TOSHKEN AXBOROT TEXNOLOGIYALARI UNIVERSITETI
URGANCH FILIALI

TABIIY VA UMUMKASBIY FANLAR KAFEDRASI

OLIY MATEMATIKA FANIDAN

O’QUV USLUBIY QO’LLANMA

5330500 – Kompyuter injiniringi (“Kompyuter injiniringi”, “AT-servisi”)
5350400 –AKT sohasida kasb ta’limi
5350100– Telekommunikatsiya texnologiyalari (Telekommunikatsiya)
5330600 – Dasturiy injiniring ta’lim yo’nalishlari uchun

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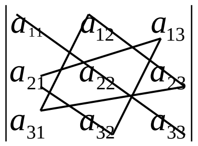
1. IKKINCHI, UCHINCHI TARTIBLI ANIQLOVCHILARNI HISOBLASH

1.1) a_{11}, a_{12}, a_{21}, a_{22} haqiqiy sonlar berilgan bo'lsin ikkinchi tartibli determinant (yoki aniqlovchi) deb, $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ kabi belgilanuvchi va $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$ tenglik bilan aniqlanuvchi songa aytiladi

Berilgan a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33} haqiqiy sonlardan tuzilgan $a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$ yig'indiga teng va

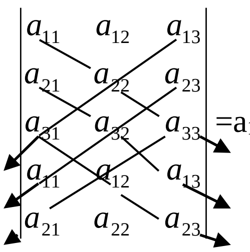
$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ kabi berilgan songa uchinchi tartibli determinant deb ataladi. Uchinchi tartibli determinantlarni uchburchaklar usulida, Sarryus usulida hamda biror satr yoki ustun elementlari bo'yicha yoyib hisoblash mumkin.

1.2) Uchburchaklar usuli:

(+) 

$a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$

1.3) Sarryus usuli:



$=a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$

1.4) Birinchi ustun elementlari bo'yicha yoyib hisoblash:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

Misol 1: $\begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix} = 3 \cdot 5 - (-4) \cdot 2 = 15 + 8 = 23$

1.5) Uchinchi tartibli determinantlarni uchrurchaklar usuli, Sarryus usuli hamda biror ixtiyoriy satr yoki ustun elementlari bo'yicha yoyib hisoblang:

Misol 2: $\begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 4 & -1 & -5 \end{vmatrix}$ $=1\cdot(-3)\cdot(-5) + 1\cdot1\cdot4 + 2\cdot(-1)\cdot1 - 1\cdot(-3)\cdot4 - 1\cdot2\cdot(-5) - 1\cdot(-1)\cdot1 =$ $= 15 + 4 - 2 + 12 + 10 + 1 = 40$

1.6) Determinantning asosiy xossalari yordamida yuqori tartibli determinantlar quyi tartibli determinantga kelteriladi.

Misol 3: det $= \begin{vmatrix} 3 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix}$ bu determinantni biror satr yoki ustunda nollar hosil qilib hisoblaymiz. Buning uchun 1-satrni (-1) ga ko'paytirib 2-satrga qo'shamiz:

$$\begin{vmatrix} 3 & 1 & 2 \\ -1 & 0 & -1 \\ 1 & 0 & 2 \end{vmatrix}$$

2 – ustun elementlari bo'yicha yoyib yozamiz:

$$\text{Det}= 1\cdot(-1)^{1+2} \begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix} =-(-2+1)=1$$

Misol 4: $\begin{vmatrix} 3 & 5 & 7 & 2 \\ 1 & 2 & 3 & 4 \\ -2 & -3 & 3 & 2 \\ 1 & 3 & 5 & 4 \end{vmatrix}$ Determinantni hisoblang.

Determinantni hisoblash uchun biror yo'l yoki ustunda nollar hosil qilamiz. Buning uchun 2-satr elementlarini (-3) ga ko'paytirib 1-satr elementlariga, 2-satrni 2 ga ko'paytirib 3-satr elementlariga qo'shamiz, 4-satr elementlaridan 2-satr elementlarini ayiramiz. Natijada berilgan determinant quyidagi ko'rinishga keladi:

$$\text{Det} = \begin{vmatrix} 0 & -1 & -2 & -10 \\ 1 & 2 & 3 & 4 \\ 0 & 1 & 9 & 10 \\ 0 & 1 & 2 & 0 \end{vmatrix}$$

Determinantni 1-ustun elementlari bo'yicha yoyib yozamiz:

$$\text{Det} = - \begin{vmatrix} -1 & -2 & -10 \\ 1 & 9 & 10 \\ 1 & 2 & 0 \end{vmatrix}$$

1- satr elementlariga 2- satr elementlarini hadma–had qo'shib, 1 – satr elementlari bo'yicha yoyib yozamiz:

$$\text{Det} = \begin{vmatrix} 0 & 7 & 0 \\ 1 & 9 & 10 \\ 1 & 2 & 0 \end{vmatrix} = 7 \cdot \begin{vmatrix} 1 & 10 \\ 1 & 0 \end{vmatrix} = -70$$

1.7) $\text{Det} = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$

n- tartibli determinantning a_{ij} elementining algebraik to'ldiruvchisi

$A_{ij} = (-1)^{i+j} M_{ij}$ formula bo'yicha hisoblanadi, bu yerda M_{ij} a_{ij} elementning minori.

Berilgan $\begin{vmatrix} 3 & -4 & 5 \\ 2 & -3 & 0 \\ 1 & 6 & 10 \end{vmatrix}$ determinantning barcha algebraik to\`ldiruvchilarini toping.

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -3 & 0 \\ 6 & 10 \end{vmatrix} = -30;$ $\quad A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 0 \\ 1 & 10 \end{vmatrix} = -20;$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -3 \\ 1 & 6 \end{vmatrix} = 15;$ $\quad A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} -4 & 5 \\ 6 & 10 \end{vmatrix} = 70;$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 3 & 5 \\ 1 & 10 \end{vmatrix} = 25;$ $\quad A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 3 & -4 \\ 1 & 6 \end{vmatrix} = -22;$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} -4 & 5 \\ -3 & 0 \end{vmatrix} = 15;$ $\quad A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 3 & 5 \\ 2 & 0 \end{vmatrix} = 10;$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 3 & -4 \\ 2 & -3 \end{vmatrix} = -1.$

Determinantning ixtiyoriy satr yoki ustun elementlarining o'z algebraik to'ldiruvchilariga ko'paytmalarining yig'indisi uning kattaligiga teng degan xossaga ko'ra, har qanday determinantni ixtiyoriy satr (ustun) bo'yicha yoyib yozish mumkin.

Tartiblari bir hil bo\`lgan 2 ta determinantni qo\`shish amali hossasini faqatgina bittadan mos satrlari (yoki ustunlari) farq qilgandagina qo\`llash mumkin:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33} \end{vmatrix}$$

Misol 5: $\begin{vmatrix} 1 & 3 & -2 \\ 8 & 9 & 5 \\ 12 & 3 & 6 \end{vmatrix} + \begin{vmatrix} 1 & -3 & -2 \\ 8 & -9 & 5 \\ 12 & -2 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -2 \\ 8 & 0 & 5 \\ 12 & 1 & 6 \end{vmatrix} =$

$= 1 \cdot (-1)^{3+2} \cdot \begin{vmatrix} 1 & -2 \\ 8 & 5 \end{vmatrix} = -(5+16) = -21.$

1.8) Bir hil n-tartibli A va B determinantlarni ko`paytirish quyidagi formula asosida amalga oshiriladi:

$$c_{ik} = \sum_{j=1}^{n} a_{ij} \cdot b_{jk} \qquad i = 1,2,\dots,n; \;\; j = 1,2,\dots,n$$

Misol: $\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} \cdot \begin{vmatrix} 1 & 7 \\ 6 & 9 \end{vmatrix} = \begin{vmatrix} 3\cdot1+2\cdot6 & 3\cdot7+2\cdot9 \\ 4\cdot1+5\cdot6 & 4\cdot7+5\cdot9 \end{vmatrix} = \begin{vmatrix} 15 & 39 \\ 34 & 73 \end{vmatrix}$

1. Mustaqil yechish uchun misollar:

Quyidagi ikkinchi tartibli determinantlarni hisoblang:

1.1. $\begin{vmatrix} -7 & 6 \\ 5 & -4 \end{vmatrix}$ 1.2. $\begin{vmatrix} 10 & -5 \\ 9 & -8 \end{vmatrix}$

1.3. $\begin{vmatrix} \sqrt{a}+\sqrt{b} & \sqrt{a}-\sqrt{b} \\ \sqrt{a}-\sqrt{b} & \sqrt{a}+\sqrt{b} \end{vmatrix}$ 1.4. $\begin{vmatrix} \sin 1^0 & \sin 89^0 \\ -\cos 1^0 & \cos 89^0 \end{vmatrix}$

1.5. $\begin{vmatrix} (x+y)/x & 2x/(x-y) \\ (y-x)/(x^2-y^2) & (y-x)/(x^2-y^2) \end{vmatrix}$

1.6. Tengsizliklarni yeching:

a) $\begin{vmatrix} x & 1 \\ -4 & x \end{vmatrix} \le \begin{vmatrix} 5 & 2 \\ 1 & x \end{vmatrix}$,

Quyidagi uchinchi tartibli determinantlarni qulay usulda hisoblang:

1.7. $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ 1.8. $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$

1.9. $\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}$ 1.10. $\begin{vmatrix} 1 & 2 & 3 \\ 8 & 1 & 4 \\ 2 & 1 & 1 \end{vmatrix}$

Determinantlarni 3-ustun elementlari bo'yicha yoyib hisoblang:

1.11. $\begin{vmatrix} 1+\cos a & 1+\sin a & 1 \\ 1-\sin a & 1+\cos a & 1 \\ 1 & 1 & 1 \end{vmatrix}$ 1.12. $\begin{vmatrix} 2\cos^2 a/2 & \sin a & 1 \\ 2\cos^2 b/2 & \sin b & 1 \\ 1 & 0 & 1 \end{vmatrix}$

2 MATRITSALAR USTIDA AMALLAR

Matritsalar ustida quyidagi chiziqli amallarni bajarish mumkin.

1. Matritsani songa ko'paytirish uchun uning barcha elementlari shu songa ko'paytiriladi. $k \neq 0$ son hamda

$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ matritsa berilgan bo'lsa, $Ak=\begin{pmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{pmatrix}$ tenglik o'rinli bo`ladi.

1. O'lchamlari bir hil bo'lgan *A* va *B* matritsalarni qo'shish uchun mos elementlari qo'shiladi:

$B=\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$ bo'lsa, $A+B=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{pmatrix}$ matritsa hosil bo'ladi.

2. Matritsalarni ko'paytirish.

Agar *A* matritsaning ustunlari soni *B* matritsaning satrlar soniga teng bo'lsa *A* ni *B* ga ko'paytirish mumkin, nxm o'lchovli $A=(a_{ik})$ matritsani mxp o'lchovli $B=(b_{ik})$ matritsaga quyidagi formula bo'yicha ko'paytiriladi.

$$c_{ik}=\sum_{j=1}^{n} a_{ij} b_{jk}$$

Amallarni bajaring:

2.1. $A=\begin{bmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{bmatrix}$ $B=\begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ *A+B* matritsani toping.

$$A+B=\begin{bmatrix} 2 & 4 & 1 \\ -1 & 0 & 2 \end{bmatrix}+\begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}=\begin{bmatrix} 2+0 & 4+2 & 1+1 \\ -1+1 & 0+1 & 2+2 \end{bmatrix}=\begin{bmatrix} 2 & 6 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

2.2. $A=\begin{bmatrix} 7 & -12 \\ -4 & 7 \end{bmatrix}$ $B=\begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$ $A{\cdot}B$ matritsani toping.

$$A{\cdot}B=\begin{bmatrix} 7 & -12 \\ -4 & 7 \end{bmatrix}.\begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}=\begin{bmatrix} 7.26+(-12).15 & 7.45+(-12).26 \\ -4.26+7.15 & -4.45+7.26 \end{bmatrix}=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Mustaqil yechish uchun misollar:

Berilgan matritsalar ustida talab qilingan amallani bajaring.

2.3. $A=\begin{bmatrix} 1 & 5 \\ 2 & -4 \end{bmatrix}$ $B=\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ *2A-B*=?

2.4. $A=\begin{bmatrix} 1 & -1 & -3 \\ 2 & 1 & 5 \end{bmatrix}$ $B=\begin{bmatrix} 0 & 3 & 2 \\ -1 & 4 & 1 \end{bmatrix}$ *3A-2B*=?

2.5. $\begin{bmatrix} 7 & 0 \\ 3 & 1 \\ -1 & 2 \end{bmatrix}-3\begin{bmatrix} 2 & \sqrt{2} \\ 1 & -1 \\ -1 & 0 \end{bmatrix}$+$\begin{bmatrix} 1 & \sqrt{18} \\ 4 & -5 \\ 3 & 1 \end{bmatrix}$

2.6. $C=(1\ \ 2\ \ 3)$, $F=\begin{bmatrix}4 & -3\\ 1 & 2\\ 0 & 2\end{bmatrix}$ $C*F=?$

2.7. $A=\begin{bmatrix}2 & 1 & -1\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}$, $B=\begin{bmatrix}1 & -1\\ 0 & 1\\ 1 & 0\end{bmatrix}$ $A*B=?$

2.8. $A=\begin{bmatrix}1 & -1 & 2\\ 2 & 3 & 4\\ -4 & 5 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & 4 & 1\\ 0 & 2 & 5\\ 1 & -1 & 4\end{bmatrix}$ $A*B=?$

2.9. $A=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$, $A^2=?$

2.10. $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 1\\ 4 & 1 & 1\end{bmatrix}$, E-birlik matritsa $2A^2+3A+5E=?$

2.11. $A=\begin{bmatrix}3 & 4 & 2\\ 1 & 0 & 5\end{bmatrix}$, $B=\begin{bmatrix}2 & 0\\ 1 & 3\\ 0 & 5\end{bmatrix}$, $C=\begin{bmatrix}1 & 3\\ 0 & 4\end{bmatrix}$ $A*B-C^2=?$

2.12. $A=\begin{bmatrix}1 & 2 & -3\\ 1 & 0 & 2\\ 4 & 5 & 3\end{bmatrix}$, $B=\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$, $C=(2\ \ 0\ \ 5)$, E- birlik matritsa $A*B*C-3E=?$

2.13. $A=\begin{pmatrix}2 & 0 & 1\\ -2 & 3 & 2\\ 4 & -1 & 5\end{pmatrix}$, $B=\begin{pmatrix}-3 & 1 & 0\\ 0 & 2 & 1\\ 0 & -1 & 3\end{pmatrix}$ $A*B=?$

2.14. $\begin{pmatrix}1 & -3 & 2\\ 3 & -4 & 1\\ 2 & -5 & 3\end{pmatrix}*\begin{pmatrix}2 & 5 & 6\\ 1 & 2 & 5\\ 1 & 3 & 2\end{pmatrix}=?$

2.15. $\begin{pmatrix}2 & -1 & 3 & -4\\ 3 & -2 & 4 & -3\\ 5 & -3 & -2 & 1\\ 3 & -3 & -1 & 2\end{pmatrix}*\begin{pmatrix}7 & 8 & 6 & 9\\ 5 & 7 & 4 & 5\\ 3 & 4 & 5 & 6\\ 2 & 1 & 1 & 2\end{pmatrix}=?$

2.16. $\begin{pmatrix}5 & 7 & -3 & -4\\ 7 & 6 & -4 & -5\\ 6 & 4 & -3 & -2\\ 8 & 5 & -6 & -1\end{pmatrix}*\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 5\\ 1 & 3 & 5 & 7\\ 2 & 4 & 6 & 8\end{pmatrix}=?$

Matritsalar ustida amallarni bajaring:

2.17. $A=\begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ $2A+5B=?$

2.18. $A=\begin{pmatrix} 3 & 5 & 7 \\ 2 & -1 & 0 \\ 4 & 3 & 2 \end{pmatrix}$ $B=\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix}$ $A+B=?$

2.19. $A=\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 5 \end{pmatrix}$ $C=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $A*C=?$

2.20. $A=\begin{pmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$ $F=\begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 0 \end{pmatrix}$ $A*F=?$

2.21. $A=\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ $B=\begin{pmatrix} 5 & 7 \\ -1 & 2 \end{pmatrix}$ $A^2\text{-}A*B+2BA=?$

2.22. $A=\begin{pmatrix} 1 & -3 & 0 \\ 2 & 5 & 1 \end{pmatrix}$ $B=\begin{pmatrix} 0 & -1 & 3 \\ 3 & 5 & 2 \\ 4 & -2 & 1 \end{pmatrix}$ $A*B=?$

2.23. $A=\begin{pmatrix} 1 & 3 & 1 \\ 2 & 0 & 4 \\ 1 & 2 & 3 \end{pmatrix}$ $B=\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ $A*B=?$ $B*A=?$

2.24. $A=\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ $A^2+A+E=?$

3. DETERMINANT XOSSALARI. MINOR VA ALGEBRAIK TO'LDIRUVCHILARGA DOIR MISOLLAR

Mustaqil yechish uchun misollar:

3.1. a) $\begin{vmatrix} 5 & 7 & -1 \\ 2 & 3 & 4 \\ 6 & 1 & 9 \end{vmatrix}$, det, A_{32} ni toping.

b) $\Delta = \begin{vmatrix} 7 & -3 & 0 & 4 \\ 2 & 1 & 1 & 5 \\ 3 & 6 & -1 & -3 \\ 8 & 1 & 1 & 1 \end{vmatrix}$ da A_{41} ni toping.

Determinantlar xossalaridan foydalanib, nollar yig'ib hisoblang:

3.2. $\begin{vmatrix} 7 & -2 & 3 \\ 0 & 0 & 1 \\ 2 & 1 & -4 \end{vmatrix}$ 3.3. $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & -1 \end{vmatrix}$

3.4. $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$ 3.5. $\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}$

3.6. $\begin{vmatrix} \sin^2 a & 1 & \cos^2 a \\ \sin^2 b & 1 & \cos^2 b \\ \sin^2 y & 1 & \cos^2 y \end{vmatrix}$ 3.7. $\begin{vmatrix} \sin^2 a & \cos 2a & \cos^2 a \\ \sin^2 b & \cos 2b & \cos^2 b \\ \sin^2 y & \cos 2y & \cos^2 y \end{vmatrix}$

3.8. $\begin{vmatrix} x & x & ax+bx \\ y & y & ay+by \\ z & z & az+bz \end{vmatrix}$ 3.9. $\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix}$

3.18. $A=\begin{vmatrix} 7 & 5 \\ 3 & 4 \end{vmatrix}$ $B=\begin{vmatrix} 2 & 9 \\ 1 & 7 \end{vmatrix}$ $A \cdot B=?$

3.19. $\begin{vmatrix} 7 & 0 & 0 \\ -8 & 1 & -1 \\ 3 & 6 & -4 \end{vmatrix}$ 3.20. $\begin{vmatrix} 4 & -1 & 1 \\ 1 & 2 & 1 \\ -3 & 1 & -2 \end{vmatrix}$

3.21. $-0{,}125\begin{vmatrix} -1/13 & 2/13 & 0 \\ -3 & 5 & 1 \\ 26 & 26 & 26 \end{vmatrix}$ 3.22. $\begin{vmatrix} 1 & 2 & -3 & 1 \\ 3 & 0 & 1 & -1 \\ 2 & 0 & 4 & 1 \\ 5 & 1 & 2 & 1 \end{vmatrix}$

3.23. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{vmatrix}$ 3.24. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$

3.25. $\begin{vmatrix} 0 & 6 & 3 & 5 & 1 \\ -3 & 2 & 4 & 1 & 0 \\ 5 & 1 & 4 & 3 & 2 \\ -3 & 8 & 7 & 6 & 1 \\ 1 & 0 & 3 & 4 & 0 \end{vmatrix}$ 3.26. $\begin{vmatrix} 2 & 3 & 4 \\ 2 & a+3 & b+4 \\ 2 & c+3 & d+4 \end{vmatrix}$

3.27. $\begin{vmatrix} 1 & -3 & -5 \\ 4 & 2 & 1 \\ 7 & 6 & -6 \end{vmatrix} + \begin{vmatrix} 1 & 3 & -5 \\ 4 & -2 & 1 \\ 7 & 6 & -6 \end{vmatrix}$ 3.28. $\begin{vmatrix} 7 & 8 \\ 5 & 6 \end{vmatrix} * \begin{vmatrix} 9 & 8 \\ 7 & 6 \end{vmatrix}$

4. MATRITSA RANGINI HISOBLASH.

TESKARI MATRITSANI TOPISH

1. $A=\begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$ (1)

A matritsaning rangi deb noldan farqli minorlarning eng yuqori tartibiga aytiladi va *rang(A)* kabi ifodalanadi.

Matritsa rangi ikki usulda topiladi:

1. Matritsa rangi ta'rifga asoslangan "minorlar ajratish" usuli;
2. Matritsa ustun va satrlarida nollar yig'ib hisoblashga asoslangan "Gauss algoritmi".

Misol 1. Matritsa rangini hisoblang:

$A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$ *A* matritsa 3x5 tartibli, demak uning rangi 3 dan yuqori bo'lmaydi.

Uchinchi tartibli minorlarni hisoblaymiz:

$M_1=\begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ 2 & -1 & 1 \end{vmatrix}=-4-10-12+12+4+10=0$ $M_2=\begin{vmatrix} 2 & -1 & -2 \\ 4 & -2 & 1 \\ 2 & -1 & 8 \end{vmatrix}=-32-2+8-8+32+2=0$

$M_3=\begin{vmatrix} 2 & -1 & 4 \\ 4 & -2 & 7 \\ 2 & -1 & 2 \end{vmatrix}=-8-14-16+16+8-14=0$ $M_4=\begin{vmatrix} -1 & 3 & -2 \\ -2 & 5 & 1 \\ -1 & 1 & 8 \end{vmatrix}=-40-3+4-10+48+1=0$

$M_5=\begin{vmatrix} 3 & -2 & 4 \\ 5 & 1 & 7 \\ 1 & 8 & 2 \end{vmatrix}=6-14+160-4+20-168=0$

Barcha uchinchi tartibli minorlar nolga teng. Ikkinchi tartibli minorlarni hisoblaymiz:

$M_1^1=\begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix}=-5+6=1$ $M_1^1 \neq 0$ $r(A)=2$

Bu usulda noldan farqli minor topilgunga qadar hisoblashlar davom etadi. Shuning uchun tartibi kattaroq matritsa rangini hisoblash bir muncha qiyinchiliklarga olib keladi.

Misol 2. Matritsa rangini elementar almashtirishlar yordamida nollar yig'ib hisoblaymiz:

$$A=\begin{pmatrix}25 & 31 & 17 & 43\\ 75 & 94 & 53 & 132\\ 75 & 94 & 54 & 134\\ 25 & 32 & 20 & 48\end{pmatrix} \sim \begin{pmatrix}25 & 31 & 17 & 43\\ 0 & 1 & 2 & 3\\ 0 & 1 & 3 & 5\\ 0 & 1 & 3 & 5\end{pmatrix} \sim \begin{pmatrix}25 & 31 & 17 & 43\\ 0 & 1 & 2 & 3\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0\end{pmatrix}$$

bu matritsaning rangi $\begin{pmatrix}31 & 17 & 43\\ 1 & 2 & 3\\ 0 & 1 & 2\end{pmatrix}$ matritsa rangiga teng.

$$\begin{vmatrix}31 & 17 & 43\\ 1 & 2 & 3\\ 0 & 1 & 2\end{vmatrix}=40\neq 0 \qquad r\begin{pmatrix}31 & 17 & 43\\ 1 & 2 & 3\\ 0 & 1 & 2\end{pmatrix}=3.$$

Demak, berilgan matritsaning rangi ham 3 ga teng. $r(A)=3$

(1) ko`rinishdagi A matritsa uchun teskari matritsa 2 usulda topiladi:

1. Klassik usuli;
2. Jordan usuli.

Misol 3. $A=\begin{pmatrix}2 & 3 & 2\\ 5 & 1 & 4\\ 1 & -2 & -1\end{pmatrix}$ matritsa uchun teskari A^{-1} matritsani klassik usulda toping.

Klassik usulda teskari matritsa $A^{-1}=\frac{1}{|A|}\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}$ (2)

formula bo'yicha hisoblanadi. Bu yerda $|A|$ berilgan matritsa determinanti. A_{ij}(i=1, 2, 3; j=1, 2, 3) transponirlangan matritsaning algebraik to'ldiruvchilari.

$|A|=\begin{vmatrix}2 & 3 & 2\\ 5 & 1 & 4\\ 1 & -2 & -1\end{vmatrix}$=-2+12-20-2+15+16=43-24=19≠0. Demak, A matritsa maxsusmas matritsa.

A^{-1} teskari matritsa mavjud. Algebraik to'ldiruvchilarini hisoblaymiz:

$A_{11}=\begin{vmatrix}1 & 4\\ -2 & -1\end{vmatrix}$=-1+8=7 $\qquad$ $A_{21}=-\begin{vmatrix}3 & 2\\ -2 & -1\end{vmatrix}$=-(-3+4)=-1

$A_{31}=\begin{vmatrix}3 & 2\\ 1 & 4\end{vmatrix}$=12-2=10 $\qquad$ $A_{12}=-\begin{vmatrix}5 & 4\\ 1 & -1\end{vmatrix}$=-(-5-4)=9

$A_{22}=\begin{vmatrix}2 & 2\\ 1 & -1\end{vmatrix}$=-2-2=-4 $\qquad$ $A_{32}=-\begin{vmatrix}2 & 2\\ 5 & 4\end{vmatrix}$=-(8-10)=2

$A_{13}=\begin{vmatrix}5 & 1\\ 1 & -2\end{vmatrix}$=-10-1=-11 $\qquad$ $A_{23}=-\begin{vmatrix}2 & 3\\ 1 & -2\end{vmatrix}$=-(-4-3)=7

$A_{33}=\begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix}=2-15=-13$

A_{ij} larni (2) formulaga qo'yamiz:

$A^{-1}=1/19\begin{pmatrix} 7 & -1 & 10 \\ 9 & -4 & 2 \\ -11 & 7 & -13 \end{pmatrix}$ teskari matritsaning to'g'ri topilganini

$AA^{-1}=E$ (3)

formula bo'yicha tekshiramiz:

$$\begin{pmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix}*1/19\begin{pmatrix} 7 & -1 & 10 \\ 9 & -4 & 2 \\ -11 & 7 & -13 \end{pmatrix}=1/19*\begin{pmatrix} 14+27-22 & -2-12+14 & 20+6-26 \\ 35+9-44 & -5-4+28 & 50+2-52 \\ 7-18+11 & -1+8-7 & 10-4+13 \end{pmatrix}==1/19$$

$$*\begin{pmatrix} 19 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & 19 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}=E$$

Demak, A^{-1} to`g`ri topilgan.

Misol 4. $A=\begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & -3 \\ 4 & 3 & -2 \end{pmatrix}$

$|A|=16\neq 0$ teskari matritsa mavjud. Teskari matritsani Jordan usulida topamiz. Berilgan matritsani birlik matritsa hisobida kengaytirib, elementar almashtirishlar bajaramiz, bu usulni to chap tomonda A matritsa o'rnida birlik matritsa hosil bo'lguncha davom ettiramiz, o'ng tomonda hosil bo'lgan matritsa berilgan matritsaga nisbatan teskari matritsa bo'ladi.

$(A|E)\sim(E|A^{-1})$ - Jordan usuli algaritmi.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 1 \\ -1 & -1 & -3 & 0 & 1 & 0 \\ 4 & 3 & -2 & 0 & 0 & 1 \end{array}\right)\sim\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & -5 & -6 & -4 & 0 & 1 \end{array}\right)\sim\left(\begin{array}{cc|cccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & -16 & 1 & 5 & 1 \end{array}\right)$$

$$\sim\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right)\sim\left(\begin{array}{ccc|ccc} 1 & 0 & 5 & -1 & -2 & 0 \\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -11/16 & -7/16 & 5/16 \\ 0 & 1 & 0 & 14/16 & 6/16 & -2/16 \\ 0 & 0 & 1 & -1/16 & -5/16 & -1/16 \end{array}\right) \quad A^{-1}=1/16\begin{pmatrix} -11 & -7 & 5 \\ 14 & 6 & -2 \\ -1 & -5 & -1 \end{pmatrix}$$

teskari matritsa to'g'ri topilganini (3) formulaga qo'yib tekshiramiz:

$$AA^{-1}=1/16\begin{pmatrix}1 & 2 & 1\\ -1 & -1 & -3\\ 4 & 3 & -2\end{pmatrix}*\begin{pmatrix}-11 & -7 & -5\\ 14 & 6 & -2\\ -1 & -5 & -1\end{pmatrix}=$$

$$=1/16\begin{pmatrix}-11+28-1 & -7+12-5 & 5-4-1\\ 11-14+3 & 7-6+15 & -5+2+3\\ -44+42+2 & -28+18+10 & 20-6+2\end{pmatrix}=$$

$$=1/16\begin{pmatrix}16 & 0 & 0\\ 0 & 16 & 0\\ 0 & 0 & 16\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$ demak, teskari matritsa to'gri topilgan.

Mustaqil yechish uchun misollar:

Berilgan kvadrat matritsaning determinantlari, normalari va ranglari topilsin:

4.1. a) $A=\begin{pmatrix}1 & 2\\ -2 & 0\end{pmatrix}$ b) $A=\begin{pmatrix}-1 & 0 & 8\\ 5 & 9 & 0\\ 0 & 4 & 3\end{pmatrix}$

c) $A=\begin{pmatrix}2 & 1 & 2\\ 1 & 1 & 1\\ 2 & 3 & 2\end{pmatrix}$ d) $A=\begin{pmatrix}2 & 3 & 4 & 0\\ 1 & 5 & 7 & 0\\ 3 & 1 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$

Quyidagi matritsalar rangini minorlar ajratish usuli bilan hisoblang:

4.2. $A=\begin{pmatrix}2 & -4 & 3 & 1 & 0\\ 1 & -2 & 1 & -4 & 2\\ 0 & 1 & -1 & 3 & 1\\ 4 & -7 & 4 & -4 & 5\end{pmatrix}$ 4.3. $A=\begin{pmatrix}2 & 1 & -2 & 3\\ -2 & 9 & -4 & 7\\ -4 & 3 & 1 & -1\end{pmatrix}$

4.4. $A=\begin{pmatrix}3 & 5 & 7\\ 1 & 2 & 3\\ 1 & 3 & 5\end{pmatrix}$ 4.5. $A=\begin{pmatrix}0 & 2 & -4\\ -1 & -4 & 5\\ 3 & 1 & 7\\ 0 & 5 & -10\\ 2 & 3 & 0\end{pmatrix}$

4.6. $A=\begin{pmatrix}2 & -1 & 3 & -2 & 4\\ 4 & -2 & 5 & 1 & 7\\ 2 & -1 & 1 & 8 & 2\end{pmatrix}$ 4.7. $A=\begin{pmatrix}1 & 2 & 1 & 3\\ 4 & -1 & -5 & -6\\ 1 & -3 & -4 & -7\\ 2 & 1 & -1 & 0\end{pmatrix}$

Quyidagi matritsalar rangini elementar almashtirish usuli bilan hisoblang:

4.8. $\begin{pmatrix} 1 & 2 & 1 & 3 & 4 \\ 3 & 4 & 2 & 6 & 8 \\ 1 & 2 & 1 & 8 & 4 \end{pmatrix}$

4.9. $\begin{pmatrix} 1 & 7 & 5 & 8 & 9 & 2 \\ 3 & 21 & 15 & 24 & 27 & 6 \\ 2 & 14 & 10 & 16 & 18 & 4 \end{pmatrix}$

4.10. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$

4.11. $\begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \end{pmatrix}$

4.12. $\begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}$

4.13. $\begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix}$

4.14. $\begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 42 & 13 & 29 & -55 & -68 \end{pmatrix}$

4.15. $\begin{pmatrix} 47 & -67 & 35 & 201 & 155 \\ 26 & 98 & 23 & -294 & 6 \\ 16 & -428 & 1 & 1284 & 52 \end{pmatrix}$

Berilgan kvadrat matritsalar uchun teskari matritsani ikki usulda toping:

4.16. $\begin{pmatrix} -1 & 1 \\ 4 & 2 \end{pmatrix}$

4.17. $\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$

4.18. $\begin{pmatrix} tg a & 1 \\ 2 & ctg a \end{pmatrix}$

4.19. $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 2 \end{pmatrix}$

4.20. $\begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}$

4.21. $\begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$

4.22. $\begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}$

Quyidagi matritsali tenglamalarni eching:

4.23. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$

4.24. $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$

5. CHIZIQLI TENGLAMALAR SISTEMASINING YECHIMI HAQIDA KRONEKER - KAPELLI TEOREMASI

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1m}x_m = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2m}x_m = b_2 \\ .... \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nm}x_m = b_n \end{cases} \qquad (1)$$

Kroneker-Kapelli teoremasi (1) chiziqli tenglamalar sistemasining birgalikda yoki birgalikda emasligini aniqlaydi.

(1) chiziqli tenglamalar sistemasining asosiy va ozod hadlar hisobiga kengaytirilgan matritsasini tuzamiz:

$$A=\begin{pmatrix} a_{11} & a_{12} & ... & a_{1m} \\ a_{21} & a_{22} & ... & a_{2m} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nm} \end{pmatrix} \qquad (2)$$

$$B=\begin{pmatrix} a_{11} & a_{12} & ... & a_{1m} & |b_1 \\ a_{21} & a_{22} & ... & a_{2m} & |b_2 \\ ... & ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nm} & |b_n \end{pmatrix} \qquad (3)$$

Teorema. Agar A matritsa rangi B matritsa rangiga teng bo'lib, noma'lumlar soniga ham teng bo'lsa, ya'ni $r(A)=r(B)=m$ bo'lsa, (1) tenglamalar sistemasi aniq bo'ladi, sistema birgalikda bo'lib yagona yechimga ega bo'ladi.

Agar $r(A)=r(B)<m$ bo'lsa, (1) sistema birgalikda bo'lib, cheksiz ko'p yechimga ega bo'ladi.

Agar $r(A)<r(B)$ bo'lsa, sistema birgalikda bo'lmaydi, sistema yechimga ega bo'lmaydi.

Misollar ko'ramiz: 5.1. Quyidagi sistemalarni birgalikda yoki birgalikda emasligini tekshiramiz:

$$a) \begin{cases} x_1 + 3x_2 + 5x_3 + 7x_4 + 9x_5 = 1 \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 5x_5 = 2 \\ 2x_1 + 11x_2 + 12x_3 + 25x_4 + 22x_5 = 4 \end{cases}$$

Buning uchun asosiy va kengaytirilgan matritsa rangini topamiz:

$$A=\begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 1 & -2 & 3 & -4 & 5 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 3 & 9 & 15 & 21 & 27 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 1 & 3 & 5 & 7 & 9 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix}$$

2- satr elementlaridan 1- satr elementlarini ayiramiz:

$$A \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix} \qquad r(A)=2$$

$$B=\left(\begin{array}{ccccc|c} 1 & 3 & 5 & 7 & 9 & 1 \\ 1 & -2 & 3 & -4 & 5 & 2 \\ 2 & 11 & 12 & 25 & 22 & 4 \end{array}\right)$$

bu matritsa rangini topish uchun yana yuqoridagi ishni takrorlaymiz, natijada B matritsa quyidagi ko'rinishni oladi.

$$B \sim \left(\begin{array}{ccccc|c} 1 & 3 & 5 & 7 & 9 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 11 & 12 & 25 & 22 & 4 \end{array}\right), \qquad B_1 = \begin{pmatrix} 7 & 9 & 1 \\ 0 & 0 & 1 \\ 25 & 22 & 4 \end{pmatrix}$$

matritsa rangini topamiz:

$$M = |B_1| = \begin{vmatrix} 7 & 9 & 1 \\ 0 & 0 & 1 \\ 25 & 22 & 4 \end{vmatrix} = 225-154=71; \qquad r(B_1) = 3$$

Demak, $r(B)=3$ bo'lib, $r(A)\neq r(B)$ va sistema birgalikda emas.

$$\text{b)} \quad \begin{cases} x_1 + 5x_2 + 4x_3 + 3x_4 = 1 \\ 2x_1 - x_2 + 2x_3 - x_4 = 0 \\ 5x_1 + 3x_2 + 8x_3 + x_4 = 1 \end{cases}$$

Sistema birgalikda yoki birgalikda emasligini tekshiring.

Ozod hadlar hisobiga kengaytirilgan matritsa tuzamiz:

$$B = \left(\begin{array}{cccc|c} 1 & 5 & 4 & 3 & 1 \\ 2 & -1 & 2 & -1 & 0 \\ 5 & 3 & 8 & 1 & 1 \end{array}\right)$$

3- satr elementlaridan 1- satr elementlarini ayiramiz:

$$B = \left(\begin{array}{cccc|c} 1 & 5 & 4 & 3 & 1 \\ 2 & -1 & 2 & -1 & 0 \\ 4 & -2 & 4 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 5 & 4 & 3 & 1 \\ 2 & -1 & 2 & -1 & 0 \\ 2 & -1 & 2 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 5 & 4 & 3 & 1 \\ 2 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$r(A)=r(B)=2$ ekanini ko'rish mumkin. Demak, sistema birgalikda.

Mustaqil yechish uchun misollar:

Berilgan chiziqli tenglamalar sistemalarining birgalikda yoki birgalikda emasligini tekshiring.

