**URGANCH DAVLAT UNIVERSITETI HUZURIDAGI**
**ILMIY DARAJA BERUVCHI**
**PhD.03/30.12.2019.FM.55.01 RAQAMLI ILMIY KENGASH**

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**BUXORO DAVLAT UNIVERSITETI**

**NURIDDINOV JAVLON ZAFAROVICH**

# O‘RAMA KO‘RINISHIDAGI INTEGRO - DIFFERENSIAL ISSIQLIK O‘TKAZUVCHANLIK TENGLAMASI UCHUN TESKARI MASALALAR

**01.01.02 – Differensial tenglamalar va matematik fizika**

**Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi**
**AVTOREFERATI**

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**Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi**

**Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences**

**Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам**

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**Falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Vazirlar Mahkamasi huzuridagi Oliy attestatsiya komissiyasida B2019.4.PhD/FM433 raqam bilan ro'yxatga olingan.**

Dissertatsiya Buxoro davlat universiteti va V.I. Romanovskiy nomidagi Matematika instituti Buxoro bo'linmasida bajarilgan.

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**KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)**

**Dissertatsiya mavzusining dolzarbligi va zarurati.** Jahon miqyosida olib borilayotgan ko‘plab ilmiy va amaliy tadqiqotlar xususiy hosilali differensial va integro–differensial tenglamalar, ular uchun qo‘yilgan to‘g‘ri va teskari masalalarni o‘rganishga olib kelinadi. Teskari masalalar astronomiyaga, kvantlarning tarqalishi nazariyasiga, geofizikaga, issiqlik fizikasiga, tibbiyotga, hamda EHM paydo bo‘lishi bilan zamonaviy ilm–fanning barcha sohalariga kirib bordi. Matematik fizikada to‘g‘ri masalalarning yechimini topish uchun tenglamaning koeffitsiyentlarini, soha chegarasini, boshlang‘ich va chegaraviy shartlarni berish lozim. Ammo amaliyotda tenglama koeffitsiyentlarini har doim ham berib bo‘lmaydi. Har doim ham tadbiqiy masalalarda boshlang‘ich va chegaraviy shartlarni, shuningdek, soha chegarasini aniqlab bo‘lmaydi. Bunday hollarda, to‘g‘ri masala yechimiga nisbatan qo‘shimcha ma’lumot kiritib, teskari masala yechimini izlash, ya’ni koeffitsiyentlar, integro–differensial tenglama holida integral had yadrosi, boshlang‘ich yoki chegaraviy shartlar, soha chegarasini topish zarurati paydo bo‘ladi. Bu kabi masalalarni yechish usullarining to‘la shakllanmaganligi bois integro–differensial issiqlik o‘tkazuvchanlik tenglamasidan yadroni aniqlash teskari masalalarini yechish muhim vazifalardan biri bo‘lib qolmoqda.

Hozirgi kunda jahon miqyosida matematik fizikaning eng tez rivojlanayotgan sohasi – teskari masalalarni tadqiq qilish usullariga alohida e’tibor qaratilmoqda. Bu soha fizika va texnika fanlaridagi eng muhim matematik muammolardan biriga aylandi. Ushbu muammoning juda keng doirada qo‘llanilishi, uning nazariyasining yangiligi va murakkabligi sababli ko‘plab olimlarning e’tiborini tortdi. So‘nggi yillarda issiqlik tarqalish jarayonlarini boshqarish jadal rivojlanmoqda, chunki har bir muhitning issiqlik o‘tkazuvchanligi va relaksatsiya funksiyasi turlicha bo‘lib, bu kattaliklar muhitning boshlang‘ich holatiga va xossalariga chambarchas bog‘liqdir. Shu sababli integro–differensial issiqlik o‘tkazuvchanlik tenglamalari uchun boshlang‘ich, boshlang‘ich-chegaraviy masalalar hamda ularga qo‘yilgan teskari masalalarni o‘rganish maqsadli ilmiy tadqiqotlar hisoblanadi.

Mamlakatimizda fundamental fanlarning ilmiy va amaliy tadbiqiga ega bo‘lgan matematik fizikaning dolzarb yo‘nalishlariga e’tibor kuchaytirildi. Bu borada parabolik tipdagi integro–differensial tenglamalardan relaksatsiya funksiyasini aniqlash bo‘yicha teskari masalalarni tadqiq etishga e’tibor qaratildi. Ushbu izlanishlar natijasida o‘zgaruvchan koeffitsiyentli parabolik tipdagi integro–differensial tenglamalar uchun teskari masalalar yechimining mavjudligi va yagonaligini isbotlashga erishildi. Differensial tenglama va matematik fizika fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish asosiy vazifa etib belgilandi[1]. Bu qaror ijrosini ta’minlashda matematik fizikaning integro–differensial

[1] O‘zbekiston Respublikasi Prezidentining 2019 yil 9 iyuldagi «Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I. Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida»gi PQ-4387-son qarori.

tenglamalar nazariyasini rivojlantirish muhim ahamiyatga ega.

Oʻzbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi PF-4947-son «Oʻzbekiston Respublikasini yanada rivojlantirish boʻyicha harakatlar strategiyasi toʻgʻrisida» Farmoni, 2019 yil 9 iyuldagi PQ-4387-son «Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qoʻllab-quvvatlash, shuningdek, Oʻzbekiston Respublikasi Fanlar Akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari toʻgʻrisida» va 2020 yil 7 maydagi PQ-4708-son «Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari toʻgʻrisida»gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

**Tadqiqotning respublika fan va texnologiyalari rivojlanishi-ning ustuvor yoʻnalishlariga bogʻliqligi.** Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. «Matematika, mexanika va informatika» ustuvor yoʻnalishi doirasida bajarilgan.

**Muammoning oʻrganilganlik darajasi.** Matematik fizikaning teskari masalalari nazariyasini rivojlantirishga A.S. Alekseev, M.M. Lavrentev, V.G. Romanov, A. Lorenzi va boshqalar oʻz hissalarini qoʻshganlar. Parabolik tipdagi tenglamalar uchun teskari masalalar V.G. Romanov, A.I. Prilepko, A.D. Iskendarov, N.Ya. Beznoshenko, F.A. Kolombo, Ya. Yannolar tomonidan qoʻyilgan va tadqiq qilingan. Parabolik tipdagi integro-differensial tenglamalardan integral hadi yadrosini aniqlash boʻyicha teskari masalalarni tadqiq etishning turli usullari L.V. Lorensi, M. Graselli, Ya. Yanno, F.A. Kolombo, D.K. Durdiyev va boshqalarning ishlarida taklif etilgan va rivojlantirilgan. Jumladan, N.Ya. Beznoshenkoning[2] ishida parabolik tipdagi tenglamalar uchun oʻzgaruvchan koeffitsiyentni aniqlash boʻyicha teskari masalalar tadqiq qilingan. A.I. Prilepko, A.B. Kostin[3] maqolasida oʻzgaruvchan koeffitsiyentli parabolik tipdagi boshlangʻich-chegaraviy masalalardan tenglamaning koeffitsiyentlarini aniqlash teskari masalasi tadqiq etilgan, yechimning mavjudligi va yagonaligi haqidagi teoremalar isbotlangan. A.D. Iskendarov tadqiqotlarida bu kabi tenglama koeffitsiyentlarini aniqlashning analitik va sonli usullari taklif etilgan. Ya. Yanno, L.V. Volfersdorf[4] ishida chegaralangan sohada integro–differensial issiqlik oʻtkazuvchanlik tenglamasi uchun integral hadi yadrosini aniqlash boʻyicha bir oʻlchamli teskari masala qaralgan va yechimning mavjudligi, yagonaligi isbotlangan hamda turgʻunlik baholari olingan.

Ushbu dissertatsiya ishida oʻzgaruvchan koeffitsiyentli integro–differensial issiqlik oʻtkazuvchanlik tenglamasi uchun koʻp oʻlchamli toʻgʻri va teskari

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[2]Н. Я. Безнощенко, Об определении коэффициента в параболическом уравнении, Дифференц. уравнения, 1974, том 10, номер 1, 24-35.

[3] Prilepko A.I., Kostin A.B. On inverse problems of determining a coefficient in parabolic equation // Siberian Math. J., 1993, vol. 34, №5, pp. 923-937.

[4] Janno J., Wolfersdorf L.V. Inverse problems for identification of memory kernels in heat flow // Ill-Posed Problems, 1996, vol. 4, №1, pp. 39–66.

masalalarning bir qiymatli yechiluvchanlik muammolari Gyolder (Hölder) fazolarida oʻrganilgan. Bunda V.G. Romanov, Ya. Yanno, L.V. Volfersdorf va D.Q. Durdiyevlarning ilmiy tadqiqotlari masalalarning qoʻyilishi jihatidan yaqin va tadqiq etishda ular tomonidan tavsiya etilgan usullardan foydalanilgan.

**Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bogʻliqligi.** Dissertatsiya Buxoro davlat universitetining rejalashtirilgan tadqiqot mavzusiga muvofiq amalga oshirildi. Buxoro davlat universitetining F-4-02 "Matematik fizikaning holatlar toʻplami cheksiz boʻlgan modellari termodinamikasi" mavzusidagi grant loyihasi doirasida bajarildi.

**Tadqiqotning maqsadi.** Oʻzgaruvchan koeffitsiyentli integro–differensial issiqlik oʻtkazuvchanlik tenglamasi uchun koʻp oʻlchamli toʻgʻri va teskari masalalarning Gyolder (Hölder) fazolarida bir qiymatli yechiluvchanlik masalalarini tadqiq etishdan iborat.

**Tadqiqotning vazifalari.** Tadqiqotning asosiy vazifalari quyidagilardan iborat:

oʻzgaruvchan koeffitsiyentli integro–differensial issiqlik tarqalish tenglamasi uchun qoʻyilgan Koshi masalasi yechimining mavjudligi va yagonaligini aniqlash;

toʻgʻri masala yechimiga qoʻyilgan qoʻshimcha shartdan foydalanib, koʻp oʻlchamli yadroni Gyolder (Hölder) fazolarida bir qiymatli yechiluvchanligini tadqiq etish;

issiqlik tarqalish integro–differensial tenglamasidan koʻp oʻlchamli ajralgan yadroni aniqlash masalasining bir qiymatli yechiluvchanligini Gyolder (Hölder) fazolarida koʻrsatish;

oʻzgaruvchan koeffitsiyentli integro–differensial issiqlik tarqalish tenglamasidan maxsus koʻrinishga ega koʻp oʻlchamli yadroni aniqlash teskari masalasi yechimining yagonaligini tadqiq etish.

**Tadqiqotning ob’ekti** ikkinchi tartibli parabolik tipdagi integro–differensial tenglamalardan iborat.

**Tadqiqotning predmeti** oʻrama koʻrinishidagi oʻzgaruvchan koeffitsiyentli integro–differensial issiqlik oʻtkazuvchanlik tenglamasi uchun koʻp oʻlchamli toʻgʻri va teskari masalalar.

**Tadqiqotning usullari.** Dissertatsiyada xususiy hosilali differensial tenglamalar va integral tenglamalar nazariyasi, shuningdek, funksional analiz metodlaridan xususan Volterra tipidagi ikkinchi tur chiziqli boʻlmagan integral tenglamalar sistemasini yechish, ketma-ket yaqinlashish, siqiluvchan akslantirish prinsipi usullaridan foydalanilgan.

**Tadqiqotning ilmiy yangiligi** quyidagilardan iborat:

oʻzgaruvchan koeffitsiyentli ikkinchi tartibli integro–differensial issiqlik tarqalish tenglamasi uchun qoʻyilgan Koshi masalasi yechimining mavjudligi va yagonaligi koʻrsatilgan;

toʻgʻri masala yechimiga nisbatan berilgan qoʻshimcha shartdan foydalanib, koʻp oʻlchamli yadroni Gyolder fazolarida bir qiymatli yechiluvchanligi isbotlangan;

issiqlik tarqalish integro–differensial tenglamasidan ko‘p o‘lchamli ajralgan yadroni aniqlash masalasining bir qiymatli yechiluvchanligi Gyolder (Hölder) fazolarida ko‘rsatilgan;

o‘zgaruvchan koeffitsiyentli integro–differensial issiqlik tarqalish tenglamasidan maxsus ko‘rinishga ega ko‘p o‘lchamli yadroni aniqlash teskari masalasi yechimining yagonaligi ko‘rsatilgan.

**Tadqiqotning amaliy natijalari** quyidagilardan iborat:

o‘zgaruvchan koeffitsiyentli integro–differensial issiqlik tarqalish tenglamasidan relaksatsiya funksiyasining mavjudlik va yagonalik shartlari topilgan;

integro–differensial issiqlik tarqalish tenglamasidan maxsus ko‘rinishga ega ko‘p o‘lchamli yadroni aniqlash teskari masalasining bir qiymatli yechiluvchanligi aniqlangan.

**Tadqiqot natijalarining ishonchliligi** differensial va integral tenglamalar nazariyasi, teskari masalalar nazariyasi, matematik analiz usullari qo‘llanilganligi, hamda, isbotlar va matematik mulohazalarning qat’iyligi bilan asoslangan.

**Tadqiqot natijalarining ilmiy va amaliy ahamiyati.** Tadqiqot natijalarining ilmiy ahamiyati matematik fizikaning integro–differensial tenglamalari uchun teskari masalalar nazariyasini yanada rivojlantirishi, ko‘p o‘lchamli yadroni aniqlash usullari qurilganligi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati seysmologiyada, neft va gaz konlarini qidirishda, issiqlik o‘tkazuvchi xotirali muhitlarda issiqlik tarqalish jarayonlarini tekshirishda tadbiq etilishi bilan izohlanadi.

