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NOCHIZIQLI ISSIQLIK TARQALISH JARAYONLARINI KOMPYUTER TAHLILI (GAMILTON-YAKOBI MODELI ASOSIDA)

5A130202 - Amaliy matematika va aborot texnologiyalari

Magistr akademik darajasini olish uchun yozilgan
DISSERTATSIYA

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MUNDARIJA

Kirish

Bugungi kunda nochiziqli masalalarni o’rganish va yechish ko’p sonli olimlarni qiziqtirayotgan dolzarb masalaga aylandi. Amaliy masalalarda esa ko’p hollarda haqqiqiy fizik jarayonlar nochiziqli hisoblanib, ularni to’g’ri ta’riflash uchun nochiziqli matematik modellardan foydalanishga to’g’ri keladi.

Tadqiqot ishining dolzarbligi. Nochiziqli effektlarni beruvchi turg’un ayirmaviy sxemalarni qurish hamda zamonaviy kompyuter texnologiyalaridan foydalangan holda, taqribiy yechimni vizuallashtirish bilan bir qatorda, chiziqlashtirish usullari ham dolzarb bo'lib hisoblanadi. Matematik modellashtirishda avtomodel hamda taqribiy-avtomodel tenglamalar, shuningdek, xususiy hosilali tenglamalar nazariyasining sifatli va analitik usullarini tadbiq etgan holda hisoblash tajribasini o'tkazish muvaffaqiyatli ekanligiga amin bo'lindi.

O'rganilayotgan matematik modellarning asosiy jihati va murakkabligi, yechimning yagona emasligidir, aynan shu jihat ularni yagona yechimli klassik masalalardan ajratib turadi.

Qayd qilingan muammolarni yengish uchun bu yerda nochiziqli parchalash usulidan foydalanildi, mos chiziqlashtirish usuli, iteratsiya jarayoni qurildi, yechimni vizuallashtirish uchun C# dasturlash tilidan foydalanildi.

Mazkur **dissertatsiya ishining ilmiy yangiligi** nochiziqli issiqlik tarqalish jarayonining Gamilton-Yakobi tenglamalari bilan ifodalanuvchi nochiziqli matematik modellarini sonli va analitik tarzda o'rganish va sonli modellashtirishdan iborat.

Dissertatsiya ishining obyekti va predmeti: nochiziqli jarayonlarning matematik modellari va ularga xos bo’lgan nochiziqli effektlar, shuningdek chiziqsiz masalalarni tadqiq etish usullari.

Dissertatsiya ishining maqsadi va vazifalari:

1. Nochiziqli jarayonlarni sonli jihatdan o'rganish, olingan yechim va front baholashlari asosida natijalarni tahlil qilish;

2. Yuqorida aytib o'tilgan masalalar uchun C# dasturlash tilida sonli sxemalar, algoritm va daturiy ta’minot ishlab chiqish;

Dissertatsiya ishining asosiy masalalari va farazlari: ikki karra nochiziqli issiqlik tarqalish jarayonlarini sonli yechishda Gamilton-Yakobi tenglamasidan foydalangan holda olingan natijalarning bu usulning samarali ekanligini ko’rsatishdan iborat.

Tadqiqot mavzusi bo’yicha adabiyotlar sharhi. Nochiziqli chegaraviy masalalarni yechish har doim bir qancha qiyinchiliklar tug’dirgan, chunki ularni analatik shaklda kamdan-kam hollarda yechib bo’linadi, yechimning yangi xossalarini o’rganish esa uzoq va mashaqqatli mehnat talab qiladi A.A. Samarskiy, V.A. Galaktionov, A.S. Kalashnikov, L.K. Martinson, R. Kershner, G.I. Barenblatt, B.F.Knerr, Chen Xinfu, Qi Y.W., Jong-Sheng Guo, Kombe Ismail, Kusano Takasi, Tomoyuki Tanigava, S.N.Dimova, M.Aripov, A.T.Haydarov, SH.A.Sadullaeva va boshqalarning ishlarida parametrlarning ma'lum bir qiymatlariga mos keluvchi avtomodel yechimlarning naqadar ahamiyatli ekanligi ko'rsatilgan [1-20]. Shu bois avtomodel va taqribiy-avtomodel yondashuv asosida turli xil jarayonlarni ta'riflovchi parabolik turdagi nochiziqli chegaraviy masalalarni o'rganishga katta e'tibor beriladi.

Dissertatsiya ishida qo’llanilgan metodikaning tavsifi: dissertatsiya ishini bajarishda sonli yechish usullari, haydash usuli, chiziqlashtirish usullari, o’zgaruvchan yo’nalishlar usuli hamda iteratsion metodlardan foydalanildi.

Dissertatsiya ishning nazariy va amaliy ahamiyati: dissertatsiya tibbiyot, biologiya, geografiya, atrof - muhitni himoya qilish va boshqa sohalarda ahamiyatli.

Dissertatsiya ishning tuzilishi: dissertatsiya ishiga kirish, 3 ta bob, xulosa, foydalanilgan adabiyotlar hamda ilovadan iborat.

I BOB. NOCHIZIQLI TIPDAGI MASALALAR

Izlanishlarni osonlashtiradigan chiziqli masalalarning muhim xossasi shundan iboratki, ular uchun superpozitsiya prinsipi qondiriladi. Bu shuni anglatadiki, chiziqli tenglamaning ikkita yechimi yig`indisi yana bir yechim bo'ladi va qo'shimcha ravishda har qanday songa yechimlarning soni ko'paytirilishi mumkin, ularning summasini ham tenglamani qondiradi. Natijada, chiziqli masala yechimining har qanday son yig'indisi ushbu masalaning yechimi bo'ladi. Bu chiziqli masalaning yechimi muayyan, oddiy, yaxshi o'rganilgan yechimlar yig`indisini shakllantirishga imkon beradi.

Chiziqli bo'lmagan tenglamalarda superpozitsiya prinsipi bajarilmaydi va chiziqli masala uchun juda yaxshi ishlab chiqilgan yig’indi shaklida yechimlar yaratishning barcha usullari endi ishlamaydi.

Shunday qilib, chiziqli bo`lmagan masalalar o`rganish uchun katta qiyinchiliklarga olib keladi. Bunda analitik usullar, odatda, ishlamaydi. Bunday holatda faqat raqamli usullarga tayanish zarur. Ilm va texnologiyaning zamonaviy muammolari bilan hosil qilingan matematik modellar, odatda, nochiziqlidir. Ushbu holat, shuningdek, hisoblash tajribasini bugungi kunda amaliy masalalarda nazariy tadqiqotlarni amalga oshirishning deyarli yagona vositasi bo'lishiga olib keladi. Biroq, biz chiziqli bo'lmagan masalalarni raqamli yechimini boshlashdan oldin, ko'rib chiqilayotgan muammolar bo'yicha noaniq parametrlarga qarab turli xil yechimlarning sifat xossalarini o'rganishimiz kerak.

Nochiziqli tipdagi masalalarni o'rganish differensial tenglamalar, matematika, mexanika, fizika, biologiya, kimyo, muhandislik, boshqaruv, navigatsiya va boshqa ko'plab sohalarda nazariy va amaliy muammolarni yechish uchun asosiy omil bo’ldi. Shunday qilib, Gamilton-Yakobi tenglamasi nazariy mexanikadan yaxshi ma'lum [14], optimal boshqaruv nazariyasi - Bellman tenglamasi [15], differensial o'yinlar nazariyasidagi Ayzeks tenglamasi [16], geometrik optikada Eykon tenglamasi [17]; Burger va Xopfning gaz va gidrodinamikada chegaraviy tenglamalari [18-20] va boshqalar.

Ushbu masalalar uchun chegaraviy qiymat yechimining klassik usuli XIX asrning birinchi yarmida O.Koshi tomonidan taklif qilingan xossalar usuli hisoblanadi. Ushbu uslub birinchi tartibli qism differensial tenglamalarning integratsiyalashuvi oddiy differensial tenglamalar tizimiga integratsiyalashuvini qisqartiradi. Birinchi tartibli qism differentsial tenglamasi uchun Koshi usuli chegara qiymatining klassik yechimining grafigi xarakteristikalarga nisbatan o'zgarmas ekanligiga asoslanadi. Ushbu uslubni qo'llashning cheklanishi, chiziqli bo'lmagan qism differensial tenglamasi holatida klassik (silliq) yechim, aniq yechimga yaqin bo’lgan yechim sifatida, faqat ma’lum bir hollardagina mavjudligi bilan izohlanadi. Shu bilan birga, masalan, o'yinlarda optimal tezlikni mazmunli vaqtga, antagonistik differentsial o'yinlarda ma'lum vaqtda optimal masofada, o`xshash bo`lmagan tarqalish frontiga ega bo'lgan, silliq bo'lmagan yoki uzilmagan funksiyalar bir xil aralash kompozit muhitdagi yorug'lik to'lqini va boshqalar o`rganiladi. Ushbu silliq bo`lmagan vazifalar global miqyosda aniqlanadi, chegara holatini qondiradi va differensiallik nuqtalarida tegishli qism differensial tenglamani qondiradi. Ular ko'rib chiqilayotgan chegara qiymatining umumiy yechimlari sifatida qaralishi mumkin. Gamilton-Yakobi tenglamasi va boshqa turdagi qism differensial tenglamalarning umumiy yechimining to'g'ri kontseptsiyasini joriy qilish zarurati XX asrning 50 - 70-yillarida faol tadqiqotlarni rag'batlantirdi. N.S Baxvalov, I.M Gelfand, S.K Godunov, O.A Ladijenskaya, O.A Oleynik, B.L Rojdestvenskiy, A.A Samarskiy, S.L Sobolev, A.N Tixonov, L.C Evans, W.H Fleming, E. Hopf, P. Lax va boshqa ko'plab taniqli matematiklar asarlarida mazkur tenglamalarnig zaif yechimlarini o'rganish bilan bog'liq muammolar o'rganildi. Ushbu kuzatuvlar umuman asosiy yechimlarning integral usullari va integral xossalariga asoslanadi.

1.1-§ Issiqlik tarqalish jarayonining matematik modellari va ularning tadqiq usullari

$Q=\left\{(t,x): t>0, x\in R^N\right\}$ sohada quyidagi ikki karra nochiziqli parabolik tenglama uchun Koshi masalasini qaraylik:

$$Au \equiv \frac{\partial u}{\partial t} + \nabla\left(u^{m-1}\left|\nabla u\right|^{p-2}\nabla u\right) + \gamma(t)u^{\beta} = 0 \quad (1.1.1)$$

$$u\,|_{t=0} = u_0(x) \geq 0, \quad x \in R^N \ (1.1.2)$$

bu yerda $\beta > 0,\ p,\ m$ - sonli parametrlar, $0 < \gamma(t) \in C(0,\infty)$, $\varepsilon = \pm 1$.

(1.1.1)-(1.1.2) – masala nochiziqli muhitda $k(u,u_x) = u^{m-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}$ koeffitsiyentli issiqlik tarqalish tenglamasining matematik modeli.

(1.1.1) tenglama bir qancha fizik jarayonlarni ifodalaydi. Masalan, quvvati $\gamma(t)u^{\beta}$ bo'lgan manba $(\varepsilon = +1)$ yoki yutilish $(\varepsilon = -1)$ bilan boradigan reaksiya-diffuziya hodisasi, issiqlik o'tkazuvchanlik, nochiziqli muhitda suyuqlik va gazlarda filtratsiya jarayonlari ([8,9-11, 12, 13]).

$u = 0$ yoki $\nabla u = 0$ bo'lganda (1.1.1) tenglama buzilgan tenglamadir. Shuning uchun ham bu tenglama klassik yechimga ega bo'lmaganligi sababli umumiy yechimni qidiramiz. Berilgan tenglamaning parametrlar qiymatlariga mos holdagi yechimlari xususiyatlarini,yechimning lokallashishini, yechim asimptotikasini o'rganishimiz lozim.Qaralayotgan tenglamaning $\beta \geq 1$, $0 < \beta < 1$, $\varepsilon = +1$, $\varepsilon = -1$, $\gamma(\mathrm{t}) = 1$, $m = 1$, $p = 2$ bo'lgandagi xususiy hollardagi yechim xossalari [8, 9-11, 12, 13] ishlarda yetarlicha o'rganilgan.

Taqribiy avtomodel yechimni qurish. Quyida (1.1.1) tenglama uchun avtomodel va taqribiy avtomodel tenglama qurish uchun nochiziqli ajratish usuli keltirilgan.

Avtomodel va taqribiy avtomodel ko'rinishga kelish uchun avval quyidagi oddiy tenglamani yechamiz:

$$\frac{d\bar{u}}{dt} = -\gamma(t)\bar{u}^{\beta} \ (1.1.3)$$

(1.1.1) tenglama uchun yechimni quyidagi ko'rinishda qidiramiz:

$$u(t,x) = \bar{u}(t)\omega(\tau(t),x) \quad (1.1.4)$$

Bunda, $\bar{u}(t)=\left(T+(\beta-1)\int_0^t\gamma(\eta)d\eta\right)^{-1/(\beta-1)}$ (1.1.1) tenglamaning yechimi, (1.1.4) ni (1.1.1) ga quysak, biz taqribiy avtomodel tenglamaga ega bo'lamiz. $\omega(\tau(t),x)=f(\xi),\ \xi=\frac{x}{\tau^{1/p}}$, quyidagi taqribiy avtomodel tenglama hosil bo'ladi:

$$\frac{d}{d\xi}\left(f^{m-1}\left|\frac{df}{d\xi}\right|^{p-2}\frac{df}{d\xi}\right)+\frac{\xi}{p}\frac{df}{d\xi}+\gamma(t)\tau(t)\bar{u}^{\beta-(m+p-2)}(f\pm f^{\beta})=0 \qquad (1.1.5)$$

$$\bar{f}(\xi)=(a-b\xi^{\gamma})_+^{\gamma_1},\ \gamma=\frac{p}{p-1},\ \gamma_1=\frac{p-1}{p+m-3},\ b=(p+m-3)(\frac{1}{p})^{\frac{1}{p-1}}.$$

$\gamma(t)\tau(t)\bar{u}^{\beta-(m+p-2)}$ yetarlicha kichik bo'lsa, bu hadni tashlab yuborilganda avtomodel tenglama hosil bo'ladi.

$$Lf=\frac{d}{d\xi}\left(f^{m-1}\left|\frac{df}{d\xi}\right|^{p-2}\frac{df}{d\xi}\right)+\frac{\xi}{p}\frac{df}{d\xi} \qquad (1.1.6)$$

Agar $\gamma(t)=(T+t)^{\alpha}$ bo'lsa, $Lf+\frac{\alpha+1}{\beta-(\alpha+1)(m+p-2)}(f\pm f^{\beta})=0$ (1.1.7) kelib chiqadi.

