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UCH QATLAMLI STERJENNING KO`NDALANG ZARBA TA’SIRIDA TEBRANISHLARI

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KIRISH

Sterjenlarda tebranish jarayonlarini tadqiq qilish va tebranish qonuniyatlarini o'rganish mexanikaning dollzarb masalalaridan biridir. Qurilish inshoatlarida va texnikada qatlamli sterjenlarning tebranish qonuniyatlarini o'rganish va amaliyotga tadqiq etish shu dolzarb masalalar sirasiga kiradi.

Uch qatlamli sterjenni ko`ndalang statik yuk ta'sirida egilishi va xususiy tebranishini o'rganish haqidagi masalalar muhim ahamiyatga ega. Shu nuqtai nazardan qaraganda dissertatsiya mavzusi doirasidagi tadqiq etilishi lozim bo'lgan uch qatlamli sterjenning ko`ndalang statik yuk ta'siri natijasida egilishi va xususiy tebranishlari haqidagi masalalar ham deformatsiyalanuvchi qattiq jism mexanikasining aktual masalasidir.

Shuni ham ta'kidlash lozimki uch qatlamli sterjenlarning zarba ta'sirida tebranishlari haqidagi masalalar, odatda juda sodda hollar uchun yechilgan. Bunday holatlar jismning elastik modeli bilan bir qatorda, deformatsiyalanuvchi qattiq jismning boshqa, mukammalroq modellarini yaratishga, muhandislik qurilmalari hisobida ishlab chiqilganiga ancha bo'lgan bo'lsada shu vaqtgacha foydalanilmagan usullardan, xususan plastiklik, qovushoq–elastiklik nazariyalari usulidan foydalanishga olib kelmoqda.

Dissertatsiya ishida tadqiqot predmeti. Uch qatlamli sterjenning ko`ndalang statik yuk ta'sirida egilishi va xususiy tebranishlarini tadqiq etish, sterjen sirtida va ko'ndalang kesmida vujudga keladigan eguvchi moment, qirquvchi kuchlarni aniqlash va olingan natijalar asosida ilmiy xulosalar keltirib chiqarishdan iborat.

Dissertatsiya ishida tadqiqot ob'ekti sifatida ikki uchi erkin sterjen olingan. Bunda, elastik materialdan yasalgan sterjenlar kesimlaridagi zo'riqishlarni aniqlash belgilangan.

Mavzuning dolzarbligi. Qatlamli sterjenlar juda ko'p muhandislik qurilmalarining tarkibiy qismini tashkil etadilar. Bundan tashqari bunday sterjenlar hozirgi zamon fani va texnikasida, qurilish konstruksiyalarining elementlaridir. Shunday holda bu sterjenlar turli xil dinamik tashqi ta’sirlar ostida ishlaydilar va ularning kesimlarida turli xil yuklanishlar vujudga keladi.

Dinamik yuklar ta’siri ostida vujudga keladigan zo’riqishlar holatlarini analitik usullar bilan aniqlash doim ham mumkin bo’lmaydi. Bunday holda masalani yechish uchun sonli usullar metodlariga murojat qilishga to’g’ri keladi. Shu nuqtai nazardan qaraganda dissertatsiya ishida qo’yilgan masala zamonaviy dolzarb masalalar qatoriga kiradi.

Ishning maqsad va vazifalari. Magistrlik dissertatsiya ishining asosiy maqsadi uzunligi bo’yicha o’zgarmas ko`ndalang zarba ta’siridagi uch qatlamli sterjenning xususiy tebranishlarini tadqiq qilish, bunda tadqiqotni aniqlashtirilgan tebranish tenglamalari asosida analitik - sonli usullar bilan yechishdan iborat. Dissertatsiya ishining asosiy maqsadidan kelib chiqib quyidagi asosiy vazifalari belgilangan:

1. Uch qatlamli sterjenning xususiy tebranishlari bo`yicha Grigolyuk-Chulkov nazariyasini o`rganish, ko`ndalang statik yuk ta’siridagi sterjenni tadqiq qilish.

2. Uch qatlamli sterjennig tebranishlari va statik yuk ta’sirida egilish tenglamalarini aniqlash**;**

3. Amaliy masalalar yechish uchun differensial tenglamalarni yechish Bubnov – Galerkin metodini o’rganish;

4. Olingan natijalar asosida ilmiy xulosalar chiqarish.

Muammoning ishlab chiqilish darajasi. Uch qatlamli sterjenning xususiy tebranishini tadqiq etish masalasi bilan tadqiqotchilar ancha vaqtlardan buyon [12,14] shug’ullanadilar. Ammo, fan va texnikaning hozirgi zamon taraqqiyot darajasi moddalarning yangidan yangi xususiyatlarini, jumladan kompozit, qatlamli, realogiya va boshqa xususiyatlarini hisobga olgan holda tadqiqotlar o’tkazishni talab etmoqda [15].

Tadqiqotning ilmiy yangiligi. Uch qatlamli sterjenning xususiy tebranishilar konkret amaliy masalalar yechish bilan tadqiq qilingan. Chunki sterjenlarning xususiy tebranishi haqidagi masalalarni tadqiq etish muhim ahamiyatga ega. Bu esa dissertatsiya ishida qaralgan va yechilishi uchun sonli usullar tadbiq etilgan masalalarning ilmiy ahamiyati ularning o'xshash amaliy masalalarni hal etishda foydalanish mumkinligidadir.

Tadqiqotning amaliy ahamiyati. Zamonaviy texnika, qurilish va boshqa sohalarda uch qatlamli sterjenlardan konstruksiyalarning asosiy elementlari sifatida foydalaniladi.

Shuning uchun ham uch qatlamli sterjenning ko`ndalang statik yuk tasiridagi egilishi va xususiy tebranishlarini tadqiq qilish, ularning tebranish chastotasi, amplitudasi, shakli va boshqa xarakteristikalarini aniqlash muhim amaliy ahamiyatga ega.

Dissertatsiya ishining tuzilishi. Magistrlik dissertatsiya ishi kirish, uchta bob, xulosa va takliflar hamda foydalanilgan adabiyotlar ro'yxatidan iborat bo'lib 71 kompyuter sahifasida bayon qilingan. Ana shu hajm doirasiga 24 ta rasm ham kiradi.

Dissertatsiya asosiy bo'limlarining qisqacha mazmuni. Dissertatsiya ishining kirish qismida ishning tadqiqot ob'yekti keltirilgan. Uch qatlamli sterjenlarning ko`ndalang statik yuk ta'sirda egilishi va xususiy tebranishlarini yangi tenglamalar asosida tadqiq qilish zamonaviy mexanikaning dolzarb mavzularidan ekanligi keltirilgan. Dissertatsiya ishining maqsadi va vazifalari belgilab berilgan.

Dissertatsiya ishining birinchi bobi "Uch qatlamli konstruktiv elment uchun asosiy tenglamalar va bog`lanishlar " deb ataladi va o'z ichiga uchta paragrafni olgan. Birinchi paragrafda uch qatlamli konstruktiv elementlar tebranishlari bo`yicha tatqiqotlar. Bunda asosiy e'tibor qatlamli sterjenlarga qaratilgan.

Ikkinchi paragrafda elastiklik nazariyasining geometrik, dinamik va fizik tenglamalari. asosiy masalalari bayon etilsin. Buning uchun avvalo chiziqli elastik muhit deformatsiyasiga oid munosabatlar qarab chiqilgan. Bunda tutash muhitlar kichik deformatsiyalari nazariyasiga tayanilgan.

Uchinchi paragrafda uch qatlamli sterjennig tebranishlari va statik yuk ta’sirida egilish tenglamalari keltirilgan. Bunda, har bir amaliy masalalarning qo’yilishida alohida – alohida e’tibor berilgan.

Dissertatsiya ishining ikkinchi bobi “Ko`ndalang statik yuk ta’siridagi uch qatlamli sterjenni tadqiq etish**”** deb nomlangan. Ushbu bobning birinchi paragrafida uchlari diafragmali va erkin tayangan uch qatlamli sterjenning egilishi haqida masala yechilgan. Ushbu paragraf doirasida uch qtlamli sterjenning egilish tenglamasini yechimlari keltirib chiqarilgan.

Bobning ikkinchi paragrafi bir uchi qistirib mahkamlangan, ikkinchi uchi erkin uch qatlamli sterjenning statik yuk ta’sirida egilishi deb nomlangan. Bunda yukning harakati davomida sterjenlarda yuzaga keladigan zo’riqishlarni aniqlash masalasi qaralgan. Natijalar asosida xulosalar chiqarilgan.

Bobning uchinchi paragrafida bir uchi qistirib mahkamlangan, ikkinchi uchi erkin lekin cheksiz katta bikrlikka ega diafragmali uch qatlamli sterjenning statik yuk ta’sirida egilishi haqidagi masala yechilgan. Bunda Grigolyuk – Chulkov tenglamasiga Bubnov- Galerkin metodini qo’llab yechish va tebranish shaklini hisoblashni sonli usullar yordamida aniqlash formulasi keltirilgan. Bu tenglamalar orqali sterjen uchlarida tebranish shakllarini aniqlash formulasi ham keltirilgan.

Dissertastiyaning uchunchi bobi “Uch qatlamli sterjenning xususiy tebranishlari” deb nomalanadi. Bu bob ham to`rta paragrafni o’z ichiga oladi. Birinchi paragrafida uch qatlamli sterjenning xususiy tebranish tenglamasi va tebranish chastotasi ketirib chiqarilgan. Bu yerda olingan natijalar asosida qirquvchi kuchlar, eguvchi momentlar, egilish holatlari grafiklari sifatida taqdim etilgan. Grafiklar sterjenning uchidan har xil uzoqlikdagi kesimlari nuqtalari uchun qurilgan. Grafiklar va boshqa sonli natijalarni tahlil qilish asosida bir nechta ilmiy xulosalar chiqarilgan. Uchinchi bobning ikkinchi paragrafi uchlari diafgramali va erkin tayangan uch qatlamli sterjenning xususiy tebranishlari, uchinchi paragrafi bir uchi qistirib mahkamlangan, ikkinchi uchi erkin uch qatlamli sterjenning xususiy tebranishi masalasi qaralgan. Ushbu boblar doirasida bir uchi erkin va ikkinchi uchi qistirib mahkamlangan sterjenlarni xususiy tebranish tenglamalari o`rganilgan. To`rtinchi paragrafi bir uchi qistirib mahkamlangan, ikkinchi erkin uchida cheksiz katta diafragma bor uch qatlamli sterjenning xususiy tebranishi deb nomlanadi.

I-BOB

UCH QATLAMLI KONSTRUKTIV ELMENTLAR UCHUN ASOSIY TENGLAMALAR VA BOG`LANISHLAR:

§ 1.1. Uch qatlamli konstruktiv elementlar tebranishlari bo`yicha tatqiqotlar

Qatlamli strukturaga ega bo`lgan sterjenlar, plastinalar va qobiqlar odatda fizik mexanik xususiyatlari turlicha bo`lgan moddalar yig`iladi. Asosiy yuk tashuvchi qatlamlar yuqori mustahkamlikga va katta bikrlikga ega materiallardan yasaladi va mexanik yuklarning asosiy qismini qabul qilishga mo`ljallangan. Monolit konstruksiyani tashkil etish uchun xizmat qiladigan bog`lovchi qatlamlar yuk tashuvchi qatlamlar orasidagi zo`riqishlarni qayta taqsimlashni taminlaydi. Qatlamlarning yana bir guruhi issiqlik, ximik, radiatsion va boshqa noxush ta'sirlardan himoya qilish uchun mo`ljallangan. Qatlamlarning bunday majmuasi qaralayotgan sistemalarning ishini, o`rab turuvchi muhitning yomon sharoitlarda ham ta'minlashga imkon beradi, nisbatan katta mustahkamlikni va bikrlikni mujasamlashtirgan massasi kichik bo`lgan konstruksiyani yaratishga imkon beradi.

Keyingi yillarda ikkita yuk tashuvchi va ularning birgalikda ishini ta'minlovchi to`ldiruvchi qatlamlardan iborat uch qatlamli konstruksiyalar juda keng tarqaldi. Uch qatlamli konstruksiyalar egilish deformatsiyasi sharoitlarda juda ratsionaldirlar. Bunda og`irlik ko`rsatkichlarning minimumini ta'minlash nuqtai nazaridan, mustahkamlik va bikrlik berilgan chegaralarda bunday konstruksiyalar juda optimal bo`ladilar. Ko`p qatlamli konstruksiyalar nazariyasini plastinalar va qobiqlar klassik nazariyasini uch qatlamli konstruksiyalar nazariyasiga umumlashtirish natijasi sifatida qarash mumkin. Ko`p hollarda konstruksiyalarning ko`p qatlamli elementlarni yupqa hisoblash noto`gri bo`ladi, chunki bu holda klassik nazariyasining asosiy gipotezalari ishlamaydi. Qatlamlar sonini oshirganda va turli to`ldiruvchilar qo`llanilganda alohida qatlamlarning ishi bilan bog`liq bo`lgan effketlar juda muhim rol o`ynay boshlaydilar. Ko`ndalang siljish va normallarni siqilishdan tashqari ko`p qatlamli konstruksiyalar ishida yuk tashuvchi qatlamlarda momentlar effketlarni, ustuvorlikni yo`qotishning mahalliy shakllarini va boshqalarni hisobga olishga to`g`ri keladi. Konstruksiyalarning uch qatlamli elementlari, shu jumladan uch qatlamli elementlarning, nazariyasi XX-asrning qirqinchi yillaridan boshlab intensiv ravishda ishlab chiqila boshladi. Uning rivojlanishiga juda ko`p olimlar katta hissa qo`shdilar.

Ularning qatorida o`zbek olimlaridan akademik T.Sh.Shirinqulov, T.R.Rashidov, M.T.O`rozboyev, X.A.Raxmatulin, K.Sh. Bobomurodov va boshqalarni ko`rsatish mumkin.

V.V.Bolotin va Yu.N.Navichkovlarning [1,2] ishlarida ko`p qatlamli konstruktiv elementlarning bikr qatlamlari elastik deformatsiyalanadi, to`ldiruvchi qatlamda esa plastik deformatsiyalar vujudga keladi deb hisoblanadi, yumshoq qatlamning qalanligi bo`yicha ko`ndalang siljish urinma kuchlanishlari tekis taqsimlangan deb faraz qilinadi. Bu bitta bikr qatlamning ikkinchi bikr qatlamga nisbatan sirpanishni hisobga olishga imkon beradi. Agar to`ldiruvchi uchun ideal elasta plastik jism modeli qabul qilgan bo`lsa, sirpanish yumshoq qatlamning butun kesim bo`yicha bir vaqtda boshlanadi deb faraz qilinadi. Shu yerning o`ziga kompozent materiallarning mexanik xususiyatlarni tavsiflash uchun kichik qatlamli muhit modeli taklif qilinadi. Kichik qatlamli muhitlar holatlarining fizik tenglamalari tarkibga chiziqli qavushoq elastik operatoplar kiritiladi.

E.I.Grigolyuk va P.P.Chulkovlarning [3] monografiyasi uch qatlamli konstruksiyalarni hisoblash metodlarini bayon qilishga baxishlangan. Qobiqlar nazariyasi oldidan uch qatlamli to`g`ri sterjenlar nazariyasi keltirilgan. Bu yerda hozirgi vaqtda bir jinsli sterjenlar uchun ishlab chiqilgan asosiy masalalar tahlil qilingan. Qavariq qobiqlar klassik nazariyasining umumlashmasi bo`lgan chekli yechilishi qavariq qobiqlarning aftirlar tamonidan yaratilgan nazariyasi bayon qilishgan. Bu nazariyaning tenglamalari silindirik, sferik, konus shaklidagi va tor shaklidagi qobiqlarning har xil tashqi ta’sirlar uchun kritik yuklarni va xususiy tebranish chastotalariga ishlatilgan. Yarim momentsiz deb atalgan uch qatlamli silindirik qobiqlar nazariyasi rivojlantirilgan, chekli egilishli qavariqmas qobiqlar uchun hisoblash nazariyasi keltirilgan. Cheli egilishli qavariq uch qatlamli qobiqlar uchun qurilgan nazariya bir jinsli qobiqlar uchun tabiiy chegaroviy shartlarni qo`yishga imkon beradi.

[4.5] ishlarda uch qatlamli plastinkalar nazariyasining yangi varianti taklif etilgan. Bu yerda nazariya siniq narmal gipotezasiga asoslangan hamda to`ldiruvchi qatlam ko`ndalang yo`nalishda siqilmaydi.

§ 1.2. Elastiklik nazariyasining geometrik, dinamik va fizik tenglamalari. Asosiy masalalar.

O‘tgan boblarda elastiklik nazariyasining asosiy tenglamalari chiqarildi. Bu tenglamalar yopiq sistemani tashkil etadi va jismga ta’sir etgan tashqi kuchlarni hisobga olgan holda uning kuchlangan-deformat-sialangan holatini aniqlashga imkon beradi. Bu tenglamalarni uch turga bo‘ladilar: *geometrik, statik (dinamik) va fizik tenglamalar*.

1^0. Geometrik tenglamalar.

Elastik jismning deformatsialangan holati deformatsia tenzori $\varepsilon_{ij} = \varepsilon_{ij}(x_k)$ komponentalari, yoki $u_i = u_i(x_k)$ ko‘chishlar bilan to‘lqin aniqlanadi. Deformatsia tenzori komponentalari va ko‘chishlar o‘zaro Koshining differensial munosabatlari bilan bog‘langan:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad (1.2.1)$$

hamda Gen-Venanning differensial munosabatlari bilan o‘zaro bog‘langan:

$$\varepsilon_{ik,j\ell} + \varepsilon_{j\ell,ik} - \varepsilon_{i\ell,jk} - \varepsilon_{jk,i\ell} = 0. \qquad (1.2.2)$$

Ma’lumki, bu munosabatlar deformatsialarning uzviylik tenglamasi deb ham yuritiladi.

2^0. Dinamik (Statik) tenglamalar.

Elastik jismning kuchlanganlik holati kuchlanish tenzorining $\sigma_{ij} = \sigma_{ij}(x_k)$

oltita komponentasi bilan to‘liq aniqlanadi. Ushbu komponentalar *uchta muvozanat* differensial tenglamalarini qanoatlantirishlari kerak:

$$\sigma_{ij,j} + \rho f_i = 0. \tag{1.2.3}$$

Agar jism harakatda bo‘lsa, simmetrik kuchlanish tenzorining olti komponentasi *uchta harakat* differensial tenglamalarini qanoatlantirishlari kerak:

$$\sigma_{ij,j} + \rho f_i = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{1.2.4}$$

Ushbu (1.2.3) - tenglamalar *statik,* (1.2.4) - tenglamalar *dinamik* tenglamalar deb yuritiladi.

3⁰. Fizik tenglamalar.

