**O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI**

**Termiz davlat universiteti**
**Matematik analiz kafedrasi**

**Mirsaburov Mirahmad, Artiqov Muxammadi, Begaliyev Olimnazar, Qurbonnazarov Abdimumin, Ibragimova Nozima**

# MATEMATIK ANALIZDAN MA’RUZALAR

**I**

**5130100-Matematika bakalavr yo‘nalishlari uchun mo‘ljallangan.**

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## Haqiqiy sonlar maydonini kiritish.

Matematikada son tushunchasi kengayib borgan.
Dastlab natural son tushunchasi:Sanoqda ishlatiladigan sonlar natural sonlar deyiladi.N={1,2,3,…,n,…}
X+2=0 tenglama natural sonlar maydonida yechimga ega emas.x=-2 yechim esa manfiy son.Natural sonlar,ularga qarama-qarshi (manfiy) sonlar va 0 soni butun sonlar qatorini tashkil etadi.Z={…,-n,…,-3,-2,-1,0,1,2,3,…,n,…}
2x-1=0 tenglama butun sonlar maydonida yechimga ega emas.x=1\2-kasr son.
Q={$\frac{m}{n}$|m∈Z,n∈N}   EKUB(m,n)=1
Q-ratsional sonlar
Kvadrati 2 ga teng ratsional son mavjud emas.
Isbot:Faraz qilamiz, teskarisini   $(m\backslash n)^2$=2
$m^2 = 2n^2$ ; m=2k; EKUB(m,n)=1
$(2k)^2 = 2n^2$; $4k^2 = 2n^2$; $2k^2 = n^2$;
n=2l; m=2k;
Farazimiz noto'g'ri ,demak $(m\backslash n)^2$ ≠2.
Kesim tushunchasi.
Kesim tushunchasi nemis matematigi Rexard Didikend tomonidan kiritilgan.
Butun ratsional sonlar maydonini ikki qismga
ajratamiz. A-quyi sinf   A'-yuqori sinf
Bu qismga ajratish quyidagi xossalarga asoslanadi.
1.Har qanday ratsional son faqat va faqat bitta sinfga tegishli bo'ladi,ya'ni
a∈A, a∉A'  yoki  a∈A', a∉A
2. Quyi sinfning ixtiyoriy elementi yuqori sinfning ixtiyoriy elementidan kichik.
∀a∈A , ∀a'∈A;  a<a' (∀-ixtiyoriy belgisi)
1-misol:1 orqali kesimga ajratamiz.
a<1,a∈A-quyi sinf;
a'≥1,a'∈A'-yuqori sinf;
Quyi sinfda eng kata element mavjud emas.
Yuqori sinfda eng kichik element mavjud va bu 1.
2-misol:
a≤1,a∈A-quyi sinf;
a'>1,a'∈A'-yuqori sinf;
Quyi sinfda eng kata element mavjud va bu 1.
Yuqori sinfda eng kichik element mavjud emas.
Kvadrati 2 dan kichik barcha musbat barcha manfiy sonlar va 0 A sinfga kiradi.
$a^2$ <2,a∈A-quyi sinf;
$a^2$>2,a'∈A'-yuqori sinf;
Quyi sinfda eng kata element mavjud emas.
Faraz qilamiz teskarisini, a-eng kata
$\left(a+\frac{1}{n}\right)^2$ <2;
$a^2 + 2\frac{a}{n} + (\frac{1}{n})^2 < 2$;

$2\frac{a}{n}+(\frac{1}{n})^2<2-a^2$;     $2\frac{a}{n}+\frac{1}{n}<2-a^2$;
$\frac{2a+1}{n}<2-a^2$;
n>$\frac{2a+1}{2-a^2}$; shunday n son  mavjud ekan, demak  farazimiz noto'g'ri
Yuqori sinfda eng kichik element mavjud emas.
Faraz qilamiz teskarisini, a'-eng katta
$\left(a'-\frac{1}{n}\right)^2$ <2;
$a'^2-2\frac{a'}{n}+(\frac{1}{n})^2<2$;
$2\frac{a'}{n}-(\frac{1}{n})^2>a'^2-2$;
$\frac{2a'-1}{n}>a'^2-2$;
n<$\frac{2a'+1}{2-a^2}$; shunday n son  mavjud ekan, demak  farazimiz noto'g'ri.

**Haqiqiy  sonlar to'plamining uzluksizligi.**

1.A :a≤1;  A' : a>1;        A\A'
2.A  :a<1;  A'  :a≥1;        A\A'
3. A  :$a^2$ <2;  A'  :$a^2$>2;        A\A'           \-kesim belgisi
Ratsional sonlar to'plamida quyi A sinfda eng katta element a ,yuqori A' sinfda eng kichik  element a'  bo'lgan kesim mavjudmi?
A :a-eng katta

a<a'
a<c<a'
c=$\frac{a+a'}{2}$
c∈A   farazimizga zid
c∈A'   farazimizga zid
Demak,bunday kesim mavjud emas.
Har qanday 3-tipdagi kesim bilan aniqlanadigan b soni irratsional sonni aniqlaydi.
A-Irratsional sonlar to'plami
Q-Ratsional sonlar to'plami
AUQ=R;   R-Haqiqiy sonlar to'plami
Didikend teoremasi:
 Haqiqiy sonlar to'plamida aniqlangan har qanday A\A' kesim b haqiqiy sonni aniqlaydi.
Haqiqiy sonlarning bu xossasiga Haqiqiy sonlar to'plamining uzluksizligi deyiladi.

To'plam chegaralari
TA'RIF:A to'plam yuqoridan chegaralangan  deyiladi, agarda ∀a∈A  a≤M bo'lsa.
Yuqori chegara tushunchasi bir qiymatli emas.(0,2)  M=2,3,4,5,6,…
To'plam yuqori chegaralarining eng kichigiga to'plamning aniq yuqori chegarasi deyiladi.
SUP X=M;
TA'RIF:A to'plam quyidan chegaralangan  deyiladi, agarda ∀a∈A  a≥m bo'lsa.

Quyi chegara tushunchasi bir qiymatli emas.(5,7)     m=4,3,2,1,0,…
To’plam quyi chegaralarining eng kattasiga to’plamning aniq quyi chegarasi deyiladi.
INF X=m;
TA’RIF:To’plam  chegaralangan deyiladi,$\forall x\in X,\exists m,M$
$m\le x\le M$ bo’lsa.
Masalan: (0,2)
SUP(0,2)=2=M
INF(0,2)=0=m
Bern Gard Boltsano teoremasi:
Agar X  to’plam yuqoridan (quyidan) chegaralangan bo’lsa,u holda to’plamning aniq yuqori chegarasi M(aniq quyi chegarasi m)mavjud.
ISBOT:1-hol.M∈X,
2-hol.M∉X,bu yerda kesim yasaymiz A’ ga X ning barcha yuqori chegaralarini kiritamiz.     A  ga esa qolgan sonlarni kiritamiz.
Didikend teoremasiga ko’ra, bu kesim b haqiqiy sonni aniqlaydi.
Ta’biiyki,b∉A
b∈ A’ bu esa yuqori chegaraning eng kichigi.
Huddi shunday to’plam quyidan hegaralangan bo’lsa, aniq quyi chegaraga ega ekanligini isbotlash mumkin.
To’plamlarning Dekart ko’paytmasi
A={a,b,c,}          B={1,2,3}                              X-dekart ko’paytma belgisi
AxB={<x,y>|x∈A,y∈B}
AxB={<a,1>,<a,2>,<a,3>,<b,1>,<b,2>,<b,3>,<c,1>,<c,2>,<c,3>}
TA’RIF:AxB dekart ko’paytmaning ixtiyoriy qismiga A va B to’plamlarda aniqlangan binar munosabat deyiladi.
Yuqoridan R={<c,1>,<c,2>,<c,3>}-AxB ning qismi
AxB ≠ BxA
TA’RIF:R binar munosabat akslantirish deyiladi,agarda shu binar munosabatdagi ixtiyoriy x(1-element) uchun yagona
y(2-element)mos kelsa.
X={a,b,c,d}                    Y={1,2,3}
R={<a,1>,<a,2>,<c,3>}-akslantirish emas.
R={<a,1>,<b,1>,<c,2>,<d,2>}-akslantirish.
TA’RIF: R binar munosabatning barcha 1-elmentlariga shu binar munosabatning aniqlanish sohasi(DomR) deyiladi.
A={a,b,c}        B={1,2,3}
R={<a,1>,<c,2>}  da  DomR=(a,c);
TA’RIF: R binar munosabatning barcha 2-elmentlariga shu binar munosabatning qiymatlar  sohasi(ImR) deyiladi.
A={a,b,c}        B={1,2,3}
R={<a,1>,<c,2>}  da      ImR=(1,2);
1.Aniqlanish sohasi X to’plamning  to’liq aks ettirsa,ushbu R binar   X ni Yga akslantirish deyiladi.
DomR=X;

2. Aniqlanish sohasi X to’plamning  qismini s ettirsa,ushbu R binar  X dan  Yga akslantirish deyiladi.
DomR=cX;

## AKSLANTIRISH VA UNING TURLARI.

**TA’RIF:**A va  B to’plamlarda aniqlangan R binar munosabat syurektiv deyiladi,agarda ∀y∈Y uchun shunday x∈DomR mavjud bo’lsaki,<x,y>∈R bo’lsin.(ImR dan element qolib ketmasin)
A={a,b,c,d}                B={1,2,3}
R={<a,1>,<b,2>,<c,1>}-syurektiv akslantirish emas.
R={<a,1>,<b,2>,<c,3>}-syurektiv akslantirish.
**TA’RIF:**A va  Bto’plamlarda aniqlangan R binar munosabat inyektiv deyiladi,agarda har bir  x∈DomR uchun yagona y∈ImR qo’yilgan bo’lsa va aksincha har bir y∈ImR  uchun yagona x∈DomR mos kelsa.
X={m,n,k}                Y={1,2,3}
R={<m,1>,<n,3>,<k,2>}-inyektiv akslantirish.
R={<m,1>,<n,3>,<k,3>}- inyektiv akslantirish emas.
TA’RIF:A va  Bto’plamlarda aniqlangan R binar munosabat biektiv deyiladi,agarda  u bir paytda ham syurektiv,ham inyektiv bo’lsa.
X={m,n,k}                Y={1,2,3}
R={<m,1>,<n,2>,<k,3>}-biektiv akslantirish.
**TA’RIF:**F akslantirish funksiya deyiladi,agarda X va Y haqiqiy sonlar to’plami R ning qismi bo’lsa.
TA’RIF:F akslantirish  funksional  deyiladi,agarda faqat X  haqiqiy sonlar to’plami R ning qismi bo’lsa.(kvadrat perimetri)
TA’RIF:F akslantirish  operator deyiladi,agarda DomR va  ImR funksiyalar to’plamidan iborat bo’lsa.(hosila jadvali)
1.f(x)=$a_n\ x^n+a_{n-1}\ x^{n-1}+a_{n-2}\ x^{n-2}+\ldots+a_0$ ;-butun ratsional funksiya
DomR=(-∞;∞);
2.f(x)=$\frac{a_n\ x^n+a_{n-1}\ x^{n-1}+a_{n-2}\ x^{n-2}+\cdots+a_0}{b_n\ x^n+b_{n-1}\ x^{n-1}+b_{n-2}\ x^{n-2}+\cdots+b_0}$ -kasr ratsional funksiya
f(x)=$\frac{x^3+x^2+2}{x^2-5x+6}$ ; DomR=R\{2,3}     ImR=(-∞;∞);
3.y=$x^m$            m>1 –darajali funksiya
4. y=$x^m$            m<1 –darajali funksiya.
5.y=$a^x$           a>1 –o’suvchi
a<1 -kamayuvchi
6.y=$\log_a x$ –logarifmik funksiya
$\log_a b = c,\qquad b = a^c$
Trigonometrik  funksiyalar.
1.$y = \sin x;$   Domy=R;    Imy=[-1,1] –toq funksiya
2.$y = \cos x;$   Domy=R;    Imy=[-1,1]  juft funksiya
3.$y = \mathrm{tg}\, x;$   Domy=R\(Πk\2);  (k=2n+1)   Imy=(-∞;∞) –toq funksiya
4.$y = \mathrm{ctg}\, x;$   Domy=R\(Πk);    Imy=(-∞;∞)  -toq funksiya

Teskari trigonometrik funksiyalar

1.y=$\sin^{-1} x$=arcsinx -toq funksiya

DomR(y)=[-1,1]; ImR(y)=[- П\2, П\2] ;

2.y=$\cos^{-1} x = arccosx.$ –juft funksiya

DomR(y)=[-1,1]; ImR(y)=[0, П];

3. $y = \mathrm{tg}^{-1} x = arctgx.$ –toq funksiya

DomR=(-∞;∞); ImR=(- П\2, П\2 );

4. 3. $y = ctg^{-1} x = arcctgx.$ –toq funksiya

DomR=(-∞;∞); ImR=(0, П);

Giperbolik funksiyalar

1. $y = \mathrm{sh}\, x = \frac{e^x - e^{-x}}{2}$ -giperbolik sinusx funksiya –toq funksiya

2. $y = \mathrm{ch}\, x = \frac{e^x + e^{-x}}{2}$ -giperbolik kosinusx funksiya –juft funksiya

3. $y = \mathrm{th}\, x = \frac{shx}{chx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ -giperbolik tangensx funksiya –toq funksiya

4. . $y = \mathrm{cth}\, x = \frac{chx}{shx} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ -giperbolik kotangensx funksiya –toq funksiya

Giperbolik funksiyalarga oid ayniyatlar

Sh(x+y)=shxchy+shychx;

Sh(x-y)=shxchy-shychx;

Ch(x+y)=chxchy-shxshy;

Ch(x-y)=chxchy+shxshy;

$(chx)^2 + (shx)^2 = ch2x;$

$(chx)^2 - (shx)^2 = 1;$

## SONLI KETMA-KETLIKLAR

TA’RIF:Natural argumentli funksiyaga sonli ketma-ketlik deyiladi.

Y=f(x), x∈N;

f(1),f(2),f(3),f(4),…,f(n),…

$x_1, x_2, x_3, x_4, \ldots, x_n, \ldots$

{

$x_n$};

1. $x_n$=1;

1,1,1,…,1,… -o’zgarmas sonli ketma-ketlik.

2. $x_n = \frac{1}{n}$;

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$

3. $x_n = \frac{(-1)^n}{n}$;

$-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \ldots, \frac{(-1)^n}{n}, \ldots$

4. $x_n = n$;

1,2,3,4,…,n,…

5. $x_n = (-1)^n$;

-1,1,-1,1,-1,1,…, $(-1)^n$,…

Sonli ketma-ketlik tabiatiga ko'ra,cheksiz sonlardan iborat va matematikada bu o'zgaruvchilar bilan ishlash ancha murakkab,chunki har bir elementning tabiati ham cheksiz.

Cheksizlikka ma'no kiritish uchun shunday savol tug'iladi.Juda kata aniqlik bilan biror haddan boshlab uning barcha hadlarini o'zgarmas son bilan almashtirish mumkinmi? Ya'ni

$x_1, x_2, x_3, x_4, \dots, x_n, \dots, x_k,$ a,a,a,a,…

Har doim ham buning imkoniyati yo'q.

