O`ZBEKISTON RESPUBLIKASI OLIY VA O`RTA MAXSUS

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Mavzu: Chegaraviy shartlarining biri spektral parametrga bog'liq bo'lgan Shturm-Liuvill masalasi uchun Ambarsumyan teoremasi

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Chegaraviy shartida spektral parametr qatnashgan chegaraviy masalalarni o`rganish – matematik fizika va mehanikada uchraydigan qiziqarli masalalardan biridir [2, 13]. [12] ishda bunday masalalarning turli hil fizik jaroyonlarga tatbiqlari topilgan. Ushbu [3, 5, 14, 24] ishlarda esa chegaraviy shartida spektral parametr qatnashgan turli hil masalalar o`rganilgan. To`lqin tenglamasida va issiqlik tarqalish tenglamasida o`zgaruvchilarni ajratsak ya'ni Fur'e usulini qo`llaydigan bo`lsak, chegaraviy shartida spektral parametr qatnashadigan Shturm-Liuvill masalasi hosil bo`ladi [12].

1929 yilda V.A.Ambarsumyan Shturm-Liuvill tenglamasi uchun qo`yilgan Neyman masalasining spektri $\lambda_n = n^2, n \geq 0$ bo`lsa, potensial nolga teng bo`lishini isbotlagan. Bu ish teskari spektral masalalar sohasidagi birinchi ish edi. Bundan so`ng, bu soha bilan G.Borg shug`ullagan. U Ambarsumyanning natijasi umumiy chegaraviy shartlarda noto`g`ri ekanini ko`rsatib bergan, hamda faqat bitta chegaraviy sharti bilan farq qiluvchi ikkita Shturm-Liuvill masalasining spektrlari bu masalalarni bir qiymatli aniqlashi haqidagi teoremani isbot qilgan ([7, 8, 16, 19]). Hozirgi kunda, bu teorema Borgning *yagonalik teoremasi* deb ataladi.

Bir qator matematiklar Ambarsumyanning natijasini boshqa masalarga o`tkazishga muaffaq bo`lganlar ([9, 10, 15, 17, 19, 22, 23, 25-28]). Masalan, grafda berilgan Shturm-Liuvill masalasi uchun Ambarsumyan teoremasining analogi [23, 27] ishlarda, matritsaviy Shturm-Liuvill masalasi uchun [22] ishda, Dirak operatori uchun [22, 25, 26] ishlarda olingan.

Mazkur bitiruv malakaviy ishda chegaraviy shartlarning ikkinchisida spektral parametr qatnashgan Shturm-Liuvill masalasi uchun Ambarsumyan teoremasi o`rganilgan.

Quyidagi

$$\begin{cases} -y'' + q(x)y = \lambda y, \qquad 0 \leq x \leq \pi \\ y'(0) = 0 \\ y'(\pi) + H(\lambda)y(\pi) = 0 \end{cases} \tag{1}$$

va

$$\begin{cases} -y'' = \lambda y, \quad 0 \le x \le \pi \\ y'(0) = 0 \\ y'(\pi) + H(\lambda) y(\pi) = 0 \end{cases} \tag{2}$$

chegaraviy masalalarning xos qiymatlarini mos ravishda $\{\lambda_n\}_{n=0}^{\infty}$ va $\{\tilde{\lambda}_n\}_{n=0}^{\infty}$ orqali belgilaymiz. Bu yerda, $q(x) \in C^1[0, \pi]$ haqiqiy funksiya bo`lib, chegaraviy shartda qatnashayotgan $H(\lambda)$ funksiya $\sqrt{\lambda}$ ga nisbatan haqiqiy koeffisiyentli ko`phad:

$$H(\lambda) = a_1\sqrt{\lambda} + a_2(\sqrt{\lambda})^2 + ... + a_m(\sqrt{\lambda})^m, \quad a_k \in R^1, \quad k = 1, 2, ..., m, \quad a_m \ne 0.$$

Teorema. Agar $\lambda_n = \tilde{\lambda}_n$, $n = 0, 1, 2, ...$ bo`lsa, u holda $q(x) \equiv 0$ bo`ladi.

Masalan, $H(\lambda) = \sqrt{\lambda}$ bo`lsa, (2) chegaraviy masala quyidagi ko`rinishni oladi:

$$\begin{cases} -y'' = \lambda y, \quad 0 \le x \le \pi \\ y'(0) = 0 \\ y'(\pi) + \sqrt{\lambda} y(\pi) = 0. \end{cases}$$

Bu masalaning xos qiymatlari ushbu

$$\tilde{\lambda}_0 = 0, \ \tilde{\lambda}_{n+1} = \left(n + \frac{1}{4}\right)^2, \ n = 0, 1, 2, ...$$

tengliklar bilan aniqlanadi.

1-§. Ambarsumyan teoremasi

V.A.Ambarsumyan 1929 yilda quyidagi teoremani isbot qilgan:

Teorema 1. Quyidagi

$$\begin{cases} -y'' + q(x)y = \lambda y & (1) \\ y'(0) = 0 & (2) \\ y'(\pi) = 0 & (3) \end{cases}$$

Shturm-Liuvill chegaraviy masalasida $q(x) \in C[0, \pi]$ haqiqiy funksiya bo`lib, (1)+(2)+(3) masalaning xos qiymatlari uchun ushbu $\lambda_n = n^2$, $n = 0, 1, 2, ...$ tenglik o`rinli bo`lsa, u xolda $q(x) \equiv 0$ bo`ladi.

Isbot. 1) Quyidagi

$$\begin{cases} -y'' + q(x)y = \lambda y\,, & 0 \le x \le \pi \\ y'(0) = hy(0) \\ y'(\pi) = -Hy(\pi) \end{cases} \tag{4}$$

Shturm-Liuvill chegaraviy masalasining xos qiymatlari uchun $q(x) \in C[0,\pi]$ haqiqiy funksiya va h, H haqiqiy sonlar bo`lganda, ushbu

$$\sqrt{\lambda_n} = n + \left\{ \frac{h+H}{\pi} + \frac{1}{2\pi}\int_0^\pi q(t)dt \right\} \cdot \frac{1}{n} + \frac{\gamma_n}{n}, \quad \{\gamma_n\} \in l_2 \tag{5}$$

asimptotik formula o`rinli ([ ]). Bizning (1)+(2)+(3) masala uchun (5) tenglik quyidagicha bo`ladi:

$$\sqrt{\lambda_n} = n + \left\{ \frac{1}{2\pi}\int_0^\pi q(t)dt \right\} \cdot \frac{1}{n} + \frac{\gamma_n}{n}, \quad \{\gamma_n\} \in l_2\,. \tag{6}$$

Bu yerga $\lambda_n = n^2$ ni qo`ysak,

$$n = n + \left\{ \frac{1}{2\pi}\int_0^\pi q(t)dt \right\} \cdot \frac{1}{n} + \frac{\gamma_n}{n},$$

ya’ni

$$\frac{1}{2\pi}\int_0^\pi q(t)dt + \gamma_n = 0 \tag{7}$$

kelib chiqadi. (7) tenglikda, $n \to \infty$ da limitga o`tsak, $\gamma_n \to 0$ bo`lishidan ushbu

$$\int_0^\pi q(t)dt = 0 \tag{8}$$

tenglik kelib chiqadi.