**Tadqiqot natijalarining joriy qilinishi.** O‘zgaruvchan koeffitsiyentli integro–differensial issiqlik o‘tkazuvchanlik tenglamalari uchun teskari masalalarga oid ilmiy natijalar asosida:

o‘zgaruvchan koeffitsiyentli integro–differensial issiqlik tarqalish tenglamasidan relaksatsiya funksiyasini aniqlashning taklif etilgan usulidan Ф-4-14 «Suyuqlik oquvchi yer osti egri chiziqli quvurning tashqi kuchlari ta’siridagi kuchlanish–deformatsiyalar holatini tadqiq qilish nazariyasini rivojlantirish va hisoblash usullarini ishlab chiqish» mavzusidagi fundamental loyihada issiqlik o‘tkazuvchanlik tenglamasidan yadroni aniqlash masalalarida foydalanilgan (Buxoro muhandislik-texnologiya institutining 2021 yil 18-maydagi 83-10/974-son ma’lumotnomasi). Ilmiy natijalarning qo‘llanilishi o‘zgaruvchan koeffitsiyentli integro–differensial issiqlik tarqalish tenglamasi uchun qo‘yilgan Koshi masalasi uchun teskari masaladan ko‘p o‘lchamli yadroni bir qiymatli yechiluvchanligini o‘rganish, issiqlik tarqalish integro–differensial tenglamasidan ko‘p o‘lchamli ajralgan yadroni aniqlash masalasining yechiluvchanligini o‘rganish imkonini bergan;

teskari masalalarni tadqiq etishning taklif etilgan usulidan АААА-А19-119032590069-3 «Geofizik va muhandislik masallarida issiqlik vazn almashish va qattiq muhitlar mexanikasi masalalarini sonli yechish va matematik modellashtirish» mavzudagi xorijiy grantda ko‘p o‘lchamli integro–differensial issiqlik o‘tkazuvchanlik tenglamasi uchun teskari masalalarni tadqiq etishda foydalanilgan (Южный

Математический Институт филиал ФГНБУ ФНЦ «Владикавказский научный центр РАН», 2021 yil 19 maydagi 37-son ma’lumotnomasi). Ilmiy natijalarning qoʻllanilishi integro–differensial issiqlik oʻtkazuvchanlik tenglamalari uchun koʻp oʻlchamli teskari masalalarning yechiluvchanligini isbotlash imkonini bergan.

**Tadqiqot natijalarining aprobatsiyasi.** Mazkur tadqiqot natijalari 6 ta ilmiy-amaliy anjumanlarda, jumladan 2 ta xalqaro va 4 ta respublika ilmiy-amaliy anjumanlarida muhokamadan oʻtkazilgan.

**Tadqiqot natijalarining e’lon qilinganligi.** Dissertatsiya mavzusi boʻyicha jami 12 ta ilmiy ish chop etilgan, shulardan, Oʻzbekiston Respublikasi Oliy Attestatsiya komissiyasining dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlar roʻyxatida 6 ta maqola, jumladan, 2 tasi xorijiy va 4 tasi respublika jurnallarida nashr etilgan.

**Dissertatsiyaning tuzilishi va hajmi.** Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar roʻyxatidan iborat. Dissertatsiyaning hajmi 96 bet.

## DISSERTATSIYANING ASOSIY MAZMUNI

**Kirish** qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning Respublika fan va texnologiyalari rivojlanishining ustuvor yoʻnalishlariga mosligi koʻrsatilgan, muammoning oʻrganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi boʻyicha ma’lumotlar keltirilgan.

Dissertatsiyaning birinchi bobi **“Oʻzgaruvchan koeffitsiyentli issiqlik tarqalish integro–differensial tenglamasi uchun qoʻyilgan toʻgʻri va teskari masalalar”** deb nomlanib oʻzgaruvchan koeffitsiyentli issiqlik tarqalish integro–differensial tenglamasi uchun qoʻyilgan toʻgʻri va teskari masala yechimining mavjudligi va yagonaligi oʻrganilgan. Bu bobning birinchi paragrafida dissertatsiyada foydalanilgan umumiy tushunchalar keltirilgan.

Birinchi bobning ikkinchi paragrafida oʻzgaruvchan koeffitsiyentli issiqlik tarqalish integro–differensial tenglamasi uchun qoʻyilgan toʻgʻri masala yechimining mavjudligi va yagonaligi oʻrganilgan.

$u(x,t)$, funksiyani $(x,t)\in\mathbb{R}^n_T$ sohada quyidagi tenglamalardan aniqlash masalasini qaraymiz:

$$u_t - a(t)\Delta u = \int_0^t k\,(x',\tau)u(x,t-\tau)d\tau, (x,t)\in\mathbb{R}^n_T, \tag{1}$$

$$u|_{t=0} = \varphi(x), x\in\mathbb{R}^n, \tag{2}$$

bu yerda $\Delta := \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$ -Laplas operatori $\mathbb{R}^n_T = \{(x,t)|x=(x',x_n)\in\mathbb{R}^n, 0<t\le T\}$, $T>0$ tayinlangan ixtiyoriy son, $a(t)\in E := \{a(t)\in C^1[0,T], 0<a_0\le a(t)\le a_1<$

$\infty\}$.

**1-Ta'rif.** *Agar ixtiyoriy* $x_1, x_2 \in \mathbb{R}^n$ *uchun*

$$|f(x_1) - f(x_2)| \le A|x_1 - x_2|^l, \qquad A = const > 0, \qquad l \in (0,1) \tag{3}$$

*tengsizlik bajarilsa,* $f(x)$ *funksiya* $\mathbb{R}^n$ *da* $l$ *ko'rsatkichli Gyolder shartini qanoatlantiradi deyiladi va (3) shartni qanoatlantiruvchi funksiyalar sinfi* $H^l(\mathbb{R}^n)$ *kabi belgilanadi.*

Xuddi shunday ko'p o'zgaruvchili funksiyalar uchun ham Gyolder funksiyalar sinfiga ta'rif berish mumkin.

**2-Ta'rif** *Agar ixtiyoriy* $\left(x^{(1)}, t_1\right), \left(x^{(2)}, t_2\right) \in \mathbb{R}_T^n$, $x^{(1)} = (x_1^{(1)}, x_2^{(1)}, \dots, x_n^{(1)})$, $x^{(2)} = (x_1^{(2)}, x_2^{(2)}, \dots, x_n^{(2)})$ *uchun*

$$\left|f(x^{(1)}, t_1) - f(x^{(2)}, t_2)\right| \le \sum_{i=1}^{n} A_i |x_i^{(1)} - x_i^{(2)}|^l + A_{n+1}|t_1 - t_2|^{l/2}$$

$$A_i = const > 0, \;\; i = 1,2, \dots n+1. \;\; l \in (0,1) \tag{4}$$

*tengsizlik bajarilsa,* $f(x,t)$ *funksiya* $\mathbb{R}_T^n$ *da* $l, l/2$ *ko'rsatkichli Gyolder shartini qanoatlantiradi deyiladi va (4) shartni qanoatlantiruvchi funksiyalar sinfi* $H^{l,l/2}(\mathbb{R}_T^n)$ *kabi belgilanadi.*

Agar $\varphi(x) \in H^l(\mathbb{R}^n)$, $f(x,t) \in H^{l,l/2}(\mathbb{R}_T^n)$ bo'lsa, $H^{l+m}(\mathbb{R}^n)$, $H^{l+m,(l+m)/2}(\mathbb{R}_T^n)$ fazolardagi normalar quyidagicha aniqlanadi:

$$|\varphi|^{l+m} = \sum_{|\alpha| \le m} \sup_{x \in \mathbb{R}^n} |D^\alpha \varphi| + \sum_{|\alpha| = m} \sup_{\substack{|x^1 - x^2| \le \rho_0 \\ x^1, x^2 \in \mathbb{R}^n}} \frac{|D^\alpha \varphi(x^1) - D^\alpha \varphi(x^2)|}{|x^1 - x^2|^l},$$

$$|f|_T^{l+m,(l+m)/2} = \sum_{2r+|s| \le m} \sup_{(x,t) \in \mathbb{R}_T^n} |\, D^r D^s f| +$$

$$+ \sum_{2r+|s| = m} \sup_{\substack{|x^1 - x^2| \le \rho_0 \\ (x,t) \in \mathbb{R}_T^n}} \frac{|D^r D^s f(x^1, t) - D^r D^s f(x^2, t)|}{|x^1 - x^2|^l} +$$

$$+ \sum_{2r+|s| = m} \sup_{\substack{|t_1 - t_2| \le \rho_1 \\ (x,t) \in \mathbb{R}_T^n}} \frac{|D^r D^s f(x, t_1) - D^r D^s f(x, t_2)|}{|t_1 - t_2|^{l/2}},$$

bu yerda $\rho_0, \rho_1 > 0$.

Berilgan $a$ va $k$ funksiyalar uchun (1) integro–differensial tenglamadan $u(x,t)$ funksiyani (2) boshlang'ich shart orqali topish masalasiga Koshi (to'g'ri) masalasi deyiladi.

(1) va (2) Koshi masalasining yechimini Puasson formulasidan foydalanib quyidagi ikkinchi tur Volterra tipidagi integral tenglamaga ekvivalent bo'lishi ko'rsatilgan

$$u(x,t)=\int\limits_{\mathbb{R}^n}\varphi(\xi)G(x-\xi;\theta(t))d\xi+\int\limits_0^{\theta(t)}\frac{d\tau}{a(\theta^{-1}(\tau))}\times$$

$$\times\int\limits_{\mathbb{R}^n}\int\limits_0^{\theta^{-1}(\tau)}k(\xi',\alpha)u(\xi,\theta^{-1}(\tau)-\alpha)G(x-\xi;\theta(t)-\tau)d\alpha d\xi, \qquad (5)$$

bu yerda $\theta(t)=\int_0^t a(s)ds$, $\theta^{-1}(t)$ funksiya $\theta(t)$ ning teskari funksiyasi, $G(x-\xi;\theta(t)-\tau)=\frac{1}{(2\sqrt{\pi(\theta(t)-\tau)})^n}e^{-\frac{|x-\xi|^2}{4(\theta(t)-\tau)}}$ esa $\frac{\partial}{\partial t}-a(t)\Delta$ o‘zgaruvchan koeffitsiyentli differensial operatorining fundamental yechimi bo‘lib, $|x|^2=x_1^2+\cdots+x_n^2$.

**1-Lemma.** *Faraz qilaylik,* $\varphi(x)\in H^{l+2}(\mathbb{R}^n), k(x',t)\in H^{l,l/2}(\overline{\mathbb{R}}_T^{n-1})$ *va* $a(t)\in E$ *bo‘lsin. U holda (5) integral tenglamaning* $H^{l+2,(l+2)/2}(\overline{\mathbb{R}}_T^n)$ *sinfga qarashli yagona* $u(x,t)$ *yechimi mavjud.*

Birinchi bobning uchinchi paragrafida o‘zgaruvchan koeffitsiyentli issiqlik tarqalish integro–differensial tenglamasi uchun qo‘yilgan teskari masala o‘rganilgan.

Ushbu paragrafda $u(x,t)$ funksiya bilan birgalikda (1) integro–differensial tenglamadan $k(x',t)$ funksiyani topish masalasi qaraladi. Buning uchun quyidagi qo‘shimcha shart kiritamiz:

$$u|_{x_n=0}=f(x',t), \qquad (x',t)\in\overline{\mathbb{R}}_T^{n-1}, \qquad (6)$$

bu yerda $f(x',t)\in H^{l+4,(l+4)/2}(\overline{\mathbb{R}}_T^{n-1})$ berilgan funksiya.

(1), (2), (6) masaladan $u(x,t)$ va $k(x',t)$ funksiyalarni topish masalasiga teskari masala deb ataladi.

(1), (2) tengliklarni $x_n$ bo‘yicha ikki marta differensiallab $\vartheta(x,t)=u_{x_nx_n}(x,t)$ belgilash kiritilsa, u holda (1), (2) masala quyidagicha ko‘rinishni oladi:

$$\vartheta_t-a(t)\Delta\vartheta=\int\limits_0^t k(x',\tau)\vartheta(x,t-\tau)d\tau, \qquad (7)$$

$$\vartheta|_{t=0}=\varphi_{x_nx_n}(x), \qquad (8)$$

$\vartheta(x,t)$ funksiya uchun $x_n=0$ dagi qo‘shimcha shart (1) va (6) dan foydalanib hosil qilingan:

$$\vartheta|_{x_n=0}=\frac{1}{a(t)}f_t-\sum_{k=1}^{n-1}\frac{\partial^2}{\partial x_k^2}f-\frac{1}{a(t)}\int\limits_0^t k(x',\tau)f(x',t-\tau)d\tau. \qquad (9)$$

(8) boshlang‘ich shart va (9) qo‘shimcha shartlardan quyidagi kelishuvchanlik sharti kelib chiqadi:

$$\varphi_{x_n x_n}(x',0)=\frac{1}{a(0)}f_t(x',0)-\sum_{i=1}^{n-1} f_{x_i x_i}(x',0). \tag{10}$$

(7), (9) tenglamalarni $t$ bo‘yicha bir marta differensiallab va $\vartheta_t(x,t)=\omega(x,t)$ belgilash kiritilsa, natijada (7), (8) va quyidagi

$$\omega_t - a(t)\Delta\omega = (\ln\, a(t))'\omega - (\ln\, a(t))'\int_0^t k(x',\tau)\vartheta(x,t-\tau)d\tau +$$

$$+\int_0^t k(x',\tau)\omega(x,t-\tau)d\tau + k(x',t)\varphi_{x_n x_n}(x), \tag{11}$$

$$\omega|_{t=0} = a(0)\Delta\varphi_{x_n x_n}(x), \tag{12}$$

$$\omega|_{x_n=0} = -\frac{a'(t)}{a^2(t)}f_t + \frac{1}{a(t)}f_{tt} - \sum_{k=1}^{n-1}\frac{\partial^2}{\partial x_k^2}f_t + \frac{a'(t)}{a^2(t)}\int_0^t k(x',\tau)f(x',t-\tau)d\tau -$$

$$-\frac{1}{a(t)}\int_0^t k(x',\tau)f_t(x',t-\tau)d\tau - \frac{1}{a(t)}k(x',t)\varphi(x',0) \tag{13}$$

tenglamalardan $\vartheta(x,t), k(x',t), \omega(x,t)$ funksiyalarni topish masalasiga keladi.

