(1;-2)$ va $B(3;2)$ hamda diagonallari kesishish nuqtasi $M(1;1)$ berilgan bo'lsa, Parallelogrammning diagonallari va qolgan ikki tomoni tenglamasini tuzing.

7. $M_0(-2;7;3)$ nuqtadan o'tib, $x-4y+5z-1=0$ tekislikka parallel tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(3;-2;4)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$x-2y+z-1=0$,$x+2y+3z+1=0$, $2x-4y+2z-1=0$

10. $\begin{cases}3x-5y+z-8=0\\ 2x+y-z+2=0\end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(1;0;-2)$ nuqtadan o'tib, $\vec{q}\{2;1;0\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(1;-3;2)$ nuqtaning $6x+3y-z-41=0$ tekislikdagi proyeksiyasini toping.

13. $\frac{x+3}{1}=\frac{y-2}{-2}=\frac{z+1}{2}$ to'g'ri chiziq va $4x+2y+2z-5=0$ tekislik orasidagi burchakni toping.

14. $\frac{x-2}{4}=\frac{y+1}{1}=\frac{z-1}{-1}, \frac{x+4}{2}=\frac{y-2}{-2}=\frac{z+2}{-3}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\sqrt{274}$. 2. *J:* $D_1(0{,}25{,}0), D_2(0,-35{,}0)$. 3. *J:* $a)\ 2x+3y-1=0$, $b)\ 3x-2y-8=0$. 4. *J:* $a)\ 4x+3y-19=0$, $b)\ 36x-23y+29=0$, $c)\ 13x-9y+7=0$. 5. *J:* $(1{,}2{,}5)$. 6. *J:* $AB: 2x-y-4=0$, $BC: x+y+5=0$, $CD: 2x-y+2=0$, $AD: x+y+1=0$. 7. *J:* $x-4y+5z+15=0$. 8. *J:* $3x-2y+4z-29=0$. 9. *J:*$d=\frac{1}{2\sqrt{6}}$. 10. *J:* $\frac{x-2}{4}=\frac{y-1}{5}=\frac{z-7}{13}$. 11. *J:* $\begin{cases} x-2y-z=0 \\ z+2=0 \end{cases}$. 12. *J:* $(7{,}0{,}1)$. 13. *J:* $arcsin\left(\frac{2}{3\sqrt{6}}\right)$. 14. *J:* $d=6$.

Variant 19

1. Uchburchak uchlari berilgan: $A(5,-6{,}2)$, $B(1{,}3,-1)$, $C(1,-1{,}2)$. Uning A uchidan BC tomoniga tushirilgan balandligini toping,

2. Piramida uchlari berilgan: $A(2{,}0{,}0)$, $B(0{,}3{,}0)$, $C(0{,}0{,}6), D(2{,}3{,}8)$. Uning hajmini va ABC yogqiga tushirilgan balandligini toping.

3. $M(-1;3)$ nuqtadan o'tib, $a)\ \frac{x-1}{2}=\frac{y+1}{3}$ to'g'ri chiziqqa parallel; c) $\frac{x-1}{2}=\frac{y+1}{3}$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(4;8)$, $B(2;-10)$, $C(-6;-2)$.a) B uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) C burchagi bissektrisasi tenglamasini tuzing.

5. $M(-4;6)$ nuqtaning $4x-5y+3=0$ to'g'ri chiziqqa proyeksiyasini toping.

6. Uchlari $A(-2;-2)$, $B(-3;1)$, $C-\left(\frac{5}{2};\frac{5}{2}\right), D(3{,}1)$ nuqtalarda bo'lgan to'rtburchak trapetsiya ekanligini ko'rsating. Shu trapetsiyasining o'rta chizig'i va diagonallari tenglamasini tuzing.

7. $M_0(-3;-2;4)$ nuqtadan o'tib, $-2y-3z+5=0$ tekislikka parallel tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(1;2;3)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$$2x-3y+6z-21=0, 4x-6y+12z+35=0, 7x-3y+z-15=0.$$

10. $\begin{cases} 2x-y+3z-5=0 \\ 4x+3y-2z+8=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(1;-1;0)$ nuqtadan o'tib, $\vec{q}\{5;-1;8\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(5;2;-1)$ nuqtaning $2x-y+3z+23=0$ tekislikdagi proyeksiyasini toping.

13. $\frac{x}{2}=\frac{y+12}{3}=\frac{z-4}{6}$ to'g'ri chiziq va $6x+15y-10z=0$ tekislik orasidagi burchakni toping.

14. $\frac{x}{1}=\frac{y+3}{2}=\frac{z-2}{1}, \frac{x-3}{1}=\frac{y+2}{2}=\frac{z-2}{1}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* 5. 2. *J:* $V=14,\ \ H=\sqrt{14}$. 3. *J:* $a)\ 3x-2y+9=0;\ \ b)\ 2x+3y-7=0$. 4. *J:* $a)\ x+y+8=0;\ \ b)\ 7x-3y-4=0;\ \ c)\ y+2=0$.
5. *J:* $(-2-1)$. 6. *J:* $AB \parallel CD, 3x+y-1=0,\ x-y=0,\ y-1=0$.
7. *J:* $x-2y-3z+11=0$. 8. *J:* $x+2y+3z-14=0$. 9. *J:* $d=5{,}5$. 10. *J:* $\frac{x}{-7}=\frac{y+1}{16}=\frac{z-1}{10}$. 11. *J:* $\frac{x-1}{5}=\frac{y+1}{-1}=\frac{z}{8}$. 12. *J:* $(1;4;7)$. 13. *J:* $arcsin(3/133)$.
14. J: $d=5\sqrt{5}/\sqrt{6}$.

Variant 20

1. ABC uchburchak uchlari berilgan: $A(5,-6,2)$, $B(1,3,-1)$, $C(1,-1,2)$.B uchidan AC tomoniga tushirilgan balandligini toping.

2. $M(2;3)$ nuqtadan o'tib, a) $0{,}5x-y+3=0$ to'g'ri chiziqqa parallel; b) $0{,}5x-y+3=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

3. Uchburchak uchlarining koordinatalari berilgan: $A(4;8)$, $B(2;-10)$, $C(-6;-2)$.a) B uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) B burchagini toping.

4. $M(1;2)$ nuqtani $2x-y-5=0$ to'g'ri chiziqqa proyeksiyasini toping.

5. Parallelogrammning bir uchidan chiquvchi ikki tomoni tenglamasi $5x-3y+28=0$ va $x-3y-4=0$ hamda shu uchga qarama-qarshi uchining koordinatasi $(10;6)$ berilgan. Parallelogrammning qolgan ikki tomoni va diagonallari tenglamasini tuzing.

6. $M(2;-1;1)$ nuqtadan o'tib, $3x-y-Z+1=0$, $x-y+2z=0$tekisliklar kesishish chizig'iga perpendikulyar tekislik tenglamasini tuzing.

7. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-1;2;3)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

8. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$3x-6y-2z+35=0, 3x-6y-2z-7=0, 4x-5y+3z-1=0.$

9. $\begin{cases} 2x+3y-16z-7=0 \\ 3x+y-17z=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

10. $M(2;-5;3)$ nuqtadan o'tib, $\vec{q}\{4;-6;9\}$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

11. $A(2;1;1)$ nuqtaning $x+y+z+5=0$ tekislikdagi proyeksiyasini toping.

12. $\frac{x-1}{0}=\frac{y}{2}=\frac{z+1}{1}$ to'g'ri chiziq va $x+y-z+1=0$ tekislik orasidagi burchakni toping.

13. $\frac{x+7}{1}=\frac{y+4}{14}=\frac{z+3}{2}, \frac{x-21}{6}=\frac{y+5}{-4}=\frac{-2}{-1}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $25/\sqrt{41}$. 2. *J:* $a)$ $0{,}5x-y+2=0$; $b)$ $2x+y-7=0$. 3. *J:* $a) x+y+8=0$; $b)$ $7x-3y-4=0$; $c)$ $arccos(4/\sqrt{41})$. 4. *J:* $(3;1)$. 5. *J:* $x-3y+8=0$; $5x-3y-32=0$; $5x-3y+4=0$; $y-1=0$. 6. *J:* $3x+7y+2z-1=0$. 7. *J:* $-x+2y+3z-14=0$. 8. *J:*$d=6$. 9. *J:* $\frac{x+1}{5}=\frac{y-3}{2}=\frac{z}{1}$. 10. *J:* $\frac{x-2}{4}=\frac{y+5}{-6}=\frac{z-3}{9}$. 11. *J:* $(1;0;-2)$. 12. *J:* $arcsin\left(\frac{1}{\sqrt{15}}\right)$. 13. *J: d=13.*

Variant 21

1. ABC uchburchak uchlari berilgan: $A(7{,}3{,}4)$, $B(1{,}0{,}6)$, $C(4{,}5,-2)$. Uning A uchidan BC tomoniga tushirilgan balandlini toping.

2. Piramida uchlari berilgan: $A(2{,}0{,}0)$, $B(2{,}3{,}8)$, $C(0{,}0{,}6)$, $D(0{,}3{,}0)$. Piramida hajmi va ACD yoqiga tushirilgan balandligini toping.

3. $M(2;3)$ nuqtadan o'tib, a) $3x-2y+2=0$ to'g'ri chiziqqa parallel;
c) $3x-2y+2=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(4;8)$, $B(2;-10)$, $C(-6;-2)$. a) A uchidan tushirilgan balandlik tenglamasini, b) C uchidan tushirilgan mediana tenglamasini, c) B burchagini toping.

5. $M(0{,}0)$ nuqtani $2x+3y=0$ to'g'ri chiziqqa proyeksiyasini toping.

6. Parallelogrammning ikki tomoni tenglamasi $y=2$ i $2x-3y+12=0$ va diagonallari kesishish nuqtasi koordinatasi $M(2;4)$ berilgan. Parallelogrammning qolgan ikki tomoni va diagonallari tenglamasini tuzing.

7. Koordinatalar boshidan o'tib, $2x-y+5z+3=0, x+3y-z-7=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(1;0;2)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:
$2x-3y+6z+28=0, 2x-3y+6z-14=0, x-4y-z+9=0.$

10. $\begin{cases} x+y-z=0 \\ 2x-y+2=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(2;-5;3)$ nuqtadan o'tib, $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z+2}{1}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(3;-1;4)$ nuqtaning $\frac{x-5}{13}=\frac{y-6}{1}=\frac{z+3}{-4}$, $\frac{x-2}{13}=\frac{y-3}{1}=\frac{z+3}{-4}$ parallel to'g'ri chiziqlar orqali o'tuvchi tekislikdagi proyeksiyasini toping.

13. $\frac{x-3}{2}=\frac{y+4}{2}=\frac{z-4}{3}$ to'g'ri chiziq va $x+2y+3z-5=0$ tekislik orasidagi burchakni toping.

14. $\frac{x-1}{2}=\frac{y}{-1}=\frac{z+1}{3}$, $\frac{x}{2}=\frac{y+1}{-1}=\frac{z-2}{3}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\frac{7}{\sqrt{2}}$. 2. *J:* $V=14, H=\sqrt{14}$. 3. *J:* $3x-2y=0,\ 2x+3y-13=0$.

4. *J:* $a) x+8y+22=0$ $\quad b)\ 11x+3y+2=0$ $c)\ arccos(3/\sqrt{15})$. 5. *J: 0.*
6. *J:* $y=6, 2x-3y+4=0, 2x-5y+16=0, 2x-y=0$. 7. *J:* $-13x+24y+10z=0$. 8. *J:* $x+2y-5=0$. 9. *J:*$d=6$. 10. *J:* $\frac{x}{1}=\frac{y-2}{2}=\frac{z-2}{3}$. 11. *J:* $\frac{x-2}{2}=\frac{y+5}{3}=\frac{z-3}{1}$. 12. *J:* $(2;-3;-5)$. 13. *J:* $arcsin(19/\sqrt{406})$. 14. *J:* $3\sqrt{5}/\sqrt{7}$.

Variant 22

1. ABC uchburchak uchlari berilgan: $A(4,-2,3),\ \ B(0,-1,3),\ C(3,-4,5)$. Uning A uchidan BC tomoniga tushirilgan balandligini toping.

2. Tetraedr hajmi $V=2$ ga teng. Uning uchta uchi koordinatalari berilgan: $A(-1,1,1),\ B(1,2,1),\ C(7,12,3)$. Uning OY o'qida yotgan D uchi kordinatasini toping.

3. $M(2;-3)$ nuqtadan o'tib, a) $3x-7y+3=0$ to'g'ri chiziqqa parallel; c) $3x-7y+3=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(2,6),\ B(4,-2),\ C(-2,-6)$.a) A uchidan tushirilgan balandlik tenglamasini, c) C uchidan tushirilgan mediana tenglamasini, c) B burchagini toping.

5. $M(4,3)$ nuqtaning $3x-4y+10=0$ to'g'ri chiziqqa proyeksiyasini toping.

6. parallelogrammning qo'shniuchlari koordinatalari $A(-3;2)$ va $B(3;6)$ va diagonallari kesishish nuqtasi $M(2,4)$ berilgan bo'lsa, uning diagonallari va qolgan ikki tomoni tenglamasini tuzing.

7. Koordinatalar boshidan o'tib, $2x-y+5z+3=0, x+3y-z-7=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-1;2;0)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$x+5y-z+10=0,\ x+2y-4=0,\ x-y+2z+10=0,\ x-y+2z-4=0.$

10. $\begin{cases} 4x-y+3z-7=0 \\ 2x+y-4z+10=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(-1;2;1)$ nuqtadan o'tib, $\begin{cases} x+y-2z-1=0 \\ x+2y-z+1=0 \end{cases}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(4;3;-1)$ nuqtaning $x+2y-y-z-3=0$ tekislikdagi proyeksiyasini toping.

21. $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-3}{2}$ to'g'ri chiziq va $x+2y-3z+4=0$ tekislik orasidagi burchakni toping.

14. $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z}{1}$, $\frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\sqrt{\frac{149}{22}}$. 2. *J:* $D_1(0,1,0)$, $D_2(0,-5,0)$. 3. *J:* $3x-7y+3=0, 7x+3y-5=0.$

4. *J:* $a)\ 3x+2y-18=0;$ $b)\ 8x-5y-14=0$; $c)\ arccos(-5/\sqrt{221})$.

5. *J: 2.* 6. *J:* $2x-3y+12=0, y=6, 2x-3y+4=0, y=2$. 7. *J:* $2x-y-z=0$. 8. *J:* $-x+2y-5=0$. 9. *J:*$d=\frac{14}{\sqrt{6}}$. 10. *J:* $\frac{x}{1}=\frac{y-2}{22}=\frac{z-3}{6}$. 11. *J:* $\frac{x+1}{3}=\frac{y-2}{-1}=\frac{z-1}{1}$. 12. *J:* $(5,-1,0)$. 13. *J:* $arcsin(5/\sqrt{406})$. 14. *J:* $d=3$.

Variant 23

1. ABC uchburchak uchlari berilgan: $A(4,-2,3)$, $B(0,-1,3)$, $C(3,-4,5)$. B uchidan AC tomoniga tushirilgan balandligini toping.

2. $M(2;-3)$ nuqtadan o'tib, $a)\ x+9y-11=0$ to'g'ri chiziqqa parallel; c) $x+9y-11=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

3. Uchburchak uchlarining koordinatalari berilgan: $A(-7{,}3)$, $B(2,-1)$, $C(-1,-5)$. a) B uchidan tushirilgan balandlik tenglamasini, b) A uchidan tushirilgan mediana tenglamasini, c) C burchagi bissektrisasi tenglamasini tuzing.

4. $M(2,-5)$ nuqtaning $4x-3y+25=0$ to'g'ri chiziqqa proyeksiyasini toping.

5. Parallelogrammning ikki tomoni tenglamasi $x-y+1=0$ $ix-2y+6=0$ va diagonallari kesishish nuqtasi $M(4;4)$ berilgan. Parallelogrammning qolgan ikki tomoni va diagonallari tenglamasini tuzing.

6. Koordinatalar boshidan o'tib, $2x-y+3z-1=0, x+2y+z=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

7. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(2;1;1)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

8. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$$x-2y+z-1=0, \qquad 2x-4y+2z-1=0, 5x-6y+7z-1=0.$$

9. $\begin{cases} 2x+y+3z-4=0 \\ x-y+2z-1=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

10. $M(2;-3;-1)$ nuqtadan o'tib, $\frac{x-4}{4}=\frac{y+1}{3}=\frac{z+3}{2}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

11. $A(3;1;-1)$ nuqtaning $x+2y+3z-30=0$ tekislikdagi proyeksiyasini toping.

12. $\frac{x+4}{3}=\frac{y-1}{2}=\frac{z-3}{4}$ to'g'ri chiziq va $2x-3y-2z+5=0$ tekislik orasidagi burchakni toping.

13. $\frac{x-9}{4}=\frac{y+2}{-3}=\frac{z}{1}$, $\frac{x}{-2}=\frac{y+7}{9}=\frac{z-2}{2}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $2\sqrt{10}$. 2. *J:* $a) x+9y+25=0,\ c)\ 9x-y-21=0$. 3. *J:* $a)\ 3x-4y-10=0$; $b)\ 4x+5y+13=0$; $c)\ x+1=0$. 4. *J:* 9,6. 5. *J:* $x-y-1=0$; $x-2y+2=0$; $3x-4y+4=0$; $x-4=0$. 6. *J:* $-7x+y+5z=0$. 7. *J:* $2x+y+z-6=0$ 8. *J:* $d=\frac{\sqrt{6}}{12}$. 9. *J:* $\frac{x}{5}=\frac{y-1}{-1}=\frac{z-1}{-3}$. 10. *J:* $\frac{x-2}{4}=\frac{y+3}{3}=\frac{z+1}{2}$. 11. *J :* (5,5,5). 12. *J:* $arcsin\left(\frac{8}{\sqrt{493}}\right)$. 13. *J:* $d=29/7$.

Variant 24

1. ABC uchburchak uchlari berilgan: $A(2,5,3)$, $B(1,2,3)$, $C(0,2,5)$. Uchburchakning C uchidan AB tomoniga tushirilgan balandligini toping.

2. Piramida uchlari berilgan: $A(3,5,4)$, $B(8,7,4)$, $C(5,10,4)$, $\mathrm{D}(4,7,8)$. Piramida hajmini va A uchidan tushirilgan balandligini toping.

3. $M(-2;-5)$ nuqtadan o’tib, a)$3x+4y+2=0$ to’g’ri chiziqqa parallel; c)$3x+4y+2=0$ to’g’ri chiziqqa perpendikulyar bo’lgan to’g’ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(2,-1)$, $B(-7,3)$, $C(-1,-5)$.a) B uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) B burchagini toping.

5. $M(-1,5)$ nuqtaning $4x+3y-5=0$ to’g’ri chiziqqa proyeksiyasini toping.

6. Parallelogrammning qo’shni uchlari koordinatalari $A(0,1)$, $B(4,5)$ va diagonallari kesishish nuqtasi $M(4,4)$ berilgan. Parallelogrammning qolgan ikki tomoni va diagonallari tenglamasini tuzing.

7. M(-1;-1;2) nuqtadan o’tib, $x+5y+9z-13=0, 3x-y-5z+1=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-1;2;1)$ ga teng bo’lsa, shu tekislik tenglamasini tuzing. Javob: $-x+2y+z-6=0$.

9. Tekisliklarning parallellarini ko’rsating va ular orasidagi masofani toping:
$6x-18y-9z-28=0, 5x-6y+7z+1=0, 4x-12y-6z-7=0.$

10. $\begin{cases} x+2y-3z-7=0 \\ 2x-y+4z+1=0 \end{cases}$ to’g’ri chiziq tenglamasini kanonik ko’rinishga keltiring.

11. $M(2;-5;3)$ nuqtadan o’tib, $\begin{cases} 2\,x-y+3z-1=0 \\ 5x+4y-z-7=0 \end{cases}$ to’g’ri chiziqqa parallel to’g’ri chiziq tenglamasini tuzing.

12. $A(1;0;2)$ nuqtaning $\frac{x-2}{-1}=\frac{y+3}{2}=\frac{z-1}{-1}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x-1}{4}=\frac{y}{12}=\frac{z-1}{-3}$ to'g'ri chiziq va $6x-3y-2z=0$ tekislik orasidagi burchakni toping.

14. $\frac{x+3}{4}=\frac{y-6}{-3}=\frac{z-3}{2}$, $\frac{x-4}{8}=\frac{y+1}{-3}=\frac{z+7}{3}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\frac{3}{\sqrt{2}}$. 2. *J:* $V=14, H=7/\sqrt{14}$. 3. *J:* $3x+4y+26=0, c)\ 4x-3y-7=0$. 4. *J:* $a)3x+4y+9=0$ $b)\ y+1=0$ $c)\ arccos\left(\frac{43}{5\sqrt{97}}\right)$. 5. *J:* 1,2. 6. *J:* $x-y+1=0, x-2y+6=0,\ x-y-1=0, x-2\,y+2=0$. 7. *J:* $2x+3y+4z-3=0$. 8. J: $-x+2y+z-6=0$. 9. *J:*$d=\frac{5}{6}$. 10. *J:* $\frac{x-1}{11}=\frac{y-3}{-10}=\frac{z}{-5}$. 11. *J:* $\frac{x-2}{-11}=\frac{y+5}{17}=\frac{z-3}{13}$. 12. *J:* $(1,-1,0)$. 13. *J:* $arcsin(6/91)$. 14. *J:* $d=13$.

Variant 25

1. ABC uchburchak uchlari berilgan: $A(-1,2,-3)$, $B(3,4,-6)$, $C(1,1,-1)$. Uchburchakning B uchidan AC tomoniga tushirilgan balandligini toping.

2. Piramida uchlari berilgan. $A(4,6,5)$, $B(6,9,4)$, $C(2,10,10)$, $D(7,5,9)$. Piramidaning hajmini va C uchidan tushirilgan balandligini toping.

3. $M(2;-3)$ nuqtadan o'tib, $a)\ 2x+3=0$ to'g'ri chiziqqa parallel; c) $2x+3=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(-2,-2)$, $B(7,-6)$, $C(1,2)$.a) B uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) B burchagini toping.

5. $M(0,6,3)$ nuqtaning $5x-12y-6=0$ to'g'ri chiziqqa proyeksiyasini toping.

6. Uchlari$A(3,-1)$, $B(2,4)$, $C(5,6)$, $D(9,3)$ nuqtalarda bo'lgan to'rtburchakning trapetsiya ekanligini ko'rsating. Shu trapetsiyaning o'rta chizig'i va diagonallari tenglamasini tuzing.

7. $M_0(1;2;3)$ nuqtadan o'tib, $x-y+z-7=0, 3x+2y-12z+5=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(2;3;4)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$$3x-6y-2z+35=0, 3x-6y-2z-1=0, x-4y-z+9=0$$

10. $\begin{cases} 2x+y-z+1=0 \\ 3x-4y+z=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(2;0;-3)$ nuqtadan o'tib, $\begin{cases} 3x-y+2z-7=0 \\ x+3y-2z-3=0 \end{cases}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(1;1;1)$ nuqtaning $\frac{x+4}{-2}=\frac{y-3}{4}=\frac{z-4}{1}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x-5}{2}=\frac{y+1}{-3}=\frac{z}{-1}$ to'g'ri chiziq va $2x+y+z=0$ tekislik orasidagi burchakni toping.

14. $\begin{cases} 2x+2y-z-10=0 \\ x-y-z-22=0 \end{cases}$ va $\frac{x+7}{3}=\frac{y-5}{-1}=\frac{z-9}{4}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:*$\frac{3\sqrt{29}}{\sqrt{38}}$. 2. *J:* $V=\frac{121}{6}$, $H=\frac{121}{3\sqrt{69}}$. 3. *J:*$a)x-2=0$, $c)y+3=0$. 4. *J:*$a)3x+4y+3=0$; $b)$ $y+2=0$; $c)$ $arccos(8.6/\sqrt{97})$. 5. *J: 3*. 6. *J:* $BC \parallel AD, 4x-6y-1=0, 7x-2y-23=0, x+7y-30=0$. 7. *J:*$2x+3y+z-11=0$. 8. *J:* $2x+3y+4z-29=0$. 9. *J:*$d=6$. 10. *J:* $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-5}{11}$. 11. *J:* $\frac{x-2}{-2}=\frac{y}{4}=\frac{z+3}{5}$. 12. *J:*$(-2,-1,3)$. 13. *J:0*. 14. *J:*$d=25$.