2) $\lambda_0 = 0$ xos qiymatga mos keluvchi xos funksiya $\varphi_0(x)$ bo`lsin. U xolda quyidagilar o`rinli:

$$\varphi_0'' = q(x)\varphi_0\,, \tag{9}$$

$$\varphi_0'(0) = 0\,, \tag{10}$$

$$\varphi_0'(\pi) = 0\,, \tag{11}$$

$$\varphi_0(x) = 0\,. \tag{12}$$

Ossilyatsiya teoremasiga ko`ra ([ ]) $\varphi_0(x)$ xos funksiya $(0,\pi)$ intervalda nolga ega emas. Agar $\varphi_0(0)=0$ yoki $\varphi_0(\pi)=0$ bo`lsa, chegaraviy shartlardan $\varphi_0(x)\equiv 0$ ziddiyat kelib chiqadi. Demak, $\varphi_0(x)\neq 0$, $x\in[0,\pi]$.

3) Quyidagi integralni ikki xil usulda hisoblaymiz:

$$J=\int_0^\pi \left(\frac{\varphi_0'}{\varphi_0}\right)' dx=\frac{\varphi_0'}{\varphi_0}\Big|_0^\pi=\frac{\varphi_0'(\pi)}{\varphi_0(\pi)}-\frac{\varphi_0'(0)}{\varphi_0(0)}=0-0=0 \qquad (13)$$

Ikkinchi tomondan, $\varphi_0''=q(x)\varphi_0$ tenglikka ko`ra

$$J=\int_0^\pi \left(\frac{\varphi_0'}{\varphi_0}\right)' dx=\int_0^\pi \frac{\varphi_0''\varphi_0-\varphi_0'^2}{\varphi_0^2}dx=\int_0^\pi \frac{q(x)\varphi_0^2-\varphi_0'^2}{\varphi_0^2}dx$$
$$=\int_0^\pi q(x)dx-\int_0^\pi \left(\frac{(\varphi_0')}{\varphi_0}\right)^2 dx \qquad (14)$$

Endi, (14) tenglikka (8) va (13) tengliklarni qo`ysak,

$$\int_0^\pi \left(\frac{\varphi_0'}{\varphi_0}\right)^2 dx=0 \qquad (15)$$

kelib chiqadi. Bunga ko`ra $\varphi_0'(x)\equiv 0$ bo`ladi. Demak, $\varphi_0(x)\equiv C\neq 0$ ekan. Buni $\varphi_0''=q(x)\varphi_0$ ga qo`ysak, $0=q(x)\cdot C$ ya'ni $q(x)\equiv 0$ kelib chiqadi.

Demak teorema isbotlandi.

2-§. Ambarsumyan teoremasining umumlashmasi.

Ambarsumyan teoremasini quyidagicha umumlashtirish mumkin: **Teorema2.** Quyidagi

$$\begin{cases} -y''+q(x)y=\lambda y, & 0\le x\le \pi \\ y'(0)=hy(0), & h\in R' \\ y'(\pi)=-Hy(0), & H\in R' \end{cases}$$

Shturm-Liuvill chegaraviy masalasida $q(x)\in C[0,\pi]$ haqiqiy funksiya va $h+H\geq 0$ bo`lib, bu masalaning xos qiymatlari uchun ushbu $\lambda_n = n^2,\ n=0,1,2,...$ tenglik o`rinli bo`lsin. U xolda $q(x)\equiv 0,\ h=0,\ H=0$ bo`ladi.

Isbot.

1) Quyidagi

$$\sqrt{\lambda_n} = n + \left\{\frac{h+H}{\pi} + \frac{1}{2\pi}\int_0^\pi q(t)dt\right\}\cdot\frac{1}{n} + \frac{\gamma_n}{n}, \qquad \{\gamma_n\}\in l_2 \tag{19}$$

asimptotik formulada $\lambda_n = n^2$ desak,

$$0 = \left\{\frac{h+H}{\pi} + \frac{1}{2\pi}\int_0^\pi q(t)dt\right\} + \gamma_n \tag{20}$$

kelib chiqadi. Bu yerda, $n\to\infty$ desak

$$\int_0^\pi q(x)dx = -2h-2H \tag{21}$$

tenglik kelib chiqadi.

2) $\lambda_0 = 0$ xos qiymatga mos keluvchi xos funksiya $\varphi_0(x)$ bo`lsin. U holda quyidagi tengliklar o`rinli:

$$\begin{cases} \varphi_0'' = q(x)\varphi_0 & (22) \\ \varphi_0'(0) = h\varphi_0(0) & (23) \\ \varphi_0'(\pi) = -H\varphi_0(\pi) & (24) \\ \varphi_0(x)\neq 0 & (25) \end{cases}$$

Ossilyatsiya teoremasiga ko`ra $\varphi_0(x)$ xos funksiya $(0,\pi)$ intervalda nolga ega emas. Agar $\varphi_0(0)=0$ yoki $\varphi_0(\pi)=0$ bo`lsa, chegaraviy shartlardan $\varphi_0(x)\equiv 0$ ziddiyat kelib chiqadi. Demak, $\varphi_0(x)\neq 0,\ x\in[0,\pi]$.

3)Quyidagi intervalni ikki xil usulda hisoblaymiz:

$$J = \int_0^\pi \left(\frac{\varphi_0'}{\varphi_0}\right)' dx = \frac{\varphi_0'}{\varphi_0}\Bigg|_0^\pi = \frac{\varphi_0'(\pi)}{\varphi_0(\pi)} - \frac{\varphi_0'(0)}{\varphi_0(0)} = -\frac{H\varphi_0(\pi)}{\varphi_0(\pi)} - \frac{h\varphi_0(0)}{\varphi_0(0)} = -H-h$$

Ikkinchi tomondan $\varphi_0'' = q(x)\varphi_0$ tenglikka ko`ra

$$J=\int_0^\pi\left(\frac{\varphi_0'}{\varphi_0}\right)'dx=\int_0^\pi\frac{\varphi_0''\varphi-\varphi_0'^2}{\varphi_0^2}dx=\int_0^\pi\frac{q(x)\varphi_0^2-\varphi_0'^2}{\varphi_0^2}x=$$
$$=\int_0^\pi q(x)dx-\int_0^\pi\left(\frac{\varphi_0'}{\varphi_0}\right)^2dx \tag{27}$$

Endi, (27) tenglikka (21) va (26) tengliklarni qo`ysak,

$$-h-H=-2h-2H-\int_0^\pi\left(\frac{\varphi_0'}{\varphi_0}\right)^2dx$$

$$\int_0^\pi\left(\frac{(\varphi_0')}{\varphi_0}\right)^2dx+h+H=0 \tag{28}$$

kelib chiqadi. Bu yerda, $h+H\ge 0$ ekanligidan bu tenglik faqat

$$\int_0^\pi\left(\frac{\varphi_0'}{\varphi_0}\right)^2dx=0 \tag{29}$$

va

$$h+H=0 \tag{30}$$

bo`lgandagina bajariladi. (29) tenglikdan $\varphi_0'(x)\equiv 0$ ya'ni $\varphi_0(x)\equiv C\neq 0$ bo`lishi kelib chiqadi. Buni (22) tenglikka qo`yib, $q(x)\equiv 0$ ekanligini, (23) va (24) tenglikka qo`yib $h=0$ va $H=0$ ekanligini topamiz.

Demak teorema isbotlandi.

3-§. Chegaraviy shartlarining biri spektral parametrga bo`g`liq bo`lgan Shturm-Liuvill masalasi.