$\{x_n\};\ \ x_n = \frac{1}{n};$ $\qquad 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n}, \dots, 0,0,0, \dots$

$\{x_n\};\ \ x_n = \frac{(-1)^n}{n};$ $\qquad -1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \dots, \frac{(-1)^n}{n}, \dots, a, a, a, \dots -?$

TA'RIF:Agar biror haddan boshlab $\{x_n\}$ elementlari a sonidan juda kichik songa farq qilsa, a soni $x_n$ ketma-ketlikning limiti deyiladi va quyidagicha yoziladi.

$\lim_{n\to\infty} x_n = a;$

TA'RIF:a soni $x_n$ ketma-ketlikning limiti deyiladi,agarda ∀ ε>0 soni uchun ∃N=N(ε) (N nomer topilsaki) n> N(ε) indeksli hadlar uchun $|x_n - a| < \varepsilon$ shart bajarilsa, $\lim_{n\to\infty} x_n = a$ deyiladi.

$|x_n - a| < \varepsilon$

$-\varepsilon < x_n - a < \varepsilon$

$a - \varepsilon < x_n - a < a + \varepsilon$

Misol: $x_n = \frac{n+2}{n};$ $\qquad 3, 2, \frac{5}{3}, \frac{3}{2}, \frac{7}{5}, \dots$ $\qquad$ a=1;

∀ ε>0 $\qquad$ ∃N=N(ε) $\qquad$ n>N(ε)

$|x_n - a| = \left|\frac{n+2}{n} - 1\right| = \left|\frac{n+2-n}{n}\right| = \frac{2}{n} < \varepsilon;$

$n > \frac{2}{\varepsilon}$; $N = E(\frac{2}{\varepsilon});$

ε=0,1 N=20; $\quad x_1, x_2, x_3, x_4, \dots, x_{20}$,1,1,1,1,…

ε=0,01 N=200; $\quad x_1, x_2, x_3, x_4, \dots, x_{200}$,1,1,1,1,…

$\lim_{n\to\infty} \frac{n+2}{n} = 1\ ;$

Misol: $x_n = \frac{1+(-1)^n}{2};$ $\qquad$ 0,1,0,1,0,1,… -limiti mavjud emas.

Misol: $x_n = \frac{1+(-1)^n}{n};$ $\qquad 0, 1, 0, \frac{1}{2}, 0, \frac{1}{3}, 0, \frac{1}{4}, \dots, \frac{1}{n}, \dots, 0,0,0, \dots$

a=0

Isbot: ∀ ε>0 $\qquad$ ∃N=N(ε) $\qquad$ n>N(ε)

$|x_n - a| = \left|\frac{1+(-1)^n}{n} - 0\right| = \left|\frac{1+(-1)^n}{n}\right| = \frac{2}{n} < \varepsilon;$

$n > \frac{2}{\varepsilon};$ $\quad N = E(\frac{2}{\varepsilon});$

ε=0,1 N=20; $\quad x_1, x_2, x_3, x_4, \dots, x_{20}$,0,0,0,0,,…

ε=0,2 N=10; $\quad x_1, x_2, x_3, x_4, \dots, x_{10}$,0,0,0,0,…

$\lim_{n\to\infty} a^{\frac{1}{n}} = 1;$ a>1 bo'lsa isbotlang.

Isbot: ∀ ε>0  ∃N=N(ε)  n>N(ε)

$|c$ -1|< ε;

$|a^{\frac{1}{n}}$-1|< ε;

$a^{\frac{1}{n}}$-1< ε;  $a^{\frac{1}{n}}$< 1+ε;  $\frac{1}{n}\lg a$<$\lg(1+\varepsilon)$;  n>$\frac{\lg a}{\lg(1+\varepsilon)}$;  $N=[\frac{\lg a}{\lg(1+\varepsilon)}]$;

$\{x_n\}$ –varianta deb yuritiladi.

TA'RIF:Agar sonli ketma-ketlik chekli limitga ega bo'lsa,bu sonli ketma-ketlik yaqinlashuvchi deyiladi.Masalan: $x_n=\frac{n+2}{n}$;

TA'RIF:Agar sonli ketma-ketlik limiti 0 bo'lsa,(ya'ni $\lim_{n\to\infty} x_n=0$) bu sonli a- ε < $x_n$ketma-ketlik cheksiz kichik miqdor deyiladi.Masalan : : $x_n=\frac{1+(-1)^n}{n}$;

Cheksiz kichik miqdorlar $\alpha_n$; $\beta_n$; ... kabi belgilanadi.

$\beta_n$=$x_n$-a; $\lim_{n\to\infty} x_n=a$;

$x_n=\frac{n}{n+1}$=$\frac{n+1-1}{n+1}$=1-$\frac{1}{n+1}$;  a=1;  $\beta_n$=-$\frac{1}{n+1}$;

LIMIT so'zi lotincha "limes" so'zidan olingan bo'lib, chek-chegara,quyuqlanish degan ma'noni anglatadi.

YAQINLASHUVCHI KETMA-KETLIKNING XOSSALARI.

1.Yaqinlashuvchi ketma-ketlikning chegaralanganligi.Tengsizliklarda limitga o'tish.

2.Yaqinlashuvchi ketma-etliklar xossalari va ular ustida amallar.

TA'RIF:Agar $\{x_n\}$ ketma-ketlik chekli limitga ega bo'lsa yoki limiti chekli bo'lsa ,yaqinlashuvchi ketma-ketlik deyiladi.

TEOREMA: Agar $\{x_n\}$ ketma-ketlik yaqinlashuvchi bo'lsa,chegaralangan bo'ladi.

Isbot: $\lim_{n\to\infty} x_n=a$

∀ ε>0  uchun  ∃N=N(ε)  n>N(ε)

$|x_n-a|$< ε;

$a-\varepsilon<x_n$<a+ ε;

Agar M=Max{ $a-\varepsilon$ , $a+\varepsilon$, $x_1, x_2, x_3, \dots, x_n$}

m=Min{ $a-\varepsilon$ , $a+\varepsilon$, $x_1, x_2, x_3, \dots, x_n$}

$x_n\geq$m  bu esa ketma-ketlik quyidan chegaralanganligini bildiradi.

m ≤ $x_n$≤M  bu esa $x_n$ ketma-ketlik chegaralanganligini bildiradi.

TEOREMA:Agar $\{x_n\}$ ketma-ketlik yaqinlashuvchi bo'lib,

$\lim_{n\to\infty} x_n=a$ bo'lsa,a>p(a<q),[p,q∈ $R$] u holda shunday n nomer topiladiki, $x_n>p(x_n>q)$ $bo'ladi.$

Isbot: $\lim_{n\to\infty} x_n=a$ ,a>p u holda $limit$ ta'rifidan ε sonining ixtiyoriyligida$n$

$\varepsilon<a-p$  ketma-ketlik limiti ta'ifga ko'ra $\forall\varepsilon>0$ son uchun shunday n nomer topilsaki,0<$\varepsilon<a-p$;

$|x_n$-a|< ε;

$-\varepsilon<x_n$-a< ε;

0<$\varepsilon<a-p$;  a- ε>p;   p<a- ε;

$-\varepsilon < x_n$-a< ε;

$-\varepsilon < x_n$-a; a- $\varepsilon < x_n$; Demak,shunday n nomer topiladiki,u uchun p $< x_n$.

TEOREMA:Agar $\{x_n\}$ va $\{y_n\}$ ketma-ketliklar yaqinlashuvchi bo'lib,

1) $\lim_{n\to\infty} x_n = a$ va $\lim_{n\to\infty} y_n = b$ bo'lsa,

2) ∀n∈N , $x_n \le y_n (x_n \ge y_n)$ bo'lsa,

U holda, a$\le b$ $(a \ge b)$ o'rinli bo'ladi.

Isbot:shartga ko'ra,

$\forall\varepsilon > 0, \exists m = m(\varepsilon) \in N, \forall n > m$ $|x_n - a| < \varepsilon$;

$\forall\varepsilon > 0, \exists m' = m'(\varepsilon) \in N, \forall n > m'$ $|y_n - b| < \varepsilon$;

Max {m,m'}=k;

$|x_n - a| < \varepsilon$; a- $\varepsilon < x_n < a + \varepsilon$;

$|y_n - b| < \varepsilon$; b- $\varepsilon < y_n < b + \varepsilon$;

2-shartga ko'ra, $x_n \le y_n$

a- $\varepsilon < x_n < a + \varepsilon$; b- $\varepsilon < y_n < b + \varepsilon$;

a+ $\varepsilon \le b + \varepsilon$; a$\le b$.

TEOREMA: :Agar $\{x_n\}$ va $\{z_n\}$ ketma-ketliklar yaqinlashuvchi bo'lib,

1) $\lim_{n\to\infty} x_n$=a, $\lim_{n\to\infty} z_n$=a bo'lsa,

2) $x_n \le y_n \le z_n$ n∈N bo'lsa,

U holda $\{y_n\}$ ketma-ketliklar yaqinlashuvchi bo'lib, , $\lim_{n\to\infty} y_n$=a bo'ladi.

Isbot: $\lim_{n\to\infty} x_n$=a, $\lim_{n\to\infty} z_n$=a shartga ko'ra,

$\forall\varepsilon > 0, \exists m = m(\varepsilon) \in N, \forall n > m$ $|x_n - a| < \varepsilon$;

$\forall\varepsilon > 0, \exists m' = m'(\varepsilon) \in N, \forall n > m'$ $|y_n - b| < \varepsilon$;

2-shartga ko'ra, a- $\varepsilon < x_n \le y_n \le z_n$ <a+ $\varepsilon$;

a- $\varepsilon < y_n$ <a+ $\varepsilon$; $|y_n - a| < \varepsilon$;

$\lim_{n\to\infty} y_n$=a bo'ladi.

TEOREMA:Agar $x_n$ ketma-ketlik limiti mavjud bo'lsa, u yagonadir.

Isbot: $x_n \to$a, faraz qilamiz $x_n \to$b, a<b;

a<r<b

N' n>N' $x_n$ >r;

N'' n>N'' $x_n$ <r;

N=max{N',N''}

Ziddiyatga duch kelamiz,farazimiz noto'g'ri,demak limit mavjud bo'lsa, u yagonadir.

Misol: $\lim_{n\to\infty} \sqrt[n]{n}$ limit hisoblansin.

$\{\sqrt[n]{n}\}$=1, $\sqrt[2]{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$,…

$\sqrt[2n]{n}$>1 $\sqrt[2n]{n}$=1+$\alpha_n$;

$\sqrt[n]{n}$=$(1 + \alpha_n)^2$;

$(1 + \alpha_n)^2$=1+n$\alpha_n$;

$\alpha_n < \sqrt{\frac{1}{n}};$

- $1 < \sqrt[n]{n} < (1 + \sqrt{\frac{1}{n}})^2;$

$\lim\limits_{n\to\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^2$=1; : $\lim\limits_{n\to\infty} \sqrt[n]{n}$ =1 bo'ladi.

Misol:limit hisoblang

$\lim\limits_{n\to\infty} \sqrt[n]{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}};$

$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} > \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n} = \frac{n}{n} = 1;$

$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} < 1 + 1 + \ldots + 1 = n;$

$1 < \sqrt[n]{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}} < n;$

$x_n < y_n < z_n;$

$\lim\limits_{n\to\infty} \sqrt[n]{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}} = 1;$

Yaqinlashuvchi ketma-ketliklar ustida amallar.

$\{x_n\}$ $va$ $\{z_n\}$ ketma-ketliklar berilgan bo'lsin.

$\{x_n\}$: $x_1, x_2, x_{3,\ldots}, x_n;$

$\{y_n\}$: $y_1, y_2, y_{3,\ldots}, y_n;$

$\{x_n \pm y_n\}$: $x_1, \pm y_1, x_2, \pm y_2, x_3, \pm y_3, \ldots, x_n, \pm y_n;$

$\{x_n y_n\}$: $x_1 y_1, x_2 y_2, x_3 y_3, \ldots, x_n y_n;$

$\{x_n / y_n\}$: $x_1 / y_1, x_2 / y_2, x_3 / y_3, \ldots, x_n / y_n;$ $(y_n \neq 0)$

$\lim\limits_{n\to\infty} x_n$=a, $\lim\limits_{n\to\infty} y_n$=b bo'lsin, u holda

$\lim\limits_{n\to\infty} (x_n\, y_n)$ =ab;

$\lim\limits_{n\to\infty} (x_n \pm y_n)$ =a ±b;

$\lim\limits_{n\to\infty} (x_n / y_n)$ =a/b; $(y_n \neq 0)$ (b≠ 0)

$\lim\limits_{n\to\infty} (c\, x_n)$ =ca;

CHEKSIZ KATTA MIQDORLAR

TA'RIF: $x_n$ miqdor cheksiz katta miqdor deyiladi,agarda ixtiyoriy E>>0 soni uchun shunday N=N(E) nomer mavjud bo'lsinki, n>N(E) hadlar uchun $|x_n|$>E bo'lsa,

$|x_n|$>E; $x_n$>E $va$ $x_n$<-E;

$Misol$: $x_n$=n; $\forall E$>>0;

$|x_n|$>E ;

|n|>E; n>E; N=N(E)=[E]+1;

$\lim_{n\to\infty} x_n$=+∞;

$Misol$: $x_n$=-n; $\forall E$>>0;

$|x_n|$>E ;

|-n|>E; n>E; N=N(E)=[E]+1;

$\lim_{n\to\infty} x_n$=-∞;

Cheksiz katta va cheksiz kichik miqdorlar orasidaginbog'lanish.

$x_n$ -cheksiz katta miqdor.

$\alpha_n=\frac{1}{x_n}$ -cheksiz kichik miqdor.

$E=\frac{1}{\varepsilon}$; $\forall\ E=\frac{1}{\varepsilon}$; N=N(E)=N($\frac{1}{\varepsilon}$);

$|x_n|$>E;-cheksiz kata miqdor;

$|\alpha_n|<\varepsilon$; -cheksiz kichik miqdor;

TA'RIF: a nuqta X to'plamning quyuqlanish nuqtasi deyiladi,agarda a nuqtaning (a-$\delta$, a+ $\delta$)atrofida X to'plamning a dan farqli hech bo'lmaganda bitta elementi bo'lsa,

TEOREMA:Agar a nuqta X to'plamning quyuqlanish nuqtasi bo'lsa,u holda bu to'plam elementlaridan a ga intiluvchi $x_n$ ketma-ketlik ajratish mumkin.

$\lim_{n\to\infty} x_n=a$; $x_n \epsilon X$;

$\delta_n \to 0$ –cheksiz kichik miqdor

(a-$\delta_1$, a+ $\delta_1$)∋ $x_1$;

(a-$\delta_2$, a+ $\delta_2$)∋ $x_2$;

(a-$\delta_3$, a+ $\delta_3$)∋ $x_3$;

...