$\tau(t)$ funksiya esa quyidagicha aniqlanadi:

$$\tau(t)=\int_0^t\bar{u}^{p+m-3}(y)dy \qquad (1.1.8)$$

Agar $\gamma(t)\tau(t)\bar{u}^{\beta-(m+p-2)}\leq 1/2$ bo'lsa,

$$L\bar{f}=-\frac{1}{2}\bar{f} \quad (1.1.9)$$

ega bo'lamiz.

$$u(t,x)\leq u_+(t,x),\qquad u_+(t,x)=\bar{u}(t)\bar{f}(\xi) \quad (1.1.10)$$

$$u_+(0,x)\leq u_0(x),\ x\in R$$

Hosil qilingan $u_+(t,x)$ esa $\gamma(t)\tau(t)\bar{u}^{\beta-(m+p-2)}\leq 1/2$ bajarilganda topilgan taqribiy avtomodel yechim bo'ladi.

Ta’rif1: Ko’rilayotgan masalaning yechimi tezlik tarqalishi cheklilik xususiyatiga eha deyiladi, agar u $|x| \geq l(t)$ da $u(t,x) \equiv 0$ bo’lsa.

Ta’rif: Agar $u(t,x) \equiv 0$bo’lsa, $\forall t > 0$ da $l(t) < +\infty$, $|x| \geq l(t)$ yechim chegaralangan bo’lsa, u holda issiqlik tarqalish fazo bo’yicha lokallashgan deyiladi.

Koshi masalasining lokallashish sharti.$\bar{u}(t) < +\infty, \forall t > 0$va $\tau(\infty) < +\infty$bo’lsin. Agar $u_0(x) \leq \bar{u}(0)\,\bar{f}(\xi)\big|_{t=0}$bo’lsa, u holda (1.1.1),(1,1,2) masalaning (1.1.10) yechimi lokallashgan bo’ladi va (1.1.1) tenglamani (1.1.6) ko’rinishga olib keladi.

Demak, (1.1.10) uchun $|x| \leq (\frac{a}{b})^{\frac{p-1}{p}}\,\tau(t)^{1/\beta-1}$.

Avtomodel yechim asimptotikasi. Quyidagi tenglama berilgan bo’lsin$\delta \neq 1$:

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\left(u^{\delta}\frac{\partial u}{\partial x}\right) \quad (1.1.11)$$

(1.1.11) tenglama uchun yechimni quyidagi ko’rinishda qidiramiz:

$$u(t,x) = f(\xi), \qquad \xi = \frac{x}{(T+t)^{1/2}} \qquad (1.1.12)$$

(1.1.12) ni (1.1.11) ga qo’yib, (1.1.1)tenglama uchun qilingan amallarni ham (1.1.11) tenglama uchun bajarganimizda quyidagi avtomodel tenglamaga ega bo’lamiz:

$$Lf = \frac{d}{d\xi}\left(f^{\delta}\frac{df}{d\xi}\right) + \frac{\xi}{2}\frac{df}{d\xi} \qquad (1.1.13)$$

$$f(0) = c > 0,\ f(d) = 0,\quad d < \infty. \qquad (1.1.14)$$

(1.1.13) avtomodel tenglama uchun yechim $\bar{f}(\xi) = (c - \frac{\delta}{4}\xi^2)^{1/\delta}$ ekanligini ko’rish qiyin emas.

(1.1.13)-(1.1.14) masalaning yechimi $\eta \to \infty$ $(\eta = -\ln\left(a - \frac{\delta}{4}\xi^2\right))$ bo'lganda $f(\xi) = \overline{f}(\xi)$ asimptotikaga ega bo'ladi.

$f(\xi) = \overline{f}(\xi)\omega(\eta)$, $\eta = -\ln\left(a - \frac{\delta}{4}\xi^2\right)$ almashtirish bajarsak, u holda (1.1.13)quyidagi ko'rinishga ega bo'ladi:

$$A = \frac{1}{\delta}\omega^{\delta}(-\frac{\omega}{\delta} + \omega_{\eta}) - \frac{2}{\xi^2\delta}\omega^{\delta}\left(a - \frac{\delta}{4}\xi^2\right)(-\frac{\omega}{\delta} + \omega_{\eta}) - \frac{1}{\delta}(-\frac{\omega}{\delta} + \omega_{\eta}) \qquad (1.1.15)$$

$\xi^2 = \frac{4(a - e^{-\eta})}{\delta}$ almashtirish orqali $A = \frac{1}{\delta}\omega^{\delta}(-\frac{\omega}{\delta} + \omega_{\eta}) = \frac{1}{2(a - e^{-\eta})}e^{-\eta}$ hosil bo'ladi.

$$\eta \to \infty,\ \omega_0^{\delta+1} - \omega_0 = 0 \qquad \Rightarrow \qquad \omega_0 = 1$$

$$\omega \to 1, \qquad f \to \overline{f} \text{ kelib chiqadi.}$$

1.2-§ Ikki karra nochiziqli parabolik tenglamalar va ularning xossalari

$Q = \{(t,x):\ 0 < t, x \in R\}$ sohada ikki karra nochiziqli kvazichiziqli tenglamani qaraylik

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(u^{m-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right) + ku\left(1 - u^{\beta}\right) \qquad (1.2.1)$$

$$u\big|_{t=0} = u_0(x)$$

u Kolmgorov-Fisher tipidagi biologik populyatsiya jarayonini ta'riflaydi, diffuziya koeffitsiyentlari esa mos ravishda $u^{m-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}$ ga teng bo'ladi, bu yerda m, p, β - musbat haqiqiy sonlar, $u = u(t,x) \geq 0$ - izlanayotgan yechim.

Bu tenglama $m = 1,\ p = 2,\ \beta = 1$holida (kubik nozihli $ku(1-u^2)$singari) (1.2.1) tenglamada kvadratik $ku(1-u)$ o'rniga Fisher tomonidan diploid populyatsiyada maqbul genning tarqalish modeli deterministik versiyasi sifatida taklif etilgan. Qayd etish joizki, nochiziqli hossalarni o'rganishda asosiy jihat turli xil yechimlarni baholash, so'ngra ular asosida masalani sonli modellashtirishdan iborat. Bunda A.A.Samarskiy, S.P.Kurdyumov, V.A.Galaktionov, L.K.Martinsonning ishlarida keltirilgan avtomodel va taqribiy-avtomodel yondashuvlar katta rol o'ynadi.([1, 3, 8])

Quyida qaralayotgan masalasining sifat hossalarini (1.2.1) uchun avtomodel tenglamani qurish yo'li bilan o'rganamiz. Avtomodel tenglamani nochiziqli parchalash usuli yordamida quramiz.([5]) (1.2.1) da $u(t,x)=e^{kt}v(\tau(t),x)$ almashtirish (1.2.1) ni

$$\frac{\partial v}{\partial \tau}=\frac{\partial}{\partial x}\left(v^{m-1}\left|\frac{\partial v}{\partial x}\right|^{p-2}\frac{\partial v}{\partial x}\right)-ke^{[(\beta-(m+p-3)k]t}v^{\beta+1}, \qquad (1.2.2)$$

$$v\big|_{t=0}=v_0(x).$$

ko'rinishga olib keladi.

$\tau(t)=\dfrac{e^{[(m+p-3)k]t}}{(m+p-3)k}$ kabi tanlab, quyidagi tenglamaga ega bo'lamiz:

$$\frac{\partial v}{\partial \tau}=\frac{\partial}{\partial x}\left(v^{m-1}\left|\frac{\partial v}{\partial x}\right|^{p-2}\frac{\partial v}{\partial x}\right)-k_1\tau^b v^{\beta+1} \qquad (1.2.3)$$

Bu yerda $k_1=k\left((m+p-3)k\right)^b$, $b=\dfrac{(\beta-m-p+3)}{(m+p-3)}$.

(1.2.3) avtomodel tenglamani olish maqsadida avvalambor $\dfrac{d\bar{v}}{d\tau}=-k_1\tau^b\bar{v}^{\beta+1}$ oddiy differencial tenglamaning $\bar{v}(\tau)=c(\tau+T_0)^{-\gamma}$ ko'rinishidagi yechimini topamiz, $T_0>0$,bu yerda $c=\left[\dfrac{\beta k_1}{b+1}\right]^{-\frac{1}{\beta}}$, $\gamma=\dfrac{b+1}{\beta}$.

So'ngra (1.2.3) tenglamaning yechimi $v(t,x)=\bar{v}(t)w(\tau,x)$ ko'rinishda izlanadi, $\tau=\tau(t)$ эса

$$\tau(\tau)=\int_0^{\tau}\bar{v}^{(m+p-3)}(t)dt=\begin{cases}\dfrac{1}{1-[\gamma(m+p-3)]}(T+\tau)^{1-[\gamma(m+p-3)]}, & агар\ 1-[\gamma(m+p-3)]\neq 0,\\ \ln(T+\tau), & агар\ 1-[\gamma(m+p-3)]=0,\\ (T+\tau), & агар\ m+p=3.\end{cases}$$

ko'rinishda tanlanadi.

U holda $w(\tau, x)$ ga nisbatan

$$\frac{\partial w}{\partial \tau} = \frac{\partial}{\partial x}\left(w^{m-1}\left|\frac{\partial w}{\partial x}\right|^{p-2}\frac{\partial w}{\partial x}\right) + \psi(\mathrm{w} - w^{\beta+1}), \qquad (1.2.4)$$

Tenglamaga ega bo'lamiz, bu yerda

$$\psi = \begin{cases} \dfrac{1}{(1-[\gamma(m+p-3)])\tau}, & агар\ 1-[\gamma(m+p-3) > 0, \\ \gamma c^{-(\gamma(m+p-3))}, & агар\ 1-[\gamma(m+p-3) = 0. \end{cases} \qquad (1.2.5)$$

Endilikda (1.2.4) uchun

$$w(\tau, x) = f(\xi), \quad \xi = |x| / \tau^{1/p} \qquad (1.2.6)$$

ko'rinishdagi avtomodel tenglamani qaraylik.

Uholda(1.2.6) ni (1.2.4) ga qo’yib $f(\xi)$ ga nisbatan:

$$L(f) = \frac{d}{d\xi}\left(f^{m-1}\left|\frac{df}{d\xi}\right|^{p-2}\frac{df}{d\xi}\right) + \frac{\xi}{p}\frac{df}{d\xi} + \mu(f - f^{\beta+1}) = 0$$

Avtomodel tenglamaga ega bo'lamiz, bu yerda $\mu = \dfrac{1}{1-[\gamma(m+p-3)]}$.

(1.2.1)tenglama uchun yuqori yechimni quraylik.

Agar $\mu = \dfrac{1}{p}$ va $\beta = \dfrac{3-(p+m)}{p-1}$ bo'lsa, u holda (1.2.1) tenglama $\bar{f}(\xi) = A(a - \xi^{\gamma})_+^{\frac{p-1}{p+m-3}}$ aniqyechimga ega bo'ladi, bu yerda $\gamma = p/(p-1)$, $(b)_+ = \max(0, b)$.

A ni $|\gamma|^{p-1} A^{m+p-3} = 1/p$ nochiziqli algebraik tenglamadan tanlaylik. a koeffitsiyentni

$$L(\bar{f}) = 0$$

dan topamiz. Bu yerda $n = \dfrac{p-1}{p+m-3}$

(1.2.1) tenglama Q sohada

$u(t, x) \le (T+t)^{-\gamma}\bar{f}(\xi)$, $\xi = |x| / \tau^{1/p}$ yuqori yechimga ega bo'ladi. Bu yerda $\bar{f}(\xi)$ yuqorida aniqlangan funksiya.

$$\frac{\partial u}{\partial t} = \nabla(|x|^n u^{(m-1)} |\nabla u^k|^{(p-2)} \nabla u^l) + \gamma(t,x)u^\beta \qquad (1.2.7)$$

Tenglamani $\Omega = \{(t,x): \quad 0 < t < T, \quad 0 < x < b\}$ sohada boshlang'ich va chegaraviy shartlari

$$u(0,x) = \psi(x) \geq 0, \; 0 \leq x \leq b \qquad (1.2.8)$$

$$\begin{cases} u(t,0) = \varphi_1(t) > 0 \\ u(t,b) = \varphi_2(t) = 0 \end{cases}, \qquad t \in [0,T] \qquad (1.2.9)$$

bo'lgan hol uchun ko'rib chiqamiz. Bu tenglama buzilgan holi uchun finit boshlang'ich funksiyali $u_0(x) \geq 0, |x| \geq 1 > 0$ Koshi masalasiga ekvivalentdir.

Teorema1. Faraz qilaylik, $\gamma_1(t)\tau(t)\bar{u}^{\beta-(m+l-1-p)} \leq \frac{S}{p}$ va $\gamma(t,x) \leq \gamma_1(t,x) > 0$

$Q = \{x \in R^N, t > 0\}$ da bo'lsin.

Bunda $u_1(t) = [T + (\beta - 1)\int_0^1 \gamma_1(\eta)d\eta]^{-\frac{1}{\beta-1}}, \; \tau(t) = \int u_1(t)^{k(p-2)+m+l-2}dt$

U holda $Q = \{x\epsilon R^N, \; t > 0\}$ sohada $u(t,x) \leq z(t,x)$bo'ladi.

Bunda $\qquad z(t,x) = u_1(t)\bar{f}(\xi),$

$\bar{f}(\xi) = (a - b\xi^{\frac{p}{p-1}})_+^{\frac{p-1}{m+l+k(p-2)-2}}$, $\xi = \varphi(x)/[\tau_1(t)]^{1/p}$, $\varphi(x) = \frac{p}{p-n}|x|^{\frac{p-n}{p}}$, $p > n$

Isbot: (1.2.7) Tenglamani quyidagicha yozamiz:

$$\frac{\partial u}{\partial t} = \nabla\left(|x|^n u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u^l\right) + \{[\gamma(t,x) - \gamma_1(t)] + \gamma_1(t)\}u^\beta$$

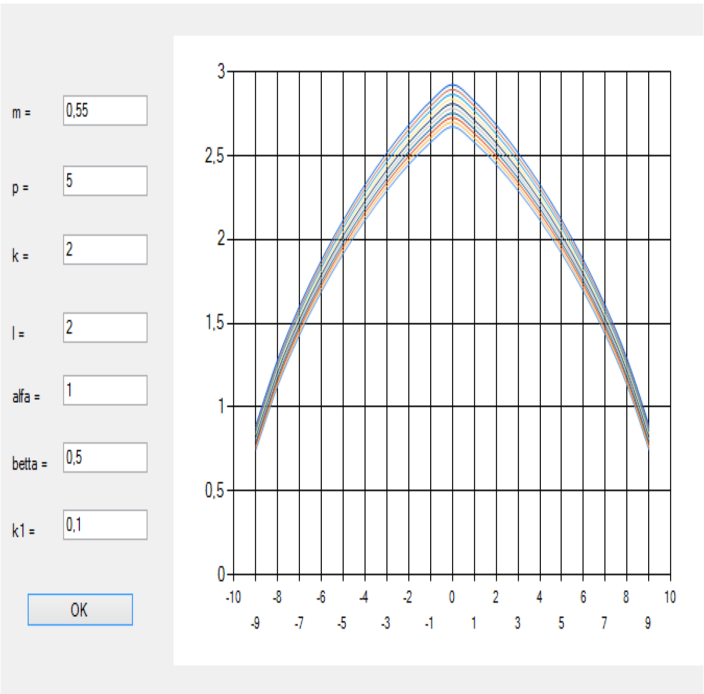
Bundan $\frac{\partial u}{\partial t} \leq \nabla\left(|x|^n u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u^l\right) + \gamma_1(t)u^\beta$ Q sohada .