Quyidagi

$$\begin{cases} -y''+q(x)y=\lambda y, \quad 0\le x\le \pi & (1)\\ y'(0)=0 & (2)\\ y'(\pi)+H(\lambda)y(\pi)=0 & (3)\end{cases}$$

chegaraviy masalasini ko`rib chiqamiz. Bu yerda, $q(x)\in C^1[0,\pi]$ haqiqiy funksiya bo`lib, (3) chegaraviy shartlarda qatnashayotgan $H(\lambda)$ funksiya $\sqrt{\lambda}$ ga nisbatan haqiqiy koeffisiyentli ko`phad:

$$H(\lambda)=a_1\sqrt{\lambda}+a_2(\sqrt{\lambda})^2+...+a_m(\sqrt{\lambda})^m, \quad a_k\in R^1, k=\overline{1,m}, a_m\neq 0 \qquad (4)$$

$c(x,\lambda)$ orqali (1) tenglamaning

$$c(0,\lambda)=1, \qquad c'(0,\lambda)=0$$

boshlang`ich shartlarini qanatlantiruvchi yechimini belgilaymiz. Bu yechim (2) chegaraviy shartni qanoatlantiradi, uni (3) chegaraviy shartga qo`ysak, (1)+(2)+(3) masalaning

$$\Delta(\lambda)\equiv c'(\pi,\lambda)+H(\lambda)c(\pi,\lambda)=0 \qquad (5)$$

xarakteristik tenglamasi kelib chiqadi.

Almashtirish operatorining xossasiga ko`ra[1] quyidagi

$$\begin{cases} -y''+q(x)y=\lambda y\\ y(0)=1\\ y(0)=h\end{cases}$$

Koshi masalasinini $\varphi(x,\lambda)$ yechimi uchun ushbu

$$\varphi(x,\lambda)=\cos\sqrt{\lambda}x+\int_0^x K(x,t)\cdot\cos\sqrt{\lambda}tdt \qquad (6)$$

[1] 212-bet A.B.Hasanov " Shturm-Liuvill chegaraviy masalalari nazariyasiga kirish" 1-qism. Toshkent " Fan" 2011-yil

tasvir o`rinli. Bu yerda, $K(x,t)$ funksiya $\lambda$ parametrga bog`liq bo`lmagan funksiya bo`lib, $q(x)\in C^1[0,\pi]$ bo`lganida $K(x,t), K_x(x,t)$ $K_t(x,t)$ funksiyalar uzluksiz va

$$K(x,\lambda)=h+\frac{1}{2}\int_0^x q(s)ds \tag{7}$$

$$K_t(x,t)|_{t=0}=0 \tag{8}$$

shartni qanoatlantiradi. Agar $q(x)\in C^1[0,\pi]$ bo`lsa, $K(x,t)$ funksiya

$$K_{xx}-q(x)K=K_{tt} \tag{9}$$

tenglamani qanoatlantiradi.

Demak, biz qarayotgan $c(x,\lambda)$ yechim uchun ushbu

$$c(x,\lambda)=\cos\sqrt{\lambda}x+\int_0^x K(x,t)\cos\sqrt{\lambda}tdt \tag{10}$$

tasvir o`rinli ekan. Bu yerda,

$$K(x,\lambda)=\frac{1}{2}\int_0^x q(s)ds \tag{11}$$

$$K_t(x,t)|_{t=0}=0 \tag{12}$$

(10) tenglikdagi integralni bo`laklab integrallasak, ushbu

$$c(x,\lambda)=\cos\sqrt{\lambda}x+\int_0^x K(x,t)d\left(\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}\right)$$

$$c(x,\lambda)=\cos\sqrt{\lambda}x+K(x,t)\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}\Bigg|_0^\pi-\frac{1}{\sqrt{\lambda}}\int_0^x K_t(x,t)\sin\sqrt{\lambda}tdt$$

$$c(x,\lambda)=\cos\sqrt{\lambda}x+K(x,x)\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}-\frac{1}{\sqrt{\lambda}}\int_0^x K_t(x,t)\sin\sqrt{\lambda}tdt \tag{13}$$

tenglik hosil bo`ladi. Agar (10) tenglikdan $x$ bo`yicha hosila olsak,

$$c'(x,\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}x+K(x,\lambda)\cos\sqrt{\lambda}x+\int_0^x K_x(x,t)\cos\sqrt{\lambda}tdt \tag{14}$$

tenglik kelib chiqadi.

Agar (13) va (14) tengliklarda $x=\pi$ deb olsak, quyidagi

$$c(\pi,\lambda)=\cos\sqrt{\lambda}\pi+K(\pi,\pi)\frac{\sin\sqrt{\lambda}\pi}{\sqrt{\lambda}}-\frac{1}{\sqrt{\lambda}}\int_0^x K_t(\pi,t)\sin\sqrt{\lambda}tdt \tag{15}$$

$$c'(\pi,\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}\pi+K(\pi,\pi)\cos\sqrt{\lambda}\pi+\int_0^x K_x(\pi,t)\cos\sqrt{\lambda}tdt \tag{16}$$

tengliklarni topamiz. Bularni (15) xarakteristik tenglamaga qo`yamiz:

$$\Delta(\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}\pi+H(\lambda)\cos\sqrt{\lambda}\pi+K(\pi,\pi)\cos\sqrt{\lambda}\pi+$$

$$\frac{H(\lambda)}{\sqrt{\lambda}}K(\pi,\pi)\sin\sqrt{\lambda}\pi+\int_0^\pi K_x(\pi,t)\cos\sqrt{\lambda}tdt-\frac{H(\lambda)}{\sqrt{\lambda}}\int_0^\pi K_t(\pi,t)\sin\sqrt{\lambda}tdt=0 \tag{17}$$

Izoh: Agar $q(x)\equiv 0$ bo`lsa, $c(x,\lambda)=\cos\sqrt{\lambda}x$ bo`ladi. Bu holda,

$$\Delta_0(\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}\pi+H(\lambda)\cos\sqrt{\lambda}\pi$$

bo`lgani uchun, $\Delta_0 = 0$ bo`ladi, ya'ni $\lambda=0$ hos qiymat bo`ladi.

Agar (1)-(4) masalada m=1 bo`lsa, ushbu

$$\begin{cases}-y''+q(x)y=\lambda y, \quad 0\le x\le\pi\\ y'(0)=0\\ y'(\pi)+a_1\sqrt{\lambda}y(\pi)=0\end{cases} \tag{18}$$

masala hosil bo`ladi, m=2 bo`lsa, ushbu

$$\begin{cases}-y''+q(x)y=\lambda y, \quad 0\le x\le\pi\\ y'(0)=0\\ y'(\pi)+[a_1\sqrt{\lambda}+a_2\lambda]y(\pi)=0\end{cases} \tag{19}$$

masala hosil bo`ladi.

Lemma1. (18) masalaning hos qiymatlarini o`sish tartibida joylashtirish mumkin, ularni $\{\lambda_n\}$ orqali belgilasak, ushbu

$$\sqrt{\lambda_n}=n+\frac{1}{\pi}arctga_1+\frac{1}{n}\left\{\frac{1}{2\pi}\int_0^\pi q(x)dx\right\}+\underline{\underline{O}}\left(\frac{1}{n^2}\right) \tag{20}$$

asimptotik formula o`rinli bo`ladi.

