

Approaches to solving the problem of risk assessment with fuzzy initial information

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Abstract—An analysis of existing methods for solving the problems of risk assessment showed that they are based on the lack of computational capabilities and the lack of necessary information about the conditions of the problem. Therefore, in such cases it is advisable to use fuzzy mathematical methods. In this paper, we consider approaches to solving the problem of risk assessment with fuzzy source information.

Index Terms—uncertainty, risk, fuzzy logic, incorrect tasks, soil fertility.

I. INTRODUCTION

For complex processes characterized by uncertainty (inaccuracy, non-stochasticity, incompleteness, fuzziness) in the initial information and situations of the external and internal environment, it is usually not possible to construct simple adequate mathematical models. Information about the parameters of such processes is expressed by experts in the form of words and sentences, i.e. in a linguistic form. In such cases, it is advisable to apply modeling, decision-making and control systems using Soft Computing technology [1], [2].

With increasing complexity of the system, there arises a difficulty associated with determining the correct set of rules and membership functions for an adequate description of the behavior of the system. Fuzzy systems suffer from deficiencies in extracting additional knowledge from the results of the experiment and adjusting fuzzy rules to improve the quality of the system's operation. When evaluating alternative options for making decisions on risk assessment in conditions of uncertainty, there arises the problem of developing fuzzy models based on fuzzy rules of inference. But there is no

universal method for constructing fuzzy evaluation models. The advantage of fuzzy logic lies in the possibility of using expert knowledge about this object in the form of if "inputs", then "exits". In the process of developing a fuzzy model of risk assessment based on the conclusions of fuzzy rules, researchers often face the problem of finding approximate solutions to ill-posed problems. It should be noted that methods designed to solve the incorrect tasks of decision support systems have been developed only for a number of particular cases of models (for example, for models based on classical logic). At the same time, there is no general approach to solving fuzzy logic problems for arbitrary fuzzy systems [6], [10].

Therefore, the analysis of risk assessment tasks in fuzzy conditions, as well as modeling and algorithmic support for solving ill-posed problems formalized in the process of this analysis, are relevant for modern decision support systems.

The problems of decision-making in the risk assessment, which led to the emergence of natural, man-made and environmental disasters, are discussed in the works of V.I.Norkin and Yu.M.Ermoliev [3], V.S.Mikhalevich and P.S.Knopov, I.V.Sergienko and V.M.Yanenko [4]. The formation of risk in the economic plan was considered in the works of A.O.Nedosekin [5]. The management problems associated with risk prevention measures are discussed in the work of Yu.M.Ermoliev [3].

In work [7] results of use of systems of fuzzy logic conclusion are resulted. It deals with the basic algorithms of the fuzzy derivation of Mamdani, Tsukimoto, Sugeno and Larsen. Basically, the object of forecasting was chosen macroeconomics and financial sphere. The forecast error is

In work [8] results of researches on forecasting of a flow of tourists to Turkey are resulted which are received with the help of fuzzy regression model and model ARIMA. The model showed its stability in the process of forecasting the flow of tourists in Turkey at different times.

The paper [9] is devoted to predicting the water level on the rivers in Italy. In the forecasting process, models based on artificial neural networks and the fuzzy logical approach of Mamdani and Takagi-Sugeno were used. The input data is the water level and the ratio of the time period in which there is a period of increasing water flow and precipitation. The results of the studies showed the reliability of the neuro-fuzzy model.

II. A MODEL OF THE RISK OF REDUCING SOIL FERTILITY

Let a sample of fuzzy experimental data be given (X_r, y_r) , $r = \overline{1, M}$; here $X_r = (x_{r,1}, x_{r,2}, \dots, x_{r,n})$ - the input n-dimensional vector and y_r - the corresponding output vector.