**2-Lemma.** *Faraz qilaylik* $\varphi(x)\in H^{l+6}(\mathbb{R}^n)$, $f(x',t)\in H^{l+4,(l+4)/2}(\overline{\mathbb{R}}_T^{n-1})$ *va* $a(t)\in E$ *shartlar bajarilsin. Bundan tashqari*

$$f(x',0)=\varphi(x',0), \qquad \varphi_{x_n x_n}(x',0)=\frac{1}{a(0)}f_t(x',0)-\sum_{i=1}^{n-1} f_{x_i x_i}(x',0)$$

*kelishuvchanlik shartlari ham o‘rinli bo‘lsin. U holda (1), (2), (6) teskari masala (7)-(9) va (11)-(13) tenglamalardan* $\vartheta(x,t)$, $k(x',t)$, $\omega(x,t)$ *funksiyalarni topishga ekvivalent.*

Ushbu paragrafning asosiy natijasi sifatida quyidagi mavjudlik va yagonalik teoremasi isbotlangan:

**1-Teorema.** *Faraz qilaylik* $a(t)\in E$, $f(x',t)\in H^{l+4,(l+4)/2}(\overline{\mathbb{R}}_T^{n-1})$, $\varphi(x)\in H^{l+6}(\mathbb{R}^n)$ *funksiyalar sinfiga qarashli bo‘lib,*

$$f(x',0)=\varphi(x',0), \qquad \varphi_{x_n x_n}(x',0)=\frac{1}{a(0)}f_t(x',0)-\sum_{i=1}^{n-1} f_{x_i x_i}(x',0)$$

*kelishuvchanlik shartlari bajarilsin. U holda shunday yetarlicha kichik musbat* $T_0>0$ *soni mavjudki,* $T\in(0,T_0]$ *lar uchun (1), (2), (6) teskari masalaning* $u(x,t)\in H^{l+2,(l+2)/2}(\overline{\mathbb{R}}_T^{n})$, $k(x',t)\in H^{l,l/2}(\overline{\mathbb{R}}_T^{n-1})$ *yechimi mavjud va yagona.*

Dissertatsiyaning ikkinchi bobi **“Issiqlik tarqalish integro–differensial tenglamasidan ko‘p o‘lchamli ajralgan yadroni aniqlash masalasi”** deb nomlangan.

Birinchi paragrafda o‘zgaruvchan koeffitsiyentli ko‘p o‘lchamli, ajralgan yadroli, ya’ni $\boldsymbol{K(x,t)=h(x_n)k(x',t)}$, $\boldsymbol{x'=(x_1,\dots,x_{n-1})}$ $\boldsymbol{h(x_n)\in H^{l+2}}(\mathbb{R})$, $\boldsymbol{k(x',t)\in H^{l,\frac{l}{2}}\left(\overline{\mathbb{R}}_T^{n-1}\right)}$ yadroli issiqlik tarqalish integro–differensial tenglamasi uchun qo‘yilgan to‘g‘ri masala yechimining mavjudligi va yagonaligi o‘rganilgan.

$u(x,t)$, funksiyani $(x,t)\in\mathbb{R}_T^n$ sohada quyidagi tenglamalardan aniqlash masalasini qaraymiz:

$$u_t-a(t)\Delta u=\int_0^t K\,(x,\tau)u(x,t-\tau)d\tau,\quad (x,t)\in\mathbb{R}_T^n, \tag{14}$$

$$u(x,0)=\varphi(x),\qquad x\in\mathbb{R}^n, \tag{15}$$

(14), (15) Koshi masalasining yechimi quyidagi Volterra tipidagi integral tenglamaga ekvivalent bo‘ladi.

$$u(x,t)=\int_{\mathbb{R}^n}\varphi\,(\xi)G(x-\xi;\theta(t))d\xi+\int_0^{\theta(t)}\frac{d\tau}{a(\theta^{-1}(\tau))}\times$$

$$\times\int_{\mathbb{R}^n}\int_0^{\theta^{-1}(\tau)} h\,(\xi_n)k(\xi',\alpha)u(\xi,\theta^{-1}(\tau)-\alpha)G(x-\xi;\theta(t)-\tau)d\alpha d\xi. \tag{16}$$

**3-Lemma.** *Faraz qilaylik,* $\varphi(x)\in H^{l+2}(\mathbb{R}^n)$, $h(x_n)\in H^{l+2}(\mathbb{R}), k(x',t)\in H^{l,l/2}(\overline{\mathbb{R}}_T^{n-1})$ *va* $a(t)\in E$ *bo‘lsin. U holda (16) integral tenglamaning* $H^{l+2,(l+2)/2}(\overline{\mathbb{R}}_T^{n})$ *sinfga qarashli yagona* $u(x,t)$ *yechimi mavjud.*

Ikkinchi paragrafda o‘zgaruvchan koeffitsiyentli issiqlik tarqalish integro–differensial tenglamasidan ko‘p o‘lchamli yadroni aniqlash masalasi tadqiq qilingan.

$u(x,t)$ funksiyadan funksiya bilan birgalikda (16) integro–differensial tenglamadan $k(x',t)$ funksiyani topish masalasi qaraladi. Buning uchun quyidagi qo‘shimcha shart kiritamiz:

$$u(x',0,t)=f(x',t),\qquad (x',t)\in\overline{\mathbb{R}}_T^{n-1}. \tag{17}$$

(12), (13), (15) teskari masalaga ekvivalent integral tenglamalar sistemasini olamiz. Buning uchun dastlab berilgan tenglamani $t$ bo‘yicha differensiallab:

$$\vartheta(x,t)=u_t(x,t) \tag{18}$$

belgilash kiritiladi va $\vartheta(x,t)$ funksiyaga nisbatan quyidagi ekvivalent masala olinadi:

$$\vartheta_t-a(t)\Delta\vartheta=(\ln\,a\,(t))'\vartheta(x,t)-(\ln\,a\,(t))'\int_0^t h\,(x_n)k(x',\tau)u(x,t-\tau)d\tau+$$

$$+\int_0^t h\,(x_n)k(x',\tau)\vartheta(x,t-\tau)d\tau+h(x_n)k(x',t)\varphi(x), \tag{19}$$

$$\vartheta|_{t=0}=a(0)\Delta\varphi(x), \tag{20}$$

$$\vartheta|_{x_n=0}=f_t(x',t),\ a(0)\Delta\varphi(x',0)=f_t(x',0). \tag{21}$$

(19), (20) tengliklarni $x_n$ bo'yicha differensiallab yangi ekvivalent masalaga kelinadi:

$$(\vartheta_{x_n})_t - a(t)\Delta\vartheta_{x_n} = (\ln\ a\,(t))'\vartheta_{x_n} - (\ln\ a\,(t))'h'(x_n)\int_0^t k\,(x',\tau) \times$$

$$\times\, u(x,t-\tau)d\tau - (\ln\ a\,(t))'h(x_n)\int_0^t k\,(x',\tau)u_{x_n}(x,t-\tau)d\tau +$$

$$+h'(x_n)\int_0^t k\,(x',\tau)\vartheta(x,t-\tau)d\tau + h(x_n)\int_0^t k\,(x',\tau)\vartheta_{x_n}(x,t-\tau)d\tau +$$

$$+h'(x_n)k(x',t)\varphi(x) + h(x_n)k(x',t)\varphi_{x_n}(x), \qquad (22)$$

$$\vartheta_{x_n}|_{t=0} = a(0)\Delta\varphi_{x_n}(x). \qquad (23)$$

(19), (20) tengliklarni $x_n$ bo'yicha ikki marta differensiallab $\omega(x,t) = \vartheta_{x_n x_n}(x,t)$ belgilash kiritilsa $\omega(x,t)$ funksiyaga nisbatan quyidagi ekvivalent masala hosil qilinadi:

$$\omega_t - a(t)\Delta\omega = (\ln\ a\,(t))'\omega - (\ln\ a\,(t))'h''(x_n)\int_0^t k\,(x',\tau)u(x,t-\tau)d\tau -$$

$$-2(\ln\ a\,(t))'h'(x_n)\int_0^t k\,(x',\tau)u_{x_n}(x,t-\tau)d\tau - (\ln\ a\,(t))'h(x_n)\int_0^t k\,(x',\tau) \times$$

$$\times\, u_{x_n x_n}(x,t-\tau)d\tau + h''(x_n)\int_0^t k\,(x',\tau)\vartheta(x,t-\tau)d\tau +$$

$$+2h'(x_n)\int_0^t k\,(x',\tau)\vartheta_{x_n}(x,t-\tau)d\tau + h(x_n)\int_0^t k\,(x',\tau)\omega(x,t-\tau)d\tau +$$

$$+h''(x_n)k(x',t)\varphi(x) + 2h'(x_n)k(x',t)\varphi_{x_n}(x) +$$

$$+h(x_n)k(x',t)\varphi_{x_n x_n}(x), \qquad (24)$$

$$\omega|_{t=0} = a(0)\Delta\varphi_{x_n x_n}(x). \qquad (25)$$

$\omega(x,t)$ funksiya uchun qo'shimcha shart (19) va (25) dan foydalanib hosil qilingan:

$$\omega|_{x_n=0} = \frac{1}{a(t)}f_{tt} - \sum_{i=1}^{n-1}\frac{\partial^2}{\partial x_i k^2}f_t - \frac{(ln\ a\,(t))'}{a(t)}f_t + \frac{(ln\,a\,(t))'}{a(t)}h''(0) \times$$

$$\times \int_0^t k\,(x',\tau)f(x',t-\tau)d\tau + 2\,\frac{(ln\,a\,(t))'}{a(t)}h'(0)\int_0^t k\,(x',\tau)u_{x_n}(x',t-\tau)d\tau +$$

$$+\frac{(ln\,a\,(t))'}{a(t)}h(0)\int_0^t k\,(x',\tau)u_{x_nx_n}(x',t-\tau)d\tau-\frac{1}{a(t)}h''(0)\int_0^t k\,(x',\tau)\times$$

$$\times f_t(x,t-\tau)d\tau-2\frac{1}{a(t)}h'(0)\int_0^t k\,(x',\tau)\vartheta_{x_n}(x',t-\tau)d\tau-$$

$$-\frac{1}{a(t)}h(0)\int_0^t k\,(x',\tau)f_{tt}(x',t-\tau)d\tau-\frac{1}{a(t)}h''(0)k(x',t)\varphi(x',0)-$$

$$-2\frac{1}{a(t)}h'(0)k(x',t)\varphi_{x_n}(x',0)-\frac{1}{a(t)}h(0)k(x',t)\varphi_{x_nx_n}(x',0). \qquad (26)$$

**4-Lemma.** *Faraz qilaylik* $a(t)\in E$, $\varphi(x)\in H^{l+6}(\mathbb{R}^n)$, $f_t(x',t)\in H^{l+4,(l+4)/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$, $h(x_n)\in H^{l+2}(\mathbb{R})$ *ushbu sinflardan. Bundan tashqari* $f(x',0)=\varphi(x',0)$, *va* $a(0)\Delta\varphi(x',0)=f_t(x',0)$ *kelishuvchanlik shartlari bajarilsin. U holda (14), (15), (17) teskari masala (18), (19) – (20), (22)–(23) va (24)–(26) masalalardan* $(\vartheta_{x_n}(x,t),u(x,t),\ u_{x_n}(x,t),u_{x_nx_n}(x,t),\vartheta(x,t),\omega(x,t),k(x',t))$ *topish masalasiga ekvivalent.*

Ushbu paragrifning asosiy natijasi sifatida quyidagi mavjudlik va yagonalik teoremasi olingan.

**2-Teorema.** *Faraz qilaylik* $a(t)\in E$, $\varphi(x)\in H^{l+6}(\mathbb{R}^n)$, $f_t(x',t)\in H^{l+4,(l+4)/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$, $h(x_n)\in H^{l+2}(\mathbb{R})$, $l\in(0,1)$ *funksiyalar sinfiga qarashli bo‘lib,* $\varphi(x',0)=f(x',0)$ *va* $a(0)\Delta\varphi(x',0)=f_t(x',0)$ *kelishuvchanlik shartlari bajarilsin. U holda shunday yetarlicha kichik musbat* $T_0>0$ *soni mavjudki,* $T\in(0,T_0]$ *lar uchun (14),(15),(17) teskari masalaning* $u(x,t)\in H^{l+2,(l+2)/2}(\overline{\mathbb{R}}_T^n)$, $k(x',t)\in H^{l,l/2}(\overline{\mathbb{R}}_T^{n-1})$ *yagona yechimi mavjud.*

Uchinchi paragrafda o‘zgaruvchan koeffitsiyentli ko‘p o‘lchamli yadroli issiqlik tarqalish integro–differensial tenglamasida integral ostida noma’lum funksiya Laplas operatori orqali berilgan bo‘lib, unga qo‘yilgan teskari masala tadqiq qilingan.