Isbot. (18) masala uchun (17) xarakteristik tenglama quyidagi ko`rinishni oladi:

$$-\sqrt{\lambda}\sin\sqrt{\lambda}\pi + a_1\sqrt{\lambda}\cos\sqrt{\lambda}\pi + K(\pi,\pi)\cos\sqrt{\lambda}\pi + a_1K(\pi,\pi)\sin\sqrt{\lambda}\pi +$$
$$+\int_0^\pi K_x(\pi,t)\cos\sqrt{\lambda}t dt - a_1\int_0^\pi K_t(\pi,t)\sin\sqrt{\lambda}t dt = 0 \qquad (21)$$

$q(x)\in C^1[0,\pi]$ b`olsin. Bu holda, $K_{xx}(x,t)$ $K_{xt}(x,t)$ $K_{tt}(x,t)$ funksiyalar uzluksiz bo`ladi. Bunga ko`ra

$$\int_0^x K_x(\pi,t)\cos\sqrt{\lambda}t dt = \int_0^x K_x(\pi,t)d\left(\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}\right) =$$
$$K_x(\pi,t)\left(\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}\right)\Bigg|_0^\pi - \frac{1}{\sqrt{\lambda}}\int_0^\pi K_{xt}(\pi,t)\sin\sqrt{\lambda}t dt = \underline{\underline{O}}\left(\frac{e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}}{\sqrt{\lambda}}\right) \qquad (22)$$

$$\int_0^x K_t(\pi,t)\sin\sqrt{\lambda}t dt = \int_0^x K_t(\pi,t)d\left(-\frac{\cos\sqrt{\lambda}t}{\sqrt{\lambda}}\right) =$$
$$= -K_t(\pi,t)\left(\frac{\cos\sqrt{\lambda}t}{\sqrt{\lambda}}\right)\Bigg|_0^\pi + \frac{1}{\sqrt{\lambda}}\int_0^\pi K_{tt}(\pi,t)\cos\sqrt{\lambda}t dt = \underline{\underline{O}}\left(\frac{e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}}{\sqrt{\lambda}}\right) \qquad (23)$$

(22) va (23) baholashlarni hisobga olib, (21) xarakteristik tenglamani quyidagi tarzda yozib olamiz:

$$\sqrt{\lambda}\sin\sqrt{\lambda}\pi = a_1\sqrt{\lambda}\cos\sqrt{\lambda}\pi + K(\pi,\pi)\cos\sqrt{\lambda}\pi +$$
$$+ a_1K(\pi,\pi)\sin\sqrt{\lambda}\pi + \underline{\underline{O}}\left(\frac{e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}}{\sqrt{\lambda}}\right) \qquad (24)$$

Demak,

$$tg\sqrt{\lambda}\,\pi = a_1 + \frac{K(\pi,\pi)}{\sqrt{\lambda}} + \frac{a_1K(\pi,\pi)}{\sqrt{\lambda}}\cdot tg\sqrt{\lambda}\,\pi + \underline{\underline{O}}\left(\frac{1}{\lambda}\right) \qquad (25)$$

Bu tenglikni ikki marta ishlatsak,

$$tg\sqrt{\lambda}\,\pi = a_1 + \frac{K(\pi,\pi)}{\sqrt{\lambda}} + \frac{a_1 K(\pi,\pi)}{\sqrt{\lambda}} \cdot$$

$$\cdot\left\{a_1 + \frac{K(\pi,\pi)}{\sqrt{\lambda}} + \frac{a_1 K(\pi,\pi)}{\sqrt{\lambda}} tg\sqrt{\lambda}\,\pi + \underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right)\right\} + \underline{\underline{O}}\left(\frac{1}{\lambda}\right),$$

ya'ni

$$tg\sqrt{\lambda}\,\pi = a_1 + \frac{K(\pi,\pi)}{\sqrt{\lambda}} + \frac{a_1^2 K(\pi,\pi)}{\sqrt{\lambda}} + \underline{\underline{O}}\left(\frac{1}{\lambda}\right) \tag{26}$$

kelib chiqadi. Bu tenglikni ushbu ko`rinishda yozib olamiz:

$$tg\sqrt{\lambda}\,\pi - a_1 = \frac{(1+a_1^2)K(\pi,\pi)}{\sqrt{\lambda}} + \underline{\underline{O}}\left(\frac{1}{\lambda}\right) \tag{27}$$

Endi quyidagi formuladan foydalanamiz:

$$tg(\alpha-\beta) = \frac{tg\alpha - tg\beta}{1 + tg\alpha tg\beta}$$

$$tg\alpha - tg\beta = tg(\alpha-\beta)\cdot(1 + tg\alpha tg\beta)$$

Bunga binoan (27) tenglikning shaklini o`zgartiramiz:

$(\alpha = \sqrt{\lambda}\pi,\ \beta = arctga_1)$

$$tg(\sqrt{\lambda}\,\pi - arctga_1)\cdot(1 + a_1 tg\sqrt{\lambda}\,\pi) = \frac{(1+a_1^2)K(\pi,\pi)}{\sqrt{\lambda}} + \underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right) \tag{28}$$

Bu yerga (26) ni qo`ysak,

$$tg\left(\sqrt{\lambda}\,\pi - arctga_1\right)\cdot\left\{1 + a_1^2 + \underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right)\right\} = \frac{(1+a_1^2)K(\pi,\pi)}{\sqrt{\lambda}} + \underline{\underline{O}}\left(\frac{1}{\lambda}\right) \tag{29}$$

tenglikni olamiz. Ushbu

$$\left(1 + a_1^2 + \underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right)\right)^{-1} = \left(1 + a_1^2\right)^{-1}\cdot\frac{1}{1 + \underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right)} = \frac{1}{1+a_1^2}\cdot\left(1 + \underline{\underline{O}}\left(\frac{1}{\lambda}\right)\right)$$

tenglikka asosan

$$tg(\sqrt{\lambda}\,\pi - arctga_1) = \frac{1}{1+a_1^2}\cdot\left(1+\underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right)\right)\left\{\frac{(1+a_1^2)K(\pi,\pi)}{\sqrt{\lambda}}+\underline{\underline{O}}\left(\frac{1}{\lambda}\right)\right\}=$$

$$=\left(1+\underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right)\right)\cdot\left\{\frac{K(\pi,\pi)}{\sqrt{\lambda}}+\underline{\underline{O}}\left(\frac{1}{\lambda}\right)\right\}=\frac{K(\pi,\pi)}{\sqrt{\lambda}}+\underline{\underline{O}}\left(\frac{1}{\lambda}\right)$$

bo`ladi. $\sqrt{\lambda_n}\pi - arctga_1 = \pi n + \delta_n$, $\left|\delta_n\right| < \pi$ belgilash kiritsak,

$$tg\delta_n = \frac{K(\pi,\pi)}{\sqrt{\lambda_n}} + \underline{\underline{O}}\left(\frac{1}{\lambda_n}\right)$$

bo`ladi. Demak,

$$\lambda_n = n + \frac{1}{\pi}arctga_1 + \frac{K(\pi,\pi)}{\pi\sqrt{\lambda_n}} + \underline{\underline{O}}(\frac{1}{\lambda_n}) \qquad (30)$$

tenglik o`rinli. Bu tenglikdan,

$$\frac{1}{\lambda_n} = \frac{1}{n} + \underline{\underline{O}}(\frac{1}{n^2})$$

kelib chiqadi. Nihoyat,

$$\sqrt{\lambda_n} = n + \frac{1}{\pi}arctga_1 + \frac{K(\pi,\pi)}{\pi n} + \underline{\underline{O}}(\frac{1}{n^2})$$

ya'ni

$$\sqrt{\lambda_n} = n + \frac{1}{\pi}arctga_1 + \frac{1}{2\pi n} + \int_0^\pi q(x)dx + \underline{\underline{O}}(\frac{1}{n^2})$$

asimptotik formula olamiz.

Lemma1 isbotlandi.