Based on fuzzy inference rules, it is required to build a model of the risk of reducing soil fertility in the following form:

$$\text{If } (z = a_{11}^1 \vee z_1 = a_{12}^1 \vee z_2 = a_{13}^1 \vee z_3 = a_{14}^1) \wedge \dots \dots \dots$$

$$(z = a_{11}^{m_1} \vee z_1 = a_{12}^{m_1} \vee z_2 = a_{13}^{m_1} \vee z_3 = a_{14}^{m_1})$$

$$\text{Then } y = \frac{\sum_{i=1}^n \mu_{f_{0i}} f_{0i}}{\sum_{i=1}^n \mu_{f_{0i}}} + \frac{\sum_{i=1}^n \mu_{f_{1i}} f_{1i}}{\sum_{i=1}^n \mu_{f_{1i}}} z + \frac{\sum_{i=1}^n \mu_{f_{2i}} f_{2i}}{\sum_{i=1}^n \mu_{f_{2i}}} z_1 + \frac{\sum_{i=1}^n \mu_{f_{3i}} f_{3i}}{\sum_{i=1}^n \mu_{f_{3i}}} z_2 + \frac{\sum_{i=1}^n \mu_{f_{4i}} f_{4i}}{\sum_{i=1}^n \mu_{f_{4i}}} z_3.$$

$$\text{If } (x_1 = a_{21}^1 \vee x_2 = a_{22}^1 \vee x_3 = a_{23}^1 \vee \dots \vee x_8 = a_{28}^1) \wedge$$

$$\dots \dots \dots (x_1 = a_{21}^{m_2} \vee x_2 = a_{22}^{m_2} \vee x_3 = a_{23}^{m_2} \vee \dots \vee x_8 = a_{28}^{m_2})$$

$$\text{Then } z = \frac{\sum_{i=1}^n \mu_{k_{0i}} k_{0i}}{\sum_{i=1}^n \mu_{k_{0i}}} + \frac{\sum_{i=1}^n \mu_{k_{1i}} k_{1i}}{\sum_{i=1}^n \mu_{k_{1i}}} x_1 + \frac{\sum_{i=1}^n \mu_{k_{2i}} k_{2i}}{\sum_{i=1}^n \mu_{k_{2i}}} x_2 + \dots + \frac{\sum_{i=1}^n \mu_{k_{7i}} k_{7i}}{\sum_{i=1}^n \mu_{k_{7i}}} x_7 + \frac{\sum_{i=1}^n \mu_{k_{8i}} k_{8i}}{\sum_{i=1}^n \mu_{k_{8i}}} x_8.$$

$$\text{If } (x_{11} = a_{31}^1 \vee x_{12} = a_{32}^1 \vee x_{13} = a_{33}^1 \vee \dots \vee x_{17} = a_{37}^1 \vee z = a_{38}^1) \wedge$$

$$\dots \dots \dots (x_{11} = a_{31}^{m_3} \vee x_{12} = a_{32}^{m_3} \vee x_{13} = a_{33}^{m_3} \vee \dots \vee x_{17} = a_{37}^{m_3} \vee z = a_{38}^{m_3})$$

$$\text{Then } z_1 = \frac{\sum_{i=1}^n \mu_{b_{0i}} b_{0i}}{\sum_{i=1}^n \mu_{b_{0i}}} + \frac{\sum_{i=1}^n \mu_{b_{1i}} b_{1i}}{\sum_{i=1}^n \mu_{b_{1i}}} x_{11} + \frac{\sum_{i=1}^n \mu_{b_{2i}} b_{2i}}{\sum_{i=1}^n \mu_{b_{2i}}} x_{12} + \dots + \frac{\sum_{i=1}^n \mu_{b_{7i}} b_{7i}}{\sum_{i=1}^n \mu_{b_{7i}}} x_{17} + \frac{\sum_{i=1}^n \mu_{b_{8i}} b_{8i}}{\sum_{i=1}^n \mu_{b_{8i}}} z.$$

$$\text{If } (x_{21} = a_{41}^1 \vee x_{22} = a_{42}^1 \vee x_{23} = a_{43}^1 \vee \dots \vee x_{27} = a_{47}^1) \wedge$$

$$\dots \dots \dots (x_{21} = a_{41}^{m_4} \vee x_{22} = a_{42}^{m_4} \vee x_{23} = a_{43}^{m_4} \vee \dots \vee x_{27} = a_{47}^{m_4})$$

$$\text{Then } z_2 = \frac{\sum_{i=1}^n \mu_{c_{0i}} c_{0i}}{\sum_{i=1}^n \mu_{c_{0i}}} + \frac{\sum_{i=1}^n \mu_{c_{1i}} c_{1i}}{\sum_{i=1}^n \mu_{c_{1i}}} x_{21} + \frac{\sum_{i=1}^n \mu_{c_{2i}} c_{2i}}{\sum_{i=1}^n \mu_{c_{2i}}} x_{22} + \dots + \frac{\sum_{i=1}^n \mu_{c_{7i}} c_{7i}}{\sum_{i=1}^n \mu_{c_{7i}}} x_{27}.$$

