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ALGEBRA VA FUNKSIONAL ANALIZ KAFEDRASI

Kichik o’lchamli Li algebralarining birinchi va ikkinchi kogomologik gruppalari mavzusidagi

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Kirish

Hozirgi kunda berilgan algebralarning strukturasini o'rganishda, ularning algebraik klassifikatsiyalari bilan birga ularning differesiallashlari, geometrik xossalari, deformatsiyalari ham keng o'rganilmoqda. Geometrik tasnifni ma'nosi shundan iboratki,agar biror algebralar ko'pxilligining to'liq tasnifi berilgan bo'lsa, bu algebralarning barchasini orbitalari yoyilmasini topish masalasi qaraladi.

Buzilish, siqish va deformatsiya tushunchalari algebraga fizikadan kirib kelgan bo'lib, xususan assotsiativ va Li algebralarining deformatsiyalari, fizik nuqtai nazardan biror fizik model boshqasini invariantlar gruppasi ta'sirining limiti yordamida hosil qilinganligini anglatadi. O'z navbatida, deformatsiya berilgan tipdagi obyektlar ko'pxilligining kichik atrofidagi lokal tuzilishini xarakterlaydi. Shuning uchun Li algebralarning deformatsiyalari, geometrik xossalari, strukturaviy nazariyalari va kogomologiyasini o'rganish juda muhimdir. Assotsiativ va Li algebralar uchun deformatsiyalar nazariyasi M.Gerstenxaber hamda A.Neyenxeys, R.V.Richardsonlar tomonidan o'tgan asrning 60-yillarida kiritilgan. Ular tomonidan bir parametrli deformatsiyalar o'rganilgan bo'lib, Li algebralarining kogomologiyasi va infinitezimal deformatsiyalari orasidagi bog'lanishlar o'rnatilgan. Li algebralarining turli deformatsiyalari A.Fialowski, M.Penkava, M.Gild, D.V.Millionchikovlar va boshqalar tomonidan o'rganilgan bo'lib, ularning bir qancha xossalari isbotlangan. Yu.B.Xakimjanov, R.M.Navarrolarning ishlarida esa filiform Li algebralari va superalgebralarining infinitezimal deformatsiyalari tasniflangan. Li algebralarining kogomologik xossalari va deformatsiyalari J.L.Lode, T.Pirashvili, D.Balovan, J.M.Lodder va A.Fialowskilar tomonidan o'rganilgan.Ushbu bitiruv malakaviy ishi yechimli kichik o'lchamli Li algebralarining birinchi va ikkinchi kogomologik gruppalarini o'rganishga bag'ishlangan. Bitiruv malakaviy ishida barcha uch o'lchamli kompleks Li algebralarining differensiallari hisoblangan. Differensiallarning ko'rinishidan foydalangan holda barcha ichki differensiallashlar topilgan. Ma'lumki, algebralarning birinchi kogomologik gruppasi differensiallashlar fazosiga faktor fazosi hisoblanadi. Differensiallashlar va ichki differensiallashlarning tasnifidan foydalangan holda birinchi kogomologik gruppalar va ularning bazislari to'liq topilgan. Bitiruv malakaviy ishining ikkinchi bobida uch va to'rt o'lchamli Li algebralarining infinitezimal deformatsiyalari topilgan. Ta'kidlash joizki, algebraning infinitezimal deformatsiyalari, uning ikkinchi gruppa kosikllari Z2(L, L) dan iborat bo'ladi. Uch va to'rt o'lchamli Li algebralarining differensiallashlar fazosi ham to'liq tasniflanib, olingan tasniflar yordamida 2-gruppa kosikllari Z2(L, L) kochegaralarining B2(L, L) o'lchamlari topilgan. Ma'lumki, Z2(L, L) va B2(L, L) ni topish 2-gruppa kogomologiya HL2(L, L) ni topish imkonini beradi, hamda H2(L, L) ning nolga teng bo'lishi algebraning qattiq bo'lishi yetarlilik sharti hisoblanadi. Bitiruv malakaviy ishida o'rganilgan algebraning ikkinchi gruppa kogomologiya noldan farqliligi ko'rsatilgan va xulosa sifatida uning qattiq emasligiga ega bo'lamiz.

Bitiruv malakaviy ish mavzusining dolzarbligi. Ma'lumki, Li algebralarining kogomologiyalarini hisoblashda algebralarning strukturaviy klassifikatsiyalari va ularning geometrik xossalari o'rganiladi. Geometrik tasnifini o'rganishning dorzarbligi shundaki, agar biror algebralarning ko'pxilligi berilgan bo'lsa, ularning orbitalarining yoyilmasi o'rganiladi.

Ishning maqsadi va vazifalari. Bitiruv malakaviy ishining mavzusi "Kichik o'lchamli algebaralarining birinchi va ikkinchi kogomologik gruppalari"bo'lib, unda qo'yilgan maqsad va vazifalar quyidagilar

1) Berilgan uch va to'rt o'lchamli Li algebralarining differensiallashlari va ichki differensiallashlari matrisaviy ko'rinishini topish;

2) Differensiallash matrisalaridan foydalanib, uch va to'rt o'lchamli algebralarning birinchi kogomologik gruppalarini hisoblab natijalarni olish;

3) Uch va to'rt o'lchamli algebralarning ikkinchi kogomologik gruppalarini hisoblab natijalarni olish.

Tadqiqot ob'ekti va predmeti: Bitiruv malakaviy ishining ob'ekti va predmeti bu Li algebralari va ularning klassifikatsiyalari, differensiallash fazosidagi matrisalari ko'rinishi, ichki differensiallashlar, algebralarning kochegara va kosikllari, kichik o'lchamli Li algebralarining birinchi va ikkinchi kogomologik gruppalaridir. Bulardan foydalanish jarayonida ham turli xil ta'rif va tushunchalardan keng foydalaniladi. Bundan umumiy natijalar va xulosalar olindi. Ushbu mavzuga oid ilmiy maqolalar o'rganildi va turli xil adabiyotlardan foydalanildi.

Olingan asosiy natijalar: Uch va to'rt o'lchamli Li algebralarning differensiallashlar fazosidagi matrisaviy ko'rinishi hisoblandi. Differensiallashlar matrisalaridan va berilgan algebralarning ichki differensiallari matrisalaridan bu algebralarning birinchi kogomologik gruppalari o'lchamining ko'rinishini, uch va to'rt o'lchamli Li algebralarining ikkinchi kosikl va ikkinchi kochegaralaridan foydalanib ushbu algebralarning ikkinchi kogomologik gruppalari o'lchamining ko'rinishi haqidagi natijalar olingan.

Bitiruv malakaviy ishning hajmi va tuzilishi: Bitiruv malakaviy ishi kirish, ikkita bob, olti paragraf, xulosa va foydalanilgan adabiyotlar ro'yxatidan, hamda 56 sahifadan iborat
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I BOB. Li algebralariga doir asosiy tushuncha.

1.1-§. Li algebrasi ta'rifi va misollar.

Ushbu bobda bitiruv malakaviy ishida foydalaniladigan asosiy tushuncha va ta'riflar keltiriladi. Li algebrasi yangi, assotsiativ va kommutativ bo'lmagan operatsiya bilan ta'minlangan. Bu turdagi algebraik sistemalarni bir nechta aksiomalar yordamida abstrakt ta'riflash mumkin

1.1.1- ta'rif. $\mathbb{F}$ maydon ustidagi G algebrada ixtiyoriy $x, y, z \in G$ elementlar uchun quyidagi ayniyatlar bajarilsa:

1. $[x, y] = -[y, x]$, -antikommutativlik ayniyati,
2. $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$, - Yakobi ayniyati,

u holda G algebraga Li algebrasi deyiladi, bu yerda $[-, -]$ G dagi ko'paytma.

1.1.2- ta'rif. F maydon ustidagi G va G' Li algebrasi izomorf deyiladi, agar barcha $x, y \in G$ lar uchun $\varphi([x, y]) = [\varphi(x) + \varphi(y)]$ munosabat bajariluvchi $\varphi : G \to G'$ vektor fazolar izomorfizmi mavjud bo'lsa. (Bu holda φ Li algebrasi izomorfizmi deyiladi).

Bizga C maydon ustida n-o'lchamli L algebra berilgan bo'lsin. L algebrada $e_1, e_2, ..., e_n$ - bazisini qaraymiz, u holda quyidagi

$$e_i e_j = \sum_{k=1}^{n} \gamma_{ij}^k e_k, 1 \leq i, j \leq k$$

ko'paytmalarga ega bo'lamiz, bu yerda $\gamma_{ij}^k \in C$ elementlar strukturaviy konstantalar deyiladi.

Demak, ixtiyoriy n o'lchamli L algebraga berilgan bazisda n^3 o'lchamli fazoda yagona nuqta mos qo'yiladi.

Ixtiyoriy L Li algebrasi uchun quyidagi ketma ketliklarni aniqlaymiz:

Quyi hosilaviy qator: $L^{[1]} = L, L^{[k+1]} = [L^{[k]}, L^{[k]}], k \geq 1$

Quyi markaziy qator: $L^1 = L, L^{k+1} = [L^k, L^1], k \geq 1$.

1.1.3-ta'rif. Agar shunday $m \in N$ soni mavjud bo'lib, $L^{[m]} = 0$ bo'lsa,u holda L Leybnits algebrasi yechimli deyiladi. Ana shunday m larning eng kichigiga L yechimli algebraning indeksi deyiladi.

1.1.4-ta'rif. Agar shunday $s \in N$ mavjud bo'lib, $L^s = 0$ bo'lsa, L Leybnits algebrasi nilpotentli deyiladi. Bunday xususiyatga ega bo'lgan minimal s soni nilpotentlik indeksi yoki L algebrasining nilindeksi deyiladi.

Ravshanki, yechimli Li algebralari nilpotent Leybnits algebralarining umumlashmasi bo'ladi, ya'ni ixtiyoriy nilpotent algebrasi yechimli bo'ladi.

1.1.5- ta'rif. Aytaylik, $d : G \to G$ chiziqli akslantirish bo'lsin. Agar ixtiyoriy $x, y \in G$ elementlar uchun quyidagi tenglik bajarilsa:

$$d([x, y]) = [d(x), y] + [x, d(y)],$$

u holda d chiziqli akslantirish G algebrada differensiallash deyiladi.

1.1.6- ta'rif. Ixtiyoriy $x \in G$ uchun $R_z(x) = [x, z]$ kabi aniqlangan $R_x : G \longrightarrow G$ akslantirish differensiallash bo'ladi va bunday differensiallashlar ichki differensiallashlar deb ataladi. Barcha ichki differensiallashlar to'plami $Inn(G)$ kabi belgilanadi.

Ma'lumki, Li algebrasining barcha differensiallashlar to'plami algebraning 1-kosikllarini, ichki differensiallashlar to'plami esa 1-kochegaralarini beradi. Birinchi kogomologik gruppa esa

$$H^1(G) = Der(G)/Inn(G)$$

kabi aniqlanadi.

1.1.7- ta'rif. Ixtiyoriy $x, y, z \in G$ uchun $\varphi : G \times G \to G$ akslantirish quyidagi shartni qanoatlantirsa,

$$\varphi(x, [y, z]) + \varphi(y, [z, x]) - \varphi(z, [x, y]) + [x, \varphi(y, z)] - [y, \varphi(z, x)] +$$

$$+ [z, \varphi(x, y)] = 0$$

φ - ikkinchi kosikl deyiladi va barcha ikkinchi kosikllar to'plami $\mathbb{Z}^2(G)$ kabi belgilanadi.

1.1.8- ta'rif. $f : G \times G \to G$ akslantirish uchun shunday $g : G \longrightarrow G$ topilib, ixtiyoriy $x, y \in G$ uchun quyidagi tenglik bajarilsa,

$$f(x, y) = g([x, y]) - [g(x), y] - [x, g(y)]$$

f - ikkinchi kochegara deyiladi va barcha ikkinchi kochegaralar to'plami $\mathbb{B}^2(G)$ kabi belgilanadi.

1.1.9- ta'rif. Ushbu ko'rinishda aniqlangan gruppa

$$H^2(G) = \mathbb{Z}^2(G)/\mathbb{B}^2(G)$$

ikkinchi kogomologik gruppa deyiladi.

$(L, [-, -])$-Leybnits algebrasining deformatsiyasi deb, $(L, [-, -])_t$-Leybnits algebrasining bir parametrli oilasiga aytiladi.

Bunda

$[-, -]_t = [-, -] + t\varphi_1 + t^2\varphi_2 + ...,$ qator orqali aniqlangan va φ_t-2-kozanjir bo'ladi. Ixtiyoriy t parametr uchun $[-, -]_t$ ko'paytma Leybnits ayniyatini bajarishi uchun

1. $[x,\varphi_1(y,z)]-[\varphi_1(x,y),z]+[\varphi_1(x,z),y]+\varphi_1(x,[y,z])-\varphi_1([x,y],z)+\varphi_1([x,z],y)=0$,

2. $[x,\varphi_n(y,z)]-[\varphi_n(x,y),z]+[\varphi_n(x,z),y]+\varphi_1(x,\varphi_{n-1}(y,z))-\varphi_1(\varphi_{n-1}(x,y),z)++\varphi_1(\varphi_{n-1}(x,z),y)+$
$...+\varphi_{n-1}(x,\varphi_1(y,z))-\varphi_{n-1}(\varphi_1(x,y),z)+\varphi_{n-1}(\varphi_1(x,z),y)+\varphi_n(x,[y,z])--\varphi_n([x,y],z)+$
$\varphi_n([x,z],y)=0$

bo'lishi zarur va yetarli.

Yuqoridagi tenglikdan ko'rinadiki φ_t-2-kosikl fazoda yotadi, ya'ni $\varphi_1\in ZL^2(L,L)$. Agar ayniyat aynan nol bo'lsa, u holda birinchi noldan farqli bo'lgan φ_i akslantirish $ZL^2(L,L)$ ga tegishli bo'ladi.

Ikkita L_t va L'_t deformatsiyalar mos ravishda ko'paytmalari μ_t,μ'_t lar yordamida aniqlangan bo'lsin. Agar shunday chiziqli $f_i=id+f_1t+f_2t^2+...$-L algebrada avtomorfizm mavjud bo'lib, bu yerda f_i-element $C^1(L,L)$ dan shunday olingan $\mu'_t(x,y)=f_t^{-1}(\mu_t(f_t(x),f_t(y)))$

o'rinli bo'lsa, u holda L_t va L'_t deformatsiyalar ekvivalent deyiladi.

Agar L_t va L'_t ekvivalent deformatsiyalar bo'lib, φ_t va φ'_t kozanjir bo'lsa , u holda $\varphi'_1-\varphi_1$ akslantirish $BL^2(L,L)$ ga tegishli bo'ladi, shuningdek deformatsiyalarni ekvivalent sinfi $HL^2(L,L)$ da yagona element aniqlaydi.

1.2-§. Kichik o'lchamli Li algebralarining tasnifi.

Ushbu paragrafda kichik o'lchamli Li algebralarining tasnifini keltiramiz.

Chiziqli Li algebralari. Agar $V-$ F maydon ustidagi chekli o'lchovli vektor fazo bo'lsa, unda $EndV$, $V \to V$ chiziqli almashtirishlar to'plamini anglatadi. To'plam F maydondagi vektor fazo sifatida n^2 o'lchamga ega $(dimV = n)$ va u ko'paytirish amaliga nisbatan halqa hisoblanadi. $EndV$, x, y elementlarning qavsi deb nomlangan $[x, y] = -[y, x]$ yangi amal bilan birgalikda Li algebrasi bo'ladi. aksioma ham o'rinli. Yangi algebraik strukturani oldingi assotsiativlikdan farqlash maqsadida $EndV$ fazoni $gl(V)$ kabi belgilab olamiz . Ushbu Li algebrasini to'la chiziqli algebra deb ataymiz. (Algebra V fazoning barcha teskarilanuvchi endomorfizmlaridan tashkil topganligi tufayli $GL(V)$ to'la chiziqli gruppa bilan uzviy bog'langan.)

Kelgusida cheksiz o'lchovli fazolar uchun ham $gl(V)$ belgilashni qo'llab ketamiz. $gl(V)$ dagi ixtiyoriy qism algebra **chiziqli Li algebrasi** deyiladi. Matritsa bilan ishlovchilar V fazodagi bazisni qayd qilib, barcha F maydondagi $n \times n$ matritsalar to'plamini $gl(V)$ bilan tenglashtirishlari mumkin va uni $gl(n, F)$ kabi belgilashlari mumkin. Keyingi ko'rsatmalar uchun standart bazisda $gl(n, F)$ algebra uchun e_{ij} matritsalardan tuzilgan ko'paytirish jadvalini yozib olamiz . $e_{ij}, e_{kl} = \delta_{jk}, e_{il}$ bo'lganligi sababli , $[e_{ij}, e_{kl}] = \delta_{jk}e_{il} - \delta_{li}e_{kj}$, $[e_{ij}, e_{kl}] = \delta_{jk}e_{il} - \delta_{li}e_{kj}$ ifodani olamiz. Ko'rinib turibdiki, barcha matritsa elementlari ± 1 yoki 0 ga teng, demak, ularning hammasi F maydonning sodda qism maydonida yotadi. Endi esa, boshqa muhim misollarni ko'rib chiqamiz. Ular 4 ta $A_l, B_l, C_l, D_l (l \geq 1)$ oilalarga mos keladi va ular klassik algebralar deyiladi. (chunki, ular klassik chiziqli Li gruppalariga mos keladi). $B_l - D_l$ $B_l - D_l$ ni misollarda $charF \neq 2$ deb hisoblaymiz.

A_l : A_l : $dimV = l+1$ bo'lsin. Nolga ega bo'lgan V fazoning endomorfizmlar to'plamini $sl(V)$ yoki V $sl(l+1, F)$ kabi belgilaymiz. (Esaltma: Matritsaning izi bu — uning barcha diagonal elementlari yig'indisi bo'lib V fazoning bazisiga bog'liq emas va shuning uchun u fazo endomorfizmi uchun aniqlangan). $Tr(xy) = Tr(yx)$ va $Tr(x + y) = Tr(x) + Tr(y)$ ekanligidan $sl(V)$ to'plam $gl(V)$ ga qism algebra bo'ladi. U holda bu algebra determinanti birga teng endomorfizmlardan tashkil topgan maxsus gruppa $SL(V)$ bilan bog'liqligi sababli , maxsus chiziqli gruppa bo'ladi. Endi uning o'lchami nechchiga teng degan savol tug'iladi. Bir tomondan $sl(V)$ - $gl(V)$ ning maxsus qism algebrasi bo'lganidan uning o'lchami $(l + 1)^2 - 1$ dan katta emas. Boshqa tomondan, nolga ega chiziqli erkli matritsalar sonini aniqlab ko'rsatish mumkin: barcha e_{ij} $(i \neq j)$ va $h_i = e_{ii} - e_{i+1,i+1}$ $(1 \leq i \leq l)$ larni olamiz: umuman olganda, qiyinchilik bilan $l + (l + 1)^2 - (l + 1)$ matritsani olamiz. Ushbu bazisni $sl(l + 1, F)$ standart ko'rinishdagi bazis sifatida qaraymiz. C_l $dimV = 2l$ va $(v_1, ..., v_{2l})$ bazis bo'lsin.

$s = \begin{pmatrix} 0 & l_l \\ -l_l & 0 \end{pmatrix}$ yordamida V fazoda kososimmetrik f formani aniqlaymiz. $f(v,\omega) = -f(\omega, v)$ shartni qanoatlantiruvchi keltirilmaydigan bichiziqli forma juft o'lchamda mavjud bo'lishini ko'rsatish mumkin.) $f(x(v),\omega) = -f(v,\omega(x))$ shartni qanoatlantiruvchi V fazodagi barcha barcha X endomorfizmlardan tashkil topgan simplektiv algebra $sp(V)$ yoki $sp(2l,F)$ kabi belgilanadi. $sp(V)$ to'plamni kommutatorga nisbatan yopiq ekanligini ko'rsatish qiyin emas.

$X = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$ $(m,n,p,q \in gl(l,F))$ uchun simplektivlik sharti matritsalar tilida $sx = -x^t s$ ko'rinishga ega (bu yerda, x^t - x ning transponerlangan matritsasi), ya'ni $n^t = n, p^t = p$ va $m^t = -q$ (Oxirgi tenglikdan $Tr(x) = 0$ kelib chiqadi). Endi $sp(2l,F)$ da bazis oson topiladi. l ta $e_{ii} - e_{l+i,l+i}$ $(1 \le i \le l)$ diagonal matritsalar olamiz. Ularga umumiy soni $l^2 - l$ ga teng bo'lgan barcha $e_{ij} - e_{l+j,l+i}$ $(1 \le i \ne j \le l)$ matritsalarni qo'shamiz. Qism matritsa n ga $e_{i,l+i}$ $(1 \le i \le l)$ va $e_{i,l+j} + e_{j,l+i}$ $(1 \le i \le j \le l)$ bazis elementlarini $l + \frac{1}{2}l(l-1)$ marta mos qo'yamiz. p soni uchun ham shu jarayon takrorlanadi. Barchasini yig'ib $dim(2l,F) = 2l^2 + l$ ni olamiz.

B_l: $dimV = 2l+1$, f esa V da $s = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & l_l \\ 0 & l_l & 0 \end{pmatrix}$ matritsali keltirilmaydigan simmetrik bichiziqli forma bo'lsin. $o(V)$ yoki $o(2l+1,F)$ ortogonal algebra V fazodagi barcha $f(x(v),\omega) = -f(v,\omega(x))$ shartni qanoatlantiruvchi endomorfizmlardan tashkil topgan. Agar x ni dagi kabi bloklarga ajratsak $x = \begin{pmatrix} a & b_1 & b_2 \\ c_1 & m & n \\ c_2 & p & q \end{pmatrix}$, unda $sx = -x^t s$ tenglik quyidagi shartlar yig'indisiga aylanadi: $a = 0$, $c_t = -b_2^t$, $c_2 = -b_1^t$, $q = -m^t$, $n^t = -n$, $p^t = -p$. (C_l xoldagi kabi bu yerdan $Tr(x) = 0$ ligi kelib chiqadi.) Bazis elementlari sifatida birinchidan , $e_{ii} - e_{l+i,l+i}$ $(2 \le i \le l+1)$ l-diogonal matritsalarini olamiz va birinchi qatori , birinchi ustuni nolga teng bo'lgan $e_{1,l+i+1} - e_{i+1,1}$ va $e_{1,i+1} - e_{l+i+1,1}$ $(1 \le i \le l)$ matritsalardan $2l$ tasini qo'shamiz. $q = -m^t$ qism matritsaga $e_{i+1,j+1} - e_{l+j+1,l+i+1}$, $(1 \le i \ne j \le l)$ matritsani , n qism matritsaga $e_{i+1,l+j+1} - e_{j+1,l+i+1}$ $(1 \le i < j \le l)$ matritsani, p qism matritsaga $e_{i+l+1,j+1} - e_{j+l+1,i+1}$ $(1 \le j < i \le l)$ matritsani mos qo'yamiz. Bazis elementlarning soni $2l^2 + l$ ga teng. (C_l kabi ishlangan.) D_l: Bunda biz boshqa ortogonal algebrani olamiz. Algebra xuddi B_l xoldagi kabi quriladi, faqat o'lchami $dimV = 2l$ juft, s esa soddaroq ko'rinishda bo'ladi $\begin{pmatrix} 0 & I_l \\ I_l & 0 \end{pmatrix}$. Xulosa qilib, $gl(n,F)$ da yana bir nechta qism algebralarni keltirib o'tamiz. $t(n,F)$ -

yuqori uchburchakli matritsalar to'plami (a_{ij}) $a_{ij} = 0$, $i > j$ va $n(n, F)$- qat'iy yuqori uchburchakli matritsalar $(a_{ij} = 0, j \leq i)$ to'plami bo'lsin. $\partial(n, F)$- esa, barcha diagonal matritsalar to'plami bo'lsin. Yuqoridagi har bir to'plam kommutatorga nisbatan yopiqligi trivial tekshiriladi. Ma'lumki, $t(n, F) = \partial(n, F) + n(n, F)$ (vektor fazolarning to'g'ri yig'indisi), xususan, $[\partial(n, F), n(n, F)] = n(n, F)$, demak, $[t(n, F), t(n, F)] = n(n, F)$. (Agar $H, K - L$ dagi qism algebra bo'lsa, unda $[H, K]$ - L da $[x, y], x \in H, y \in K$ kommutatorga tortilgan qism fazoni anglatadi).

Quyidagi teoremalarda uch va to'rt o'lchamli Li algebralarining tasnifi keltirilgan.

Teorema 1.2.1[1] Barcha uch o'lchamli kompleks Li algebralari quyidagi algebralardan biriga izomorf bo'ladi :

$G_0:$ $abel$

$G_1:$ $[e_1, e_2] = e_3;$

$G_2:$ $[e_1, e_2] = e_1;$

$G_3:$ $[e_1, e_2] = e_2,$ $[e_1, e_3] = e_2 + e_3;$

$G_4:$ $[e_1, e_2] = e_2,$ $[e_1, e_3] = \lambda e_3,$ $\lambda \in C^*, |\lambda| \leq 1;$

$G_5:$ $[e_1, e_2] = e_3,$ $[e_1, e_3] = -2e_1,$ $[e_2, e_3] = 2e_2$

Teoema 1.2.2[1] Barcha to'rt o'lchamli kompleks Li algebralari quyidagi algebralardan biriga izomorf bo'ladi :

$L_0:$ $abel$

$L_1:$ $[e_1, e_2] = e_3;$

$L_2:$ $[e_1, e_2] = e_1;$

$L_3:$ $[e_1, e_2] = e_2,$ $[e_1, e_3] = e_2 + e_3;$

$L_4:$ $[e_1, e_2] = e_2,$ $[e_1, e_3] = \lambda e_3,$ $\lambda \in C^*, |\lambda| \leq 1;$

$L_5:$ $[e_1, e_2] = e_1,$ $[e_3, e_4] = e_3;$

$L_6:$ $[e_1, e_2] = e_3,$ $[e_1, e_3] = -2e_1,$ $[e_2, e_3] = 2e_2;$

$L_7:$ $[e_1, e_2] = e_3,$ $[e_1, e_3] = e_4;$

$L_8:$ $[e_1, e_2] = e_2,$ $[e_1, e_3] = e_3,$ $[e_1, e_4] = \alpha e_4$ $\alpha \in C^*$

$L_9:$ $[e_1, e_2] = e_3,$ $[e_1, e_3] = e_4,$ $[e_1, e_4] = \alpha e_2 - \beta e_3 - e_4$

$\alpha \in C^*,$ $\beta \in C;$

$L_{10}:$ $[e_1, e_2] = e_3,$ $[e_1, e_3] = e_4,$ $[e_1, e_4] = \alpha(e_2 + e_3),$ $\alpha \in C^*;$

$L_{11}:$ $[e_1, e_2] = e_3,$ $[e_1, e_3] = e_4,$ $[e_1, e_4] = e_2;$

$L_{12}:$ $[e_1, e_2] = \frac{1}{3}e_2 + e_3,$ $[e_1, e_3] = \frac{1}{3}e_3,$ $[e_1, e_4] = \frac{1}{3}e_4;$

$L_{13}:$ $[e_1, e_2] = e_2,$ $[e_1, e_3] = e_3,$ $[e_1, e_4] = 2e_4,$ $[e_2, e_3] = e_4;$

$L_{14}: \ [e_1, e_2] = e_3, \ [e_1, e_3] = e_2, \ [e_2, e_3] = e_4;$

$L_{15}: \ [e_1, e_2] = e_3, \ [e_1, e_3] = -\alpha e_2 + e_3, [e_1, e_4] = e_4, [e_2, e_3] = e_4, \ \alpha \in C.$

Keyingi paragraflarda ushbu algebralarni o'rganamiz.

1.3-§. Li algebralarining differensiallashlari va ichki differensiallashlar.