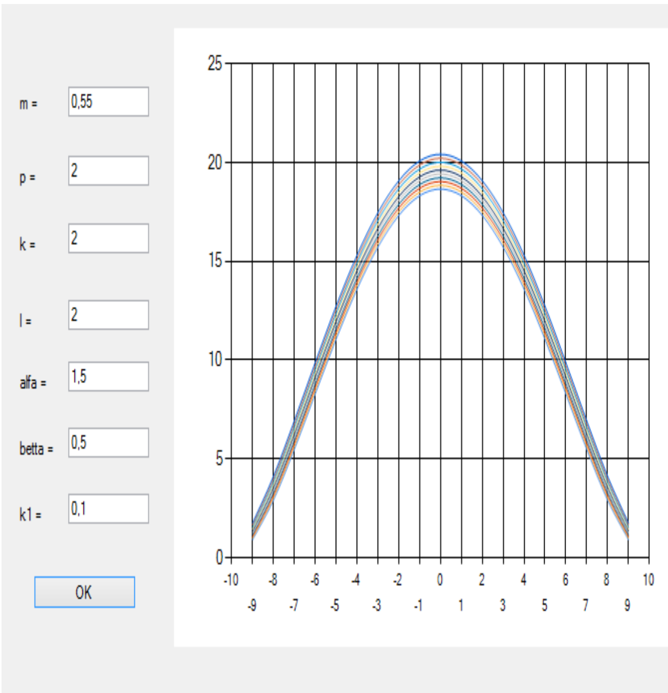
$\frac{\partial \omega}{\partial t} = \nabla\left(|x|^n u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u^l\right) + \gamma_1(t)u^\beta$ tenglamaga nisbatan solishtirish teoremasini ishtatish maqsadida, chiziqsiz ajratish algoritmini ishtatib $\omega(t,x) = u_1(t)\bar{f}(\xi)$ funksiyani topamiz.

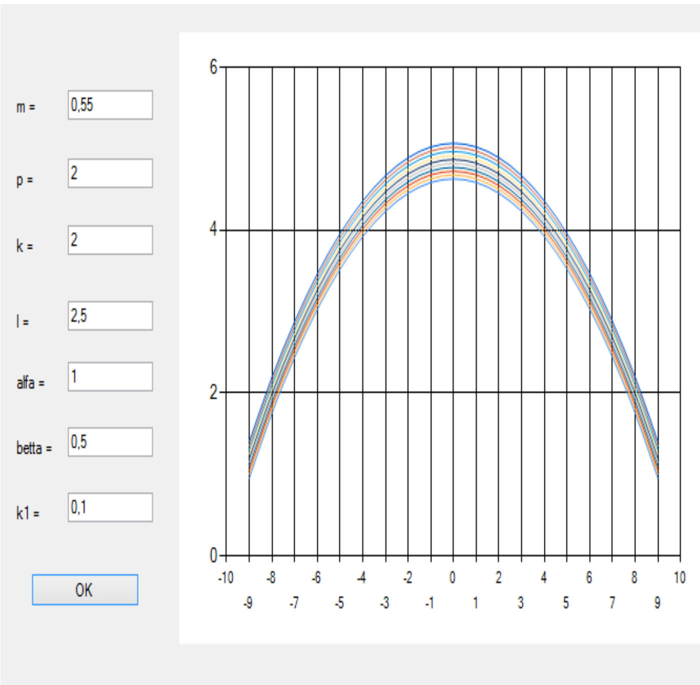
Bundan solishtirish teoremasi asosida $u(t,x) \leq \omega(t,x)$ Q sohada ekanligini tekshirish qiyin emas. Teorema1 isbotlandi.

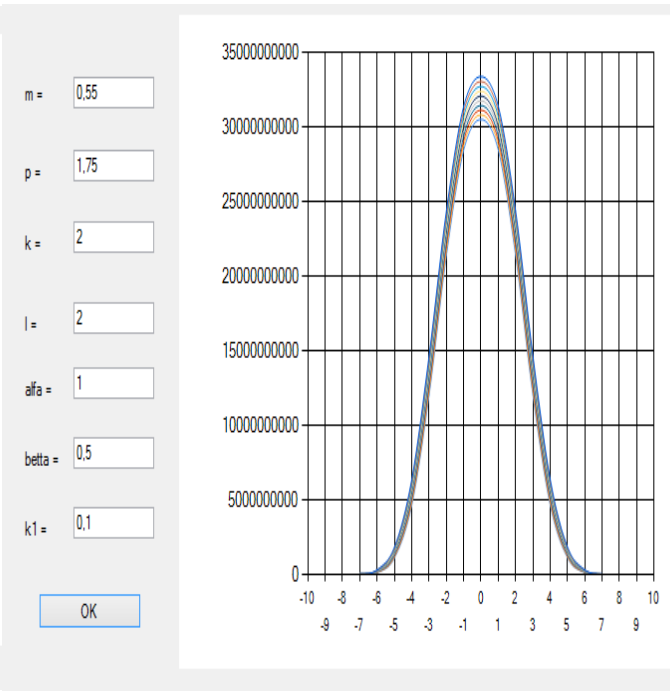
Bundan $\gamma_1(t)$ ning xususiy hollarida ($\gamma_1(t) = const, \; \gamma_1(t) = t^\alpha$ va $m, p \; va \; l$ larning xususiy qiymatida) Fujita, Samarskiy, Kurdyumov, Galaktionov va Mixaylovlarning olgan natijalari kelib chiqadi ([1,3,10,12]).

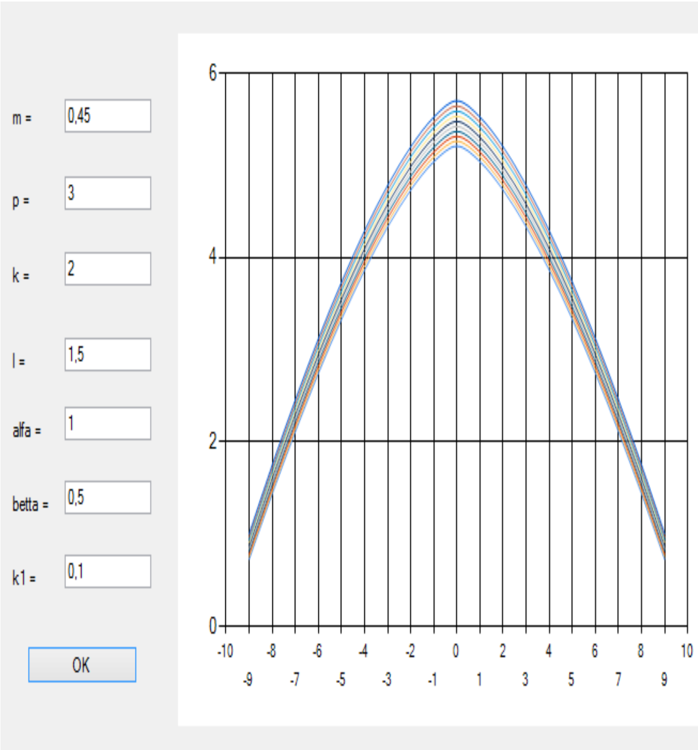
Bizning bakalavrdagi ishimizda quyidagicha hisoblash eksperimentlari o’tkazilgan va natijalar quyida keltirilgan [26].







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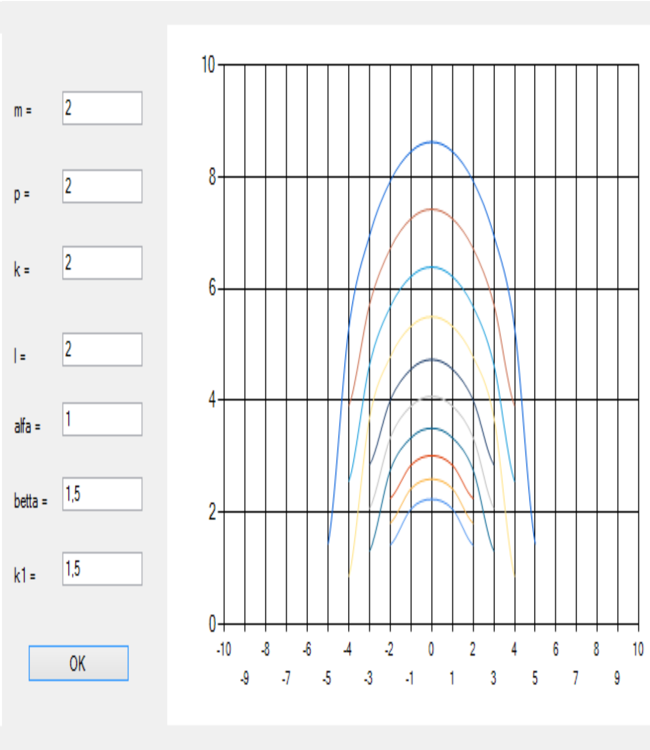
k = 2
l = 1,5
alfa = 1
betta = 0,5
k1 = 0,1
OK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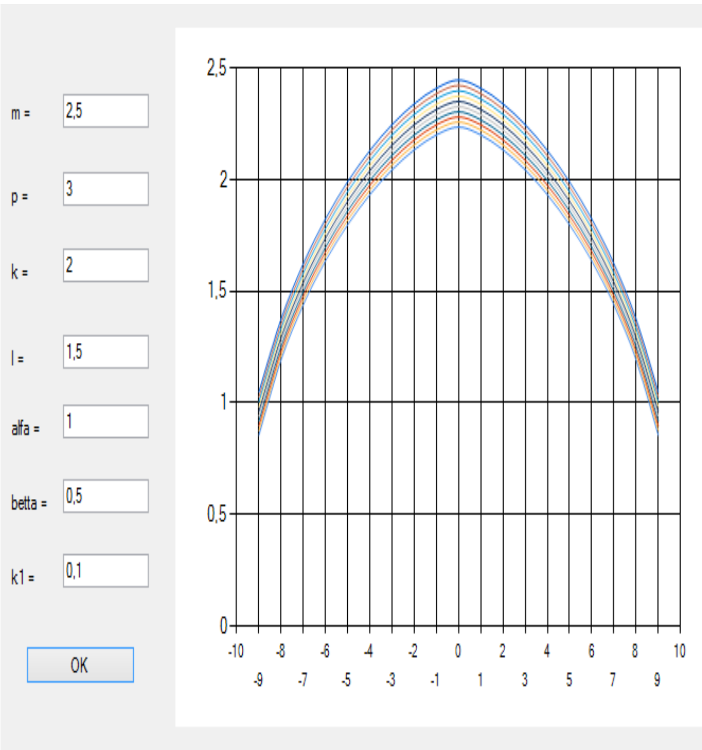
p = 2
k = 2
l = 2
alfa = 1
betta = 1,5
k1 = 1,5
OK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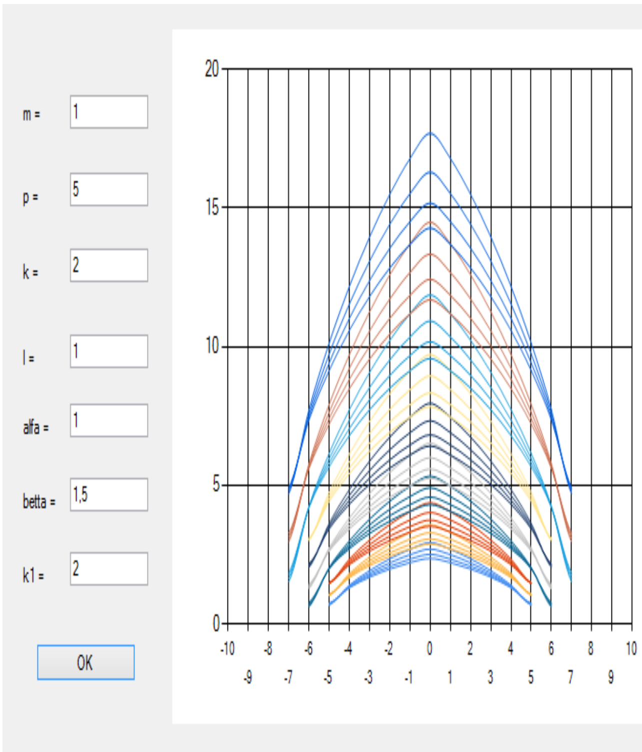
p = 3
k = 2
l = 1,5
alfa = 1
betta = 0,5
k1 = 0,1
OK


m = 1
p = 5
k = 2
l = 1
alfa = 1
betta = 1,5
k1 = 2
OK

I bob bo'yicha xulosa

Nochiziqli tipdagi masalalarni o'rganish differensial tenglamalar, matematika, mexanika, fizika, biologiya, kimyo, muhandislik, boshqaruv, navigatsiya va boshqa ko'plab sohalarda nazariy va amaliy muammolarni yechish uchun asosiy omil bo'ldi. Dissertatsiya mavzusiga asosan nochiziqli jarayonlarni o'rganish asosida matematik modelning harorat tarqalish tezligining chekliligi shartlari, lokallashishi, global yechimlarning mavjudligi ko'rsatilgan, o'z navbatida bu ilgari bu sohadagi global yechimlarning mavjudligini umumlashtiradi. Mazkur dissertatsiya ishining birinchi bobda ayrim nochiziqli jarayonlarning matematik modellari va ularga xos bo'lgan nochiziqli effektlar, shuningdek chiziqsiz masalalarni tadqiq etish usullari va ta'riflari o'rganilgan va keltirilgan.

II BOB. IKKI KARRA NOCHIZIQLI PARABOLIK TENGLAMALAR VA ULARNING GAMILTON-YAKOBI TENGLAMASI BILAN IFODALANISHI

2.1-§ Ikki karra nochiziqli parabolik tenglamaning avtomodel yechimi

$Q=\{(t,x):t>0,x\in R^N\}$ sohada quyidagi ikki karra nochiziqli parabolik tenglama uchun Koshi masalasini qaraylik:

$$\frac{\partial u}{\partial t}=\frac{\partial u}{\partial x}\left(u^{m-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}\frac{\partial u}{\partial x}\right)+\gamma(t)u^{\beta} \qquad (2.1.1)$$

$$u|_{t=0}=u_0(x)\ge 0,\quad x\in R^N \quad (2.1.2)$$

bu yerda $\beta>0,\ p,m$ - sonli parametrlar, $0<\gamma(t)\in C(0,\infty)$, $\varepsilon=\pm 1.$

(2.1.1)-(2.1.2) – masala nochiziqli muhitda $k(u,u_x)=u^{m-1}\left|\frac{\partial u}{\partial x}\right|^{p-2}$ koeffitsiyentli issiqlik tarqalish tenglamasining matematik modeli.

(2.1.1) tenglama bir qancha fizik jarayonlarni ifodalaydi. Masalan, quvvati $\gamma(t)u^{\beta}$ bo’lgan manba $(\varepsilon=+1)$ yoki yutilish $(\varepsilon=-1)$ bilan boradigan reaksiya-diffuziya hodisasi, issiqlik o’tkazuvchanlik, nochiziqli muhitda suyuqlik va gazlarda filtratsiya jarayonlari ([8,9-11, 12, 13]).