Lemma2. (19) masalaning xos qiymatlarini o`sish tartibida joylashtirish mumkin, ularni $\{\lambda_n\}$ orqali belgilasak, ushbu

$$\sqrt{\lambda_n} = n + \frac{1}{2} + \frac{1}{(2n+1)\pi}\left\{\int_0^\pi q(x)dx - \frac{2}{a_2}\right\} + \underline{\underline{O}}(\frac{1}{n^2}) \qquad (32)$$

asimptotik formula o`rinli.

Isbot. (19) masala uchun (17) xarakteristik tenglama quyidagi ko`rinishni oladi:

$$
\begin{gathered}
-\sqrt{\lambda}\sin\sqrt{\lambda}\pi+(a_1\sqrt{\lambda}+a_2\lambda)\cos\sqrt{\lambda}\pi+K(\pi,\pi)\cos\sqrt{\lambda}\pi+ \\
+(a_1+a_2\sqrt{\lambda})K(\pi,\pi)\sin\sqrt{\lambda}\pi+\int_0^x K_x(\pi,t)\cos\sqrt{\lambda}t dt- \\
-(a_1+a_2\sqrt{\lambda})\cdot\int_0^x K_t(\pi,t)\sin\sqrt{\lambda}t dt=0
\end{gathered} \tag{33}
$$

$q(x)\in C^1[0,\pi]$ bo`lgani uchun $K_{xx}(x,t)$ $K_{xt}(x,t)$ $K_{tt}(x,t)$ funksiyalar uzluksiz bo`ladi. Bunga ko`ra

$$
\int_0^x K_x(\pi,t)\cos\sqrt{\lambda}t dt=\underline{\underline{O}}\left(\frac{e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}}{\sqrt{\lambda}}\right) \tag{34}
$$

$$
\int_0^x K_t(\pi,t)\sin\sqrt{\lambda}t dt=\underline{\underline{O}}\left(\frac{e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}}{\sqrt{\lambda}}\right) \tag{35}
$$

(34) va (35) baholashlarni hisobga olib, (33) xarakteristik tenglamani quyidagi tarzda yozib olamiz:

$$
\begin{gathered}
-\sqrt{\lambda}\sin\sqrt{\lambda}\pi+a_1\sqrt{\lambda}\cos\sqrt{\lambda}\pi+a_2\lambda\cos\sqrt{\lambda}\pi+K(\pi,\pi)\cos\sqrt{\lambda}\pi+ \\
+a_1K(\pi,\pi)\sin\sqrt{\lambda}\pi+a_2\sqrt{\lambda}K(\pi,\pi)\sin\sqrt{\lambda}\pi+\underline{\underline{O}}\left(\frac{e^{\left\|\operatorname{Im}\sqrt{\lambda}\right\|\pi}}{\sqrt{\lambda}}\right)+\underline{\underline{O}}\left(e^{\left\|\operatorname{Im}\sqrt{\lambda}\right\|\pi}\right)=0,
\end{gathered}
$$

$$
\begin{gathered}
-\sqrt{\lambda}\sin\sqrt{\lambda}\pi+a_1\sqrt{\lambda}\cos\sqrt{\lambda}\pi+a_2\lambda\cos\sqrt{\lambda}\pi+a_2\sqrt{\lambda}K(\pi,\pi)\sin\sqrt{\lambda}\pi+ \\
+\underline{\underline{O}}\left(e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}\right)=0\ ,
\end{gathered}
$$

$$
-\sin\sqrt{\lambda}\pi+a_1\cos\sqrt{\lambda}\pi+a_2K(\pi,\pi)\sin\sqrt{\lambda}\pi+a_2\sqrt{\lambda}\cos\sqrt{\lambda}\pi+\underline{\underline{O}}\left(\frac{e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}}{\sqrt{\lambda}}\right)=0\ ,
$$

$$a_2\sqrt{\lambda}\cos\sqrt{\lambda}\pi = \sin\sqrt{\lambda}\pi - a_1\cos\sqrt{\lambda}\pi - a_2\sin\sqrt{\lambda}\pi + \underline{\underline{O}}\left(\frac{e^{\left\|\operatorname{Im}\sqrt{\lambda}\right\|\pi}}{\sqrt{\lambda}}\right) = 0 \ ,$$

$$a_2\sqrt{\lambda}ctg\sqrt{\lambda}\pi = 1 - a_1 ctg\sqrt{\lambda}\pi - a_2 K(\pi,\pi) + \underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right) \ ,$$

Demak,

$$ctg\sqrt{\lambda}\,\pi = \frac{1}{\sqrt{\lambda}}\left(\frac{1}{a_2} - K(\pi,\pi)\right) - \frac{a_1}{a_2\sqrt{\lambda}}ctg\sqrt{\lambda}\,\pi + \underline{\underline{O}}\left(\frac{1}{\lambda}\right) \tag{36}$$

Bu tenglikni ikki marta ishlatsak,

$$ctg\sqrt{\lambda}\,\pi = -\frac{1}{\sqrt{\lambda}}\cdot\left(\frac{1}{a_2} - K(\pi,\pi)\right) -$$

$$-\frac{a_1}{a_2\sqrt{\lambda}}\cdot\left\{\frac{1}{a_2\sqrt{\lambda}} - \frac{K(\pi,\pi)}{\sqrt{\lambda}} - \frac{a_1}{a_2\sqrt{\lambda}}ctg\sqrt{\lambda}\,\pi + \underline{\underline{O}}\left(\frac{1}{\lambda}\right)\right\} + \underline{\underline{O}}\left(\frac{1}{\lambda}\right)$$

ya’ni

$$ctg\sqrt{\lambda}\,\pi = \frac{1}{\sqrt{\lambda}}\cdot\left(\frac{1}{a_2} - K(\pi,\pi)\right) - \underline{\underline{O}}\left(\frac{1}{\lambda}\right) \tag{37}$$

kelib chiqadi. Bu tenglikni o`ng tomoni nolga intilgani uchun chap tomoni ham nolga intiladi, bu hol esa

$$\sqrt{\lambda_n}\,\pi = \left(n + \frac{1}{2}\right)\pi + \delta_n \ , \left|\delta_n\right| < \frac{\pi}{2} \tag{38}$$

bo`lganda bajariladi. Bunga ko`ra,

$$tg\delta_n = \frac{1}{\sqrt{\lambda_n}}\cdot\left(K(\pi,\pi) - \frac{1}{a_2}\right) + \underline{\underline{O}}\left(\frac{1}{\lambda_n}\right) \tag{39}$$

bo`ladi. $tgx \sim x, (x \to 0)$ bo`lgani uchun

$$\delta_n = \frac{1}{\sqrt{\lambda_n}}\cdot\left(K(\pi,\pi) - \frac{1}{a_2}\right) + \underline{\underline{O}}\left(\frac{1}{\lambda_n}\right)$$

bo`ladi. (38) ga ko`ra

$$\frac{1}{\sqrt{\lambda_n}}=\frac{1}{n+\frac{1}{2}+\frac{\delta_n}{\pi}}+\frac{1}{n+\frac{1}{2}}\cdot\frac{1}{1+\frac{2\delta_n}{\pi(2n+1)}}=\frac{1}{n+\frac{1}{2}}\cdot\left(1-\frac{2\delta_n}{\pi(2n+1)}+...\right)=\frac{1}{n+\frac{1}{2}}+\underline{\underline{O}}(\frac{1}{n^2}),$$

$$\frac{1}{\lambda_n}=\left(\frac{1}{n+\frac{1}{2}}+\underline{\underline{O}}(\frac{1}{n^2})\right)^2=\frac{1}{\left(n+\frac{1}{2}\right)^2}+\underline{\underline{O}}(\frac{1}{n^3})+\underline{\underline{O}}(\frac{1}{n^4})=\underline{\underline{O}}(\frac{1}{n^2}),$$