Kuchlanish tenzorining σ_{ij} komponentalari deformatsiya tenzorining ε_{ij} komponentalari bilan Guk qonuni vositasida bog‘langan

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} \tag{1.2.5}$$

yoki u_i ko‘chish komponentalari bilan

$$\sigma_{ij} = \lambda\theta\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \tag{1.2.6}$$

ko‘rinishda bog‘langan. Bu yerda

$$\theta = \varepsilon_{ii} = u_{i,i} = div\vec{u}.$$

Ba’zi hollarda Guk qonunini (1.2.5) ga teskari shaklda, ya’ni ε_{ij} larga nisbatan yechilgan ko‘rinishda ishlatishga to‘g‘ri kelishi mumkin:

$$\varepsilon_{ij} = \frac{1}{E}\left[(1+\nu)\sigma_{ij} - \nu\delta_{ij}\sum\right], \tag{1.2.7}$$

bu yerda:

$$\sum = \sigma_{ii} = \sigma_{11} + \sigma_{22} + \sigma_{33}.$$

Yuqorida sanab o‘tilgan (1.2.1) (1.2.3), (1.2.5), (1.2.7) formulalar elastiklik nazariyasi *statik masalalarining asosiy tenglamalari deb* yuritiladi. Elastiklik nazariyasi *dinamik masalalarining asosiy tenglamalari deb* (1.2.1) - (1.2.2), (1.2.4) (1.2.5) va (1.2.7) tenglamalarga aytiladi.

Asosiy masalalar. Chiziqli-elastik jismning holatini uning V hajmining ichki nuqtalarida aniqlovchi asosiy tenglamalarga uning S sirtidagi shartlarni qo‘shish kerak. Bu shartlar *chegaraviy shartlar* deyiladi va ular tashqi berilgan F_i sirt kuchlari bilan yoki jism sirti nuqtalarining berilgan $u_i|_S$ ko‘chishlari bilan aniqlanadi.

Chegaraviy shartlarning berilishiga qarab elastiklik nazariyasining uch asosiy masalalarini bir-biridan farqlaydilar.

1⁰. Birinchi tur asosiy masala.

Birinchi tur asosiy masalada f_i massaviy va F_i sirt kuchlari jismning butun sirtida berilganda jism egallagan V hajmning ichki nuqtalarida kuchlanish tenzori komponentalari $\sigma_{ij}(x_k)$ larni hamda V - hajmning ichki nuqtalari va jism S sirti nuqtalarida ko‘chish vektorining $u_i(x_k)$ komponentalarini aniqlash talab etiladi. Demak, bu holda chegaraviy shartlar:

$$\sigma_{ij} n_j|_S = F_i \qquad (1.2.8)$$

ko‘rinishda bo‘ladi. Bu yerda F_i - sirt kuchi $\vec{F}$ ning komponentalari; $n_j - S$ sirtning qaralayotgan nuqtasida tashqi normali bo‘yicha yo‘nalgan birlik $\vec{n}$ vektori komponentalari.

Bu holda izlanayotgan to‘qqiz noma’lumlar (oltita σ_{ij} kuchlanishlar va uchta u_i ko‘chishlar) (1.2.3) yoki (1.2.4), (1.2.5) tenglamalarni, hamda (1.2.8) chegaraviy shartlarni qanoatlantirishlari kerak.

2⁰. Ikkinchi tur asosiy masala.

Ikkinchi tur asosiy masalada f_i massaviy kuchlar va jismning S sirtida $u_i(x_k)|_S$ ko‘chishlar ma’lum bo‘lganda, jism egallangan V hajm ichidagi nuqtalarda $u_i(x_k)$ ko‘chishlarni va kuchlanish tenzori komponentalari $\sigma_{ij}(x_k)$ larni aniqlash talab etiladi. Demak, bu holda chegaraviy shartlar

$$u_i = u_i|_S \qquad (1.2.9)$$

ko‘rinishida bo‘ladi. Izlanuvchi $u_i(x_k)$ va $\sigma_{ij}(x_k)$ funksiyalar (1.2.3) yoki (1.2.4), (1.2.5) hamda (1.2.9) chegaraviy shartlarni qanoatlantirishlari kerak.

3^0. Uchinchi tur asosiy masala.

Chegaraviy shartlar aralash xarakterga ega bo‘lishlari mumkin. Birinchi tur asosiy masalada jismning butun sirtida kuchlanishlar, ikkinchi tur asosiy masalada jismning butun sirtida ko‘chishlar beriladi. Shunday masalalar ham uchrashi mumkinki. Bunda jism sirtining ma’lum qismida kuchlanishlar, qolgan qismida esa ko‘chishlar berilishi mumkin. bunday holda *masala aralash masala* deyiladi. Faraz qilaylik, jism S sirtining S_σ qismida kuchlanishlar, S_u qismida esa ko‘chishlar berilgan bo‘lsin. Tabiiyki,

$$S = S_\sigma + S_u \,.$$

Uchinchi tur asosiy masalada jism sirtining S_σ qismida berilgan tashqi sirt kuchlari - $F_{i,}$ va qolgan S_u qismida berilgan $u_i(x_k)\big|_S$ ko‘chishlar, hamda umumiy holda, berilgan f_i massaviy kuchlar bo‘yicha jism egallagan V sohaning ichki nuqtalarida $u_i(x_j)$ ko‘chishlarni hamda $\sigma_{ij}(x_j)$ kuchlanishlarni aniqlash talab etiladi.

Izlanayotgan to‘qqiz noma’lum funksiyalar bu holda (1.2.3) yoki (1.2.4), (1.2.5) tenglamalarni hamda

$$\begin{aligned} \sigma_{ij} n_j \big|_{S_\sigma} &= F_i \\ u_i \big|_{S_u} &= u_i \end{aligned} \qquad (1.2.10)$$

chegaraviy shartlarni qanoatlantirishlari kerak.

Yuqoridagi uch asosiy masaladan tashqari, elastiklik nazariyasining *to‘g‘ri va teskari masalalarini* ham farqlaydilar.

Elastiklik nazariyasining to‘g‘ri masalasida yuqorida keltirilgan uch asosiy masaladan birini tashqi kuchlar berilgan holda yechish, ya’ni jismning kuchlangan - deformatsialangan holatini aniqlovchi $u_i(x_k)$ va $\sigma_{ij}(x_k)$ funksialarni jism egallagan V sohaning ichki nuqtalari uchun aniqlash talab etiladi. Ammo ta’kidlash lozimki, elastiklik nazariyasining to‘g‘ri masalasini yechish juda katta matematik qiyinchiliklarga olib keladi.

Elastiklik nazariyasining teskari masalasida $u_i = u_i(x_k)$ ko‘chishlar yoki $\sigma_{ij} = \sigma_{ij}(x_k)$ kuchlanishlar uzluksiz funksiyalar sifatida beriladi. Asosiy (1.2.1) (1.2.2), (1.2.3) yoki (1.2.4) hamda (1.2.5) tenglamalardan qolgan funksiyalar va berilgan u_i ko‘chishlarni yoki kuchlanishlarni yuzaga keltiruvchi tashqi kuchlarni aniqlash talab etiladi.

Teskari masalani yechish to‘g‘ri masalani yechishga nisbatan ancha oson kechadi. Agar bunda ko‘chishlar berilgan bo‘lsa masala nisbatan juda oson yechiladi. σ_{ij} kuchlanishlar berilgan holda u_i ko‘chishlarni aniqlash uchun (1.2.1) tenglamalarni integrallashga to‘g‘ri keladi va σ_{ij} kuchlanishlarni uzviylik tenglamalari qanoatlanadigan qilib berishga to‘g‘ri keladi. Lekin baribir bunday masalani yechish to‘g‘ri masalani yechishga nisbatan oson.

Кўчишлардаги tenglamalar. Elastiklik nazariyasining to‘g‘ri masalasini, asosiy o‘zgarmaslar sifatida birinchi navbatda, yoki $u_i(x_k)$ ko‘chishlarni yoki $\sigma_{ij}(x_k)$ larni qabul qilib yechish qulay. To‘g‘ri masalani yechishning ana shu ikki yo‘li *ko‘chishlarga nisbatan yechim* yoki *kuchlanishlarga nisbatan yechim* deyiladi. Bunday holatlarda asosiy tenglamalar ham ko‘chishlarga nisbatan yoki kuchla-nishlarga nisbatan yozilishlari kerak.

Quyida biz asosiy tenglamani (muvozanat tenglamalarini) ko‘chishlarga nisbatan keltirib chiqaramiz. Buning uchun (1.2.6) Guk qonuni yordamida (1.2.3) kuchlanishlarga nisbatan muvozanat tenglamalaridan kuchlanish tenzorining $\sigma_{ij}(x_k)$ komponentalarini chiqarib tashlash zarur bo‘ladi.

Guk qonunining (1.2.6) ifodasidan x_j koordinata bo‘yicha hosila olamiz:

$$\sigma_{ij,j} = \lambda\delta_{ij}\theta_{,j} + \mu(u_{i,jj} + u_{j,ij}), \tag{1.2.11}$$

lekin

$$\theta_{,j}\cdot\delta ij=\theta_{,i};\quad u_{j,ij}=u_{j,ji}\left(\frac{\partial^2 u_j}{\partial x_i\partial x_j}=\frac{\partial^2 u_j}{\partial x_j\partial x_i}\right) \tag{1.2.12}$$

$$u_{j,j}=\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}=div\vec{u}=\theta$$

hamda

$$u_{i,jj}=\frac{\partial^2 u_i}{\partial x_1^2}+\frac{\partial^2 u_i}{\partial x_2^2}+\frac{\partial^2 u_i}{\partial x_3^2}=\nabla^2 u_i \tag{1.2.13}$$

bu yerda ∇^2 orqali Laplas operatori belgilangan. Endi (1.2.12) va (1.2.13) ifodalardan foydalanib (1.2.11) - Guk qonunini quyidagicha yozish mumkin:

$$\sigma_{ij,j}=\lambda\delta_{ij}\theta_{,j}+\mu(\nabla^2 u_i+\theta_{,i})$$

yoki

$$\sigma_{ij,j}=\mu\nabla^2 u_i+(\lambda+\mu)\theta_{,i}. \tag{1.2.14}$$

Lekin

$$\lambda+\mu=\frac{Ev}{(1+v)(1-2v)}+\frac{E}{2(1+v)}=\frac{E}{2(1+v)(1-2v)}=\frac{\mu}{1-2v},$$

$$\sigma_{ij,j}=\mu\nabla^2 u_i+\frac{\mu}{1-2v}\theta_{,i}.$$

yoki nihoyat:

$$\sigma_{ij,j}=\mu\left(\nabla^2 u_i+\frac{1}{1-2v}\theta_{,i}\right) \tag{1.2.15}$$

olingan (1.2.15) ifodani (1.2.4) muvozanat tenglamalariga qo‘ysak,

$$\nabla^2 u_i+\frac{1}{1-2v}\theta_{,i}=-\frac{\rho}{\mu}f_i \tag{1.2.16}$$

elastik muvozanatning ko‘chishlarga nisbatan tenglamalariga ega bo‘lamiz. Bu tenglamalar uchta differensial tenglamalar sistemasini aniqlaydi:

$$\begin{aligned}
&\nabla^2 u_1+\frac{1}{1-2v}\frac{\partial\theta}{\partial x_1}=-\frac{\rho}{\mu}f_1;\\
&\nabla^2 u_2+\frac{1}{1-2v}\frac{\partial\theta}{\partial x_2}=-\frac{\rho}{\mu}f_2;\\
&\nabla^2 u_3+\frac{1}{1-2v}\frac{\partial\theta}{\partial x_3}=-\frac{\rho}{\mu}f_3.
\end{aligned} \tag{1.2.17}$$

olingan (1.2.16) yoki undan kelib chiquvchi (1.2.17) tenglamalar *Lame tenglamalari deyiladi.*

Lame tenglamalarini bitta vektor tenglama ko‘rinishida ishlatish ancha qulay. Buning uchun (1.2.16) tenglamani $\vec{э}_i$ bazis vektoriga ko‘paytirish yetarli.

$$\nabla^2 u_i \cdot \vec{э}_i + \frac{1}{1-2\nu}\theta_{,i}\vec{э}_i = -\frac{\rho}{\mu} f_i \vec{э}_i,$$

lekin

$$\theta_{,i}\vec{э} = \frac{\partial\theta}{\partial x_1}\vec{э}_1 + \frac{\partial\theta}{\partial x_2}\vec{э}_2 + \frac{\partial\theta}{\partial x_3}\vec{э}_3 = grad\theta,$$

hamda

$$\theta = div\vec{u}, \qquad \vec{u}_i \vec{э}_i = \vec{u}$$

bo‘lganliklari uchun

$$\nabla^2\vec{u} + \frac{1}{1-2\nu} grad\, div\vec{u} = -\frac{\rho}{\mu}\vec{f}_i \qquad (1.2.18)$$

vektor tenglamaga ega bo‘lamiz. Ushbu tenglamani

$$\nabla^2\vec{u} = grad\, div\vec{u} - rot\, rot\vec{u}$$

ekanligini hisobga olib

$$\frac{2(1-\nu)}{1-2\nu} grad\, div\,\vec{u} - rot\, rot\,\vec{u} = -\frac{\rho}{\mu}\vec{f} \qquad (1.2.19)$$

kabi ba‘zida ishlatish qulay bo‘lgan shaklda yozish mumkin. Ko‘p masalalarda massaviy kuchlarni hisobga olmaslik yoki nolga teng deb hisoblash mumkin. Bu holda Lamening (1.2.16) tenglamalari

$$\nabla^2 u_i + \frac{1}{1-2\nu}\theta_{,i} = 0 \qquad (1.2.20)$$

ko‘rinishni oladi. Bu tenglamani x_i koordinata bo‘yicha differensiallab,

$$\nabla^2 u_{i,i} + \frac{1}{1-2\nu}\theta_{,ii} = 0$$

tenglamaga ega bo‘lamiz. Lekin

$$u_{i,i} = \theta \text{ va } \theta_{,ii} = \nabla^2\theta$$

bo‘lganliklari uchun bu tenglamadan

$$\nabla^2\theta + \frac{1}{1-2\nu}\nabla^2\theta = \frac{2(1-\nu)}{1-2\nu}\nabla^2\theta = 0$$

yoki

$$\nabla^2\theta = 0 \qquad (1.2.21)$$

ifodani olamiz. Bu ifoda massaviy kuchlar nolga teng yoki o‘zgarmas bo‘lganlarida θ - hajmiy deformatsia Laplas tenglamasini qanoatlantirishini, va demak, garmonik funksiya ekanligini ko‘rsatadi.

Endi (1.2.20) ga ∇^2 - Laplas operatori bilan ta’sir etamiz

$$\nabla^2\nabla^2 u_i - \frac{1}{1-2\nu}\nabla^2\theta_{,i} = 0,$$

lekin

$$\nabla^2\theta_{,i} = (\nabla^2\theta)_{,i}$$

bo‘lganligidan

$$\nabla^2\nabla^2 u_i = 0, \qquad (1.2.22)$$

ya’ni ko‘chish vektorining u_i komponentalari bigarmonik funksiyasi ekan. Lekin bu narsa u_i ko‘chishlar ixtiyoriy bigarmonik funksiya ekanligini anglatmaydi, chunki ular avvalo tartibi past bo‘lgan Lame tenglamalarini ham qanoatlantirishlari kerak.

Agar masala ko‘chishlarga nisbatan yechilayotgan bo‘lsa, chegaraviy shartlar ham ko‘chishlar orqali ifodalanishi kerak. Ikkinchi tur asosiy masalada bu sohada muammo yo‘q. Ammo birinchi tur masalada (1.2.8) chegaraviy shartlarni ko‘chishlar orqali yozish kerak.

Bu ishni uddalash uchun yana (1.2.6) Guk qonunidan foydalanamiz. Uning ikkala tomonini ham n_j ga ko‘paytiramiz:

$$\sigma_{ij}n_j = \lambda\delta_{ij}n_j\theta + \mu(u_{i,j}n_j + u_{j,i}n_j)$$

lekin

$$n_j\delta_{ij} = n_i;$$

$$u_{i,j}n_j = \frac{\partial u_i}{\partial x_1}n_1 + \frac{\partial u_i}{\partial x_2}n_2 + \frac{\partial u_i}{\partial x_3}n_3 = \frac{\partial u_i}{\partial n},$$

ya'ni $u_{i,j}n_j$ ko'paytma $u_i(x_k)$ funksiyadan jism sirtining shu nuqtasidagi normali bo'yicha hosilasidan iborat bo'lganligi uchun yuqoridagi tenglik:

$$\sigma_{ij}n_j = \lambda\,\theta\,n_i + \mu\left(\frac{\partial u_i}{\partial n} + u_{j,i}n_j\right)$$

ko'rinishni oladi. U holda (1.2.8) chegaraviy shartlarni quyidagicha yozish mumkin:

$$\left[\lambda\theta\,n_i + \mu\left(\frac{\partial u_i}{\partial n} + \frac{\partial u_j}{\partial x_i}n_j\right)\right]_S = F_i \qquad (1.2.23)$$

Shunday qilib (1.2.16) Lame tenglamalari, birinchi tur asosiy masalada (1.2.23) chegaraviy shartlar bilan va ikkinchi tur asosiy masalada (1.2.9) chegaraviy shartlar bilan, ko'chish vektorining hamma uchta u_i komponentalarini to'liq aniqlaydi. Ko'chish komponentalari u_i lar ma'lum bo'lgach, (5.1) formulalar bilan deformatsiya tenzorining ε_{ij} komponentalari va (1.2.6) Guk qonuni formulalari bilan kuchlanish tenzorining hamma σ_{ij} komponentalari hisoblanadi. Boshqacha aytganda, jism istalgan nuqtasining kuchlangan-deformatsiyalangan holati to'liq aniqlanadi.

Elastiklik nazariyasining uchinchi asosiy masalasi ko'chishlarga nisbatan yechilayotgan bo'lsa, (5.10) chegaraviy shartlarni

$$\left[\lambda\theta n_i + \mu\left(\frac{\partial u_i}{\partial n} + \frac{\partial u_j}{\partial x_i}n_j\right)\right]_{S_\sigma} = F_i \qquad (1.2.24)$$

$$u_i\Big|_{S_u} = u_i$$

ko'rinishda foydalanish zarur bo'ladi. Agar jism muvozanatda emas balki harakatda bo'lsa, uning harakat tenglamalari (1.2.4) ko'rinishda bo'ladi. Ko'rinib turibdiki, ushbu harakat tenglamalari ko'chishlarga nisbatan quyidagi uch shakldа yozilishi mumkin:

$$\nabla^2 u_i + \frac{1}{1-2\nu}\theta_{,i} = -\frac{\rho}{\mu}\left(f_i - \frac{\partial^2 \vec{u}_i}{\partial t^2}\right);$$
$$\nabla^2 \vec{u} + \frac{1}{1-2\nu} grad\, div\, \vec{u} = -\frac{\rho}{\mu}\left(\vec{f} - \frac{\partial^2 \vec{u}_i}{\partial t^2}\right) \qquad (1.2.25)$$
$$\frac{2(1-\nu)}{1-2\nu} grad\, div\, \vec{u} - rot\, rot\, \vec{u} = -\frac{\rho}{\mu}\left(\vec{f} - \frac{\partial^2 \vec{u}_i}{\partial t^2}\right).$$

Keltirilgan ushbu tenglamalarda, tabiiyki, u_i ko‘chishlar x_k koordinatalardan tashqari, t vaqtning ham funksiyalari bo‘lishlari, ya‘ni

$$u_i = u_i(x_k, t)$$

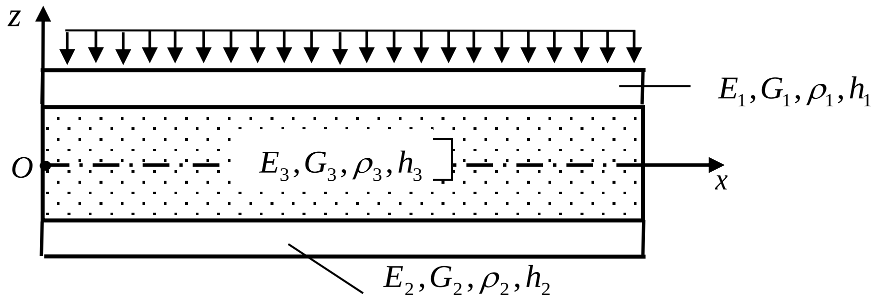
bo‘lishi kerak.

