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Kvazi Novolterra kubik stoxostik operatorning ikki o’lchovli simpleksdagi traektoriyasi

Mutaxassislik: 5A130101 – Matematika (Matematik analiz)

Magistr

akademik darajasini olish uchun yozilgan

MAGISTRLIK DISSERTATSIYA

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MUNDARIJA

KIRISH

Inson, uning har tomonlama uyg'un kamol topishi va farovonligi, shaxs mafaatlarini ro'yobga chiqarishning sharoitlarini va ta'sirchan mexanizmlarini yaratish, eskirgan tafakkur va ijtimoiy hulq atvorning andozalarini o'zgartirish respublikada amalga oshirilayotgan islohotlarning asosiy maqsadi va harakatlantiruvchi kuchidir. Xalqning boy intellektual merosi va umumbashariy qadriyatlar asosida, zamonaviy madaniyat, iqtisodiyot, fan, texnika va texnologiyalarning yutuqlari asosida kadrlar tayyorlashning mukammal tizimini shakllantirish O'zbekiston taraqqiyotining muhim shartidir.

Tadqiqot masalasining dolzarbligi: Hozirgi fan va texnika rivojlanayotgan bir vaqtda biomatematika muammolarni yechishda muhim ahamiyatga ega bo'lgan, kubik stoxastik operatorlar nazariyasi, operatorlar nazariyasining jadal rivojlanishidagi markaziy bo'limlaridan biri hisoblanadi. Bu fanning rivojlanishida bir qancha olimlarning ishlarini alohida ta'kidlab o'tish mumkin.

Jumladan: Kolmogorov A.N., Fomin S.V., Axiezer N.I., Vallander S.S., Devaney R.L., G'anixo'jayev N.N., G'anixo'jayev R.N., Sarimsoqov A.T., va boshqalar. Bularning ishlarida kvadrat stoxastik operatorlarning dinamikasini to'la o'rganish masalalar o'rganilgan. Kvazi novolterra kvadratik stohastik operatorlar dinamikasini o'rganish masalalari U. Jamilov , U.A Roziqov, va boshqalarning ilmiy ishlarida o'rganilgan.

Kvazi novolterra kubik stoxastik operatorlarning dinamikasini o'rganish masalalar nisbatan kam o'rganilgan.

Bunga Roziqov U.A., Hamrayev A.Y. larning ishlarini ta'kidlab o'tishimiz lozim.

Ushbu magistrlik dissertatsiyasida, kubik stoxastik operatorni

$$x'_l = \sum_{i,j,k=1}^{n} P_{ijk,l} x_i x_j x_k, \qquad l \in \{1, 2, ..., n\}$$

bunda $P_{jik,l} = P_{kij,l} = ... = P_{ijk,l} \geq 0, \quad \sum_{l=1}^{n} P_{ijk,l} = 1.$ (*)

Kvazi novolterra kubik opertori

$$P_{ijk,l} = 0,\ da\ \ l \in \{i, j, k\} \quad (**)$$

Shart ostida (*) operatorni

$$S^{n-1} = \left\{ x = (x_1, x_2, ..., x_n) \in R^n : x_i \geq 0, \sum_{i=1}^{n} x_i = 1 \right\} \text{ simpleksda}$$

$P_{ijk,l} \neq 0,\ i \neq j,\ j \neq k, i \neq k,\ \ l \in \{i, j, k\}$, shartni (**) ga e'tiborga olganimizda

Kvazi novolterra kubik stoxastik operatorni hosil qilamiz

Bu operatorni ushbu disertatsiya ishimizda *n=3* bo'lgan holatda qo'zg'almas nuqtasining yagonaligini, operatorning invariant qism to'plamlarini hamda medianadagi traektoriya holati (traektoriya) o'rganildi.

Tadqiqotda qo'llanilgan uslublarning qisqacha tavsifi: Tadqiqot muammosi o'rganilayotgan kubik operatorlarni o'rganish analizning zamonaviy metodlari yordamida hal etiladi.

Tadqiqotning maqsadi: Mazkur magistrlik dissertatsiya ishi kvazi novolterra kubik operatorlarni ikki o'lchovli simpleksda o'rganishdan iborat.

Tadqiqot vazifalari:

- operatorning simpleksdagi holatini to'la o'rganish ;
- kubik operatorlarini kvazi novolterra kubik operator sifatida dinamikasini sohada o'zgarishi.

Tadqiqot obyekti: Ushbu magistrlik dissertatsiyasi kvazi novolterra kubik operatorlami ikki o'lchovli simpleks sohada o'rganishga bag'ishlangan.

Tadqiqot predmeti: Funksional analiz, matematik analizning asosiy elementlari.

Himoyaga olib chiqilayotgan asosiy holatlar: Qo'yilgan masala yetarli darajada o'rganilmagan va 3-bobda keltirilgan teoremaning isbotlanganligi yangi natijadir.

Tadqiqotning ilmiy yangiligi: Olingan natijaning hammasi yangi bo'lib, biomatematik nuqtai nazardan yaxshi talqin qilingan.

Tadqiqotning ilmiy va amaliy ahamiyati: Ushbu magistrlik dissertatsiyasining amaliy ahamiyati bugungi zamonaviy matematikadagi kubik operatorlarni dinamikasini to'la o'rganish juda ko'p murrakabliklarga va qiyinchiliklarga olib kelmoqda. Bunday qiyinchiliklarni bartaraf etish kubik stoxastik operatorlarni kvazi sharti ostida, novolterra kubik stoxastik operatorlar sifatida o'rganish asosida o'tishidir. Biroq volterra kubik stoxastik operatorlarni o'rganish va ular o'rtasida umumiy nazariyani qurish muammolar ham bor.

Natijalarning e'lon qilinganligi: Biz konkret olingan kvazi novolterra kubik operatorning trayektoriyasini o'rganib, teorema isbotladik. Bu isbotlagan teoremamizni maqola sifatida Namangan Davlat Universiteti tomonidan tashkil etilgan "Yosh matematiklarning yangi teoremalari" xalqaro konferensiyaga berdik.

Dissertatsiya tarkibining qisqacha tavsifi: Ushbu magistrlik dissertatsiyasi kirish, uch bob, xulosa va foydalanilgan adabiyotlar ro’yxatidan iborat bo'lib, kubik operatorlarni, kvazi novolterra kubik operatorini ikki o’lchovli simpleks fazoda dinamikasini to'la tadqiq qilish masalasini o’rganishga bag’ishlangan.

Ushbu magistrlik dissertatsiyasining kirish qismida dissertatsiya mavzusiga aloqador ishlarning obzori, shu bilan birga dissertatsiyaning qisqacha mazmuni keltirilgan.

Birinchi bob uchta paragrafdan iborat bo’lib, birinchi paragrafda dissertatsiya ishida qo’llaniladigan dinamik sistemalar va ularning xossalari keltirilgan. Ikkinchi paragrafda kubik operatorlaming aniqlanishi keltirilgan.
Uchinchi paragrafda kubik operatorlaming qo’zg’olmas nuqtalari haqidagi ma’lumotlar keltirilgan.

Ikkinchi bob esa uchta paragrafdan iborat bo’lib, birinchi paragrafda volterra va novolterra kubik stoxastik operatorlarning aniqlanishi keltirilgan. Ikkinchi paragrafda S^2 simpleksda aniqlangan volterra kubik operatorining aniqlanishi keltirilgan. Uchinchi paragrafda S^2 simpleksda aniqlangan novolterra kubik operatorning dinamikasi keltirilgan.

Uchinchi bob esa uchta paragrafdan iborat bo'lib, S^2 simpleksda aniqlangan kvazi novolterra kubik operatorning dinamikasi medianada o'rganilgan.

Ikkinchi va uchinchi paragraflarda trayektoriyani medianada, o'rganish keltirilgan natija teorema sifatida keltirilib isbotlangan.

I BOB. DINAMIK SISTEMALAR VA KUBIK OPERATORLARNING ANIQLANISHI

1.1-§. Dinamik sistemalar

Birinchidan, bir dinamik tizim nima? Javob juda oddiy: ilmiy kalkulyator olib, biror raqamini kiritaylik. Keyin qayta-qayta funksiya tugmachasini birini bosib boshlaylik. Bu iterativ tartibi alohida dinamik tizimiga bir misol. Biz qayta "exp" tugmasini bosaylik. Misol uchun, agar, boshlang'ich qiymat berilgan bo’lsa, bizdan raqamlar ketma-ketligini hisoblash talab etiladi.

$$x, e^x, e^{e^x}, \; e^{e^{e^x}}, \ldots$$

Bu ko'rsatkichli funksiya hisoblanadi. Ya'ni "exp" ning keyingi darajalari tajriba qayta-qayta amalga oshirilgan sari juda tez o'sib boradi. Buni biz x_0 boshlang'ich qiymatni kiritgan holimizda yaqqol ko'ramiz. "exp" funksiya murakkibligiga qaramasdan tartiblangan bo'lib, tahlil qilish ancha oson. Aslida esa betartiblik ko'p bo'lgan juda ko'p dinamik tizimlar uchraydi. Birinchi misol kabi ishlaymiz, ya'ni, dastlab x ning qiymatlari bir qator qabul qilinadi. Bir qarashda lozim bo'lgan simvolni topolmasligimiz mumkin. Shunday bo'lsada, qiymatlar natijasini kutib turganimiz sari hisob aniqligi ortib dinamik tizimiga muhim ta'sir ko'rsatadi.

Bu nuqtalarda biz iteratsiya vazifalari o'zga dinamik tizimlarni ko'plab boshqa turlari ham borligini ham ta'kidlab o'tishimiz lozim. Misol uchun, uzluksiz va diskret dinamik tizimlardan farqli o'laroq bir qancha misollar ko'ramiz.

Endi bir necha "amaliy" misollarni ko'rib chiqaylik. Dinamik tizimlar fizika klassik mexanikasi, matematik iqtisodiyot va biologiya ayirmali tenglamalari uchun ilm-fanning barcha tarmoqlarida sodir bo'ladi. Biz birinchi muvaffaqqiyatli chatishish biomatematikadagi biologik oddiy bir modelni tasvirlab berishiga turtki bo'lib xizmat qilishini ko'ramiz.

Biomatematikada biologik turlarning mu'lum bir turlarini to'plashda chatishishni uzoq muddatli nasl bera olishidan manfaatdormiz. Berilgan aniq kuzatilgan yoki emperik belgilangan parametrlarni (yirtqichlarning soni jiddiyligi, oziq-ovqat va boshqalar) mavjudligini, biomatematika tanlamalarini bayon qilish uchun matematik modelni olish lozim. Bunday gen bir yilda bir marta o'lchanadi, qachonki, sifatida yoki belgisida bir marta yoki doimiy diskret o'zgartirish qabul qilishiga qarab bir differensial tenglama yoki ayirmali tenglama bo'lishi mumkin.

Har ikki holda ham biomatematika a'zolari dastlabki gen bilan nima sodir bo’lishiga qarab tanlashadi. Vaqt turlarning yo'q bo'lishiga olib keladi, davom etish kabi gen nol bo'lishga ham moyil. Yoki gen vaqti-vaqti bilan yoki hatto tasodifiy tanlanma qiladi. Bir gen uzoq vaqtli nasl bera olishi mumkin.

Bir necha oddiy biologik modeller boshlang'ich hisob kurslarda uchraydi. Misol uchun, ko'rsatkichli o'sishi yoki parchalanish tenglamasi differensial tenglama hisoblanadi. Ushbu model biz qabul qilish belgilangan vaqtda gen turiga to'g'ri proporsional bo'ladi va tezligi boshqa bir tur o'zgarishlarga bog'liq. Bu albatta juda sodda model bo'lib, o'lim darajasi va hokazo to'lib toshgan omillar inobatga olinmaydi. Biroq, bu model tez hal bo'ladi, ayniqsa, oddiy differrensial tenglama ishlab chiqish talab qilinsa.

Dinarnik sistemalar ikki yirik toifaga bo'linadi - uzluksiz vaqtli va diskret vaqtli. Mavzuni ta'riflashda soddaroq diskret dinamik sistemalardan boshlaymiz. X -ixtiyoriy to'plam bo'lsin. Uning har bir elementiga boshqa bir tayin elementini mos qo'yuvchi qonun (akslantirish) dinamik sistema hosil qiladi. Bu qonunni f bilan belgilaylik.

Bu holat matematikada

$$f : X \to X$$

tarzida yoziladi va « f akslantirish X ni o'ziga o'tkazadi» deb o'qiladi. Bunda x elementga mos keladigan elementni $f(x)$ kabi belgilash qabul qilingan.

Agar chalkashlik keltirib chiqarmasa, qavslar tashlab, ya'ni x elementga mos element fx shaklida yozilaveradi.

Bu yerda shuni ta'kidlash lozimki, akslantirishlar- matematikaning hamma sohalarida uchraydigan asosiy tushunchalardan.

Dinamik sistemalarning bosh xususiyati bu - akslantirishni qayta va qayta qo'llaganda nima sodir bo'lishini o'rganish: x elementga $f(x)$ mos kelgach, u yana X to'plamning elementi bo’lgani uchun, « fx ga nima mos keladi?» degan savol qo'yish mumkin. Ravshanki, agar, fx ga, qavslar bilan yozilsa, $f(f(x))$ qavslarsiz yozilganda esa ffx mos keladi. Bunda yonma-yon kelgan akslantirishlarni, algebradagi kabi qisqalik uchun daraja ko'rinishida yozish qabul qilingan: f^2x . Bu yana X ning elementi bo'lgani uchun unga f akslantirishni qo'llab $f\left(f\left(f(x)\right)\right) = f^3x$ ni, so'ng f^4x ni va hokazo tuzish mumkin. Natijada

$$x, f(x), f^2(x), f^3(x), f^4(x), \ldots$$

ketma-ketlik hosil bo'ladi.

U x elementning orbitasi yoki (aniqrog'i yarim trayektoriyasi) deb ataladi va odatda $O(x)$ yoki O_x kabi belgilanadi. Shunday qilib, orbita bu - X to'plamning biror elementiga akslantirishni ketma-ket qo'llash natijasidir.

Agar $x = f^n(x)$ desak, u holda $\{x_n\}$ orbita hosil bo'ladigan ketma-ketlikdir.

$$x_n = f(x_n),\ x_0 = x \qquad (1.1.1)$$

shartlarni qanoatlantiradi. Bu shartlarning birinchisi dinamik sistema tenglamasi deyiladi, ikkinchisi esa boshlang'ich shart deb ataladi. Diskret tenglamani qanoatlantiruvchi barcha $\{x_n\}$ketma-ketliklar majmuasi uning umumiy yechimi deb ataladi. Boshlang'ich shart ana shu majmuadan tayin yechimni ajratib beradi. Dinamik sistemaning tayin boshlang'ich shartni qanoatlantiruvchi yechimini topish Koshi masalasi ham deyiladi.

Endi yuqorida bayon qilinganlarni o'zimizga yaxshi tanish misollarda talqin etaylik.

Arifmetik progressiya - dinamik sistemalarning eng sodda namunasidir.

Bunda X to'plam sifatida barcha haqiqiy sonlar to'plami R ni, yoki ratsional sonlar to'plami Q ni, yoki butun sonlar to'plami Z ni, hatto ayrim shartlarda natural sonlar to'plami N ni olish mumkin. Bunda f akslantirish

$$f(x) = x + d$$

formula bilan beriladi. Bunda, albatta, x_0 kabi d progressiya ayirmasi ham qaralayotgan X to'plamga tegishli bo'lishi lozim. Bu dinamik sistema tenglamasini yozsak, odatdagi

$$x_{n+1} = x_n + d \qquad (1.1.2)$$

ko'rinishni oladi. (1.1.2) tenglamaning barcha yechimlari $x_n = x + nd$ formula bilan beriladi (bunda x - ixtiyoriy son). U maktab matematikasida chiqariladigan arifmetik progressiyaning n - hadi uchun

$$x_n = x + (n-1)d$$

formuladan biroz farq qilmoqda: keyingi formulada progressiya x_1 haddan boshlanadi, biz esa dinamik sistemani x_0 haddan boshlayapmiz.

(1.1.2) dinamik sistema xossalariga kelsak, $d = 0$ bo'lgan hol qiziq emas, $d \neq 0$ bo'lganda esa shunday tasdiq o'rinlidir: n ortishi bilan x_n cheksizlikka intiladi.

Ikkinchi misol sifatida geometrik progressiyani qarash tabiiy. U mana bunday dinamik sistema degani:

$$f(x) = qx$$

(buyerda x —boshlang'ich had, q —maxraj).

Tenglamasi:

$$x_{n+1} = qx_n \qquad (1.1.3)$$

Bu safar maxraj $q=0$ va $q=1$ bo'lgan hollar , $q=-1$ bo'lgan hol ham qiziq emas. Shuning uchun maxrajni 0 hamda ± 1 dan farqli deb hisoblay miz. Birinchi kuzatuv shunday: Boshlang'ich had $x_0=0$ bo 'lsa, barcha had 0 ga teng.

Ya'ni, 0 nuqtaga f akslantirishni qo'llasak, u joyidan qo'zg'almaydi. Bunday xossaga ega nuqta dinamik sistemaning qo'zg'almas nuqtasi deyiladi. Shunday qilib, 0 soni (1.1.3) uchun qo'zg'almas nuqtadir.