5.2. $\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + x_2 + 2x_3 = 1 \\ x_1 + x_2 + 3x_3 = 2 \end{cases}$

5.3. $\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + x_2 + 2x_3 = 1 \\ 2x_1 + 2x_2 + 4x_3 = 1 \end{cases}$

5.4. $\begin{cases} 2x + 3y + 2z = 9 \\ x + 2y - 3z = 14 \\ 3x + 4y + z = 16 \end{cases}$

5.5. $\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ x_1 + 4x_2 - 3x_3 = 7 \end{cases}$

5.6. $\begin{cases} 2x_1 - x_2 = 3 \\ 3x_1 - 5x_2 = 1 \\ 4x_1 - 7x_2 = 1 \end{cases}$

5.7. $\begin{cases} x_1 + 3x_2 + 5x_3 - 2x_4 = 3 \\ x_1 + 4x_2 - 2x_3 + 3x_4 = 2 \\ -x_1 - 2x_2 - 12x_3 - 7x_4 = -4 \\ 3x_1 + 11x_2 + x_3 - 4x_4 = 7 \end{cases}$

5.8. $\begin{cases} 3x_1 + 2x_2 = 4 \\ x_1 - 4x_2 = -1 \\ 7x_1 + 10x_2 = 12 \\ 5x_1 + 6x_2 = 8 \\ 3x_1 - 16x_2 = -5 \end{cases}$

5.9. $\begin{cases} x_1 + 5x_2 + 4x_3 = 1 \\ 2x_1 + 10x_2 + 8x_3 = 3 \\ 3x_1 + 15x_2 + 12x_3 = 5 \end{cases}$

5.10. $\begin{cases} x_1 - 3x_2 + 2x_3 = -1 \\ x_1 + 9x_2 + 6x_3 = 3 \\ x_1 + 3x_2 + 4x_3 = 1 \end{cases}$

5.11. $\begin{cases} x_1 + 2x_2 - 4x_3 = 18 \\ 3x_1 - x_2 + 4x_3 = 4 \\ 2x_1 + x_2 + 5x_3 = 0 \end{cases}$

5.12. $\begin{cases} 2x_1 + x_2 + 3x_3 = 5 \\ x_1 - 3x_2 + 5x_3 = 4 \\ 2x_1 + x_2 + 5x_3 = 0 \end{cases}$

5.13. $\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + 4x_2 - 3x_3 = 2 \end{cases}$

5.14. $\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + 4x_2 - 3x_3 = 5 \end{cases}$

5.15 $\begin{cases} x_1 - 3x_2 + 5x_3 + 7x_4 + 9x_5 = 1 \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 5x_5 = 2 \\ 2x_1 + 11x_2 + 12x_3 + 25x_4 + 22x_5 = 4 \end{cases}$

5.16. $\begin{cases} x_1 + 2x_2 + 3x_3 = 14 \\ 3x_1 + 2x_2 + x_3 = 10 \\ x_1 + x_2 + x_3 = 6 \\ 2x_1 + 3x_2 - x_3 = 5 \\ x_1 + x_2 = 3 \end{cases}$

5.17. $\begin{cases} 3x_1 + 2x_2 = 4 \\ x_1 - 4x_2 = -1 \\ 7x_1 + 10x_2 = 12 \\ 5x_1 + 6x_2 = 8 \\ 3x_1 - 16x_2 = -5 \end{cases}$

5.18. $\begin{cases} 3x_1 - 5x_2 + 2x_3 + 4x_4 = 2 \\ 7x_1 - 4x_2 + x_3 + 3x_4 = 5 \\ 5x_1 + 7x_2 - 4x_3 - 6x_4 = 3 \end{cases}$

6. CHIZIQLI TENGLAMALAR SISTEMASINI KRAMER HAMDA TESKARI MATRITSA USULI BILAN YECHISH

1. Chiziqli tenglamalar sistemasini yechishning Kramer formulasi determinantlardan foydalanib sistema yechimini topishdir.

Sistema yechimi Kramer formulalari deb atalgan quyidagi formulalar bo'yicha topiladi:

$$x_1 = \frac{\Delta_1}{\Delta}, \qquad x_2 = \frac{\Delta_2}{\Delta}, \qquad x_3 = \frac{\Delta_3}{\Delta}, \ \dots, \ x_n = \frac{\Delta_n}{\Delta}.$$

Bu yerda Δ noma'lumlar oldidagi koeffitsiyentlardan tuzilgan kvadrat matritsa determinanti, Δ_1, Δ_2, Δ_3, …, Δ_n lar asosiy matritsada mos ravishda 1, 2, 3, …, n-ustun elementlarini ozod hadlar bilan almashtirishdan hosil bo'lgan determinantlar. Shuni ta'kidlash kerakki, sistemada noma'lumlar va tenglamalar soni teng bo'lgan hollarda Kramer formulasini qo'llash maqsadga muvofiq.

Agar $\Delta \neq 0$ bo'lsa, sistema yagona yechimga ega bo'ladi.

Agar $\Delta=0$ bo'lib, Δ_1, Δ_2, Δ_3 lardan kamida bittasi noldan farqli bo'lsa sistema yechimga ega emas.

Agar $\Delta=0$ bo'lib, $\Delta_1=\Delta_2=\Delta_3=\ldots=\Delta_n=0$ bo'lsa, sistema aniqmas, cheksiz ko'p yechimga ega bo'ladi. Formulani 3 noma'lumli 3 ta chiziqli tenglamalar sistemasi misolida keltiramiz:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \qquad (1)$$

sistema uchun

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \qquad \Delta_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \qquad \Delta_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

Buni misollarda ko'ramiz: 6.1-misol.

a) $\begin{cases} x_1 + 2x_2 + x_3 = 8 \\ 3x_1 + 2x_2 + x_3 = 10 \\ 4x_1 + 3x_2 - 2x_3 = 4 \end{cases}$ sistemani Kramer formulasi bilan yeching.

$$\Delta=\begin{vmatrix}1 & 2 & 1\\ 3 & 2 & 1\\ 4 & 3 & -2\end{vmatrix}$$ = -4+8+9-8-3+12=14

Δ≠0 bo'lgani uchun sistema aniq yagona yechim Kramer formulalari yordamida topiladi.

$$\Delta_1=\begin{vmatrix}8 & 2 & 1\\ 10 & 2 & 1\\ 4 & 3 & -2\end{vmatrix}$$ = -32+8+30-8+40-24=14

$$\Delta_2=\begin{vmatrix}1 & 8 & 1\\ 3 & 10 & 1\\ 4 & 4 & -2\end{vmatrix}$$ = -20+32+12-40-4+48=28

$$\Delta_3=\begin{vmatrix}1 & 2 & 8\\ 3 & 2 & 10\\ 4 & 3 & 4\end{vmatrix}$$ = 8+80+72-64-24-30=42

$$x_1=\frac{14}{14}=1,\quad x_2=\frac{28}{14}=2,\quad x_3=\frac{42}{14}=3. \qquad (1;2;3)$$

b) $\begin{cases}4x_1+2x_2+3x_3=-2\\ 3x_1+8x_2-\ x_3=\ 8\\ 9x_1+\ x_2+8x_3=\ 0\end{cases}$ sistemani Kramer formulasi yordamida yeching.

$$\Delta=\begin{vmatrix}4 & 2 & 3\\ 2 & 8 & -1\\ 9 & 1 & 8\end{vmatrix}$$ =256+6-18-216-32+4=266-266=0

Δ=0 Kramer teoremasiga ko'ra, sistema yoki aniqmas, yoki birgalikdamas. Δ_1 ni hisoblaymiz:

$$\Delta_1=\begin{vmatrix}-2 & 2 & 3\\ 8 & 8 & -1\\ 0 & 1 & 8\end{vmatrix}$$ = -128+24-128-2= -234≠0

Δ=0, Δ_1≠0 bo'lgani uchun Kramer teoremasiga ko'ra sistema aniqlanmagan.

c) $\begin{cases}-2x_1+\ x_2-\ x_3=\ 7\\ 4x_1+5x_2-3x_3=-5\\ x_1+3x_2-2x_3=\ 1\end{cases}$ Kramer formulasiga ko'ra yeching.

$$\Delta=\begin{vmatrix}-2 & 1 & -1\\ 4 & 5 & -3\\ 1 & 3 & -2\end{vmatrix}$$ = 20-3-12+5+8-18=33-33=0

$\Delta=0$, demak sistema yoki aniqmas, yoki birgalikdamas. Δ_1, Δ_2, Δ_3 larni hisoblaymiz:

$$\Delta_1=\begin{vmatrix}7 & 1 & -1\\ -5 & 5 & -3\\ 1 & 3 & -2\end{vmatrix} = -70+15-3+5-10+63=83-83=0$$

$$\Delta_2=\begin{vmatrix}-2 & 7 & -1\\ 4 & -5 & -3\\ 1 & 1 & -2\end{vmatrix} = -20-21-4-5+56-6=56-56=0$$

$$\Delta_3=\begin{vmatrix}-2 & 1 & 7\\ 4 & 5 & -5\\ 1 & 3 & 1\end{vmatrix} = -10-5+84-35-4-30=84-84=0$$

$\Delta=0$, $\Delta_1=\Delta_2=\Delta_3=0$ bo'lgani uchun sistema aniqmas, cheksiz ko'p yechimga ega.

Sistemani Gauss algoritmi bilan yechamiz:

$$\left(\begin{array}{ccc|c}-2 & 1 & -1 & 7\\ 4 & 5 & -3 & -5\\ 1 & 3 & -2 & 1\end{array}\right)\sim\left(\begin{array}{ccc|c}-2 & 1 & -1 & 7\\ 0 & 7 & -5 & 9\\ 0 & \frac{7}{2} & -\frac{5}{2} & \frac{9}{2}\end{array}\right)\sim\left(\begin{array}{ccc|c}-2 & -1 & -1 & 7\\ 0 & 7 & -5 & 9\\ 0 & 0 & 0 & 0\end{array}\right)$$

berilgan tenglama $\begin{cases}-2x_1+\ x_2=\ x_3+7\\ 4x_1+5x_2=3x_3-5\\ \qquad x_3\in R\end{cases}$ sistemaga teng kuchli.

Bu tenglamani Kramer formulasi bilan yechish mumkin.

$$\Delta=\begin{vmatrix}-2 & 1\\ 4 & 5\end{vmatrix} = -10-4= -14$$

$$\Delta_1=\begin{vmatrix}x_3+7 & 1\\ 3x_3-5 & 5\end{vmatrix} =5(x_3+7)-3x_3+5=5x_3+35-3x_3+5=2x_3+40=2(x_3+20)$$

$$\Delta_2=\begin{vmatrix}-2 & x_3+7\\ 4 & 3x_3-5\end{vmatrix} = -2(3x_3-5)-4(x_3+7) = -6x_3+10-4x_3-28 =$$

$$= -10x_3-18 = -2(5x_3+9)$$

$$x_1=\frac{2(x_3+20)}{-14}=-\frac{x_3+20}{7},\qquad x_2=\frac{-2(5x_3+9)}{-14}=\frac{5x_3+9}{7}$$

Sistema yechimi $\left(-\frac{x_3+20}{7};\ \frac{5x_3+9}{7};\ x_3\right)$ bo'ladi.

2. Chiziqli tenglamalar sistemasini teskari matritsa usulida yechish. Berilgan (1) sistemani

$$AX=B \qquad (2)$$

matritsa ko'rinishida yozib olamiz.

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \qquad X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad B=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

(2) tenglamani har ikki tomonini chapdan A^{-1} teskari matritsaga ko'paytiramiz.

$A^{-1}\cdot AX = A^{-1}\cdot B,$ $\qquad A^{-1}\cdot A = E$ bo'lgani uchun

$$X = A^{-1}\cdot B \qquad (3)$$

tenglik hosil bo'ladi.

(3) formula bilan topilgan X ustun matritsa sistemaning yechimi bo'ladi.

1-misolni a)-sini shu usul bilan yechamiz:

$$\begin{cases} x_1+2x_2+x_3=8 \\ 3x_1+2x_2+x_3=10 \\ 4x_1+3x_2-2x_3=4 \end{cases}$$

$A=\begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 4 & 3 & -2 \end{pmatrix}$ matritsa uchun teskari matritsa mavjud, chunki

$\Delta = |A| = 14 \neq 0.$ $\qquad A^{-1}=\frac{1}{14}\begin{pmatrix} -7 & 7 & 0 \\ 10 & -6 & 2 \\ 1 & 5 & -4 \end{pmatrix}$

$$X = A^{-1}\cdot B = \frac{1}{14}\begin{pmatrix} -7 & 7 & 0 \\ 10 & -6 & 2 \\ 1 & 5 & -4 \end{pmatrix}\cdot\begin{pmatrix} 8 \\ 10 \\ 4 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} -56+70 \\ 80-60+8 \\ 8+50-16 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix};$$

Javob: $(1;2;3)$.

Mustaqil yechish uchun misollar:

Quyidagi tenglamalar sistemasini Kramer va teskari matritsa usulida yeching.

6.2. $\begin{cases} x+y-3z=-1 \\ 2x-y+z=2 \\ 3x+2y-4z=1 \end{cases}$ $\qquad$ 6.3. $\begin{cases} x_1+3x_2-4x_3=-1 \\ x_1-5x_2+x_3=7 \\ 2x_1+x_2-3x_3=3 \end{cases}$

6.4. $\begin{cases} 3x_1+x_2-x_3=2 \\ 2x_1-3x_2+x_3=-1 \\ x_1-x_2+2x_3=5 \end{cases}$ $\qquad$ 6.5. $\begin{cases} 2x_1+2x_2-x_3+x_4=4 \\ 4x_1+3x_2-x_3+2x_4=6 \\ 8x_1+5x_2-3x_3+4x_4=12 \\ 3x_1+3x_2-2x_3+2x_4=6 \end{cases}$

6.6. $\begin{cases} 2x_1 + 5x_2 + 4x_3 + x_4 = 20 \\ x_1 + 3x_2 + 2x_3 + x_4 = 11 \\ 2x_1 + 10x_2 + 9x_3 + 7x_4 = 40 \\ 3x_1 + 8x_2 + 9x_3 + 2x_4 = 37 \end{cases}$ 6.7. $\begin{cases} 2x - y - 6z + 3t + 1 = 0 \\ 7x - 4y + 2z - 15t + 32 = 0 \\ x - 2y - 4z + 9t - 5 = 0 \\ x - y + 2z - 6t + 8 = 0 \end{cases}$

6.8. $\begin{cases} 2x + y + 4z + 8t = -1 \\ x + 3y - 6z + 2t = 3 \\ 3x - 2y + 2z - 2t = 8 \\ 2x + y - 2z = 4 \end{cases}$ 6.9. $\begin{cases} 3x + 2y + z = 5 \\ x + y - z = 0 \\ 4x - y + 5z = 3 \end{cases}$

6.10. $\begin{cases} 2x_1 + x_2 - x_3 = 5 \\ 3x_1 - x_2 + 2x_3 = -5 \\ 7x_1 + x_2 - x_3 = 10 \end{cases}$ 6.11. $\begin{cases} 2x - 3y + z = 2 \\ x + 5y - 4z = -5 \\ 4x + y - 3z = -4 \end{cases}$

6.12. $\begin{cases} 2x - 4y + 3z = 1 \\ x - 2y + 4z = 3 \\ 3x - y + 5z = 2 \end{cases}$ 6.13. $\begin{cases} 2x - y + z = 2 \\ 3x + 2y + 2z = -2 \\ x - 2y + z = 1 \end{cases}$

6.14. $\begin{cases} x + 2y + 3z = 5 \\ 2x - y - z = 1 \\ x + 3y + 4z = 6 \end{cases}$ 6.15. $\begin{cases} 2x_1 - x_2 + x_3 = 4 \\ 3x_1 + 2x_2 - x_3 = 1 \\ x_1 + x_2 - 2x_3 = -3 \end{cases}$

7. CHIZIQLI TENGLAMALAR SISTEMASINI GAUSS VA GAUSS-JORDAN USULLARI BILAN YECHISH

1. Gaussning klassik usuli - bu berilgan sistemaning umumiy yechimini topishdan iborat bo'lib, bunda sistemaning tenglamalari ustida elementar almashtirishlar bajarib berilgan sistema trapetsiyali yoki uchburchakli ko'rinishga keltiriladi. So'ng oxirgi tenglamadan boshlab noma'lumlar ketma-ket topiladi.

7.1-misol. a) $\begin{cases} x_1 + 2x_2 - 4x_3 = -4 \\ 3x_1 - 2x_2 + x_3 = 11 \\ 4x_1 - 5x_2 + x_3 = 9 \end{cases} \sim \begin{cases} x_1 + 2x_2 - 4x_3 = -4 \\ -8x_2 + 13x_3 = 23 \\ -13x_2 + 17x_3 = 25 \end{cases} \sim \begin{cases} x_1 + 2x_2 - 4x_3 = -4 \\ -8x_2 + 13x_3 = 23 \\ -\frac{33}{8}x_3 = -\frac{99}{8} \end{cases}$

$x_3=3$, $x_2=2$, $x_1=4$ Javob: $(4;2;3)$.

2. Gauss-Jordan usuli noma'lumlarni ketma-ket yo'qotish Gauss usuli va teskari matritsa qurish Jordan algoritmiga asoslangan. Gauss-Jordan usuliga sxema ko'rinishida quyidagicha yoziladi: $(A|B) \sim (E|X)$.

$(A|B)$-asosiy matritsani ozod hadlar hisobiga kengaytirilgan matritsa.

E - birlik matritsa. X - tenglama yechimini ifodalovchi ustun matritsa.

b) $\begin{cases} x_1 + x_2 - 6x_3 - 4x_4 = 6 \\ 3x_1 - x_2 - 6x_3 - 4x_4 = 2 \\ 2x_1 + 3x_2 + 9x_3 + 2x_4 = 6 \\ 3x_1 + 2x_2 + 3x_3 + 8x_4 = -7 \end{cases}$

Sistemani Gauss-Jordan usuli bilan yeching.

$$\left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 3 & -1 & -6 & -4 & 2 \\ 2 & 3 & 9 & 2 & 6 \\ 3 & 2 & 3 & 8 & -7 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & -4 & 12 & 8 & -16 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & -1 & 21 & 20 & -25 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 1 & 21 & 10 & -6 \\ 0 & -1 & 21 & 20 & -25 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 0 & 24 & 12 & -10 \\ 0 & 0 & 18 & 18 & -21 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 0 & 1 & 1/2 & -5/12 \\ 0 & 0 & 18 & 18 & -21 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -3 & -2 & 2 \\ 0 & 1 & -3 & -2 & 4 \\ 0 & 0 & 1 & 1/2 & -5/12 \\ 0 & 0 & 0 & 9 & -27/2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1/2 & 3/4 \\ 0 & 1 & 0 & -1/2 & 11/4 \\ 0 & 0 & 1 & 1/2 & -5/12 \\ 0 & 0 & 0 & 1 & -3/2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1/3 \\ 0 & 0 & 0 & 1 & -3/2 \end{array}\right)$$

Javob: (0; 2; 1/3; -3/2).

c) $\begin{cases} x_1 - x_2 + x_3 = 1 \\ x_1 + x_2 - 2x_3 = 3 \\ 2x_1 + 2x_2 - 4x_3 = 6 \end{cases}$ Berilgan sistema birgalikda, chunki

$$r\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 2 & -4 \end{pmatrix} = r\left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 2 & -4 & 6 \end{array}\right)$$

Sistema cheksiz ko'p yechimga ega, umumiy yechimni Gauss-Jordan usuli yordamida topamiz:

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 2 & -4 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 2 & -3 & 2 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & -3/2 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 2 \\ 0 & 1 & -3/2 & 1 \end{array}\right)$$

$$\begin{cases} x_1 - \dfrac{1}{2}x_3 = 2 \\ x_2 - \dfrac{3}{2}x_3 = 1 \end{cases} \Rightarrow \begin{cases} x_1 = \dfrac{1}{2}x_3 + 2 \\ x_2 = \dfrac{3}{2}x_3 + 1 \end{cases}$$

Javob: $\left(\frac{1}{2}x_3+2;\ \frac{3}{2}x_3+1;\ x_3\right)$ $\quad x_3\in R$.

Mustaqil yechish uchun misollar:

Quyidagi tenglamalar sistemasini Gauss, Gauss-Jordan usuli bilan yeching:

7.2. $\begin{cases} x_1-x_2+3x_3=3\\ 2x_1+3x_2-4x_3=-1\\ 3x_1+2x_2-x_3=2\end{cases}$ 7.3. $\begin{cases} x_1+2x_2+3x_3=0\\ 2x_1-3x_2-x_3=1\\ 3x_1+x_2+4x_3=-1\end{cases}$

7.4. $\begin{cases} x_1-3x_2+2x_3+x_4=2\\ 2x_1+x_2+4x_3+3x_4=1\\ x_1+5x_2-x_3+x_4=-4\\ 3x_1-x_2+6x_3+5x_4=0\end{cases}$ 7.5. $\begin{cases} 2x_1+x_2+3x_3-4x_4=3\\ x_1-2x_2+x_3-3x_4=-1\\ 3x_1+4x_2-5x_3+x_4=4\\ 2x_1-4x_2+2x_3-6x_4=5\end{cases}$

7.6. $\begin{cases} x_1-2x_2+x_3=4\\ x_1+3x_2+x_3=0\end{cases}$ 7.7. $\begin{cases} 3x_1+x_2=0\\ -x_1+2x_2=5\\ 2x_1-4x_2=1\end{cases}$

7.8. $\begin{cases} 3x_1-2x_2-5x_3+x_4=3\\ 2x_1-3x_2+x_3+5x_4=-3\\ x_1+2x_2-x_3-4x_4=-1\\ x_1-x_2-4x_3+9x_4=22\end{cases}$ 7.9. $\begin{cases} x_1+x_2+x_3+x_4+x_5=15\\ x_1+2x_2+3x_3+4x_4+5x_5=35\\ x_1+3x_2+6x_3+10x_4+15x_5=70\\ x_1+4x_2+10x_3+20x_4+35x_5=126\\ x_1+5x_2+15x_3+35x_4+70x_5=210\end{cases}$

7.10. $\begin{cases} x+2y+3z=5\\ 2x-y-z=1\\ x+3y+4z=6\end{cases}$ 7.11. $\begin{cases} 2x_1-x_2+x_3-x_4=5\\ x_1+2x_2-2x_3+3x_4=-6\\ 3x_1+x_2-x_3+2x_4=-1\end{cases}$

7.12. $\begin{cases} x_1-2x_2-5x_3=1\\ 4x_1+x_2-2x_3=-3\\ -x_1+3x_2+7x_3=2\end{cases}$ 7.13. $\begin{cases} -x_1+2x_2-x_3=4\\ 3x_1+x_2-2x_3=1\\ 4x_1-x_2+x_3=-3\end{cases}$

7.14. $\begin{cases} x_1-3x_2=-5\\ -x_1+x_2=1\\ 4x_1-x_2=2\end{cases}$ 7.15. $\begin{cases} x_1-x_2-3x_3=6\\ -2x_1+2x_2+6x_3=-9\end{cases}$

7.16. $\begin{cases} x_1+2x_2+x_3=8\\ x_2+3x_3+x_4=15\\ 4x_1+x_3+x_4=11\\ x_1+x_2+5x_4=23\end{cases}$ 7.17. $\begin{cases} -x_1+x_2+x_3-x_4=-2\\ x_1+2x_2-2x_3-x_4=-5\\ 2x_1-x_2-3x_3+2x_4=-1\\ x_1+2x_2+3x_3-6x_4=-10\end{cases}$

8. n – O'LCHOVLI ARIFMETIK FAZO. VEKTORLAR SISTEMASI. VEKTORNI VEKTORLAR SISTEMASI BO'YICHA YOYISH

1. n – o'lchovli arifmetik fazo deb, mumkin bo'lgan n ta $x_1, x_2, ...x_n$ haqiqiy sonlarning tartiblangan tizimlari to'plamiga aytiladi va R_n kabi belgilanadi.

$x=(x_1, x_2, ...x_n)$ – R_n fazoning arifmetik vektori yoki nuqtasi deyiladi, $x_1, x_2, ...x_n$ haqiqiy sonlar x vektorning koordinatalari yoki komponentlari deyiladi. Komponentlar soni arifmetik vektor yoki nuqta o'lchovi hisoblanadi.

Oxyz koordinatalar sistemasida har qanday x vektorni $\overline{a} = a_x \overline{i} + a_y \overline{j} + a_z \overline{k}$ ko'rinishda yozish mumkin. Vektorning bu ko'rinishda yozilishi uning koordinata o'qlari bo'yicha yoyib yozishdir. a_x, a_y, a_z vektorning koordinata o'qlaridagi proyeksiyalari. i, j, k - birlik vektorlar.

a vetor moduli yoki uzunligi $|\overline{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ formula bo'yicha hisoblanadi.

$\vec{a}$ voktor yo`nalishi vektorning koordinata o'qlari *Ox, Oy, Oz* bilan hosil qilgan burchaklar bilan aniqlanadi, bu burchaklar kosinuslari:

$\cos\alpha = \dfrac{a_x}{|\overline{a}|}$, $\cos\beta = \dfrac{a_y}{|\overline{a}|}$, $\cos\gamma = \dfrac{a_z}{|\overline{a}|}$ formula bilan hisoblanadi, bunda

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

tenglik o'rinli bo'ladi.

Uchlari $A(x_1; y_1; z_1)$, $B(x_2; y_2; z_2)$ nuqtalar bilan berilgan $\overline{AB}$ vektor koordinatasi

$\overline{AB} = (x_2 - x_1; y_2 - y_1; z_2 - z_1)$

ga teng bo'ladi.

$$\text{Cos}\alpha = \frac{x_2 - x_1}{|\overline{AB}|}; \quad \cos\beta = \frac{y_2 - y_1}{|\overline{AB}|}; \quad \cos\gamma = \frac{z_2 - z_1}{|\overline{AB}|}$$

1-misol. *ABC* uchburchakda, *AN* to'g'ri chiziq *BAC* burchak bissektrissasi hisoblanadi, *N* nuqta *BC* tomonda yotadi. Agar $|\overline{AB}| = \overline{b}$, $|\overline{AC}| = \overline{c}$ bo'lsa, $\overline{AN}$ vektor uzunligini toping.

Δ *ABC* dan $\overline{BC} = \overline{c} - \overline{b}$ uchburchakdan ichki burchak bissektrissasining xossasiga ko'ra $BN:NC = b:c$ yoki $|BN|:|BC| = b:(b+c)$; $\dfrac{BN}{c-b} = \dfrac{b}{b+c}$, bundan $BN = \dfrac{b(\overline{c} - \overline{b})}{b+c}$

$\overline{AN} = \overline{AB} + \overline{BN}$ bo'lgani uchun $\overline{AN} = \overline{b} + \dfrac{b}{b+c}(\overline{c} - \overline{b}) = \dfrac{b\overline{c} + c\overline{b}}{b+c}$ hosil bo'ladi.

2- misol. $A(1; 3; 2)$, $B(5; 8; -1)$ nuqtalar berilgan bo'lsa $\overline{a} = \overline{AB}$ vektorni toping.

AB vektorning proyeksiyalari

$a_x=x_2-x_1=5-1=4;$ $a_y=y_2-y_1=8-3=5;$ $a_z=z_2-z_1=-1-2=-3$ formulalar bo'yicha hisoblanadi. Demak, $\overline{AB}=4\bar{i}+5\bar{j}-3\bar{k}$ ko'rinishda yoziladi.

n –o'lchovli arifmetik vektorlar ustida quyidagi chiziqli amallarni bajarish mumkin.

$\bar{x}=(x_1, x_2, ...x_n)$, $\bar{y}=(y_1; y_2;.....y_n)$ n- o'lchovli vektorlar va $\lambda>0$ haqiqiy son belirgan bo'lsin.

1) Vektorlarni qo'shish uchun mos koordinatalari qo'shiladi:

$$\bar{x}+\bar{y}=(x_1+y_1;\ x_2+y_2;\ ....;\ x_n+y_n)$$

2) $\bar{x}$ vektorni λ songa ko'paytirish uchun berilgan vektorning har bir koordinatasini λ soniga ko'paytiriladi:

$$\lambda\bar{x}=(\lambda x_1; \lambda x_2; ....; \lambda x_n)$$

3) $\bar{x}$; $\bar{y}$ vektorlarning skalyar ko'paytirish uchun mos koordinatalari ko'paytirilib, yig'indisi olinadi:

$$(x\cdot y)=x_1y_1+x_2y_2+\ldots+x_ny_n)$$

4) Vektorlar uzunliklari $|\bar{x}|=\sqrt{x_1^2+x_2^2+.....+x_n^2}$ formula bo'yicha topiladi.

5) $\bar{x}$; $\bar{y}$ vekorlar orasidagi burchak

$$\cos\varphi=\frac{(\bar{x};\bar{y})}{|\bar{x}||\bar{y}|}=\frac{x_1y_1+x_2y_2+......+x_ny_n}{\sqrt{x_1^2+x_2^2+....+x_n^2}\sqrt{y_1^2+y_2^2+......+y_n^2}}$$

formula bilan topiladi; $(\varphi\epsilon[0;\pi])$

Misollar:

$\bar{x}=(3; -4; 2;5)$, $\bar{y}=(-1; 3 ; -7; 2)$ vektorlar berilgan:

a) $3\bar{x}+2\bar{y}$ vektorni;

b) $\bar{x}*\bar{y}$ skalyar ko'paytmasini;

c) $\bar{x}$, $\bar{y}$ vetorlar orasidagi burchakni toping.

Yechish:

a) $3\bar{x}+2\bar{y}=(9; -12; 6; 15)+(-2; 6; -14; 4)=(7; -6; -8; 19)$;

b) $\bar{x}*\bar{y}=-3-12-14+10=-19$;

c) $\cos\varphi=\frac{(\bar{x};\bar{y})}{|\bar{x}||\bar{y}|}$; $|\bar{x}|=\sqrt{9+16+4+25}=\sqrt{54}$; $|\bar{y}|=\sqrt{1+9+49+4}=\sqrt{63}$;

$\cos\varphi=\frac{-19}{\sqrt{54}\sqrt{63}}$; $\varphi=-\arccos\frac{-19}{\sqrt{54}\sqrt{63}}=-\arccos\frac{-19}{9\sqrt{42}}$.

2. Vektorlar sistemasi. Vektorni vektorlar sistemasi bo'yicha yoyish.

n-o'lchovli *m* ta vektordan iborat vektorlar *n*-o'lchovli vektorlar sistemasini tashkil etadi.

$$\begin{cases} \overline{a_1}(a_{11}; a_{12}; \ldots\ldots a_n) \\ \overline{a_2}(a_{21}; a_{22}; \ldots. a_{2n}) \\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots \\ \overline{a_m}(a_{m1}; a_{m2}; \ldots. a_{mn}) \end{cases}$$

$\overline{a_1}, \overline{a_2}, \ldots, \overline{a_n}$ vektorlar sistemasi va $\lambda_1, \lambda_2, \ldots\ldots \lambda_m$ haqiqiy sonlar berilgan bo'lsin.

$\lambda_1\overline{a_1} + \lambda_2\overline{a_2} + \ldots\ldots + \lambda_m\overline{a_m}$ vektorga $\overline{a_1}, \overline{a_2}, \ldots, \overline{a_n}$ vektorning $\lambda_1, \lambda_2, \ldots\ldots \lambda_m$ koeffitsientli chiziqli kombinatsiyasi deyiladi.

Vektorlar sistemasi va $\overline{b}(b_1; b_2; \ldots\ldots; b_m)$ vektor berilgan bo'lsa $\overline{b}$ vektorni sistema vektorlari bo'yicha yoyish uchun $\sum_{j=1}^{m} \overline{a_j} x_j = \overline{b}$ chiziqli tenglamalar sistemasining yechimlaridan birini ko'rsatish yetarli.

Agar chiziqli tenglamalar sistemasi birgina yechimga ega bo'lsa, $\overline{b}$ vektor sistema vektorlari bo'yicha birgina usulda, agar cheksiz ko'p yechimga ega bo'lsa, cheksiz ko'p usulda yoyiladi, agar yechimga ega bo'lmasa $\overline{b}$ vektorni sistema vektorlari bo'yicha yoyib bo'lmaydi.

2- misol.

$\overline{b}(3;-1;4;5)$ vektorni

$\overline{a}_1(2;1;3;2),\ \overline{a_2}(1;-2;4;-4),\ \overline{a_3}(3;1;-5;2),\ \overline{a_4}(-4;-3;1;-6)$ vektorlar sistemasi bo'yicha yoying.

Buning uchun $\overline{b} = \overline{a_1}x_1 + \overline{a_2}x_2 + \overline{a_3}x_3 + \overline{a_4}x_4$ vektor tenglama tuzib, uni Gauss - Jordan usulida yechamiz: $(A|B) \sim (E|X)$

$(A|B) = \left(\begin{array}{cccc|c} 2 & 1 & 3 & -4 & 3 \\ 1 & -2 & 1 & -3 & -1 \\ 3 & 4 & -5 & 1 & 4 \\ 2 & -4 & 2 & -6 & 5 \end{array}\right)$ berilgan vektorlar sistemasi koordinatalaridan tuzilgan matritsani ozod hadlar ustuni hisobiga kengaytirilgan matritsa. *A* matritsa o'rnida birlik matritsa hosil qilish uchun 2-satr elementlarini (-2) ga ko'paytirib 1-satrga, (-3) ga ko'paytirib 3-satrga,

(-2) ga ko'paytirib 4-satrga qoshamiz: $\begin{pmatrix} 1 & 3 & 2 & -1 & \vline & 4 \\ 1 & -2 & 1 & -3 & \vline & -1 \\ 0 & 10 & -8 & 10 & \vline & 7 \\ 0 & 0 & 0 & 0 & \vline & 7 \end{pmatrix}$ bundan sistemaning yechimga ega emasligi ko'rinadi: $7 \neq 0$.

Demak, $\overline{b}$ vektorni $\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}$ vektorlar sistemasi bo'yicha yoyish mumkin emas.

3-misol. $\overline{b}(5;1;6)$ vektorni

$\overline{a_1}(1;2;1), \overline{a_2}(2;-1;3), \overline{a_3}(3;-1;4)$ vektorlar sistemasi bo'yicha yoying.

Vektor tenglama tuzib Gauss - Jordan usulida yechamiz:

$$\overline{b} = \overline{a_1}x_1 + \overline{a_2}x_2 + \overline{a_3}x_3$$

$$\begin{pmatrix} 1 & 2 & 3 & \vline & 5 \\ 2 & -1 & -1 & \vline & 1 \\ 1 & 3 & 4 & \vline & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & \vline & 5 \\ 0 & -5 & -7 & \vline & -9 \\ 0 & 1 & 1 & \vline & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & \vline & 3 \\ 0 & 1 & 7/5 & \vline & 9/5 \\ 0 & 0 & 1 & \vline & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & \vline & 3 \\ 0 & 1 & 7/5 & \vline & 9/5 \\ 0 & 0 & -2/5 & \vline & -4/5 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 1 & \vline & 3 \\ 0 & 1 & 7/5 & \vline & 9/5 \\ 0 & 0 & 1 & \vline & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & \vline & 1 \\ 0 & 1 & 0 & \vline & -1 \\ 0 & 0 & 1 & \vline & 2 \end{pmatrix}; \quad \begin{matrix} x_1 = 1 \\ x_2 = -1 \\ x_3 = 2 \end{matrix} \qquad \overline{b} = \overline{a_1} - \overline{a_2} + 2\overline{a_3}$$

Mustaqil yechish uchun misollar:

8.1. $\overline{r} = \overline{OM} = 2\overline{i} + 3\overline{j} + 6\overline{k}$ vektorni yasang va uning radius vektori uzunligini hamda yo'nalishini aniqlang. $\cos^2\alpha + \cos^2\beta + + \cos^2\gamma = 1$ formula bo'yicha tekshiring.

8.2. *M* nuqtaning radius vektori *0x* o'q bilan 45^0 va *0y* o'q bilan 60^0 burchak hosil etadi. Vektorning uzunligi *r*=6. Agar *M* ning applikatasi manfiy bo'lsa, uning koordinatalarini aniqlang va $\overline{OM} = \overline{r}$ vektorni $\overline{i}, \overline{j}, \overline{k}$ lar orqali ifodalang.

8.3. *x0y* tekislikda *A*(4;2), *B*(2;3), *C*(0;5) nuqtalar berilgan va $\overline{OA} = \overline{a},\ \overline{OB} = \overline{b},\ \overline{OC} = \overline{c}$ vektorlar yasalgan. $\overline{a}$ vektor $\overline{b}$ *va* $\overline{c}$ vektorlar bo'yicha topilsin.

8.4. Parallelogrammning ketma-ket uchta *A*(1; -2; 3), *B*(3; 2; 1), *C*(6; 4; 4) uchlari berilgan. Uning to'rtinchi uchini toping.

8.5. Uchlari *A*(2; -1; 3), *B*(1;1;1), *C*(0;0;5) nuqtalarda bo'lgan uchburchak *ABC* ning burchaklari aniqlansin.

8.6. $\overline{a} = 2\overline{i} + \overline{j}$ va $\overline{b} = -\overline{2j} + \overline{k}$ vektorlarda yasalgan parallelogramm dioganallari orasidagi burchak topilsin.

8.7. $\overline{a} = \overline{i} + \overline{j} + 2\overline{k}$ va $\overline{b} = \overline{i} - \overline{j} + 4\overline{k}$ vektorlar berilgan. $\text{Pr}_{\overline{b}}\,\overline{a}$ va $\text{Pr}_{\overline{a}}\,\overline{b}$ aniqlansin.

8.8. 1) Agar *m* va *n* o’zaro 30^0 burchak tashkil etuvchi birlik vektorlar bo’lsa, $(m+n)^2$ hisoblansin.