Elastik jismning harakati (1.2.25) tenglamalardan biri yordamida yechilayotganda (1.2.23), (1.2.9) yoki (1.2.24) chegaraviy shartlardan tashqari vaqtning $t = 0$ payti uchun boshlang‘ich shartlar ham qo‘yilishi kerak. Boshqacha aytganda, $t = 0$ bo‘lganda u_i ko‘chishlar va ularning vaqt bo‘yicha birinchi tartibli hosilalari $\left(\frac{\partial u_i}{\partial t} - tezliklar\right)$ berilgan qiymatlarga ega bo‘lishlari kerak:

$$u_i(x_k, t)\Big|_{t=0} = u_i^{(0)}; \quad \frac{\partial u_i(x_k, t)}{\partial t}\Big|_{t=0} u_i^{(01)}. \qquad (1.2.26)$$

§ 1.3. Uch qatlamli sterjennig tebranishlari va statik yuk ta’sirida egilish tenglamalari.

Foydalanilgan adabiyotlar ro`yxatidagi [1] tadqiqotda to`plangan kuch bilan yuklangan uch qatlamli konsol stejenning egilishlari boshlang`ich parametrlar metodini qo`llab aniqlangan. Quyida biz [2] ilmiy ishda keltirilgan usuldan foydalanib uch qatlamli stejenning chetlari turlicha mahkamlangan hollar uchun ko`chish va egilish funksiyalarini aniqlovchi sodda va qulay formullarni keltirib chiqaramiz. Quyida uch qatlamli sterjen uchlar har xil mahkamlangan hollarda uning xususiy tebranishlari o’rganiladi



2.1-rasm. Tadqiqot obe'kti

Uch qatlamli sterjen statik ravishda qo`yilgan yuk ta'siri ostida bo`lsa

$$D\left(1-\frac{\upsilon h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\frac{\partial^4\chi(x,t)}{\partial x^4}+\rho hb\left(1-\frac{h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\chi(x,t)=Q(x,t)$$

(1.3.1)

harakat tenglamalarida dinamik hadlar tushib qoladi va u muvozanat tenglamasiga aylanadi.

Bu yerda

$$D\left(1-\frac{\upsilon h^2}{\beta}\frac{d^2}{dx^2}\right)\frac{d^4\chi(x)}{dx^4}=P \qquad (1.3.2)$$

Hosil bo`lgan tenglama uch qatlamli sterjenning ko`ndalang egilishini tavsiflaydi.

$\chi(x,t)$ – ko'chish funksiyasi;

D – sterjen materialining bikrligi;

$P(x,t)$ – tashqi dinamik yuk;

υ – umumlashgan ko'chish;

h,b – sterjen devori qalinligi va eni;

t – vaqt;

x – bo'ylama koordinata;

Yuqoridagi (1.3.1) tenglamada ko`chishlar $\chi(x)$ funksiya orqali ifodalanganligi, va demak momentlar va ko`ngalang kuchlar ham shu funksiya orqali ifodalanganliklari uchun (1.3.2) tenglamani asosiy hal qiluvchi tenglama sifatida qabul qilish mumkin.

Endi o`lchamsiz parametrlarni

$$\xi = \frac{x}{\ell}; \quad \chi = \frac{\chi}{\ell},\ k = \frac{h^2}{\beta \ell^2} \tag{1.3.3}$$

formulalar bilan kiritib.

$$\frac{D}{\ell^3}\left(1 - \upsilon k \frac{d^2}{d\xi^2}\right)\frac{d^4 \chi(\xi)}{d\xi^4} = P \tag{1.3.4}$$

tenglamaga ega bo`lamiz. Oxirgi oltinchi tartibli bir jinslimas differensial tenglamani yechamiz. Buning uchun tenglamaning bir jinsli qismini ajratib olamiz

$$\left(1 - \upsilon k \frac{d^2}{d\xi^2}\right)\frac{d^4 \chi(\xi)}{d\xi^4} = 0 \tag{1.3.5}$$

Hosil qilingan (1.3.5) differensial tenglamani yechimi

$$\frac{d^4 \chi(\xi)}{d\xi^4} = 0 \ \text{ va } \ \chi(\xi) - \upsilon k \frac{d^2 \chi(\xi)}{d\xi^2} = 0$$

tenglamalar yechimlarining yig`indisiga;

$$\frac{d^4 \chi(\xi)}{d\xi^4} \neq 0 \ \text{ va } \ \chi(\xi) - \upsilon k \frac{d^2 \chi(\xi)}{d\xi^2} = 0$$

tenglamalar yechimlarining yig`indisiga;

$$\frac{d^4 \chi(\xi)}{d\xi^4} = 0 \ \text{ va } \ \chi(\xi) - \upsilon k \frac{d^2 \chi(\xi)}{d\xi^2} \neq 0$$

tenglamalar yechimlarining yig`indisiga teng bo`ladi.

Ammo, (1.3.4)-birjinslimas differensial tenglamaning ko`rinishidan kelib chiqqan holda biz ikkinchi holdan foydalanamiz, ya'ni

$$\chi(\xi) - \upsilon k \frac{d^2 \chi(\xi)}{d\xi^2} = 0; \tag{1.3.6}$$

$$\frac{d^4 \chi(\xi)}{d\xi^4} = 1 \ \left(\frac{d^4 \chi(\xi)}{d\xi^4} \neq 0 \ \text{ yoki } \ \frac{d^4 \chi(\xi)}{d\xi^4} = C = const \ \text{ xususiy holda } C = 1 \right) \tag{1.3.7}$$

tenglamalar yechimlarini topamiz. Avvalo (1.3.6) tenglamani xarakteristikalar usuli bilan yechamiz. Ya'ni yechimni

$$\chi(\xi) = Ce^{\lambda\xi} \tag{1.3.8}$$

ko`rinishda izlaymiz. Ushbu (1.3.8) yechimni (1.3.6) tenglamaga qo`yib

$Ce^{\lambda\xi} - \upsilon k\lambda^2 Ce^{\lambda\xi} = 0$ tenglamaga ega bo`lamiz.

Bundan

$$\upsilon k\lambda^2 - 1 = 0 \qquad (1.3.9)$$

Xarakteristik tenglamani olamiz. Bu tenglamaning ildizlari

$$\lambda = \pm\frac{1}{\sqrt{k\upsilon}} \qquad (1.3.10)$$

U holda ma’lumki [3] (1.3.6) tenglamaning umumiy yechimi

$$\chi_1(\xi) = C_1 e^{\frac{\xi}{\sqrt{k\upsilon}}} + C_2 e^{-\frac{\xi}{\sqrt{k\upsilon}}} \qquad (1.3.11)$$

ko`rinishda bo`ladi.

Endi (1.3.7) tenglamaning yechimini topamiz. Uni bir marta integrallab

$$\frac{d^3\chi(\xi)}{d\xi^3} = \xi + C_3 \qquad (1.3.12)$$

ifodaga ega bo`lamiz va (2.12) ketma-ket integrallaymiz va

$$\frac{d^2\chi(\xi)}{d\xi^2} = \frac{\xi^2}{2} + C_3\xi + C_4$$

$$\frac{d\chi(\xi)}{d\xi} = \frac{\xi^3}{6} + C_3\frac{\xi^2}{2} + C_4\xi + C_5$$

$$\chi_2(\xi) = \frac{\xi^4}{24} + C_3\frac{\xi^3}{6} + C_4\frac{\xi^2}{2} + C_5\xi + C_6 \qquad (1.3.13)$$

yechimga ega bo`lamiz. Demak (2.5) tenglamaning umumiy yechimi

$$\chi(\xi) = \chi_1(\xi) + \chi_2(\xi) = C_1 e^{\frac{\xi}{\sqrt{k\upsilon}}} + C_2 e^{-\frac{\xi}{\sqrt{k\upsilon}}} + \frac{\xi^4}{24} + C_3\frac{\xi^3}{6} + C_4\frac{\xi^2}{2} + C_5\xi + C_6 \qquad (1.3.14)$$

bo`ladi. U holda (2.14) asosida (2.6) bir jinslimas tenglamaning umumiy yechimni sifatida

$$\chi(\xi) = \frac{Pl^3}{D}\left(\chi_1(\xi) + \chi_2(\xi)\right) \qquad (1.3.15)$$

yechimni olsak u (1.3.6) tenglamani qanoatlantirmaydi. Shuning uchun (1.3.11) yechimni $k^2\upsilon^2$ - o`zgarmas songa ko`paytirib

$$\chi_1(\xi) = C_1 k^2\upsilon^2 e^{\frac{\xi}{\sqrt{k\upsilon}}} + C_2 k^2\upsilon^2 e^{-\frac{\xi}{\sqrt{k\upsilon}}}$$

ko`rinishda olamiz.

Bu tenglamaning umumiy echimi quyidagicha.

$$\chi(\xi)=\frac{P\ell^3}{D}\left(C_1k^2\upsilon^2e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+C_2k^2\upsilon^2e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\xi^4}{24}+C_3\frac{\xi^3}{6}+C_4\frac{\xi^2}{2}+C_5\xi+C_6\right) \quad (1.3.16)$$

Bu yerda C_1, C_2, C_3, C_4, C_5, C_6 lar chegaraviy shartlardan topiladi.

II-BOB
KO`NDALANG STATIK YUK TA'SIRIDAGI UCH QATLAMLI STERJENNI TADQIQ ETISH

Quyida o`rtasiga qattiq jism tushishi natijasida zarbaga uchraydigan uch qatlamli sterjenning tebranishlari qaraladi. Ma'lumki, bir jinsli sterjenlar hisobida Bubnov-Galerkin usuli qo`llanilagi. Bu usul juda qulay hisoblash formulasiga olib keladi. Uch qatlamli stejenlarni zarba, harakatlanuvchi yuk, zarba to`lqini ta'siri kabi dinamik yuklanishlarga hisoblashda yuqori garmonikalar, ya'ni tebranish chastotasining katta qiymatlari ham muhim rol o`ynaydi.

Odatda chastotalarni va uch qatlamli sterjenning dinamik xaraktestikalarini aniqlashda tebranish shakllarini trigonometrik va giperbolik funksiyalar yordamida tasvirlab, juda katta transsendent tenglamalarni yechish muammosiga duch kelamiz. Shu sababli, uch qatlamli sterjennig xususiy tebranish shaklini muvaffaqiyatli tanlash, hisoblash jarayonini ancha yengillashtiradi. Bunda xususiy chastotalar va tebranish shakllarini topish masalasi ko`p mehnat talab qiluvchi, murakkab muammodan iborat va doimo o`ziga xos qiziqish uyg`otadi.

Bunda zarbali va harakatlanuvchi yuk ta'siridagi tebranishlar shaklini hisoblashda talab etilgan aniqlikni qo`lga kiritish uchun, tebranishlar xususiy shakllarining yetarli soni bilan chegaralanish kifoya. Qaralayotgan sterjenni dinamik yuklanish ta'siriga hisoblashda ko`chish funksiyasi statik egilishning maksimal qiymati bilan ko`chishining dinamiklik koeffitsiyenti ko`paytmasiga teng qilib tanlanadi.

§ 2.1. Uchlari diafragmali va erkin tayangan uch qatlamli sterjenning egilishi

Uchlari diafragmali va erkin tayangan uch qatlamli sterjenni qaraymiz. Bu holda masalaning chegaraviy shartlari [1] ilmiy tadqiqot ishi natijalari asosida sterjenning $x = 0$ *va* $x = l$ uchlarida quyidagi ko`rinishda bo`ladi

$$\left\{\left(1-\frac{h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\chi=0,\left(1-\frac{\upsilon h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2\chi}{\partial x^2}=0,\qquad \frac{\partial^3\chi}{\partial x^3}=0\right.$$

Bu yerda o`lchamsiz parametrlarga (2.3) formulalar bilan o`tib, $\xi=0$ va $\xi=1$ bo`lganda

$$\begin{cases}\left(1-k\dfrac{\partial^2}{\partial\xi^2}\right)\chi(\xi)=0,\\[2mm] \left(1-k\upsilon\dfrac{\partial^2}{\partial\xi^2}\right)\dfrac{\partial^2\chi(\xi)}{\partial\xi^2}=0,\qquad \dfrac{\partial^3\chi(\xi)}{\partial\xi^3}=0,\end{cases}\tag{2.1.1}$$

chegaraviy shartlarga ega bo`lamiz. Umumiy (1.3.16) yechimni (1.3.17) chegaraviy shartlarga qo`yib C_1, C_2, C_3, C_4, C_5, C_6 o`zgarmaslarni aniqlaymiz. Buning uchun $\chi^{II}(\xi)$, $\chi^{III}(\xi)$, $\chi^{IV}(\xi)$ hosilalarini hisoblab olamiz

$$\begin{cases}\chi^{I}(\xi)=\dfrac{P\ell^3}{D}\left(C_1k\upsilon\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-C_2k\upsilon\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\dfrac{\xi^3}{6}+C_3\dfrac{\xi^2}{2}+C_4\xi+C_5\right)\\[2mm] \chi^{II}(\xi)=\dfrac{P\ell^3}{D}\left(C_1k\upsilon e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+C_2k\upsilon e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\dfrac{\xi^2}{2}+C_3\xi+C_4\right)\\[2mm] \chi^{III}(\xi)=\dfrac{P\ell^3}{D}\left(C_1\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-C_2\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi+C_3\right)\\[2mm] \chi^{VI}(\xi)=\dfrac{P\ell^3}{D}\left(C_1e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+C_2e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+1\right)\\[2mm] \chi^{V}(\xi)=\dfrac{P\ell^3}{D}\left(C_1\dfrac{\sqrt{k\upsilon}}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-C_2\dfrac{\sqrt{k\upsilon}}{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}\right)\end{cases}\tag{2.1.2}$$

Hosilalarning topilgan ifodalarini (2.2.1) chegaraviy shartlarga qo`yib ushbu

$$\begin{cases}C_1k^2\upsilon e^{\frac{\xi}{\sqrt{k\upsilon}}}(\upsilon-1)+C_2k^2\upsilon e^{-\frac{\xi}{\sqrt{k\upsilon}}}(\upsilon-1)+\dfrac{\xi^4}{24}+C_3\left(\dfrac{\xi^3}{6}-\xi k\right)+C_4\left(\dfrac{\xi^2}{2}-k\right)+C_5\xi+C_6-\dfrac{k\xi^2}{2}=0,\\[2mm] C_1k\upsilon e^{\frac{\xi}{\sqrt{k\upsilon}}}+C_2k\upsilon e^{-\frac{\xi}{\sqrt{k\upsilon}}}+\dfrac{\xi^2}{2}+C_3\xi+C_4-k\upsilon(C_1e^{\frac{\xi}{\sqrt{k\upsilon}}}+C_2e^{-\frac{\xi}{\sqrt{k\upsilon}}}+1)=0,\\[2mm] C_1\sqrt{k\upsilon}e^{\frac{\xi}{\sqrt{k\upsilon}}}-C_2\sqrt{k\upsilon}e^{-\frac{\xi}{\sqrt{k\upsilon}}}+\xi+C_3=0.\end{cases}$$

yoki

$$\begin{cases} C_1 k^2 \upsilon e^{\frac{\xi}{\sqrt{k\nu}}}(\nu-1)+C_2 k^2 \upsilon e^{-\frac{\xi}{\sqrt{k\nu}}}(\upsilon-1)+\dfrac{\xi^4}{24}+C_3\left(\dfrac{\xi^3}{6}-\xi k\right)+C_4\left(\dfrac{\xi^2}{2}-k\right)+C_5\xi+C_6-\dfrac{k\xi^2}{2}=0, \\ \dfrac{\xi^2}{2}+C_3\xi+C_4-k\upsilon=0, \\ C_1\sqrt{k\upsilon}e^{\frac{\xi}{\sqrt{k\upsilon}}}-C_2\sqrt{k\upsilon}e^{-\frac{\xi}{\sqrt{k\upsilon}}}+\xi+C_3=0. \end{cases} \qquad (2.1.3)$$

ifodalarga ega bo`lamiz. Bu yerdan $\xi=0$ bo`lganda

$$\begin{cases} C_1 k^2 \upsilon(\upsilon-1)+C_2 k^2 \upsilon(\upsilon-1)-C_4 k+C_6=0 \\ C_4-k\upsilon=0 \\ C_3+C_2\sqrt{k\upsilon}-C_1\sqrt{k\upsilon}=0 \end{cases} \qquad (2.1.4)$$

tenglamalarni va $\xi=1$ bo`lganda

$$\begin{cases} C_1 k^2 \upsilon e^{\frac{1}{\sqrt{k\upsilon}}}(\upsilon-1)+C_2 k^2 \upsilon e^{-\frac{1}{\sqrt{k\upsilon}}}(\upsilon-1)+C_3\left(\dfrac{1}{6}-k\right)+C_4\left(\dfrac{1}{2}-k\right)+C_5+C_6+\dfrac{1}{24}-\dfrac{k}{2}=0 \\ C_3+C_4-k\upsilon+\dfrac{1}{2}=0 \\ C_1\sqrt{k\upsilon}e^{\frac{1}{\sqrt{k\upsilon}}}-C_2\sqrt{kve}^{-\frac{1}{\sqrt{k\upsilon}}}+1+C_3=0. \end{cases} \qquad (2.1.5)$$

tenglamalarni olamiz.

Endi (2.1.4) ning 2-chi tenglamasidan C_4 ni topamiz.

$$C_4=k\upsilon \qquad (2.2.6)$$

va (2.1.5) ning 2-chi tenglamasidan C_3 ni topib.

$$C_3=-C_4+kv-\frac{1}{2}$$

C_4 o`rniga uning (2.1.6) qiymatidan foydalangan holda

$$C_3=-\frac{1}{2} \qquad (2.1.7)$$

ga ega bo`lamiz.