Endi 0 dan farqli x_0 boshlang'ich nuqtalarni qaraylik. Agar $|q|>1$ bo'lsa, nomer o'sishi bilan x_n ning absolyut qiymati ham kattalashib, cheksizlikka intiladi - bu xossa dinamik sistemalar lug'atida «qo'zg'almas nuqta 0 turg'un emas» deb ifodalanadi. Atama shunday izohlanadi: boshlang'ich nuqta 0 bo'lsa, u joyidan qo'zg'almaydi, bordi-yu, 0 «biroz turtib yuborilsa», ya'ni x_0 qiymat qanchalik kichik bo'lmasin 0 dan farq qilsa, x_n ning qiymati 0 dan uzoqlashib boradi.

Nihoyat, $q<1$ bo'lsa, geometrik progressiya cheksiz kamayuvchi deyilishi ma'lum. Sababi ravshan: n ortishi bilan x_n ning absolyut qiymati kichiklashib, 0 ga intiladi. Bunday holda 0 *turg'un qo'zg'almas nuqta* deyiladi.

Ko'rilgan uch holni - arifmetik progressiya hamda turg'un va turg'un bo'lmagan geometrik progressiyalar dinamikasini grafikda ham tasvirlash mumkin.

Endi uncha tanish bo'lmagan progressiyani qarashga o'taylik. Ma'lumki, $y=kx+b$ funksiya chiziqli deb ataladi — uning grafigi to'g'ri chiziqdan iborat. Bu formula ayni paytda $f(x)=kx+b$ akslantirishni,demak, dinamik sistemani ham aniqlaydi. Uning tenglamasi

$$\lim_{t\to\infty} P(t)=+\infty \quad x_{n+1}=kx_n+b \qquad (1.1.4)$$

Bunda $b=0$ bo'lsa, geometrik progressiya, $k=1$ bo'lganda esa arifmetik progressiya hosil bo'ladi. Shunday qilib, (1.1.4) dinamik sistema bir yo'la ham arifmetik, ham geometrik progressiyani umumlashtirar ekan.

Biroq bu urnumlashtirish u qadar uzoqqa bormaydi. Haqiqatan, $k \neq 1$bo'lsa,

$$\overline{x_n} = \frac{b}{1-k} \qquad (1.1.5)$$

o'zgarmas ketma-ketlik (1.1.4) tenglamani qanoatlantiradi. Boshqacha qilib aytganda, (1.1.5) «ketma-ketlik» (1.1.4) tenglamaning xususiy yechimi bo'ladi, dinamik sistemalar tilida esa $\frac{b}{1-k}$ nuqta qo'zg'almas nuqtadir.

Quyidagi

$$\frac{dP}{dt} = kP \qquad (1.1.6)$$

tenglamani qaraymiz.

Ma'lumki, bu uzluksiz tipdagi dinamik sistemaning matematik modelini tashkil etadi.

Bu differensial tenglamani umumiy yechimi

$$P(t) = P_0 e^{kt} \qquad (1.1.7)$$

bu yerda $P_0 = P(0)$, ya'ni $P_0 -$ boshlang'ich shartni ifodalovchi kattalik.

(1.1.7) ifodani asimptotik o'rganganimizda uning limitik qiymati k parametrga bevosita bog'liqdir, ya'ni $k \geq 0$ bo'lganda

$$\lim_{t \to \infty} P(t) = +\infty$$

$k < 0$ bo'lganda esa,

$$\lim_{t \to \infty} P(t) = 0$$

demakdir.

(1.1.6) differensial tenglamani quyidagi ko'rinishda yozib olaylik.

$$P_{n+1} = kP_n \qquad (1.1.8)$$

Boshlang'ich dinamik vaziyat $P_0 = P(0)$. U holda

$$P_1 = kP_0$$
$$P_2 = kP_1 = k^2 P_0$$
$$P_3 = kP_2 = k^3 P_0$$
$$\dots$$
$$P_1 = kP_{n-1} = k^n P_0$$

Bundan esa bu vaziyatlarni asimptotik o'rganaylik.

$k > 1$ bo'lganida

$$\lim_{n \to \infty} P_n = +\infty$$

$0 < k < 1$ bo'lganida

$$\lim_{n \to \infty} P_n = 0$$

Bu ifodalarni $x_0, f(x_0), f(f(x_0)),\dots$ — ifodalar bilan taqqoslab, $x = P_0,$

$f(x) = kx$ uchun olganimizda

$$f(x) = P_1,\; f(f(x)) = k^2 x = P_2,\; f(f(f(x))) = P_3,\; \dots.$$

ketma-ketlikni olishimiz mumkin.

Xuddi shunday analizni quyidagi

$$\frac{dP}{dt} = kP(L - P) \qquad (1.1.9)$$

jarayon uchun o'rganaylik. $k > 0$ hol uchun olaylik,

$$1.\ agar\ \ P = L\ bo'lsa\ \ \frac{dP}{dt} = 0$$
$$2.\ agar\ \ P > L\ bo'lsa\ \ \frac{dP}{dt} < 0$$
$$3.\ agar\ \ P < L\ bo'lsa\ \ \frac{dP}{dt} > 0$$

(1.1.9) ning umumiy yechimi,

$$P(t)=\frac{LPe^{Lkt}}{L-P_0+Pe^{Lkt}} \qquad (1.1.10)$$

ko'rinishda bo'ladi.

(1.1.10) ni yuqoridagi uch holat bo'yicha alohida analiz qilinadi. (1.1.9) ni diskret tipdagi dinamik sistema uchun yozadigan bo'lsak,

$$P_{n+1}=kP_n(1-P_n) \qquad (1.1.11)$$

funksiya uchun $x = P_0$ olganimizda

$$f(x)=kx(1-x)$$

jarayon uchun

$$P_1=f(x)$$
$$P_2=f(f(x))$$
$$P_3=f(f(f(x)))$$

deb olishimiz mumkin.

Bir o'zgaruvchiga bog'liq dinamik sistema quyidagi ko'rinishda bo'lsin.

$$Q(x)=a_nx^n+a_{n-1}x^{n-1}+...+a_0 \qquad (1.1.12)$$

Boshlang'ich vaziyat tanlanganda, masalan, x_0 qolgan vaziyat quyidagi ko'rinishda olinadi.

$$x_1=x_0-\frac{Q(x_0)}{Q'(x_0)}$$
$$x_2=x_1-\frac{Q(x_1)}{Q'(x_1)} \qquad (1.1.13)$$
$$...$$
$$x_n=x_{n-1}-\frac{Q(x_{n-1})}{Q'(x_{n-1})}$$

(1.1.13) jarayon mavjud bo'lishi uchun $Q'(x_0)\neq 0$ bo'lishi kerakligi ravshan.

(1.1.13) ni umumiy ko'rinishda

$$N(x) = x - \frac{Q(x)}{Q'(x)} \qquad (1.1.14)$$

kabi ifodalab, $N(x)$, x ni funksiyasi sifatida o’rganish holi amaliy jihatdan qulayroqdir.

Endi yuqorida ko'rgan

$$f(x) = kx(1-x)$$

funksiyamizni quyidagicha analitik analiz qilaylik $(k = \mu)$

I.2-§. Kubik operatorning aniqlanishi

Fizikaviy dunyoni o'rganish uchun qurol sifatida matematikadan foydalanib, biz judayam qadrli intellektual tajribani olishimiz mumkin. Bir tomondan fizik (biologik) jarayonlarni bilish bizga yangi teoremalarni va ularning yechimlarini ko'rsatadi.

Boshqa tomondan matematik analiz tashqi olamning yangi ma'nosi va strukturasini beradi. Bu struktura va mantiqni bilish "jismlar tabiatini" tashkil etadi.

Ko'plab masalalar dinamik sistemalar nazariyasi bilan yechiladi.

$E=\{1,2,...,n\}$ bo'lsin, u holda

$$S^{n-1}=\left\{\left(x_1,x_2,x_3,...,x_n\right)\in\mathbb{R}^n : x_i\geq 0,\sum_{i=1}^{n}x_i=1,\ i=\overline{1,...,n}\right\} \tag{1.2.1}$$

S^{n-1} ko'phadni $n-1$ o'lchovli simpleks deb atagan edik. Har bir element $x\in S^{n-1}$ da ehtimollik dunyosi deb hisoblanadi va uni n ta elementdan tashkil topgan biologik (fizik, ijtimoiy va hokazo) sistema deb olish mumkin.

Bu sistema uchun asosiy vazifalardan biri sistemaning evolyutsiyasini o'rganish hisoblanadi. Odatda bu bir qancha qonunlar bilan aniqlanadi. Matematik genetikada vujudga kelgan muammolarni yechish uchun kvadratik operatorlar ishlatiladi, hozirgi kunda bu nazariyalar judayam rivojlangan.

E biologik sistemadagi irsiy ko'payishga doir misolni qaraylik. Bu yerda E sistema n ta har xil $1,2,...,n$ elementdan tashkil topgan. Har xil i,j,k ota-onalar uchun l ehtimollik aniqlaydi (Elementlar o'rta hisobda avlod sifatida qaraladi, ya'ni (i,j,k) uchlikning har xil turidan yangi l tur hosil bo'ladi.)

Bu ehtimollikni $P_{ijk,l}$ orqali belgilaymiz. U holda

$$P_{ijk,l}\geq 0,\ \sum_{i,j,k=1}^{n}P_{ijk,l}=1$$

o'rinli va i,j,k o'zgarishiga qaramay $P_{ijk,l}$ qiymat o'zgarmaydi

Doimiy populyatsiyada $x = (x_1, x_2, ..., x_n)$ bir qiymatli ehtimollikni aniqlaydi. Shunday qilib, $x \in S^{n-1}$ quyidagi ehtimollik

$$x_l' = \sum_{i,j,k=1}^{n} P_{ijk,l} x_i x_j x_k, \quad l = 1, 2, ..., n \qquad (1.2.2)$$

avlod uchun to'la ehtimollik bo'ladi.

Faraz qilaylik,

$$W : S^{n-1} \to S^{n-1}$$

(1.2.2) tenglik bilan ifodalanadi. W operator kubik operator deb ataladi. Shunday qilib, agar populyatsiya holati x vaziyatda bo'lsa, keyingi avlodi holati quyidagicha bo'ladi

$$x' = Wx.$$

(1.2.2) kubik operatori uchun $l \notin \{i, j, k\}$ da $P_{ijk,l} = 0$ o'rinli bo'lsa, bu operatorga Volterra kubik stoxastik operatori deyiladi va quyidagi tengliklar o'rinli. Volterra kubik stoxastik operatorining umumiy ko'rinishini $n = 3$ holi uchun quyidagicha yozishimiz mumkin.

$$x_1' = \sum_{i,j,k=1}^{3} P_{ijk,1} x_i x_j x_k = \sum_{i,j=1}^{3} \left(P_{ij1,1} x_i x_j x_1 + P_{ij2,1} x_i x_j x_2 + P_{ij3,1} x_i x_j x_3 \right) =$$

$$= \sum_{i=1}^{3} (P_{i11,1} x_i x_1^2 + P_{i12,1} x_i x_1 x_2 + P_{i13,1} x_i x_1 x_3 + P_{i21,1} x_i x_2 x_i + P_{i22,1} x_i x_2^2 +$$

$$+ P_{i23,1} x_i x_2 x_3 + P_{i31,1} x_i x_3 x_1 + P_{i32,1} x_i x_3 x_2 + P_{i33,1} x_i x^2{}_3 \Big) =$$

$$= P_{111,1} x_1^3 + P_{112,1} x_1^2 x_2 + P_{113,1} x_1^3 x_3 + P_{121,1} x_1^2 x_2 + P_{122,1} x_1 x_2^2 +$$

$$+ P_{123,1} x_1 x_2 x_3 + P_{131,1} x_1^2 x_3 + P_{132,1} x_1 x_3 x_2 + P_{133,1} x_1 x_3^2 + P_{211,1} x_2 x_1^2 +$$

$$+ P_{212,1} x_1 x_2^2 + P_{213,1} x_2 x_1 x_3 + P_{221,1} x_1 x_2^2 + P_{222,1} x_2^3 + P_{223,1} x_2^2 x_3 +$$

$$+ P_{231,1} x_2 x_3 x_1 + P_{232,1} x_2^2 x_3 + P_{233,1} x_2 x_3^2 + P_{311,1} x_3 x_1^2 + P_{312,1} x_3 x_1 x_2 +$$

$$+ P_{133,1} x_3^2 x_1 + P_{321,1} x_3 x_2 x_1 + P_{322,1} x_3 x_2^2 + P_{323,1} x_2 x_3^2 + P_{331,1} x_1 x_3^2 +$$

$$+ P_{332,1} x_3^2 x_1 + P_{333,1} x_3^3 = P_{111,1} x_1^3 + P_{222,1} x_2^3 + P_{333,1} x_3^3 + 3P_{112,1} x_1^2 x_2 +$$

$$+ 3P_{113,1} x_1^2 x_3 + 3P_{122,1} x_1 x_2^2 + 3P_{133,1} x_1 x_3^2 + 3P_{223,1} x_2^2 x_3 + 3P_{322,1} x_3 x_2^2 + 6P_{123,1} x_1 x_2 x_3$$

$$x_2' = \sum_{i,j,k=1}^{3} P_{ijk,2} x_i x_j x_k = \sum_{i,j=1}^{3} \left(P_{ij1,2}\, x_i x_j x_1 + P_{ij2,2} x_i x_j x_2 + P_{ij3,2} x_i x_j x_3 \right) =$$
$$= \sum_{i=1}^{3} (P_{i11,2} x_i x_1^2 + P_{i12,2} x_i x_1 x_2 + P_{i13,2} x_i x_1 x_3 + P_{i21,2} x_i x_2 x_i + P_{i22,2} x_i x_2^2 +$$
$$+ P_{i23,2} x_i x_2 x_3 + P_{i31,2} x_i x_3 x_1 + P_{i32,2} x_i x_3 x_2 + P_{i33,2} x_i\, x^2{}_3 \Big) =$$
$$= P_{111,2} x_1^3 + P_{112,2} x_1^2 x_2 + P_{113,2} x_1^3 x_3 + P_{121,2} x_1^2 x_2 + P_{122,2} x_1 x_2^2 +$$
$$+ P_{123,2} x_1 x_2 x_3 + P_{131,2} x_1^2 x_3 + P_{132,2} x_1 x_3 x_2 + P_{133,2} x_1 x_3^2 + P_{211,2} x_2 x_1^2 +$$
$$+ P_{212,2} x_1 x_2^2 + P_{213,2} x_2 x_1 x_3 + P_{221,2} x_1 x_2^2 + P_{222,2} x_2^3 + P_{223,2} x_2^2 x_3 +$$
$$+ P_{231,2} x_2 x_3 x_1 + P_{232,2} x_2^2 x_3 + P_{233,2} x_2 x_3^2 + P_{311,2} x_3 x_1^2 + P_{312,2} x_3 x_1 x_2 +$$
$$+ P_{133,2} x_3^2 x_1 + P_{321,2} x_3 x_2 x_1 + P_{322,2} x_3 x_2^2 + P_{323,2} x_2 x_3^2 + P_{331,2} x_1 x_3^2 +$$
$$+ P_{332,2} x_3^2 x_1 + P_{333,2} x_3^3 = P_{111,2} x_1^3 + P_{222,2} x_2^3 + P_{333,2} x_3^3 + 3P_{112,2} x_1^2 x_2 +$$
$$+ 3P_{113,2} x_1^2 x_3 + 3P_{122,2} x_1 x_2^2 + 3P_{133,2} x_1 x_3^2 + 3P_{223,2} x_2^2 x_3 + 3P_{233,2} x_2 x_3^2 + 6P_{123,2} x_1 x_2 x_3$$

$$x_3' = \sum_{i,j,k=1}^{3} P_{ijk,3} x_i x_j x_k = \sum_{i,j=1}^{3} \left(P_{ij1,3}\, x_i x_j x_1 + P_{ij2,3} x_i x_j x_2 + P_{ij3,3} x_i x_j x_3 \right) =$$
$$= \sum_{i=1}^{3} (P_{i11,3} x_i x_1^2 + P_{i12,3} x_i x_1 x_2 + P_{i13,3} x_i x_1 x_3 + P_{i21,3} x_i x_2 x_i + P_{i22,3} x_i x_2^2 +$$
$$+ P_{i23,3} x_i x_2 x_3 + P_{i31,3} x_i x_3 x_1 + P_{i32,3} x_i x_3 x_2 + P_{i33,3} x_i\, x^2{}_3 \Big) =$$
$$= P_{111,3} x_1^3 + P_{112,3} x_1^2 x_2 + P_{113,3} x_1^3 x_3 + P_{121,3} x_1^2 x_2 + P_{122,3} x_1 x_2^2 +$$
$$+ P_{123,3} x_1 x_2 x_3 + P_{131,3} x_1^2 x_3 + P_{132,3} x_1 x_3 x_2 + P_{133,3} x_1 x_3^2 + P_{211,3} x_2 x_1^2 +$$
$$+ P_{212,3} x_1 x_2^2 + P_{213,3} x_2 x_1 x_3 + P_{221,3} x_1 x_2^2 + P_{222,3} x_2^3 + P_{223,3} x_2^2 x_3 +$$
$$+ P_{231,3} x_2 x_3 x_1 + P_{232,3} x_2^2 x_3 + P_{233,3} x_2 x_3^2 + P_{311,3} x_3 x_1^2 + P_{312,3} x_3 x_1 x_2 +$$
$$+ P_{133,3} x_3^2 x_1 + P_{321,3} x_3 x_2 x_1 + P_{322,3} x_3 x_2^2 + P_{323,3} x_2 x_3^2 + P_{331,3} x_1 x_3^2 +$$
$$+ P_{332,3} x_3^2 x_1 + P_{333,3} x_3^3 = P_{111,3} x_1^3 + P_{222,3} x_2^3 + P_{333,3} x_3^3 + 3P_{112,3} x_1^2 x_2 +$$
$$+ 3P_{113,3} x_1^2 x_3 + 3P_{122,3} x_1 x_2^2 + 3P_{133,3} x_1 x_3^2 + 3P_{223,3} x_2^2 x_3 + 3P_{233,3} x_2 x_3^2 + 6P_{123,3} x_1 x_2 x_3$$