$u=0$ yoki $\nabla u=0$ bo’lganda (2.1.1) tenglama buzilgan tenglamadir. Shuning uchun (2.1.1) tenglama klassik yechimga ega bo’lmaganligi sababli umumiy yechimni qidiramiz. Berilgan tenglamaning parametrlar qiymatlariga mos holdagi yechimlari xususiyatlarini, yechimning lokallashishini, yechim asimptotikasini o’rganishimiz lozim.(2.1.1) tenglamaning $\beta\ge 1,$ $0<\beta<1,\ \varepsilon=+1,\ \varepsilon=-1,\ \gamma(\mathrm{t})=1,\ m=1,\ p=2$ bo’lgandagi xususiy hollardagi yechim xossalari [8, 9-11, 12, 13] ishlarda yetarlicha o’rganilgan.

Quyida (2.1.1) tenglama uchun avtomodel va taqribiy avtomodel tenglama qurish uchun nochiziqli ajratish usuli keltirilgan.

Avtomodel va taqribiy avtomodel ko’rinishga kelish uchun avval quyidagi oddiy tenglamani yechamiz:

$$\frac{d\bar{u}}{dt} = -\gamma(t)\bar{u}^{\beta} \quad (2.1.3)$$

(2.1.1) tenglama uchun yechimni quyidagi ko'rinishda qidiramiz:

$$u(t,x) = \bar{u}(t)\omega(\tau(t),x) \qquad (2.1.4)$$

Bunda, $\bar{u}(t) = \left(T + (\beta - 1)\int_0^t \gamma(\eta)d\eta \right)^{-1/(\beta-1)}$ (2.1.1) tenglamaning yechimi, (2.1.4) ni (2.1.1) ga quysak, biz taqribiy avtomodel tenglamaga ega bo'lamiz. $\omega(\tau(t),x) = f(\xi),\ \xi = \frac{x}{\tau^{1/p}}$, quyidagi taqribiy avtomodel tenglama hosil bo'ladi:

$$\frac{d}{d\xi}\left(f^{m-1}\left|\frac{df}{d\xi}\right|^{p-2}\frac{df}{d\xi} \right) + \frac{\xi}{p}\frac{df}{d\xi} + \gamma(t)\tau(t)\bar{u}^{\beta-(m+p-2)}(f \pm f^{\beta}) = 0 \qquad (2.1.5)$$

$$\bar{f}(\xi) = (a - b\xi^{\gamma})_{+}^{\gamma_1},\ \gamma = \frac{p}{p-1},\ \gamma_1 = \frac{p-1}{p+m-3},\ b = (p+m-3)(\frac{1}{p})^{\frac{1}{p-1}}.$$

$\gamma(t)\tau(t)\bar{u}^{\beta-(m+p-2)}$ yetarlicha kichik bo'lsa, bu hadni tashlab yuborilganda avtomodel tenglama hosil bo'ladi.

$$Lf = \frac{d}{d\xi}\left(f^{m-1}\left|\frac{df}{d\xi}\right|^{p-2}\frac{df}{d\xi} \right) + \frac{\xi}{p}\frac{df}{d\xi} \qquad (2.1.6)$$

Agar $\gamma(t) = (T+t)^{\alpha}$ bo'lsa, $Lf + \frac{\alpha+1}{\beta-(\alpha+1)(m+p-2)}(f \pm f^{\beta}) = 0$ (2.1.7) kelib chiqadi.

$\tau(t)$ funksiya esa quyidagicha aniqlanadi:

$$\tau(t) = \int_0^t \bar{u}^{p+m-3}(y)dy \qquad (2.1.8)$$

Agar $\gamma(t)\tau(t)\bar{u}^{\beta-(m+p-2)} \le 1/2$ bo'lsa,

$$L\bar{f} = -\frac{1}{2}\bar{f} \quad (2.1.9)$$

ega bo'lamiz.

$$u(t,x) \le u_{+}(t,x), \qquad u_{+}(t,x) = \bar{u}(t)\bar{f}(\xi) \quad (2.1.10)$$

$$u_+(0,x) \le u_0(x), \ \ x \in R$$

Hosil qilingan $u_+(t,x)$ esa $\gamma(t)\tau(t)\bar{u}^{\beta-(m+p-2)} \le 1/2$ bajarilganda topilgan taqribiy avtomodel yechim bo'ladi.

2.2-§ Gamilton-Yakobi tenglamasining yechimlari orqali nochiziqli ikkinchi tartibli parabolik tenglamalarning ifodalanishi

Fan va texnikada juda ko'p sonli matematik modellar nochiziqli qoidalarga asoslangan holda boradi. Ularning xususiyatlarini o'rganishda yangicha yondashuvlar va usullar lozim. Chiziqli bo'lmagan jarayonlarni o'rganish yangi muammolarning doimiy manbai bo'lib kelgan va bu esa o'z navbatida matematik analiz, differensial tenglamalar va boshqa bir necha sohalarda yangi metodlarning kirib kelishiga turtki bo'la oldi.

Chiziqli masalalardan chiziqli bo'lmagan masalalarning farqli jihatini ko'rsatib beruvchi eng muhim xususiyatlaridan biri – bu xususiyatlari paydo bo'lishi ehtimoli, hatto umuman silliq berilganlar bo'lsa ham, yoki yana ham aniqroq qilib aytganda, mavjudlik, yagonalik teoremalari uchun va kichik vaqt oralig'ida o'rnatilgan doimiy bog'liqliklar uchun berilganlar sinfida. Nochiziqli masalalarda hattoki, silliq koeffitsiyentlarda ham keskinlashish rejimida chegaralanmagan yechim hosil bo'lishi mumkin. ([1])

Mazkur dissertatsiya ishida $Q_T = \{ \ (t,x): \ 0 < t < T, \ x \in R_+ \}$ sohada ikki karra nochiziqli issiqlik tarqalish tenglamasi uchun quyidagi masalaning yechimi xossalari o'rganilgan:

$$L(u) = -\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(u^{m-1} \left| \frac{\partial u^k}{\partial x} \right|^{p-2} \frac{\partial u^l}{\partial x} \right) = 0 \qquad (2.2.1)$$

$$\begin{gathered} u|_{t=0} = u_0(x) \ge 0, u|_{x=0} = (T-t)^{-\alpha}, \ 0 < t < T, \ \alpha > 0 \\ u|_{t=0} = u_0(x) \ge 0, u|_{x=0} = (T+t)^{-\alpha}, \ t > 0, \ \alpha > 0 \end{gathered} \qquad (2.2.2)$$

bu yerda, $m, \mathrm{p}, \mathrm{k}, \mathrm{l} \in R$ - nochiziqli muhit xossalarini xarakterlovchi sonli parametrlar.

(2.2.2) dagi birinchi chegaraviy shart keskinlashish rejimi (режим с обострением) deb ataladi.

(2.2.1), (2.2.2) masala issiqlikning tarqalish jarayonini, suyuqlik va gazlarda filtiratsiya, $\left(u^{m-1}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u^l}{\partial x}\right)$ koeffitsiyentli nochiziqli muhitda diffuziya masalalarini ifodalaydi. Xususan, (1) tenglama $(m+\text{p-}3)>0$ bo'lganda sekin diffuziya tenglamasi, $2<(m+\text{ p})<3$ tez diffuziya tenglamasi, $0<(m+\text{ p})<2$ holda esa o'ta tez diffuziya hodisasi, shuningdek, ($m=1$) p-Laplas tenglamasi va $\text{p}=2$ bo'lganda pufakli muhit tenglamalarini (УПС) mos holda ifodalaydi.

[1] adabiyotda (2.2.1) tenglamaning $\text{p}=2$ bo'lgan xususiy holi uchun Gamilton-Yakobi tenglamasini yechish orqali issiqlik tarqalish jarayoni evolyutsiyasi asimptotik bosqichida yechimlarni ifodalash imkoniyati bo'ladi.

(2.2.1), (2.2.2) masalaning yechimi xusussiyatlarini o'rganishda, ular (2.2.1) tenglamaning o'rniga quyida keltirilgan Gamilton-Yakobi tenglamasini qo'yishgan.

$$\frac{\partial u}{\partial t}=lk^{p-2}u^{m+l-3}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\left(\frac{\partial u}{\partial x}\right)^2 \qquad (2.2.3)$$

Keyinchalik qilingan bir qator izlanishlarda (masalan, [2]) esa, yanada umumiyroq $\frac{\partial u}{\partial t}=\frac{\partial u}{\partial x}\left(k(u)\frac{\partial u}{\partial x}\right)$tenglama uchun, quyidagi $\frac{\partial u}{\partial t}=\frac{k(u)}{u}\left(\frac{\partial u}{\partial x}\right)^2$ Gamilton-Yakobi tenglamasini o'rganish orqali bu yondashuv yanada rivojlantirilgan.

Yuqorida keltirilgan ishlarni rivojlantirish maqsadidagi mazkur dissertatsiya ishida (2.2.2) boshlang'ich va chegaraviy shartlar bilan berilgan quyidagi Gamilton-Yakobi tenglamasining yechimlari xossalarini o'rganishga asoslangan holda (2.2.1) ikki karra nochiziqli parabolik tenglamaning yechimlari xossalarini o'rganish taklif qilingan.

$$\frac{\partial u}{\partial t}=lk^{p-2}u^{m+l-3}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\left(\frac{\partial u}{\partial x}\right)^2$$

(2.2.1) tenglama quyidagi ko'rinishdagi avtomodel yechimga ega ekanligini qayd qilib o'tamiz.

$$u(t,x) = (T-t)^{-\alpha} f(\xi),\ \xi = x[\tau(t)]^{-1/p},$$

bunda $\tau(t) = (T-t)^{1-(m+l+k(p-2)-2))/\alpha} / [1-(m+l+k(p-2)-2))/\alpha]$ (2.2.4)

shuningdek, $f(\xi)$ funksiya quyidagi oddiy differensial tenglamani qanoatlantiradi:

$$\frac{d}{d\xi}\left(f^{m-1}\left|\frac{df^k}{d\xi}\right|^{p-2}\frac{df^l}{d\xi}\right) + \frac{\xi}{p}\frac{df}{d\xi} = 0$$

Shuningdek, (2.2.3) Gamilton-Yakobi tenglamasi esa quyidagi ko'rinishdagi avtomodel yechimga ega ekanligini qayd qilib o'tamiz.

$$u(t,x) = (T+t)^{-\alpha} f(\xi),\ \xi = x[\tau(t)]^{-1/p},$$

bunda, $\tau(t) = (T+t)^{(1-(m+l+k(p-2)-2))/\alpha} / [(1-(m+l+k(p-2)-2))/\alpha]$

$f(\xi)$ funksiya quyidagi avtomodel tenglamani qanoatlantiradi:

$$f^{m+l+k(p-2)-2}\left|\frac{df}{d\xi}\right|^{p-2}\left(\frac{df}{d\xi}\right)^2 + \frac{\xi}{p}\frac{df}{d\xi} + \frac{\alpha}{1-\alpha(m+l+k(p-2)-1)} f = 0 \qquad (2.2.5)$$

$\alpha = 2(p-1)+m$ bo'lganda funksiya ko'rinishi quyidagicha bo'ladi:

$$f(\xi) = A\left(c - \xi^{\frac{p}{p-1}}\right)_+^{\frac{p-1}{m+l+k(p-2)-2}}, \qquad (2.2.6)$$

bunda,

$$(n)_+ = \max(0,n)$$

Quyidagi funksiyani keltiramiz:

$$z(t,x) = A(T+t)^{-\alpha}\left(c - x(T+t)^{-1/p}\right)^{p/(p-1)}\Big)_+^{(p-1)/(m+l+k(p-2)-2)},\ x\in R_+,\ c>0$$

(2.2.7)

buyerda, $z(t,x)$- funksiya (2.2.3) Gamilton-Yakobi tenglamasining yechimi.

Teorema1. (2.2.6) ifodadagi doimiy uchun $A^{m+l+k(p-2)-3} > (m+l+k(p-2)-2)]^{p-1} / p^p$,

$m+l+k(p-2)-2>0, u \quad \dfrac{\alpha}{1-\alpha(m+l+k(p-2)-1)} \le 1/p$ hamda

$u_0(x) \le z(0,x) = AT^{-\alpha}(c-(xT^{-1/p})^{p/(p-1)})_+^{(p-1)/(m+l+k(p-2)-2)}, \ x \in R_+, \ c>0$ lar o'rinli bo'lsin. U holda (2.2.1), (2.2.2) masalaning umumlashgan yechimi uchun Q_∞ sohada $u(t,x) \le z(t,x)$ tengsizlik o'rinli bo'ladi hamda yechim ta'sirning chekli tezlikda tarqalish(KCPB) xususiyatga ega bo'ladi. Bunda $z(t,x)$- funksiya (2.2.5) formula orqali keltirilgan Gamilton-Yakobi tenglamasining yechimi.

Teorema2. (1) tenglamada m+l+k(p-2)-2>0. bo'lsin. U holda, (2.2.1), (2.2.2) tenglamaning yechimi fazoda lokallashgan deyiladi, agar

$1-(m+l+k(p-2)-1)\alpha<0$ bajarilsa,

hamda lokallashmagan deyiladi, agar $1-(m+l+k(p-2)-1)\alpha \ge 0$ bo'lsa.

Global yechim mavjudligi sharti va (2.2.1) tenglama uchun Koshi masalasi yechimining buzilish sharti [4] da o'rganilgan.

II bob bo'yicha xulosa

Mazkur dissertatsiya ishining II bobida jarayon xossalarini Gamilton-Yakobi tenglamalari bilan tasvirlanuvchi modellar orqali ham ifodalash mumkinligi ko'rsatildi. Chunki Gamilton-Yakobi tenglamalari modellari birinchi tartibli tenglamalar bilan ifodalanadigan nochiziqli tenglamalardir, boshlang'ich matematik modeli esa ikkinchi tartibli tenglamalar bilan tasvirlanadi. Nochiziqli jarayon xossalarini o'rganishda, birinchi tartibli tenglamalar ikkinchi tartibli tenglamalarga qaraganda osonroq. Shu bilan birgalikda Gamilton-Yakobi tenglamasining yangi xossalari ham o'rganildi.