Yuqoridagi (2.1.4) va (2.1.5) sistemalarning 3-chi tenglamalaridagi C_3 larni unung o`rniga topilgan (2.1.7) qiymatini olib borib qo`yamiz

$$\begin{cases} C_1\sqrt{k\upsilon}-C_2\sqrt{k\upsilon}=-C_3 \\ C_1\sqrt{k\upsilon e}^{-\frac{1}{\sqrt{k\upsilon}}}-C_2\sqrt{k\upsilon e}^{\frac{1}{\sqrt{k\upsilon}}}-1=C_3 \end{cases}$$

yoki

$$\begin{cases} C_1\sqrt{k\upsilon} - C_2\sqrt{k\upsilon} = \dfrac{1}{2} \\ C_1\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}} - C_2\sqrt{k\upsilon}e^{\frac{1}{\sqrt{k\upsilon}}} = -\dfrac{1}{2} \end{cases} \qquad (2.1.8)$$

Bu sistemani birinchi tenglamasini $e^{-\frac{1}{\sqrt{kv}}}$ ga ko`paytirib

$$\begin{cases} C_1\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}} - C_2\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}} = \dfrac{1}{2}e^{-\frac{1}{\sqrt{k\upsilon}}} \\ C_1\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}} - C_2\sqrt{k\upsilon}e^{\frac{1}{\sqrt{k\upsilon}}} = -\dfrac{1}{2} \end{cases}$$

Undan ikkinchi tenglamani hadma-had ayiramiz, u holda

$$C_1 = -\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}} - 1\right)\sqrt{k\upsilon}} \qquad (2.1.9)$$

Topilgan (2.1.9) ifodani (2.1.8) sistemaning ikkinchi tenglamasiga qo`yib C_2 ni topamiz:

$$-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}} - 1\right)\sqrt{k\upsilon}} - C_2\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}} = -\frac{1}{2}$$

yoki

$$C_2 = -\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}} - 1\right)\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}}} \qquad (2.1.10)$$

Endi (2.1.3) tenglamalar sistemasining 1-chi tenglamasidan C_6 ni topamiz

$$C_1k^2\upsilon(\upsilon-1) + C_2k^2\upsilon(\upsilon-1) - C_4k + C_6 = 0$$

bu yerda C_1, C_2 va C_4 lar o`rniga ularning (2.1.6), (2.1.9) va (2.1.10) ifodalarni qo`yamiz

$$-\frac{1}{2\sqrt{k\upsilon}\left(e^{\frac{1}{\sqrt{k\upsilon}}} - 1\right)}k^2\upsilon(\upsilon-1) - \frac{1}{2\sqrt{k\upsilon}\left(e^{\frac{1}{\sqrt{k\upsilon}}} - 1\right)e^{-\frac{1}{\sqrt{k\upsilon}}}}k^2\upsilon(\upsilon-1) - k^2\upsilon = -C_6$$

Bu yerdan

$$C_6 = k^2\upsilon\left(1+(\upsilon-1)\frac{1+e^{\frac{1}{\sqrt{k\upsilon}}}}{2\sqrt{k\upsilon}\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)}\right) \qquad (2.1.11)$$

Endi (2.1.5) ni 1-chi tenglamasidan C_5 ni topib olamiz

$$C_5 = -C_1k^2\upsilon e^{\frac{1}{\sqrt{k\upsilon}}}(\upsilon-1)-C_2k^2\upsilon e^{-\frac{1}{\sqrt{k\upsilon}}}(\upsilon-1)-C_3\left(\frac{1}{6}-k\right)+C_4\left(\frac{1}{2}-k\right)-C_6+\frac{1}{24}-\frac{k}{2}$$

yoki

$$C_5 = (-C_1-C_2)k^2ve^{\frac{1}{\sqrt{kv}}}(v-1)-C_3\left(\frac{1}{6}-k\right)+C_4\left(\frac{1}{2}-k\right)-C_6+\frac{1}{24}-\frac{k}{2}$$

tenglamani soddalashtirib quyidagi ko`rinishda yozib olamiz

$$C_5 = \left(\frac{e^{\frac{1}{\sqrt{k\upsilon}}}}{2\sqrt{k\upsilon}\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)}+\frac{e^{\frac{1}{\sqrt{k\upsilon}}}}{2\sqrt{k\upsilon}\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)e^{-\frac{1}{\sqrt{k\upsilon}}}}\right)k^2\upsilon(\upsilon-1)+\frac{1}{2}\left(\frac{1}{6}-k\right)+k\upsilon\left(\frac{1}{2}-k\right)-$$

$$-\left(1+(\upsilon-1)\frac{1+e^{\frac{1}{\sqrt{k\upsilon}}}}{2\sqrt{k\upsilon}\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)}\right)k^2\upsilon+\frac{1}{24}-\frac{k}{2}$$

yoki

$$C_5 = \frac{k\upsilon}{2}-k-2k^2\upsilon+\frac{3}{24} \qquad (2.1.12)$$

Shunday qilib diafragmali uchlari erkin tayangan uch qatlamli sterjenning P=const tekis taqsimlangan statik yuk ta'sirida egilishida $\chi(\xi)$ funksiyasi quyidagi formula bilan hisoblanadi

$$\chi(\xi)=\frac{P\ell^3}{D}\left(\begin{array}{l}\left(-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}}\right)k^2\upsilon^2 e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\left(-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}}}\right)k^2\upsilon^2 e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\\ +\frac{\xi^4}{24}-\frac{1}{2}\frac{\xi^3}{6}+k\upsilon\frac{\xi^2}{2}+\left(\frac{k\upsilon}{2}-k-2k^2\upsilon+\frac{3}{24}\right)\xi+k^2\upsilon\left(1+(\upsilon-1)\frac{1+e^{\frac{1}{\sqrt{k\upsilon}}}}{2\sqrt{k\upsilon}\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)}\right)\end{array}\right). \qquad (2.1.13)$$

Sterjenning o`rta qatlamga nisbatan hisoblangan to`liq momenti

$$M=-D\left(1-\frac{\upsilon h^2}{\beta}\frac{d^2}{dx^2}\right)\frac{d^2\chi}{dx^2}. \qquad (2.1.14)$$

Endi o`lchamsiz parametrlarni (1.3.3) formulalar orqali kiritib

$$M=-D\left(1-\upsilon k\frac{d^2}{d\xi^2}\right)\frac{d^2\chi(\xi)}{d\xi^2} \qquad (2.1.15)$$

tenglamaga ega bo`lamiz. Buning uchun $\chi^{II}(\xi)$, $\chi^{III}(\xi)$, $\chi^{IV}(\xi)$, $\chi^{V}(\xi)$ hosilalarini hisoblab olamiz

$$\begin{cases}\chi^{II}(\xi)=\frac{P\ell^3}{D}\left(-\frac{k\upsilon e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}}-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}}}k\upsilon e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\xi^2}{2}-\frac{1}{2}\xi+k\upsilon\right)\\ \chi^{III}(\xi)=\frac{P\ell^3}{D}\left(-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}}\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}}}\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi-\frac{1}{2}\right)\\ \chi^{VI}(\xi)=\frac{P\ell^3}{D}\left(-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}}}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+1\right)\\ \chi^{V}(\xi)=\frac{P\ell^3}{D}\left(-\frac{\sqrt{k\upsilon}}{k\upsilon}\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\sqrt{k\upsilon}}{k\upsilon}\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}}}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}\right)\end{cases} \qquad (2.1.16)$$

(2.1.13) tenglamani topilgan (2.1.15) tenglamaga qo`yamiz

$$M=-Pl^3\left(-\frac{1}{2b\sqrt{k\upsilon}}k\upsilon e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}-\frac{1}{2b\sqrt{k\upsilon e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}}}k\upsilon e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+\frac{\xi^2}{2}-\frac{1}{2}\xi+\upsilon k+\frac{1}{2b\sqrt{k\upsilon}}k\upsilon e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\frac{1}{2b\sqrt{k\upsilon e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}}}k\upsilon e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\upsilon k\right)$$

yoki

$$M=-Pl^3\left(\frac{\xi^2}{2}-\frac{1}{2}\xi\right) \qquad (2.1.17)$$

Ko`ndalang kuch

$$Q=-D\left(1-\frac{\upsilon h^2}{\beta}\frac{d^2}{dx^2}\right)\frac{d^3\chi}{dx^3}$$

yoki

$$Q=-D\left(1-\upsilon k\frac{d^2}{d\xi^2}\right)\frac{d^3\chi(\xi)}{d\xi^3} \qquad (2.1.18)$$

Topilgan (2.1.16) tenglamadagi 3-chi va 5-chi hosilalarini qo`yamiz

$$Q=-Pl^3\left(-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}}\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon e^{-\frac{1}{\sqrt{k\upsilon}}}}}\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi-\frac{1}{2}\right)-$$

$$-Pl^3\left(k\upsilon\left(-\frac{\sqrt{k\upsilon}}{k\upsilon}\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\sqrt{k\upsilon}}{k\upsilon}\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon e^{-\frac{1}{\sqrt{k\upsilon}}}}}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}\right)\right)$$

yoki

$$Q=-Pl^3\left(\xi-\frac{1}{2}\right) \qquad (2.1.19)$$

To`ldiruvchi qatlamdagi ko`ndalang kuch

$$Q_3=-D\frac{d^3\chi}{dx^3}$$

yoki

$$Q_3=-D\frac{d^3\chi(\xi)}{d\xi^3}. \qquad (2.1.20)$$

Topilgan (2.1.16) tenglamalarning uchinchi tartibli hosilasini olib qo`yamiz

$$Q_3 = -Pl^3\left(-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}}\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}}}\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi-\frac{1}{2}\right) \quad (2.1.21)$$

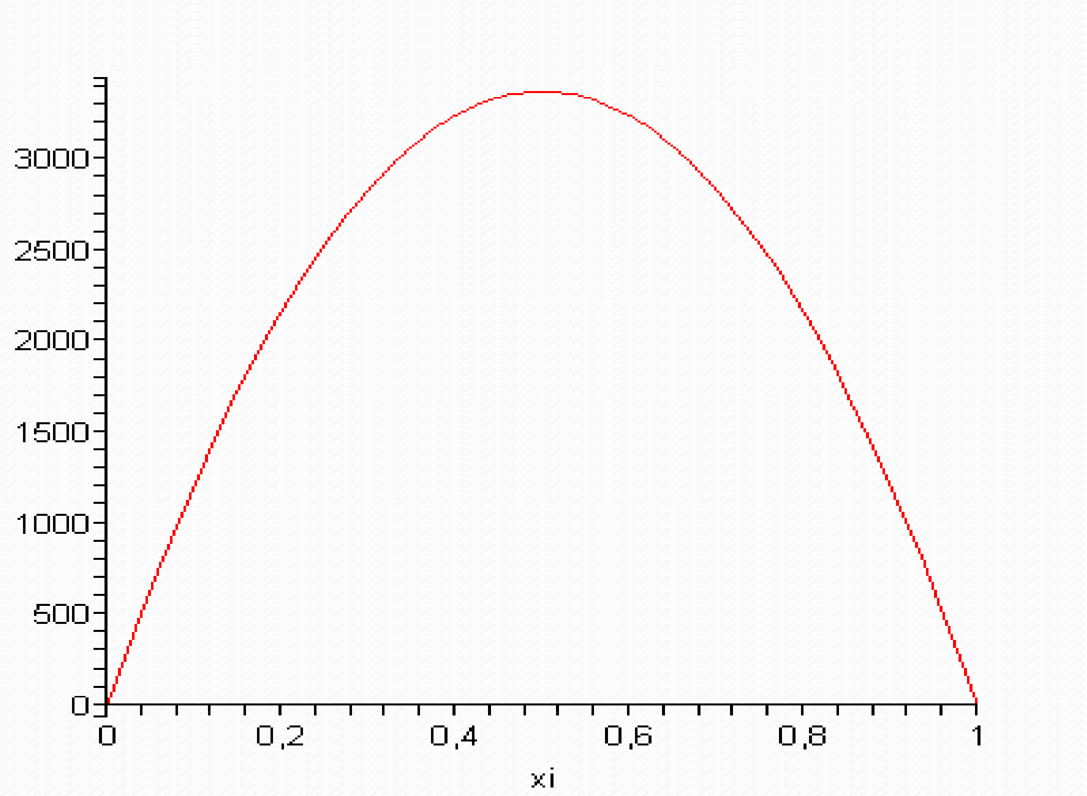
To`ldiruvchi qatlamdagi eguvchi moment

$$H = -D\frac{d^2\chi}{dx^2} \qquad (2.1.22)$$

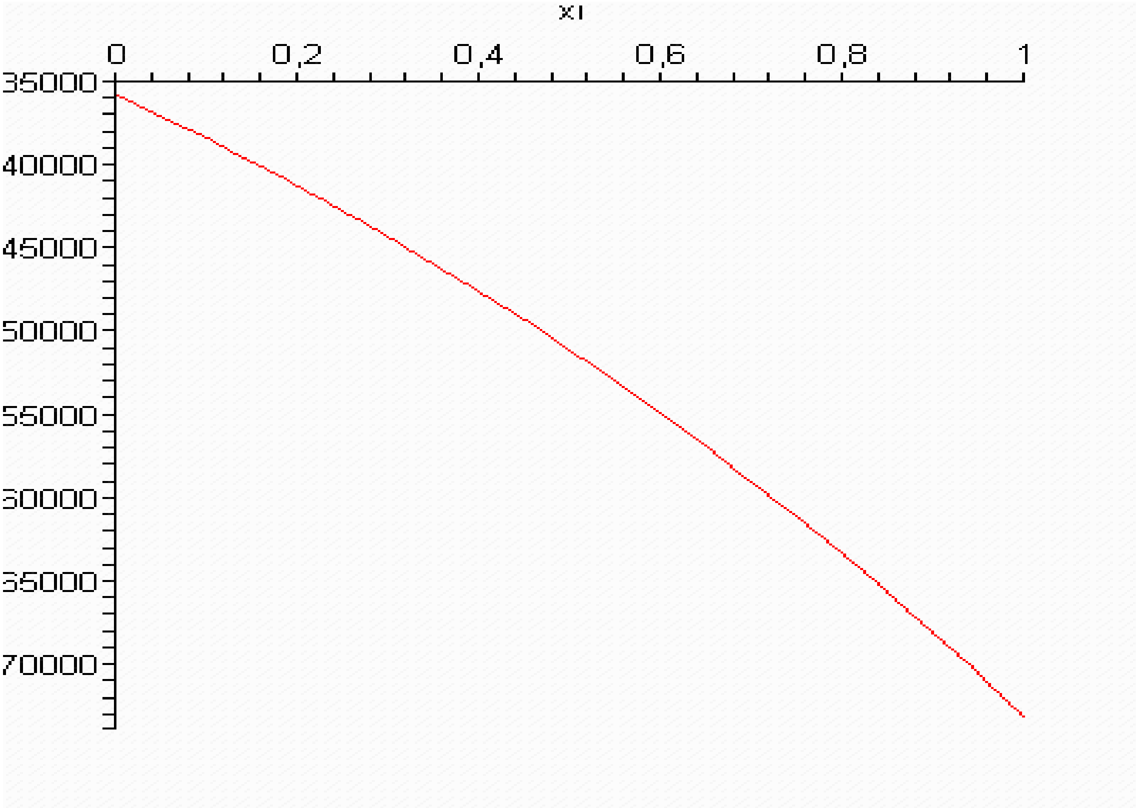
yuqoridagi (2.1.16) tenglamalarning ikkinchi tartibli hosilasini qo`yamiz;

$$H = -Pl^3\left(-\frac{k\upsilon e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}}-\frac{1}{2\left(e^{\frac{1}{\sqrt{k\upsilon}}}-1\right)\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}}}k\upsilon e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\xi^2}{2}-\frac{1}{2}\xi+k\upsilon\right)$$

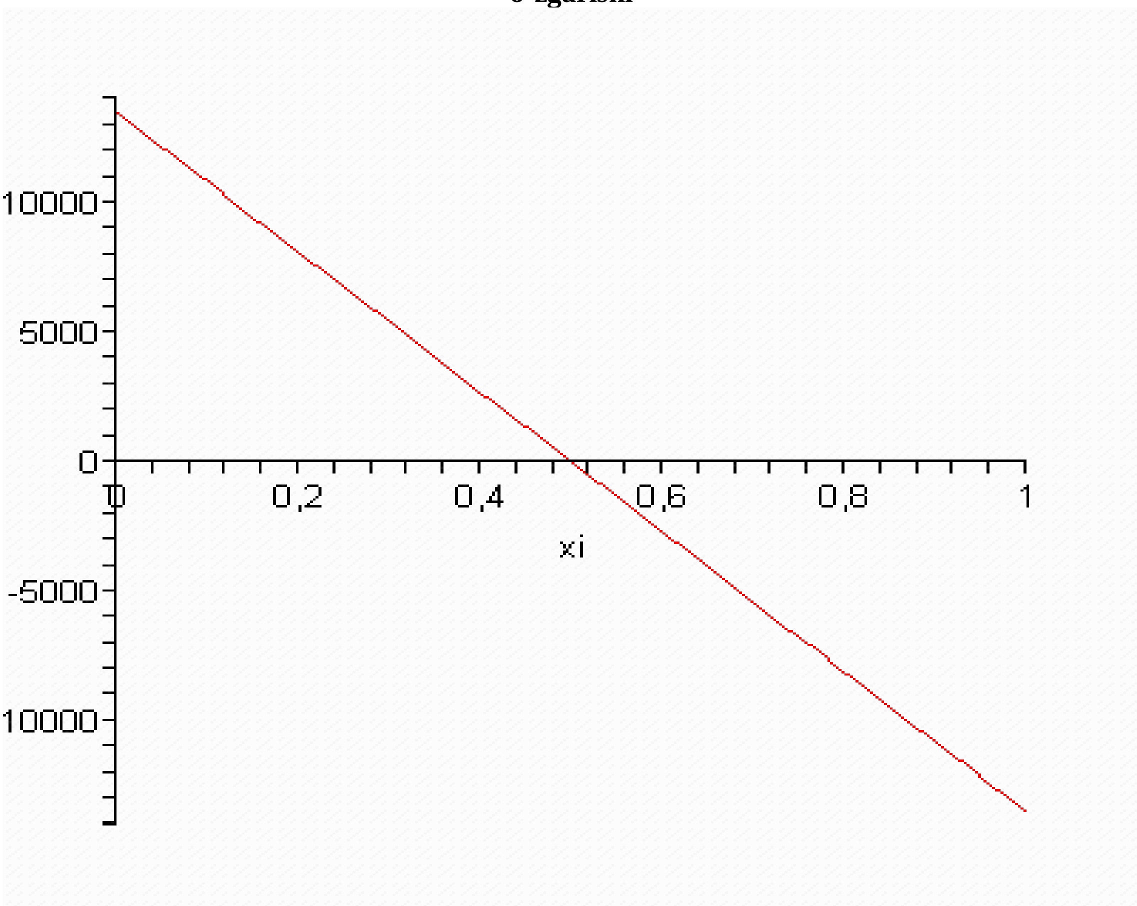
Oxirgi formulalar yordamida qaralayotgan uch qatlamli sterjen kesimlaridagi zo`riqishlar hisoblanadi. Ushbu ishda sonli hisoblashlar «Maple 9.5» programmasi yordamida amalga oshirildi.