(a-$\delta_n$, a+ $\delta_n$)∋ $x_n$;

a-$\delta_n < x_n <$ a+ $\delta_n$; - $\delta_n < x_n - a < \delta_n$; $|x_n - a| < \delta_n$;

$\delta_n \to 0$;

$\lim_{n\to\infty} x_n$ =a;

TA'RIF:X to'pamning +∞ quyuqanish nuqtasi deyiladi,∀Δ>>0 son uchun (Δ,+∞) oraliqda hech bo'lmaganda 1 ta element mavjud bo'lsa.

TA'RIF:X to'pamning -∞ quyuqanish nuqtasi deyiladi,$\forall\Delta$>>0 son uchun (-∞, $-\Delta$) oraliqda hech bo'lmaganda 1 ta element mavjud bo'lsa.

FUNKSIYA LIMITI TUSHUNCHASI

Geyne ta'rifi: $\lim\limits_{x\to a} f(x)$ =A; Domf(x)=X;

$x_n \to$a, f($x_1$),f($x_2$),f($x_3$),…,f($x_n$) $\to A$;

$z_n \to$a, f($z_1$),f($z_2$),f($z_3$),…,f($z_n$) $\to A$;

F(x)=sin$\frac{1}{x}$ ,x$\neq$ 0;

$x_n = \frac{1}{\frac{\pi}{2}+2\pi n} \to 0$; $\lim\limits_{n\to\infty} f(x)$ =1;

$x_n = \frac{1}{\frac{\pi}{6}+2\pi n} \to 0$; $\lim\limits_{n\to\infty} f(x)$ =0,5;

$x = 0$ quyuqlanish nuqtada sin$\frac{1}{x}$ funksiya limitga ega emas.

$M$isol:f(x)=sinx;

Dom{sinx}=$(-\infty, +\infty)$;

$x_n$=$\frac{\pi}{6}$+2$\pi$n$\to$+∞; $\lim\limits_{n\to\infty} f(x)$ =0,5;

$x_n$=$\frac{\pi}{2}$+2$\pi$n$\to$+∞; $\lim\limits_{n\to\infty} f(x)$ =1 ;

$f(x)$ =sinx funksiyaning cheksiz uzoqlashgan nuqtada limiti mavjud emas.

$Fu$nksiya limitining "$\varepsilon - \delta$" tildagi ta'rifi:

Agar $\forall\varepsilon > 0$ *soni uchun* $\exists\delta = \delta(\varepsilon)$ topilsaki,|x-a|< $\delta$ tengsizlikni qanoatlantiruvchi x lar uchun |f(x)-A|< $\varepsilon$ bo'lsa,A soniga f(x) funksiyaning x=a nuqtadagi limiti deyiladi va quyidagicha yoziladi.

$\lim\limits_{x\to\infty} f(x)$ =A ;

$\forall\varepsilon > 0$ , $\exists\delta = \delta(\varepsilon)$ ,|x-a|< $\delta$ ,|f(x)-A|< $\varepsilon$ ;

*AJOYIB LIMITLAR*

$\lim\limits_{x\to 0}\frac{sinx}{x}$ =1; 0<x<$\frac{\pi}{2}$;

Bir tomonli limitlar.

1. $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a+0} f(x)$ =f(a+0); -o'ng limit deyiladi.
2. $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a-0} f(x)$ =f(a-0); -chap limit deyiladi.

$\lim\limits_{x\to a-0} f(x)$ =A; ; $\lim\limits_{x\to -0} a^{\frac{1}{x}}$ =0;

$\forall\varepsilon > 0$ , $\exists\delta = \delta(\varepsilon)$ ,a-x< $\delta$ ,|f(x)-A|< $\varepsilon$ ;

$\lim_{x\to a+0} f(x)$ =A; $\lim_{x\to +0} a^{\frac{1}{x}}$ =+∞;

$\forall \varepsilon > 0$ , $\exists \delta = \delta(\varepsilon)$ ,x-a< $\delta$ ,|f(x)-A|< $\varepsilon$ ;

$0 + 0$ →+0 –o'ng limit

$0 - 0$ →-0 –chap limit

MONOTON KETMA-KETLIKLAR VA ULARNING LIMITI.

$x_n$ ketma-ketlik berildi.Shu ketma-ketlikning limiti mavjudmi? degan savol qiziqtiradi.Shunday alomatlar borki,shu alomatlarga ko'ra berilgan ketma-etlik limitga ega bo'ladi.

TEOREMA:Agar $x_n$ ketma-ketlik monoton o'suvchi bo'lib, yuqoridan chegaralangan bo'lsa, u limitga ega, chegaralangan bo'lmasa uning limiti +∞;

Isbot:Shartga ko'ra,monoton o'suvchi

$x_{n+1}$>$x_n$; $x_n \leq a$; a=sup{$x_n$}

$\forall \varepsilon > 0$,∃ $x_N$>a- $\varepsilon$;

n>N $x_n$ > $x_N$>a- $\varepsilon$;

a- $\varepsilon < x_n$ < a∓$\varepsilon$ $\lim_{n\to +\infty} x_n$ =a;

2) $\forall e > 0$,∃ $x_N$>>E;

n>N $x_n > x_N > E$;

$\lim_{n\to +\infty} x_n$ =+∞;

TEOREMA:Agar $x_n$ ketma-ketlik monoton kamayuvchi bo'lib, quyidan chegaralangan bo'lsa, u limitga ega, chegaralangan bo'lmasa uning limiti -∞;

Misol: $x_n = \frac{c^n}{n!}$ ,c>1;

$x_{n+1} = \frac{c^{n+1}}{(n+1)!} = \frac{c}{n+1} x_n$;

$x_{n+1} < x_n$ -monoton kamayuvchi;

$x_n$ >0; $\lim_{n\to +\infty} x_n$ =a;

$\lim_{n\to +\infty} x_{n+1} = \lim_{n\to +\infty} \frac{c}{n+1} x_n$ ;

a=0x a;

$\lim_{n\to +\infty} x_n = \lim_{n\to +\infty} \frac{c^n}{n!}$ =0;

Ichm-ich joylashgan oraliqlar haqida

Lemma: $x_n$ ketma-ketlik monoton o'suvchi, $y_n$ ketma-ketlik monoton kamayuvchi bo'lib,ixtiyoriy n da $x_n \leq y_n$ bajarilsa,hamda $\lim_{n\to +\infty} (y_n - x_n)$ =0 bo'lsa, u holda bu ketma-ketlikar limitlari mavjud va ular o'zaro teng.

Isbot:∀n, $x_n < y_n$ <$y_1$ (kamayuvchi)
$x_n < y_1$;(yuqoridan chegaralangan)
$\lim_{n\to+\infty} x_n$=a;
$y_n$>$x_n$>$x_1$ (o'suvchi)
$y_n$>$x_1$;(quyidan chegaralangan)
$\lim_{n\to+\infty} y_n$=b;
$\lim_{n\to+\infty}(y_n - x_n) = \lim_{n\to+\infty} y_n - \lim_{n\to+\infty} x_n = 0;$
b-a=0 b=a;
ICHMA-ICH JOYLASHGAN ORALIQLAR
$[a_n, b_n] \subset \ldots [a_3, b_3] \subset [a_2, b_2] \subset [a_1, b_1];$ (1)
Agar ichma-ich joylashgan (1) kesmalar uchun $\lim_{n\to+\infty}(b_n - a_n)$=0 bo'lsa, u holda uning chegaralari umumiy limitga ega bo'ladi.
$\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} b_n$=c;
Bu ikki ketma-ketliklarga tegishli yagona nuqta mavjud.
2 ta deb faraz qilaylik,u holda $\lim_{n\to+\infty} a_n = k;$ $\lim_{n\to+\infty} b_n$=c;
k-c$\neq 0$ Teorema shartiga zid.
$x_n=(1+\frac{1}{n})^n$ $x_{n+1}=(1+\frac{1}{n+1})^{n+1}$
$x_n=(1+\frac{1}{n})^n=1+\frac{n}{n}+\frac{(n-1)}{n2!}+\frac{(n-1)(n-2)}{n^2 3!}+\ldots+\frac{(n-1)!}{n^{n-1}n!};$
$x_{n+1}=(1+\frac{1}{n+1})^{n+1}$ da ham huddi shunday almashtirish bajarsak bo'ladi.
$x_{n+1} > x_n;$ -monoton o'suvchi
$x_n<1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots+\frac{1}{n!}<2+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots+\frac{1}{2^n}<1+\frac{1}{1-\frac{1}{2}}<e$;-yuqoridan chegaralangan ;
$\lim_{n\to+\infty}(1+\frac{1}{n})^n$=e;
FUNKSIYA TUSHUNCHASI.
Reja:
1.Funksiya ta'rifi,berilish usullari.
2.Funksiyaning chegaralanganligi.Davriy funksiyalar.Juft va toq funksiyalar.
3.Elementar funksiyalar.
4.Teskari funksiya.Murakkab funksiyalar.
Y=kx+b;-chiziqli funksiya.
Y=a$x^2$+bx+c; -kvadrat funksiya.
X={a,b,c}={b,a,c}={c,a,b}-tartiblanmagan to'plam
<x,y,z>≠<y,x,z> -tartiblangan to'plam
X={a,b,c} N(X)=3;
Y={k,l} N(Y)=2;T
Ta'rif:A va B to'plamlar dekart ko'paytmasining ixtiyoriy qismiga binar munosabat deyiladi.
Ta'rif:X va Y to'plamlarda aniqlangan R binar munosabat akslantirish deyiladi,$\forall x \in DomR$ $uchun$ ∃ yagona $y \in ImR$ mavjud bo'lsa.
Funksiyaning chegaralanganligi

1.y=f(x) funksiya yuqoridan chegaralangan deyiladi,shunday M soni mavjud bo'lsaki,f(x)≤M bajarilsa;
2. y=f(x) funksiya quyidan chegaralangan deyiladi,shunday m soni mavjud bo'lsaki,f(x)≥m bajarilsa;
3. y=f(x) funksiya chegaralangan deyiladi,shunday m,M sonlari mavjud bo'lsaki,m≤f(x)≤M bajarilsa;
Y={x}=x-[x]; -chegaralangan funksiya
Funksiyaning juft va toqligi.
1.Ta'rif:y=f(x) funksiya juft deyiladi,f(-x)=f(x) teng bo'lsa,grafigi OY ga nisbatan simmetrik.masalan:y=$x^2$;
2.Ta'rif:y=f(x) funksiya toq deyiladi,f(-x)=-f(x) teng bo'lsa grafigi koordinatalar boshiga nisbatan simmetrik.masalan:y=4x;
Funksiyaning davriyligi
Ta'rif:y=f(x) funksiya davriy deyiladi,agarda ∃T>0 soni mavjud bo'lsaki, ixtiyoriy x uchun f(x+T)=f(x);
Sin(x+T)=sinx;
Sin(x+T)-sinx=0;
$2sin\frac{x+T-x}{2}cos\frac{x+T+x}{2}$=0;
Sin$\frac{T}{2}$=0;T=2$\pi$;
Trigonometrik funksiyalar davri:
Y=sinx T=2$\pi$;
Y=cosx T=2$\pi$;
Y=tgx T=$\pi$;
Y=ctgx T=$\pi$;
Funksiyaning monotonligi
Ta'rif:y=f(x) funksiya monoton o'suvchi deyiladi,$x_1 > x_2(x_1 , x_2 \in DomR)$lar uchun f($x_1$)>f($x_2$) shart bajarilsa.
Ta'rif:y=f(x) funksiya monoton kamayuvchi deyiladi,$x_1 > x_2$ ($x_1 , x_2 \in DomR$)lar uchun f($x_1$)<f($x_2$) shart bajarilsa.
F(x)=$x^3$; Domf(x)=(-∞, ∞) $x_1 > x_2$;
f($x_1$) −f($x_2$)=( $x_1 - x_2$)($x_1{}^2$+$x_1x_2 + x_2{}^2$);
1. Domf(x)=(-∞, 0) F(x)=$x^3$ -o'suvchi
2. Domf(x)=(0,∞) F(x)=$x^3$ -o'suvchi
Murakkab funksiyalar superpozitsiyasi
y=f(x), x=k(t), y=f(k(t));
secx=$\frac{1}{cosx}$; cosecx=$\frac{1}{sinx}$;

YAQINLASHISH PRINSIPI
Bizga $x_n$ ketma-ketlik berilgan bo'lsin.Shu ketma-ketlikning limiti mavjudmi? Degan savolga javob beramiz.
Lemma:Agar $x_n$ sonli ketma-ketlik chegaralangan bo'lsa,undan doimo yaqinlashuvchi ketma-ketlik ajratish mumkin.
$n_1, n_2, n_3,\ldots, n_k,\ldots$∈N; m≤ $x_n$ ≤M;(chegaralangan)

[m,M]=[m,$\frac{m+M}{2}$]U[$\frac{m+M}{2}$,M]

$[m_1, M_1]$ $M_1$-$m_1$ $=\frac{M-m}{2}$ ; $x_{n1}$;

$[m_2, M_2]$ $M_1$-$m_1$ $=\frac{M_1-m_1}{2}$ ; $x_{n2}$;

…

$[m_k, M_k]$ $M_k$-$m_k$ $=\frac{M_{k-1}-m_{k-1}}{2}$ ; $x_{nk}$;

$m_1$<$m_2$ $< m_3$ -monoton o'suvchi;

$M_1 > M_2 > M_3$ -monoton kamayuvchi;

SONLI KETMA-KETLIK YAQINLASHISHINING KOSHIY PRINSIPI$\infty\varepsilon > 0$ *soni uchun* ∃N=N($\varepsilon$) mavjud bo'lishi kerakki,∀n,n'>N lar uchun

$|x_n - x_{n\prime}|< \varepsilon$ bo'lishi zarur va yetarlidir.

Isbot: $|x_n - x_{n\prime}|< \varepsilon$

$x_{n\prime} - \varepsilon < x_n < x_{n\prime} + \varepsilon$ ;

Zaruriyligi:Faraz qilaylik, $x_n$ ketma-ketlik yaqinlashuvchi bo'lsin.

$\lim_{n\to\infty} x_n$=a;

$\forall\varepsilon > 0, \exists N, n > N$ $|x_n - a|< \varepsilon$;

$n' > N$ $|x_{n\prime} - a|< \varepsilon$;

$|x_n - x_{n'}|<|(x_n - a) + (a - x_{n'})| <|x_n - a|+ |x_{n'} - a|< \varepsilon$;

Yetarliligi:Faraz qilaylik ,teorema sharti bajarilsin.n' ni fiksirlaymiz(aniq qiymat beramiz).

$\forall\ n > N$

$x_{n\prime} - \varepsilon < x_n < x_{n\prime} + \varepsilon$ ;

m<$x_n$<M –chegaralangan.

Bundan Boltsano-Koshiy teoremasiga ko'ra, undn qism ketma-ketlik ajratish mumkin.

$\{x_n\}$ $\lim_{n\to\infty} x_{nk}$=c;

nk>N $|x_{nk} - c| < \varepsilon$;

$|x_n - x_{nk}| < \varepsilon$;

$x_n - c| < |x_n - x_{nk} + x_{nk} - c|<x_{nk} - c| + |x_n - x_{nk}|$<2 $\varepsilon$;

Bunday xossaga ega bo'lgan sonli ketma-ketliklar fundamental ketma-ketliklar deyiladi.