III BOB. GAMILTON-YAKOBI TENGLAMASINING YECHIMI BAHOSI

3.1-§ Nochiziqli ikkinchi tartibli issiqlik tarqalish tanglamasi va Gamilton-Yakobi tenglamasi yechim xossalarini solishtirish

Mazkur ishda $Q_T = \{\ (t,x):\ 0 < t < T,\ x \in R_+\}$ sohada ikki karra nochiziqli issiqlik tarqalish tenglamasi uchun quyidagi masalaning yechimi xossalari o'rganilgan:

$$L(u) = -\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(u^{m-1}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u^l}{\partial x}\right) = 0 \qquad (3.1.1)$$

$$\begin{array}{l} u|_{t=0} = u_0(x) \ge 0, u|_{x=0} = (T-t)^{-\alpha},\ 0 < t < T,\ \alpha > 0 \\ u|_{t=0} = u_0(x) \ge 0, u|_{x=0} = (T+t)^{-\alpha},\ t > 0,\ \alpha > 0 \end{array} \qquad (3.1.2)$$

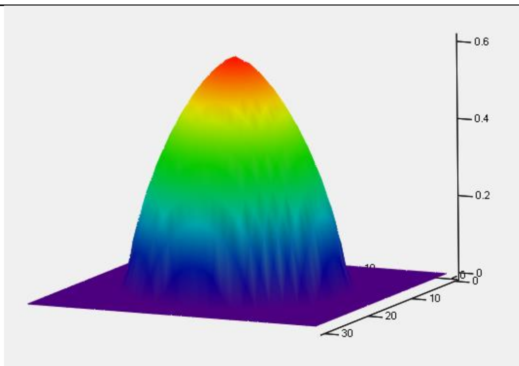
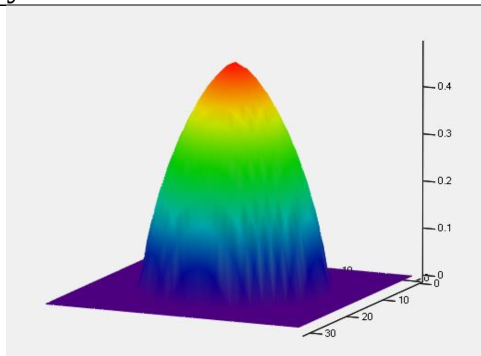
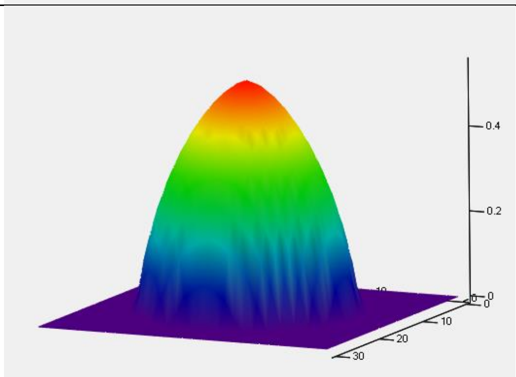
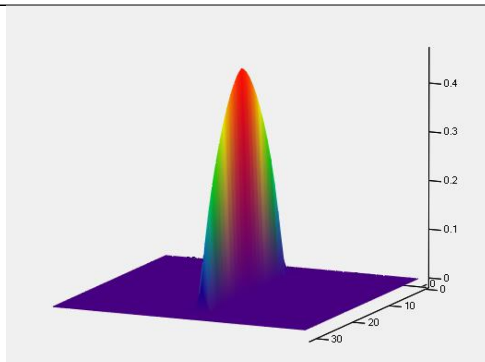
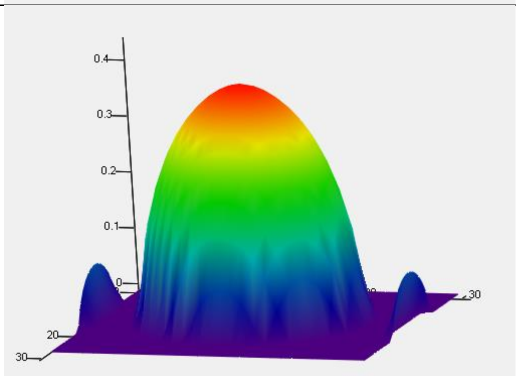
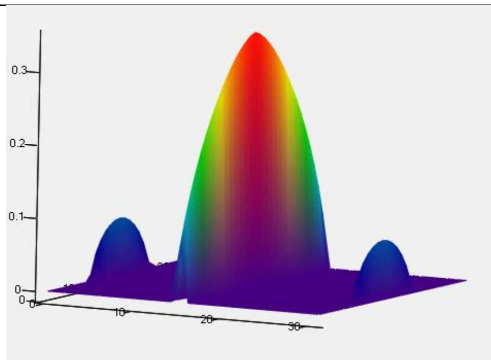
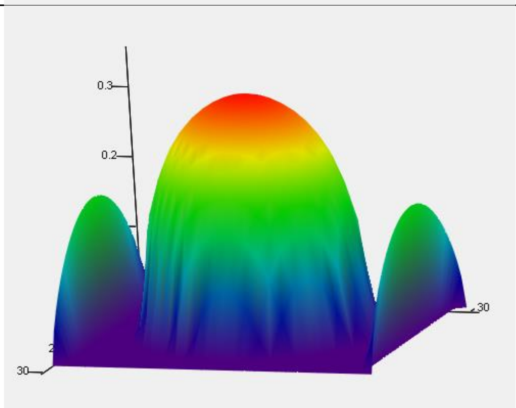
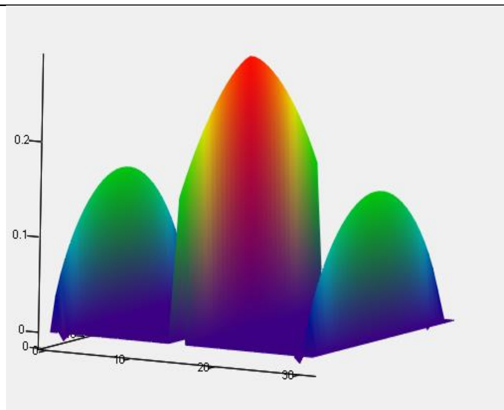
bu yerda, $m, \mathrm{p}, \mathrm{k}, \mathrm{l} \in R$- nochiziqli muhit xossalarini xarakterlovchi sonli parametrlar.

(3.1.2) dagi birinchi chegaraviy shartkeskinlashish rejimi(режимсобострением) deb ataladi.

(3.1.1), (3.1.2) masala issiqlikning tarqalish jarayonini, suyuqlik va gazlarda filtiratsiya, $\left(u^{m-1}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u^l}{\partial x}\right)$ koeffitsiyentli nochiziqli muhitda diffuziya masalalarini ifodalaydi. Xususan, (1) tenglama $(m+\ \mathrm{p}\text{-}3)>0$ bo'lganda sekin diffuziya tenglamasi, $2 < (m+\ \mathrm{p}) < 3$tez diffuziya tenglamasi, $0 < (m+\ \mathrm{p}) < 2$ holda esa o'ta tez diffuziya hodisasi, shuningdek, ($m=1$) p-Laplas tenglamasi va $\mathrm{p}=2$ bo'lganda pufakli muhit tenglamalarini (УПС) mos holda ifodalaydi.

[1] adabiyotda (3.1.1) tenglamaning $\mathrm{p}=2$ bo'lgan xususiy holi uchun Gamilton-Yakobi tenglamasini yechish orqali issiqlik tarqalish jarayoni evolyutsiyasi asimptotik bosqichida yechimlarni ifodalash imkoniyati bo'ladi.

(3.1.1) tenglamani sonli usullardan foydalangan holda yechish orqali nochiziqli ikkinchi tartibli issiqlik tarqalish tanglamasi va Gamilton-Yakobi tenglamasi yechim xossalarini solishtirildi. Natijalar grafik va vizual ko'rinishda berilishi orqali solishtirish yaqqol ko’rsatilgan. Shuningdek, natijalaar sonli parametrlarning turli qiymatlarida bir necha hollarda quyida keltirilgan.

p=3, m=1,7, k=1,75, l=1,5, a=0,75, T=1,5		
	Gamilton-Yakobi tenglamasi yechimi	Nochiziqli ikkinchi tartibli issiqlik tarqalish tanglamasi yechimi
t=1		
t=2		
t=6		
t=7		

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0,490985438653378 0,469419972570485 0,407660416307938 0,306944648750096 0,150786508385884 0 0 0
0,387548897882992 0,367502138715496 0,306944648750096 0,19938874533211 0 0 0 0
0,244044215728346 0,222086504345284 0,150786508385884 0 0 0 0 0
0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0

1-qatlam
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0,449467066984432 0,43330767502446 0,383123397026435 0,2955581174502 0,148342774555237 0 0 0
0,36342168049491 0,346779594089055 0,294097484214615 0,195405932020612 0 0 0 0
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2-qatlam
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0,228016564614712 0,209476711195901 0,145976183203796 0 0 0 0 0
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0 0 0 0 0 0 0 0

3-qatlam
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0,220882699876129 0,203742499628748 0,143683727291236 0 0 0 0 0
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0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

4-qatlam
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0,362988823293112 0,354328529926423 0,324268743100247 0,265059304575171 0,141462492639055 0 0 0,0833285101745874
0,308313066278044 0,297688597296261 0,261178665843917 0,184160031270211 0 0 0 0,0364360880470368
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0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

5-qatlam
0,384252912534223 0,373942474828212 0,348550976438619 0,303104598423133 0,22015370443064 0 0 0,154816110839084
0,37060440986467 0,363368146489313 0,338720541092402 0,292141163796316 0,200332537067338 0 0 0,149932955130199
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0,208063339335682 0,19325883955404 0,13930966262331 0 0 0 0 0,0662745765622975
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

6-qatlam

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0,202279078543001 0,188455759525073 0,13722252103612 0 0 0 0 0,115861363894765
0 0 0 0 0 0 0 0,0750013487052267
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

7-qatlam
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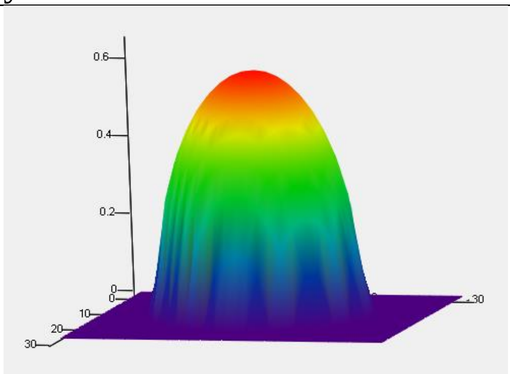
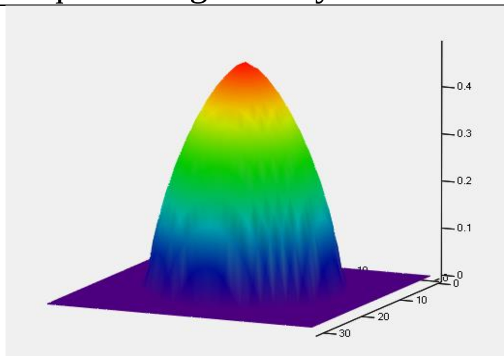
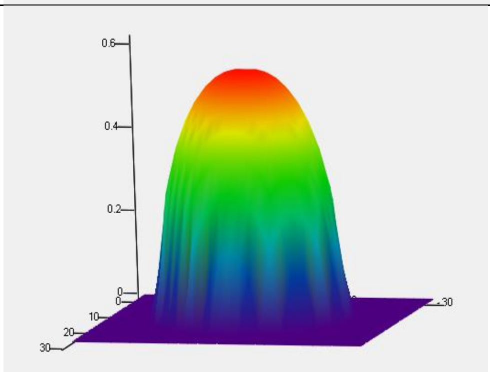
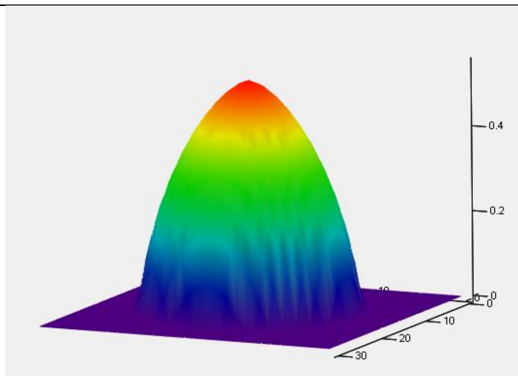
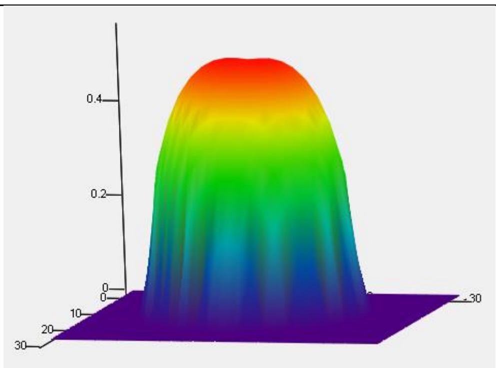
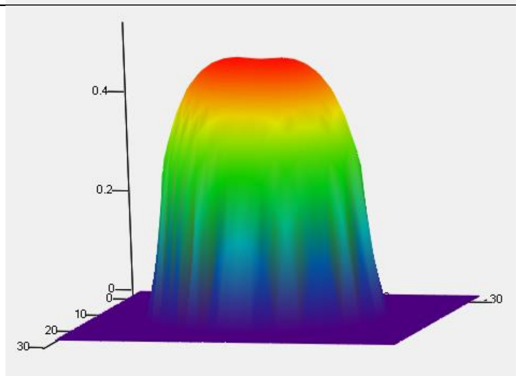
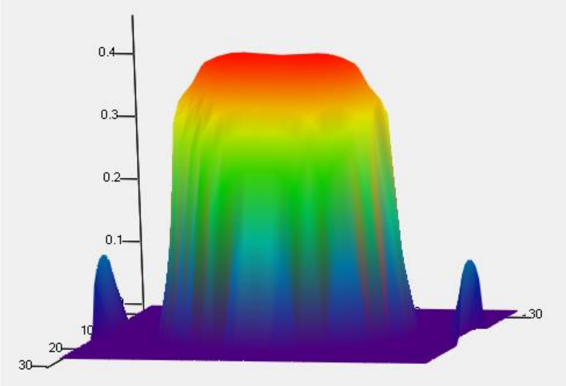
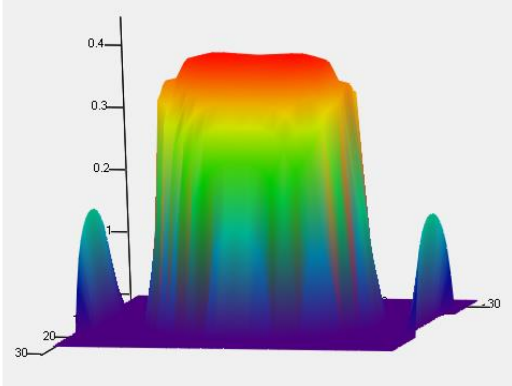
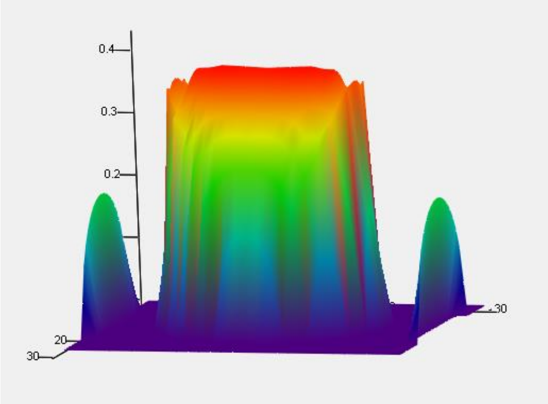
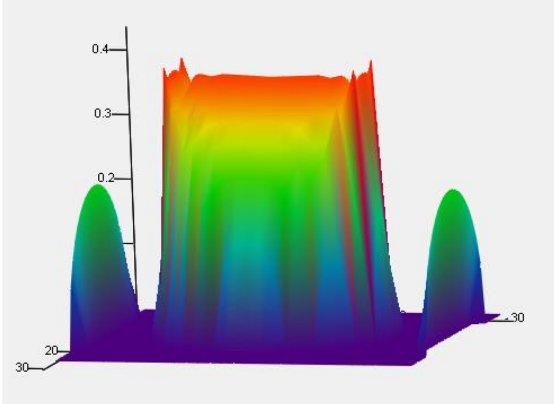
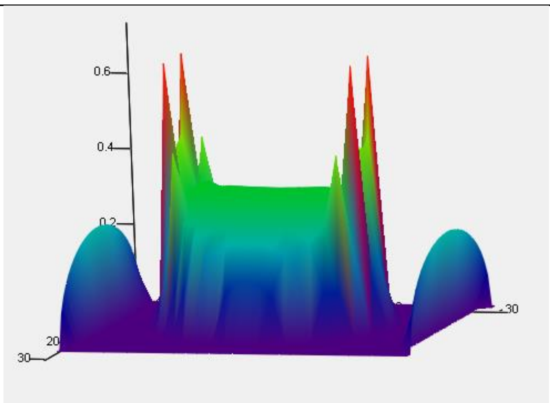
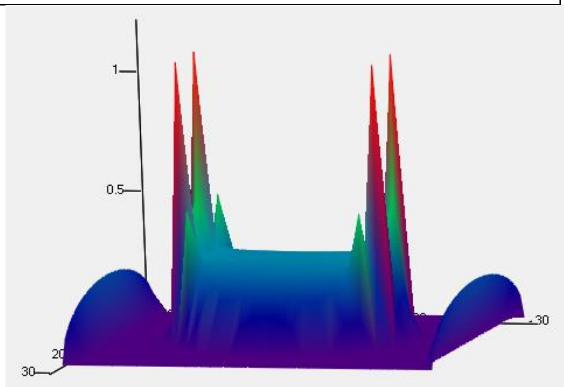
0 0 0 0 0 0 0 0,114615478489739
0 0 0 0 0 0 0 0,0748060044672083
0 0 0 0 0 0 0 4,5440990435436E-12