(1.2.3)

(1.2.3) uchun $P_{ijk,l} = 0,\ da\ \ l \notin \{i, j, k\}$ shart o'rinli bo'lsa, biz (1.2.2) operatorga Volterra kubik stoxastik operatori deyiladi va (1.2.3) uchun quyidagi tengliklar o'rinli.

$$\begin{cases} x_1' = P_{111,1}x_1{}^3 + 3P_{112,1}x_1{}^2 x_2 + 3P_{122,1}x_1 x_2{}^2 + 3P_{113,1}x_1{}^2 x_3 + 3P_{133,1}x_1 x_3{}^2 + 6P_{123,1}x_1 x_2 x_3 \\ x_2' = P_{222,2}x_2{}^3 + 3P_{112,2}x_1{}^2 x_2 + 3P_{122,2}x_1 x_2{}^2 + 3P_{113,2}x_1{}^2 x_3 + 3P_{133,2}x_1 x_3{}^2 + 6P_{123,2}x_1 x_2 x_3 \\ x_3' = P_{333,3}x_3{}^3 + 3P_{112,3}x_1{}^2 x_2 + 3P_{122,3}x_1 x_2{}^2 + 3P_{113,3}x_1{}^2 x_3 + 3P_{133,3}x_1 x_3{}^2 + 6P_{123,1}x_1 x_2 x_3 \end{cases}$$

II bobda Volterra kubik stoxastik operatori to'la o'rganilgan.

1.3-§. Kubik operatorning qo'zg'almas nuqtalari

Teorema.1.3.1.(Brouver). Kompakt to'plamni kompakt to'plamga akslantiradigan uzluksiz akslantirish kamida bitta qo'zg'almas nuqtaga ega.

Isbot: Ma'lumki, agar $K \subset X$ to'plamning istalgan ochiq qoplamasidan chekli qism qoplama ajratish mumkin bo'lsa, u holda K kompakt to'plam deyiladi. Yevklid fazosida esa chegaralangan yopiq to'plamni kompakt to'plam deymiz.

Ta'rif.1.3.1 Agar $W: S^{n-1} \to S^{n-1}$ akslantirish uchun shunday $x \in S^{n-1}$ nuqta mavjud bo'lib, $W(x) = x$ tenglik bajarilsa, x nuqta W akslantirishning qo'zg'almas nuqtasi deyiladi.

W operatorning qo'zg'almas nuqtasi deganda $W(\lambda) = \lambda$ tenglamani ildizlariga aytamiz va

$$Fix(W) = \{x \in S^{n-1} : W(x) = x)$$

bilan barcha qo'zg'almas nuqtalari to'plamini belgilaymiz. S^{n-1} simpleks to'plam kompakt bo'lgani uchun

$$\lambda^{(0)}, \lambda^{(1)} = W(\lambda^{(0)}), \ \lambda^{(2)} = W^2\left(\lambda^{(0)}\right), \ldots$$

$$\lambda^{(0)} = \left(x^{(0)}, y^{(0)}, z^{(0)}\right) \in S^{n-1}$$

ketma-ketlikdan yaqinlashuvchi qismiy ketma-ketlik ajratishimiz mumkin. Demak, limit nuqtalar to'plamini $w\left(\lambda^0\right)$ bilan belgilasak, $w\left(\lambda^0\right) \neq \varnothing$

Qo'zg'almas nuqtalar uch turga bo'linadi:

1) O'ziga tortuvchi;
2) O'zidan itaruvchi;
3) O'tuvchi(egar).

I Bobning xulosasi

Ushbu bobda dinamik sistemalar va ularning xossalari keltirilgan.

Dinamik sistemalarning bosh xususiyati bu - - akslantirishni qayta va qayta qo'llaganda nima sodir bo'lishini o'rganish. Kubik operatorning aniqlanishi va dinamikasini keltirdik.

Dinamik sistemalarni o'rganish uchun Kvazi kubik Volterra, Novolterra operatorlarni dinamik sistemada o'rganish ancha salmoqli bo'ladi.

II BOB. VOLTERRA KUBIK STOXASTIK OPERATORNING ANIQLANISHI

2.1 -§. Volterra kubik stoxastik operatorlarini aniqlanishi

Tarif.2.1.1. Kubik stoxastik operator deb quyidagi ko'rinishdagi operatorga aytiladi.

$$x'_l = \sum_{i,j,k=1}^{n} P_{ijk,l} x_i x_j x_k , \qquad l \in \{1,2,...,n\} = E \qquad (2.1.1)$$

bunda

$$P_{ijk,l} = P_{jik,l} = ... = P_{kij,l} \geq 0 , \qquad \sum_{l=1}^{n} P_{ijk,l} = 1$$

Tarif. 2.1.2. Kubik operatorni Volterra kubik operatori deyiladi, agar

$$P_{ijk,l} = 0, \qquad \forall l \notin \{i, j, k\}, \ \forall i, \ j \ , k \ , \ l \in E \qquad (2.1.2)$$

Shart bajarilsa.

Shunday sonlarki (2.1.2)- uchun (2.1.1) – bajariladi.

Agar quyidagi

$$P \equiv P(W) = \left\{ P_{ijk,l} \right\}_{ijk,l=1}^{n} \qquad (2.1.4)$$

Kubik matritsa bo'lsa , bu yerda

$$W\left(x^{(n)}\right) = x^{(n+1)} \qquad n = 0,1,2,...$$

simpleksda o'zgaruvchi operator.

$$S^{n-1} = \left\{ \left(x_1, x_2, x_3, ..., x_n\right) \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^{n} x_i = 1, \quad i = \overline{1,...,n} \right\} \qquad (2.1.5)$$

Masalaning qo'yilishi: (2.1.1) kubik stoxastik operatorni, (2.1.2) ga ko'ra Volterra kubik stoxastik operator sifatida, $n = 3$ hollarida dinamikasini to'la o'rganishdan iborat.

$$\begin{cases} x_1' = P_{111,1}x_1{}^3 + 3P_{112,1}x_1{}^2x_2 + 3P_{122,1}x_1x_2{}^2 + 3P_{113,1}x_1{}^2x_3 + 3P_{133,1}x_1x_3{}^2 + 6P_{123,1}x_1x_2x_3 \\ x_2' = P_{222,2}x_2{}^3 + 3P_{112,2}x_1{}^2x_2 + 3P_{122,2}x_1x_2{}^2 + 3P_{113,2}x_1{}^2x_3 + 3P_{133,2}x_1x_3{}^2 + 6P_{123,2}x_1x_2x_3 \\ x_3' = P_{333,3}x_3{}^3 + 3P_{112,3}x_1{}^2x_2 + 3P_{122,3}x_1x_2{}^2 + 3P_{113,3}x_1{}^2x_3 + 3P_{133,3}x_1x_3{}^2 + 6P_{123,1}x_1x_2x_3 \end{cases}$$

2.2-§. S^2 simpleksda aniqlangan volterra kubik operatorining to'la dinamikasi

Quyidagi ko'rinishdagi $W_\varepsilon : S^2 \to S^2$ operatorni qaraymiz.

$$\begin{cases} x' = x\left(x^2 + \left(2+3\varepsilon\right)xy + \left(1-3\varepsilon\right)y^2 + \left(1-3\varepsilon\right)z^2 + \left(2+3\varepsilon\right)xz + 2yz\right) \\ y' = y\left(y^2 + \left(2+3\varepsilon\right)xy + \left(1-3\varepsilon\right)x^2 + \left(2+3\varepsilon\right)yz + \left(1-3\varepsilon\right)z^2 + 2xz\right) \\ z' = z\left(z^2 + \left(2+3\varepsilon\right)yz + \left(1-3\varepsilon\right)y^2 + \left(1-3\varepsilon\right)x^2 + \left(2+3\varepsilon\right)xz + 2xy\right) \end{cases} \qquad (2.2.1)$$

$$-\frac{2}{3} \le \varepsilon \le \frac{1}{3}.$$

Faraz qilaylik, $\lambda_0 = \left(x_0, y_0, z_0\right) \in S^2$ boshlang'ich nuqta bo'lsin.Bunga (2.2.1) operatorni ta'sir ettirib quyidagi ko'rinishdagi ketma-ketlikka o'zgaradi:

$$\lambda_n = W_\varepsilon\left(\lambda_{n-1}\right), \quad n = 1,2,....$$

Asosiy masalada maqsadimiz $\{\lambda_n\}$ ning traektoriyasini o'rganishdan iborat, ya'ni $\lambda_n = W_n^{(n)}(\lambda_0) = W_\varepsilon\left(W_\varepsilon^{(n-1)}(\lambda_0)\right)$ ixtiyoriy $\lambda_0 \in S^2$

1. Operatorning qo'zg'olmas nuqtalari haqida.

Qo'zg'olmas nuqtalarni $W_\varepsilon(\lambda) = \lambda$ ning yechimlariga aytamiz.

a) Ma'lumli $M_1(1,0,0),\ M_2(0,1,0),\ M_3(0,0,1)$ lar (2.2.1) operatorning qo'zg'olmas

nuqtalari bu simpleksning uchlaridir.

b) Faraz qilaylik $x = 0,\ y \ne 0, z \ne 0$ u holda (2.2.1) operatordan

$$\begin{cases} 1 = y^2 + (2+3\varepsilon)yz + (1-3\varepsilon)z^2 \\ 1 = z^2 + (2+3\varepsilon)yz + (1-3\varepsilon)y^2 \end{cases}$$

Bu sistema 1 ta yagona $y = z = \frac{1}{2}$ ya'ni $N_1\left(0, \frac{1}{2}, \frac{1}{2}\right)$ ga teng.

Analogik holda bu ikkalasi ham $N_2\left(\frac{1}{2}, 0, \frac{1}{2}\right)$ *va* $N_3\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ ham olish mumkin.

c) Ixtiyoriy $xyz \neq 0$ bo'lsin (2.2.1) operatordan

$$\begin{cases} 1 = x^2 + (2+3\varepsilon)xy + (1-3\varepsilon)y^2 + (1-3\varepsilon)z^2 + (2+3\varepsilon)xz + 2yz \\ 1 = y^2 + (2+3\varepsilon)xy + (1-3\varepsilon)x^2 + (1-3\varepsilon)z^2 + (2+3\varepsilon)yz + 2xz \\ 1 = z^2 + (2+3\varepsilon)yz + (1-3\varepsilon)y^2 + (1-3\varepsilon)x^2 + (2+3\varepsilon)xz + 2xy \end{cases}$$

Bu sistema faqat yagona $x = y = z = \frac{1}{3}$ ko'rinishdagi yechimga ega $C\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

Demak (2.2.1) operatorning qo'zg'olmas nuqtalarini quyidagilar

$$T = \{M_1, M_2, M_3, N_1, N_2, N_3, C\}$$

Eslatma: Agar $\varepsilon = 0$ bo'lsa barcha nuqtalar qo'zg'olmas nuqtalar bo'lib qoladi.

$$x' = x, y' = y, z' = z$$

2. Operatorning invariant qism to'plamlari:

Tushunarliki $S^2 = M_1M_2M_3$ simpleksning barcha qirralari invariantdir. Xuddi shunday (2.2.1) operatorning M_1N_1, M_2N_2, M_3N_3 medianalari ham invariantdir.

Haqiqatdan ham, $x = y$ ekanligida (2.2.1) operatordan $x' = y'$ kelib chiqadi.

Quyidagi to'plamlarni qaraylik

$$G_1 = \{(x,y,z) \in S^2 : x > y > z > 0\}, \quad G_2 = \{(x,y,z) \in S^2 : x > z > y > 0\},$$
$$G_3 = \{(x,y,z) \in S^2 : y > x > z > 0\}, \quad G_4 = \{(x,y,z) \in S^2 : y > z > x > 0\},$$
$$G_5 = \{(x,y,z) \in S^2 : z > x > y > 0\}, \quad G_6 = \{(x,y,z) \in S^2 : z > y > x > 0\}$$

Lemma.2.2.1 G_i , $i = 1,..,6$ to'plamlarni har biri S^2 da W_ε operatorga nisbatan qism to'plamlarni tashkil qiladi.

Isbot: Simmetrik x, y, z larga nisbatan o'rganish yetarli: $W_\varepsilon(G_1) \subseteq G_1$.

Qaraylik

$$x'-y'=x\left(x^2+(2+3\varepsilon)xy+(1-3\varepsilon)y^2+(1-3\varepsilon)z^2+(2+3\varepsilon)xz+2yz\right)-$$
$$-y\left(y^2+(2+3\varepsilon)xy+(1-3\varepsilon)x^2+(2+3\varepsilon)yz+(1-3\varepsilon)z^2+2xz\right)=$$
$$=(x-y)\left(x^2+(2+6\varepsilon)xy+y^2+(1-3\varepsilon)z^2+(2+3\varepsilon)z(x+y)\right)>0$$

Demak, $x'>y'$. Anologik holda $y'>z'$ ko'rinishda ham yozish mumkin.Lemma isbotlandi.

3. Qirradagi operatorning holati.

Faraz qilaylik $\lambda\in M_1M_2$ bo'lsin (qolgan holatlar analogik holda o'rganiladi). (2.2.1) operatorning M_1M_2 qirrada o'rgansak. $(z=0)$

$$\begin{cases} x'=x\left(x^2+(2+3\varepsilon)xy+(1-3\varepsilon)y^2\right) \\ y'=y\left(y^2+(2+3\varepsilon)xy+(1-3\varepsilon)x^2\right) \end{cases} \qquad (2.2.2)$$

$x+y=1$ ni (2.2.2) ga qo'llasak $f_\varepsilon(x)=x'=x\left(-6\varepsilon x^2+9\varepsilon x+(1-3\varepsilon)\right)$ hosil bo'ladi, bu yerda f_ε $x\in[0,1]$ da o'suvchi funksiyani ifodalaydi $\varepsilon\in\left[-\frac{2}{3},\frac{1}{3}\right]$

Bu holatda $Fix(f_\varepsilon)=\{x\in[0,1]:f_\varepsilon(x)=x\}=\left\{0,\frac{1}{2},1\right\}$ qo'zg'olmas nuqtalari to'plamini olamiz.

Bu qo'zg'olmas nuqtalarning xarakterini o'rganish uchun $|f'_\varepsilon(a)|$ ni $a\in Fix(f_\varepsilon)$ holatda o'rganamiz, bu holatda $x^{(n)}=f_\varepsilon\left(x^{(n-1)}\right)$ $n\geq 1$ bunda $x^{(0)}\in[0,1]$ quyidagilarni ko'ramiz.

$$\lim_{x\to\infty}x^{(n)}=\begin{cases} 0, & x^{(0)}\in\left[0,\frac{1}{2}\right), \varepsilon\in\left(0,\frac{1}{3}\right] \\ \frac{1}{2}, & x^{(0)}\in(0,1)\setminus\left\{\frac{1}{2}\right\}, \varepsilon\in\left[-\frac{2}{3},0\right) \\ 1, & x^{(0)}\in\left(\frac{1}{2},1\right), \varepsilon\in\left(0,\frac{1}{3}\right] \end{cases}$$

4. Medianalarda operatorning holati:

Faraz qilaylik, $x=y\neq z$ (boshqa holatlar analogik holda o'rganiladi).