2) Agar $|\overline{a}| = 2\sqrt{2},\ |\overline{b}| = 4$ hamda ($\overline{a} \wedge \overline{b}$)= 135^0 bo’lsa, $(a-b)^2$ hisoblansin.

8.9. O’zaro komplanar $\overline{a}, \overline{b}, \overline{c}$ vektorlar berilgan bo’lib, $|\overline{a}| = 3, |\overline{b}| = 2, |\overline{c}| = 5$ va ($\overline{a} \wedge \overline{b}$)=$60^0$, ($\overline{b} \wedge \overline{c}$)=$60^0$ bo’lsa, $\overline{u} = \overline{a} + \overline{b} - \overline{c}$ vektor yasalsin, $|\overline{u}| = \sqrt{(a+b+c)^2}$ formula bo’yicha uning moduli hisoblansin.

8.10. Teng yonli *OACB* trapetsiyada *M* va *N* nuqtalar mos ravishda *BC*=2, *AC*=2 tomonlarning o’rtalari. Trapetsiyaning o’tkir burchagi 60^0 ga teng. $\overrightarrow{OM}$ *va* $\overrightarrow{ON}$ vektorlar orasidagi burchak aniqlansin.

8.11. $\overline{a}(2;-1;3;4), \overline{b}(5;2;-2;6)$ vektorlar berilgan:

a) 2 $\overline{a}$, 5 $\overline{a} + 3\overline{b}, \overline{a} - 2\overline{b}$ vektorlarni;

b) $(\overline{a}, \overline{b})$; $(3\overline{a} + b, \overline{a} - 2\overline{b})$ skalyar ko’paytmalarini;

c) $\overline{a}$ *va* $\overline{b}$ vektor orasidagi burchakni toping.

Quyidagi $\overline{b}$ vektorlarni berilgan $\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}$ vektorlar sistemasi bo’yicha yoyish mumkin yoki mumkin emasligini ko’rsating va yoying:

8.12. $\overline{b} = (-4;9); \overline{a_1} = (1;-3); \overline{a_2} = (2;-5);$

8.13. $\overline{b} = (8;-3;-10;10); \overline{a_1} = (1;0;4;3); \overline{a_2} = (1;1;-4;5);$
$\overline{a_3} = (1;-2;0;3); \overline{a_4} = (-2;3;1;4)$

8.14. $\overline{b} = (3;-1;4;5); \overline{a_1} = (2;1;3;2); \overline{a_2} = (1;-2;4;-4);$
$\overline{a_3} = (3;1;-5;2); \overline{a_4} = (-4;-3;1;-6)$

8.15. $\overline{b} = (9;-2;-3); \overline{a_1} = (1;-1;2); \overline{a_2} = (-5;-1;-4); \overline{a_3} = (4;1;5)$

8.16. λ ning qanday qiymatlarida $\overline{b} = (1;3;5)$ vektorni

$\overline{a_1} = (3;2;5), \overline{a_2} = (2;4;7), \overline{a_3} = (5;6;\lambda)$

vektorlar orqali yoyish mumkin?

8.17. $A(2;2;0)$ va $B(0;-2;5)$ nuqtalar berilgan. $\overline{AB}=\overline{u}$ vektor yasalsin, uning uzunligi va yo'nalishi aniqlansin.

8.18. 1) $(a+b)^2$;

2) $(a+b)^2+(a-b)^2$ ifodalardagi qavslar ochilsin va hosil bo'lgan formulalarning geometrik ma'nosi aniqlansin.

8.19. Agar $\overline{m}$ va $\overline{n}$ oralaridagi burchak 60^0 ga teng birlik vektorlar bo'lsa, $\overline{a}=2\overline{m}+\overline{n}$ va $\overline{b}=\overline{m}-2\overline{n}$ vektorlardan yasalgan parallelogramm dioganallarining uzunliklari aniqlansin.

8.20. Muntazam tetraedrning bir uchidan o'tkazilgan ikki tekis burchagining bissektrisalari orasidagi burchak aniqlansin.

8.21. $\overrightarrow{OA}=\overline{a}$ va $\overrightarrow{OB}=\overline{b}$ vektorlar berilgan. $|\overline{a}|=2; |\overline{b}|=4$ va $(a\hat{\ }b)=60^0$. Uchburchak *OAB* ning *OM* medianasi bilan *OA* tomoni orasidagi bursak aniqlansin.

8.22. Tomonlari 6 va 4 *sm* bo'lgan to'g'ri to'rtburchak uchidan qarshi tomonlarini teng ikkiga bo'luvchi to'g'ri chiziqlar orasidagi burchaklar topilsin.

8.23. $\overline{m}$ va $\overline{n}$ lar o'zaro 120^0 burchak tashkil etuvchi birlik vektorlar bo'lsa, $\overline{a}=2\overline{m}+4\overline{n}$ va $\overline{b}=\overline{m}-\overline{n}$ vektorlar orasidagi burchak topilsin.

8.24. $\overline{a}(1;-3;2;0)$, $\overline{b}(4;-2;1;3)$, $\overline{c}(5;-3;2;1)$, $\overline{d}(1;2;2;-3)$ vektorlar uchun quyidagilarni hisoblang:

a) vektorlarning ortogonallarini aniqlang;

b) $(\overline{a}\,\hat{}\,\overline{b}), (\overline{b}\,\hat{}\,\overline{c}), (\overline{b}\,\hat{}\,\overline{d})$ larni hisoblang.

9. CHIZIQLI BOG'LIQ VA CHIZIQLI ERKLI VEKTORLAR SISTEMASI

N o'lchovli *m* ta vektorlardan iborat vektorlar sistemasi berilgan bo'lsin.

$$\begin{cases} \overline{a_1}(a_{11}a_{12}\dots\dots a_{1n}) \\ \overline{a_2}(a_{21}a_{22}\dots\dots a_{2n}) \\ ---------- \\ \overline{a_m}(a_{m1}a_{m2}\dots\dots a_{mn}) \end{cases} \qquad (1)$$

(1) Vektorlar sistemasi chiziqli erkli yoki chiziqli bog'liq ekanini aniqlash uchun berilgan vektorlar sistemasi vektorlaridan vektor tenglama tuzamiz:

$$\overline{a_1}x_1+\overline{a_2}x_2+\dots\dots+\overline{a_m}x_m=\overline{\theta} \qquad (2)$$

bu erda $\overline{\theta}$ - n o'lchovli nol vektor. (1) Tenglama m noma'lumli n ta bir jinsli chiziqli tenglamalar sistemasi. Bu sistema aniq bo'lib, yagona nol yechimga ega bo'lsa, berilgan vektorlar sistemasi o'zaro chiziqli bog'liq bo'lmagan yoki chiziqli erkli vektorlar sistemasi bo'ladi.

Agar sistema aniqmas bo'lib, nol yechimdan tashqari nolmas yechimlarga ega bo'lsa, vektorlar sistemasi chiziqli bog`liq sistema bo'ladi; bunda x_1, x_2,…, x_m lardan kamida bittasi noldan farqli bo'lsa, $\overline{a_1}, \overline{a_2}, \ldots, \overline{a_m}$ lardan birini qolgan vektorlar orqali chiziqli ifodalash mumkin, bu esa sistema chiziqli bog'liq ekanini ko'rsatadi. (1) sistemaning chiziqli bog'liq yoki chiziqli erkli ekanini topish uchun vektorlar koordinatalaridan matritsa tuzamiz. Agar $r(A)=m$ bo'lsa, sistema chiziqli erkli, agar $r(A)<m$ bo'lsa, chiziqli bog'liq bo'ladi.

Misol-1. $\overline{a_1}(1;4;5),\ \overline{a_2}(2;-1;1),\ \overline{a_3}(-1;1;3)$ vektorlarning chiziqli bog'liq yoki chiziqli erkli ekanini aniqlang.

$A=\begin{pmatrix} 1 & 2 & -1 \\ 4 & -1 & 1 \\ 5 & 1 & 3 \end{pmatrix}$ matritsa rangini aniqlaymiz.

$M=\begin{vmatrix} 1 & 2 & -1 \\ 4 & -1 & 1 \\ 5 & 1 & 3 \end{vmatrix}$=-3+10-4-5-24-1=-27≠0

$r(A)=3, \qquad r(A)=m=3.$

Vektorlar sistemasi chiziqli erkli.

Misol-2. $\overline{a_1}(1;3;2),\ \overline{a_2}(2;7;3),\ \overline{a_3}(-1;2;-7)$ vektorlarning chiziqli bog'liq yoki chiziqli erkli ekanini aniqlang:

$A=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{pmatrix}$; $M=\begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{vmatrix}$=-49+8-9+14+42-6=64-64=0

$M_1=\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$=7-6=1≠0 $r(A)=2$,

vektorlar soni $m=3$. $r(A)\neq m$. Vektorlar sistemasi chiziqli bogliq.

Mustaqil yechish uchun misollar:

Vektorlar sistemasining chiziqli bog'liq yoki chiziqli bog'liq emasligini aniqlang.

9.1. $\overline{a_1}=(1;2;3), \quad \overline{a_2}=(3;6;7)$

9.2. $\overline{a_1} = (5;4;3)$, $\overline{a_2} = (3;3;2)$, $\overline{a_3} = (8;1;3)$

9.3. $\overline{a_1} = (4;-2;6)$, $\overline{a_2} = (6;-3;9)$

9.4. $\overline{a_1} = (4;-5;2;6)$, $\overline{a_2} = (2;-2;1;3)$, $\overline{a_3} = (6;-3;3;9)$, $\overline{a_4} = (4;-1;5;6)$

9.5. $\overline{a_1} = (2;-3;1)$, $\overline{a_2} = (3;-1;5)$, $\overline{a_3} = (1;-4;3)$

9.6. $\overline{a_1} = (1;0;0;2;5)$, $\overline{a_2} = (0;1;0;3;4)$, $\overline{a_3} = (0;0;1;4;7)$, $\overline{a_4} = (2;-3;4;11;12)$

9.7. $\overline{a_1} = (1;1;1)$, $\overline{a_2} = (0;1;1)$, $\overline{a_3} = (0;0;1)$

9.8. $\overline{a_1} = (3;2;1)$, $\overline{a_2} = (2;-3;0)$, $\overline{a_3} = (-3;-2;13)$

9.9. $\overline{a_1} = (3;2;1)$, $\overline{a_2} = (2;3;1)$, $\overline{a_3} = (-1;-4;-1)$.

9.10. $\overline{a} = (1;-3;4)$, $\overline{b} = (2;-1;0)$

9.11. $\overline{a_1} = (1;3;1;0)$, $\overline{a_2} = (-2;1;-3;-1)$, $\overline{a_3} = (4;0;5;1)$, $\overline{a_4} = (3;2;-1;-4)$

9.12. $\overline{x_1} = (-3;-1;5)$, $\overline{x_2} = (6;-3;15)$, $\overline{x_3} = (0;-5;25)$;

9.13. $\overline{a_1} = (1;1;1;1)$, $\overline{a_2} = (1;2;1;2)$, $\overline{a_3} = (3;2;-1;-4)$

9.14. $\overline{x_1} = (1;-1;1;0)$, $\overline{x_2} = (1;1;-1;1)$, $\overline{x_3} = (-1;3;-3;1)$

9.15. λ qanday qiymatlarida vektor $\overline{x} = (2;\ 3;\ 4)$ ni quyidagi $\overline{a_1}, \overline{a_2}, \overline{a_3}$ vektorlar orqali yoyish mumkin?

$$\overline{a_1} = (1;\ 2;\ 3),\quad \overline{a_2} = (3;\ 1;\ -1),\quad \overline{a_3} = (2;\ 1;\ \lambda)$$

Quyidagi vektorlar sistemasini chiziqli bog'liq yoki chiziqli bog'liq emasligini aniqlang:

9.16. $\overline{a} = (-3;-2)$; $\overline{b} = (2;7)$

9.17. $\overline{a} = (1;-4;3)$; $\overline{b} = (2;-7;6)$

9.18. $\overline{a} = (1;-2;-3)$, $\overline{b} = (2;1;-1)$, $\overline{c} = (3;7;4)$

19. $\overline{a_1} = (2;0;-1;1)$, $\overline{a_2} = (3;8;1;5)$, $\overline{a_3} = (1;0;1;-3)$, $\overline{a_4} = (-1;0;1;-3)$

9.20. $\overline{a_1} = (1;2;0)$; $\overline{a_2} = (3;-1;1)$; $\overline{a_3} = (0;1;1)$

9.21. $\overline{x_1} = (1;1;1;1)$, $\overline{x_2} = (1,-1,-1;1)$, $\overline{x_3} = (1;-1;1;1)$, $\overline{x_4} = (1;1;-1;-1)$

9.22. $\overline{x_1} = (4;-5;2;6)$, $\overline{x_2} = (2;-2;1;3)$, $\overline{x_3} = (6;-3;3;9)$, $\overline{x_4} = (4;-1;5;6)$

9.23-9.25 misollar uchun

$\overline{a_1} = (4;1;3;-2)$, $\overline{a_2} = (1;2;-3;2)$, $\overline{a_3} = (16;9;1;3)$, $\overline{a_4} = (0;1;2;3)$, $\overline{a_5} = (1;-1;15;0)$

vektorlar berilgan bo`lsin.

9.23. $\overline{a_1}, \overline{a_2}, \overline{a_3}, \overline{a_4}, \overline{a_5}$ vektorlar uchun quyidagi kombinatsiyani toping:

a) $\frac{1}{2}\overline{a_1} + 3\overline{a_2} - \frac{1}{2}\overline{a_4} + \overline{a_5}$ b) $\overline{a_2} - 5\overline{a_3} + \overline{a_4} + 2\overline{a_5}$

c) $[\overline{a_1} \times \overline{a_2}], [\overline{a_1} \times \overline{a_3}], [\overline{a_1} \times \overline{a_4}]$

10. VEKTORLAR SISTEMASINING RANGI VA BAZISI. VEKTORLAR SISTEMASIDA ELEMENTAR ALMASHTIRISHLAR. KANONIK BAZIS

$a_1, a_2, \ldots, a_n$ vektorlar sistemasi berilgan bo`lsin. Berilgan *vektorlar sistemasining bazisi* deb uning chiziqli bog'liq bo'lmagan shunday bir qismiga aytiladiki, bunda berilgan sistemaning har bir vektori bazis vektorlari orqali yoyilishi mumkin bo'ladi. Berilgan vektorlar sistemasining ixtiyoriy bazisi tarkibidagi vektorlar soniga uning *rangi* deyiladi.

1. Misol. Quyidagi vektorlar sistemasining bazislaridan birini quring va rangini aniqlang:

$a_1(1;2;-1;3)$, $a_2(0;3;4;1)$, $a_3(-2;-1;6;-5)$, $a_4(5;1;2;-4)$

Yechish: $a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = \theta$ vektor tenglama umumiy yechimini Gauss-Jordan usulida quramiz:

$$\left(\begin{array}{cccc|c} 1 & 0 & -2 & 5 & 0 \\ 2 & 3 & -1 & 1 & 0 \\ -1 & 4 & 6 & 2 & 0 \\ 3 & 1 & -5 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 5 & 0 \\ 0 & 3 & 3 & -9 & 0 \\ 0 & 4 & 4 & 7 & 0 \\ 0 & 1 & 1 & -19 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 5 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -19 & 0 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

Yechilgan sistemadan x_1, x_2, x_4 - yechilgan noma'lumlar, x_3 esa erkin noma'lum ekanligi ko'rinib turibdi. Demak, berilgan vektorlar sistemasining bazisi a_1, a_2 va a_4 vektorlar sistemasi bo'lib, sistemaning rangi bazisidagi vektorlar soni 3 ga teng.

Agar berilgan ikkita n o'lchovli a_1 va a_2 vektorlarning skalyar ko'paytmasi nolga teng bo'lsa, a_1 va a_2 vektorlar o'zaro *ortogonal vektorlar* deyiladi.

n o'lchovli nolmas vektorlardan tarkib topgan vektorlar sistemasi berilgan bo'lib, sistema vektorlarining har qanday ikki jufti o'zaro ortogonal bo'lsa, u holda sistemaga *ortogonal vektorlar sistemasi* deyiladi.

2. Misol. Quyidagi vektorlar sistemasi ortogonalmi?

$a_1(0;5;-2)$, $a_2(29;-2;-5)$, $a_3(2;4;10)$

Yechish:

$(a_1 * a_2) = 0 - 10 + 10 = 0$

$(a_1 * a_3)$=0+20-20=0

$(a_2 * a_3)$=58-8-50=0

Berilgan vektorlar sistemasi ortogonal vektolar sistemasi ekan.

Teng o`lchovli *n* ta $a_1, a_2, \ldots a_k$ chiziqli erkli vektorlar sistemasi ustida *ortogonal vektorlar sistemasini* qurish, ya'ni mos ravishda $b_1, b_2, \ldots b_k$ ortogonal sistema bilan almashtirish mumkin. Buning uchun *Shmidt formulalari*dan foydalanamiz:

$b_1=a_1$

$$\mathrm{b}_{\mathrm{t}} = a_t - \sum_{i=1}^{t-1} \frac{(b_i \cdot a_t)}{(b_i \cdot b_i)} b_i \qquad t\epsilon\{2;3;\ldots k\}$$

3. Misol. $a_1(1;1;1)$, $a_2(0;1;1)$, $a_3(0;0;1)$ vektorlar sistemasi ustida ortogonal sistema quring. rang (a_1,a_2,a_3)=3 chiziqli erkli sistema ekan.

$b_1=a_1(1;1;1)$

$$b_2 = a_2 - \frac{(b_1 \cdot a_2)}{(b_1 \cdot b_1)} b_1 = (0;1;1) - \frac{2}{3}(1;1;1) = \left(-\frac{2}{3};\frac{1}{3};\frac{1}{3}\right)$$

$$b_3 = a_3 - \frac{(b_1 \cdot a_3)}{(b_1 \cdot b_1)} b_1 - \frac{(b_2 \cdot a_3)}{(b_2 \cdot b_2)} b_2 = (0;0;1) - \frac{1}{3}(1;1;1) - \frac{1/3}{2/3}\left(-\frac{2}{3};\frac{1}{3};\frac{1}{3}\right) =$$

$$= \left(0;-\frac{1}{2};\frac{1}{2}\right)$$

Berilgan vektorlar sistemasi ustida qurilgan ortogonal sistema vektorlarini butun koordinatali vektorlarga aylantirib, (1;1;1); (-2;1;1); (0;-1;1) natijani olamiz.

Nolmas *b vektorning normallangan* yoki *birlik vektori* deb, $\frac{b}{|b|}$ vektorga aytiladi.

Har bir vektori normallangan, ya'ni birlik vektor ko'rinishiga keltirilgan ortoganal sistemaga *ortonormallangan vektorlar sistemasi* deyiladi.

4. Misol. Yuqoridagi misolda topilgan ortonormal $b_1(1;1;1)$; $b_2(-2;1;1)$; $b_3(0;-1;1)$ sistemaning har bir vektorini birlik ko'rinishga keltiramiz.

$$\frac{b_1}{|b_1|} = \frac{1}{\sqrt{1^2+1^2+1^2}}(1;1;1) = \left(\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right)$$

$$\frac{b_2}{|b_2|} = \frac{1}{\sqrt{(-2)^2+1^2+1^2}}(-2;1;1) = \left(-\frac{2}{\sqrt{6}};\frac{1}{\sqrt{6}};\frac{1}{\sqrt{6}}\right)$$

$$\frac{b_3}{|b_3|} = \frac{1}{\sqrt{0^2 + (-1)^2 + 1^2}}(0;-1;1) = \left(0; -\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right)$$

n o`lchovli birlik $e_1(1;0;0;\ldots;0), e_2(0;1;0;\ldots;0), \ldots, e_n(0;0;0;\ldots;1)$ vektorlar *kanonik bazis*ni tashkil qiladi.

Mustaqil yechish uchun misollar:

Quyida berilgan vektorlar sistemasining bazislaridan birini quring va ranglarini aniqlang:

10.1. a_1=(1;-2;-5), a_2=(3;4;-1), a_3=(2;-3;0)

10.2. a_1=(1;1;-2;-5), a_2=(3;4;-1;2), a_3=(4;1;-2;3), a_4=(5;2;-3;1)

10.3. $e_1; e_2; e_3$ bazisda a_1=(1;1;0), a_2=(1;-1;1), a_3=(-3;5;6) vektorlar berilgan. $a_1; a_2; a_3$ vektorlar bazisni tashkil qilishini ko`rsating.

10.4. $e_1; e_2; e_3$ bazisda vektor b=(4;-4;5) berilgan. Shu vektorni quyidagi $a_1; a_2; a_3$ bazisda ifodalang: a_1=(1;1;0), a_2=(1;-1;1), a_3=(-3;5;-6)

10.5. $e_1; e_2; e_3$ bazisda berilgan a=(1;2;0), b=(3;-1;1), c=(0;1;1) vektorlar o`zlari bazis tashkil qilishini ko`rsating.

10.6. $e_1; e_2; e_3$ bazisda quyidagi a, b, c vektorlar berilgan: $a=e_1+e_2+e_3$, $b=2e_2+3e_3$, $c=e_2+5e_3$. a, b, c vektorlar bazis tashkil qilishini isbotlang. Vektor $d=2e_1-e_2+e_3$ ni a, b, c bazisdagi koordinatalarini toping.

Quyidagi vektorlar sistemasining bazislarini toping:

10.7. a_1=(1;2;0;0); a_2=(1;2;3;4); a_3=(3;6;0;0);

10.8. a_1=(1;2;3;4); a_2=(2;3;4;5); a_3=(3;4;5;6); a_4=(4;5;6;7);

Berilgan vektorlar sistemasining rangi va barcha bazislari topilsin:

10.9. a_1=(1;2;0;0); a_2=(1;2;3;4); a_3=(3;6;0;0);

10.10. a_1=(1;2;3;4); a_2=(2;3;4;5); a_3=(3;4;5;6); a_4=(4;5;6;7);

10.11. a_1=(2;1;-3;1); a_2=(4;2;-6;2); a_3=(6;3;-9;3); a_4=(1;1;1;1);

Vektorlar juftliklari o'zaro ortoganalmi:

10.12. a_1(4;-5) va a_2(1;0);

10.13. a_1(4;1;2) va a_2(-1;0;2);

10.14. a_1(2;0;4;-1) va a_2(1;2;3;4);

10.15. a_1(1;3;2;-3) va a_2(1;1;1;2)?

Quyida berilgan chiziqli erkli vektorlar sistemalari ustida ortogonal va ortonormallangan vektorlar sistemalari qurilsin:

10.16. a_1(1;0) va a_2(1;1)

17. $a_1(1;1;1;0)$, $a_2(0;1;1;1)$, $a_3(0;0;1;1)$

Quyida berilgan vektorlar sistemasining rangi va bazislari topilsin:

10.18. $a_1=(5;2;-3;1)$; $a_2=(4;1;-2;3)$; $a_3=(1;1;-1;2)$; $a_4=(3;4;-1;2)$

10.19. $a_1=(2;-1;3;5)$; $a_2=(4;-3;1;3)$; $a_3=(3;-2;3;4)$; $a_4=(4;-1;15;17)$; $a_5=(7;-6; -7;0)$

10.20. $a_1=(2;1;-3;1)$; $a_2=(4;2;-6;2)$; $a_3=(6;3;-9;3)$; $a_4=(1;1;1;1)$

10.21. $a_1=(1;2;3)$; $a_2=(2;3;4)$; $a_3=(3;2;3)$; $a_4=(4;3;4)$ $a_5=(1;1;1)$

10.22. $a_1=(5;2;-3;1)$; $a_2=(4;1;-2;3)$; $a_3=(1;1;-1;-2)$; $a_4=(3;4;-1;2)$

10.23. $a_1=(2;-1;3;5)$; $a_2=(4;-3;1;3)$; $a_3=(3;-2;3;4)$; $a_4=(4;-1;15;17)$; $a_5=(-7;-6;-7;0)$

Quyida berilgan chiziqli erkli vektorlar sistemalari ustida ortogonal va ortonormallangan vektorlar sistemalari qurilsin:

10.24. $a_1(1;1)$, $a_2(0;2)$

10.25. $a_1(1;0;1;0)$, $a_2(0;1;1;1)$, $a_3(1;1;0;1)$

10.26. $a_1(1;1;1;1)$, $a_2(1;1;1;0)$, $a_3(1;0;1;1)$

11. VEKTOR KO`RINISHIDA YOZILGAN CHIZIQLI TENGLAMALAR SISTEMASINING BIRGALIKDALIK VA ANIQLIK SHARTLARI. FUNDAMENTAL YECHIMLAR

m ta noma'lumli n ta chiziqli *bir jinsli tenglamalar sistemasi* vektor shaklda berilgan bo'lsin:

$$a_1x_1+a_2x_2+\ldots+a_mx_m=\theta$$

$rang(a_1a_2,\ldots,a_m)=rang(a_1,a_2,\ldots,a_m,\theta)$ bo'lgani uchun sistema har doim *birgalikda*. $Rang(a_1,a_2,\ldots,a_m)=m$ munosabat o'rinli bo'lsa, sistema *aniq* va yagona nol yechimga ega.

$Rang(a_1,a_2,\ldots,a_m)<m$ munosabat o'rinli bo'lsa, sistema *aniqmas* va trivial yechimdan tashqari nolmas yechimlarga ham ega bo'ladi. Ushbu holda, har bir nolmas yechim m o'lchovli vektor sifatida qaralishi mumkin.

Bir jinsli chiziqli tenglamalar sistemasining fundamental yechimlari sistemasi yoki tizimi deb, uning chiziqli bog'liq bo'lmagan nolmas F_1, $F_2,\ldots,F_k$ yechimlariga aytiladiki, sistemaning har bir yechimi ushbu yechimlarning chiziqli kombinatsiyasi ko'rinishida aniqlanishi mumkin.

Agar $rang(a_1,a_2,\ldots,a_m)=r<m$ bo'lsa, sistema o'zining fundamental yechimlari tizimi mavjudligi bilan xarakterlanadi va tizim har biri m o'lchovli $m-r$ ta nolmas vektorlardan tarkib topadi.

Bir jinsli sistemaning fundamental yechimlari tizimi quyidagicha quriladi:

1. Bir jinsli sistemaning umumiy yechimi quriladi;

2. m-r o'lchovli m-r ta vektorlardan iborat chiziqli erkli vektorlar siatemasi, masalan: $e_1(1;0;\ldots;0)$, $e_2(0;1;0;\ldots;0)$,…, $e_{m\text{-}r}(0;0;\ldots;1)$ tanlanadi;

3. Umumiy yechim erkli noma'lumlari o'rniga e_1 vektor mos koordinatalarini qo'yib, bazis noma'lumlar aniqlanadi va mos ravishda F_1 fundamental yechim quriladi. Shuningdek, e_2, e_3, …, e_{m-r} vektorlardan foydalanib, mos ravishda F_2, F_3, …, F_{m-r} fundamental yechimlar quriladi.

1. Misol. Bir jinsli sistemaning fundamental yechimlari tizimidan birini quring va uning umumiy yechimini vektor shaklda aniqlang:

$$\begin{cases} 4x_1 + 7x_2 + 2x_3 + 3x_4 = 0 \\ x_1 + 3x_2 - x_3 + 2x_4 = 0 \\ 2x_1 + x_2 + 4x_3 - x_4 = 0 \end{cases}$$

Sistemaning umumiy yechimini Gayss-Jordan usulida quramiz:

$$\left(\begin{array}{cccc|c} 4 & 7 & 2 & 3 & 0 \\ 1 & 3 & -1 & 2 & 0 \\ 2 & 1 & 4 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & -5 & 6 & -5 & 0 \\ 1 & 3 & -1 & 2 & 0 \\ 0 & -5 & 6 & -5 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & 1 & -1{,}2 & 1 & 0 \\ 1 & 0 & 2{,}6 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

m=4, r=2 bo'lgani uchun m-r=2 ta chiziqli erkli $e_1(1;0)$ va $e_2(0;1)$ sistemani tanlaymiz. $e_1(1;0)$ vektor koordinatalarini umumiy yechimning mos erkli nomalumlari o'rniga qo'yib, bazis nomalumlarni aniqlaymiz va $F_1(-2{,}6;1{,}2;1;0)$ fundamental echimni quramiz. $e_2(0;1)$ vektor yordamida $F_2(1;-1;0;1)$ fundamental yechimni quramiz. Boshqacha qilib aytganda kengaytirilgan matritsadagi koeffitsiyentlarni sistemaga qo'yamiz:

$$\begin{cases} x_2 - 1{,}2x_3 + x_4 = 0 \\ x_1 \quad 2{,}6x_3 - x_4 = 0 \end{cases} \qquad \begin{cases} x_1 = -2{,}6x_3 + x_4 \\ x_2 = 1{,}2x_3 - x_4 \\ x_3 = x_3 \\ x_4 = x_4 \end{cases}$$

Fundamental yechimlar $F_1(4{,}6;1{,}2;1;0)$ va $F_2(1;-1;0;1)$ quriladi.
Umumiy yechimni tuzamiz:

$$X = \lambda_1 F_1 + \lambda_2 F_2 = \lambda_1 \begin{pmatrix} -2{,}6 \\ 1{,}2 \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

Bu yerda λ_1 va λ_2 lar ixtiyoriy haqiqiy sonlar.

m ta nomalumli n ta chiziqli bir jinsli bo'lmagan tenglamalar sistemasi vektor shaklda berilgan bo'lsin:

$$a_1x_1 + a_2x_2 + \ldots + a_mx_m = b \qquad (b \neq \theta)$$

Sistemaning umumiy yechimini vektor shaklda yozish mumkin:

$X=F_0+\lambda_1 F_1+\lambda_2 F_2+\ldots+\lambda_{m-r} F_{m-r}$

Bu yerda F_0 - bir jinslimas sistemaning xususiy yechimlaridan biri, $F_1, F_2, \ldots, F_{m-r}$ - berilgan sistemaga mos ravishdagi

$a_1x_1+a_2x_2+\ldots+a_mx_m=\theta$

bir jinsli tenglamalar sistemasining fundamental yechimlari tizimi, $\lambda_1, \lambda_2, \ldots, \lambda_{m-r}$ -ixtiyoriy haqiqiy sonlar.

2. Misol. Berilgan sistema umumiy yechimini vektor shaklda quring:

$$\begin{cases} 4x_1+7x_2+2x_3+3x_4=8 \\ x_1+3x_2-x_3+2x_4=3 \\ 2x_1+x_2+4x_3-x_4=2 \end{cases}$$

$$\left(\begin{array}{cccc|c} 4 & 7 & 2 & 3 & 8 \\ 1 & 3 & -1 & 2 & 3 \\ 2 & 1 & 4 & -1 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & -5 & 6 & -5 & -4 \\ 1 & 3 & -1 & 2 & 3 \\ 0 & -5 & 6 & -5 & -4 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & 1 & -1{,}2 & 1 & 0{,}8 \\ 1 & 0 & 2{,}6 & -1 & 0{,}6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$F_0(0{,}6;\ 0{,}8;\ 0;\ 0)$ sistemaning xususiy yechimlaridan birini qurdik.

Sistema umumiy yechimi vektor shaklini yozamiz:

$$X=F_0+\lambda_1 F_1+\lambda_2 F_2=\begin{pmatrix} 0{,}6 \\ 0{,}8 \\ 0 \\ 0 \end{pmatrix}+\lambda_1\begin{pmatrix} -2{,}6 \\ 1{,}2 \\ 1 \\ 0 \end{pmatrix}+\lambda_2\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

bu yerda λ_1 va λ_2 lar ixtiyoriy haqiqiy sonlar.

Mustaqil yechish uchun misollar:

Bir jinsli tenglamalar sistemasini yeching:

11.1. $\begin{cases} x_1+2x_2-x_3-x_4=0 \\ -2x_1-4x_2+2x_3+2x_4=0 \\ 3x_1+6x_2-3x_3-3x_4=0 \end{cases}$

11.2. $\begin{cases} x_1-2x_2+x_3=0 \\ x_1+2x_2-2x_3=0 \\ 3x_1+2x_2-3x_3=0 \end{cases}$

11.3. $\begin{cases} x_1-7x_2+5x_3-3x_4=0 \\ 2x_1-3x_2+7x_3-x_4=0 \\ x_1-x_2+x_3-2x_4=0 \\ 5x_2+x_3=0 \end{cases}$

11.4. $\begin{cases} x_1+2x_2+4x_3-3x_4=0 \\ 3x_1+5x_2+6x_3-4x_4=0 \\ 4x_1+5x_2-2x_3+3x_4=0 \\ 3x_1+8x_2+24x_3-19x_4=0 \end{cases}$

Bir jinsli bo’lmagan chiziqli tenglamalar sistemalarining fundamental yechimlarini toping:

11.5. $\begin{cases} x_1 - 2x_2 + 3x_3 + x_4 = 1 \\ 2x_1 - 3x_2 - x_3 - x_4 = -4 \end{cases}$

11.6. $\begin{cases} 2x_1 - x_2 + x_3 + x_4 = 1 \\ x_2 - x_3 + 2x_4 = 2 \\ 2x_2 - 2x_3 + 3x_4 = 3 \end{cases}$

11.7. $\begin{cases} x_1 + 2x_2 - x_3 = 5 \\ 2x_1 - x_2 - 3x_3 = 4 \end{cases}$

11.8. $\begin{cases} 3x_1 + x_2 - x_3 - 2x_4 = -4 \\ x_1 - x_2 - x_3 + 2x_4 = 1 \end{cases}$

11.9. $\begin{cases} x_1 + x_2 + x_3 + x_4 = 2 \\ 2x_2 - 2x_3 + 2x_4 = 2 \\ x_1 - x_3 + x_4 = 2 \end{cases}$

Sistemani yeching:

11.10. $\begin{cases} 3x - 2y - z = 0 \\ 2x - y + 3z = 0 \\ -3y - 4z = 0 \end{cases}$

11.11. $\begin{cases} 3x + 2y - z = 0 \\ 2x - y + 3z = 0 \\ x + y - z = 0 \end{cases}$

11.12. $\begin{cases} 3x_1 - 2x_2 + 3x_3 + 3x_4 = 0 \\ 3x_1 - 2x_2 - x_3 + x_4 = 0 \\ x_1 - x_2 + 2x_3 + 5x_4 = 0 \end{cases}$

11.13. $\begin{cases} x_1 - 2x_2 + 4x_3 - 3x_4 = 0 \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0 \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0 \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0 \end{cases}$

Sistemalarni fundamental yechimlarini va umumiy yechimini toping:

11.14. $\begin{cases} 3x_1 - x_2 + 2x_3 + 3x_4 = 18 \\ -x_1 - x_2 + 2x_4 = 0 \\ x_1 + x_2 + 3x_3 - 2x_4 = 0 \end{cases}$

11.15. $\begin{cases} 3x + 5y + 2z = 0 \\ 5x + 2y + 3z = 0 \end{cases}$

11.16. $\begin{cases} 2x_1 + x_2 - 4x_3 - x_4 + x_5 = 0 \\ x_1 + 2x_2 + x_3 + 2x_4 + x_5 = 0 \\ 3x_1 - x_2 - 5x_3 + x_4 + 2x_5 = 0 \end{cases}$

11.17. $\begin{cases} x_1 - 3x_2 - x_3 + 4x_4 - x_5 = 7 \\ 2x_1 - x_2 - 3x_3 + x_4 + 4x_5 = -3 \\ 3x_1 - 2x_2 - 2x_3 + 5x_4 + 3x_5 = 4 \end{cases}$

11.18. $\begin{cases} x_1 + 2x_2 + 3x_3 = 2 \\ x_1 - x_2 + x_3 = 0 \\ x_1 + 3x_2 - x_3 = -2 \\ 3x_1 + 4x_2 + 3x_3 = 0 \end{cases}$

12. CHIZIQLI FAZO. EVKLID FAZO.
ORTOGONAL MATRITSA

12.1. $a_1(0; 1; -3)$, $a_2(3; 5; 0)$, $a_3(1; 2; -1)$ vektorlar sistemalariga tortilgan chiziqli qism osti fazosining bazislaridan birini, o`lchamini hamda ortonormallangan bazisini topamiz:

Buning uchun $a_1x_1+a_2x_2+a_3x_3=\theta$ vektor tenglama umumiy yechimini Gauss-Jordan usulida quramiz:

$$\left(\begin{array}{ccc|c} 0 & 3 & 1 & 0 \\ 1 & 5 & 2 & 0 \\ -3 & 0 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 3 & 1 & 0 \\ 1 & 5 & 2 & 0 \\ 0 & 15 & 5 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 3 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

x_1, x_3 noma'lumlar umumiy yechimning bazis noma'lumlari. Demak, mos ravishda, a_1, a_3 vektorlar tizimi berilgan sistemaning bazislaridan birini tashkil etadi. Tizim 2 ta vektordan tarkib topgani uchun, berilgan vektorlar sistemasining o`lchami 2 ga teng.