Variant 26

1. ABC uchburchak uchlari berilgan: $A(-1,2,-3)$, $B(3,4,-6)$, $C(1,1,-1)$. Uchburchakning B uchidan AC tomoniga tushirilgan balandligini toping.
2. Tetraedr uchlari berilgan: $A(1,3,6)$, $B(2,2,1)$, $C(-1,0,1)$, $D(-4,6,-3)$. Tetraedr hajmini va D uchidan tushirilgan balandligini toping.
3. $M(2;-3)$ nuqtadan o'tib, a)$16x-24y-7=0$ to'g'ri chiziqqa parallel; b) $16x-24y-7=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.
4. Uchburchak uchlarining koordinatalari berilgan: $A(7,-6)$, $B(-2,-2)$, $C(1,2)$. a) B uchidan tushirilgan balandlik tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) B burchagini toping.
5. $x+2y+5=0$ va $x+2y+3=0$ to'g'ri chiziqlar orasidagi masofani toping.
6. Uchlari $A(6,-3)$, $B(3,3)$, $C(5,6)$, $D(10,3)$ nuqtalarda bo'lgan to'rtburchakning trapetsiya ekanligini ko'rsating. Shu trapetsiyaning o'rta chizig'i va diagonallari tenglamasini tuzing.
7. $M_0(3;-1;5)$ nuqtadan o'tib, $3x-2y+2z+7=0$, $5x-4y+3z+1=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.
8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-2;1;3)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.
9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$$x-2y+3z-1=0, \qquad 3x+y-z+5=0, \qquad 2x-4y+6z+3=0.$$

10. $\begin{cases} x+y+3z-2=0 \\ 3x-2y+7z-1=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.
11. $M(1;1;1)$ nuqtadan o'tib, OX o'qiga parallel to'g'ri chiziq tenglamasini tuzing.
12. $A(2;3;1)$ nuqtaning $\frac{x+7}{1}=\frac{y+2}{2}=\frac{z+2}{3}$ to'g'ri chiziqdagi proyeksiyasini toping.
13. $\begin{cases} x+y-2z+3=0 \\ x+2y-3z-1=0 \end{cases}$ to'g'ri chiziq va $2x-y-z=0$ tekislik orasidagi burchakni toping.
14. $\frac{x+1}{1}=\frac{y}{1}=\frac{z-1}{2}$, $\frac{x}{1}=\frac{y+1}{3}=\frac{z-2}{4}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J: 3.* 2. *J:* $V = \frac{70}{3}$, $H = 2\sqrt{14}$. 3. *J:* $a)\ 2x - 3y - 13 = 0, b)3x + 2y = 0$. 4. *J:*$a)\ 3x - 4y - 2 = 0$; $b)\ 4x + 5y + 2 = 0$; $c)$ $\arccos(11\sqrt{97}/485)$. 5. *J:* $8/\sqrt{5}$. 6. *J:* $BC \parallel AD, 6x - 4y - 27 = 0, y = 3, 9x + y - 51 = 0$. 7. *J:* $2x + y - 2z - 15 = 0$. 8. *J:* $2x + y + 3z - 29 = 0$. 9. *J:*$d = \frac{5}{2\sqrt{14}}$. 10. *J:* $\frac{x-1}{13} = \frac{y-1}{2} = \frac{z}{-5}$. 11. *J:* $\begin{cases} y - 1 = 0 \\ z - 1 = 0. \end{cases}$ 12. *J:* $(-5,2,4)$. 13. *J: 0.* 14. *J:* $d = \sqrt{3}/3$.

Variant 27

1. Tetraedr uchlari berilgan: $A(-4,2,6)$, $B(2,-3,0)$, $C(-10,5,8)$, $D(-4,6,-3)$. Tetraedrning D uchidan tushirilgan balandligini va hajmini toping.

2. $M(2;-4)$ nuqtadan o'tib, a)$2x - 3y + 6 = 0$ to'g'ri chiziqqa parallel; c)$2x - 3y + 6 = 0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

3. Uchburchak uchlarining koordinatalari berilgan: $A(-5,3)$, $B(3,4)$, $C(7,-3)$.a) A uchidan tushirilgan balandlik tenglamasini, c) C uchidan tushirilgan mediana tenglamasini, c) B burchagini toping.

4. $x - 3 = 0$ va $2x + 5 = 0$ to'g'ri chiziqlar orasidagi masofani toping.

5. Parallelograammning ikki qo'shni tomoni tenglamasi $2x - 3y + 3 = 0$ va $2x - y + 9 = 0$ hamda diagonallari kesishish nuqtasi (2;5) berilgan. Parallelogrammning qolgan ikki tomoni tenglamasi va diagonallari tenglamasini tuzing.

6. $M_0(1;1;-2)$ nuqtadan o'tib, $2x + 3z = 0$, $x - y + z - 1 = 0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

7. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(4;1;5)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

8. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$$3x + 2y - z - 6 = 0, \qquad 9x + 6y - 3z = 0, \qquad x + 4y - 5z + 9 = 0.$$

9. $\begin{cases}3x - y + 4z - 7 = 0, \\ x + 5y - 3z + 1 = 0.\end{cases}$ to’g’ri chiziq tenglamasini kanonik ko’rinishga keltiring.

10. $M(2;4;5)$ nuqtadan o’tib, OX o’qiga parallel to’g’ri chiziq tenglamasini tuzing.

11. $A(1;2;1)$ nuqtaning $\frac{x+2}{3} = \frac{y}{-1} = \frac{z+1}{2}$ to’g’ri chiziqdagi proyeksiyasini toping.

12. $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z}{1}$ to’g’ri chiziq va $x - 2y + z - 5 = 0$ tekislik orasidagi burchakni toping.

13. $\frac{x-2}{4} = \frac{y+1}{1} = \frac{z-1}{-1}$, $\frac{x+4}{2} = \frac{y-2}{-2} = \frac{z+2}{-3}$ to’g’ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $V = 56/3$, $H = 4$. 2. *J:*$a)\ 2x - 3y + 16 = 0$, $b)3x + 2y - 2 = 0$. 3. *J:* $a)\ 4x - 7y + 41 = 0$; $b)\ x - 16y - 55 = 0$; $c)$ $\arccos(2{,}5/\sqrt{221})$.
4. *J:* 5,5. 5. *J:* $2x - 3y + 11 = 0$, $2x - y + 1 = 0$, $x - y + 3 = 0$, $y - 1 = 0$.
6. *J:* $3x + y - 2z - 8 = 0$. 7. *J:* $4x + y + 5z - 42 = 0$. 8. *J:*$d = \frac{6}{\sqrt{14}}$. 9. *J:* $\frac{x}{-17} = \frac{y-1}{13} = \frac{z-2}{-16}$. 10. *J:*$\begin{cases}y - 4 = 0 \\ z - 5 = 0.\end{cases}$ 11. *J:* $(-0{,}5; -0{,}5; 2)$. 12. *J:* $arc\,sin\,1/6$.
13. *J:* $d = 6$.

Variant 28

1. ABC uchburchak uchlari berilgan. $A(-2,1,1)$, $B(2,3,-2)$, $C(0,0,3)$. Uchburchakning A uchidan BC tomoniga tushirilgan balandligini toping.

2. Tetraedr uchlari berilgan: $A(1,1,2)$, $B(-1,1,3)$, $C(2,-2,4)$, $D(-1,0,-2)$. Tetraedrning B uchidan tushirilgan balandligini va hajmini toping.

3. $M(2;-1)$ nuqtadan o’tib, a)$3x - 5y - 7 = 0$ to’g’ri chiziqqa parallel; c)$3x - 5y - 7 = 0$ to’g’ri chiziqqa perpendikulyar bo’lgan to’g’ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(-2,0)$, $B(2,6)$, $C(4,2)$. a) AC tomon tenglamasini, c) B uchidan tushirilgan balandlik tenglamasini, c) B uchidan tushirilgan medianasi tenglamasini tuzing.

5. $3x-4y+5=0$ va $3x-4y+1=0$ to'g'ri chiziqlar orasidagi masofani toping.

6. Parallelogrammning ikki qo'shni uchlari $A(-6,-3)$, $B(-4,1)$ va diagonallari kesishish nuqtasi $M(-2,1)$ berilgan. Parallelogrammning qolgan ikki tomoni va diagonallari tenglamasini tuzing.

7. $M(2;-1;1)$ nuqtadan o'tib, $x+5y+9z-13=0$, $3x-y-5z+1=0$ tekisliklar kesishish chizig'i ga perpendikulyar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(1;-2;1)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:
$3x+4y-5=0,\ 5x-7y+8z=0,\ 6x+8y-16=0.$

10. $\begin{cases} x-3y+z-5=0 \\ 2x+4y-2z+10=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(1;0;-3)$ nuqtadan o'tib, OY o'qiga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(2;0;0)$ nuqtaning $\frac{x-30}{1}=\frac{y-2}{3}=\frac{z+2}{4}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x+2}{3}=\frac{y}{-1}=\frac{z-1}{2}$ to'g'ri chiziq va $4x+2y+2z-5=0$ tekislik orasidagi burchakni toping.

14. $\frac{x+7}{1}=\frac{y+4}{14}=\frac{z+3}{-2}$, $\frac{x-21}{6}=\frac{y+5}{-4}=\frac{z-2}{-1}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\frac{3\sqrt{29}}{\sqrt{38}}$. 2. *J:* $V=35/6,\ \ H=\sqrt{5}$. 3. *J:* $a)\ 3x-5y-11=0, c)5x+3y-1=0$. 4. *J:* $a)\ x-3y+2=0$; $b)\ 5x-y-4=0$; $c)\ 3x+y-12=0$. 5. *J: 0,8.* 6. *J:* $2x-3y+3=0,\ 2x-y+9=0,\ 2x-3y+11=0,\ 2x-y+1=0$. 7. J: $x+y+z-3=0$. 8. *J:* $x-2y+z-6=0$. 9. *J:*$d=0{,}6$. 10. *J:* $\frac{x}{1}=\frac{y}{2}=\frac{z-5}{5}$. 11. *J:* $\begin{cases} x-1=0 \\ z+3=0. \end{cases}$ 12. *J:* $(-2,-1{,}3)$. 13. *J:* $arcsin\left(\frac{\sqrt{7}}{2\sqrt{3}}\right)$. 14. *J:* $d=13$.

Variant 29

1. ABC uchburchak uchlari berilgan: $A(-2,1,1)$, $B(2,3,-2)$, $C(0,0,3)$. Uchburchakning B uchidan AC tomoniga tushirilgan balandligini toping.

2. Tetraedr uchlari berilgan: $A(2,3,1)$, $B(4,1,-2)$, $C(6,3,7)$, $D(7,5,-3)$. Tetraedrning hajmini va ACD yoqiga tushirilgan balandligini toping.

3. $M(-1;-4)$ nuqtadan o'tib, a)$\frac{x}{4}+\frac{y}{3}=1$ to'g'ri chiziqqa parallel; c)$\frac{x}{4}+\frac{y}{3}=1$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4. Uchburchak uchlarining koordinatalari berilgan: $A(1,1)$, $B(2,4)$, $C(-3,1)$.a) AC tomon tenglamasini, c) A uchidan tushirilgan mediana tenglamasini, c) B uchidan tushirilgan balandlik tenglamasini tuzing

5. $5x-12y+52=0$ va $10x-24y-39=0$ to'g'ri chiziqlar orasidagi masofani toping.

6. Uchlari $A(4,12)$, $B(6,5)$, $C(3,2)$, $D(-3,5)$ nuqtalarda bo'lgan to'rtburchak trapetsiya ekanligini ko'rsating. Shu trapetsiyaning o'rta chizig'i va diagonallari tenglamasini tuzing.

7. $4x-y+3z-1=0$, $x+5y-z+2=0$ tekisliklar kesishish chizig'idan o'tib, $2x-y+5z-3=0$ tekislikka perpendikulayar tekislik tenglamasini tuzing.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(2;1;3)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping:

$$2x-2y+z+3=0, \qquad 6x-6y+3z-12=0, \qquad 4x-y+2z-4=0.$$

10. $\begin{cases} 2x+3y-4z-3=0 \\ 3x-5y+z+5=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(2;-5;3)$ nuqtadan o'tib, OZ o'qiga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(0;0;-1)$ nuqtaning $\frac{x+2}{-3}=\frac{y-1}{-2}=\frac{z-9}{1}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x}{2}=\frac{y}{-2}=\frac{z-1}{1}$ to'g'ri chiziq va $x-2y+2z-5=0$ tekislik orasidagi burchakni toping.

14. $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z}{1},\frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: 1. *J:* $\sqrt{29}$. 2. *J:* $V=70/3,\ H=70\sqrt{581}$. 3. *J:* $a)\ 3x+4y+19=0, b)4x-3y-8=0$. 4. *J:* $a)\ y-1=0;\quad b)\ x-y+2=0;\ c)\ x-2=0$. 5. *J:* 5,5. 6. *J:* $BC\ \parallel AD, 2x-2y-7=0,\ y=5, 10x-y-28=0$. 7. *J:* $7x+14y+5=0$. 8. *J:* $2x+y+3z-14=0$. 9. *J:*$d=\frac{7}{3}$. 10. *J:* $\frac{x}{17}=\frac{y-1}{-14}=\frac{z}{19}$. 11. *J:*$\begin{cases} x-2=0 \\ y+5=0. \end{cases}$ 12. *J:* (1,3,8). 13. *J:*$arc\,sin(8/9)$. 14. *J:* $d=3$.

Variant 30

1. $\mathrm{A}(2,0,-2), \mathrm{B}(1,2,3), \mathrm{C}(3,-3,1)$ nuqtalar berilgan. ABC uchburchak yuzini toping.

2. Tetraedr uchlari berilgan. $A(7,2,4)$, $B(7,-1,-2)$, $C(3,3,1)$, $D(-4,2,1)$. Uning hajmini va ABC yoqiga tushirilgan balandligini toping.

3. $M_0(-3,-1)$ nuqtadan o'tib, a) AB to'g'ri chiziqqa parallel, c) AB to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing. Bu yerda: $A(-2;6)$, $B(3;-1)$.

4. Uchburchak uchlarining koordinatalari berilgan: $A(0,0)$, $B(1,6)$, $C(4,2)$. a) AC tomon tenglamasini, b) B uchidan tushirilgan mediana tenglamasini, c) B uchidan tushirilgan balandlik tenglamasini tuzing .

5. $3x+y-3\sqrt{10}=0$ va $6x+2y+5\sqrt{10}=0$ to'g'ri chiziqlar orasidagi masofani toping.

6. Parallelogrammning ikki qo'shni uchlari $A(0;0)$, $B(-2;5)$ va diagonallari kesishish nuqtasi $M(1;3)$ berilgan. Parallelogrammning qolgan ikki tomoni va diagonallari tenglamasini tuzing.

7. $A(1;-1;3)$ va $B(1;2;4)$ nuqtalardan o'tib, $2x-3y+z+1=0$ tekislikka perpendikulyar tekislik tenglamasi tuzilsin.

8. Koordinatalar boshidan tekislikka tushirilgan perpendikulyar asosidagi nuqta $P(-5;0;3)$ ga teng bo'lsa, shu tekislik tenglamasini tuzing.

9. Tekisliklarning parallellarini ko'rsating va ular orasidagi masofani toping: $4x-3y+4=0,\ 8x-6z-12=0,\ 2x-z+3=0.$

10. $\begin{cases} 5x-2y+3z+1=0 \\ x+3y-5z-1=0 \end{cases}$ to'g'ri chiziq tenglamasini kanonik ko'rinishga keltiring.

11. $M(3;-1;1)$ nuqtadan o'tib, OZ o'qiga parallel to'g'ri chiziq tenglamasini tuzing.

12. $A(0;3;0)$ nuqtaning $\frac{x-4}{2}=\frac{y-6}{1}=\frac{z+1}{-3}$ to'g'ri chiziqdagi proyeksiyasini toping.

13. $\frac{x+1}{0}=\frac{y-5}{-1}=\frac{z}{2}$ to'g'ri chiziq va $x+2y+2z-5=0$ tekislik orasidagi burchakni toping.

14. $\frac{x+3}{4}=\frac{y-6}{-3}=\frac{z-3}{2}$, $\frac{x-4}{8}=\frac{y+1}{-3}=\frac{z+7}{3}$ to'g'ri chiziqlar orasidagi masofani toping.

Javoblar: **1**. *J:* $0{,}5\sqrt{506}$. **2.** *J:* $V=\frac{43}{2},\ H=\frac{43}{\sqrt{105}}$. **3.** *J:* $a)\ \frac{x+3}{5}=\frac{y+1}{-7},\ b)5x-7y+8=0$. **4**. *J:*$a)\ x-2y=0;\ \ b)\ 5x+y-11=0;\ \ c)\ 2x+y-8=0$. **5**. *J:* 5,5. **6**. *J:* $5x+2y=0,\ x-2y+12=0,\ 5x+2y-24=0,\ x-2y=0$. **7.** *J:* $3x+y-3z+7=0$. **8.** *J:* $-5x+3z-34=0$. **9.** *J:*$d=2$. **10.** *J:* $\frac{x}{1}=\frac{y}{28}=\frac{z-1}{17}$.

11. *J:* $\begin{cases} x-3=0 \\ y+1=0. \end{cases}$ **12.** *J:* $(2;5;2)$. **13.** *J:* $arc\,sin\,(2/3\sqrt{5})$. **14**. *J:* $d=13$.

MATEMATIK ANALIZ ELEMENTLARI

1-§. To'plamlar va ular ustida amallar.

To`plam tushunchasi matematikaning boshlang'ich tushunchalaridan biri bo'lib, unga ta'rif berilmaydi, balki, uni misollar orqali tushintiriladi.

To`plam tushunchasi to`plamlar nazariyasining asoschisi bo`lgan nemis matematigi Georg Kantor tomonidan (1845-1908 y.) kiritilgan.

1-ta'rif. To`plamni tashkil qiluvchi obektlar uning ***elementlari*** deyiladi.

To`plamlar lotin yoki grek alifbosining katta harflari $A, B, C, D, \ldots$ lar bilan, uning elementlari esa, shu alifboning kichik harflari $a, b, c, d, \ldots$ lar bilan belgilanadi.

To’plam ikki turga bo’linadi: chekli to’plam va cheksiz to’plam.

2-ta'rif. CHekli elementlardan tashkil topgan to`plam ***chekli to`plam***,cheksiz elementlardan tashkil topgan to’plam esa aksincha, ***cheksiz to`plam*** deyiladi.

«a element A to`plamga tegishli» bo’lsa $a \in$ A, tegishli bo’lmasa, $a \notin$ A kabi belgilanadi.

3-ta'rif. Birorta ham elementga ega bo`lmagan to`plam ***bo`sh to`plam*** deyiladi va u $\emptyset$ kabi belgilanadi.

4-ta'rif. Agar B to`plamning har bir elementi A to`plamning ham elementi bo`lsa, B to`plam A to`plamning ***qismi*** yoki ***qismiy to’plami*** deyiladi va B $\subset$ A kabi belgilanadi.

5-ta'rif. Agar A to`plam B to’plamning qismi, B to’plam A to’plamning qismi,ya’ni A $\subset$ B va B $\subset$ A bo’lsa, u holda A va B to’plamlar bir biriga ***teng to’plamlar*** deyiladi va A $=$ B kabi belgilanadi.

6-ta'rif. Agar B to`plamning barcha elementlari A to`plamning elementi bo`lib, shu bilan birga A to’plamda yana B ga tegishli bo`lmagan elementlar ham bor bo`lsa, B to`plam A to`plamning ***xos qism to`plami*** deyiladi.

7-ta'rif. A to`plamning o`zi va bo`sh to`plam, A to`plamning ***xosmas qism to`plami*** deyiladi.

8-ta'rif. A va B to`plamlarning barcha elementlaridan tashkil topgan to’plam A va B to’plamlarning ***yig’indisi (birlashmasi)*** deyiladi va A $\cup$ B kabi yoziladi.

9-ta'rif. A va B to`plamlarning barcha umumiy elementlaridan tashkil topgan to’plam A va B to’plamlarning ***ko’paytmasi (kesishmasi)*** deyiladi va $A \cap B$ kabi yoziladi.

10-ta’rif. A to`plamning B to`plamga tegishli bo’lmagan elementlaridan tashkil topgan to’plam A to’plamdan B to’plamning ***ayirmasi*** deyiladi va $A \backslash B$ kabi yoziladi.

11-ta’rif. A to’plamaning B to’plamga tegishli bo’lmagan elementlaridan va B to’plamning A to’plamga tegishli bo`lmagan elementlaridan tashkil topgan to`plam A va B to`plamlarning ***simmetrik ayirmasi*** deyiladi va $A\Delta B$ kabi yoziladi.

Ta’rifdan ko`rinadiki, simmetrik ayirma $A\Delta B = (A\backslash B) \cup (B\backslash A)$ dan iborat ekan.

12-ta’rif. Birinchi elementi A to’plamdan, ikkinchi elementi B to’plamdan olingan $(a; b)(a \in A, b \in B)$ ko’rinishdagi juftliklardan tuzilgan to’plamga A va B to’plamlarning ***Dekart ko’paytmasi*** yoki ***to’g’ri ko’paytmasi*** deyiladi va $A \times B$ kabi yoziladi.

13-ta’rif. Agar A to’plam B to`plamning qismi ya’ni A ⊂ B bo’lsa, ushbu $B\backslash A = \{x: x \in B, x \notin A\}$ to`plam A to’plamni B to’plamga ***to’ldiruvchi to’plam*** deyiladi va CA yoki $C_B A$ kabi belgilanadi.

To`plamlar ustida amallarning xossalari.

1^0-xossa. Kommutativlik;

$$A \cap B = B \cap A \qquad (1)$$

$$A \cup B = B \cup A \qquad (2)$$

2^0-xossa. Assotsiativlik;

$$(A \cup B) \cup C = A \cup (B \cup C) \qquad (3)$$

$$(A \cap B) \cap C = A \cap (B \cap C) \qquad (4)$$

3^0-xossa. Distributivlik;

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \qquad (5)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \qquad (6)$$

4^0-xossa. Idempotentlik;

$$A \cup A = A \tag{7}$$

$$A \cap A = A \tag{8}$$

Misollar.

Quyidagi munosabatlarni o'rinli bo'lishini ko'rsating:

1. $A \cap B \subset A \subset A \cup B$.
2. $A \cap (A \cup B) = A$.
3. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
4. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
5. $A \backslash (B \cap C) = (A \backslash B) \cup (A \backslash C)$.
6. $A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$.
7. $A \cup (CA \cap B) = A \cup B$.
8. $A \backslash (B \backslash C) = (A \backslash B) \cup (A \cap C)$.
9. $(A \backslash B) \cup (B \backslash A) = (A \cup B) \backslash (A \cap B)$.
10. $(A \backslash B) \cup (B \backslash A) = (A \cup B) \cap (CA \cup CB)$.
11. $A = \{1,2,3,4,5,6,7,8\}$ va $B = \{2,4,6,8,10,12\}$ bo'lsin. U holda $A \cup B =?$ va $A \cap B =?$ ni toping.
12. $A = \{1,2,3,4,5,6,7,8\}$ va $B = \{2,4,6,8,10,12\}$ bo'lsin. U holda $A \backslash B =?$ va $B \backslash A =?$ ni toping.
13. $A = \{1,3,5,7,9\}$ va $B = \{2,4,6,8,10\}$ bo'lsin. U holda $A \Delta B =?$ ni toping.
14. $A = \{2n - 1\ toq\ sonlar\}$ va $B = \{N - natural\ sonlar\}$ bo'lsin. U holda $CA =?$ ni toping.
15. $A = \{1,2,3,4,5\}$ va $B = \{2,4,6,8\}$ bo'lsin. U holda $A \times B =?$ ni toping.

2§. Sonli ketma-ketliklar.

1-ta'rif. Natural sonlar to'plamida berilgan funksiya, ya'ni $x_n = f(n),\ n \in N$ funksiya ***sonli ketma-ketlik*** deb ataladi. Boshqacha so'z bilan aytganda, sonlarning biror qoida yoki qonun bilan ketma-ket kelishi sonli ***ketma-ketlik*** deyiladi va $\{x_n\}$ bilan belgilanadi.

$$\{x_n\} = \{x_{1,}\ x_2, \quad \dots, x_n \dots\} \qquad (1)$$

2-ta'rif. Ketma-ketlikning n -hadi, x_n $-$uning ***umumiy hadi*** deyiladi.