(2.2.1) operatorni M_1N_1 medianada o'rgansak, $2x+z=1$ ekanligidan quyidagi ko'rinishga ega bo'lamiz $g_\varepsilon(x)=x\left(-18\varepsilon x^2+15\varepsilon x+(1-3\varepsilon)\right)$

Bu yerda g_ε funksiyamiz $x\in[0,1]$ boladi agarda $\varepsilon\in\left[-\frac{2}{3},\frac{1}{3}\right]$ bo'lsa .

g_ε funksiya o'suvchi bo'ladi $\varepsilon\in\left[-\frac{2}{3},0\right), x\in\left(-\frac{1}{3},1\right]$, va

$\varepsilon\in\left(0,\frac{1}{3}\right],\ x\in\left(0,\frac{1}{3}\right)$. g_ε kamayuvchi bo'ladi $\varepsilon\in\left[-\frac{2}{3},0\right), x\in\left(0,\frac{1}{3}\right)$ va

$\varepsilon\in\left(0,\frac{1}{3}\right], x\in\left(\frac{1}{3},1\right]$.

bo'lsa.

$$Fix(g_\varepsilon)=\left\{x\in[0,1]:g_\varepsilon(x)=x\right\}=\left\{0,\frac{1}{3},\frac{1}{2}\right\}$$

kabi qo'zg'almas nuqtalarini olamiz. $g_\varepsilon(x)$ ning qo'zg'almas nuqtalarining xarakterini aniqlash uchun $\left|g'_\varepsilon(a)\right|$ ni tekshiramiz. $a\in Fix(g_\varepsilon)$ uchun $x^{(n)}=g_\varepsilon(x)(x^{(n-1)}),\ n\geq 1$, uchun $x^{(0)}\in[0,1]$ bundan

$$\lim_{x\to\infty}x^{(n)}=\begin{cases}0, & x^{(0)}\in\left[0,\frac{1}{2}\right)\setminus\left\{\frac{1}{3}\right\},\ \varepsilon\in\left(0,\frac{1}{3}\right]\\ \frac{1}{3}, & x^{(0)}\in(0,1)\setminus\left\{\frac{1}{3},\frac{1}{2}\right\},\varepsilon\in\left[-\frac{2}{3},0\right)\\ \frac{1}{2}, & x^{(0)}\in\left(\frac{1}{2},1\right],\varepsilon\in\left(0,\frac{1}{3}\right]\end{cases}$$

hosil bo'ladi.

2.3.§. Operatorning qism to'plamlarda traektoriyaning holati

G_i , $i=1,..,6$ ***da Traektoriyaning holati.***

Faraz qilaylik $\lambda=(x,y,z)$ va $\varphi(\lambda)=xyz$ bo'lsin.

Lemma.2.3.1. Ixtiyoriy $\lambda\in S^2$ uchun quyidagi tengsizlik bajariladi.

$$\varphi\left(W_\varepsilon(\lambda)\right)\le\varphi(\lambda)$$

Isbot.

$$\varphi\left(W_\varepsilon(\lambda)\right)=x'y'z'=\varphi(\lambda)\left(x^2+\left(2+3\varepsilon\right)xy+\left(1-3\varepsilon\right)y^2+\left(1-3\varepsilon\right)z^2+\left(2+3\varepsilon\right)xz+2yz\right)$$

$$\left(y^2+2+3\varepsilon\right)xy+\left(1-3\varepsilon\right)y^2+\left(1-3\varepsilon\right)z^2+\left(2+3\varepsilon\right)yz+2xz\le$$

$$\le\left(z^2+\left(2+3\varepsilon\right)yz+\left(1-3\varepsilon\right)y^2+\left(1-3\varepsilon\right)x^2+\left(2+3\varepsilon\right)xz+2xy\right)\le$$

$$\le\varphi(\lambda)\left(\frac{3+6\varepsilon(xy+yz+xz-x^2-y^2-z^2}{3}\right)^3=\varphi(\lambda)\left(\frac{3+6\varepsilon\alpha(x,y)}{3}\right)^3$$

Bu yerda $z=1-x-y$, $\alpha(x,y)=3x+3y-3x^2-3y^2-3xy-1$ ekanligidan

$\max\alpha(x,y)=\alpha\left(\frac{1}{3};\frac{1}{3}\right)=0$ kelib chiqadi.

Lemma.2.3.2. Ixtiyoriy $\lambda\in S^2$ uchun tenglik bajariladi.

$$\lim_{x\to\infty}\varphi\left(W_\varepsilon^{(n)}(\lambda)\right)=0$$

Isbot. Lemma2 ga ko'ra $\varphi\left(W_\varepsilon^{(n+1)}(\lambda)\right)\le\varphi\left(W_\varepsilon^{(n)}(\lambda)\right)$, yoki $\varphi\left(W_\varepsilon^{(n)}\right)$ monoton kamayadi, agar $\lambda\in S^2\setminus\bigcup_{i=1}^{6}G_i$ bo'lsa..

Tushunarliki $0<\varphi(\lambda)<\frac{1}{27}$ bundan $\varphi\left(W_\varepsilon^{(n)}\right)$ yaqinlashuvchi bo'adi . Aytaylik

$$\lim_{x\to\infty}\varphi\left(W_\varepsilon^{(n)}(\lambda)\right)=\mu>0$$

U holda

$$\lim_{x\to\infty}\frac{\varphi\left(W_{\varepsilon}^{(n+1)}(\lambda)\right)}{\varphi\left(W_{\varepsilon}^{(n)}(\lambda)\right)}=$$

$$=\lim_{x\to\infty}\Big(\left(x^{(n)}\right)^2+(2+3\varepsilon)x^{(n)}y^{(n)}+(1-3\varepsilon)\left(y^{(n)}\right)^2+(1-3\varepsilon)\left(z^{(n)}\right)^2+(2+3\varepsilon)x^{(n)}z^{(n)}+$$
$$+2y^{(n)}z^{(n)})\Big(\left(y^{(n)}\right)^2+(2+3\varepsilon)x^{(n)}y^{(n)}+(1-3\varepsilon)\left(x^{(n)}\right)^2+(2+3\varepsilon)y^{(n)}z^{(n)}+(1-3\varepsilon)\left(z^{(n)}\right)^2+$$
$$+2x^{(n)}z^{(n)})\Big(\left(z^{(n)}\right)^2+(2+3\varepsilon)y^{(n)}z^{(n)}+(1-3\varepsilon)\left(y^{(n)}\right)^2+(1-3\varepsilon)\left(x^{(n)}\right)^2+(2+3\varepsilon)x^{(n)}z^{(n)}+$$
$$+2x^{(n)}y^{(n)})=1$$

Qaraymiz (Lemma2 ga ko'ra)

$$\Big(\left(x^{(n)}\right)^2+(2+3\varepsilon)x^{(n)}y^{(n)}+(1-3\varepsilon)\left(y^{(n)}\right)^2+(1-3\varepsilon)\left(z^{(n)}\right)^2+(2+3\varepsilon)x^{(n)}z^{(n)}+$$
$$+2y^{(n)}z^{(n)})\Big(\left(y^{(n)}\right)^2+(2+3\varepsilon)x^{(n)}y^{(n)}+(1-3\varepsilon)\left(x^{(n)}\right)^2+(2+3\varepsilon)y^{(n)}z^{(n)}+(1-3\varepsilon)\left(z^{(n)}\right)^2+$$
$$+2x^{(n)}z^{(n)})\Big(\left(z^{(n)}\right)^2+(2+3\varepsilon)y^{(n)}z^{(n)}+(1-3\varepsilon)\left(y^{(n)}\right)^2+(1-3\varepsilon)\left(x^{(n)}\right)^2+(2+3\varepsilon)x^{(n)}z^{(n)}+$$
$$+2x^{(n)}y^{(n)})\le 1$$

yoki

$$\left(1+3\varepsilon\left(y^{(n)}+z^{(n)}-2\left(y^{(n)}\right)^2-2\left(z^{(n)}\right)^2-2y^{(n)}z^{(n)}\right)\right)\cdot$$
$$\cdot\left(1+3\varepsilon\left(x^{(n)}+z^{(n)}-2\left(x^{(n)}\right)^2-2\left(z^{(n)}\right)^2-2x^{(n)}z^{(n)}\right)\right)\cdot$$
$$\cdot\left(1+3\varepsilon\left(y^{(n)}+x^{(n)}-2\left(x^{(n)}\right)^2-2\left(y^{(n)}\right)^2-2y^{(n)}x^{(n)}\right)\right)\le 1$$

bundan

$$\lim_{x\to\infty}\max\Big\{|3\varepsilon|\left|2\left(y^{(n)}\right)^2+2\left(z^{(n)}\right)^2+2y^{(n)}z^{(n)}-y^{(n)}-z^{(n)}\right|,$$
$$|3\varepsilon|\left|2\left(x^{(n)}\right)^2+2\left(z^{(n)}\right)^2+2z^{(n)}-x^{(n)}-z^{(n)}\right|,$$
$$|3\varepsilon|\left|2\left(x^{(n)}\right)^2+2\left(y^{(n)}\right)^2+2x^{(n)}y^{(n)}-x^{(n)}-y^{(n)}\right|\Big\}=0$$

bndan $\lim_{x\to\infty} x^{(n)} = \lim_{x\to\infty} y^{(n)} = \lim_{x\to\infty} z^{(n)} = \frac{1}{3}$ hosil bo'ladi. $\mu < \frac{1}{27}$ ga ko'ra ziddiyotga kelamiz. Lemma isbotlandi.

Lemma.2.3.3. Faqat qo'zg'almas nuqtalar chegarada ichgi nuqtaga intiladi.

Isbot. Faraz qilaylik λ' ichki Traektoriyaning limiti bo'lsin. $\lambda \in \bigcup_{i=1}^{6} G_i$ bunda

$$\lambda' = \lim_{x\to\infty} W_\varepsilon^{(n+1)}(\lambda) = \lim_{x\to\infty} W_\varepsilon\left(W_\varepsilon^{(n)}(\lambda)\right) = W_\varepsilon\left(\lim_{x\to\infty} W_\varepsilon^{(n)}(\lambda)\right) = W_\varepsilon(\lambda'),$$

λ' qo'zg'almas nuqta ekanligi kelib chiqadi. $\lambda = (x, y, z) \neq C,\ xyz > 0$ Lemma3 dan kelib chiqadiki chegaradagi ichkari nuqtalariga intiladi, quyidagicha belgilash kiritamiz. $G_{12} = G_1 \cup G_2,\ G_{34} = G_3 \cup G_4,\ G_{56} = G_5 \cup G_6$

Lemma2.3.4. Quyidagi tengliklar o'rinli.

a) $$\lim_{x\to\infty} W_\varepsilon^{(n)}(\lambda) = \begin{cases} (1,0,0), & agar\ \lambda \in G_{12} \\ (0,1,0), & agar\ \lambda \in G_{34} \\ (0,0,1), & agar\ \lambda \in G_{56} \end{cases}$$

$$\varepsilon \in \left(0; \frac{1}{3}\right].$$

b) $$\lim_{x\to\infty} W_\varepsilon^{(n)}(\lambda) = C = \left(\frac{1}{3}; \frac{1}{3}; \frac{1}{3}\right)$$

$$\varepsilon \in \left[-\frac{2}{3}, 0\right), \quad \lambda \in \bigcup_{n=1}^{6} G_i, \quad i = 1, 2, ..., 6$$

Isbot. Faraz qilaylik $\lambda \in G_{12}$ (qolganlari analitik holatda isbotlanadi) $\lambda = (x, y, z), \quad x = \max\{x, y, z\}.$

Isbotlash kerak $x^{(n)} > x^{(n-1)},\ \varepsilon \in \left(0; \frac{1}{3}\right]$ yoki $x^{(n)} < x^{(n-1)},\ \varepsilon \in \left[-\frac{2}{3}; 0\right)$ bo'lganda yetarliki

$$x^2+(2+3\varepsilon)xy+(1-3\varepsilon)y^2+(1-3\varepsilon)z^2+(2+3\varepsilon)xz+2yz=$$

$$=x^2+y^2+z^2+2xy+2xz+2yz+3\varepsilon(y(x-y)+z(x-z))=$$

$$=\begin{cases}>1, & \varepsilon\in\left(0;\frac{1}{3}\right]\\ <1, & \varepsilon\in\left[-\frac{2}{3};0\right)\end{cases}$$

Tushunarliki $y(x-y)+z(x-z)>0$, $x'>x,\ \varepsilon\in\left(0;\frac{1}{3}\right]$ (yoki

$x'<x,\ \varepsilon\in\left(-\frac{2}{3};0\right]$)

Iteratsiya orqali quyidagilarni olish mumkin $x^{(n)}>x^{(n-1)},\ \varepsilon\in\left(0;\frac{1}{3}\right]$

($x^{(n)}<x^{(n-1)},\ \varepsilon\in\left(-\frac{2}{3};0\right]$) shunday qilib $x^{(n)}$ yaqinlashuvchi. Chunki

$x^{(n)}$ o'suvchi $\left(x^{(n)},y^{(n)},z^{(n)}\right)$ ning traektoriyasi $x^{(n)}>y^{(n)}>z^{(n)}$ ga ko'ra

$(x^{(n)}+y^{(n)}+z^{(n)}=1)$ yaqinlashuvchi Lemma4 ga ko'ra quyidagilar

o'rinli $\lim\limits_{x\to\infty}x^{(n)}=1$ $\left(\lim\limits_{x\to\infty}x^{(n)}=\frac{1}{3}\right)$. Lemma isbotlandi.

Olingan barcha natijalarni quyidagi teorema orqali bilish mumkin.

Teorema.2.3.1 Ixtiyoriy $\lambda=(x,y,z)\in S^2$ (2) operatorning traektoriyasi quyidagicha bo'ladi:

$$\lim_{x\to\infty} W_{\varepsilon}^{(n)}(\lambda)=\begin{cases}\lambda, \quad agar\ \lambda\in T\\ \left(0,\frac{1}{2},\frac{1}{2}\right), agar\ \lambda\in\left\{y=z>\frac{1}{3}\right\}\\ \left(\frac{1}{2},0,\frac{1}{2}\right), agar\ \lambda\in\left\{x=z>\frac{1}{3}\right\}\\ \left(\frac{1}{2},\frac{1}{2},0\right), agar\ \lambda\in\left\{x=y>\frac{1}{3}\right\}\\ (1,0,0), \quad agar\ \lambda\in\{x>y\geq z\}\cup\{x>z\geq y\}\\ (0,1,0), agar\ \lambda\in\{y>x\geq z\}\cup\{y>z\geq x\}\\ (0,0,1), agar\ \lambda\in\{z>x\geq y\}\cup\{z>y\geq x\}\end{cases}$$

$$\varepsilon\in\left(0;\frac{1}{3}\right]$$

a)
$$\lim_{x\to\infty} W_{\varepsilon}^{(n)}(\lambda)=\begin{cases}\left(0,\frac{1}{2},\frac{1}{2}\right), agar\ \lambda\in\left\{x\in S^2;\ x=0\right\}\\ \left(\frac{1}{2},0,\frac{1}{2}\right), agar\ \lambda\in\left\{x\in S^2;\ y=0\right\}\\ \left(\frac{1}{2},\frac{1}{2},0\right), agar\ \lambda\in\left\{x\in S^2;\ z=0\right\}\\ C,\ agar\ \lambda\in \operatorname{int} S^2\end{cases}$$

$$\varepsilon\in\left[-\frac{2}{3},0\right)$$

III-bob. S^2 SIMPLEKSDA ANIQLANGAN KVAZI NOVOLTERRA KUBIK OPERATORNING DINAMIKASI

3.1-§. Qo'zg'olmas nuqtalarning mavjudligi

Bizga $S^{n-1}=\left\{x\in Ў^n : x\geq 0,\ \sum_{i=1}^{n} x_i = 1\right\}$ $(n-1)-$o'lchovli simpleksni o'ziga akslantiruvchi

$$V:\ x_l^{'}=\sum_{ij,k=1}^{n} P_{ijk,l}x_i x_j x_k \qquad (3.1.1)$$

$$\sum_{l=1}^{n} P_{ijk,l}=1,\quad P_{ijk,l}=P_{jik,l}=P_{kji,l}=P_{kij,l}=P_{jki,l}=P_{ikj,l}\geq 0 \qquad (3.1.2)$$

Operator berilgan bo'lsin. Ya'ni

$$V: S^{n-1}\to S^{n-1}$$

(3.1.1), (3.1.2) ni kubik stoxastik operator deb ataymiz.

$P_{ijk,l}=0$ $\quad l\in\{i,j,k\}$ shartni qanoatlantiruvchi (3.1.1), (3.1.2) operatorni novolterra kubik operatori deb ataladi.

(3.1.1), (3.1.2)-ni *n=3* (ya'ni ikki o'lchovli simpleks) da o'rganamiz. Novolterra kubik operator S^2-ni yana S^2-ga o'tkazmaydi. Shu sababli (3.1.1), (3.1.2) operator koeffitsentlariga qo'shimcha shartlar kiritamiz

$P_{ijk,l}=1,\quad l=i,\ l\neq j,\ l\neq k,\ yoki\ l=j,\ l\neq i,\ l\neq k,\ yoki\ l=k, l\neq i, l\neq j$

va $P_{iij,l}=1,$ koeffitsentni $l\neq i, l\neq j$ shartni kiritib quyidagi operatorni hosil qilamiz.

$$V:\begin{cases} x^{'}=\alpha_1 y^3+\beta_1 z^3+3y^2z+3yz^2+2xyz \\ y^{'}=\gamma_1 x^3+\beta_2 z^3+3x^2z+3xz^2+2xyz \\ z^{'}=\gamma_2 x^3+\alpha_2 z^3+3x^2y+3xy^2+2xyz \end{cases} \qquad (3.1.3)$$

Bu yerda

$$\alpha_1+\alpha_2=\beta_1+\beta_2=\gamma_1+\gamma_2=1 \qquad (3.1.4)$$
$$\alpha_i\geq 0,\ \beta_i\geq 0,\ \gamma_i\geq 0,\ i=1,2$$

(3.1.3), (3.1.4)-ni $S^2=\{(x,y,z)\in R^3;\ x\ge 0, y\ge 0, z\ge 0,\ x+y+z=1\}$ simpleksda aniqlangan kvazi novolterra kubik operator deb ataymiz.