Quyidagi tenglamalardan $u(x,t)$, $k(x',t)$ funksiyalarni aniqlash masalasi ko‘rib chiqilgan:

$$u_t=a(t)\Delta u-\int_0^t k\,(x',t-\tau)a(\tau)\Delta u(x,\tau)d\tau,\quad (x,t)\in\mathbb{R}_T^n, \qquad (27)$$

$$u(x,0)=\varphi(x),\ x\in\mathbb{R}^n, \qquad (28)$$

$$u(x',0,t)=f(x',t),\qquad (x',t)\in\mathbb{R}_T^{n-1}. \qquad (29)$$

bu yerda $a(t)>0$ yetarlicha silliq funksiya.

(27) integro-differensial tenglamani $a(t)\Delta u$ ga nisbatan ikkinchi tur Volterra integral tenglamasi ko‘rinishida yozib olinadi.

$$a(t)\Delta u = \int_0^t k\,(x', t-\tau)a(\tau)\Delta u(x,\tau)d\tau + u_t \qquad (30)$$

Integral tenglamalar nazariyasidan (30) integral tenglamaning yechimi quyidagi ko‘rinishda bo‘ladi:

$$u_t - a(t)\Delta u = -\int_0^t r\,(x', t-\tau)u_\tau(x,\tau)d\tau. \qquad (31)$$

(31) tenglamada $r(x',t)$ funksiya $k(x',t)$ funksiyaning rezolventasi, $r(x',t)$ va $k(x',t)$ funksiyalar orasidagi bog‘lanish quyidagi tenglik bilan aniqlanadi:

$$r(x',t) = k(x',t) + \int_0^t k\,(x', t-\tau)r(x',t)d\tau, \quad (x,t) \in \mathbb{R}_T^n. \qquad (32)$$

Endi (31), (28), (29) masaladan $u(x,t)$ va $r(x',t)$ funksiyalarni topish masalasini ko‘rib chiqamiz. Qidirilayotgan funksiyalarni topgandan so‘ng (30) integral tenglamadan $k(x',t)$ funksiyani topish mumkin bo‘ladi.

(31), (28) Koshi masalasida $u(x,t)$ funksiyani $x_n$ bo‘yicha ikki marta differensiallab yangi $\vartheta^{(1)}(x,t)$ funksiyani $\vartheta^{(1)}(x,t) = u_{x_n x_n}(x,t)$ belgilash yordamida kiritib, $\vartheta^{(1)}(x,t)$ funksiyaga nisbatan yordamchi masala olinadi:

$$\vartheta_t^{(1)} - a(t)\Delta\vartheta^{(1)} = -\int_0^t r\,(x', t-\tau)\vartheta_\tau^{(1)}(x,\tau)d\tau, \qquad (33)$$

$$\vartheta^{(1)}(x,0) = \varphi_{x_n x_n}(x). \qquad (34)$$

$$\vartheta^{(1)}(x',0,t) = \frac{1}{a(t)} f_t(x',t) - \sum_{i=1}^{n-1} f_{x_i x_i}(x',t) +$$

$$+\frac{1}{a(t)}\int_0^t r\,(x', t-\tau)f_\tau(x',\tau)d\tau. \qquad (35)$$

(34), (35) shartlardan quyidagi kelichuvchanlik sharti olinadi:

$$\varphi_{x_n x_n}(x',0) = \frac{1}{a(0)} f_t(x',0) - \sum_{i=1}^{n-1} f_{x_i x_i}(x',0). \qquad (36)$$

(32), (35) tengliklarni $t$ bo‘yicha differensiallab, $\vartheta^{(2)}(x,t) := \vartheta_t^{(1)}(x,t)$ va $h(x',t) := r_t(x',t)$ belgilash kiritilsa $\vartheta^{(2)}(x,t)$ funksiyaga nisbatan ekvivalent masalaga kelinadi:

$$\vartheta_t^{(2)} - a(t)\Delta\vartheta^{(2)} = a'(t)\Delta\vartheta^{(1)} - r(x',0)\vartheta^{(2)} -$$

$$-\int_0^t h\,(x', t-\tau)\vartheta^{(2)}(x,\tau)d\tau, \qquad (37)$$

$$\vartheta^{(2)}(x,0)=a(0)\Delta\varphi_{x_nx_n}(x), \tag{38}$$

$$\vartheta^{(2)}(x',0,t)=-\frac{a'(t)}{a^2(t)}f_t(x',t)+\frac{1}{a(t)}f_{tt}(x',t)-\sum_{i=1}^{n-1}f_{tx_ix_i}(x',t)-$$

$$-\frac{a'(t)}{a^2(t)}\int_0^t r\,(x',t-\tau)f_\tau(x',\tau)d\tau+\frac{1}{a(t)}\int_0^t h\,(x',\tau)f_\tau(x',t-\tau)d\tau+$$

$$+\frac{1}{a(t)}r(x',0)f_t(x',t). \tag{39}$$

(37), (39) tengliklarni $t$ bo‘yicha differensiallab $\vartheta(x,t):=\vartheta_t^{(2)}(x,t)$ belgilash kiritilsa, $\vartheta(x,t)$ funksiyaga nisbatan ekvivalent masalaga kelinadi:

$$\vartheta_t-a(t)\Delta\vartheta=2a'(t)\Delta\vartheta^{(2)}+a''(t)\Delta\vartheta^{(1)}-r(x',0)\vartheta-$$

$$-h(x',t)a(0)\Delta\varphi_{x_nx_n}(x)-\int_0^t h\,(x',\tau)\vartheta(x,t-\tau)d\tau, \tag{40}$$

$$\vartheta(x,0)=\Psi(x), \tag{41}$$

$$\vartheta(x',0,t)=F(x',t)+\left(2\frac{(a'(t))^2}{a^3(t)}-\frac{a''(t)}{a^2(t)}\right)\int_0^t r\,(x',t-\tau)f_\tau(x',\tau)d\tau-$$

$$-2\frac{a'(t)}{a^2(t)}\int_0^t h\,(x',\tau)f_\tau(x',t-\tau)d\tau-\frac{1}{a(t)}\int_0^t h\,(x',\tau)f_{tt}(x',t-\tau)d\tau+$$

$$+\frac{1}{a(t)}h(x',t)f_t(x',0), \tag{42}$$

bu yerda

$$\Psi(x)=a^2(0)\Delta^2\varphi_{x_nx_n}(x)+a'(0)\Delta\varphi_{x_nx_n}(x)-r(x',0)a(0)\Delta\varphi_{x_nx_n}(x),$$

$$F(x',t)=\left(-\frac{a''(t)}{a^2(t)}+2\frac{(a'(t))^2}{a^3(t)}\right)f_t(x',t)+\frac{1}{a(t)}f_{ttt}(x',t)-\sum_{i=1}^{n-1}f_{ttx_ix_i}(x',t)-$$

$$-2\frac{a'(t)}{a^2(t)}r(x',0)f_t(x',t)+\frac{1}{a(t)}r(x',0)f_{tt}(x',t)-2\frac{a'(t)}{a^2(t)}f_{tt}(x',t).$$

**5-Lemma.** *(27)– (29) teskari masala (37), (38) va (40)–(42) masalalardan* $\vartheta^{(2)}(x,t),\vartheta(x,t),h(x',t),r(x,'t)$ *funksiyalarni aniqlash masalasiga ekvivakent.*

Ushbu paragrafning asosiy natijasi sifatida quyidagi mavjudlik va yagonalik teoremasi keltirilgan:

**3-Teorema.** *Faraz qilaylik* $a(t)>0$ yetarlicha silliq funksiya, $\varphi(x)\in H^{l+8}(\mathbb{R}^n)$, $f(x',t)\in H^{l+6,(l+6)/2}(\bar{\mathbb{R}}_T^{n-1})$ funksiyalar sinfiga qarashli bo‘lib, $f(x',0)=\varphi(x',0)$ *va* $\varphi_{x_nx_n}(x',0)=\frac{1}{a(0)}f_t(x',0)-\sum_{i=1}^{n-1}f_{x_ix_i}(x',0)$. *kelishuvchanlik shartlari, hamda* $|f_t(x',0)|>f_0=const>0$ *bo‘lsin. U holda* *shunday yetarlicha kichik musbat $T_0 > 0$ soni mavjudki, $T \in (0, T_0]$ lar uchun (31), (28), (29) teskari masalaning $u(x,t) \in H^{l+2,(l+2)/2}\left(\overline{\mathbb{R}}_T^n\right)$, $h(x',t) \in H^{l,l/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$, $l \in (0,1)$sinflarga tegishli yagona yechimi mavjud.*

Uchinchi bob **"Parabolik tipdagi integro–differensial tenglama uchun teskari masalalar"** deb nomlanib, tenglamadagi yadro $L = \Delta + c(x)$ differensial operator bilan o'rama shaklda berilgan.

Bu bobning birinchi va ikkinchi paragraflarida quyidagi masala qaralgan: $u(x,t)$ va $k(x',t)$ funksiyalarni $(x,t) \in \mathbb{R}_T^n$ sohada quyidagi berilgan tenglamalardan aniqlash masalasini ko'rib chiqamiz:

$$u_t - Lu = -\int_0^t k\,(x',\tau)Lu(x,t-\tau)d\tau, \quad (x,t) \in \mathbb{R}_T^n, \tag{43}$$

$$u|_{t=0} = \varphi(x), \quad x \in \mathbb{R}^n, \tag{44}$$

$$u|_{x_n=0} = f(x',t), \qquad (x',t) \in \mathbb{R}_T^{n-1}. \tag{45}$$

Quyidagi lemma o'rinli.

**6-Lemma.** *Faraz qilaylik $k(x',t) \in H^{l+2,(l+2)/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$ bo'lsin. U holda (41)–(43) masala yechimi*

$$u_t(x,t) = Lu - \int_0^t r\,(x',t-\tau)u_\tau(x,\tau)d\tau, \tag{46}$$

*integral tenglama, hamda (44), (45) boshlang'ich va qo'shimcha shartlardan $u(x,t)$ va $r(x',t)$ funksiyalarni topish masalasiga ekvivalent. Bu yerda $r(x',t)$ funksiya $k(x',t)$ funksiyaning rezolventasi va ular quyidagi integral tenglama bilan bog'langan*:

$$k(x',t) = r(x',t) - \int_0^t r\,(x',t-\tau)k(x',\tau)d\tau. \tag{47}$$

**7-Lemma.** *Quyidagi tenglamalardan*

$$\vartheta_t - L\vartheta - 2c_{x_n}\vartheta_{x_n}^{(2)} - c_{x_n x_n}\vartheta^{(2)} + r(x',0)\vartheta + h(x',t)[L\varphi_{x_n x_n}(x) +$$
$$+2c_{x_n}\varphi_{x_n}(x) + c_{x_n x_n}\varphi(x)] + \int_0^t h\,(x',t-\tau)\vartheta(x,\tau)d\tau = 0 \tag{48}$$

$$\vartheta|_{t=0} = Y_{x_n x_n}(x) \tag{49}$$

$$\vartheta(x',0,t) = f_{ttt}(x',t) - \Delta_{x'}f_{tt}(x',t) - c(x',0)f_{tt}(x',t) +$$
$$+r(x',0)f_{tt}(x',t) + h(x',t)L\varphi|_{x_n=0} + \int_0^t f_{tt}\,(x',t-\tau)h(x',\tau)d\tau. \tag{50}$$

*bu yerda*

$$\vartheta(x,t) = u_{ttx_n x_n}(x,t), \quad \vartheta^{(2)} = u_{tt},$$

$$\triangle_{x'} = \sum_{i=1}^{n-1} \frac{\partial^2}{\partial x_i^2}, \quad Y_{x_n x_n}(x) = \frac{\partial^2}{\partial x_n^2}\left(L^2\varphi(x) - r(x',0)L\varphi(x)\right)$$

$$r(x',0) = \frac{L^2\varphi(x',0) - f_{tt}(x'0)}{L\varphi(x',0)}. \qquad (51)$$

*$\vartheta(x,t), h(x',t)$ funksiyalarni topish masalasi $(46), (44), (45)$ masalaga ekvivalent.*

Ushbu paragrafda quyidagi mavjudlik va yagonalik teoremasi isbotlangan.

**4-Teorema.** *Faraz qilaylik, $\varphi(x) \in H^{l+8}(\mathbb{R}^n)$, $|L\varphi(x',0)|^l \geq const > 0$, $c(x) \in H^{l+4}(\mathbb{R}^n)$, $f(x',t) \in H^{l+6,(l+6)/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$, $l \in (0,1)$ va $f(x',0) = \varphi(x',0)$, $f_t(x',0) = L\varphi(x',0)$ kelishuvchanlik shartlari bajarilsin. U holda shunday yetarlicha kichik musbat $T_0 > 0$ soni mavjudki, $T \in (0, T_0]$ lar uchun (46), (44), (45) teskari masalaning $u(x,t) \in H^{l+2,(l+2)/2}\left(\overline{\mathbb{R}}_T^n\right)$, $h(x',t) \in H^{l,l/2}\left(\overline{\mathbb{R}}_T^n\right)$ sinflarga qarashli yagona yechimi mavjud.*

Uchinchi bobning uchinchi paragrafida o‘zgaruvchan koeffitsiyentli parabolik tipdagi integro–differensial tenglama maxsus ko‘rinishdagi

$$k(x,t) = \sum_{i=0}^{N} c_i\,(x) b_i(t)$$

yadrosining yagonaligi o‘rganilgan.

