MATHEMATICAL MODELING OF FLUID MOTION IN PIPES

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Abstract: The article provides information on the mathematical modeling of the incompressible viscous fluid in the pipe. The study shows laminar and turbulent regimes of fluid motion, as well as the physical meaning of these regimes. Consider a straight round pipe with a diameter constant along the entire length. The flow rate on the walls of the pipe due to adhesion is zero, in the middle of the pipe, it has the greatest value. A cylinder with a characteristic length and a characteristic radius inside the liquid whose axis coincides with the axis of the pipe is considered and the flow of the liquid through the cylinder is studied. The calculation formulas for calculating the maximum flow velocity in the cylinder, the volume of liquid passing through the cross-section of the pipe, the coefficient of resistance to friction in the pipe along the flow length, and the maximum value of the tangential stress are derived. The results of comparison of empirical and semi-empirical formulas for calculating the coefficient of resistance to friction are presented.

Keywords: Reynolds number, laminar flow, turbulent flow, parabolic flow, the friction force is the integral coordinate of the pipe, viscosity, density, bulk flow velocity, average speed, maximum speed, radius, Hook, Gegen, Poisal, Darcy-Weisbach, Blasius, Nikuradse, the volume of fluid resistance coefficient.

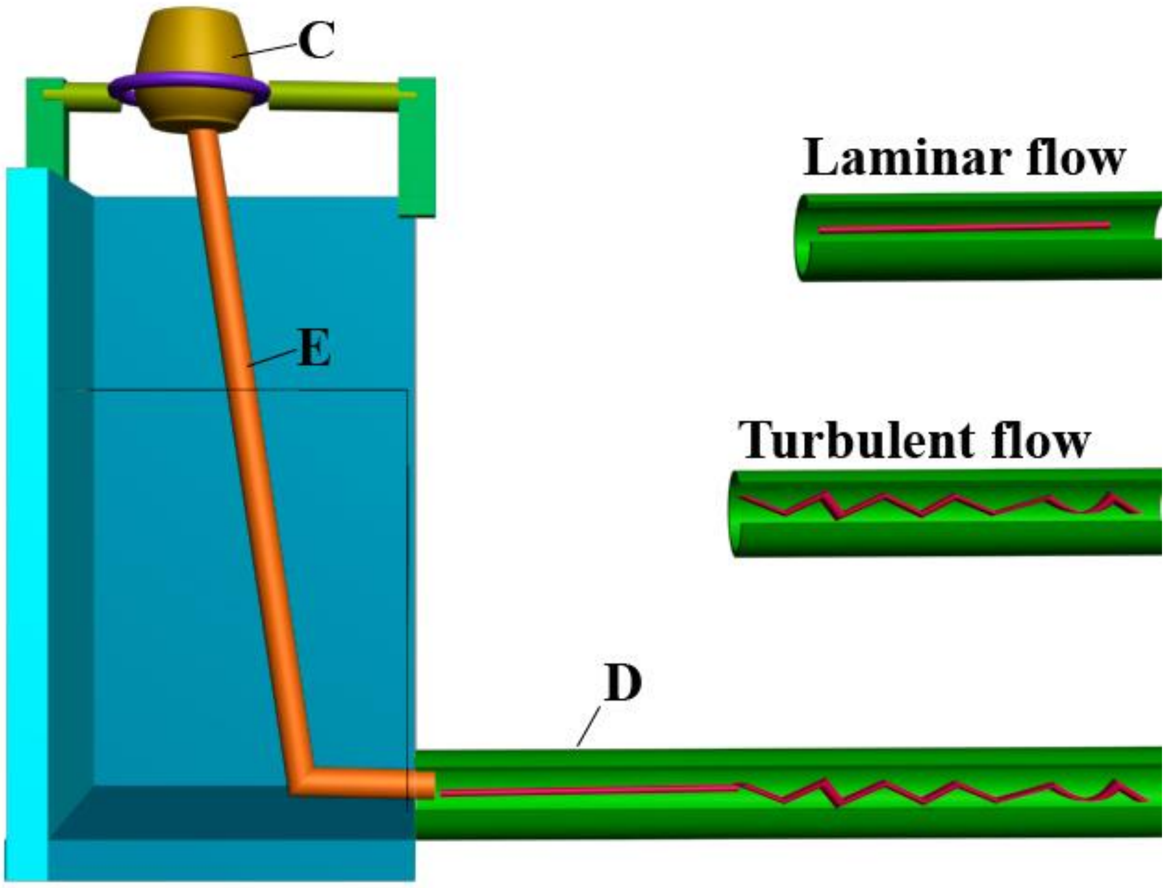
The presence of viscosity in liquids resists the movement of liquid layers relative to each other. In other words, in laminar (layered) flows due to the viscosity there is internal friction, it is expressed by the number of tangential stresses at the boundaries of the layers, or is characterized by the number of tangent forces relating to the unit area. Individual concentric layers of liquid relative to each other move so that the velocity of the liquid is directed in the direction of the main axis. The movement of this type of fluid is called laminar flow [1-12].