(3.1.3), (3.1.4)-ni qo'zg'olmas nuqtalarini o'rganamiz.

Ma'lumki $V(x)=x$ yechimi x ga qo'zg'olmas nuqta deb ataladi.

(3.1.3)-dan

$$\begin{cases}\alpha_1 y^3+\beta_1 z^3+3y^2z+3yz^2+2xyz=x\\ \gamma_1 x^3+\beta_2 x^3+3x^2z+3xz^2+2xyz=y\\ \gamma_2 x^3+\alpha_2 y^3+3x^2y+3xy^2+2xyz=z\end{cases}\qquad(3.1.5)$$

Lemma3.1.1: Agar $(x_0,y_0,z_0)\in S^2$ nuqta (3.1.5) ning musbat yechimi bo'lsa, u holda (u_0,v_0) nuqta quyidagi tenglamalar sistemasining yechimi bo'ladi:

$$\begin{cases}\dfrac{\alpha_1 v_0{}^3+\beta_1+3v_0^2+3v_0+2u_0v_0}{\gamma_2 u_0^3+\alpha_2 v_0^3+3u_0^2v_0+3u_0v_0^2+2u_0v_0}=u_0\\ \dfrac{\gamma_1 u_0{}^3+\beta_2+3u_0^2+3u_0+2u_0v_0}{\gamma_2 u_0^3+\alpha_2 v_0^3+3u_0^2v_0+3u_0v_0^2+2u_0v_0}=v_0\end{cases}\qquad(3.1.6)$$

Bu yerda $u_0=\dfrac{x_0}{z_0}$, $v_0=\dfrac{y_0}{z_0}$.

Isbot: $(x_0,y_0,z_0)\in S^2$ nuqta (3.1.5) ning yechimi bo'lsin, ya'ni

$$\begin{cases}\alpha_1 y_0{}^3+\beta_1 z_0{}^3+3y_0{}^2z_0+3y_0z_0{}^2+2x_0y_0z_0=x_0\\ \gamma_1 x_0{}^3+\beta_2 x_0{}^3+3x_0{}^2z_0+3x_0z_0{}^2+2x_0y_0z_0=y_0\\ \gamma_2 x_0{}^3+\alpha_2 y_0{}^3+3x_0{}^2y_0+3x_0y_0{}^2+2x_0y_0z_0=z_0\end{cases}$$

$u_0=\dfrac{x_0}{z_0}$, $v_0=\dfrac{y_0}{z_0}$ dan foydalansak

$$\begin{cases}\alpha_1 v_0{}^3z_0^3+\beta_1 z_0{}^3+3v_0{}^2z_0^3+3v_0z_0{}^3+2u_0v_0z_0^3=x_0\\ \gamma_1 u_0{}^3z_0^3+\beta_2 z_0{}^3+3u_0{}^2z_0^3+3u_0z_0{}^3+2u_0v_0z_0^3=y_0\\ \gamma_2 u_0{}^3z_0^3+\alpha_2 v_0{}^3z_0^3+3u_0{}^2v_0z_0^3+3u_0v_0{}^2z_0^3+2u_0v_0z_0^3=z_0\end{cases}$$

Tenglik hosil bo’ladi. Natijada quyidagi sistemani hosil qilamiz.

$$\begin{cases} \dfrac{\alpha_1 v_0{}^3 + \beta_1 + 3v_0^2 + 3v_0 + 2u_0 v_0}{\gamma_2 u_0^3 + \alpha_2 v_0^3 + 3u_0^2 v_0 + 3u_0 v_0^2 + 2u_0 v_0} = u_0 \\ \dfrac{\gamma_1 u_0{}^3 + \beta_2 + 3u_0^2 + 3u_0 + 2u_0 v_0}{\gamma_2 u_0^3 + \alpha_2 v_0^3 + 3u_0^2 v_0 + 3u_0 v_0^2 + 2u_0 v_0} = v_0 \end{cases}$$

Bu esa (u_0, v_0)nuqta (3.1.6) tenglamalar sistemasining yechimi ekanligini ko’rsatadi.

Lemma.3.1.2: Agar $(u_0, v_0)\; u_0 > 0, v_0 > 0$ nuqta (3.1.6) tenglamalar sistemasini yechimi bo’lsa u holda $(u_0 z_0, v_0 z_0, z_0) \in S^2$ nuqta (3.1.5) tenglamaning yechimi bo’ladi bu yerda

$$z_0 = \frac{1}{\gamma_2 u_0^3 + \alpha_2 v_0^3 + 3u_0^2 v_0 + 3u_0 v_0^2 + 2u_0 v_0}$$

Isbot: Agar $(x_0, y_0, z_0) \in S^2$ nuqta (3.1.6) ning yechimi bo’lsa, u holda $(u_0 z_0, v_0 z_0, z_0) \in S^2$ nuqta (3.1.5) tenglamalar sistemasining yechimi bo’lishini ko’rsatamiz.

Faraz qilaylik $(u_0, v_0)\; u_0 > 0, v_0 > 0$ nuqta (3.1.6) tenglamalar sistemasining yechimi va $x_0 = u_0 z_0\,, y_0 = v_0 z_0$ $z_0,\; x_0 = u_0 z_0$ *va* $y_0 = v_0 z_0$

$$x_0 = \frac{u_0}{\gamma_2 u_0^3 + \alpha_2 v_0^3 + 3u_0^2 v_0 + 3u_0 v_0^2 + 2u_0 v_0}$$

$$y_0 = \frac{v_0}{\gamma_2 u_0^3 + \alpha_2 v_0^3 + 3u_0^2 v_0 + 3u_0 v_0^2 + 2u_0 v_0}$$

Agar $\gamma_2 x_0{}^3 + \alpha_2 y_0{}^3 + 3x_0{}^2 y_0 + 3x_0 y_0{}^2 + 2x_0 y_0 z_0$ ifodaga $z_0,\; x_0 = u_0 z_0$ *va* $y_0 = v_0 z_0$ ni qo’ysak u holda (u_0, v_0) juftlik (3.1.6) sistemaning yechimi ya’ni

$$\frac{\alpha_1 {v_0}^3+\beta_1+3v_0^2+3v_0+2u_0v_0}{\gamma_2u_0^3+\alpha_2v_0^3+3u_0^2v_0+3u_0v_0^2+2u_0v_0}=u_0$$

Hisoblardan so'ng

$$\alpha_1 {y_0}^3+\beta_1 {z_0}^3+3{y_0}^2z_0+3y_0{z_0}^2+2x_0y_0z_0=x_0$$
$$\gamma_1 {x_0}^3+\beta_2 {x_0}^3+3{x_0}^2z_0+3x_0{z_0}^2+2x_0y_0z_0=y_0$$

ekanligi kelib chiqadi. Demak

$\left(u_0z_0, v_0z_0, z_0\right)\in S^2$ nuqta (3.1.5)-sistemaning yechimi ekan

3.2-§. S^2 simpleksda aniqlangan Kvazi novolterra kubik operatoming qo'zg'almas nuqtasining yagonaligi

Ko'plab masalalar dinamik sistemalar nazariyasi bilan yechiladi. Biz quyidagi

$$S^{(n-1)} = \left\{ (x_1, x_2, x_3, ..., x_n) \in \mathbb{R}^n : x_i \geq 0, \sum_{i=1}^{n} x_i = 1, \quad i = \overline{1,...,n} \right\} \qquad (3.2.1)$$

S^{n-1} to'plamni $n-1$ o'lchovli simpleks deb ataymiz.

Ta'rif *3.2.1* Kubik operatorni Kvazi novolterra kubik stahastik operator deyiladi, agar $P_{ijk,l} = 0,\ da\ l \in \{i, j, k\}$ $\qquad (3.2.2)$

$$P_{ijk,l} = P_{jik,l} = ... = P_{kij,l} \geq 0\ , \qquad \sum_{i,j,k=1}^{m} P_{ijk,l} = 1 \qquad (3.2.3)$$

bo'lsa, bu yerda $P_{ijk,l} \neq 0, i \neq j, j \neq k, i \neq k,\ \ l \in \{i, j, k\}$ shart asosida (3.2.2) uchun,

(3.2.3) bajariladi.

Biz ushbu dissertatsiya ishimizda stoxostik operatorning qism operatori bo'lgan kvazi kubik novolterra operatorini $n = 3$ holida o'rganishni maqsad qilib olganmiz.

Bizga $n-1$ o'lchovli simpleksni o'ziga o'tkazuvchi W operatori berilgan bo'lsin, ya'ni

$$S^{n-1} = \left\{ (x_1, x_2, x_3, ..., x_n) \in \mathbb{R}^n, \sum_{i=1}^{n} x_i = 1,\ x_i \geq 0,\ i = \overline{1,...,n} \right\}$$

ni $n-1$o'lchovli simpleks deb ataymiz,

$W : S^{n-1} \to S^{n-1}$ operatorini quyidagicha

$$W(x_l) = x_l' = \sum_{i,j,k=1}^{n} P_{ijk,l} x_i x_j x_k \qquad (3.2.4)$$

Bu yerda $\begin{cases} \sum\limits_{i,j,k=1}^{n} P_{ijk,l} = 1 \\ P_{ijk,l} = P_{jik,l} = ... = P_{kij,l} \geq 0 \end{cases} \qquad (3.2.5)$

(3.2.4), (3.2.5) ni kubik stoxastik operator deeyiladi.

Buni $n=3$ holatda o’rgansak.

$W:S^2 \to S^2$ ga o’tkazadi , ya’ni

$$W:\begin{cases} x_1' = P_{111,1}x_1^{\ 3} + P_{222,1}x_2^{\ 3} + P_{333,1}x_3^{\ 3} + 3P_{122,1}x_1x_2^{\ 2} + 3P_{112,1}x_1^{\ 2}x_2 + \\ \quad + 3P_{133,1}x_1x_3^{\ 2} + 3P_{113,1}x_1^{\ 2}x_3 + 3P_{223,1}x_2^{\ 2}x_3 + 3P_{233,1}x_2x_3^{\ 2} + 6P_{123,1}x_1x_2x_3 \\ x_2' = P_{111,2}x_1^{\ 3} + P_{222,2}x_2^{\ 3} + P_{333,2}x_3^{\ 3} + 3P_{122,2}x_1x_2^{\ 2} + 3P_{112,2}x_1^{\ 2}x_2 + \\ \quad + 3P_{133,2}x_1x_3^{\ 2} + 3P_{113,2}x_1^{\ 2}x_3 + 3P_{223,2}x_2^{\ 2}x_3 + 3P_{233,2}x_2x_3^{\ 2} + 6P_{123,2}x_1x_2x_3 \\ x_3' = P_{111,3}x_1^{\ 3} + P_{222,3}x_2^{\ 3} + P_{333,3}x_3^{\ 3} + 3P_{122,3}x_1x_2^{\ 2} + 3P_{112,3}x_1^{\ 2}x_2 + \\ \quad + 3P_{133,3}x_1x_3^{\ 2} + 3P_{113,3}x_1^{\ 2}x_3 + 3P_{223,3}x_2^{\ 2}x_3 + 3P_{233,3}x_2x_3^{\ 2} + 6P_{123,3}x_1x_2x_3 \end{cases} \qquad (3.2.6)$$

$$P_{ijk,l} = 0,\ l \notin \{i, j, k\} \qquad (3.2.7)$$

$(3.2.6)$ operatorga $(3.2.7)$ shartni ta’sir ettirsak

$$W:\begin{cases} x_1' = P_{111,1}x_1^{\ 3} + 3P_{122,1}x_1x_2^{\ 2} + 3P_{112,1}x_1^{\ 2}x_2 + 3P_{113,1}x_1^{\ 2}x_3 + 3P_{133,1}x_1x_3^{\ 2} + 6P_{123,1}x_1x_2x_3 \\ x_2' = P_{222,2}x_2^{\ 3} + 3P_{112,2}x_1^{\ 2}x_2 + 3P_{122,2}x_1x_2^{\ 2} + 3P_{223,2}x_2^{\ 2}x_3 + 3P_{233,2}x_2x_3^{\ 2} + 6P_{123,2}x_1x_2x_3 \\ x_3' = P_{333,3}x_3^{\ 3} + 3P_{113,3}x_1^{\ 2}x_3 + 3P_{133,3}x_1x_3^{\ 2} + 3P_{223,3}x_2^{\ 2}x_3 + 3P_{233,3}x_2x_3^{\ 2} + 6P_{123,3}x_1x_2x_3 \end{cases} \qquad (3.2.8)$$

$(3.2.8)$ ga Volterra kubik stoxostik operator deyiladi.biz bu operator bilan yuqoridagi **2.3-§** da bir misol orqali tanishdik. Endi

$$P_{ijk,l} = 0,\ l \in \{i, j, k\} \qquad (3.2.9)$$

$(3.2.9)$ shartni $(3.2.6)$ ga qo’llasak

$$\mathrm{W}:\begin{cases} x_1' = P_{222,2}x_2^{\ 3} + P_{333,1}x_3^{\ 3} + P_{223,1}x_2^{\ 2}x_3 + 3P_{233,1}x_2x_3^{\ 2} \\ x_2' = P_{111,2}x_1^{\ 3} + P_{333,2}x_3^{\ 3} + 3P_{133,2}x_1x_3^{\ 2} + 3P_{113,2}x_1^{\ 2}x_3 \\ x_3' = P_{222,3}x_2^{\ 3} + P_{111,3}x_1^{\ 3} + 3P_{122,3}x_1x_2^{\ 2} + 3P_{112,3}x_1^{\ 2}x_2 \end{cases} \qquad (3.2.10)$$

$(3.2.10)$ ga Novolterra kubik stoxostik operator deyiladi. Bu operator o’zini o’ziga o’tkazmaydi. Ya’ni

$$\left(x'_1+x'_2+x'_3\right)=\left(x_1+x_2+x_3\right)^3 \qquad (3.2.11)$$

Tenglik $\left(x'_1+x'_2+x'_3\right)=1$ va $\left(x_1+x_2+x_3\right)=1$ bo'lganligi uchun (3.2.11) bajarilishi shart edi . Shu sababli $(3.2.9)$ shartga qo'shimcha qilib

$$P_{ijk,l}\neq 0,\ i\neq j,\ j\neq k,\ i\neq k,\ \sum_{l=1}^{n}P_{ijk,l}=1 \qquad (3.2.9)'$$

Shartni olsak va (3.2.6) operatorga qo'llasak quydagi operator hosil bo'ladi

$$W:\begin{cases} x'_1=P_{222,1}x_2^3+P_{333,1}x_3^3+3P_{133,1}x_1x_3^2+3P_{223,1}x_2^2x_3+6P_{123,1}x_1x_2x_3 \\ x'_2=P_{111,2}x_1^3+P_{333,2}x_3^3+3P_{133,2}x_1x_3^2+3P_{113,2}x_1^2x_3+6P_{123,2}x_1x_2x_3 \\ x'_3=P_{111,3}x_1^3+P_{222,3}x_2^3+3P_{122,3}x_1x_2^2+3P_{112,3}x_1^2x_2+6P_{123,3}x_1x_2x_3 \end{cases} \qquad (3.2.12)$$

Hosil bo'lgan bu operator (3.2.11) tenglikni bajaradi va $(3.2.12)$ ga Kvazi Novolterra Kubik stoxostik operator deyiladi.Soddalik uchun quydagicha belgilashlardan foydalandik

$P_{111,2}=a_1,\ P_{111,3}=a_2,\ P_{222,1}=b_1,\ P_{222,3}=b_2,\ P_{333,1}=c_1,\ P_{333,2}=c_2,\ P_{123,1}=d_1,\ P_{123,2}=d_2,\ P_{123,3}=d_3$

$$W:\begin{cases} x'_1=b_1x_2^3+c_1x_3^3+3x_2x_3^2+3x_2^2x_3+6d_1x_1x_2x_3 \\ x'_2=a_1x_1^3+c_2x_3^3+3x_1x_3^2+3x_1^2x_3+6d_2x_1x_2x_3 \\ x'_3=a_2x_1^3+b_2x_2^3+3x_1x_2^2+3x_1^2x_2+6d_3x_1x_2x_3 \end{cases} \quad (3.2.13)$$

Hosil bo'lgan (3.2.13) tenglamalar sistemasi bizga tanish bo'lgan parametrga bogliq kubik tenglamalar sistemasidir.