$\{x_n\},\ \{y_n\}$ sonli ketma-ketlik va $a \in R$ o'zgarmas son berilgan bo'lsin. Ular ustida quidagi amallarni bajarish mumkin:

a) Songa ko'paytirish:

$$\{ax_n\} = \{ax_1\,;\ ax_2; \dots; ax_n \dots\} \qquad (2)$$

b) Qo'shish (ayirish):

$$\{x_n\} \pm \{y_n\} = \{x_n \pm y_n\} = \{x_1 \pm y_1; x_2 \pm y_2; \dots; x_n \pm y_n \dots\} \qquad (3)$$

c). Ko'paytirish:

$$\{x_n\} \cdot \{y_n\} = \{x_n \cdot y_n\} = \{x_1 \cdot y_1; x_2 \cdot y_2; \dots; x_n \cdot y_n \dots\} \qquad (4)$$

d) Bo'lish:

$$\frac{\{x_n\}}{\{y_n\}} = \left\{\frac{x_n}{y_n}\right\} = \left\{\frac{x_1}{y_1}; \frac{x_2}{y_2}; \dots; \frac{x_n}{y_n}\right\} \qquad (5).$$

Bu yerda, $y_n \neq 0$.

3-ta'rif. Shunday M soni mavjud bo'lsaki, istalgan $n \in N$ uchun $x_n \leq M$ tengsizlik bajarilsa, $\{x_n\}$ ketma-ketlik ***yuqoridan chegaralangan*** ketme-ketlik deyiladi.

4-tarif. Shunday m soni mavjud bo'lsaki, istalgan $n \in N$ uchun $x_n \geq m$ tengsizlik bajarilsa, $\{x_n\}$ ketma-ketlik ***quyidan chegaralangan*** ketma-ketlik deyiladi.

5-ta'rif. Ham yuqoridan, ham quyidan chegaralangan ketma-ketlik, ***chegaralangan ketma-ketlik*** deyiladi.

6-ta'rif. Agar $\forall\ n \in N$ uchun $x_n \leq x_{n+1}$ $(x_n < x_{n+1})$ tengsizlik o'rinli bo'lsa, $\{x_n\}$ ***o'suvchi (qat'iy o'suvchi) ketma-ketlik*** deyiladi.

7-ta'rif. Agar $\forall\ n \in N$ uchun $x_n \geq x_{n+1}$ $(x_n > x_{n+1})$ tengsizlik o'rinli bo'lsa, $\{x_n\}$ ***kamayuvchi (qat'iy kamayuvchi) ketma-ketlik*** deyiladi.

8-ta'rif. O'suvchi va kamayuvchi ketme-ketliklar ***monoton ketma-ketliklar*** deyiladi.

9-ta'rif. Agar ixtiyoriy $\varepsilon > 0$ son olinganda ham shunday natural $n_0 = n_0(\varepsilon)$ soni topilsaki, barcha $n > n_0$ natural sonlar uchun

$$|x_n - a| < \varepsilon$$

tengsizlik bajarilsa, a o'zgarmas son $\{x_n\}$ ketma-ketlikning ***limiti*** deyiladi va $\lim\limits_{n\to\infty} x_n = a$ kabi belgilanadi.

10-ta'rif. Agar $\{x_n\}$ ketma-ketlik chekli limitga ega bo'lsa, u ***yaqinlashuvchi*** aks holda (limit mavjud bo'lmasa yoki cheksiz bo'lsa) ***uzoqlashuvchi ketma-ketlik*** deyiladi.

11-ta'rif. Agar $\lim\limits_{n\to\infty} x_n = 0$ bo'lsa, u holda $\{x_n\}$ ***cheksiz kichik miqdor*** deb ataladi.

12-ta'rif. Agar $\forall\, M > 0$ son olinganda ham shunday $n_0 \in N$ son topilsaki, $\forall\, n > n_0$ uchun $|x_n| > M$ tengsizlik bajarilsa, $\{x_n\}$ ***cheksiz katta miqdor*** deb ataladi.

Agar $\forall\, M > 0$ son olinganda ham shunday $n_0 \in N$ son topilsaki, $\forall\, n > n_0$ uchun $x_n > M$ $(x_n < -M)$ tengsizlik bajarilsa, u holda $\{x_n\}$ ketma-ketlikning limiti $+\infty(-\infty)$ deb olinadi va $\lim\limits_{n\to\infty} x_n = \infty, (\lim\limits_{n\to\infty} x_n = -\infty)$ kabi belgilanadi.

Yaqinlashuvchi $\{x_n\}$ va $\{y_n\}$ ketma-ketliklar berilgan bo'lib, $\lim\limits_{n\to\infty} x_n = a$ va $\lim\limits_{n\to\infty} y_n = b$ bo'lsin.

U holda

1^0. $\lim\limits_{n\to\infty}(x_n \pm y_n) = \lim\limits_{n\to\infty} x_n \pm \lim\limits_{n\to\infty} y_n = a \pm b$

2^0. $\lim\limits_{n\to\infty}(x_n \cdot y_n) = \lim\limits_{n\to\infty} x_n \cdot \lim\limits_{n\to\infty} y_n = a \cdot b$

3^0. $\lim\limits_{n\to\infty} \dfrac{x_n}{y_n} = \dfrac{\lim\limits_{n\to\infty} x_n}{\lim\limits_{n\to\infty} y_n} = \dfrac{a}{b}$ $\qquad (b \neq 0)$ bo'ladi.

4^0. Agar $\forall\, n \in N$ uchun $x_n \leq y_n$ bo'lsa, u holda $a \leq b$ bo'ladi.

5^0. Agar $\{x_n\}$ ketma-ketlik yaqinlashuvchi bo'lsa, u chegaralangan bo'ladi.

Eslatma! Ketma-ketlikni chegarlanganligidan uning yaqinlashuvchiligi kelib chiqavermaydi.

Misol: $\{x_n\} = \{(-1)^n\}$ .CHegarlangan, ammo yaqinlashuvchi emas.

Misollar.

I. Quydagi ketma-ketliklarni chegaralanganligini isbotlang.

1). $x_n = 1 + \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{(n-1)n}, \qquad (n \geq 2)$

2). $x_n = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}}, \qquad (n \geq 1)$

3). $x_n = \frac{n^2}{2^n}$

4). $x_n = \frac{n}{n^2 + 1}$

II. Quydagi ketma-ketliklarning chegaralanmaganligini isbotlang.

1). $x_n = \frac{2^n}{n^2}$

2). $x_n = (-1)^n \cdot n$

3). $x_n = n!$

III. Sonli ketma-ketliklarning limitini hisoblang.

1). $\lim\limits_{n\to\infty} \frac{n+1}{n} = ?$ $\qquad j: 1$

2). $\lim\limits_{n\to\infty} \frac{n^2 + 3n - 1}{2n^2 + 5n + 4} = ?$ $\qquad j: 0{,}5$

3). $\lim\limits_{n\to\infty} \frac{\sqrt{4n+1}}{\sqrt{n+2}} = ?$ $\qquad j: 2$

4). $\lim\limits_{n\to\infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 - 1}\right) = ?$ $\qquad j: 0{,}5$

5). $\lim\limits_{n\to\infty} \frac{1}{2n+1} = ?$ $\qquad j: 0$

3§. Funksiya va uning limiti.

1-ta'rif. Agar D to'plamdan olingan har bir x songa biror qoida yoki qonunga ko'ra E to'plamdan olingan yagona $y \in R$ son mos qo'yilgan bo'lsa, D to'plamda ***funksiya berilgan*** deyiladi va $f: x \to y$ yoki $y = f(x)$ ko'rinishda belgilanadi.

Bu yerda $\boldsymbol{x}$ ***−erkli o'zgaruvchi (funksiya argumenti),*** $\boldsymbol{y}$ ***−erksiz o'zgaruvchi (***$\boldsymbol{x}$ ***−o'zgaruvchining funksiyasi)*** deyiladi.

D −to'plam funksiyaning ***aniqlanish sohasi (to'plami)***, E to'plam funksiyaning ***qiymatlar sohasi (to'plami)*** deyiladi.

Funksiya uch xil usulda beriladi.

a) analitik usul (formula ko'rinishda);

b) jadval usuli;

c) grafik usul.

2-ta'rif. Agar shunday o'zgarmas M (o'zgarmas m) son topilsaki, $\forall x \in D$ uchun $f(x) \leq M$ $(f(x) \geq m)$ tengsizlik bajarilsa, $f(x)$ funksiya D to'plamda ***yuqoridan (quyidan) chegaralangan*** deb ataladi.

Agar $f(x)$ funksiya ham yuqoridan, ham quyidan chegaralangan bo'lsa, ya'ni shunday M va m sonlar topilsaki, $\forall x \in D$ uchun

$$m \leq f(x) \leq M$$

tengsizlik bajarilsa, $f(x)$ funksiya D to'plamda ***chegaralangan*** deyiladi.

3-ta'rif. Agar $x_1 < x_2$ tengsizlikni qanoatlantiruvchi $\forall x_1, x_2 \in D$ uchun

$$f(x_1) \leq f(x_2) \ \ (f(x_1) < f(x_2))$$

tengsizlik o'rinli bo'lsa, $f(x)$ funksiya D to'plamda ***o'suvchi (qat'iy o'suvchi)*** deyiladi.

4-ta'rif. Agar $x_1 < x_2$ tengsizlikni qanoatlantiruvchi $\forall x_1, x_2 \in D$ uchun

$$f(x_1) \geq f(x_2) \ \ (f(x_1) > f(x_2))$$

tengsizlik o'rinli bo'lsa, $f(x)$ funksiya D to'plamda ***kamayuvchi (qat'iy kamayuvchi)*** deyiladi.

5-ta'rif. O'suvchi va kamayuvchi funksiyalar ***monoton funksiyalar*** deyiladi.

6-ta'rif. Agar shunday o'zgarmas $\mathrm{T}(\mathrm{T} \neq 0)$ soni mavjud bo'lsaki,

$\forall x \in \mathrm{D}$ uchun $f(x+T)=f(x)$ tenglik o'rinli bo'lsa, $f(x)$ funksiya ***davriy funksiya*** deyiladi va bu shartni qanoatlantiruvchi musbat T ning eng kichigi (agar u mavjud bo'lsa) ***funksiyaning davri*** deyiladi.

7-ta'rif. Agar $\forall x \in \mathrm{D}$ uchun $f(-x)=f(x)$ tenglik bajarilsa, $f(x)$ ***juft funksiya***, $f(-x)=-f(x)$ tenglik bajarilsa, $f(x)$ toq funksiya deyiladi, aks holda, ya'ni

$f(-x) \neq \begin{cases} f(x) \\ -f(x) \end{cases}$ bo'lsa, funksiya juft ham emas, toq ham emas deyiladi.

Funksiyaning limiti.

8-ta'rif. (Geyne ta'rifi). Agar D to'plamning nuqtalaridan tuzilgan x_0 ga intiluvchi (x_0 nuqta bu to'plamga tegili bo'lishi ham, tegishli bo'lmasligi ham mumkin) har qanday $\{x_n\}$ $(x_n \neq x_0,\ n=1,2,3,\dots)$ ketma-ketlik olinganda ham funksiya qiymatlaridan tuzilgan mos $\{f(x_n)\}$ ketma-ketlik hamma vaqt yagona A (chekli yoki cheksiz) soniga intilsa, shu A soniga $f(x)$ funksiyaning x_0 nuqtadagi limiti deyiladi va

$$\lim_{x \to x_0} f(x) = A$$

kabi belgilanadi.

9-ta'rif. (Koshi ta'rifi) Agar $\forall \varepsilon > 0$ soni uchun shunday $\exists \delta > 0$ son topilsaki, argument x ning $0 < |x - x_0| < \delta$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida $|f(x) - A| < \varepsilon$ tengsizlik bajarilsa, A son $f(x)$ funksiyaning $\boldsymbol{x_0}$ ***nuqtadagi limiti*** deyiladi.

10-ta'rif. Agar D to'plamning nuqtalaridan tuzilgan va har bir hadi x_0 dan katta (kichik) bo'lib, x_0 ga intiluvchi har qanday $\{x_n\}$ $(x_n \neq x_0,\ n=1,2,3,\dots)$ ketma-ketlik olinganda ham funksiya qiymatlaridan tuzilgan mos $\{f(x_n)\}$ ketma-ketlik hamma vaqt yagona A soniga intilsa, shu A soniga $f(x)$ funksiyaning x_0 nuqtadagi o'ng (chap) limiti deyiladi va

$$\lim_{x \to x_0+0} f(x) = A \qquad \left(\lim_{x \to x_0-0} f(x) = A\right)$$

kabi belgilanadi.

Funksiyaning o'ng va chap limitlari, uning bir tomonli limitlari deyiladi.

Teorema. $f(x)$ funksiya x_0 nuqtada A limitga ega bo'lishi uchun uning shu nuqtada o'ng va chap limitlari mavjud bo'lib, ular $\lim_{x\to x_0+0} f(x) = \lim_{x\to x_0-0} f(x)$ teng bo'lishlari zarur va yetarli.

Birinchi ajoyib limit

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \quad yoki \quad \left(\lim_{x\to 0}\frac{x}{\sin x} = 1\right)$$

Ikkinchi ajoyib limit

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \quad yoki \quad \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

Muxum limitlar:

1. $\lim_{x\to 0}\frac{a^x - 1}{x} = \ln a$

2. $\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$

3. $\lim_{x\to 0}\frac{(1+x)^\alpha - 1}{x} = \alpha$

4. $\lim_{x\to 0}\frac{arc\sin x}{x} = 1$

5. $\lim_{x\to 0}\frac{\operatorname{arctg} x}{x} = 1$

6. $Agar \lim_{x\to x_0} u(x) = 1, \quad \lim_{x\to x_0} v(x) = \infty\ bo'lsa, u\ holda$

$$\lim_{x\to x_0} u^v = \lim_{x\to x_0}\left(\left(1+(u-1)\right)^{\frac{1}{u-1}}\right)^{(u-1)v} = e^{\lim_{x\to x_0}(u-1)v}$$

bo'ladi.

Misollar.

Quyidagi funksiyalarning aniqlanish va qiymatlar sohasini toping?

1. $y = 3x - 1$
2. $y = \sqrt{x-4}$
3. $y = \sqrt[3]{x^2 + x - 5}$
4. $y = \frac{5}{x+4}$
5. $y = \sqrt{3x-1} + \sqrt{x+2}$
6. $y = 5 + sin3x$
7. $y = \lg(9 - x^2)$
8. $y = |x| + 15$
9. $y = lgsinx$

$10.\ y = \sqrt{\dfrac{x+7}{x-6}}$

$11.\ y = \sqrt{cosx + 3}$

$12.\ y = \sqrt{sin2x - 4}$

$13.\ y = \dfrac{1}{x^3 - 8}$

$14.\ y = e^{\frac{1}{x}}$

Quyidagi funksiyalarni juft yoki toqligini tekshiring?

$1.\ f(x) = x^2 sinx$

$2.\ f(x) = x^2 - x + 1$

$3.\ f(x) = |x| + 7$

$4.\ f(x) = x^3 cos^2 x$

$5.\ f(x) = \sqrt{x^3 + |x| + 4}$

$6.\ f(x) = e^{tg\alpha}$

$7.\ f(x) = 2^x + 2^{-x}$

$8.\ f(x) = \lg(x^4 + x^2 - 1)$

$9.\ f(x) = \dfrac{|sinx|}{1 + cosx}$

$10.\ f(x) = |tg\alpha - 1|.$

Funksiyaning limitini hisoblang.

$1.\ \lim\limits_{x\to 2} \dfrac{x^2 - 4}{x + 2};$ $\quad j: 0$

$2.\ \lim\limits_{x\to 3} \dfrac{x - 3}{x^2 - x - 6};$ $\quad j: 0{,}2$

$3.\ \lim\limits_{x\to \infty} \dfrac{x^2 + x - 1}{3x^2 - 2x + 7};$ $\quad j: \dfrac{1}{3}$

$4.\ \lim\limits_{x\to \infty} \dfrac{x^2 - 1}{x + 1};$ $\quad j: \infty$

$5.\ \lim\limits_{x\to \infty} \dfrac{x^2 - 3x + 6}{x^3 + 1};$ $\quad j: 0$

$6.\ \lim\limits_{x\to \infty} \dfrac{5x^3 + 6x}{1 - 3x^3}$ $\quad j: -\dfrac{5}{3}$

$7.\ \lim\limits_{x\to 0} \dfrac{\sqrt{1+x} - \sqrt{1-x}}{x}$ $\quad j: 1$

8. $\lim\limits_{x\to 2}\dfrac{(x-2)^2}{x^3-8}$; $j: 0$

9. $\lim\limits_{x\to -1}\dfrac{x^3-2x-1}{x^5-2x-1}$; $j: \dfrac{1}{3}$

10. $\lim\limits_{x\to 1}\dfrac{x^{100}-2x+1}{x^{50}-2x+1}$; $j: 2\dfrac{1}{24}$

11. $\lim\limits_{x\to 0}\dfrac{sin10x}{x}$ $j: 10$

12. $\lim\limits_{x\to 0}\dfrac{sin\frac{x}{4}}{x}$; $j: 0.25$

13. $\lim\limits_{x\to 0}\dfrac{tgx}{x}$; $j: 1$

14. $\lim\limits_{x\to 0}\dfrac{1-cos2x}{xsinx}$; $j: 2$

15. $\lim\limits_{x\to 0}\dfrac{1-cosx}{x^2}$; $j: 0.5$

16. $\lim\limits_{x\to 1}\left(\dfrac{1}{x-1}-\dfrac{2}{x^2-1}\right)$; $j: 0.5$

17. $\lim\limits_{x\to \infty}\left(\sqrt{x^2+3x}-x\right)$ $j: 1.5$

18. $\lim\limits_{x\to 2}\left(\dfrac{1}{x-2}-\dfrac{4}{x^2-4}\right)$; $j: 0.25$

19. $\lim\limits_{x\to \infty}\left(x-\sqrt{x^2-a^2}\right)$; $j: 0$

20. $\lim\limits_{x\to \frac{\pi}{2}}\left(\dfrac{sinx}{cos^2x}-tg^2x\right)$; $j: 0.5$

21. $\lim\limits_{x\to 0}\dfrac{tgx-sinx}{sin^3x}$; $j: 0.5$

23. $\lim\limits_{x\to 0}\dfrac{sin5x-sin3x}{sinx}$; $j: 2$

24. $\lim\limits_{x\to 0} xctg2x$; $j: 0.5$

25. $\lim\limits_{x\to \frac{\pi}{2}}(sinx)^{tgx}$ $j: 1$

26. $\lim\limits_{x\to \infty}\left(\dfrac{x+2}{2x-1}\right)^{x^2}$; $j: 0$

27. $\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{x}$; $\quad j: 1$

28. $\lim\limits_{x\to\infty} x\cdot(\ln(x+1)-lnx)$; $\quad j: 1$

29. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4}-2}{sin5x}$; $\quad j: 0.05$

30. $\lim\limits_{x\to 0}\dfrac{1+xsinx-cos2x}{sin^2x}$; $\quad j: 3$

31. $\lim\limits_{n\to\infty}\left(\dfrac{1+2+3+\cdots+n}{n+2}-\dfrac{n}{2}\right)$; $\quad j: -0.5$

32. $\lim\limits_{x\to -2}\dfrac{x^2+x-2}{x^2+2x}$; $\quad j: 1.5$

33. $\lim\limits_{x\to\infty}\left(1-\dfrac{2}{x}\right)^x$; $\quad j: e^{-2}$

34. $\lim\limits_{x\to 0}(1+2x)^{\frac{1}{x}}$; $\quad j: e^2$

35. $\lim\limits_{x\to\infty}\left(\dfrac{x}{x+1}\right)^x$; $\quad j: e^{-1}$

36. $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+1}\right)^{2x}$; $\quad j: e^{-2}$

4-§. Funksiyaning uzluksizligi.

$y=f(x)$ funksiyaning limiti to’g’risida quyidagilarni aytish mumkin:

1^0. $x\to x_0$ da $f(x)$ funksiyaning limiti mavjud, chekli va

$$\lim_{x\to x_0} f(x)=f(x_0);$$

2^0. $x\to x_0$ da $f(x)$ funksiyaning limiti mavjud, chekli va

$$\lim_{x\to x_0} f(x)=A\neq f(x_0);$$

3^0. $x\to x_0$ da $f(x)$ funksiyaning limiti mavjud va $\lim\limits_{x\to x_0} f(x)=\infty$;

4^0. $x\to x_0$ da $f(x)$ funksiyaning limiti mavjud emas.

Yuqoridagilardan 1-hol muhimdir.

1-ta'rif: Agar $x \to x_0$ da $f(x)$ funksiyaning limiti mavjud va u $f(x_0)$ ga teng, ya'ni $\lim\limits_{x \to x_0} f(x) = f(x_0)$ bo'lsa, $f(x)$ funksiya x_0 nuqtada uzluksiz deb ataladi.

2-ta'rif: (Geyne ta'rifi). Agar D to'plamning nuqtalaridan tuzilgan x_0 ga intiluvchi har qanday $\{x_n\}$ $(x_n \neq x_0,\ n = 1, 2, 3, \dots)$ ketma-ketlik olinganda ham funksiya qiymatlaridan tuzilgan mos $\{f(x_n)\}$ ketma-ketlik hamma vaqt $f(x_0)$ ga intilsa, $f(x)$ funksiya x_0 nuqtada ***uzluksiz*** deyiladi

3-ta'rif. (Koshi ta'rifi). Agar $\forall \varepsilon > 0$ soni uchun shunday $\exists \delta > 0$ son topilsaki, argument x ning $|x - x_0| < \delta$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida $|f(x) - f(x_0)\ | < \varepsilon$ tengsizlik bajarilsa, $f(x)$ funksiya $\boldsymbol{x_0}$ nuqtada ***uzluksiz*** deyiladi.

4-ta'rif. Agar $x \to x_0 + 0$ $(x \to x_0 - 0)$ da $f(x)$ funksiyaning o'ng (chap) limiti mavjud va $f(x_0)$ ga teng, ya'ni $\lim\limits_{x \to x_0 + 0} f(x) = f(x_0)$ ($\lim\limits_{x \to x_0 - 0} f(x) = f(x_0)$) bo'lsa, $f(x)$ funksiya x_0 nuqtada ***o'ngdan (chapdan) uzluksiz*** deyiladi.

Yuqorida keltirilgan ta'rifdan ko'rinadiki, agar $f(x)$ funksiya x_0 nuqtada ham o'ngdan, ham chapdan bir vaqtda uzluktsiz bo'lsa, funksiya shu ***nuqtada uzluksiz*** deyiladi.

5-ta'rif. Agar $f(x)$ funksiya D to'plamning har bir nuqtasida uzluksiz bo'lsa, funksiya D ***to'plamda uzluksiz*** deyiladi.

$f(x)$ funksiya $(a; b)$ intervalning har bir nuqtasida uzluksiz bo'lsa, funksiya shu ***intervalda uzluksiz*** deyiladi.

$f(x)$ funksiya $(a; b)$ intervalning har bir nuqtasida uzluksiz bo'lib, a nuqtada o'ngdan, b nuqtada esa chapdan uzluksiz bo'lsa, funksiya $[a; b]$ segmentda uzluksiz bo'ladi.

1^0 Funksiya ning chegaralanganligi haqidagi teorema.

Agar $y = f(x)$ funksiya $[a; b]$ kesmada uzluksiz bo`lsa, u shu kesmada chegaralangan funksiyadir, ya'ni shunday o`zgarmas chekli $m;\ M$ sonlar mavjudki, barcha $x \in [a; b]$ qiymatlar uchun $m \leq f(x) \leq M$ tengsizlik o`rinli.

2^0 Funksiyaning eng kichik va eng katta qiymatining mavjudligi haqidagi teorema.

Agar $y = f(x)$ funksiya $[a;b]$ kesmada uzluksiz bo`lsa, u holda berilgan funksiya shu kesmada o`zining eng kichik va eng katta qiymatiga erishadi, ya'ni shunday x_1; $x_2 \in [a;b]$ mavjudki, barcha $x \in [a;b]$ uchun
$f(x_1) \leq f(x)\ va\ f(x) \leq f(x_2)$ tengsizliklar o`rinli bo`ladi.

3^0 Oraliq qiymat haqidagi teorema.