p=2; m=1,55; k=1,25; l=2,5; a=0,75; T=1,5.

	Gamilton-Yakobi tenglamasi yechimi	Nochiziqli ikkinchi tartibli issiqlik tarqalish tanglamasi yechimi
t=1		
t=2		
t=4		

t=8		
t=9		
t=12		

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1-qatlam
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0,494800718465663 0,446799462420619 0,378912901950439 0,277461669958834 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

2-qatlam
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3-qatlam
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4-qatlam
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5-qatlam
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6-qatlam
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7-qatlam
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8-qatlam
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9-qatlam
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10-qatlam
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11-qatlam
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12-qatlam
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0,379332398662053 0,380190799757842 0,380993014159968 0,381766699596545 0,382952474104154
0,376045002283618 0,387190110204326 0,336443019339634 0,472207358694267 0 0 0 0 0 0 0,237154154003211
0,370229611488261 0,371147072277993 0,371989010482021 0,373087155014681 0,371060983817534
0,370645977902081 0,346488211826082 0,358615748386578 0,212507150567953 0 0 0 0 0 0 0,230150575133674
0,357110989994111 0,358115378482043 0,359066582683289 0,359895845687837 0,354160104501479
0,357061516356036 0,311321784177871 0,394465606044612 0 0 0 0 0 0 0 0,220802856804111
0,338248792874941 0,339334950506133 0,340594809311975 0,338544275664995 0,333174771753626
0,310348343034464 0,298814439053149 0,126963708194511 0 0 0 0 0 0 0 0,208780405933473
0,310224240722697 0,31185964072507 0,311347513313377 0,30548908661376 0,287081267002509 0,263581528988016
0,126963708194511 0 0 0 0 0 0 0 0 0,193560315020962
0,264637815784916 0,266084054708459 0,260124875505259 0,239351131158859 0,212507150567953 0 0 0 0 0 0 0 0 0 0
0,174264612023599
0,168818306545355 0,162076050371991 0,126963708194511 0 0 0 0 0 0 0 0 0 0 0 0 0,149240793894731
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0,114600542151664
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0,053016335474118
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

3.2-§ Ayirmali sxemalar va yechish usuli

O'zgaruvchan yo'nalishlar usuli

O'zgaruvchan yo'nalishlar metodini ko'rib chiqamiz (метод переменных направлений). Bu metod samarali tejamkor metodlar sinfiga mansub bo'lib, ikki o'lchamli issiqlik tarqalishi tenglamalarini yechishda qo'l keladi ([25]). Issiqlik tarqalishi tenglamasi chegaraviy masalalarini aynan shu metod yordamida ayirmali sxemalarini ko'ramiz.

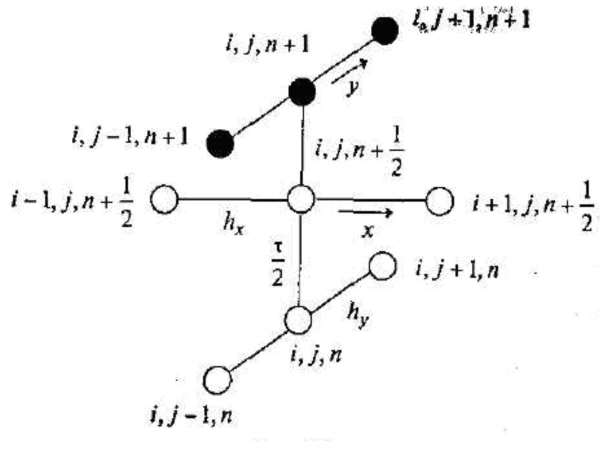
$$u_t = u_{xx} + u_{yy} + f(x,y,t), \qquad (x,y,t) \in (0,L)\times(0,M)\times(0,T), \qquad (3.2.1)$$

$$u(x,y,0) = \psi(x,y), \qquad (x,y) \in [0,L]\times[0,M], \quad \text{(boshlang'ich shart)} \quad (3.2.2)$$

$$u(x,y,t) = \varphi(x,y,t), \qquad (x,y) \in \Gamma, 0 < t \le T \text{,(chegaraviy shart)} \qquad (3.2.3)$$

Bu yerda $\Omega = (0,L)\times(0,M)$, Γ- berilgan soha chegarasi Ω (прямоугольника); *f(x,y,t)* – issiqlik manbaining zichligi; $\psi(x,y)$, $\varphi(x,y,t)$- berilgan funksiyalar; *L,M,T*— berilgansonlar.

Berilgan tenglama ikkita ayirmali shema birlashmasi yordamida approksimatsiya qilinadi, ularning har biri ma'lum fazoviy yo'nalishga ega bo'ladi. A operator A_1va A_2operatorlar yig'indisi deb olsak, yig'indining har bir operatori masala darajasiga ko'ra oshkor var oshkormas sxemalar yordamida approksimatsiya qilinadi [16]. Shu sababli ham $t = t^{n+1}$qatlamni aniqlashimiz uchun qo'shimcha (oraliq)$t = t^{n+\frac{1}{2}}$qatlam kerak bo'ladi. Yani $t = t^n$qatlamdan $t = t^{n+1}$qatlamga o'tishimiz uchun oraliq$t = t^{n+\frac{1}{2}}$qatlamdan foydalanamiz. Bu metodni realizatsiyaqilishda olti nuqtali shablondan foydalaniladi (9-rasm).



9-rasm. O'zgaruvchan yo'nalish usulidagi shablon

Vaqt bo'yicha n qatlamdan n+1 qatlamga o'tish ikki bosqichda amalga oshiriladi.

*Birinchi bosqich.*Vaqt bo'yicha $\frac{\tau}{2}$qadam bilan $\left(n+\frac{1}{2}\right)$- qatlam nuqtalari aniqlanadi. Vaqt bo'yicha xususiy hosila quyidagi formula yordamida approksimatsiya qilinadi:

$$\hat{u}_t(x_i, y_j, t^n) = \frac{\hat{u}^{n+1}_{i,j} - \hat{u}^n_{i,j}}{\tau}$$

bunda x bo'yicha hosila $\left(n+\frac{1}{2}\right)$- qatlamda, y bo'yicha hosila esa n- qatlamda approksimatsiya qilinadi. Boshlang'ich funksiya*f(x,y,t)* to'rdagi tasviri bilan almashtiriladi. Mos ravishda ayirmali sxemamiz quyidagicha ko'rinishda bo'ladi:

$$\frac{\hat{u}^{n+\frac{1}{2}}_{i,j} - \hat{u}^n_{i,j}}{\frac{\tau}{2}} = \frac{\hat{u}^{n+\frac{1}{2}}_{i+1,j} - 2\hat{u}^{n+\frac{1}{2}}_{i,j} + \hat{u}^{n+\frac{1}{2}}_{i-1,j}}{h_x^2} + \frac{\hat{u}^n_{i,j+1} - 2\hat{u}^n_{i,j} + \hat{u}^n_{i,j-1}}{h_y^2} + f^{n+\frac{1}{2}}_{i,j}, \quad \begin{matrix} 1 \le i \le I-1, \\ 1 \le j \le J-1, \end{matrix} \qquad (3.2.4)$$

bu yerda, $f^{n+\frac{1}{2}}_{i,j} = f\left(x_i, y_j, t^n + \frac{1}{2}\tau\right)$. Yoki ayirmali sxema natijasini quyidagicha yozishimiz mumkin:

$$\frac{\tau}{2h_x^2}\hat{u}^{n+\frac{1}{2}}_{i-1,j} - \left(1+\frac{\tau}{h_x^2}\right)\hat{u}^{n+\frac{1}{2}}_{i,j} + \frac{\tau}{2h_x^2}\hat{u}^{n+\frac{1}{2}}_{i+1,j} = F^n_{i,j}, \quad \begin{matrix} 1 \le i \le I-1, \\ 1 \le j \le J-1, \end{matrix}$$

bu yerda, $F^n_{i,j} = -\frac{\tau}{2} f^{n+\frac{1}{2}}_{i,j} - \frac{\tau}{2h_y^2}\left(\hat{u}^n_{i,j+1} - 2\hat{u}^n_{i,j} + \hat{u}^n_{i,j-1}\right) - \hat{u}^n_{i,j}.$ $f^{n+\frac{1}{2}}_{i,j}$ (3.2.4) o'rniga $f^n_{i,j}$ dan ham foydalansak bo'ladi. Har bir $j = 1,...,J-1$uchun uch dioganalli chiziqli algebraik tenglamalar sistemasini yechish talab qilinadi, har bir tenglamada uchta $\hat{u}^{n+\frac{1}{2}}_{i-1,j}, \hat{u}^{n+\frac{1}{2}}_{i,j}, \hat{u}^{n+\frac{1}{2}}_{i+1,j}$noma'lum qatnashgan bo'lib qolgan qiymatlar n- qatlamdan olinadi va hisoblash jarayoni amalga oshiriladi. Qisqacha aytganda qaralayotgan ayirmali sxema x bo'yicha oshkormas va y bo'yicha oshkor sxema hisoblanadi. Oraliq qatlam tugun nuqtalari progonka metodi yordamida x yo'nalishi bo'yicha hisoblab topiladi.

Ikkinchi bosqich. $\left(n+\frac{1}{2}\right)$- qatlamdan $\frac{\tau}{2}$ qadam bilan n-qatlamga o'tish. Vaqt bo'yicha xususiy hosila quyidagi formula bo'yicha $\hat{u}_t(x_i, y_j, t^n) = \frac{\hat{u}^{n+1}_{i,j} - \hat{u}^n_{i,j}}{\tau}$ approksimatsiya qilinadi. Shuningdek x bo'yicha hosila $\left(n+\frac{1}{2}\right)$- qatlamda, y bo'yicha hosila n-qatlamda approksimatsiya qilinadi. Mos ravishda ayirmali sxema quyidagicha ko'rinishda bo'ladi:

$$\frac{\hat{u}^{n+1}_{i,j} - \hat{u}^{n+\frac{1}{2}}_{i,j}}{\frac{\tau}{2}} = \frac{\hat{u}^{n+\frac{1}{2}}_{i+1,j} - 2\hat{u}^{n+\frac{1}{2}}_{i,j} + \hat{u}^{n+\frac{1}{2}}_{i-1,j}}{h_x^2} + \frac{\hat{u}^{n+1}_{i,j+1} - 2\hat{u}^{n+1}_{i,j} + \hat{u}^{n+1}_{i,j-1}}{h_y^2} + f^{n+1}_{i,j} \qquad (3.2.5)$$

Natijani quyidagicha ko'rinishda yozish mumkin:

$$\frac{\tau}{2h_y^2}\hat{u}^{n+1}_{i,j-1} - \left(1+\frac{\tau}{h_y^2}\right)\hat{u}^{n+1}_{i,j} + \frac{\tau}{2h_y^2}\hat{u}^{n+1}_{i,j+1} = \hat{O}^{n+\frac{1}{2}}_{i,j}, \quad \begin{matrix} 1 \le i \le I-1, \\ 1 \le j \le J-1, \end{matrix}$$

bu yerda $\hat{O}^{n+\frac{1}{2}}_{i,j} = -\frac{\tau}{2} f^n_{i,j} - \frac{\tau}{2h_x^2}\left(\hat{u}^{n+\frac{1}{2}}_{i-1,j} - 2\hat{u}^{n+\frac{1}{2}}_{i,j} + \hat{u}^{n+\frac{1}{2}}_{i+1,j}\right) - \hat{u}^{n+\frac{1}{2}}_{i,j}$. $f^n_{i,j}$ (3.2.5) o’rniga $f^{n+\frac{1}{2}}_{i,j}$ dan foydalansak ham bo'ladi.

Har bir $i = 1,...,I-1$ uchun uch dioganalli chiziqli algebraik tenglamalar sistemasini yechish talab etiladi. Har bir tenglama uchta $\hat{u}^{n+1}_{i,j-1}, \hat{u}^{n+1}_{i,j}, \hat{u}^{n+1}_{i,j+1}$ noma'lumdan iborat bo'lib, qolgan qiymatlar $\left(n+\frac{1}{2}\right)$- qatlamdan olinadi. Endi bu sxemamiz x bo'yicha oshkor va y bo'yicha oshkormas sxema hisoblanib, noma'lumlar y yo'nalishi bo'yicha progonka metodi yordamida topiladi.

(3.2.4)- va (3.2.5)- ayirmali sxemalarimiz progonka metodini qo'llashimiz uchun turg’un bo'lib, yechimga ega va u yagonadir.