$$\text{If } (x_{31} = a_{51}^1 \vee x_{32} = a_{52}^1 \vee x_{33} = a_{53}^1 \vee \dots \vee x_{37} = a_{57}^1 \vee z_2 = a_{58}^1) \wedge$$

$$\dots \dots \dots (x_{31} = a_{51}^{m_5} \vee x_{32} = a_{52}^{m_5} \vee x_{33} = a_{53}^{m_5} \vee \dots \vee x_{37} = a_{57}^{m_5} \vee z_2 = a_{58}^{m_5}) \wedge$$

$$\text{Then } z_3 = \frac{\sum_{i=1}^n \mu_{d_{0i}} d_{0i}}{\sum_{i=1}^n \mu_{d_{0i}}} + \frac{\sum_{i=1}^n \mu_{d_{1i}} d_{1i}}{\sum_{i=1}^n \mu_{d_{1i}}} x_{31} + \frac{\sum_{i=1}^n \mu_{d_{2i}} d_{2i}}{\sum_{i=1}^n \mu_{d_{2i}}} x_{32} + \dots + \frac{\sum_{i=1}^n \mu_{d_{7i}} d_{7i}}{\sum_{i=1}^n \mu_{d_{7i}}} x_{37} + \frac{\sum_{i=1}^n \mu_{d_{8i}} d_{8i}}{\sum_{i=1}^n \mu_{d_{8i}}} z_2.$$

In general, it is required to build a model based on fuzzy inference rules:

$$\bigcup_{p=1}^{k_j} \left(\bigcap_{i=1}^n x_i = a_{i,jp} - \text{with weight } w_{jp} \right) \rightarrow y_j = b_{m_0} + b_{m_1} x_1^j + \dots + b_{m_n} x_n^j.$$

III. APPROXIMATE SOLUTION OF INCORRECT PROBLEMS

In the process of constructing the model, it is necessary to find such values of the coefficients of the rules $B = (b_{ij}), i = \overline{1, n}, j = \overline{1, m}$, over which the minimum of the following expression is reached:

$$\sum_{r=\overline{1, M}} (y_r - y_r^f) \rightarrow \min \tag{1}$$

where $- y_r^f - (X_r)$ is the result of fuzzy inference rules with parameter B in the r-th row of the selection. The input vector $X_r = (x_{r,1}, x_{r,2}, \dots, x_{r,n})$ corresponds to the following result of fuzzy inference:

$$y_r^f = \frac{\sum_{j=\overline{1, m}} \mu_{d_j}(X_r) \cdot d_j}{\sum_{j=\overline{1, m}} \mu_{d_j}(X_r)} \text{ or } y_r^f = \frac{\int \mu_{d_j}(X_r) \cdot d_j dd_j}{\int \mu_{d_j}(X_r) dd_j}; \tag{2}$$

here $d_j = b_{j0} + b_{j1}x_{11} + b_{j2}x_{12} + \dots + b_{jn}x_{1n}$ is the output of the j-rule; $- \mu_{d_j}(x_r)$ the membership function corresponding to each experimental information:

$$\mu_{d_j}(X_r) = \mu_{j1}(x_{r1}) \cdot \mu_{j1}(x_{r2}) \cdot \mu_{j1}(x_{r3}) \cdot \dots \cdot \mu_{j1}(x_{rn}) \vee \mu_{j2}(x_{r1}) \cdot \mu_{j2}(x_{r2}) \cdot \mu_{j2}(x_{r3}) \cdot \dots \cdot \mu_{j2}(x_{rn}) \vee \dots \vee \mu_{jm}(x_{r1}) \cdot \mu_{jm}(x_{r2}) \cdot \mu_{jm}(x_{r3}) \cdot \dots \cdot \mu_{jm}(x_{rn});$$

$$\beta_{jr} = \frac{\mu_{d_j}(X_r) \cdot d_j}{\sum_{j=\overline{1, m}} \mu_{d_j}(X_r)} \text{ or } \beta_{jr} = \frac{\mu_{d_j}(X_r) \cdot d_j}{\int \mu_{d_j}(X_r) \partial d}.$$