Agar $y = f(x)$ funksiya $[a;b]$ kesmada uzluksiz bo'lib, shu bilan birga $m\ \ va\ M$ lar funksiyaning $[a;b]$ dagi eng kichik va eng katta qiymatlari bo`lsa, u holda bu funksiya shu kesmada $m\ \ va\ M$ orasidagi barcha oraliq qiymatlarni qabul qiladi, ya'ni $m < \mu < \ M$ shartni qanoatlantiruvchi istalgan $\mu$ son uchun kamida bitta shunday $c \in [a;b]$ nuqta mavjudki, uning uchun $f(c) = \mu$ tenglik to`g`ri bo`ladi.

4^0 Funksiyaning nolga aylanishi haqidagi teorema.

Agar $y = f(x)$ funksiya $[a;b]$ kesmada uzluksiz va kesmaning oxirlarida turli ishorali qiymatlarni qabul qilsa, u holda $[a;b]$ kesmada kamida bitta shunday nuqta mavjudki, bu nuqtada funksiya qiymati nolga aylanadi.

Teorema 1. Agar $f(x)\ va\ g(x)$ funksiyalar D to'plamda aniqlangan bo'lib, ularning har biri x_0 nuqtada uzluksiz bo'lsa, $f(x) \pm g(x)$; $f(x) \cdot\ g(x)$, $\frac{f(x)}{g(x)}$ $(g(x) \neq 0, \forall x \in D)$ funksiyalar ham shu nuqtada uzluksiz bo'ladi.

Xossa. Asosiy elementar funksiyalar o`zlari aniqlangan barcha nuqtalarda uzluksizdir.

6-ta'rif. Agar $x \to x_0$ da $f(x)$ funksiyaning limiti mavjud, chekli bo'lib, $\lim\limits_{x \to x_0} f(x) = A \neq f(x_0);\ (2^0 - hol),\quad \lim\limits_{x \to x_0} f(x) = \infty;\ (3^0 -\ hol)\ yoki$ funksiyaning limiti mavjud bo'lmasa (4^0-hol), unda $f(x)$ funksiya x_0 ***nuqtada uzilishga ega*** deyiladi.

2^0-hol. Bartaraf qilish (yo'qotish) mumkin bo'lgan uzilish deyiladi.

4^0-hol. Bunda funksiyaning x_0 nuqtadagi bir tomonli limitlariga nisbatan uchta hol bo'ladi:

a) $x \to x_0$ da $f(x)$ funksiyaning o'ng va chap limitlari mavjud va chekli bo'lib, ular bir-biriga teng emas:

$$\lim_{x \to x_0+0} f(x) \neq \lim_{x \to x_0-0} f(x)$$

Funksiyaning x_0 nuqtadagi bunday uzilishi ***birinchi tur uzilish*** deyiladi.

b) $x \to x_0$ da $f(x)$ funksiyaning o'ng va chap limilaridan hech bo'lmaganda biri mavjud emas.

Funksiyaning x_0 nuqtadagi bunday uzilishi ***ikkinchi tur uzilish*** deyiladi.

c) $x \to x_0$ da $f(x)$ funksiyaning o'ng va chap limitlaridan biri cheksiz yoki o'ng va chap limitlar turli ishorali cheksiz.

Funksiyaning x_0 nuqtadagi bunday uzilishi ham ***ikkinchi tur uzilish*** deyiladi.

3^0-hol. $x \to x_0$ da $f(x)$ funksiyaning limiti cheksiz bo'lsin. Bu holda funksiyaning x_0 nuqtadagi o'ng va chap limitlari ham cheksiz bo'ladi.

Funksiyaning x_0 nuqtadagi bunday uzilishi ham ***ikkinchi tur uzilish*** deyiladi.

Teorema 2. Agar $f(x)$ funksiya D oraliqda o'suvchi (kamayuvchi) bo'lsa, u faqat birinchi tur uzilishga ega bo'ladi.

Misollar.

Quyidagi funksiyalarning uzluksizligini ko'rsating:

1. $y = f(x) = sinx.$
2. $y = f(x) = \cos\left(x + \frac{\pi}{3}\right).$
3. $y = f(x) = x^2 + 1.$
4. $y = f(x) = |x|.$
5. $y = f(x) = \sqrt[3]{x}.$

Quyidagi funksiyalarning uzilish nuqtalarini aniqlang va grafiklarini chizing:

6. $y = f(x) = \frac{1}{sinx}.$
7. $y = f(x) = \frac{|x+1|}{x+1}.$
8. $y = f(x) = x - [x]\,.\,([x] -\ x\ ning\ butun\ qismi).$

9. $y = f(x) = \dfrac{x}{cosx}$.

10. $y = f(x) = \dfrac{5}{x^3 - x^2}$.

Mustaqil yechish uchun misollar.
Limitlarni hisoblang:
1-variant.

1. $\lim\limits_{x\to -2} \dfrac{2x^2 - 5x + 3}{x^2 - 1}$;

2. $\lim\limits_{x\to 1} \dfrac{2x^2 - 5x + 3}{x^2 - 1}$;

3. $\lim\limits_{x\to 0} \dfrac{2 - \sqrt{4 + x^2}}{x^2}$;

4. $\lim\limits_{x\to \infty} \dfrac{2x^2 - 5x + 3}{x^3 - 1}$;

5. $\lim\limits_{x\to \infty} \dfrac{2x^2 - 5x + 3}{x - 1}$;

6. $\lim\limits_{x\to \infty} \dfrac{2x^2 - 5x + 3}{x^2 - 1}$;

7. $\lim\limits_{x\to 2} \left(\dfrac{1}{x - 2} - \dfrac{1}{x^2 - 3x + 2}\right)$;

8. $\lim\limits_{x\to -\infty} \dfrac{x}{x - \sqrt{x^2 + 4}}$;

9. $\lim\limits_{x\to \infty} \dfrac{(x - 1)(x - 2)(x - 3)(x - 4)}{(5x - 1)^4}$;

10. $\lim\limits_{n\to \infty} \dfrac{\sqrt{n + 2} + \sqrt[3]{n^3 + 1}}{\sqrt{4n^2 + 1}}$;

11. $\lim\limits_{x\to 0} \dfrac{arc \sin 6x}{\sin 7x}$;

12. $\lim\limits_{x\to 0} \dfrac{1 - cos^2 6x}{x^4}$;

13. $\lim\limits_{x\to 2} \dfrac{sin(x^2 - 4)}{x^2 - 4x + 4}$;

14. $\lim\limits_{x\to +\infty} \left(\dfrac{x + 2}{x - 3}\right)^{4x}$;

15. $\lim\limits_{n\to 1} (3x - 2)^{\frac{4x}{x-1}}$;

16. $\lim\limits_{n\to \pm\infty} \left(\dfrac{2x + 1}{3x + 1}\right)^{x}$;

17. $\lim\limits_{x\to \infty} (3x + 1) \cdot ln\dfrac{x + 1}{x}$;

18. $\lim\limits_{x\to +\infty} (2x^4 + 1)\, tg\left(\dfrac{1}{x^4 + 1}\right)$;

2-variant.

1. $\lim\limits_{x\to -1} \dfrac{2x^2 - x - 6}{x^2 - 4}$;

2. $\lim\limits_{x\to 2} \dfrac{2x^2 - x - 6}{x^2 - 4}$;

3. $\lim\limits_{x\to 0} \dfrac{x}{1 - \sqrt{x + 1}}$;

4. $\lim\limits_{x\to \infty} \dfrac{2x^2 - x - 6}{x^2 - 4}$;

5. $\lim\limits_{x\to \infty} \dfrac{2x^2 - x - 6}{x + 1}$;

6. $\lim\limits_{x\to +\infty} \dfrac{2x^2 - x - 6}{x^3 - 4}$;

7. $\lim\limits_{x\to -\infty} \dfrac{2x + 1}{x + \sqrt{4x^2 - 3}}$;

8. $\lim\limits_{x\to 2} \left(\dfrac{x}{x - 2} - \dfrac{1}{x^2 - 4}\right)$;

9. $\lim\limits_{x\to +\infty} \dfrac{(2x^2 - 1)^2}{(x + 1)(x + 2)(x + 3)(x - 4)}$;

10. $\lim\limits_{n\to +\infty} \dfrac{n + \sqrt{4n^4 + 1}}{n^2 + 1}$;

11. $\lim\limits_{x\to 1} \dfrac{x^2 - 1}{sin(x - 1)}$;

12. $\lim\limits_{x\to 0} \dfrac{\arcsin 5x}{\sin 4x}$;

13. $\lim\limits_{x\to+\infty}\left(\frac{x+3}{x-4}\right)^{2x}$;

14. $\lim\limits_{x\to1}(3x-2)^{\frac{x}{x-1}}$;

15. $\lim\limits_{x\to\pm\infty}\left(\frac{3x-1}{5x+2}\right)^{x}$;

16. $\lim\limits_{x\to\pm\infty}\left(\frac{2x^2+3x-1}{x^2+1}\right)^{x}$;

17. $\lim\limits_{x\to2}\left(\frac{e^{x-2}-1}{x^2-4}\right)$;

18. $\lim\limits_{x\to+\infty}(2x+1)\cdot ln\frac{x+1}{x}$;

3-variant.

1. $\lim\limits_{x\to1}\frac{3x^2-5x+2}{x^2-2}$;

2. $\lim\limits_{x\to1}\frac{3x^2-5x+2}{x^2-1}$;

3. $\lim\limits_{x\to-1}\frac{x+1}{\sqrt{3-x}-2}$;

4. $\lim\limits_{x\to\infty}\frac{3x^2-5x+2}{x^2-1}$;

5. $\lim\limits_{x\to\infty}\frac{3x^2-5x+2}{x^3-1}$;

6. $\lim\limits_{x\to+\infty}\frac{3x^2-5x+2}{x-1}$;

7. $\lim\limits_{x\to-\infty}\frac{x+1}{\sqrt{x^2+x}-x}$;

8. $\lim\limits_{x\to0}\left(\frac{1}{x}-\frac{1}{x^2-x}\right)$;

9. $\lim\limits_{x\to+\infty}\frac{(2x+1)(x-1)(x+2)(x-4)}{(2x-1)^4}$;

10. $\lim\limits_{x\to0}\frac{sin4x}{\sqrt{1+x}-1}$;

11. $\lim\limits_{x\to1}\frac{arcsin(x^2-1)}{x^2-x}$;

12. $\lim\limits_{x\to\infty}\frac{\sqrt{x^4-x+\sqrt[3]{8x^6+2}}}{\sqrt[4]{16x^8-1}}$;

13. $\lim\limits_{x\to0}\frac{1-cos4x}{x}$;

14. $\lim\limits_{x\to\infty}\left(1+\frac{4}{x-2}\right)^{x}$;

15. $\lim\limits_{x\to\infty}\left(\frac{x+5}{x+7}\right)^{x+1}$;

16. $\lim\limits_{x\to1}\left(\frac{x^2+1}{3x^2+1}\right)^{\frac{1}{x-1}}$;

17. $\lim\limits_{x\to1}\frac{e^{x-1}-1}{x^2-1}$;

18. $\lim\limits_{x\to+\infty}(x+4)ln\frac{x-1}{x}$;

4-variant.

1. $\lim\limits_{x\to1}\frac{5x^2+13x-6}{x+3}$;

2. $\lim\limits_{x\to0,4}\frac{5x^2+13x-6}{x+3}$;

3. $\lim\limits_{x\to-3}\frac{5x^2+13x-6}{x+3}$;

4. $\lim\limits_{x\to\infty}\frac{5x^2+13x-6}{x+3}$;

5. $\lim\limits_{x\to\infty}\frac{5x^2+13x-6}{x^2+3}$;

6. $\lim\limits_{x\to\infty}\frac{5x^2+13x-6}{3x^3+x-6}$;

7. $\lim\limits_{x\to2}\left(\frac{1}{x-2}-\frac{1}{x^2-4}\right)$;

8. $\lim\limits_{x\to-\infty}\frac{2x+\sqrt{9x^2+x}}{x+4}$;

9. $\lim\limits_{x\to\infty}\frac{(2x-3)(3x+1)(x-4)(1-3x)}{x^4+x-1}$;

10. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+6x-1}-x\right)$;

11. $\lim\limits_{x\to2}\frac{tg(x^2-4)}{x-2}$;

12. $\lim\limits_{x\to-1}\frac{sin(x^2-1)}{x^2+3x+2}$;

13. $\lim\limits_{x\to 4}(x-3)^{\frac{1}{x-4}}$;

14. $\lim\limits_{x\to\infty}\left(1+\frac{2}{x+1}\right)^{x}$;

15. $\lim\limits_{x\to\infty}\left(\frac{3x+2}{3x-5}\right)^{\sqrt{x^2+1}}$;

16. $\lim\limits_{x\to\pm\infty}\left(\frac{3x+2}{5x+3}\right)^{x}$;

17 $\lim\limits_{x\to\infty}\left(\frac{sin\frac{2}{x}}{e^{\frac{1}{x}}-1}\right)$;

18. $\lim\limits_{x\to\infty} xln\left(\frac{2x+3}{2x+5}\right)$;

5-variant.

1. $\lim\limits_{x\to -1,5}\frac{2x^2-3x-9}{x-3}$;
2. $\lim\limits_{x\to 4}\frac{2x^2-3x-9}{x-3}$;
3. $\lim\limits_{x\to 3}\frac{2x^2-3x-9}{x-3}$;
4. $\lim\limits_{x\to\infty}\frac{2x^2-3x-9}{x-3}$;
5. $\lim\limits_{x\to\infty}\frac{2x^2-3x-9}{x^2-3}$;
6. $\lim\limits_{x\to\infty}\frac{2x^2-3x-9}{x^3-3x+1}$;
7. $\lim\limits_{x\to 1}\left(\frac{2}{x-1}-\frac{6}{x^2+x-2}\right)$;
8. $\lim\limits_{x\to -\infty}\frac{x+\sqrt{9x^2+1}}{2x-4}$;
9. $\lim\limits_{x\to\infty}\left(\sqrt{x^4-x+1}-x^2\right)$;
10. $\lim\limits_{x\to\infty}\frac{3x^5-1}{(x^2+1)(x+2)(x-3)(2x-5)}$;
11. $\lim\limits_{x\to 0}\frac{sin3x+sinx}{x^2}$;
12. $\lim\limits_{x\to 0}\frac{arctg2x}{sin3x}$;
13. $\lim\limits_{x\to\infty}\left(1+\frac{2}{x-5}\right)^{2x}$;
14. $\lim\limits_{x\to 3}\left(\frac{2x-1}{x+2}\right)^{\frac{6}{x^2-9}}$;
15. $\lim\limits_{x\to 6}(x-5)^{\frac{1}{x-6}}$;
16. $\lim\limits_{x\to\pm\infty}\left(\frac{2x+1}{x+2}\right)^{x}$;
17. $\lim\limits_{x\to\infty}(2x+1)ln\left(\frac{x^2+x}{x^2+1}\right)$;
18. $\lim\limits_{x\to 0}\left(\frac{e^{3x}-1}{sin^2 2x}\right)$;

6-variant.

1. $\lim\limits_{x\to -3}\frac{5x^2-3x-2}{x-2}$;
2. $\lim\limits_{x\to 2}\frac{5x^2-3x-2}{x-2}$;
3. $\lim\limits_{x\to 1}\frac{5x^2-3x-2}{x-1}$;
4. $\lim\limits_{x\to\infty}\frac{5x^2-3x-2}{x-2}$;
5. $\lim\limits_{x\to\infty}\frac{5x^2-3x-2}{x^2-2x+1}$;
6. $\lim\limits_{x\to\infty}\frac{5x^2-3x-2}{x^3-1}$;
7. $\lim\limits_{x\to -1}\left(\frac{3}{x+1}-\frac{x}{x^2-1}\right)$;
8. $\lim\limits_{x\to\infty}\left(\sqrt{x(x-2)}-\sqrt{x^2-3}\right)$;
9. $\lim\limits_{x\to\infty}\frac{(x^2+1)(2x-3)(x+5)}{(x+1)^4}$;
10. $\lim\limits_{x\to -\infty}\frac{2x-\sqrt{16x^2+1}}{3x+4}$;
11. $\lim\limits_{x\to 0}\frac{sin3x-sinx}{x}$;
12. $\lim\limits_{x\to 0}\frac{x\,tg\,3x}{1-\cos 2x}$;

13. $\lim\limits_{x\to\infty} \frac{\sin(3+x)}{x+3}$;

14. $\lim\limits_{x\to\infty} \left(1+\frac{3}{x+1}\right)^{x}$;

15. $\lim\limits_{x\to 2} \left(\frac{2x+1}{3x-1}\right)^{\frac{1}{x-2}}$;

16. $\lim\limits_{x\to\pm\infty} \left(\frac{2x+1}{3x-2}\right)^{2x}$;

17. $\lim\limits_{x\to 0} \left(\frac{e^{x^2-1}-1}{\sqrt{x}-1}\right)$;

18 $\lim\limits_{x\to\infty} x\, ln\left(\frac{x-3}{x+1}\right)$;

7-variant.

1 $\lim\limits_{x\to 1} \frac{7x^2-6x-16}{x-2}$;

2. $\lim\limits_{x\to 2} \frac{7x^2-6x-16}{x-2}$;

3. $\lim\limits_{x\to 2} \frac{7x^2-6x-16}{(x-2)^3}$;

4. $\lim\limits_{x\to\infty} \frac{7x^2-6x-16}{x-2}$;

5. $\lim\limits_{x\to\infty} \frac{7x^2-6x-16}{x^2-2}$;

6. $\lim\limits_{x\to\infty} \frac{7x^2-6x-16}{x^3-2}$;

7. $\lim\limits_{x\to\infty} \frac{(x^2+4)(2x-3)(x+1)}{(4x-1)^4}$;

8. $\lim\limits_{x\to -5} \left(\frac{1}{x+5}-\frac{4}{x^2-25}\right)$;

9. $\lim\limits_{x\to -\infty} \frac{5x-\sqrt{9x^2-1}}{2x+5}$;

10. $\lim\limits_{x\to\infty} \left(\sqrt{x^2+3x+2}-\sqrt{x^2-3}\right)$;

11. $\lim\limits_{x\to 0} \frac{\sin 2x-\sin 6x}{x}$;

12. $\lim\limits_{x\to 2} \frac{arc\,\sin(x^2-4)}{x-2}$;

13. $\lim\limits_{x\to\infty} \left(1-\frac{5}{x+1}\right)^{x}$;

14. $\lim\limits_{x\to 3} \left(\frac{2x-1}{x+2}\right)^{\frac{1}{2x-6}}$;

15. $\lim\limits_{x\to 9} (x-8)^{\frac{1}{x-9}}$;

16. $\lim\limits_{x\to\pm\infty} \left(\frac{2x-4}{x+5}\right)^{3x-1}$;

17. $\lim\limits_{x\to\infty} (2x+1)ln\left(\frac{x^2+x}{x^2+1}\right)$;

18. $\lim\limits_{x\to 0} \left(\frac{e^{x^2}-1}{e^{2x^2}-1}\right)$;

8-variant.

1. $\lim\limits_{x\to -2,5} \frac{2x^2+3x-5}{x+1}$;

2. $\lim\limits_{x\to 1} \frac{2x^2+3x-5}{x+1}$;

3. $\lim\limits_{x\to 0} \frac{\sqrt{1+3x^2}-1}{x+x^2}$;

4. $\lim\limits_{x\to\infty} \frac{2x^2+3x-5}{x^3+1}$;

5. $\lim\limits_{x\to\infty} \frac{2x^2+3x-5}{x+1}$;

6. $\lim\limits_{x\to\infty} \frac{2x^2+3x-5}{x^2+1}$;

7. $\lim\limits_{x\to\infty} \frac{(x-1)^5}{(x^2+1)(2x+1)^3}$;

8. $\lim\limits_{x\to 1} \left(\frac{1}{x-1}-\frac{2}{x^2-3x+2}\right)$;

9. $\lim\limits_{x\to -\infty} \frac{\sqrt{9x^2-x}-x}{3x+2}$;

10. $\lim\limits_{x\to\infty} \left(\sqrt{x(x+5)}-x\right)$;

11. $\lim\limits_{x\to 1} \frac{\sin\pi(1-x)}{1-x}$;

12. $\lim\limits_{x\to 0} \frac{cos 3x-\cos x}{\sin x^2}$;

13. $\lim\limits_{x\to\infty}(3x^4+1)\,tg\left(\frac{3}{x^4+1}\right)$;

14. $\lim\limits_{x\to\infty}\left(1-\frac{5}{2x+3}\right)^x$;

15. $\lim\limits_{x\to\infty}\left(\frac{3x-5}{3x+8}\right)^{2x-3}$;

16. $\lim\limits_{x\to2}(3x-5)^{\frac{2x}{x^2-4}}$;

17. $\lim\limits_{x\to\pm\infty}\left(\frac{3x-1}{2x+3}\right)^{2x}$;

18. $\lim\limits_{x\to\infty}(3x^2+1)\,ln\left(\frac{x^2+3}{x^2+1}\right)$;

9-variant.

1. $\lim\limits_{x\to-1,5}\frac{2x^2-x-6}{x-2}$;

2. $\lim\limits_{x\to1}\frac{2x^2-x-6}{x-2}$;

3. $\lim\limits_{x\to2}\frac{2x^2-x-6}{x-2}$;

4. $\lim\limits_{x\to\infty}\frac{2x^2-x-6}{x-2}$;

5. $\lim\limits_{x\to\infty}\frac{2x^2-x-6}{4x^2+5x+6}$;

6. $\lim\limits_{x\to\infty}\frac{2x^2-x-6}{x^3-x+1}$;

7. $\lim\limits_{x\to\infty}\frac{(x^2+1)(1-2x)(1-x)^2}{(x-1)^5}$;

8. $\lim\limits_{x\to\infty}\left(\frac{x^3}{5x^2+1}-\frac{3x^2}{5x+1}\right)$;

9. $\lim\limits_{x\to-\infty}\frac{\sqrt{9x^2+1}-x}{x+1}$;

10. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+6x^2-1}-x^2\right)$;

11. $\lim\limits_{x\to0}\frac{sin^2\,2x}{x^2}$;

12. $\lim\limits_{x\to0}\frac{\cos4x-\cos x}{arc\sin3x^2}$;

13 $\lim\limits_{x\to\infty}\left(1+\frac{3}{2x+5}\right)^x$;

14. $\lim\limits_{x\to2}(3x-5)^{\frac{2}{x^2-4}}$;

15. $\lim\limits_{x\to3}\left(\frac{2x-1}{x+2}\right)^{\frac{1}{2x-6}}$;

16. $\lim\limits_{x\to\pm\infty}\left(\frac{x^2+x}{2x^2+3}\right)^{x^2}$;

17 $\lim\limits_{x\to\infty}(x-5)[\ln(x-3)-\ln x]$;

18. $\lim\limits_{x\to\infty}x(e^{\frac{1}{x}}-1)$;

10-variant.

1. $\lim\limits_{x\to-0,5}\frac{2x^2-5x-3}{x-3}$;

2. $\lim\limits_{x\to1}\frac{2x^2-5x-3}{x-3}$;

3. $\lim\limits_{x\to3}\frac{2x^2-5x-3}{x-3}$;

4. $\lim\limits_{x\to\infty}\frac{2x^2-5x-3}{x-3}$;

5. $\lim\limits_{x\to\infty}\frac{2x^2-5x-3}{(x-3)^2}$;

6. $\lim\limits_{x\to\infty}\frac{2x^2-5x-3}{x^3-1}$;

7. $\lim\limits_{x\to\infty}\frac{(2x-1)^5}{(x^2+1)(x-1)(2x-1)^2}$;

8. $\lim\limits_{x\to3}\left(\frac{2}{3x-x^2}-\frac{1}{9-x^2}\right)$;

9. $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2-3}\right)$;

10. $\lim\limits_{x\to-\infty}\frac{x-\sqrt{x^2+3}}{x+3}$;

11. $\lim\limits_{x\to1}\frac{\sin(x^2-1)}{\sqrt{2-x}-x}$;

12. $\lim\limits_{x\to0}\frac{1-cos6x}{1-cos2x}$;

13. $\lim\limits_{x\to 1}(3x-2)^{\frac{1}{x-1}}$;

14. $\lim\limits_{x\to\infty}\left(1-\frac{3}{1+2x}\right)^{x}$;

15. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{2x+4}\right)^{2x}$;

16. $\lim\limits_{x\to\pm\infty}\left(\frac{x^2+3x-1}{0{,}5x^2+1}\right)^{x}$;

17. $\lim\limits_{x\to\infty}(2x-1)[\ln(3x-1)-\ln(3x+1)]$;

18. $\lim\limits_{x\to 3}\frac{(e^{x-3}-1)}{x^2-9}$;

11-variant.