Boshlang’ich va chegaraviy shartlar quyidagicha ifodalanadi:

$$\hat{u}^0_{i,j} = \psi_{i,j}, \ 0 \le i \le I, \ 0 \le j \le J, \quad \text{(nolinchiqatlam)}$$

$$\hat{u}^n_{0,j} = \varphi^n_{0,j}, \quad \hat{u}^n_{I,j} = \varphi^n_{I,j}, \ 0 \le j \le J, 1 \le n \le N, \text{(Oxo'qigaperpendikulyartekislik)}$$

$$\hat{u}^n_{i,0} = \varphi^n_{i,0}, \quad \hat{u}^n_{i,J} = \varphi^n_{i,J}, \ 0 \le i \le I, 1 \le n \le N, \text{(Oyo'qigaperpendikulyartekislik)}$$

bu yerda $\varphi^n_{i,j} = \varphi(x_i, y_j, t^n), \psi_{i,j} = \psi(x_i, y_j)$.

Oraliq qatlam $\hat{u}_{i,j}^{n+\frac{1}{2}}$ da chegaraviy qiymatlar (x=0 va$x$=L) quyidagicha hisoblanadi. $i=0\ (x=0)\,(\,i=I\ (x=L).)$ da

$$\hat{u}_{0,j}^{n+\frac{1}{2}}=\frac{1}{2}\left(\hat{u}_{0,j}^{n+1}+\hat{u}_{0,j}^{n}\right)-\frac{\tau}{4h_y^2}\left(\hat{u}_{0,j-1}^{n+1}-2\hat{u}_{0,j}^{n+1}+\hat{u}_{0,j+1}^{n+1}\right)+\frac{\tau}{4h_y^2}\left(\hat{u}_{0,j-1}^{n}-2\hat{u}_{0,j}^{n}+\hat{u}_{0,j+1}^{n}\right).$$

Bizga (3.2.1) - (3.2.3) da berilgan masalamizning to'rdagi tasviri quyidagicha bo'ladi:

$$\Delta_1 \overset{k+\frac{1}{2}}{y}=\frac{1}{h_x^2}\left[d_{ij+1}(y_{ij+1}-y_{ij})-d_{ij}(y_{ij}-y_{ij-1})\right]-\frac{V_1}{h_x}\left(y_{ij+1}-y_{ij}\right)$$

$$\Delta_2 \overset{k}{y}=\frac{1}{h_y^2}\left[d_{i+1j}(y_{i+1j}-y_{ij})-d_{ij}(y_{ij}-y_{i-1j})\right]-\frac{V_2}{h_y}\left(y_{i+1j}-y_{ij}\right)$$

$$\Delta_3 \overset{k+1}{y}=\frac{1}{h_y^2}\left[d_{i+1j}(y_{i+1j}-y_{ij})-d_{ij}(y_{ij}-y_{i-1j})\right]-\frac{V_2}{h_y}\left(y_{i+1j}-y_{ij}\right)$$

$$\frac{\overset{k+\frac{1}{2}}{y_{ij}}-\overset{k}{y_{ij}}}{\frac{\tau}{2}}=\Delta_1 \overset{k+\frac{1}{2}}{y}+\Delta_2 \overset{k}{y} \qquad (3.2.6)$$

$$\frac{\overset{k+1}{y_{ij}}-\overset{k+\frac{1}{2}}{y_{ij}}}{\frac{\tau}{2}}=\Delta_3 \overset{k+1}{y}+\Delta_1 \overset{k+\frac{1}{2}}{y} \qquad (3.2.7)$$

Hosil qilgan ayirmali sxemalarimiz chiziqli va yagona echimga ega. Bu sxema koeffitsiyentlari A_i, C_i, B_i bo'lgan chiziqli algebraik tenglamalar sistemasiga keladi. Uning qisqacha ko'rinishi quyidagicha yoziladi:

$$A_i y_{i-1,j}-C_i y_{i,j}+B_i y_{i+1,j}=-F_i,\ i=1,2,\dots,n-1; \qquad j=1,\ 2,\dots,\ m \ (3.2.8)$$

$$y_{0,j}=\varphi_4(t,x) \qquad j=1,\ 2,\ \dots,\ m\ , \ (3.2.9)$$

$$y_{n,j}=\varphi_5(t,x,b) \qquad j=1,\ 2,\ \dots,\ m\ , \qquad (3.2.10)$$

$$y_{i,0}=u_0(x_i) \qquad i=0,1,\ 2,\ \dots,\ n\ . \ (3.2.11)$$

(3.2.8) - (3.2.11) algebraik tenglamalar sistemamizda hosil bo'ladigan matritsamiz uch diaganalli va quyidagicha:

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ A_1 & -C_1 & B_1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & A_i & -C_i & B_i & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & A_{n-1} & -C_{n-1} & B_{n-1} \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{bmatrix}$$

Bu tenglamalar sistemasining yechimi progonka usulini qo'llagan holda topiladi va yechim aniqligini oshirish maqsadida iteratsiya qo'llaniladi. Odatda iteratsiyaning yaqinlashish chegarasi sifatida $\max\limits_i \left| \overset{(s+1)}{y_i} - \overset{(s)}{y_i} \right| \le \varepsilon$ shart kiritiladi. Iteratsiyaning boshlang'ich qiymati sifatida vaqt bo'yicha bitta oldingi qadam qiymatlari olinadi.

Progonka (haydash) usuli

Yuqorida hosil qilgan algebraik tenglamalar sistemasini yechish progonka metodi yordamida amalga oshiriladi. Bu metod quyidagi rekkurent formula yordamida amalga oshiriladi [16,18].

$$y_{i,j} = \alpha_{i+1} y_{i+1,j} + \beta_{i+1} \quad (12)$$

bunda α_i va β_i lar noma'lum koeffitsiyentlar. $y_{i-1,j} = \alpha_i y_{i,j} + \beta_i$ ifodani (3.2.8) ifodaga qo'yib quyidagi formulani qosil qilamiz:

$$(A_i \alpha_i - C_i) y_{i,j} + A_i \beta_i + B_i y_{i+1,j} = -F_i .$$

Hosil bo'lgan ifodada (3.2.6) rekkurent formuladan foydalansak

$$[(A_i \alpha_i - C_i)\alpha_{i+1} + B_i] y_{i+1,j} + A_i \beta_i + (A_i \alpha_i - C_i)\beta_{i+1} = -F_i .$$

ifoda kelib chiqadi. Bu tenglama barcha $y_{i,j}$ larda o'rinli bo'ladi, agar

$$(A_i \alpha_i - C_i)\alpha_{i+1} + B_i = 0, \quad A_i \beta_i + (A_i \alpha_i - C_i)\beta_{i+1} + F_i = 0$$

shart bajarilsa.

Bu ifodalardan α_{i+1} ва β_{i+1}koeffitsiyentlar uchun quyidagi rekkurent formula hosil bo'ladi:

$$\alpha_{i+1} = \frac{B_i}{C_i - \alpha_i A_i}, \qquad i = 1, 2, ..., n-1 \quad (3.2.13)$$

$$\beta_{i+1} = \frac{A_i \beta_i + F_i}{C_i - \alpha_i A_i}, \qquad i = 1, 2, ..., n-1 \qquad (3.2.14)$$

Агар α_i, β_i koeffitsiyentlar va $y_{n,j}$ ning qiymati ma'lum bo'lsa, u holda $y_{i,j}$ larning qiymatlarini ketma-ket hisoblab topa olamiz. α_i, β_i koeffitsiyentlarni chapdan o'ngga harakatlangan holda aniqlaymiz, $y_{i,j}$ larni esa aksincha o'ngdan chapga tomon ketma-ket aniqlaymiz.

α, β, y funksiyalarning har biri uchun Koshi masalasini yechishimiz kerak bo'ladi, chunki bu funksiyalarning boshlang'ich qiymatlari bizga noma'lum. Buning uchun chegaraviy shartlardan ham foydalanamiz. (3.2.8) ifodadan $i = 0$ bo'lganda quyidagiga ega bo'lamiz:

$$y_{0,j} = \alpha_1 y_{1,j} + \beta_1$$

ikkinchi tomondan esa $y_{0,j} = \psi_1(t_j)$

Bu ifodalardan ko'rinadiki

$$\alpha_1 = 0 \qquad (3.2.15)$$

$$\beta_1 = \psi_1(t_j) \qquad (3.2.16)$$

Shunga ko'ra α_i ва β_i funksiyalar uchun Koshi masalasi hosil bo'ladi: α учун (3.2.7), (3.2.9), β учун (3.2.15), (3.2.16). Bu formulalar to'g'ri progonka (прямой прогонки) formulalari deb yuritiladi.

α_i ва β_i funksiyalarning $i = 1, 2, ..., n$ dagi barcha qiymatlari hisoblangach, $y_{n,j} = \psi_2(t_j)$ chegaraviy qiymatlari ham hisoblanadi. Endi bizga barcha boshlang'ich qiymatlar ma'lum, (3.2.8) dan foydalanib $y_{i,j}$ noma'lumlarni topsak bo'ladi.

Bu metod $|C_i| \geq |A_i| + |B_i|$shart bajarilganda turg'un bo'lib, yechimga ega bo'ladi. Hosil qilgan ayirmali sxemamizda esa bu shart bajariladi.

(2.1.1) tenglamani sonli yechishda quyidagi ayirmali sxemalardan foydalanilgan:

$$\overset{s}{A_i}\,\overset{s+1}{\overline{y}_{i-1}} - \overset{s}{C_i}\,\overset{s+1}{\overline{y}_i} + \overset{s}{B_i}\,\overset{s+1}{\overline{y}_{i+1}} = -\overset{s}{F_i}$$

$$a_{i+1} = \frac{1}{2}\left[\left(y_i^{j+1}\right)^{m-1}\left|\frac{(y_{i+1}^{j+1})^k - (y_i^{j+1})^k}{h}\right|^{p-2} + \left(y_{i+1}^{j+1}\right)^{m-1}\left|\frac{(y_i^{j+1})^k - (y_{i-1}^{j+1})^k}{h}\right|^{p-2}\right]$$

$$a_i = \frac{1}{2}\left[\left(y_{i-1}^{j+1}\right)^{m-1}\left|\frac{(y_i^{j+1})^k - (y_{i-1}^{j+1})^k}{h}\right|^{p-2} + \left(y_i^{j+1}\right)^{m-1}\left|\frac{(y_{i-1}^{j+1})^k - (y_{i-2}^{j+1})^k}{h}\right|^{p-2}\right]$$

$$A_i = 2\cdot k^{p-2}\cdot\frac{y_i^{m+k(p-1)-p}}{h}\cdot\left(\frac{y_i - y_{i-1}}{h}\right)^{p-2}$$

$$B_i = 2\cdot k^{p-2}\cdot\frac{y_{i+1}^{m+k(p-1)-p}}{h}\cdot\left|\frac{y_{i+1} - y_i}{h}\right|^{p-2}$$

$$C_i = A_i + B_i + 1 \quad F_i = -\tau\cdot y_i^{\beta} + y_i$$

III bob bo'yicha xulosa

Dissertatsiya ishining III bobida Gamilton-Yakobi tenglamasining yechimlari orqali nochiziqli ikkinchi tartibli parabolik tenglamalarning ifodalanishi va ularning yechim xossalari solishtirilgan. Shu bilan birgalikda Gamilton-Yakobi tenglamasining yangi xossalari ham o'rganiladi va yechimlar xossalari kompyuter orqali tahlil qilinadi. Masalani sonli yechishning ayirmali sxemasi, yechish algoritmi, o'zgaruvchan yo'nalishlar usuli hamda haydash usulidan foydalanib sonli tajribalar o'tkazilgan. Natijalar animatsiya va grafik tasvirda berildi.

XULOSA

Olingan natijalar quyidagi xulosalarga olib keladi:

- Dissertatsiya mavzusiga asosan nochiziqli jarayonlarni o’rganish asosida matematik modelning harorat tarqalish tezligining chekliligi shartlari, lokallashishi, global yechimlarning mavjudligi ko’rsatilgan, o’z navbatida bu ilgari bu sohadagi global yechimlarning mavjudligini umumlashtiradi. Dissertatsiyani amalga oshirishda jarayon xossalarini Gamilton-Yakobi tenglamalari bilan tasvirlanuvchi modellar orqali ham ifodalash mumkinligi ko’rsatildi. Chunki Gamilton-Yakobi tenglamalari modellari birinchi tartibli tenglamalar bilan ifodalanadigan nochiziqli tenglamalardir, boshlang’ich matematik modeli esa ikkinchi tartibli tenglamalar bilan tasvirlanadi.Nochiziqli jarayon xossalarini o’rganishda, birinchi tartibli tenglamalar ikkinchi tartibli tenglamalarga qaraganda osonroq. Shu bilan birgalikda Gamilton-Yakobi tenglamasining yangi xossalari ham o’rganiladi va yechimlar xossalari kompyuter orqali tahlil qilinadi;
- Yuqori yechim sifatida taklif etilgan Gamilton-Yakobi tenglamasining yuqori yechimligi ko’rsatilib masala sonli modellashtirilgan;
- Masalani sonli yechishning ayirmali sxemasi, yechish algoritmi, o’zgaruvchan yo’nalishlar usuli hamda haydash usulidan foydalanib sonli tajribalar o’tkazilgan;
- Natijalar animatsiya va grafik tasvirda berildi;
- Programma mahsulotlari Visual Studio 2017 muhitida C# dasturlash tili asosida yaratilgan;
- Keltirilgan yondashuv asosida bajarilgan hisoblashlar mazkur dissertatsiya ishining effektiv ekanliginni ko’rsatdi, ya’ni taqribiy yechim yuqori yechimga tez yaqinlashishi ko’rsatildi;
- Yaratilgan programmalar parametrlarning boshqa qiymatlarida ham hisoblashlar o’tqazish imkoniyatini beradi va yechimning xossalari to’g’risida tegishli xulosalarga kelish mumkin.