The flow of real liquids in many cases differs sharply from laminar flows. They have such a special property, which is called turbulence. In real flows, which occurs in pipes, channels and in the boundary layer with increasing values of the Reynolds number, the transition of the laminar flow to the turbulent one is clearly observed. Such transition of a laminar flow into a turbulent one is called turbulence and they play fundamental importance throughout hydrodynamics. Initially, such a transition was found in currents occurring in straight pipes and channels.

In a straight pipe with a smooth wall and a constant cross section, each particle of the liquid moves along a straight path at small Reynolds numbers. Due to the presence of viscosity of the liquid particles close to the wall move more slowly than away from the wall. The flow moves in the form of ordered layers moving relative to each other. However, observations show that at large Reynolds numbers, the flow passes into an unordered state or into a turbulent flow. There is a strong mixing in the liquid, this can be seen if you enter into the liquid moving in the pipe paint. Initially, the practical observation of this experience is carried out by O. Reynolds (1883-1912) in which the paint is introduced into the liquid[1].

When the flow of laminar paint move in a well-defined trickle and as the flow becomes turbulent the paint spreads over the entire pipe and stains the weight of the liquid. This shows that, in a turbulent flow to the fluid moving along the axis of the pipe, a transverse movement acts, or a movement perpendicular to the axis of the pipe. This lateral movement causes mixing of the dye throughout the fluid [2-10].

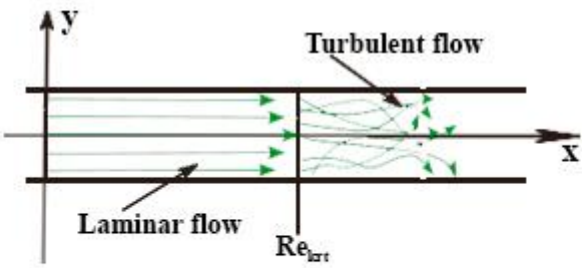
The device for this experiment is shown in figure 1.



Rice. 1. The experience in which the liquid is introduced into the paint

The experiment begins with passing through the pipe d liquid with low speeds. At the same time, paint is supplied from the tank C through the tube E. This produces the following picture, the tinted trickle has the form of a straight horizontal line, and the rest of the moving fluid remains unpainted. Therefore, in this case, the particles of the tinted trickle are not mixed with the rest of the liquid, and the fluid flow mode in the pipe d is laminar. With a gradual increase in the speed in the pipe D, there comes a moment when the tinted trickle disappears and the entire moving liquid becomes uniformly colored. This indicates that the liquid particles in the flow are mixed, i.e. in the pipe D there is a turbulent regime.

When the incompressible viscous fluid moves starting at the same value of the Reynolds number $\mathrm{Re} = \rho UL/\mu$, the laminar flow passes into a turbulent one, the same value of the Reynolds number is called the critical Reynolds number, where ρ - density, μ - viscosity of the liquid, U - the maximum velocity of the main flow, L - the characteristic scale of the length.



Rice. 2. The transition form laminar flow to turbulent

From Fig. 2. it is seen that at, $\mathrm{Re} < \mathrm{Re}_{krt}$, laminar flow, and $\mathrm{Re}_{krt} < \mathrm{Re}$ and the flow goes into turbulent mode.

We will consider the flow of liquid in a straight round pipe with a diameter constant along the entire length and consider the cylinder length L, and the radius y inside the liquid whose axis coincides with the axis of the pipe.

The flow rate on the walls of the pipe due to adhesion is zero, in the middle of the pipe, it has the greatest value. At points of the cylindrical surfaces having axes coincident with the axis of the pipe, the flow velocity is constant. Individual concentric layers slide one over the other so that the velocity everywhere has an axial direction and the motion of this kind is called laminar flow. At a sufficiently large distance from the inlet to the pipe, the distribution of flow velocities along the radius does not depend on the coordinate in the longitudinal direction.