$\lim_{x\to 0}(1 + x)^{\frac{1}{x}}$=e;

Isbot:1)x>0

$x_{nk} \to 0$; n=[$\frac{1}{x_{nk}}$] –butun qismi

n<$\frac{1}{x_{nk}}$<n+1;

$\frac{1}{n+1} < x_{nk}$<$\frac{1}{n}$;

$1 + \frac{1}{n+1} < 1 + x_{nk}$<$1 + \frac{1}{n}$;

$(1+\frac{1}{n+1})^n < (1+x_{nk})^{\frac{1}{-}}<(1+\frac{1}{n})^{n+1}$;

$\lim_{x\to 0}(1+x)^{\frac{1}{x}}$=e;

2)x<0

$x_{nk}$<0, $x_{nk} = -y_{nk}$ almashtirish bajaramiz va yuqoridagi ishlarni bajarib,

$\lim_{x\to 0}(1+x)^{\frac{1}{x}}$=e ga ega bo'lamiz .

CHEKSIZ KICHIK MIQDORLAR SHKALASI.

Ixtiyoriy argumentli funksiya uchun limit mavjud bo'lishining Koshiy prinsipi

Teorema:f(x) funksiyaning x→a limiti mavjud bo'lishi uchun berilgan

$\forall\varepsilon > 0$ $soni\ uchun$ $\exists\ \delta = \delta(\varepsilon)$>0 soni mavjud bo'lishi kerakki,|x-a|< $\delta$ ,

|x'-a|< $\delta$ lar uchun |f(x)-f(x')|< $\varepsilon$ bo'lishi zarur va yetarlidir.

Zaruriyligi:Faraz qilamiz y=f(x) ning x=a nuqtadagi limiti mavjud bo'lsin.

$\forall\varepsilon > 0$ ; $\exists\ \delta = \delta(\varepsilon)$>0;

|x-a|< $\delta$,|f(x)-A|<$\frac{\varepsilon}{2}$;

|x'-a|< $\delta$,|f(x')-A|<$\frac{\varepsilon}{2}$;

|f(x)-f(x')|= |f(x)-A+A-f(x')|< |f(x')-A|+|f(x)-A|<$\frac{\varepsilon}{2}+\frac{\varepsilon}{2} < \varepsilon$;

Yetarliligi: x→a $\{x_n\}$ ketma-ketlik olamiz $\lim_{n\to\infty} x_n$=a;

$\forall\delta > 0$ ; $\exists\ N$;n>N;n'>N;

|$x_n$-a|< $\delta$; |$x_{n'}$-a|< $\delta$;

{f($x_n$)}; {f($x_{n'}$)}; |f(x)-f(x')|< $\varepsilon$;

Cheksiz kichik miqdorlar shkalasi

$\alpha(x),\beta(x),\gamma(x)$; x→a

$\lim_{x\to a} \alpha(x)$=0;

$\lim_{x\to a} \beta(x)$=0;

$\lim_{x\to a} \gamma(x)$=0;

1.Agar $\lim_{x\to 0} \frac{\alpha(x)}{\beta(x)}$=k;(k$\neq$ 0) shart bajarilsa,berilgan $\alpha(x)$ va $\beta(x)$ lar bir xil tartibli cheksiz kichik miqdorlar deyiladi.

$\alpha(x)=(sinx)^2$; $\beta(x)$=1-cosx -bir xil tartibli miqdorlar

$\lim_{x\to 0} \frac{\alpha(x)}{\beta(x)}$=2;

2. Agar $\lim_{x\to 0} \frac{\alpha(x)}{\beta(x)}$=0 shart bajarilsa,berilgan $\alpha(x)$(suratdagi) cheksiz kichik miqdor $\beta(x)$(maxrajdagi) cheksiz kichik miqdorga nisbatan yuqori tartibli cheksiz kichik miqdor deyiladi.

$\alpha(x)=sinx^2$; $\beta(x)$=x $\alpha(x)$ cheksiz kichik miqdor $\beta(x)$ ga nisbatan yuqori tartibli.Quyidagicha yoziladi: $\alpha(x)$=0($\beta(x)$);

$\lim_{x\to 0} \frac{\alpha(x)}{\beta(x)}$=0;

3. Agar $\lim\limits_{x\to 0}\frac{\alpha(x)}{\beta(x)}$=? mavjud bo'lmasa ,bunday cheksiz kichik miqdorlar taqqoslanmaydi deyiladi.

$\alpha(x)=xsin\frac{1}{x}$; $\beta(x)$=x -taqqoslanmaydigan cheksiz kichik miqdorlar.

$\lim\limits_{x\to 0}\frac{\alpha(x)}{\beta(x)}$ –mavjud emas.

4. Agar $\lim\limits_{x\to a}\frac{\alpha(x)}{\beta(x)}$=1 shart bajarilsa,berilgan $\alpha(x)$ va $\beta(x)$ cheksiz kichik miqdorlar ekvivalent miqdorlar deyiladi.

Agar $\gamma(x)$cheksiz kichik miqdor $\alpha(x)$ $va$ $\beta(x)$ cheksiz kichik miqdolarga nisbatan yuqori tartibli bo'lsa, $\alpha(x)$ $va$ $\beta(x)$ cheksiz kichik miqdolar ekvivalent miqdorlar deyiladi.Quyidagicha belgilanadi: $\alpha(x)\sim\beta(x)$;

sinx$\sim x$;1-cosx$\sim\frac{x^2}{2}$;$a^x$-1$\sim xlna$;

Ekvivalent cheksiz kichik miqdorlar bosh qismi

$\lim\limits_{x\to a}\frac{\alpha(x)}{c(\beta(x))^k}$=1 Maxajdagi ifoda suratdagi cheksiz kivhik miqdorning bosh qismi deyiladi.

$\frac{x^2}{2}$ ifoda cosx ning bosh qismi

FUNKSIYA UZLUKSIZLIGI TUSHUNCHASI

Y=f(x)

X=Dom(f(x))=D(f(x);

Im(f(x))=E(f(x));

$x_0\in X$;

1.f($x_0$); 2. $\lim\limits_{x\to x_0}(f(x))$=A; 3. f($x_0$)=A;

y=f(x) funksiya $x_0$ nuqtada uzluksiz deyiladi.

$\lim\limits_{x\to x_0} f(x)$= f( $\lim\limits_{x\to x_0} x$) ;

Uzluksizlikning "$\varepsilon-\delta$" tildagi ta'rifi

Y=f(x) funksiya x=$x_0$ nuqtada uzluksiz deyiladi,agarda $\forall\varepsilon$>0 kichik soni uchun $\exists\delta=\delta(\varepsilon)$>0 soni topilsaki,|x-$x_0$|< $\delta$ ni qanoatlantiruvchi x lar uchun |f(x)-f($x_0$ )|< $\varepsilon$ tenglik o'rinli bo'lsa.

Y=$x^2$; [1,3]

$\forall\varepsilon$>0; $\exists\delta=\delta(\varepsilon)$; ,|x-$x_0$|< $\delta$

|f(x)-f($x_0$ )|=| $x^2$-${x_0}^2$|=,|x-$x_0$||x+$x_0$|=6 $\delta$ < $\varepsilon$; $\delta=\frac{\varepsilon}{6}$;

MONOTON FUNKSIYA UZLUKSIZLIGI

Ta'rif:Agar f(x-0)=f(x) bo'lsa,funksiya chapdan uzluksiz deyiladi.

Ta'rif:Agar f(x+0)=f(x) bo'lsa,funksiya o'ngdan uzluksiz deyiladi.

Ta'rif:Funksiya X to'plamda uzluksiz deyiladi,agarda u X to'plamning har bir nuqtasida uzluksiz bo'lsa.

Teorema:Agar y=f(x) funksiya X oraliqda aniqlangan bo'lib,x$\in X$ uning qiymatlar to'plami biror-bir Y oraliqda joylashgan bo'lib,uni tutash to'ldirsa, u holda bu funksiya X to'plamda uzluksizdir.

$\forall\boldsymbol{y_1}\in\boldsymbol{Y}$ $\exists\boldsymbol{x_1}\in\boldsymbol{X}$

$\boldsymbol{y_1 = f(x_1)}$ **ya'ni funksiya syurektiv**

$\forall \boldsymbol{x_0} \in \boldsymbol{X}$ **,** $\boldsymbol{y_0}$**=f(**$\boldsymbol{x_0}$**);**

$\forall \boldsymbol{x_1} \in \boldsymbol{X}$ **,** $\boldsymbol{y_1}$**=f(**$\boldsymbol{x_1}$**);** $\boldsymbol{y_1 = y_0 + \varepsilon}$;

$\boldsymbol{x_0 < x < x_1}$

**f(**$\boldsymbol{x_0}$**)<f(x)< f(**$\boldsymbol{x_1}$**)** $\boldsymbol{= y_0 + \varepsilon =}$ **f(**$\boldsymbol{x_0}$**)+** $\boldsymbol{\varepsilon}$;

$\boldsymbol{-\varepsilon < f(x) - f(x_0) < \varepsilon}$;

$\boldsymbol{\delta = x_1 - x_0}$**;** $\boldsymbol{|x - x_0| < \delta}$

$|f(x) - f(\boldsymbol{x_0})| < \boldsymbol{\varepsilon}$;

$\boldsymbol{y_1 = f(x)}$; $\boldsymbol{y_2}$**=k(x) ; funksiyalar** $\boldsymbol{x_0}$ **nuqtada uzluksiz bo'lsa,**

**f(x)±k(x) –uzluksiz;**

**f(x)k(x) –uzluksiz;**

$\frac{f(x)}{g(x)}$ -uzluksiz;g(x)≠0;

1-misol: $\lim\limits_{x \to 0} \frac{a^x - 1}{x} = \ln a$;

Isbot:

$a^x$-1=y; x→ 0; y→ 0;

$$\lim_{y \to 0} \frac{y}{\log_a(1+y)} = \lim_{x \to 0} \frac{1}{\frac{\log_a(1+y)}{y}} = \frac{1}{\log_a e} = \ln a;$$

2-misol: $\lim\limits_{x \to 0} \frac{(1+x)^m - 1}{x} = m$;

Isbot:

$(1+x)^m - 1 = y$; $(1+x)^m = y+1$; $m = \frac{\ln(1+y)}{\ln(1+x)}$; x→ 0; y→ 0;

$$\lim_{x \to 0} \frac{1}{\frac{\ln(1+y)}{y}}\, m\, \frac{\ln(1+x)}{x} = m;$$

Uzilish nuqtalarini sinflash (klassifikatsiyalash)

$\lim\limits_{x \to x_0} f(x) = f(x_0)$

1-tur uzilish:

$f(x_0 - 0)$ –chap limit mavjud,chekli

$f(x_0)$ –nuqtadagi qiymati mavjud,chekli

$f(x_0 + 0)$ o'ng limit mavjud,chekli

$f(x_0) = f(x_0 - 0) = f(x_0 + 0)$ shu tengliklardan birortasi bajarilmasa,1-tur uzilishga ega deyiladi.

1.Y=f(x)=signx;

x>0 f(x)=1;

x=0, f(x)=0;

x<0, f(x)=-1;

f(-0)=-1; f(+0)=1; f(0)=0; - 1-tur uzilishga ega.

2.y=E(x)=[x];

f(n-0)=n-1; f(n)=n; f(n+0)=n ; 1-tur uzilishga ega.

f(n)=f(n+0)=n bo'lgani uchun ushbu [x] funksiya o'ngdan uzluksiz.

W=f(x+0)-f(x-0) –sakrash deyiladi.

2-tur uzilish:

$f(x_0 - 0) = \pm\infty$ *yoki* mavjud bo'lmasa;

f($x_0$) =$\pm\infty$ $yoki$ mavjud bo'lmasa;
f($x_0+0$)=$\pm\infty$ $yoki$ mavjud bo'lmasa;
ushbu shartlardan birortasi bajarilsa berilgan funksiya 2-tur uzilishga ega deyiladi.
y=sin$\frac{1}{x}$ ;
$x_n=\frac{1}{\frac{\pi}{4}+2\pi n}\to 0$;f($x_n$)$\to\frac{\sqrt{2}}{2}$;
$z_n=\frac{1}{\frac{\pi}{2}+2\pi n}\to 0$; f($z_n$) $\to 1$; $\frac{\sqrt{2}}{2}$
$\neq 1$; $limit\ mavjud\ emas, shuning\ uchun\ 2-tur$ uzilishga ega.
2.y=$\frac{1}{x^3}$;
f(-0)=-∞;
f(+0)=+∞; -2-tur uzilishga ega.
KESMADA UZLUKSIZ FUNKSIYA XOSSALARI.
1.Funksiyaning nolga aylanishi nhaqidagi teorema.(Boltsano-Koshiyning 1-teoremasi)Agar y=f(x) funksiya [a,b] kesmada uzluksiz bo'lib,kesma chegaralarida turli ishorali qiymatlar qabul qilsa,ya'ni f(a)f(b)<0 bo'lsa [a,b] kesmada shunday c nuqta mavjudki,f(c)=0 tenglik o'rinli bo'ladi.
Isbot:Aniqlik uchun f(a)<0 va f(b)>0 deb olamiz va kesmani teng ikiga bo'lamiz.
[a,$\frac{a+b}{2}$],[$\frac{a+b}{2},b$] f($\frac{a+b}{2}$) $=0$; $c=\frac{a+b}{2}$;
$\frac{a+b}{2}=a_1$;f($\frac{a_1+b_1}{2}$)=0; c=$\frac{a_1+b_1}{2}$;
[$a_n, b_n$]C…C[$a_2, b_2$]C[$a_1, b_1$]C[a,b]
$a_1<a_2<a_3<\cdots<a_n\to c$; f($a_n$)$\to f(c)$;
$b_n<\ldots<b_3<b_2<b_1\to c$; f($b_n$)$\to f(c)$;
f($a_n$)<0, f($b_n$)>0; c$\leq 0$, $c\geq 0$;
1-misol:f(x)=[x]-0,5, [0,1]
F(x)=0 -? Bunday nuqta mavjud emas.
2-misol:y=$x^2$; [1,2]
f(1)f(2)<0 shart bajarilmaganligi uchun f(c)=0 bo'ladigan c nuqta mavjud emas.
Boltsano-Koshiyning 2-teoremasi.
Agar y=f(x) funksiya [a,b] kesmada uzluksiz bo'lib,f(a)=A,f(b)=B bo'lsa,(A<B) $\forall C\in[A,B]$ uchun $\exists c\in(a,b)$ mavudki,f(c)=C tenglik o'rinli bo'ladi.
Isbot:k(x)=f(x)-C funksiya kiritamiz.
k(a)=A-C>0; k(b)=B-C<0; Boltsano-Koshiyning 1-teoremasiga ko'ra,
$\exists c\in(a,b)$ $k(c)=0$;
k(c)=f(c)-C=0; f(c)=C;
Bu teorema o'rta(oraliq) qiymat haqidagi teorema deb ataladi.
Teskari funksiya mavjudligi haqidagi teorema:
y=f(x) funksiya biror X oraliqda uzluksiz va monoton o'suvchi(monoton kamayuvchi ) bo'lsa, u holda uning qiymatlar to'plami Y da monoton o'suvchi(monoton kamayuvchi) bir qiymatli uzluksiz x=g(y) funksiya mavjud.
Isbot:Domf(x)=X; Imf(x)=Y;
x, y=f(x)
y uchun ∃x, x=g(y);