Bu yerda

$a_1+a_2=1,\ b_1+b_2=1,\ c_1+c_2=1,\ d_1+d_2+d_3=1$

Biz bu kiritilgan 9 ta parameter yordamida kubik tenglamalar sistemasini yechish oson emasligini yaxshi bilamiz shu sababli (3.2.13) kubik tenglamani quydagicha xususiy holda o'rganish mening disertatsiya ishim ya'ni

$a_1=a_2=\frac{1}{2},\ b_1=b_2=\frac{1}{2},\ c_1=c_2=\frac{1}{2},\ d_1=d_2=d_3=\frac{1}{3}$ bo'lgan holda o'rganamiz va quydagi tenglamalar sistemasini hosil qilamiz

$$W:\begin{cases} x_1' = \frac{1}{2}x_2^3 + \frac{1}{2}x_3^3 + 3x_2x_3^2 + 3x_2^2x_3 + 2x_1x_2x_3 \\ x_2' = \frac{1}{2}x_1^3 + \frac{1}{2}x_3^3 + 3x_1^2x_3 + 3x_1x_3^2 + 2x_1x_2x_3 \\ x_3' = \frac{1}{2}x_1^3 + \frac{1}{2}x_2^3 + 3x_1x_2^2 + 3x_1^2x_2 + 2x_1x_2x_3 \end{cases} \qquad (3.2.14)$$

Bu operator quydagi tengliklarni qanoatlantiradi

$$x_1 + x_2 + x_3 = 1,\ x_1' + x_2' + x_3' = 1$$

ya'ni

$$\left(x'_1 + x'_2 + x'_3\right) = \left(x_1 + x_2 + x_3\right)^3$$

Tenglik bajariladi

Bu tenglamalar sistemasining qo'zgolmas nuqtalarini toppish uchun

$$W\left(x_l'\right) = x_l$$

Tenglikni ishlash kerak . Ya'ni

$$\begin{cases} x_1 = \frac{1}{2}x_2^3 + \frac{1}{2}x_3^3 + 3x_2x_3^2 + 3x_2^2x_3 + 2x_1x_2x_3 \\ x_2 = \frac{1}{2}x_1^3 + \frac{1}{2}x_3^3 + 3x_1^2x_3 + 3x_1x_3^2 + 2x_1x_2x_3 \\ x_3 = \frac{1}{2}x_1^3 + \frac{1}{2}x_2^3 + 3x_1x_2^2 + 3x_1^2x_2 + 2x_1x_2x_3 \end{cases} \qquad (3.2.15)$$

Bu sistemaning yechimi $N\left(\frac{1}{3};\frac{1}{3};\frac{1}{3}\right)$ nuqta. Hisoblab topiladi.

Bu esa $x_1 = \frac{1}{3};\ x_2 = \frac{1}{3};\ x_3 = \frac{1}{3}$ qo'zg'almas nuqtalardir.

Teorem:3.2.1 (3.2.15) sistemaning qo'zg'almas nuqtalari yagona.

Isbot: (3.2.15) sistemaning uchunchi tengligiga x_3 ni $1-x_1-x_2$ ga almashtiramiz $\frac{1}{2}x_1^3 + \frac{1}{2}x_2^3 + 3x_1^2x_2 + 3x_1x_2^2 + 2x_1x_2(1-x_1-x_2) = 1-x_1-x_2$

bu ifodani x_1 ga nisbatan kubik tenglama ko'rinishi

$$x_1^3+(2x_2)x_1^2+2(x_2+1)^2x_1+(x_2^3+2x_2-2)=0$$

bu kubik tenglamani x_1 ga nisbatan yechamiz. Buning uchun quyidagi

$\lambda^3+A\lambda^2+B\lambda+C=0$ kubik tenglamani yechish formulalaridan foydalanamiz.

$A\equiv 3a,\quad B\equiv 3b,\ \alpha\equiv a^2-b,\quad \beta=2a^3-3ab+C$ almashtirishlardan keyin quydagi shartlar asosida kubik tenglamaning uchtadan yechimlari hosil bo'ladi([7] adabiyot).

$\alpha>0 \quad \beta=0$ $\qquad \begin{cases}\lambda_1=a\\ \lambda_2=(3\alpha)^{1/2}-a\\ \lambda_3=-(3\alpha)^{1/2}-a\end{cases}$,

$|\beta|\le 2\alpha^{3/2}$ $\qquad \begin{cases}\lambda_1=2\alpha^{1/2}\sin\phi-a\\ \lambda_2=-2\alpha^{1/2}\sin(\pi/3+\phi)-a\\ \lambda_3=2\alpha^{1/2}\sin(\pi/3-\phi)-a\end{cases}$,

$\phi=(1/3)\sin^{-1}\left\{\beta/\left[2\alpha^{3/2}\right]\right\}$

$-\pi/6\le\phi\le\pi/6$

$|\beta|>2\alpha^{3/2}$ $\qquad \begin{cases}\lambda_1=-2\alpha^{1/2}\cosh\psi-a\\ \lambda_2=-\alpha^{1/2}\cosh\psi-a+i(3\alpha)^{1/2}\sinh\psi\end{cases}$,

$\psi=(1/3)\cosh^{-1}\left\{|\beta|/\left[2\alpha^{3/2}\right]\right\}$

$|\beta|<-2\alpha^{3/2}$ $\qquad \begin{cases}\lambda_1=-2\alpha^{1/2}\cosh\psi-a\\ \lambda_2=-\alpha^{1/2}\cosh\psi-a+i(3\alpha)^{1/2}\sinh\psi\\ \lambda_2=-\alpha^{1/2}\cosh\psi-a-i(3\alpha)^{1/2}\sinh\psi\end{cases}$,

$\alpha=0 \qquad -\infty<\beta<\infty$ $\qquad \begin{cases}\lambda_1=-\beta^{1/3}\cosh\psi-a\\ \lambda_2=\beta^{1/3}/2-a+3i\beta^{2/3}/4\\ \lambda_3=\beta^{1/3}/2-a-3i\beta^{2/3}/4\end{cases}$,

$\alpha<0 \qquad -\infty<\beta<\infty$ $\qquad \begin{cases}\lambda_1=-2(-\alpha)^{1/2}\sinh\theta-a\\ \lambda_2=(-\alpha)^{1/2}\sinh\theta-a+i(-3\alpha)^{1/2}\cosh\theta\\ \lambda_2=(-\alpha)^{1/2}\sinh\theta-a-i(-3\alpha)^{1/2}\cosh\theta\end{cases}$

$\theta=(1/3)\sinh^{-1}\left\{|\beta|/\left[2(-\alpha^{3/2})\right]\right\}$

Misolimizga mos shartlardan foydalansak quyidagi yechim hosil bo'ladi.

$$x_1=-2\sqrt{\frac{2}{9}x_2^2+\frac{4}{3}x_2+\frac{2}{3}}sh\theta-\frac{2}{3}x_2$$

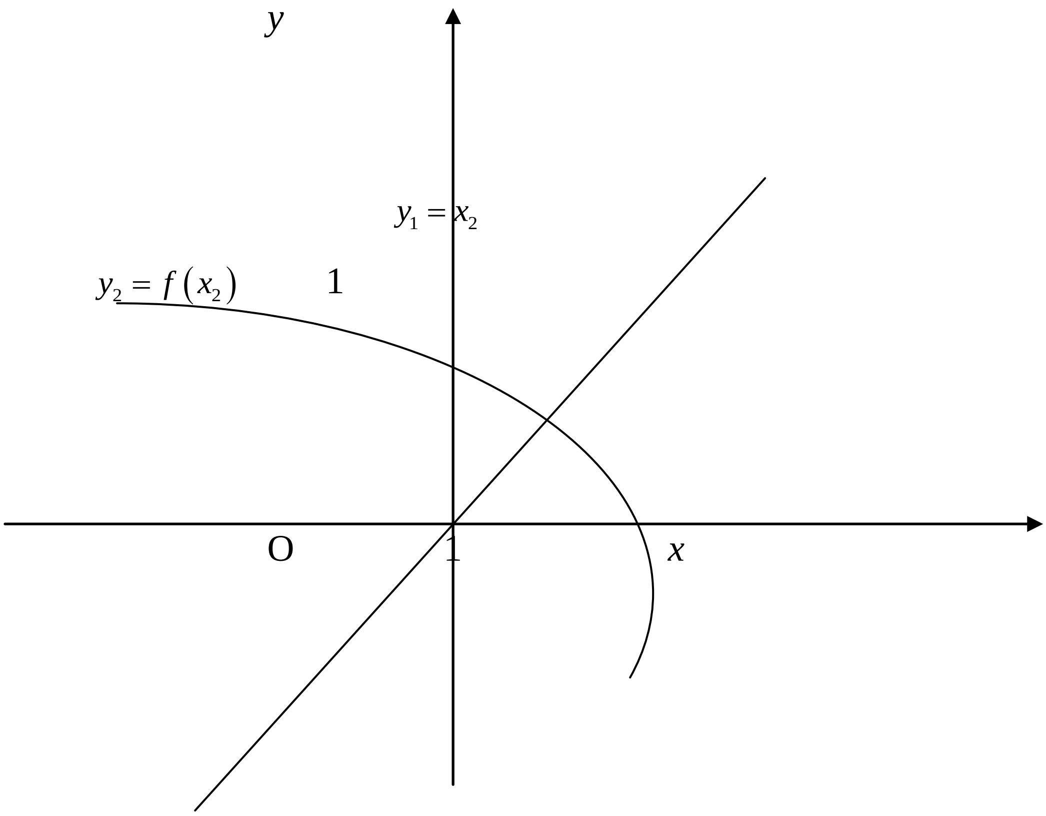
Bu yechimni $x_2=1-2x_1$ ga qo'ysak $x_2=f(x_2)$ ko'rinishdagi tenglama hosil bo'ladi.

Ya'ni

$$x_2 = -3 - 12\sqrt{\frac{2}{9}x_2{}^2 + \frac{4}{3}x_2 + \frac{2}{3}sh\theta}$$

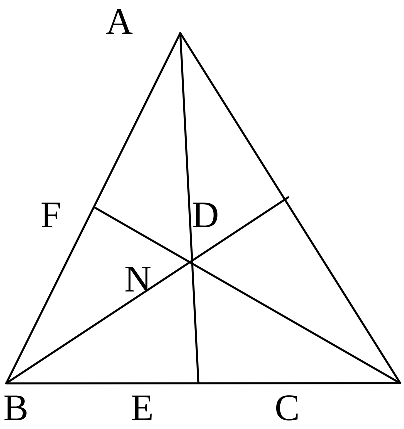
Bu tenglamaning yagonaligini ko'rsatish uchun $f\left(x_2\right) = -3 - 12\sqrt{\frac{2}{9}x_2{}^2 + \frac{4}{3}x_2 + \frac{2}{3}sh\theta}$ funksiya orqali $f(0)$, $f(1)$, $f'(x_2)$, $f''(x_2)$ larni tekshiramiz va tekshirishlar orqali $f(0) > 0$, $f(1) < 0$, $f'(x_2) < 0$, $f''(x_2) < 0$ ekanligini ko'rishimiz mumkin.

Bundan $y_1 = x_2$ *va* $y_2 = f\left(x_2\right)$ funksiyalarning bir nuqtada kesishishi kelib chiqadi va yagona qo'zg'almas nuqtaga ega ekanligi isbotlanadi.



3.3-§. Trayektoriyaning medianadagi holati va invariant qism to’plamlari

Lemma3.3.1: Simpleksning BD, AE va CF medianalari invariantdir.



BD mediana $(x_1 = x_2,\ x_3 = 1 - 2x_1)$ $(x_1,\ x_1, 1 - 2\mathrm{x}_1)$

AE median $(x_2 = x_3,\ x_1 = 1 - 2x_2)$ $(\ 1 - 2\mathrm{x}_2,\ x_2, x_2)$

CF mediana $(x_1 = x_3,\ x_2 = 1 - 2x_3)$ $(x_3,\ 1 - 2\mathrm{x}_{3,}\ x_3)$

Isbot:

$$W: \begin{cases} x_1' = \frac{1}{2}x_2^{\ 3} + \frac{1}{2}x_3^{\ 3} + 3x_2x_3^{\ 2} + 3x_2^{\ 2}x_3 + 2x_1x_2x_3 \\ x_2' = \frac{1}{2}x_1^{\ 3} + \frac{1}{2}x_3^{\ 3} + 3x_1^{\ 2}x_3 + 3x_1x_3^{\ 2} + 2x_1x_2x_3 \\ x_3' = \frac{1}{2}x_1^{\ 3} + \frac{1}{2}x_2^{\ 3} + 3x_1x_2^{\ 2} + 3x_1^{\ 2}x_2 + 2x_1x_2x_3 \end{cases} \qquad (3.3.1)$$

birinchi va ikkinchi tenglikni ayirsak quyidagilar hosi bo’ladi

$$x_1' - x_2' = \frac{1}{2}x_2^{\ 3} + \frac{1}{2}x_3^{\ 3} + 3x_2x_3^{\ 2} + 3x_2^{\ 2}x_3 + 2x_1x_2x_3 - \frac{1}{2}x_1^{\ 3} - \frac{1}{2}x_3^{\ 3} - 3x_1^{\ 2}x_3 - 3x_1x_3^{\ 2} - 2x_1x_2x_3$$

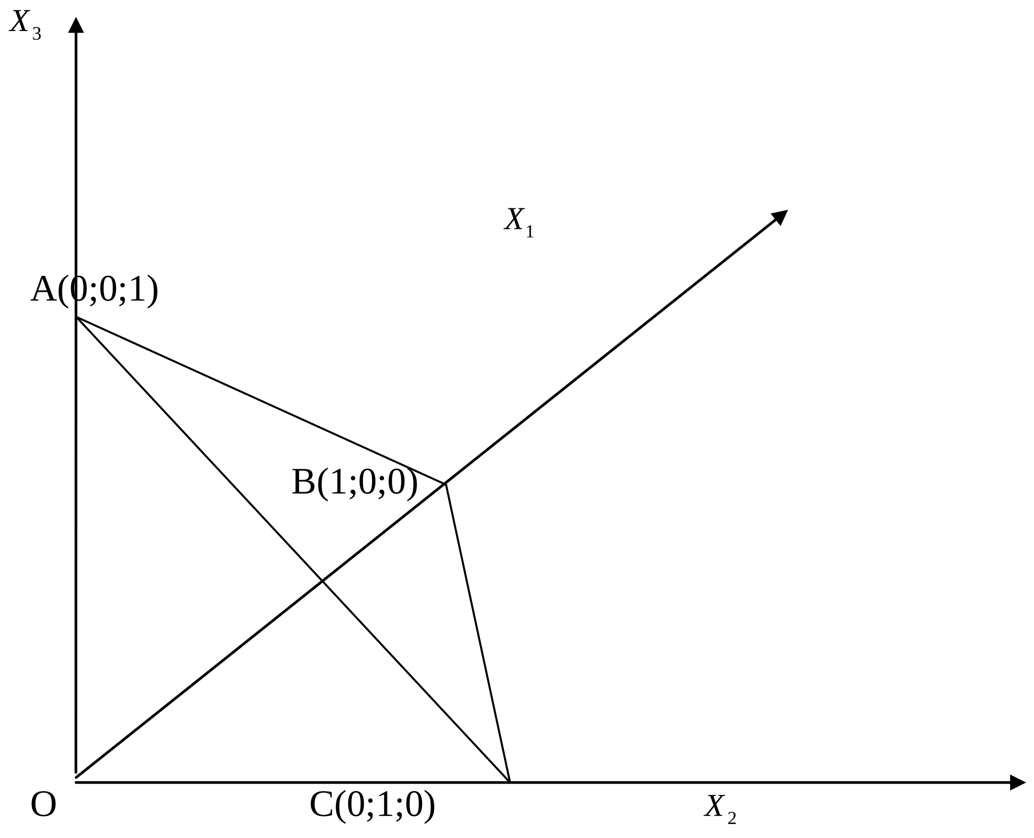
$$x_1^{'} - x_1^{'} = \frac{1}{2}(x_2 - x_1)(x_2^2 + x_1x_2 + x_1^2) + 3x_3(x_2 - x_1)(x_2 + x_1) + 3x_3^2(x_2 - x_1)$$

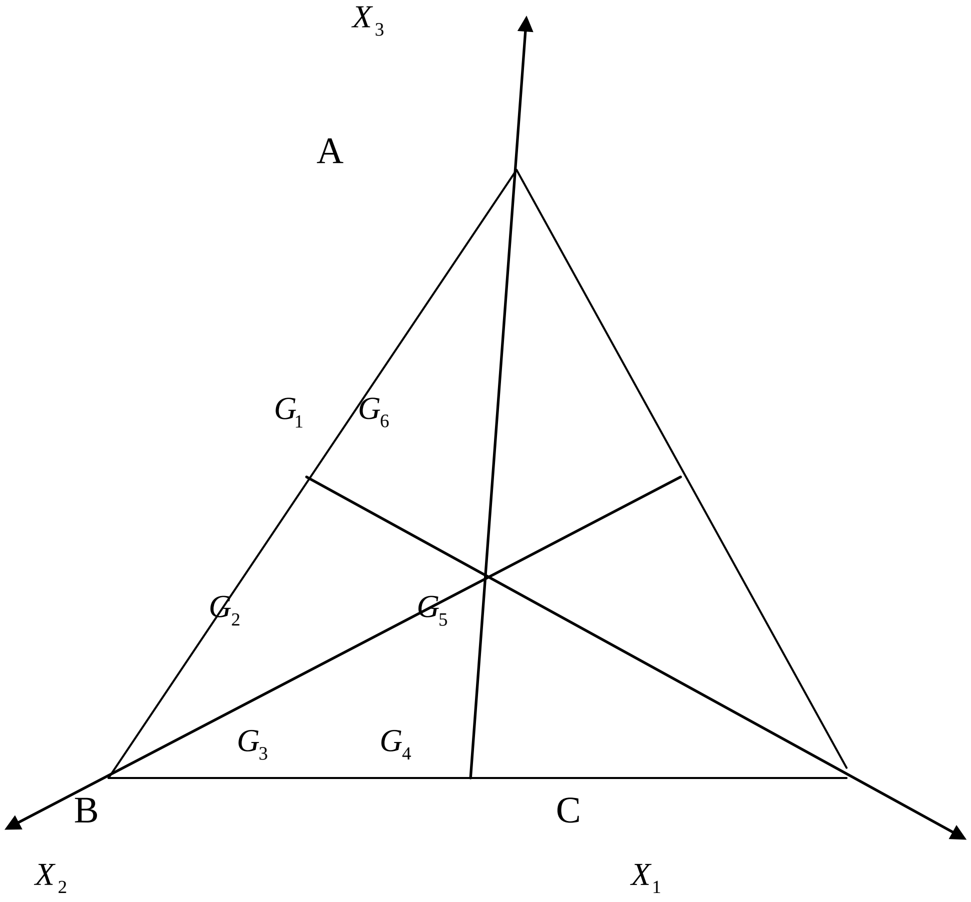
$$x_1^{'} - x_1^{'} = (x_2 - x_1)\left(\frac{1}{2}x_2^2 + \frac{1}{2}x_1x_2 + \frac{1}{2}x_1^2 + 3x_2x_3 + 3x_1x_3 + 3x_3^2\right)$$

$$x_1^{'} = x_2^{'} \qquad\qquad x_2^{'} = x_1^{'}$$

Demak $x_1 = x_2$ bo’lganda W operator ta’sirida $x_1^{'} = x_2^{'}$ bo’lar ekan. Bundan ko’rinadiki mediana ustidan olingan ixtiyoriy nuqta W tasirida yana o’ziga qaytib tushadi. Boshqa medianalar ham shu tarzda analitik tekshiriladi (isbotlanadi)

Lemma.3.3.2: G_{14}, G_{25}, G_{36} To’plamlar o’zaro invariant to’plamlar.