Bazisni tashkil qiluvchi $a_1(0; 1; -3)$ va $a_3(1; 2; -1)$ vektorlarni ortogonallaymiz:

$$b_1 = a_1(0;1;-3)$$

$$b_2 = a_3 - \frac{(b_1 \cdot a_3)}{(b_1 \cdot b_1)} b_1 = (1;2;-1) - \frac{0\cdot 1 + 1\cdot 2 + (-3)\cdot(-1)}{0\cdot 0 + 1\cdot 1 - 3\cdot(-3)}(0;1;-3) = (1;2;-1) - \frac{5}{10}(0;1;-3) = (1;\frac{3}{3};\frac{1}{2})$$

hosil bo`lgan ortogonal sistema vektorlarini butun koordinatali vektorlarga aylantirib, $b_1(0; 1; -3)$ va $b_2(2; 3; 1)$ ni olamiz. Bu ortogonal sistemaning har bir vektorini birlik ko`rinishga keltiramiz, ya'ni ortonormallashtiramiz:

$$\frac{b_1}{|b_1|} = \frac{(0;1;-3)}{\sqrt{0^2+1^2+(-3)^2}} = (0;\frac{1}{\sqrt{10}};-\frac{3}{\sqrt{10}})$$

$$\frac{b_2}{|b_2|} = \frac{(2;3;1)}{\sqrt{2^2+3^2+1^2}} = \left(\frac{2}{\sqrt{14}};\frac{3}{\sqrt{14}};\frac{1}{\sqrt{14}}\right)$$

12.2. $x(3; -2; 4)$ vektor e_1, e_2, e_3 bazisda berilgan. Vektorning

$$\begin{cases} e_1^{'} = e_1 + 2e_2 - 3e_3 \\ e_2^{'} = e_1 + e_2 + e_3 \\ e_3^{'} = 2e_1 - e_2 + 2e_3 \end{cases}$$

bazisdagi koordinatalarini topamiz:

Koeffitsientlar matritsasi P ning transponirlangan matritsasi P^T ni hosil qilamiz:

$$P = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 1 & 1 \\ 2 & -1 & 2 \end{pmatrix} \qquad P^T = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & -1 \\ -3 & 1 & 2 \end{pmatrix}$$

U holda x vektorning dastlabki bazisdagi koordinatalari uning yangi bazisdagi koordinatalari orqali (matritsa shaklida $x=P^Tx`$) quyidagicha ifodalanadi:

$$\begin{cases} x_1 = x_1^{`} + x_2^{`} + 2x_3^{`} \\ x_2 = 2x_1^{`} + x_2^{`} - x_3^{`} \\ x_3 = -3x_1^{`} + x_2^{`} + 2x_3^{`} \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 2 & | & 3 \\ 2 & 1 & -1 & | & -2 \\ -3 & 1 & 2 & | & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & | & 3 \\ 0 & -1 & -5 & | & 8 \\ 0 & 4 & 8 & | & 13 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -3 & | & -5 \\ 0 & 1 & 5 & | & 8 \\ 0 & 0 & -12 & | & -19 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & | & -1/4 \\ 0 & 1 & 0 & | & 1/12 \\ 0 & 0 & 1 & | & 19/12 \end{pmatrix}$$

Demak, dastlab berilgan *x(3; -2; 4)* vektorning yangi bazisdagi koordinatalari: $x\left(-\frac{1}{4};\frac{1}{12};\frac{19}{12}\right)$

Ta'rif: $P \cdot P^T = P \cdot P^{-1} = E$ shartni bajaruvchi *P* matritsaga ortogonal matritsa deyiladi.

12.3. Quyidagi matritsa ortogonal matritsa bo'lishini tekshiramiz :

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & Cos\alpha & -Sin\alpha \\ 0 & Sin\alpha & Cos\alpha \end{pmatrix} \qquad P^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & Cos\alpha & Sin\alpha \\ 0 & -Sin\alpha & Cos\alpha \end{pmatrix}$$

$$P \cdot P^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E$$

Demak, berilgan *P* matritsa ortogonal matritsa bo'ladi.

Mustaqil yechish uchun misollar:

Quyidagi vektorlar sistemalariga tortilgan chiziqli qism osti fazosining bazislaridan birini , o'lchamini va ortonormallangan bazisini toping:

12.4. $a_1(3; -1; 2)$, $a_2(1;4;-1)$, $a_3(7;2;3)$

12.5. $x(2;-1)$ vektor e_1, e_2, bazisda berilgan . Vektorning $e_1\grave{}=e_1-3e_2$; $e_2\grave{}=2e_1+e_2$ bazisdagi koordinatalarini toping .

Quyidagi matritsalardan ortogonallarini ajrating:

12.6. $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 0.5 \\ 4 & -1 & 3 \end{pmatrix}$ 12.7. $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ 12.8. $\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$

12.9. Quyida berilgan ikki vektorlar sistemalaridan har biri bazis bo'la olishini isbotlang. Ushbu bazislarda berilgan aynan bir vektorning koordinatalari orasida munosabatlarni o'rnating:

a) $e_1(1;2)$, $e_2(1;1)$; $e_1\grave{}(1;1)$, $e_2\grave{}(3;4)$

b) $e_1(1;3)$, $e_2(2;3)$, $e_1'(1;0)$, $e_2'(0;-3)$

c) $e_1(2;3)$, $e_2(2;4)$, $e_1'(0;-1)$, $e_2'(6;11)$

12.10. R_3 da *i, j, k* bazisdan fazoni *Oy* ordinata o'qi atrofida α burchakka burgandagi bazisga o'tish matritsasini quring.

Quyidagi vektorlar sistemalariga tortilgan chiziqli qism osti fazosining bazislaridan birini , o'lchamini va ortonormallangan bazisini toping:

12.11. $a_1(1;2;-1;3)$, $a_2(0;3;4;1)$, $a_3(-2;-1;6;-5)$, $a_4(5;4;2;-4)$

12.12. $x(3;-2)$ vektor e_1, e_2 bazisda berilgan vektorning $e_1\grave{}=2e_1-e_2$; $e_2\grave{}=e_1+e_2$ bazisdagi koordinatalarini toping .

12.13. $x(1;2;-2)$ vektor e_1, e_2, e_3 bazisda berilgan vektorning $e_1\grave{}=e_1+e_2-e_3$;

$e_2` = 2e_1 - e_2 + e_3$ bazisdagi koordinatalarini toping .

Quyidagi matritsalardan ortogonallarini ajrating:

12.14. $\begin{pmatrix} Sin\alpha & 0 & Cos\alpha \\ 0 & 1 & 1 \\ -Cos\alpha & 0 & Sin\alpha \end{pmatrix}$ 12.15. $\begin{pmatrix} tg\alpha & tg\alpha \\ -ctg\alpha & ctg\alpha \end{pmatrix}$

Quyida berilgan ikki vektorlar sistemalaridan har biri bazis bo'la olishini isbotlang. Ushbu bazislarda berilgan aynan bir vektorning koordinatalari orasida munosabatlarni o'rnating:

12.16. $e_1(2;1;-1)$, $e_2(3;1;2)$, $e_3(1;0;4)$

$e_1`(1;1;-1)$, $e_2`(2;3;-2)$, $e_3`(3;4;-4)$

13. CHIZIQLI OPERATOR

13.1. Agar R^3da chiziqli $\tilde{A}$ operator $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda o'zining $A = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{bmatrix}$ matritsasi bilan berilgan bo'lsa, $\vec{x} = 4\vec{e}_1 - 3\vec{e}_2 + \vec{e}_3$ vektorning *y=A(x)* aksini toping.

Y=AX formulaga binoan, $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{bmatrix} * \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ -13 \\ -18 \end{bmatrix}$

Demak, $y = 10e_1 - 13e_2 - 18e_3$

13.2. $\vec{e}_1, \vec{e}_2$ bazisda $\tilde{A}$ operator $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ matritsaga ega.

$e_1 = e_1 - 2e_2, \quad e_2 = 2e_2 + e_2$ bazisida $\tilde{A}$ operatorining matritsasini toping. O'tish matritsasi $C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ ning teskari matrisasi $C^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

Demak, $B = C^{-1}AC = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 8 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}$

13.3. Chizqli $\tilde{A}$ operator $A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning hos qiymatlari va hos vektorlarini toping.

Xarakteristik tenglama tuzamiz:

$$|A - \lambda E| = \begin{vmatrix} 1-\lambda & 4 \\ 9 & 1-\lambda \end{vmatrix} = 0, \quad \lambda^2 - 2\lambda - 35 = 0; \quad \lambda_1 = -5, \quad \lambda_2 = 7$$

λ_1=-5 ga tegishli $X^{(1)}=(X_1,X_2)$ hos vektorni topamiz. Buning uchun quyidagi tenglamani echamiz:

$$\lambda_1 = -5 \quad (A - \lambda E) \cdot x = 0 \qquad \begin{pmatrix} 6 & 4 \\ 9 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x_2 = -1{,}5x_1$$

agar $x_1=C$ deb olsak $x_2=-1.5C$, $X^{(1)}=(C;-1.5C)$ vektorlar har qanday $C \neq 0$ uchun *A* operatorini hos qiymati λ_1=-5 ga tegishli hos vector bo'ladi. Huddi shunday λ_2=7 hos qiymati uchun A operatorni hos vektorlarni $X^{(2)} = \left(\frac{2}{3}C_1, C_1\right)$, $C_1 \neq 0$ vektorlar tashkil etadi.

13.4. Chiziqli operatorning $A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsasini diogonal ko'rinishiga keltiring.

$A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matrisa bilan berilgan chiziqli operatorning hos qiymatlari va hos vektorlarning 3-misolda topilgan: λ_1=-5 λ_2=7

$X^{(1)} = (C; -1,5C)$; $X^{(2)} = \left(\frac{2}{3}C_1, C_1\right)$; $X^{(1)}$ *va* $X^{(2)}$ vektorning koordinatalari proportsional emas, shuning uchun $X^{(1)}$ va $X^{(2)}$ vektorlar chiziqli erkli. Demak, $X^{(1)}$ va $X^{(2)}$ bazisda *A*-matritsaning diagonal ko'rinishi:

$A^* = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ *ёки* $A^* = \begin{pmatrix} -5 & 0 \\ 0 & 7 \end{pmatrix}$. Buni tekshirish uchun bazis vektorlar sifatida $X^{(1)}$=(2; -3), $X^{(2)}$=(4; 6) vektorlarni olsak, yangi bazisga o'tkazuvchi o'tish matritsa *C* ning ko'rinish: $\begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix}$ bo'ladi. Dioganal matrtisa:

$$A^* = C^{-1}AC = \begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix} = \frac{1}{24} \begin{pmatrix} 6 & -4 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix} = \frac{1}{24} \begin{pmatrix} -30 & 20 \\ 21 & 14 \end{pmatrix}$$
$$\begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix} = \frac{1}{24} \begin{pmatrix} -120 & 0 \\ 0 & 168 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 0 & 7 \end{pmatrix}$$

Mustaqil yechish uchun misollar:

13.5. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli $\tilde{A}$ operator $A = \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix}$ matritsa bilan berilgan, $x=4e_1-3e_2$ bo'lsa, $y=A(x)$ ni toping.

13.6. $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator $\begin{pmatrix} -1 & 0 & 2 \\ 2 & 1 & -7 \\ 3 & 0 & 1 \end{pmatrix}$ matritsa bilan berilgan $x=2e_1-4e_2-e_3$ bo'lsa, $y=A(x)$ ni toping.

13.7. $\vec{e}_1, \vec{e}_2$ bazisda $\tilde{A}$ operatorlar $A = \begin{pmatrix} 2 & 4 \\ -3 & 3 \end{pmatrix}$ matritsaga ega.

$e_1' = e_2 - 2e_1$, $e_2' = 2e_1 - 4e_2$ bazisda $\tilde{A}$ operatorning matritsasini toping.

Berilgan matritsalarning hos qiymatlari va hos vektorlarini toping:

13.8. $A=\begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix}$

13.9. $A=\begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}$

13.10. $A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}$

13.11. $\vec{e}_1, \vec{e}_2, \vec{e}_3$, bazisdan $\vec{e}_2, \vec{e}_3, \vec{e}_1$ bazisga o'tih matrisasini toping.

13.12. $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$ bazisda $\tilde{A}$ operatorining matritsasi

$$A=\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & -1 & 2 \\ 2 & 5 & 0 & 1 \\ 1 & 2 & 1 & 3 \end{pmatrix}$$ berilgan. Ushbu operatorning

1) $\vec{e}_1, \vec{e}_3, \vec{e}_2, \vec{e}_4$ bazisdagi matritsasini toping;

2) $e_1;\ e_1+e_2;\ e_1+e_2+e_3;\ e_1+e_2+e_3+e_4$ bazisdagi matritsasini toping.

O'zlarining matritsalari bilan berilgan chiziqli operatorlarning hos qiymatlari va hos vektorlarini toping:

13.13. $A=\begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$;

13.14. $A=\begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix}$;

13.15. $A=\begin{pmatrix} 1 & -3 & 3 \\ 2 & 6 & 3 \\ -1 & -4 & 8 \end{pmatrix}$;

13.16. $A=\begin{pmatrix} 7 & -12 & 6 \\ 10 & -19 & 10 \\ 12 & -24 & 13 \end{pmatrix}$;

13.17. $A=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

Chiziqli operatorning $\tilde{A}$ matritsasini diogonal ko'rinishiga keltiring:

13.18. $A=\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ -2 & -2 & 1 \end{pmatrix}$

13.19. $A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -6 & 1 & 7 & -1 \end{pmatrix}$

14. KVADRATIK FORMALAR

14.1. $L(x_1, x_2, x_3) = 4x_1^2-12x_1x_2-10x_1x_3+x_2^2-3x_3^2$ kvadratik formaning *A* matritsasini tuzing. Kvadratik formaning matritsasini topamiz:

$$L = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 4 & -6 & -5 \\ -6 & 1 & 0 \\ -5 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad A = \begin{pmatrix} 4 & -6 & -5 \\ -6 & 1 & 0 \\ -5 & 0 & -3 \end{pmatrix}$$

14.2. $L(x_1, x_2) = 2x_1^2+4x_1x_2-3x_2^2$ kvadratik forma berilgan.

$x_1=2y_1-3y_2;$ $x_2=y_1+y_2;$ chiziqli almashtirish orqali hosil bo'lgan

$L(y_1, y_2)$ kvadratik formani toping.

Berilgan kvadratik formaning matritsasi $A=\begin{pmatrix} 2 & 2 \\ 2 & -3 \end{pmatrix}$ chiziqli almashtirish matritsasi $C=\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$ bo'ladi.

Qidirilayotgan kvadratik formaning matritsasini quyidagicha:

$$A' = C^t \cdot A \cdot C = \begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 13 & -17 \\ -17 & 3 \end{pmatrix}$$

kvadratik formaning ko'rinishi:

$L(y_1, y_2)=13y_1^2-34y_1y_2+3y_2^2$

14.3. Kvadratik formani – kanonik ko'rinishga keltiring.

$L(x_1, x_2, x_3) = x_1^2-3x_1x_2+4x_1x_3+2x_2x_3+x_3^2 = x_1^2-x_1(3x_2-4x_3)+2x_2x_3+x_3^2$

x_1 o'zgaruvchining kvadrati o'rnida turgan koeffitsiyenti nol'dan farqli bo'lgani uchun, x_1 o'zgaruvchining to'liq kvadratini topamiz:

$$L = \left[x_1^2 - 2x_1\left(\frac{1}{2}(3x_2 - 4x_3)\right) + \left(\frac{1}{2}(3x_2 - 4x_3)\right)^2 \right] - \left(\frac{1}{2}(3x_2 - 4x_3)\right)^2 + 2x_2x_3 + x_3^2 =$$

$$\left(x_1 - \frac{3}{2}x_2 + 2x_3\right)^2 - \frac{9}{4}x_2^2 + 8x_2x_3 - 3x_3^2$$

endi o'zgaruvchi x_2 uchun kvadratini topamiz:

$$L = \left(x_1 - \frac{3}{2}x_2 + 2x_3\right)^2 - \frac{9}{4}\left(x_2 - \frac{16}{9}x_3\right)^2 + \frac{37}{9}x_3^2,$$

Demak, no'ldan farqli chiziqli almashtirish

$$y_1 = x_1 - \frac{3}{2}x_2 - 2x_3$$

$$y_2 = x_2 - \frac{16}{9}x_3$$

$y_3=y_3$ berilgan kvadratik formani kanonik ko'rinishga keltiradi:

$$L(y_1,\ y_2,\ y_3)=y_1^2-\frac{9}{4}y_2^2+\frac{37}{9}y_3^2$$

14.4. Kvadratik forma $L=13x_1^2-6x_1x_2+5x_2^2$ musbat aniqlangan kvadratik forma ekanligini isbotlang.

Kvadratik formaning matritsasi $A=\begin{pmatrix}13 & -3\\ -3 & 5\end{pmatrix}$ bo'ladi.

Xarakteristik tenglama tuzamiz:

$$|A-\lambda E|=\begin{vmatrix}13-\lambda & -3\\ -3 & 5-\lambda\end{vmatrix} \quad \text{yoki} \quad \lambda^2-18\lambda+56=0$$

ya'ni $\lambda_1=14,\quad \lambda_2=4$ xarakteristik tenglamaning yechimlari musbat bo'lgani uchun, *L*-musbat aniqlangan kvadratik forma bo'ladi.

Mustaqil yechish uchun misollar:

14.5. Kvadratik formani matritsa ko'rinishida yozing:

$$L=2x_1^2+3x_2^2-x_3^2+4x_1x_2-6x_1x_3+10x_2x_3$$

14.6. Kvadratik formaning matritsasini toping:

$$L(x_1,\ x_2,\ x_3)=(x_1\ \ x_2\ \ x_3)\begin{pmatrix}-1 & 0 & 2\\ 2 & 4 & 1\\ 3 & 0 & -1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

14.7. Kvadratik forma $L(x_1,\ x_2)=3x_1^2-x_2^2+4x_1x_2$, berilgan.

$x_1=2y_1-y_2$, $x_2=y_1-y_2$, chiziqli almashtirish orqali hosil bo'lgan kvadratik formani toping.

14.8. $x_1^2+4x_2^2+3x_3^2+2x_1x_2$

14.9. $-2x_2^2-x_1^2-x_1x_3+2x_2x_3-2x_3^2$

14.10. $x_1^2+26x_2^2+10x_1x_2$

15. TEKISLIKDAGI TO'G'RI CHIZIQ TENGLAMALARI. TO'G'RI CHIZIQ NORMAL TENGLAMASI. NUQTADAN CHIZIQQACHA BO'LGAN MASOFA

1^0. *Tekislikdagi* $A(x_1;\ u_1)$ va $B(x_2;\ u_2)$ nuqtalar orasidagi masofa:

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad (1)$$

2^0. Tenglikda *yo'naltirilgan kesmaning* yoki *boshi* $A(x_1;u_1)$ va *oxiri* $B(x_2;u_2)$ bo'lgan $\overline{AB}$ *vektorning* koordinata o'qlaridagi *proyektsiyalari:*

$$\Pr_x\overline{AB}=X=x_2-x_1,\qquad \Pr_u\overline{AB}=U=u_2-u_1 \qquad (2)$$

3^0. *Kesmani berilgan nisbatta bo'lish:* $A(x_1;u_1)$ va $B(x_2;u_2)$ nuqtalar berilgan *AB* kesmani *AN:NB*= λ nisbatda bo'luvchi *N(x;u)* nuqtaning koordinatalari ushbu:

$$x=\frac{x_1+\lambda x_2}{1+\lambda}, \qquad u=\frac{y_1+\lambda y_1}{1+\lambda} \qquad (3)$$

formulalar bilan aniqlanadi. Xususiy holda kesmani teng ikkiga, ya'ni $\lambda=1:1=1$ nisbatda bo'lganda

$$\text{x}=\frac{x_1+x_2}{2}, \quad \text{u}=\frac{y_1+y_2}{2} \qquad (4)$$

4^0. Uchlari $A(x_1;\ u_1)$, $B(x_2;\ u_2)$, $C(x_3;\ u_3)$, ..., $F(x_n;\ u_n)$ nuqtalarda bo'lgan *ko'pburchak yuzi:*

$$S=\pm\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}+\begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix}+\ldots+\begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right] \qquad (5)$$

ga teng.

5^0. *To'g'ri chiziqning burchak koeffitsientli tenglamasi:*

$$u=kx+b \qquad (6)$$

k parametr to'g'ri chiziqning *Ox* o'qqa og'ish burchagi α ning tangensiga teng bo\`lib ($k=tg\alpha$), to'g'ri chiziqning burchak koeffitsenti, ba'zan qiyaligi deyiladi. *b* parametr boshlang'ich ordinata yoki *Oy* o'q ajratgan kesma kattaligi .

6^0. *To'g'ri chiziqning umumiy tenglamasi:*

$$Ax+By+C=0 \qquad (A^2+B^2\neq 0) \qquad (7)$$

Xususiy hollar:

a) *C*=0 bo'lsa, $y=-\frac{A}{B}x$ to'g'ri chiziq koordinatalar boshidan o'tadi;

b) *B*=0 bo'lsa, $x=-\frac{C}{A}=a$ to'g'ri chiziq *0*x o'qqa parallel bo'ladi;

c) *A*=0 bo'lsa, $y=-\frac{C}{B}=b$ to'g'ri chiziq *0y* o'qqa parallel bo'ladi;

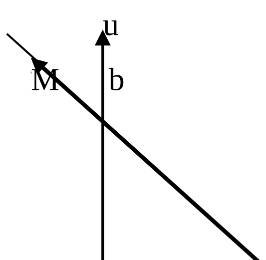
d) *B*=*C*=0 bo'lsa, *Ax*=0 yoki *x*=0 - to'g'ri chiziq *0y* o'qdan iborat;

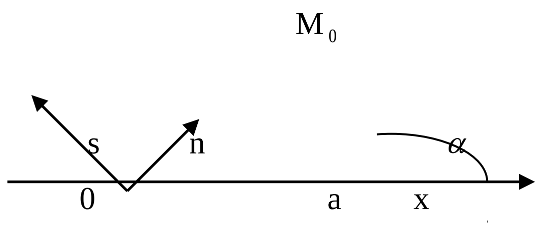
e) *A*=*C*=0 bo'lsa, *By*=0 yoki *u*=0 - to'g'ri chiziq *0x* o'qdan o'tadi.

7^0. *To'g'ri chiziqning o'qlardan ajratgan kesmalari bo'yicha tenlamasi*:

$$\frac{x}{a}+\frac{y}{b}=1 \qquad (8)$$

Bu yerda *a* va *b* - to'g'ri chiziqning o'qlardan kesgan kesmalarining kattaliklari.





8^0. *To'g'ri chiziqning vektor parametrli tenglamasi:*

$$\overline{M_0M} = ts \qquad (9)$$

Bu yerda *M(x;y)* to'g'ri chiziqning ixtiyoriy nuqtasi $\overline{M_0M}$ $(x-x_0;\ y-y_0)$ vektor va *s(m;n)* yo'naltiruvchi vektori o'zaro kollinear, t-ixtiyoriy haqiqiy son yoki parametr.

9^0. (9) tenglamani koordinatalarda

$$\begin{cases} x - x_0 = tm \\ y - y_0 = tn \end{cases} \qquad (10)$$

ifodalab, *to'g'ri chiziqning parametrli tenglamasini* hosil qilish mumkin.

10^0. (10) tenglamalarda *t* parametr yo'qotilsa, to'g'ri chiziqning kanonik teglamasi hosil bo'ladi:

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} \qquad (11)$$

11^0. Agar $|\overline{a}| = P$ $(P \geq 0)$, $\quad v = \frac{\overline{a}}{P} = (\text{Cos}\,\alpha,\ \text{Cos}\,\beta)$ $\quad \overline{a}$ normal radius vektorninig birlik vektori bo'lib, to'g'ri chiziqning ixtiyoriy *M(x;y)* nuqtasining mos radius vektori $\overline{r}$ (x;y) bo'lsa, u holda $\overline{r}$ radius vektorning $\overline{a}$ yoki $\overline{v}$ vektordagi sonli proyektsiyasi *P* ga teng:

$$\text{P}r_v\overline{r} = P, \quad \text{yoki} \quad |v|\text{P}r_v\overline{r} = P, \quad \text{yoki} \quad (r\cdot v) = P\ (P \geq 0) \qquad (12)$$

Bu tenglama to'g'ri chiziqning *vektor ko'rinishdagi tenglamasi* deyiladi.

(12) tenglama koordinatalarda

$$x\text{Cos}\,\alpha + y\text{Cos}\,\beta = P \quad \text{yoki} \quad x\text{Cos}\,\alpha + y\text{Sin}\,\alpha = P\ (P \geq 0) \qquad (13)$$

ko'rinishni oladi. Bunda α - $\overline{a}$ yoki $\overline{v}$ vektorning *Ox* o'qining musbat yo'nalishi bilan hosil qilgan burchak kattaligi. (13) shakldagi tenglama to'g'ri chiziqning *normal tenglamasi* deyiladi.

12^0. (7) shakldagi tenlamadan (13) shakldagi tenglamaga o'tish uchun umumiy ko'rinishdagi tenglama normallovchi ko'paytuvchi deb ataladigan $\mu = \pm\frac{1}{\sqrt{A^2 + B^2}}$ songa ko'paytiriladi, bunda "+" yoki "–" ishoradan *C* ozod had ishorasining qarama–qarshisi tanlanadi, aks holda $P = -\mu\,\text{C} \geq 0$ munosabat bajarilmaydi.

Masala: $3x+4y-8=0$ tenglamani normal ko'rinishga keltiring .

Berilgan umumiy shakldagi tenglama uchun normallovchi ko'paytuvchi

$$\mu = \pm\frac{1}{\sqrt{3^2+4^2}} = \frac{1}{5}.$$

Tenglamani, $\mu = \frac{1}{5}$ ga ko'paytiramiz, natijada to'g'ri chiziq tenglamasi quyidagi ko'rinishda normal holga keltiriladi:

$$\frac{3}{5}x + \frac{4}{5}y = \frac{8}{5}.$$

13^0. $y=k_1x+b_1$ to'g'ri chiziqdan $y=k_2x+b_2$ to'g'ri chiziqqacha soat strelkasiga qarshi yo'nalishda hisoblanuvchi φ *burchak*

$$tg\varphi = \frac{k_2-k_1}{1+k_1\,k_2} \qquad (14)$$

formula bilan aniqlanadi.

14^0. $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ tenglamalar bilan berilgan to'g'ri chiziqlar uchun (14) formula quyidagi ko'rinishga ega bo'ladi:

$$tg\varphi = \frac{A_1B_2-A_2B_1}{A_1A_2+B_1B_2} \qquad (15)$$

yoki $$Cos\varphi = \frac{(n_1\cdot n_2)}{|n_1|\cdot|n_2|} = \frac{A_1A_2+B_1B_2}{\sqrt{A_1^2+B_1^2}\bullet\sqrt{A_2^2+B_2^2}} \qquad (16)$$

15^0. To'g'ri chiziqning *parallellik* sharti:

$$k_1=k_2 \quad \text{yoki} \quad \frac{A_1}{A_2}=\frac{B_1}{B_2} \qquad (17)$$

16^0. To'g'ri chiziqning *perpendikulyarlik* sharti:

$$k_1\bullet k_2=-1 \quad \text{yoki} \quad A_1A_2+B_1B_2=0 \qquad (18)$$

17^0. Berilgan $A(x_1;u_1)$ nuqtadan o'tuvchi *to'g'ri chiziqlar dastasining tenglamasi:* $u-y_1=k(x-x_1)$ (19)

18^0. Berilgan ikki $A(x_1;u_1)$ va $B(x_2;u_2)$ *nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi:*

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \qquad (20)$$

19^0. Parallel bo'lmagan ikki $A_1x+B_1u+S_1=0$ va $A_2x+B_2u+S_2=0$ to'g'ri chiziqning *kesishish nuqtasini* topish uchun ularning tenglamalarini birgalikda yechish bilan

$$x=\frac{\begin{vmatrix}-C_1 & B_1\\ -C_2 & B_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}}, \quad u=\frac{\begin{vmatrix}A_1 & -C_1\\ A_2 & -C_2\end{vmatrix}}{\begin{vmatrix}A_1 & B_1\\ A_2 & B_2\end{vmatrix}} \qquad (21)$$

ni hosil qilamiz.

20^0. *$(x_0; u_0)$ nuqtadan to'g'ri chiziqqacha bo'lgan d masofani topish uchun* to'g'ri chiziq normal tenglamasining chap tomonidagi o'zgaruvchi koordinatalar o'rniga *$(x_0; u_0)$* koordinatalarni qo'yib, hosil bo'lgan sonning absolyut qiymatini olamiz, ya'ni

$$d=|x_0\cos\beta+y_0\sin\beta-P| \qquad (22)$$

yoki $$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}} \qquad (23)$$

21^0. *$Ax+Bu+C=0$* va $A_1x+B_1y+C_1=0$ to'g'ri chiziqlar orasidagi burchaklar *bissektrissalarining* tenglamalari:

$$\frac{Ax+By+C}{\sqrt{A^2+B^2}}=\pm\frac{A_1x+B_1y+C_1}{\sqrt{A_1^2+B_1^2}} \qquad (24)$$

22^0. Berilgan ikki to'g'ri chiziqning kesishish nuqtasidan o'tuvchi to'g'ri chiziqlar *dastasining tenglamasi:*

$$\alpha(Ax+By+C)+\beta(A_1x+B_1y+C_1)=0 \qquad (25)$$

$\alpha=1$ deb olish mumkin, u holda biz (25) dastadan berilgan to'g'ri chiziqlardan ikkinchisini yo'qatgan bo'lamiz, ya'ni u vaqtda (25) dan ikkinchi to'g'ri chiziqning tenglamasini hosil qila olmaymiz.

Mustaqil yechish uchun misollar:

15.1. $\frac{x+2\sqrt{5}}{4}+\frac{y-2\sqrt{5}}{2}=0$ to'g'ri chiziq berilgan. To`g`ri chiziqning

a) umumiy tenglamasi,

b) burchak koeffitsientli tenglamasi,

c) kesmalarga nisbatan tenglamasini yozing.

15.2. *$4x+3u-36=0$* to'g'ri chiziq, koordinata o'qlari bilan hosil qilgan uchburchakning yuzini toping.

15.3. To'g'ri chiziq koordinata o'qlaridan teng kesmalar ajratadi. Agar to'g'ri chiziq koordinata o'qlari bilan hosil qilgan uchburchak yuzi 8 *kv.birl.* bo'lsa, to'g'ri chiziq tenglamasini yozing.

15.4. $A(2;5)$ nuqtadan o'tuvchi va ordinata o'qida $b=7$ kesma ajratuvchi to'g'ri chiziq tenglamasini yozing.

15.5. Agar to'g'ri chiziq koordinata o'qlaridan teng kesmalar ajratsa va to'g'ri chiziqni koordinata o'qlari orasidagi kesmasi $5\sqrt{2}$ ga teng bo'lsa, to'g'ri chiziq tenglamasini yozing.

15.6. $u=-2$, $u=4$ to'g'ri chiziqlar $3x-4u-5=0$ to'g'ri chiziqni A va B nuqtalarda kesib o'tadi. $\overline{AB}$ vektorni uzunligi va uni koordinata o'qlaridagi proyektsiyalarini toping.

15.7. To'g'i chiziqlar orasidagi burchakni toping:

1) $\begin{cases} y = 2x-3 \\ y = \frac{1}{2}x+1 \end{cases}$ 2) $\begin{cases} 5x- \ y+7=0 \\ 2x-3y+1=0 \end{cases}$ 3) $\begin{cases} 2x+y=0 \\ y=3x-4 \end{cases}$

15.8. $3x-2u+7=0$, $6x-4u-9=0$, $6x+4u-5=0$, $2x+3u-6=0$ to'g'ri chiziqlar orasidan parallel va perpendikulyar to'g'ri chiziqlarni aniqlang.

15.9. $A(2;3)$ nuqtadan o'tuvchi to'g'ri chiziqlar dastasini yozing. Bu dastadan Ox o'qi bilan 1) 45^0, 2) 60^0, 3) 135^0, 4) 0^0 burchaklar tashkil etuvchi to'g'ri chiziqni toping.

15.10. $A(-2;5)$ nuqta va $2x-u=0$ to'g'ri chiziqni yasang. A nuqtadan o'tuvchi va

1) berilgan to'g'ri chiziqqa parallel

2) berilgan to'g'ri chiziqqa perpendikulyar to'g'ri chiziq tenglamasini yozing.

15.11. $2x-5u-10=0$ to'g'ri chiziqni koordinata o'qlari bilan kesishish nuqtalariga perpendikulyar qo'yilgan. Ularning tenglamasini yozing.

15.12. $A(-1;3)$ va $B(4;-2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini yozing.

15.13. Uchlari $A(-2;0)$, $B(4;-2)$ va $C(4;2)$ bo'lgan uchburchakka BD balandlik va BE mediana o'tkazilgan. AC tomon, BE mediana va BD balandlik tenglamalarini yozing.

15.14. Uchburchak tomonlari quyidagi tenglamalar bilan berilgan:

$x+3u=0$, $x=3$, $x-2u+3=0$. Uchburchakni burchaklari va uchlarini toping.

15.15. Kvadrat tomonlaridan birining tenglamasi $x+3u-7=0$ va diogonallari kesishgan nuqta $P(0;-1)$ berilgan. Kvadratning qolgan uchta tomon tenglamalarini yozing.

15.16. Romb tomonlaridan birining tenglamasi $5x+2u-9=0$. Agar romb diogonallari $O(0;0)$ da kesishgan bo'lib, ulardan birining tenglamasi $u=2x$ bo'lsa, pombning qolgan uchta tomon tenglamasini yozing.

15.17. Uchburchak tomonlarining o'rtasi berilgan $P(1;2)$ - AB tomonining o'rtasi, $R(-4;3)$ - BC tomonining o'rtasi, $Q(5;-1)$ - AC tomonining o'rtasi, CF balandlik va AR mediana kesishgan nuqta topilsin.

15.18. Rombning ikki qarama-qarshi uchlarining koordinatalari berilgan, $A(1;-4)$ $C(-1;3)$. Romb diogonallarining tenglamasini yozing.

15.19. Agar $A(-5;5)$ va $B(3;1)$ uchburchakning uchlari, $D(2;5)$ esa balandliklari kesishgan nuqta bo’lsa, uchburchak tomonlarining tenglamasini yozing.

15.20. $2x+2u-5=0$ to’g’ri chiziq Ox o’qining musbat yo’nalishi bilan qanday burchak hosil qiladi?

15.21. Ou o’qidan $b=1$ birlikka teng kesma ajratuvchi Ox o’qining musbat yo’nalishi bilan $\alpha=\frac{2\pi}{3}$ burchak hosil qiluvchi to’g’ri chiziq tenglamasini yozing.

15.22. Koordinata boshidan va $A(-2;-3)$ nuqtadan o’tuvchi to’g’ri chiziq tenglamasini yozing.

15.23. $M(-3;-4)$ nuqtadan o’tuvchi koordinata o’qlariga parallel to’g’ri chiziqlar tenglamasini yozing.

15.24. $O(0;0)$ va $A(-3;0)$ nuqtalar berilgan OA kesmada parallelogramm yasalgan, uning diogonallari $B(0;2)$ nuqtada kesishadi. Parallelogramm tomonlari va diogonallari tenglamasini yozing.

15.25. Tomonlari 8 *sm* va 2 *sm* bo’lgan teng yonli trapetsiyaning o’tkir burchagi 45^0. Trapetsiyaning katta asosi Ox o’qida yotsa, Ou o’qi esa trapetsiyaning simmetriya o’qi bo’lsa, trapetsiyaning tomonlari tenglamasini yozing.

15.26. Agar to’g’ri chiziq koordinata o’qlari bilan hosil qilgan uchburchak yuzi 6 *kv.b.* bo’lsa va to’g’ri chiziq (-4;6) nuqtadan o’tsa, uning tenglamasini yozing.

15.27. To’g’ri chiziqlar orasidagi burchakni toping:

a) $\begin{cases} 3x+2y=0 \\ 6x+4y+9=0 \end{cases}$ b) $\begin{cases} 3x-4y=0 \\ 8x+6y=11 \end{cases}$

15.28. Uchlari $A(-2;0)$, $B(2;4)$ va $C(4;0)$ bo’lgan uchburchak berilgan. Uchburchak tomonlari, AE medianasi, BD balandlik tenglamalarini, AE mediana uzunligini toping.

15.29. Tomonlari $x+u=4$, $3x-u=0$, $x-3u-8=0$ tenglamalar bilan berilgan uchburchakni burchaklari, uchlari va uchburchakni yuzini toping.

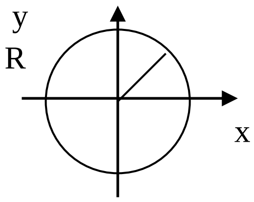
15.30. Koordinatalar boshidan $2x+u=a$ to’g’ri chiziq bilan teng yonli uchburchak hosil qiluvchi ikki o’zaro perpendikulyar to’g’ri chiziq o’tkazilgan. Shu uchburchakning yuzini toping.

Ko’rsatma: $2x+u=3$ *bilan* $u=kx$ *va* $u=-\frac{x}{k}$ *to’g’ri chiziqlarning kesishgan nuqtalari* M *va* N *ning koordinatalarini topgandan so’ng* $OM=ON$ *tenglikdan* k *ni topish kerak.*

16. IKKINCHI TARTIBLI EGRI CHIZIQLAR

1[0]. Markazi koordinata boshida, radiusi *R* bo’lgan *aylana* tenglamasi(1-rasm):

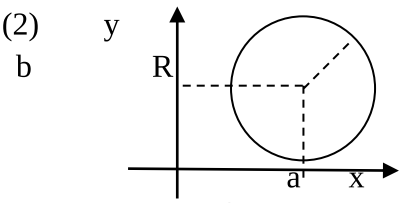
$$x^2 + y^2 = R^2 \qquad (1)$$



1- rasm.

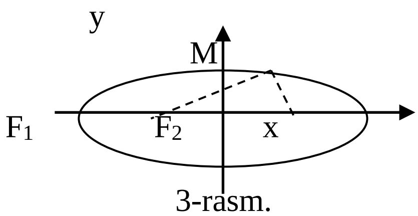
2[0]. Markazi *(a;b)* nuqtada, radiusi *R* bo’lgan *aylana* tenglamasi (2-rasm):

$$(x-a)^2 + (y-b)^2 = R^2 \qquad (2)$$



2-rasm.