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Elektron resurslar

1. **http://www.ziyonet.uz**
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ILOVA

```
using AxGLLib;
using System;
using System.Collections.Generic;
using System.ComponentModel;
using System.Data;
using System.Drawing;
using System.Linq;
using System.Text;
using System.Threading.Tasks;
using System.Windows.Forms;
namespace IkkiUlchovli
{
    public partial class Form1 : Form
    {
        public Form1()
        {
            InitializeComponent();
            T = 2;
            N = 15;
            tau = T / N;
            hawX = L / N;
            hawY = L / N;
            u = new double[N + 1, N + 1, N + 1];
            u2 = new double[2 * (N + 1), 2 * (N + 1)];
            u_n = new double[N + 1, N + 1, N + 1];
            u_n12 = new double[N + 1, N + 1, N + 1];
            u_n1 = new double[N + 1, N + 1, N + 1];
            r = new double[N + 1, N + 1];
        }
```

```
double p, m, l, k, a, b, gamma1, betta, alfa, gamma, Coeff1, T, L, tau,
hawX, hawY;
double[,] r;
double[,,] u_n;
double[,,] u_n12;
double[,,] u_n1;
double[,,] u;
double[,] u2;
int N,j, d;
private void timer1_Tick_1(object sender, EventArgs e)
{
    j = j + 1; d = d + 1;
    if (j > N - 1) timer1.Enabled = false;
    for (int i = 0; i < N + 1; i++)
    {
        for (int kk = 0; kk < N + 1; kk++)
      u2[i, kk] = u[j, N - i, N - kk];
    }
    for (int i = 0; i < N + 1; i++)
    {
        for (int kk = 1; kk < N + 1; kk++)
                    u2[i, N + kk] = u2[i, N - kk];
    }
    for (int i = 1; i < N + 1; i++)
    {
        for (int kk = 0; kk < 2 * N + 1; kk++)
            u2[N + i, kk] = u2[N - i, kk];
    }
    axOpenGL1.RemoveSurfaces();
    axOpenGL1.AddSurface(u2);
```

```
        }
         private void button2_Click(object sender, EventArgs e)
        {
            j = -1; d = -1;
            timer1.Enabled = true;
        }
          private double UIJ(double xx, double tt)
        {
            double res = Math.Pow(T + tt, alfa);
            double a1 = xx * Math.Pow(T + tt, -gamma);
            a1 = a - b * Math.Pow(a1, p / (p - 1));
            if (a1 > 0)
                res = res * Math.Pow(a1, (p - 1) / (m + l + k * (p - 2) - 2));
            else res = 0;
            return res;
        }
        public double aiX(int ii, int jj, int tt)
        {
            double res;
            if (jj != 0)
                res = l * Math.Pow(u_n[tt, ii, jj], (k - 1.0) * (p - 2.0)) *
                    Math.Pow(Math.Abs(u_n[tt, ii, jj] - u_n[tt, ii, jj - 1]) / hawX, p -
2.0) * (tau / 2) *
                    Math.Pow(k, p - 2.0) * Math.Pow(u_n[tt, ii, jj], m + l - 2) /
Math.Pow(hawX, 2.0);
            else res = 0;
            return res;
        }
        public double biX(int ii, int jj, int tt)
        {
```

```
            double res = l * Math.Pow(k, p - 2.0) * Math.Pow(u_n[tt, ii, jj + 1], (k -
1.0) * (p - 2.0)) *
                Math.Pow(u_n[tt, ii, jj + 1], m + l - 2) * Math.Pow(Math.Abs(u_n[tt,
ii, jj + 1] - u_n[tt, ii, jj]) / hawX, p - 2.0) *
                (tau / 2) / Math.Pow(hawX, 2.0);
            return res;
        }
        public double ciX(int ii, int jj, int tt)
        {
            double res = aiX(ii, jj, tt) + biX(ii, jj, tt) + 1;
            return res;
        }
        public double fiX(int ii, int jj, int tt)
        {
            double res= 0;
            if (jj != 0)
             res = -tau * Math.Pow(u_n[tt, ii, jj], betta) / 2 - (u_n[tt, ii, jj - 1] - 2 *
u_n[tt, ii, jj] + u_n[tt, ii, jj + 1]) *
                tau / (2 * hawY * hawY) + u_n[tt, ii, jj];
            return res;
        }
        public double aiY(int ii, int jj, int tt)
        {
            double res;
            if (jj != 0)
                res = l * Math.Pow(u_n12[tt, ii, jj], (k - 1.0) * (p - 2.0)) *
                    Math.Pow(Math.Abs(u_n12[tt, ii, jj] - u_n12[tt, ii, jj - 1]) / hawY, p
- 2.0) * tau *
                    Math.Pow(k, p - 2.0) * Math.Pow(u_n12[tt, ii, jj], m + l - 2) /
Math.Pow(hawY, 2.0);
```

```
            else res = 0;
            return res;
        }
        public double biY(int ii, int jj, int tt)
        {
            double res = l * Math.Pow(k, p - 2.0) * Math.Pow(u_n12[tt, ii, jj + 1], (k
- 1.0) * (p - 2.0)) *
                Math.Pow(u_n12[tt, ii, jj + 1], m + l - 2) *
Math.Pow(Math.Abs(u_n12[tt, ii, jj + 1] - u_n12[tt, ii, jj]) / hawY, p - 2.0) *
                tau / Math.Pow(hawY, 2.0);
            return res;
        }
        public double ciY(int ii, int jj, int tt)
        {
            double res = aiY(ii, jj, tt) + biY(ii, jj, tt) + 1;
            return res;
        }
        public double fiY(int ii, int jj, int tt)
        {
            double res = -tau * Math.Pow(u_n12[tt, ii, jj], betta) / 2 - (u_n12[tt, ii-1,
jj] - 2 * u_n12[tt, ii, jj]
                + u_n12[tt, ii+1, jj]) * tau / (2 * hawX * hawX) + u_n12[tt, ii, jj];
            return res;
        }
        private void button1_Click(object sender, EventArgs e)
        {
            p = Convert.ToDouble(textBox1.Text);
            m = Convert.ToDouble(textBox2.Text);
            l = Convert.ToDouble(textBox3.Text);
            k = Convert.ToDouble(textBox4.Text);
```

```
gamma1 = (p - 1) / (m + l + k * (p - 2) - 2);
betta = Convert.ToDouble(textBox5.Text);
alfa = -1 / (betta - 1);
gamma = alfa * (2 - m - l - k * (p - 2) - 1) / p;

Coeff1 = (m + l + k * p - 2 * k - 2) / (p - 1) * Math.Pow(Math.Pow(k, p -
2) * l * (m + p - 1), 1 / (p - 1));
a = Convert.ToDouble(textBox6.Text);
double m1 = Math.Pow(gamma, 1 / (p - 1));
double m2 = p / (Coeff1 * (p - 1));
b = m2 * m1;
L = Math.Sqrt(Math.Pow(a / b, 2 * (p - 1) / p) * Math.Pow((T + T), 2 *
gamma) / 2);
double[] alfaC = new double[N + 1];
double[] bettaC = new double[N + 1];
double[] A = new double[N + 1];
double[] C = new double[N + 1];
double[] B = new double[N + 1];
double[] F = new double[N + 1];
for (int i = 0; i < N + 1; i++)
{
   for (int j = 0; j < N + 1; j++)
      r[i, j] = Math.Sqrt(Math.Pow(hawX * i, 2) + Math.Pow(hawY * j,
2));
}
for (int i = 0; i < N + 1; i++)
{
   for (int j = 0; j < N + 1; j++)
      u[0, i, j] = UIJ(r[i, j], 0);
}
```

```
for (int t = 0; t < N + 1; t++)
{
    for (int j = 0; j < N + 1; j++)
    {
        u[t, 0, j] = UIJ(r[0, j], t * tau);
        u[t, N, j] = UIJ(r[N, j], t * tau);
    }
}
for (int t = 0; t < N + 1; t++)
{
    for (int i = 0; i < N + 1; i++)
    {
        u[t, i, 0] = UIJ(r[i, 0], t * tau);
        u[t, i, N] = UIJ(r[i, N], t * tau);
    }
}
for (int i = 0; i < N + 1; i++)
    for (int j = 0; j < N + 1; j++)
        u_n[0, i, j] = u[0, i, j];

for (int kk = 1; kk < N + 1; kk++)
{
    for (int i = 1; i < N; i++)
    {
        for (int j = 1; j < N; j++)
        {
            // 0->1/2
            u_n12[kk, 0, j] = (1 / 2) * (u[kk, 0, j] + u[kk - 1, 0, j]) - ((u[kk, 0,
j - 1] - 2 * u[kk, 0, j] + u[kk, 0, j + 1]) * tau / (4 * hawY * hawY)) +
```

```
                    ((u[kk - 1, 0, j - 1] - 2 * u[kk - 1, 0, j] + u[kk - 1, 0, j + 1]) * tau
/ (4 * hawY * hawY));
                u_n12[kk, N, j] = (1 / 2) * (u[kk, N, j] + u[kk - 1, N, j]) - ((u[kk,
N, j - 1] - 2 * u[kk, N, j] + u[kk, N, j + 1]) * tau / (4 * hawY * hawY)) +
                    ((u[kk - 1, N, j - 1] - 2 * u[kk - 1, N, j] + u[kk - 1, N, j + 1]) *
tau / (4 * hawY * hawY));
                alfaC[0] = 0;
                bettaC[0] = u[kk, 0, j];
                for (int j1 = 0; j1 < N; j1++)
                    alfaC[j1 + 1] = biX(i, j1, kk) / (ciX(i, j1, kk) - aiX(i, j1, kk) *
alfaC[j1]);
                for (int j1 = 0; j1 < N; j1++)
                    bettaC[j1 + 1] = (aiX(i, j1, kk) * bettaC[j1] + fiX(i, j1, kk)) /
                        (ciX(i, j1, kk) - aiX(i, j1, kk) * alfaC[j1]);
                for (int j1 = N - 1; j1 >= 0; j1--)
                    u_n12[kk, i, j1] = alfaC[j1 + 1] * u_n12[kk, i, j1 + 1] +
bettaC[j1 + 1];
                //1/2->1
                u_n1[kk, 0, j] = u[kk, 0, j];
                u_n1[kk, N, j] = u[kk, N, j];
                alfaC[0] = 0;
                bettaC[0] = u[kk, 0, j];
                for (int j1 = 0; j1 < N; j1++)
                    alfaC[j1 + 1] = biY(i, j1, kk) / (ciY(i, j1, kk) - aiY(i, j1, kk) *
alfaC[j1]);
                for (int j1 = 0; j1 < N; j1++)
                    bettaC[j1 + 1] = (aiY(i, j1, kk) * bettaC[j1] + fiY(i, j1, kk)) /
                        (ciY(i, j1, kk) - aiY(i, j1, kk) * alfaC[j1]);
                for (int j1 = N - 1; j1 >= 0; j1--)
```

```
                    u_n1[kk, i, j1] = alfaC[j1 + 1] * u_n1[kk, i, j1 + 1] + bettaC[j1
+ 1];
                for (int j1 = 0; j1 < N + 1; j1++)
                    u[kk, i, j1] = u_n1[kk, i, j1];
            }
        }
    }
}
    }
}

using System;
using System.Collections.Generic;
using System.ComponentModel;
using System.Data;
using System.Drawing;
using System.Linq;
using System.Text;
using System.Threading.Tasks;
using System.Windows.Forms;
using System.IO;
namespace DvuxMerniyGamiltonYakobi
{
publicpartialclassForm1 : Form
    {
constint nmax = 15;
int j, d, i;
double[,,] u = newdouble[nmax + 1, nmax + 1, nmax + 1];
double[,] u2 = newdouble[2*nmax + 1, 2*nmax + 1];
```

```
double a, b, tau, p, m, k, l, alfa, T, gamma, gamma1, betta, xmax, hx, hy,
Coeff1;
private void button4_Click(object sender, EventArgs e)
        {
            timer1.Enabled = false;
        }

Private void button3_Click(object sender, EventArgs e)
        {
StreamWriter sw;
FileInfo ff = newFileInfo("result.txt");
if (ff.Exists == true)
            {
                ff.Delete();
            }

            sw = ff.AppendText();

            sw.WriteLine("Parametrlar p={0}; m={1}; k={2}; l={3}; a={4};
T={5}.", p.ToString(), m.ToString(), k.ToString(), l.ToString()
                , a.ToString(), T.ToString());
            sw.WriteLine();
for (int i = 0; i < nmax + 1; i++)
            {
                sw.WriteLine(i.ToString() + "-qatlam");
for (int j = 0; j < nmax + 1; j++)
                {
for (int k = 0; k < nmax + 1; k++)
                    {
                        sw.Write(u[i, j, k].ToString() + ' ');
```

```
                    }
                    sw.WriteLine();
                }
                sw.WriteLine();
            }
            sw.Close();
}
Private void timer1_Tick(object sender, EventArgs e)
{
            j = j + 1; d = d + 1;

if (j > nmax - 1) timer1.Enabled = false;
for (int i = 0; i < nmax + 1; i++)
            {
for (int kk = 0; kk < nmax + 1; kk++)
                {
                    u2[i, kk] = u[j, nmax - i, nmax - kk];
                }
            }
for (int i = 0; i < nmax + 1; i++)
            {
for (int kk = 1; kk < nmax + 1; kk++)
                {
                    u2[i, nmax + kk] = u2[i, nmax - kk];
}
            }
for (int i = 1; i < nmax + 1; i++)
            {
for (int kk = 0; kk < 2 * nmax + 1; kk++)
{
```

```
                u2[nmax + i, kk] = u2[nmax - i, kk];
              }
            }
            axOpenGL1.RemoveSurfaces();
            axOpenGL1.AddSurface(u2);
        }
Private void button2_Click(object sender, EventArgs e)
        {
            j = -1; d = -1;
            timer1.Enabled = true;
        }
public Form1()
        {
            InitializeComponent();
        }
Private double uxy(double xx, double yy, double tt)
        {
double res = Math.Pow(T + tt, alfa);
double a1 = Math.Pow(xx * xx + yy * yy, 0.5) * Math.Pow(T + tt, -gamma);
            a1 = a - b * Math.Pow(a1, p / (p - 1));
if (a1 > 0)
              res = res * Math.Pow(a1, (p - 1) / (m + l + k * (p - 2) - 2));
else res = 0;
return res;
        }
Private void button1_Click(object sender, EventArgs e)
        {
            p = Convert.ToDouble(textBox1.Text);
            m = Convert.ToDouble(textBox2.Text);
            l = Convert.ToDouble(textBox3.Text);
```

```
            k = Convert.ToDouble(textBox4.Text);
            gamma1 = (p - 1) / (m + l + k * (p - 2) - 2);
            betta = Convert.ToDouble(textBox5.Text);
            alfa = -1 / (betta - 1);
            gamma = alfa * (2 - m - l - k * (p - 2) - 1) / p;
            Coeff1 = (m + l + k * p - 2 * k - 2) / (p - 1) * Math.Pow(Math.Pow(k, p -
2) * l * (m + p - 1), 1 / (p - 1));
            a = Convert.ToDouble(textBox6.Text);
double m1 = Math.Pow(gamma, 1 / (p - 1));
double m2 = p / (Coeff1 * (p - 1));
            b = m2 * m1;
            T = 1.5;
            xmax = Math.Sqrt(Math.Pow(a / b, 2 * (p - 1) / p) * Math.Pow((T + T), 2
* gamma) / 2);
            tau = T / nmax;
            hx = xmax / nmax;
            hy = xmax / nmax;
for (int i = 0; i < nmax + 1; i++)
            {
for (int j = 0; j < nmax + 1; j++)
                {
                    u[0, i, j] = uxy(i * hy, j * hx, 0);
                    u[i, j, 0] = uxy(0, j * hy, i * tau);
                    u[i, j, nmax] = uxy(nmax * hx, j * hy, i * tau);
                    u[i, 0, j] = uxy(j * hx, 0, i * tau);
                    u[i, nmax, j] = uxy(j * hx, nmax * hy, i * tau);
                }
            }

for (int t = 1; t < nmax + 1; t++)
```

```
            {
for (i = 0; i < nmax + 1; i++)
                {
for (int j = 0; j < nmax; j++)
                    {
                        u[t, i, j] = u[t - 1, i, j] + tau * (Math.Pow(u[t - 1, i, j], l + m - 1 +
(k - 1) * (p - 2))
                            * l * Math.Pow(k, p - 2) * Math.Pow(Math.Abs(u[t - 1, i, j + 1]
- u[t - 1, i, j])/hx, p - 2)
                            * Math.Abs(u[t - 1, i, j + 1] - u[t - 1, i, j])/(hx*hx)
                            -Math.Pow(u[t-1, i, j], betta));
}
                }
            }
        }
 }}
```