x’>x’’,  g(y’)>g(y’’),          bunda y’=f(x’) va y’’=f(x’’)
teskarisi    g(y’)<g(y’’),
y=f(x) –monoton  o’suvchi (kamayuvchi)
Funksiya chegaralanganligi haqida teorema(Karl-Veyershtrassning 1-teoremasi):
Agar y=f(x)  funksiya [a,b] kesmada uzluksiz b’lsa, u  chegaralangandir.
$\exists m$,M      m≤f(x) $\leq M$;
Isbot:Teskarisini faraz qilamiz,funksiya chegaralangan bo’lmasin.
$\forall n, x_n \epsilon [a,b]$             f($x_n$)<n
f($x_1$)<1; f($x_2$)<2; f($x_3$)<3;…; f($x_n$)<n;
f($x_0$)<∞;    bu esa uzluksizlik shartiga zid.
m≤f(x) $\leq M$;-chegaralangan.
Misol:y=$\frac{1}{x}$ ,     (0,1]
$\forall n$ , $\frac{1}{x}$>n;$x < \frac{1}{n}$   bu funksiya chegaralanmagan.
Karl-Veyershtrassning 2-teoremasi:
Y=f(x) funksiya  [a,b] da uzluksiz bo’lsa, u o’zining eng katta va eng kichik qiymatlariga erishadi.
Sup f(x)=M;
Inf f(x)=m;
$\exists x$, x’$\epsilon [a,b]$       f(x)=M;     f(x’)=m;
Isbot: Sup f(x)=M; Inf f(x)=m;
Teskarisini faraz qilamiz,
k(x)=$\frac{1}{M-f(x)} \leq \mu$;     f(x)=M-$\frac{1}{\mu}$;               M-$\frac{1}{\mu} < M$; $-$aniq yuqori chegara.
Tekis uzluksizlik tushunchasi
Ta’rif:y=f(x)  funksiya X oraliqda tekis uzluksiz deyiladi,agarda
$\forall \varepsilon > 0$ $soni\ uchun$ $\exists \delta = \delta(\varepsilon)$(yagona) topilsaki,|x’-x’’|< $\delta$ tengsizlikni qanoatlantiruvchi x’,x’’ lar uchun |f(x’)-f(x’’)|< $\varepsilon$ tengsizlik o’rinli bo’lsa.
Tekis uzluksiz bo’lmagan funksiya ta’rifi
$\forall \varepsilon > 0$ , $\exists \delta = \delta(\varepsilon)$(yagona)
|x’-x’’|< $\delta$;   (f(x’)-f(x’’))> $\varepsilon$
y=sin$\frac{1}{x}$;                      [0,$\frac{2}{\pi}$]
|x’-x’’|=$\frac{1}{n(2n+1)\pi}$;
|f(x’)-f(x’’)|=|0-($\pm$1)|=1;
$\varepsilon$=1,     (f(x’)-f(x’’))> $\varepsilon$;  -tekis uzluksiz emas.
KANTOR TEOREMASI
Agar y=f(x) funksiya [a,b] kesmada uzluksiz bo’lsa,u shu kesmada tekis uzluksizdir.
Isbot:Teskarisini faraz qilamiz,ya’ni [a,b] kesmada uzluksiz funksiya tekis uzluksiz emas. $\forall \varepsilon > 0$ , $\exists \delta$  topiladiki, |x’-x’’|< $\delta$;
| f(x’)-f(x’’)|$\geq \varepsilon$;
$\delta_n \to 0$, $x'_n$ , $x''_n$          |f($x'_n$ )-f($x''_n$ )| $\geq \varepsilon$;
|x’-x’’|<$\delta_n$;
$x'_n \to x_0$ ; $x''_n \to x_0$ ;

f($x'_n$) → $f(x_0)$ ; f($x''_n$) → $f(x_0)$ ;
| f(x')-f(x'')|≥ $\varepsilon$ ifoda farazimiz noto'g'riligini ifodalaydi;
| f(x')-f(x'')|< $\varepsilon$;
HOSILA TUSHUNCHASI
Ta'rif:y=f(x) funksiya x=x' nuqtadagi hosilasi deb,x' nuqtadagi $\Delta x$ argument orttirmasiga mos funksiya orttirmasi $\Delta f(x) =$ f($x' + \Delta x$)-f(x) ni argument orttirmasiga nisbati $\Delta x \to 0$ intilgandagi chekli limitiga aytiladi. $\lim\limits_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x}$=f'(x')
Y=f(x) funksiya X to'plamda aniqlangan bo'lsin.
x ga $\Delta x$ orttirma beramiz,x$\epsilon X$; $x + \Delta x \,\epsilon$X;
$\Delta x$ argument orttirmasi.
$\Delta f(x) =$ f($x + \Delta x$)-f(x) funksiya orttirmasi
Elementar funksiyalarning hosilalari
1.y=c=const; $x_0$,
Domf(x)=(-∞; +∞)
y= f($x_0$)=c;
y=f($x_0 + \Delta x$)=c;
$\Delta y =$ f($x_0 + \Delta x$) −f($x_0$)=c-c=0;
$\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{0}{\Delta x}$=0; y'=(c)'=0;
2.y=$x^m$; $x_0$,
y=f($x_0$)= ${x_0}^m$;
y=f($x_0 + \Delta x$) $= (x_0 + \Delta x)^m$;
$\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{(x_0+\Delta x)^m-(x_0)^m}{\Delta x} = {x_0}^{m-1} \lim\limits_{\Delta x \to 0} \frac{(1+k)^m-1}{k}$=m${x_0}^{m-1}$; k=$\frac{\Delta x}{x_0}$;
Misol:y=$x^3$; y'=?
y'=3$x^{3-1} = 3x^2$;
3.y=$a^x$;
y=f($x_0$)=$a^x$;
y=f($x_0 + \Delta x$)= $a^{x_0+\Delta x}$;
$\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{a^{x_0+\Delta x}-a^{x_0}}{\Delta x} = a^{x_0} \lim\limits_{\Delta x \to 0} \frac{a^{x_0+\Delta x}-1}{\Delta x} = a^{x_0}$lna;
Y'=($a^x$)'=$a^x$lna; ($e^x$)'=$e^x$lne=$e^x$;
4.y=sinx;
y=f($x_0$)=$sinx$;
y=f($x_0 + \Delta x$)=sin($x + \Delta x$);
$\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{\sin(x+\Delta x)-\sin x}{\Delta} = \lim\limits_{\Delta x \to 0} \frac{2sin\frac{\Delta x}{2}cos\frac{2x+\Delta x}{2}}{\frac{2\Delta x}{2}} = \lim\limits_{\Delta x \to 0} cos\frac{2x+\Delta x}{2}$=cosx;
Hosila jadvali

| | |
|---|---|
| 1.(c)'=0;c=const | 11.(arcsinx)'=$\frac{1}{\sqrt{1-x^2}}$; |
| 2.($x^m$)'=m$x^{m-1}$; | 12.(arccosx)'=$-\frac{1}{\sqrt{1-x^2}}$; |
| 3.( $a^x$)'=$a^x$lna; | 13.(arctgx)'=$\frac{1}{1+x^2}$ ; |

4.($e^x$)’=$e^x$;

5.(lnx)’=$\frac{1}{x}$;

6.$\log_a x = \frac{1}{xlna}$;

7.(sinx)’=cosx;

8.(cosx)’=-sinx;

9.(tgx)’=$\frac{1}{(cosx)^2}$;

10. .(ctgx)’=$-\frac{1}{(sinx)^2}$;

14.(arcctgx)’=$-\frac{1}{1+x^2}$ ;

15.(shx)’=chx;

16.(chx)’=-shx;

17.(thx)’=$\frac{1}{(chx)^2}$;

18.(cthx)’=$-\frac{1}{(shx)^2}$; x $\neq 0$;

HOSILANING GEOMETRIK VA FIZIK MA’NOLARI.TESKARI FUNKSIYA HOSILASI. MURAKKAB FUNKSIYA HOSILASI.

Aylana bilan to’g’ri chiziq 3 ta holatda bo’lishi mumkin.

1.kesishmaydi.

2.2 ta nuqtada kesishadi-kesuvchi.

3.1 ta nuqtada kesishadi-urinma.

Bu ta’rifni boshqa hollarda qo’llab bo’lmaydi.

Urinma ta’rifiga aniqlik kiritamiz.y=f(x) chiziq olamiz,unda M va$M_0$ nuqtalarni belgilaymiz.

Ta’rif: $MM_0$ kesuvchining M nuqta y=f(x) funksiy grafigi bo’yicha $M_0$ nuqtaga intilgandagi $MM_0$ kesuvchining limit vaziyatiga y=f(x) funksiyaning $M_0$ nuqtaga o’tkazilgan urinmasi deyiladi.

Masala:y=$\frac{x^2}{2}$;$x_0$=1;

y’=x;

y’(1)=1=tg$\alpha$; $\alpha = \frac{\pi}{4}$;

Hosilaning fizik(mexanik) ma’nosi

S=S(t)

Tezlik tushunchasini kiritish uchun S va t ni ki

$\Delta S = S(t+\Delta t) - S(t)$; $\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = S'(t) = V(t)$;

Umuman olganda,funksiya hosilasi uning o’zgarish tezligini ifodalaydi.

Teskari funksiya hosilasi

Teorema:y=f(x) funksiya quyidagi shartlarni qanoatlantirsa:

1.u monoton o’suvchi(kamayuvchi) qat’iy uzluksiz

2.x, f(x)$\neq 0$; u holda x ga mos y=f(x) nuqtada x=g(y) teskari funksiyasi mavjud bo’lsin va uning hosilasi quyidagicha hisoblanadi.$x'_y = \frac{1}{y'_x}$;

$\Delta x \to 0$; $\Delta y \to 0$

$x'_y = \lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = \frac{1}{\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}} = \frac{1}{y'_x}$;

1.y=arcsinx; x=siny;

D(x)=[-1,1];

I(y)=[-$\frac{\pi}{2}$, $\frac{\pi}{2}$];

$x'_y = \frac{1}{y'_x}$;  $y'_x = \frac{1}{x'_y} = \frac{1}{cosy} = \frac{1}{\pm\sqrt{1-(siny)^2}} = \frac{1}{\sqrt{1-x^2}}$;

2. y=arccosx;  x=cosy;

D(x)=[-1,1];

I(y)=[$-\frac{\pi}{2}, \frac{\pi}{2}$];

$x'_y = \frac{1}{y'_x}$;  $y'_x = \frac{1}{x'_y} = \frac{1}{siny} = \frac{1}{\pm\sqrt{1-(cosy)^2}} = -\frac{1}{\sqrt{1-x^2}}$;

y=arctgx va y=arcctgx funksiyalaning hosilalari ham yuqoridagi usulda hisoblanadi.

Funksiya orttirmasi uchun formula

Teorema:y=f(x) funksiya x nuqtada f'(x) chekli hosilaga ega bo'lsa,u holda funksiya orttirmasi uchun shu holda ushbu tenglik o'rinli.

$\Delta f(x) = f'(x)\Delta x + \alpha\Delta x;$

Isbot:Teorema shartiga ko'ra, $\lim_{\Delta x\to 0}\frac{\Delta f(x)}{\Delta x}$=f'(x);

$\frac{\Delta f(x)}{\Delta x}$-f'(x)= $\alpha\Delta x;$

$\Delta f(x) = f'(x)\Delta x + \alpha\Delta x;$

Agar y=f(x) funksiya nuqtada uzluksiz bo'lsa,u holda bu funksiyaning shu nuqtadagi uzluksiz bo'lishi zarur.

Agar u=u(x),v=v(x) funksiyalar x nuqtada chekli hosilaga ega bo'lsa,u holda

1.(u$\pm v$)=u'$\pm v'$;

2.( u$v$)=u'v';

3.$\left(\frac{u}{v}\right)' = \frac{u'v-v'u}{v^2}$,v$\neq 0$;

Murakkab funksiya hosilasi

Teorema:Agar u=u(x) ,x,u'(x) chekli hosilaga ega bo'lsin.y=f(u) ko'rinishida berilgan murakkab funksiya hosilasi quyidagi formula asosida hisoblanadi.

y'=(f(u))'=f'(u)u'(x);

Isbot:$\Delta u(x) = u'\Delta x + \alpha\Delta x;$

$\Delta f(u) = f'\Delta u + \gamma\Delta u;$

$\Delta x \to 0, \Delta y \to 0, \Delta u \to 0;$

$\lim_{\Delta x\to 0}\frac{\Delta f(x)}{\Delta x}$=f'(u)u'(x);

BIR TOMONLI HOSILA TUSHUNCHASI

y=f(x)  [a,b]  x$\in (a,b)$

(x-$\delta, x+\delta) \in X$ ichki nuqtasi deyiladi.

x=a

$\boldsymbol{\lim_{\Delta x\to +0}\frac{f(a+\Delta x)-f(a)}{\Delta x}}$=f'(a+0)- o'ng hosila

x=b

$\boldsymbol{\lim_{\Delta x\to -0}\frac{f(b+\Delta x)-f(b)}{\Delta x}}$=f'(b-0)- chap hosila

f(x)=$\begin{cases} x^2+1, x\in[0,1] \\ x+1, x\in[1,2] \end{cases}$

f'(+0)=$2x|_{x=0}$=0 –o'ng hosila

f’(-0)=1;-chap hosila
f’(1-0)= $2x|_{x=1}$=2;
f’(1+0)=1;
Cheksiz hosilalar
$(x-1)^2+y^2=1$ ;
$\lim_{\Delta x\to+0}\frac{\Delta y}{\Delta x}=+\infty=f'(+0);$
$\lim_{\Delta x\to-0}\frac{\Delta y}{\Delta x}=+\infty=f'(-0);$
$\lim_{\Delta x\to+0}\frac{\Delta y}{\Delta x}=-\infty=f'(+0);$
$\lim_{\Delta x\to-0}\frac{\Delta y}{\Delta x}=-\infty=f'(-0);$
Misol: y=$x^{\frac{1}{3}}$;
y’=$\frac{1}{3}x^{\frac{-2}{3}}$; $\lim_{\Delta x\to-0}\frac{\Delta y}{\Delta x}=+\infty;$
Misol:y=|x|;
y’=$\begin{cases}1, x>0\\ -1, x<0\end{cases}$
f’(+0)=1 –o’ng hosila
f’(-0)=-1 –chap hosila 2-tur uzilishga ega.
Misol:y=$x\sin\frac{1}{x}$ x$\neq$ 0;
y’= $\sin\frac{1}{x}$ - $\frac{1}{x}$ cos$\frac{1}{x}$ ;
$\lim_{\Delta x\to-0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to-0}\sin\frac{1}{\Delta x}-$ limit avjud emas.
Funksiyaning 1- tartibli hosilasi hech qachon 1-tur uzilishga ega bo’lmaydi.
Misol:f(x)=$\begin{cases}x^2,\ x\epsilon(0,2)\\ 5,\ x=2\\ x+2,\ x\epsilon(2,4)\end{cases}$
f(2-0)=4, f(2+0)=4, f(2)=5; -1-tur uzilish
Misol: f(x)=$\begin{cases}\frac{\sin x}{x}, x\neq 0\\ f(0)=5\end{cases}$
f(-0)=1, f(+0)=1, f(0)=5; -1-tur uzilish
Misol: f(x)=$\begin{cases}\frac{\sin x}{x}, x\neq 0\\ f(0)=1\end{cases}$
f(-0)=f(+0)= f(0)=1; -uzluksiz f(0)=1-uzluksizlikni yo’qotuvchi nuqta
FUNKSIYA DIFFERENSIALI TUSHUNCHASI
Y=f(x) x $\epsilon$X; x+Δ x $\epsilon$X; f(x)$\epsilon C\ X$;
Δy=$\Delta f(x)=f$(x + Δ x)-f(x)=AΔ x+O(Δ x) (1)
Ta’rif:Agar y=f(x) funksiyaning x nuqtadagi orttirmasini (1) ko’rinishda yozish mumkin bo’lsa,u holda f(x) funksiya x nuqtada differensiallanuvchi deyiladi.
Diffrentio(lotincha)-ayirma
Δy=$\Delta f(x)-$ AΔ x = O(Δ x) ;
df(x) =A(Δ x) Δ x −erkli,x ga bog’liq emas.