Ba'zi chiziqli algebralar, algebralarni differensiallashda tabiiyroq yuzaga keladi. Algebrada differensiallash sifatida , oddiy ko'paytmaning differensiallash shartini qanoatlantiruvchi $d : G \to G$, chiziqli akslantirishni tushunamiz.

$$d(x,y) = d(x)y + xd(y)$$

Uch va to'rt o'lchamli Li algebralarining differensiallashlari haqidagi natijani keltiramiz:

Lemma 1.3.1 G_1, G_2, G_3 va G_5 uch o'lchamli Li algebralarning differensiallashlarining matritsaviy ko'rinishi quyidagicha bo'ladi:

$$Der(G_1) := \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ 0 & 0 & \alpha_1+\beta_2 \end{pmatrix}, Der(G_2) := \begin{pmatrix} \alpha_1 & 0 & 0 \\ \beta_1 & 0 & \beta_3 \\ 0 & 0 & \gamma_3 \end{pmatrix},$$

$$Der(G_3) := \begin{pmatrix} 0 & \alpha_2 & \alpha_3 \\ 0 & \gamma_3 & 0 \\ 0 & \gamma_2 & \gamma_3 \end{pmatrix}, \; Der(G_5) := \begin{pmatrix} \alpha_1 & 0 & \alpha_3 \\ 0 & \alpha_2 & 0 \\ 0 & -2\alpha & 0 \end{pmatrix}.$$

Isbot: Yuqoridagi G_1, G_2, G_3 va G_5 algebralarning differensialini 1.1.3-ta'rifdagi $d(x,y) = d(x)y + xd(y)$ formuladan foydalanib hisoblaymiz.

i. $G_1 := [e_1, e_2] = e_3$

$$d(e_1) = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3$$

$$d(e_2) = \beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3$$

$$d(e_3) = d[e_1,e_2] = [d(e_1)e_2] + [e_1 d(e_2)] = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3)e_2 + e_1(\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3) =$$

$= \alpha_1 e_3 + \beta_2 e_3 = (\alpha_1 + \beta_2)e_3 = 0$; $d(e_3) = (\alpha_1 + \beta_2)e_3$

$$Der(G_1) := \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ 0 & 0 & \alpha_1+\beta_2 \end{pmatrix}.$$

ii. $G_2 =: [e_1, e_2] = e_1$

$$d(e_1) = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3$$

$$d(e_2) = \beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3$$

$$d(e_3) = \gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3$$

$$d(e_1) = [d(e_1,e_2)] = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3)e_2 + e_1(\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3) = \alpha_1 e_1 + \beta_2 e_1 = (\alpha_1 + \beta_2)e_1$$

$$d(e_1,e_3) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3)e_3 + e_3(\gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3) = \gamma_2 e_1 = 0, \; \gamma_2 = 0$$

$d(e_2,e_3)=(\beta_1e_1+\beta_3e_3)e_3+e_2(\gamma_1e_1+\gamma_3e_3)=\gamma_1e_1=0\,,\ \gamma_1=0$

$d(e_1)=\alpha_1e_1\ \ d(e_2)=\beta_1e_1+\beta_3e_3\ \ d(e_3)=\gamma_3e_3\ \ Der(G_2):=\begin{pmatrix}\alpha_1 & 0 & 0\\ \beta_1 & 0 & \beta_3\\ 0 & 0 & \gamma_3\end{pmatrix}.$

iii $G_3:\ [e_1,e_2]=e_2\ \ ,[e_1,e_3]=e_2+e_3$

$d(e_1)=\alpha_1e_1+\alpha_2e_2+\alpha_3e_3\ \ d(e_2)=\beta_1e_1+\beta_2e_2+\beta_3e_3\ \ d(e_3)=\gamma_1e_1+\gamma_2e_2+\gamma_3e_3$

$d(e_1,e_2)=(\alpha_1e_1+\alpha_2e_2+\alpha_3e_3)e_2+(\beta_1e_1+\beta_2e_2+\beta_3e_3)e_1=\alpha_1e_2+\beta_2e_2+\beta_3(e_2+e_3)=$
$=(\alpha_1+\beta_2+\beta_3)e_2+\beta_3e_3$

$\beta_1=0\ \ \beta_2=\alpha_1+\beta_2+\beta_3\ \ -\alpha_1=\beta_3\ \ \alpha_1=-\beta_3$

$d(e_1,e_3)=(-\beta_3e_1+\alpha_2e_2+\alpha_3e_3)e_3+(\gamma_1e_1+\gamma_2e_2+\gamma_3e_3)e_1=-\beta_3(e_2+e_3)+\ \ \gamma_2e_2+\gamma_3(e_2+e_3)=(-\beta_3+\gamma_2+\gamma_3)e_2+(-\beta_3+\gamma_3)e_3$

$\beta_2e_2+\beta_3e_3+\gamma_1e_1+\gamma_2e_2+\gamma_3e_3=\gamma_1e_1+(\beta_2+\gamma_2)e_2+(\beta_3+\gamma_3)e_3\ \ \gamma_1=0\ \ \beta_2+\gamma_2=-\beta_3+\gamma_2+\gamma_3$
$\beta_2=\gamma_3-\beta_3\ \ \beta_2=\gamma_3$

$d(e_1)=\alpha_2e_2+\alpha_3e_3$

$d(e_2)=\gamma_3e_2$

$d(e_3)=\gamma_2e_2+\gamma_3e_3\ \ Der(G_3):=\begin{pmatrix}0 & \alpha_2 & \alpha_3\\ 0 & \gamma_3 & 0\\ 0 & \gamma_2 & \gamma_3\end{pmatrix}.$

iv. $G_5:[e_1,e_2]=e_3\,,[e_1,e_3]=-2e_1\,,[e_2,e_3]=2e_2$

$d(e_1)=\alpha_1e_1+\alpha_2e_2+\alpha_3e_3\ \ d(e_2)=\beta_1e_1+\beta_2e_2+\beta_3e_3\ \ d(e_3)=\gamma_1e_1+\gamma_2e_2+\gamma_3e_3$

$d(e_1,e_2)=(\alpha_1e_1+\alpha_2e_2+\alpha_3e_3)e_2+(\beta_1e_1+\beta_2e_2+\beta_3e_3)e_1=\alpha_1e_3-2\alpha_3e_2+\beta_2e_3-2\beta_3e_1=$
$=(\alpha_1+\beta_2)e_3-2\alpha_3e_2-2\beta_3e_1$

$\gamma_1=-2\beta_3\ \ \gamma_2=-2\alpha_3\ \ \gamma_3=\alpha_1+\beta_2$

$d(e_1,e_3)=(\alpha_1e_1+\alpha_2e_2+\alpha_3e_3)e_3+(\gamma_1e_1+\gamma_2e_2+\gamma_3e_3)e_1=-2\alpha_1e_1+2\alpha_2e_2-2\alpha_3e_3$

$d(e_2,e_3)=(\beta_1e_1+\beta_2e_2+\beta_3e_3)e_3+(-2\beta_3e_1-2\alpha_3e_2)=-2\beta_1e_1+2\beta_2e_2+\beta_3e_3+(-2\beta_3e_1-2\alpha_3e_2)=-2\beta_1e_1+2\beta_2e_2-2\beta_3e_3$

$2\beta_1=-2\beta_1\ \ 2\beta_2=2\beta_2\ \ 2\beta_3=-2\beta_3$

$d(e_1)=\alpha_1e_1+\alpha_3e_3\ \ d(e_2)=\beta_2e_2\ \ d(e_3)=-2\alpha_3e_2$

$Der(G_5):=\begin{pmatrix}\alpha_1 & 0 & \alpha_3\\ 0 & \alpha_2 & 0\\ 0 & -2\alpha & 0\end{pmatrix}.$

Lemma 1.3.2 L_1, L_3, L_{11} va L_{13} to'rt o'lchamli Li algebralarning differensiallashlarining matritsaviy ko'rinishi quyidagicha bo'ladi:

$$Der(L_1) := \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ 0 & 0 & \alpha_1+\beta_2 & 0 \\ 0 & 0 & \beta_3 & \beta_4 \end{pmatrix}, \ Der(L_3) := \begin{pmatrix} 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \beta_2 & 0 & \beta_4 \\ 0 & \gamma_2 & \beta_2 & \gamma_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{pmatrix},$$

$$Der(L_11) := \begin{pmatrix} 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \beta_2 & \beta_3 & \beta_4 \\ 0 & \beta_4 & \beta_2 & \beta_4 \\ 0 & \beta_4 & \beta_4 & \beta_2 \end{pmatrix}, \ Der(L_13) := \begin{pmatrix} 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \beta_2 & \beta_3 & \alpha_3 \\ 0 & 0 & 0 & -\alpha_2 \\ 0 & 0 & 0 & \beta_2+\gamma_3 \end{pmatrix}.$$

Isbot: Yuqoridagi L_1 , L_3 , L_{11} va L_{13} algebralarning differensialini 1.1.3-ta'rifdagi $d(x,y) = d(x)y + xd(y)$ formuladan foydalanib hisoblaymiz.

i. L_1 :

$[e_1, e_2] = e_3$

$d(e_1) = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4$

$d(e_2) = \beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4$

$d(e_3) = \gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3 + \gamma_4 e_4$

$d(e_4) = \delta_1 e_1 + \delta_2 e_2 + \delta_3 e_3 + \delta_4 e_4$

$d(e_1, e_2) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_2 + (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4)e_1 = \alpha_1 e_3 + \beta_2 e_3 = (\alpha_1 + \beta_2)e_3$

$\gamma_1 = 0 \quad \gamma_2 = 0 \quad \gamma_3 = \alpha_1 + \beta_2 \quad \gamma_4 = 0$

$d(e_1, e_4) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_4 + (\delta_1 e_1 + \delta_2 e_2 + \delta_3 e_3 + \delta_4 e_4)e_1 = \delta_2 e_3 = 0$

$= \delta_2 = 0$

$d(e_2, e_3) = 0$

$d(e_2, e_4) = (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4)e_4 + (\delta_1 e_1 + \delta_2 e_2 + \delta_3 e_3 + \delta_4 e_4)e_2 = \delta_1 e_3 = 0$

$\delta_1 = 0$

$d(e_1) = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4$

$d(e_2) = \beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4$

$d(e_3) = (\alpha_1 + \beta_2)e_3$

$d(e_4) = \delta_3 e_3 + \delta_4 e_4$

$$Der(L_1) := \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ 0 & 0 & \alpha_1+\beta_2 & 0 \\ 0 & 0 & \beta_3 & \beta_4 \end{pmatrix}, \ Der(L_1) = 10\,.$$

ii.

$L_3: \ [e_1, e_1] = e_2, \quad [e_1, e_3] = e_2 + e_3$

$d(e_1, e_2) = d(e_1)e_2 + e_1 d(e_2) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_2 + (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4)e_1 = \alpha_1 e_2 + \beta_2 e_2 + \beta_3(e_2 + e_3) = (\alpha_1 + \beta_2 + \beta_3)e_2 + \beta_3 e_3$

$\beta_1 = 0, \quad \beta_2 = \alpha_1 + \beta_2 + \beta_3, \quad \alpha_1 = -\beta_3 \quad \beta_3 = \beta_3$

$d(e_1, e_3) = (-\beta_3 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_3 + (\gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3 + \gamma_4 e_4) = -\beta_3(e_2 + e_3) + \gamma_2 e_2 + \gamma_3(e_2 + e_3) = (-\beta_3 + \gamma_3 + \gamma_2)e_2 + (-\beta_3 + \gamma_3)e_3$

$\beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4 + \gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3 + \gamma_4 e_4 = \gamma_1 e_1 + (\beta_2 + \gamma_2)e_2 + (\beta_3 + \gamma_3)e_3 + (\beta_4 + \gamma_4)e_4$

$\gamma_1 = 0, \quad \beta_2 + \gamma_2 = -\beta_3 + \gamma_3 + \gamma_2, \quad \gamma_3 = \beta_2 + \beta_3 \quad \gamma_3 \beta_2$

$d(e_1, e_4) = (-\beta_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_4 + (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4)e_1 = \beta_2 e_2 + \beta_3 e_2 + \beta_3 e_3 = (\beta_2 + \beta_3)e_2 + \beta_3 e_3 = 0$

$d(e_2, e_3) = 0$

$d(e_1) = -\beta_3 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4$

$d(e_2) = \beta_3 e_2 + \beta_4 e_4$

$d(e_3) = \gamma_2 e_2 + \beta_3 e_3 + \gamma_4 e_4$

$d(e_4) = \delta_1 e_1 + \delta_2 e_2 + \delta_3 e_3 + \delta_4 e_4$

$$\begin{pmatrix} 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \beta_2 & 0 & \beta_4 \\ 0 & \gamma_2 & \beta_2 & \gamma_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{pmatrix}, Der(L_3) = 10\,.$$

iii. $L_{11}: \quad [e_1, e_2] = e_3\,, \ [e_1, e_3] = e_4\,, \ [e_1, e_4] = e_2\,.$

$d(e_1, e_2) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_2 + (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4)e_1 = \alpha_1 e_3 + \beta_2 e_3 + \beta_3 e_4 + \beta_4 e_2 = \beta_4 e_2 + (\alpha_1 + \beta_2)e_3 + \beta_4 e_4$

$\gamma_1 = 0 \quad \gamma_2 = \beta_4 \quad \gamma_3 = \alpha_1 + \beta_2 \quad \gamma_4 = \beta_4$

$d(e_1, e_3) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_3 + (\beta_4 e_2 + (\alpha_1 + \beta_2)e_3 + \beta_4 e_4)e_1 = \alpha_1 e_4 + \beta_4 e_3 + (\alpha_1 + \beta_2)e_4 + \beta_4 e_2 = \beta_4 e_2 + \beta_4 e_3 + (2\alpha_1 + \beta_2)e_4 + \beta_4 e_4$

$\delta_1 = 0 \quad \delta_2 = \beta_4 \quad \delta_3 = \beta_4 \quad \delta_4 = 2\alpha_1 + \beta_2$

$d(e_1, e_4) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_4 + (\beta_4 e_2 + \beta_4 e_3 + (2\alpha_1 + \beta_2)e_4)e_1 = \alpha_1 e_2 + \beta_4 e_3 + \beta_4 e_4 + (2\alpha_1 + \beta_2)e_2 = (3\alpha_1 + \beta_2)e_2 + \beta_4 e_3 + \beta_4 e_4$

$\beta_1 = 0 \quad \beta_2 = 3\alpha_1 + \beta_2 \quad \beta_3 = \beta_4 \quad \alpha_1 = 0$

$d(e_1) = \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4$

$d(e_2) = \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4$

$d(e_3) = \beta_4 e_2 + \beta_2 e_3 + \beta_4 e_4$

$d(e_4) = \beta_4 e_2 + \beta_4 e_3 + \beta_2 e_4$

$$Der(L_{11}) = \begin{pmatrix} 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \beta_2 & \beta_3 & \beta_4 \\ 0 & \beta_4 & \beta_2 & \beta_4 \\ 0 & \beta_4 & \beta_4 & \beta_2 \end{pmatrix}, \; Der(L_{11}) = 6\,.$$

iv. $L_{13} : [e_1, e_2] = e_2\,, [e_1, e_3] = e_3\,, [e_1, e_4] = 2e_4\,, [e_2, e_3] = e_4$

$d(e_1, e_2) = (\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_2 + (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4) = \alpha_1 e_2 - \alpha_3 e_4 + \beta_2 e_2 + \beta_3 e_3 + 2\beta_4 e_4 = (\alpha_1 + \beta_2)e_2 + \beta_3 e_3 + (2\beta_4 - \alpha_3)e_4$

$\beta_1 = 0 \quad \beta_2 = \alpha_1 + \beta_2 \quad \alpha_1 = 0 \quad \beta_4 = 2\beta_4 - \alpha_3$

$d(e_1, e_3) = (\alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_4 + (\gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3 + \gamma_4 e_4)e_1 = \alpha_2 e_4 + \gamma_2 e_2 + \gamma_3 e_3 + 2\gamma_4 e_4 = \gamma_2 e_2 + \gamma_3 e_3 + (\alpha_2 + 2\gamma_4)e_4 \quad \gamma_1 = 0 \quad \gamma_4 = \alpha_2 + 2\gamma_4 \quad \gamma_4 = -\alpha_2$

$d(e_1, e_4) = (\alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4)e_4 + (\beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4)e_1 = \beta_2 e_2 + \beta_3 e_3 + 2\beta_4 e_4$

$\beta_1 = 0\,, \beta_2 = 0 \;\;, \beta_3 = 0$

$d(e_2, e_3) = (\beta_2 e_2 + \beta_3 e_3 + \alpha_3 e_4)e_3 + (\gamma_2 e_2 + \gamma_3 e_3 - \alpha_2 e_4)e_2 = \beta_2 e_4 + \gamma_3 e_4 = (\beta_2 + \gamma_3)e_4$

$\beta_4 = \beta_2 + \gamma_3$

$d(e_1) = \alpha_2 e_2 + \alpha_3 e_3 + \alpha_4 e_4$

$d(e_2) = \beta_2 e_2 + \beta_3 e_3 + \alpha_3 e_4$

$d(e_3) = -\alpha_2 e_4$

$d(e_4) = (\beta_2 + \gamma_3)e_4$

$$Der(L_{13}) = \begin{pmatrix} 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \beta_2 & \beta_3 & \alpha_3 \\ 0 & 0 & 0 & -\alpha_2 \\ 0 & 0 & 0 & \beta_2 + \gamma_3 \end{pmatrix}, \; Der(L_{13}) = 6\,.$$

Endi ichki differensial ta'rifdan foydalanib, G_1, G_2, G_3 va G_5 algebralarning ichki differensiallarini topamiz: $ad_x = R_x : L \to L\,,\; ad_x(y) = [y, x]$

$$ad_x([y, z]) = [[y, z], x] = -[[x, y], z] - [[z, x], y] = [[y, x], z] + [y, [z, x] =$$

$= [ad_x(y), z] + [y, ad_x(z)]\,,\; ad_x$ - differensiali.

$$InnD(L) = \{ad_x | x \in L\}$$

i $G_1 : [e_1, e_2] = e_3$

$ad_{e_1}(e_1) = 0\,,\; ad_{e_1}(e_2) = -e_3\,, ad_{e_1}(e_3) = 0$

$ad_{e_2}(e_1) = e_3\,,\; ad_{e_2}(e_2) = 0\,,\; ad_{e_2}(e_3) = 0$

$ad_{e_3}(e_1) = 0\,,\; ad_{e_3}(e_2) = 0\,,\; ad_{e_3}(e_3) = 0$

$$E_1=\begin{pmatrix}0&0&0\\0&0&-1\\0&0&0\end{pmatrix},\ E_2=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix},\ E_3=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\ -\alpha_1E_1+\alpha_2E_2=\begin{pmatrix}0&0&\alpha_2\\0&0&\alpha_1\\0&0&0\end{pmatrix}$$

,

$$\begin{pmatrix}\alpha_1&\alpha_2&\alpha_3\\\beta_1&\beta_2&\beta_3\\0&0&\alpha_1+\beta_2\end{pmatrix}\supset\begin{pmatrix}0&0&\alpha_2\\0&0&\alpha_1\\0&0&0\end{pmatrix},\ Inn(G_1)=2\,.$$

ii. $G_2:[e_1,e_2]=e_1$

$ad_{e_1}(e_1)=0\,,\ ad_{e_1}(e_2)=-e_1\,,ad_{e_1}(e_3)=0$

$ad_{e_2}(e_1)=e_1\,,\ ad_{e_2}(e_2)=0\,,\ ad_{e_2}(e_3)=0$

$ad_{e_3}(e_1)=0\,,\ ad_{e_3}(e_2)=0\,,\ ad_{e_3}(e_3)=0$

$$E_1=\begin{pmatrix}0&0&0\\-1&0&0\\0&0&0\end{pmatrix},\ E_2=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},\ E_3=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix},\ -\alpha_1E_1+\alpha_2E_2=\begin{pmatrix}\alpha_2&0&0\\\alpha_1&0&0\\0&0&0\end{pmatrix}$$

,

$$\begin{pmatrix}\alpha_1&0&0\\\beta_1&0&\beta_3\\0&0&\gamma_3\end{pmatrix}\supset\begin{pmatrix}\alpha_2&0&0\\\alpha_1&0&0\\0&0&0\end{pmatrix}\ ,Inn(G_2)=2\,.$$

iii. $G_3:[e_1,e_2]=e_2,[e_1,e_3]=e_2+e_3$

$ad_{e_1}(e_1)=0\,,\ ad_{e_1}(e_2)=-e_2\,,\ ad_{e_1}(e_3)=-e_2-e_3\,,$

$ad_{e_2}(e_1)=e_2\,,\ ad_{e_2}(e_2)=0\,,\ ad_{e_2}(e_3)=0\,,$

$ad_{e_3}(e_1)=e_2+e_3\,,\ ad_{e_3}(e_2)=0\,,\ ad_{e_3}(e_3)=0\,,$

$$E_1=\begin{pmatrix}0&0&0\\0&-\lambda_1&0\\0&0&-\lambda_2-\lambda_3\end{pmatrix}\quad E_2=\begin{pmatrix}0&0&0\\\lambda_2&0&0\\0&0&0\end{pmatrix}\quad E_2=\begin{pmatrix}\lambda_2+\lambda_3&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

$$\begin{pmatrix}0&\alpha_2&\alpha_3\\0&\gamma_3&0\\0&\gamma_2&\gamma_3\end{pmatrix}\supset\begin{pmatrix}\lambda_2+\lambda_3&0&0\\\lambda_2&\lambda_1&0\\0&0&\lambda_2+\lambda_3\end{pmatrix},\ Inn(G_3)=3\,.$$

iv. $G_5:[e_1,e_2]=e_3,[e_1,e_3]=-2e_1,[e_2,e_3]=2e_2$

$ad_{e_1}(e_1)=0\,,\ ad_{e_1}(e_2)=-e_3\,,\ ad_{e_1}(e_3)=2e_1\,,$

$ad_{e_2}(e_1)=e_3\,,\ ad_{e_2}(e_2)=0\,,\ ad_{e_2}(e_3)=-2e_2\,,$

$ad_{e_3}(e_1)=-2e_1\,,\ ad_{e_3}(e_2)=2e_2\,,\ ad_{e_3}(e_3)=0$

$$E_1=\begin{pmatrix}0&0&0\\0&0&-1\\2&0&0\end{pmatrix}\quad E_2=\begin{pmatrix}0&0&1\\0&0&0\\0&-2&0\end{pmatrix}\quad E_3=\begin{pmatrix}-2&0&0\\0&2&0\\0&0&0\end{pmatrix}$$

$$-\lambda_1 E_1 - \lambda_2 E_2 - \lambda_3 E_3 = \begin{pmatrix} 2\lambda_3 & 0 & \lambda_2 \\ 0 & 2\lambda_3 & 0 \\ 2\lambda_1 & 2\lambda_2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_1 & 0 & \alpha_3 \\ 0 & \alpha_2 & 0 \\ 0 & -2\alpha_3 & 0 \end{pmatrix} \supset \begin{pmatrix} 2\lambda_3 & 0 & \lambda_2 \\ 0 & 2\lambda_3 & 0 \\ 2\lambda_1 & 2\lambda_2 & 0 \end{pmatrix}, \; Inn(G_5) = 3\,.$$

Xuddi shunga o'xshash L_1, L_3, L_{11} va L_{13} to'rt o'lchovli algebralarning ham ichki differensiallarini topamiz:

i. $L_1 : [e_1, e_2] = e_3$

$ad_{e_1}(e_1) = 0\,,\; ad_{e_1}(e_2) = -e_3\,, ad_{e_1}(e_3) = 0\,, ad_{e_1}(e_4) = 0\,,$

$ad_{e_2}(e_1) = e_3\,, ad_{e_2}(e_2) = 0\,, ad_{e_2}(e_3) = 0\,, ad_{e_2}(e_4) = 0\,,$

$ad_{e_3}(e_1) = 0\,, ad_{e_3}(e_2) = 0\,, ad_{e_3}(e_3) = 0\,, ad_{e_3}(e_4) = 0\,,$

$ad_{e_4}(e_1) = 0\,, ad_{e_4}(e_2) = 0\,, ad_{e_4}(e_3) = 0\,, ad_{e_4}(e_4) = 0\,,$

$$E_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; E_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; E_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; E_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$-\lambda_1 E_1 + \lambda_2 E_2 = \begin{pmatrix} 0 & \lambda_2 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ 0 & 0 & \alpha_1 + \beta_2 & 0 \\ 0 & 0 & \beta_3 & \beta_4 \end{pmatrix} \supset \begin{pmatrix} 0 & \lambda_2 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$Inn(L_1) = 2\,.$

ii. $L_3 : [e_1, e_2] = e_2, [e_1, e_3] = e_2 + e_3$

$ade_1(e_1) = 0,\;\; ade_1(e_2) = -e_2,\;\; ade_1(e_3) = -e_2 - e_3,\;\; ade_1(e_4) = 0$

$ade_2(e_1) = e_2,\;\; ade_2(e_2) = 0,\;\; ade_2(e_3) = 0,\;\; ade_2(e_4) = 0$

$ade_3(e_1) = e_2 + e_3,\;\; ade_3(e_2) = 0,\;\; ade_3(e_3) = 0,\;\; ade_3(e_4) = 0$

$ade_4(e_1) = 0,\;\; ade_4(e_2) = 0,\;\; ade_4(e_3) = 0,\;\; ade_4(e_4) = 0$

$$E_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad E_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad E_3 = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad E_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$-\lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3 = \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 & 0 \\ 0 & -\lambda_1 & 0 & 0 \\ 0 & -\lambda_2 & -\lambda_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 & 0 \\ 0 & -\lambda_1 & 0 & 0 \\ 0 & -\lambda_2 & -\lambda_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \subset \begin{pmatrix} 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \beta_2 & 0 & \beta_4 \\ 0 & \gamma_2 & \beta_2 & \gamma_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{pmatrix}, Inn(L_3) = 3$$

$L_{11} : [e_1, e_2] = e_3, [e_1, e_3] = e_4, [e_1, e_4] = e_2$

$ad_{e_1}(e_1) = 0, ad_{e_1}(e_2) = -e_3, ad_{e_1}(e_3) = -e_4, ad_{e_1}(e_4) = -e_2\,,$

$ad_{e_2}(e_1) = e_3, ad_{e_2}(e_2) = 0, ad_{e_2}(e_3) = 0, ad_{e_2}(e_4) = 0\,,$

$ad_{e_3}(e_1) = e_4, ad_{e_3}(e_2) = 0, ad_{e_3}(e_3) = 0, ad_{e_3}(e_4) = 0\,,$

$ad_{e_4}(e_1) = e_2, ad_{e_4}(e_2) = 0, ad_{e_4}(e_3) = 0, ad_{e_4}(e_4) = 0\,.$

$$E_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \end{pmatrix},\ E_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},\ E_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},\ E_4 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$-\lambda_1 E_1 + \lambda_2 E_2 + \lambda_3 E_3 + \lambda_4 E_4 = \begin{pmatrix} 0 & \lambda_4 & \lambda_2 & \lambda_3 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \\ 0 & \lambda_1 & 0 & 0 \end{pmatrix}$$

$$,\ \begin{pmatrix} 0 & \lambda_4 & \lambda_2 & \lambda_3 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \\ 0 & \lambda_1 & 0 & 0 \end{pmatrix} \subset \begin{pmatrix} 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \beta_2 & \beta_3 & \beta_4 \\ 0 & \beta_4 & \beta_2 & \beta_4 \\ 0 & \beta_4 & \beta_4 & \beta_2 \end{pmatrix}$$

iv. $L_{13} : [e_1, e_2] = e_2, [e_1, e_3] = e_3, [e_1, e_4] = 2e_4, [e_2, e_3] = e_4$

$ad_{e_1}(e_1) = 0, ad_{e_1}(e_2) = -e_2, ad_{e_1}(e_3) = -e_3, ad_{e_1}(e_4) = -2e_4$

$ad_{e_2}(e_1) = e_2, ad_{e_2}(e_2) = 0, ad_{e_2}(e_3) = -e_4, ad_{e_2}(e_4) = 0$

$ad_{e_3}(e_1) = e_3, ad_{e_3}(e_2) = e_4, ad_{e_3}(e_3) = 0, ad_{e_3}(e_4) = 0$

$ad_{e_4}(e_1) = 2e_4, ad_{e_4}(e_2) = 0, ad_{e_4}(e_3) = 0, ad_{e_4}(e_4) = 0$

$$E_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}, \; E_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; E_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; E_4 = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$-\lambda_1 E_1 - \lambda_2 E_2 + \lambda_3 E_3 + \lambda_4 E_4 = \begin{pmatrix} 0 & \lambda_2 & \lambda_3 & 2\lambda_4 \\ 0 & \lambda_1 & 0 & \lambda_3 \\ 0 & 0 & \lambda_1 & \lambda_2 \\ 0 & 0 & 0 & 2\lambda_1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \lambda_2 & \lambda_3 & 2\lambda_4 \\ 0 & \lambda_1 & 0 & \lambda_3 \\ 0 & 0 & \lambda_1 & \lambda_2 \\ 0 & 0 & 0 & 2\lambda_1 \end{pmatrix} \subset \begin{pmatrix} 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \beta_2 & 0 & \beta_4 \\ 0 & \gamma_2 & \beta_2 & \gamma_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{pmatrix}, Inn(L_{13}) = 4\,.$$

II bob. Li algebralarining kogomologik guruppalari

2.1-§. Uch o'lchamli Li algebralarining birinchi va ikkinchi kogomologik guruppalari.