1. $\lim\limits_{x\to -2}\frac{6x^2-5x-1}{x-1}$;

2. $\lim\limits_{x\to -\frac{1}{6}}\frac{6x^2-5x-1}{6x+1}$;

3. $\lim\limits_{x\to 1}\frac{6x^2-5x-1}{x-1}$;

4. $\lim\limits_{x\to\infty}\frac{6x^2-5x-1}{x-1}$;

5. $\lim\limits_{x\to\infty}\frac{6x^2-5x-1}{(x-1)^3}$;

6. $\lim\limits_{x\to\infty}\frac{6x^2-5x-1}{x^2-1}$;

7. $\lim\limits_{x\to\infty}\frac{(x^2+1)(3x+1)(1-4x)}{(2x-1)^4}$;

8. $\lim\limits_{x\to 1}\left(\frac{x^2}{x-1}-\frac{x^3}{x^2-1}\right)$;

9. $\lim\limits_{x\to +\infty}\left(\sqrt[3]{x^3+6x^2}-x\right)$;

10. $\lim\limits_{x\to\infty}\frac{x^2-\sqrt{4x^6+1}}{x^3+\sqrt{x^6-2}}$;

11. $\lim\limits_{x\to 1}\frac{\sin(x^2-1)}{x-1}$;

12. $\lim\limits_{x\to 0}\frac{1-\cos x}{\sqrt{1-x^2}-1}$;

13. $\lim\limits_{x\to\infty}\frac{sin^2 2x}{x^2}$;

14. $\lim\limits_{x\to\infty}\left(\frac{3x-1}{3x+4}\right)^{3x}$;

15. $\lim\limits_{x\to 1}(3-2x)^{\frac{1}{x-1}}$;

16. $\lim\limits_{x\to 1}\left(\frac{2x}{4x-1}\right)^{\frac{3}{x-1}}$;

17. $\lim\limits_{x\to\infty}x[\ln(2x-1)-\ln(2x+3)]$;

18. $\lim\limits_{x\to\infty}\frac{e^{x^2-1}-1}{\sqrt{3+x}-2}$;

12-variant.

1. $\lim\limits_{x\to -1}\frac{2x^2-3x-2}{x-2}$;

2. $\lim\limits_{x\to -\frac{1}{2}}\frac{2x^2-3x-2}{x-2}$;

3. $\lim\limits_{x\to 2}\frac{2x^2-3x-2}{x-2}$;

4. $\lim\limits_{x\to\infty}\frac{2x^2-3x-2}{x-2}$;

5. $\lim\limits_{x\to\infty}\frac{2x^2-3x-2}{x^3+8}$;

6. $\lim\limits_{x\to\infty}\frac{2x^2-3x-2}{x^2-4x+4}$;

7. $\lim\limits_{x\to\infty}\frac{(2x-1)^6}{(x^3-1)(x^2+2x)(2x+1)}$;

8. $\lim\limits_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{x^2+x-2}\right)$;

9. $\lim\limits_{x\to\infty}\frac{\sqrt{9x^3+1}-x}{\sqrt{x^3+1}-x}$;

10. $\lim\limits_{x\to 0}\frac{sin^2 2x}{x^2}$;

11. $\lim\limits_{x\to -\infty}\left(\sqrt{x^4+6x^2+1}-x^2\right)$;

12. $\lim\limits_{x\to 0}\frac{tg\,6x^2}{cos3x-cosx}$;

13. $\lim\limits_{x\to\infty}\left(1-\dfrac{3}{1+3x}\right)^{x}$;

14. $\lim\limits_{x\to\pm\infty}\left(\dfrac{x-1}{2x-3}\right)^{x^2}$;

15. $\lim\limits_{x\to\infty}\left(\dfrac{x-1}{x+4}\right)^{x}$;

16. $\lim\limits_{x\to 0}(1+3x)^{\frac{2}{x}}$;

17. $\lim\limits_{x\to 3}\dfrac{4}{x-3}(\ln(3x-7)-\ln(2x-4))$;

18. $\lim\limits_{x\to 0}\dfrac{e^{3x}-1}{\sqrt{1-3x}-1}$;

13-variant.

1. $\lim\limits_{x\to 2}\dfrac{2x^2+3x-5}{x-1}$;

2. $\lim\limits_{x\to -2,5}\dfrac{2x^2+3x-5}{x-1}$;

3. $\lim\limits_{x\to 1}\dfrac{2x^2+3x-5}{x-1}$;

4. $\lim\limits_{x\to\infty}\dfrac{2x^2+3x-5}{x-1}$;

5. $\lim\limits_{x\to\infty}\dfrac{2x^2+3x-5}{x^3-1}$;

6. $\lim\limits_{x\to\infty}\dfrac{2x^2+3x-5}{(x-1)^2}$;

7. $\lim\limits_{x\to\infty}\dfrac{(2x-1)^5}{(x^2+1)(x-1)^3}$;

8. $\lim\limits_{x\to 1}\left(\dfrac{1}{x+2}-\dfrac{x}{x^2+8}\right)$;

9. $\lim\limits_{x\to -\infty}\dfrac{\sqrt[6]{x^6+1}+2x}{\sqrt[3]{x^3-1}+x}$;

10. $\lim\limits_{x\to\infty}\dfrac{\sqrt{4x^3+x}+x}{\sqrt{x^3+3}-x}$;

11. $\lim\limits_{x\to 2}\dfrac{\sin(x^2-2x)}{x-2}$;

12. $\lim\limits_{x\to\infty}(x+1)sin\dfrac{4}{x}$;

13. $\lim\limits_{x\to\infty}\left(1+\dfrac{5}{2x+1}\right)^{x}$;

14. $\lim\limits_{x\to\infty}\left(\dfrac{2x+1}{2x+3}\right)^{x+2}$;

15. $\lim\limits_{x\to 0}(1-5x)^{\frac{5}{x}}$;

16. $\lim\limits_{x\to\pm\infty}\left(\dfrac{2x+1}{x+3}\right)^{x+2}$;

17. $\lim\limits_{x\to\infty}x[\ln(1+5x)-\ln(4+5x)]$;

18. $\lim\limits_{x\to 3}\dfrac{e^{x-3}-1}{x^2-9}$;

14-variant.

1. $\lim\limits_{x\to -0,8}\dfrac{5x^2-x-4}{x-1}$;

2. $\lim\limits_{x\to 2}\dfrac{5x^2-x-4}{x-1}$;

3. $\lim\limits_{x\to 1}\dfrac{5x^2-x-4}{x-1}$;

4. $\lim\limits_{x\to\infty}\dfrac{5x^2-x-4}{x-1}$;

5. $\lim\limits_{x\to\infty}\dfrac{5x^2-x-4}{5x^2+2x-1}$;

6. $\lim\limits_{x\to\infty}\dfrac{5x^2-x-4}{x^3-1}$;

7. $\lim\limits_{x\to\infty}\dfrac{(x^2+1)(3-x^2)(2x+3)}{(2x-1)^5}$;

8. $\lim\limits_{x\to 1}\left(\dfrac{1}{x-1}-\dfrac{1}{x^2+5x-6}\right)$;

9. $\lim\limits_{x\to -\infty}\dfrac{\sqrt{x^2+1}-x}{\sqrt{9x^2-x+1}}$;

10. $\lim\limits_{x\to\infty}\left(\sqrt{4x^4+12x^2-1}-2x^2\right)$;

11. $\lim\limits_{x\to 0}\dfrac{arctg^2x}{1-cosx}$;

12. $\lim\limits_{x\to\infty}\dfrac{sin3x}{3x}$;

13. $\lim\limits_{x\to 2}(3-x)^{\frac{1}{x-2}}$;

14. $\lim\limits_{x\to\infty}\left(1+\frac{2}{5x-4}\right)^{x}$;

15. $\lim\limits_{x\to\infty}\left(\frac{5x-1}{5x+4}\right)^{x}$;

16. $\lim\limits_{x\to 1}\left(\frac{x^2-x+5}{4-x}\right)^{\frac{3}{x-1}}$;

17. $\lim\limits_{x\to 3}\frac{4}{x-3}[\ln(3x-7)-\ln(2x-4)]$;

18. $\lim\limits_{x\to 1}\frac{e^{x-1}-1}{\sqrt{x}-1}$;

15-variant.

1. $\lim\limits_{x\to -1}\frac{2x^2+5x+2}{x+2}$;

2. $\lim\limits_{x\to -\frac{1}{2}}\frac{2x^2+5x+2}{x+2}$;

3. $\lim\limits_{x\to -2}\frac{2x^2+5x+2}{x+2}$;

4. $\lim\limits_{x\to\infty}\frac{2x^2+5x+2}{x+2}$;

5. $\lim\limits_{x\to\infty}\frac{2x^2+5x+2}{(x+2)^2}$;

6. $\lim\limits_{x\to\infty}\frac{2x^2+5x+2}{x^3+2x^2+1}$;

7. $\lim\limits_{x\to\infty}\frac{4(2x^2+1)(x-1)(1-3x)}{(2x-1)^4}$;

8. $\lim\limits_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^2}\right)$;

9. $\lim\limits_{x\to -\infty}\frac{x-\sqrt{4x^2-3}}{2x+\sqrt{4x^2+x}}$;

10. $\lim\limits_{x\to\infty}x\left(\sqrt{x^2+1}-x\right)$;

11. $\lim\limits_{x\to 1}\frac{1-\cos x}{x\sin x}$;

12. $\lim\limits_{x\to 2}\frac{x^2+4x-12}{\sin(x^2-4)}$;

13. $\lim\limits_{x\to 2}(3-x)^{\frac{1}{x-2}}$;

14. $\lim\limits_{x\to\infty}\left(1-\frac{4}{2x-3}\right)^{x}$;

15. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{2x-5}\right)^{3x}$;

16. $\lim\limits_{x\to\pm\infty}\left(\frac{2x+1}{3x-5}\right)^{2x}$;

17. $\lim\limits_{x\to\infty}(2x+1)[ln(x^2+x)-ln(x^2+1)]$;

18. $\lim\limits_{x\to 0}\frac{\sqrt{1+3x}-1}{e^{x^2}-1}$;

16-variant.

1. $\lim\limits_{x\to -0{,}5}\frac{-2x^2+3x-1}{x-1}$;

2. $\lim\limits_{x\to 2}\frac{-2x^2+3x-1}{x-1}$;

3. $\lim\limits_{x\to 1}\frac{-2x^2+3x-1}{x-1}$;

4. $\lim\limits_{x\to\infty}\frac{-2x^2+3x-1}{x-1}$;

5. $\lim\limits_{x\to\infty}\frac{-2x^2+3x-1}{(x-1)^2}$;

6. $\lim\limits_{x\to\infty}\frac{-2x^2+3x-1}{2x^3-3x+1}$;

7. $\lim\limits_{x\to 1}\left(\frac{1}{x^2-2x}-\frac{1}{x^2-3x+2}\right)$;

8. $\lim\limits_{x\to -\infty}\left(\sqrt{x^2+x-1}-\sqrt{x^2-x+1}\right)$;

9. $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x^3+1}+2x}{\sqrt{4x^2-1}-x+1}$;

10. $\lim\limits_{x\to\infty}(2x+1)\sin(2x+1)$;

11. $\lim\limits_{x\to-0,5}\dfrac{\sin(2x+1)}{2x+1}$;

12. $\lim\limits_{x\to\pi}\dfrac{arc\ tg(\pi-x)}{\pi-x}$;

13. $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+1}\right)^{2x+1}$;

14. $\lim\limits_{x\to4\pm0}\left(\dfrac{x-2}{2x+1}\right)^{\frac{2}{x-4}}$;

15. $\lim\limits_{x\to\infty}\left(1-\dfrac{3}{2x-5}\right)^{x}$;

16. $\lim\limits_{x\to1}(5-4x)^{\frac{1}{x^2-1}}$;

17. $\lim\limits_{x\to\infty}(2-x)[\ln(x+3)-\ln(x-4)]$;

18. $\lim\limits_{x\to0}\dfrac{e^{sin^2x}-e^{sinx}}{x}$;

17-variant.

1. $\lim\limits_{x\to3}\dfrac{-3x^2+9x-6}{x-1}$;

2. $\lim\limits_{x\to2}\dfrac{-3x^2+9x-6}{x-1}$;

3. $\lim\limits_{x\to1}\dfrac{-3x^2+9x-6}{x-1}$;

4. $\lim\limits_{x\to\infty}\dfrac{-3x^2+9x-6}{x-1}$;

5. $\lim\limits_{x\to\infty}\dfrac{-3x^2+9x-6}{x^3-1}$;

6. $\lim\limits_{x\to\infty}\dfrac{-3x^2+9x-6}{x^2-1}$;

7. $\lim\limits_{x\to\infty}\dfrac{(1-x)^5}{(x^2+3x-1)(x+1)^2}$;

8. $\lim\limits_{x\to3}\left(\dfrac{2}{3x-x^2}-\dfrac{1}{9-x^2}\right)$;

9. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^4-2x}+x^2}{\sqrt{4x^4-1}-x}$;

10. $\lim\limits_{x\to2}\dfrac{\sin(x-2)}{x-2}$;

11. $\lim\limits_{x\to0}\dfrac{1-cosx}{x^2-x}$;

12. $\lim\limits_{x\to\infty}\dfrac{\sin(x-2)}{x-2}$;

13. $\lim\limits_{x\to\infty}\left(1-\dfrac{8}{x-4}\right)^{x}$;

14. $\lim\limits_{x\to\infty}\left(\dfrac{x+1}{x-7}\right)^{2x+1}$;

15. $\lim\limits_{x\to1}(7-6x)^{\frac{1}{x-1}}$;

16. $\lim\limits_{x\to\infty}\left(\dfrac{3x^2+x+1}{x^2-x}\right)^{x^2}$;

17. $\lim\limits_{x\to\infty}(x-1)[\ln(x+2)-\ln(x+3)]$;

18. $\lim\limits_{x\to e}\dfrac{\ln x-1}{x-e}$;

18-variant.

1. $\lim\limits_{x\to2}\dfrac{5x^2-4x-1}{x-1}$;

2. $\lim\limits_{x\to-0,2}\dfrac{5x^2-4x-1}{x-1}$;

3. $\lim\limits_{x\to1}\dfrac{5x^2-4x-1}{x-1}$;

4. $\lim\limits_{x\to\infty}\dfrac{5x^2-4x-1}{x-1}$;

5. $\lim\limits_{x\to\infty}\dfrac{5x^2-4x-1}{(x-1)^2}$;

6. $\lim\limits_{x\to\infty}\dfrac{5x^2-4x-1}{(x-1)^3}$;

7. $\lim\limits_{x\to\infty}\dfrac{(x^2+5)(1-2x)(x-1)}{(2x+1)^4}$;

8. $\lim\limits_{x\to3}\left(\dfrac{1}{x^2-9}-\dfrac{1}{x^2-3x}\right)$;

9. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{5x^2+2x}-1}{\sqrt{20x^2+1}-4}$;

10. $\lim\limits_{x\to-2}\dfrac{\sin(x+2)}{x+2}$;

11. $\lim\limits_{x\to\infty}(x+2)\sin(x+2)$;

12. $\lim\limits_{x\to0}\dfrac{\sin 3x-\sin x}{tg\, 2x}$;

13. $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x+3}\right)^{2x+1}$;

14. $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{2x-3}\right)^{x}$;

15. $\lim\limits_{x\to2}(9-4x)^{\frac{1}{x-2}}$;

16. $\lim\limits_{x\to\pm\infty}\left(\dfrac{x^2-x+1}{2x^2+3x+1}\right)^{x}$;

17. $\lim\limits_{x\to\infty}(x+2)[\ln(x-3)-\ln(x+7)]$;

18. $\lim\limits_{x\to0}\dfrac{e^{2x}-1}{3x}$;

19-variant.

1. $\lim\limits_{x\to1}\dfrac{-2x^2+3x-1}{x-0{,}5}$;

2. $\lim\limits_{x\to2}\dfrac{-2x^2+3x-1}{x-0{,}5}$;

3. $\lim\limits_{x\to0{,}5}\dfrac{-2x^2+3x-1}{x-0{,}5}$;

4. $\lim\limits_{x\to\infty}\dfrac{-2x^2+3x-1}{x-0{,}5}$;

5. $\lim\limits_{x\to\infty}\dfrac{-2x^2+3x-1}{(x-0{,}5)^2}$;

6. $\lim\limits_{x\to\infty}\dfrac{-2x^2+3x-1}{x^3-0{,}5^3}$;

7. $\lim\limits_{x\to\infty}\dfrac{(2x-1)^5}{(x^2+1)(x-1)(x+1)^2}$;

8. $\lim\limits_{x\to0}\left(\dfrac{1}{x^2-x}-\dfrac{x^3}{x^2+2x}\right)$;

9. $\lim\limits_{x\to-\infty}\dfrac{x+\sqrt{4x^2+1}}{x+1}$;

10. $\lim\limits_{x\to\infty}\dfrac{\sqrt{9x^4-x}+x^2}{\sqrt{9x^4+x}-x^2}$;

11. $\lim\limits_{x\to0}\dfrac{\sin 3x+\sin x}{x}$;

12. $\lim\limits_{x\to0}\dfrac{arc\sin 2x}{3x}$;

13. $\lim\limits_{x\to\infty}(2x-1)\sin(2x-1)$;

14. $\lim\limits_{x\to\infty}\left(1-\dfrac{3}{2x+3}\right)^{x}$;

15. $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+1}\right)^{2x-3}$;

16. $\lim\limits_{x\to\pm\infty}\left(\dfrac{x^2+2x-3}{2x^2+5x+1}\right)^{x-1}$;

17. $\lim\limits_{x\to\infty}x^2[\ln(x+1)-\ln(x+3)]$;

18. $\lim\limits_{x\to3}\dfrac{e^x-e^3}{x-3}$;

20-variant.

1. $\lim\limits_{x\to-2}\dfrac{6x^2-7x+1}{x-1}$;

2. $\lim\limits_{x\to\frac{1}{6}}\dfrac{6x^2-7x+1}{x-1}$;

3. $\lim\limits_{x\to1}\dfrac{6x^2-7x+1}{x-1}$;

4. $\lim\limits_{x\to\infty}\dfrac{6x^2-7x+1}{x-1}$;

5. $\lim\limits_{x\to\infty}\dfrac{6x^2-7x+1}{x^3-1}$;

6. $\lim\limits_{x\to\infty}\dfrac{6x^2-7x+1}{x^2+x+1}$;

7. $\lim\limits_{x\to1}\left(\dfrac{2}{x+1}-\dfrac{1}{x^3+1}\right)$;

8. $\lim\limits_{x\to\infty} \dfrac{(x+3)^2(x+1)(x-1)(2x-3)}{(x-3)^5}$;

9. $\lim\limits_{x\to+\infty} \left(\sqrt{x^2+x}-x\right)$;

10. $\lim\limits_{x\to\infty} \dfrac{\sqrt{1+x^2}+2x}{\sqrt{4x^2+1}}$;

11. $\lim\limits_{x\to\frac{2}{3}} \dfrac{\sin(3x-2)}{3x-2}$;

12. $\lim\limits_{x\to0} \dfrac{\sin x+sin3x}{tg\, x}$;

13. $\lim\limits_{x\to\infty} \left(1-\dfrac{5}{2x+3}\right)^x$;

14. $\lim\limits_{x\to\infty} \left(\dfrac{x^2+x+1}{x^2+3}\right)^{2x+1}$;

15. $\lim\limits_{x\to1} (8-7x)^{\frac{1}{x-1}}$;

16. $\lim\limits_{x\to\pm\infty} \left(\dfrac{3x-5}{x+1}\right)^{8x}$;

17. $\lim\limits_{x\to\infty} \dfrac{[\ln(x+1)-\ln(2x-2)]}{x-3}$;

18. $\lim\limits_{x\to1} \dfrac{e^{x^2}-1}{\sin 2x}$;

21-variant.

1. $\lim\limits_{x\to-2} \dfrac{3x^2+x-4}{x-1}$;

2. $\lim\limits_{x\to-\frac{4}{3}} \dfrac{3x^2+x-4}{x-1}$;

3. $\lim\limits_{x\to1} \dfrac{3x^2+x-4}{x-1}$;

4. $\lim\limits_{x\to\infty} \dfrac{3x^2+x-4}{x-1}$;

5. $\lim\limits_{x\to\infty} \dfrac{3x^2+x-4}{(x-1)^2}$;

6. $\lim\limits_{x\to\infty} \dfrac{3x^2+x-4}{x^3-1}$;

7. $\lim\limits_{x\to\infty} \dfrac{(3x-1)^4}{(x+2)(x^3+x-1)}$;

8. $\lim\limits_{x\to-1} \left(\dfrac{2}{x^2+x}-\dfrac{1}{x^2+1}\right)$;

9. $\lim\limits_{x\to-\infty} \left(\sqrt{x(x+2)}-x\right)$;

10. $\lim\limits_{x\to\infty} \dfrac{x+\sqrt{9x^2-1}}{2x-\sqrt{4x^2+1}}$;

11. $\lim\limits_{x\to0} \dfrac{arc\, tg\, 3x}{\sin 3x-\sin x}$;

12. $\lim\limits_{x\to\infty} \dfrac{\sin(x+5)}{x+5}$;

13. $\lim\limits_{x\to\infty} \left(1-\dfrac{9}{3-2x}\right)^x$

14. $\lim\limits_{x\to\infty} \left(\dfrac{x+1}{x+3}\right)^{2x+1}$;

15. $\lim\limits_{x\to1} (6-5x)^{\frac{3}{x-1}}$;

16. $\lim\limits_{x\to\pm\infty} \left(\dfrac{x^2+x-3}{2x^2+1}\right)^{3x}$;

17. $\lim\limits_{x\to1} \dfrac{e^{2x}-e^2}{x-1}$;

18. $\lim\limits_{x\to0} \dfrac{1}{x}\ln(1+2x)$;

22-variant.

1. $\lim\limits_{x\to-2} \dfrac{2x^2+7x+3}{x+3}$;

2. $\lim\limits_{x\to-\frac{1}{2}} \dfrac{2x^2+7x+3}{x+3}$;

3. $\lim\limits_{x\to-3} \dfrac{2x^2+7x+3}{x+3}$;

4. $\lim\limits_{x\to\infty} \dfrac{2x^2+7x+3}{x+3}$;

5. $\lim\limits_{x\to\infty} \dfrac{2x^2+7x+3}{3x^2+7x+3}$;

6. $\lim\limits_{x\to\infty} \dfrac{2x^2+7x+3}{3x^3+x+1}$;

7. $\lim\limits_{x\to\infty} \frac{(x^2+1)(x-1)(1+2x)^2}{(2x-1)^5}$;

8. $\lim\limits_{x\to 1} \left(\frac{1}{x^2-5x+6} - \frac{1}{x-2}\right)$;

9. $\lim\limits_{x\to-\infty} x(\sqrt{4+x^2}) - x$;

10. $\lim\limits_{x\to\infty} \frac{x+\sqrt{9x^2+1}}{3-2x}$;

11. $\lim\limits_{x\to\infty} \frac{\sin(x+2)}{4(x+2)}$;

12. $\lim\limits_{x\to 0} \frac{\sin 7x - \sin x}{x\, tg\, 2x}$;

13. $\lim\limits_{x\to\infty} \left(1 - \frac{4}{3-2x}\right)^x$;

14. $\lim\limits_{x\to\infty} \left(\frac{x}{x+1}\right)^x$;

15. $\lim\limits_{x\to 2} (10-9x)^{\frac{2}{x-1}}$;

16. $\lim\limits_{x\to\pm\infty} \left(\frac{x+2}{3x-1}\right)^{2x^2}$;

17. $\lim\limits_{x\to 0} \frac{1-e^{2x}}{\sin 2x}$;

18. $\lim\limits_{x\to\infty} x[\ln(2+x) - \ln x]$;

23-variant.