3[0]. *Ellips* (3-rasm):



3-rasm.

Fokus deb ataluvchi $F_1(-c;0)$ va $F_2(c;0)$ nuqtalardan $|F_1M|+|F_2M|=2a$ masofaga teng ixtiyoriy *M(x;y)* nuqtalar to’plami ellips deyeladi. F_1M va F_2M kesmalar *fokal radiuslar* deyiladi, hamda

$$|F_1M| = \sqrt{(x+c)^2 + y^2}$$

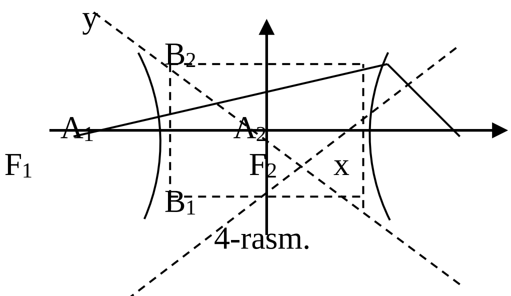
$$|F_2M| = \sqrt{(x-c)^2 + y^2} \qquad (3)$$

ga teng. Ellipsning *kanonik tenglamasi*:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (4)$$

bunda $b = \sqrt{a^2 - c^2}$. Ellipsning *kichik yarim o’qi b*, *katta yarim o’qi a*. *Markazi* esa *O(0;0)* – koordinata boshi. Ellipsning *uchlari (-a;0), (a;0), (0;-b), (0;b)*. Ellipsning *simmetriya markazi O(0;0), simmetriya o’qlari Ox, Oy* o’qlar. Ellipsning *ekstsentrisiteti* $\varepsilon = \frac{c}{a} < 1$ ga aytiladi.

4[0]. *Giperbola* (4-rasm):



4-rasm.

Fokuslar $F_1(-c;0)$ va $F_2(c;0)$ gacha bo’lgan masofalar ayirmasi.

$$\left\| F_1M \right| - \left| F_2M \right\| = 2a$$

ga teng ixtiyoriy *M(x;y)* nuqtalar to’plamiga *giperbola* deyiladi.
Kanonik tenglamasi:

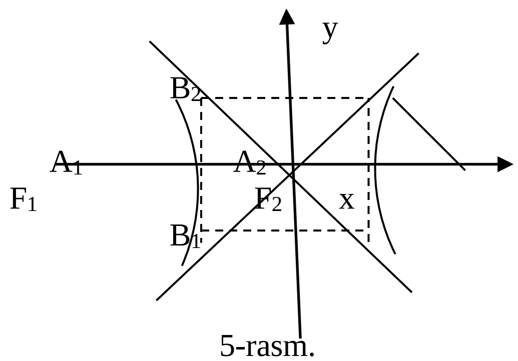
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (5)$$

bunda $b = \sqrt{c^2 - a^2}$. *Haqiqiy uchlari*: *A_1(-a;0), A_2(a;0); mavhum uchlari:B_1(0;-b), B_2(0;b).* Giperbolaning asimtotalari: $y = \frac{b}{a}x$ (I va III choraklardan o'tadi) va

$y = -\frac{b}{a}x$ (II va IV choraklardan o'tadi).

Yarim o'qlari teng, ya'ni $a = b$ giperbolaga *teng tomonli giperbola* deyiladi (5-rasm) va $x^2 - y^2 = a^2$ ko'rinishida ifodalanadi.

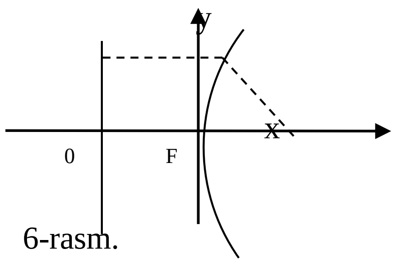
Ekstrisentrisiteti: $\varepsilon = \frac{c}{a} > 1$



5-rasm.

5^0. *Parabola* (6-rasm): *Fokusi* $F(\frac{p}{2};0)$ dan va *direktrisasi* $x = -\frac{p}{2}$ to'g'ri chizig'igacha teng masofada yotuvchi ixtiyoriy *M(x;y)* nuqtalar to'plamiga *parabola* deyiladi. Parabolaning *kanonik tenglamasi:*

$$y^2 = 2px \qquad (6)$$



6-rasm.

Parabolaning uchi koordinata boshi *O(0;0). Fokusdan direktrisa* to'g'ri chizig'igacha bo'lgan masofa *p* ga teng.

Mustaqil yechish uchun misollar:

16.1. *A(-4;6)* nuqta berilgan. Diametri *OA* kesma bo'lgan aylana tenglamasini tuzing.

16.2. *A(-6;0)* nuqtadan o'tuvchi va *Oy* o'qiga koordinatalar boshida urinuvchi aylana tenglamasini tuzing.

16.3. $x^2+y^2+4x-6y=0$ aylananing *Oy* o'qi bilan kesishgan nuqtalariga o'tkazilgan radiuslari orasidagi burchak topilsin.

16.4. *A (-1;3), B (0;2)* va *C (1;-1)* nuqtalardan o'tuvchi aylana tenglamasi yozilsin. *Ko`rsatma*: Izlanayotgan aylananing tenglamasini $x^2+y^2+mx+ny+p=0$ ko`rinishida yozib, undagi x va y lar o`rniga berilgan har bir nuqtaning koordinatalarini qo`ygandan so`ng m, n va p larni topish kerak.

16.5. *A(4;4)* nuqtadan va $x^2+y^2+4x-4y=0$ aylana bilan $y=-x$ to'g'ri chiziqning kesishgan nuqtalaridan o'tuvchi aylana tenglamasi yozilsin.

Ellips.

16.6. Katta o'qi 8 va kichik o'qi 6 bo'lgan ellipsning tenglamasini tuzing. Ellips tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ dan masalani shartiga ko'ra topamiz. $2a=8, 2b=6;$ ya'ni $a=4, b=3$. Bularni ellips tenglamasiga qo'yamiz. $\frac{x^2}{16}+\frac{y^2}{9}=1$

16.7. $4x^2+9y^2=36$ ellips tenglamasidan uning o'qlari, fokuslari va ekstsentrisitetini toping.

$4x^2+9y^2=36$

Tenglamani ikkala tomonini 36 ga bo`lamiz. $\frac{x^2}{9}+\frac{y^2}{4}=1$ $a^2=9;$ $a=+3;$ $b^2=4;$ $b=+2;$

$c^2=a^2-b^2$ dan $c^2=9-4=5;$ $c=+\sqrt{5}$; $\varepsilon=\frac{\sqrt{5}}{3}$.

Demak, $2a=6;$ $2b=4;$

$F_1(\sqrt{5},0);$ $F_2(-\sqrt{5},0);$ $\varepsilon=\frac{\sqrt{5}}{3}\prec 1$

16.8. Katta yarim o'qi $a=5$ va c parametri

1) 4.8; 2) 4; 3) 3; 4)1.4; 5) 0

Berilgan ellipsni kanonik tenglamasini yozing. Har bir ellipsni chizing va ularning ekstsentrisitetini toping.

16.9. Yer fokuslaridan birida Quyosh joylashgan ellips bo`yicha harakat qiladi. Quyoshdan Yergacha bo'lgan eng kichik masofa taxminan 147.5 *million km* ga, eng katta masofa 152.5 *million km* ga teng bo'lsa, Yer orbitasining katta yarim o'qi va ekstsentrisiteti topilsin.

16.10. Ekstsentrisiteti $\varepsilon=\frac{3}{4}$ bo'lgan va $M=(-4;\sqrt{21})$ nuqtadan o'tuvchi ellips tenglamasini yozing va M nuqtaning fokal radius-vektorlarini toping.

16.11. Koordinata o'qlariga nisbatan simmetrik bo'lgan ellips $M(2;\sqrt{3})$ va $B(0;2)$ nuqtalaridan o'tadi. Uning tenglamasi yozilsin va M nuqtadan fokuslarigacha bo'lagan masofa topilsin.

16.12. $9x^2+25y^2=225$ ellipsda shunday $M(x;y)$ nuqta topilsinki, undan o'ng fokusgacha bo'lgan masofa chap fokusgacha bo'lgan masofadan 4 marta katta bo'lsin.

16.13. Agar ellipsning fokuslari orasidagi masofa uning katta va kichik yarim o'qlarining uchlari orasidagi masofaga teng bo'lsa, uning ekstsentrisiteti topilsin.

Giperbola.

16.14. Fokuslari orasidagi masofa $2\sqrt{11}$ bo'lib, o'zi (9;-4) nuqtadan o'tgan giperbola tenglamasini tuzing.

Shartga asosan $2c=2\sqrt{11}$, bundan $c=\sqrt{11}$. Giperbola (9;-4) nuqtadan o`tganligi uchun bu nuqta giperbola tenglamasini qanoatlantiradi, ya'ni

$$\frac{9^2}{a^2}-\frac{(-4)^2}{b^2}=1$$

$81b^2-16a^2=a^2b^2$

$a^2+b^2=c^2=11$ buni ellips tenglamasiga qo`yamiz.

$81b^2-16(11-b^2)=(11-b^2)b^2$

$b^4-86b^2-176=0$

$b_1^2=2;$ $b_2^2=-83$

$a^2=11-b^2=9$

Demak, giperbola tenglamasi quyidagicha bo`ladi: $\frac{x^2}{9}-\frac{y^2}{2}=1$

16.15. $16x^2-2y^2=400$ giperbola tenglamasi berilgan. Uning o'qlari, fokuslari, ekstsentrisitetini toping va asimptotasinining tenglamasini tuzing.

16.16. Giperbolaning ekstsentrisiteti $\sqrt{2}$ ga teng va $M\ (2a;\ a\sqrt{3})$ nuqtadan o'tadi. Giperbolani sodda teglamasini tuzing.

16.17. Giperbolani fokuslari $F_1(-\sqrt{7};0)$ va $F_2\ (\sqrt{7};0)$ nuqtalarda joylashgan. Agar Giperbola $A(2;0)$ nuqtadan o'tsa, uning asimptotalari tenglamasini tuzing.

16.18. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning fokusidan asimptotalarigacha bo'lgan masofalar va asimptotalari orasidagi burchak topilsin.

16.19. Biror uchidan fokuslarigacha bo'lgan masofalari 9 va 1 ga teng bo'lgan giperbolaning kanonik tenglamasi yozilsin.

16.20. $M\left(6;\frac{3}{2}\sqrt{5}\right)$ nuqtadan o'tuvchi, koordinata o'qlariga nisbatan simmetrik bo'lgan giperbolaning haqiqiy yarim o'qi $a=4$. Giperbolaning chap fokusidan asimptotalariga tushirilgan perpendikulyarning tenglamalari yozilsin.

Parabola.

16.21. Parabola (3;5) nuqtadan o'tadi. Uning kanonik tenglamasini yozing.

Parabola (3;5) nuqtadan o'tganligi uchun tenglamasini qanoatlantiradi.

$y^2=2px$ $\quad$ $x=3$ $\quad$ $y=5$

$25=2\cdot p\cdot 3$ $\quad$ $25=6p$ $\quad$ $p=\frac{25}{6}$

Demak, $y^2=2\cdot\frac{25}{6}x$ $\quad$ $y^2=\frac{25}{3}x$ - parabolaning kanonik tenglamasi.

16.22. 1) (0;0) va (1;-3) nuqtalardan o'tuvchi va Ox o'qqa nisbatan simmetrik;

2) (0;0) va (2;-4) nuqtalardan o'tuvchi va Oy o'qqa nisbatan simmetrik bo'lgan parabola tenglamasi yozilsin.

16.23. Agar parabola $x=y$ to'g'ri chiziq va $x^2+6x+y^2=0$ aylananing kesishish nuqtalaridan o'tsa, uning tenglamasi va direktrisasini yozing.

16.24. $y^2=6x$ parabolada fokal radius vektor 4.5 ga teng bo'lgan nuqtani toping.

16.25. $A(-1;3)$, $B(0;2)$ va $C(1;-1)$ nuqtalardan o'tuvchi aylana tenglamasini yozing.

16.26. Ellips $M(2\sqrt{3};\sqrt{6})$ va $A(6;0)$ nutalardan o'tadi. Uning tenglamasini, ekstsentrisiteti va M nuqtadan fokuslargacha bo'lgan masofani yozing.

16.27. $x^2+4y^2=4$ ellipsning, markazi shu ellipsning "yuqori" uchida bo'lgan va uning fokuslaridan o'tuvchi aylana bilan umumiy nuqtalari topilsin.

16.28. $y^2=a^2+x^2$ giperbola fokuslari koordinatalarini va asimptotalari orasidagi burchakni toping.

16.29. Uchlari $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellipsning fokuslarida, fokuslari esa uning uchlarida bo'lgan giperbola tenglamasini yozing.

16.30. 1) (0;0) va (-1;2) nuqtalardan o'tuvchi va Ox o'qiga simmetrik.

2) (0;0) va (2;4) nuqtalardan o'tuvchi va Oy o'qiga simmetrik bo'lgan parabola tenglamasini yozing.

16.31. Markazi $y^2=2px$ parabolaning fokusida bo'lib, parabola direktrisasiga urinuvchi aylana tenglamasi yozilsin. Parabola va aylananing kesishgan nuqtalari topilsin.

17. FAZODA TEKISLIK TENGLAMALARI

1^0. Uch o'lchovli *Oxyz* koordinatalar sistemasida berilgan tekislik tenglamasi:

$Ax+By+Cz+D=0 \quad (A^2+B^2+C^2 \neq 0)$ (1)

$\overline{N}$ *(A;B;C)* tekislikka perpendikulyar bo'lgan *normal vector* deyiladi.

2^0. $M_1(x_1;y_1;z_1)$ *nuqtadan o'tuvchi* va $\overline{N}(A;B;C)$ vektorga perpendikulyar *tekislik tenglamasi*:

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0 \qquad (2)$$

3^0. $Ax+By+Cz+D=0$ tenglamaning maxsus hollari:

1) D=0 bo'lganda, $Ax+By+Cz=0$ tekislik koordinatalar boshidan o`tadi;
2) C=0 bo'lgan, $Ax+By+=D=0$ tekislik Oz o'qiga parallel;
3) $C=D$=0 bo'lganda, $Ax+By=0$ tekislik Oz o'qidan o'tadi;
4) $B=C$=0 bo'lganda, $Ax+D$=0 tekislik yOz tekislikka parallel;
5) Koordinata tekisliklarining tenglamalari: x=0, y=0 va z=0.

4^0. *Tekislikning koordinata o'qlaridan ajratgan kesmalar bo'yicha tenglamasi*: $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

(3)

5^0. *Ikki tekislik orasidagi burchak*:

$$Cos\alpha = \pm\frac{AA_1+BB_1+CC_1}{\sqrt{A^2+B^2+C^2}\bullet\sqrt{A_1^2+B_1^2+C_1^2}} = \pm\frac{(\overline{N}\bullet\overline{N}_1)}{|\overline{N}|\bullet|\overline{N}_1|} \qquad (4)$$

formuladan topiladi, bunda $\overline{N}$ va $\overline{N}_1$ mos ravishda $Ax+By+Cz+D=0$ va $A_1x+B_1y+C_1z+D_1=0$ tekisliklarga normal vektorlari.

Parallellik sharti: $\frac{A}{A_1}=\frac{B}{B_1}=\frac{C}{C_1}$ (5)

Perpendikulyarlik sharti: $AA_1+BB_1+CC_1=0$ (6)

6^0. $M_0(x_0;y_0;z_0)$ *nuqtadan* o'tuvchi Ax+By+Cz+D=0 *tekislikkacha* bo'lgan *masofa*:

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}} \qquad (7)$$

7^0. Berilgan ikki tekislikning *kesishgan chizig'idan* o'tuvchi barcha *tekisliklar dastasining tenglamasi* quyidagicha yoziladi:

$$\alpha(Ax+By+Cz+D)+\beta(A_1x+B_1y+C_1z+D)=0 \qquad (8)$$

8^0. Bir to'g'ri chiziqda yotmaydigan *uchta* $(x_1;y_1;z_1)$, $(x_2;y_2;z_2)$ va $(x_3;y_3;z_3)$ *nuqtadan o'tuvchi tekislik tenglamalari*:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0 \qquad (9)$$

Mustaqil yechish uchun misollar:

17.1. $M_1(0;-1;3)$ va $M_2(1;3;5)$ nuqtalar berilgan, M_1 nuqtadan o'tuvchi va $N=\overline{M_1}\,\overline{M_2}$ vektorga perpendikulyar tekislik tenglamasi yozilsin.

17.2. $M(a;a;0)$ nuqtadan o'tuvchi va $\overline{OM}$ vektorga perpendikulyar tekislik tenglamasi yozilsin.

17.3. $A(a;-\frac{a}{2};a)$ va $B(0;\frac{a}{2};0)$ nuqtadan teng uzoqlikda bo'lgan nuqtalar geometrik o'rnining tenglamasi yozilsin.

17.4. $M_1(0;1;3)$ va $M_2(2;4;5)$ nuqtalardan o'tuvchi va Ox o'qqa parallel tekislik tenglamasi yozilsin.

17.5. Ox o'qdan va $M(0;-2;3)$ nuqtadan o'tuvchi tekislik tenglamasi yozilsin.

17.6. Oz o'qdan va $M(2;-4;3)$ nuqtadan o'tuvchi tekislik tenglamasi yozilsin.

17.7. Oy o'qqa parallel, Ox va Oz o'qlardan a va c kesmalar ajratuvchi tekislik tenglamasi yozilsin.

17.8. $M(2;-1;3)$ nuqtadan o'tuvchi va koordinata o'qlaridan teng kesmalar ajratuvchi tekislik tenglamasi yozilsin.

17.9. $M(-4;0;4)$ nuqtadan o'tuvchi va Ox va Oy o'qlaridan $a=4$ va $b=3$ kesmalar ajratuvchi tekislikning tenglamasi yozilsin.

17.10. 1) x-2y+2z-8=0 va x+z-6=0

2) x+2z-6=0 va x+2y-4=0

tekisliklar orasidagi burchak topilsin.

17.11. (2;2;-2) nuqtadan o'tuvchi va x-2y-3z=0 tekislikka parallel tekislik topilsin.

17.12. (-1;-1;2) nuqtadan o'tuvchi va x-2y+z-4=0 hamda x+2y-2z+4=0 tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.

17.13. $M(-1;2;3)$ nuqtadan OM ga perpendikulyar tekislik tenglamasi yozilsin.

17.14. Oy o'qdan va (4;0;3) nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.

17.15. Oz o'qqa parallel hamda $M_1(2;2;0)$ va $M_2(4;0;0)$ nuqtalardan o'tuvchi tekislikning tenglamasi yozilsin.

17.16. $M(1;-3;5)$ nuqtadan o'tuvchi va Oy va Oz o'qlardan Ox o'qdagidan ko'ra ikki marta katta kesma ajratuvchi tekislik tenglamasi yozilsin.

17.17. (0;0;*a*) nuqtadan o'tuvchi va x-y-z=0 hamda 2y=x tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.

17.18. M_1(-1;-2;0) va M_2(1;1;2) nuqtalardan o'tuvchi hamda x+2y+2z-4=0 tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

17.19. M_1(1;-1;2), M_2(2;1;2) va M_3(1;1;4) nuqtalardan o'tuvchi tekislikning tenglamasi yozilsin.

17.20. *Oz* o'qdan $2x+y-\sqrt{5}z=0$ tekislik bilan 60^0 burchak tashkil etuvchi tekislik tenglamasi tuzilsin.

17.21. (5;1;-1) nuqtadan x-2y-2z+4=0 tekislikkacha bo`lgan masofa topilsin.

17.22. (4;3;0) nuqtadan M_1(1;3;0), M_2(4;-1;2) va M_3(3;0;1) nuqtalardan o'tuvchi tekislikkacha bo'lgan masofa topilsin.

17.23. 4x+3y-5z-8=0 va 4x+3y-5z+12=0 parallel tekisliklar orasidagi masofa topilsin. *Ko'rsatma. Birinchi tekislikda ihtiyoriy, masalan (2;0;0) nuqta olib, undan ikkinchi tekislikkacha bo'lgan masofa topilsin.*

17.24. 2x-y+3z-9=0; x+2y+2z-3=0; 3x+y-4z+6=0 tekisliklarning kesishgan nuqtasi topilsin.

17.25. (2;-1;1) nuqtadan o'tuvchi va 3x+2y-z+4=0 va x+y+z-3=0 tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.

18. FAZODA TO'G'RI CHIZIQ TENGLAMASI

1^0. *A(a;b;c)* nuqtadan o'tuvchi va *P(m;n;p)* vektorga parallel bo'lgan to'g'ri chiziq tenglamalari. *N(x;y;z)*-to'g'ri chiziqning ixtiyoriy nuqtasi bo'lsin $\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-c}{p}$

(1)

Bu tenglamalar to'g'ri chiziqning *kanonik* tenglamalari deyiladi. *P(m;n;p)* vektor to'g'ri chiziqning yo'naltiruvchi vektori deyiladi.

2^0. (1) tenglamadagi har bir nisbatni t parametrga tenglab, to'g'ri chiziqning $\begin{cases} x = mt+a \\ y = nt+b \\ z = pt+c \end{cases}$

(2)

ko'rinishdagi *parametric tenglamalariga* ega bo'lamiz.

3^0. Ikki nuqta $(x_1;y_1;z_1)$ va $(x_2;y_2;z_2)$ dan o'tuvchi to'g'ri chiziq tenglamalari:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \qquad (3)$$

4^0. To'g'ri chiziqning *umumiy tenglamalari*:

$$\begin{cases} Ax + By + Cz + D = 0 \\ A_1x + B_1y + C_1z + D_1 = 0 \end{cases} \quad (4)$$

5^0. *Ikki to'g'ri chiziq orasidagi burchak*

$$Cos\varphi = \frac{m \bullet m_1 + n \bullet n_1 + p \bullet p_1}{\sqrt{m^2 + n^2 + p^2} \bullet \sqrt{m_1^2 + n_1^2 + p_1^2}} \quad (5)$$

6^0. $\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p}$ *to'g'ri chiziq bilan Ax+By+Cz+D=0 tekislik orasidagi burchak*

$$Sin\varphi = \frac{Am + Bn + Cp}{\sqrt{A^2 + B^2 + C^2} \bullet \sqrt{m^2 + n^2 + p^2}} \quad (6)$$

Parallellik sharti: *Am+Bn+Cp=0* (7)

Perpendikulyarlik sharti: $\frac{A}{m} = \frac{B}{n} = \frac{C}{p}$ (8)

7^0. *Tekislik bilan to'g'ri chiziqning kesishgan nuqtasi* (2) ko'rinishidagi to'g'ri chiziqning parametrik tenglamalari tekislikning *Ax+By+Cz+D=0* tenglamasidagi *x, y, z* larning *t* ga nisbatan yozilgan qiymatlarini qo'yamiz. Hosil bo'lgan tenglamadan t_0 ni, so'ngra kesishgan nuqta koordinatalari x_0, y_0, z_0 ni topamiz.

8^0. *Ikki to'g'ri chiziqning bir tekislikda yotish sharti:*

$$\begin{vmatrix} a-a_1 & b-b_1 & c-c_1 \\ m & n & p \\ m_1 & n_1 & p_1 \end{vmatrix} = 0 \quad (9)$$

Mustaqil yechish uchun misollar:

18.1. *x=4, y=3* to'g'ri chiziq yasalsin va uning yo'naltiruvchi vektori topilsin.

18.2. 1) $\begin{cases} y = 3 \\ z = 2 \end{cases}$ 2) $\begin{cases} y = 2 \\ z = x + 1 \end{cases}$ 3) $\begin{cases} x = 4 \\ z = y \end{cases}$

To'g'ri chiziqlar yasalsin va ularning yo'naltiruvchi vektorlari aniqlansin.

18.3. *A(-1; 2; 3)* va *B(2; 6; -2)* nuqtalardan o'tuvchi to'g'ri chiziq tenglamalari yozilsin va uning yo'naltiruvchi kosinuslari topilsin.

18.4. *A(2; -1; 3)* va *B(2; 3; 3)* nuqtalardan o'tuvchi to'g'ri tenglamalari yozilsin.

18.5. 1) *(-2; 1; -1)* nuqtadan o'tuvchi va $P\{1; -2; 3\}$ vektorga parallel bo'lgan;

2) *A(3; -1; 4)* va *B(1; 1; 2)* nuqtalardan o'tuvchi to'g'ri chiziqning tenglamalari yozilsin.

18.6. *y=3x-1, 2z=-3x+2* to'g'ri chiziq bilan *2x+y+z-4=0* tekislik orasidagi burchak topilsin.

18.7. $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z-1}{3}$ to'g'ri chiziq *2x+y-z=0* tekislikka parallel ekanligi, $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z+3}{3}$ to'g'ri chiziq esa shu tekislik ustida yotishi ko'rsatilsin.

18.8. *(-1; 2; -3)* nuqtadan o'tuvchi va *x=2, y-z=1* to'g'ri chiziqqa perependikulyar tekislikning tenglamasi yozilsin.

18.9. $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+1}{3}$ to'g'ri chiziqdan va *(3; 4; 0)* nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.

18.10. $\frac{x-1}{1}=\frac{y+1}{2}=\frac{z+2}{2}$ to'g'ri chiziqdan o'tuvchi va *2x+3y-z=4* tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

18.11. (*a; b; c*) nuqtadan o'tuvchi va: 1) *Oz* o'qqa parallel; 2) *Oz* o'qqa perpendikulyar bo'lgan to'g'ri chiziq tenglamalari yozilsin.

18.12. *x=2z-1; y= -2z+1* to'g'ri chiziq bilan *(1; -1; -1)* nuqta va koordinatalar boshidan o'tuvchi to'g'ri chiziq orasidagi burchak topilsin.

18.13. *(2; -3; 4)* nuqtadan *Oz* o'qqa tushirilgan perpendikulyarning tenglamalari yozilsin.

Ko'rsatma. Izlangan to'g'ri chiziq (0; 0; 4) nuqtadan ham o'tadi.

18.14. *N(2; -3; 4)* nuqtadan $\frac{x+1}{3}=\frac{y+2}{4}=\frac{z-1}{5}$ to'g'ri chiziqqacha bo'lgan masofa topilsin.

Ko'rsatma. A(-1; -2; 1) - to'g'ri chiziqdagi nuqta; $P\{3;4;5\}$ *- to'g'ri chiziqning yo'naltiruvchi vektori. U vaqtda* $d = AN\sin\alpha = \frac{AN\left|P \bullet \overline{AN}\right|}{P \bullet AN} = \frac{\left|P \bullet \overline{AN}\right|}{P}$

18.15. $\begin{cases} 2x+y+8z-16=0 \\ x-2y-z+2=0 \end{cases}$ to'g'ri chiziq tenglamalari:

1) proektsiyalari bo'yicha; 2)kanonik ko'rinishda yozilsin. To'g'ri chiziqning koordinatalar tekisliklaridagi izlari topilsin, to'g'ri chiziq va uning proektsiyalari yasalsin.

18.16. *A(0; -4; 0)* nuqtadan o'tuvchi va $P\{1;2;3\}$ vektorga parallel to'g'ri chiziq tenglamalari yozilsin; to'g'ri chiziqning *xOz* tekisligidagi izi topilsin.

18.17. *x*=3, *z*=5 to'g'ri chiziqning yo'naltiruvchi vektori topilsin.

18.18. *(2; -3; 4)* nuqtadan *Oy* o'qqa tushirilgan perpendikulyarning tenglamalari yozilsin.

18.19. $\begin{cases} 2x-y-7=0 \\ 2x-z+5=0 \end{cases}$ *va* $\begin{cases} 3x-2y+8=0 \\ z=3x \end{cases}$

to'g'ri chiziqlar orasidagi burchak topilsin.

18.20. $\frac{x-3}{2}=\frac{y}{1}=\frac{z-1}{2}$ va $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z}{2}$ parallel to'g'ri chiziqlardan o'tuvchi tekislikning tenglamasi yozilsin.

18.21. *x=2t-1, y=t+2, z=1-t* to'g'ri chiziqning *3x-2y+z=3* tekislik bilan kesishgan nuqtasi topilsin.

18.22. $\frac{x}{2}=\frac{y-1}{1}=\frac{z+1}{2}$ to'g'ri chiziqning *x+2y+3z-29=0* tekislik bilan kesishgan nuqtasi topilsin.

18.23. $\left.\begin{array}{l} x = \ \ z-2 \\ y = 2z+1 \end{array}\right\}$ va $\frac{x-2}{3}=\frac{y-4}{1}=\frac{z-2}{1}$

to'g'ri chiziqlarning kesishuvchi ekanligi ko'rsatilsin va ular yotgan tekislikning tenglamasi yozilsin.

18.24. *(2; 1; 0)* nuqtadan *x=3z-1; y=2z* to'g'ri chiziqqa tushirilgan perpendikulyarning tenglamalari yozilsin.

19. TO'PLAMLARGA DOIR MISOLLAR

n o'lchovli haqiqiy arifmetik fazoda ikki $M(x_1;x_2;....x_n)$ va $N(y_1;y_2;....y_n)$ nuqtalar orasidagi masofa:

$$d(M;N)=\sqrt{\sum_{i=1}^{n}(x_i-y_i)^2}\ .$$

formula asosida hisoblanadi va quyidagi xossalarga bo'ysinadi:

1) Har qanday *M* va *N* nuqtalar uchun *d(M;N)=d(N;M)*;

2) *d(M;N)≥0*, yani agar *M≠ N* bo'lsa , $d(M;N)>0$ va agar *M=N bo`lsa, d(M,N) =0*;

3) Har qanday *M, N* va *K* nuqtalar uchun *uchburchak tengsizligi* deb aytiluvchi *d(M; N)≤d(M;K)+ d(K; N)* munosabatlar o'rinli.

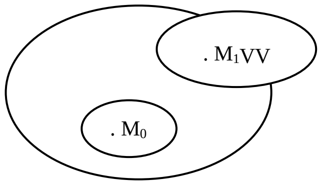
n o'lchovli haqiqiy arifmetik fazoda $M_0(x_{10};x_{20};...;x_{n0})$ nuqta va *r>0* son berilgan bo'lsin. *Rn fazoda M0 nuqtaning r atrofi* deb, *M0* markazdan *r* sondan kichik masofada yotuvchi mumkin bo'lgan barcha *M(x1;x2;……xn)* nuqtalar to'plamiga aytiladi va *Sr(M0)* yozuv bilan belgilanadi:

Sr(M0)={M(x1;x2……xn)Є $R_n | d$ *(M;M0)‹ r}.*

Rn fazoda *n*-o‘lchovli nuqtalarning biror-bir *V* to‘plami berilgan bo‘lsin.*V* to‘plamga tegishli har qanday *M(x1;x2;……xn)* nuqtaning har bir koordinatasi uchun *|x1|≤A, |x2|≤A,… …,|xn|≤A* munosabatlarni qanoatlantiruvchi *A›0* son mavjud bo‘lsa, *V* nuqtalar to‘plamiga *Rn fazoda chegaralangan to‘plam* deyiladi.

n o‘lchovli *V nuqtalar to*‘*plamining ichki nuqtasi* deb, o‘zining biror-bir atrofi bilan *V* to‘plamga tegishli nuqtaga aytiladi.

V nuqtalar to‘*plamining chegaraviy nuqtasi* deb, har qanday atrofi to‘plamga tegishli va tegishli bo‘lmagan nuqtalardan iborat nuqtaga aytiladi. 1-rasm.



1-rasmdagi M_0 nuqta *V* to‘plamning ichki, M_1 nuqta esa chegaraviy nuqtasidir.

V nuqtalar to‘plamining barcha chegaraviy nuqtalari to’plamiga uning *chegarasi* deyiladi.

n o‘lchovli *V nuqtalar to*‘*plamining quyuqlashish nuqtasi* deb, har bir atrofi *V* to’plamning cheksiz ko‘p nuqtalarini o‘z ichiga oluvchi $M_0 \in V$ nuqtaga aytiladi.

Har bir *n*-o‘lchovli quyuqlashish nuqtasi o‘ziga tegishli nuqtalar to‘plamiga R_n *fazoda yopiq to*‘*plam* deyiladi. Har bir *n* o‘lchovli nuqtasi ichki nuqta bo‘ladigan nuqtalar to‘plamiga esa R_n *da ochiq to*‘*plam* deyiladi.

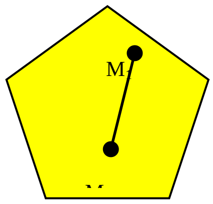
R_n fazoda chegaralangan va yopiq *n* o‘lchovli nuqtalar to‘plamiga *ixcham (kompakt) nuqtalar to*‘*plami* deyiladi.

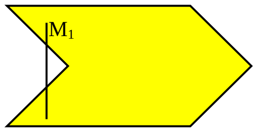
R_n fazoda [*MN*] *kesma* yoki *M* va *N nuqtalarning chiziqli qavariq kombinatsiyasi* deb, quyidagi to‘plamga aytiladi:

$$[MN]=\{P \in Rn \mid P=\alpha M+(1-\alpha)N, \qquad \alpha \in [0;1]\}.$$

Har qanday ikki M_1 va M_2 nuqtalari qaralmasin, ularni tutashtiruvchi [M_1M_2] kesma ham *V* to’plamga tegishli bo‘lsa, *V* nuqtalar to‘plamiga R_n *fazoda qavariq nuqtalar to*‘*plami* deyiladi.

2-rasmda qavariq nuqtalar to‘plami, 3-rasmda qavariq bo‘lmagan nuqtalar to‘plami tasvirlangan.





2-rasm 3-rasm

V qavariq to‘plamning chetki nuqtasi deb, o‘zidan tashqari to‘plam nuqtalarining chiziqli qavariq kombinatsiyasi shaklida yoyilmaydigan yoki shuning o‘zi, uchlari to‘plamga tegishli biror-bir kesmaning o‘rta nuqtasi bo‘la olmaydigan nuqtaga aytiladi.

Mustaqil yechish uchun masqlar:

Quyidagi sohalar bilan chegaralangan to‘plamlar qavariq to‘plam bo‘ladimi?

19.1. $\begin{cases} 3x_1 + 3x_2 = 8 \\ 6x_1 + 6x_2 = 17 \end{cases}$

19.2. $\begin{cases} x_1 + x_2 \le 2 \\ -x_1 + x_2 \le 2 \end{cases}$

19.3. $\begin{cases} 2x_1 + 5x_2 \le 20 \\ 5x_1 + 6x_2 \le 30 \\ x_1 \ge 0,\ x_2 \ge 0 \end{cases}$

19.4. $\begin{cases} -2x_1 + x_2 \le 1 \\ x_1 - 2x_2 \ge 1 \\ x_1 \ge 0,\ x_2 \ge 0 \end{cases}$

19.5. $\begin{cases} x_1^2 + x_2^2 \le 6 \\ x_1 - \frac{1}{3}x_2^2 + 1 \le 0 \end{cases}$

19.6. $x_1^2 + 3x_2^2 \le 5$

Quyidagi to‘plamlar qavariqmi?

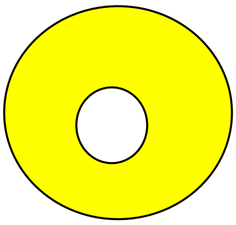
19.7. Markazsiz doira. Markazsiz shar.

19.8. Kesma va bu kesmada yotmaydigan nuqta.

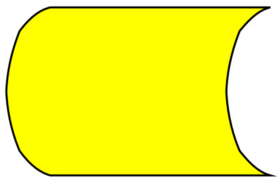
19.9. *R*3- fazoda umumiy nuqtaga ega bo‘lgan 2 ta tetraedr.

19.10. *R*2- fazoda umumiy tomonga ega bo‘lgan 2 ta uchburchak.

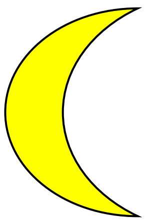
19.11.



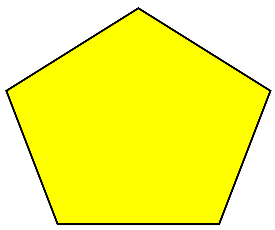
19.12.



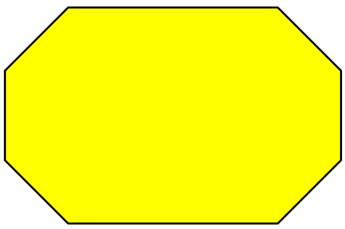
19.13.



19.14.

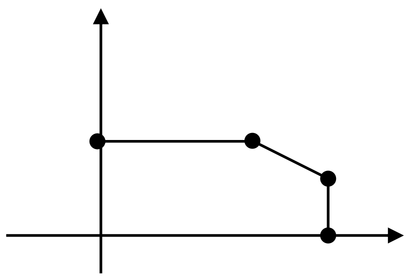


19.15.

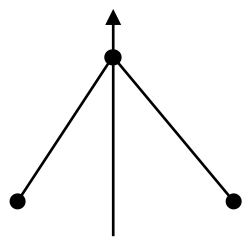


Quyida qavariq to‘plamlar berilgan. Harflar bilan berilgan nuqtalarning qaysilari chetki, ichki va chegaraviy nuqta ekanligini aniqlang:

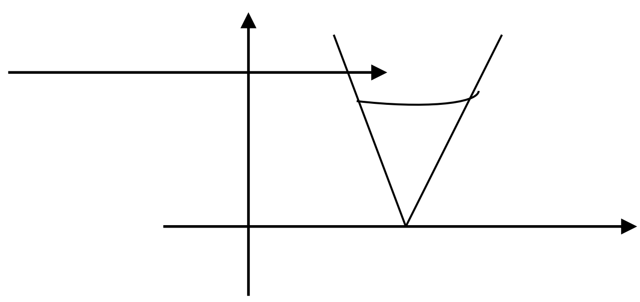
19.16.