TEOREMA:y=f(x) funksiya x nuqtada differensiallashuvchi bo'lishi uchun uning shu nuqtadagi chekli hosilasini mavjud bo'lishi zarur va yetarlidir.
Zaruriyligi:Funksiya differensiallanuvchi bo'lsin.

$\Delta y = A\Delta x + O(\Delta x) \qquad |\Delta x$

$\frac{\Delta y}{\Delta x}$=A+$\frac{O(\Delta x)}{\Delta x}$;

$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}$ (A+$\frac{O(\Delta x)}{\Delta x}$); f'(x)=A;

Yetarliligi:chekli hosila mavjud.y=f'(x)

$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = f'(x);$

$\frac{\Delta y}{\Delta x} - f'(x) = \alpha; \qquad \alpha \to 0, \Delta x \to 0;$

$\Delta y = f'(x)\Delta x + \alpha\Delta x$ f'(x)=A,

$\Delta y = A\Delta x + O(\Delta x)$

Dy=df(x) =f'(x)dx;

f(x)=x, df(x)=dx=$\Delta x$;

DIFFERENSIALLASH QOIDALARI

1.d(c)=0;c=const

2.d($x^m$)=m$x^{m-1}$dx;

3.d( $a^x$)=$a^x$lnadx;

4.($d\ e^x$)=$e^x dx$;

5.d(lnx)=$\frac{1}{x}dx$;

6.$d(\log_a x) = \frac{1}{xlna}dx$;

7.d(sinx)=cosxdx;

8.d(cosx)=-sinxdx;

9.d(tgx)=$\frac{1}{(cosx)^2}dx$;

10.d(ctgx)=$-\frac{1}{(sinx)^2}dx$;

11.d(arcsinx)=$\frac{1}{\sqrt{1-x^2}}dx$;

12.d(arccosx)=$-\frac{1}{\sqrt{1-x^2}}dx$;

13.d(arctgx)=$\frac{1}{1+x^2}$ dx;

14.d(arcctgx)=$-\frac{1}{1+x^2}dx$ ;

15.d(shx)=chxdx;

16.d(chx)=-shxdx;

17.d(thx)=$\frac{1}{(chx)^2}dx$;

18.d(cthx)=$-\frac{1}{(shx)^2}dx$; x $\neq 0$;

SODDA XOSSALARI

1.d(cu)=cdu,c=const;

2.d(u$\pm v$)=du$\pm dv$;

3.d(uv)=vdu+udv;

4.d($\frac{u}{v}$) = $\frac{vdu-udv}{v^2}$,v$\neq 0$;

Differensialning geometrik ma'nosi.

tga=k=f'(x)

$\Delta MNK\ da$ tga=$\frac{KN}{MK}$=f'(x)

KN=f'(x)MK

KN=f'(x) $\Delta x$

KN=df(x);

Mavzu:1-tartibli differensial ko'rinishning invariantligi.

y=f(x)   x=k(t)

y=f(k(t))

y'=f'(x) k'(t);   dy=f'(x)k'(t)dt;

df(x)=f'(x)dx, x-erkli bo'lsa,

x-erksiz bo'lsa ham   df(x)=f'(x)dx;

Bu funksiya differensiali ko'rinishining invariantligi

y=$\sqrt{1-x^2}$; -1<x<1

y'=$\frac{-x}{\sqrt{1-x^2}}$;

x=sint,   -$\frac{\pi}{2}<t<\frac{\pi}{2}$,   1- va 4- choraklar;

y=$\sqrt{1-(sint)^2}$=$\sqrt{(cost)^2}$=|cost|=cost;

dx=costdt, dy=-sintdt,

y'=$\frac{dy}{dx}$=$\frac{-sintdt}{costdt}$=$\frac{-sint}{cost}$=$\frac{-x}{\sqrt{1-x^2}}$;

$\frac{df}{dx}=\frac{df}{du}\frac{du}{dx}$,   u=u(x);

$\frac{df}{dx}=\frac{df}{du}\frac{du}{dv}\frac{dv}{dx}$   u=u(x);  v=v(x);

Taqribiy hisoblashlarda:

$\Delta f(x)=f'(x)dx+O(\Delta x)$

f(x+$\Delta x$)-f(x)=f'(x) $\Delta x$

f(k)=f(x)+f'(x) $\Delta x$;

Misol:y=$\sqrt{x}$;

$\sqrt{2,3}$ =$\sqrt{2,25+0,05}$=$\sqrt{2.25}$+$\frac{0.05}{2\sqrt{2,25}}$=$\frac{91}{60}$;

YUQORI TARTIBLI HOSILA VA DIFFERENSIALLAR

Y=f(x) funksiya berilgan bo'lsin.Bu funksiyada y'=f'(x) hosila olaylik.Bu o'z navbatida x ning funksiyasi  y'=f'(x)=k(x) . Bunda k(x) dan ham hosila olish mumkin.Bu esa y=f(x) ning 2-tartibli hosilasi deyiladi.

Misol:y=sinx+$x^3$;

y'=cosx+$3x^2$;

y''= - sinx+6x;

y'''= - cosx+6;

y''''=sinx;

Yuqori tartibli hosilalar uchun umumiy formulalar

1.y=$x^m$;   y'=m$x^{m-1}$;   y''=m(m-1) $x^{m-2}$;  …

$y^{(n)}$=m(m-1)(m-2)…(m-(n-1)) $x^{m-n}$;

Misol:y=$x^7$;

$y^{(5)}$ = 7.6.5.4.3 $x^{7-5}$== 7.6.5.4.3 $x^2$;

$y^{(7)}$ = 7.6.5.4.3.2.1$x^{7-7}$ = 7! ;

2.y=lnx;

y'=$\frac{1}{x}$;   y''=$\frac{-1}{x^2}$;   …

$y^{(n)}$=$\frac{(-1)^{(n-1)}\,(n-1)!}{x^n}$;

3.y=sinx;

Y'=cosx=sin($\frac{\pi}{2}+x$), y''=-sinx= sin(2 $\frac{\pi}{2}+x$), …

$y^{(n)} = \sin\left(n\frac{\pi}{2}+x\right)$;

4.y=cosx; $y^{(n)} = \cos\left(n\frac{\pi}{2}+x\right)$;

5.y=$a^x$;

y'=$a^x lna$, y''=$a^x(lna)^2$, …

$y^{(n)} = a^x(lna)^n$;

6.y=arcctgx;

y'= $(cosy)^2$=cosy sin($\frac{\pi}{2}+y$),

y''= $(cosy)^2$ sin($\frac{2\pi}{2}+2y$), …

$y^{(n)} = (cosy)^n$ sin($\frac{n\pi}{2}+ny$);

LEYBNITS FORMULALARI

Y=u(x)v(x)=uv

Y'=u'v+uv';

Y''=u''v+u'v'+u'v'+uv''=u''v+2u'v'+uv'';`

Y'''=u'''v+3u''v'+3u'v''+uv''';

$y^{(n)} = (uv)^n \sum_{k=0}^{n} C_n^k\, u^{(n-k)}v^k$;

$C_n^k = \frac{n!}{k!(n-k)!}$;

YUQORI TARTIBLI DIFFERENSIALLAR

Y=f(x), dy=f'(x)dx,

dx-o'zgarmas x ga bog'liq emas,agar x erkli o'zgaruvchi bo'lsa,

d($x^n$)=n$x^{n-1}$dx;

d$x^n$=dx dx dx dx…dx –n ta;

$d^2y$=y'' d$x^2$; $d^3y$=y''' d$x^3$; … $d^ny$=$y^{(n)}$ d$x^n$;

1.x-erkli o'zgaruvchi bo'lsa, $d^ny$=$y^{(n)}$ d$x^n$;

2.x-erksiz o'zgaruvchi bo'lsa,x=k(t), $d^2y$=$y^{(2)}$ d$x^2 + y'd^2x$;

Misol:y= $x^3$+lnx;

$d^2y = \left(6x - \frac{1}{x^2}\right),\ dx^2$;

Misol:y=$x^2$;

1.x-erkli o'zgaruvchi bo'lsa, dy=2xdx, $d^2y = 2dx^2$;

2.x-erksiz o'zgaruvchi bo'lsa,x=$t^2$,y=$t^4$;

$d^2y = 2(2tdt)(2tdt) = 8t^2dt^2$;

$d^2$f(x)=2$dx^2$+2x$d^2x = 2(2tdt)(2tdt) + 2t^2 2\ dt^2$=122$t^2\ dt^2$;

YUQORI TARTIBLI DIFFERENSIALLARNING SODDA XOSSALARI

1.$d^n\big(cf(x)\big) = cd^n\big(f(x)\big); c = const$;

2. $d^n(f(x) \pm g(x))$=$d^n\big(f(x)\big) \pm d^n\big(g(x)\big)$;

3. $d^n\big(f(x)g(x)\big) = \sum_{k=0}^{n} C_n^k\, d^kf(x)d^{n-k}g(x)\big)$;

Misol:F(x)=$|x|^3$ ikkinchi tartibli hosilasini toping.

F(x)=$\begin{cases} x^3, agar\ x > 0 \\ -x^3, agar\ x < 0 \end{cases}$

F’(x)=$\begin{cases} 3x^2, agar\ x > 0 \\ -3x^2, agar\ x < 0 \end{cases}$ F’(x)= $3x^2\ signx$;

F’’(x)=$\begin{cases} 6x, agar\ x > 0 \\ -6x, agar\ x < 0 \end{cases}$ F’’(x)=6xsignx=6|x|;

DIFFERENSIAL HISOBNING ASOSIY TEOREMALARI

1.PIER FERMA ((1602-1665)fransuz matematigi) teoremasi:

Y=f(x) funksiya biror X to’plamda aniqlangan ,uzluksiz va uning ichki c$\epsilon X$ nuqtasida o’zining eng kata yoki eng kichik qiymatiga erishsa,agar o’sha nuqtada f’(c) mavjud bo’lsa,u holda f’(c)=0 bo’lishi zarurdir.

Isbot:Faraz qilaylik f(c)-eng kata qiymat

f(x)$\leq f(c)$

$\lim_{x\to c-0} \frac{f(x)-f(c)}{x-c}$=f’(c)$\geq 0$;

$\lim_{x\to c+0} \frac{f(x)-f(c)}{x-c}$=f’(c)$\leq 0$;

Misol:y=$x^3$;

y’=3$x^2$=0,x=0;

Misol:y=$x^2$, [1,2]

Maxf(x)=f(2)=4;

f’(x)= $2x|_{x=2} = 4 \neq 0$;

Minf(x)=f(1)=1;

f’(x)= $2x|_{x=1} = 1 \neq 0$;

2.MISHEL ROLL ((1652-1719)fransuz matematigi) teoremasi:

Y=f(x) funksiya uchun quyidagilar

1)y=f(x) funksiya [a,b] kesmada aniqlangan va uzluksiz bo’lsin.

2)f(x) funksiya (a,b)intervalda chekli hosilaga ega.

3)f(a)=f(b) shartlar o’rinli bo’lsa, u holda (a,b) intervalda shunday c nuqta borki,f’(c)=0 bo’ladi.

Isbot:c-(a,b) intevalning ichki nuqtasi

1-hol:M-eng kata qiymat,m-eng kichik qiymat

Agar M=m bo’lsa, m$\leq f(x) \leq M$; f(x)=M=const, f’(x)=0;

2-hol:m$\neq M$ u holda min va max ning hech bo’lmaganda bittasini ichki nuqtada qabul qiladi.Ferma teoremasiga, ko’ra,f’(c)=0;

Berilgan shartlar juda muhim:

1-misol:f(x)=x-E(x)= $\begin{cases} x, x\epsilon[0,1) \\ 0, x = 1 \end{cases}$

f’(x)=1, x$\epsilon(0,1)$

f(0)=f(1)=0; f’(c)=0, c-mavjud emas.

Demak,1-shart juda muhim ekan.

2-misol:y=$\begin{cases} x, & 0 \leq x \leq 0{,}5 \\ 1-x, & 0{,}5 \leq x \leq 1 \end{cases}$

1)f(x) $\epsilon[0,1]$

2)f’(x), f’(0,5-0)=1, f’(0,5+0)=-1;

3)f(0)=f(1)=0;

f'(x)= $\begin{cases} 1, & 0 < x < 0{,}5 \\ -1, & 0{,}5 < x < 1 \end{cases}$ f'(c)=0, c-nuqta mavjud emas.

Demak, 2-shart muhim ekan.

3.y=x, [0,1]

1)f(x) funksiya [0,1] da uzluksiz,

2)f'(x)=1, (0,1)

3)f(0)=0, f(1)=1,

f'(x)=1 f'(c)=0, c-nuqta mavjud emas.

Demak,3-shart muhim ekan.

3.LAGRANJ teoremasi:

Y=f(x) funksiya uchun quyidagilar :

1)y=f(x) funksiya [a,b] kesmada aniqlangan va uzluksiz bo'lsin.

2)f(x) funksiya (a,b)intervalda chekli hosilaga ega degan shartlar o'rinli bo'lsa,

U holda c∈(a,b) nuqta uchun f'(c)=$\frac{f(b)-f(a)}{b-a}$ o'rinli bo'ladi.

Isbot:Isbotlash uchun quyidagi F(x) funksiyani tuzib olamiz.

F(x)=f(x)-f(a)-$\frac{f(b)-f(a)}{b-a}(x-a)$,

F(x) funksiyani Roll teoremasida tekshiramiz:

1)F(x) funksiya [a,b] da uzluksiz;

2)F(x),(a,b) F'(x)=f'(x)- $\frac{f(b)-f(a)}{b-a}$;

3)F(a)=F(b);

Roll teoremasiga ko'ra,F'(c)=0

F'(c)=f'(c)- )- $\frac{f(b)-f(a)}{b-a} = 0$ tenglikdan

f'(c)=$\frac{f(b)-f(a)}{b-a}$ kelib chiqadi.