1. $\lim\limits_{x\to 1} \frac{2x^2-x-15}{x-3}$;

2. $\lim\limits_{x\to 2,5} \frac{2x^2-x-15}{x-3}$;

3. $\lim\limits_{x\to 3} \frac{2x^2-x-15}{x-3}$;

4. $\lim\limits_{x\to\infty} \frac{2x^2-x-15}{x-3}$;

5. $\lim\limits_{x\to\infty} \frac{2x^2-x-15}{(x-3)^3}$;

6. $\lim\limits_{x\to\infty} \frac{2x^2-x-15}{(x-3)^2}$;

7. $\lim\limits_{x\to\infty} \frac{(3x+1)^4}{(x^2+1)(3x-3)(x+5)}$;

8. $\lim\limits_{x\to 2} \left(\frac{1}{x-2} - \frac{1}{x^2-4}\right)$;

9. $\lim\limits_{x\to-\infty} \frac{1+\sqrt{4x^2+2}}{2x+1}$;

10. $\lim\limits_{x\to\infty} \frac{1+\sqrt{4n^2+2}}{2n+1}$;

11. $\lim\limits_{x\to 2} \frac{\sin(2x-4)}{x-2}$;

12. $\lim\limits_{x\to 0} \frac{arc\, tg\, 3x}{\sin 2x}$;

13. $\lim\limits_{x\to\infty} \left(1 - \frac{4}{4x-1}\right)^x$;

14. $\lim\limits_{x\to\infty} \left(\frac{x+3}{x-7}\right)^{2x-3}$;

15. $\lim\limits_{x\to 1} (1-2x)^{\frac{1}{x^2}}$;

16. $\lim\limits_{x\to\pm 1} \left(\frac{2x+3}{x-7}\right)^{2x-3}$;

17. $\lim\limits_{x\to\infty} (2x+3)[\ln(x^2+x) - \ln(x^2+1)]$;

18. $\lim\limits_{x\to 0} \frac{e^{2x}-1}{\sin x}$;

24-variant.

1. $\lim\limits_{x\to 3} \frac{2x^2-9x+4}{x-4}$;

2. $\lim\limits_{x\to-\frac{1}{2}} \frac{2x^2-9x+4}{x-4}$;

3. $\lim\limits_{x\to 4} \frac{2x^2-9x+4}{x-4}$;

4. $\lim\limits_{x\to\infty} \frac{2x^2-9x+4}{x-4}$;

5. $\lim\limits_{x\to\infty} \frac{2x^2-9x+4}{(x-4)^2}$;

6. $\lim\limits_{x\to\infty} \frac{2x^2-9x+4}{x^3-3x^2+1}$;

7. $\lim\limits_{x\to\infty} \dfrac{(2x-1)^6}{(2x^2+1)(x^3-1)(x+3)}$;

8. $\lim\limits_{x\to 1} \left(\dfrac{1}{x-1} - \dfrac{2}{x^2+x-2}\right)$;

9. $\lim\limits_{x\to\infty} \dfrac{1+\sqrt{9x^2-x}}{2x-3}$;

10. $\lim\limits_{x\to-\infty} \dfrac{3x-\sqrt{x^2+1}}{\sqrt{x^2+x}+1}$;

11. $\lim\limits_{x\to 2} \dfrac{sin(x^2-4)}{(x-2)^2}$;

12. $\lim\limits_{x\to 0} \dfrac{1-\cos 4x}{x\, tg\, 2x}$;

13. $\lim\limits_{x\to\infty} \dfrac{\sin(2x+6)}{4x+12}$;

14. $\lim\limits_{x\to\infty} \left(1-\dfrac{4}{3x+1}\right)^x$;

15. $\lim\limits_{x\to\infty} \left(\dfrac{x+3}{x+4}\right)^{2x+3}$;

16. $\lim\limits_{x\to\pm\infty} \left(\dfrac{x+3}{3x+4}\right)^{2x+3}$;

17. $\lim\limits_{x\to\infty} (2x-1)[\ln(x^2+4)-\ln(x^2+1)]$;

18. $\lim\limits_{x\to 0} \dfrac{e^x-e^{-x}}{\sin x}$;

25-variant.

1. $\lim\limits_{x\to 3} \dfrac{2x^2+3x-2}{x+2}$;

2. $\lim\limits_{x\to-\frac{1}{2}} \dfrac{2x^2+3x-2}{x+2}$;

3. $\lim\limits_{x\to-2} \dfrac{2x^2+3x-2}{x+2}$;

4. $\lim\limits_{x\to\infty} \dfrac{2x^2+3x-2}{x+2}$;

5. $\lim\limits_{x\to\infty} \dfrac{2x^2+3x-2}{(x+2)^2}$;

6. $\lim\limits_{x\to\infty} \dfrac{2x^2+3x-2}{(x+2)^3}$;

7. $\lim\limits_{x\to\infty} \dfrac{(x^3+1)(2x-1)(x+3)}{(1-2x)^5}$;

8. $\lim\limits_{x\to-1} \left(\dfrac{1}{x^2+2x+1} - \dfrac{1}{x+1}\right)$;

9. $\lim\limits_{x\to-\infty} \dfrac{3x-\sqrt{4+9x^2}}{3x+4}$;

10. $\lim\limits_{x\to 1} \dfrac{\sin 2(x-1)}{x^2-1}$;

11. $\lim\limits_{x\to 0} \dfrac{1-\cos x}{arc\sin 2x}$;

12. $\lim\limits_{x\to\infty} \dfrac{\sin 2(x-1)}{x^2-1}$;

13. $\lim\limits_{x\to\infty} \left(1+\dfrac{5}{2x-3}\right)^x$;

14. $\lim\limits_{x\to\infty} \left(\dfrac{2x-1}{2x+3}\right)^{2x+1}$;

15. $\lim\limits_{x\to 0} (1-7x)^{\frac{3}{x}}$;

16. $\lim\limits_{x\to\pm\infty} \left(\dfrac{2x-1}{x+3}\right)^{2x+1}$;

17. $\lim\limits_{x\to\infty} (3x-1)[\ln(2x-3)-\ln(2x+5)]$;

18. $\lim\limits_{x\to 0} \dfrac{e^{x^2}-\cos x}{x^2}$;

26-variant.

1. $\lim\limits_{x\to 1} \dfrac{-2x^2+3x-1}{x-0{,}5}$;

2. $\lim\limits_{x\to 2} \dfrac{-2x^2+3x-1}{x-0{,}5}$;

3. $\lim\limits_{x\to 0{,}5} \dfrac{-2x^2+3x-1}{x-0{,}5}$;

4. $\lim\limits_{x\to\infty} \dfrac{-2x^2+3x-1}{x-0{,}5}$;

5. $\lim\limits_{x\to\infty}\dfrac{-2x^2+3x-1}{(x-0{,}5)^2}$;

6. $\lim\limits_{x\to\infty}\dfrac{-2x^2+3x-1}{x^3-0{,}5^3}$;

7. $\lim\limits_{x\to\infty}\dfrac{(1-x)^5}{(x^3+3x-1)(x+1)^2}$;

8. $\lim\limits_{x\to 3}\left(\dfrac{2}{3x-x^2}-\dfrac{1}{9-x^2}\right)$;

9. $\lim\limits_{x\to -\infty}\dfrac{\sqrt{x^4-2x}+x^2}{\sqrt{4x^4+1}-x}$;

10. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^4-2x}+x^2}{\sqrt{4x^4+1}-x}$;

11. $\lim\limits_{x\to 2}\dfrac{\sin(x-2)}{x-2}$;

12. $\lim\limits_{x\to 0}\dfrac{arc\ tg^2 x}{1-\cos x}$;

13. $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{5x-4}\right)^x$;

14. $\lim\limits_{x\to\infty}\left(\dfrac{5x-1}{5x+4}\right)^x$;

15. $\lim\limits_{x\to 2}(3-x)^{\frac{1}{x-2}}$;

16. $\lim\limits_{x\to\pm\infty}\left(\dfrac{x^2-x+5}{4-x}\right)^{\frac{3}{x-1}}$;

17. $\lim\limits_{x\to\infty} x[\ln(x-4)-\ln(x-1)]$;

18. $\lim\limits_{x\to 1}\dfrac{e^{x-1}-1}{\sqrt{x}-1}$;

27-variant.

1. $\lim\limits_{x\to -2}\dfrac{6x^2-5x-1}{x-1}$;

2. $\lim\limits_{x\to -\frac{1}{6}}\dfrac{6x^2-5x-1}{6x+1}$;

3. $\lim\limits_{x\to 1}\dfrac{6x^2-5x-1}{x-1}$;

4. $\lim\limits_{x\to\infty}\dfrac{6x^2-5x-1}{x-1}$;

5. $\lim\limits_{x\to\infty}\dfrac{6x^2-5x-1}{(x-1)^3}$;

6. $\lim\limits_{x\to\infty}\dfrac{6x^2-5x-1}{x^2-1}$;

7. $\lim\limits_{x\to\infty}\dfrac{(x^2+1)(1-2x)(1-x)^2}{(x-1)^5}$;

8. $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{5x^2+1}-\dfrac{3x^2}{5x+1}\right)$;

9. $\lim\limits_{x\to -\infty}\dfrac{\sqrt{9x^2+1}-x}{x+1}$;

10. $\lim\limits_{x\to\infty}\left(\sqrt{x^4+6x^2-1}-x^2\right)$;

11. $\lim\limits_{x\to 0}\dfrac{sin^2 2x}{x^2}$;

12. $\lim\limits_{x\to 2}\dfrac{arc\ \sin(x^2-4)}{x-2}$;

13. $\lim\limits_{x\to\infty}\left(1-\dfrac{5}{x+1}\right)^x$;

14. $\lim\limits_{x\to 2}\left(\dfrac{2x+1}{3x-1}\right)^{\frac{1}{x-2}}$;

15. $\lim\limits_{x\to 8}(x-7)^{\frac{1}{x-8}}$;

16. $\lim\limits_{x\to\pm\infty}\left(\dfrac{2x+1}{3x-2}\right)^{2x}$;

17. $\lim\limits_{x\to\infty}(2x+1)ln\left(\dfrac{x^2+x}{x^2+1}\right)$;

18. $\lim\limits_{x\to 0}\left(\dfrac{e^{3x}-1}{sin^2 2x}\right)$.

FUNKSIYA HOSILASI.

1-§. Funksiya hosilasi.

$y=f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo'lsin. $(a;b)$ intervalga tegishli ixtiyoriy x_0 nuqtani olamiz. Bu nuqtaga funksiyaning $y_0=f(x_0)$ qiymati mos keladi. Boshqa x nuqtani olamiz, $x\in(a;b)$. Unga funksiyaning $y=f(x)$ qiymati mos keladi. $x-x_0$ ayirma x argumentning x_0 nuqtadagi orttirmasi deyiladi va Δx bilan belgilanadi. $f(x)-f(x_0)$ ayirma funksiya orttirmasi deyiladi va Δy bilan belgilanadi, ya'ni

$$\Delta x = x - x_0, \quad \Delta y = f(x) - f(x_0)\,.$$

Bundan, $x=x_0+\Delta x\,; \Delta y=f(x_0+\Delta x)-f(x_0)$ (1)

Δx va Δy orttirmalarni egri chiziq bo'ylab harakatlanayotgan nuqta koordinatalarining o'zgarishi deb ataladi.

$y=f(x)$ funksiya $(a;b)$ ochiq intervalda berilgan bo'lsin. Bu intervalda x_0 nuqta olib, unga shunday Δx $(\Delta x>0,\quad \Delta x<0)$ ortirma beraylikki, $x_0+\Delta x\in(a;b)$ bo'lsin. Natijada $f(x)$ funksiya ham x_0 nuqtada

$$\Delta y=\Delta f(x_0)=f(x_0+\Delta x)-f(x_0)$$

ortirmaga ega bo'ladi.

1-ta'rif. Agar funksiya ortirmasi Δy ning argument ortirmasi Δx ga nisbatining argument ortirmasi nolga intilgandagi limiti

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

mavjud va chekli bo'lsa, bu limit $f(x)$ funksiyaning $\boldsymbol{x_0}$ ***nuqtadagi hosilasi*** deyiladi va

$f'(x_0)\quad yoki\quad y'_{/x=x_0}$ kabi belgilanadi.

Demak,

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \qquad (1)$$

$\forall\, x\in(a;b)$ uchun

$$f'(x)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} \qquad (2)$$

a). Hosilaning geometrik ma'nosi.

$y = f(x)$ funksiyaning x_0 nuqtadagi hosilasi uning grafigiga $M_0\big(x_0\,;f(x_0)\big)$ nuqtada o’tkazilgan urinmaning OX o’qining musbat yo’nalishi bilan hosil qilgan burchagining tangensiga teng. U holda $y = f(x)$ funksiyaning grafigiga $M_0\big(x_0\,;f(x_0)\big)$ nuqtada o’tkazilgan urinma tenglamasi

$$y = f(x_0) + f'(x_0)(x - x_0) \qquad (3)$$

bo’ladi.

b). Hosilaning mexanik ma’nosi.

To’g’ri chiziqli notekis harakatda nuqtaning t vaqt ichida bosib o’tgan yo’li $S = S(t)$ ga teng bo’lsin. U holda t_0 vaqt oralig’ida bosib o’tilgan yo’l $S = S(t_0)$ bo’lib, $t = t_0 + \Delta t$ vaqt ichida bosib o’tilgan yo’l esa, $S(t_0 + \Delta t)$ bo’ladi. Δt vaqt ichida bosib o’tilgan yo’l esa

$$\Delta S = S(t_0 + \Delta t) - S(t_0)$$

ga teng.

$\frac{\Delta S}{\Delta t}$ nisbat Δt vaqt oralig’idagi ***o’rtacha tezlik*** deyiladi. Bu nisbatning $\Delta t \to 0$ dagi limiti nuqtaning t_0 vaqtdagi tezligini aniqlaydi, bu ***oniy tezlik*** deyiladi. Demak, bosib o’tilgan yo’ldan vaqt bo’yicha olingan hosila tezlikni ifodalar ekan, ya’ni

$$S'_t = v \qquad (4)$$

Huddi shunday mulohaza yuritib, v tezlikdan vaqt bo’yicha olingan hosila, a −tezlanish ekanligini bildiradi,ya’ni

$$v'_t = a \qquad (5)$$

Hosila hisoblashning sodda qoidalari.

$f(x)\ va\ g(x)$ funksiyalar $x \in (a;b)$ nuqtada $f'(x)\ va\ g'(x)$ hosilalarga ega bo’lsin. C −o’zgarmas son.($C = const$).

1. $\big(C\,f(x)\big)' = C\,f'(x);$
2. $[f(x) \pm g(x)]' = f'(x) \pm g'(x);$
3. $[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x);$
4. $\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}; \qquad (g(x) \neq 0)$

bo'ladi.

Murakkab funksiyaning hosilasi.

$u = f(x)$ funksiya $(a; b)$ oraliqda, $y = F(u)$ funksiya esa $(c; d)$ oraliqda berilgan bo'lsa, $y = F\big(f(x)\big)$ funksiya ***x* *o'zgaruvchining murakkab funksiyasi*** deyiladi.

Teorama. Agar $u = f(x)$ funksiya $x_0 \in (a; b)$ nuqtada $f'(x_0)$ hosilaga ega bo'lib, $y = F(u)$ funksiya esa x_0 nuqtaga mos $u_0\big(u_0 = f(x_0)\big)$ nuqtada $F'(u_0)$ hosilaga ega bo'lsa, u holda murakkab funksiya $y = F\big(f(x)\big)$ ham x_0 nuqtada hosilaga ega va

$$y'(x_0) = F'(u_0) \cdot f'(x_0)$$

formula o'rinli.

$\forall x \in (a; b)$ uchun $y = F(f(x))$ murakkab funksiyaning hosilasi

$$y' = F'\big(f(x)\big) \cdot f'(x) \qquad (6)$$

ga teng bo'ladi.

Hosila jadvali.

1. $(C)' = 0, \quad (C = \ const);$
2. $(x)' = 1\ ;$
3. $(x^n)' = nx^{n-1};$
4. $\left(\sqrt{x}\right)' = \dfrac{1}{2\sqrt{x}};$
5. $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2};$
6. $(e^x)' = e^x;$
7. $(a^x)' = a^x \cdot lna, \ \ (a > 0, a \neq 1);$
8. $(lnx)' = \dfrac{1}{x};$
9. $(\log_a x)' = \dfrac{1}{x \cdot lna};$
10. $(\sin x)' = \cos x;$
11. $(\cos x)' = -\sin x;$

12. $(tgx)' = \frac{1}{cos^2x}, \quad \left(x \neq \frac{\pi}{2} + \pi k, k \in Z\right);$

13. $(ctgx)' = -\frac{1}{sin^2x}, \quad (x \neq \pi k, k \in Z);$

14. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, (-1 < x < 1);$

15. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}, (-1 < x < 1);$

16. $(arctgx)' = \frac{1}{1+x^2};$

17. $(arcctgx)' = -\frac{1}{1+x^2};$

18. $(\mathrm{sh}\, x)' = \left(\frac{e^x - e^{-x}}{2}\right)' = chx;$

19. $(\mathrm{ch}\, x)' = \left(\frac{e^x + e^{-x}}{2}\right)' = shx;$

20. $(thx)' = \left(\frac{shx}{chx}\right)' = \frac{1}{ch^2x};$

21. $(ctx)' = \left(\frac{chx}{shx}\right)' = -\frac{1}{sh^2x};$

22. $(secx)' = \left(\frac{1}{cosx}\right)' = \frac{sinx}{cos^2x};$

23. $(cosex) = \left(\frac{1}{sinx}\right)' = -\frac{cosx}{sin^2x};$

Misollar.

Quyidagi funksiyalarni hosilasini toping;

1. $y = x^2 + 3x - 1;$ $\quad J: y' = 2x + 3.$
2. $y = \sqrt[3]{x} + 5;$ $\quad J: y' = \frac{1}{3\sqrt[3]{x}}.$
3. $y = x \cdot \sqrt[3]{x};$ $\quad J: y' = \frac{4\sqrt[3]{x}}{3}$
4. $y = \log_3 x + x^3\ ;$ $\quad J: y' = \frac{1}{x \cdot ln3} + 3x^2.$
5. $y = 7^x + e^x;$ $\quad J: y' = 7^x \ln 7 + e^x.$

6. $y = sinx - 2^{x+1}$; $J: y' = cosx - 2^{x+1} \cdot ln2.$

7. $y = tgx + \sqrt{x}$; $J: y' = \dfrac{1}{cos^2x} + \dfrac{1}{2\sqrt{x}}$

8. $y = x\ sinx$; $J: y' = sinx + x\ cosx.$

9. $y = x^2\ cosx$; $J: y' = 2x\ cosx - x^2\ sinx.$

10. $y = \dfrac{x}{sinx}$; $J: y' = \dfrac{sinx - x\ cosx}{sin^2x}$

11. $y = \dfrac{5^x}{x}$; $J: y' = \dfrac{5^x \cdot ln5 \cdot\ x - 5^x}{x^2}$

12. $y = e^x\ arcsinx$; $J: y' = e^x\ \left(arcsinx + \frac{1}{\sqrt{1-x^2}}\right).$

13. $y = \dfrac{x^2 + \sqrt{x}}{cosx}$; $J: y' = \dfrac{\left(2x + \frac{1}{2\sqrt{x}}\right) \cdot cosx + (x^2 + \sqrt{x}) \cdot sinx}{cos^2x}$

14. $y = \sin 2x + 3$; $J: y = 2\ cos2x.$

15. $y = cos^2x + sin3x$; $J: y' = -sin2x + 3cos3x.$

16. $y = ln3x - \sqrt[4]{x}$; $J: y' = \dfrac{1}{x} - \dfrac{1}{4\sqrt[4]{x^3}}$

17. $y = tg2x\ ctg2x$; $J: y' = 0.$

18. $y = sin^2 3x + e^{sinx}$; $J: y' = 3\ sin6x + cosx\ e^{sinx}$

19. $y = \sqrt{x^2 + 1}$; $J: y' = \dfrac{x}{\sqrt{x^2 + 1}}.$

20. $y = (x + 1)^{100}$; $J: y = 100(x + 1)^{99}.$

21. $y = (3x - 4)^{50}$; $J: y' = 150(3x - 4)^{49}.$

22. $y = \dfrac{1}{(2x + 5)^4}$; $J: y' = -\dfrac{8}{(2x + 5)^5}$

23. $y = tg^3 3x$; $J: y' = \dfrac{9sin^2 3x}{cos^4 3x}$

24. $y = cos2x\ e^{sinx}$; $J: y' = e^{sinx}\ cosx(cos2x - 2sinx).$

25. $y = \sqrt[3]{cos^6x + 2}$; $J: y' = -\dfrac{6cos^5x\ sinx}{3\sqrt[3]{(cos^6x + 2)^2}}$

26. $y = ctg^2 3x$; $J: y' = -\dfrac{6\ cos3x}{sin^3 3x}$

27. $y = x\, lnx;$ $J: y' = lnx + 1$

28. $y = \ln sinx - \frac{1}{2} sin^2 x;$ $J: y' = ctgx\, cos^2 x$

29. $y = x^3 \cdot 3^x;$ $J: x^2 3^x (3 + x\, ln3)$

30. $y = e^{-x^2} + 7x;$ $J: y' = -2xe^{-x^2} + 7$

31. $y = \ln\left(x + \sqrt{a^2 + x^2}\right), a = const;$ $J: y' = \frac{1}{\sqrt{a^2 + x^2}}$

32. $y = ln\sqrt{\frac{1 + 2x}{1 - 2x}};$ $J: y' = \frac{2}{1 - 4x^2}$

33. $y = x^x;$ $J: y' = x^x(\ln x + 1)$

34. $y = \frac{1 + e^x}{1 - e^x};$ $J: y' = \frac{2e^x}{(1 - e^x)^2}$

35. $y = ln\frac{e^x}{x^2 + 1};$ $J: y' = \frac{(x - 1)^2}{x^2 + 1}$

36. $y = x^2 + arctgx;$ $J: y' = 2x - \frac{1}{1 + x^2}$

37. $y = arcsin\sqrt{x};$ $J: y' = \frac{1}{2\sqrt{x - x^2}}$

38. $y = \arccos(1 - 2x);$ $J: y' = \frac{1}{\sqrt{x(1 - x)}}$

39. $y = x\, arccosx - \sqrt{1 - x^2};$ $J: y' = arccosx$

40. $y = arctg2x + 4;$ $J: y' = \frac{2}{1 + 4x^2}$

2-§. Aniqmasliklarni ochish. Lopital qoidalari.

$\frac{0}{0}, \quad \frac{\infty}{\infty}, \quad 0 \cdot \infty, \quad \infty - \infty, \quad 0^0, \quad \infty^0, \quad 1^\infty$

ko’rinishdagi aniqmasliklarni ochishda, tegishli funksiyalarning hosilalari mavjud bo’lganda, berilgan aniqmasliklarni ochish masalasi yengillashadi. Odatda hosilalardan foydalanib aniqmasliklarni ochish. Lopital qoidalari deyiladi.

1^0. $\frac{0}{0}$ ko’rinishidagi aniqmaslik.

Teorema 1. $(a;b)$ intervalda aniqlangan, uzluksiz $f(x)\ va\ g(x)$ funksiyalar uchun quyidagi shartlar:

1. $\lim_{x\to x_0} f(x)=0,\ \lim_{x\to x_0} g(x)=0;$

2. $(a;b)$ intervalda chekli $f'(x),\ g'(x)$ hosilalar mavjud va $g'(x)\neq 0;$

3. $\lim_{x\to x_0}\dfrac{f'(x)}{g'(x)}=k\,;$

(chekli yoki cheksiz) bajarilsa, u holda

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}$$

tenglik o'rinli bo'ladi.

Bu teorema $(c;+\infty)$ intervalda aniqlangan, uzluksiz $f(x)\ va\ g(x)$ funksiyalar uchun ham o'rinli, ya'ni

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=\lim_{x\to\infty}\frac{f'(x)}{g'(x)}$$

2^0. $\frac{\infty}{\infty}$ ko'rinishidagi aniqmaslik.

Teorema 2. $(a;b)$ intervalda aniqlangan, uzluksiz $f(x)\ va\ g(x)$ funksiyalar uchun quyidagi shartlar:

1. $\lim_{x\to x_0} f(x)=\infty,\ \lim_{x\to x_0} g(x)=\infty;$

2. $(a;b)$ intervalda chekli $f'(x),\ g'(x)$ hosilalar mavjud va $g'(x)\neq 0;$

3. $\lim_{x\to x_0}\dfrac{f'(x)}{g'(x)}=k\,;$

(chekli yoki cheksiz) bajarilsa, u holda

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}$$

tenglik o'rinli bo'ladi.

Bu teorema $(c;+\infty)$ intervalda aniqlangan, uzluksiz $f(x)\ va\ g(x)$ funksiyalar uchun ham o'rinli, ya'ni

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=\lim_{x\to\infty}\frac{f'(x)}{g'(x)}$$

3^0. Boshqa ko'rinishdagi aniqmasliklar.