Isbot: $G_{14} = G_1 \cup G_4 \qquad G_{25} = G_2 \cup G_5 \qquad G_{36} = G_3 \cup G_6$

$$G_1 = \left\{ \left(x_1, x_2, x_3 \right) \in S^2, \ x_1 < x_2 < x_3 \right\}$$

$$G_2 = \left\{ \left(x_1, x_2, x_3 \right) \in S^2, \ x_1 < x_3 < x_2 \right\}$$

$$G_3 = \left\{ \left(x_1, x_2, x_3 \right) \in S^2, \ x_3 < x_1 < x_2 \right\}$$

$$G_4 = \left\{ \left(x_1, x_2, x_3 \right) \in S^2, \ x_3 < x_2 < x_1 \right\}$$

$$G_5 = \left\{ \left(x_1, x_2, x_3 \right) \in S^2, \ x_2 < x_3 < x_1 \right\}$$

$$G_6 = \left\{ \left(x_1, x_2, x_3 \right) \in S^2, \ x_2 < x_1 < x_3 \right\}$$

$x_1 > x_2$ nuqta olsak va W operatorni ta'sir ettirsak $x_1' < x_2'$ hosil bo'ladi.

$x_2 > x_3$ nuqta olsak va W operatorni ta'sir ettirsak $x_2' < x_3'$ hosil bo'ladi.

$x_1 > x_3$ nuqta olsak va W operatorni ta'sir ettirsak $x_1' < x_3'$ hosil bo'ladi.

Bundan ko'rinadiki $x_1 < x_2 < x_3$ (G_1) dan olingan W operatorni ta'sirda

$x_3 < x_2 < x_1 \ (G_4)$ ga tushadi va aksincha $x_3 < x_2 < x_1 \ (G_4)$ dan olingan ixtiyoriy nuqta W ta'sirida $x_1 < x_2 < x_3 \ (G_1)$ ga qaytib tushadi va hakoza…

Boshqa hollar ham yuqoridagi kabi analitik tekshiriladi va isbotlanadi.

III Bobning xulosasi:

Ushbu bob dissertatsiyaning asosini tashkil etadi. Biz ushbu bobda S^2 fazoda Kvazi novolterra W kubik operatorni qaraymiz ya'ni

$$W: S^2 = S^2$$

Operator W ko'rinishi quyidagicha

$$W:\begin{cases} x_1' = \frac{1}{2}x_2^3 + \frac{1}{2}x_3^3 + 3x_2x_3^2 + 3x_2^2x_3 + 2x_1x_2x_3 \\ x_2' = \frac{1}{2}x_1^3 + \frac{1}{2}x_3^3 + 3x_1^2x_3 + 3x_1x_3^2 + 2x_1x_2x_3 \\ x_3' = \frac{1}{2}x_1^3 + \frac{1}{2}x_2^3 + 3x_1x_2^2 + 3x_1^2x_2 + 2x_1x_2x_3 \end{cases}$$

$$x_1 + x_2 + x_3 = 1, \quad x_1' + x_2' + x_3' = 1$$

Yuqoridagi operatorning dinamikasini o'rganib torema isbotladik

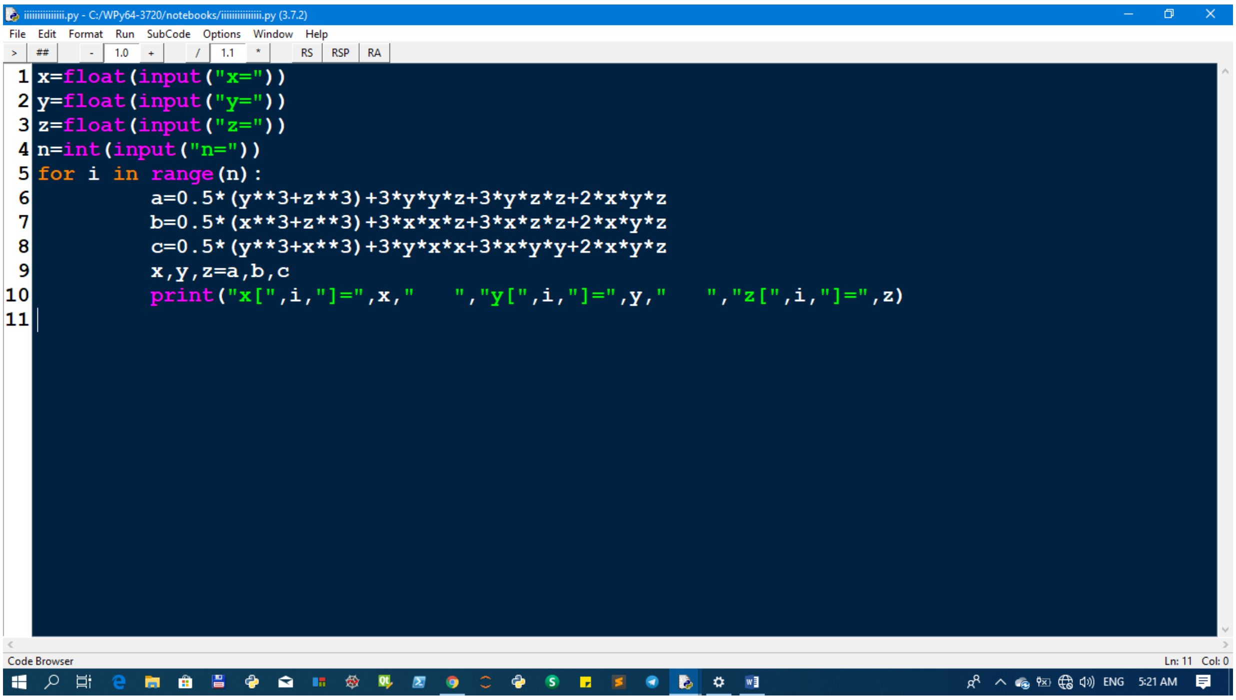
Xulosa

O'zini-o'ziga o'tkazuvchi-operatordan olingan natijalar ancha salmoqlidir. Xususan, chiziqli operatorlar ustida esa erishilgan matematik model haqida, yoki undan ham ko'proq desak mubolag'a bo'lmaydi.

Bundan tashqari bu operator diskret tipdagi dinamik sistemaning matematik modeli sifatida o'rganganimizda biomatematikaning ya'ni alohida bir tur chatishishlari natijasi sifatida urg'u berganimizda bu olingan natija shuni bildiradiki, har bir sohadagi boshlang'ich nuqta holatiga uning trayektoriyasiga bevosita bog'liq bo'lar ekan, faqat qo'zg'almas nuqtadan tashqari albatta.

Qo'zg'almas nuqtani biologik nuqtai nazardan mazmunlasak, tur chatishishi o'ziga qaytishini ifodalaydi, bu qaytish ma'lum ma'nodagi davriy ham bo'lishi mumkin. Bu davriy qo'zg'almas nuqtalarni o'rganish bu dissertatsiyaga kirmadi.

ILOVA




```
x=float(input("x="))
y=float(input("y="))
z=float(input("z="))
n=int(input("n="))
for i in range(n):
        a=0.5*(y**3+z**3)+3*y*y*z+3*y*z*z+2*x*y*z
        b=0.5*(x**3+z**3)+3*x*x*z+3*x*z*z+2*x*y*z
        c=0.5*(y**3+x**3)+3*y*x*x+3*x*y*y+2*x*y*z
        x,y,z=a,b,c
        print("x[",i,"]=",x,"   ","y[",i,"]=",y,"   ","z[",i,"]=",z)
```

```
=========== RESTART: C:/Python/notebooks/iiiiiiiiiiiii.py ===========
x=0.5
y=0.5
z=0
n=30
x[ 0 ]= 0.0625    y[ 0 ]= 0.0625    z[ 0 ]= 0.875
x[ 1 ]= 0.4957275390625    y[ 1 ]= 0.4957275390625    z[ 1 ]= 0.008544921875
x[ 2 ]= 0.07151977122794051    y[ 2 ]= 0.07151977122794051    z[ 2 ]= 0.856960457544119
x[ 3 ]= 0.4943361785456487    y[ 3 ]= 0.4943361785456487    z[ 3 ]= 0.011327642908702515
x[ 4 ]= 0.07443163678524707    y[ 4 ]= 0.07443163678524707    z[ 4 ]= 0.8511367264295057
x[ 5 ]= 0.49384139688984446    y[ 5 ]= 0.49384139688984446    z[ 5 ]= 0.012317206220310534
x[ 6 ]= 0.07546411471307937    y[ 6 ]= 0.07546411471307937    z[ 6 ]= 0.8490717705738396
x[ 7 ]= 0.4936605341386042    y[ 7 ]= 0.4936605341386042    z[ 7 ]= 0.012678931722786748
x[ 8 ]= 0.07584113342993366    y[ 8 ]= 0.07584113342993366    z[ 8 ]= 0.8483177331401183
x[ 9 ]= 0.4935937791143482    y[ 9 ]= 0.4935937791143482    z[ 9 ]= 0.012812441771260372
x[ 10 ]= 0.0759802348310881    y[ 10 ]= 0.0759802348310881    z[ 10 ]= 0.8480395303376941
x[ 11 ]= 0.4935690535174944    y[ 11 ]= 0.4935690535174944    z[ 11 ]= 0.012861892964621948
x[ 12 ]= 0.07603174973636903    y[ 12 ]= 0.07603174973636903    z[ 12 ]= 0.8479365005260944
x[ 13 ]= 0.4935598834463342    y[ 13 ]= 0.4935598834463342    z[ 13 ]= 0.01288023310382893
x[ 14 ]= 0.07605085424988743    y[ 14 ]= 0.07605085424988743    z[ 14 ]= 0.8478982914897173
x[ 15 ]= 0.49355648086383613    y[ 15 ]= 0.49355648086383613    z[ 15 ]= 0.012887038240804229
x[ 16 ]= 0.07605794286556872    y[ 16 ]= 0.07605794286556872    z[ 16 ]= 0.847884114174292
x[ 17 ]= 0.493555217986273    y[ 17 ]= 0.493555217986273    z[ 17 ]= 0.012889563743742376
x[ 18 ]= 0.07606057350141102    y[ 18 ]= 0.07606057350141102    z[ 18 ]= 0.8478788521460431
x[ 19 ]= 0.49355474820522705    y[ 19 ]= 0.49355474820522705    z[ 19 ]= 0.01289050103614118
x[ 20 ]= 0.07606154934194106    y[ 20 ]= 0.07606154934194106    z[ 20 ]= 0.8478768936559036
x[ 21 ]= 0.4935545641792477    y[ 21 ]= 0.4935545641792477    z[ 21 ]= 0.01289084866086176
x[ 22 ]= 0.07606190705696846    y[ 22 ]= 0.07606190705696846    z[ 22 ]= 0.8478761169441362
x[ 23 ]= 0.4935544088815415    y[ 23 ]= 0.4935544088815415    z[ 23 ]= 0.012890975411150753
x[ 24 ]= 0.07606199960360277    y[ 24 ]= 0.07606199960360277    z[ 24 ]= 0.847875380315624
x[ 25 ]= 0.49355356828216373    y[ 25 ]= 0.49355356828216373    z[ 25 ]= 0.01289100200531625
x[ 26 ]= 0.07606167196580797    y[ 26 ]= 0.07606167196580797    z[ 26 ]= 0.8478710717877099
x[ 27 ]= 0.49354620971184443    y[ 27 ]= 0.49354620971184443    z[ 27 ]= 0.012890827827841199
x[ 28 ]= 0.07605829256144069    y[ 28 ]= 0.07605829256144069    z[ 28 ]= 0.8478331574736678
x[ 29 ]= 0.493480064245668    y[ 29 ]= 0.493480064245668    z[ 29 ]= 0.012889106875604538
```