Demak,

$$\delta_n=\frac{1}{n+\frac{1}{2}}\cdot\left(K(\pi,\pi)-\frac{1}{a_2}\right)+\underline{\underline{O}}\left(\frac{1}{n^2}\right),$$

Bu ifodani (38) tenglikka qo`ysak, ushbu

$$\sqrt{\lambda_n}\pi=\left(n+\frac{1}{2}\right)\pi+\frac{1}{n+\frac{1}{2}}\cdot\left(K(\pi,\pi)-\frac{1}{a_2}\right)+\underline{\underline{O}}\left(\frac{1}{n^2}\right),$$

$$\sqrt{\lambda_n}=n+\frac{1}{2}+\frac{1}{\left(n+\frac{1}{2}\right)\pi}\cdot\left(K(\pi,\pi)-\frac{1}{a_2}\right)+\underline{\underline{O}}\left(\frac{1}{n^2}\right), \qquad (40)$$

formula kelib chiqadi. Agar (11) tenglikdan

$$K(\pi,\pi)=\frac{1}{2}\int_0^\pi q(x)dx \qquad (41)$$

ekanligini topib, buni (40) ga qo`ysak,

$$\sqrt{\lambda_n}=n+\frac{1}{2}+\frac{1}{(2n+1)\pi}\left\{\int_0^\pi q(x)dx-\frac{2}{a_2}\right\}+\underline{\underline{O}}(\frac{1}{n^2}) \qquad (42)$$

hosil bo`ladi. Lemma2 isbotlandi.

Lemma3. (1)-(4) masalada $m\geq 3$ bo`lsin. U holda bu masalaning xos qiymatlarini o`sish tartibida joylashtirish mumkin, ularni $\{\lambda_n\}$ orqali belgilasak, ushbu

$$\sqrt{\lambda_n}=n+\frac{1}{2}+\frac{1}{(2n+1)\pi}\int_0^\pi q(x)dx+\underline{\underline{O}}(\frac{1}{n^2}) \qquad (43)$$

asimptotik formula o`rinli bo`ladi.

Isbot. (1)-(4) masala uchun (17) xarakteristik tenglama quyidagi ko`rinishni oladi:

$$
\begin{gathered}
-\sqrt{\lambda}\sin\sqrt{\lambda}\pi+\left[a\sqrt{\lambda}_1+a_2(\sqrt{\lambda})^2+\ldots a_m(\sqrt{\lambda})^m\right]\cos\sqrt{\lambda}\pi+ \\
+K(\pi,\pi)\cos\sqrt{\lambda}\pi+\left[a_1+a_2\sqrt{\lambda}+\ldots+a_m(\sqrt{\lambda})^{m-1}\right]K(\pi,\pi)\sin\sqrt{\lambda}\pi+ \\
+\int_0^x K_x(\pi,t)\cos\sqrt{\lambda}tdt- \\
-\left[a_1+a_2\sqrt{\lambda}+\ldots+a_m(\sqrt{\lambda})^{m-1}\right]\int_0^x K_t(\pi,t)\sin\sqrt{\lambda}tdt=0
\end{gathered}
\tag{44}
$$

Bu yerga (34) va (35) baholashlarni qo`yamiz:

$$
\begin{gathered}
-\sqrt{\lambda}\sin\sqrt{\lambda}\pi+\left[a_1\sqrt{\lambda}+a_2(\sqrt{\lambda})^2+\ldots+a_m(\sqrt{\lambda})^m\right]\cos\sqrt{\lambda}\pi+K(\pi,\pi)\cos\sqrt{\lambda}\pi+ \\
+\left[a_1+a_2\sqrt{\lambda}+\ldots+a_m(\sqrt{\lambda})^{m-1}\right]K(\pi,\pi)\sin\sqrt{\lambda}\pi+ \\
+\underline{\underline{O}}\left(\frac{e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}}{\sqrt{\lambda}}\right)+\underline{\underline{O}}\left(e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}\right)+\underline{\underline{O}}\left(\sqrt{\lambda}e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}\right)+\ldots+\underline{\underline{O}}\left(\sqrt{\lambda}^{m-2}e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}\right)=0,
\end{gathered}
$$

$$
\begin{gathered}
\left[a_{m-1}\sqrt{\lambda}^{m-1}+a_m(\sqrt{\lambda})^m\right]\cos\sqrt{\lambda}\pi+a_m(\sqrt{\lambda})^{m-1}K(\pi,\pi)\sin\sqrt{\lambda}\pi+ \\
+\underline{\underline{O}}\left(\sqrt{\lambda}^{m-2}e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}\right)=0,
\end{gathered}
$$

$$
\left[a_{m-1}+a_m(\sqrt{\lambda})\right]\cos\sqrt{\lambda}\pi++a_mK(\pi,\pi)\sin\sqrt{\lambda}\pi+\underline{\underline{O}}\left(\frac{e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}}{\sqrt{\lambda}}\right)=0,
\tag{45}
$$

Bu tenglamani quyidagi ko`rinishda yozib olamiz:

$$
a_m(\sqrt{\lambda})\cos\sqrt{\lambda}=-a_{m-1}\cos\sqrt{\lambda}\pi-a_mK(\pi,\pi)\sin\sqrt{\lambda}\pi+\underline{\underline{O}}\left(\frac{e^{\left|\operatorname{Im}\sqrt{\lambda}\right|\pi}}{\sqrt{\lambda}}\right)=0,
$$

$$ctg\sqrt{\lambda}\pi = -\frac{a_{m-1}}{a_m\sqrt{\lambda}}ctg\sqrt{\lambda}\pi - \frac{K(\pi,\pi)}{\sqrt{\lambda}} + \underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right) \tag{46}$$

Bu tenglikni ikki marta ishlatsak,

$$ctg\sqrt{\lambda}\,\pi = -\frac{a_{m-1}}{a_m\sqrt{\lambda}}\cdot\left\{\frac{a_{m-1}}{a_m\sqrt{\lambda}}ctg\sqrt{\lambda}\,\pi - \frac{K(\pi,\pi)}{\sqrt{\lambda}} + \underline{\underline{O}}\left(\frac{1}{\lambda}\right)\right\} - \frac{K(\pi,\pi)}{\sqrt{\lambda}} + \underline{\underline{O}}\left(\frac{1}{\lambda}\right),$$

ya’ni

$$ctg\sqrt{\lambda}\,\pi = -\frac{K(\pi,\pi)}{\sqrt{\lambda}} + \underline{\underline{O}}\left(\frac{1}{\lambda}\right) \tag{47}$$

kelib chiqadi. Bu tenglikdan xuddi lemma2ning isbotlagandek qilib, (43) formulani keltirib chiqaramiz. Lemma3 isbotlandi.

(1)-(4) masalaning xos qiymatlarini $\left\{\lambda_n\right\}_{n=0}^{\infty}$ orqali belgilasak, ushbu

$$\begin{cases} -y'' + \tilde{q}(x)\,y = \lambda y, \quad 0 \le x \le \pi & (1') \\ y'(0) = 0 & (2') \\ y'(\pi) + Hy(\lambda)y(\pi) = 0 & (3') \end{cases}$$

masalaning xos qiymatlarini $\left\{\tilde{\lambda}_n\right\}_{n=0}^{\infty}$ orqali belgilaymiz. Bu yerda, $\tilde{q}(x) \in C^1[0,\pi]$ haqiqiy funksiya.