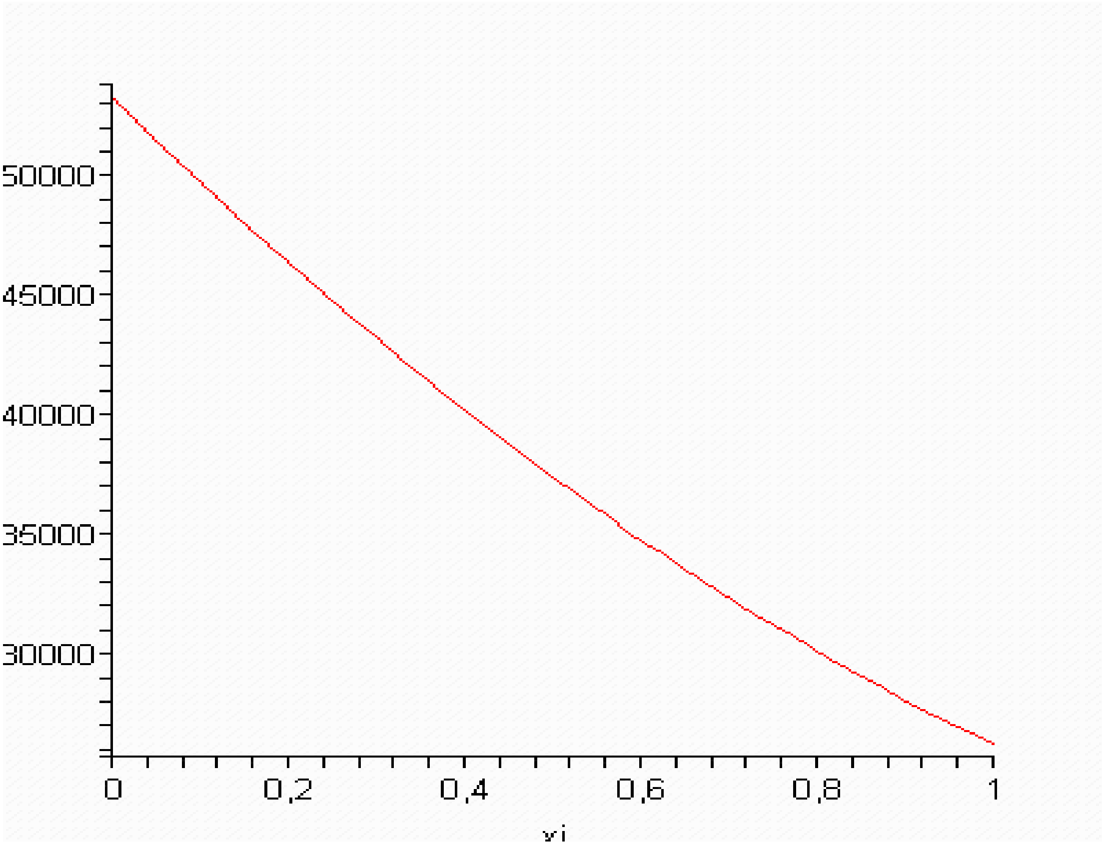
2.1-rasm sterjen kesimlarida eguvchi momentning koordinatadan o`zgarishi



2.2-rasm sterjen kesimlarida to`ldiruvchi qatlamdagi eguvchi momentning koordinatadan o`zgarishi



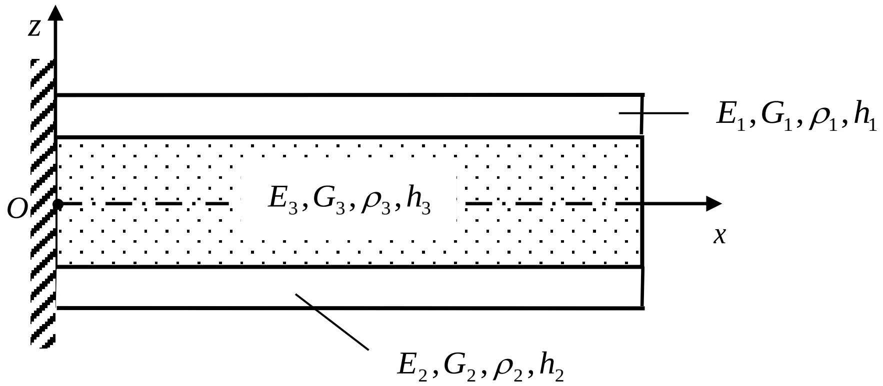
2.3-rasm sterjen kesimlaridagi qirquvchi kuchning koordinatadan o`zgarishi



2.4-rasm to`ldiruvchi qatlamda qirquvchi kuchning koordinatadan bog`liq o`zgarishi

§ 2.2. Bir uchi qistirib mahkamlangan, ikkinchi uchi erkin uch qatlamli sterjenning statik yuk ta'sirida egilishi

Bir uchi qistirib mahkamlangan, ikkinchi uchi erkin uch qatlamli sterjenni qaraymiz. Bu holda masalaning chegaraviy shartlari [1] ilmiy tadqiqot ishi natijalari asosida sterjenning $x=0$ *va* $x=l$ uchlarida quyidagi ko`rinishda bo`ladi



2.2-rasm.

$x=0$ bo`lganda

$$\left\{ \left(1-\frac{h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\chi = 0, \qquad \frac{\partial \chi}{\partial x} = 0, \qquad \frac{\partial^3 \chi}{\partial x^3} = 0 \right.$$

$x=l$ bo`lganda

$$\left\{ \frac{\partial^2 \chi}{\partial x^2} = 0, \qquad \frac{\partial^4 \chi}{\partial x^4} = 0, \qquad \left(1-\upsilon\frac{h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\frac{d^3\chi}{dx^3} = 0, \right.$$

Bu yerda o`lchamsiz parametrlarga (1.3.3) formulalar bilan o`tib, quyidagi

$$\chi(\xi) - k\chi''(\xi) = 0, \;\; \chi'(\xi) = 0, \;\; \chi'''(\xi) = 0, \quad \text{dan} \quad \xi = 0. \qquad (2.2.1)$$

$$\chi''(\xi) = 0, \;\; \chi^{IV}(\xi) = 0, \;\; \chi'''(\xi) - k\upsilon\chi^{V}(\xi) = 0, \text{dan} \quad \xi = 1. \qquad (2.2.2)$$

chegaraviy shartlarga ega bo`lamiz. Umumiy (1.3.16) yechimni (2.2.1) va (2.2.2) chegaraviy shartlarga qo`yib C_1, C_2, C_3, C_4, C_5, C_6 o`zgarmaslarni aniqlaymiz. Buning uchun $\chi^{II}(\xi)$, $\chi^{III}(\xi)$, $\chi^{IV}(\xi)$ hosilalarini hisoblab olamiz

$$\begin{cases} \chi^{I}(\xi)=\dfrac{P\ell^3}{D}\left(C_1 k\upsilon\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2 k\upsilon\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\dfrac{\xi^3}{6}+C_3\dfrac{\xi^2}{2}+C_4\xi+C_5\right) \\ \chi^{II}(\xi)=\dfrac{P\ell^3}{D}\left(C_1 k\upsilon e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+C_2 k\upsilon e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\dfrac{\xi^2}{2}+C_3\xi+C_4\right) \\ \chi^{III}(\xi)=\dfrac{P\ell^3}{D}\left(C_1\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\xi+C_3\right) \\ \chi^{VI}(\xi)=\dfrac{P\ell^3}{D}\left(C_1 e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+C_2 e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+1\right) \\ \chi^{V}(\xi)=\dfrac{P\ell^3}{D}\left(C_1\dfrac{\sqrt{k\upsilon}}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2\dfrac{\sqrt{k\upsilon}}{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}\right) \end{cases} \qquad (2.2.3)$$

(2.2.1) tenglamadagi hosilalarni o`rniga topilga (2.2.3) hosilalardan foydalanamiz.

$$\begin{cases} C_1 k^2\upsilon e^{\frac{\xi}{\sqrt{k\upsilon}}}(\upsilon-1)+C_2 k^2\upsilon e^{-\frac{\xi}{\sqrt{k\upsilon}}}(\upsilon-1)+\dfrac{\xi^4}{24}+C_3\left(\dfrac{\xi^3}{6}-k\xi\right)+C_4\left(\dfrac{\xi^2}{2}-k\right)+C_5\xi+C_6-\dfrac{k\xi^2}{2}=0, \\ C_1 k\upsilon\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2 k\upsilon\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\dfrac{\xi^3}{6}+C_3\dfrac{\xi^2}{2}+C_4\xi+C_5=0, \\ C_1\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\xi+C_3=0 \end{cases} \qquad (2.2.4)$$

tenglamalarga ega bo`lamiz. Bu yerdan $\xi=0.$ bo`lganda

$$\begin{cases} C_1 k^2\upsilon(\upsilon-1)+C_2 k^2\upsilon(\upsilon-1)-C_4 k+C_6=0, \\ C_1 k\upsilon\sqrt{k\upsilon}-C_2 k\upsilon\sqrt{k\upsilon}+C_5=0, \\ C_1\sqrt{k\upsilon}-C_2\sqrt{k\upsilon}+C_3=0 \end{cases} \qquad (2.2.5)$$

Yuqoridagi (2.2.2) tenglamadagi hosilalarni o`rniga topilga (2.2.3) hosilalardan foydalanamiz.

$$\begin{cases} C_1 k\upsilon e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+C_2 k\upsilon e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\dfrac{\xi^2}{2}+C_3\xi+C_4=0, \\ C_1 e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+C_2 e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+1=0, \\ C_1\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\xi+C_3-k\upsilon\left(C_1\dfrac{\sqrt{k\upsilon}}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2\dfrac{\sqrt{k\upsilon}}{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}\right)=0. \end{cases}$$

tenglamalarga ega bo`lamiz. Bu yerdan $\xi=1$ bo`lganda

$$\begin{cases} C_1 k\upsilon e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} + C_2 k\upsilon e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + \frac{1}{2} + C_3 + C_4 = 0 \\ C_1 e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} + C_2 e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + 1 = 0 \\ C_1 \sqrt{k\upsilon} e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} - C_2 \sqrt{k\upsilon} e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + 1 + C_3 - k\upsilon\left(C_1 \frac{\sqrt{k\upsilon}}{k\upsilon} e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} - C_2 \frac{\sqrt{k\upsilon}}{k\upsilon} e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} \right) = 0 \end{cases} \qquad (2.2.6)$$

Endi (2.2.6) ning 3-chi tenglamasidan C_3 ni topamiz.

$$C_3 = -1 \qquad (2.2.7)$$

yuqoridagi (2.2.7) ning 3-chi tenglamasidagi C_3 ning topilgan qiymatini o`rniga qo`yamiz va (2.2.6) tenglamaning 2-chi ifodasini

$$\begin{aligned} & C_1 - C_2 = \frac{1}{\sqrt{k\upsilon}} \\ & C_1 e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} + C_2 e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + 1 = 0 \end{aligned} \qquad (2.2.8)$$

Bu sistemani birinchi tenglamasini $e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}$ ga ko`paytirib

$$\begin{aligned} & C_1 e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} - C_2 e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} = \frac{1}{\sqrt{k\upsilon}} e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} \\ & C_1 e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} + C_2 e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + 1 = 0 \end{aligned}$$

undan ikkinchi tenglamani hadma-had qo`shamiz, u holda

$$C_1 = \frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} - \sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} \right)}. \qquad (2.2.9)$$

Topilgan (2.2.9) ifodani (2.2.8) sistemaning birinchi tenglamasiga qo`yib C_2 ni topamiz.

$$\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} - \sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} \right)} - C_2 = \frac{1}{\sqrt{k\upsilon}}$$

yoki

$$C_2 = -\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} + \sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}. \qquad (2.2.10)$$

Endi (2.2.5) tenglamalar sistemasining 2-chi tenglamasidan C_5 ni topamiz.

$$C_1 k\upsilon\sqrt{k\upsilon} - C_2 k\upsilon\sqrt{k\upsilon} = -C_5$$

Bu yerda C_1 va C_2 lar o`rniga ularning (2.2.9) va (2.2.10) ifodalarni qo`yamiz

$$\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} - \sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)} k\upsilon\sqrt{k\upsilon} + \frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} + \sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)} k\upsilon\sqrt{k\upsilon} = -C_5$$

yoki

$$C_5 = -k\upsilon \qquad (2.2.11)$$

Yuqoridagi (2.2.6) tenglamalar sistemasining 1-chi tenglamasidan C_4 ni topamiz.

$$C_1 k\upsilon e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} + C_2 k\upsilon e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + \frac{1}{2} + C_3 = -C_4$$

bu yerda C_1, C_2 va C_3 lar o`rniga ularning (2.2.9), (2.2.10) va (2.2.7)ifodalarni qo`yamiz

$$\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} - \sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)} k\upsilon e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} - \frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}} + \sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)} k\upsilon e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}} + \frac{1}{2} - 1 = -C_4$$

u holda

$$C_4 = \frac{1}{2} + k\upsilon \qquad (2.2.12)$$

Endi (2.2.5) tenglamalar sistemasining 1-chi tenglamasidan C_6 ni topamiz.

$$C_1 k^2\upsilon(\upsilon - 1) + C_2 k^2\upsilon(\upsilon - 1) + \frac{\xi^4}{24} - C_4 k = -C_6$$

bu yerda C_1, C_2 va C_4 lar o`rniga ularning (2.2.9), (2.2.10) va (2.2.12)ifodalarni qo`yamiz

$$\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}k^2\upsilon(\upsilon-1)-\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}k^2\upsilon(\upsilon-1)-\left(\frac{1}{2}+\sqrt{k\upsilon}\right)k=-C_6$$

yoki

$$C_6=\frac{k}{2}+k\sqrt{k\upsilon}-\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}-2\sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}k^2\upsilon(\upsilon-1) \qquad (2.2.13)$$

Shunday qilib bir uchi qistirib mahkamlangan, ikkinchi uchi erkin uch qatlamli sterjenning P=const tekis taqsimlangan statik yuk ta'sirida egilishida $\chi(\xi)$ funksiyasi quyidagi formula bilan hisoblanadi.

$$\chi(\xi)=\frac{P\ell^3}{D}\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}k^2\upsilon^2e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}k^2\upsilon^2e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}\right)+$$
$$+\frac{P\ell^3}{D}\left(\frac{\xi^4}{24}-\frac{\xi^3}{6}+\left(\frac{1}{2}+k\upsilon\right)\frac{\xi^2}{2}-k\upsilon\xi+\frac{k}{2}+k\sqrt{k\upsilon}-\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}-2\sqrt{k\upsilon}}{\sqrt{k\upsilon}\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}k^2\upsilon(\upsilon-1)\right). \qquad (2.2.14)$$

Bu yerdan hosilalarni hisoblab olamiz.

$$\chi^{I}(\xi)=\frac{P\ell^3}{D}\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}k\upsilon e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}k\upsilon e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\xi^3}{6}-\frac{\xi^2}{2}+\left(\frac{1}{2}+\sqrt{k\upsilon}\right)\xi-k\upsilon\right)$$

$$\chi^{II}(\xi)=\frac{P\ell^3}{D}\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\xi^2}{2}-\xi+\left(\frac{1}{2}+\sqrt{k\upsilon}\right)\right)$$

$$\chi^{III}(\xi)=\frac{P\ell^3}{D}\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi-1\right)$$

$$\chi^{IV}(\xi)=\frac{P\ell^3}{D}\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\frac{\sqrt{k\upsilon}}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{\sqrt{k\upsilon}}{k\upsilon}\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi\right) \qquad (2.2.15)$$

$$\chi^{V}(\xi)=\frac{P\ell^3}{D}\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\frac{1}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{1}{k\upsilon}\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}\right)$$

Sterjenning o`rta qatlamga nisbatan hisoblangan to`liq momenti

$$M=-D\left(1-\upsilon k\frac{d^2}{d\xi^2}\right)\frac{d^2\chi(\xi)}{d\xi^2} \qquad (2.2.16)$$

(2.2.15) dagi topilgan hosilalarni (2.58) tenglamaga qo`yamiz.

$$M=-Pl^3\left(\begin{array}{l}\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\xi^2}{2}-\xi+\left(\frac{1}{2}+\sqrt{k\upsilon}\right)-\\ -\upsilon k\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+1\right)\end{array}\right)$$

yoki

$$M=-Pl^3\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}(1-\upsilon k)-\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}(1-\upsilon k)+\frac{\xi^2}{2}-\xi+\left(\frac{1}{2}+\sqrt{k\upsilon}\right)-\upsilon k\right) \qquad (2.2.17)$$

Ko`ndalang kuch

$$Q=-D\left(1-\upsilon k\frac{d^2}{d\xi^2}\right)\frac{d^3\chi(\xi)}{d\xi^3}. \qquad (2.2.18)$$

Topilgan (2.2.15) tenglamadagi 3-chi va 5-chi tartibli hosilalarini (2.60) ga qo`yamiz.

$$Q=-Pl^3\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi-1\right)+$$

$$+Pl^3\upsilon k\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\frac{1}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{1}{k\upsilon}\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}\right)$$

yoki

$$Q=-Pl^3(\xi-1) \qquad (2.2.19)$$

To`ldiruvchi qatlamdagi ko`ndalang kuch

$$Q_3=-D\frac{d^3\chi(\xi)}{d\xi^3}. \qquad (2.2.20)$$

Topilgan (2.2.14) tenglamalarning uchinchi tartibli hosilasini olib qo`yamiz.