============ RESTART: C:/WPy64-3720/notebooks/iiiiiiiiiiiiii.py ============

x=0.2

y=0.3

z=0.5

n=30

x[0]= 0.49599999999999994 y[0]= 0.3365 z[0]= 0.16749999999999998

x[1]= 0.162535867 y[1]= 0.2846452589374999 z[1]= 0.5528188740624999

x[2]= 0.5425001404065174 y[2]= 0.3306025440014288 z[2]= 0.12689731559205314

x[3]= 0.12218726266016278 y[3]= 0.26461849806274085 z[3]= 0.6131942392770944

x[4]= 0.5915086011379473 y[4]= 0.32114198939320354 z[4]= 0.08734940946884315

x[5]= 0.08445521929781183 y[5]= 0.24222350657562092 z[5]= 0.6733212741265493

x[6]= 0.6352434409667478 y[6]= 0.3097525082473365 z[6]= 0.05500405078586185

x[7]= 0.05523293573304402 y[7]= 0.22225427875684417 z[7]= 0.7225127855099502

x[8]= 0.6669493957477632 y[8]= 0.29951913396257734 z[8]= 0.03353147028917457

x[9]= 0.03688560445645937 y[9]= 0.20874856224105037 z[9]= 0.7543658333010357

x[10]= 0.685800736776012 y[10]= 0.29233498926991575 z[10]= 0.02186427394970849

x[11]= 0.027288312616716513 y[11]= 0.20187921447775387 z[11]= 0.7708324728924384

x[12]= 0.6957201662177053 y[12]= 0.28787550087220076 z[12]= 0.01640433287082044

x[13]= 0.022812404088116828 y[13]= 0.1993287258247803 z[13]= 0.7778588699692822

x[14]= 0.7008990198855032 y[14]= 0.28503073753491276 z[14]= 0.014070242226122194

x[15]= 0.020800133657122757 y[15]= 0.1989375567908434 z[15]= 0.7802623084916482

x[16]= 0.7038938170381409 y[16]= 0.2829799745703425 z[16]= 0.01312620521035994

x[17]= 0.01986009771147569 y[17]= 0.19948281907318968 z[17]= 0.7806570736718648

x[18]= 0.7059354501156073 y[18]= 0.2812990773870128 z[18]= 0.012765443866970589

x[19]= 0.019368294973972973 y[19]= 0.20040047989158974 z[19]= 0.7802311392432121

x[20]= 0.7075587197093379 y[20]= 0.27979746835270475 z[20]= 0.012643554264303913

x[21]= 0.019063026666454083 y[21]= 0.2014519566376363 z[21]= 0.7794842436751487

x[22]= 0.7089854564415067 y[22]= 0.2783936071011458 z[22]= 0.012618617396857403

x[23]= 0.018837352280154773 y[23]= 0.20253901451907072 z[23]= 0.7786166760354383

x[24]= 0.71029691522737 y[24]= 0.2770494267843114 z[24]= 0.012632786637516014

x[25]= 0.018647191333326325 y[25]= 0.20361366802163172 z[25]= 0.7776765278994652

x[26]= 0.7114369382552406 y[26]= 0.27571417629337136 z[26]= 0.012661058975480035

x[27]= 0.018467702716642605 y[27]= 0.20457930781969938 z[27]= 0.7763896158656628

x[28]= 0.7115755062232586 y[28]= 0.2740564194032387 z[28]= 0.012678905570140748

x[29]= 0.018226847517092054 y[29]= 0.20469822391331302 z[29]= 0.7720159772135573

```
============ RESTART: C:/WPy64-3720/notebooks/iiiiiiiiiiiii.py ============
x=0
y=0
z=1
n=30
x[ 0 ]= 0.5     y[ 0 ]= 0.5     z[ 0 ]= 0.0
x[ 1 ]= 0.0625     y[ 1 ]= 0.0625     z[ 1 ]= 0.875
x[ 2 ]= 0.4957275390625     y[ 2 ]= 0.4957275390625     z[ 2 ]= 0.008544921875
x[ 3 ]= 0.07151977122794051     y[ 3 ]= 0.07151977122794051     z[ 3 ]= 0.856960457544119
x[ 4 ]= 0.4943361785456487     y[ 4 ]= 0.4943361785456487     z[ 4 ]= 0.011327642908702515
x[ 5 ]= 0.07443163678524707     y[ 5 ]= 0.07443163678524707     z[ 5 ]= 0.8511367264295057
x[ 6 ]= 0.49384139688984446     y[ 6 ]= 0.49384139688984446     z[ 6 ]= 0.012317206220310534
x[ 7 ]= 0.07546411471307937     y[ 7 ]= 0.07546411471307937     z[ 7 ]= 0.8490717705738396
x[ 8 ]= 0.4936605341386042     y[ 8 ]= 0.4936605341386042     z[ 8 ]= 0.012678931722786748
x[ 9 ]= 0.07584113342993366     y[ 9 ]= 0.07584113342993366     z[ 9 ]= 0.8483177331401183
x[ 10 ]= 0.4935937791143482     y[ 10 ]= 0.4935937791143482     z[ 10 ]= 0.012812441771260372
x[ 11 ]= 0.0759802348310881     y[ 11 ]= 0.0759802348310881     z[ 11 ]= 0.8480395303376941
x[12]=0.4935690535174944 | y[ 12 ]= 0.4935690535174944 z[ 12 ]= 0.012861892964621948
x[ 13 ]= 0.07603174973636903     y[ 13 ]= 0.07603174973636903     z[ 13 ]= 0.8479365005260944
x[ 14 ]= 0.4935598834463342     y[ 14 ]= 0.4935598834463342     z[ 14 ]= 0.01288023310382893
x[ 15 ]= 0.07605085424988743     y[ 15 ]= 0.07605085424988743     z[ 15 ]= 0.8478982914897173
x[ 16 ]= 0.49355648086383613     y[ 16 ]= 0.49355648086383613     z[ 16 ]= 0.012887038240804229
x[ 17 ]= 0.07605794286556872     y[ 17 ]= 0.07605794286556872     z[ 17 ]= 0.847884114174292
x[ 18 ]= 0.493555217986273     y[ 18 ]= 0.493555217986273     z[ 18 ]= 0.012889563743742376
x[ 19 ]= 0.07606057350141102     y[ 19 ]= 0.07606057350141102     z[ 19 ]= 0.8478788521460431
x[ 20 ]= 0.49355474820522705     y[ 20 ]= 0.49355474820522705     z[ 20 ]= 0.01289050103614118
x[ 21 ]= 0.07606154934194106     y[ 21 ]= 0.07606154934194106     z[ 21 ]= 0.8478768936559036
x[ 22 ]= 0.4935545641792477     y[ 22 ]= 0.4935545641792477     z[ 22 ]= 0.01289084866086176
x[ 23 ]= 0.07606190705696846     y[ 23 ]= 0.07606190705696846     z[ 23 ]= 0.8478761169441362
x[ 24 ]= 0.4935544088815415     y[ 24 ]= 0.4935544088815415     z[ 24 ]= 0.012890975411150753
x[ 25 ]= 0.07606199960360277     y[ 25 ]= 0.07606199960360277     z[ 25 ]= 0.847875380315624
x[ 26 ]= 0.49355356828216373     y[ 26 ]= 0.49355356828216373     z[ 26 ]= 0.01289100200531625
x[ 27 ]= 0.07606167196580797     y[ 27 ]= 0.07606167196580797     z[ 27 ]= 0.8478710717877099
x[ 28 ]= 0.49354620971184443     y[ 28 ]= 0.49354620971184443     z[ 28 ]= 0.012890827827841199
x[ 29 ]= 0.07605829256144069     y[ 29 ]= 0.07605829256144069     z[ 29 ]= 0.8478331574736678
```

```
============ RESTART: C:/WPy64-3720/notebooks/iiiiiiiiiiiii.py ============
x=0.1
y=0.2
z=0.7
n=30
x[ 0 ]= 0.5814999999999999     y[ 0 ]= 0.368     z[ 0 ]= 0.05050000000000001
x[ 1 ]= 0.0699278138125     y[ 1 ]= 0.17566990699999996     z[ 1 ]= 0.7544022791874998
x[ 2 ]= 0.6056944443304079     y[ 2 ]= 0.3638386066820887     z[ 2 ]= 0.030466948987502695
x[ 3 ]= 0.050637352743998075     y[ 3 ]= 0.15976524717249516     z[ 3 ]= 0.7895974000835048
x[ 4 ]= 0.6202445548318217     y[ 4 ]= 0.3597691716373792     z[ 4 ]= 0.019986273530792915
x[ 5 ]= 0.040398648316030655     y[ 5 ]= 0.15203835529733703     z[ 5 ]= 0.8075629963866139
x[ 6 ]= 0.6284680461612154     y[ 6 ]= 0.3562754900508123     z[ 6 ]= 0.015256463787916844
x[ 7 ]= 0.035503667531723876     y[ 7 ]= 0.14946400214820477     z[ 7 ]= 0.8150323303199051
x[ 8 ]= 0.6335023716765584     y[ 8 ]= 0.35321120215172225     z[ 8 ]= 0.013286426171220462
x[ 9 ]= 0.03313993255443892     y[ 9 ]= 0.14939939809098962     z[ 9 ]= 0.8174606693530749
x[ 10 ]= 0.6371351427848354     y[ 10 ]= 0.35037340854746424     z[ 10 ]= 0.012491448663210829
x[ 11 ]= 0.0318486447599586     y[ 11 ]= 0.15040836523692616     z[ 11 ]= 0.8177429899896467
x[ 12 ]= 0.6401840821272603     y[ 12 ]= 0.3476447739195986     z[ 12 ]= 0.01217114391273543
x[ 13 ]= 0.030993469093605182     y[ 13 ]= 0.1518525828494512     z[ 13 ]= 0.8171539479357267
x[ 14 ]= 0.6429891514702004     y[ 14 ]= 0.34497173168050554     z[ 14 ]= 0.012039116485643276
x[ 15 ]= 0.030316669495885667     y[ 15 ]= 0.15347062847115459     z[ 15 ]= 0.8162127009420074
x[ 16 ]= 0.645685530817368     y[ 16 ]= 0.3423326219207539     z[ 16 ]= 0.011981843989020977
x[ 17 ]= 0.02971698875155064     y[ 17 ]= 0.15515822671661428     z[ 17 ]= 0.8151247747132638
x[ 18 ]= 0.6483252036769469     y[ 18 ]= 0.3397199002904179     z[ 18 ]= 0.011954866576921701
x[ 19 ]= 0.02915519579277821     y[ 19 ]= 0.15687356073622538     z[ 19 ]= 0.8139711551038585
x[ 20 ]= 0.6509269629843216     y[ 20 ]= 0.3371319113883441     z[ 20 ]= 0.01194086052594414
x[ 21 ]= 0.028616239388861826     y[ 21 ]= 0.15859912934796525     z[ 21 ]= 0.8127838359592133
x[ 22 ]= 0.6534956055971423     y[ 22 ]= 0.3345689225202787     z[ 22 ]= 0.011933085972597762
x[ 23 ]= 0.02809431272181776     y[ 23 ]= 0.16032618491331166     z[ 23 ]= 0.8115723446520045
x[ 24 ]= 0.6560220512231159     y[ 24 ]= 0.33202770020470557     z[ 24 ]= 0.011928775587278423
x[ 25 ]= 0.027586113567820272     y[ 25 ]= 0.1620431107204124     z[ 25 ]= 0.8103063581403243
x[ 26 ]= 0.6584157091418393     y[ 26 ]= 0.32946572010073205     z[ 26 ]= 0.011925330491702562
x[ 27 ]= 0.02707999249633489     y[ 27 ]= 0.16368017098994966     z[ 27 ]= 0.8086602277347222
x[ 28 ]= 0.6598671635969657     y[ 28 ]= 0.32648718071709887     z[ 28 ]= 0.011907836993248308
x[ 29 ]= 0.0264792089217673     y[ 29 ]= 0.164628470458597     z[ 29 ]= 0.803687919334764
```

```
============ RESTART: C:/WPy64-3720/notebooks/iiiiiiiiiiiii.py ============
x=0.7
y=0.1
z=0.2
n=30
x[ 0 ]= 0.0505     y[ 0 ]= 0.5814999999999999     z[ 0 ]= 0.368
x[ 1 ]= 0.7544022791874998     y[ 1 ]= 0.06992781381249999     z[ 1 ]= 0.17566990699999993
x[ 2 ]= 0.030466948987502 68     y[ 2 ]= 0.6056944443304079     z[ 2 ]= 0.3638386066820887
x[ 3 ]= 0.7895974000835048     y[ 3 ]= 0.05063735274399807     z[ 3 ]= 0.1597652471724951
x[ 4 ]= 0.0199862735307929     y[ 4 ]= 0.6202445548318217     z[ 4 ]= 0.3597691716373792
x[ 5 ]= 0.8075629963866139     y[ 5 ]= 0.04039864831603064     z[ 5 ]= 0.15203835529733697
x[ 6 ]= 0.01525646378791683     y[ 6 ]= 0.6284680461612153     z[ 6 ]= 0.35627549005081227
x[ 7 ]= 0.8150323303199047     y[ 7 ]= 0.035503667531723855     z[ 7 ]= 0.1494640021482047
x[ 8 ]= 0.013286426171220445     y[ 8 ]= 0.6335023716765577     z[ 8 ]= 0.35321120215172175
x[ 9 ]= 0.8174606693530716     y[ 9 ]= 0.033139932554438783     z[ 9 ]= 0.14939939809098907
x[ 10 ]= 0.012491448663210681     y[ 10 ]= 0.6371351427848277     z[ 10 ]= 0.3503734085474599
x[ 11 ]= 0.8177429899896168     y[ 11 ]= 0.031848644759957426     z[ 11 ]= 0.15040836523692075
x[ 12 ]= 0.012171143912734102     y[ 12 ]= 0.6401840821271902     z[ 12 ]= 0.34764477391956045
x[ 13 ]= 0.817153947935458     y[ 13 ]= 0.03099346909359499     z[ 13 ]= 0.15185258284940134
x[ 14 ]= 0.012039116485631409     y[ 14 ]= 0.6429891514695664     z[ 14 ]= 0.34497173168016526
x[ 15 ]= 0.8162127009395925     y[ 15 ]= 0.030316669495795964     z[ 15 ]= 0.1534706284707006
x[ 16 ]= 0.011981843988914633     y[ 16 ]= 0.6456855308116372     z[ 16 ]= 0.3423326219177154
x[ 17 ]= 0.8151247746915593     y[ 17 ]= 0.029716988750759357     z[ 17 ]= 0.1551582267124829
x[ 18 ]= 0.01195486657596673     y[ 18 ]= 0.6483252036251579     z[ 18 ]= 0.3397199002632804
x[ 19 ]= 0.8139711549087948     y[ 19 ]= 0.029155195785791305     z[ 19 ]= 0.15687356069863162
x[ 20 ]= 0.011940860517359462     y[ 20 ]= 0.6509269625163487     z[ 20 ]= 0.3371319111459686
x[ 21 ]= 0.8127838342062012     y[ 21 ]= 0.028616239327142298     z[ 21 ]= 0.1585991290058988
x[ 22 ]= 0.011933085895385932     y[ 22 ]= 0.6534956013687647     z[ 22 ]= 0.33456892035548397
x[ 23 ]= 0.8115723288984124     y[ 23 ]= 0.028094312176473457     z[ 23 ]= 0.16032618180118835
x[ 24 ]= 0.011928774892622944     y[ 24 ]= 0.6560220130205936     z[ 24 ]= 0.3320276808695389
x[ 25 ]= 0.8103062165791759     y[ 25 ]= 0.02758610874850485     z[ 25 ]= 0.16204308241135554
x[ 26 ]= 0.01192532424161024     y[ 26 ]= 0.658415364064697     z[ 26 ]= 0.32946554742701273
x[ 27 ]= 0.8086589562734986     y[ 27 ]= 0.02707994991830367     z[ 27 ]= 0.163679913634656
x[ 28 ]= 0.01190778082505058     y[ 28 ]= 0.6598640510628141     z[ 28 ]= 0.3264856407063872
x[ 29 ]= 0.8036765466171854     y[ 29 ]= 0.026478834223383072     z[ 29 ]= 0.1646261408564532
```

```
=========== RESTART: C:/WPy64-3720/notebooks/iiiiiiiiiiiii.py ===========
x=0.4
y=0.3
z=0.3
n=30
x[ 0 ]= 0.26099999999999995     y[ 0 ]= 0.36950000000000005     z[ 0 ]= 0.3695
x[ 1 ]= 0.42440428212500014     y[ 1 ]= 0.2877978589375     z[ 1 ]= 0.28779785893750004
x[ 2 ]= 0.237168039610383     y[ 2 ]= 0.3814159801948087     z[ 2 ]= 0.3814159801948087
x[ 3 ]= 0.4574193733739268     y[ 3 ]= 0.2712903133130372     z[ 3 ]= 0.2712903133130372
x[ 4 ]= 0.20709649493193957     y[ 4 ]= 0.3964517525340321     z[ 4 ]= 0.3964517525340321
x[ 5 ]= 0.5012836980219589     y[ 5 ]= 0.24935815098902608     z[ 5 ]= 0.24935815098902608
x[ 6 ]= 0.1708738610002365     y[ 6 ]= 0.4145630694998984     z[ 6 ]= 0.41456306949989835
x[ 7 ]= 0.5574686617800638     y[ 7 ]= 0.22126566911001788     z[ 7 ]= 0.22126566911001794
x[ 8 ]= 0.13041549599846314     y[ 8 ]= 0.434792252000918     z[ 8 ]= 0.43479225200091787
x[ 9 ]= 0.6246735987055267     y[ 9 ]= 0.18766320064768502     z[ 9 ]= 0.18766320064768507
x[ 10 ]= 0.09026202704333838     y[ 10 ]= 0.4548689864796761     z[ 10 ]= 0.454868986479676
x[ 11 ]= 0.6961566773388584     y[ 11 ]= 0.1519216613346065     z[ 11 ]= 0.15192166133460655
x[ 12 ]= 0.05667952533087961     y[ 12 ]= 0.47166023734666745     z[ 12 ]= 0.4716602373466673
x[ 13 ]= 0.7597081501177273     y[ 13 ]= 0.12014592497745784     z[ 13 ]= 0.12014592497745792
x[ 14 ]= 0.03407302146350262     y[ 14 ]= 0.4829634893772134     z[ 14 ]= 0.4829634893772132
x[ 15 ]= 0.8044665732988953     y[ 15 ]= 0.09776671367744603     z[ 15 ]= 0.09776671367744616
x[ 16 ]= 0.021920120245061646     y[ 16 ]= 0.4890399408581505     z[ 16 ]= 0.4890399408581502
x[ 17 ]= 0.8291965987497054     y[ 17 ]= 0.08540170356719083     z[ 17 ]= 0.08540170356719096
x[ 18 ]= 0.01645552144391074     y[ 18 ]= 0.49177224810417564     z[ 18 ]= 0.4917722481041753
x[ 19 ]= 0.840470415383735     y[ 19 ]= 0.07976481878652542     z[ 19 ]= 0.07976481878652555
x[ 20 ]= 0.014247346652388186     y[ 20 ]= 0.49287640610898925     z[ 20 ]= 0.4928764061089889
x[ 21 ]= 0.8450535650126381     y[ 21 ]= 0.07747345579926833     z[ 21 ]= 0.07747345579926847
x[ 22 ]= 0.013399297164780728     y[ 22 ]= 0.4933010663347129     z[ 22 ]= 0.4933010663347125
x[ 23 ]= 0.8468210180826115     y[ 23 ]= 0.07659163571307     z[ 23 ]= 0.07659163571307015
x[ 24 ]= 0.01308053128394537     y[ 24 ]= 0.49346616864875475     z[ 24 ]= 0.49346616864875437
x[ 25 ]= 0.8475141383667743     y[ 25 ]= 0.07626223393719614     z[ 25 ]= 0.0762622339371963
x[ 26 ]= 0.012962912771916654     y[ 26 ]= 0.4935764552114834     z[ 26 ]= 0.493576455211483
x[ 27 ]= 0.8480237751996667     y[ 27 ]= 0.07616186731578686     z[ 27 ]= 0.07616186731578702
x[ 28 ]= 0.012930652062753036     y[ 28 ]= 0.49405611988109194     z[ 28 ]= 0.49405611988109155
x[ 29 ]= 0.8504766445832161     y[ 29 ]= 0.07632764744797113     z[ 29 ]= 0.07632764744797128
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