Then [2] can be rewritten in the following form:

$$y_r^f = \sum_{j=1,m} \beta_{r,j} \cdot d_j = \sum_{j=1,m} (\beta_{r,j} \cdot b_{j,0} + \beta_{r,j} \cdot b_{j,1} \cdot x_{r,1} + \beta_{r,j} \cdot b_{j,2} \cdot x_{r,2} + \dots + \beta_{r,j} \cdot b_{j,n} \cdot x_{r,n})$$

We introduce the following notation:

$$Y^f = (y_1^f, y_2^f, \dots, y_M^f)^T; \\ Y = (y_1, y_2, \dots, y_M)^T;$$

$$A = \begin{bmatrix} \beta_{1,1}, \dots, \beta_{1,m}, x_{1,1} \cdot \beta_{1,1}, \dots, x_{1,n} \cdot \beta_{1,1}, \dots, x_{1,t} \cdot \beta_{1,m} \\ \vdots \\ \beta_{M,1}, \dots, \beta_{M,m}, \dots, x_{M,n} \cdot \beta_{M,1}, \dots, x_{M,t} \cdot \beta_{M,m} \end{bmatrix}$$

Then we rewrite [1] in the following matrix form: find a vector B such that condition

$$E = (Y - Y^f)^T \cdot (Y - Y^f) \rightarrow \min \quad (3)$$

The solution of problem [3] corresponds to the solution of the following equation:

$$Y = A \cdot B \quad (4)$$

In the process of developing fuzzy risk assessment models based on fuzzy inference rules, in cases where the problem [4] does not satisfy the correctness condition, often face the problem of finding an approximate solution to ill-posed problems.

The results of the subsequent mathematical analysis depend to a large extent on how adequately the initial information about the subject of the study is used in modeling, i.e. what is the degree of adequacy of the model. In connection with this, the main tasks of developing models of weakly formalized processes are [8]:

- Analysis of compact and noncompact classes of correctness. Indication of the possibility of obtaining fuzzy and fuzzy-stable solutions to ill-posed problems formalized in the process of constructing a risk assessment model using different membership functions;

- development of algorithms for solving unstable problems formalized in the process of constructing a model for estimating and predicting risk based on fuzzy sets.

The fuzzy solution of equations $Az = u$ is the primary information, expressed with the help of a fuzzy set $\bigcup_{\alpha} A_{\alpha}$

and having the following properties:

- * given the operator A and the initial data z ;
- * $\forall \alpha \in (0, 1], A_{\alpha} = \{z : \mu_A(z) \geq \alpha\}$;
- $\exists \varepsilon(\alpha) > 0, \sup_{z \in A_{\alpha}} \rho_z(A(z), A_{\alpha}) < \varepsilon(\alpha) < \infty$. where z is

the interval between the sets $A(z)$ and A .

The search for solutions of equation

$Az = u$ is reduced to the problem of finding a fuzzy solution of this equation.

Suppose that in the linear regression model $y = a_0 + a_1x_1 + a_2x_2 + \dots + a_nx_n$ - i - the estimation coefficients a_i and the input data i are given in a fuzzy form

We find the parameters of the model whose membership function is given in a Gaussian form. Let be the a_i - Gaussian fuzzy number in the linear fuzzy regression model given with the parameter (\tilde{a}_i, c_i) . Here \tilde{a}_i is the center of the fuzzy number, c_i - the latitude of the interval, $c_i > 0$.

Suppose that x_i - the input data, which are Gaussian fuzzy numbers. Let the membership functions of the input data be given in the following form:

$$\mu(x) = \begin{cases} e^{-\frac{1}{2} \frac{(x-\tilde{a})^2}{c_1^2}}, & x \leq a, \\ e^{-\frac{1}{2} \frac{(x-\tilde{a})^2}{c_2^2}}, & x > a. \end{cases}$$

Then such fuzzy numbers are defined with three parameters: (c_1, \tilde{a}, c_2) ; here \tilde{a}_i is the center of the fuzzy number; c_1 - latitude of the left interval; c_2 - the latitude of the right interval.

In this case, the problem is formed as follows: find parameters (\tilde{a}_i, c_i) of the coefficients i such that the following conditions are satisfied:

- let y_k in the equation correspond to the found interval with a degree not lower than α , $0 < \alpha < 1$;
- the latitude of the interval with degree α is minimal.