Lemma4. Agar $\lambda_n = \tilde{\lambda}_n, n = 0,1,2...$ bo`lsa, u holda ushbu

$$\int_0^\pi \left[q(x) - \tilde{q}(x)\right]dx = 0 \tag{48}$$

tenglik o`rinli.

Isbot. 1) m=1 bo`lsin. Bu holda lemma1ga ko`ra ushbu

$$\sqrt{\lambda_n} = n + \frac{1}{\pi}arctga_1 + \frac{1}{n}\left\{\frac{1}{2\pi}\int_0^\pi q(x)dx\right\} + \underline{\underline{O}}\left(\frac{1}{n^2}\right),$$

$$\sqrt{\tilde{\lambda}_n} = n + \frac{1}{\pi} arctga_1 + \frac{1}{n}\left\{\frac{1}{2\pi}\int_0^{\pi} q(x)dx\right\} + \underline{\underline{O}}(\frac{1}{n^2})$$

formulalar o`rinli. Bunga ko`ra

$$2\pi n\left(\sqrt{\lambda_n} - \sqrt{\tilde{\lambda}_n}\right) = \int_0^{\pi}\left[q(x) - \tilde{q}(x)\,dx\right] + \underline{\underline{O}}(\frac{1}{n}) \quad (49)$$

Teorema shartiga ko`ra $\lambda_n = \tilde{\lambda}_n$, demak ushbu

$$\int_0^{\pi}\left[q(x) - \tilde{q}(x)\right]dx = \underline{\underline{O}}\left(\frac{1}{n}\right)$$

tenglik o`rinli. $n \to \infty$ da limitga o`tsak,

$$\int_0^{\pi}\left[q(x) - \tilde{q}(x)\right]dx = 0$$

kelib chiqadi.

2) m=2 bo`lsin. Bu holda lemma2ga ko`ra ushbu

$$\sqrt{\lambda_n} = n + \frac{1}{2} + \frac{1}{(2n+1)\pi}\left\{\int_0^{\pi} q(x)dx - \frac{2}{a_2}\right\} + \underline{\underline{O}}(\frac{1}{n^2})$$

$$\sqrt{\tilde{\lambda}_n} = n + \frac{1}{2} + \frac{1}{(2n+1)\pi}\left\{\int_0^{\pi} \tilde{q}(x)\,dx - \frac{2}{a_2}\right\} + \underline{\underline{O}}(\frac{1}{n^2})$$

formulalar o`rinli bo`ladi. Bunga ko`ra

$$(2n+1)\pi \cdot \left(\sqrt{\lambda_n} - \sqrt{\tilde{\lambda}_n}\right) = \int_0^{\pi}\left[q(x) - \tilde{q}(x)\,dx\right] + \underline{\underline{O}}(\frac{1}{n}) \quad (50)$$

Bu yerda, $\lambda_n = \tilde{\lambda}_n$ deb, $n \to \infty$ da limitga o`tsak, (48) kelib chiqadi.

3) $m \geq 3$ bo`lsin. Bu holda lemma3ga asosan (50) tenglikni olamiz. Bundan (48) kelib chiqadi. Lemma4 isbotlandi.

4-§Chegaraviy shartlarining biri spektral parametrga bo`g`liq bo`lgan Shturm-Liuvill masalasi uchun Ambarsumyan teoremasi.

Quyidagi

$$\begin{cases} -y''+q(x)y=\lambda y, \quad 0\le x\le \pi \\ y'(0)=0 \\ y'(\pi)+H(\lambda)y(\pi)=0 \end{cases} \qquad (1)$$

va

$$\begin{cases} -y''=\lambda y, \quad 0\le x\le \pi \\ y'(0)=0 \\ y'(\pi)+H(\lambda)y(\pi)=0 \end{cases} \qquad (2)$$

chegaraviy masalasining xos qiymatlarini mos ravishda $\left\{\lambda_n\right\}_{n=0}^{\infty}$ va $\left\{\tilde{\lambda}_n\right\}_{n=0}^{\infty}$ orqali belgilaymiz. Bu yerda, $q(x)\in C^1[0,\pi]$ haqiqiy funksiya bo`lib, chegaraviy shartda qatnashayotgan $H(\lambda)$ funksiya $\sqrt{\lambda}$ ga nisbatan haqiqiy koeffisiyentli ko`phad:

$$H(\lambda)=a_1\sqrt{\lambda}+a_2(\sqrt{\lambda})^2+...+a_m(\sqrt{\lambda})^m, \qquad a_k\in R^1, \quad k=\overline{1,m}, \ a_m\ne 0$$

Teorema. Agar $\lambda_n=\tilde{\lambda}_n, n=0,1,2...$ bo`lsa, u holda $q(x)\equiv 0$ bo`ladi.

Isbot. $\tilde{q}(x)\equiv 0$ bo`lgani uchun lemma4ga ko`ra

$$\int_0^{\pi} q(x)dx=0$$

bo`ladi.

$\lambda=0$ soni (2) masalaning xos qiymati bo`ladi,chunki $H(0)=0$ bo`lgani uchun $y(x)\equiv 1$ funksiya (2) masalaning noldan farqli yechimi bo`ladi, ya’ni xos $y(x)\equiv 1$ funksiya $\lambda=0$ xos qiymatga mos keluvchi xos funksiya bo`ladi. (1) va (2) masalalarning xos qiymatlari ustma-ust tushganlari uchun $\lambda=0$ son (1) masalaning ham xos qiymati bo`ladi.

Bu xos qiymatga mos keluvchi xos funksiyani $\varphi_0(x)$ orqali belgilaymiz. U holda ushbu

$$\begin{cases} \varphi_0''=q(x)\varphi_0 \\ \varphi_0'(0)=0 \\ \varphi_0'(\pi)=0 \\ \varphi_0(x)\neq 0 \end{cases} \tag{4}$$

tengliklar o`rinli bo`ladi. Agar $\varphi_0(0)=0$ yoki $\varphi_0(\pi)=0$ bo`lsa, chegaraviy shartlardan $\varphi_0(x)\equiv 0$ ziddiyat kelib chiqadi. Demak, $\varphi_0(0)\neq 0$ va $\varphi_0(\pi)\neq 0$ bo`ladi.

$\varphi_0(x)$ funksiya $(0,\pi)$ intervalda biror $x_0\in(0,\pi)$ ildizga ega bo`lsa, ya'ni $\varphi_0(x_0)=0$ bo`lsa, $\varphi_0'(x_0)\neq 0$ bo`ladi,chunki agar $\varphi_0'(x_0)=0$ desak,differensial tenglamalar kursidagi yagonalik teoremasiga ko`ra $\varphi_0(x)\equiv 0$ ziddiyat kelib chiqadi. (4) ga $\varphi_0''(x_0)=0$ bo`ladi. $\varphi_0(x)$ ni x_0 nuqta atrofida Teylor formulasiga yoysak,

$$\varphi_0(x)=\varphi_0(x_0)+\varphi_0'(x_0)\cdot(x-x_0)+\frac{\varphi_0''(x_0)}{2}\cdot(x-x_0)^2+\frac{\varphi_0'''(\varsigma)}{6}\cdot(x-x_0)^3$$

$$\varphi_0(x)=\varphi_0'(x_0)\cdot(x-x_0)+\frac{\varphi_0'''(\varsigma)}{6}\cdot(x-x_0)^3 \tag{5}$$

bo`ladi. $\varphi_0'''(x\ )$ mavjud, chunki $q(x)\in C^1[0,\pi]$ va $\varphi_0''(x)=q(x)\varphi_0(x)$. $\varphi_0''(x\ )$ning differensiallanuvchiligiga asosan

$$\varphi_0''(x)=\varphi_0''(x_0)+\varphi_0'''(x_0)\cdot(x-x_0)+\overline{\overline{O}}(x-x_0),$$

ya'ni

$$\varphi_0''(x)=\varphi_0'''(x_0)\cdot(x-x_0)+\overline{\overline{O}}(x-x_0). \tag{6}$$