The movement of fluid in the pipe occurs under the influence of the pressure drop in the direction of the pipe axis, but in each cross-section perpendicular to the pipe axis, the pressure can be considered as constant. The movement of each fluid element is accelerated due to the pressure drop and slowed down due to the shear stress caused by friction [2-12].

In the direction of the main axis on the cylinder are the forces of pressure $p_1\pi y^2$ and $p_2\pi y^2$, applied to the input and output bases of the cylinder , respectively, as well as the tangential force $2\pi yL\tau$, acting on the side surface of the cylinder. It is required to determine the maximum velocity of the flow in the cylinder, the volume of fluid flowing through the cross section of the pipe, the coefficient of resistance of the pipe to friction along the length of the flow, as well as the maximum value of the tangential voltage.

Equating the forces of the acting fluid in the cylinder, we obtain as a condition of equilibrium in the direction of motion equation (Fig.3.)

$$p_1 \pi y^2 = p_2 \pi y^2 + 2\pi y L \tau \,. \qquad (1)$$

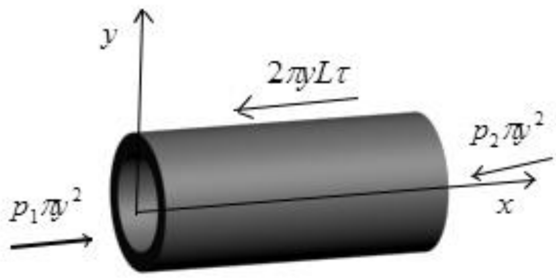


Fig. 3.

The projection of the internal friction force is taken with a plus sign, because the velocity gradient is negative (the velocity of the layer decreases with increasing radius r)

From the formula (1) we determine the tangent stress τ

$$\tau = \frac{p_1 - p_2}{L} \cdot \frac{y}{2} \,. \qquad (2)$$

In this case, the flow velocity u decreases with increasing coordinate y and is zero at $y = r$. Therefore, on the basis of the law of friction $\tau = \mu \frac{du}{dy}$ Hooke should take that $\tau = -\mu \frac{du}{dy}$. Substituting this expression in (2), we obtain

$$-\mu \frac{du}{dy} = \frac{p_1 - p_2}{L} \cdot \frac{y}{2} \,,$$

from here, you can see that

$$\frac{du}{dy} = -\frac{p_1 - p_2}{\mu L} \cdot \frac{y}{2} \,. \qquad (3)$$

Now, given that $y = r$ with velocity $u(y) = 0$ and integrating equation (3) with this initial condition we have

$$u(y) = -\frac{p_1 - p_2}{4\mu L} y^2 + C \,, \qquad (4)$$

to determine the constant C of equation (4), use the condition $u(r) = 0$ at $y = r$, or

$$u(r) = -\frac{p_1 - p_2}{4\mu L} r^2 + C = 0 \,,$$

from here you can see that

$$C = \frac{p_1 - p_2}{4\mu L} r^2 \,. \qquad (5)$$

Substituting the value of the constant C from (5) to equation (4) we have

$$u(y) = -\frac{p_1 - p_2}{4\mu L} y^2 + \frac{p_1 - p_2}{4\mu L} r^2,$$

And in turn, we obtain an equation to determine the flow rate of the following formula

$$u(y) = \frac{p_1 - p_2}{4\mu L}(r^2 - y^2). \qquad (6)$$

Thus, we have a parabolic velocity distribution along the radius of the pipe (Fig. 4.). The greatest value of speed is in the middle of the pipe ($y = 0$), where it is

$$u_{\max} = \frac{p_1 - p_2}{4\mu L} r^2. \qquad (7)$$

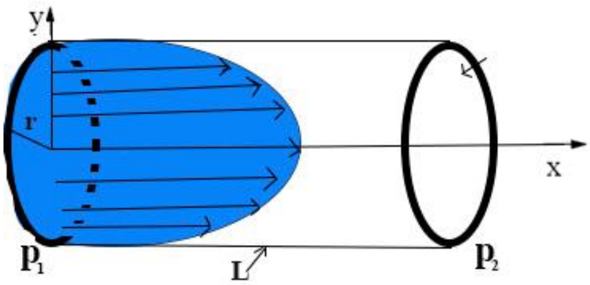


Fig. 4.