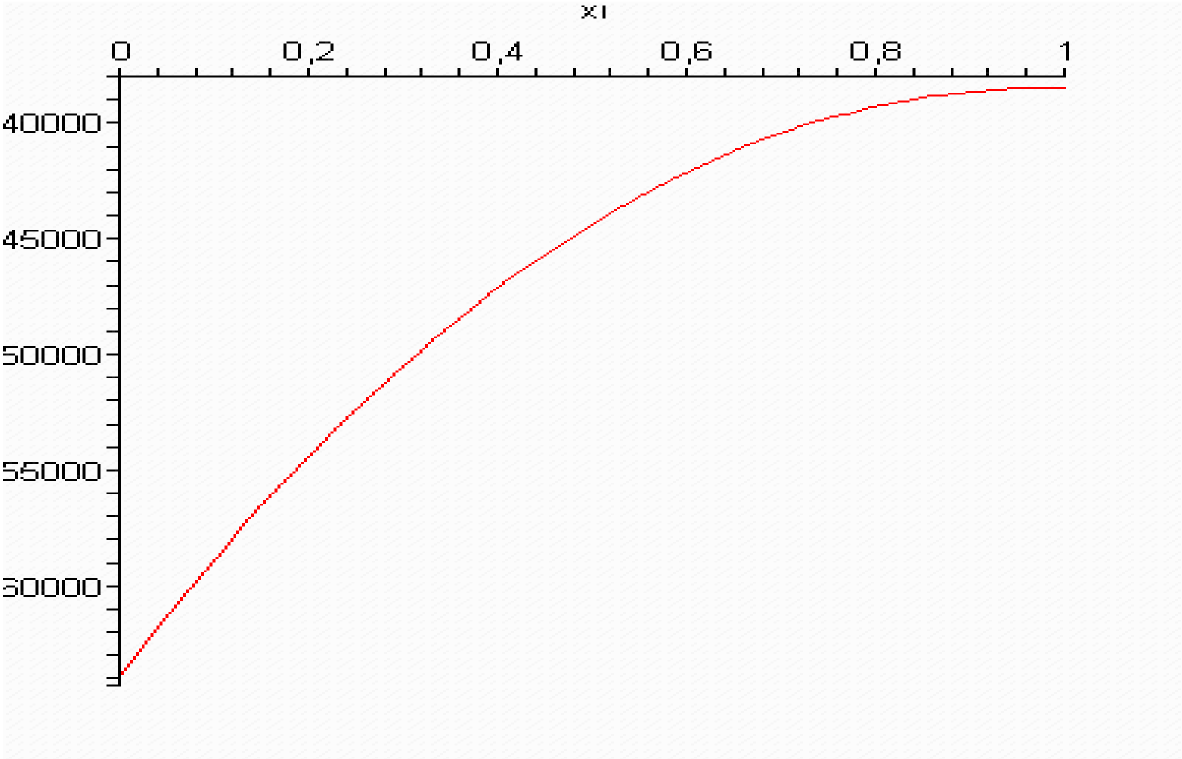
$$Q_3=-Pl^3\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi-1\right) \qquad (2.2.21)$$

To`ldiruvchi qatlamdagi eguvchi moment

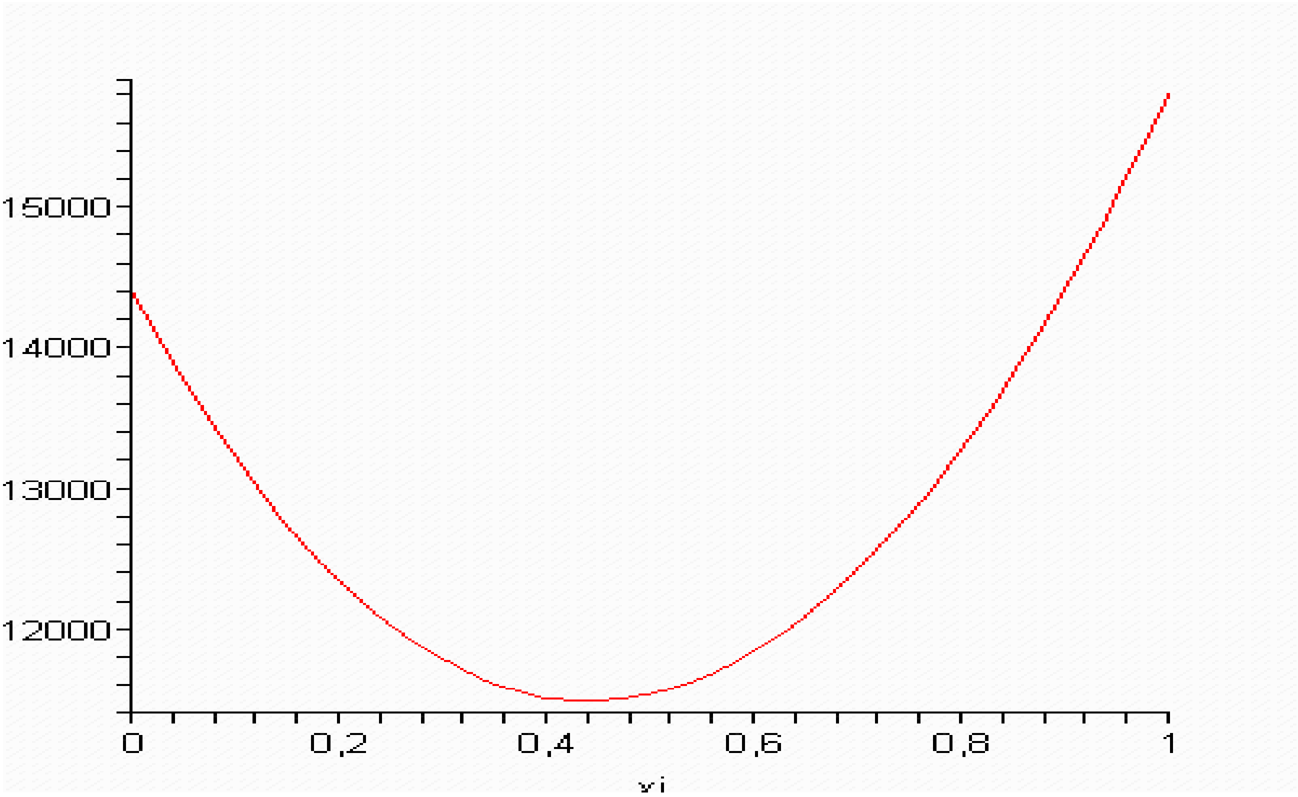
$$H=-Pl^3\frac{d^2\chi(\xi)}{d\xi^2} \qquad (2.2.22)$$

Yuqoridagi (2.2.15) tenglamalarning ikkinchi tartibli hosilasini qo`yamiz;

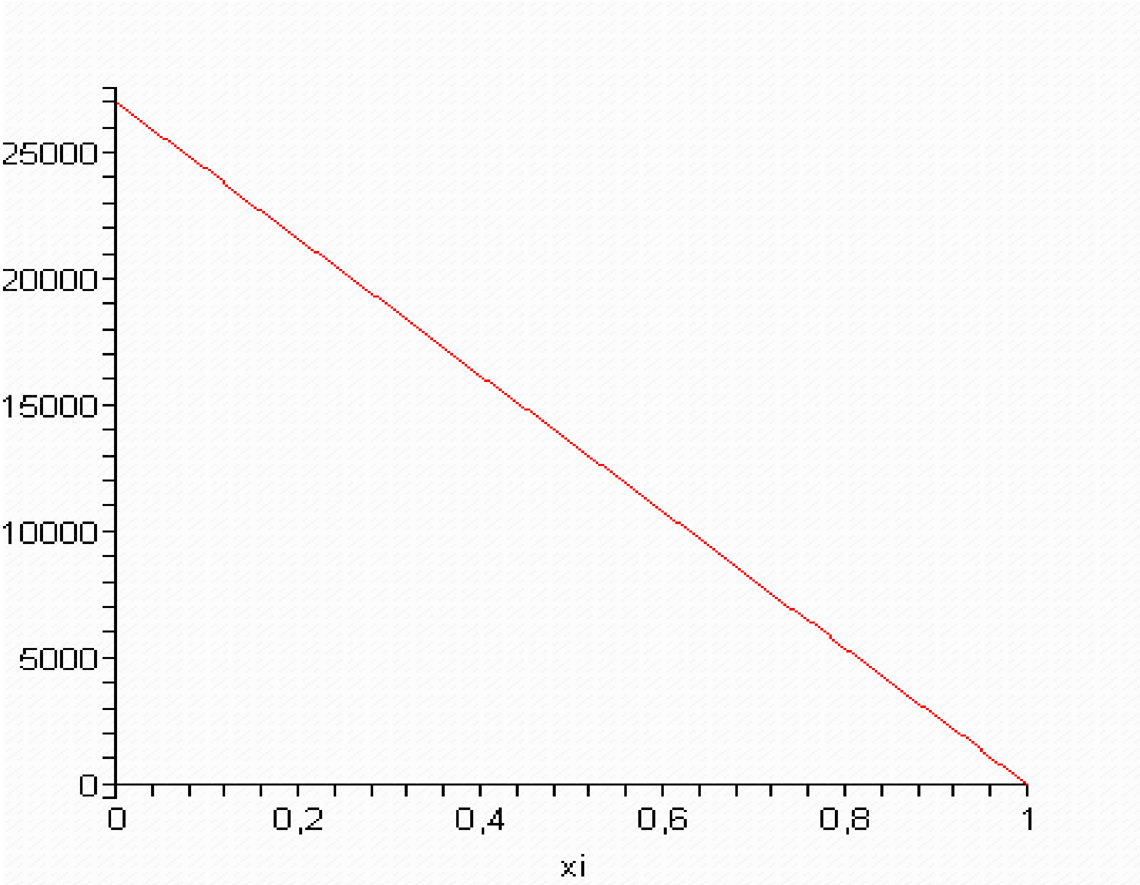
$$H=-Pl^3\left(\frac{e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}-\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}+\sqrt{k\upsilon}}{\left(e^{-\frac{\sqrt{k\upsilon}}{k\upsilon}}+e^{\frac{\sqrt{k\upsilon}}{k\upsilon}}\right)}\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\xi^2}{2}-\xi+\left(\frac{1}{2}+\sqrt{k\upsilon}\right)\right) \qquad (2.2.23)$$



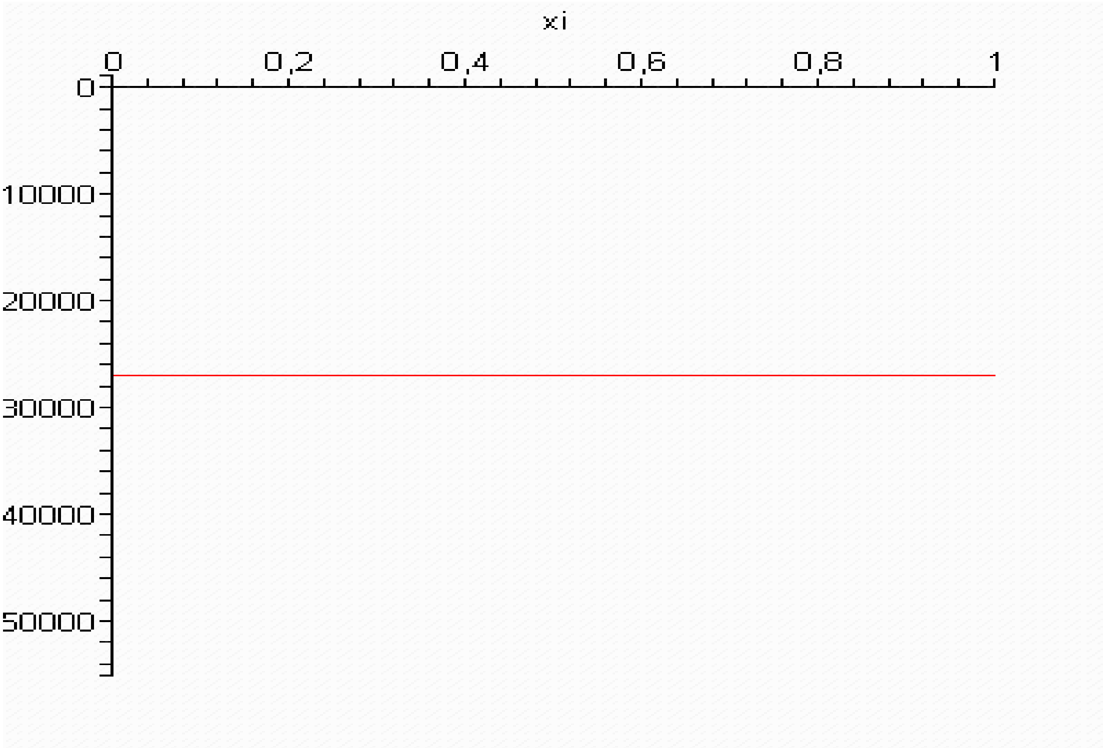
2.5-rasm sterjen kesimlarida eguvchi momentning koordinatadan o`zgarishi



2.6-rasm sterjen kesimlarida to`ldiruvchi qatlamdagi eguvchi momentning koordinatadan o`zgarishi



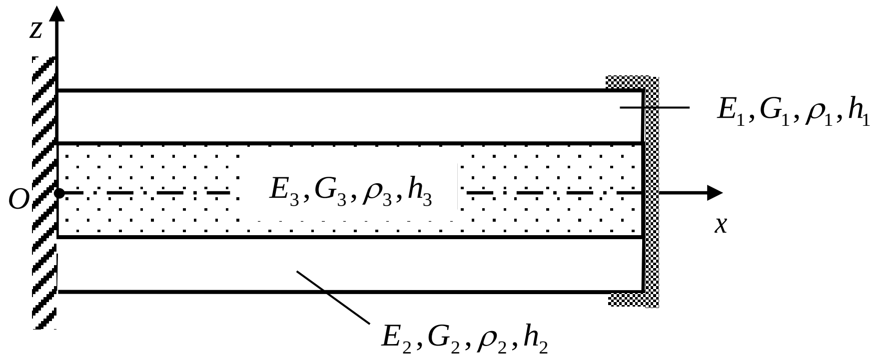
2.7-rasm sterjen kesimlaridagi qirquvchi kuchning koordinatadan o`zgarishi



2.8-rasm to`ldiruvchi qatlamda qirquvchi kuchning koordinatadan bog`liq o`zgarishi

§ 2.3. Bir uchi qistirib mahkamlangan, ikkinchi uchi erkin lekin cheksiz katta bikrlikka ega diafragmali uch qatlamli sterjenning statik yuk ta’sirida egilishi

Bir uchi qistirib mahkamlangan, ikkinchi uchi erkin lekin cheksiz katta bikrlikka ega diafragmali uch qatlamli sterjenni qaraymiz. Bu holda masalaning chegaraviy shartlari [1] ilmiy tadqiqot ishi natijalari asosida sterjenning $x = 0$ *va* $x = l$ uchlarida quyidagi ko`rinishda bo`ladi



2.3-rasm.

$x = 0$ bo`lganda

$$\left\{\left(1-\frac{h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\chi = 0, \qquad \frac{\partial \chi}{\partial x} = 0, \qquad \frac{\partial^3 \chi}{\partial x^3} = 0\right.$$

$x = l$ bo`lganda

$$\left\{\left(1-\upsilon\frac{h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\frac{d^2\chi}{dx^2} = 0, \qquad \frac{\partial^3 \chi}{\partial x^3} = 0, \qquad \frac{\partial^5 \chi}{\partial x^5} = 0.\right.$$

Bu yerda o`lchamsiz parametrlarga (1.3.1) formulalar bilan o`tib, quyidagi

$$\chi(\xi) - k\chi^{II}(\xi) = 0, \quad \chi^{I}(\xi) = 0, \quad \chi^{III}(\xi) = 0, \quad \text{dan} \quad \xi = 0. \tag{2.3.1}$$

$$\chi^{II}(\xi) - k\upsilon\chi^{IV}(\xi) = 0, \quad \chi^{III}(\xi) = 0, \quad \chi^{V}(\xi) = 0, \quad \text{dan} \quad \xi = 1. \tag{2.3.2}$$

chegaraviy shartlarga ega bo`lamiz. (2.3.1) tenglamadagi hosilalarni o`rniga topilgan.

$$\begin{cases}\chi^{I}(\xi)=\dfrac{P\ell^3}{D}\left(C_1k\upsilon\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2k\upsilon\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\dfrac{\xi^3}{6}+C_3\dfrac{\xi^2}{2}+C_4\xi+C_5\right)\\ \chi^{II}(\xi)=\dfrac{P\ell^3}{D}\left(C_1k\upsilon e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+C_2k\upsilon e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\dfrac{\xi^2}{2}+C_3\xi+C_4\right)\\ \chi^{III}(\xi)=\dfrac{P\ell^3}{D}\left(C_1\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\xi+C_3\right)\\ \chi^{VI}(\xi)=\dfrac{P\ell^3}{D}\left(C_1e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+C_2e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+1\right)\\ \chi^{V}(\xi)=\dfrac{P\ell^3}{D}\left(C_1\dfrac{\sqrt{k\upsilon}}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2\dfrac{\sqrt{k\upsilon}}{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}\right)\end{cases}\qquad(2.3.3)$$

hosilalardan foydalanamiz.

$$\begin{cases}C_1k^2\upsilon e^{\frac{\xi}{\sqrt{k\upsilon}}}(\upsilon-1)+C_2k^2\upsilon e^{-\frac{\xi}{\sqrt{k\upsilon}}}(\upsilon-1)+\dfrac{\xi^4}{24}+C_3\left(\dfrac{\xi^3}{6}-k\xi\right)+C_4\left(\dfrac{\xi^2}{2}-k\right)+C_5\xi+C_6-\dfrac{k\xi^2}{2}=0,\\ C_1k\upsilon\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2k\upsilon\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\dfrac{\xi^3}{6}+C_3\dfrac{\xi^2}{2}+C_4\xi+C_5=0,\\ C_1\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}-C_2\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\xi}{k\upsilon}}+\xi+C_3=0\end{cases}\qquad(2.3.4)$$

tenglamalarga ega bo`lamiz. Bu yerdan $\xi=0.$ bo`lganda

$$\begin{cases}C_1k^2\upsilon(\upsilon-1)+C_2k^2\upsilon(\upsilon-1)-C_4k+C_6=0,\\ C_1k\upsilon\sqrt{k\upsilon}-C_2k\upsilon\sqrt{k\upsilon}+C_5=0,\\ C_1\sqrt{k\upsilon}-C_2\sqrt{k\upsilon}+C_3=0\end{cases}\qquad(2.3.5)$$

Yuqoridagi (2.3.2) tenglamadagi hosilalarni o`rniga topilga (2.3.3) hosilalarni o`rniga qo`yamiz.

$$C_1k\upsilon e^{\frac{\xi}{\sqrt{k\upsilon}}}+C_2k\upsilon e^{-\frac{\xi}{\sqrt{k\upsilon}}}+\frac{\xi^2}{2}+C_3\xi+C_4-k\upsilon(C_1e^{\frac{\xi}{\sqrt{k\upsilon}}}+C_2e^{-\frac{\xi}{\sqrt{k\upsilon}}}+1)=0,$$

$$C_1\sqrt{k\upsilon}e^{\frac{\xi}{\sqrt{k\upsilon}}}-C_2\sqrt{k\upsilon}e^{-\frac{\xi}{\sqrt{k\upsilon}}}+\xi+C_3=0,$$

$$C_1\frac{\sqrt{k\upsilon}}{k\upsilon}e^{\frac{\xi}{\sqrt{k\upsilon}}}-C_2\frac{\sqrt{k\upsilon}}{k\upsilon}e^{-\frac{\xi}{\sqrt{k\upsilon}}}=0$$

tenglamalarga ega bo`lamiz.