KOSHIY teoremasi:

Berilgan y=f(x) va g=g(x) funksiyalar uchun quyidagilar :

1)f(x) va g(x) funksiyalar [a,b] da uzluksiz;

2) f(x) va g(x) funksiyalar (a,b) da chekli hosilaga ega;

3) (a,b) da g'(x) $\neq 0$ o'rinli bo'lsa,

U holda c∈(a,b) nuqta uchun $\frac{f'(c)}{g'(c)}=\frac{f(b)-f(a)}{g(b)-g(a)}$ o'rinli bo'ladi.

Isbot:g(b) $\neq g(a)$ shart nega kiritilmadi?

g(b)=g(a) bo'lsa,Roll teoremasiga ko'ra,g'(c)=0 bo'ladi. Bu esa 3-shartda kiritilgan.

Isbotlash uchun F(x) funksiya kiritamiz.

F(x)=f(x)-f(b)- $\frac{f(b)-f(a)}{g(b)-g(a)}$(g(x)-g(a));

F(x) funksiyani Roll teoresida tekshiramiz.

1)F(x) funksiya [a,b] da uzluksiz,

2)F'(x)=f'(x)- $\frac{f(b)-f(a)}{g(b)-g(a)}$g'(x),

3)F(a)=F(b),

Roll teoremasiga ko'ra, F'(x)=0,

F'(x)= f'(x)-$\frac{f(b)-f(a)}{g(b)-g(a)}$g'(x) tenglik o'rinli ekanligidan,

$\frac{f'(c)}{g'(c)}=\frac{f(b)-f(a)}{g(b)-g(a)}$ tenglik o'rinli bo'ladi.

TEYLOR FORMULASI

P(x)=$a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_nx^n$ ko'phadni biror ikkihad ya'ni x+$x_0$ ning darajalari bo'yicha yoyish Teylor formulasi asosida tushuntiriladi.U quyidagicha amalga oshiriladi.

P($x_0$)= $A_0$;
P'($x_0$)= $A_1$;
P''($x_0$)=2! $A_2$;
P'''($x_0$)=3! $A_3$;
…
$P^{(n)}$ ($x_0$)=$n!$ $A_n$;

P(x)= $A_0+A_1(x-x_0)+A_2(x-x_0)^2+A_3(x-x_0)^3+\cdots+A_n(x-x_0)^n$ ;

Misol:P(x)=2-3x+$x^2+x^3$ ko'phadni x+1 ning darajalari bo'yicha yoying

P(x)=2-3x+$x^2+x^3$, $P(-1)=5;$
P'(x)=-3+2x+3$x^2$, $P'(-1)=-2;$
P''(x)=2+6x, P''(-1)=-4;
P'''(x)=6, P'''(-1)=6;

P(x)=5-2(x+1)-$\frac{4}{2!}(x+1)^2$+$\frac{6}{3!}(x+1)^3$=5-2(x+1)-2$(x+1)^2+(x+1)^3$;

Qoldiq hadli Teylor formulasi

P(x)=
$A_0+A_1(x-x_0)+A_2(x-x_0)^2+A_3(x-x_0)^3+\cdots+A_n(x-x_0)^n+R(x);$

Qoldiq hadni R(x) ko'rinishida belgilash noqulay,shuning uchun uning bir necha hollarni ko'ramiz.

1.$R(x)=\frac{(x-x_0)^{n+1}f^{n+1}(c)}{(n+1)!}$ - Lagranj ko'rinishidagi qo'shimcha hadli Teylor formulasi

2. $R(x)=\frac{(x-x_0)^{n+1}f^{n+1}(x_0+\theta(x-x_0))(1-\theta)^n}{n!}$ – Koshiy ko'rinishidagi qo'shimcha hadli Teylor formulasi (0< $\theta<1$)

3.R(x)=O($(x-x_0)^n$)- Peano ko'rinishidagi qo'shimcha hadli Teylor formulasi

ELEMENTAR FUKSIYALARNI MAKLOREN QATORIGA YOYISH

Kolen Makloren (1698-1746)

Bruk Teylor (1685-1731)

Funsiyalarni Maklorn qatoriga yoyish Teylor formulasining xususiy holi bo'lib, $x_0=0$ bo'lganda o'rinli bo'ladi.

1.y=$e^x$ funksiyani Makloren qatoriga yoying

y=1+x+$\frac{x^2}{2!}$+$\frac{x^3}{3!}$+$\frac{x^4}{4!}$+…+$\frac{x^n}{n!}$+R(x);

2.y=sinx funksiyani Makloren qatoriga yoying

$$y^{(n)}=\begin{cases} n=2m, \sin(m\pi)=0 \\ n=2m-1, -\cos(m\pi)=(-1)^{m-1} \end{cases}$$

y=x-$\frac{x^3}{3!}+\frac{x^5}{5!}$-$\frac{x^7}{7!}$+$\frac{x^9}{9!}$-$\frac{x^{11}}{11!}$+…+$\frac{(-1)^{m-1}x^{2m-1}}{(2m-1)!}$+R(x);

3. y=cosx funksiyani Makloren qatoriga yoying

$$y^{(n)}=\begin{cases} n=2m, \cos(m\pi)=(-1)^m \\ n=2m-1, \sin(m\pi)=0 \end{cases}$$

y=1-$\frac{x^2}{2!}+\frac{x^4}{4!}$-$\frac{x^7}{7!}$+$\frac{x^6}{6!}$-$\frac{x^8}{8!}$+…+$\frac{(-1)^m x^{2m}}{(2m)!}$+R(x);

4.y=ln(1+x) funksiyani Makloren qatoriga yoying

y=x-$\frac{x^2}{2!}+\frac{x^3}{3!}$-$\frac{x^4}{4!}$+…+$\frac{(-1)^{n-1}x^n}{n}$+R(x);

5.y=arctgx funksiyani Makloren qatoriga yoying

y=x-$\frac{x^3}{3!}+\frac{x^5}{5!}$-$\frac{x^7}{7!}$+$\frac{x^9}{9!}$-$\frac{x^{11}}{11!}$+…+$\frac{(-1)^{m-1}x^{2m-1}}{(2m-1)!}$+r(x);

FUNKSIYA O’ZGARISHINI O’RGANISH

1.Funksiyaning o’zgarmaslik sharti:y=f(x) funksiya X to’plamda aniqlangan va uzluksiz hamda soha ichida f’(x) chekli hosilaga ega bo’lsin.Bu funksiya X to’plamda uzluksiz bo’lishi uchun ,uning hosilasi f’(x)=0 bo’lishi yetarli.

Isbot:faraz qilaylik,k nuqta ichki nuqta bo’lsin.f’(k)=0

[k,x] dan olingan c ichki nuqta uchun Lagranj teoremasiga ko’ra,$\frac{f(x)-f(k)}{x-k}$=f’(c)=0

Bundan esa f(x)=f(k)=const kelib chiqadi.

Natija:Berilgan f(x) va g(x) funksiyalar X to’plamda chekli hosilaga ega va f’(x)=g’(x) bo’lsa,u holda bu fuksiyalar o’zgarmas songa farq qiladi.Haqiqatdan ham,

f’(x)-g’(x)=0

(f(x)-g(x))’=0;    f(x)-g(x)=const;    f(x)=c+g(x);   c=const;

Funksiyaning monotonlik sharti

Teorema:y=f(x) funksiya X to’plamda aniqlangan, uning ichki nuqtalarida chekli hosilaga ega.f(x) funksiya x to’plamda monoton o’suvchi bo’lishi uchun f’(x)>0 bo’lishi yetarli.

Isbot:X to’plamdan x>k nuqtalar olamiz.

[k,x] oraliqda Lagranj formulasini qo’llasak, $\frac{f(x)-f(k)}{x-k}$=f’(c)>0;

Zaruriylik sharti muhim emas.

Funksiya ekstremumining zaruriylik sharti

Ta’rif:y=f(x) funksiya X to’plamda aniqlangan bo’lib,k$\epsilon X(k-ichki\ nuqta,$
$\exists\delta>0,(k-\delta,k+\delta)\ \epsilon X$

1.Bu funksiya k nuqtada max erishadi deyiladi,agada $x\epsilon(k-\delta,k+\delta)$
f(x)$\leq f(k)$ shart bajarilsa;

2.Bu funksiya k nuqtada min erishadi deyiladi,agada $x\epsilon(k-\delta,k+\delta)$
f(x)$\geq f(k)$ shart bajarilsa;

f’(x)=0 –statsionar nuqtalar

Ekstremum vaziyat statsionar,hosilalar cheksiz,hosilalar mavjud bo’lmagan nuqtalarda kuzatiladi.

Hosila nol degan so’z ekstremum vaziyat degan so’z emas

$y=x^3$,   y’=$3x^2$, y’(0)=0    lekin x=0 nuqta ekstremum nuqta emas.

Funksiya ekstremumining yetarli shartlari

Y=f(x) funksiya uchun f'(k-0)>0,f'(k+0)<0 shart bajarilsa,funksiya k nuqtada max ga erishadi.
Y=f(x) funksiya uchun f'(k-0)<0,f'(k+0)>0 shart bajarilsa,funksiya k nuqtada min ga erishadi.
Y=f(x) funksiya uchun f''(k)$\neq 0$ shart bajarilsa funksiya k nuqtada ekstremumga erishadi.Agar f''(k)>0 bo'lsa,funksiya k nuqtada min ga erishadi deyiladi, agar f''(k)<0 bo'lsa,funksiya k nuqtada max ga erishadi deyiladi.
Y=f(x) funksiya uchun f'(k)=0, f''(k)=0, …,$f^{m-1}(k) = 0,\ f^{m}(k) \neq 0$; shartlar o'rinli bo'lsa,
a)m=2t bo'lsa,
1) $f^{2t}(k)$>0 , f(x)-f(k)>0 , f(x)>f(k) – k nuqtada min ga erishadi.
2) $f^{2t}(k)$<0 , f(x)-f(k)<0 , f(x)<f(k) –k nuqtada max ga erishadi.
b)m=2t-1 bo'lsa, $f^{m}(k) \neq 0$ bo'lganda,
ekstremum nuqta bo'lmasligi ham mumkin.
Misol:y=$x^5$, k=0;
$y' = 5x^4$, y'(0)=0;
Y''=20$x^3$, y''(0)=0;
$y^{(3)} = 60x^2$, $y^{(3)}(0) = 0$;
$y^{(4)} = 120x$, $y^{(4)}(0) = 0$;
$y^{(5)} = 120$, $y^{(5)}(0) = 120 \neq 0$; ammo k=0 ekstremum nuqta emas,burilish nuqtasi.
Misol:y=$x^4$, x=0;
$y' = 4x^3$, y'(0)=0;
Y''=12$x^2$, y''(0)=0;
$y^{(3)} = 24x$, $y^{(3)}(0) = 0$;
$y^{(4)}$ =24, $y^{(4)}(0)$=24$\neq 0$; x=0 nuqta ekstremum nuqtasi.
FUNKSIYANING BOTIQLIK VA QAVARIQLIK ORALIQLARI.BURILISH NUQTALARI,ASIMTOTALARI
Ta'rif:Faraz qilaylik,f(x) funksiya (a,b) oraliqda uzluksiz
Agar funksiya grafigi uning ixtiyoriy nuqtasiga o'tkazilgan urinmadan pastda yotsa
(yuqorida yotsa) funksiya grafigi (a,b) oraliqda qavariq(botiq) deyiladi.
Funksiya hosilalarida qanday shart bajarilsa funksiya grafigi qavariq bo'ladi?
Teorema(Yetarli shart) Agar (a,b) intervalda f(x) funksiya 2-tartibli hosilaga ega bo'lib,shu intervalning ixtiyoriy nuqtasida u manfiy bo'lsa,u holda funksiya grafigi qavariq bo'ladi.
Isbot:f(x) funksiya (a,b) da uzluksiz, $f^{(2)}(x) < 0$
f'(x)=tg$\alpha$, $\alpha, \frac{\pi}{2}, -\frac{\pi}{2}$, $\alpha$ –kamayuvchi
f'(x)=tg$\alpha$ –kamayuvchi
(f'(x))'<0, $f^{(2)}(x) < 0$
Yetarliligi:f''(x)<0
(f'(x))'<0, f'(x) –kamayuvchi
Kamayuvchi funksiya grafigi urinmadan pastda yotadi. F(x)-qavariq.

F'(x)=0 1-tur kritik nuqtalar (statsionar)
F''(x)=0 2-tur kritik nuqtalar
Misol:y=$x^3$-3$x^2$+2x+1;
f'(x)=3$x^2$-6x+2, f''(x)=6x-6=6(x-1);
1)f''(x)<0 x$\in (-\infty,1)$ qavariq;
2)f''(x)>0 x$\in (1,\infty)$ botiq;
Misol:y=$x^2$,y'=2x, y''=2>0 x$\in R$ da botiq;
Misol: y=$-x^2$,y'=-2x, y''=-2<0 x$\in R$ da qavariq;
Ta'rif:y=f(x) funksiya uchun ichki k,t nuqtalar berilgan bo'lsin
f($\frac{k+t}{2}$) > $\frac{f(k)+f(t)}{2}$ bo'lsa, f(x)-qavariq;
f($\frac{k+t}{2}$) < $\frac{f(k)+f(t)}{2}$ bo'lsa, f(x)-botiq;
Ta'rif:Funksiyaning qavariq va botiqlik oraliqlari o'zgargan nuqtalar funiyaning burilish nuqtalari deyiladi.
Teorema:Agar f(x) funksiya (a,b) oraliqda uzluksiz hosilaga ega bo'lib,k nuqta burilish nuqtasi bo'lsa,u holda f''(k)=0 bo'lishi zarur.
Isbot:x=k nuqta burilish nuqtasi bo'lsin.k nuqta (a,b) oraliqqa tegishli,bundan esa
(a,k) da f''(x)<0
(k,b) da f''(x)>0 Boltsano-Koshiy teoremasiga ko'ra,f''(x)=0
Y=$x^4$;
Y''=12$x^2$; y''(0)=0;-yetarli emas.
x=0 –burilish nuqtasi;
Y=$x^3$;
Y''=6x; y''(0)=0;
x=0 -burilish nuqtasi;
Teorema:(yetarli shart)Agar x nuqtada 2-tartibli hosila 0 ga teng bo'lib,shu nuqta atrofida f''(x) 2-tartibli hosilasini nuqtao'zgartirsa,u holda bu nuqta burilish nuqtasidir.
Funksiya asimtotalari
Ta'rif:y=kx+l (k$\neq$ 0) y=f(x) funksiyaning og'ma asimtotasi deyiladi.
x$\to\infty$ f(x)-kx-l=$\alpha(x)$;
kx=f(x)-l- $\alpha(x)$;
k=$\lim\limits_{x\to\infty}\frac{f(x)}{x}$ -limit mavjud bo'lsa,
l=f(x)-kx- $\alpha(x)$;
l=$\lim\limits_{x\to\infty}(f(x)-kx-\alpha(x))$=$\lim\limits_{x\to\infty}(f(x)-kx)$;
Misol:y=$\frac{x^2+1}{x-2}$ ushbu funksiyaning og'ma asimtotasini toping
k=$\lim\limits_{x\to\infty}\frac{f(x)}{x}$=$\lim\limits_{x\to\infty}\frac{x^2+1}{x(x-2)}$=1;
l= $\lim\limits_{x\to\infty}(f(x)-kx)$= $\lim\limits_{x\to\infty}(\frac{x^2+1}{x-2}-x)$=2;
Demak,o'gma asimtota ko'rinishi y=x+2;
2.Gorizontal asimtota: $\lim\limits_{x\to\infty} f(x)$=b,y=b -gorizontal asimtota;

Misol: $y=\frac{1}{x^2+1}$; $\lim\limits_{x\to\infty}\frac{1}{x^2+1}=0$; y=0 –gorizontal asimtota;

3.Vertikal asimtota: $\lim\limits_{x\to a} f(x)=+\infty$; $x=a$ –vertikal asimtota;

Misol: $y=\frac{x^2+1}{x-2}$; $\lim\limits_{x\to 2}\frac{x^2+1}{x-2}=\infty$; x=2 -vertikal asimtota;

ANIQMASLIKLARNI OCHISHDA LOPITAL QOIDALAR

Teoema:f(x) va g(x) funksiyalar ushbu shartlani qanoatlantirsin.