The total amount Q of liquid flowing through the pipe section (fluid flow) is defined as the volume of the paraboloid of rotation (Fig.4.) and acreage is defined as follows. Equation (6) multiply and divide by r^2,

$$u(y) = \frac{p_1 - p_2}{4\mu L}(r^2 - y^2)\cdot\frac{r^2}{r^2},$$

from here, you can see that

$$u(y) = \frac{p_1 - p_2}{4\mu L} r^2 \left(\frac{r^2 - y^2}{r^2}\right) = u_{\max}\left(1 - \frac{y^2}{r^2}\right). \qquad (8)$$

The total liquid flow through a pipe with a circular cross section on the basis of the Gagen-Poiseuille formula is determined as follows [1,3,7,8,11,12]

$$Q = \int_0^r u(y) 2\pi y dy = 2\pi u_{\max} \int_0^r \left(y - \frac{y^3}{r^2}\right) dy = 2\pi u_{\max}\left[\frac{y^2}{2} - \frac{y^4}{4r^2}\right]_0^r,$$

or given the formula (7), for the flow of liquid have the formula

$$Q = 2\pi \cdot \frac{p_1 - p_2}{4\mu L} \cdot r^2 \cdot \frac{r^2}{4} = \frac{\pi (p_1 - p_2) r^4}{8\mu L}. \qquad (9)$$

Enter the average flow rate, the values of which are determined by the cross section of the pipe as follows:

$$\bar{u} = \frac{Q}{\pi r^2}. \qquad (10)$$

Equation (10) with the formula (9) is written as

$$\bar{u} = \frac{(p_1 - p_2) r^2}{8\mu L},$$

by comparing the function $\bar{u}(y)$ with the maximum speed $u_{\max}$ determined by the formula (7) it can be seen that $\bar{u}(y)=\frac{1}{2}u_{\max}$ or the average speed of the laminar flow in the pipe is half the maximum speed (Fig. 4).
Determine the pressure difference (p_1-p_2)

$$p_1-p_2=\frac{8\mu L\bar{u}}{r^2},$$

from here we have

$$p_1-p_2=\frac{8\mu L\bar{u}}{r^2}=\frac{32\mu\bar{u}}{2r}\cdot\frac{L}{2r}=\frac{32\mu\bar{u}}{D}\cdot(\frac{L}{D}), \qquad (11)$$

here $D=2r$ is the diameter of the pipe.
The pressure loss along the flow length is determined by Darcy-Weisbach equation

$$p_1-p_2=\frac{\lambda}{2}\cdot\rho\bar{u}^2(\frac{L}{D}), \qquad (12)$$

here, λ- is the hydraulic loss ratio along the length of the pipe or the resistance coefficient of the pipe. From the last equation we have

$$\lambda=\frac{(p_1-p_2)}{\frac{1}{2}\rho\bar{u}^2}\cdot(\frac{D}{L}) \qquad (13)$$

Substituting p_1-p_2 the value of the formula (11) in the equation (13) we obtain, for the resistance coefficient of the pipe following formula

$$\lambda=\frac{32\mu\bar{u}}{D}\cdot(\frac{L}{D})\cdot\frac{2}{\rho\bar{u}^2}\cdot(\frac{D}{L})=\frac{64\mu}{\rho\bar{u}D}$$

or from here you can see that

$$\lambda=\frac{64}{\mathrm{Re}}, \qquad (14)$$

Here is $\mathrm{Re}=\frac{\rho\bar{u}D}{\mu}$- Reynolds number.
From the formula (12), we have

$$\frac{p_1-p_2}{L}=\frac{\lambda}{D}\frac{\rho}{2}\bar{u}^2. \qquad (15)$$

The tangential stress reaches its maximum value in the pipe wall, here this stress is determined by the formula.

$$\tau_0=\frac{(p_1-p_2)}{L}\cdot\frac{r}{2}, \qquad (16)$$

this formula takes place regardless of which mode (laminar or turbulent) the flow is located. Thus, the tangential stress on the pipe wall is determined by measuring the pressure reduction experimentally.
Substituting the value of $\frac{p_1-p_2}{L}$ from formula (15) to formula (16) we have the following formula

$$\tau_0=\frac{\lambda}{8}\rho\bar{u}^2. \qquad (17)$$

This is the formula for calculating the maximum tangent stress.

One of the methods for calculating turbulent flows is the use of empirical and semi-empirical formulas. To illustrate this, we give two best approximations for smooth pipes, and such indicate their limits of application by Reynolds number.