$a)\ \lim_{x\to x_0} f(x)=0, \qquad \lim_{x\to x_0} g(x)=\infty$

bo'lganda $f(x)\cdot g(x)$ ifoda $0\cdot\infty$ ko'rinishidagi aniqmaslik bo'lib, uni quyidagicha

$$f(x)\cdot g(x)=\frac{f(x)}{\frac{1}{g(x)}}=\frac{g(x)}{\frac{1}{f(x)}}$$

yozish orqali $\frac{0}{0}$ $yoki$ $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslikka keltirish mumkin.

$b)\ \lim_{x\to x_0} f(x)=\infty, \qquad \lim_{x\to x_0} g(x)=\infty$

bo'lganda $f(x)-g(x)$ ifoda $\infty-\infty$ ko'rinishdagi aniqmaslik bo'lib, uni ham quyidagicha

$$f(x)-g(x)=\frac{\frac{1}{g(x)}-\frac{1}{f(x)}}{\frac{1}{f(x)}\cdot\frac{1}{g(x)}}$$

yozish orqali $\frac{0}{0}$ ko'rinishdagi aniqmaslikka keltirish mumkin.

$c)\ \lim_{x\to x_0} f(x)=1;\ (0);\ (\infty); \quad \lim_{x\to x_0} g(x)=\infty;\ (0);\ (0)$

ga intilganda $y=[f(x)]^{g(x)}$ darajali-ko'rsatkichli ifoda $1^{\infty},\ 0^{0},\ \infty^{0}$ ko'rinishdagi aniqmasliklar hosil bo'ladi. Bu ko'rinishdagi aniqmasliklar-ni ochish uchun avvalo $y=[f(x)]^{g(x)}$ ifoda logarifmlanadi.

$$ln\, y=g(x)\cdot l\, nf(x).$$

$x\to x_0$ da $g(x)\cdot l\, nf(x)$ ifoda $0\cdot\infty$ ko'rinishdagi aniqmaslikni ifodalaydi.

Eslatma: *Lopital qoidasini misollarda ketma-ket bir necha marta ham qo'llash mumkin.*

Misollar.

Lopital qoidasidan foydalanib funksiya limitini hisoblang.

1. $\lim_{x\to 0} \frac{sinx}{4x};$
2. $\lim_{x\to a} \frac{x^3-a^3}{x-a};$
3. $\lim_{x\to 0} \frac{x-sinx}{x^3};$

4. $\lim\limits_{x\to 0} \dfrac{e^x - 1}{sin2x}$;

5. $\lim\limits_{x\to 0} \dfrac{3^x - 1}{x}$;

6. $\lim\limits_{x\to \infty} \dfrac{lnx}{x}$;

7. $\lim\limits_{x\to \infty} \dfrac{e^x}{x^2}$;

8. $\lim\limits_{x\to 0} x \cdot lnx$;

9. $\lim\limits_{x\to 0} x^x$;

10. $\lim\limits_{x\to \infty} \dfrac{x^2 + 3x - 1}{3x^2 - 7x + 9}$;

11. $\lim\limits_{x\to \infty} (\sqrt{x+1} - \sqrt{x-1})$;

12. $\lim\limits_{x\to \infty} (x+1)^{\frac{1}{x}}$;

13. $\lim\limits_{x\to 0} (1+x)^{lnx}$, $\quad (x > 0)$;

14. $\lim\limits_{x\to 2} \left(\dfrac{1}{x^2 - 4} - \dfrac{1}{x-2}\right)$;

15. $\lim\limits_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{sinx}\right)$.

Mustaqil yechish uchun misollar.

Variant 1

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 4x^3 - 5x + 4$

2) $y = 3\sqrt{x} + 2ctgx$

3) $y = 5sinx + x^3 \cdot e^x$

4) $y = 2x^4 \cdot arcsinx$

5) $y = 2^x \cdot lnx$

6) $y = e^x \cdot ln2 + \sqrt[3]{x}lnx$

7) $y = -\frac{2}{x^2} + \frac{x^2}{2}$

8) $y = \frac{lnx}{x^3} + \frac{x^4}{2e^x}$

9) $y = x^3 tgx + \frac{tgx}{x^3}$

10) $y = 2^x ln2 + x^2\sqrt{2}$

II. Murakkab funksiya hosilasini toping.

1) $y = \frac{1}{3x+2}$

2) $y = sin5x$

3) $y = (2x^3 + 3)^4$

4) $y = tg^6(1 - x)$

5) $y = sin(3x + 2)^3$

6) $y = ln^3 sinx + 2^{sinx}$

7) $y = e^x \cdot arctge^x$

8) $y = \frac{sinx}{cos^2x} + \ln\frac{1+sinx}{cosx}$

9) $y = (x + 2)^{sinx}$

10) $y = x^2 \cdot e^{x^2} \cdot lnx$

III. $y = \frac{x^3}{3}$ egri chiziqqa $x = -1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 3t^2 + 2$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,2]$ vaqt oralig'idagi o'rtacha tezligini va $t = 1$ c ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to1}\frac{x^3-3x^2+2}{x^3-4x^2+3}$ 2) $\lim\limits_{x\to\infty}\frac{\pi-2arctgx}{e^{\frac{3}{x}}-1}$ 3) $\lim\limits_{x\to0}\frac{lnx}{1+2lnsinx}$

Variant 2

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 5x^3 + 10x - 3$

2) $y = 4\sqrt{x} + 2ctgx$

3) $y = 3sinx + x^5 \cdot e^x$

4) $y = 2x^3 \cdot arctgx$

5) $y = 3^x \cdot lnx$

6) $y = e^x ln3 + lnx \cdot \sqrt{x}$

7) $y = -\frac{3}{x^3} + \frac{x^3}{3}$

8) $y = \frac{lnx}{x^5} + \frac{x^4}{2e^x}$

9) $y = x^4 tgx + \frac{tgx}{x^4}$

10) $y = 3^x ln2 + x^3 \cdot \sqrt{2}$

II. Murakkab funksiya hosilasini toping.

1) $y = cos5x$

2) $y = \frac{2}{3x-2}$

3) $y = (2x^3 - 3)^4$

4) $y = tg^5(1 - x)$

5) $y = sin(2x + 3)^3$

6) $y = ln^3 cosx$

7) $y = e^{2x} \cdot sin^3 e^x$

8) $y = \frac{arctg2x}{1-x^2} + \frac{sin^2x}{cosx}$

9) $y = (cos2x)^{lnx}$

10) $y = x^3 \cdot e^{x^3} \cdot lnx$

III. $y = \frac{x^3}{3}$ egri chiziqqa $x = 1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 3t^2 + 3$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2\ c$ ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x \to 1} \frac{x-1}{x^n - 1}$ 2) $\lim\limits_{x \to 1} (1 - x) \cdot tg\frac{\pi x}{2}$ 3) $\lim\limits_{x \to 1} \left(\frac{2}{x^2 - 1} - \frac{1}{x - 1}\right)$

Variant 3

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 2x^3 - 4x + 3$
2) $y = 5\sqrt{x} + 3tgx$
3) $y = 4cosx + e^x \cdot x^6$
4) $y = 4x^3 \cdot 3^x$
5) $y = x^4 \cdot arcsinx$
6) $y = 2^x \cdot lnx - e^x \cdot ln2$
7) $y = \frac{x^3}{3} + \frac{3}{x^3}$
8) $y = \frac{lnx}{x^4} + \frac{x^4}{2e^x}$
9) $y = x^5 \cdot arctgx - \frac{1+x^2}{2e^x}$
10) $y = 4^x ln2 + x^4 \cdot \sqrt{2}$

II. Murakkab funksiya hosilasini toping.

1) $y = cos3x$
2) $y = \frac{2}{4x-3}$
3) $y = (5x^2 + 1)^4$
4) $y = sin^5(1 - 2x)$
5) $y = ctg(2 + 3x)^3$
6) $y = ln^2(2x + 1)$
7) $y = e^x \cdot \sqrt{1 - e^x} - arcsine^x$
8) $y = lntg\frac{x}{2} - \frac{x}{sinx}$
9) $y = (cos3x)^{lnx}$
10) $y = x^5 \cdot e^{x^5} \cdot lnx$

III. $y = \frac{x^3}{3}$ egri chiziqqa $x = 2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 4t^2 - 1$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [2,5]$ vaqt oralig'idagi o'rtacha tezligini va $t = 3\ c$ ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 0}\frac{x-sinx}{x^3}$ 2) $\lim\limits_{x\to 1}\frac{lnx}{1-x^3}$ 3) $\lim\limits_{x\to 0} xlnx$

Variant 4

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 3x^3 + 2x - 4$

2) $y = 6\sqrt{x} + 3cosx$

3) $y = 2arctgx + 2^x \cdot x^5$

4) $y = 5x^4 \cdot e^x$

5) $y = \sqrt[3]{x} \cdot tgx$

6) $y = \frac{1}{x} \cdot ln2 + \frac{lnx}{\sqrt{3}}$

7) $y = \frac{\sqrt{x}}{3} + \frac{3}{\sqrt{x}}$

8) $y = \frac{arcsinx}{1-x^2}$

9) $y = \sqrt{2} \cdot sinx + \frac{1-x^2}{e^x}$

10) $y = 5^x ln5 + x^5 \cdot \sqrt{5}$

II. Murakkab funksiya hosilasini toping.

1) $y = cos3x$

2) $y = \frac{3}{2x-5}$

3) $y = (4x^3 + 2x)^5$

4) $y = cos^4(1 - 3x)$

5) $y = tg(1 + 2x)^3$

6) $y = sin^3\sqrt{x}$

7) $y = arctg^4 e^{2x}$

8) $y = lnsinx - \frac{x}{cosx}$

9) $y = (x + 3)^{\sqrt{x}}$

10) $y = x^6 \cdot e^{x^6}$ ·

III. $y = 4 - x^2$ egri chiziqqa $x = 2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 4t^2 + 3$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 4$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to\frac{\pi}{6}}\frac{1-2sinx}{cos3x}$ 2) $\lim\limits_{x\to\infty}\frac{\pi-2arctgx}{e^{\frac{2}{x}}-1}$ 3) $\lim\limits_{x\to 0}\frac{lnsin2x}{lnsin5x}$

Variant 5

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = \frac{1}{3}x^3 - 4x + 1$

2) $y = 7\sqrt{x} + 3tgx$

3) $y = 4arccosx + \sqrt[3]{x} \cdot e^x$

4) $y = 3lnx \cdot 2^x$

5) $y = \sqrt[3]{x} \cdot ctgx$

6) $y = \frac{3^x}{ln3} + \sqrt{3}lnx$

7) $y = \frac{x^2}{4} - \frac{4}{x^2}$

8) $y = \frac{arctgx}{1-x}$

9) $y = e^x \cdot sinx + \frac{1+x^2}{lnx}$

10) $y = x^3 \cdot \sqrt{3} + 3^x \cdot ln3$

II. Murakkab funksiya hosilasini toping.

1) $y = tg3x$

2) $y = \frac{5}{4x-3}$

3) $y = (x^3 - 3x)^4$

4) $y = sin^5(2 - 3x)$

5) $y = arctg\sqrt{x}$

6) $y = cos^3 e^x$

7) $y = lntg\frac{x}{2} - x \cdot ctg2x$

8) $y = \frac{cos2x}{sin^2x} - \sqrt{tgx}$

9) $y = \left(ln(x + 2)\right)^{\sqrt{x}}$

10) $y = e^{x^3} \cdot tg3x \cdot x^3$

III. $y = 4 - x^2$ egri chiziqqa $x = 1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 3t^2 + 5$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,4]$ vaqt oralig'idagi o'rtacha tezligini va $t = 3$ c ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 0} \frac{e^{2x}-1}{arcsin3x}$ 2) $\lim\limits_{x\to \frac{\pi}{4}} \frac{ctgx-1}{sin4x}$ 3) $\lim\limits_{x\to 1} \frac{tg\frac{\pi x}{2}}{sin4x}$

Variant 6

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = \frac{2}{3}x^3 - 3x + 2$

2) $y = 8\sqrt{x} + 3tgx$

3) $y = 5arccosx + \frac{3}{x} \cdot 2^x$

4) $y = 4lnx \cdot (x^2 + 1)$

5) $y = \frac{cosx}{3x^5}$

6) $y = \frac{e^x}{ln2} + \sqrt{2}lnx$

7) $y = \frac{x^2}{5} - \frac{5}{x^2}$

8) $y = \frac{arctgx}{1+x}$

9) $y = e^x \cdot tgx + \frac{1-x^2}{cosx}$

10) $y = 3^x \cdot lnx - x^5\sqrt{5}$

II. Murakkab funksiya hosilasini toping.

1) $y = ctg3x$

2) $y = \frac{6}{5x-2}$

3) $y = (x^4 + 2x)^3$

4) $y = cos^5(3 - 4x)$

5) $y = arctg\sqrt{x}$

6) $y = sin^3 2^x$

7) $y = e^{2x} \cdot arctge^{2x}$

8) $y = \frac{sin2x}{cos^3x} + lntgx$

9) $y = (2x + 3)^{\sqrt{x}}$

10) $y = 2^x cos4x \cdot x^5$

III. $y = x^2 - 4$ egri chiziqqa $x = 2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 4t^2 + 5$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1{,}4]$ vaqt oralig'idagi o'rtacha tezligini va $t = 3\ c$ ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to5} \frac{\sqrt[3]{x}-\sqrt[3]{5}}{\sqrt{x}-\sqrt{5}}$

2) $\lim\limits_{x\to0} \frac{lncosax}{lncosbx}$

3) $\lim\limits_{x\to\frac{\pi}{2}} \left(\frac{x}{ctgx} - \frac{\pi}{2cosx}\right)$

Variant 7

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = \frac{4}{3}x^3 + 2x - 1$

2) $y = 9\sqrt{x} + 5sinx$

3) $y = \frac{1}{2}arctgx + \frac{4}{x} \cdot 3^x$

4) $y = (x^2 - 4)lnx$

5) $y = 2^x \cdot e^x$

6) $y = \frac{2^x}{ln2} + 2^3lnx$

7) $y = \frac{x^3}{6} - \frac{6}{x^3}$

8) $y = \frac{cosx}{3x^6} + 3tgx$

9) $y = e^x \cdot ctgx + \frac{1+x^2}{e^x}$

10) $y = 5^x \cdot ln5 + x^5 \cdot \sqrt{5}$

II. Murakkab funksiya hosilasini toping.

1) $y = sin5x$

2) $y = \frac{7}{6x+5}$

3) $y = (x^4 + 3x)^3$

4) $y = tg^6(2 - 5x)$

5) $y = arcctg\frac{1}{x}$

6) $y = cos^3e^{3x}$

7) $y = ln^4 sinx + 2^{cosx}$

8) $y = \frac{tg4x}{e^{3x}} - \sqrt{arcsin2x}$

9) $y = \left(\frac{2}{x}\right)^{\ln(x-3)}$

10) $y = 3^x \cdot sin4x \cdot x^5$

III. $y = x^2 - 4$ egri chiziqqa $x = -2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 5t^2 + 3$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2\ c$ ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - sinx}$ 2) $\lim\limits_{x\to 0} \frac{lnx}{1+2lnsinx}$ 3) $\lim\limits_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$

Variant 8

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = \frac{5}{3}x^3 + 3x - 2$

2) $y = \frac{\sqrt{x}}{2} + 5cosx$

3) $y = 3arcctgx + \frac{5}{x} \cdot 5^x$

4) $y = (x^2 - 9)lnx$

5) $y = 3tgx \cdot x^3$

6) $y = \frac{3^x}{ln3} + \frac{lnx}{3}$

7) $y = \frac{x^3}{3} + \frac{3}{x^3}$

8) $y = \frac{1+x^2}{arctgx} + 2ctgx$

9) $y = e^x \cdot sinx + \frac{3x^5}{lnx}$

10) $y = 7^x \cdot ln7 + x^7\sqrt{7}$

II. Murakkab funksiya hosilasini toping.

1) $y = cos5x$

2) $y = \frac{8}{4x+3}$

3) $y = (x^4 - 3x)^5$

4) $y = sin^4(3 - 2x)$

5) $y = arcsin\frac{1}{x}$

6) $y = tg^3 2x + 2^{cosx}$

7) $y = e^{3x} \cdot sinx + arctg2x$

8) $y = \frac{sin\sqrt{x}}{cos2x} + ln(1 + x^2)$

9) $y = (5x - 3)^{\sqrt{x}}$

10) $y = 4^x \cdot cos2x \cdot x^4$

III. $y = x^2 - 4$ egri chiziqqa $x = -4$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = t^3 + 2t$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 1$ c ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 0}\frac{cos5x - cos3x}{sin^2x}$ 2) $\lim\limits_{x\to 1}\frac{tg\frac{\pi x}{2}}{\ln(1-x)}$ 3) $\lim\limits_{x\to 0}\left(\frac{1}{x} - \frac{1}{e^x-1}\right)$

Variant 9

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = \frac{7}{3}x^3 + 4x - 3$

2) $y = \frac{\sqrt{x}}{3} + 5tgx$

3) $y = 3arcsinx + \frac{6}{x}\cdot e^x$

4) $y = (x^2 + 9)lnx$

5) $y = 3ctgx \cdot x^6$

6) $y = \frac{5^x}{ln5} + e^5 \cdot lnx$

7) $y = \frac{x^4}{4} - \frac{4}{x^4}$

8) $y = \frac{arccosx}{1-x^2} + \frac{2-x^3}{tgx}$

9) $y = e^x \cdot cosx + \frac{4x^3}{lnx}$

10) $y = 5^x ln5 + x^5\sqrt{5}$

II. Murakkab funksiya hosilasini toping.

1) $y = tg5x$

2) $y = \frac{9}{3x+5}$

3) $y = (x^4 + 5x)^3$

4) $y = sin^5(4 - 3x)$

5) $y = arccos\frac{2}{x}$

6) $y = ctg^3x^2$

7) $y = ln(2 - x^2) + 3^{\sqrt{x}}$

8) $y = \frac{tg3x}{\sqrt{1+x^2}} + lncos2x$

9) $y = (sin3x)^{\ln(x+2)}$

10) $y = 5^x \cdot sin2x \cdot x^5$

III. $xy = 4$ egri chiziqqa $x = 1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = t^3 + \frac{3}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [4,5]$ vaqt oralig'idagi o'rtacha tezligini va $t = 4$ c ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim_{x\to 0}\frac{1-cos6x}{sin^2x}$ 2) $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$ 3) $\lim_{x\to\infty}\frac{ln^2x}{x^3}$

Variant 10

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y=\frac{8}{3}x^3+5x-2$

2) $y=\frac{\sqrt{x}}{4}+5ctgx$

3) $y=7arccosx+\frac{7}{x}\cdot 2^x$

4) $y=(x^3+5)lnx$

5) $y=4sinx\cdot x^4$

6) $y=\frac{3^x}{ln3}-e^3\cdot lnx$

7) $y=\frac{x^5}{5}+\frac{5}{x^5}$

8) $y=\frac{arctgx}{1+x^2}$

9) $y=\frac{3-x^4}{tgx}$

10) $y=2^x\cdot\sqrt{x}-\frac{lnx}{x}$

II. Murakkab funksiya hosilasini toping.

1) $y=ctg5x$

2) $y=\frac{7}{4x+3}$

3) $y=(x^4+6x)^5$

4) $y=cos^4(3-2x)$

5) $y=arctg\frac{3}{x}$

6) $y=sin^3\sqrt{x}$

7) $y=lnsinx+e^x\cdot cos2x$

8) $y=\frac{tg2x}{\sqrt{1+x^2}}+2^{\sqrt{x}}\cdot arcsin3x$

9) $y=(arctg2x)^{3x}$

10) $y=e^{x^2}\cdot\sqrt{x}\cdot tg3x$

III. $xy=4$ egri chiziqqa $x=2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S=t^3+\frac{2}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t\in[4,5]$ vaqt oralig'idagi o'rtacha tezligini va $t=3$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim_{x\to -2}\frac{x^4-5x^2+4}{x^4-3x^2-4}$ 2) $\lim_{x\to 0}\frac{sinx-xcosx}{sin^3x}$ 3) $\lim_{x\to 0}\frac{x^{-1}}{ctgx}$

Variant 11

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 2 + 4x - x^3$

2) $y = \frac{1}{2}sinx + 3arccosx$

3) $y = \frac{5}{x} \cdot 2^x + x^4 \cdot tgx$

4) $y = (x^4 - 5x) \cdot lnx$

5) $y = 3cosx \cdot \sqrt{x}$

6) $y = \frac{5^x}{ln5} + \frac{7}{lnx}$

7) $y = \frac{x^6}{6} - \frac{6}{x^6}$

8) $y = \frac{arctgx}{1+x^2} + \frac{5+x^4}{ctgx}$

9) $y = 3^x \cdot \sqrt{x} + 2xlnx$

10) $y = \frac{\sqrt{x}+x^3}{2+x^2}$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{x+1}$

2) $y = sin\frac{x}{2}$

3) $y = \frac{4}{5x-3}$

4) $y = (x^5 + 2x)^3$

5) $y = sin^3(1 - 3x^2)$

6) $y = arcsin\frac{2}{x}$

7) $y = tg^4\sqrt{x}$

8) $y = arcctg2x + lncosx$

9) $y = (arcsin3x)^{x^2}$

10) $y = e^{\sqrt{x}} \cdot x^2 \cdot ctg2x$

III. $xy = 2$ egri chiziqqa $x = 1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = t^3 + \frac{4}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [2,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to-1}\frac{x^3+3x^2-2}{x^3+4x^2-3}$ 2) $\lim\limits_{x\to0}\frac{lntgx}{lntg2x}$ 3) $\lim\limits_{x\to0}\left(\frac{1}{2x^2} - \frac{1}{2xtgx}\right)$

Variant 12

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 3 + 2x - x^3$

2) $y = \frac{1}{3}sinx + 2arcsinx$

3) $y = \frac{6}{x} \cdot 3^x + 5tgx \cdot \sqrt{x}$

4) $y = (x^4 + 6x) \cdot lnx$

5) $y = e^x \cdot cosx + \sqrt{2} \cdot x^3$

6) $y = \frac{x^7}{7} + \frac{7}{x^7}$

7) $y = \frac{7^x}{ln7} - \frac{lnx}{7}$

8) $y = \frac{cosx}{2e^x} - \frac{3x}{1+x^2}$

9) $y = x^2 arccosx + \frac{e^x}{1-x^2}$

10) $y = 2\sqrt{x} \cdot 2^x + 4\sqrt[3]{x}$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{x-1}$

2) $y = sin\frac{x}{3}$

3) $y = \frac{4}{6x-5}$

4) $y = (x^5 + 3x)^4$

5) $y = 4sin^3(1 - x^2)$

6) $y = arctg\frac{3}{x}$

7) $y = e^{cos^2 c}$

8) $y = ctg^3\sqrt{x+2}$

9) $y = (arcsin2x)^{2x+1}$

10) $y = 2^{\sqrt{x}} \cdot x^5 \cdot ctg3x$

III. $xy = 4$ egri chiziqqa $x = 2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = t^3 + \frac{5}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [2,4]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim_{x\to 2}\frac{5x^2-4-x^4}{3x^2+4-x^4}$ 2) $\lim_{x\to 0}\frac{lnx}{1+2lnsinx}$ 3) $\lim_{x\to\infty}\frac{a^x}{x}$

Variant 13

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 4 + 3x - x^3$

2) $y = \frac{1}{3}cosx + 3arccosx$

3) $y = 6e^x \cdot x^3 + \sqrt{x} \cdot ctgx$

4) $y = (x^3 + 3x^2) \cdot 2^x$

5) $y = 2lnx + 3 \cdot \sqrt[3]{x}$

6) $y = \frac{8^x}{ln8} - \frac{lnx}{8}$

7) $y = \frac{x^8}{8} - \frac{8}{x^8}$

8) $y = \frac{sinx}{1+x^2} + \frac{3x}{lnx}$

9) $y = x^3 arcsinx - \frac{e^x}{1-x^2}$

10) $y = 2arctgx + \sqrt[3]{2} \cdot e^x$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{x-2}$