The interval of the estimate with degree α is the following: $d_{\alpha} = y_2 - y_1$. 1 and 2 can be found from the system

$$\begin{cases} \alpha = \exp\left(-\frac{1}{2} \frac{(y_2 - \tilde{a})^2}{c_2^2}\right), \\ \alpha = \exp\left(-\frac{1}{2} \frac{(y_1 - \tilde{a})^2}{c_1^2}\right). \end{cases}$$

Hence $y_2 = c_2 \sqrt{-2 \ln \alpha} + \tilde{a}$, $y_1 = c_1 \sqrt{-2 \ln \alpha} + \tilde{a}$, $d_{\alpha} = -2 \ln \alpha (c_2 + c_1)$.

Condition a) is written in the form

$$\mu(y_k) \geq \alpha \Rightarrow \begin{cases} y_k \leq \tilde{a}_k + c_{2k} \sqrt{-2 \ln \alpha}, \\ y_k \geq \tilde{a}_k - c_{1k} \sqrt{-2 \ln \alpha}. \end{cases}$$

The problem takes the following form:

$$\min \sum_{k=1}^m d_{\alpha}^k = \min \sum_{k=1}^m (c_{2k} + c_{1k}) \sqrt{-2 \ln \alpha},$$

$$\begin{cases} y_k \leq \tilde{a}_k + c_{2k} \sqrt{-2 \ln \alpha}, \\ y_k \geq \tilde{a}_k - c_{1k} \sqrt{-2 \ln \alpha}. \end{cases}$$

To find the model parameters, the membership function of which is bell-shaped, it is required to solve the following linear programming problem:

$$\begin{cases} \sum_{k=1}^m (c_{1k} + c_{2k}) \sqrt{\frac{1-\alpha}{\alpha}} \rightarrow \min, \\ y_k \leq \tilde{a}_k + c_{2k} \sqrt{\frac{1-\alpha}{\alpha}}, \\ y_k \geq \tilde{a}_k + c_{1k} \sqrt{\frac{1-\alpha}{\alpha}}. \end{cases}$$

After finding the parameters $\tilde{a}_k, c_{1k}, c_{2k}$, the type of the specified fuzzy model is determined.

The problem of optimization of weakly formalizable processes is solved on the basis of a fuzzy-multiple approach. The solutions of optimization optimization and risk forecasting problems are obtained and analyzed.

The task of forecasting the risk of reducing soil fertility is solved on the basis of an unclear approach.

The fertility of the soil is characterized by such generally accepted components of fertility as the moisture reserve, the amount of humus, nitrogen, phosphorus.

IV. COMPUTATIONAL EXPERIMENT

On the basis of experimental data, a numerical expression of the dependence of the risk of reducing the fertility of the soil on its components:

$$y = \frac{\sum_{i=1}^n \mu_{a_{0i}} a_{0i}}{\sum_{i=1}^n \mu_{a_{0i}}} + \frac{\sum_{i=1}^n \mu_{a_{1i}} a_{1i}}{\sum_{i=1}^n \mu_{a_{1i}}} x_1 + \frac{\sum_{i=1}^n \mu_{a_{2i}} a_{2i}}{\sum_{i=1}^n \mu_{a_{2i}}} x_2 + \dots + \frac{\sum_{i=1}^n \mu_{a_{7i}} a_{7i}}{\sum_{i=1}^n \mu_{a_{7i}}} x_7 + \frac{\sum_{i=1}^n \mu_{a_{8i}} a_{8i}}{\sum_{i=1}^n \mu_{a_{8i}}} x_8 \quad (5)$$

The parameters of the model are defined as fuzzy subsets, i.e. they are given through the membership functions of the corresponding subsets:

$$a_i = (\mu_{a_i}, (a'_i, a''_i));$$

where

$$\begin{aligned} \mu_{a_0} &= e^{25 \cdot 10^{-2}(x+0.93)^2}, & a_0 &\in [-0, 95; -0, 91]; \\ \mu_{a_1} &= e^{25 \cdot 10^{-2}(x+0.25)^2}, & a_1 &\in [-0, 27; -0, 23]; \\ \mu_{a_2} &= e^{25 \cdot 10^{-8}(x+0.002)^2}, & a_2 &\in [-0, 0022; -0, 0018]; \\ \mu_{a_3} &= e^{25 \cdot 10^{-8}(x-0.004)^2}, & a_3 &\in [0, 0038; 0, 0042]; \\ \mu_{a_4} &= e^{25 \cdot 10^{-6}(x-0.004)^2}, & a_4 &\in [0, 0028; 0, 0032]; \\ \mu_{a_5} &= e^{25 \cdot 10^{-2}(x+0.49)^2}, & a_5 &\in [-0, 51; -0, 47]; \\ \mu_{a_6} &= e^{25 \cdot 10^{-2}(x-0.13)^2}, & a_6 &\in [0, 11; 0, 15]; \\ \mu_{a_7} &= e^{25 \cdot 10^{-4}(x+0.05)^2}, & a_7 &\in [-0, 052; -0, 048]; \\ \mu_{a_8} &= e^{25 \cdot 10^{-4}(x+0.04)^2}, & a_8 &\in [0, 038; 0, 042]. \end{aligned}$$