Ushbu paragrafda uch o'lchamli Li algebralarining birinchi va ikkinchi kogomologik guruppalari haqidagi natijalarni bayon qilamiz.

Teorema 2.1.1. Uch o'lchamli Li algebralarining birinchi kogomologik gruppalarining o'lchami quyidagicha bo'ladi:

$$DimH^1(G_1) = 4,\ DimH^1(G_2) = 2,\ DimH^1(G_3) = 1,\ DimH^1(G_5) = 0.$$

Isbot: Teoremani isbotlash uchun yuqoridagi keltirilgan algebralarning differensiallashlari va ichki differensiallashlaridagi natijalarni olamiz. Bizga 1.1.4-dagi ta'rifdan ma'lumki, $H^1(G) = Der(G)/Inn(G)$.

i. G_1 algebra uchun yuqorida keltirilgan differensiali $Der(G_1) = 6$ ga teng, ichki differensiali esa, $Inn(G_1) = 2$ ga teng. Bundan kelib chiqadiki,

$H^1(G_1) = Der(G_1)/Inn(G_1) = 4$.

Demak, $DimH^1(G_1) = 4$.

ii. G_2 algebra uchun ham yuqoridagilar o'rinli bo'lib, unda $Der(G_2) = 4$ ga teng,ichki differensiali esa, $Inn(G_2) = 2$ ga teng. Bundan kelib chiqadi,

$H^1(G_2) = Der(G_2)/Inn(G_2) = 2$.

Demak, $DimH^1(G_2) = 2$.

iii. G_3 algebrada differensiali $Der(G_3) = 4$, ichki differensiali $Inn(G_3) = 1$. Bundan kelib chiqadi, $H^1(G_3) = Der(G_3)/Inn(G_3) = 1$.

Demak, $DimH^1(G_3) = 2$.

iv. G_5 algebrada differensiali $Der(G_5) = 3$, ichki differensiali $Inn(G_5) = 3$. Ya'ni $Der(G_5) = Inn(G_5)$ ga teng. Bundan kelib chiqadi,

$H^1(G_5) = Der(G_5)/Inn(G_5) = 0$. Demak, $DimH^1(G_5) = 0$.

Isboti tugadi.

Endi uch o'lchamli Li algebralarning ikkinchi gruppa kogomologiyasi to'g'risidagi natijani keltiramiz:

Teorema 2.1.2. Uch o'lchamli Li algebralarining ikkinchi kogomologik gruppalarining o'lchami quyidagicha bo'ladi:

$$DimH^2(G_1) = 10,\ DimH^2(G_2) = 5,\ DimH^2(G_3) = 3,\ DimH^2(G_5) = 0.$$

Isbot. Bizga ma'lumki,1.1.7- ta'rifga ko'ra ikkinchi kogomologik gruppa

$H^2(G) = Z^2(G)/B^2(G)$ ga teng. Endi uch o'lchamli algebralarning $Z^2(G)$ larini hisoblaymiz. Buning uchun quyidagi formuladan foydalanamiz.

$\varphi(e_i, e_j) = \sum\limits_{t=1}^{n} \delta_{i,j}^t e_t$

$$[e_i, \varphi(e_j, e_k)] - [e_j, \varphi(e_i, e_k)] + [e_k, \varphi(e_i, e_j)] - \varphi([e_i, e_j], e_k) + \varphi([e_i, e_k], e_j) - \varphi([e_j, e_k], e_i) = 0$$

$\varphi(e_1, e_1) = \delta_{1,1}^1 e_1 + \delta_{1,1}^2 e_2 + \delta_{1,1}^3 e_3$

$\varphi(e_1, e_2) = \delta_{1,2}^1 e_1 + \delta_{1,2}^2 e_2 + \delta_{1,2}^3 e_3$

$\varphi(e_1, e_3) = \delta_{1,3}^1 e_1 + \delta_{1,3}^2 e_2 + \delta_{1,3}^3 e_3$

$\varphi(e_2, e_1) = \delta_{2,1}^1 e_1 + \delta_{2,1}^2 e_2 + \delta_{2,1}^3 e_3$

$\varphi(e_2, e_2) = \delta_{2,2}^1 e_1 + \delta_{2,2}^2 e_2 + \delta_{2,2}^3 e_3$

$\varphi(e_2, e_3) = \delta_{2,3}^1 e_1 + \delta_{2,3}^2 e_2 + \delta_{2,3}^3 e_3$

$\varphi(e_3, e_1) = \delta_{3,1}^1 e_1 + \delta_{3,1}^2 e_2 + \delta_{3,1}^3 e_3$

$\varphi(e_3, e_2) = \delta_{3,2}^1 e_1 + \delta_{3,2}^2 e_2 + \delta_{3,2}^3 e_3$

$\varphi(e_3, e_3) = \delta_{3,3}^1 e_1 + \delta_{3,3}^2 e_2 + \delta_{3,3}^3 e_3$

i. $G_1 : [e_1, e_2] = e_3$

1) $e_1, e_1, e_1 :$

$[e_1, \varphi(e_1, e_1)] - [e_1, \varphi(e_1, e_1)] + [e_1, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_1) + \varphi([e_1, e_1], e_1) - \varphi([e_1, e_1], e_1)$
$= [e_1, \delta_{1,1}^1 e_1 + \delta_{1,1}^2 e_2 + \delta_{1,1}^3 e_3] = \delta_{1,1}^2 e_3 = 0 \ \Rightarrow \ \delta_{1,1}^2 = 0$

2) $e_1, e_1, e_2 :$

$[e_1, \varphi(e_1, e_2)] - [e_1, \varphi(e_1, e_2)] + [e_2, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_2) + \varphi([e_1, e_2], e_1) - \varphi([e_1, e_2], e_1 =$

$= [e_2, \delta_{1,1}^1 e_1 + \delta_{1,1}^3 e_3] = -\delta_{1,1}^1 e_3 = 0 \Rightarrow \delta_{1,1}^1 = 0$

3) $e_1, e_1, e_3 :$

$[e_1, \varphi(e_1, e_3)] - [e_1, \varphi(e_1, e_3)] + [e_3, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_3) + \varphi([e_1, e_3], e_1) - \varphi([e_1, e_3], e_1) =$
0

4) $e_1, e_2, e_1 :$

$[e_1, \varphi(e_2, e_1)] - [e_2, \varphi(e_1, e_1)] + [e_1, \varphi(e_1, e_2)] - \varphi([e_1, e_2], e_1) + \varphi([e_1, e_1], e_2) - \varphi([e_2, e_1], e_1) =$
$= [e_1, \delta_{2,1}^1 e_1 + \delta_{2,1}^2 e_2 + \delta_{2,1}^3 e_3] - [e_2, \delta_{1,1}^3 e_3] + [e_1, \delta_{1,2}^1 e_1 + \delta_{1,2}^2 e_2 + \delta_{1,2}^3 e_3] - \varphi(e_3, e_1) + \varphi(e_3, e_1) =$
$= \delta_{2,1}^2 e_3 + \delta_{1,2}^2 e_3 = (\delta_{2,1}^2 + \delta_{1,2}^2) e_3 = 0 \ \Rightarrow \ \delta_{2,1}^2 = -\delta_{1,2}^2$

5) $e_1, e_2, e_2 :$

$[e_1, \varphi(e_2, e_2)] - [e_2, \varphi(e_1, e_2)] + [e_2, \varphi(e_1, e_2)] - \varphi([e_1, e_2], e_2) + \varphi([e_1, e_2], e_2) - \varphi([e_2, e_2], e_1) =$
$= [e_1, \delta_{2,2}^1 e_1 + \delta_{2,2}^2 e_2 + \delta_{2,2}^3 e_3] = \delta_{2,2}^2 e_3 = 0 \ \Rightarrow \ \delta_{2,2}^2 = 0$

6) $e_1, e_2, e_3 :$

$[e_1,\varphi(e_2,e_3)]-[e_2,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_3)+\varphi([e_1,e_3],e_2)-\varphi([e_2,e_3],e_1)=$
$=\delta^2_{2,3}e_3+\delta^1_{1,3}e_3-\delta^1_{3,3}e_1-\delta^2_{3,3}e_2-\delta^3_{3,3}e_3=-\delta^1_{3,3}e_1-\delta^2_{3,3}e_2+(\delta^2_{2,3}+\delta^2_{1,3}-\delta^3_{3,3})e_3=0\ \Rightarrow \delta^1_{3,3}=0,\delta^2_{3,3}=0,\delta^3_{3,3}=\delta^2_{2,3}+\delta^2_{1,3}$

7) $e_1,e_3,e_1:$

$[e_1,\varphi(e_3,e_1)]-[e_3,\varphi(e_1,e_1)]+[e_1,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_1)+\varphi([e_1,e_1],e_3)-\varphi([e_3,e_1],e_1)=$
$=[e_1,\delta^1_{3,1}e_1+\delta^2_{3,1}e_2+\delta^3_{3,1}e_3]+[e_1,\delta^1_{1,3}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3]=\delta^2_{3,1}e_3+\delta^2_{1,3}e_3=(\delta^2_{3,1}+\delta^2_{1,3})e_3=0$
$\Rightarrow\ \delta^2_{3,1}=-\delta^2_{1,3}$

8) $e_1,e_3,e_2:$

$[e_1,\varphi(e_3,e_2)]-[e_3,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_2)+\varphi([e_1,e_2],e_3)-\varphi([e_3,e_2],e_1)=$
$=[e_1,\delta^1_{3,2}e_1+\delta^2_{3,2}e_2+\delta^2_{3,2}e_3]=\delta^2_{3,2}e_3=0\ \Rightarrow\ \delta^2_{3,2}=0$

9) $e_1,e_3,e_3:$

$[e_1,\varphi(e_3,e_3)]-[e_3,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_3)]-\varphi\left([e_1,e_3],e_3\right)+\varphi\left([e_1,e_3],e_3\right)-\varphi\left([e_3,e_3],e_1\right)=$
$=\left[e_1,\delta^1_{3,3}e_1+\delta^2_{3,3}e_2+\delta^3_{3,3}e_3\right]=\delta^2_{3,3}e_3=0\ \Rightarrow\ \delta^2_{3,3}=0$

10) $e_2,e_1,e_1:$

$[e_2,\varphi(e_1,e_1)]-[e_1,\varphi(e_2,e_1)]+[e_1,\varphi(e_2,e_1)]-\varphi([e_2,e_1],e_1)+\varphi([e_2,e_1],e_1)-\varphi([e_1,e_1],e_2)=0$

11) $e_2,e_1,e_2:$

$[e_2,\varphi(e_1,e_2)]-[e_1,\varphi(e_2,e_2)]+[e_2,\varphi(e_1,e_2)]-\varphi([e_2,e_1],e_2)+\varphi([e_2,e_2],e_1)-\varphi([e_1,e_2],e_2)=$
$=[e_2,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]+[e_2,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]=-\delta^1_{1,2}e_3-\delta^1_{1,2}e_3=-2\delta^1_{1,2}e_3=0\ \Rightarrow$
$\delta^1_{1,2}=0$

12) $e_2,e_1,e_3:$

$[e_2,\varphi(e_1,e_3)]-[e_1,\varphi(e_2,e_3)]+[e_3,\varphi(e_2,e_1)]-\varphi([e_2,e_1],e_3)+\varphi([e_2,e_3],e_1)-\varphi([e_1,e_3],e_2)=$
$[e_2,\delta^1_{1,3}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3]-[e_1,\delta^1_{2,3}e_1+\delta^2_{2,3}e_2+\delta^3_{2,3}e_3]+\varphi(e_3,e_3)=-\delta^1_{1,3}e_3-\delta^2_{2,3}e_3+\delta^3_{3,3}e_3=$
$-(\delta^1_{1,3}-\delta^2_{2,3}+\delta^3_{3,3})e_3=0\ \Rightarrow\ \delta^3_{3,3}=\delta^2_{2,3}+\delta^2_{1,3}$

13) $e_2,e_2,e_1:$

$[e_2,\varphi(e_2,e_1)]-[e_2,\varphi(e_2,e_1)]+[e_1,\varphi(e_2,e_2)]-\varphi([e_2,e_2],e_1)+\varphi([e_2,e_1],e_2)-\varphi([e_2,e_1],e_2)=0$

14) $e_2,e_2,e_2:$

$[e_2,\varphi(e_2,e_2)]-[e_2,\varphi(e_2,e_2)]+[e_2,\varphi(e_2,e_2)]-\varphi([e_2,e_2],e_2)+\varphi([e_2,e_2],e_2)-\varphi([e_2,e_2],e_2)=$
$=[e_2,\delta^1_{2,2}e_1+\delta^2_{2,2}e_2+\delta^3_{2,2}e_3]=-\delta^1_{2,2}e_3=0\ \Rightarrow \delta^1_{2,2}=0$

15) $e_2,e_2,e_3:$

$[e_2,\varphi(e_2,e_3)]-[e_2,\varphi(e_2,e_3)]+[e_3,\varphi(e_2,e_2)]-\varphi\left([e_2,e_2],e_3\right)+\varphi\left([e_2,e_3],e_2\right)-\varphi\left([e_2,e_3],e_2\right)=0$

16) $e_2, e_3, e_1:$

$[e_2,\varphi(e_3,e_1)]-[e_3,\varphi(e_2,e_1)]+[e_1,\varphi(e_2,e_3)]-\varphi([e_2,e_3],e_1)+\varphi([e_2,e_1],e_3)-\varphi([e_3,e_1],e_2)=$ $[e_2,\delta_{3,1}^1e_1+\delta_{3,1}^2e_2+\delta_{3,1}^3e_3]+[e_1,\delta_{2,3}^1e_1+\delta_{2,3}^2e_2+\delta_{2,3}^3e_3]-\varphi(e_3,e_3)=-\delta_{3,1}^2e_3+\delta_{2,3}^2e_3-\delta_{3,3}^3e_3=(-\delta_{3,1}^2+\delta_{2,3}^2-\delta_{3,3}^3)e_3=0 \;\Rightarrow\; \delta_{3,3}^3=\delta_{2,3}^2-\delta_{3,1}^3$

17) $e_2, e_3, e_2:$

$[e_2,\varphi(e_3,e_2)]-[e_3,\varphi(e_2,e_2)]+[e_2,\varphi(e_2,e_3)]-\varphi([e_2,e_3],e_2)+\varphi([e_2,e_2],e_3)-\varphi([e_3,e_2],e_2)=$ $=[e_2,\delta_{3,2}^1e_1+\delta_{3,2}^2e_2+\delta_{3,2}^3e_3]+[e_2,\delta_{2,3}^1e_1+\delta_{2,3}^2e_2+\delta_{2,3}^3e_3]=-\delta_{3,2}^1e_3-\delta_{2,3}^1e_3==-(\delta_{3,2}^1+\delta_{2,3}^1)e_3=0 \;\Rightarrow\; \delta_{3,2}^1=-\delta_{2,3}^1$

18) $e_2, e_3, e_3:$

$[e_2,\varphi(e_3,e_3)]-[e_3,\varphi(e_2,e_3)]+[e_3,\varphi(e_2,e_3)]-\varphi\left([e_2,e_3],e_3\right)+\varphi\left([e_2,e_3],e_3\right)-\varphi\left([e_3,e_3],e_2\right)=0$

19) $e_3, e_1, e_1:$

$[e_3,\varphi(e_1,e_1)]-[e_1,\varphi(e_3,e_1)]+[e_1,\varphi(e_3,e_1)]-\varphi\left([e_3,e_1],e_1\right)+\varphi\left([e_3,e_1],e_1\right)-\varphi\left([e_1,e_1],e_3\right)=0$

20) $e_3, e_1, e_2:$

$[e_3,\varphi(e_1,e_2)]-[e_1,\varphi(e_3,e_2)]+[e_2,\varphi(e_3,e_1)]-\varphi\left([e_3,e_1],e_2\right)+\varphi\left([e_3,e_2],e_1\right)-\varphi\left([e_1,e_2],e_3\right)=$ $-\left[e_1,\delta_{3,2}^1e_1+\delta_{3,2}^2e_2+\delta_{3,2}^3e_3\right]+\left[e_2,\delta_{3,1}^1e_1+\delta_{3,1}^2e_2+\delta_{3,1}^3e_3\right]-\varphi\left(e_3,e_3\right)=-\delta_{3,2}^2e_3-\delta_{3,1}^1e_3--\delta_{3,3}^3e_3=-(\delta_{3,2}^2+$ $\delta_{3,1}^1+\delta_{3,3}^3)e_3=0 \;\Rightarrow\; \delta_{3,3}^3=-\delta_{3,2}^2-\delta_{3,1}^1$

21) $e_3, e_1, e_3:$

$[e_3,\varphi(e_1,e_3)]-[e_1,\varphi(e_3,e_3)]+[e_3,\varphi(e_3,e_1)]-\varphi\left([e_3,e_1],e_3\right)+\varphi\left([e_3,e_3],e_1\right)-\varphi\left([e_1,e_3],e_3\right)=0$

22) $e_3, e_2, e_1:$

$[e_3,\varphi(e_2,e_1)]-[e_2,\varphi(e_3,e_1)]+[e_1,\varphi(e_3,e_2)]-\varphi\left([e_3,e_2],e_1\right)+\varphi\left([e_3,e_1],e_2\right)-\varphi\left([e_2,e_1],e_3\right)=$ $=-\left[e_2,\delta_{3,1}^1e_1+\delta_{3,1}^2e_2+\delta_{3,1}^3e_3\right]+\left[e_1,\delta_{3,2}^1e_1+\delta_{3,2}^2e_2+\delta_{3,2}^3e_3\right]+\varphi\left(e_3,e_3\right)=\delta_{3,1}^1e_3+\delta_{3,2}^2e_3--\delta_{3,3}^3e_3=$ $(\delta_{3,1}^1+\delta_{3,2}^2-\delta_{3,3}^3)e_3=0 \;\Rightarrow\; \delta_{3,3}^3=\delta_{3,1}^1+\delta_{3,2}^2$

23) $e_3, e_2, e_2:$

$[e_3,\varphi(e_2,e_2)]-[e_2,\varphi(e_3,e_2)]+[e_2,\varphi(e_3,e_2)]-\varphi\left([e_3,e_2],e_2\right)+\varphi\left([e_3,e_2],e_2\right)-\varphi\left([e_2,e_2],e_3\right)=0$

24) $e_3, e_2, e_3:$

$[e_3,\varphi\left(e_2,e_3\right)]-[e_2,\varphi\left(e_3,e_3\right)]+[e_3,\varphi\left(e_3,e_2\right)]-\varphi\left([e_3,e_2],e_3\right)+\varphi\left([e_3,e_3],e_2\right)-\varphi\left([e_2,e_3],e_3\right)=0$

25) $e_3, e_3, e_1:$

$[e_3,\varphi(e_3,e_1)]-[e_3,\varphi(e_3,e_1)]+[e_1,\varphi(e_3,e_3)]-\varphi\left([e_3,e_3],e_1\right)+\varphi\left([e_3,e_1],e_3\right)-\varphi\left([e_3,e_1],e_3\right)=$

0

26) e_3, e_3, e_2 :

$[e_3, \varphi(e_3, e_2)] - [e_3, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_2) + \varphi([e_3, e_2], e_3) - \varphi([e_3, e_2], e_3) =$

0

27) e_3, e_3, e_3 :

$[e_3, \varphi(e_3, e_3)] - [e_3, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_3) + \varphi([e_3, e_2], e_3) - \varphi([e_3, e_3], e_3) =$

0

$\varphi(e_1, e_1) = \delta_{1,1}^3 e_3$

$\varphi(e_1, e_2) = \delta_{1,2}^2 e_2 + \delta_{1,2}^3 e_3$

$\varphi(e_1, e_3) = \delta_{1,3}^1 e_1 + \delta_{1,3}^3 e_3$

$\varphi(e_2, e_1) = \delta_{2,1}^1 e_1 - \delta_{1,2}^2 e_2 + \delta_{2,1}^3 e_3$

$\varphi(e_2, e_2) = \delta_{2,2}^3 e_3$

$\varphi(e_2, e_3) = \delta_{2,3}^1 e_1 + \delta_{2,3}^3 e_3$

$\varphi(e_3, e_1) = \delta_{3,1}^2 e_2 + \delta_{3,1}^3 e_3$

$\varphi(e_3, e_2) = -\delta_{2,3}^1 e_1 + \delta_{3,2}^3 e_3$

$\varphi(e_3, e_3) = 0$

Ikkinchi kosikllar to'plami $Z^2(G_1) = 13$ ga teng .Ikkinchi kochegaralar to'plami esa $B^2(G_1) = n^2 - Der(G_1)$ ko'rinishda aniqlanadi.Bunga ko'ra , $B^2(G_1) = 3$. Demak, $dimH^2(G_1) = Z^2(G_1)/B^2(G_1) \cong 13/3 = 10$.

ii. $G_2 : [e_1, e_2] = e_1$

1) e_1, e_1, e_1 :

$[e_1, \varphi(e_1, e_1)] - [e_1, \varphi(e_1, e_1)] + [e_1, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_1) + \varphi([e_1, e_1], e_1) - \varphi([e_1, e_1], e_1) =$
$[e_1, \delta_{1,1}^1 e_1 + \delta_{1,1}^2 e_2 + \delta_{1,1}^3 e_3] = \delta_{1,1}^2 e_1 = 0, \Rightarrow \delta_{1,1}^2 = 0$

2) e_1, e_1, e_2 :

$[e_1, \varphi(e_1, e_2)] - [e_1, \varphi(e_1, e_2)] + [e_2, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_2) + \varphi([e_1, e_2], e_1) - \varphi([e_1, e_2], e_1) =$
$\left[e_2, \delta_{1,1}^1 e_1 + \delta_{1,1}^3 e_3\right] = -\delta_{1,1}^1 e_1 = 0, \Rightarrow \delta_{1,1}^1 = 0$

3) e_1, e_1, e_3 :

$[e_1, \varphi(e_1, e_3)] - [e_1, \varphi(e_1, e_3)] + [e_3, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_3) + \varphi([e_1, e_3], e_1) - \varphi([e_1, e_3], e_1) =$

0

4) e_1, e_2, e_1 :

$[e_1, \varphi(e_2, e_1)] - [e_2, \varphi(e_1, e_1)] + [e_1, \varphi(e_1, e_2)] - \varphi([e_1, e_2], e_1) + \varphi([e_1, e_1], e_2) - \varphi([e_2, e_1], e_1) =$
$\delta_{2,1}^2 e_1 + \delta_{1,2}^1 e_1 = (\delta_{2,1}^2 + \delta_{1,2}^1) e_1 = 0, \ \delta_{2,1}^2 = -\delta_{1,2}^1$

5) e_1, e_2, e_2 :

$[e_1,\varphi(e_2,e_2)]-[e_2,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_2)]-\varphi\left([e_1,e_2],e_2\right)+\varphi\left([e_1,e_2],e_2\right)\ -\varphi\left([e_2,e_2],e_1\right)=$ $\left[e_1,\delta^1_{2,2}e_1+\delta^2_{2,2}e_2+\delta^3_{2,2}e_3\right]=\delta^2_{2,2}e_1=0\,,\ \delta^2_{2,2}=0$

6) $e_1,e_2,e_3:$

$[e_1,\varphi(e_2,e_3)]-[e_2,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_2)]-\varphi\left([e_1,e_2],e_3\right)+\varphi\left([e_1,e_3],e_2\right)\ -\varphi\left([e_2,e_3],e_1\right)=$ $\left[e_1,\delta^1_{2,3}e_1+\delta^2_{2,3}e_2+\delta^3_{2,3}e_3\right]-\left[e_2,\delta^1_{1,3}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3\right]-\varphi\left(e_1,e_3\right)=\delta^2_{2,3}e_1+\delta^1_{1,3}e_1-\delta^1_{1,3}e_1-$ $\delta^2_{1,3}e_2-\delta^3_{1,3}e_3=\delta^2_{2,3}e_1-\delta^2_{1,3}e_2-\delta^3_{1,3}e_3=0\,,\ \delta^2_{2,3}=0\,,\ \delta^2_{1,3}=0\,,\delta^3_{1,3}=0$

7) $e_1,e_3,e_1:$

$[e_1,\varphi(e_3,e_1)]-[e_3,\varphi(e_1,e_1)]+[e_1,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_1)+\varphi([e_1,e_1],e_3)-\ \varphi([e_3,e_1],e_1)=$ $[e_1,\delta^1_{3,1}e_1+\delta^2_{3,1}e_2+\delta^3_{3,1}e_3]=\delta^2_{3,1}e_1=0\,,\ \delta^2_{3,1}=0$

8) $e_1,e_3,e_2:$

$[e_1,\varphi(e_3,e_2)]-[e_3,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_3)]-\varphi\left([e_1,e_3],e_2\right)+\varphi\left([e_1,e_2],e_3\right)\ -\varphi\left([e_3,e_2],e_1\right)=$ $\left[e_1,\delta^1_{3,2}e_1+\delta^2_{3,2}e_2+\delta^3_{3,2}e_3\right]+\left[e_2,\delta^1_{1,3}e_1\right]+\varphi\left(e_1,e_3\right)\ =\delta^2_{3,2}e_1-\delta^1_{1,3}e_1+\delta^1_{1,3}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3=$ $\delta^2_{3,2}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3=0\,,\ \delta^2_{3,2}=0\,,\ \delta^2_{1,3}=0\,,\ \delta^3_{1,3}=0$

9) $e_1,e_3,e_3:$

$[e_1,\varphi(e_3,e_3)]-[e_3,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_3)+\varphi([e_1,e_3],e_3)-\ \varphi([e_3,e_3],e_3)=$ $[e_1,\delta^1_{3,3}e_1+\delta^2_{3,3}e_2+\delta^3_{3,3}e_3]=\delta^2_{3,3}e_1=0\,,\ \delta^2_{3,3}=0$

10) $e_2,e_1,e_1:$

$[e_2,\varphi(e_1,e_1)]-[e_1,\varphi(e_2,e_1)]+[e_1,\varphi(e_2,e_1)]-\varphi\left([e_2,e_1],e_1\right)+\varphi\left([e_2,e_1],e_1\right)\ -\varphi\left([e_1,e_1],e_2\right)=$ $\left[e_2,\delta^1_{2,1}e_1+\delta^2_{2,1}e_2+\delta^3_{2,1}e_3\right]=-\delta^2_{2,1}e_1=0\,,\ \delta^2_{2,1}=0$

11) $e_1,e_2,e_1:$

$[e_2,\varphi(e_1,e_2)]-[e_1,\varphi(e_2,e_2)]+[e_2,\varphi(e_2,e_1)]-\varphi\left([e_2,e_1],e_2\right)+\varphi\left([e_2,e_2],e_1\right)\ -\varphi\left([e_1,e_2],e_2\right)=$ $\left[e_2,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3\right]+\left[e_2,\delta^1_{2,1}e_1+\delta^2_{2,1}e_2+\delta^3_{2,1}e_3\right]=-\delta^1_{1,2}e_1-\delta^1_{2,1}e_1=$

$-(\delta^1_{1,2}+\delta^1_{2,1})e_1=0\,,\ \delta^1_{1,2}=-\delta^1_{2,1}$

12) $e_2,e_1,e_3:$

$[e_2,\varphi(e_1,e_3)]-[e_1,\varphi(e_2,e_3)]+[e_3,\varphi(e_2,e_1)]-\varphi\left([e_2,e_1],e_3\right)+\varphi\left([e_2,e_3],e_1\right)-\ \varphi\left([e_1,e_3],e_2\right)=$ 0

13) $e_2,e_2,e_1\ :$

$[e_2,\varphi(e_2,e_1)]-[e_2,\varphi(e_2,e_1)]+[e_1,\varphi(e_2,e_2)]-\varphi\left([e_2,e_2],e_1\right)+\varphi\left([e_2,e_2],e_1\right)-\ \varphi\left([e_2,e_1],e_2\right)=$ 0

14) $e_2,e_2,e_2:$

$[e_2,\varphi(e_2,e_2)]-[e_2,\varphi(e_2,e_2)]+[e_2,\varphi(e_2,e_2)]-\varphi([e_2,e_2],e_2)+\varphi([e_2,e_2],e_2)-\ \varphi([e_2,e_2],e_2)=$

$=[e_2,\delta^1_{2,2}e_1+\delta^3_{2,2}e_3]=-\delta^1_{2,2}e_1=0\,,\ \delta^1_{2,2}=0$

15) $e_2,e_2,e_3:$

$[e_2,\varphi(e_2,e_3)]-[e_2,\varphi(e_2,e_3)]+[e_3,\varphi(e_2,e_2)]-\varphi\left([e_2,e_2],e_3\right)+\varphi\left([e_2,e_3],e_2\right)-\varphi\left([e_2,e_3],e_2\right)=0$