Bir o‘lchamli issiqlik o‘tkazuvchanlik integro–differensial tenglamasi uchun Koshi masalasini qaraymiz.

$$u_t - a(t)u_{xx} = \int_0^t k\,(x, t-\tau) u(x,y,\tau) d\tau, \quad (x,y) \in \mathbb{R}^2, t \in (0,T], \qquad (52)$$

$$u|_{t=0} = \varphi(x,y), \qquad (x,y) \in \mathbb{R}^2, \qquad (53)$$

bu yerda $T > 0$-tayinlangan o‘zgarmas son, $y \in \mathbb{R}$ parametr va $a(t) > 0$ yetarlicha silliq funksiya. Teskari masala (52), (53) to‘g‘ri masala yechimi haqida

$$u|_{x=y} = f(y,t), \qquad (y,t) \in \mathbb{R} \times [0,T] \qquad (54)$$

qo‘shimcha shart bo‘yicha $k(x,t)$, $x \in \mathbb{R}$, $t > 0$ funksiyani aniqlashdan iborat. Bu yerda barcha $y \in \mathbb{R}$ va $t \in [0,T]$ larda $f(y,t)$ – berilgan funksiya.

Faraz qilaylik, $k(x,t)$ va uning $k_x$, $k_t$ hosilalari ixtiyoriy $T > 0$ da $B(D_T), [D_T := \{(x,t): x \in \mathbb{R}; 0 \leq t \leq T\}]$ sinfga qarashli, $\varphi(x;y) \in B^4(\mathbb{R}^2)$, $B^4(\mathbb{R}^2)$- $\mathbb{R}^2$ da to‘rtinchi tartibli hosilasi bilan chegaralangan, uzluksiz funksiyalar sinfi.

Ushbu paragrafning asosiy natijasi quyidagi yagonalik teoremasidan iborat.

**5-Teorema.** *Faraz qilaylik, $\varphi(x,y) \in B^4(\mathbb{R}^2)$, $\{f(y,t), f_t(y,t), f_{tt}(y,t), f_{tyy}(y,t)\} \in B(D_T)$ bo‘lib, $\inf_{(x,y)\in\mathbb{R}^2} |\varphi(x,y)| \geq \beta_0 > 0$, bo‘lsin, bu yerda $\beta_0$ –ma’lum son. U holda, (52)-(54) teskari masalaning*

$$k(x,t)=\sum_{i=0}^{N} c_i\,(x) b_i(t), \quad c_i(x)\in B^2(\mathbb{R}^2),\, b_i(t)\in C^1([0,T])$$

*ko'rinishga ega yechimi $D_T$ sohada bir qiymatli aniqlangan.*

## XULOSA

Dissertatsiyada o'zgaruvchan koeffitsiyentli parabolik tipdagi integro–differensial tenglamalarda integral hadning yadrosini aniqlash uchun ko'p o'lchamli teskari masalalarning yechimi mavjudliga va yagonaligi o'rganilgan. Integro–differensial tenglamalarni ikkinchi tur Volterra tipidagi tenglamalar sistemasiga keltirib tadqiq qilingan. O'rama ko'rinishdagi o'ng tomonida integral operator bo'lgan parabolik integro–differensial tenglamalarning keng sinfni uchun teskari masalalar ko'rib chiqilgan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

o'zgaruvchan koeffitsiyentli integro–differensial issiqlik tarqalish tenglamasi uchun qo'yilgan Koshi masalasi yechimining mavjudligi ko'rsatilgan;

to'g'ri masala yechimiga qo'yilgan qo'shimcha shartdan foydalanib, ko'p o'lchamli yadroni Gyolder (Hölder) fazolarida bir qiymatli yechiluvchanligini isbotlangan;

issiqlik tarqalish integro–differensial tenglamasidan ko'p o'lchamli ajralgan yadroni aniqlash masalasining bir qiymatli yechiluvchanligini Gyolder (Hölder) fazolarida ko'rsatilgan;

o'zgaruvchan koeffitsiyentli integro–differensial issiqlik tarqalish tenglamasidan maxsus ko'rinishga ega ko'p o'lchamli yadroni aniqlash teskari masalasi yechimining yagonaligi ko'rsatilgan.

**SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREE**
**PhD.03/30.12.2019.FM.55.01 URGENCH STATE UNIVERSITY**

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**BUKHARA STATE UNIVERSITY**

**NURIDDINOV JAVLON ZAFAROVICH**

# INVERSE PROBLEMS FOR AN INTEGRO-DIFFERENTIAL HEAT EQUATION WITH AN INTEGRAL TERM OF CONVOLUTION TYPE

**01.01.02–Differential Equations and Mathematical Physics**

**ABSTRACT OF DISSERTATION**
**for the doctor of philosophy (PhD) on physical and mathematical sciences**

**Bukhara - 2021**

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Cabinet of Ministers of the Republic of Uzbekistan under number **B2019.4.PhD/FM433**.

The dissertation was performed at the Bukhara branch of the Institute of Mathematics named after V.I. Romanovskiy at the Academy of Sciences of the Republic of Uzbekistan and Bukhara State University.

The abstract of the dissertation is posted in three languages (uzbek, English, russian (resume)) on the website (www.ik-mat.urdu.uz) and the «Ziyonet» Information and educational portal (www.ziyonet.uz).

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## INTRODUCTION (Doctor of Philosophy (PhD) dissertation annotation)

**Actuality and demand of the theme of dissertation.** Many scientific and applied studies around the world lead to the study of special derivative differential and integro-differential equations, the correct and inverse problems for them. Inverse problems pervaded astronomy, quantum propagation theory, geophysics, thermal physics, medicine, and all areas of modern science with the advent of computers. In order to solve the right problems in mathematical physics, it is necessary to give the coefficients of the equation, the boundary of the field, the initial and boundary conditions. But in practice, it is not always possible to give the coefficients of the equation. It is not always possible to define the initial and boundary conditions, as well as the boundaries of the field, in practical matters. In such cases, it is necessary to search for the solution of the inverse problem by entering additional information about the direct problem, that is, to find the coefficients, integral terms in the form of integro-differential equations, initial or boundary conditions, field boundaries. Due to the incompleteness of methods for solving such problems, one of the most important tasks is to solve the inverse problem of determining the kernel from the integro-differential thermal conductivity equation.

Today, the world's fastest-growing field of Mathematical Physics is the study of inverse problems. This field has become one of the most important mathematical problems in physics and engineering. The wide application of this problem has attracted the attention of many scientists due to the novelty and complexity of its theory. In recent years, the management of heat conductivity processes has been developing rapidly, as the thermal conductivity and relaxation function of each medium are different, and these magnitudes are closely related to the initial state and properties of the medium. Therefore, for integro-differential thermal conductivity equations, the study of initial, initial-boundary value problems, and their inverse problems is a targeted research.

In our country, special attention is paid to the current trends in mathematical physics, which have a scientific and practical application of fundamental sciences. In this regard, attention was paid to the study of inverse problems in determining the relaxation function from the integro-differential equations of parabolic type. As a result of these studies, it was possible to prove the existence and uniqueness of the solution of inverse problems for integro-differential equations of parabolic type with variable coefficients. The main task is to conduct research at the level of international standards in the priority areas of differential equations and mathematical physics.[1] The development of the theory of integro-differential equations in mathematical physics is important in ensuring the implementation of this decision.

Decree of the President of the Republic of Uzbekistan № PF-4947 of February 7, 2017 "On the strategy of actions for further development of the Republic of Uzbekistan", № PP-4387 of July 9, 2019 "On Mathematics Education and Science On

[1] Decree of President of the Republic of Uzbekistan at the “On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I.Romanovsky of the Academy of Sciences of the Republic of Uzbekistan” PQ-4387 dated July 9, 2019.

measures to support the further development of the state, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I.Romanovsky of the Academy of Sciences of the Republic of Uzbekistan "and № 7 PP-4708 of May 7, 2020 This dissertation research will to some extent serve in the implementation of the tasks set out in the decisions "On measures to improve the quality of education and development of research" and other normative legal acts related to this activity.

**Connection of research to priority directions of development of science and technologies of the Republic.** This work was performed in accordance with the priority areas of science and technology development in the Republic of Uzbekistan IV, "Mathematics, Mechanics and Computer Science".

**The degree of scrutiny of the problem.** In developing the theory of inverse problems of mathematical physics, A.S. Alekseev, M.M. Lavrentev, V.G. Romanov, A. Lorenzi and others contributed. Inverse problems for equations of parabolic type V.G. Romanov, A.I. Prilepko, A.D. Iskendarov, N.Ya. Beznoshenko, F.A. Colombo, Ya. Yanno posted and researched. Various methods for studying the inverse problems of determining the kernel of an integral term from integro-differential equations of the parabolic type have been developed by L.V. Lorenzi, M. Graselli, Ya. Yanno, F.A. Colombo, D.K. Durdiev et al. proposed and developed. N.Ya. Beznoshenko,[2] studied the inverse problems of determining the coefficient of variability for equations of parabolic type. A.I. Prilepko, A.B. Kostin[3] explores the inverse problem of determining the coefficients of an equation from the initial-boundary problems of the parabolic type with variable coefficients, and proves the theorems on existence and uniqueness of the solution. A.D. Iskendarov's research proposed analytical and numerical methods for determining the coefficients of such equations. Ya. Yanno, L.V. Volfersdor[4] in his work, a one-dimensional inverse problem for determining the core of the integral term for the integro-differential thermal conductivity equation in a limited area is considered, and the existence, uniqueness of the solution is proved, and stability estimates are obtained.

In this dissertation, the problems of one-valued solvability of multidimensional direct and inverse problems for integro-differential thermal conductivity equations with variable coefficients are studied in Hölder spaces. Thus V.G. Romanov, Ya. Yanno, L.V. Volfersdorf and D.Q. Durdiyev's research close in terms of problem-solving and uses the methods recommended by them.

**Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out.** The dissertation was conducted in

[2] N. Ya. Beznoshchenko, On the determination of the coefficient in a parabolic equation, Differ. Equations, 1974, volume 10, № 1. -P.24-35

[3] Prilepko A.I., Kostin A.B. On inverse problems of determining a coefficient in parabolic equation // Siberian Math. J., 1993, vol. 34, №5.-P. 923-937.

[4] Janno J., Wolfersdorf L.V. Inverse problems for identification of memory kernels in heat flow // Ill-Posed Problems, 1996, vol. 4, №1. –P.39–66.

accordance with the planned research topic of Bukhara State University. Implemented within the framework of the grant project of Bukhara State University F-4-02 "Thermodynamics of models of mathematical physics with infinite set of cases".

**The aim of research work**. For the integro-differential thermal conductivity equation with a variable coefficient, the study of one-valued solvability problems in Hölder spaces of multidimensional direct and inverse problems.

**Research problems.**The main objectives of the study are:

-determine the existence and uniqueness of the solution of the Cauchy problem for the equation of integro-differential heat conductivity with variable coefficients;

-to study the one-valued solubility of a multidimensional kernel in Hölder spaces using an additional condition for solving the direct problem;

-to show the one-valued solvability of the problem of determining a multidimensional degeneration kernel from the integro-differential equation of heat conductivity in Hölder spaces;

-to study the uniqueness of the solution of the inverse problem of determining a multidimensional kernel with a special form from the equation of differential heat-differential heat conductivity.

**The research object** consists of integro-differential equations of the second order parabolic type.

**The research subject** multidimensional direct and inverse problems for the integro-differential thermal conductivity equation with variable coefficients in the form of windings.

**Research methods.** The dissertation uses the theory of special differential equations and integral equations, as well as methods of functional analysis, in particular, the solution of the second type of nonlinear integral equations of the Volterra type, the method of series approximation, fixed point theorem.

**Novelty of the research are followings:**

-the existence and uniqueness of the solution of the Cauchy problem for the second-order integro-differential heat conductivity equation with variable coefficients;

-the one-valued solubility of a multidimensional kernel in Golder spaces was studied using an additional condition given for the solution of the correct problem;

the one-valued solvability of the problem of determining a multidimensional degeneration kernel from the integro-differential equation of heat conductivity was learned in Hölder spaces;

-the uniqueness of the solution of the inverse problem of determining a multidimensional kernel with a special form from the equation of differential heat conductivity with variable coefficients is shown.

**The scientific novelty of the research** is as follows:

-the conditions for the existence and uniqueness of the relaxation function are found in the integro-differential heat conductivity equation with variable coefficients;

-the one-valued solvability of the inverse problem of determining a multidimensional kernel with a special appearance from the integro-differential heat conductivity equation has been determined.

**Reliability of research results** based on the theory of differential and integral equations, the theory of inverse problems, the application of methods of mathematical analysis, as well as the solidity of proofs and mathematical considerations.

**The reliability of the results of the study.** The scientific significance of the results of the research is explained by the further development of the theory of inverse problems for the integro-differential equations of mathematical physics, the development of methods for determining the multidimensional kernel.