The mass status of the humus in the soil is a function of the state of the system in percent; x_1 - volume of soil, g/sm^3 ; x_2 - depth of plowing, sm; x_3 - rate of phosphorus input, kg/ha; x_4 - rate of potassium intake, kg/ha; x_5 - nitrogen content in soil, %; x_6 - share of organic carbon in soil, %; x_7 - average temperature per day, %; x_8 - soil moisture, %.

According to equation (5), with an increase in the proportion of organic carbon in the soil composition, soil moisture, phosphorus and potassium rates, and also the amount of humus increase by an average of [0.11, 0.15]; [0.038, 0.042]; [0.0038; 0.0042]; [0,028,0,032], respectively. The increase in the volumetric mass of the soil, the amount of nitrogen in it, and the depth of plowing per unit reduce the proportion of humus in the soil composition by an average of [0.23, 0.27]; [0.47, 0.51] and [0.0018; 0.0022], respectively.

The fuzzy approach allows you to rely on any a priori information and obtain a fuzzy solution for a given level of accuracy of the source data.

A program has been created to solve this problem and the final values of the solution of the problem.

The value of the regularization parameter: 2.61934474110603E-0010.

On the basis of the proposed method, an approximating model for estimating and predicting risk in fuzzy conditions has been created with the verification of the solution for stability without consideration (Fig.1.2). The forecast error for the first model was 0.05-3.5 %, and for the second model it was 5.5-50.33 %.

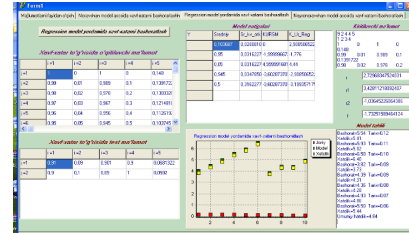


Fig. 1. The program for solving the task of risk assessment without checking for stability

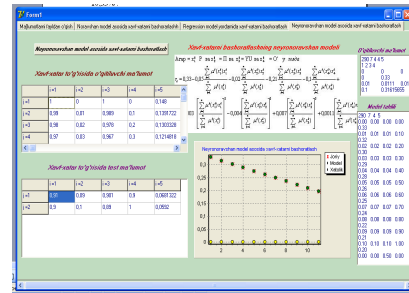


Fig. 2. The program for solving the problem of risk assessment with a test of sustainability

An algorithm for solving the problem of parametric programming based on fuzzy current information is developed.

To solve the problems of estimation, forecasting of risk and decision-making in weakly formalized systems, a program was created in the DELPHI-7 environment (Figure 3) and the results of solving the multicriterial optimization problem.

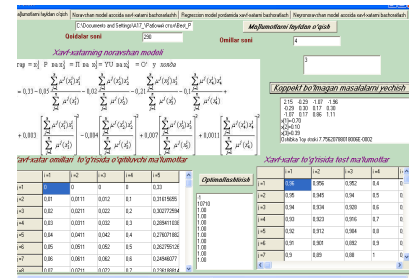


Fig. 3. Program for solving multicriterial optimization problems

V. CONCLUSION

The approach based on the expert definition of models of non-linear optimization problems in the form of fuzzy values allows the decision-maker to understand the meaning of the objective function and the constraints (semantics) of the optimization problem for weakly formalizable processes.

It is shown that the solution of the problem of optimization of weakly formalized processes on the basis of the fuzzy-multiple approach makes it possible to obtain optimal solutions for the problems of risk assessment and forecasting. Descriptions of objective functions and constraints of nonlinear optimization problems in the form of fuzzy expressions allow us to describe the problem in the form of "soft" models and obtain effective solutions.

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