19.17.



19.18.



Quyidagi sohalar bilan chegaralangan qavariq to‘plamlarni tekislikda tasvirlang. To‘plamning chetki nuqtalarini toping:

19.19. $\begin{cases} 2x_1 + 3x_2 \le 6 \\ 2x_1 + 3x_2 \ge 6 \end{cases}$

19.20. $\begin{cases} x_1 + 3x_2 \le 3 \\ x_1 + 2x_2 \ge 3 \\ 0 \le x_1 \le 5 \end{cases}$

19.21. $\begin{cases} 3x_1 + 4x_2 \le 12 \\ 4.5x_1 + 6x_2 \ge 24 \end{cases}$

19.22. $\begin{cases} 3x_1 + 4x_2 \le 12 \\ x_1 \ge 0,\ x_2 \ge 0 \end{cases}$

20. SONLI KETMA-KETLIKLARNING LIMITI

Har bir n natural songa aniq bir x_n haqiqiy sonni mos qo‘yuvchi qonun berilgan bo‘lsa, R_1 haqiqiy sonlar o‘qida $x_1, x_2, ..., x_n, ...,$ yoki $\{x_n\}$ *nuqtalar (sonlar) ketma-ketligi* berilgan deyiladi.

Masalan, har bir n natural songa $x_n=\frac{3(n+1)}{2n}$ son mos qo‘yilgan bo‘lsa,

$$3;\ \frac{9}{4};\ 2;\ \frac{15}{8};\ \ldots;\ \frac{3(n+1)}{2n};\ \ldots$$

sonlar ketma-ketligi berilganligini anglatadi.

a sonning har qanday oldindan tayinlangan ε atrofi uchun $\{x_n\}$ sonli ketma-ketlikning shunday bir N tartib raqamini (ε ga bog‘liq ravishda) tanlash mumkin bo‘lsaki, barcha $n>N$ tartib raqamli hadlari $|x_n - a| < \varepsilon$ tengsizlikni qanoatlantirsa, a soni $\{x_n\}$ *sonli ketma-ketlikning limiti* deyiladi.

n o‘lchovli haqiqiy fazoda har bir k natural songa aniq bir n o‘lchovli M_k nuqtani mos qo‘yuvchi qonuniyat o‘rnatilgan bo‘lsa, R_n fazoda cheksiz *n o‘lchovli nuqtalarning ketma-ketligi* berilgan deyiladi va $M_1, M_2, ..., M_k, ...,$ yoki $\{M_k\}$ ko‘rinishda yoziladi.

Masalan, har bir k natural songa ikki o‘lchovli $M_k\ (2k;\ \frac{3}{k})$ nuqta mos qo‘yilgan bo‘lsin. Bu esa, R_2 haqiqiy koordinatalar tekisligida

$$M_1(2;\ 3),\ M_2(4;\ \frac{3}{2}),\ M_3(6;\ 1),\ ...,\ M_n(2k;\ \frac{3}{k}),\ ...$$

nuqtalar ketma-ketligi berilganligini anglatadi.

n o‘lchovli M_0 nuqtaning har qanday ε atrofida berilgan nuqtalar ketma-ketligining biror-bir mos tartib raqamidan boshlab, barcha hadlari tegishli bo‘lsa, ya‘ni har qanday oldindan tayinlanadigan $\varepsilon>0$ uchun K tartib raqamni (ε ga bog‘liq ravishda) ko‘rsatish mumkin bo‘lsaki, barcha $k>K$ tartib raqamli hadlar $M_k \in S_\varepsilon(M_0)$ bo‘lsa, M_0 nuqtaga $\{M_k\}$ *nuqtalar ketma-ketligining limiti* deyiladi va $\lim\limits_{k\to\infty}\{M_k\} = M_0$ ko‘rinishida yoziladi.

Mustaqil yechish uchun misollar:

20.1. Umumiy hadi orqali berilgan ketma-ketlikning birinchi beshta hadini yozing:

a) $x_n = \frac{1}{2n+1}$ b) $x_n = \frac{n+2}{n^3+1}$

c) $x_n = \frac{n}{2n+1}$ d) $x_n = (-1)^n \frac{n+1}{n^2}$.

20.2. Ketma-ketlikning berilgan hadlari orqali umumiy hadining formulasini yozing:

a) $1; \frac{1}{2}; \frac{1}{6}; \frac{1}{24}; ...$ b) $1; 2\frac{1}{4}; 2\frac{7}{9}; 3\frac{1}{16}; 3\frac{6}{25}; ...$

c) 2; 10; 26; 82; 242; 730; …

20.3. Sonli ketma-ketlik chegaralanganligini isbotlang:

a) $x_n = \frac{n^2+1}{n^2+2}$

Isbot: $\frac{n^2+1}{n^2+2} = 1 - \frac{1}{n^2+2}$ va $0 < \frac{1}{n^2+2} \le \frac{1}{2}$ shuning uchun $\frac{1}{2} < x_n < 1$.

b) $x_n = \frac{(-1)^n \ n+1}{\sqrt{n^2+2}}$ c) $x_n = Sin\ n$

d) $x_n = (1-(-1)^n)$

20.4. Sonli ketma-ketlik monotonligini isbotlang:

a) $x_n = \lg n - \lg(n-1),\ (n>1)$

Isbot: $x_n = \lg n - \lg(n-1) = \lg\frac{n}{n-1}$

$x_{n+1} - x_n = \lg\frac{n+1}{n} - \lg\frac{n}{n-1} = \lg\frac{n^2-1}{n^2} = \lg(1-\frac{1}{n^2}) < 0$. Demak,

$x_{n+1} - x_n < 0,\ \ x_{n+1} < x_n$, shuning uchun bu ketma-ketlik monoton kamayuvchi.

b) $x_n = 3^n - 2^n$ c) $x_n = \sqrt{n^2-1}$

d) $x_n = \sum_{k=1}^{n} k$.

20.5. Ketma-ketlik limiti ta'rifidan foydalanib, quyidagilarni isbotlang:

a) $\lim_{n\to\infty}\frac{n}{n+1} = 1$

Isbot: ixtiyoriy $\varepsilon > 0$ son olamiz, $|x_n - 1| = \left|\frac{n}{n+1} - 1\right| = \frac{1}{n+1}$; $|x_n - 1| < \varepsilon$ tengsizlikni qanoatlantiruvchi *n* larni topish uchun $\frac{1}{n+1} \prec \varepsilon$ tengsizlikni yechamiz. $n > \frac{1-\varepsilon}{\varepsilon}$. Shunday qilib, $\frac{1-\varepsilon}{\varepsilon}$ sonining butun qismi $N=\left[\frac{1-\varepsilon}{\varepsilon}\right]$ bo‘ladi, u holda $|x_n-1|<\varepsilon$ tengsizlik barcha $n>N$ larda bajariladi. ε -ixtiyoriy son bo‘lgani uchun $\lim_{n\to\infty}\frac{n}{n+1}=1.$

Agar $\varepsilon=0.01$ bo‘lsa, $N=\left[\frac{1-0.01}{0.01}\right]=99,$ $n>99$ larda $|x_n-1|<0.01$ bo‘ladi.

b) $\lim_{n\to\infty}\frac{4n-1}{2n+1}=2$ c) $\lim_{n\to\infty}\frac{3n+1}{5n-1}=\frac{3}{5}$

d) $\lim_{n\to\infty}\frac{2n-1}{2-3n}=-\frac{2}{3}$ qaysi *n* dan boshlab, $\left|\frac{2n-1}{2-3n}-\left(-\frac{2}{3}\right)\right|<0.0001$ tengsizlik o‘rinli bo‘ladi?

Quyidagi limitlarni toping:

20.6. $\lim_{n\to\infty}\frac{3n^3+2}{4n^3-1}$ 20.7. $\lim_{n\to\infty}\frac{2n^3+3}{n^3+n-1}$

20.8. $\lim_{n\to\infty}\frac{(n+1)^3}{5n^3+1}$ 20.9. $\lim_{n\to\infty}\frac{\sqrt[3]{n^2+n}}{4+n}$

20.10. $\lim_{n\to\infty}\frac{\sqrt{2n^4+n^2+1}}{2n+n^2-1}$ 20.11. $\lim_{n\to\infty}\frac{n!}{(n+1)!-n!}$

20.12. $\lim_{n\to\infty}\frac{1+\frac{1}{2}+...+\frac{1}{2^n}}{1+\frac{1}{3}+...+\frac{1}{3^n}}$ 20.13. $\lim_{n\to\infty}\frac{3+6+9+...+3n}{n^2+4}$

20.14. $\lim_{n\to\infty}\sqrt{n}(\sqrt{n+2}-\sqrt{n-3})$ 20.15. $\lim_{n\to\infty}\frac{(2n+1)!+(2n+2)!}{(2n+3)!-(2n+2)!}$

20.16. $\lim_{n\to\infty}\sqrt{n^3+8}(\sqrt{n^3+2}-\sqrt{n^3-1})$ 20.17. $\lim_{n\to\infty}\left(\frac{1}{3}+\frac{1}{15}+...+\frac{1}{4n^2-1}\right)$

20.18. $\lim_{n\to\infty}(\frac{1}{2n}Cos\,n^3-\frac{3n}{6n+1})$ 20.19. $\lim_{n\to\infty}\left(\frac{n}{n^2+1}Sin\,n!+\frac{2n^2}{1-9n^2}\right)$

20.20. R^2 fazoda quyidagi ketma-ketliklarning limiti $a\in R^2$ ekanligini isbotlang:

a) $\{x^{(n)}\}=\left\{\frac{1}{n},\frac{1}{n}\right\}$ $a=(0,0),$ $\forall\varepsilon>0$ son olaylik.

$$\rho(x^n, a) = \rho\left(\left(\frac{1}{n}, \frac{1}{n}\right), (0,0)\right) = \sqrt{\left(\frac{1}{n} - 0\right)^2 + \left(\frac{1}{n} - 0\right)^2} = \sqrt{\frac{2}{n^2}} < \varepsilon, \quad n > \frac{\sqrt{2}}{\varepsilon}; \; N = \left[\frac{\sqrt{2}}{\varepsilon}\right]$$

bo‘ladi, u holda $\rho(x^{(n)}, a) < \varepsilon$ tengsizlik barcha $n > N$ larda bajariladi. Ta’rifga ko‘ra $\lim\limits_{n\to\infty}\left(\frac{1}{n}, \frac{1}{n}\right) = (0,0) = a$.

b) $\{x^{(n)}\} = \left\{\left(\frac{3}{n}, \frac{1}{n^2}\right)\right\}$, $\quad a = (0, 0)$

c) $\{x^{(n)}\} = \left\{\left(\frac{3n}{2n-1}, \frac{2-n}{2+n}\right)\right\}$, $\quad a = \left(\frac{3}{2}, -1\right)$

R^2 fazoda ketma-ketliklar limitini toping:

20.21. $x^{(n)} = \left(\left(\frac{n-1}{n}\right)^5, \frac{n^3+27}{n^4-15}\right)$

20.22. $x^{(n)} = \left(\frac{9n+5}{4n}, \frac{1}{2n+4}\right)$

20.23. $x^{(n)} = \left(\frac{3n^4-2}{\sqrt{n^8+3n}}, \sqrt{n^2+8n} - \sqrt{n^2+4n}\right)$

20.24. Berilgan ketma-ketliklarning 5 ta hadini va umumiy hadi formulasini yozing:

a) $x_1 = 1;\; x_{n+1} = n!$ b) $x_1 = 1;\; x_{n+1} = x_n + 3$

20.25. Ketma-ketlik chegaralanmaganligini isbotlang:

a) $x_n = n^{(-1)^n}$ b) $x_n = (-1)^n n$

c) $x_n = n \cdot \log_{\frac{1}{2}} n$ d) $x_n = tg\; n$

20.26. Ketma-ketlik limitini ta’rifidan foydalanib isbotlang:

a) $\lim\limits_{n\to\infty} \frac{4n-1}{5n+1} = \frac{4}{5}$ b) $\lim\limits_{n\to\infty} \frac{\sqrt{n^2+n}}{n} = 1$

c) $\lim\limits_{n\to\infty} \frac{3n-1}{5n+1} = \frac{3}{5}$, qaysi n dan boshlab $\left|\frac{3n-1}{5n+1} - \frac{3}{5}\right| < 0.001$ tengsizlik o`rinli bo‘ladi?

Quyidagi limitlarni toping:

20.27. $\lim\limits_{n\to\infty} \frac{3n^3-4}{n^3+6}$

20.28. $\lim\limits_{n\to\infty} \frac{n^2+n+1}{(n+1)^2}$

20.29. $\lim\limits_{n\to\infty} \frac{(n+1)^4 + (n-1)^4}{n^4+10}$

20.30. $\lim\limits_{n\to\infty} \frac{\sqrt[3]{4n^3+2n-1}}{2n+2}$

20.31. $\lim\limits_{n\to\infty} \frac{\sqrt{2n^2+1}}{\sqrt[4]{n^4+3n-1}}$

20.32. $\lim\limits_{n\to\infty} \frac{(n+1)!+n!}{(n+2)!}$

20.33. $\lim\limits_{n\to\infty}\frac{1+2+...+n}{n^2}$

20.34. $\lim\limits_{n\to\infty}\left(\frac{1}{n}Cos\frac{n\pi}{2}+1\right)$

20.35. $\lim\limits_{n\to\infty}\frac{1+3+...+(2n-1)}{n\sqrt{n^2+1}}$

20.36. $\lim\limits_{n\to\infty}\frac{\log_a(n+1)!}{\log_a n!}$ $(n>2,\ a>1)$

20.37. $\lim\limits_{n\to\infty}\frac{1+a+a^2+...+a^{n-1}}{1+b+b^2+...+b^{n-1}}$ $(b>a>0)$

20.38. $\lim\limits_{n\to\infty}\frac{(n+k)!+n!}{(n+k)!-n!}$

20.39. $\lim\limits_{n\to\infty}\sqrt{n}(\ln(n+2\sqrt{n}+1)-\ln n)$

20.40. $\lim\limits_{n\to\infty}(\sqrt{2+4+6+...+2n}-\sqrt{1+3+5+...+(2n-1)})$

21. FUNKSIYA. FUNKSIYA ANIQLANISH SOHASI, QIYMATLAR TO‘PLAMI, JUFT-TOQLIGIGA DOIR MISOLLAR

y=f(x) (y=f(M)=f(x₁, x₂,..., xₙ)) funksiya berilgan *R (Rₙ)* fazoning qism osti to‘plamiga uning *aniqlanish sohasi* deyiladi va *D(f)* yoki *D(y)* yozuv bilan ifodalanadi.

y=f(x) (y=f(M)) funksiya o‘z aniqlanish sohasi *D(f)* ning har bir nuqtasida qabul qilishi mumkin bo‘lgan barcha qiymatlari to‘plamiga esa uning *qiymatlari to‘plami* yoki *o‘zgarish sohasi* deyiladi. Funksiya qiymatlar to‘plami R_1 haqiqiy sonlar to‘plamining qism osti to‘plami bo‘lib, *E(f)* yoki *E(y)* belgilar bilan yoziladi.

Agar har qanday $\pm x\in V$ lar uchun *f(-x)=f(x)* tenglik o‘rinli bo‘lsa, bir o‘zgaruvchili *y=f(x)* funksiya *V* to‘plamda *juft funksiya* deyildi. Juft funksiya grafigi *0y* ordinata o‘qiga nisbatan simmetrikdir.

Agar har qanday $\pm x\in V$ lar uchun *f(-x)=-f(x)* munosabat o‘rinli bo‘lsa, *y=f(x)* funksiya *V* to‘plamda *toq funksiya* deyiladi. Toq funksiya grafigi esa koordinatalar boshiga nizbatan simmetrikdir.

y=f(x) funksiya uchun shunday bir musbat *t* son mavjud bo‘lsaki, funksiyaning aniqlanish sohasiga tegishli har qanday *x* va *x+t* nuqtalari uchun *f(x+t)=f(x)* tenglik bajarilsa, *y=f(x)* funksiya *davriy funksiya* deyiladi. *t* son esa funksiya davri deb yuritiladi. Amalda funksiya davrlari ichidan eng kichigi *T* ni topish masalasi qo‘yiladi.

Mustaqil yechish uchun misollar:

Quyidagi funksiyalarning aniqlanish sohasini toping:

21.1. $f(x)=\sqrt{1-2x}+3\arcsin\frac{3x-1}{2}$

Birinchi qo‘shiluvchi $1-2x \geq 0$ da haqiqiy qiymatlarni qabul qiladi, ikkinchi qo‘shiluvchi esa $-1 \leq \frac{3x-1}{2} \leq 1$ bo‘lganda. Shunday qilib, funksiyaning aniqlanish sohasini topish uchun

$$\begin{cases} 1-2x \geq 0 \\ \frac{3x-1}{2} \leq 1 \\ \frac{3x-1}{2} \geq -1 \end{cases}$$ tengsizliklar sistemasini yechib topamiz

$$x \leq \frac{1}{2}, \quad x \leq 1, \quad x \geq -\frac{1}{3}$$

Demak, $D(f) = \left[-\frac{1}{3}; \frac{1}{2}\right]$.

21.2. $y = \lg(x^2 - 4x + 3)$

21.3. $y = \arcsin(3x-4)$

21.4. $y = \frac{1}{\lg(1-x)} + \sqrt{x+2}$

21.5. $y = \sqrt{Sin(x)} - \sqrt{9-x^2}$

21.6. $y = \arcsin\frac{1+x^2}{2x}$

21.7. $y = x^{\frac{1}{\lg x}}$

21.8. $y = tg\sqrt{16-x^2}$

21.9. $y = \frac{1}{\sqrt{|x| - 2|x-1|}}$

21.10. $f(x) = \ln Cos(x)$.

Quyidagi funksiyalarning qiymatlar to‘plamini toping:

21.11. $y = 1 + 2^{x+1}$,

$y = 2^{x+1} = 2 \cdot 2^x$ ko‘rsatkichli funksiya, uning qiymatlar to‘plami $y \in (0; +\infty)$, demak berilgan funksiyaning qiymatlar to‘plami $(1; +\infty)$ bo‘ladi yoki berilgan funksiyaning qiymatlar to‘plami uning teskari funksiyasi $x = \log_2(y-1) - 1$ ning aniqlanish sohasi $y > 1$ bilan ustma-ust tushadi, shuning uchun $E(y) = (1; +\infty)$.

21.12. $y = Sin(x) - Cos(x)$

21.13. $y = x + \frac{1}{x}$

21.14. $y = \sqrt{-x^2 - x + 2}$

21.15. $y = \frac{x+1}{x-2}$

21.16. $y = \frac{1}{x^2+1} + 1$

21.17. $y = \frac{1}{\arcsin(1-x)}$.

Quyidagi funksiyalarni juft yoki toqligini tekshiring:

21.18. $y = 2^x + 2^{-x}$,

$f(-x) = 2^{-x} + 2^x = f(x)$ bo‘lganligi uchun bu juft funksiyadir.

21.19. $y = \dfrac{1}{x + \dfrac{1}{x + \dfrac{1}{x}}}$

21.20. $y = Sin\sqrt{x}$

21.21. $y = |x| - 5e^{x^2}$

21.22. $y = \dfrac{|Sinx|}{1 - Cosx}$

21.23. $y = Sin(arcCos\,x)$

21.24. $y = \left|\dfrac{10^x + 1}{10^x - 1}\right|$

21.25. $y = \lg\dfrac{x+3}{x-3}$

21.26. $f(x) = \begin{cases} 1, & agar\ x - ratsional\, son\, bo`lsa \\ -1, & agar\ x - irratsiond\ son\, bo`lsa \end{cases}$

Funksiyalarni asosiy davrini toping:

21.27. $y = Sin6x + tg\,4x$

Birinchi qo‘shiluvchi uchun asosiy davr $\dfrac{2\pi}{6} = \dfrac{\pi}{3}$ bo‘ladi, ikkinchi qo‘shiluvchi uchun davr $\dfrac{\pi}{4}$ bo`ladi. $\dfrac{\pi}{3}$ va $\dfrac{\pi}{4}$ sonlarining eng kichik umumiy bo‘luvchisi π bo‘lgan funksiyaning asosiy davri bo‘ladi. $f(x+T) = f(x); T = \pi$.

21.28. $y = 2$

21.29. $y = Sin\dfrac{3}{2}x + 1$

21.30. $y = Sin\,x - Cos\,x$

21.31. $y = Sin2x - 2tg\left(\dfrac{x}{2}\right)$

21.32. $y = Cos^2 x$

21.33. $y = x - [x]$.

Quyidagi 2 o‘zgaruvchili funksiyalarni aniqlanish sohasini toping:

21.34. $f(x, y) = \arcsin(x + y)$,

$f(x, y)$ funksiya ma’noga ega bo‘lishi uchun x va y lar ushbu $-1 \le x + y \le 1$ munosabatda bo‘lishi lozim. Bu tengsizliklarni tekislikning $x + y + 1 = 0$ va $x + y - 1 = 0$ to‘g‘ri chiziqlar orasidagi nuqtalarning koordinatalari qanoatlantiradi.

$D(f) = \{ (x, y) \in R^2;\ |x + y| \le 1\}$

21.35. $f(x,y)=\sqrt{x+y}$

21.36. $f(x,y)=\sqrt{-x}+\sqrt{y}$

21.37. $f(x,y)=\arccos\frac{x^2+y^2}{9}$

21.38. $f(x,y)=\sqrt{x-\sqrt{y}}$

21.39. $f(x,y)=1+\sqrt{-(x-y)^2}$

21.40. $f(x,y)=\frac{1}{x-1}+\frac{1}{y}$

21.41. $f(x,y)=\frac{\sqrt{4x-y^2}}{\lg(1-x^2-y^2)}$

21.42. $f(x,y)=\sqrt{x^2-4}+\sqrt{1-y^2}$.

Funksiyalarning aniqlanish sohasini toping:

21.43. $y=\sqrt{x^2+x-2}$

21.44. $y=\log_2(-x)$

21.45. $y=\arccos\left(\frac{x}{2}-1\right)$

21.46. $y=\sqrt{3^x-5^x}$

21.47. $y=\frac{1}{xe^x}$

21.48. $y=Sin\sqrt{x-3}$

Funksiyalarning qiymatlar sohasini toping:

21.49. $y=Sin^2x-Cos^2x$

21.50. $y=-x^2-5x+6$

21.51. $y=1-|x|$

21.52. $y=\frac{2-x}{x+3}$

21.53. $y=\lg\frac{2}{\sqrt{4-x^2}}$

21.54. $y=\sqrt{Sin x-1}$

Funksiyalarni juft yoki toqligini tekshiring:

21.55. $y=x^2Sin\frac{1}{x}$

21.56. $y=\lg Cos\,x$

21.57. $y=\frac{16^x-1}{4^x}$

21.58. $y=x^4Sin7x$

Funksiyalarni asosiy davrini toping:

21.59. $y=-2Cos\frac{x}{3}+1$

21.60. $y=Sin\frac{x}{3}+ctg\frac{x}{4}$

21.61. $y=2^{Sin2x}\cdot 2^{Cos2x}$

21.62. $y=\log_2 Sin\ x$

Funksiyalarni aniqlanish sohasini toping:

21.63. $f(x,y)=\sqrt{x}+\sqrt{y}$

21.64. $f(x,y)=\sqrt{1-x^2-y^2}$

22. FUNKSIYANING LIMITI

Ta'rif. *y=f(x)* funksiya berilgan bo'lsin. Har qanday ε >0 son uchun shunday bir $\delta > 0$ son tanlash mumkin bo'lsaki, V to'plamga tegishli va $|x - x_0| < \delta$ munosabatlarni qanoatlantiruvchi har bir x uchun $|\mathrm{f(x) - b}| < \varepsilon$ tengsizlik bajarilsa b soni *f(x) funksiyaning* $x \to x_0$ *dagi limiti* deyiladi va $\lim\limits_{x \to x_0} f(x) = b$ kabi yoziladi.

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 1^∞, 0^0, ∞^0 ko'rinishdagi aniqmasliklarni ochishda quyidagi ajoyib limitlardan foydalaniladi:

1. $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

2. $\lim\limits_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

3. $\lim\limits_{x \to 0} \frac{\ln(1 + x)}{x} = 1$

4. $\lim\limits_{x \to 0} \frac{a^x - 1}{x} = \ln a,\ a > 0$

5. $\lim\limits_{x \to 0} \frac{(1 + x)^P - 1}{x} = P$

6. $\lim\limits_{x \to 0} \frac{tgx}{x} = 1$

7. $\lim\limits_{x \to 0} \frac{\arcsin x}{x} = 1$

8. $\lim\limits_{x \to 0} \frac{arctgx}{x} = 1$

9. $\lim\limits_{x \to 0} \frac{\log_a(1 + x)}{x} = \log_a e$

10. $\lim\limits_{x \to 0} \frac{e^x - 1}{x} = 1$

11. $\lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$

Ta'rif. Har qanday oldindan tayinlanadigan ε >0 son uchun M_0 nuqtaning δ atrofi $S_\delta(M_0)$ni ko'rsatish mumkin bo'lsaki, barcha $M \in S_\delta(M_0) \cap V$, $M \neq M_0$ nuqtalar uchun $|f(M) - b| < \varepsilon$ tengsizlik o'rinli bo'lsa, u holda b soni *f(M) funksiyaning* $M \to M_0$ *dagi limiti* deyiladi va

$$b = \lim_{M \to M_0} f(M) \quad \text{yoki} \quad b = \lim_{\substack{x_1 \to x_1^0 \\ x_2 \to x_2^0 \\ \cdots \\ x_n \to x_n^0}} f(M)$$

ko'rinishda yoziladi.

Mustaqil yechish uchun misollar:

Funksiya limiti ta’rifidan foydalanib quyidagilarni isbotlang:

22.1. $\lim\limits_{x\to 3}(3x-5)=4$

Ixtiyoriy $\varepsilon>0$ son uchun shungay $\delta>0$ topilib, $|x-3|<\delta$ tengsizlikni qanoatlantiruvchi barcha x lar uchun $|(3x-5)-4|<\varepsilon$ tengsizlik o‘rinli bo‘lishini ko‘rsatishimiz kerak. Ixtiyoriy $\varepsilon>0$ son olaylik. $|(3x-5)-4|=|3x-9|=|3(x-3)|=3|x-3|<\varepsilon$ $|x-3|<\frac{\varepsilon}{3}$.

Agar $\delta<\frac{\varepsilon}{3}$ deb olsak, $|x-3|<\delta$ tengsizlikni qanoatlantiruvchi x lar uchun $|(3x-5)-4|<\varepsilon$ tengsizlik o‘rinli bo‘ladi.

Shu bilian $\lim\limits_{x\to 3}(3x-5)=4$ ekanligi isbotlandi.

22.2. $\lim\limits_{x\to 1}(4x-1)=3$

22.3. $\lim\limits_{x\to\frac{\pi}{2}} Sinx=1$

22.4. $\lim\limits_{x\to 2}(x^2-1)=3$, δ ning qanday qiymatlarida $0<|x-2|<\delta$ tengsizlikdan $|(x^2-1)-3|<0.001$ tengsizlik kelib chiqadi?

$\frac{0}{0},\frac{\infty}{\infty}$ ko‘rinishidagi aniqmasliklarni oching:

22.5. $\lim\limits_{x\to 2}\frac{x-2}{x^2-3x+2}=\lim\limits_{x\to 2}\frac{x-2}{(x-2)(x-1)}=\lim\limits_{x\to 2}\frac{1}{x-1}=1$

22.6. $\lim\limits_{x\to 3}\frac{x^2-9}{x^2-2x-3}$

22.7. $\lim\limits_{x\to \pi}\frac{tgx}{\sin 2x}$

22.8. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\cos 2x}$

22.9. $\lim\limits_{x\to 0}\frac{x}{\sqrt{1+3x}-1}$

22.10. $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

22.11. $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}$

22.12. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

22.13. $\lim\limits_{x\to \pi}\frac{\sqrt{1-tgx}-\sqrt{1+tgx}}{Sin2x}$

22.14. $\lim\limits_{x\to\infty}\frac{5x^3-7x}{1-2x^3}$

22.15. $\lim\limits_{x\to\infty}\frac{x^3-1}{x^2+1}$

22.16. $\lim\limits_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}$ 22.17. $\lim\limits_{x\to-2}\frac{3x+6}{x^3+8}$

22.18. $\lim\limits_{x\to-1}\frac{x^2-x-2}{x^3+1}$ 22.19. $\lim\limits_{x\to\pi+0}\frac{\sqrt{1+Cosx}}{Sinx}$

22.20. $\lim\limits_{x\to7}\frac{2-\sqrt{x-3}}{x^2-49}$ 22.21. $\lim\limits_{x\to1}\frac{x^4-2x^3+2x^2-2x+1}{3x^4-5x^3+2x^2-x+1}$

22.22. $\lim\limits_{x\to5}\frac{\sqrt{6-x}-1}{3-\sqrt{4+x}}$ 22.23. $\lim\limits_{x\to1}\frac{3x-2-\sqrt{4x^2-x-2}}{x^2-3x+2}$

22.24. $\lim\limits_{x\to0}\frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{x^2-x}$

$\lim\limits_{x\to0}\frac{Sinx}{x}=1$ ajoyib limitdan foydalanib quyidagi limitlarni toping:

22.25. $\lim\limits_{x\to0}\frac{Sin6x}{Sin7x}=\lim\limits_{x\to0}\frac{\frac{Sin6x}{6x}6x}{\frac{Sin7x}{7x}7x}=\lim\limits_{x\to0}\frac{6x}{7x}=\frac{6}{7}$

22.26. $\lim\limits_{x\to0}\frac{Sin4x}{x}$ 22.27. $\lim\limits_{x\to0}\frac{1-Cos2x}{xSinx}$

22.28. $\lim\limits_{h\to0}\frac{Sin(x+h)-Sin(x-h)}{h}$ 22.29. $\lim\limits_{x\to0}\frac{1-Cosx}{x^2}$

22.30. $\lim\limits_{x\to0}\frac{Sin4x}{\sqrt{x+1}-1}$ 22.31. $\lim\limits_{x\to0-0}\frac{\sqrt{1-Cos2x}}{x}$

22.32. $\lim\limits_{x\to0}\frac{2x\cdot Sinx}{\sec x-1}$ 22.33. $\lim\limits_{x\to0}\frac{1-Cos2x+tg^2x}{xSinx}$

22.34. $\lim\limits_{x\to-2}\frac{\arcsin(x+2)}{x^2+2x}$ 22.35. $\lim\limits_{x\to0}\frac{1-Cos5x}{1-Cos3x}$

22.36. $\lim\limits_{x\to\frac{\pi}{2}}\frac{Cos5x}{Sin4x}$ 22.37. $\lim\limits_{x\to0}\frac{\sqrt{1+Cosx}-\sqrt{2Cosx}}{\sqrt{3+Cosx}-2\sqrt{Cosx}}$

22.38. $\lim\limits_{x\to\frac{\pi}{2}}\frac{1-Sinx}{(\frac{\pi}{2}-x)}$ 22.39. $\lim\limits_{x\to\frac{\pi}{2}}\frac{Cos\frac{x}{2}-Sin\frac{x}{2}}{Cosx}$

Limitlarni toping:

22.40. $\lim\limits_{x\to1}\frac{x^2-x-2}{x^3+1}$ 22.41. $\lim\limits_{x\to3}\frac{9-x^2}{\sqrt{3x}-3}$

22.42. $\lim\limits_{x\to a}\dfrac{\sqrt{ax}-x}{x-a}$ 22.43. $\lim\limits_{x\to\infty}\dfrac{5x^2-3x+2}{2x^2+4x+1}$

22.44. $\lim\limits_{x\to\infty}\dfrac{5x^2+2^{\frac{1}{x}}}{1-x^2}$ 22.45. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{Sin2x-Cos2x-1}{Cosx-Sinx}$

22.46. $\lim\limits_{x\to 0}\dfrac{Sin^2\frac{x}{2}}{x^2}$ 22.47. $\lim\limits_{x\to 0}\dfrac{Sin3x}{\sqrt{x+2}-\sqrt{2}}$

22.48. $\lim\limits_{x\to\frac{1}{5}}\dfrac{\arcsin(1-2x)}{4x^2-1}$ 22.49. $\lim\limits_{x\to 0}\dfrac{1-Cosmx}{x^2}$

22.50. $\lim\limits_{x\to 2}\left[\dfrac{Sin(x-2)}{x^2-4}+2^{\frac{-1}{(x-2)^2}}\right]$ 22.51. $\lim\limits_{x\to 0}\dfrac{tgx-Sinx}{x^3}$

22.52. $\lim\limits_{h\to 0}\dfrac{Cos(x+h)-Cos(x-h)}{h}$ 22.53. $\lim\limits_{x\to x_0}\dfrac{tgx-tgx_0}{x-x_0}$

22.54. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{Sinx-Cosx}{\pi-4x}$ 22.55. $\lim\limits_{x\to\infty}\dfrac{2x^4+3x^2+5x-6}{x^3+3x^2+7x-1}$

22.56. $\lim\limits_{x\to\infty}\dfrac{(2x^3+4x+5)(x^2+x+1)}{(x+2)(x^4+2x^3+7x^2+x-1)}$

22.57. $\lim\limits_{x\to+\infty}(\sqrt{x^2+3x}-x)=\lim\limits_{x\to+\infty}\dfrac{\left(\sqrt{x^2+3x}-x\right)\left(\sqrt{x^2+3x}+x\right)}{\sqrt{x^2+3x}+x}=\lim\limits_{x\to+\infty}\dfrac{x^2+3x-x^2}{\sqrt{x^2+3x}+x}=\dfrac{3}{2}$

22.58. $\lim\limits_{x\to 1}\left(\dfrac{1}{x-1}-\dfrac{2}{x^2-1}\right)$ 22.59. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2-x}\right)$

22.60. $\lim\limits_{x\to 2}\left(\dfrac{1}{x-2}-\dfrac{12}{x^3-8}\right)$ 22.61. $\lim\limits_{x\to 0}\left(\dfrac{1}{Sin^2x}-\dfrac{1}{4Sin^2\frac{x}{2}}\right)$

22.62. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-4x}\right)$ 22.63. $\lim\limits_{x\to-2}\left(\dfrac{1}{x+2}+\dfrac{4}{x^2-4}\right)$

22.64. $\lim\limits_{x\to\pi}Sin2x\cdot ctgx$

22.65. $\lim\limits_{x\to\infty}\left(1-\dfrac{1}{x^2}\right)^x$; Bu limitni hisoblashda $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$ ajoyib limitdan foydalanamiz:

$$\lim_{x\to\infty}\left(1-\frac{1}{x^2}\right)^x=\lim_{x\to\infty}\left[\left(1+\frac{1}{-x^2}\right)^{-x^2}\right]^{\frac{x}{-x^2}}=e^{\lim\limits_{x\to\infty}\left(\frac{-x}{x^2}\right)}=e^{-\lim\limits_{x\to\infty}\frac{1}{x}}=e^0=1$$

22.66. $\lim\limits_{x\to\infty}\left(1+\frac{5}{x}\right)^{2x}$ 22.67. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{2x-1}\right)^{x}$

22.68. $\lim\limits_{x\to 0}(1-4x)^{\frac{1-x}{x}}$ 22.69. $\lim\limits_{x\to\infty}\left(\frac{x+8}{x-2}\right)^{x}$

22.70. $\lim\limits_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+1}\right)^{x}$ 22.71. $\lim\limits_{x\to\infty}\left(\frac{3x+1}{3x-1}\right)^{3x+1}$

22.72. $\lim\limits_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{2-5x}$

Ko‘p o‘zgaruvchili funksiyalarning limitini toping:

22.73. $\lim\limits_{\substack{x\to 0\\ y\to 3}}\frac{Sin(x^2\cdot y)}{x^2}$

$$\lim_{\substack{x\to 0\\ y\to 3}}\frac{\sin(x^2\cdot y)}{x^2}=\lim_{\substack{x\to 0\\ y\to 3}}\frac{\sin(x^2\cdot y)}{x^2\cdot y}\cdot y=\lim_{\substack{x\to 0\\ y\to 3}}y=3$$

22.74. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{a-\sqrt{a^2-xy}}{xy}$, $a\neq 0$

22.75. $\lim\limits_{\substack{x\to 3\\ y\to 0}}\frac{tg(xy)}{y}$ 22.76. $\lim\limits_{\substack{x\to 0\\ y\to 0}}(x^2+y^2)\cdot Sin\frac{1}{xy}$

22.77. $\lim\limits_{\substack{x\to\infty\\ y\to a}}\left(1+\frac{y}{x}\right)^{x}$ 22.78. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{\sqrt{1+x^2y^2}-1}{x^2+y^2}$

22.79. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\left(1+x^2+y^2\right)^{\frac{1}{x^2+y^2}}$

23. FUNKSIYA UZLUKSIZLIGI. UZLUKLI FUNKSIYALAR

Ta’rif. Agar x_0 nuqtaning biror atrofida (x_0 nuqtaning o’zida ham) $y=f(x)$ funksiya aniqlangan bo'lsa va agar

$$\lim_{\Delta x\to 0}\left[f\left(x_0+\Delta x\right)-f\left(x_0\right)\right]=0 \qquad (1)$$

bo‘lsa, $x=x_0$ *qiymatda (yoki x_0 nuqtada) funksiya uzluksiz* deyiladi. (1)ifodaning uzluksizlik shartini bunday yozish mumkin:

$$\lim_{\Delta x\to 0} f\left(x_0+\Delta x\right)=f\left(x_0\right) \quad \text{yoki} \quad \lim_{x\to 0} f\left(x\right)=f\left(x_0\right).$$

x_0 nuqtada uzluksiz $f(x)$ va $g(x)$ funksiyalar bo‘lsa, u holda x_0 nuqtada quyidagi funksiyalar ham uzluksiz bo‘ladi:

a) $f(x)+g(x)$

b) $k\,f(x)$ (k-o‘zgarmas)

c) $f\left(x\right)\cdot g\left(x\right)$

d) $\dfrac{f(x)}{g(x)}$ $(g(x_0)\neq 0)$

Agar $f(x)$ funksiya $\left[a;b\right]$ kesmada uzluksiz bo‘lsa va kesmaning chetki nuqtalarida turli ishorali qiymatlarga erishsa ($f(a)\cdot f(b)<0$), u holda $(a;\ b)$ internalga tegishli kamida bitta c nuqta topiladiki, $f(c)=0$ tenglik bajariladi.