In 1911 Blasius obtained an empirical formula for the resistance coefficient of smooth pipes (it is valid in the range $2320 \div 4 \cdot 10^5$):

$$\lambda = \frac{0.3164}{\mathrm{Re}^{0.25}}, \qquad (18)$$

this formula is known as the Blasius law of resistance.
The formula of Nikuradse (it is applied on the interval $\mathrm{Re} = 1 \cdot 10^5 \div 1 \cdot 10^6$):

$$\lambda = 0.0032 + \frac{0.221}{\mathrm{Re}^{0.237}}. \qquad (19)$$

The range of variation of the reduced your critical Reynolds number, is in good agreement with the critical Reynolds number for plane-parallel flows $\mathrm{Re}_{krt} = 5770$ [2,13-20].
Results obtained by formulas (14),(18),(19) shown in Fig. 5.

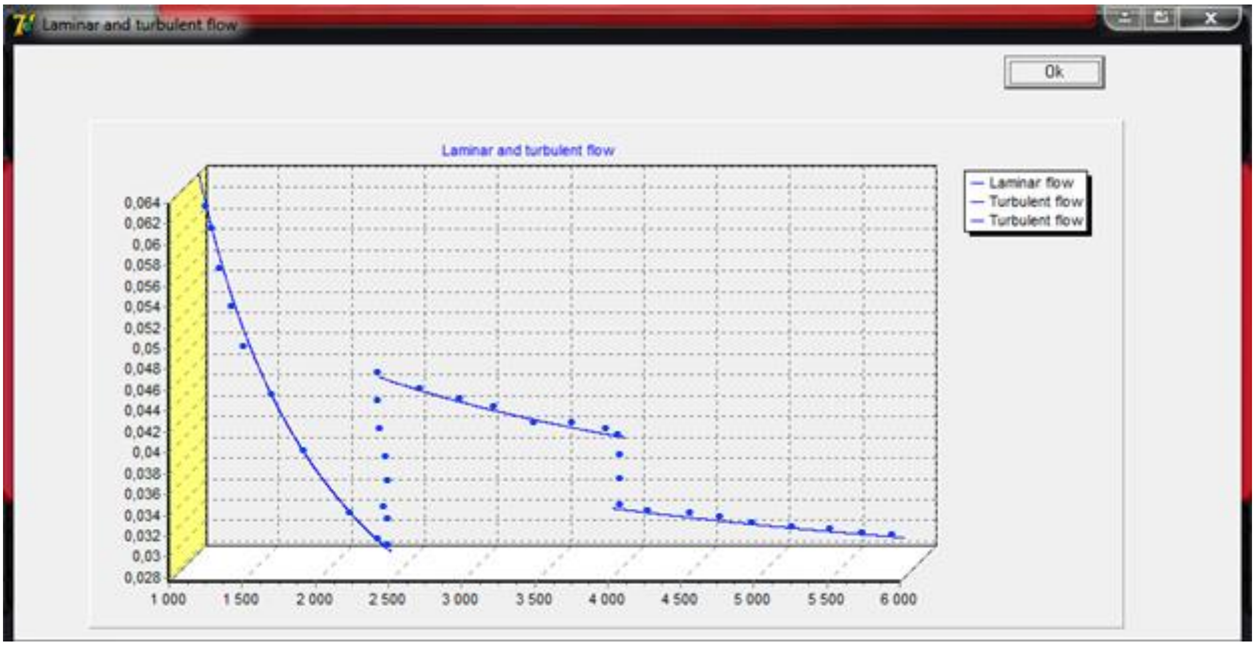


Fig.5.Resistance coefficient of smooth pipes. 1-laminar flow (Poiseuille), 2-turbulent flow (Blasius), 3-turbulent flow (Nikuradze), • - experimental data

For fig.5. experimental and calculated results illustrating the dependence of the resistance coefficient of the pipe on the Reynolds number for smooth pipes are presented. The comparison of the results obtained shows that for small Reynolds numbers the theoretical formula (14) is confirmed by experimental data. The coincidence of the calculated and experimental results is observed before $\mathrm{Re} = 2000 \div 2320$, with a further increase in the Reynolds number due to the active inclusion of turbulence mechanisms. Resistance is increasing. The subsequent dependence of the parameter λ on the Reynolds number is very different from the results obtained for the laminar flow.
Thus, it is shown that the motion of incompressible viscous flows in the channels, pipes and in the boundary layer can be laminar and turbulent modes and also, the physical meaning of these modes is indicated.
This process is illustrated by the experience with a painted stream proposed by O. Reynolds. The formulas for calculating the maximum velocity of the liquid flow in

the cylinder located inside the pipe, the volume of liquid flowing through the cross-section of the pipe, the coefficient of resistance of the pipe to friction along the length of the flow, as well as for the maximum value of the tangential stress are derived.

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