The practical significance of the results of the study is explained by its application in seismology, exploration of oil and gas fields, the study of heat transfer processes in thermally conductive storage media.

**Implementation of the research results.** Based on the scientific results of the inverse problems for the equations of integro-differential thermal conductivity with variable coefficients:

-Fundamentals of the proposed method for determining the relaxation function from the equation of variable coefficients of integro-differential heat conductivity F-4-14 In the project, the equation of thermal conductivity was used to determine the core (reference of the Bukhara Institute of Engineering and Technology No. 83-10 / 974 of May 18, 2021). Application of scientific results The study of the one-valued solubility of a multidimensional kernel from the inverse problem for the Cauchy problem for the integro-differential heat conductivity equation with variable coefficients, the multidimensional degeneration kernel from the integro-differential heat equation allowed to study the solubility of the problem of identification;

- AAAA-A19-119032590069-3 for a multidimensional integrated-differential thermal conductivity equation in a foreign grant on the topic "Numerical solution and mathematical modeling of heat transfer and solid state mechanics in geophysical and engineering problems". From the proposed method of studying inverse problems used in the study of inverse issues (Southern Mathematical Institute, branch of the Federal State Budgetary Scientific Institution FSC "Vladikavkaz Scientific Center of the Russian Academy of Sciences", reference №37 of May 19, 2021). The application of scientific results has made it possible to prove the solvability of multidimensional inverse problems for integro-differential thermal conductivity equations.

**Approbation of the research results.** The results of this research were discussed at 6 scientific conferences, including 2 international and 4 national scientific conferences.

**Publications of the research results.** A total of 12 scientific articles on the topic of the dissertation were published. Among them 6 articles in the list of scientific publications recommended for publication of the main scientific results of dissertations of the Higher Attestation Commission of the Republic of Uzbekistan, 2 in foreign and 4 in national journals were published.

**The structure and volume of the dissertation** The dissertation consists of an introduction part, three chapters, a conclusion and a list of references. The volume of the dissertation is 96 pages.

## THE MAIN CONTENT OF THE DISSERTATION

**In introduction** the motivation of research theme and correspondence to the priority research areas of science and technology of the Republic are given, we present a review of international research on the theme of the dissertation and degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state scientific novelty and practical results of the research. Moreover, we give the theoretical and practical importance of the obtained results, and also give information on the implementation of the research results, the published works and the structure of dissertation.

The first chapter of the dissertation is called "Direct and inverse problems for the integro-differential equation of heat caduchivity conductivity with variable coefficients." and it is studied existence and uniqueness of the solution of the problem. The first paragraph of this chapter outlines the general concepts used in the dissertation.

The second paragraph of the first chapter studied the existence and uniqueness of the solution of the direct problem for the integro-differential equation of heat conduction with variable coefficients.

Consider the problem of determining a function $u(x,t),\ (x,t) \in \mathbb{R}_T^n$ from the following equations:

$$u_t - a(t)\Delta u = \int_0^t k\,(x',\tau)u(x,t-\tau)d\tau, (x,t) \in \mathbb{R}_T^n, \qquad (1)$$

$$u|_{t=0} = \varphi(x), x \in \mathbb{R}^n, \qquad (2)$$

where $\Delta := \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ is Laplas operator, $\mathbb{R}_T^n = \{(x,t) | x = (x', x_n) \in \mathbb{R}^n, 0 < t \leq T\}$, $T > 0$ is arbitrary positive number, $a(t) \in E := \{a(t) \in C^1[0,T], 0 < a_0 \leq a(t) \leq a_1 < \infty\}$.

***Definition 1***. *If the inequality*

$$|f(x_1) - f(x_2)| \leq A|x_1 - x_2|^l, \qquad A = const > 0, \qquad l \in (0,1) \qquad (3)$$

*is satisfied for arbitrary* $x_1, x_2 \in \mathbb{R}^n$, *then the function* $f(x)$ *satisfies the Holder condition with l in* $\mathbb{R}^n$ *and the class of functions satisfying the condition (3) is signed by* $H^l(\mathbb{R}^n)$.

Similarly, for functions with multidimensional, the Hölder function class can be defined, as:

***Definition 2***. *If the inequality*

$$\left|f(x^{(1)},t_1)-f(x^{(2)},t_2)\right| \le \sum_{i=1}^{n} A_i|x_i^{(1)}-x_i^{(2)}|^l + A_{n+1}|t_1-t_2|^{l/2}$$
$$A_i = const > 0, \;\; i=1,2,\dots n+1. \; l \in (0,1) \qquad (4)$$

*is satisfied for arbitrary* $(x^{(1)},t_1),(x^{(2)},t_2) \in \mathbb{R}_T^n$, $x^{(1)} = (x_1^{(1)},x_2^{(1)},\dots,x_n^{(1)})$, $x^{(2)} = (x_1^{(2)},x_2^{(2)},\dots,x_n^{(2)})$ *, then the function* $f(x,t)$ *satisfies the Holder condition with* $l, l/2$ *in* $\mathbb{R}_T^n$ *and the class of functions satisfying the condition (3) is signed by* $H^{l,l/2}(\mathbb{R}_T^n)$ *.*

If $(x) \in H^l(\mathbb{R}^n)$, $f(x,t) \in H^{l,l/2}(\mathbb{R}_T^n)$, the norms in spaces $H^{l+m}(\mathbb{R}^n)$, $H^{l+m,(l+m)/2}(\mathbb{R}_T^n)$ are defined as follows:

$$|\varphi|^{l+m} = \sum_{|\alpha|\le m} \sup_{x\in\mathbb{R}^n} |D^\alpha \varphi| + \sum_{|\alpha|=m} \sup_{\substack{|x^1-x^2|\le\rho_0 \\ x^1,x^2\in\mathbb{R}^n}} \frac{|D^\alpha\varphi(x^1)-D^\alpha\varphi(x^2)|}{|x^1-x^2|^l},$$

$$|f|_T^{l+m,(l+m)/2} = \sum_{2r+|s|\le m} \sup_{(x,t)\in\mathbb{R}_T^n} |D^r D^s f| +$$

$$+ \sum_{2r+|s|=m} \sup_{\substack{|x^1-x^2|\le\rho_0 \\ (x,t)\in\mathbb{R}_T^n}} \frac{|D^r D^s f(x^1,t)-D^r D^s f(x^2,t)|}{|x^1-x^2|^l} +$$

$$+ \sum_{2r+|s|=m} \sup_{\substack{|t_1-t_2|\le\rho_1 \\ (x,t)\in\mathbb{R}_T^n}} \frac{|D^r D^s f(x,t_1)-D^r D^s f(x,t_2)|}{|t_1-t_2|^{l/2}},$$

where $\rho_0, \rho_1 > 0$.

When $k(t), h(x,t), \varphi(x), \mu_1(t), \mu_2(t)$ are given functions then this problem (1)-(2) is called as a direct problem.

For the given functions $a$ and $k$, the problem of finding the function $u\,(x,t)$ from the integro-differential (1) equation by the initial condition (2) is called the Cauchy problem.

We replace the Cauchy problem (1)-(2) with the equivalent Volterra integral equation:

$$u(x,t) = \int_{\mathbb{R}^n} \varphi\,(\xi) G(x-\xi;\theta(t))d\xi + \int_0^{\theta(t)} \frac{d\tau}{a(\theta^{-1}(\tau))} \times$$

$$\times \int_{\mathbb{R}^n} \int_0^{\theta^{-1}(\tau)} k\,(\xi',\alpha) u(\xi,\theta^{-1}(\tau)-\alpha) G(x-\xi;\theta(t)-\tau) d\alpha d\xi, \qquad (5)$$

where $\theta^{-1}$ is the inverse function of $\theta(t)$ function $\theta(t) = \int_0^t a(s)ds$, $G(x-\xi;\theta(t)-\tau) = \frac{1}{(2\sqrt{\pi(\theta(t)-\tau)})^n} e^{-\frac{|x-\xi|^2}{4(\theta(t)-\tau)}}$ is a fundamental solution of a differential operator $\frac{\partial}{\partial t} - a(t)\Delta$ with a variable coefficient, $|x|^2 = x_1^2 + \cdots + x_n^2$.

**Lemma 1.** *Let* $\varphi(x) \in H^{l+2}(\mathbb{R}^n), k(x',t) \in H^{l,l/2}(\overline{\mathbb{R}}_T^{n-1})$ *and* $a(t) \in E$ *. Then there exists a unique solution to the integral equation (5) that* $u(x,t) \in H^{l+2,(l+2)/2}(\overline{\mathbb{R}}_T^{n})$.

The third paragraph of the first chapter studied the inverse problem for the integro-differential equation of heat conduction with variable coefficients.

In this section we consider the problem of finding the function $k(x',t)$ from the integro-differential equation (1) together with the function $u(x,t)$. To do this, we include the following additional conditions:

$$u|_{x_n=0} = f(x',t), \qquad (x',t) \in \overline{\mathbb{R}}_T^{n-1}, \tag{6}$$

where $f(x',t) \in H^{l+4,(l+4)/2}(\overline{\mathbb{R}}_T^{n-1})$ is given function.
The problem of finding the functions $u(x,t)$ and $k(x',t)$ from the problem (1), (2), (6) is called the inverse problem.

We denote by $\vartheta$ the function $u_{x_n x_n}$, i.e. $\vartheta(x,t) = u_{x_n x_n}(x,t)$. We differentiate two times the equalities equation (1), (2) with respect to $x_n$, we obtain:

$$\vartheta_t - a(t)\Delta\vartheta = \int_0^t k\,(x',\tau)\vartheta(x,t-\tau)d\tau, \tag{7}$$

$$\vartheta|_{t=0} = \varphi_{x_n x_n}(x), \tag{8}$$

For the function $\vartheta(x,t)$, the additional condition at $x_n = 0$ is taken using (1) and (6):

$$\vartheta|_{x_n=0} = \frac{1}{a(t)} f_t - \sum_{k=1}^{n-1} \frac{\partial^2}{\partial x_k^2} f - \frac{1}{a(t)} \int_0^t k\,(x',\tau) f(x',t-\tau)d\tau. \tag{9}$$

From the initial condition (8) and the additional condition (9) arises the following condition:

$$\varphi_{x_n x_n}(x',0) = \frac{1}{a(0)} f_t(x',0) - \sum_{k=1}^{n-1} f_{x_i x_i}(x',0). \tag{10}$$

We differentiate equations (7), (9) by $t$ once and introduce the notation $\vartheta_t(x,t) = \omega(x,t)$, resulting in the problem of finding the functions $\vartheta(x,t), k(x',t), \omega(x,t)$ from equations (7), (8) and we obtain

$$\omega_t - a(t)\Delta\omega = (\ln\, a\,(t))'\omega - (\ln\, a\,(t))' \int_0^t k\,(x',\tau)\vartheta(x,t-\tau)d\tau +$$

$$+ \int_0^t k\,(x',\tau)\omega(x,t-\tau)d\tau + k(x',t)\varphi_{x_n x_n}(x), \tag{11}$$

$$\omega|_{t=0} = a(0)\Delta\varphi_{x_n x_n}(x), \tag{12}$$

$$\omega|_{x_n=0} = -\frac{a'(t)}{a^2(t)} f_t + \frac{1}{a(t)} f_{tt} - \sum_{k=1}^{n-1} \frac{\partial^2}{\partial x_k^2} f_t + \frac{a'(t)}{a^2(t)} \int_0^t k\,(x',\tau) f(x', t-\tau) d\tau -$$

$$-\frac{1}{a(t)} \int_0^t k\,(x',\tau) f_t(x', t-\tau) d\tau - \frac{1}{a(t)} k(x',t)\varphi(x',0) \tag{13}$$

**Lemma 2.** *Let* $\varphi(x) \in H^{l+6}(\mathbb{R}^n)$, $f(x',t) \in H^{l+4,\frac{l+4}{2}}\left(\overline{\mathbb{R}}_T^{n-1}\right)$, $a(t) \in E$ *and the matching conditions*

$$f(x',0) = \varphi(x',0), \qquad \varphi_{x_n x_n}(x',0) = \frac{1}{a(0)} f_t(x',0) - \sum_{k=1}^{n-1} f_{x_i x_i}(x',0)$$

*are met. Then problem (1),(2),(6)(17) is equivalent to the auxiliary problem of determining the functions* $\vartheta(x,t)$, $k(x',t)$, $\omega(x,t)$ *from the (7)-(9)and (11)-(13) equations.*

The following theorem of existence and uniqueness is valid as the main result of this paragraph:

**Theorem 1.** *Suppose that* $a(t) \in E$, $(x',t) \in H^{l+4,(l+4)/2}(\overline{\mathbb{R}}_T^{n-1})$ , $\varphi(x) \in H^{l+6}(\mathbb{R}^n)$ *function are from the classes, and crossing assignments should be completed*

$$f(x',0) = \varphi(x',0), \qquad \varphi_{x_n x_n}(x',0) = \frac{1}{a(0)} f_t(x',0) - \sum_{k=1}^{n-1} f_{x_i x_i}(x',0)$$

*then there exists* $T_0 > 0$ *the numbers are all small enough to be positive number* $T \in (0, T_0)$ *and for inverse problem (1)-(3), there is unique solution* $u(x,t) \in H^{l+2,(l+2)/2}(\overline{\mathbb{R}}_T^n)$, $k(x',t) \in H^{l,l/2}(\overline{\mathbb{R}}_T^{n-1})$.