Agar $f(x)$ funksiya x_0 nuqtada uzluksiz bo‘lmasa, funksiya x_0 nuqtada urilgan yoki x_0 nuqta uning *urilish nuqtasi* deyiladi.

$y=f(x)$ funksiyaning x_0 nuqtada chapdan va o‘ngdan limitlari mavjud bo‘lib, o‘zaro teng bo‘lmasa, ya’ni

$$\lim_{x\to x_0-0} f\left(x\right)=f\left(x_0-0\right)\neq f\left(x_0+0\right)=\lim_{x\to x_0+0} f\left(x\right),$$

u holda x_0 nuqta *funksiyaning birinchi tur urilish nuqtasi* deyiladi.

Agar x_0 nuqtada funksiyaning chapdan va o‘ngdan limitlari $f(x_0-0)$ va $f(x_0+0)$ lar o‘zaro teng bo‘lib, funksiyaning x_0 nuqtasida erishadigan qiymati $f(x_0)$ dan farq qilsa, unda x_0 nuqta *bartaraf etilishi mumkin uzilish nuqtasi* deb ataladi.

$y=f(x)$ funksiyaning x_0 nuqtada chapdan yoki o‘ngdan limitlarining biri mavjud bo‘lmasa (xususan, cheksiz bo‘lsa), u holda x_0 nuqta *funksiyaning ikkinchi tur uzilish nuqtasi* deyiladi.

23.1. Quyidagi fuksiyalarning ko‘rsatilgan nuqtalarida bir tomonli limitlarini toping:

a) $f(x)=\begin{cases} x+1, & agar\ \ 0<x<1 \\ 3x+1, & agar\ 1\le x\le 3 \end{cases}$ $\qquad x=1$ nuqtasida

$$f(1-0)=\lim_{x\to 1-0} f(x)=\lim_{x\to 1-0}(x+1)=2$$

$$f(1+0)=\lim_{x\to 1+0} f(x)=\lim_{x\to 1+0}(3x+1)=4$$

b) $f(x)=\begin{cases} 3x, & agar\ -1\le x\le 1 \\ 2x, & agar\ 1<x\le 3 \end{cases}$ x=1 x=2 nuqtalarda

c) $y=\{x\}$, $\{x\}-x$ ning kasr qismi; x=1, x=2, x=3 nuqtalarda

d) $f(x)=\dfrac{3x+1}{x-1}$, $x=1$ nuqtada

Mustaqil yechish uchun misollar:

23.2. Quyidagi funksiyalarning uzluksizligini ta'rifga binoan isbotlang.

a) $f(x)=x^2+x-2$ barcha $x\in(-\infty;+\infty)$ larda

$$\lim_{\Delta x\to 0}(f(x+\Delta x)-f(x))=\lim_{\Delta x\to 0}((x+\Delta x)^2+(x+\Delta x)-2-(x^2+x-2))=$$

$$=\lim_{\Delta x\to 0}(2x\cdot\Delta x+\Delta x^2+\Delta x)=0$$

Demak, *f(x)* barcha $x\in(-\infty;+\infty)$ larda uzluksiz.

b) $f(x)=\sin(3x+2)$, barcha $x\in(-\infty;+\infty)$ larda

c) $f(x)=\dfrac{1}{x+1}$, barcha $(-1;+\infty)$ larda

Quyidagi funksiyalarning uzilish nuqtalari va ularning turlarini aniqlang. Grafiklarini yasang:

23.3. $f(x)=\begin{cases} x^2, & agar\ -\infty<x\le 0 \\ (x-1)^2, & agar\ 0<x\le 2 \\ 5-x, & agar\ 2<x<\infty \end{cases}$

f(x) funksiya $(-\infty;0)$, $(0;2)$ va $(2;+\infty)$ intervallarda aniqlangan va uzluksiz bo'lgan elementar funksiyalar bilan berilgan. Demak, faqat $x_1=0$ *va* $x_2=2$ nuqtalarda uzulishga ega bo'lishi mumkin.

$x_1=0$ nuqta uchun chap va o'ng limitlarni hisoblaymiz:

$$\lim_{x\to 0-0} f(x)=\lim_{x\to 0-0} x^2=0;$$

$$\lim_{x\to 0+0} f(x)=\lim_{x\to 0+0}(x-1)^2=1;\quad f(0)=0$$

Bu esa $x_1=0$ nuqtada *f(x)* fuksiya birinchi tur uzilishga ega bo'lishini bildiradi. $x_2=2$ nuqta uchun:

$$\lim_{x\to 2-0} f(x)=\lim_{x\to 2-0}(x-1)^2=1$$

$$\lim_{x\to 2+0} f(x)=\lim_{x\to 2+0}(5-x)=3 \quad f(2)=1$$

bo‘ladi.

$x_2=2$ nuqtada funksiya 1-tur uzilishga ega bo‘ladi.

23.4. $y=\dfrac{4}{x-2}$

23.5. $f(x)=\begin{cases}\dfrac{x}{2}, & x\neq 2\\ 0, & x=2\end{cases}$

23.6. $y=1+\dfrac{|x+1|}{x+1}$

23.7. $y=2^{\frac{1}{x}}$

23.8. $f(x)=\begin{cases}0.5x^2, & agar\ |x|<2\\ 2.5, & agar\ |x|=2\\ 3, & agar\ |x|>2\end{cases}$

23.9. $f(x)=\begin{cases}Sinx, & x<0\\ x, & 0\le x\le 2\\ 0, & x>2\end{cases}$

23.10. $f(x)=\begin{cases}-1, & x<0\\ Cosx, & 0\le x\le \pi\\ 1-x, & x>\pi\end{cases}$

23.11. $f(x)=\begin{cases}-x, & x\le 1\\ \dfrac{2}{x-1}, & x>1\end{cases}$

23.12. $f(x)=\begin{cases}-\dfrac{1}{x}, & x<0\\ 1, & 0\le x<1\\ x, & 1\le x\le 2\\ 3, & 2<x\le 3\end{cases}$

Funksiyalarning uzilish nuqtalarini toping va uzilish turlarini aniqlang:

23.13. $y=\dfrac{4}{4-x^2}$

23.14. $y=5^{\frac{4}{1-x}}+1$

23.15. $y=\dfrac{x+5}{x-3}$

23.16. $f(x)=9^{\frac{1}{2-x}}$

Quyidagi tenglamalar ko‘rsatilgan kesmalarda yechimga ega ekanligini ko‘rsating:

23.17. a) $x^3+3x+1=0;\ [-1;0]$ kesmada. Bu funksiya $[-1;0]$ da uzluksiz. Kesmaning uchlaridagi qiymatlari $f(1)=-3$, $f(0)=1$ bo‘lib, turli ishorali. Boltsano – Koshi teoremasiga binoan (-1; 0) da biror C nuqta topib $f(x)=c^3+3c+1=0$ bo‘lib, c berilgan tenglamalarning yechimi bo‘ladi.

b) $x^5-6x^2+3x-7=0;\quad [0;2]$

c) $3Sin^3x-5Sinx+1=0;\quad \left[0;\dfrac{\pi}{2}\right]$

d) $Cos^4x + 3Cosx + 1 = 0; \quad [0;\pi]$

23.18. Quyidagi funksiyalar ko‘rsatilgan kesmalarda chegaralangan ekanligini isbotlang:

a) $f(x) = Sinx \cdot Cos^2x - \sqrt{x+1}, \quad [0;10]$

$y = Sinx,\ y = Cos^2x\ \ va\ \ y = \sqrt{x+1}$ funksiyalarning har biri $[0;10]$ da uzluksiz bo‘lganligi uchun, $f(x) = Sinx \cdot Cos^2x - \sqrt{x+1}$ funksiya ham [0;10] da uzluksiz. Shuning uchun Veyershtrass teoremasiga binoan *f(x)* funksiya [0;10] da chegaralangan.

b) $f(x) = \dfrac{\sqrt{x^2+3x+1}}{x-1}$, [2;7]

c) $f(x) = \sqrt{x^2+3x+1} \cdot Cos^7x, \quad [0;2\pi]$

23.19. Bir tomonlama limitlarini toping:

a) $f(x) = \begin{cases} x^2, & agar\ -1 < x \le 2 \\ 2x+1, & agar\ \ 2 < x < 3 \end{cases}$ $\quad x = 2$ nuqtada

b) *y=E(x),* *E(x) – x* ning butun qismi

x=-2, *x*=0 , *x*=1 nuqtalarda

c) $f(x) = \dfrac{1}{x-2}$, $x = 2$ nuqtada.

23.20. Funksiyalarning uzluksizligini ta’rifga binoan izbotlang:

a) $f(x) = x^3 - 3$

b) $f(x) = Cos(2x+1)$

Quyidagi funksiyalarning uzilish nuqtalari va uzilish turlarini aniqlang:

23.21. $y = \dfrac{x}{x+2}$

23.22. $y = 2 - \dfrac{|x|}{x}$

23.23. $f(x) = \begin{cases} 2, & agar\ \ x = 0\ va\ x = \pm 2 \\ 4 - x^2, & agar\ |x| < 2 \\ 4, & agar\ \ |x| > 2 \end{cases}$

23.24. $y = \dfrac{1}{1 + 2^{\frac{1}{x}}}$

23.25. $y = 2^{\frac{1}{x-2}}$

24. HOSILA

y=f(x) funksiya grafigining $M_0(x_0;y_0)$ nuqtasiga o‘tkazilgan urinma tenglamasi

$$y-y_0=f`(x_0)(x-x_0)$$

normal tenglamasi

$$y-y_0=-\frac{1}{f`(x_0)}(x-x_0)$$

ko`rinishga ega bo‘ladi.

$M_0(x_0;y_0)$ nuqtada kesishuvchi $y=f_1(x)$ va $y=f_2(x)$ egri chiziqlar orasidagi burchak

$$tg\varphi=\frac{f_2`(x_0)-f_1`(x_0)}{1+f_1`(x_0)f_2`(x_0)}$$

formula orqali topiladi.

Funksiya differensiali dy=y`dx orqali hisoblanadi.

Agar Δx yetarlicha kichik miqdor bo`lsa, u holda $\Delta y=dy$ bo‘ladi va $f(x+\Delta x)\approx f(x)+f`(x)\Delta x$ taqribiy formula o‘rinlidir.

$y=f[g(x)]$ murakkab funksiya differensiali: $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$ yoki x_0 nuqtadagi hosilasi: $y`(x_0)=f`(u_0)\cdot g`(x_0)$ bo‘ladi.

Hosilalar jadvali:

Funksiyalar	*Hosilalar*
$y=C$ (bunda *C*-const)	$y`=0$
$y=x$	$y`=1$
$y=Cu$ (bunda *C*-const)	$y`=Cu`$
$y=u\pm v$	$y`=u`\pm v`$
$y=u\cdot v$	$y`=u`v+uv`$
$y=\frac{u}{v}$	$y`=\frac{u`v-uv`}{v^2}$
$y=\frac{C}{u}$, (bunda *C*-const)	$y`=-\frac{C}{u^2}u`$
$y=u^n$	$y`=nu^{n-1}u`$
$y=\sqrt{u}$	$y`=\frac{1}{2\sqrt{u}}u`$
$y=a^u$	$y`=a^u\ln a\cdot u`\quad a>1,\ a\neq 1$

$y=\log_a u$	$y` = \frac{1}{u}u`\log_a e = \frac{u`}{u\ln a}$
$y=\ln u$	$y` = \frac{1}{u}u`$
y=Sin u	$y` = Cosu \cdot u`$
$y = Cosu$	$y` = -Sinu \cdot u`$
$y = tgu$	$y` = \frac{1}{Cos^2u}u`$
$y = ctgu$	$y` = -\frac{1}{Sin^2u}u`$
$y = Secu$	$y` = Secu \cdot tg\,u \cdot u`$
$y = Cosecu$	$y` = -Cosecu \cdot ctg\,u \cdot u`$
$y = \arcsin u$	$y` = \frac{1}{\sqrt{1-u^2}} \cdot u`$
$y = \arccos u$	$y` = -\frac{1}{\sqrt{1-u^2}} \cdot u`$
$y = arctgu$	$y` = \frac{1}{1+u^2} \cdot u`$
$y = arcctgu$	$y` = -\frac{1}{1+u^2} \cdot u`$
$y = u \cdot v$	$y^{(n)} = u^{(n)}v + C_n^1 u^{(n-1)}v` + C_n^2 u^{(n-2)}v`` + ... + uv^{(n)}$
$y = [f(x)]^{\varphi(x)}$	$y` = [f(x)]^{\varphi(x)}\left\{\varphi`(x)\ln f(x) + \varphi(x)\frac{f`(x)}{f(x)}\right\}$
$y = shu$	$y` = chu \cdot u`$
$y = chu$	$y` = shu \cdot u`$
$y = thu$	$y` = \frac{1}{ch^2u} \cdot u`$
$y = cthu$	$y` = -\frac{1}{sh^2u} \cdot u`$

Hosila ta’rifidan foydalanib, *y=f(x)* funksiyalar uchun *y`* hosilasini toping:

24.1. $y = 2x^3 + 5x^2 - 7x - 4$ funksiya orttirmasini topamiz:

$$\Delta y = (2(x+\Delta x)^3 + 5(x+\Delta x)^2 - 7(x+\Delta x) - 4) - (2x^3 + 5x^2 - 7x - 4) =$$
$$= 6x^2\Delta x + 6x\Delta x^2 + 2\Delta x^3 + 10x\Delta x + 5\Delta x^2 - 7\Delta x$$

$\Delta x \to 0$ intilganda quyidagi limitni topamiz:

$\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} (6x^2 + 6x\Delta x + 2\Delta x^2 + 10x + 5\Delta x - 7) = 6x^2 + 10x - 7$ Shunday qilib, ta'rifga ko'ra hosila $y` = 6x^2 + 10x - 7$.

Mustaqil yechish uchun misollar:

24.2. a) $y = \frac{1}{x}$ b) $y = \sqrt{x}$

c) $y = \frac{1}{x^2}$ d) $y = \frac{1}{\sqrt{x}}$

Differensiallash qoida va formulalaridan foydalanib, quyidagi funksiyalarning hosilasini toping:

24.3. $y = x + \frac{1}{x^2} - \frac{1}{5x^5}$ 24.4. $y = 3x^2 + 5\sqrt[3]{x^5} - \frac{4}{x^3}$

24.5. $y = x^3 Sinx$ 24.6. $y = Sinx \cdot \ln x$

24.7. $y = \frac{x^4 + 1}{x^4 - 1}$ 24.8. $y = x^2 ctgx$

24.9. *y=x arccos x* 24.10. $y = e^x arctgx$

24.11. $y = \frac{Cosx}{x^2}$ 24.12. $y = \frac{x^2}{x^2 + 1}$

24.13. $y = 3x^3 \ln x - x^3$ 24.14. $y = \frac{Cosx}{1 - Sinx}$

24.15. $y = \frac{\sqrt{x}}{\sqrt{x} + 1}$ 24.16. $y = x^2 \log_3 x$

24.17. $y = \frac{\ln x}{Sinx} + x\, ctgx$ 24.18. $y = \frac{x\, tgx}{1 + x^2}$

24.19. $y = arctgx - arcctgx$

24.20. a) $y = |\ln x|$ funksiya *x*=1 da hosilaga egami? Tekshiring.

b) $y = |x|$ funksiyani *x*=0 da bir tomonli hosilalarini toping. Bu funksiya *x*=0 da hosilaga egami?

Quyidagi masalalarda egri chiziqlarga o'tkazilgan urinmalarning tenglamalari yozilsin va egri chiziqlar hamda urinmalari yasalsin:

24.21. $x^2 + 2xy^2 + 3y^4 = 6$ egri chiziqqa M(1,-1) nuqtada.

Egri chiziq tenglamasidan *y*` hosilani topamiz:

$2x+2y^2+4xyy`+12y^3y`=0$, ya'ni $y`=-\dfrac{x+y^2}{2xy+6y^3}$.

Demak $y`(-1;1)=-\dfrac{1+(-1)^2}{2\cdot 1(-1)+6(-1)^3}=\dfrac{1}{4}$.

Urinma tenglamasi $\qquad y+1=\dfrac{1}{4}(x-1)$

$$x-4y+5=0$$

Normal tenglamasi $\qquad y+1=-4(x-1)$

$$4x+y-3=0.$$

24.22. $y=\dfrac{x^3}{3}$ $\qquad x=-1$ nuqtada;

24.23. $y=\dfrac{8}{4+x^2}$ $\qquad x=2$ nuqtada;

24.24. $y=Sinx$ $\qquad x=\pi$ nuqtada;

24.25. $y=x^2+2x-1$ parabolaning $y=2x^2$ parabola bilan kesishgan nuqtasida o'tkazilgan urinma va normal tenglamalarini yozing.

24.26. $y=x^2$ *va* $y^2=x$ parabolalar qanday burchak ostida kesishadi?

24.27. $y=Sinx$ sinusoida Ox o'qini qanday burchak ostida kesib o'tadi?

24.28. $y=x-x^3$ *va* $y=5x$ chiziqlar orasidagi burchakni toping.

24.29. $y=1+Sinx$, $y=1$ chiziqlar orasidagi burchakni toping.

24.30. $y=2x^3+5x^2$ funksiyaning orttirmasini va differensialini toping.

24.31. $f(x)=x^3-7x^2+8$ funksiya uchun $\Delta x=0.01$ bo`lsa, $\Delta f(5)$ *va* $df(5)$ ni hisoblang.

24.32. Quyidagi funksiyalarning differensialini toping:

a) $y=\dfrac{3}{4}x\sqrt[3]{x}$ $\qquad$ b) $y=(x^2+2x+2)e^{-x}$

c) $y=x^2Sinx+2xCosx-2Sinx$ $\qquad$ d) $y=\dfrac{2^{3x}}{3^{2x}}$

Quyidagilarni taqribiy hisoblang:

24.33. *arcsin* 0.51

y=arcsin x funksiya uchun *x*=0.5 va $\Delta x = 0{,}01$ deb olamiz. Taqribiy hisoblash formulasiga ko‘ra topamiz. $\arcsin(x + \Delta x) \approx \arcsin x + (\arcsin x)`\cdot\Delta x$

$$\arcsin 0.51 \approx \arcsin 0.5 + \frac{1}{\sqrt{1-(0.5)^2}} \cdot 0.01 = \frac{\pi}{6} + 0.011 = 0.513.$$

24.34. $\sqrt[3]{1.02}$ 24.35. $\sqrt[5]{33}$

24.36. $Sin29^o$ 24.37. $arctg1.05$

Funksiyalarning hosilasini toping:

24.38. $y = (1+\sqrt[3]{x})^2$ 24.39. $y = \frac{3}{\sqrt[3]{x}} - \frac{2}{\sqrt{x}}$

24.40. $y = x^{\frac{3}{2}} Sinx$ 24.41. $y = \sqrt[5]{x} arctgx$

24.42. $y = \frac{x^7 - 5x^4 + 1}{x^2 + 1}$ 24.43. $y = \frac{Cosx}{1 + 2Sinx}$

24.44. $y = x^2 - 3x + 5$ parabolaning $M_0(2;3)$ nuqtasida o‘tkazilgan urinma va normal tenglamasini tuzing.

24.45. $\frac{x^2}{9} - \frac{y^2}{8} = 1$ gipirbolaga $M_0(-9;-8)$ nuqtasida o‘tkazilgan urinma tenglamasini tuzing.

24.46. $y = x^2$ parabolaning qaysi nuqtasida o‘tkazilgan urinma

a) $y = 4x - 5$ to‘g‘ri chiziqqa parallel;

b) $2x - 6y + 5 = 0$ to‘g‘ri chiziqqa perpendikulyar bo‘ladi?

24.47. $2y = x^2$ *va* $2y = 8 - x^2$ egri chiziqlar qanday burchak ostida kesishadi?

Funksiyalar differensialini toping:

24.48. $y = arctg\frac{x}{a}$; $dy - ?$ 24.49. $y = e^{2x}\arcsin x$; $dy - ?$

24.50. $y = x\ln x$; $dy - ?$ 24.51. $y = \frac{(x+1)^2}{(x+2)^3(x+3)^4}$ $dy - ?$

Quyidagilarni taqribiy hisoblang:

24.52. $tg46^o$ 24.53. $Cos31^o$

24.54. $\ln tg 47^o 15`$ 24.55. $\sqrt[4]{15.8}$

25. YUQORI TARTIBLI HOSILALAR

$y=f(x)$ funksiya uchun birinchi tartibli hosilasi y' aniqlangan bo‘lsin. Birinchi hosiladan olingan hosila *ikkinchi tartibli hosila* yoki boshlang‘ich funksiyaning *ikkinchi hosilasi* deyiladi va y'' yoki $f''(x)$ simvol bilan belgilanadi:

$$y''=(y')'=f``(x).$$

Ikkinchi hosiladan olingan hosila *uchinchi tartibli hosila* yoki boshlang‘ich funksiyaning *uchinchi hosilasi* deyiladi va y''' yoki $f'''(x)$ simvol bilan belgilanadi:

Umuman $f(x)$ funksiyaning *n-tartibli hosilasi* deb uning (*n-1*)-tartibli hosilasidan (birinchi tartibli) hosilasiga aytiladi va $y^{(n)}$ yoki $f^{(n)}(x)$ simvol bilan belgilanadi:

$$y^{(n)}=\left(y^{(n-1)}\right)'=f^{(n)}(x).$$

Bunda ushbu formulalar o‘rinli:

$$(U+V)^{(n)}=U^{(n)}+V^{(n)};$$

$$(CU)^{(n)}=C\cdot U^{(n)};$$

$$(U\cdot V)^{(n)}=U^{(n)}V+nU^{(n-1)}V'+\frac{n(n-1)}{1\cdot 2}U^{(n-2)}V''+...+UV^{(n)}$$ (Leybnits formulasi).

Yuqori tartibli differensiallar ham shunday ta’riflanadi:

n-tartibli differensial deb (*n-1*)-tartibli differensialning birinchi differensialiga aytiladi:

$$d^n y=d(d^{n-1}y)=\left[f^{(n-1)}(x)dx^{n-1}\right]'dx$$

$$d^n(y)=f^{(n)}(x)dx^n.$$

Mustaqil yechish uchun misollar:

Murakkab funksiya hosilasini toping:

25.1. $y=\frac{1}{3}Sin^3\sqrt{x}-\frac{2}{5}Sin^5\sqrt{x}+\frac{1}{7}Sin^7\sqrt{x}$

$$y`=\frac{1}{3}\cdot 3Sin^2\sqrt{x}Cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}-\frac{2}{5}\cdot 5Sin^4\sqrt{x}\cdot Cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}+\frac{1}{7}\cdot 7Sin^6\sqrt{x}\cdot Cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}=$$

$$=\frac{1}{2\sqrt{x}}Sin^2\sqrt{x}\cdot Cos\sqrt{x}\cdot\left(1-2Sin^2\sqrt{x}+Sin^4\sqrt{x}\right)=\frac{1}{2\sqrt{x}}\cdot Sin^2\sqrt{x}\cdot Cos^5\sqrt{x}$$

25.2. $y=\ln\left(2x^3+3x^2\right)$

25.3. $y=\sqrt{4x+Sin4x}$

25.4. $y=\sqrt{\frac{x}{2}-Sin\frac{x}{2}}$

25.5. $y=e^{\frac{x}{a}}\cdot Cos\frac{x}{a}$

25.6. $y=\sqrt{1-3x^2}$

25.7. $y=Cos^3\left(\frac{x}{3}\right)$

25.8. $y=-ctg^2\frac{x}{2}-2\ln\left(Sin\frac{x}{2}\right)$

25.9. $y=\arccos\frac{9-x^2}{9+x^2}$

25.10. $y=1-e^{Sin^2 3x}\cdot Cos^2 3x$

25.11. $y=e^{\sqrt{2x}}\left(\sqrt{2x}-1\right)$

25.12. $y=-\mathrm{cos}ec^2\left(\frac{x}{2}\right)$

25.13. $y=\arcsin\sqrt{1-0.2x^2}$

25.14. $y=\frac{1}{\sqrt{1-mx^2}}$

25.15. $y=\frac{Sinx}{1+\ln Sinx}$

25.16. $y=arctg(x+1)+\frac{x+1}{x^2+2x+2}$

25.17. $y=\ln tg\frac{x}{2}+Cosx+\frac{1}{3}Cos^3x$

25.18. $y=\ln\left(1-\frac{1}{x}\right)+\frac{1}{x}$

25.19. $y=\ln\ln x(\ln\ln\ln x-1)$

25.20. $y=tg^3tgx+3tgtgx$

25.21. $y=2^{Cos3x-3Cosx}$

25.22. $y=\frac{x^2e^{x^2}}{x^2+1}$

25.23. $y=arctg\frac{2x^4}{1-x^2}$

25.24. $y=\arccos\sqrt{1-2^x}$

25.25. $y=\log_2 Sin^2x$

25.26. $y=xe^x(Sinx-Cosx)+e^xCosx$

25.27. $y=\log_x e$

25.28. $y=x^x$

25.29. $y=\sqrt[3]{x\sqrt{x\sqrt{x}}}$

25.30. $y=\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$

Oshkormas ko‘rinishda berilgan funksiyalar hosilasini toping:

25.31. $x^3+\ln y-x^2e^y=0$ $\qquad 3x^2+\frac{y`}{y}-x^2e^y y`-2xe^y=0,\quad ya'ni\quad y`=\frac{\left(2xe^y-3x^2\right)y}{1-x^2ye^y}$

25.32. $Ax^2 2Bxy+Cy^2+2Dx+2Ey+F=0$

25.33. $x^4-6x^2y^2+9y^4-5x^2+15y^2-100=0$

25.34. $x^y-y^x=0$

25.35. $xSiny+ySinx=0$

25.36. $e^x+e^y-2^{xy}-1=0$

25.37. $Sin\left(y-x^2\right)-\ln\left(y-x^2\right)+2\sqrt{y-x^2}-3=0$

25.38. $\frac{y}{x}+e^{\frac{y}{x}}-3\sqrt{\frac{y}{x}}=0$

Parametrik ko‘rinishda berilgan quyidagi funksiyalarni differensiallang:

25.39. $\begin{cases} x = aCost \\ y = aS\text{int} \end{cases}$

25.40. $\begin{cases} x = aCos^3 t \\ y = aSin^3 t \end{cases}$

25.41. $\begin{cases} x = \dfrac{3at}{1+t^2} \\ y = \dfrac{3at^2}{1+t^3} \end{cases}$

25.42. $\begin{cases} x = cht \\ y = sht \end{cases}$

25.43. Funksiyalarning 2-tartibli hosilalari topilsin:

1) $y = Sin^2 x$; 2) $y = tgx$; 3) $y = \sqrt{1+x^2}$.

25.44. Quyidagi funksiyalarning 3-tartibli hosilalari topilsin:

1) $y = x\ln x$; 2) $s = t \cdot e^{-t}$; 3) $y = arctg\dfrac{x}{a}$.

Quyidagi funksiyalarning n-tartibli hosilalari topilsin:

25.45. $y = \ln x$

$y` = \dfrac{1}{x} = x^{-1}$; $y`` = -1 \cdot x^{-2}$; $y``` = 1 \cdot (-2) \cdot x^{-3}$; $y^{(IV)} = 1 \cdot 2 \cdot 3 \cdot x^{-4}$;

$y^{(n)} = 1 \cdot 2 \cdot 3 \cdot ... \cdot (n-1) \cdot (-1)^{n-1} \cdot x^{-n} = (-1)^{n-1} \cdot \dfrac{(n-1)!}{x^n}$.

25.46. $y = e^{-\frac{x}{a}}$;

25.47. $y = \sqrt{x}$;

25.48. $y = x^n$;

25.49. $y = Sinx$;

25.50. $y = Cos^2 x$;

25.51. $\begin{cases} x = \ln t \\ y = 1/t \end{cases}$

Quyidagi funksiyalarning 1, 2, 3-tartibli differensiallarini toping:

25.52. $y = xSinx$

25.53. $y = x(\ln x - 1)$

Funksiyalarning hosilasini toping:

25.54. $y = \dfrac{3}{4} \cdot 4x^3 \sqrt{x}$

25.55. $y = (x^2 + 2x + 2)e^{-x}$

25.56. $y = \ln(2x^3 + 3x^2)$

25.57. $y = \ln tg\dfrac{2x+1}{4}$

25.58. $y = \dfrac{x}{2}\sqrt{a^2 - x^2} + \dfrac{a^2}{2}\arcsin\dfrac{x}{a}$

25.59. $y = arctg\sqrt{4x^2 - 1}$

25.60. $y = e^{-x} - Sin e^{-x} Cos e^{-x}$ 25.61. $y = \ln\frac{x^5}{x^5+2}$

25.62. $y = \frac{x - e^{2x}}{x + e^{2x}}$ 25.63. $y = 3x\ Sin^3 x + 3Cosx - Cos3x$

25.64. $y = \ln\frac{\sqrt{x^2+2x}}{x+1}$ 25.65. $y = \frac{\ln x}{x^5} + \frac{1}{5x^5}$

25.66. $y = 2\left(tg\sqrt{x} - \sqrt{x}\right)$ 25.67. $y = \log_a\left(x + \sqrt{x^2+9}\right)$

25.68. $y = \log_{Cosx} Sinx$ 25.69. $y = x^{1/\ln x}$

25.70. $y = x^2 e^{x^2} \ln x$

25.71. $\left(u^v\right)' = vu^{v-1}u' + u^v v' \ln u$ ekanini ko‘rsating.

Quyidagi tenglamalardan $y`$ ni toping:

25.72. 1) $x^2 + y^2 = a^2$; 2) $y^2 = 2px$ 3) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

25.73. 1) $x^2 + xy + y^2 = 6$; 2) $x^2 + y^2 - xy = 0$

25.74. 1) $x^{2/3} + y^{2/3} = a^{2/3}$; 2) $e^y - e^{-x} + xy = 0$

25.75. $e^x Sinu - e^{-y} Cosx = 0$ 25.76. $x = y + arctgy$

25.77. 1) $x^2 + y^2 = a^2$; 2) $ax + ay - xy = c$; 3) $x^m y^n = 1$ tenglamalardan $y``$ topilsin.

25.78. Tenglamalardan $\frac{d^2y}{dx^2}$ topilsin:

1) $\begin{cases} x = aCost \\ y = aS\mathrm{int} \end{cases}$ 2) $\begin{cases} x = t^2 \\ y = \frac{t^3}{3} - t \end{cases}$ 3) $\begin{cases} x = a(t - S\mathrm{int}) \\ y = a(1 - Cost) \end{cases}$

25.79. Funksiyalarning 2-tartibli hosilalari topilsin:

1) $y = e^{-x^2}$; 2) $y = ctgx$; 3) $\arcsin\frac{x}{2}$

Funksiyalarning n-tartibli hosilalari topilsin:

25.80. $y = x\sqrt[n]{x}$ 25.81. $y = \frac{1}{2x+1}$

25.82. $y = 5 - 3Cos^2 x$ 25.83. $y = 2^x + 2^{-x}$

26. LOPITAL QOIDASI

Roll teoremasi: $y = f(x)$ funksiya [*a;b*] kesmada aniqlangan va uzluksiz bo‘lsin. Agar funksiya (*a;b*) intervalda differensiallanuvchi bo`lib, *f(a)=f(b)* tenglik o‘rinli bo‘lsa, u holda *(a;b)* intervalga tegishli hech bo`lmaganda bitta shunday bir *c* nuqta topiladiki, $f`(c) = 0$ bo‘ladi.

Lagranj teoremasi: $y = f(x)$ funksiya [*a;b*] kesmada aniqlangan va uzluksiz bo‘lib, (*a;b*) intervalda differensiallanuvchi bo‘lsa, u holda *(a;b)* intervalga tegishli hech bo‘lmaganda bitta shunday bir *c* nuqta topiladiki, $f(b) - f(a) = f`(c)(b-a)$ munosabat o‘rinli bo‘ladi.

Teylor-Makloren formulasi

$$f(x) = f(a) + \frac{f`(a)}{1!}(x-a) + \frac{f``(a)}{2!}(x-a)^2 + ... + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x) \text{ ifoda}$$

Teylor formulasi, $R_n(x)$ Teylor formulasining qoldiq hadi.

Teylor formulasining *a=0* dagi hususiy ko’rinishi

$$f(x) = f(0) + \frac{f`(0)}{1!}x + \frac{f``(0)}{2!}x^2 + ... + \frac{f^{(n)}(0)}{n!}x^n + R_n(x)$$

Makloren formulasi deyiladi.

Bu formula funksiyaning erkli o‘zgaruvchi *x* ning darajalari bo‘yicha yoyilmasini beradi.

Roll, Lagranj, Koshi teoremalariga doir misollar.

1. 26.1. $f(x) = \frac{x^4}{4}$ funksiya uchun [-1;1] segmentda Roll teoremasini tatbiq etish mumkinmi?

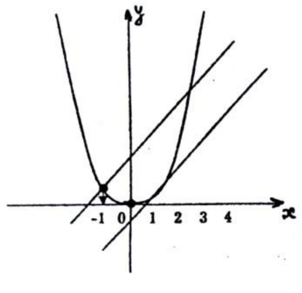
a) *f(x)* funksiya uchun Roll teoremasining birinchi sharti bajariladi: *f(x)* funksiya [-1;1]da uzluksiz.

b) *f(x)* funksiya uchun ikkinchi shart ham bajariladi. $f`(x) = x^3$ hosila mavjud.

a) *f(x)* funksiya uchun *f(-1)=f(1)=1/4* tenglik o‘rinli. Demak, *f `(c)=0* bo‘ladigan nuqta mavjud: $f`(x)=x^3=0$, *x=c=0* da o‘rinli.

$$f`(c)=f`(0)=0$$

26.2. $y=x^2$ parabolaning qaysi nuqtasiga o‘tkazilgan urinma *A*(-1;1) va *B*(3;9) nuqtalarnui birlashtiruvchi vatarga parallel bo‘ladi?



a= -1; *b*=3

AB vatarning burchak koeffitsienti $K = \frac{9-1}{3+1} = 2$

f '(x)=2x; 2x=2 tenglik faqat *x=1* bo'lganda o'rinli, demak *x=1* nuqtaga o'tkazilgan urinma vatarga parallel.

Mustaqil yechish uchun misollar:

26.3. Roll teoremasini $f(x)=\sqrt[3]{x^2}$ funksiyaga [-1;1] segmentda tatbiq qilish mumkinmi?

26.4. $f(x)=x^2-6x+100$ funksiya uchun Roll teoremasi shartlari *x* ning qanday qiymatlarida qanoatlantiradi?

a=1, b=5.

26.5. [*a, b*] segmentda $f(x)=x^2$ funksiya uchun Lagranj formulasi yozilsin va *C(x,y)* nuqta topilsin

26.6. [1;4] segmentda $f(x)=\sqrt{x}$ funksiya uchun Lagranj formulasi yozilsin va *C(x,y)* nuqta topilsin.

26.7. $y=2x-x^2$ egri chiziqning *AB* yoyi ustida shunday *M(x,y)* nuqtani topingki, bu nuqtaga o'tkazilgan urinma *AB* vatarga parallel bo'lsin: *A*(1;1) *B*(3;-3)

26.8. $f(x)=x^3$ va $\varphi(x)=x^2$ funksiyalar uchun Koshining

$\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)} = \frac{f`(c)}{\varphi`(c)}$ formulasi yozilsin hamda *c* topilsin.

Quyidagi funksiyalarni Makloren formulasi bo'yicha yozing:

26.9. $f(x) = e^x$

26.10. $f(x) = Sinx$

26.11. $f(x)=ln(1+x)$

26.12. $\sqrt{e^x}$ ni Makloren formulasiga ko'ra $x=1$ da hisoblang (4 ta hadini olib).