2) $y = cos\frac{x}{3}$

3) $y = \frac{5}{6x+7}$

4) $y = (x^5 + 3x^2)^3$

5) $y = 5sin^3(2 - x)$

6) $y = arcctg\frac{3}{x}$

7) $y = e^{sin^2 x}$

8) $y = ctg^3\sqrt{2x-1}$

9) $y = (3x+1)^{cos2x}$

10) $y = 3^{\sqrt{x}} \cdot x^3 \cdot cos3x$

III. $xy = 3$ egri chiziqqa $x = 1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = t^3 + \frac{6}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [2,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 3$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to\infty} \frac{\frac{\pi}{2} - arctgx}{\frac{1}{2}ln\frac{x-1}{x+1}}$ 2) $\lim\limits_{x\to 0} \frac{lntg2x}{lntgx}$ 3) $\lim\limits_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{lnx}\right)$

Variant 14

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 5 + 4x - 2x^3$

2) $y = \frac{1}{3}sinx + 2arctgx$

3) $y = 3 \cdot 3^x \cdot x^4 + 2tgx$

4) $y = (x^4 + 2x^3) \cdot e^x$

5) $y = 2cosx \cdot \sqrt[3]{x}$

6) $y = \frac{3}{x^3} - \frac{x^3}{3}$

7) $y = \frac{3^x}{ln3} - \frac{lnx}{3}$

8) $y = \frac{cosx}{2-x} - \frac{\sqrt{x}}{sinx}$

9) $y = x^5 arccosx + \frac{1+x^2}{ctgx}$

10) $y = 4\sqrt{x} + 2lnx$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{x+3}$

2) $y = sin\frac{x}{4}$

3) $y = \frac{6}{3x-2}$

4) $y = (x^5 - 3x^2)^3$

5) $y = 4sin^3(3-x)$

6) $y = arcsin\frac{\sqrt{x}}{2}$

7) $y = e^{ln^2 x}$

8) $y = tg^3\sqrt{4x+1}$

9) $y = (arccos3x)^{\frac{1}{x}}$

10) $y = 4^{\sqrt{x}} \cdot x^4 \cdot tg4x$

III. $xy = 3$ egri chiziqqa $x = 2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = t^3 + \frac{6}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,2]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2\,c$ ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 0} \frac{1-cos4x}{sin^2 2x}$ 2) $\lim\limits_{x\to \frac{\pi}{2}} \frac{tgx}{tg3x}$ 3) $\lim\limits_{x\to 1} \left(\frac{2}{x^2-1} - \frac{1}{x-1}\right)$

Variant 15

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 6 + 5x - 2x^3$

2) $y = \frac{1}{4}tgx - 5arcsinx$

3) $y = 4 \cdot 4^x \cdot x^4 + e^x \cdot \sqrt{x}$

4) $y = (x^3 - x^5) \cdot lnx$

5) $y = x^3 \cdot arcsinx + 4\sqrt[3]{x}$

6) $y = \frac{5}{x^3} - \frac{x^3}{5}$

7) $y = \frac{5^x}{ln5} - \frac{lnx}{5}$

8) $y = \frac{arccosx}{x^2-2x} + \frac{4lnx}{\sqrt{x}}$

9) $y = \frac{cosx}{3-x} + \frac{2}{x}$

10) $y = (2 + x^2)ctgx + \sqrt{3} \cdot x^3$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{x - 3}$

2) $y = tg\frac{x}{4}$

3) $y = \frac{5}{7x+3}$

4) $y = (x^6 - 3x^4)^3$

5) $y = 4sin^3(3 - x)$

6) $y = arccose^x$

7) $y = 2^{lnx}$

8) $y = ctg^2\sqrt{5x + 2}$

9) $y = (arctg4x)^{\sqrt{x}}$

10) $y = 5^{\sqrt{x}} \cdot x^5 \cdot lgx$

III. $xy = 3$ egri chiziqqa $x = -2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = t^3 - \frac{6}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2\,c$ ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim_{x\to 0}\frac{1-sinx}{x-tgx}$ 2) $\lim_{x\to 0}\frac{lnsin2x}{lnsinx}$ 3) $\lim_{x\to\infty}\left(x\cdot sin\frac{a}{x}\right)$

Variant 16

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 2 + 4x - 3x^3$

2) $y = \frac{1}{4}ctgx - 7arccosx$

3) $y = 5\cdot 5^x\cdot x^5 + e^x\cdot 2\sqrt[3]{x}$

4) $y = (x^2 - x^4)\cdot 2^x$

5) $y = x^3\cdot arctgx + 3^x lnx$

6) $y = \frac{5}{x^4} - \frac{x^4}{5}$

7) $y = \frac{7^x}{ln7} - \frac{lnx}{7}$

8) $y = \frac{sinx}{5-x} + \frac{3}{x^2}$

9) $y = \frac{6lnx}{\sqrt{x}} + \frac{arcsinx}{x^2+3x}$

10) $y = (3 - x^2)cosx + \sqrt{3}$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{1-x}$

2) $y = ctg\frac{x}{4}$

3) $y = \frac{7}{5x-3}$

4) $y = (x^5 + 4x^3)^4$

5) $y = 6cos^3(5-2x)$

6) $y = arctg^5 e^x$

7) $y = 3^{\ln(x+2)}$

8) $y = ctg^4\sqrt{4x-3}$

9) $y = (3x-2)^{sin2x}$

10) $y = 4^{\sqrt{x}}\cdot x^3\cdot \ln(x+1)$

III. $y = x^3 + 1$ egri chiziqqa $x = 1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = t^3 + \frac{2}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [2,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 3$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim_{x\to\infty}\frac{\frac{\pi}{2}-arctgx}{\frac{1}{2}ln\frac{x-1}{x+1}}$ 2) $\lim_{x\to\frac{\pi}{2}}\frac{tg4x}{tg6x}$ 3) $\lim_{x\to 2}\left(\frac{2}{x^2-4} - \frac{1}{x-2}\right)$

Variant 17

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 3 + 5x - 4x^3$

2) $y = \frac{1}{5}sinx + 7arctgx$

3) $y = 6 \cdot x^6 \cdot 6^x + 2\sqrt[3]{x}$

4) $y = (x^3 - x^4) \cdot e^x$

5) $y = x^4 \cdot arccosx + 3lnx$

6) $y = \frac{7}{x^5} - \frac{x^5}{7}$

7) $y = \frac{2^x}{3\sqrt{x}} + \frac{5-x^4}{sinx}$

8) $y = \frac{cosx}{6-3x} - \frac{2}{x^3}$

9) $y = \frac{2arcsinx}{x^3+3x} + \frac{10^x}{\sqrt{x}}$

10) $y = 2lnx - \sqrt{3}tgx$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{2 - x}$

2) $y = sin\frac{x}{5}$

3) $y = \frac{7}{6x+5}$

4) $y = (x^5 + 5x^4)^3$

5) $y = arcctge^{2x}$

6) $y = arctg^5 e^x$

7) $y = 3^{\ln(x-5)}$

8) $y = tg^5\sqrt{3x + 4}$

9) $y = (arccos3x)^{\sqrt{x+3}}$

10) $y = 5^{\sqrt{x}} \cdot x^5 \cdot \ln(x + 2)$

III. $y = x^3 + 1$ egri chiziqqa $x = -1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 2t^3 + \frac{3}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,2]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to\infty} \frac{arctgx-\frac{\pi}{2}}{\frac{1}{2}ln\frac{x-1}{x+1}}$

2) $\lim\limits_{x\to\frac{\pi}{2}} \frac{tg6x}{tg4x}$

3) $\lim\limits_{x\to 3} \left(\frac{3}{x^2-9} - \frac{1}{x-3}\right)$

Variant 18

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 7 + 6x - 2x^3$

2) $y = \frac{1}{5}cosx - 6arccosx$

3) $y = -\frac{3}{x^4} \cdot 4^x$

4) $y = (x^4 - x^5)lnx$

5) $y = e^x \cdot \log_3 x + 3\sqrt[3]{x}$

6) $y = \frac{8}{x^4} - \frac{x^4}{8}$

7) $y = \frac{10^x}{ln10} - \frac{lnx}{10}$

8) $y = \frac{tgx}{4-2x} + \frac{3}{x}$

9) $y = x^4 arctgx - \frac{3+x^3}{3ctgx}$

10) $y = arcsinx \cdot (x^3 - 3x)$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{x-4}$

2) $y = cos\frac{x}{5}$

3) $y = \frac{7}{8x-4}$

4) $y = (x^5 - 5x)^3$

5) $y = 4tg^3(3 - x^2)$

6) $y = arctg\sqrt{x}$

7) $y = 3^{sin2x}$

8) $y = sin^3 lnx$

9) $y = (arcctg5x)^{x^3}$

10) $y = 5^x \cdot x^5 \cdot \ln(5x+2)$

III. $y = x^3 - 1$ egri chiziqqa $x = 2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 3t^3 + \frac{5}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [2{,}3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 3$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 0}\frac{sinx-x}{tgx-x}$ 2) $\lim\limits_{x\to 0}\frac{lnsin2x}{lnsin4x}$ 3) $\lim\limits_{x\to\infty}\left(x \cdot sin\frac{a}{2x}\right)$

Variant 19

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 8 + 7x - 3x^2$

2) $y = \frac{1}{5}tgx - \log_2 x$

3) $y = -7arccosx + 4cosx$

4) $y = \frac{4}{x^5} \cdot 5^x$

5) $y = (x^6 - 6x) \cdot lnx$

6) $y = \frac{6}{x^3} - \frac{x^3}{6}$

7) $y = \frac{lnx}{sinx} + \frac{3cosx}{3-x^3}$

8) $y = \frac{9^x}{ln9} + \frac{5}{x^2}$

9) $y = \frac{arcsinx}{x^5-5x} + \frac{4-x^4}{3ctgx}$

10) $y = 6 \cdot 6^x \cdot \sqrt{x} - \sqrt[3]{x}$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{x+4}$

2) $y = tg\frac{x}{5}$

3) $y = \frac{8}{9x+3}$

4) $y = (x^6 - 6x)^3$

5) $y = 5ctg^3(4 - x^2)$

6) $y = arcctg\sqrt{x}$

7) $y = 3^{cos2x}$

8) $y = ln^3 sin5x$

9) $y = \left(\sqrt{x}\right)^{sin3x}$

10) $y = 4\sqrt{3x} \cdot e^{2x} \cdot cos5x$

III. $y = x^3 - 1$ egri chiziqqa $x = -2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 2t^2 + \frac{4}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2$ c ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim_{x\to 0} \frac{1-cos2x}{sin^2x}$

2) $\lim_{x\to \frac{\pi}{4}} \frac{tg2x}{tg6x}$

3) $\lim_{x\to -1} \left(\frac{2}{x^2-1} - \frac{1}{x-1}\right)$

Variant 20

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 9 + 8x - 4x^3$

2) $y = \frac{1}{5}ctgx + \log_3 x$

3) $y = 5arctgx + 3^x$

4) $y = \frac{5}{x^9} \cdot 9^x$

5) $y = (x^7 - 7x) \cdot lnx$

6) $y = \frac{8}{x^4} - \frac{x^4}{8}$

7) $y = \frac{e^x}{ln2} + \frac{2}{x^2}$

8) $y = \frac{lnx}{cosx} + \frac{arcsinx}{\sqrt{x}}$

9) $y = \frac{x^6-6x}{arcctgx}$

10) $y = 3\sqrt[3]{x} + \sqrt[3]{3} \cdot e^x$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{2x-1}$

2) $y = ctg\frac{x}{5}$

3) $y = \frac{9}{4-3x}$

4) $y = (x^7 - 7x)^3$

5) $y = 3sin^3(1 - x^2)$

6) $y = arcsinlnx$

7) $y = 2^{cos^2x}$

8) $y = ln^3\sqrt{2x+1}$

9) $y = (sin3x)^{\sqrt{1-2x}}$

10) $y = 5e^{3x} \cdot sin2x \cdot \sqrt{1+2x}$

III. $y = \sqrt{x}$ egri chiziqqa $x = 4$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 5t^2 + \frac{5}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [2,4]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2\ c$ ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 0} \frac{e^{2x}-1}{arcsin3x}$

2) $\lim\limits_{x\to 1} \frac{tg\frac{\pi x}{2}}{\ln(1-x)}$

3) $\lim\limits_{x\to 2}(x-2)\cdot ctg\pi(x-2)$

Variant 21

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 10 + 9x - 5x^3$

2) $y = \frac{1}{6} + 5\log_5 x$

3) $y = 6arcctgx + 10^x$

4) $y = (x^8 + 8x)\cdot lnx$

5) $y = \frac{10}{x^{10}}\cdot 10^x$

6) $y = \frac{10}{x^5} - \frac{x^5}{10}$

7) $y = \frac{e^x}{ln5} + \frac{lnx}{\sqrt{5}}$

8) $y = \frac{sinx}{lnx} + \frac{arccosx}{\sqrt{x}}$

9) $y = \frac{5-x^3}{tgx} + \frac{2\cdot 3^x}{\sqrt{x}}$

10) $y = 3^x\cdot lnx + \sqrt[3]{3}\cdot e^x$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{2x+1}$

2) $y = sin\frac{x}{6}$

3) $y = \frac{5}{6-3x}$

4) $y = (x^4 - 4x)^5$

5) $y = 2cos^4(2 - x^2)$

6) $y = arcsin^3 lnx$

7) $y = 5^{tg^2x}$

8) $y = ln^3 sinx$

9) $y = \left(\frac{1}{x^2}\right)^{sin3x}$

10) $y = 2^x\cdot\sqrt{3x}\cdot sin5x$

III. $y = \sqrt[3]{x}$ egri chiziqqa $x = 1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 6t^2 + \frac{6}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [2,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 3\ c$ ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 0}\frac{e^{3x}-1}{arcsin2x}$

2) $\lim\limits_{x\to 2}\frac{tg(\pi x/4)}{\ln(2-x)}$

3) $\lim\limits_{x\to 3}(x-3)ctg\pi(x-$

3)

Variant 22

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 7 + 8x - 6x^3$

2) $y = \frac{1}{6}cosx + \log_2 x$

3) $y = -7arcsinx + \sqrt[3]{x^2}$

4) $y = \frac{2}{x^2}\cdot 2^x + x^2cosx$

5) $y = (x^4 - 4x)\cdot lnx$

6) $y = \frac{x^3}{3} + \frac{3}{x^3}$

7) $y = \frac{e^x}{ln2} + \frac{lnx}{e^x}$

8) $y = \frac{x\sqrt{x}}{lnx} + \frac{sinx}{x^2-2x^3}$

9) $y = \frac{arctgx}{x^2-3} - \frac{3^x}{2\sqrt{x}}$

10) $y = \sqrt{2}\cdot arcsinx + 3tgx$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{2x-3}$

2) $y = cos\frac{x}{6}$

3) $y = \frac{3}{1-6x}$

4) $y = (x^3 - 2x)^4$

5) $y = 3tg^4(3 + x^2)$

6) $y = arcsin^3e^x$

7) $y = 3^{ctg^2x}$

8) $y = ln^4cos2x$

9) $y = (sin4x)^{\frac{1}{x^2}}$

10) $y = \sqrt[4]{x}\cdot e^{4x}\cdot sin4x$

III. $y = \sqrt[3]{x}$ egri chiziqqa $x = -1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 3t^2 + \frac{3}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t\in[1,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 0}\frac{e^{4x}-1}{arcsin3x}$

2) $\lim\limits_{x\to 4}\frac{tg\frac{\pi x}{8}}{\ln(4-x)}$

3) $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{2x}{ctgx} - \frac{\pi}{cosx}\right)$

Variant 23

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 5 + 6x - 4x^3$

2) $y = \frac{1}{6}tgx + \log_5 x$

3) $y = -4arccosx + 3^x \cdot ln3$

4) $y = \frac{3}{x^3} \cdot 3^x + x\sqrt{x}$

5) $y = (x^5 + 5x) \cdot lnx$

6) $y = \frac{x^4}{4} + \frac{4}{x^4}$

7) $y = \frac{e^x}{ln3} - \frac{lnx}{\sqrt{3}}$

8) $y = \frac{tgx}{\sqrt{x}} + \frac{3sinx}{x^3 - 3x^2}$

9) $y = \frac{arctgx}{x^2 + 2x} - 5^x \cdot \sqrt[3]{x}$

10) $y = \sqrt{2} \cdot arcsinx + x^3 tgx$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{3x + 1}$

2) $y = tg\frac{x}{6}$

3) $y = \frac{4}{2 - 3x}$

4) $y = (x^3 + x^2)^5$

5) $y = 4ctg^3(5 + x^2)$

6) $y = arccos^3\sqrt{x}$

7) $y = 4^{sin^3 x}$

8) $y = ln^5 tg3x$

9) $y = (3x + 1)^{sin5x}$

10) $y = 5^x \cdot e^{5x} \cdot \sqrt{5x}$

III. $y = \sqrt[3]{x - 1}$ egri chiziqqa $x = 2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 4t^2 + \frac{4}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,2]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x \to 0} \frac{1 - cos4x}{sin^2 x}$ 2) $\lim\limits_{x \to 0} \left(\frac{2}{x} - \frac{1}{1 - e^x}\right)$ 3) $\lim\limits_{x \to \infty} \frac{ln^2 x}{2x^3}$

Variant 24

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 4 + 5x - 2x^3$

2) $y = \frac{1}{6}ctgx + \log_3 x$

3) $y = -4arctgx + \sqrt[4]{x^3}$

4) $y = \frac{4}{x^4} \cdot 4^x + 2e^x$

5) $y = (x^3 + 3x^2) \cdot lnx$

6) $y=\frac{x^5}{5}-\frac{5}{x^5}$

7) $y=\frac{2^x}{ln2}+\frac{lnx}{\sqrt{2}}$

8) $y=\frac{ctgx}{\sqrt{x}}+\frac{3cosx}{x^4-2x^2}$

9) $y=\frac{3\sqrt{x}}{arcsinx}+x^2lnx$

10) $y=x\sqrt{x}+\sqrt{2}tgx$

II. Murakkab funksiya hosilasini toping.

1) $y=\sqrt{3x-2}$

2) $y=ctg\frac{x}{6}$

3) $y=\frac{5}{3-4x}$

4) $y=(x^4+2x^3)^5$

5) $y=5sin^4(1+\sqrt{x})$

6) $y=arctg^2\frac{1}{x}$

7) $y=5^{\ln(x+1)}$

8) $y=ln^3ctg5x$

9) $y=(arcsin2x)^{\ln(3x+1)}$

10) $y=3^{\sqrt{x}}\cdot x^5\cdot cos5x$

III. $y=\sqrt[3]{x+1}$ egri chiziqqa $x=-2$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S=3t^2+\frac{2}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t\in[2,4]$ vaqt oralig'idagi o'rtacha tezligini va $t=4$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to5}\frac{\sqrt[3]{x}-\sqrt[3]{5}}{\sqrt{x}-\sqrt{5}}$

2) $\lim\limits_{x\to\frac{\pi}{4}}\frac{ctgx-1}{sin4x}$

3) $\lim\limits_{x\to\infty}x\cdot sin\frac{x}{2}$

Variant 25

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y=3+4x-5x^3$

2) $y=\frac{1}{6}ctgx+\log_3 x$

3) $y=-4arctgx+\sqrt[4]{x^3}$

4) $y=\frac{4}{x^4}\cdot4^x+2e^x$

5) $y=(x^3+3x^2)\cdot lnx$

6) $y=\frac{x^6}{6}+\frac{6}{x^6}$

7) $y=\frac{3^x}{ln3}+\frac{\sqrt[3]{x}}{e^3}$

8) $y=\frac{\sqrt{x}}{sinx}+\frac{5tgx}{x+\sqrt{x}}$

9) $y=\frac{arctgx}{x^3-2x^2}-x^3\cdot lnx$

10) $y=\sqrt{2}arcsinx+4^xln4$

II. Murakkab funksiya hosilasini toping.

1) $y = \sqrt{3x+2}$

2) $y = sin\frac{x}{7}$

3) $y = \frac{6}{5-3x}$

4) $y = (x^3 - 3x^2)^4$

5) $y = 4cos^3 lnx$

6) $y = arcctg^3 lnx$

7) $y = 6^{ln^3 x}$

8) $y = ctg^4\sqrt{3x}$

9) $y = (arcsin3x)^{\sqrt{x-2}}$

10) $y = \sqrt[3]{x} \cdot e^{3x} \cdot 2^{sinx}$

III. $y = lnx$ egri chiziqqa $x = 1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 5t^2 + \frac{3}{t}$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,5]$ vaqt oralig'idagi o'rtacha tezligini va $t = 3$ *c* ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 4}\frac{\sqrt[3]{x}-\sqrt[3]{4}}{\sqrt{x}-2}$ 2) $\lim\limits_{x\to\frac{\pi}{4}}\frac{tgx-1}{cos2x}$ 3) $\lim\limits_{x\to 1}\left(\frac{2}{lnx}-\frac{3x}{lnx}\right)$

Variant 26

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 2x^3 - 3x + 4$

2) $y = 6\sqrt{x} + 3ctgx$

3) $y = 2sinx + x^4 \cdot e^x$

4) $y = 3x^2 \cdot arcsinx$

5) $y = 4^x \cdot lnx$

6) $y = e^x ln5 + \sqrt[4]{x}lnx$

7) $y = -\frac{2}{x^3} + \frac{x^2}{2}$

8) $y = \frac{lnx}{x^4} + \frac{x^3}{2e^x}$

9) $y = x^3 tgx + \frac{tgx}{x^3}$

10) $y = 3^x ln3 + x^5\sqrt{2}$

II. Murakkab funksiya hosilasini toping.

1) $y = \frac{1}{5x+2}$

2) $y = sin6x$

3) $y = (2x^4 + 3)^4$

4) $y = tg^5(1-x)$

5) $y = sin(3x+2)^4$

6) $y = ln^4 sinx + 4^{sinx}$

7) $y = e^x \cdot arctge^x$

8) $y = \frac{sinx}{cos^3 x} + ln\frac{1+sinx}{cosx}$

9) $y = (x+5)^{sinx}$

10) $y = x^3 \cdot e^{x^2} lnx$

III. $y = \frac{x^3}{2}$ egri chiziqqa $x = -1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing.

IV. Jism $S = 2t^2 + 3$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,2]$ vaqt oralig'idagi o'rtacha tezligini va $t = 1$ c ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 1} \frac{x^3-2x^2+1}{x^3-5x^2+4}$ 2) $\lim\limits_{x\to \infty} \frac{\pi-2arctgx}{e^{\frac{4}{x}}-1}$ 3) $\lim\limits_{x\to 0} \frac{lnx}{1+3lnsinx}$

Variant 27

I. Funksiya hosilasini, hosila olish qoidasi va formulasidan foydalanib hisoblang.

1) $y = 6x^3 + 5x - 3$

2) $y = 3\sqrt{x} + 5tgx$

3) $y = 3cosx + x^5 e^x$

4) $y = 2x^4 \cdot arcctgx$

5) $y = 5^x \cdot lnx$

6) $y = e^x ln2 + lnx \cdot \sqrt[3]{x}$

7) $y = -\frac{3}{x^3} + \frac{x^3}{3}$

8) $y = \frac{lnx}{x^5} + \frac{x^4}{2e^x}$

9) $y = x^3 ctgx + \frac{tgx}{x^3}$

10) $y = 2^x ln2 + x^5 \cdot \sqrt{2}$

II. Murakkab funksiya hosilasini toping.

1) $y = cos3x$

2) $y = \frac{2}{2x-3}$

3) $y = (2x^4 - 3)^2$

4) $y = tg^6(1 - x)$

5) $y = sin(3x + 1)^2$

6) $y = ln^4 cosx$

7) $y = e^{3x} \cdot sin^2 e^x$

8) $y = \frac{arctg4x}{1-x^3} + \frac{sin^5 x}{cosx}$

9) $y = (cos3x)^{lnx}$

10) $y = x^3 \cdot e^{x^3} \cdot lnx$

III. $y = \frac{x^3}{4}$ egri chiziqqa $x = -1$ nuqtadan o'tkazilgan urinma tenglamasini tuzing, egri chiziq va urinma tenglamasini chizing

IV. Jism $S = 5t^2 + 3$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanyapti. Jismning$t \in [1,3]$ vaqt oralig'idagi o'rtacha tezligini va $t = 2\ c$ ondagi tezligini toping.

V. Lopital qoidasidan foydalanib, limitni hisoblang.

1) $\lim\limits_{x\to 1} \frac{x-1}{x^n-1}$ 2) $\lim\limits_{x\to 1}(1-x)\cdot tg\frac{\pi x}{2}$ 3) $\lim\limits_{x\to 1}\left(\frac{3}{x^2-1} - \frac{1}{x-1}\right)$

Foydalanilgan adabiyotlar

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