1.f(x) ,g(x) (a,b]

2. $\lim\limits_{x\to a} f(x)=0$, $\lim\limits_{x\to a} g(x)=0$

3.f'(x) va g'(x) lar mavjud va $g'(x)\neq 0$;

4. $\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}=$k bo'lsa,u holda

$\lim\limits_{x\to a}\frac{f(x)}{g(x)}=$k bo'ladi.

Isbot:f(a)=0, g(a)=0 f(x),g(x)∈C[a,b]

[a,x] oraliqda Koshiy teoremasiga ko'ra, $\frac{f(x)-f(a)}{g(x)-g(a)}=\frac{f'(c)}{g'(c)}$;

$\lim\limits_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)}=\lim\limits_{x\to a}\frac{f(x)}{g(x)}=\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}=$k;

Misol: $\lim\limits_{x\to 0}\frac{\sin x}{x}=\lim\limits_{x\to 0}\frac{(\sin x)'}{x'}=\lim\limits_{x\to 0}\frac{\cos x}{1}=1$;

$\lim\limits_{x\to 0}\frac{a^x-1}{x}=\lim\limits_{x\to 0}\frac{a^x\ln a}{1}=\ln a$;

Teorema: f(x) va g(x) funksiyalar ushbu shartlani qanoatlantirsin.

1.f(x) ,g(x) [a,+∞)

2. $\lim\limits_{x\to\infty} f(x)=0$, $\lim\limits_{x\to\infty} g(x)=0$

3.f'(x) va g'(x) lar mavjud va $g'(x)\neq 0$;

4. $\lim\limits_{x\to\infty}\frac{f'(x)}{g'(x)}=$k bo'lsa,u holda

$\lim\limits_{x\to\infty}\frac{f(x)}{g(x)}=$k bo'ladi.

Isbot: $\lim\limits_{x\to\infty}\frac{f(x)}{g(x)}$ tenglikda $x=\frac{1}{t}$, $t\to 0$ belgilash kiritamiz;

$\lim\limits_{t\to 0}\frac{f(\frac{1}{t})}{g(\frac{1}{t})}=\lim\limits_{t\to 0}\frac{f'(\frac{1}{t})(-\frac{1}{t^2})}{g'(\frac{1}{t})(-\frac{1}{t^2})}==\lim\limits_{t\to 0}\frac{f'(\frac{1}{t})}{g'(\frac{1}{t})}=\lim\limits_{x\to\infty}\frac{f'(x)}{g'(x)}$;

$\frac{\infty}{\infty}$ tipidagi aniqmasliklar uchun Lopital qoidasi

Teorema: f(x) va g(x) funksiyalar ushbu shartlani qanoatlantirsin.

1.f(x) ,g(x) (a,b] aniqlangan;

2. $\lim\limits_{x\to a} f(x)=\infty$, $\lim\limits_{x\to a} g(x)=\infty$;

3.(a,b] da f'(x) va g'(x) lar mavjud va $g'(x)\neq 0$;

4. $\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}=$k bo'lsa,u holda

$\lim\limits_{x\to a}\frac{f(x)}{g(x)}=$k bo'ladi.

Isbot:4-tenglikni "$\varepsilon-\delta$" tilga o'tkazamiz.

$\forall\varepsilon>0, \exists\delta=\delta(\varepsilon)>0$

|x-a|< $\delta$ ,| $\frac{f'(x)}{g'(x)}-k$|< $\varepsilon$;

a<x<x’;

[a,x] da qo’llaymiz,chunki uzluksiz emas

[x,x’]da $\exists c\epsilon(x,x')$ Koshiy teoremasiga ko’ra,

$\frac{f(x')-f(x)}{g(x')-g(x)}=\frac{f'(c)}{g'(c)}$;

|$\frac{f'(c)}{g'(c)}-k$|< $\varepsilon$ , |$\frac{f(x')-f(x)}{g(x')-g(x)}-k$| < $\varepsilon$;

|$\frac{f(x)}{g(x)}-k$|=|$\frac{f(x')-kg(x')}{g(x)}+\left[1-\frac{g(x')}{g(x)}\right]\left[\frac{f(x')-f(x)}{g(x')-g(x)}-k\right]$|;

$\lim_{x\to a} g(x)=\infty$;

|$\frac{f(x)}{g(x)}-k$|< $\varepsilon$ ;

Misol: $\lim_{x\to a} g(x)=\infty$;

$\lim_{x\to\infty}\frac{lnx}{x^m}=\lim_{x\to\infty}\frac{\frac{1}{x}}{mx^{m-1}}=0$; $x^m$ funksiya lnx funksiyaga nisbatan tezroq cheksizlikka cheksizlikka.

Misol: $\lim_{x\to\infty}\frac{x^m}{a^x}=\lim_{x\to\infty}\frac{mx^{m-1}}{a^x lna}=\lim_{x\to\infty}\frac{m(m-1)x^{m-2}}{a^x lnalna}=\ldots=0$;

$a^x$ funksiya $x^m$ ga nisbatan tezroq cheksizlikka intiladi.

$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}=\lim_{x\to a}\frac{f''(x)}{g''(x)}=\ldots=\lim_{x\to a}\frac{f(x)^{(k)}}{g(x)^{(k)}}$;

Aniqmasliklarning boshqa ko’rinishlari

1.$\frac{0}{0}$ ko’rinishidagi aniqmaslik:

$\lim_{x\to a} f(x)=0$; $\lim_{x\to a} g(x)=0$;

$\lim_{x\to a}\frac{f'(x)}{g'(x)}$ =k bo’lsa, . $\lim_{x\to a}\frac{f(x)}{g(x)}$ =k bo’ladi.

2.0 $\infty$ ko’rinishidagi aniqmaslik:

$\lim_{x\to a} f(x)=0$; $\lim_{x\to a} g(x)=\infty$;

Bu yerda f(x)=$\frac{1}{k(x)}$ *almashtirish bajarib* $\frac{\infty}{\infty}$ ko’rinishiga keltiramiz,yoki

g(x)=$\frac{1}{t(x)}$ *almashtirish bajarib* $\frac{0}{0}$ ko’rinishiga keltiramiz.

Misol: $\lim_{x\to 0} x^m$lnx=$\lim_{x\to 0}\frac{lnx}{x^{-m}}=\lim_{x\to 0}\frac{x^m}{-m}=0$;

3. $\infty$ - $\infty$ ko’rinishidagi aniqmaslik:

$\lim_{x\to a} f(x)=\infty$; $\lim_{x\to a} g(x)=\infty$;

Bu yerda f(x)=$\frac{1}{\frac{1}{f(x)}}$ va g(x)=$\frac{1}{\frac{1}{g(x)}}$ almashtirish bajarib, $\frac{0}{0}$ ko’rinishiga keltiramiz;

Misol: $\lim_{x\to 0}((ctgx)^2-\frac{1}{x^2})$= :$\lim_{x\to 0}((\frac{cosx}{sinx})^2-\frac{1}{x^2})$; ushbu ifoda soddalashtirishlardan so’ng quyidagi ko’rinishga keladi.

2$\lim_{x\to 0}\frac{-xsinx}{(sinx)^2+2xsinxcosx}$=2$\lim_{x\to 0}\frac{-1}{\frac{sinx}{x}+2cosx}=\frac{-2}{3}$;

4.$1^\infty$,$0^0$ ko’rinishidagi aniqmaslik:

a) $\lim_{x\to a} f(x)=1;\quad \lim_{x\to a} g(x)=\infty;$

b) $\lim_{x\to a} f(x)=0;\quad \lim_{x\to a} g(x)=0;$

$\lim_{x\to a} f(x)^{g(x)}=\lim_{x\to a} e^{g(x)\ln f(x)}$=exp{$\lim_{x\to a} g(x)\ln f(x)$};0 $\infty$ $dan$ foydalanamiz.

Misol: $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{1-\cos x}}$=exp{$\lim_{x\to 0}\frac{x\cos x-\sin x}{x(\sin x)^2}$}=exp{-$\frac{1}{3}$}=$e^{-\frac{1}{3}}$;

BOSHLANG'ICH FUNKSIYA TUSHUNCHASI

F(x) funksiya X to'plamda aniqlangan f'(x)=0 bo'lsa f(x) funksiyaning o'zgarmasligini isbotlang

Isbot:[x',x] oraliqda Lagranj teoremasini qo'llaymiz.

$\frac{f(x)-f(x')}{x-x'}$=f'(x')=0

f(x)-f(x')=0; f(x)=f(x'); x'-fiksrlangan nuqta,f(x') –o'zgarmas son;

$\int f(x)dx = F(x) + C;$

f(x) –integral osti funksiyasi

f(x)dx-integral osti ifodasi

ushbu belgi Lagranj tomonidan kiritilgan.

Xossalari:

1. $\int f'(x)dx = f(x) + C;$

2. $\int df(x) = f(x) + C;$

Yuzani hisoblash masalasi:

[a,b] kesmada f(x) $\geq$ 0 funksiya berilgan bo'lsin.

Y=f(x), x=a, x=b, y=0 chiziqlar bilan chegaralangan egri chiziqli trapetsiya yuzini hisoblash maalasi

[a,b] ga tegishli x o'zgaruvchi kiritamiz.Bo'yalgan yuzani f(x) deb belgilaymiz,unga

$\Delta x$ orttirma beriladi.

$\Delta f(x)$=f(x+$\Delta x$) $- f(x)$;

maxf(x)=M; minf(x)=m;

m$\Delta x \leq \Delta f(x) \leq$M$\Delta x$;

$\Delta x \to$0 ;

$M \to$f(x) ;

$m \to$f(x) ;

$\lim_{\Delta x\to 0}\frac{\Delta f(x)}{\Delta x} = f$(x)=P'(x);

P(x) funksiya f(x) uchun boshlang'ich funksiya bo'lsin

F(x) funksiya f(x) ning biror boshlang'ich funksiya bo'lsin

P(x)=F(x)+C

P(a)=0

P(a)=F(a)+C=0

C=-F(a);

P(x)=F(x)-F(a)

P(b)=F(b)-F(a) ; -Nyuton-Leybits formulasi

Misol:y=a$x^2$,a>0

$F(x)=\frac{ax^3}{3}$;

$P(x)=F(x)-F(0)=\frac{ax^2 x}{3}=\frac{yx}{3}$;

Integrallar jadvali

1. $\int 0dx = c$;
2. $\int 1dx = x + c$;
3. $\int \frac{1}{x}dx = \ln|x| + c$;
4. $\int x^m dx = \frac{x^{m+1}}{m+1} + c$;
5. $\int a^x dx = \frac{a^x}{lna} + c$;
6. $\int e^x dx = e^x + c$;
7. $\int cosxdx = sinx + c$;
8. $\int sinxdx = -cosx + c$;
9. $\int \frac{1}{(sinx)^2}dx = tgx + c$;
10. $\int \frac{1}{(cosx)^2}dx = -ctgx + c$;

Boshlang'ich funksiya va aniq integral tushunchasi.

Biz o'tgan darslarda X to'plamda aniqlangan f(x) funksiya hosilasini hisoblash masalasini kiritdik.Endi bu masalaga teskari masalani o'rganamiz.Ya'ni funksiya hosilasi berilgan ,unga ko'ra f(x)funksiyani tiklash masalasi

Ta'rif:X oraliqda f(x) funksiya F(x) funksiyaning hosilasiga teng bo'lsa,F(x) funksiya f(x) funksiyaning boshlang'ich funksiyasi deyiladi.

Teorema:Agar X to'plamda F(x) funksiya f(x) funksiyaning boshlang'ich funksiyasi bo'lsa,u holda (F(x)+C) ham f(x) funksiyaningboshlang'ich funksiyasi bo'ladi.

(F(x)+C)'=F'(x)+(C)'=f(x);

F'(x)=f(x);

Intergral o'zgaruvchisini almashtirib integrallash.

Bizga f(x) funksiya va uning boshlang'ichi F(x) berilgan bo'lsin.x=k(t) bo'lsa,

.$\int f(k(t)k'(t)dx = F(k(t) + C$;

Misol:$\int (sinx)^3 cosxdx = \int (sinx)^3 dsinx$=|sinx=t|=$\int t^3 dt=\frac{t^4}{4}$+C=$\frac{(sinx)^4}{4}$+C;

Mazkur misolda integral o'zgaruvchisi x ni sinx=t bilan almashtirdik.

Misol: $\int (sinx)^3 dx = \int (sinx)^2 sin\,xdx = -\int (sinx)^2 dcosx$=|cosx=t|=

$=-\int (1-t^2)dt = -$(t - $\frac{t^3}{3}$)+C=$\frac{(cosx)^3}{3}$ –cosx+C;

Misol:$\int e^{x^2} xdx$=0,5 $\int e^{x^2} dx^2$=$|x^2 = t|$=0,5 $\int e^t dt = e^t + C = 0{,}5e^{x^2} + C$;

Misol: $\int tg\,xdx = \int \frac{sinx}{cosx}dx = -\int \frac{1}{cosx}dcosx$=|cosx=t|=$-\int \frac{1}{t}dt$=-ln|t|+C=

=-ln|cosx|+C;

Ctgx funksiya integrali ham huddi yuqoridagidek hisoblanadi.

$\int ctg\,xdx$ ==ln|sinx|+C;

Misol: $\int \frac{xdx}{1+x^4}$ da $x^2$=t o'zgartirish kiritamiz.

$\int \frac{xdx}{1+x^4}$ =0,5arctg$x^2$+C;

Misol: $\int \frac{lnxdx}{x}$=$\int lnx\,dln$x=|lnx=t|=0,5$t^2$+C=0,5$(lnx)^2$+C;

$\int \frac{dx}{xlnx}$ funksiya integrali ham yuqoridagidek usulda hisoblanadi.

$\int \frac{dx}{xlnx}$ =ln(lnx)+C;