16) $e_2,e_3,e_1:$

$[e_2,\varphi(e_3,e_1)]-[e_3,\varphi(e_2,e_1)]+[e_1,\varphi(e_2,e_3)]-\varphi\left([e_2,e_3],e_1\right)+\varphi\left([e_2,e_1],e_3\right)- -\varphi\left([e_3,e_1],e_2\right)=$ $\left[e_2,\delta^1_{3,1}e_1+\delta^3_{3,1}e_3\right]-\delta^1_{1,3}e_1=-\delta^1_{3,1}e_1-\delta^1_{1,3}e_1=-(\delta^1_{3,1}+\delta^1_{1,3})e_1=0\ \ \delta^1_{3,1}=-\delta^1_{1,3}$

17) $e_2,e_3,e_2:$

$[e_2,\varphi(e_3,e_2)]-[e_3,\varphi(e_2,e_2)]+[e_2,\varphi(e_2,e_3)]-\varphi\left([e_2,e_3],e_2\right)+\varphi\left([e_2,e_2],e_3\right)-\ \varphi\left([e_3,e_2],e_2\right)=$ $\left[e_2,\delta^1_{3,2}e_1+\delta^3_{3,2}e_3\right]+\left[e_2,\delta^1_{2,3}e_1+\delta^3_{2,3}e_3\right]=-\delta^1_{3,2}e_1-\delta^1_{2,3}e_1==-(\delta^1_{3,2}+\delta^1_{2,3})e_1=0\,,\ \delta^1_{3,2}=-\delta^1_{2,3}$

18) $e_2,e_3,e_3:$

$[e_2,\varphi(e_3,e_3)]-[e_3,\varphi(e_2,e_3)]+[e_3,\varphi(e_2,e_3)]-\varphi([e_2,e_3],e_3)+\varphi([e_2,e_3],e_3)-\varphi([e_3,e_3],e_2)=$ $=-\delta^1_{3,3}e_1=0\,,\ \delta^1_{3,3}=0$

19) $e_3,e_1,e_1:$

$[e_3,\varphi(e_1,e_1)]-[e_1,\varphi(e_3,e_1)]+[e_1,\varphi(e_3,e_1)]-\varphi([e_3,e_1],e_1)+\varphi([e_3,e_1],e_1)-\varphi([e_1,e_1],e_3)=$ $=0$

20) $e_3,e_1,e_2:$

$[e_3,\varphi(e_1,e_2)]-[e_1,\varphi(e_3,e_2)]+[e_2,\varphi(e_3,e_1)]-\varphi([e_3,e_1],e_2)+\varphi([e_3,e_2],e_1)-\varphi([e_1,e_2],e_3)=$ $=[e_2,\delta^1_{3,1}e_1+\delta^3_{3,1}e_3]-\delta^1_{1,3}e_1=-\delta^1_{3,1}e_1-\delta^1_{1,3}e_1=-(\delta^1_{3,1}+\delta^1_{1,3})e_1=0\,,\ \delta^1_{3,1}=-\delta^1_{1,3}$

21) $e_3,e_1,e_3:$

$[e_3,\varphi(e_1,e_3)]-[e_1,\varphi(e_3,e_3)]+[e_3,\varphi(e_3,e_1)]-\varphi([e_3,e_1],e_3)+\varphi([e_3,e_3],e_1)-\varphi([e_1,e_3],e_3)=$ $=0$

22) $e_3,e_2,e_1:$

$[e_3,\varphi(e_2,e_1)]-[e_2,\varphi(e_3,e_1)]+[e_1,\varphi(e_3,e_2)]-\varphi([e_3,e_2],e_1)+\varphi([e_3,e_1],e_2)-\varphi([e_2,e_1],e_3)=$ $=-[e_2,\delta^1_{3,1}e_1+\delta^3_{3,1}e_3]+\delta^1_{1,3}e_1=\delta^1_{3,1}e_1+\delta^1_{1,3}e_1=(\delta^1_{3,1}+\delta^1_{1,3})e_1=0\,,\ \delta^1_{3,1}=-\delta^1_{1,3}$

23) $e_3,e_2,e_2:$

$[e_3,\varphi(e_2,e_2)]-[e_2,\varphi(e_3,e_2)]+[e_2,\varphi(e_3,e_2)]-\varphi([e_3,e_2],e_2)+\varphi([e_3,e_2],e_2)-\varphi([e_2,e_2],e_3)=$ $=0$

24) $e_3,e_2,e_3:$

$[e_3,\varphi(e_2,e_3)]-[e_2,\varphi(e_3,e_3)]+[e_3,\varphi(e_3,e_2)]-\varphi([e_3,e_2],e_3)+\varphi([e_3,e_3],e_2)-\varphi([e_2,e_3],e_3)=$ $=0$

25) $e_3,e_3,e_1:$

$[e_3,\varphi(e_3,e_1)]-[e_3,\varphi(e_3,e_1)]+[e_1,\varphi(e_3,e_3)]-\varphi([e_3,e_3],e_1)+\varphi([e_3,e_1],e_3)-\varphi([e_3,e_1],e_3)=$ $=0$

26) $e_3,e_3,e_2:$

$[e_3,\varphi(e_3,e_2)]-[e_3,\varphi(e_3,e_2)]+[e_2,\varphi(e_3,e_3)]-\varphi([e_3,e_3],e_2)+\varphi([e_3,e_2],e_3)-\varphi([e_3,e_2],e_3)=$
$=0$

27) $e_3,e_3,e_3:$

$[e_3,\varphi(e_3,e_3)]-[e_3,\varphi(e_3,e_3)]+[e_3,\varphi(e_3,e_3)]-\varphi([e_3,e_3],e_3)+\varphi([e_3,e_3],e_3)-\varphi([e_3,e_3],e_3)=$
$=0$

$\varphi(e_1,e_1)=\delta^3_{1,1}e_3$

$\varphi(e_1,e_2)=\delta^3_{1,2}e_3$

$\varphi(e_1,e_3)=\delta^1_{1,3}e_1$

$\varphi(e_2,e_1)=\delta^3_{2,1}e_3$

$\varphi(e_2,e_2)=\delta^3_{2,2}e_3$

$\varphi(e_2,e_3)=\delta^1_{2,3}e_1+\delta^3_{2,3}e_3$

$\varphi(e_3,e_1)=-\delta^1_{1,3}e_1+\delta^3_{3,1}e_3$

$\varphi(e_3,e_2)=-\delta^1_{2,3}e_1+\delta^3_{3,2}e_3$

$\varphi(e_3,e_3)=\delta^3_{3,3}e_3$

Ikkinchi kosikllar to'plami $dimZ^2(G_2)=10$ ga teng,kochegaralar to'plami esa, $dimB^2(L_2)=9-4=5$ teng.Ikkinchi kogomologik gruppa quyidagi ko'rinishda aniqlanadi: $dimH^2(G_2)=Z^2(G_2)/B^2(G_2)\cong 5$.

iii. $G_3:[e_1,e_2]=e_2,[e_1,e_3]=e_2+e_3$

1. $e_1,e_1,e_1:$

$[e_1,\varphi(e_1,e_1)]-[e_1,\varphi(e_1,e_1)]+[e_1,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_1)+\varphi([e_1,e_1],e_1)-\varphi([e_1,e_1],e_1)=$
$[e_1,\delta^1_{1,1}e_1+\delta^2_{1,1}e_2+\delta^3_{1,1}e_3]=\delta^2_{1,1}e_2+\delta^3_{1,1}e_2+\delta^3_{1,1}e_3=0\quad \delta^2_{1,1}=0\quad \delta^3_{1,1}=0$

2. $e_1,e_1,e_2:$

$[e_1,\varphi(e_1,e_2)]-[e_1,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_2)+\varphi([e_1,e_2],e_1)-\varphi([e_1,e_2],e_1)=$
$[e_2,\delta^1_{1,1}e_1]=-\delta^1_{1,1}e_1=0\Rightarrow\delta^1_{1,1}=0$

3. $e_1,e_1,e_3:$

$[e_1,\varphi(e_1,e_3)]-[e_1,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_3)+\varphi([e_1,e_3],e_1)-\varphi([e_1,e_3],e_1)=$
0

4. $e_1,e_2,e_1:$

$[e_1,\varphi(e_2,e_1)]-[e_2,\varphi(e_1,e_1)]+[e_1,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_1)+\varphi([e_1,e_1],e_2)-\varphi([e_2,e_1],e_1)=$
$[e_1,\delta^1_{2,1}e_1+\delta^2_{2,1}e_2+\delta^3_{2,1}e_3]+[e_1,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]=\delta^2_{2,1}e_2+\delta^3_{2,1}e_2+\delta^3_{2,1}e_3+\delta^2_{1,2}e_2+\delta^3_{1,2}e_2+\delta^3_{1,2}e_3=$
$(\delta^2_{2,1}+\delta^3_{2,1}+\delta^2_{1,2}+\delta^3_{1,2})e_2+(\delta^3_{2,1}+\delta^3_{1,2})e_3=0$

$\delta^3_{2,1}=-\delta^3_{1,2}$, $\delta^2_{2,1}=\delta^2_{1,2}$

5. $e_1,e_2,e_2:$

$[e_1,\varphi(e_2,e_2)]-[e_2,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_2)+\varphi([e_1,e_2],e_2)-\varphi([e_2,e_2],e_1)=$

$\delta^3_{2,2}=0 \quad \delta^2_{2,2}=0$

6. $e_1,e_2,e_3:$

$[e_1,\varphi(e_2,e_3)]-[e_2,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_3)+\varphi([e_1,e_3],e_2)-\varphi([e_2,e_3],e_1)=$ $[e_1,\delta^1_{2,3}e_1+\delta^2_{2,3}e_2+\delta^3_{2,3}e_3]-[e_2,\delta^1_{1,3}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3]+[e_3,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]-\varphi(e_2,e_3)+$ $\varphi(e_2,e_2)+\varphi(e_2,e_3)=\delta^2_{2,3}e_2+\delta^3_{2,3}e_2+\delta^3_{2,3}e_3+\delta^1_{1,3}e_2-\delta^1_{1,2}e_2-\delta^1_{1,2}e_3-\delta^1_{3,2}e_1-\delta^1_{3,2}e_2-\delta^1_{3,2}e_3+\delta^1_{1,2}e_1$

$\delta^3_{2,3}=-\delta^1_{1,3} \quad \delta^3_{3,2}=\delta^1_{1,2} \quad \delta^1_{2,2}=\delta^1_{2,2}=\delta^1_{2,3}-\delta^1_{3,2}$

7. $e_1,e_3,e_1:$

$\delta^2_{3,1}=-\delta^2_{1,3} \quad \delta^3_{3,1}=-\delta^3_{1,3}$

8. $e_1,e_3,e_2:$

$[e_1,\varphi(e_3,e_2)]-[e_3,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_2)+\varphi([e_1,e_2],e_3)-\varphi([e_3,e_2],e_1)=$ $[e_1,\delta^2_{3,2}e_2+\delta^3_{3,2}e_3]-[e_3,\delta^1_{1,2}e_1]+[e_2,\delta^1_{1,3}e_1]-\delta^3_{2,2}e_3-\delta^1_{3,2}-\delta^2_{3,2}e_2-\delta^3_{3,2}e_3+\delta^1_{2,3}e_1+\delta^2_{2,3}+\delta^3_{2,3}e_3=$ $\delta^2_{3,2}e_2+\delta^3_{3,2}e_2+\delta^3_{3,2}e_3+\delta^1_{1,2}e_2+\delta^2_{1,2}e_3-\delta^1_{1,3}e_2-\delta^3_{2,2}e_3-\delta^1_{3,2}e_1-\delta^2_{3,2}e_2-\delta^3_{3,2}e_3+\delta^1_{2,3}e_1+\delta^2_{2,3}e_2+\delta^3_{2,3}e_3=$ $(\delta^2_{3,2}+\delta^1_{1,2}-\delta^1_{1,3}+\delta^2_{2,3})e_2+(\delta^2_{1,2}-\delta^3_{2,2}+\delta^3_{2,3})e_3+(\delta^1_{2,3}-\delta^1_{3,2})e_1$

$\delta^2_{3,2}=\delta^1_{1,3}-\delta^1_{1,2}-\delta^2_{2,3} \quad \delta^2_{1,2}=\delta^3_{2,2}-\delta^3_{2,3} \quad \delta^1_{2,3}=\delta^1_{3,2}$

9. $e_1,e_3,e_3:$

$[e_1,\varphi(e_3,e_3)]-[e_3,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_3)+\varphi([e_1,e_3],e_3)-\varphi([e_3,e_3],e_1)=$ $[e_1,\delta^1_{3,3}e_1+\delta^2_{3,3}e_2+\delta^3_{3,3}e_3]=\delta^2_{3,3}e_2+\delta^3_{3,3}e_2+\delta^3_{3,3}e_3=(\delta^2_{3,3}+\delta^3_{3,3})e_2+\delta^3_{3,3}e_3=0\,,$

$\delta^2_{3,3}=-\delta^3_{3,3} \quad \delta^3_{3,3}=0$

10. $e_2,e_1,e_1:$

$[e_2,\varphi(e_1,e_1)]-[e_1,\varphi(e_2,e_1)]+[e_1,\varphi(e_2,e_1)]-\varphi([e_2,e_1],e_1)+\varphi([e_2,e_1],e_1)-\varphi([e_1,e_1],e_2)=$ $[e_2,\delta^1_{1,1}e_1+\delta^2_{1,1}e_2+\delta^3_{1,1}e_3]=-\delta^2_{1,1}e_2-\delta^3_{1,1}e_2-\delta^3_{1,1}e_3=-(\delta^2_{1,1}+\delta^3_{1,1})e_2-\delta^3_{1,1}e_3=0$

$\delta^2_{1,1}=-\delta^3_{1,1} \quad \delta^3_{1,1}=0$

11. $e_2,e_1,e_2:$

$[e_2,\varphi(e_1,e_2)]-[e_1,\varphi(e_2,e_2)]+[e_2,\varphi(e_2,e_1)]-\varphi([e_2,e_1],e_2)+\varphi([e_2,e_2],e_1)-\varphi([e_1,e_2],e_2)=$ $[e_2,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]+[e_2,\delta^1_{2,1}e_1+\delta^2_{2,1}e_2+\delta^3_{2,1}e_3]=-\delta^1_{1,2}e_2-\delta^1_{2,1}e_2=-(\delta^1_{1,2}+\delta^1_{2,1})e_2=0\,,$ $\delta^1_{1,2}=-\delta^1_{2,1}$

12. $e_2,e_1,e_3:$

$[e_2,\varphi(e_1,e_3)]-[e_1,\varphi(e_2,e_3)]+[e_3,\varphi(e_2,e_1)]-\varphi([e_2,e_1],e_3)+\varphi([e_2,e_3],e_1)-\varphi([e_1,e_3],e_2)=$ $[e_2,\delta^1_{1,3}e_1]-[e_1,\delta^3_{2,3}e_2+\delta^3_{2,3}e_3]+[e_3,\delta^1_{2,1}e_1]+\delta^1_{2,3}e_1+\delta^2_{2,3}e_2+\delta^3_{2,3}e_3-\delta^1_{2,2}e_1-\delta^1_{3,2}e_1-\delta^2_{3,2}e_2-\delta^3_{3,2}e_3=$ $-\delta^1_{1,3}e_2-\delta^2_{2,3}e_2-\delta^3_{2,3}e_2-\delta^3_{2,3}e_3-\delta^1_{2,1}e_2-\delta^1_{2,1}e_3+\delta^1_{2,3}e_1+\delta^2_{2,3}e_2+\delta^3_{2,3}e_3-\delta^1_{2,2}e_1-\delta^1_{3,2}e_1-\delta^2_{3,2}e_2-\delta^3_{3,2}e_3=$ $(-\delta^1_{1,3}-\delta^3_{2,3}-\delta^1_{2,1}-\delta^2_{3,2})e_2+(-\delta^1_{2,1}-\delta^3_{3,2})e_3+(\delta^1_{2,3}-\delta^1_{2,2}-\delta^1_{3,2})e_1=0$

$\delta^1_{1,3}=-\delta^3_{2,3}-\delta^1_{2,1}-\delta^2_{3,2}$

$\delta^1_{2,1} = -\delta^3_{3,2}$

$\delta^1_{2,3} = \delta^1_{2,2} + \delta^1_{3,2}$

13. $e_2, e_2, e_1:$

$[e_2, \varphi(e_2, e_1)] - [e_2, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_1) + \varphi([e_2, e_1], e_2) - \varphi([e_2, e_1], e_2) = 0$

14. $e_2, e_2, e_2:$

$[e_2, \varphi(e_2, e_2)] - [e_2, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_2) + \varphi([e_2, e_2], e_2) - \varphi([e_2, e_2], e_2) = [e_2, \delta^1_{2,2} e_1] = -\delta^1_{2,2} e_2 = 0 \Rightarrow \ \delta^1_{2,2} = 0$

15. $e_2, e_2, e_3:$

$[e_2, \varphi(e_2, e_3)] - [e_2, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_3) + \varphi([e_2, e_3], e_2) - \varphi([e_2, e_3], e_2) = 0$

16. $e_2, e_3, e_1:$

$[e_2, \varphi(e_3, e_1)] - [e_3, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_1) + \varphi([e_2, e_1], e_3) - \varphi([e_3, e_1], e_2) = [e_2, \delta^1_{3,1} e_1] - [e_3, \delta^1_{2,1} e_1] + [e_1, \delta^2_{2,3} e_2] - \varphi(e_2, e_3) + \varphi(e_3, e_2) = -\delta^1_{3,1} e_2 + \delta^1_{2,1} e_2 + \delta^1_{2,1} e_3 + \delta^2_{2,3} e_2 + \delta^3_{2,3} e_2 + \delta^3_{2,3} e_3 - \delta^1_{2,3} e_1 - \delta^2_{2,3} e_2 - \delta^3_{2,3} e_3 + \delta^1_{3,2} e_1 + \delta^2_{3,2} e_2 + \delta^3_{3,2} e_3 = (\delta^1_{2,1} - \delta^1_{3,1} + \delta^3_{2,3} + \delta^2_{3,2}) e_2 + (\delta^1_{2,1} + \delta^3_{3,2}) e_3 + (\delta^1_{3,2} - \delta^1_{2,3}) e_1 = 0$

$\delta^1_{2,1} = \delta^1_{3,1} - \delta^3_{2,3} + \delta^2_{3,2} \quad \delta^1_{2,1} = -\delta^3_{3,2} \quad \delta^1_{3,2} = \delta^1_{2,3}$

17. $e_2, e_3, e_2:$

$[e_2, \varphi(e_3, e_2)] - [e_3, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_2) + \varphi([e_2, e_2], e_3) - \varphi([e_3, e_2], e_2) = [e_2, \delta^1_{3,2} e_1 + \delta^1_{3,2} e_2 + \delta^1_{3,2} e_3] + [e_2, \delta^1_{2,3} e_1 + \delta^1_{2,3} e_2 + \delta^1_{2,3} e_3] = -\delta^1_{3,2} e_2 - \delta^1_{2,3} e_2 = -(\delta^1_{3,2} + \delta^1_{2,3}) e_2 = 0$

$\delta^1_{3,2} = -\delta^1_{2,3}$

18. $e_2, e_3, e_3:$

$[e_2, \varphi(e_3, e_3)] - [e_3, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_3) + \varphi([e_2, e_3], e_3) - \varphi([e_3, e_3], e_2) = [e_2, \delta^1_{3,3} e_1] = -\delta^1_{3,3} e_2 = 0$

$\delta^1_{3,3} = 0$

19. $e_3, e_1, e_1:$

$[e_3, \varphi(e_1, e_1)] - [e_3, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_1) + \varphi([e_3, e_1], e_1) - \varphi([e_1, e_1], e_3) = 0$

20. $e_3, e_1, e_2:$

$[e_3, \varphi(e_1, e_2)] - [e_1, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_2) + \varphi([e_3, e_2], e_1) - \varphi([e_1, e_2], e_3) = [e_3, \delta^1_{1,2} e_1] -$

$-[e_1, \delta^2_{3,2} e_2 + \delta^3_{3,2} e_3] + [e_2, \delta^1_{3,1} e_1] + \varphi(e_3, e_2) - \varphi(e_2, e_3) = -\delta^1_{1,2} e_2 - \delta^1_{1,2} e_3 - \delta^2_{3,2} e_2 -$

$-\delta^3_{3,2} e_2 - \delta^3_{3,2} e_3 - \delta^1_{3,1} e_2 + \delta^2_{3,2} e_2 + \delta^3_{3,2} e_3 - \delta^1_{2,3} e_1 - \delta^2_{2,3} e_2 - \delta^3_{2,3} e_3 =$

$(-\delta^1_{1,2} - \delta^3_{3,2} - \delta^1_{3,1} - \delta^2_{2,3})e_2 + (-\delta^1_{1,2} - \delta^3_{2,3})e_3 - \delta^1_{2,3}e_1 = 0$

$\delta^1_{1,2} = -\delta^3_{3,2} - \delta^1_{3,1} - \delta^2_{2,3} \quad \delta^1_{1,2} = -\delta^3_{2,3}$

21. e_3, e_1, e_3 :

$[e_3, \varphi(e_1, e_3)] - [e_1, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_3) + \varphi([e_3, e_3], e_1) - \varphi([e_1, e_3], e_3) = [e_3, \delta^1_{1,3}e_1]+$

$+[e_3, \delta^1_{3,1}e_1] + \varphi(e_2, e_3) - \varphi(e_2, e_3) = -\delta^1_{1,3}e_2 - \delta^1_{1,3}e_3 - \delta^1_{3,1}e_2 - \delta^1_{3,1}e_3 =$

$(-\delta^1_{1,3} - \delta^1_{3,1})e_2 + (-\delta^1_{1,3} - \delta^1_{3,1})e_3 = 0$

$\delta^1_{1,3} = -\delta^1_{3,1}$

22. e_3, e_2, e_1 :

$[e_3, \varphi(e_2, e_1)] - [e_2, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_1) + \varphi([e_3, e_1], e_2) - \varphi([e_2, e_1], e_3) = [e_3, \delta^1_{2,1}e_1]-$

$-[e_2, \delta^1_{3,1}e_1] + [e_1, \delta^2_{3,2}e_2 + \delta^3_{3,2}e_3] - \varphi(e_3, e_2) + \varphi(e_2, e_3) = -\delta^1_{2,1}e_2 - \delta^1_{2,1}e_3 + \delta^1_{3,1}e_2 + \delta^2_{3,2}e_2+$

$+\delta^3_{3,2}e_2 + \delta^3_{3,2}e_3 - \delta^2_{3,2}e_2 - \delta^3_{3,2}e_3 + \delta^2_{2,3}e_2 + \delta^3_{2,3}e_3 =$

$= (-\delta^1_{2,1} + \delta^1_{3,1} + \delta^3_{3,2} + \delta^2_{2,3}e_2)e_2 + (-\delta^1_{2,1} + \delta^3_{2,3})e_3$

$\delta^1_{2,1} = \delta^1_{3,1} + \delta^3_{3,2} + \delta^2_{2,3}$

$\delta^1_{2,1} = \delta^3_{2,3}$

23. e_3, e_2, e_2 :

$[e_3, \varphi(e_2, e_2)] - [e_2, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_2) + \varphi([e_3, e_2], e_2) - \varphi([e_2, e_2], e_3) = 0$

24. e_3, e_2, e_3 :

$[e_3, \varphi(e_2, e_3)] - [e_2, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_3) + \varphi([e_3, e_3], e_2) - \varphi([e_2, e_3], e_3) = 0$

25. e_3, e_3, e_1 :

$[e_3, \varphi(e_3, e_1)] - [e_3, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_1) + \varphi([e_3, e_1], e_3) - \varphi([e_3, e_1], e_3) = 0$

26. e_3, e_2, e_2 :

$[e_3, \varphi(e_3, e_1)] - [e_3, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_1) + \varphi([e_3, e_1], e_3) - \varphi([e_3, e_1], e_3) = 0$

27. e_3, e_3, e_3 :

$[e_3, \varphi(e_3, e_3)] - [e_3, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_3) + \varphi([e_3, e_3], e_3) - \varphi([e_3, e_3], e_3) = 0$

$\varphi(e_1, e_1) = 0$

$\varphi(e_1, e_2) = \delta^1_{1,2}e_1$

$\varphi(e_1,e_3)=\delta^1_{1,3}e_1+\delta^2_{1,3}e_2$

$\varphi(e_2,e_1)=-\delta^1_{1,2}e_1$

$\varphi(e_2,e_2)=0$

$\varphi(e_2,e_3)=-\delta^1_{3,2}e_1-\delta^1_{1,3}e_2+\delta^3_{2,3}e_3$

$\varphi(e_3,e_1)=-\delta^1_{1,3}e_1-\delta^2_{1,3}e_2$

$\varphi(e_3,e_2)=-\delta^1_{2,3}e_1-\delta^2_{2,3}e_2$

$\varphi(e_3,e_3)=0$

Ikkinchi kosikllar to'plami $dimZ^2(G_3)=8$ ga teng,kochegaralar to'plami esa,

$dimB^2(G_3)=9-4=5$ teng.Ikkinchi kogomologik gruppa quyidagi ko'rinishda aniqlanadi:

$dimH^2(G_3)=Z^2(G_3)/B^2(G_3)\cong 3\,.$

iv. $L_3:\ [e_1,e_2]=e_2\,,\quad [e_1,e_3]=-2e_1\,,\quad [e_2,e_3]=2e_2$

1. $e_1,e_1,e_1:$

$[e_1,\varphi(e_1,e_1)]-[e_1,\varphi(e_1,e_1)]+[e_1,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_1)+\varphi([e_1,e_1],e_1)-\varphi([e_1,e_1],e_1)=$
$=[e_1,\delta^1_{1,1}e_1+\delta^2_{1,1}e_2+\delta^3_{1,1}e_3]=\delta^2_{1,1}e_3-2\delta^3_{1,1}e_1=0$

$\delta^2_{1,1}=0\quad \delta^3_{1,1}=0$

2. $e_1,e_1,e_2:$

$[e_1,\varphi(e_1,e_2)]-[e_1,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_2)+\varphi([e_1,e_2],e_1)-\varphi([e_1,e_2],e_1)=$
$[e_2,\delta^1_{1,1}e_1]=-\delta^1_{1,1}e_3=0\Rightarrow\delta^1_{1,1}=0$

3. $e_1,e_1,e_3:$

$[e_1,\varphi(e_1,e_3)]-[e_1,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_3)+\varphi([e_1,e_3],e_1)-\varphi([e_1,e_3],e_1)=$
0

4. $e_1,e_2,e_1:$

$[e_1,\varphi(e_2,e_1)]-[e_2,\varphi(e_1,e_1)]+[e_1,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_1)+\varphi([e_1,e_1],e_2)-\varphi([e_2,e_1],e_1)=$
$=[e_1,\delta^1_{2,1}e_1+\delta^2_{2,1}e_2+\delta^3_{2,1}e_3]+[e_1,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]-\delta^1_{3,1}e_1-$
$-\delta^2_{3,1}e_2-\delta^3_{3,1}e_3+\delta^1_{3,1}e_1+\delta^2_{3,1}e_2+\delta^3_{3,1}e_3=\delta^2_{2,1}e_3-2\delta^3_{2,1}e_1+\delta^2_{1,2}e_3-$
$-2\delta^3_{1,2}e_1=(\delta^2_{2,1}+\delta^2_{1,2})e_3-2(\delta^3_{2,1}+\delta^3_{1,2})e_1=0$

$\delta^2_{2,1}=-\delta^2_{1,2}$

$\delta^3_{2,1}=0\quad \delta^2_{1,2}=0$

5. $e_1,e_2,e_2:$

$[e_1,\varphi(e_2,e_2)]-[e_2,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_2)+\varphi([e_1,e_2],e_2)-\varphi([e_2,e_2],e_1)=$
$[e_1,\delta^1_{2,2}e_1+\delta^2_{2,2}e_2+\delta^3_{2,3}e_3]=\delta^2_{2,2}e_3-2\delta^3_{2,2}e_1=0\quad \delta^2_{2,2}=0\quad \delta^3_{2,2}=0$