(5) va (6) ga ko`ra ushbu

$$\lim_{x\to x_0}\frac{\varphi_0''(x)}{\varphi_0(x)}=\lim_{x\to x_0}\frac{\varphi_0'''(x_0)\cdot(x-x_0)+\overline{\overline{O}}(x-x_0)}{\varphi_0'(x_0)\cdot(x-x_0)+\frac{\varphi_0'''(\varsigma)}{6}\cdot(x-x_0)^3}=$$

$$=\lim_{x\to x_0}\frac{\varphi_0'''(x_0)+\overline{\overline{O}}(1)}{\varphi_0'(x_0)+\frac{\varphi_0'''(\varsigma)}{6}\cdot(x-x_0)^2}=\frac{\varphi_0'''(x_0)}{\varphi_0'(x_0)}. \tag{7}$$

tenglik o`rinli. Demak, agar $x_0$ son $\varphi_0(x)$ ning ildizi bo`lsa, ushbu

$$\lim_{x\to x_0}\frac{\varphi_0''(x)}{\varphi_0(x)}==\frac{\varphi_0'''(x_0)}{\varphi_0'(x_0)}.$$

limit chekli bo`ladi.

$\varphi_0(x)$ funksiya $(0,\pi)$ intervalda cheksiz ildizga ega bo`la bo`lmaydi, agar teskarisini faraz qilsak, Bolsano-Veyershtrass teoremasiga asosan ulardan yaqinlashuvchi $\{x_k\}, x_k\to a, (k\to\infty)$ ketma-ketlik tanlash mumkin. $\varphi_0(x_k)=0,\ k=\overline{1,\infty}$ ekanligidan $\varphi_0(a)=0$ va

$$\varphi_0'(a)=\lim_{x_k\to a}\frac{\varphi_0(x_k)-\varphi_0(a)}{x_k-a}=0.$$

bo`ladi. Yagonalik teoremasiga ko`ra $\varphi_0(x)\equiv 0$ ziddiyat kelib chiqadi.

Demak, $\varphi_0(x)$ funksiya $(0,\pi)$ oraliqda cheklita ildizga ega bo`lishi mumkin ekan. Bu ildizlar karrasiz ekanligi yuqorida ko`rsatildi. Bu ildizlarni

$$x_1, x_2, ..., x_n \in (0,\pi)$$

orqali belgilaymiz. Yuqorida ta'kidlanganikdek, ushbu

$$\lim_{x_k\to a}\frac{\varphi_0''(x\ \ )}{\varphi_0(x\ \ )}, (k=\overline{1,n})$$

limit chekli bo`ladi.

Endi quyidagi integralni ko`rib chiqamiz:

$$J=\int_0^{\pi}\frac{\varphi_0''(x)}{\varphi_0(x)}dx \tag{8}$$

Birinchi tomondan bu yerda, $\varphi_0''(x)=q(x)\varphi_0(x)$ ni qo`ysak va (3) ni inobatga olsak,ushbu

$$J=\int_0^{\pi}\frac{q(x)\varphi_0(x)}{\varphi_0(x)}dx=\int_0^{\pi}q(x)dx=0 \tag{9}$$

tenglik hosil bo`ladi.

Ikkinchi tomondan (8) ni bo`laklab integrallaymiz:

$$J=\int_0^{\pi}\frac{1}{\varphi_0(x)}d(\varphi_0'(x))=\left.\frac{\varphi_0'(x)}{\varphi_0(x)}\right|_0^{\pi}+\int_0^{\pi}\left(\frac{\varphi_0'(x)}{\varphi_0(x)}\right)^2dx.$$

(9) va (4) chegaraviy shartlarni e`tiborga olsak,

$$\int_0^{\pi}\left(\frac{\varphi_0'(x)}{\varphi_0(x)}\right)^2dx=0$$

kelib chiqadi. Bunga ko`ra $\varphi_0'(x)\equiv 0$ bo`ladi ya'ni $\varphi_0(x)\equiv C$,$(C\neq 0)$. Buni

$$-\varphi_0''(x)+q(x)\varphi_0(x)=0$$

tenglikka qo`ysak, $q(x)\equiv 0$ kelib chiqadi. Teorema isbotlandi.

Agar $H(\lambda)=\sqrt{\lambda}$ bo`lsa, ushbu

$$\begin{cases}-y''=\lambda y, & 0\le x\le\pi\\ y'(0)=0\\ y'(\pi)+\sqrt{\lambda}y(\pi)=0\end{cases}$$

masala hosil bo`ladi. Bu masalaning xos qiymatlarini topamiz. Buning uchun $y''+\lambda y=0$ tenglamaning $\varphi(0,\lambda)=1,\varphi'(0,\lambda)=0$ boshlang`ich shartlarni qanoatlantiruvchi yechimini topamiz.

$$\varphi(x,\lambda)=\cos\sqrt{\lambda}x$$

Bu yechimni ikkinchi chegaraviy shartga qo`yib, ushbu

$$-\sqrt{\lambda}\sin\sqrt{\lambda}\pi+\sqrt{\lambda}\cos\sqrt{\lambda}\pi=0$$

xarakteristik tenglamani hosil qilamiz. Bu tenglamani yechamiz:

$$-\sqrt{\lambda}(\sin\sqrt{\lambda}\pi-\cos\sqrt{\lambda}\pi)=0,$$

$$\sqrt{\lambda}(\frac{1}{\sqrt{2}}\sin\sqrt{\lambda}\pi-\frac{1}{\sqrt{2}}\cos\sqrt{\lambda}\pi)=0,$$

$$\sqrt{\lambda}\sin\left(\sqrt{\lambda}\pi-\frac{\pi}{4}\right)=0,$$

$$\lambda_0=0,\quad \sqrt{\lambda}\pi-\frac{\pi}{4}=\pi n,\ n=\overline{0,\infty}.$$

Demak, xos qiymatlar ushbu

$$\lambda_0=0,\quad \lambda_{n+1}=\left(n+\frac{1}{4}\right)^2,\ n=\overline{0,\infty}.$$

tengliklar bilan aniqlanar ekan.

Natija1. Agar (1) va (2)masalalarda $H(\lambda)=\sqrt{\lambda}$ bo`lib,ushbu $\{0\}\cup\left\{\left(m_k+\frac{1}{4}\right)^2\right\}_{k=1}^{\infty}$ to`plamning biror qismi bo`lsa, u holda $q(x)\equiv 0$ bo`ladi. Bu yerda, $\{m_k\}_{k=1}^{\infty}$ ayrim natural sonlardan tuzilgan biror ketma-ketlikni bildiradi.

Xulosa

Shturm_Liuvill masalalari teskari spektral masalalarni yechishda muxim ro`l o`ynaydi. Yagonalik teoremalari ilk bor 1929-yilda V.A.Ambarsumyan tomonidan o`rganila boshlandi.

Ushbu bituruv malakaviy ishda klassik yagonalik teoremalari ya'ni Ambarsumyan teoremalarining chegaraviy shartlarida spektral parametr qatnashgan holda analogi o`rganildi va bu ishda 2011-yilda chop qilingan "Ambarzumyan's theorems with eigenparameter in the boundary conditions " ilmiy maqola atroflicha o`rganib chiqildi.

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