The second chapter of the dissertation is called **"The problem of determining the multidimensional degenerated kernel from the integro-differential equation of heat conduction",** the first paragraph of which is devoted to the integro-differential equation of multidimensional kernel heat conduction with variable coefficients. the existence and uniqueness of the direct problem solution is studied.

We consider the problem of determining the function $\boldsymbol{u(x,t)}$ from the following equations:

$$u_t - a(t)\Delta u = \int_0^t K\,(x,\tau) u(x, t-\tau) d\tau, \quad (x,t) \in \mathbb{R}_T^n, \tag{14}$$

$$u(x,0) = \varphi(x), \qquad x \in \mathbb{R}^n, \tag{15}$$

The solution of the Cauchy problem (14), (15) satisfies the integral equation

$$u(x,t) = \int\limits_{\mathbb{R}^n} \varphi(\xi) G(x-\xi; \theta(t)) d\xi + \int\limits_0^{\theta(t)} \frac{d\tau}{a(\theta^{-1}(\tau))} \times$$

$$\times \int\limits_{\mathbb{R}^n} \int\limits_0^{\theta^{-1}(\tau)} h(\xi_n) k(\xi', \alpha) u(\xi, \theta^{-1}(\tau) - \alpha) G(x-\xi; \theta(t) - \tau) d\alpha d\xi. \qquad (16)$$

**Lemma 3.** *Let* $\varphi(x) \in H^{l+2}(\mathbb{R}^n)$, $h(x_n) \in H^{l+2}(\mathbb{R}), k(x',t) \in H^{l,l/2}(\overline{\mathbb{R}}_T^{n-1})$ *and* $a(t) \in E$ *. Then there exists a unique solution to the integral equation (16) that* $u(x,t) \in H^{l+2,(l+2)/2}(\overline{\mathbb{R}}_T^n)$

The second paragraph examines the problem of determining a multidimensional kernel from the integro-differential equation of heat conduction with variable coefficients.

The problem of finding the functions $u(x,t)$ and $k(x',t)$ from the integro-differential equation (16) is considered. We introduce following additional condition:

$$u(x', 0, t) = f(x', t), \qquad (x', t) \in \overline{\mathbb{R}}_T^{n-1}. \qquad (17)$$

We obtain a system of integral equations equivalent to the inverse problem (12), (13), (15).

To do this, differentiate the original equation by $t$:

$$\vartheta(x,t) = u_t(x,t) \qquad (18)$$

is introduced and the following equivalent problem is obtained with respect to the function $\vartheta(x,t)$:

$$\vartheta_t - a(t)\Delta\vartheta = (\ln\, a(t))'\vartheta(x,t) - (\ln\, a(t))' \int\limits_0^t h(x_n) k(x', \tau) u(x, t-\tau) d\tau +$$

$$+ \int\limits_0^t h(x_n) k(x', \tau) \vartheta(x, t-\tau) d\tau + h(x_n) k(x', t) \varphi(x), \qquad (19)$$

$$\vartheta|_{t=0} = a(0)\Delta\varphi(x), \qquad (20)$$

$$\vartheta|_{x_n=0} = f_t(x', t),\ a(0)\Delta\varphi(x', 0) = f_t(x', 0). \qquad (21)$$

Differentiating equations (19), (20) by $x_n$ to the equivalent problem:

$$(\vartheta_{x_n})_t - a(t)\Delta\vartheta_{x_n} = (\ln\, a(t))'\vartheta_{x_n} - (\ln\, a(t))' h'(x_n) \int\limits_0^t k(x', \tau) \times$$

$$\times u(x, t-\tau) d\tau - (\ln\, a(t))' h(x_n) \int\limits_0^t k(x', \tau) u_{x_n}(x, t-\tau) d\tau +$$

$$+h'(x_n)\int_0^t k(x',\tau)\vartheta(x,t-\tau)d\tau + h(x_n)\int_0^t k(x',\tau)\vartheta_{x_n}(x,t-\tau)d\tau +$$

$$+h'(x_n)k(x',t)\varphi(x) + h(x_n)k(x',t)\varphi_{x_n}(x), \qquad (22)$$

$$\vartheta_{x_n}|_{t=0} = a(0)\Delta\varphi_{x_n}(x). \qquad (23)$$

Now let us denote by $\omega(x,t)$ the function $\vartheta_{x_n x_n}(x,t)$ . Differentiating (19), (20) twice with respect to $x_n$ we obtain the following Cauchy problem with respect to $\omega$:

$$\omega_t - a(t)\Delta\omega = (\ln\ a(t))'\omega - (\ln\ a(t))'h''(x_n)\int_0^t k(x',\tau)u(x,t-\tau)d\tau -$$

$$-2(\ln\ a(t))'h'(x_n)\int_0^t k(x',\tau)u_{x_n}(x,t-\tau)d\tau - (\ln\ a(t))'h(x_n)\int_0^t k(x',\tau) \times$$

$$\times u_{x_n x_n}(x,t-\tau)d\tau + h''(x_n)\int_0^t k(x',\tau)\vartheta(x,t-\tau)d\tau +$$

$$+2h'(x_n)\int_0^t k(x',\tau)\vartheta_{x_n}(x,t-\tau)d\tau + h(x_n)\int_0^t k(x',\tau)\omega(x,t-\tau)d\tau +$$

$$+h''(x_n)k(x',t)\varphi(x) + 2h'(x_n)k(x',t)\varphi_{x_n}(x) +$$

$$+h(x_n)k(x',t)\varphi_{x_n x_n}(x), \qquad (24)$$

$$\omega|_{t=0} = a(0)\Delta\varphi_{x_n x_n}(x). \qquad (25)$$

An additional condition for the function $\omega(x,t)$is created using (19) and (25):

$$\omega|_{x_n=0} = \frac{1}{a(t)}f_{tt} - \sum_{i=1}^{n-1}\frac{\partial^2}{\partial x_i k^2}f_t - \frac{(ln\ a(t))'}{a(t)}f_t + \frac{(ln\,a(t))'}{a(t)}h''(0) \times$$

$$\times\int_0^t k(x',\tau)f(x',t-\tau)d\tau + 2\frac{(ln\,a(t))'}{a(t)}h'(0)\int_0^t k(x',\tau)u_{x_n}(x',t-\tau)d\tau +$$

$$+\frac{(ln\,a(t))'}{a(t)}h(0)\int_0^t k(x',\tau)u_{x_n x_n}(x',t-\tau)d\tau - \frac{1}{a(t)}h''(0)\int_0^t k(x',\tau) \times$$

$$\times f_t(x,t-\tau)d\tau - 2\frac{1}{a(t)}h'(0)\int_0^t k(x',\tau)\vartheta_{x_n}(x',t-\tau)d\tau -$$

$$-\frac{1}{a(t)}h(0)\int_0^t k(x',\tau)f_{tt}(x',t-\tau)d\tau - \frac{1}{a(t)}h''(0)k(x',t)\varphi(x',0) -$$

$$-2\frac{1}{a(t)}h'(0)k(x',t)\varphi_{x_n}(x',0)-\frac{1}{a(t)}h(0)k(x',t)\varphi_{x_nx_n}(x',0). \quad (26)$$

**Lemma 4.** *Suppose that* $a(t)\in E$, $\varphi(x)\in H^{l+6}(\mathbb{R}^n)$, $f_t(x',t)\in H^{l+4,(l+4)/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$, $h(x_n)\in H^{l+2}(\mathbb{R})$ *and matching conditions* $f(x',0)=\varphi(x',0)$, *va* $a(0)\Delta\varphi(x',0)=f_t(x',0)$ *are met. Then the problem (14), (15), (17) is equivalent to the problem of determination function* $(\vartheta_{x_n}(x,t),u(x,t),u_{x_n}(x,t),u_{x_nx_n}(x,t),\vartheta(x,t),\omega(x,t),k(x',t))$ *from the integral equations (18), (19) – (20), (22)–(23) and (24)–(26).*

The following theorem of existence and uniqueness is valid as the main result of this paragraph:

**Theorem 2.** *If the conditions* $a(t)\in E$, $\varphi(x)\in H^{l+6}(\mathbb{R}^n)$, $f_t(x',t)\in H^{l+4,(l+4)/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$, $h(x_n)\in H^{l+2}(\mathbb{R})$, $l\in(0,1)$ *and matching conditions* $\varphi(x',0)=f(x',0)$, $a(0)\Delta\varphi(x',0)=f_t(x',0)$ *are met, then there exists sufficiently small number* $T_0>0$, $T\in(0,T_0]$ *that the solution to the integral equations (14),(15),(17) in the class of functions* $u(x,t)\in H^{l+2,(l+2)/2}(\overline{\mathbb{R}}_T^n)$, $k(x',t)\in H^{l,l/2}(\overline{\mathbb{R}}_T^{n-1})$ *exists and unique.*

In the third paragraph, in the integro-differential equation of multidimensional kernel heat conductive with a variable coefficient, the Laplace operator gives an unknown function under the integral, and the inverse problem is studied.

Consider the problem of determining functions $u(x,t)$, $k(x',t)$ from the equations:

$$u_t=a(t)\Delta u-\int_0^t k\,(x',t-\tau)a(\tau)\Delta u(x,\tau)d\tau,\quad (x,t)\in\mathbb{R}_T^n, \quad (27)$$

$$u(x,0)=\varphi(x),\ x\in\mathbb{R}^n, \quad (28)$$

$$u(x',0,t)=f(x',t),\qquad (x',t)\in\mathbb{R}_T^{n-1}. \quad (29)$$

where $a(t)>0$ is sufficiently smooth function.

Rewriting the equation (27) in the form of Volterra integral equation with respect to $a(t)\Delta u$ :

$$a(t)\Delta u=\int_0^t k\,(x',t-\tau)a(\tau)\Delta u(x,\tau)d\tau+u_t \quad (30)$$

From the theory of integral equations (30) the solution of the integral equation is as follows:

$$u_t-a(t)\Delta u=-\int_0^t r\,(x',t-\tau)u_\tau(x,\tau)d\tau. \quad (31)$$

In (31) $r(x',t)$ is resolvent of the kernel $k(x',t)$ and satisfies in integral equation:

$$r(x',t) = k(x',t) + \int_0^t k\,(x',t-\tau)r(x',t)d\tau, \quad (x,t) \in \mathbb{R}_T^n. \tag{32}$$

In the sequel we investigate the problem of determining functions $u(x,t)$, $r(x',t)$ satisfying the equations (31), (28), (29). The after solving this problem $k(x',t)$ can be found (30).

We introduce new function $\vartheta^{(1)}(x,t)$ by formula $\vartheta^{(1)}(x,t) = u_{x_n x_n}(x,t)$. Then the straightaway differentiation of equations (31), (28) with respect to $x_n$ twice leads us to the following relations for $\vartheta^{(1)}(x,t)$:

$$\vartheta_t^{(1)} - a(t)\Delta\vartheta^{(1)} = -\int_0^t r\,(x',t-\tau)\vartheta_\tau^{(1)}(x,\tau)d\tau, \tag{33}$$

$$\vartheta^{(1)}(x,0) = \varphi_{x_n x_n}(x). \tag{34}$$

$$\vartheta^{(1)}(x',0,t) = \frac{1}{a(t)} f_t(x',t) - \sum_{i=1}^{n-1} f_{x_i x_i}(x',t) + \\ + \frac{1}{a(t)}\int_0^t r\,(x',t-\tau)f_\tau(x',\tau)d\tau. \tag{35}$$

from (34), (35) it follows the matching condition:

$$\varphi_{x_n x_n}(x',0) = \frac{1}{a(0)} f_t(x',0) - \sum_{i=1}^{n-1} f_{x_i x_i}(x',0). \tag{36}$$

We carry out the next converting of problem. Denoting for this purpose the derivative of $\vartheta^{(1)}(x,t)$ with respect to t by $\vartheta^{(2)}(x,t)$, i.e. $\vartheta^{(2)}(x,t) := \vartheta_t^{(1)}(x,t)$ and $h(x',t) := r_t(x',t)$, from (32), (35) we get:

$$\vartheta_t^{(2)} - a(t)\Delta\vartheta^{(2)} = a'(t)\Delta\vartheta^{(1)} - r(x',0)\vartheta^{(2)} - \\ - \int_0^t h\,(x',t-\tau)\vartheta^{(2)}(x,\tau)d\tau, \tag{37}$$

$$\vartheta^{(2)}(x,0) = a(0)\Delta\varphi_{x_n x_n}(x), \tag{38}$$

$$\vartheta^{(2)}(x',0,t) = -\frac{a'(t)}{a^2(t)} f_t(x',t) + \frac{1}{a(t)} f_{tt}(x',t) - \sum_{i=1}^{n-1} f_{t x_i x_i}(x',t) - \\ - \frac{a'(t)}{a^2(t)}\int_0^t r\,(x',t-\tau)f_\tau(x',\tau)d\tau + \frac{1}{a(t)}\int_0^t h\,(x',\tau)f_\tau(x',t-\tau)d\tau +$$

$$+\frac{1}{a(t)}r(x',0)f_t(x',t). \tag{39}$$

Introducing also function $\vartheta(x,t)$ as $\vartheta(x,t):=\vartheta_t^{(2)}(x,t)$, in this way, we obtain the final problem of determining $\vartheta(x,t)$ satisfy the equation:

$$\vartheta_t - a(t)\Delta\vartheta = 2a'(t)\Delta\vartheta^{(2)} + a''(t)\Delta\vartheta^{(1)} - r(x',0)\vartheta -$$

$$-h(x',t)a(0)\Delta\varphi_{x_nx_n}(x) - \int_0^t h(x',\tau)\vartheta(x,t-\tau)d\tau, \tag{40}$$

$$\vartheta(x,0) = \Psi(x), \tag{41}$$

$$\vartheta(x',0,t) = F(x',t) + \left(2\frac{(a'(t))^2}{a^3(t)} - \frac{a''(t)}{a^2(t)}\right)\int_0^t r(x',t-\tau)f_\tau(x',\tau)d\tau -$$

$$-2\frac{a'(t)}{a^2(t)}\int_0^t h(x',\tau)f_\tau(x',t-\tau)d\tau - \frac{1}{a(t)}\int_0^t h(x',\tau)f_{tt}(x',t-\tau)d\tau +$$

$$+\frac{1}{a(t)}h(x',t)f_t(x',0), \tag{42}$$

where

$$\Psi(x) = a^2(0)\Delta^2\varphi_{x_nx_n}(x) + a'(0)\Delta\varphi_{x_nx_n}(x) - r(x',0)a(0)\Delta\varphi_{x_nx_n}(x),$$

$$F(x',t) = \left(-\frac{a''(t)}{a^2(t)} + 2\frac{(a'(t))^2}{a^3(t)}\right)f_t(x',t) + \frac{1}{a(t)}f_{ttt}(x',t) - \sum_{i=1}^{n-1} f_{ttx_ix_i}(x',t) -$$

$$-2\frac{a'(t)}{a^2(t)}r(x',0)f_t(x',t) + \frac{1}{a(t)}r(x',0)f_{tt}(x',t) - 2\frac{a'(t)}{a^2(t)}f_{tt}(x',t).$$