Bu yerdan $\xi = 1$ bo`lganda

$$\frac{1}{2} + C_3 + C_4 - k\upsilon = 0,$$
$$C_1\sqrt{k\upsilon}e^{\frac{1}{\sqrt{k\upsilon}}} - C_2\sqrt{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}} + 1 + C_3 = 0, \qquad (2.3.6)$$
$$C_1\frac{\sqrt{k\upsilon}}{k\upsilon}e^{\frac{1}{\sqrt{k\upsilon}}} - C_2\frac{\sqrt{k\upsilon}}{k\upsilon}e^{-\frac{1}{\sqrt{k\upsilon}}} = 0$$

ifodalarga ega bo'lamiz.

Endi (2.3.5), (2.3.6) olti no'malumli olti tenglamalar sistemasini yechib quyidagilarni olamiz:

Buning uchun (2.3.5) ni 2-tenglamasidan C_5 ni, 3-tenglamasidan C_3 ni topamiz.

$$C_5 = (C_2 - C_1)k\upsilon\sqrt{k\upsilon} \qquad (2.3.7)$$

$$C_3 = (C_2 - C_1)\sqrt{k\upsilon} \qquad (2.3.8)$$

Endi (2.3.6) ni 3-tenglamasidan C_1 ni topamiz,

$$C_1 = C_2 e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}} \qquad (2.3.9)$$

(2.3.9) ni (2.3.7), (2.3.8) larga olib borib qo'ysak C_5 va C_3 larni C_2 ga bog'liq qiymatlariga ega bo'lamiz:

$$C_5 = C_2(1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}})k\upsilon\sqrt{k\upsilon} \qquad (2.3.10)$$

$$C_3 = C_2(1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}})\sqrt{k\upsilon} \qquad (2.3.11)$$

Endi (2.3.6) ni 1-tenglamasidan C_4 ni topamiz va (2.3.9), (2.3.11) larni olib borib qo'ysak C_4 ni C_2 ga bog'liq qiymatiga ega bo'lamiz:

$$C_4 = k\upsilon - k\upsilon\sqrt{k\upsilon}\, C_2(1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}) - \frac{1}{2} \qquad (2.3.12)$$

(2.3.5) ni 1-tenglamasidan C_6 ni topamiz va (2.3.9), (2.3.12) larni olib borib qo'ysak C_6 ni C_2 ga bog'liq ifodasiga ega bo'lamiz:

$$C_6 = k^2\upsilon - \frac{k}{2} - k^2\upsilon C_2\left(\sqrt{k\upsilon}\,(1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}) + (1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}})(\upsilon - 1)\right) \qquad (2.3.13)$$

Endi (2.3.6) ni 2-tenglamasiga (2.3.9) va (2.3.11) larni olib borib qo'ysak C_2 ni topib olamiz:

$$C_2 = \frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}} - 1\right)\sqrt{k\upsilon}}. \qquad (2.3.14)$$

Topilgan (2.3.14) ifodani (2.3.9) dagi C_2 o\`rniga qo\`yamiz

$$C_1 = \frac{e^{-2\frac{\sqrt{kv}}{kv}}}{\left(e^{-2\frac{\sqrt{kv}}{kv}} - 1\right)\sqrt{k\upsilon}}. \qquad (2.3.15)$$

Endi (2.3.14) ni (2.3.10) dagi C_2 o\`rniga qo\`yamiz

$$C_5 = \frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}} - 1\right)\sqrt{k\upsilon}}(1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}})k\upsilon\sqrt{kv}$$

shu ifodani soddalashtiramiz

$$C_5 = -k\upsilon \qquad (2.3.16)$$

(2.3.14) ni yuqoridagi (2.3.11) dagi C_2 o\`rniga qo\`yamiz

$$C_3 = \frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}} - 1\right)\sqrt{k\upsilon}}(1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}})\sqrt{k\upsilon}$$

u holda

$$C_3 = -1 \qquad (2.3.17)$$

Topilgan C_4 ham C_2 orqali soddalashtirib olamiz.

$$C_4 = k\upsilon - \sqrt{k\upsilon}\,\frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}} - 1\right)\sqrt{k\upsilon}}(1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}) - \frac{1}{2}$$

yoki

$$C_4 = k\upsilon + \frac{1}{2} \qquad (2.3.18)$$

C_6 ham shu ifodalar kabi C_2 orqali soddalashtirib olamiz.

$$C_6 = k^2\upsilon - \frac{k}{2} - k^2\upsilon \frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}} - 1\right)\sqrt{k\upsilon}} \left(\sqrt{k\upsilon}\,(1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}) + (1 - e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}})(\upsilon - 1)\right)$$

yoki

$$C_6 = k^2\upsilon - \frac{k}{2} + k^2\upsilon\left(1 + \frac{\upsilon - 1}{\sqrt{k\upsilon}}\right) \qquad (2.3.19)$$

Shunday qilib bir uchi qistirib mahkamlangan, ikkinchi uchi erkin lekin cheksiz katta bikrlikka ega diafragmali uch qatlamli sterjenni qaraymiz. P=const tekis taqsimlangan statik yuk ta’sirida egilishida $\chi(\xi)$ funksiyasi quyidagi formula bilan hisoblanadi.

$$\chi(\xi) = \frac{P\ell^3}{D}\left(\begin{array}{l} \dfrac{e^{-2\frac{\sqrt{kv}}{kv}}}{\left(e^{-2\frac{\sqrt{kv}}{kv}} - 1\right)\sqrt{k\upsilon}} k^2\upsilon^2 e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}} + \dfrac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}} - 1\right)\sqrt{k\upsilon}} k^2\upsilon^2 e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}} + \\ + \dfrac{\xi^4}{24} - \dfrac{\xi^3}{6} + \left(k\upsilon + \dfrac{1}{2}\right)\dfrac{\xi^2}{2} - k\upsilon\xi + k^2\upsilon - \dfrac{k}{2} + k^2\upsilon\left(1 + \dfrac{\upsilon - 1}{\sqrt{k\upsilon}}\right) \end{array}\right)$$

(2.3.20)

Sterjenning o`rta qatlamga nisbatan hisoblangan to`liq momenti

$$M = -D\left(1 - \upsilon k \frac{d^2}{d\xi^2}\right)\frac{d^2\chi(\xi)}{d\xi^2} \qquad (2.3.21)$$

tenglamaga ega bo`lamiz. Buning uchun $\chi^{I}(\xi)$ $\chi^{II}(\xi)$, $\chi^{III}(\xi)$, $\chi^{IV}(\xi)$, $\chi^{V}(\xi)$ hosilalarini hisoblab olamiz.

$$\chi^{I}(\xi) = \frac{P\ell^3}{D}\left(\frac{e^{-2\frac{\sqrt{kv}}{kv}}}{\left(e^{-2\frac{\sqrt{kv}}{kv}} - 1\right)} k\upsilon e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}} - \frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}} - 1\right)\sqrt{k\upsilon}} k\upsilon e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}} + \frac{\xi^3}{6} - \frac{\xi^2}{2} + \left(k\upsilon + \frac{1}{2}\right)\xi - k\upsilon\right)$$

$$\chi^{II}(\xi) = \frac{P\ell^3}{D}\left(\frac{e^{-2\frac{\sqrt{kv}}{kv}}}{\left(e^{-2\frac{\sqrt{kv}}{kv}} - 1\right)} \sqrt{k\upsilon} e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}} + \frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}} - 1\right)\sqrt{k\upsilon}} \sqrt{k\upsilon} e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}} + \frac{\xi^2}{2} - \xi + \left(k\upsilon + \frac{1}{2}\right)\right)$$

$$\chi^{III}(\xi)=\frac{P\ell^3}{D}\left(\frac{e^{-2\frac{\sqrt{kv}}{kv}}}{\left(e^{-2\frac{\sqrt{kv}}{kv}}-1\right)}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}-1\right)\sqrt{k\upsilon}}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi-1\right)$$

$$\chi^{IV}(\xi)=\frac{P\ell^3}{D}\left(\frac{e^{-2\frac{\sqrt{kv}}{kv}}}{\left(e^{-2\frac{\sqrt{kv}}{kv}}-1\right)}\frac{\sqrt{k\upsilon}}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\sqrt{k\upsilon}}{k\upsilon}\frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}-1\right)\sqrt{k\upsilon}}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+1\right)$$

$$\chi^{V}(\xi)=\frac{P\ell^3}{D}\left(\frac{e^{-2\frac{\sqrt{kv}}{kv}}}{\left(e^{-2\frac{\sqrt{kv}}{kv}}-1\right)}\frac{1}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{1}{k\upsilon}\frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}-1\right)\sqrt{k\upsilon}}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}\right) \qquad (2.3.22)$$

Topilgan (2.3.22) tenglamadagi 2-chi va 4-chi tartibli hosilalarini (2.3.21) tenglamani o`rniga qo`yamiz.

$$M=-Pl^3\left(\frac{e^{-2\frac{\sqrt{kv}}{kv}}}{\left(e^{-2\frac{\sqrt{kv}}{kv}}-1\right)}\sqrt{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}-1\right)\sqrt{k\upsilon}}\sqrt{k\upsilon}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\xi^2}{2}-\xi+\left(k\upsilon+\frac{1}{2}\right)\right)+$$

$$+Pl^3\upsilon k\left(\frac{e^{-2\frac{\sqrt{kv}}{kv}}}{\left(e^{-2\frac{\sqrt{kv}}{kv}}-1\right)}\frac{\sqrt{k\upsilon}}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\frac{\sqrt{k\upsilon}}{k\upsilon}\frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}-1\right)\sqrt{k\upsilon}}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+1\right)$$

yoki

$$M=-Pl^3\left(\frac{\xi^2}{2}-\xi+\frac{1}{2}\right) \qquad (2.3.22)$$

Ko`ndalang kuch

$$Q=-D\left(1-\upsilon k\frac{d^2}{d\xi^2}\right)\frac{d^3\chi(\xi)}{d\xi^3}. \qquad (2.3.23)$$

Endi (2.3.22) tenglamadagi 3-chi va 5-chi tartibli hosilalarini (2.3.23) tenglamaga qo`yamiz.

$$Q=-Pl^{3}\left(\left(\frac{e^{-2\frac{\sqrt{k\nu}}{k\nu}}}{\left(e^{-2\frac{\sqrt{k\nu}}{k\nu}}-1\right)}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}-1\right)\sqrt{k\upsilon}}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}+\xi-1\right)-\right.$$
$$\left.-\upsilon k\left(\frac{e^{-2\frac{\sqrt{k\nu}}{k\nu}}}{\left(e^{-2\frac{\sqrt{k\nu}}{k\nu}}-1\right)}\frac{1}{k\upsilon}e^{\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}-\frac{1}{k\upsilon}\frac{1}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}-1\right)\sqrt{k\upsilon}}e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}\right)\right)$$

ya'ni

$$Q=-Pl^{3}(\xi-1) \tag{2.3.24}$$

To`ldiruvchi qatlamdagi ko`ndalang kuch

$$Q_{3}=-D\frac{d^{3}\chi(\xi)}{d\xi^{3}}. \tag{2.3.25}$$

Topilgan (2.3.22) tenglamalarning uchinchi tartibli hosilasini olib (2.3.25) tenglamaga qo`yamiz.

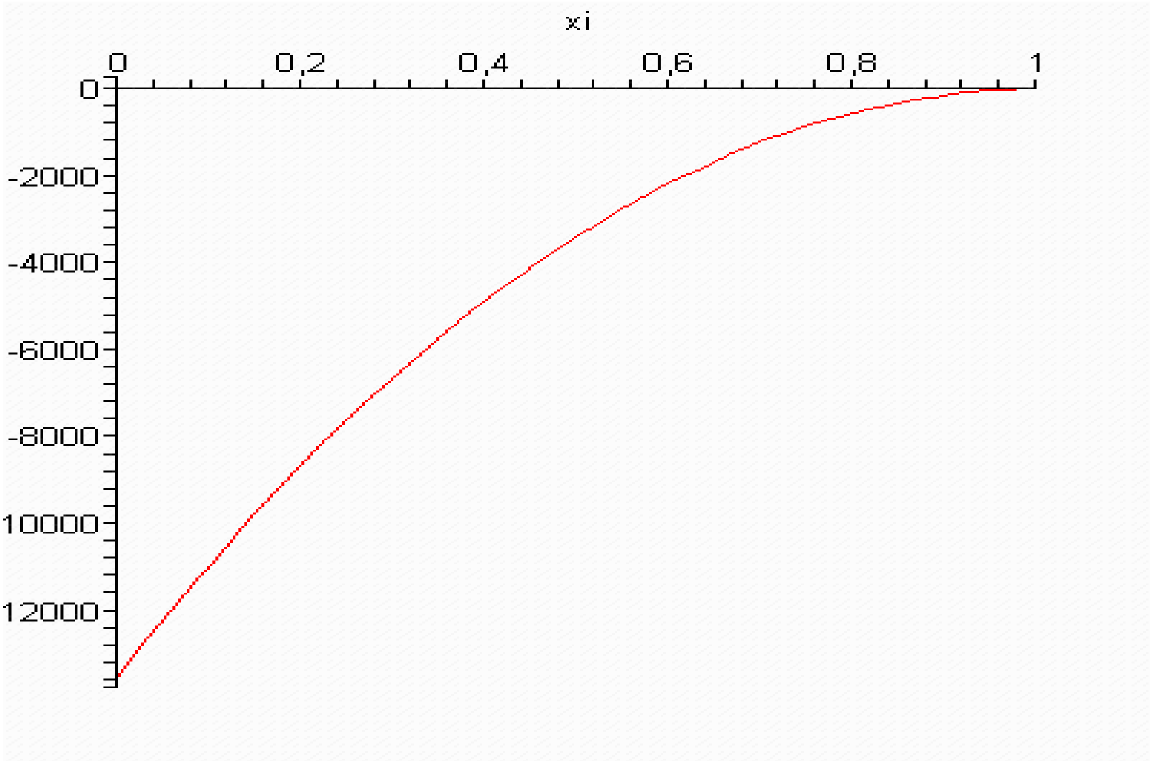
$$Q_{3}=-Pl^{3}\left(\frac{e^{-\frac{\sqrt{k\nu}}{k\nu}}}{\left(e^{-2\frac{\sqrt{k\nu}}{k\nu}}-1\right)}-\frac{e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}-1\right)\sqrt{k\upsilon}}+\xi-1\right) \tag{2.3.26}$$

To`ldiruvchi qatlamdagi eguvchi moment

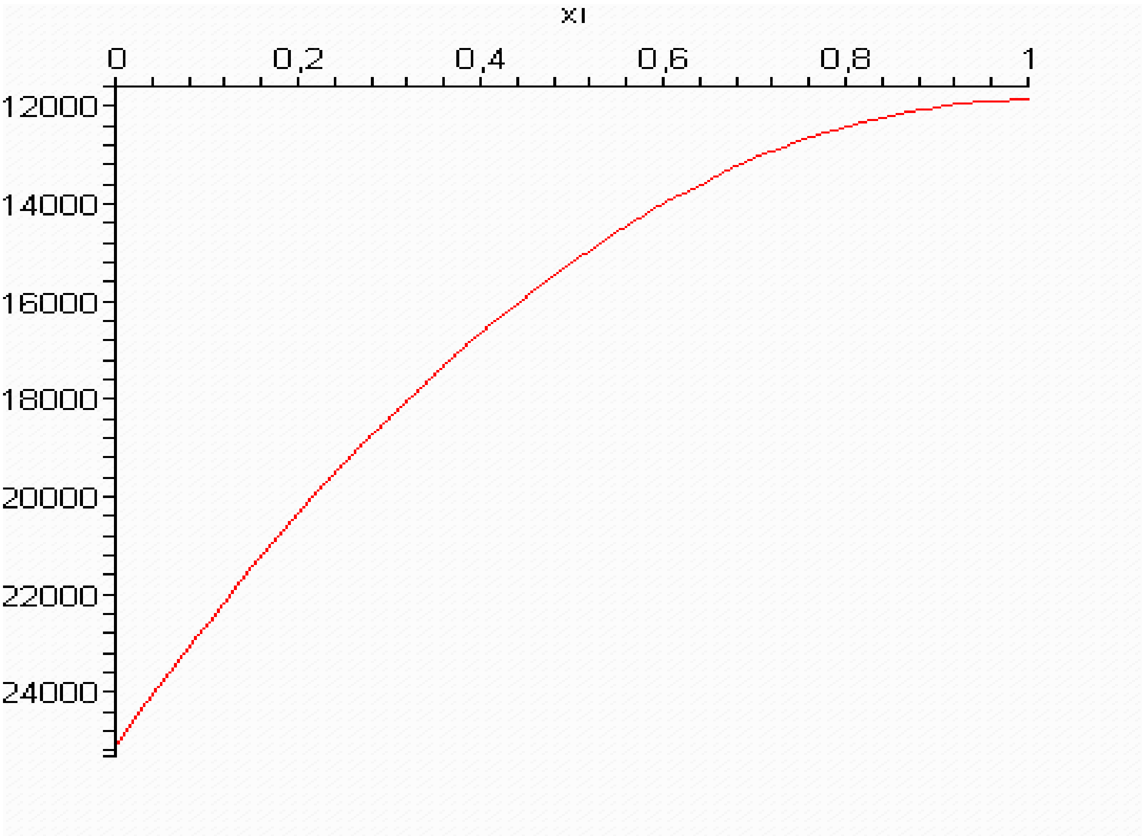
$$H=-Pl^{3}\,\frac{d^{2}\chi(\xi)}{d\xi^{2}} \tag{2.3.27}$$

Yuqoridagi (2.3.22) tenglamalarning ikkinchi tartibli hosilasini (2.3.27) qo`yamiz;