6. $e_1,e_2,e_3:$

$[e_1,\varphi(e_2,e_3)]-[e_2,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_3)+\varphi([e_1,e_3],e_2)-\varphi([e_2,e_3],e_1)=$ $[e_1,\delta^1_{2,3}e_1+\delta^2_{2,3}e_2+\delta^3_{2,3}e_3]-[e_2,\delta^1_{1,3}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3]+[e_3,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]-\delta^1_{3,3}e_1-$ $\delta^2_{3,3}e_2-\delta^3_{3,3}e_3-2\delta^1_{1,2}e_1-2\delta^2_{1,2}e_2-2\delta^3_{1,2}e_3+2\delta^1_{2,1}e_1--2\delta^2_{1,2}e_2-2\delta^3_{1,2}e_3=\delta^2_{2,3}e_3-2\delta^3_{2,3}e_1+$ $\delta^1_{1,3}e_3-2\delta^3_{1,3}e_2-2\delta^1_{1,2}e_1+2\delta^2_{1,2}e_2-\delta^1_{3,3}e_1-\delta^2_{3,3}e_2-\delta^3_{3,3}e_3-2\delta^1_{1,2}e_1-4\delta^2_{1,2}e_2-4\delta^3_{1,2}e_3+2\delta^1_{2,1}e_1=$ $(\delta^2_{2,3}+\delta^1_{1,3}-\delta^3_{3,3}-4\delta^3_{1,2})e_3+(-2\delta^3_{2,3}-2\delta^1_{1,2}-\delta^1_{3,3}-2\delta^1_{1,2}+2\delta^1_{2,1})e_1-(2\delta^3_{1,3}+2\delta^2_{1,2}+\delta^2_{3,3}+4\delta^2_{1,2})?_2=0$

$\delta^2_{2,3}=-\delta^1_{1,3}\quad \delta^2_{3,3}=0$

$2\delta^1_{1,2}-2\delta^1_{2,1}+2\delta^1_{1,2}+2\delta^3_{2,3}=4\delta^1_{1,2}-2\delta^1_{2,1}+2\delta^3_{2,3}$

7. $e_1,e_3,e_1:$

$[e_1,\varphi(e_3,e_1)]-[e_3,\varphi(e_1,e_1)]+[e_1,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_1)+\varphi([e_1,e_1],e_3)-\varphi([e_3,e_1],e_1)=$ $[e_1,\delta^1_{3,1}e_1+\delta^2_{3,1}e_2]+[e_1,\delta^1_{1,3}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3]=\delta^2_{3,1}e_3-2\delta^1_{3,1}e_1+\delta^2_{1,3}e_3-2\delta^3_{1,3}e_1=(\delta^2_{3,1}+\delta^2_{1,3})e_3-$ $2(\delta^3_{3,1}+\delta^3_{1,3})e_1$

$\delta^2_{3,1}=-\delta^2_{1,3}\quad \delta^3_{3,1}=0$

8. $e_1,e_3,e_2:$

$[e_1,\varphi(e_3,e_2)]-[e_3,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_2)+\varphi([e_1,e_2],e_3)-\varphi([e_3,e_2],e_1)=$ $[e_1,\delta^1_{3,2}e_1+\delta^2_{3,2}e_2]-[e_3,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]+[e_2,\delta^1_{1,3}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3]+2\delta^1_{1,2}e_1+2\delta^2_{1,2}+2\delta^3_{1,2}e_3+$ $4\delta^1_{1,2}e_1-2\delta^1_{2,1}e_1+2\delta^3_{2,3}e_1-2\delta^3_{1,3}e_2-6\delta^2_{1,2}e_2+\delta^3_{3,3}e_3-2\delta^1_{2,1}e_1+\delta^2_{1,2}e_2+2\delta^3_{1,3}e_3=\delta^2_{3,2}e_3-2\delta^3_{3,2}e_1-$ $2\delta^1_{1,2}e_1+2\delta^2_{1,2}e_2-\delta^1_{1,3}e_3+2\delta^3_{1,3}e_2+6\delta^1_{1,2}e_1-2\delta^2_{1,2}e_2+4\delta^3_{1,2}e_3-4\delta^1_{2,1}e_1+2\delta^3_{2,3}e_1-2\delta^3_{1,3}e_2+\delta^3_{3,3}e_3=$ $\delta^2_{3,2}e_3-2\delta^3_{3,2}e_1+4\delta^1_{1,2}e_1-\delta^1_{1,3}e_3+4\delta^3_{1,2}e_3-4\delta^1_{2,1}e_1+2\delta^3_{2,3}e_1+\delta^3_{3,3}e_3=(\delta^2_{3,2}-\delta^1_{1,3}+4\delta^3_{1,2}+\delta^3_{3,3})e_3+$ $(-2\delta^3_{3,2}+4\delta^1_{1,2}-4\delta^1_{2,1}+2\delta^3_{2,3})e_1=0$

$\delta^3_{3,2}=2\delta^1_{1,2}-2\delta^1_{2,1}+\delta^3_{2,3}\quad \delta^2_{3,2}=\delta^1_{1,3}$

9. $e_1,e_3,e_3:$

$[e_1,\varphi(e_3,e_3)]-[e_3,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_3)+\varphi([e_1,e_3],e_3)-\varphi([e_3,e_3],e_1)=$ $[e_1,(\delta^1_{1,2}-2\delta^1_{2,1}+2\delta^3_{2,3})e_1+(-2\delta^3_{1,3}-6\delta^2_{1,2})e_2+\delta^3_{3,3}e_3]=-2\delta^3_{1,3}e_3-6\delta^2_{1,2}e_3-2\delta^3_{3,3}e_1=-(\delta^3_{1,3}+$ $3\delta^2_{1,2})e_3-2\delta^3_{3,3}e_1=0$

$\delta^3_{1,3}=-3\delta^2_{1,2}\quad \delta^3_{3,3}=0$

10. $e_2,e_1,e_1:$

$[e_2,\varphi(e_1,e_1)]-[e_1,\varphi(e_2,e_1)]+[e_1,\varphi(e_2,e_1)]-\varphi([e_2,e_1],e_1)+\varphi([e_2,e_1],e_1)-\varphi([e_1,e_1],e_2)=$ 0

11. $e_2,e_1,e_2:$

$[e_2,\varphi(e_1,e_2)]-[e_1,\varphi(e_2,e_2)]+[e_2,\varphi(e_2,e_1)]-\varphi([e_2,e_1],e_2)+\varphi([e_2,e_2],e_1)-\varphi([e_1,e_2],e_2)=$ $[e_2,\delta^1_{1,2}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]+[e_2,\delta^1_{2,1}e_1-\delta^2_{1,2}e_2-\delta^3_{1,2}e_3]+\delta^1_{3,2}e_1+\delta^1_{1,3}e_2+\delta^1_{1,3}e_2-4\delta^3_{1,2}e_2+2\delta^1_{1,2}e_3-$ $2\delta^1_{2,1}e_3+\delta^3_{2,3}e_3-\delta^1_{3,2}e_1-\delta^1_{1,3}e_2+4\delta^3_{1,2}e_2-2\delta^1_{1,2}e_3+2\delta^1_{2,1}e_3-\delta^3_{2,3}e_3=-\delta^1_{1,2}e_3+2\delta^3_{1,2}e_2-\delta^1_{2,1}e_3-$ $2\delta^3_{1,2}e_2=(-\delta^1_{1,2}-\delta^1_{2,1})e_3$

$\delta^1_{2,1} = -\delta^1_{1,2}$

12. e_2, e_1, e_3 :

$[e_2, \varphi(e_1, e_3)] - [e_1, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_1)] - \varphi([e_2, e_1], e_3) + \varphi([e_2, e_3], e_1) - \varphi([e_1, e_3], e_2) = [e_2, \delta^1_{1,3} e_1] - [e_1, (-\delta^1_{1,3} + 4\delta^3_{1,2}) e_2 + \delta^3_{2,3} e_3] + [e_3, -\delta^1_{1,2} e_1 - \delta^2_{1,2} e_2 - \delta^3_{1,2} e_3] + (4\delta^1_{1,2} - 2\delta^1_{2,1} + 2\delta^3_{2,3}) e_1 + (-\delta^3_{1,3} - 6\delta^2_{1,2}) e_2 - 2\delta^1_{1,2} e_1 - 2\delta^2_{1,2} e_2 - 2\delta^3_{1,2} e_3 + 2\delta^1_{1,2} e_1 + 2\delta^2_{1,2} e_2 + 2\delta^3_{1,2} e_3 = -\delta^1_{1,3} e_3 - 6\delta^2_{1,2} e_2 + \delta^1_{1,3} e_3 - 4\delta^3_{1,2} e_3 + 2\delta^3_{2,3} e_1 - 2\delta^2_{1,2} e_2 + 4\delta^1_{1,2} e_1 - 2\delta^1_{2,1} e_1 + 2\delta^3_{2,3} e_1 - 2\delta^3_{1,3} e_2 - 6\delta^2_{1,2} e_2 = -10\delta^2_{1,2} e_2 - 4\delta^3_{1,2} e_3 + 4\delta^3_{2,3} e_1 + 2\delta^1_{1,2} e_1 - 2\delta^1_{2,1} e_1 - 2\delta^3_{1,3} e_2 = (-10\delta^2_{1,2} - 2\delta^3_{1,3}) e_2 - 4\delta^3_{1,2} e_3 + (4\delta^3_{2,3} + 2\delta^1_{1,2} - 2\delta^1_{2,1}) e_1 = 0$

$\delta^3_{1,3} = 0$

$\delta^1_{1,2} = \delta^1_{2,1} - 2\delta^3_{2,3}$

13. e_2, e_2, e_1 :

$[e_2, \varphi(e_2, e_1)] - [e_2, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_1) + \varphi([e_2, e_1], e_2) - \varphi([e_2, e_1], e_2) = 0$

14. e_2, e_2, e_2 :

$[e_2, \varphi(e_2, e_2)] - [e_2, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_2) + \varphi([e_2, e_2], e_2) - \varphi([e_2, e_2], e_2) = [e_2, \delta 2, 2^1 e_1] = -\delta 2, 2^1 e_3 = 0 \Rightarrow \delta 2, 2^1 = 0$

15. e_2, e_2, e_3 :

$[e_2, \varphi(e_2, e_3)] - [e_2, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_3) + \varphi([e_2, e_3], e_2) - \varphi([e_2, e_3], e_2) = 0$

16. e_2, e_3, e_1 :

$[e_2, \varphi(e_3, e_1)] - [e_3, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_1) + \varphi([e_2, e_1], e_3) - \varphi([e_3, e_1], e_2) = [e_2, \delta^1_{3,1} e_1 - \delta^2_{1,3} e_2 - \delta^3_{1,3} e_3] - [e_3, -\delta^1_{1,2} e_1 - \delta^2_{1,2} e_2] + [e_1, -\delta^1_{1,3} e_2] + 2\delta^1_{1,2} e_1 + 2\delta^2_{1,2} e_2 - 4\delta^1_{1,2} e_1 + 2\delta^1_{2,1} e_1 - 2\delta^3_{2,3} e_1 + 2\delta^3_{1,3} e_2 + 6\delta^2_{1,2} e_2 - 2\delta^1_{1,2} e_1 - 2\delta^2_{1,2} e_2 = -\delta^1_{3,1} e_3 - 2\delta^3_{1,3} e_2 + 2\delta^1_{1,2} e_1 - 2\delta^2_{1,2} e_2 - \delta^1_{1,3} e_3 - 2\delta^3_{2,3} e_1 - 4\delta^1_{1,2} e_1 + 2\delta^1_{2,1} e_1 - 2\delta^3_{2,3} e_1 + 2\delta^3_{1,3} e_2 + 6\delta^2_{1,2} e_2 = -\delta^1_{3,1} e_3 - 4\delta^1_{1,2} e_1 + 4\delta^2_{1,2} e_2 - \delta^1_{1,3} e_3 - 4\delta^3_{2,3} e_1 + 2\delta^1_{2,1} e_1 = (-\delta^1_{3,1} - \delta^1_{1,3}) e_3 + 4\delta^2_{1,2} e_2 + (-\delta^1_{1,2} - 4\delta^3_{2,3} + 2\delta^1_{2,1}) e_1 = 0$

$\delta^1_{3,1} = -\delta^1_{1,3} \quad \delta^2_{1,2} = 0 \quad \delta^1_{2,1} = 2\delta^1_{1,2} + 2\delta^3_{2,3}$

17. e_2, e_3, e_2 :

$[e_2, \varphi(e_3, e_1)] - [e_3, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_2) + \varphi([e_2, e_2], e_3) - \varphi([e_3, e_2], e_2) = [e_2, \delta^1_{3,2} e_1 + (2\delta^1_{1,2} - 2\delta^1_{2,1} + \delta^3_{2,3}) e_3] + [e_2, \delta^1_{2,3} e_1 + \delta^3_{2,3} e_3] = -\delta^1_{3,2} e_3 + 4\delta^1_{1,2} e_2 - 4\delta^1_{2,1} e_2 + 2\delta^3_{2,3} e_2 - \delta^1_{2,3} e_3 + 2\delta^3_{2,3} e_2 = -\delta^1_{3,2} e_3 - \delta^1_{2,3} e_3 + 4\delta^1_{1,2} e_2 - 4\delta^1_{2,1} e_2 + 4\delta^3_{2,3} e_2 = (-\delta^1_{3,2} - \delta^1_{2,3}) e_3 + (4\delta^1_{1,2} - 4\delta^1_{2,1} + 4\delta^3_{2,3}) e_2 = 0$

$\delta^1_{2,3} = -\delta^1_{3,2} \quad \delta^1_{2,1} = -\delta^1_{1,2} - \delta^3_{2,3}$

18. e_2, e_3, e_3 :

$[e_2, \varphi(e_3, e_3)] - [e_3, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_3) + \varphi([e_2, e_3], e_3) - \varphi([e_3, e_3], e_2) = [e_2, \delta^1_{3,3} e_1] = -\delta^1_{3,3} e_3 = 0 \Rightarrow \delta^1_{3,3} = 0$

19. $e_3, e_1, e_1:$

$[e_3, \varphi(e_1, e_1)] - [e_3, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_1) + \varphi([e_3, e_1], e_1) - \varphi([e_1, e_1], e_3) = 0$

20. $e_3, e_1, e_2:$

$[e_3, \varphi(e_1, e_2)] - [e_1, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_2) + \varphi([e_3, e_2], e_1) - \varphi([e_1, e_2], e_3) = [e_3, \delta^1_{1,2}e_1] - [e_1, \delta^1_{1,3}e_2 + (2\delta^1_{1,2} - 2\delta^1_{2,1} + \delta^3_{2,3})e_3] + [e_2, -\delta^1_{3,1}e_1] - 2\delta^1_{1,2}e_1 - 2\delta^1_{2,1}e_1 = -\delta^1_{1,2}e_3 - \delta^1_{1,3}e_3 - 2(2\delta^1_{1,2} - 2\delta^1_{2,1} + \delta^3_{2,3})e_1 + \delta^1_{3,1}e_3 - 2\delta^1_{1,2}e_1 - 2\delta^1_{2,1}e_1 = -\delta^1_{1,2}e_3 - \delta^1_{1,3}e_3 - 4\delta^1_{1,2}e_1 + 4\delta^1_{2,1}e_1 - 2\delta^3_{2,3}e_1 + \delta^1_{3,1}e_3 - 2\delta^1_{1,2}e_1 - 2\delta^1_{2,1}e_1 = (-\delta^1_{1,2} - \delta^1_{1,3} + \delta^1_{3,1})e_3 + (-6\delta^1_{1,2} + 2\delta^1_{2,1} - 2\delta^3_{2,3})e_1 = 0$

$\delta^1_{1,2} = \delta^1_{3,1} - \delta^1_{1,3}$

$\delta^1_{2,1} = \delta^3_{2,3} + 3\delta^1_{1,2}$

21. $e_3, e_1, e_3:$

$[e_3, \varphi(e_1, e_3)] - [e_1, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_3) + \varphi([e_3, e_3], e_1) - \varphi([e_1, e_3], e_3) = [e_3, \delta^1_{1,3}e_1 + \delta^2_{1,3}e_2] + [e_3, \delta^1_{3,1}e_1 + \delta^2_{3,1}e_2] - 2\delta^1_{1,3}e_1 - 2\delta^2_{1,3}e_3 + 2\delta^2_{1,3}e_2 = 2\delta^1_{1,3}e_1 - 2\delta^2_{1,3}e_2 + 2\delta^1_{3,1}e_1 - 2\delta^2_{3,1}e_2 = (2\delta^1_{1,3} + 2\delta^1_{3,1})e_1 - (2\delta^2_{1,3} + 2\delta^2_{3,1})e_2 = 0$

$\delta^1_{3,1} = -\delta^1_{1,3} \quad \delta^2_{3,1} = -\delta^2_{1,3}$

22. $e_3, e_2, e_1:$ $[e_3, \varphi(e_2, e_1)] - [e_2, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_1) + \varphi([e_3, e_1], e_2) - \varphi([e_2, e_1], e_3) = [e_3, -\delta^1_{1,2}e_1] - [e_2, -\delta^1_{1,3}e_1] + [e_1, \delta^1_{1,3}e_2 + 2\delta^1_{1,2}e_3 - 2\delta^1_{2,1}e_3 + \delta^3_{2,3}e_3] - 2\delta^1_{1,2}e_1 + 2\delta^1_{1,2}e_1 = 2\delta^1_{1,2}e_1 - \delta^1_{1,3}e_3 + \delta^1_{1,3}e_3 - 4\delta^1_{1,2}e_1 + 4\delta^1_{2,1}e_1 - 2\delta^3_{2,3}e_1 == (4\delta^1_{2,1} + 2\delta^1_{1,2} - 2\delta^3_{2,3})e_1 = 0 \quad \delta^1_{1,2} = \delta^3_{2,3} - 2\delta^1_{2,1}$

23. $e_3, e_2, e_2:$

$[e_3, \varphi(e_2, e_2)] - [e_2, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_2) + \varphi([e_3, e_2], e_2) - \varphi([e_2, e_2], e_3) = 0$

24. $e_3, e_3, e_1:$

$[e_3, \varphi(e_2, e_3)] - [e_2, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_3) + \varphi([e_3, e_3], e_2) - \varphi([e_2, e_3], e_3) = [e_3, \delta^1_{2,3}e_1 - \delta^1_{1,3}e_2] + [e_3, \delta^1_{3,2}e_1 + \delta^1_{1,3}e_2] = -2\delta^1_{2,3}e_1 + 2\delta^1_{1,3}e_2 - 2\delta^1_{3,2}e_1 - 2\delta^1_{1,3}e_2 = 0$

$\delta^1_{2,3} = -\delta^1_{3,2}$

25. $e_3, e_2, e_2:$

$[e_3, \varphi(e_3, e_1)] - [e_3, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_1) + \varphi([e_3, e_1], e_3) - \varphi([e_3, e_1], e_3) = 0$

26. $e_3, e_3, e_2:$

$[e_3, \varphi(e_3, e_2)] - [e_3, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_2) + \varphi([e_3, e_2], e_3) - \varphi([e_3, e_2], e_3) = 0$

27. $e_3, e_3, e_3:$

$[e_3, \varphi(e_3, e_3)] - [e_3, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_3) + \varphi([e_3, e_3], e_3) - \varphi([e_3, e_3], e_3) =$

0

$\varphi(e_1, e_1) = 0$

$\varphi(e_1, e_2) = \delta^1_{1,2}e_1$

$\varphi(e_1, e_3) = \delta^1_{1,3}e_1 + \delta^2_{1,3}e_2$

$\varphi(e_2, e_1) = -\delta^1_{1,2}e_1$

$\varphi(e_2, e_2) = 0$

$\varphi(e_2, e_3) = -\delta^1_{3,2}e_1 - \delta^1_{1,3}e_2 + \delta^3_{2,3}e_3$

$\varphi(e_3, e_1) = -\delta^1_{1,3}e_1 - \delta^2_{1,3}e_2$

$\varphi(e_3, e_2) = \delta^1_{3,2}e_1 + \delta^1_{1,3}e_2 + (3\delta^1_{1,2} - 2\delta^1_{2,1})e_3$

$\varphi(e_3, e_3) = 0$

Ikkinchi kosikllar to'plami $dimZ^2(G_5) = 6$ ga teng,kochegaralar to'plami esa, $dimB^2(G_5) = 9-3 = 6$ teng.Ikkinchi kogomologik gruppa quyidagi ko'rinishda aniqlanadi:

$dimH^2(G_5) = Z^2(G_5)/B^2(G_5) \cong 0\,.$

Teorema isbotlandi.

2.2-§. To'rt o'lchamli Li algebralarining birinchi kogomologik gruppalari.

Teorema 2.2.1. To'rt o'lchamli Li algebralarining birinchi kogomologik gruppalarining o'lchami quyidagicha bo'ladi:

$$DimH^1(L_1) = 8,\ DimH^1(L_3) = 6,\ DimH^1(L_{11}) = 2,$$

$$DimH^1(L_{13}) = 2.$$

Isbot. To'rt o'lchamli algebralarning birinchi kogomologik gruppalarini hisoblash uch o'lchamli algebralarning birinchi kogomologik gruppalarini hisoblash bilan bir xil bo'ladi.

i. L_1 algebra uchun yuqorida keltirilgan differensiali $Der(L_1) = 10$ ga teng, ichki differensiali esa, $Inn(L_1) = 2$ ga teng. Bundan kelib chiqadiki,

$H^1(L_1) = Der(L_1)/Inn(L_1) = 8$.

Demak, $DimH^1(L_1) = 8$.

ii. L_3 algebra uchun ham yuqoridagilar o'rinli bo'lib, unda $Der(L_3) = 9$ ga teng,ichki differensiali esa, $Inn(L_3) = 3$ ga teng. Bundan kelib chiqadi,

$H^1(L_3) = Der(L_3)/Inn(L_3) = 6$.

Demak, $DimH^1(L_3) = 6$.

iii. L_{11} algebrada differensiali $Der(L_{11}) = 6$, ichki differensiali $Inn(L_{11}) = 4$. Bundan kelib chiqadi, $H^1(L_{11}) = Der(L_{11})/Inn(L_{11}) = 2$.

Demak, $DimH^1(L_{11}) = 2$.

iv. L_{13} algebrada differensiali $Der(L_{13}) = 6$, ichki differensiali $Inn(L_{13}) = 4$. Bundan kelib chiqadi,

$H^1(L_{13}) = Der(L_{13})/Inn(L_{13}) = 2$. Demak, $DimH^1(L_{13}) = 2$.

Isboti tugadi.

2.3-§. To'rt o'lchamli Li algebralarining ikkinchi kogomologik gruppalari.

Ushbu paragrafda to'rt o'lchamli Li algebralarining ikkinchi kogomologik gruppalari keltiriladi.

Teorema 2.3.1. To'rt o'lchamli Li algebrasining ikkinchi kogomologik gruppasining o'lchami quyidagicha bo'ladi: $DimH^2(L_1) = 33$

Isbot. To'rt o'lchamli Li algebralari uchun 2.1-paragrafda foydalanilgan ayniyat o'rinli bo'ladi:

$\varphi(e_i, e_j) = \sum\limits_{t=1}^{n} \delta_{i,j}^t e_t$

$[e_i, \varphi(e_j, e_k)] - [e_j, \varphi(e_i, e_k)] + [e_k, \varphi(e_i, e_j)] - \varphi([e_i, e_j], e_k) + \varphi([e_i, e_k], e_j) -$

$\varphi([e_j, e_k], e_i) = 0$

$\varphi(e_1, e_1) = \delta_{1,1}^1 e_1 + \delta_{1,1}^2 e_2 + \delta_{1,1}^3 e_3 + \delta_{1,1}^4 e_4$

$\varphi(e_1, e_2) = \delta_{1,2}^1 e_1 + \delta_{1,2}^2 e_2 + \delta_{1,2}^3 e_3 + \delta_{1,2}^4 e_4$

$\varphi(e_1, e_3) = \delta_{1,3}^1 e_1 + \delta_{1,3}^2 e_2 + \delta_{1,3}^3 e_3 + \delta_{1,3}^4 e_4$

$\varphi(e_1, e_4) = \delta_{1,4}^1 e_1 + \delta_{1,4}^2 e_2 + \delta_{1,4}^3 e_3 + \delta_{1,4}^4 e_4$

$\varphi(e_2, e_1) = \delta_{2,1}^1 e_1 + \delta_{2,1}^2 e_2 + \delta_{2,1}^3 e_3 + \delta_{2,1}^4 e_4$

$\varphi(e_2, e_2) = \delta_{2,2}^1 e_1 + \delta_{2,2}^2 e_2 + \delta_{2,2}^3 e_3 + \delta_{2,2}^4 e_4$

$\varphi(e_2, e_3) = \delta_{2,3}^1 e_1 + \delta_{2,3}^2 e_2 + \delta_{2,3}^3 e_3 + \delta_{2,3}^4 e_4$

$\varphi(e_2, e_4) = \delta_{2,4}^1 e_1 + \delta_{2,4}^2 e_2 + \delta_{2,4}^3 e_3 + \delta_{2,4}^4 e_4$

$\varphi(e_3, e_1) = \delta_{3,1}^1 e_1 + \delta_{3,1}^2 e_2 + \delta_{3,1}^3 e_3 + \delta_{3,1}^4 e_4$

$\varphi(e_3, e_2) = \delta_{3,2}^1 e_1 + \delta_{3,2}^2 e_2 + \delta_{3,2}^3 e_3 + \delta_{3,2}^4 e_4$

$\varphi(e_3, e_3) = \delta_{3,3}^1 e_1 + \delta_{3,3}^2 e_2 + \delta_{3,3}^3 e_3 + \delta_{3,3}^4 e_4$

$\varphi(e_3, e_4) = \delta_{3,4}^1 e_1 + \delta_{3,4}^2 e_2 + \delta_{3,4}^3 e_3 + \delta_{3,4}^4 e_4$

$\varphi(e_4, e_1) = \delta_{4,1}^1 e_1 + \delta_{4,1}^2 e_2 + \delta_{4,1}^3 e_3 + \delta_{4,1}^4 e_4$

$\varphi(e_4, e_2) = \delta_{4,2}^1 e_1 + \delta_{4,2}^2 e_2 + \delta_{4,2}^3 e_3 + \delta_{4,2}^4 e_4$

$\varphi(e_4, e_3) = \delta_{4,3}^1 e_1 + \delta_{4,3}^2 e_2 + \delta_{4,3}^3 e_3 + \delta_{4,3}^4 e_4$

$\varphi(e_4, e_4) = \delta_{4,4}^1 e_1 + \delta_{4,4}^2 e_2 + \delta_{4,4}^3 e_3 + \delta_{4,4}^4 e_4$

i. $[e_1, e_2] = e_3$

1. e_1, e_1, e_1 :

$[e_1, \varphi(e_1, e_1)] - [e_1, \varphi(e_1, e_1)] + [e_1, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_1) + \varphi([e_1, e_1], e_1) - \varphi([e_1, e_1], e_1) = [e_1, \delta_{1,1}^1 e_1 + \delta_{1,1}^2 e_2 + \delta_{1,1}^3 e_3 + \delta_{1,1}^4 e_4] = \delta_{1,1}^2 e_3 = 0 \Rightarrow \delta_{1,1}^2 = 0$

2. e_1, e_1, e_2 :

$[e_1, \varphi(e_1, e_2)] - [e_1, \varphi(e_1, e_2)] + [e_2, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_2) + \varphi([e_1, e_2], e_1) - \varphi([e_1, e_2], e_1) = [e_2, \delta_{1,1}^1 e_1 + \delta_{1,1}^3 e_3 + \delta_{1,1}^4 e_4] = -\delta_{1,1}^1 e_3 = 0 \Rightarrow \delta_{1,1}^1 = 0$

3. e_1, e_1, e_3 :

$[e_1, \varphi(e_1, e_3)] - [e_1, \varphi(e_1, e_3)] + [e_3, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_3) + \varphi([e_1, e_3], e_1) -$
$-\varphi([e_1, e_3], e_1) = 0$

4. e_1, e_1, e_4 :

$[e_1, \varphi(e_1, e_4)] - [e_1, \varphi(e_1, e_4)] + [e_3, \varphi(e_1, e_1)] - \varphi([e_1, e_1], e_4) + \varphi([e_1, e_4], e_1) -$
$-\varphi([e_1, e_4], e_1) = 0$

5. e_1, e_2, e_1 :

$[e_1, \varphi(e_2, e_1)] - [e_2, \varphi(e_1, e_1)] + [e_1, \varphi(e_1, e_2)] - \varphi([e_1, e_2], e_1) + \varphi([e_1, e_1], e_2) - \varphi([e_2, e_1], e_1) = [e_1, \delta^1_{2,1}e_1 + \delta^2_{2,1}e_2 + \delta^3_{2,1}e_3 + \delta^4_{2,1}e_4] + [e_1, \delta^1_{1,2}e_1 + \delta^2_{1,2}e_2 + \delta^3_{1,2}e_3 + \delta^4_{1,2}e_4] = \delta^2_{2,1}e_3 + \delta^2_{1,2}e_3 = (\delta^2_{2,1} + \delta^2_{1,2})e_3 = 0$

$\delta^2_{2,1} = -\delta^2_{1,2}$

6. e_1, e_2, e_2 :

$[e_1, \varphi(e_2, e_2)] - [e_2, \varphi(e_1, e_2)] + [e_2, \varphi(e_1, e_2)] - \varphi([e_1, e_2], e_2) + \varphi([e_1, e_2], e_2) - \varphi([e_2, e_2], e_1) = [e_1, \delta^1_{2,2}e_1 + \delta^2_{2,2}e_2 + \delta^3_{2,2}e_3 + \delta^4_{2,2}e_4] = \delta^2_{2,2}e_3 = 0 \Rightarrow \delta^2_{2,2} = 0$