**Lemma 5.** *The problem 27)– (29) is equivalent to the problem of determination function $\vartheta^{(2)}(x,t), \vartheta(x,t), h(x',t), r(x,'t)$ from the integral equations (37), (38) and (40)–(42).*

The following theorem of existence and uniqueness is valid as the main result of this paragraph:

**Theorem 3.** *If the conditions $\varphi(x) \in H^{l+8}(\mathbb{R}^n)$, $f(x',t) \in H^{l+6,(l+6)/2}(\bar{\mathbb{R}}_T^{n-1})$, $|f_t(x',0)| > f_0 = const > 0$ and matching conditions $f(x',0) = \varphi(x',0)$ , $\varphi_{x_nx_n}(x',0) = \frac{1}{a(0)}f_t(x',0) - \sum_{i=1}^{n-1} f_{x_ix_i}(x',0)$ are met, then there exists sufficiently small number $T_0 > 0$, $T \in (0,T_0]$ that the solution to the integral equations (31), (28), (29) in the class of functions $uu(x,t) \in H^{l+2,(l+2)/2}\left(\overline{\mathbb{R}}_T^n\right)$, $h(x',t) \in H^{l,l/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$, $l \in (0,1)$ exists and unique.*

The third chapter is colled **"Inverse problems for integro-differential equations of parabolic type",** in which the kernel of the equation is given in the form of convulution with a differential operator $L = \Delta + c(x)$.

The first and second paragraphs of this chapter solution the following problems: We consider an inverse problem of determining functions $u(x,t)$, $k(x',t)$ $(x,t) \in \mathbb{R}_T^n$ that satisfy the following equations:

$$u_t - Lu = -\int_0^t k\,(x',\tau) Lu(x,t-\tau)d\tau, \quad (x,t) \in \mathbb{R}_T^n, \tag{43}$$

$$u|_{t=0} = \varphi(x), \quad x \in \mathbb{R}^n, \tag{44}$$

$$u|_{x_n=0} = f(x',t), \qquad (x',t) \in \mathbb{R}_T^{n-1}. \tag{45}$$

**Lemma 6. Let** $k(x',t) \in H^{l+2,(l+2)/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$. *Then the problem (41)–(43) is equivalent to the problem of finding functions* $u(x,t)$, $r(x',t)$ *that satisfy equation*

$$u_t(x,t) = Lu - \int_0^t r\,(x',t-\tau) u_\tau(x,\tau)d\tau, \tag{46}$$

*and conditions (44), (45), where* $r(x',t)$ *is resolvent of kernel* $k(x',t)$ *and In equation (46) kernels* $k(x',t)$ *and* $r(x',t)$ *are related by the formula*:

$$k(x',t) = r(x',t) - \int_0^t r\,(x',t-\tau) k(x',\tau)d\tau. \tag{47}$$

**7-Lemma.** *Problem* $(46),(44),(45)$ *is equivalent to the following auxiliary problem for functions* $\vartheta(x,t), h(x',t)$

$$\vartheta_t - L\vartheta - 2c_{x_n}\vartheta_{x_n}^{(2)} - c_{x_nx_n}\vartheta^{(2)} + r(x',0)\vartheta + h(x',t)[L\varphi_{x_nx_n}(x) +$$

$$+2c_{x_n}\varphi_{x_n}(x) + c_{x_nx_n}\varphi(x)] + \int_0^t h\,(x',t-\tau)\vartheta(x,\tau)d\tau = 0 \tag{48}$$

$$\vartheta|_{t=0} = Y_{x_nx_n}(x) \tag{49}$$

$$\vartheta(x',0,t) = f_{ttt}(x',t) - \Delta_{x'} f_{tt}(x',t) - c(x',0) f_{tt}(x',t) +$$

$$+r(x',0) f_{tt}(x',t) + h(x',t)L\varphi|_{x_n=0} + \int_0^t f_{tt}\,(x',t-\tau)h(x',\tau)d\tau. \tag{50}$$

*where*

$$\vartheta(x,t) = u_{ttx_nx_n}(x,t), \quad \vartheta^{(2)} = u_{tt},$$

$$\triangle_{x'} = \sum_{i=1}^{n-1} \frac{\partial^2}{\partial x_i^2}, \quad Y_{x_nx_n}(x) = \frac{\partial^2}{\partial x_n^2}\left(L^2\varphi(x) - r(x',0)L\varphi(x)\right)$$

$$r(x',0) = \frac{L^2\varphi(x',0) - f_{tt}(x'0)}{L\varphi(x',0)}. \tag{51}$$

The following existence and uniqueness theorem is proved in this paragraph.

**4-Teorema.** *If conditions* $\varphi(x) \in H^{l+8}(\mathbb{R}^n)$, $|L\varphi(x',0)|^l \geq const > 0$, $c(x) \in H^{l+4}(\mathbb{R}^n)$, $f(x',t) \in H^{l+6,(l+6)/2}\left(\overline{\mathbb{R}}_T^{n-1}\right)$, $l \in (0,1)$ *and equalities* $f(x',0) = \varphi(x',0)$, $f_t(x',0) = L\varphi(x',0)$ *are satisfied then for sufficiently small number* $T_0 > 0$ $T \in (0,T_0]$ *the unique solution of integral equations (46), (44), (45) in the class of functions* $u(x,t) \in H^{l+2,(l+2)/2}\left(\overline{\mathbb{R}}_T^{n}\right)$, $h(x',t) \in H^{l,l/2}\left(\overline{\mathbb{R}}_T^{n}\right)$.

The third paragraph of the third chapter studies the uniqueness of the kernel

$$k(x,t) = \sum_{i=0}^{N} c_i\,(x) b_i(t)$$

in a special form of the integro-differential equation of parabolic type with variable coefficients.

We consider the Cauchy problem for the one-dimentional integro-differential heat equation:

$$u_t - a(t)u_{xx} = \int_0^t k\,(x,t-\tau)u(x,y,\tau)d\tau, \quad (x,y) \in \mathbb{R}^2, t \in (0,T], \tag{52}$$

$$u|_{t=0} = \varphi(x,y), \qquad (x,y) \in \mathbb{R}^2, \tag{53}$$

where $T > 0$ is fixed positive constant, $y \in \mathbb{R}$ parameter of the problem and $a(t) > 0$ is enough smooth positive function.

Consider the following problem: find the kernel $k(x,t)$, $x \in \mathbb{R}, t > 0$ of the integral term equation (52), if the solution to the problem (52), (53) is known $x = y$:

$$u|_{x=y} = f(y,t), \qquad (y,t) \in \mathbb{R} \times [0,T] \tag{54}$$

where $f(y,t)$ –given function.

The following uniqueness theorem is proved in this paragraph.

**5-Teorema.** Let $(x,y) \in B^4(\mathbb{R}^2)$, $\{f(y,t), f_t(y,t), f_{tt}(y,t), f_{tyy}(y,t)\} \in B(D_T)$, $\inf_{(x,y)\in\mathbb{R}^2} |\varphi(x,y)| \geq \beta_0 > 0$ *holds, where* $\beta_0$ *is given number. Then, the function* $k(x,t)$ *representable in the form*

$$k(x,t) = \sum_{i=0}^{N} c_i\,(x) b_i(t), \quad c_i(x) \in B^2(\mathbb{R}^2),\ b_i(t) \in C^1([0,T])$$

*is uniquely determined in the domain* $D_T$ .

## CONCLUSION

In the dissertation is studied the uniqness and exists multidimensional inverse problems for determining the kernel of the integral term in integro-differential equations of parabolic type. Methods for solving inverse integro-differential equations are obtained by reducing them to a system of equations of the second type Voltaire type. Inverse problems are considered for a wide class of parabolic integro-differential equations with the integral operator on the right side of the winding view.

The main research results are as follows:

It is shown that there is a unique solution to the Cauchy problem for the integro–differential heat conduction equation with variable coefficient.

Using an additional condition to solve the direct problem for multidimensional kernel one-valued solubility was studied in Hölder spaces.

The exists and uniqueness solution is shown of the problem of determining a multidimensional degenerated kernel from the integro–differential equation of heat conduction in the Hölder spaces.

The uniqueness of the solution to the inverse problem of determining a multidimensional kernel with a special form the integro–differential equation of a heat conduction with variable coefficient was determined.

**НАУЧНЫЙ СОВЕТ PhD.03/30.12.2019.FM.55.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ УРГЕНЧСКОМ ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ**

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**БУХАРСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ**

**НУРИДДИНОВ ЖАВЛОН ЗАФАРОВИЧ**

# ОБРАТНЫЕ ЗАДАЧИ ДЛЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ТЕПЛОПРОВОДНОСТИ С ИНТЕГРАЛЬНЫМ ЧЛЕНОМ ТИПА СВЁРТКИ

**01.01.02 – Дифференциальные уравнения и математическая физика**

**А В Т О Р Е Ф Е Р А Т**
**диссертации доктора философии (PhD) по физико-математическим наукам**

**Бухара – 2021**

**Тема диссертации доктора философии (Doctor of Philosophy) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Кабинете Министров Республики Узбекистан за B2019.4.PhD/FM433**

Диссертация выполнена в Бухарском отделении Института Математики имени В.И.Романовского Академии Наук Республики Узбекистан и Бухарском государственном университете.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (www.ik-mat.urdu.uz) и на Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

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## ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

**Целью исследования** - построение методов определения ядра из интегро-дифференциального уравнения теплопроводности с переменным коэффициентом, исследование однозначных задач разрешимости в пространствах Гёльдера многомерных прямых и обратных задач.

**Объект исследования** - интегро-дифференциальные уравнения параболического типа второго порядка.

**Новизна исследования заключается в следующем:**

существование и единственность решения задачи Коши для интегро-дифференциального уравнения теплопроводности второго порядка с переменными коэффициентами;

однозначная разрешимость многомерного ядра в пространствах Гёльдера исследовалась с использованием дополнительного условия, заданного для решения правильной задачи;

изучена однозначная разрешимость задачи определения ядра многомерного вырождения из интегро-дифференциального уравнения теплопроводности в пространствах Гёльдера;

показана единственность решения обратной задачи определения многомерного ядра специального вида из уравнения дифференциальной теплопроводности с переменными коэффициентами.

**Внедрение результатов исследования.** На основе научных результатов обратные задачи для интегро-дифференциальной уравнений теплопроводности с переменными коэффициентами:

-Основы предлагаемого метода определения функции релаксации по уравнению переменных коэффициентов интегро-дифференциальной уравнении теплопроводности в проекте №Ф-4-14 «Развитие теории и разработка методов исследования динамического напряженно – деформированного состояния криволинейных участков тонкостенных подземных трубопроводов с протекающей жидкостью при воздействии динамических нагрузок» (справка Бухарского ИТИ, Техника и технологии № 83-10 / 974 от 18.05.2021 г.). Применение научных результатов обратная задача для задачи Коши, поставленная для интегро-дифференциального уравнения тепловыделения с переменным коэффициентом, позволила изучить однозначную разрешимость многомерного ядра;

- предлагаемый метод исследования обратных задач использован в зарубежном гранте АААА-А19-119032590069-3 «Математическое моделирование и численное решение задач механики сплошной среды и тепломассообмена в геофизических и инженерных задачах» (Южный математический институт, филиал ФГБНУ ФНЦ «Владикавказский научный центр РАН», справка №37 от 19 мая 2021 г.) Из предложенного метода исследования обратных задач, применяемого при исследовании обратных задач позволило доказать разрешимость многомерных обратных задач для интегро-дифференциальных уравнений теплопроводности.

**Структура и объем диссертации.** Диссертация состоит из введения, трех глав, заключения и списка использованной литературы. Объем диссертации составляет 96 страницы.

## E’LON QILINGAN ISHLAR ROʻYXATI
## LIST OF PUBLISHED WORKS
## СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

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