Lopital qoidasi

1) $\lim\limits_{x\to 0}\frac{f(x)}{\varphi(x)} = \left\{\frac{0}{0}\right\} = \lim\limits_{x\to 0}\frac{f`(x)}{\varphi`(x)} = \left\{\frac{0}{0}\right\} = \lim\limits_{x\to 0}\frac{f``(x)}{\varphi``(x)} = \ldots$

2) $\lim\limits_{x\to\infty}\frac{f(x)}{\varphi(x)} = \left\{\frac{\infty}{\infty}\right\} = \lim\limits_{x\to\infty}\frac{f`(x)}{\varphi`(x)} = \left\{\frac{\infty}{\infty}\right\} = \lim\limits_{x\to\infty}\frac{f``(x)}{\varphi``(x)} = \ldots$

Quyidagi funksiyalar limitini hisoblang:

26.13. $\lim\limits_{x\to 0}\frac{Sinax}{Sinbx} = \left\{\frac{0}{0}\right\} = \lim\limits_{x\to 0}\frac{aCosax}{bCosax} = \frac{a}{b}$

26.14. $\lim\limits_{x\to 4}\frac{x^2-16}{x^2-6x+8}$ 26.15. $\lim\limits_{x\to a}\frac{e^x-e^a}{x-a}$

26.16. $\lim\limits_{x\to 0}\frac{tgx-x}{x-Sinx}$ 26.17. $\lim\limits_{x\to 1}\frac{1-4Sin^2\frac{\pi x}{6}}{1-x^2}$

26.18. $\lim\limits_{x\to 0}\frac{x-Sinx}{x^3}$ 26.19. $\lim\limits_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$

26.20. $\lim\limits_{x\to\infty}(1-x)^{\frac{1}{x}}$ $y=(1-x)^{\frac{1}{x}}$ deb belgilab, tenglikning ikkala qismini ligarifmlaymiz $lny=\frac{1}{x}\ln|1-x|=\frac{\ln|1-x|}{x}$. Endi limitga o'tamiz

$$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{\ln|1-x|}{x}=\left\{\frac{\infty}{\infty}\right\}=\lim_{x\to\infty}\frac{-1/|1-x|}{1}=0.\ \ln y=0,\quad y=1$$

26.21. $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{tgx}$ 26.22. $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{Sinx}$

26.23. $\lim\limits_{x\to\frac{\pi}{2}}(tgx)^{2Cosx}$ 26.24. $\lim\limits_{x\to 0}(Sinx)^{Sinx}$

26.25. $\lim\limits_{x\to 0}x\ln x$

26.26. $f(x)=\sqrt[3]{8x-x^2}$ funksiya uchun *a*=0, *b*=8 bo'lganda Roll teoremasi shartlari *x* ning qanday qiymatlarida bajariladi?

26.27. [-1;2] segmentda 4/*x* va $1-\sqrt[3]{x^2}$ funksiyalariga Lagranj teoremasini tatbiq qilish mumkin emasligi ko'rsatilsin.

26.28. Tenglamalari $x=t^2$, $y=t^3$ parametrik ko'rinishda berilgan egri chiziqning *AB* yoyida shunday *M* nuqtani topingki, bu nuqtada o'tkazilgan urinma *AB* vatarga parallel bo'lsin, *A* va *B* nuqtalarga *t=1*, *t=3* qiymatlar mos keladi.

26.29. $y=x^3-3x$ egri chiziqning *AB* yoyining qaysi nuqtasida o'tkazilgan urinma *AB* vatarga parallel bo'ladi: *A*(0;0), *B*(3;8)

26.30. Quyidagi funksiyalar uchun Lagranj formulasi yozilsin va *C(x,y)* nuqta topilsin.

1) [0;1] segmentda *f(x) = arctgx*

2) [0;1] segmentda *f(x) = arcSinx*

3) [1;2] segmentda *f(x) = lnx*

26.31. Quyidagi funksiyalar uchun Koshi formulasi yozilsin va *C* nuqta topilsin:

1) [0; π/2] segmentda *Sinx* va *Cosx* 2) [1;4] segmentda x^2 va $\sqrt{x}$

Quyidagi funksiyalarni Makloren formulasi bo'yicha yozing:

26.32. $f(x)=cosx$ 26.33. $f(x)=(1+x)^a$

Quyidagi funksiyalar limitini toping:

26.34. $\lim\limits_{x\to\infty}\frac{\pi-2arctgx}{e^{\frac{3}{x}}-1}$ 26.35. $\lim\limits_{x\to 0}\frac{2-(e^x+e^{-x})Cosx}{x^4}$

26.36. $\lim\limits_{x\to 0}\frac{e^{3x}-3x-1}{Sin^2 5x}$ 26.37. $\lim\limits_{x\to 0}\frac{Sin3x-3xe^x+3x}{arctgx-Sinx-\frac{x^3}{6}}$

26.38. $\lim\limits_{x\to 0}\frac{\ln x}{1+2\ln Sinx}$ 26.39. $\lim\limits_{x\to 1}\frac{\ln(x-1)}{ctg\pi x}$

26.40. $\lim\limits_{x\to 0}(1-Cosx)ctgx$ 26.41. $\lim\limits_{x\to 0}\left(\frac{1}{x^2}-ctg^2x\right)$

26.42. $\lim\limits_{x\to\infty}\left(x+2^x\right)^{\frac{1}{x}}$ 26.43. $\lim\limits_{x\to 0}\left(\frac{tgx}{x}\right)^{\frac{1}{x^2}}$

27. FUNKSIYANI HOSILA YORDAMIDA TO‘LA TEKSHIRISH

Funksiyani tekshirish va grafigini yasash quyidagi umumiy sxema bo‘yicha bajariladi:

1) Funksiyaning aniqlanish sohasi topiladi.

2) Funksiya juft $\left(f(-x)=f(x),\ \pm x\in D(f)\right)$, toqligi $\left(f(-x)=-f(x),\ \pm x\in D(f)\right)$ yoki juft ham emas, toq ham emasligi aniqlanadi. Agar funksiyaning juft yoki toqligi aniqlansa, funksiyani musbat yoki manfiy haqiqiy sonlar yarim o‘qida tekshirish yetarli.

Agar funksiya juft funksiya bo‘lsa, bu funksiyaning grafigi *Oy* o‘qiga nisbatan simmetrik, toq bo‘lsa koordinata boshiga nisbatan simmetrik bo‘ladi.

3) Davriy yoki davriymasligi aniqlanadi. Davriy funksiyani bir davr oralag‘ida tekshirish yetarli.

4) Funksiya grafigining koordinata o‘qlari bilan kesishish nuqtalari topiladi. *Ox* o‘qi bilan kesishish nuqtalari $\begin{cases} y=f(x) \\ y=0 \end{cases}$ sistema, *Oy* o‘qi bilan kesishish nyqtalari esa $\begin{cases} y=f(x) \\ x=0 \end{cases}$ sistemani yechish bilan topiladi. Funksiya grafigining asimptotalari quriladi.

5) Uzilish nuqtalari aniqlanadi va ularning atrofida funksiyaning o‘zini tutishi tekshiriladi. Funksiyanig og’ma asimptotasi

($k=\lim\limits_{x\to\infty}\frac{f(x)}{x}$ $b=\lim\limits_{x\to\infty}\left[f(x)-kx\right],$ $y=kx+b$) tekshiriladi

6) Funksiyaning o‘sish va kamayish intervallari, maksimum va minimum nyqtalari topiladi.

7) Funksiya grafigining qavariqligi va egilish nuqtalari topiladi.

8) Yig‘ilgan ma’lumotlar jadval ko‘rinishida tuziladi.

9) Funksiya grafigi yasaladi.

27.1. Quyidagi berilgan funksiyani tekshirib, grafigini chizing:

$f(x)=\dfrac{x^2+1}{x^2-1}$ berilgan funksiya $D=\{(-\infty;-1)\cup(-1;1)\cup(1;+\infty)\}$ to‘plamda aniqlangan.

Bu funksiya uchun $f(-x)=f(x)$ bo‘lganidan u juftdir va uni $[0;+\infty]$ oraliqda tekshirish kifoya.

Funksiyaning birinchi va ikkinchi tartibli hosilalari:

$$f`(x)=\frac{-4x}{(x^2-1)^2} \qquad f``(x)=\frac{4(1+3x^2)}{(x^2-1)^3}$$

Birinchi tartibli hosila $[0;+\infty)$ oraliqning $x=1$ nuqtasidan boshqa barcha nuqtalarida aniqlangan va $x=0$ nuqtada nolga aylanadi. Ikkinchi tartibli hosilaning $x=0$ nuqtadagi qiymati $f``(0)=-4<0$, shuning uchun $f(x)$ funksiya $x=0$ nuqtada maksimumga ega va bu maksimum qiymat $f(0)=-1$ bo‘ ladi.

Endi $(0;1)$ va $(1;+\ \infty)$ da $f`(x)<0$ bo‘lganidan bu to‘plamda $f(x)$ ning kamayuvchiligi kelib chiqadi. So‘ngra:

$$\lim_{x\to-1-0}\frac{x^2+1}{x^2-1}=+\infty \qquad \lim_{x\to-1+0}\frac{x^2+1}{x^2-1}=-\infty$$

$$\lim_{x\to1-0}\frac{x^2+1}{x^2-1}=-\infty \qquad \lim_{x\to1+0}\frac{x^2+1}{x^2-1}=+\infty$$

bo‘lgani uchun $x=\pm1$ (funksiyaning ikkinchi tur uzilish nuqtalari) to‘g‘ri chiziqlar vertical asimptotalar ekanligini va

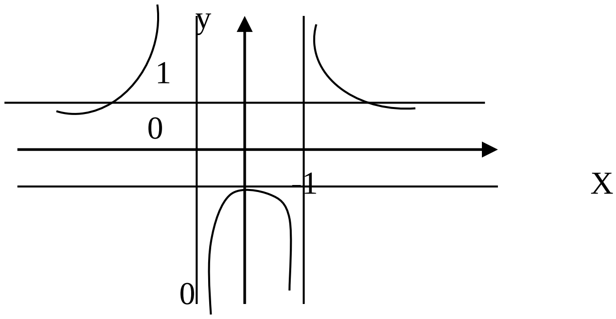
$$k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^2+1}{x^2-1}\cdot\frac{1}{x}=0$$

$$b=\lim_{x\to\infty}[f(x)-kx]=\lim_{x\to\infty}\frac{x^2+1}{x^2-1}=1$$

limitlarga ko‘ra $y=1$ gorizontal to‘g‘ri chiziq $f(x)$ funksiya grafigining asimptotasi ekanligini hosil qilamiz.

Endi $1+3x^2=0$ tenglama xaqiqiy sonlar o‘qida yechimga ega bo‘lmaganligi sababli funksiyaning ikkinchi tartibli hosilasi nolga teng bo‘lmasligi, ya’ni egilish nuqtasi yo‘qligi kelib ciqadi. Ikkinchi tartibli hosilaning qiymatlari $[0;\ 1)$ da $f``(x)>0$, $(1;\ +\ \infty)$ da $f``(x)<0$. Demak, funksiya grafigi $(-1;\ 1)$ da qavariq, hamda $(-\infty;\ -1)$ va $(1;\ +\infty)$ da botiq bo‘ladi.

	$(-\infty;-1)$	-1	(-1; 0)	0	(0; 1)	1	$(1;+\infty)$
$f'(x)$	+		+	0	-		-
$f''(x)$	+		-	-4	-		+
$f(x)$	↗		↗	-1	↘		↘
	∪	II tur uzilish	∩	max	∩	II tur uzilish	∪



Mustaqil yechish uchun misollar:

Quyidagi funksiyalar grafiklarining asimptotalarini toping:

27.2. $y=\frac{x^2-2x+3}{x+2}$ $\lim\limits_{x\to-2}\frac{x^2-2x+3}{x+2}=\infty$, ya’ni x=-2 to‘g‘ri chiziq vertical asimptotadir. $k=\lim\limits_{x\to\infty}\frac{x^2-2x+3}{x(x+2)}=1$ $b=\lim\limits_{x\to\infty}[\frac{x^2-2x+3}{x+2}-x]=-4$ demak $y=kx+b$ formulaga ko‘ra $y=x-4$ to‘g‘ri chiziq og‘ma asimptotadir.

27.3. $y=\frac{2x}{x-1}$ 27.4. $y=\frac{x}{2x-1}+x$

27.5. $y=\frac{\ln x}{x}$ 27.6. $y=\frac{1}{x}+4x^2$

27.7. $y=2\sqrt{x^2+4}$ 27.8. $y=\frac{x}{x^2+1}$

27.9. $y=x+\frac{Sinx}{x}$ 27.10. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Quyidagi berilgan funksiyalarni tekshirib, grafiklarini chizing:

27.11. $y=\sqrt[3]{1-x^3}$ 27.12. $y=Sin^2x$

27.13. $y=\ln x-\ln(x-1)$ 27.14. $y=\frac{x^3}{x^2-4}$

27.15. $y = 16x(x-1)^3$

27.16. $y = 2Sinx + Cos2x$ $([0,\pi))$ oraliqda

27.17. $y = \frac{x^2}{x-2}$

27.18. $y = (x-1)\sqrt{x}$

27.19. $y = \ln\frac{x}{x-1}$

27.20. $y=Sin2x-x$ $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda

27.21. $y=2x\pm ctgx$ (0, π) oraliqda

27.22. $y=x+e^{-x}$

27.23. $y=ln(x+\sqrt{x^2+1})$

27.24. $y = \frac{x^3}{(x-2)^2}$

Quyidagi funksiyalar grafiklarining asimptotalarini toping:

27.25. $y=2x-\frac{Cosx}{x}$

27.26. $y=\frac{\ln^2 x}{x} - 3x$

27.27. $y = x^2 \cdot e^{-x}$

27.28. $y=0.5x+arctgx$

27.29. $y=-xarctgx$

28. ANIQMAS INTEGRALNI INTEGRALLASH USULLARI

Integrallash amali differensiallashga teskari amal bo'lgani uchun asosiy integrallash formulalarini bevosita topish mumkin. Barcha formulalarda $u = u(x)$ x ning differensiallanuvchi funksiyasi deb belgilanadi. Ixtiyoriy formulani o'ng tomonidan hosila olib tekshirish mumkin.

Asosiy integrallash jadvali:

1. $\int du = u + C$
2. $\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq 1$
3. $\int \frac{du}{u} = \ln u + C$
4. $\int a^u du = \frac{a^u}{\ln a} + C$
5. $\int e^u du = e^u + C$
6. $\int sinudu = -cos\, u + C$
7. $\int cosu\, du = sinu + C$
8. $\int \frac{du}{cos^2 u} = tgu + C$
9. $\int \frac{du}{sin^2 u} = -ctgu + C$
10. $\int tgudu = -ln|cosu| + C$
11. $\int ctgudu = -ln|sinu| + C$
12. $\int \frac{du}{u^2+a^2} = \frac{1}{a} arctg\frac{u}{a} + C$
13. $\int \frac{du}{u^2-a^2} = \frac{1}{2} ln\left|\frac{u-a}{u+a}\right| + C$
14. $\int \frac{du}{\sqrt{a^2-u^2}} = arcsin\frac{u}{a} + C$
15. $\int \frac{du}{\sqrt{u^2+\lambda}} = ln\left|u + \sqrt{u^2+\lambda}\right| + C.$

Aniqmas integral $I = \int f(x)dx$ ko'rinishida yoziladi. Asosiy masala $y=f(x)$ funksiya uchun boshlang‘ich *F(x)* funksiyani topishdan iborat.

Boshlang‘ich funksiya *F(x)* uch usulda topiladi:

I. Bevosita integrallash. Bu usulda boshlang‘ich funksiya integrallar jadvalidagi formulalar orqali amalga oshiriladi.

Misol keltiramiz:

28.1. $\int (\ln x)^4 \frac{dx}{x}$; $\frac{dx}{x}$ ifodani diferensialning tarifiga ko‘ra *d(lnx)* kabi yozish mumkin. Shuning uchun $\int (ln\,x)^4 \frac{dx}{x} = \int (ln\,x)^4 d(ln\,x)$ tenglik o‘rinli, bu ifoda *lnx* ga nisbatan darajaning integrali, demak,

$$\int (ln\,x)^4 \frac{dx}{x} = \frac{ln^5 x}{5} + C.$$

28.2.

$$\int \frac{2-x^4}{1+x^2} dx = \int \frac{1-x^4+1}{1+x^2} dx = \int (1-x^2)dx + \int \frac{dx}{1+x^2} = x - \frac{x^3}{3} + arctg\,x + C$$

Bu yerda

$$\int dx = x + C, \quad \int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad \int \frac{dx}{1+x^2} = arctg\,x + C$$

formulalardan foydalandik.

II. Aniqmas integralni yangi o‘zgaruvchi kiritish usuli bilan integrallash.

Bu usulga ko‘ra integral ostida biror funksiyani yangi o‘zgaruvchi kiritish bilan integral jadvaldagi formula ko‘rinishiga keltiriladi.

28.3. A) $J = \int \left(2sin\frac{x}{2} + 3\right)^2 cos\frac{x}{2}\,dx$ aniqmas integralni yangi o‘zgaruvchi kiritish usuli bilan integrallang.

Buning uchun integral ostida shunday yangi o‘zgaruvchi kiritish kerakki, bu ifodani differensiallasak qolgan ifoda kelib chiqishi kerak.

$$J = \int (2\sin\frac{x}{2}+3)^2 \cos\frac{x}{2} dx = \left| \begin{array}{l} 2\sin\frac{x}{2}+3 = t \;\; differiansiallaymiz \\ 2\cos\frac{x}{2} * \frac{1}{2} dx = dt \\ \cos\frac{x}{2} dx = dt \end{array} \right| =$$

$$= \int t^2 dt = \frac{t^3}{3} + C = \frac{(2\sin\frac{x}{2}+3)^3}{3} + C$$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ formuladan foydalandik.

$$B)\ \int \frac{\sin 4x dx}{\cos^4 2x + 4} = \left| \begin{array}{l} \cos^2 2x = t \\ -2\cos 2x * \sin 2x * 2dx = dt \\ -2\sin 4x dx = dt \\ \sin 4x dx = -\frac{dt}{2} \end{array} \right| =$$

$$= -\frac{1}{2}\int \frac{dt}{t^2 + 2^2} = -\frac{1}{2} arctg \frac{t}{2} + C = -\frac{1}{2} arctg \frac{\cos^2 2x}{2} + C;$$

Bu yerda $\int \frac{dx}{a^2+x^2} = \frac{1}{a} arctg\ \frac{x}{a} + C$ formuladan foydalandik.

C) $J = \int \sqrt{4 - x^2} dx$ aniqmas integralni integrallang.

Bu integral trigonometrik almashtirish yordamida integrallanadi:

J=

$$\int \sqrt{4 - x^2} dx = \left| \begin{array}{l} x = 2\sin t \\ dx = 2\cos t dt \end{array} \right| = \int \sqrt{4 - 4\sin^2 t} * 2\cos t dt = 4\int \cos^2 t dt = 2t + \sin 2t + C$$

x=2sint dan *sint*=$\frac{x}{2}$, *t=arcsin* $\frac{x}{2}$ ni oxirgi tenglikka qo'yamiz

$$J = 2arcsin\frac{x}{2} + sin2arcsin\frac{x}{2} + C;$$

III. Aniqmas integralni bo'laklab integrallash.

Aniqmas integralni bo'laklab integrallash formulasi $\int udv = uv - \int vdu.$ Bu formulaga ko'ra $\int f(x)dx$ integral ostidagi ifoda ikki bo'lakka bo'linadi.

Shunday ikki bo'lakka bo'linadiki $\int udv$ ni hisoblash berilgan integralni hisoblashga qaraganda qulayroq bo'lsin.

$$28.4. \int x^2 arctgdxdx = \left| \begin{array}{l} u = arctgx, du = \frac{dx}{1+x^2} \\ x^2 dx = dv, v = \frac{x^3}{3} \end{array} \right| = \frac{x^3}{3} arctgx - \int \frac{x^3}{3} \cdot \frac{dx}{1+x^2} =$$

$$= \frac{x^3}{3} arctgx - \frac{1}{3}\int \left(x - \frac{x}{x^2+1} \right) dx = \frac{x^3}{3} arctgx - \frac{x^2}{6} + \frac{1}{6}\ln(x^2+1) + C.$$

Mustaqil yechish uchun misollar:

Quyidagi integrallarni toping:

28.5. $\int x\sqrt{x} dx$;

28.6. $\int \frac{dx}{\sqrt[5]{x}}$;

28.7. $\int \frac{2-\sqrt{1-x^2}}{\sqrt{1-x^2}} dx$;

28/.8. $\int \left(x^2 + 2x + \frac{1}{x}\right) dx$;

28.9. $\int \frac{x-2}{x^3}dx$;

28.10. $\int\left(\sqrt{x}+\sqrt[3]{x}\right)dx$;

28.11. $\int \frac{\left(\sqrt{x}-1\right)^3}{x}dx$;

28.12. $\int \frac{x-1}{\sqrt[3]{x^2}}dx$;

28.13. $\int \frac{\cos 2x}{cos^2 x\, sin^2 x}dx$;

28.14. $\int ctg^2 x\, dx$;

28.15. $\int \frac{dx}{\sin^2 x \cos^2 x}$;

O‘zgaruvchini almashtirish usulidan foydalanib integrallarni toping:

28.16. $\int x\, cos(x^2)dx$;

28.17. $\int \frac{e^{\sqrt{2x-1}}}{\sqrt{2x-1}}dx$;

28.18. $\int x^3(1-2x^4)^3dx$;

28.19. $\int x\sqrt{a-x}dx,\quad a-x=t^2$;

28.20. $\int \frac{dx}{x\ln x}$;

28.21. $\int \frac{cosx\, dx}{\sqrt{1+2sin^2 x}}$;

28.22. $\int \frac{\sin 2x}{\sqrt{2+cos^2 x}}dx$;

Bo‘laklab integrallash usulidan foydalanib integrallarni toping:

28.23. $\int (x+1)e^x dx$;

28.24. $\int e^{2x}\cos x\, dx$;

28.25. $\int \frac{\ln x}{x^3}dx$;

28.26. $\int x^2 e^{3x}dx$;

28.27. $\int x\, cosxdx$;

28.28. $\int ln^2 xdx$;

28.29. $\int \frac{x\cos x\, dx}{sin^3 x}$;

28.30. $\int \frac{arcsin\frac{x}{2}}{\sqrt{2^2-x^2}}dx$;

29. ANIQ INTEGRAL

Aniq integral $I=\int_a^b f(x)dx$ ko’rinishida yoziladi va Nyuton-Leybnits formulasiga ko’ra hisoblanadi:

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b)-F(a)$$

F(x) funksiya *y=f(x)* uchun boshlang’ich funksiya, *a*-integralning quyi, *b*-yuqori chegaralari.

Aniq integralni ham xuddi aniqmas integralga o’xshab integrallab, so’ngra Nyuton-Leybnits formulasiga ko’ra hisoblanadi.

Aniq integralni yordamida egri chiziqlar bilan chegaralangan yuzalarni, aylanma jismlar hajmini, egri chiziq yuyining uzunligini va h.k.larni hisoblash mumkin.

Quyidagi aniq integralni hisoblang:

29.1. $\int_0^1 \sqrt{1+t}dt = \int_0^1 (1+t)^{\frac{1}{2}} d(t+1) = \frac{2}{3}(1+t)^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}(\sqrt{8}-1)$

29.2 $\int_0^{\pi/2} \sin^3 x dx = \int_0^{\pi/2} (1-\cos^2 x)\sin x dx = \left| -\sin x dx \overset{\cos x = t}{=} dt \;\; \begin{array}{c|c} x\,0 & \pi/2 \\ \hline t\,1 & 0 \end{array} \right| = -\int_1^0 (1-t^2)dt = \left(t - \frac{t^3}{3}\right)\Big|_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$

29.3. $\int_0^1 xe^{-x}dx = \begin{bmatrix} u = x, & du = dx \\ dv = e^{-x}dx, & v = -e^{-x} \end{bmatrix} = -xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}dx = (-xe^{-x} - e^{-x})\Big|_0^1 = 1 - \frac{2}{e}$

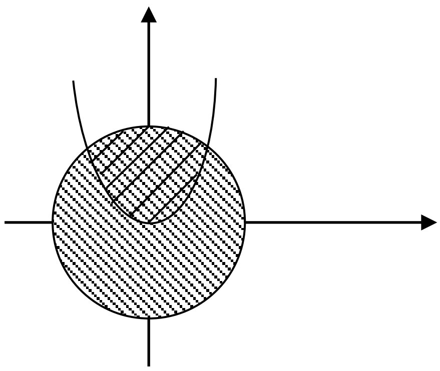
29.4. $\int_{\frac{\sqrt{2}}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}dx = \begin{bmatrix} x = \sin t \\ dx = \cos t dt \\ t = \arcsin x \\ \frac{x}{t}\;\; \frac{\sqrt{2}/2}{\pi/4}\;\; \frac{1}{\pi/2} \end{bmatrix} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sqrt{1-\sin^2 t}}{\sin^2 t}\cos t dt = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos^2 t}{\sin^2 t}dt = \int_{\pi/4}^{\pi/2} \frac{1-\sin^2 t}{\sin^2 t}dt =$

$= (-ctgt - t)\Big|_{\pi/4}^{\pi/2} = \frac{\pi}{2} + 1 + \frac{\pi}{4} = 1 - \frac{\pi}{4}$

29.5. $x^2+y^2=8$ aylana $y=x^2/2$ bilan ikki qismga bo'lingan. Ikkala qismini yuzasini toping.

Yechish: Grafiklar kesishish nuqtalarini topamiz:

$\frac{1}{2}x^2 = \sqrt{8-x^2}$



$\frac{x^4}{4} = 8 - x^2$

$\frac{x^4}{4} = 8 - x^2$

$-32 + 4x^2 + x^4 = 0$

$x_1 = 2, \;\; x_2 = -2,$

$S_d = \pi r^2 = 8\pi \qquad r = 2\sqrt{2}$

$S_1 = \int_{-2}^{2} (\sqrt{8-x^2})dx = \left(4\arcsin\frac{x}{\sqrt{8}} + \frac{1}{2}x\sqrt{8-x^2} - \frac{x^3}{6}\right)\Big|_{-2}^{2} = 2\Pi + \frac{4}{3}$ (kv.birl.)

$S_2 = S_d - S_1 = 8\pi - \left(2\pi + \frac{4}{3}\right) = 6\pi - \frac{4}{3}$ (kv.birl.)

29.6. $y=2-x^2$ va $y^3=x^2$ egri chiziqlar bilan chegaralangan figuraning yuzini toping:

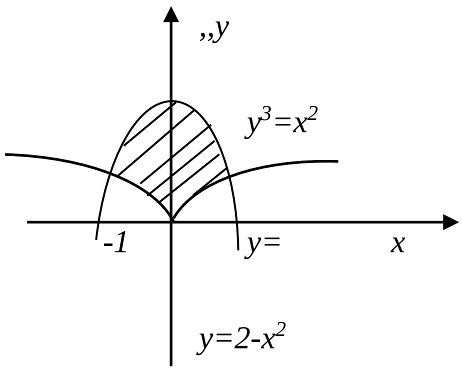
Yechish: Egri chiziqlar kesishish nuqtalarini topamiz:

$2-x^2=\sqrt[3]{x^2}$

$8-12x^2+6x^4-x^6=x^2$

$x^6-6x^4+13x^2-8=0$

$x_1=-1,\ \ x_2=1,$



$$S=\int_{-1}^{1}\left(2-x^2-\sqrt[3]{x^2}\right)dx=\left(2x-\frac{x^3}{3}-\frac{3}{5}\sqrt[3]{x^5}\right)\Big|_{-1}^{1}=\left(2-\frac{1}{3}-\frac{3}{5}\right)-\left(-2+\frac{1}{3}+\frac{3}{5}\right)=\frac{32}{15}$$

(kv.birl.)

29.7. $y=x^2$ va $x=y^2$ parabolalar bilan chegaralangan figurani Ox o‘qi atrofida aylantirishdan hosil bo‘lgan jism hajmini hisoblang:

Yechish: $\begin{cases} y=x^2 \\ x=y^2 \end{cases}$ sistemasidan kesishish nuqtalarini topamiz:

$x_1=0,\ x_2=1,\ y_1=0,\ y_2=1$

$$V=V_1+V_2=\pi\int_0^1 xdx-\pi\int_0^1 x^4dx=\pi\left(\frac{x^2}{2}-\frac{x^5}{5}\right)\Big|_0^1=\pi\left(\frac{1}{2}-\frac{1}{5}\right)=\frac{3}{10}\pi\ \ (kub.birlik)$$

29.8. $y^2=\frac{2}{3}(x-1)^3$ yarim kubik parabolaning $y^2=\frac{x}{3}$ parabola ichki qismi bilan chegaralangan yoy uzunligini hisoblang:

Yechish: Egri chiziqlarning kesishish nuqtasini aniqlaymiz: $\frac{2}{3}(x-1)^3=\frac{x}{3}$

$$x=2,\ \partial a\ \ y=\sqrt{\frac{2}{3}},\ \ chunki\ \ y=\sqrt{\frac{2}{3}(x-1)}(x-1),\ \ \ u\ \ holda\ \ y^{`}=\sqrt{\frac{3}{2}}\sqrt{x-1},$$

$$L=2\int_1^2\sqrt{1+\frac{3}{2}(x-1)}dx=2\frac{1}{\sqrt{2}}\int_1^2\sqrt{3x-1}dx=\frac{2\sqrt{2}}{9}(5\sqrt{5}-2\sqrt{2})$$

Mustaqil yechish uchun misollar:

Quyidagi aniq integralni hisoblang:

29.9. $\int_1^5 \sqrt{x-1}dx$

29.10. $\int_1^2 \frac{dx}{x^2-4x+5}$

29.11. $\int_{-1}^4 \frac{x}{\sqrt{x+5}}dx$

29.12. $\int_1^e \frac{\sqrt[3]{1+\ln x}}{x}dx$

29.13. $\int_0^1 \frac{xdx}{\sqrt{1-x^2}}$

29.14. $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

29.15. $\int_0^{\pi/4} \sin 4xdx$

29.16. $\int_1^e x^2 \ln xdx$

29.17. $\int_0^{\pi} e^x \sin xdx$

29.18. $\int_0^1 \arcsin xdx$

29.19. $\int_0^1 \ln(x+1)dx$

29.20. $\int_0^{\pi/2} \sin x\cos^2 xdx$

29.21. $\int_0^1 \frac{dx}{e^x+1}$

29.22. $\int_0^4 \frac{dx}{1+\sqrt{2x+1}}$

29.23. $\int_0^1 \frac{x^2dx}{\sqrt{4-x^2}}$

29.24. $\int_0^1 \sqrt{1+x^2}dx$

29.25. $\int_1^3 \frac{dx}{x+x^2}$

29.26. $\int_1^2 \frac{dx}{2x-1}$

29.27. $\int_1^e \frac{dx}{x(1+\ln^2 x)}$

29.28. $\int_1^e x\arcsin xdx$

29.29. $\int_0^{\pi/2} x^2 \cos xdx$

Berilgan chiziqlar bilan chegaralangan figuralar yuzalarini hisoblang:

29.30. $y=4-x^2$ va Ox o'q bilan

29.31. $y=(x-1)^2$ va $x^2-\frac{y^2}{2}=1$

29.32. $y=x^2+1 \;\; va \; y=3-x$ chiziqlar bilan chegaralangan figuraning yuzini toping.

Egri chiziqlar yoylari uzunliklari hisoblansin:

29.33. $y=1-\ln\cos x,\ \ x=0$ dan $x=\dfrac{\pi}{6}$ gacha

29.34. $x=8\sin t+6\cos t, y=6\sin t-8\cos t, t=0$ dan $t=\dfrac{\pi}{2}$ gacha

29.35. $x=\dfrac{1}{3}t^3-t, y=t^2+2, t=0$ dan $t=3$ gacha

29.36. $x^2-y^2=4, y=\pm 2$ chiziqlar bilan chegaralangan figurani *Oy* o‘qi atroqida aylantirishdan hosil bo‘lgan jismning hajmini toping.

29.37. $y=\dfrac{1}{1+x^2}, x=\pm 1, y=0$ chiziqlar bilan chegaralangan figurani *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini toping.

29.38. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini hisoblang.

30. HOSMAS INTEGRALLAR.

I. Xosmas integrallar ikki hil turda bo‘ladi:

1-tur xosmas integrallar chegaralari cheksiz bo‘lgan integrallardir.

$$\int_a^{+\infty} f(x)dx=\lim_{b\to\infty}\int_a^b f(x)dx \quad (1)$$

Bu tenglikda o‘ng tomonda turgan limit mavjud bo‘lsa, u holda xosmas integral $\int_a^{+\infty} f(x)dx$ yaqinlashuvchi integrallar deyiladi.

Agar ko‘rsatilgan limit cheksizga teng bo‘lsa yoki mavjud bo‘lmasa xosmas integral uzoqlashuvchi deyiladi.

$\int_{-\infty}^{b} f(x)dx, \int_{-\infty}^{\infty} f(x)dx$ xosmas integrallar ham shunday aniqlanadi.

2-tur xosmas integrallar.

Agar $y=f(x)$ funksiya $[a;b]$ kesmaning *x=a* nuqtasida, *x=b* nuqtasida yoki $[a;b]$ ga tegishli biror *x=c* nuqta atrofida aniqlanmagan bo‘lsa, bunday funksiyadan olingan integrallar 2-tur xosmas integrali deyiladi.

$y=f(x)$ funksiya $[a;b]$ oraliqda aniqlangan va uzliksiz bo‘lib, *x=b* nuqta atrofida chegaralanmagan funksiya bo‘lsa, u holda $\lim_{e\to 0}\int_a^{b-s} f(x)dx$ limitga $f(x)$ funktsiyaning 2-tur xosmas integrali deyiladi va $\int_a^b f(x)dx = \lim_{\varepsilon\to 0}\int_a^{b-\varepsilon} f(x)dx$ tenglik bilan aniqlanadi. Agar o‘ng tomonda turgan limit mavjud bo‘lsa, xosmas integral yaqinlashuvchi deyiladi. Agar limit mavjud bo‘lmasa, xosmas integral uzoqlashuvchi deyiladi.

Boshqa 2-tur integrallar ham xuddi shunday aniqlanadi.

Misollar ko‘ramiz:

Xosmas integrallarni aniqlang:

30.1. $I = \int_0^{+\infty}\cos x dx$ xosmas integrallarni taqribiy hisoblang va yaqinlashishini tekshiring:

$I = \lim_{b\to+\infty}\int_0^{+b}\cos x dx = \lim_{b\to+\infty}\sin x\Big|_0^b = \lim_{b\to+\infty}(\sin b - \sin 0) = \lim_{b\to+\infty}\sin b$ demak xosmas integral uzoqlashuvi.

30.2. $I = \int_0^{+\infty}\frac{dx}{1+x^2}$ xosmas integralni hisoblang va yaqinlashishni tekshiring:

$I = \lim_{b\to+\infty}\int_0^{+b}\frac{dx}{1+x^2} = \lim_{b\to+\infty} arctgx\Big|_0^b = \lim_{b\to+\infty}(arctgb - arctg0) = \lim_{b\to+\infty} arctgb = \frac{\pi}{2}$ xosmas integral yaqinlashuvchi.

30.3. $\int_0^1\frac{dx}{x}$; $f(x) = \frac{1}{x}$ funksiya *x=0* nuqtada chegaralanmagan.

$\int_0^1\frac{dx}{x} = \lim_{a\to+0}\int_0^1\frac{dx}{x} = \lim_{a\to+0}\ln x\Big|_a^1 = \lim_{a\to+0}(\ln 1 - \ln a) = \lim_{a\to+0}\ln a$`. Bu limit mavjud emas. Berilgan xosmas integral uzoqlashuvchi.

30.4. $\int_0^1\frac{dx}{\sqrt{1-x^2}}$ xosmas integralni hisoblang va yaqinlashishini tekshiring:

$f(x) = \frac{1}{\sqrt{1-x^2}}$ funksiya *x=1* nuqtada uzulishga ega

$$\int_0^1\frac{dx}{\sqrt{1-x^2}} = \lim_{\varepsilon\to 0}\int_0^{1-\varepsilon}\frac{dx}{\sqrt{1-x^2}} = \lim_{\varepsilon\to 0}\arcsin\Big|_0^{1-\varepsilon} = \lim_{\varepsilon\to 0}(\arcsin(1-\varepsilon) - \arcsin 0) = \frac{\pi}{2}$$

30.5. $I = \int_0^2\frac{dx}{x^2-4x+3}$ xosmas integralni hisolang.

$f(x)=\frac{1}{x^2-4x+3}$ *x=1* nuqtada uzilishga ega, shuning uchun

$$I=\int_0^2\frac{dx}{x^2-4x+3}=\int_0^1\frac{dx}{x^2-4x+3}+\int_1^2\frac{dx}{x^2-4x+3};$$

$$\int_0^1\frac{dx}{x^2-4x+3}=\lim_{\varepsilon\to0}\int_0^{1-\varepsilon}\frac{dx}{(x-2)^2-1}=\lim_{\varepsilon\to0}\frac{1}{2}\ln\frac{x-3}{x-1}\Bigg|_0^{1-\varepsilon}=\frac{1}{2}\lim_{\varepsilon\to0}(\ln\frac{2+\varepsilon}{\varepsilon}-\ln3)=\frac{1}{2}\lim_{\varepsilon\to0}\ln\frac{2+\varepsilon}{\varepsilon}=\infty$$

Integral uzoqlashuvchi, demak berilgan xosmas integral ham uzoqlashuvchi.

Quyidagi xosmas integralni hisoblang va ularni yaqinlashuvchi yoki uzoqlashuvchiligini aniqlang:

30.8. $\int_1^{\infty}\frac{dx}{\sqrt{x}}$

30.9. $\int_0^{\infty}xe^{-x^2}dx$

30.10. $\int_1^{\infty}\frac{dx}{x^2\sqrt{x^2-1}}$

30.11. $\int_0^{\infty}x^2e^{\frac{x}{2}}dx$

30.12. $\int_2^{6}\frac{dx}{\sqrt[3]{(4-x)^2}}$

30.13. $\int_1^{\infty}\frac{arctgxdx}{x^2}$

30.14. $\int_0^{2}\frac{dx}{(x-1)^2}$

30.15. $\int_1^{\infty}\frac{\sin xdx}{x^2}$

30.16. $\int_1^{\infty}x^2e^{-x^3}dx$

30.17. $\int_1^{e}\frac{dx}{x\ln x}$

30.18. $\int_{-\infty}^{+\infty}\frac{xdx}{1+x^2}$

30.19. $\int_1^{3}\frac{dx}{\sqrt{4x-x^2-3}}$

30.20. $\int_{-\infty}^{+\infty}\frac{dx}{(x^2+1)(x^2+4)}$