7. e_1, e_2, e_3 :

$[e_1, \varphi(e_2, e_3)] - [e_2, \varphi(e_1, e_3)] + [e_3, \varphi(e_1, e_2)] - \varphi([e_1, e_2], e_3) + \varphi([e_1, e_3], e_2) - \varphi([e_2, e_3], e_1) = [e_1, \delta^1_{2,3}e_1 + \delta^2_{2,3}e_2 + \delta^3_{2,3}e_3 + \delta^4_{2,3}e_4] - [\delta^1_{1,3}e_1 + \delta^2_{1,3}e_2 + \delta^3_{1,3}e_3 + \delta^4_{1,3}e_4] - \varphi(e_3, e_3) = \delta^2_{2,3}e_3 + \delta^1_{1,3}e_3 - \delta^1_{3,3}e_1 - \delta^2_{3,3}e_2 - \delta^3_{3,3}e_3 - \delta^4_{3,3}e_4 = (\delta^2_{2,3} + \delta^1_{1,3} - \delta^3_{3,3})e_3 - \delta^1_{3,3}e_1 - \delta^2_{3,3}e_2 - \delta^4_{3,3}e_4 = 0 \Rightarrow \delta^3_{3,3} = \delta^2_{2,3} + \delta^1_{1,3}; \delta^1_{3,3} = 0, \delta^4_{3,3} = 0$

8. e_1, e_2, e_4 :

$[e_1, \varphi(e_2, e_4)] - [e_2, \varphi(e_1, e_4)] + [e_4, \varphi(e_1, e_2)] - \varphi([e_1, e_2], e_4) + \varphi([e_1, e_4], e_2) - \varphi([e_2, e_4], e_1) = [e_1, \delta^1_{2,4}e_1 + \delta^2_{2,4}e_2 + \delta^3_{2,4}e_3 + \delta^4_{2,4}e_4] - [e_2, \delta^1_{1,4}e_1 + \delta^2_{1,4}e_2 + \delta^3_{1,4}e_3 + \delta^4_{1,4}e_4] - \varphi(e_3, e_4) = \delta^2_{2,4}e_3 + \delta^1_{1,4}e_3 - \delta^1_{3,4}e_1 - \delta^2_{3,4}e_2 - \delta^3_{3,4}e_3 - \delta^4_{3,4}e_4 = (\delta^2_{2,4} + \delta^1_{1,4} - \delta^3_{3,4})e_3 - \delta^1_{3,4}e_1 - \delta^2_{3,4}e_2 - \delta^4_{3,4}e_4 = 0 \Rightarrow \delta^3_{3,4} = \delta^2_{2,4} + \delta^1_{1,4};$

$\delta^1_{3,4} = 0 \quad \delta^2_{3,4} = 0 \quad \delta^4_{3,4} = 0$

9. e_1, e_3, e_1 :

$[e_1, \varphi(e_3, e_1)] - [e_3, \varphi(e_1, e_1)] + [e_1, \varphi(e_1, e_3)] - \varphi([e_1, e_3], e_1) + \varphi([e_1, e_1], e_3) - \varphi([e_3, e_1], e_1) = [e_1, \delta^1_{3,1}e_1 + \delta^2_{3,1}e_2 + \delta^3_{3,1}e_3 + \delta^4_{3,1}e_4] + [e_1, \delta^1_{1,3}e_1 + \delta^2_{1,3}e_2 + \delta^3_{1,3}e_3 + \delta^4_{1,3}e_4] = \delta^2_{3,1}e_3 + \delta^2_{1,3}e_3 = (\delta^2_{3,1} + \delta^2_{1,3})e_3 = 0 \Rightarrow \delta^2_{3,1} = -\delta^2_{1,3}$

10. e_1, e_3, e_2 :

$[e_1, \varphi(e_3, e_2)] - [e_3, \varphi(e_1, e_2)] + [e_2, \varphi(e_1, e_3)] - \varphi([e_1, e_3], e_2) + \varphi([e_1, e_2], e_3) - \varphi([e_3, e_2], e_1) = [e_1, \delta^1_{3,2}e_1 + \delta^2_{3,2}e_2 + \delta^3_{3,2}e_3 + \delta^4_{3,2}e_4] + [e_2, \delta^1_{1,3}e_1 + \delta^2_{1,3}e_2 + \delta^3_{1,3}e_3 + \delta^4_{1,3}e_4] + \varphi(e_3, e_3) = \delta^2_{3,2}e_3 - \delta^1_{1,3}e_3 + \delta^3_{3,3}e_3 = (\delta^2_{3,2} - \delta^1_{1,3} + \delta^3_{3,3})e_3 = 0 \Rightarrow \delta^3_{3,3} = \delta^1_{1,3} - \delta^2_{3,2}$

11. e_1, e_3, e_3 :

$[e_1, \varphi(e_3, e_3)] - [e_3, \varphi(e_1, e_3)] + [e_3, \varphi(e_1, e_3)] - \varphi([e_1, e_3], e_3) + \varphi([e_1, e_3], e_3)-$

$-\varphi([e_3, e_3], e_1) = 0$

12. $e_1, e_3, e_4 :$

$[e_1, \varphi(e_3, e_4)] - [e_3, \varphi(e_1, e_4)] + [e_4, \varphi(e_1, e_3)] - \varphi([e_1, e_3], e_4) + \varphi([e_1, e_4], e_3)-$

$-\varphi([e_3, e_4], e_1) = 0$

13. $e_1, e_4, e_1 :$

$[e_1, \varphi(e_4, e_1)] - [e_4, \varphi(e_1, e_1)] + [e_4, \varphi(e_1, e_4)] - \varphi([e_1, e_4], e_1) + \varphi([e_1, e_1], e_4) - \varphi([e_4, e_1], e_1) = [e_1, \delta^1_{4,1} e_1 + \delta^2_{4,1} e_2 + \delta^3_{4,1} e_3 + \delta^4_{4,1} e_4] + [e_1, \delta^1_{1,4} e_1 + \delta^2_{1,4} e_2 + \delta^3_{1,4} e_3 + \delta^4_{1,4} e_4] = \delta^2_{4,2} e_3 - \delta^1_{1,4} e_3 + \delta^3_{3,4} e_3 = (\delta^2_{4,2} - \delta^1_{1,4} + \delta^3_{3,4}) e_3 = 0 \Rightarrow \delta^2_{4,2} = \delta^1_{1,4} - \delta^3_{3,4}$

14. $e_1, e_4, e_2 :$

$[e_1, \varphi(e_4, e_2)] - [e_4, \varphi(e_1, e_2)] + [e_2, \varphi(e_1, e_4)] - \varphi([e_1, e_4], e_2) + \varphi([e_1, e_2], e_4) - \varphi([e_4, e_2], e_2) = [e_1, \delta^1_{4,2} e_1 + \delta^2_{4,2} e_2 + \delta^3_{4,2} e_3 + \delta^4_{4,2} e_4] + [e_2, \delta^1_{1,4} e_1 + \delta^2_{1,4} e_2 + \delta^3_{1,4} e_3 + \delta^4_{1,4} e_4] \varphi(e_3, e_4) = \delta^2_{4,2} e_3 - \delta^1_{1,4} e_3 + \delta^3_{3,4} e_3 = (\delta^2_{4,2} - \delta^1_{1,4} + \delta^3_{3,4}) e_3 = 0 \Rightarrow \delta^2_{4,2} = \delta^1_{1,4} - \delta^3_{3,4}$

15. $e_1, e_4, e_3 :$

$[e_1, \varphi(e_4, e_3)] - [e_4, \varphi(e_1, e_3)] + [e_3, \varphi(e_1, e_4)] - \varphi([e_1, e_4], e_3) + \varphi([e_1, e_3], e_4) - \varphi([e_4, e_3], e_1) = [e_1, \delta^1_{4,3} e_1 + \delta^2_{4,3} e_2 + \delta^3_{4,3} e_3 + \delta^4_{4,3} e_4] = \delta^2_{4,3} e_3 = 0$

16. $e_1, e_4, e_4 :$

$[e_1, \varphi(e_4, e_4)] - [e_4, \varphi(e_1, e_4)] + [e_4, \varphi(e_1, e_4)] - \varphi([e_1, e_4], e_4) + \varphi([e_1, e_4], e_4) - \varphi([e_4, e_4], e_1) = [e_1, \delta^1_{4,4} e_1 + \delta^2_{4,4} e_2 + \delta^3_{4,4} e_3 + \delta^4_{4,4} e_4] = \delta^2_{4,4} e_3 = 0 \Rightarrow \delta^2_{4,4} = 0$

17. $e_2, e_1, e_1 :$

$[e_2, \varphi(e_1, e_1)] - [e_1, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_1)] - \varphi([e_2, e_1], e_1) + \varphi([e_2, e_1], e_1)-$

$-\varphi([e_1, e_1], e_2) = 0$

18. $e_2, e_1, e_2 :$

$[e_2, \varphi(e_1, e_2)] - [e_1, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_1)] - \varphi([e_2, e_1], e_2) + \varphi([e_2, e_2], e_1) - \varphi([e_1, e_2], e_2) = [e_2, \delta^1_{1,2} e_1 + \delta^2_{1,2} e_2 + \delta^3_{1,2} e_3 + \delta^4_{1,2} e_4] + [e_2, \delta^1_{2,1} e_1 + \delta^2_{2,1} e_2 + \delta^3_{2,1} e_3 + \delta^4_{2,1} e_4] = -\delta^1_{1,2} e_3 - \delta^1_{2,1} e_3 = -(\delta^1_{1,2} + \delta^1_{2,1}) e_3 = 0 \Rightarrow \delta^1_{2,1} = -\delta^1_{1,2}$

19. $e_2, e_1, e_3 :$

$[e_2, \varphi(e_1, e_3)] - [e_1, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_1)] - \varphi([e_2, e_1], e_3) + \varphi([e_2, e_3], e_1) - \varphi([e_1, e_3], e_2) = [e_2, \delta^1_{1,3} e_1 + \delta^2_{1,3} e_2 + \delta^3_{1,3} e_3 + \delta^4_{1,3} e_4] - [e_1, \delta^1_{2,3} e_1 + \delta^2_{2,3} e_2 + \delta^3_{2,3} e_3 + \delta^4_{2,3} e_4] + \varphi(e_3, e_3) = -\delta^1_{1,3} e_3 - \delta^2_{2,3} e_3 = (\delta^3_{3,3} - \delta^2_{2,3} - \delta^1_{1,3}) e_3 = 0$

$\delta^3_{3,3} = \delta^2_{2,3} + \delta^1_{1,3}$

20. $e_2, e_1, e_4 :$

$[e_2, \varphi(e_1, e_4)] - [e_1, \varphi(e_2, e_4)] + [e_4, \varphi(e_2, e_1)] - \varphi([e_2, e_1], e_4) + \varphi([e_2, e_4], e_1) - \varphi([e_1, e_4], e_2) =$

$[e_2, \delta^1_{1,4}e_1+\delta^2_{1,4}e_2+\delta^3_{1,4}e_3+\delta^4_{1,4}e_4]-[e_1, \delta^1_{2,4}e_1+\delta^2_{2,4}e_2+\delta^3_{2,4}e_3+\delta^4_{2,4}e_4]+\varphi(e_3, e_4) = -\delta^2_{2,4}e_2+\delta^3_{2,4}e_3 = -\delta^1_{1,4}e_2 - \delta^2_{2,4}e_2 - \delta^3_{2,4}e_2 - \delta^1_{1,4}e_3 - \delta^2_{2,4}e_3 + \delta^3_{3,4}e_3 = (\delta^3_{3,4} - \delta^2_{2,4} - \delta^1_{1,4})e_3 = 0 \Rightarrow \delta^3_{3,4} = \delta^2_{2,4} + \delta^1_{1,4}$

21. e_2, e_2, e_1 :

$[e_2, \varphi(e_2, e_1)] - [e_2, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_1) + \varphi([e_2, e_1], e_2)-$

$-\varphi([e_2, e_1], e_2) = 0$

22. e_2, e_2, e_2 :

$[e_2, \varphi(e_2, e_2)]-[e_2, \varphi(e_2, e_2)]+[e_2, \varphi(e_2, e_2)]-\varphi([e_2, e_2], e_2)+\varphi([e_2, e_2], e_2)-\varphi([e_2, e_2], e_2) = [e_2, \delta 2, 2^1 e_1 + \delta 2, 2^3 e_3 + \delta 2, 2^3 e_3 + \delta 2, 2^4 e_4] = -\delta^1_{2,2}e_1 = 0 \Rightarrow \delta^1_{2,2} = 0$

23. e_2, e_2, e_3 :

$[e_2, \varphi(e_2, e_3)] - [e_2, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_3) + \varphi([e_2, e_3], e_2)-$

$-\varphi([e_2, e_3], e_2) = 0$

24. e_2, e_2, e_4 :

$[e_2, \varphi(e_2, e_4)] - [e_2, \varphi(e_2, e_4)] + [e_4, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_4) + \varphi([e_2, e_4], e_2)-$

$-\varphi([e_2, e_4], e_2) = 0$

25. e_2, e_3, e_1 :

$[e_2, \varphi(e_3, e_1)]-[e_3, \varphi(e_2, e_1)]+[e_1, \varphi(e_2, e_3)]-\varphi([e_2, e_3], e_1)+\varphi([e_2, e_1], e_3)-\varphi([e_3, e_1], e_2) = [e_2, \delta^1_{3,1}e_1 + \delta^2_{3,1}e_2 + \delta^3_{3,1}e_3 + \delta^4_{3,1}e_4] + [e_1, \delta^1_{2,3}e_1 + \delta^2_{2,3}e_2 + \delta^3_{2,3}e_3 + \delta^4_{2,3}e_4] - \varphi(e_3, e_3) = -\delta^1_{3,1}e_3 + \delta^2_{2,3}e_3 - \delta^3_{3,3}e_3 = (\delta^2_{2,3} - \delta^1_{3,1} - \delta^3_{3,3})e_3 = 0$

$\delta^2_{2,3} = \delta^1_{3,1} + \delta^3_{3,3}$

26. e_2, e_3, e_2 :

$[e_2, \varphi(e_3, e_1)]-[e_3, \varphi(e_2, e_2)]+[e_2, \varphi(e_2, e_3)]-\varphi([e_2, e_3], e_2)+\varphi([e_2, e_2], e_3)-\varphi([e_3, e_2], e_2) = [e_2, \delta^1_{3,2}e_1 + \delta^2_{3,2}e_2 + \delta^3_{3,2}e_3 + \delta^4_{3,2}e_4] + [e_2, \delta^1_{2,3}e_1 + \delta^2_{2,3}e_2 + \delta^3_{2,3}e_3 + \delta^4_{2,3}e_4] = -\delta^1_{3,2}e_3 - \delta^1_{2,3}e_3 = -(\delta^1_{3,2} + \delta^1_{2,3})e_3 = 0 \Rightarrow \delta^1_{3,2} = -\delta^1_{2,3}$

27. e_2, e_3, e_3 :

$[e_2, \varphi(e_3, e_3)] - [e_3, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_3) + \varphi([e_2, e_3], e_3)-$

$-\varphi([e_3, e_3], e_2) = 0$

28. e_2, e_3, e_4 :

$[e_2, \varphi(e_3, e_4)] - [e_3, \varphi(e_2, e_4)] + [e_4, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_4) + \varphi([e_2, e_4], e_3)-$

$-\varphi([e_3, e_4], e_2) = 0$

29. e_2, e_4, e_1 :

$[e_2, \varphi(e_4, e_1)]-[e_4, \varphi(e_2, e_1)]+[e_1, \varphi(e_2, e_4)]-\varphi([e_2, e_4], e_1)+\varphi([e_2, e_1], e_4)-\varphi([e_4, e_1], e_2) = [e_2, \delta^1_{4,1}e_1 + \delta^2_{4,1}e_2 + \delta^3_{4,1}e_3 + \delta^4_{4,1}e_4] + [e_1, \delta^1_{2,4}e_1 + \delta^2_{2,4}e_2 + \delta^3_{2,4}e_3 + \delta^4_{2,4}e_4] - \varphi(e_3, e_4) = -\delta^1_{4,1}e_3 - \delta^2_{2,4}e_3 - \delta^3_{3,4}e_3 = (\delta^2_{2,4} - \delta^1_{4,1} - \delta^3_{3,4})e_3 = 0 \Rightarrow \delta^2_{2,4} = \delta^1_{4,1} + \delta^3_{3,4}$

30. $e_2, e_4, e_2:$

$[e_2, \varphi(e_4, e_2)] - [e_4, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_4)] - \varphi([e_2, e_4], e_2) + \varphi([e_2, e_2], e_4) - \varphi([e_4, e_2], e_2) = [e_2, \delta^1_{4,2}e_1 + \delta^2_{4,2}e_2 + \delta^3_{4,2}e_3 + \delta^4_{4,2}e_4] + [e_2, \delta^1_{2,4}e_1 + \delta^2_{2,4}e_2 + \delta^3_{2,4}e_3 + \delta^4_{2,4}e_4] = -\delta^1_{4,2}e_3 - \delta^1_{2,4}e_3 = -(\delta^1_{4,2} + \delta^1_{2,4})e_3 = 0$

$\delta^1_{4,2} = -\delta^1_{2,4}$

31. $e_2, e_4, e_3:$

$[e_2, \varphi(e_4, e_3)] - [e_4, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_4)] - \varphi([e_2, e_4], e_3) + \varphi([e_2, e_3], e_4) -$
$-\varphi([e_4, e_3], e_2) = 0$

32. $e_2, e_4, e_4:$

$[e_2, \varphi(e_4, e_4)] - [e_4, \varphi(e_2, e_4)] + [e_4, \varphi(e_2, e_4)] - \varphi([e_2, e_4], e_4) + \varphi([e_2, e_4], e_4) -$
$-\varphi([e_4, e_4], e_2) = 0$

33. $e_3, e_1, e_1:$

$[e_3, \varphi(e_1, e_1)] - [e_3, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_1) + \varphi([e_3, e_1], e_1) -$
$-\varphi([e_1, e_1], e_3) = 0$

34. $e_3, e_1, e_2:$

$[e_3, \varphi(e_1, e_2)] - [e_1, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_2) + \varphi([e_3, e_2], e_1) - \varphi([e_1, e_2], e_3) = -[e_1, \delta^1_{3,2}e_1 + \delta^2_{3,2}e_2 + \delta^3_{3,2}e_3 + \delta^4_{3,2}e_4] + [e_2, \delta^1_{3,1}e_1 + \delta^2_{3,1}e_2 + \delta^3_{3,1}e_3 + \delta^4_{3,1}e_4] - \varphi(e_3, e_3) = -\delta^2_{3,2}e_3 - \delta^1_{3,1}e_3 - \delta^3_{3,3}e_3 = -(\delta^2_{3,2} + \delta^1_{3,1} + \delta^3_{3,3})e_3 = 0$

$\delta^2_{3,2} = -\delta^1_{3,1} - \delta^3_{3,3}$

35. $e_3, e_1, e_3:$

$[e_3, \varphi(e_1, e_3)] - [e_1, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_3) + \varphi([e_3, e_3], e_1) -$
$-\varphi([e_1, e_3], e_3) = 0$

36. $e_3, e_1, e_4:$

$[e_3, \varphi(e_1, e_4)] - [e_1, \varphi(e_3, e_4)] + [e_4, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_4) + \varphi([e_3, e_4], e_1) -$
$-\varphi([e_1, e_4], e_3) = 0$

37. $e_3, e_2, e_1:$ $[e_3, \varphi(e_2, e_1)] - [e_2, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_1) + \varphi([e_3, e_1], e_2) - \varphi([e_2, e_1], e_3) = -[e_2, \delta^1_{3,1}e_1 + \delta^2_{3,1}e_2 + \delta^3_{3,1}e_3 + \delta^4_{3,1}e_4] + [e_1, \delta^1_{3,2}e_1 + \delta^2_{3,2}e_2 + \delta^3_{3,2}e_3 + \delta^4_{3,2}e_4] + \varphi(e_3, e_3) = \delta^1_{3,1}e_3 + \delta^2_{3,2}e_3 + \delta^3_{3,3}e_3 = (\delta^1_{3,1} + \delta^2_{3,2} + \delta^3_{3,3})e_3 = 0 \Rightarrow \delta^2_{3,2} = -\delta^1_{3,1} - \delta^3_{3,3}$

38. $e_3, e_2, e_2:$

$[e_3, \varphi(e_2, e_2)] - [e_2, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_2) + \varphi([e_3, e_2], e_2) -$
$-\varphi([e_2, e_2], e_3) = 0$

39. $e_3, e_2, e_3:$

$[e_3, \varphi(e_2, e_3)] - [e_2, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_3) + \varphi([e_3, e_3], e_2) -$

$-\varphi([e_2,e_3],e_3)=0$

40. $e_3,e_2,e_4:$

$[e_3,\varphi(e_2,e_4)]-[e_2,\varphi(e_3,e_4)]+[e_4,\varphi(e_3,e_2)]-\varphi([e_3,e_2],e_4)+\varphi([e_3,e_4],e_2)-$

$-\varphi([e_2,e_4],e_3)=0$

41. $e_3,e_3,e_1:$

$[e_3,\varphi(e_3,e_1)]-[e_3,\varphi(e_3,e_1)]+[e_1,\varphi(e_3,e_3)]-\varphi([e_3,e_3],e_1)+\varphi([e_3,e_1],e_3)-$

$-\varphi([e_3,e_1],e_3)=0$

42. $e_3,e_3,e_2:$

$[e_3,\varphi(e_3,e_2)]-[e_3,\varphi(e_3,e_2)]+[e_2,\varphi(e_3,e_3)]-\varphi([e_3,e_3],e_2)+\varphi([e_3,e_2],e_3)-$

$-\varphi([e_3,e_2],e_3)=0$

43. $e_3,e_3,e_3:$

$[e_3,\varphi(e_3,e_3)]-[e_3,\varphi(e_3,e_3)]+[e_3,\varphi(e_3,e_3)]-\varphi([e_3,e_3],e_3)+\varphi([e_3,e_3],e_3)-$

$-\varphi([e_3,e_3],e_3)=0$

44. $e_3,e_3,e_4:$

$[e_3,\varphi(e_3,e_4)]-[e_3,\varphi(e_3,e_4)]+[e_4,\varphi(e_3,e_3)]-\varphi([e_3,e_3],e_4)+\varphi([e_3,e_4],e_3)-$

$-\varphi([e_3,e_4],e_3)=0$

45. $e_4,e_3,e_1:$

$[e_4,\varphi(e_3,e_1)]-[e_3,\varphi(e_4,e_1)]+[e_1,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_1)+\varphi([e_4,e_1],e_3)-$

$-\varphi([e_3,e_1],e_4)=0$

46. $e_4,e_3,e_2:$

$[e_4,\varphi(e_3,e_2)]-[e_3,\varphi(e_4,e_2)]+[e_2,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_2)+\varphi([e_4,e_2],e_3)-\varphi([e_3,e_2],e_4)=$
$[e_2,\delta^1_{4,3}e_1+\delta^2_{4,3}e_2+\delta^3_{4,3}e_3+\delta^4_{4,3}e_4]=-\delta^1_{4,3}e_3=0\Rightarrow\delta^1_{4,3}=0$

47. $e_4,e_3,e_3:$

$[e_4,\varphi(e_3,e_3)]-[e_3,\varphi(e_4,e_3)]+[e_3,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_3)+\varphi([e_4,e_3],e_3)-$

$-\varphi([e_3,e_3],e_4)=0$

48. $e_4,e_3,e_4:$

$[e_4,\varphi(e_3,e_4)]-[e_3,\varphi(e_4,e_4)]+[e_4,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_4)+\varphi([e_4,e_4],e_3)-$

$-\varphi([e_3,e_4],e_4)=0$

49. $e_4,e_1,e_1:$

$[e_4,\varphi(e_1,e_1)]-[e_1,\varphi(e_4,e_1)]+[e_1,\varphi(e_4,e_1)]-\varphi([e_4,e_1],e_1)+\varphi([e_4,e_1],e_1)-$

$-\varphi([e_1,e_1],e_4)=0$

50. $e_4,e_1,e_2:$

$[e_4,\varphi(e_1,e_2)]-[e_1,\varphi(e_4,e_2)]+[e_2,\varphi(e_4,e_1)]-\varphi([e_4,e_1],e_2)+\varphi([e_4,e_2],e_1)-\varphi([e_1,e_2],e_4)=-[e_1,\delta_{4,2}^1e_1+\delta_{4,2}^2e_2+\delta_{4,2}^3e_3+\delta_{4,2}^4e_4]+[e_2,\delta_{4,1}^1e_1+\delta_{4,1}^2e_2+\delta_{4,1}^3e_3+\delta_{4,1}^4e_4]-\varphi(e_3,e_4)=-\delta_{4,2}^2e_3-\delta_{4,1}^1e_3-\delta_{3,4}^3e_3=-(\delta_{4,2}^2+\delta_{4,1}^1+\delta_{3,4}^3)e_3=0$

$\delta_{4,2}^2=-\delta_{4,1}^1-\delta_{3,4}^3$

51. $e_4,e_1,e_3:$

$[e_4,\varphi(e_1,e_3)]-[e_1,\varphi(e_4,e_3)]+[e_3,\varphi(e_4,e_1)]-\varphi([e_4,e_1],e_3)+\varphi([e_4,e_3],e_1)-$
$-\varphi([e_1,e_3],e_4)=0$

52. $e_4,e_1,e_4:$

$[e_4,\varphi(e_1,e_4)]-[e_1,\varphi(e_4,e_4)]+[e_4,\varphi(e_4,e_1)]-\varphi([e_4,e_1],e_4)+\varphi([e_4,e_4],e_1)-$
$-\varphi([e_1,e_4],e_4)=0$

53. $e_4,e_2,e_1:$

$[e_4,\varphi(e_2,e_1)]-[e_2,\varphi(e_4,e_1)]+[e_1,\varphi(e_4,e_2)]-\varphi([e_4,e_2],e_1)+\varphi([e_4,e_1],e_2)-\varphi([e_2,e_1],e_4)=-[e_2,\delta_{4,1}^1e_1+\delta_{4,1}^2e_2+\delta_{4,1}^3e_3+\delta_{4,1}^4e_4]+[e_1,\delta_{4,2}^1e_1+\delta_{4,2}^2e_2+\delta_{4,2}^3e_3+\delta_{4,2}^4e_4]+\varphi(e_3,e_4)=-\delta_{4,1}^1e_3+\delta_{4,2}^2e_3+\delta_{3,4}^3e_3=(\delta_{3,4}^3+\delta_{4,2}^2-\delta_{4,1}^3)e_3=0$

$\delta_{4,2}^2=\delta_{4,1}^1-\delta_{3,4}^3$

54. $e_4,e_2,e_2:$

$[e_4,\varphi(e_2,e_2)]-[e_2,\varphi(e_4,e_2)]+[e_2,\varphi(e_4,e_2)]-\varphi([e_4,e_2],e_2)+\varphi([e_4,e_2],e_2)-$
$-\varphi([e_2,e_2],e_4)=0$

55. $e_4,e_2,e_3:$

$[e_4,\varphi(e_2,e_3)]-[e_2,\varphi(e_4,e_3)]+[e_3,\varphi(e_4,e_2)]-\varphi([e_4,e_2],e_3)+\varphi([e_4,e_3],e_2)-$
$-\varphi([e_2,e_3],e_4)=0$

56. $e_4,e_2,e_4:$

$[e_4,\varphi(e_2,e_4)]-[e_2,\varphi(e_4,e_4)]+[e_4,\varphi(e_4,e_2)]-\varphi([e_4,e_2],e_4)+\varphi([e_4,e_4],e_2)-\varphi([e_2,e_4],e_4)=-[e_2,\delta_{4,4}^1e_1+\delta_{4,4}^3e_3+\delta_{4,4}^4e_4]=\delta_{4,4}^1e_3=0\Rightarrow\delta_{4,4}^1=0$

57. $e_4,e_3,e_1:$

$[e_4,\varphi(e_3,e_1)]-[e_3,\varphi(e_4,e_1)]+[e_1,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_1)+\varphi([e_4,e_1],e_3)-$
$-\varphi([e_3,e_1],e_4)=0$

58. $e_4,e_3,e_2:$

$[e_4,\varphi(e_3,e_2)]-[e_3,\varphi(e_4,e_2)]+[e_2,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_2)+\varphi([e_4,e_2],e_3)-$
$-\varphi([e_3,e_2],e_4)=0$

59. $e_4,e_3,e_3:$

$[e_4,\varphi(e_3,e_3)]-[e_3,\varphi(e_4,e_3)]+[e_3,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_3)+\varphi([e_4,e_3],e_3)-$
$-\varphi([e_3,e_3],e_4)=0$

60. e_4, e_3, e_4 :

$[e_4, \varphi(e_3, e_4)] - [e_3, \varphi(e_4, e_4)] + [e_4, \varphi(e_4, e_3)] - \varphi([e_4, e_3], e_4) + \varphi([e_4, e_4], e_3) -$

$-\varphi([e_3, e_4], e_4) = 0$

61. e_4, e_4, e_1 :

$[e_4, \varphi(e_4, e_1)] - [e_4, \varphi(e_4, e_1)] + [e_1, \varphi(e_4, e_4)] - \varphi([e_4, e_4], e_1) + \varphi([e_4, e_1], e_4) -$

$-\varphi([e_4, e_1], e_4) = 0$

62. e_4, e_4, e_2 :

$[e_4, \varphi(e_4, e_2)] - [e_4, \varphi(e_4, e_2)] + [e_2, \varphi(e_4, e_4)] - \varphi([e_4, e_4], e_2) + \varphi([e_4, e_2], e_4) -$

$-\varphi([e_4, e_2], e_4) = 0$

63. e_4, e_4, e_3 :

$[e_4, \varphi(e_4, e_3)] - [e_4, \varphi(e_4, e_3)] + [e_3, \varphi(e_4, e_4)] - \varphi([e_4, e_4], e_3) + \varphi([e_4, e_3], e_4) -$

$-\varphi([e_4, e_3], e_4) = 0$

64. e_4, e_4, e_4 :

$[e_4, \varphi(e_4, e_4)] - [e_4, \varphi(e_4, e_4)] + [e_4, \varphi(e_4, e_4)] - \varphi([e_4, e_4], e_4) + \varphi([e_4, e_4], e_4) -$

$-\varphi([e_4, e_4], e_4) = 0$

$\varphi(e_1, e_1) = \delta^3_{1,1} e_3 + \delta^4_{1,1} e_4\,,\ \varphi(e_1, e_2) = \delta^1_{1,2} e_1 + \delta^2_{1,2} e_2 + \delta^3_{1,2} e_3 + \delta^4_{1,2} e_4\,,$

$\varphi(e_1, e_3) = \delta^1_{1,3} e_1 + \delta^2_{1,3} e_2 + \delta^3_{1,3} e_3 + \delta^4_{1,3} e_4\,,\ \varphi(e_1, e_4) = \delta^1_{1,4} e_1 + \delta^2_{1,4} e_2 + \delta^3_{1,4} e_3 + \delta^4_{1,4} e_4\,,$

$\varphi(e_2, e_1) = -\delta^1_{1,2} e_1 - \delta^2_{1,2} e_2 + \delta^3_{2,1} e_3 + \delta^4_{2,1} e_4\,,\ \varphi(e_2, e_2) = \delta^3_{2,2} e_3 + \delta^4_{2,2} e_4\,,$

$\varphi(e_2, e_3) = \delta^1_{2,3} e_1 + \delta^3_{2,3} e_3 + \delta^4_{2,3} e_4\,,\ \varphi(e_2, e_4) = \delta^1_{2,4} e_1 + \delta^2_{2,4} e_2 + \delta^3_{2,4} e_3 + \delta^4_{2,4} e_4\,,$

$\varphi(e_3, e_1) = \delta^1_{1,3} e_1 - \delta^2_{1,3} e_2 + \delta^3_{3,1} e_3 + \delta^4_{3,1} e_4\,,\ \varphi(e_3, e_2) = -\delta^1_{2,3} e_1 + \delta^3_{3,2} e_3 + \delta^4_{3,2} e_4\,,$

$\varphi(e_3, e_3) = \delta^3_{3,3} e_3\,,\ \varphi(e_3, e_4) = \delta^3_{3,4} e_3\,,$

$\varphi(e_4, e_1) = \delta^1_{4,1} e_1 - \delta^2_{1,4} e_2 + \delta^3_{4,1} e_3 + \delta^4_{4,1} e_4\,,\ \varphi(e_4, e_2) = -\delta^1_{2,4} e_1 + (\delta^1_{1,4} - \delta^3_{3,4}) e_2 + \delta^3_{4,2} e_3 + \delta^4_{4,2} e_4\,,$

$\varphi(e_4, e_3) = \delta^3_{4,3} e_3 + \delta^4_{4,3} e_4\,,\ \varphi(e_4, e_4) = \delta^3_{4,4} e_3 + \delta^4_{4,4} e_4\,.$

Bu to'rt o'lchamli algebrada $dim B^2(L_1) = 16 - 10 = 6$ ga teng, $dim Z^2(L_1) = 39$ ga teng. Demak, $dim H^2 = 33$. Teorema isbotlandi.

Teorema 2.3.2. To'rt o'lchamli Li algebrasining ikkinchi kogomologik gruppasining o'lchami quyidagicha bo'ladi:

$Dim H^1(L_3) = 21\,.$

Isbot. Bu teorema uchun ham 2.3.1-teoremada foydalangan ayniyat o'rinli bo'ladi va quyidagi algebrani hisoblaymiz:

$L_{13}:\ [e_1, e_2] = e_2\,, \quad [e_1, e_3] = e_2 + e_3$

1. e_1, e_1, e_1 :

$[e_1,\varphi(e_1,e_1)]-[e_1,\varphi(e_1,e_1)]+[e_1,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_1)+\varphi([e_1,e_1],e_1)-\varphi([e_1,e_1],e_1)=$ $[e_1,\delta^1_{1,1}e_1+\delta^2_{1,1}e_2+\delta^3_{1,1}e_3]=\delta^2_{1,1}e_2+\delta^3_{1,1}e_2+\delta^3_{1,1}e_3=(\delta^2_{1,1}+\delta^3_{1,1})e_2+\delta^3_{1,1}e_3=0$

$\delta^3_{1,1}=0\quad \delta^2_{1,1}=0$

2. $e_1,e_1,e_2:$

$[e_1,\varphi(e_1,e_2)]-[e_1,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_2)+\varphi([e_1,e_2],e_1)-\varphi([e_1,e_2],e_1)=$ $[e_2,\delta^1_{1,1}e_1+\delta^4_{1,1}e_4]=-\delta^1_{1,1}e_2=0\Rightarrow\delta^1_{1,1}=0$

3. $e_1,e_1,e_3:$

$[e_1,\varphi(e_1,e_3)]-[e_1,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_3)+\varphi([e_1,e_3],e_1)-$
$-\varphi([e_1,e_3],e_1)=0$

4. $e_1,e_1,e_4:$

$[e_1,\varphi(e_1,e_4)]-[e_1,\varphi(e_1,e_4)]+[e_3,\varphi(e_1,e_1)]-\varphi([e_1,e_1],e_4)+\varphi([e_1,e_4],e_1)-$
$-\varphi([e_1,e_4],e_1)=0$

5. $e_1,e_2,e_1:$

$[e_1,\varphi(e_2,e_1)]-[e_2,\varphi(e_1,e_1)]+[e_1,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_1)+\varphi([e_1,e_1],e_2)-\varphi([e_2,e_1],e_1)=$ $[e_1,\delta^2_{2,1}e_2+\delta^3_{2,1}e_3]-[e_1,\delta^2_{1,2}e_2+\delta^3_{1,2}e_3]-\delta^1_{2,1}e_1-\delta^2_{2,1}e_2-\delta^3_{2,1}e_3-\delta^4_{2,1}e_4+\delta^1_{2,1}e_1+\delta^2_{2,1}e_2+\delta^3_{2,1}e_3+\delta^4_{2,1}e_4=$ $\delta^2_{2,1}e_2+\delta^3_{2,1}e_2+\delta^3_{2,1}e_3-\delta^2_{1,2}e_2-\delta^3_{1,2}e_2-\delta^3_{1,2}e_3=(\delta^2_{2,1}+\delta^3_{2,1}-\delta^2_{1,2}-\delta^3_{1,2})e_2+(\delta^3_{2,1}-\delta^3_{1,2})e_3=0$

$\delta^2_{2,1}=\delta^2_{1,2}+\delta^3_{1,2}-\delta^3_{2,1}\quad \delta^3_{2,1}=\delta^3_{1,2}\quad \delta^2_{1,2}=\delta^2_{2,1}$

6. $e_1,e_2,e_2:$

$[e_1,\varphi(e_2,e_2)]-[e_2,\varphi(e_1,e_2)]+[e_2,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_2)+\varphi([e_1,e_2],e_2)-\varphi([e_2,e_2],e_1)=$ $[e_1,\delta^2_{2,2}e_2+\delta^3_{2,2}e_3]=\delta^2_{2,2}e_3+\delta^3_{2,2}e_2+\delta^3_{2,2}e_3=(\delta^2_{2,2}+\delta^3_{2,2})e_3+\delta^3_{2,2}e_3=0$

$\delta^2_{2,2}=0\quad \delta^3_{2,2}=0$

7. $e_1,e_2,e_3:$

$[e_1,\varphi(e_2,e_3)]-[e_2,\varphi(e_1,e_3)]+[e_3,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_3)+\varphi([e_1,e_3],e_2)-\varphi([e_2,e_3],e_1)=$ $[e_1,\delta^2_{2,3}e_2+\delta^3_{2,3}e_3]-[e_2,\delta^1_{1,3}e_1]+[e_3,\delta^1_{1,2}e_1]-\delta^1_{2,3}e_1-\delta^2_{2,3}e_2-\delta^3_{2,3}e_3-\delta^4_{2,3}e_4+\delta^1_{2,2}e_1+\delta^4_{2,2}e_4+$ $\delta^1_{3,2}e_1+\delta^2_{3,2}e_2+\delta^3_{3,2}e_3=\delta^2_{2,3}e_2+\delta^3_{2,3}e_2+\delta^3_{2,3}e_3+\delta^1_{1,3}e_2-\delta^1_{1,2}e_2-2\delta^1_{1,2}e_3-\delta^1_{2,3}e_1-\delta^2_{2,3}e_2-\delta^3_{2,3}e_3-$ $\delta^4_{2,3}e_4+\delta^1_{2,2}e_1+\delta^4_{2,2}e_4+\delta^1_{3,2}e_1+\delta^2_{3,2}e_2+\delta^3_{3,2}e_3+\delta^3_{3,2}e_4=(\delta^3_{2,3}+\delta^1_{1,3}-\delta^1_{1,2}+\delta^2_{3,2})e_2+(\delta^3_{3,2}-\delta^1_{1,2})e_3+$ $(\delta^1_{2,2}-\delta^1_{2,3}+\delta^1_{3,2})e_1+(\delta^4_{2,2}-\delta^+_{2,3}\delta^3_{3,2})e_4$

$\delta^3_{2,3}=\delta^1_{1,2}-\delta^1_{1,3}-\delta^2_{3,2}\qquad \delta^3_{3,2}=\delta^1_{1,2}$

$2\delta^1_{2,2}=\delta^1_{2,3}-\delta^1_{3,2}\qquad \delta^4_{2,2}=\delta^4_{2,3}-\delta^3_{3,2}\qquad \delta^1_{2,3}=\delta^1_{3,2}$

8. $e_1,e_2,e_4:$

$[e_1,\varphi(e_2,e_4)]-[e_2,\varphi(e_1,e_4)]+[e_4,\varphi(e_1,e_2)]-\varphi([e_1,e_2],e_4)+\varphi([e_1,e_4],e_2)-\varphi([e_2,e_4],e_1)=$ $[e_1,\delta^2_{2,4}e_2+\delta^3_{2,4}e_3]-[e_2,\delta^1_{1,4}e_1]-\delta^1_{2,4}e_1-\delta^2_{2,4}e_2-\delta^3_{2,4}e_3-\delta^4_{2,4}e_4=(\delta^2_{2,4}+\delta^3_{2,4})e_2+\delta^3_{2,4}e_3+\delta^1_{1,4}e_2-$ $\delta^1_{2,4}e_1-\delta^2_{2,4}e_2-\delta^3_{2,4}e_3+\delta^4_{2,4}e_4=(\delta^3_{2,4}+\delta^1_{1,4})e_2-\delta^1_{2,4}e_1-\delta^4_{2,4}e_4=0$

$\delta_{2,4}^3 = -\delta_{1,4}^1$

$2\delta_{2,4}^1 = 0 \qquad \delta_{2,4}^4 = 0$

9. $e_1, e_3, e_1:$

$[e_1, \varphi(e_3, e_1)] - [e_3, \varphi(e_1, e_1)] + [e_1, \varphi(e_1, e_3)] - \varphi([e_1, e_3], e_1) + \varphi([e_1, e_1], e_3) - \varphi([e_3, e_1], e_1) = [e_1, \delta_{3,1}^2 e_2 + \delta_{3,1}^3 e_3] + [e_1, \delta_{1,3}^2 e_2 + \delta_{1,3}^3 e_3] = \delta_{3,1}^2 e_2 + \delta_{3,1}^3 e_2 + \delta_{3,1}^3 e_3 + \delta_{1,3}^2 e_2 + \delta_{1,3}^3 e_2 + \delta_{1,3}^3 e_3 = (\delta_{3,1}^2 + \delta_{3,1}^3 + \delta_{1,3}^3) e_2 + (\delta_{3,1}^3 + \delta_{1,3}^3) e_3 = 0$

$\delta_{3,1}^2 = -\delta_{3,1}^3 - \delta_{1,3}^2 - \delta_{1,3}^3 \quad \delta_{3,1}^3 = -\delta_{1,3}^3 \quad \delta_{1,3}^3 = -\delta_{3,1}^3 \quad \delta_{3,1}^2 = -\delta_{1,3}^2$

10. $e_1, e_3, e_2:$

$[e_1, \varphi(e_3, e_2)] - [e_3, \varphi(e_1, e_2)] + [e_2, \varphi(e_1, e_3)] - \varphi([e_1, e_3], e_2) + \varphi([e_1, e_2], e_3) - \varphi([e_3, e_2], e_1) = [e_1, \delta_{3,2}^2 e_2 + \delta_{3,2}^3 e_3] - [e_3, \delta_{1,2}^1 e_1] + [e_2, \delta_{1,3}^1 e_1] - \varphi(e_2, e_2) - \varphi(e_3, e_2) + \varphi(e_2, e_3) = +\delta_{3,2}^2 e_2 + \delta_{3,2}^3 e_2 + \delta_{3,2}^3 e_3 + \delta_{1,2}^1 e_2 + \delta_{1,2}^1 e_3 - \delta_{1,3}^1 e_2 - \delta_{2,2}^1 e_1 - \delta_{2,2}^4 e_4 - \delta_{3,2}^1 e_1 - \delta_{3,2}^1 e_2 - \delta_{3,2}^3 e_3 - \delta_{3,2}^4 e_4 + \delta_{2,3}^1 e_1 + \delta_{2,3}^2 e_2 + \delta_{2,3}^3 e_3 + \delta_{2,3}^4 e_4 = (\delta_{3,2}^3 + \delta_{1,2}^1 - \delta_{1,3}^1 + \delta_{2,3}^2) e_2 + (\delta_{1,2}^1 + \delta_{2,3}^3) e_3 + (\delta_{2,3}^1 - \delta_{2,2}^1 - \delta_{3,2}^1) e_1 + (\delta_{2,3}^4 - \delta_{3,2}^4 - \delta_{2,2}^4) e_4$

$\delta_{3,2}^3 = \delta_{1,3}^1 - \delta_{1,2}^1 - \delta_{2,3}^2 \quad \delta_{1,2}^1 = -\delta_{2,3}^3 \quad \delta_{2,3}^1 = \delta_{2,2}^1 + \delta_{3,2}^1 \quad \delta_{2,3}^4 = \delta_{3,2}^4 + \delta_{2,2}^4$

11. $e_1, e_3, e_3:$

$[e_1, \varphi(e_3, e_3)] - [e_3, \varphi(e_1, e_3)] + [e_3, \varphi(e_1, e_3)] - \varphi([e_1, e_3], e_3) + \varphi([e_1, e_3], e_3) - \varphi([e_3, e_3], e_1) = [e_1, \delta_{3,3}^2 e_2 + \delta_{3,3}^3 e_3] = \delta_{3,3}^2 e_2 + \delta_{3,3}^3 e_2 + \delta_{3,3}^3 e_3 = (\delta_{3,3}^2 + \delta_{3,3}^3) e_2 + \delta_{3,3}^3 e_3 = 0$

$\delta_{3,3}^2 = -\delta_{3,3}^3 \quad \delta_{3,3}^3 = 0$

12. $e_1, e_4, e_2:$

$[e_1, \varphi(e_4, e_1)] - [e_4, \varphi(e_1, e_1)] + [e_4, \varphi(e_1, e_4)] - \varphi([e_1, e_4], e_1) + \varphi([e_1, e_1], e_4) - \varphi([e_4, e_1], e_1) = [e_1, \delta_{4,1}^2 e_2 + \delta_{4,1}^3 e_3] + [e_1, \delta_{1,4}^2 e_2 + \delta_{1,4}^3 e_3] = (\delta_{4,1}^2 + \delta_{4,1}^3 + \delta_{1,4}^2 + \delta_{1,4}^3) e_2 + (\delta_{4,1}^3 + \delta_{1,4}^3) e_3 = 0$

$\delta_{4,1}^2 = -\delta_{4,1}^3 - \delta_{1,4}^2 - \delta_{1,4}^3 \quad \delta_{4,1}^3 = -\delta_{1,4}^3 \quad \delta_{4,1}^2 = -\delta_{1,4}^2$

13. $e_1, e_4, e_2:$

$[e_1, \varphi(e_4, e_2)] - [e_4, \varphi(e_1, e_2)] + [e_2, \varphi(e_1, e_4)] - \varphi([e_1, e_4], e_2) + \varphi([e_1, e_2], e_4) - \varphi([e_4, e_2], e_2) = [e_1, \delta_{4,2}^2 e_2 + \delta_{4,2}^3 e_3] - [e_2, \delta_{1,4}^1 e_1] + \delta_{2,4}^1 e_1 + \delta_{2,4}^2 e_2 + \delta_{2,4}^3 e_3 + \delta_{2,4}^4 e_4 = \delta_{4,2}^2 e_2 + \delta_{4,2}^3 e_2 + \delta_{4,2}^3 e_3 + \delta_{1,4}^1 e_2 + \delta_{2,4}^1 e_1 + \delta_{2,4}^2 e_2 + \delta_{2,4}^3 e_3 + \delta_{2,4}^4 e_4 = (\delta_{4,2}^2 + \delta_{4,2}^3 + \delta_{1,4}^1 + \delta_{2,4}^2) e_2 + (\delta_{4,2}^3 + \delta_{2,4}^3) e_3 + \delta_{2,4}^1 e_1 + \delta_{2,4}^4 e_4 = 0$

$\delta_{4,2}^2 = -\delta_{4,2}^3 - \delta_{1,4}^1 - \delta_{2,4}^2 \quad \delta_{4,2}^3 = -\delta_{2,4}^3 \quad \delta_{2,4}^1 = 0 \quad \delta_{2,4}^4 = 0$

14. $e_1, e_4, e_3:$

$[e_1, \varphi(e_4, e_3)] - [e_4, \varphi(e_1, e_3)] + [e_3, \varphi(e_1, e_4)] - \varphi([e_1, e_4], e_3) + \varphi([e_1, e_3], e_4) - \varphi([e_4, e_3], e_1) = [e_1, \delta_{4,3}^2 e_2 + \delta_{4,3}^3 e_3] + [e_3, \delta_{1,4}^1 e_1] + \delta_{2,4}^2 e_2 + \delta_{2,4}^3 e_3 + \delta_{3,4}^1 e_1 + \delta_{3,4}^2 e_2 + \delta_{3,4}^3 e_3 + \delta_{3,4}^4 e_4 = (\delta_{4,3}^2 + \delta_{4,3}^3 - \delta_{1,4}^1 + \delta_{2,4}^2 + \delta_{3,4}^2) e_2 + (\delta_{4,3}^3 - \delta_{1,4}^1 + \delta_{2,4}^3 + \delta_{3,4}^3) e_3 + \delta_{3,4}^1 e_1 + \delta_{3,4}^4 e_4 = 0$

$\delta_{4,3}^2 = -\delta_{1,4}^1 - \delta_{4,3}^3 - \delta_{2,4}^2 + \delta_{3,4}^2 \quad \delta_{4,3}^3 = \delta_{1,4}^1 - \delta_{2,4}^3 - \delta_{3,4}^3 \quad \delta_{3,4}^1 = 0 \quad \delta_{3,4}^4 = 0$

15. $e_1, e_4, e_4:$

$[e_1, \varphi(e_4, e_4)] - [e_4, \varphi(e_1, e_4)] + [e_4, \varphi(e_1, e_4)] - \varphi([e_1, e_4], e_4) + \varphi([e_1, e_4], e_4) - \varphi([e_4, e_4], e_1) =$

$[e_1, \delta^2_{4,4}e_2 + \delta^3_{4,4}e_3] = (\delta^2_{4,4} + \delta^3_{4,4})e_2 + \delta^3_{4,4}e_3 = 0$

$\delta^2_{4,4} = -\delta^3_{4,4} \quad \delta^3_{4,4} = 0$

16. $e_2, e_1, e_1:$

$[e_2, \varphi(e_1, e_1)] - [e_1, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_1)] - \varphi([e_2, e_1], e_1) + \varphi([e_2, e_1], e_1) -$

$-\varphi([e_1, e_1], e_2) = 0$

17. $e_2, e_1, e_2:$

$[e_2, \varphi(e_1, e_2)] - [e_1, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_1)] - \varphi([e_2, e_1], e_2) + \varphi([e_2, e_2], e_1) - \varphi([e_1, e_2], e_2) =$
$[e_2, \delta^1_{1,2}e_1] + [e_2, \delta^1_{2,1}e_1] + \delta^1_{2,2}e_1 + \delta^4_{2,2}e_4 - \delta^1_{2,2}e_1 - \delta^4_{2,2}e_4 = \delta^1_{1,2}e_2 - \delta^1_{2,1}e_2 = (\delta^1_{1,2} - \delta^1_{2,1})e_2 = 0$

$\delta^1_{1,2} = \delta^1_{2,1}$

18. $e_2, e_1, e_3:$

$[e_2, \varphi(e_1, e_3)] - [e_1, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_1)] - \varphi([e_2, e_1], e_3) + \varphi([e_2, e_3], e_1) - \varphi([e_1, e_3], e_2) =$
$[e_2, \delta^1_{1,3}e_1] - [e_1, \delta^2_{2,3}e_2 + \delta^3_{2,3}e_3] + [e_3, \delta^1_{2,1}e_1] + \delta^1_{2,3}e_1 + \delta^2_{2,3}e_2 + \delta^3_{2,3}e_3 + \delta^4_{2,3}e_4 - \delta^1_{2,2}e_1 - \delta^4_{2,2}e_4 -$
$\delta^1_{3,2}e_1 - \delta^2_{3,2}e_2 - \delta^3_{3,2}e_3 - \delta^4_{3,2}e_4 = -\delta^1_{1,3}e_2 - \delta^2_{2,3}e_2 - \delta^3_{2,3}e_2 - \delta^3_{2,3}e_3 + \delta^1_{2,1}e_2 + \delta^1_{2,1}e_3 + \delta^1_{2,3}e_1 + \delta^2_{2,3}e_2 +$
$\delta^3_{2,3}e_3 + \delta^4_{2,3}e_4 - \delta^1_{2,2}e_1 - \delta^4_{2,2}e_4 - \delta^1_{3,2}e_1 - \delta^2_{3,2}e_2 - \delta^3_{3,2}e_3 - \delta^4_{3,2}e_4 = (\delta^1_{2,1} - \delta^1_{1,3} - \delta^2_{3,2})e_2 + (\delta^1_{2,1} -$
$\delta^3_{2,3} - \delta^3_{3,2})e_3 + (\delta^1_{2,3} - \delta^2_{2,2} - \delta^1_{3,2})e_1 + (\delta^4_{2,3} - \delta^4_{2,2} - \delta^4_{3,2})e_4 = 0$

$\delta^1_{2,1} = \delta^1_{1,3} + \delta^2_{3,2} \quad \delta^1_{2,1} = \delta^3_{2,3} + \delta^3_{3,2} \quad \delta^1_{2,3} = \delta^1_{2,2} + \delta^1_{3,2} \quad \delta^4_{2,3} = \delta^4_{2,2} + \delta^4_{3,2}$

19. $e_2, e_1, e_4:$

$[e_2, \varphi(e_1, e_4)] - [e_1, \varphi(e_2, e_4)] + [e_4, \varphi(e_2, e_1)] - \varphi([e_2, e_1], e_4) + \varphi([e_2, e_4], e_1) - \varphi([e_1, e_4], e_2) =$
$[e_2, \delta^1_{1,2}e_1] - [e_1, \delta^2_{2,4}e_2 + \delta^3_{2,4}e_3] + \delta^2_{2,4}e_2 + \delta^3_{2,4}e_3 = -\delta^1_{1,4}e_2 - \delta^2_{2,4}e_2 - \delta^3_{2,4}e_2 - \delta^3_{2,4}e_3 + \delta^2_{2,4}e_2 + \delta^3_{2,4}e_3 =$
$(-\delta^1_{1,4} - \delta^3_{2,4})e_2$

$-\delta^1_{1,4} = \delta^3_{2,4} \quad \delta^3_{2,4} = -\delta^1_{1,4}$

20. $e_2, e_2, e_1:$

$[e_2, \varphi(e_2, e_1)] - [e_2, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_1) + \varphi([e_2, e_1], e_2) -$

$-\varphi([e_2, e_1], e_2) = 0$

21. $e_2, e_2, e_2:$

$[e_2, \varphi(e_2, e_2)] - [e_2, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_2) + \varphi([e_2, e_2], e_2) - \varphi([e_2, e_2], e_2) =$
$[e_2, \delta 2, 2^1 e_1] = -\delta^1_{2,2}e_2 = 0 \Rightarrow \delta^1_{2,2} = 0$

22. $e_2, e_2, e_3:$

$[e_2, \varphi(e_2, e_3)] - [e_2, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_3) + \varphi([e_2, e_3], e_2) -$

$-\varphi([e_2, e_3], e_2) = 0$

23. $e_2, e_2, e_3:$

$[e_2, \varphi(e_2, e_4)] - [e_2, \varphi(e_2, e_4)] + [e_4, \varphi(e_2, e_2)] - \varphi([e_2, e_2], e_4) + \varphi([e_2, e_4], e_2) -$

$-\varphi([e_2, e_4], e_2) = 0$

24. e_2, e_3, e_1 :

$[e_2, \varphi(e_3, e_1)] - [e_3, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_1) + \varphi([e_2, e_1], e_3) - \varphi([e_3, e_1], e_2) = [e_2, \delta^1_{3,1} e_1] - [e_3, \delta^1_{2,1} e_1] + [e_1, \delta^2_{2,3} e_2 + \delta^3_{2,3} e_3] - \delta^1_{2,3} e_1 - \delta^2_{2,3} e_2 - \delta^3_{2,3} e_3 - \delta^4_{2,3} e_4 + \delta^4_{2,2} e_4 + \delta^1_{3,2} e_1 + \delta^2_{3,2} e_2 + \delta^3_{3,2} e_3 + \delta^4_{3,2} e_4 = -\delta^1_{3,1} e_2 + \delta^1_{2,1} e_2 + \delta^1_{2,1} e_3 + \delta^2_{2,3} e_2 + \delta^3_{2,3} e_2 + \delta^3_{2,3} e_3 - \delta^1_{2,3} e_1 - \delta^2_{2,3} e_2 - \delta^3_{2,3} e_3 - \delta^4_{2,3} e_4 + \delta^4_{2,2} e_4 + \delta^1_{3,2} e_1 + \delta^2_{3,2} e_2 + \delta^3_{3,2} e_3 + \delta^4_{3,2} e_4 = (\delta^1_{2,1} - \delta^1_{3,1} + \delta^3_{2,3} + \delta^2_{3,2}) e_2 + (\delta^1_{2,1} + \delta^3_{3,2}) e_3 + (\delta^1_{3,2} - \delta^1_{2,3}) e_1 + (\delta^4_{2,2} + \delta^4_{3,2} - \delta^4_{2,3}) e_4$

$\delta^1_{2,1} = \delta^1_{3,1} - \delta^3_{2,3} - \delta^2_{3,2}$ $\delta^1_{2,1} = -\delta^3_{3,2}$ $\delta^1_{3,2} = \delta^1_{2,3}$ $\delta^4_{2,2} = \delta^4_{2,3} - \delta^4_{3,2}$

25. e_2, e_3, e_2 :

$[e_2, \varphi(e_3, e_1)] - [e_3, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_2) + \varphi([e_2, e_2], e_3) - \varphi([e_3, e_2], e_2) = [e_2, \delta^1_{3,2} e_1] + [e_2, \delta^1_{2,3} e_1] = -\delta^1_{3,2} e_2 - \delta^1_{2,3} e_2 = -(\delta^1_{3,2} + \delta^1_{2,3}) e_2 = 0$

$\delta^1_{3,2} = -\delta^1_{2,3}$ $-\delta^1_{2,3} = \delta^1_{2,3}$ $\delta^1_{3,2} = 0$ $\delta^1_{2,3} = 0$

26. e_2, e_3, e_3 :

$[e_2, \varphi(e_3, e_3)] - [e_3, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_3) + \varphi([e_2, e_3], e_3) - \varphi([e_3, e_3], e_2) = [e_2, \delta^1_{3,3} e_1] = -\delta^1_{3,3} e_2 = 0 \Rightarrow \delta^1_{3,3} = 0$

27. e_2, e_3, e_4 :

$[e_2, \varphi(e_3, e_4)] - [e_3, \varphi(e_2, e_4)] + [e_4, \varphi(e_2, e_3)] - \varphi([e_2, e_3], e_4) + \varphi([e_2, e_4], e_3) -$

$-\varphi([e_3, e_4], e_2) = 0$

28. e_2, e_4, e_1 :

$[e_2, \varphi(e_4, e_1)] - [e_4, \varphi(e_2, e_1)] + [e_1, \varphi(e_2, e_4)] - \varphi([e_2, e_4], e_1) + \varphi([e_2, e_1], e_4) - \varphi([e_4, e_1], e_2) = [e_2, \delta^1_{4,1} e_1] + [e_1, \delta^2_{2,4} e_2 + \delta^3_{2,4} e_3] - \delta^2_{2,4} e_2 - \delta^3_{2,4} e_3 = -\delta^1_{4,1} e_1 + \delta^2_{2,4} e_2 + \delta^3_{2,4} e_2 + \delta^3_{2,4} e_3 - \delta^2_{2,4} e_2 - \delta^3_{2,4} e_3 = (\delta^3_{2,4} - \delta^1_{4,1}) e_2$

$\delta^3_{2,4} = \delta^1_{4,1}$

29. e_2, e_4, e_2 :

$[e_2, \varphi(e_4, e_2)] - [e_4, \varphi(e_2, e_2)] + [e_2, \varphi(e_2, e_4)] - \varphi([e_2, e_4], e_2) + \varphi([e_2, e_2], e_4) - \varphi([e_4, e_2], e_2) = [e_2, \delta^1_{4,2} e_1] = -\delta^1_{4,2} e_2 = 0$

$\delta^1_{4,2} = 0$

30. e_2, e_4, e_3 :

$[e_2, \varphi(e_4, e_3)] - [e_4, \varphi(e_2, e_3)] + [e_3, \varphi(e_2, e_4)] - \varphi([e_2, e_4], e_3) + \varphi([e_2, e_3], e_4) - \varphi([e_4, e_3], e_2) = [e_2, \delta^1_{4,3} e_1] = -\delta^1_{4,3} e_2 = 0$

$\delta^1_{4,3} = 0$

31. e_2, e_4, e_3 :

$[e_2, \varphi(e_4, e_4)] - [e_4, \varphi(e_2, e_4)] + [e_4, \varphi(e_2, e_4)] - \varphi([e_2, e_4], e_4) + \varphi([e_2, e_4], e_4) - \varphi([e_4, e_4], e_2) = [e_2, \delta^1_{4,4} e_1] = -\delta^1_{4,4} e_2 = 0$

$\delta^1_{4,4} = 0$

32. $e_3, e_1, e_1:$

$[e_3, \varphi(e_1, e_1)] - [e_3, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_1) + \varphi([e_3, e_1], e_1) -$

$-\varphi([e_1, e_1], e_3) = 0$

33. $e_3, e_1, e_2:$

$[e_3, \varphi(e_1, e_2)] - [e_1, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_2) + \varphi([e_3, e_2], e_1) - \varphi([e_1, e_2], e_3) = [e_3, \delta^1_{1,2}e_1] - [e_1, \delta^2_{3,2}e_2 + \delta^3_{3,2}e_3] + [e_2, \delta^1_{3,1}e_1] + \delta^4_{2,2}e_4 + \delta^2_{3,2}e_2 + \delta^3_{3,2}e_3 + \delta^4_{3,2}e_4 - \delta^2_{2,3}e_2 - \delta^3_{2,3}e_3 - \delta^4_{2,3}e_4 = -\delta^1_{1,2}e_2 - \delta^1_{1,2}e_3 - \delta^2_{3,2}e_2 - \delta^3_{3,2}e_2 - \delta^3_{3,2}e_3 - \delta^1_{3,1}e_2 + \delta^4_{2,2}e_4 + \delta^2_{3,2}e_2 + \delta^3_{3,2}e_3 + \delta^4_{3,2}e_4 - \delta^2_{2,3}e_2 - \delta^3_{2,3}e_3 - \delta^4_{2,3}e_4 == (-\delta^1_{1,2} - \delta^3_{3,2} - \delta^1_{3,1} - \delta^2_{2,3})e_2 + (-\delta^1_{1,2} - \delta^3_{2,3})e_3 + (\delta^4_{2,2} + \delta^4_{3,2} - \delta^4_{2,3})e_4$

$\delta^1_{1,2} = -\delta^3_{3,2} - \delta^1_{3,1} - \delta^2_{2,3}$

$\delta^1_{1,2} = -\delta^3_{2,3}$

$\delta^4_{2,2} = \delta^4_{2,3} - \delta^4_{3,2}$

34. $e_3, e_1, e_3:$

$[e_3, \varphi(e_1, e_3)] - [e_1, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_3) + \varphi([e_3, e_3], e_1) - \varphi([e_1, e_3], e_3) = [e_3, \delta^1_{1,3}e_1] + [e_3, \delta^1_{3,1}e_1] = -\delta^1_{1,3}e_2 - \delta^1_{1,3}e_3 - \delta^1_{3,1}e_2 - \delta^1_{3,1}e_3 = -(\delta^1_{1,3} + \delta^1_{3,1})e_2 - (\delta^1_{1,3} + \delta^1_{3,1})e_3 = 0$

$\delta^1_{1,3} = -\delta^1_{3,1} \quad \delta^1_{1,3} = -\delta^1_{3,1}$

35. $e_3, e_1, e_4:$

$[e_3, \varphi(e_1, e_4)] - [e_1, \varphi(e_3, e_4)] + [e_4, \varphi(e_3, e_1)] - \varphi([e_3, e_1], e_4) + \varphi([e_3, e_4], e_1) - \varphi([e_1, e_4], e_3) = [e_3, \delta^1_{1,4}e_1] - [e_1, \delta^2_{3,4}e_2 + \delta^3_{3,4}e_3] + \delta^2_{2,4}e_2 + \delta^3_{2,4}e_3 + \delta^2_{3,4}e_2 + \delta^3_{3,4}e_3 = -\delta^1_{1,4}e_2 - \delta^1_{1,4}e_3 - \delta^2_{3,4}e_2 - \delta^3_{3,4}e_2 - \delta^3_{3,4}e_3 + \delta^2_{2,4}e_2 + \delta^3_{2,4}e_3 + \delta^2_{3,4}e_2 + \delta^3_{3,4}e_3 = (\delta^2_{2,4} - \delta^1_{1,4} - \delta^3_{3,4})e_2 + (\delta^3_{2,4} - \delta^1_{1,4})e_3 = 0$

$\delta^2_{2,4} = \delta^1_{1,4} + \delta^3_{3,4}$

$\delta^3_{2,4} = \delta^1_{1,4}$

36. $e_3, e_2, e_1:$ $[e_3, \varphi(e_2, e_1)] - [e_2, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_1) + \varphi([e_3, e_1], e_2) - \varphi([e_2, e_1], e_3) = [e_3, \delta^1_{2,1}e_1] - [e_2, \delta^1_{3,1}e_1] + [e_1, \delta^2_{3,2}e_2 + 2\delta^3_{3,2}e_3] - \varphi(e_3, e_2) + \varphi(e_2, e_3) = -\delta^1_{2,1}e_2 - \delta^1_{2,1}e_3 + \delta^1_{3,1}e_2 + \delta^2_{3,2}e_2 + \delta^3_{3,2}e_2 + \delta^3_{3,2}e_3 - \delta^2_{3,2}e_2 - \delta^3_{3,2}e_3 - \delta^4_{3,2}e_4 + \delta^2_{2,3}e_2 + \delta^3_{2,3}e_3 + \delta^4_{2,3}e_4 = (-\delta^1_{2,1} + \delta^1_{3,1} + \delta^3_{3,2} + \delta^2_{2,3})e_2 + (-\delta^1_{2,1} + \delta^3_{2,3})e_3 + (\delta^4_{2,3}) = 0$

$\delta^1_{3,1} = \delta^1_{2,1} - \delta^3_{3,2} - \delta^2_{2,3} \quad \delta^3_{2,3} = \delta^1_{2,1} \quad \delta^4_{2,3} = \delta^4_{3,2}$

37. $e_3, e_2, e_2:$

$[e_3, \varphi(e_2, e_2)] - [e_2, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_2) + \varphi([e_3, e_2], e_2) -$

$-\varphi([e_2, e_2], e_3) = 0$

38. $e_3, e_2, e_3:$

$[e_3, \varphi(e_2, e_3)] - [e_2, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_3) + \varphi([e_3, e_3], e_2) -$

$-\varphi([e_2, e_3], e_3) = 0$

39. e_3, e_2, e_4 :

$[e_3, \varphi(e_2, e_4)] - [e_2, \varphi(e_3, e_4)] + [e_4, \varphi(e_3, e_2)] - \varphi([e_3, e_2], e_4) + \varphi([e_3, e_4], e_2)-$

$-\varphi([e_2, e_4], e_3) = 0$

40. e_3, e_3, e_1 :

$[e_3, \varphi(e_3, e_1)] - [e_3, \varphi(e_3, e_1)] + [e_1, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_1) + \varphi([e_3, e_1], e_3)-$

$-\varphi([e_3, e_1], e_3) = 0$

41. e_3, e_3, e_2 :

$[e_3, \varphi(e_3, e_2)] - [e_3, \varphi(e_3, e_2)] + [e_2, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_2) + \varphi([e_3, e_2], e_3)-$

$-\varphi([e_3, e_2], e_3) = 0$

42. e_3, e_3, e_3 :

$[e_3, \varphi(e_3, e_3)] - [e_3, \varphi(e_3, e_3)] + [e_3, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_3) + \varphi([e_3, e_3], e_3)-$

$-\varphi([e_3, e_3], e_3) = 0$

43. e_3, e_3, e_4 :

$[e_3, \varphi(e_3, e_4)] - [e_3, \varphi(e_3, e_4)] + [e_4, \varphi(e_3, e_3)] - \varphi([e_3, e_3], e_4) + \varphi([e_3, e_4], e_3)-$

$-\varphi([e_3, e_4], e_3) = 0$

44. e_4, e_3, e_1 :

$[e_4, \varphi(e_3, e_1)] - [e_3, \varphi(e_4, e_1)] + [e_1, \varphi(e_4, e_3)] - \varphi([e_4, e_3], e_1) + \varphi([e_4, e_1], e_3) - \varphi([e_3, e_1], e_4) = -[e_3, \delta^1_{4,1} e_1] + [e_1, \delta^2_{4,3} e_2 + \delta^3_{4,3} e_3] + \varphi(e_2, e_4) + \varphi(e_3, e_4) = \delta^1_{4,1} e_2 + \delta^1_{4,1} e_3 + \delta^2_{4,3} e_2 + \delta^3_{4,3} e_2 + \delta^3_{4,3} e_3 + \delta^2_{2,4} e_2 + \delta^3_{2,4} e_3 + +\delta^2_{3,4} e_2 + \delta^3_{3,4} e_3 = (\delta^1_{4,1} + \delta^2_{4,3} + \delta^3_{4,3} + \delta^2_{2,4} + \delta^2_{3,4}) e_2 + (\delta^1_{4,1} + \delta^3_{4,3} + \delta^3_{2,4} + \delta^3_{3,4}) e_3 = 0$

$\delta^1_{4,1} = -\delta^2_{4,3} - \delta^3_{4,3} - \delta^2_{2,4} - \delta^2_{3,4} \quad \delta^1_{4,1} = -\delta^3_{4,3} - \delta^3_{2,4} - \delta^3_{3,4}$

45. e_4, e_3, e_2 :

$[e_4, \varphi(e_3, e_2)] - [e_3, \varphi(e_4, e_2)] + [e_2, \varphi(e_4, e_3)] - \varphi([e_4, e_3], e_2) + \varphi([e_4, e_2], e_3)-$

$-\varphi([e_3, e_2], e_4) = 0$

46. e_4, e_3, e_3 :

$[e_4, \varphi(e_3, e_3)] - [e_3, \varphi(e_4, e_3)] + [e_3, \varphi(e_4, e_3)] - \varphi([e_4, e_3], e_3) + \varphi([e_4, e_3], e_3)-$

$-\varphi([e_3, e_3], e_4) = 0$

47. e_4, e_3, e_4 :

$[e_4, \varphi(e_3, e_4)] - [e_3, \varphi(e_4, e_4)] + [e_4, \varphi(e_4, e_3)] - \varphi([e_4, e_3], e_4) + \varphi([e_4, e_4], e_3)-$

$-\varphi([e_3, e_4], e_4) = 0$

48. e_4, e_1, e_1 :

$[e_4, \varphi(e_1, e_1)] - [e_1, \varphi(e_4, e_1)] + [e_1, \varphi(e_4, e_1)] - \varphi([e_4, e_1], e_1) + \varphi([e_4, e_1], e_1)-$

$-\varphi([e_1, e_1], e_4) = 0$

49. e_4, e_1, e_2 :

$[e_4,\varphi(e_1,e_2)]-[e_1,\varphi(e_4,e_2)]+[e_2,\varphi(e_4,e_1)]-\varphi([e_4,e_1],e_2)+\varphi([e_4,e_2],e_1)-\varphi([e_1,e_2],e_4)=$ $-[e_1,\delta^2_{4,2}e_2+\delta^3_{4,2}e_3]+[e_2,\delta^1_{4,1}e_1]-\varphi(e_2,e_4)-\delta^2_{4,2}e_2-\delta^3_{4,2}e_2-\delta^3_{4,2}e_3-\delta^1_{4,1}e_2-\delta^2_{2,4}e_2-\delta^3_{2,4}e_3=$ $(-\delta^2_{4,2}-\delta^3_{4,2}-\delta^1_{4,1}-\delta^2_{2,4})e_2+(-\delta^3_{4,2}-\delta^3_{2,4})e_3=0$

$\delta^2_{4,2}=-\delta^3_{4,2}-\delta^1_{4,1}-\delta^2_{2,4}\quad\delta^3_{4,3}=-\delta^3_{2,4}$

50. $e_4,e_1,e_3:$

$[e_4,\varphi(e_1,e_3)]-[e_1,\varphi(e_4,e_3)]+[e_3,\varphi(e_4,e_1)]-\varphi([e_4,e_1],e_3)+\varphi([e_4,e_3],e_1)-\varphi([e_1,e_3],e_4)=$ $[e_1,\delta^2_{4,3}e_2+\delta^3_{4,3}e_3]+[e_3,\delta^1_{4,1}e_1]-\varphi(e_2,e_4)-\varphi(e_3,e_4)=\delta^2_{4,3}e_2+\delta^3_{4,3}e_2+\delta^3_{4,3}e_3-\delta^1_{4,1}e_2-\delta^1_{4,1}e_3-$ $\delta^2_{2,4}e_2-\delta^3_{2,4}e_3-\delta^2_{3,4}e_2-\delta^3_{3,4}e_3=(\delta^2_{4,3}+\delta^3_{4,3}-\delta^1_{4,1}-\delta^2_{2,4}-\delta^2_{3,4})e_2+(\delta^3_{4,3}-\delta^1_{4,1}-\delta^3_{2,4}-\delta^3_{3,4})e_3=0$

$\delta^2_{4,3}=\delta^1_{4,1}-\delta^3_{4,3}+\delta^2_{2,4}+\delta^2_{3,4}\quad\delta^3_{4,3}=\delta^1_{4,1}+\delta^3_{2,4}+\delta^3_{3,4}$

51. $e_4,e_1,e_4:$

$[e_4,\varphi(e_1,e_4)]-[e_1,\varphi(e_4,e_4)]+[e_4,\varphi(e_4,e_1)]-\varphi([e_4,e_1],e_4)+\varphi([e_4,e_4],e_1)-$

$-\varphi([e_1,e_4],e_4)=0$

52. $e_4,e_2,e_1:$

$[e_4,\varphi(e_2,e_1)]-[e_2,\varphi(e_4,e_1)]+[e_1,\varphi(e_4,e_2)]-\varphi([e_4,e_2],e_1)+\varphi([e_4,e_1],e_2)-\varphi([e_2,e_1],e_4)=$ $-[e_2,\delta^1_{4,1}e_1]+[e_1,\delta^2_{4,2}e_2+\delta^3_{4,2}e_3]+\varphi(e_2,e_4)=\delta^1_{4,1}e_2+\delta^2_{4,2}e_2+\delta^3_{4,2}e_2+\delta^3_{4,2}e_3+\delta^2_{2,4}e_2+\delta^3_{2,4}e_3=$ $(\delta^1_{4,1}+\delta^2_{4,2}+\delta^3_{4,2}+\delta^2_{2,4})e_2+(\delta^3_{4,2}+\delta^3_{2,4})e_3=0$

53. $e_4,e_2,e_2:$

$[e_4,\varphi(e_2,e_2)]-[e_2,\varphi(e_4,e_2)]+[e_2,\varphi(e_4,e_2)]-\varphi([e_4,e_2],e_2)+\varphi([e_4,e_2],e_2)-$

$-\varphi([e_2,e_2],e_4)=0$

54. $e_4,e_1,e_4:$

$[e_4,\varphi(e_2,e_3)]-[e_2,\varphi(e_4,e_3)]+[e_3,\varphi(e_4,e_2)]-\varphi([e_4,e_2],e_3)+\varphi([e_4,e_3],e_2)-$

$-\varphi([e_2,e_3],e_4)=0$

55. $e_4,e_1,e_4:$

$[e_4,\varphi(e_2,e_4)]-[e_2,\varphi(e_4,e_4)]+[e_4,\varphi(e_4,e_2)]-\varphi([e_4,e_2],e_4)+\varphi([e_4,e_4],e_2)-$

$-\varphi([e_2,e_4],e_4)=0$

56. $e_4,e_3,e_1:$

$[e_4,\varphi(e_3,e_1)]-[e_3,\varphi(e_4,e_1)]+[e_1,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_1)+\varphi([e_4,e_1],e_3)-\varphi([e_3,e_1],e_4)=$ $[e_3,\delta^1_{4,1}e_1]+[e_1,\delta^2_{4,3}e_2+\delta^3_{4,3}e_3]+\varphi(e_2,e_4)+\varphi(e_3,e_4)=-\delta^1_{4,1}e_2-\delta^1_{4,1}e_3+\delta^2_{4,3}e_2+\delta^3_{4,3}e_2+\delta^3_{4,3}e_3+$ $\delta^2_{2,4}e_2+\delta^3_{2,4}e_3+\delta^2_{3,4}e_2+\delta^3_{3,4}e_3=-\delta^1_{4,1}e_2-\delta^1_{4,1}e_3+\delta^2_{4,3}e_2+\delta^3_{4,3}e_2+\delta^3_{4,3}e_3+\delta^2_{2,4}e_2+\delta^3_{2,4}e_3+\delta^2_{3,4}e_2+\delta^3_{3,4}e_3=$ $(-\delta^1_{4,1}+\delta^2_{4,3}+\delta^3_{4,3}+\delta^2_{2,4}+\delta^2_{3,4})e_2+(-\delta^1_{4,1}+\delta^3_{4,3}+\delta^3_{2,4}+\delta^3_{3,4})e_3=0$

$\delta^1_{4,1}=\delta^2_{4,3}+\delta^3_{4,3}+\delta^2_{2,4}+\delta^2_{3,4}\quad\delta^1_{4,1}=\delta^3_{4,3}+\delta^3_{2,4}+\delta^3_{3,4}$

57. $e_4,e_3,e_2:$

$[e_4,\varphi(e_3,e_2)]-[e_3,\varphi(e_4,e_2)]+[e_2,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_2)+\varphi([e_4,e_2],e_3)-$

$-\varphi([e_3,e_2],e_4)=0$

58. $e_4,e_3,e_3:$

$[e_4,\varphi(e_3,e_3)]-[e_3,\varphi(e_4,e_3)]+[e_3,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_3)+\varphi([e_4,e_3],e_3)-$

$-\varphi([e_3,e_3],e_4)=0$

59. $e_4,e_3,e_4:$

$[e_4,\varphi(e_3,e_4)]-[e_3,\varphi(e_4,e_4)]+[e_4,\varphi(e_4,e_3)]-\varphi([e_4,e_3],e_4)+\varphi([e_4,e_4],e_3)-$

$-\varphi([e_3,e_4],e_4)=0$

60. $e_4,e_4,e_1:$

$[e_4,\varphi(e_4,e_1)]-[e_4,\varphi(e_4,e_1)]+[e_1,\varphi(e_4,e_4)]-\varphi([e_4,e_4],e_1)+\varphi([e_4,e_1],e_4)-$

$-\varphi([e_4,e_1],e_4)=[e_1,\delta^2_{4,4}e_2]=\delta^2_{4,4}e_2=0$

$\delta^2_{4,4}=0$

61. $e_4,e_4,e_2:$

$[e_4,\varphi(e_4,e_2)]-[e_4,\varphi(e_4,e_2)]+[e_2,\varphi(e_4,e_4)]-\varphi([e_4,e_4],e_2)+\varphi([e_4,e_2],e_4)-$

$-\varphi([e_4,e_2],e_4)=0$

62. $e_4,e_4,e_3:$

$[e_4,\varphi(e_4,e_3)]-[e_4,\varphi(e_4,e_3)]+[e_3,\varphi(e_4,e_4)]-\varphi([e_4,e_4],e_3)+\varphi([e_4,e_3],e_4)-$

$-\varphi([e_4,e_3],e_4)=0$

63. $e_4,e_4,e_4:$

$[e_4,\varphi(e_4,e_4)]-[e_4,\varphi(e_4,e_4)]+[e_4,\varphi(e_4,e_4)]-\varphi([e_4,e_4],e_4)+\varphi([e_4,e_4],e_4)-$

$-\varphi([e_4,e_4],e_4)=0$

64. $e_1,e_3,e_4:$

$[e_1,\varphi(e_3,e_4)]-[e_3,\varphi(e_1,e_4)]+[e_4,\varphi(e_1,e_3)]-\varphi([e_1,e_3],e_4)+\varphi([e_1,e_4],e_3)-$

$-\varphi([e_3,e_4],e_1)=[e_1,\delta^2_{3,4}e_2+\delta^3_{3,4}e_3]-[e_3,\delta^1_{1,4}e_1]-\varphi(e_3,e_4)=(\delta^3_{3,4}+\delta^1_{1,4}-\delta^3_{2,4})e_3(\delta^1_{1,4}-\delta^2_{2,4})e_2=0$

$\delta^3_{3,4}=\delta^3_{2,4}-\delta^1_{1,4}\quad\delta^1_{1,4}=\delta^2_{2,4}$

$\varphi(e_1,e_1)=\delta^4_{1,1}e_4\quad\varphi(e_1,e_2)=-\delta^3_{2,3}e_1+\delta^2_{1,2}e_2+\delta^3_{1,2}e_3+\delta^4_{1,2}e_4$

$\varphi(e_1,e_3)=\delta^1_{1,3}e_1+\delta^2_{1,3}e_2+\delta^3_{1,3}e_3+\delta^4_{1,3}e_4\quad\varphi(e_1,e_4)=\delta^2_{1,4}e_2+\delta^3_{1,4}e_3+\delta^4_{1,4}e_4$

$\varphi(e_2,e_1)=\delta^3_{2,3}e_1+\delta^2_{1,2}e_2+\delta^3_{2,1}e_3+\delta^4_{2,1}e_4\quad\varphi(e_2,e_2)=(\delta^4_{2,3}-\delta^4_{3,2})e_4$

$\varphi(e_2,e_3)=\delta^2_{2,3}e_2+\delta^3_{2,3}e_3+\delta^4_{2,3}e_4\quad\varphi(e_2,e_4)=0$

$\varphi(e_3,e_1)=-\delta^1_{1,3}e_1-\delta^2_{1,3}e_2-\delta^3_{1,3}e_3+\delta^4_{3,1}e_4\quad\varphi(e_3,e_2)=\delta^2_{3,2}e_2-\delta^3_{2,3}e_3+\delta^4_{2,3}e_4$

$\varphi(e_3,e_3)=\delta^4_{3,3}e_4\quad\varphi(e_3,e_4)=0$

$\varphi(e_4,e_1)=\delta^2_{4,1}e_2+\delta^3_{4,1}e_3+\delta^4_{4,1}e_4\quad\varphi(e_4,e_2)=\delta^4_{4,2}e_4$

$\varphi(e_4,e_3)=\delta^4_{4,3}e_4\quad\varphi(e_4,e_4)=\delta^4_{4,4}e_4\,.$

Bu to'rt o'lchamli algebrada $dimB^2(L_3) = 16 - 11 = 5$ ga teng, $dimZ^2(L_3) = 26$ ga teng. Demak, $dimH^2 = 21$. Teorema isbotlandi.

Teorema 2.3.3. Quyidagi to'rt o'lchamli Li algebrasi uchun $Z^2(L_{11}) = 6$, $B^2(L_{11}) = 4$ teng bo'lsa, u holda

$$DimH^2(L_{11}) = Z^2(L_{11})/B^2(L_{11}) \cong 2$$

.

Teorema 2.3.4. Quyidagi to'rt o'lchamli Li algebrasi uchun $Z^2(L_{13}) = 3$, $B^2(L_{13}) = 2$ teng bo'lsa, u holda

$$DimH^2(L_{13}) = Z^2(L_{13})/B^2(L_{13}) \cong 1$$

.

Ushbu teoremalarning isboti yuqoridagilari kabi isbotlanadi.

Xulosa

"Kichik o'lchamli Li algebralarining birinchi ikkinchi kogmologik gruppalari"nomli bitiruv malakaviy ishi 2 ta bob 6 paragrafdan iborat. Ushbu bitiruv malakaviy ishidan quyidagilarni xulosa qilish mumkin:

1.Birinchi bob "Li algebralariga doir asosiy tushunchalar"bo'lib, bu algebralarning berilishi, ularning differensiallari, ichki differensiallarini matritsaviy ko'rinishda olingan.

2.Ikkinchi bobi "Li algebralarining kogomologik gruppalari"ga bag'ishlangan bo'lib, unda uch va to'rt o'lchamli Li algebralarning birinchi va ikkinchi kogomologik gruppalari hisoblangan.

FOYDALANILGAN ADABIYOTLAR.

1. Khudoyberdiyev A.Kh., Omirov B.A. Infinitesimal deformations of null-filiform Leibniz superalgebras . // Journal of Geometry and Physics. Netherlands. 2013. Vol.74.-P. 370-380.(№ 39. Impact Factor Search. IF=0.797).

2. Burde D.,Steinhoff Ch. Classification of orbit closures of 4-dimensional complex Lie algebras. Journal of algebra. 1999

3. Nijenhuis A., Richardson R. W. Cohomology and deformations in graded Lie algebras. Bull. Amer. Math. Soc. 1966. Vol. 72. P.1-29.

4. Fialowski A., Millionschikov D.V. Cohomology of gradied Lie algebras of maximal class. Journal of Algebra. 2006. Vol. 296. P.157-176.

5. Khakimjanov Yu., Navarro R.M., Deformations of filiform Lie algebras and superalgebras. J. Geom. Phys. 2010. Vol. 60. P.1156-116.

6. Gerstenhaber M.On the deformation of rings and algebras. Ann. of Math. 1964. Vol. 79. N2. P.59-103.

7. Хамфрис Дж. Введение в теорию алгебр Ли и их представлений. Москва 2003. c.12-16.

8. J.C. Ndogmo and P.Winternitz, Solvable Lie algebras with abelian nilradicals, J.Phys. A 27 (1994),405-423.

9. J.M. Ancochea Bermudez, R.Campoamor-Stursberg, and L.Garcia Vergnolle, Classification of Lie algebras witz naturally graded quasi-filiform nilradicals, J.Geom. Phys. 61 (2011), 2168-2186.

10. S.Tremblay and P.Winternitz, Solvable Lie algebras with triangular nilradicals, J.Phys. A 31 (1998), 789-806.

11. G.Leger, E.Luks, Cohomology theorems for Borel-like solvabl Lie algebras in arbitrary characteristic, Canad. J.Math. 24 (1972), 1019-1026.