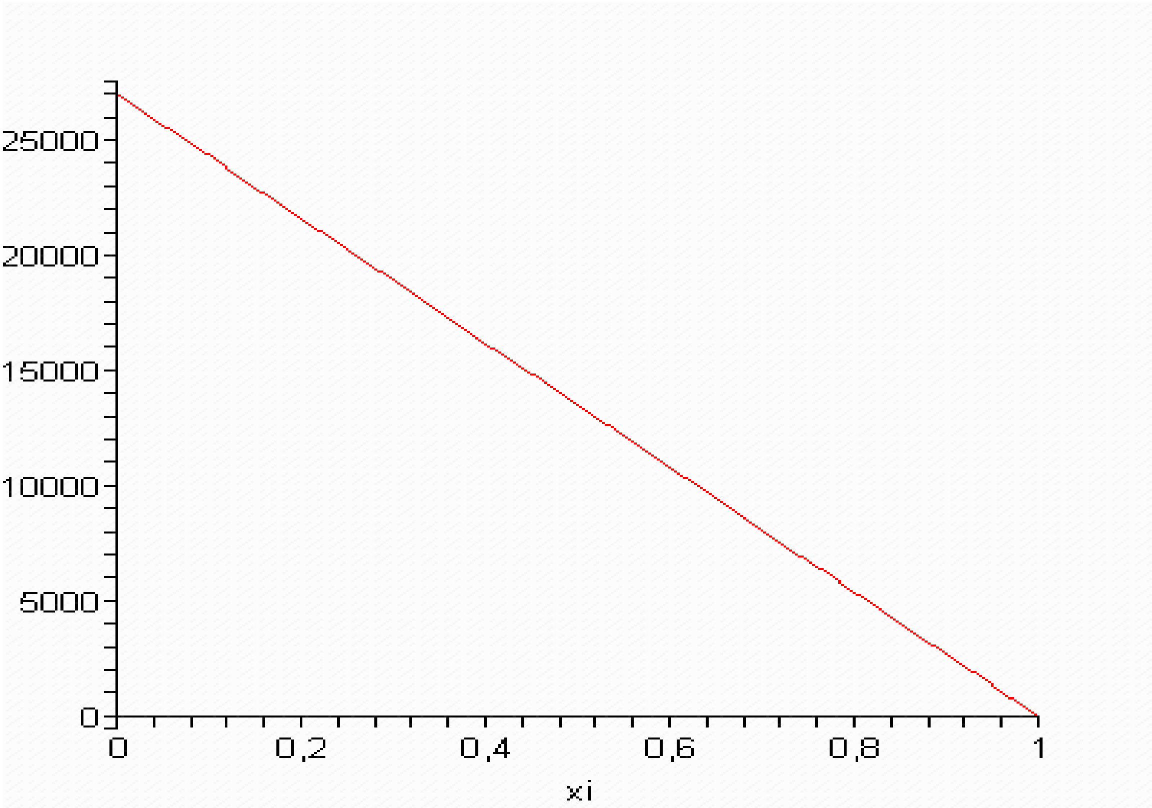
$$H=-Pl^{3}\left(\frac{e^{-\frac{\sqrt{k\nu}}{k\nu}}}{\left(e^{-2\frac{\sqrt{k\nu}}{k\nu}}-1\right)}\sqrt{k\upsilon}+\frac{e^{-\frac{\sqrt{k\upsilon}\,\xi}{k\upsilon}}}{\left(e^{-2\frac{\sqrt{k\upsilon}}{k\upsilon}}-1\right)}+\frac{\xi^{2}}{2}-\xi+\left(k\upsilon+\frac{1}{2}\right)\right) \tag{2.3.28}$$



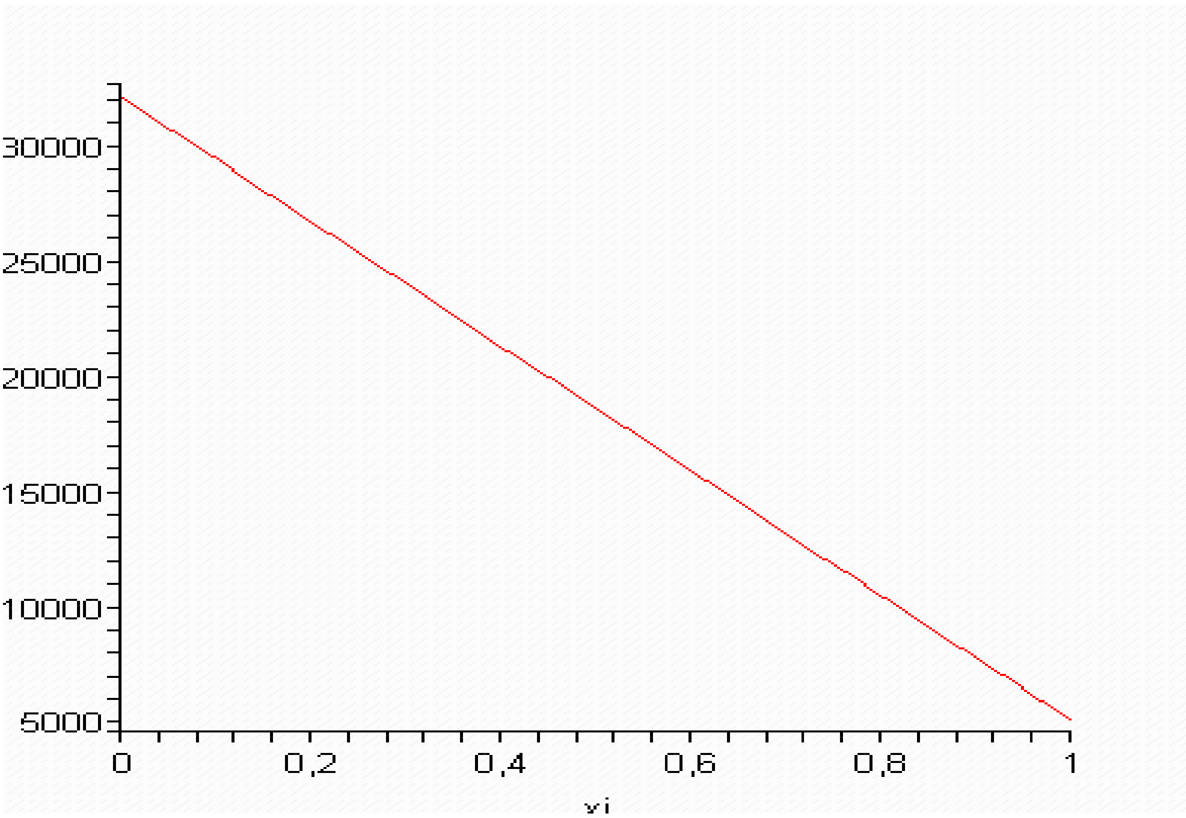
2.9-rasm sterjen kesimlarida eguvchi momentning koordinatadan o`zgarishi



2.10-rasm sterjen kesimlarida to`ldiruvchi qatlamdagi eguvchi momentning koordinatadan o`zgarishi



2.11-rasm sterjen kesimlaridagi qirquvchi kuchning koordinatadan o`zgarishi



2.12-rasm to`ldiruvchi qatlamda qirquvchi kuchning koordinatadan bog`liq o`zgarishi

III-BOB

UCH QATLAMLI STERJENNING XUSUSIY TEBRANISHLARI

§ 3.1. Uch qatlamli sterjenning xususiy tebranish tenglamasi va tebranish chastotasi.

Uch qatlamli sterjenning xususiy tebranishlari, tebranish shakli va chastotalarini hisoblash bir qator [5,70,117] ishlarda tadqiq etilgan. Xususiy chastotalarni hisoblash uchun [114,115] ishlarda muvozanat differensial tenglamalarining aniq yechimidan foydalanilgan. Hisoblashning taqribiy usullari, xususan Bubnov-Galerkin va Rits usullari, xususiy chastotalarni topish uchun [29,70] ishlarda foydalanilgan.

Ba’zi ishlarda ko`chish funksiyasi darajali qator ko`rinishida tasvirlangan va xususiy tebrnishlar chastotasining qiymatlari topilgan. Ushbu [41] usul bilan biz ham uchlari har xil mahkamlangan uch qatlamli sterjenning xususiy tebranish chastotalari va shakllarini hisoblaymiz.

Uch qatlamli sterjenning xususiy tebranish tenglamasini topish uchun

$$D\left(1-\frac{\upsilon h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\frac{\partial^4\chi(x,t)}{\partial x^4}+\rho hb\frac{\partial}{\partial t^2}\left(1-\frac{h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\chi(x,t)=Q(x,t) \quad (3.1.1)$$

$Q(x,t)=0$ deb hisoblash kifoya,

$$D\left(1-\frac{\upsilon h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\frac{\partial^4\chi(x,t)}{\partial x^4}+\rho hb\frac{\partial}{\partial t^2}\left(1-\frac{h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\chi(x,t)=0, \quad (3.1.2)$$

bu yerda

$\chi(x,t)$ – ko’chish funksiyasi;

D – sterjen materialining bikrligi;

υ – umumlashgan ko’chish;

h – sterjen qalinligi;

b – sterjen eni

t – vaqt;

x – bo’ylama koordinata;

Bu tenglamada o`lchamsiz koordinatalarga

$$\xi = \frac{x}{\ell}; \; k = \frac{h^2}{\beta \ell^2}; \; \omega^{*2} = \frac{\rho h b \omega^2 l^2}{D}, \tag{3.1.3}$$

formulalar yordamida o`tamiz. U holda (3.1.3) dan

$$D\left(1 - \upsilon k \frac{\partial^2}{\partial \xi^2}\right)\frac{\partial^4 \chi(\xi,t)}{\partial \xi^4} + \rho h b \frac{\partial}{\partial t^2}\left(1 - k \frac{\partial^2}{\partial \xi^2}\right)\chi(\xi,t) = 0 \tag{3.1.4}$$

ni olamiz.

Olingan (3.1.4) tenglamani yechish uchun $\chi(x,t)$ – ko'chish funksiyasini

$$\chi(\xi,t) = X(\xi)\sin(\omega t) \tag{3.1.5}$$

ko`rinishda izlaymiz. U holda (3.1.4) tenglama

$$\upsilon k X^{VI}(\xi) - X^{IV}(\xi) + \omega^{*2}\left[X(\xi) - kX^{II}(\xi)\right] = 0 \tag{3.1.6}$$

ko`rinishga keladi.

Erkin tebranishlar chastotasi va formasini aniqlashga Bubnov-Galerkin usuldan foydalanamiz. Variatsion tengamaning umumiy ko`rinishi qo`yidagicha.

$$\int_0^1 \left(\upsilon k X^{VI}(\xi) - X^{IV}(\xi) + \omega^{*2}\left[X(\xi) - kX^{II}(\xi)\right]\right)\delta w \, d\xi = 0 \tag{3.1.7}$$

ma'lumki

$$w(\xi,t) = \left(1 - \frac{h^2}{\beta}\frac{\partial^2}{\partial x^2}\right)\chi(x,t). \tag{3.1.8}$$

Bu yerda o`lchamsiz parametrlarga (3.1.3) formulalar bilan o`tib (3.1.7) ga qo`yib, topamiz.

$$\int_0^1 \left(\upsilon k X^{VI}(\xi) - X^{IV}(\xi) + \omega^{*2}\left[X(\xi) - kX^{II}(\xi)\right]\right)\delta\left(X(\xi) - kX^{II}(\xi)\right)d\xi = 0$$

(3.1.9)

ko`chish funksiyasini quyidagicha tasvirlaymiz.

$$X(\xi) = \sum_{i=1}^{m} c_i \sin(i\pi\xi) \tag{3.1.10}$$

(3.1.10) ning oltinchi tartibgacha bo`lgan hosilalarini topamiz.

$$\begin{aligned}
X^{I}(\xi) &= i\pi\sum_{i=1}^{m} c_i \cos(i\pi\xi), \\
X^{II}(\xi) &= -i^2\pi^2\sum_{i=1}^{m} c_i \sin(i\pi\xi), \\
X^{III}(\xi) &= i^3\pi^3\sum_{i=1}^{m} c_i \cos(i\pi\xi), \\
X^{IV}(\xi) &= -i^4\pi^4\sum_{i=1}^{m} c_i \sin(i\pi\xi), \\
X^{V}(\xi) &= i^5\pi^5\sum_{i=1}^{m} c_i \cos(i\pi\xi), \\
X^{VI}(\xi) &= -i^6\pi^6\sum_{i=1}^{m} c_i \sin(i\pi\xi),
\end{aligned} \tag{3.1.11}$$

topilgan (3.1.11) hosilalarni (3.1.9) tenglamaning o`rniga qo`yib yechib olamiz.

$$\int_0^1 \left\{ -\sum_{i=1}^{m} C_i i^4\pi^4\left(1+k\upsilon i^2\pi^2\right)\sin(i\pi\xi) + \omega^{*2}\sum_{i=1}^{m} C_i\left(1+ki^2\pi^2\right)\sin(i\pi\xi)\right\}\cdot\left(1+km^2\pi^2\right)\sin(m\pi\xi)d\xi = 0$$

Bu sistema noldan farqli yechimlarga ega bo`lishi uchun koeffitsiyentlar matrisasining determinati nolga teng bo`lishi zarur, ya’ni

$$\sum_{n=1}^{N} C_n \int_0^1 \left[n^4\pi^4\left(1+k\upsilon n^2\pi^2\right) - \omega^{*2}\left(1+kn^2\pi^2\right)\right]\sin(n\pi\xi)\left(1+km^2\pi^2\right)\sin(m\pi\xi)d\xi = 0 \tag{3.1.12}$$

Ammo bu integral faqat $n = m$ bo`lganagina noldan farqli bo`lishi mumkin. Qolgan hollarda $\sin k\pi x$ funksiyalar sistemasining (k=1,2,3…) ortaganalligi uchun nolga teng. U holda

$$\sum_{n=1}^{N} C_n \int_0^1 \omega^{*2}\left(1+kn^2\pi^2\right)^2 \sin^2(n\pi\xi)d\xi - \int_0^1 n^4\pi^4\left(1+k\upsilon n^2\pi^2\right)\left(1+km^2\pi^2\right)\sin^2(n\pi\xi)d\xi = 0 .$$

$$\omega^{*2} = \frac{\int_0^1 n^4\pi^4\left(1+k\upsilon n^2\pi^2\right)\left(1+km^2\pi^2\right)\sin^2(n\pi\xi)d\xi}{\int_0^1 \omega^{*2}\left(1+kn^2\pi^2\right)^2 \sin^2(n\pi\xi)d\xi} \tag{3.1.13}$$

Bu yerdan integrallashni amalga oshirib uch qatlamli sterjen xususiy chastotalarini aniqlash formulasiga ega bo`lamiz.

$$\omega^{*2} = \frac{n^4\pi^4\left(1+km^2\pi^2\right)}{1+kn^2\pi^2} \tag{3.1.14}$$

§ 3.2. Uchlari diafgramali va erkin tayangan uch qatlamli sterjenning xususiy tebranishlari.

Erkin tayangan uch qatlamli sterjenni qaraymiz. Sterjen uchlarini boshqa turdagi mahlamlanishlar uchun $\chi_i(\xi)$ larni quyidagicha tanlab olamiz

$$\chi_i(\xi) = \sum_{j=1}^{i+6} a_{i,j} \xi^{j-1} \tag{3.2.1}$$

bu yerda

$$\varphi_i(\xi) = \sum_{j=1}^{i+6} a_{i,j} \xi^{j-1} + a_{n,n+6} \xi^{j+5}, \quad a_{n,n+6} = 1 \tag{3.2.2}$$

Bu yechimni chegaraviy shartlarga qo`yib hisoblaymiz.

Uchlari diafragmali va erkin tayangan uch qatlamli sterjenni qaraymiz. Bu holda masalaning chegaraviy shartlari [1] ilmiy tadqiqot ishi natijalari asosida sterjenning $x = 0$ *va* $x = l$ uchlarida quyidagi ko`rinishda bo`ladi

$$\left(1 - \frac{h^2}{\beta} \frac{\partial^2}{\partial x^2}\right)\chi = 0, \ \left(1 - \frac{\upsilon h^2}{\beta} \frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 \chi}{\partial x^2} = 0, \qquad \frac{\partial^3 \chi}{\partial x^3} = 0. \tag{3.2.3}$$

Olingan bu (3.2.3) chegaraviy shartlarda (3.1.3) yordamida o`lchamsiz parametrlarga o`tib

$$\chi(\xi) - k\chi''(\xi) = 0, \ \chi''(\xi) - k\upsilon\chi^{IV}(\xi) = 0, \ \chi'''(\xi) = 0, \quad \text{dan} \quad \xi = 0, \quad \xi = 1. \tag{3.2.4}$$

Yuqoridagi (3.2.2) tenglamani $i = 1$ bo`lgan holda qarab chiqamiz. U holda (3.2.2)

$$\chi_1(\xi) = a_{11} + a_{12}\xi + a_{13}\xi^2 + a_{14}\xi^3 + a_{15}\xi^4 + a_{16}\xi^5 + \xi^6 \tag{3.2.5}$$

ko`rinishga ega bo`ladi. Hosil qilingan (3.2.5) ni (3.2.3) chegaraviy shartlarga qo`yib noma'lum koeffitsientlar $a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}$ larni aniqlaymiz. Olingan (3.2.3) ni ketma-ket hosilalarni aniqlaymiz.

$$\begin{cases} \chi_1^I(\xi) = a_{12} + 2a_{13}\xi + 3a_{14}\xi^2 + 4a_{15}\xi^3 + 5a_{16}\xi^4 + 6\xi^5; \\ \chi_1^{II}(\xi) = 2a_{13} + 6a_{14}\xi + 12a_{15}\xi^2 + 20a_{16}\xi^3 + 30\xi^4; \\ \chi_1^{III}(\xi) = 6a_{14} + 24a_{15}\xi + 60a_{16}\xi^2 + 120\xi^3; \\ \chi_1^{IV}(\xi) = 24a_{15} + 120a_{16}\xi + 360\xi^2. \end{cases}$$

Topilgan hosilalarni (3.2.4) tenglamaga $\xi = 0$ bo`lganga qo`yamiz. U holda

$$\begin{cases} a_{11} - 2ka_{13} = 0 \\ a_{12} - 12k\upsilon a_{15} = 0 \\ a_{14} = 0 \end{cases} \qquad (3.2.6)$$

hosil bo`ladi.

Endi $\xi = 1$ bo'lganda (3.2.4) tenglamani yechamiz.

$$\begin{cases} a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + 1 - k(2a_{13} + 6a_{14} + 12a_{15} + 20a_{16} + 30) = 0 \\ 2a_{13} + 6a_{14} + 12a_{15} + 20a_{16} + 30 - k\upsilon(24a_{15} + 120a_{16} + 360) = 0 \\ 6a_{14} + 24a_{15} + 60a_{16} + 120 = 0 \end{cases} \qquad (3.2.7)$$

(3.2.7) tenglamaga topilgan (3.2.6) tenglamaning qiymatlarini qo`yib hisoblaymiz.

$$\begin{cases} 2ka_{13} + a_{12} + a_{15} + a_{16} + 1 - k(2a_{13} + 12a_{15} + 20a_{16} + 30) = 0 \\ 2a_{13} + 12a_{15} + 20a_{16} + 30 - k\upsilon(24a_{15} + 120a_{16} + 360) = 0 \\ 24a_{15} + 60a_{16} + 120 = 0 \end{cases} \qquad (3.2.8)$$

(3.2.8) tenglamalar sistemasini 3-tenglamasidan a_{15} ni aniqlaymiz.

$$2a_{15} + 5a_{16} = -10;$$

yoki

$$a_{15} = -5 - \frac{5}{2}\mathrm{a}_{16}; \qquad (3.2.9)$$

(3.2.8) tenglamaning birinchi va ikkinchi tenglamasini ayiramiz va topilgan (3.2.6), (3.2.9) ni qo`yamiz va a_{16} ni quyidagicha topamiz.

$$a_{16} = -3 \qquad (3.2.10)$$

(3.2.10) ni (3.2.9) ga qo`yib

$$a_{15} = \frac{5}{2}; \qquad (3.2.11)$$

hosil qilamiz. Endi (3.2.11) va (3.2.10) ni (3.2.8) ning birinchi tenglamasiga qo`yib a_{13} ni aniqlaymiz

$$a_{13} = 30k\upsilon\,. \qquad (3.2.12)$$

Endi (3.2.12) ni (3.2.6) ning birinchi tenglamasiga qo`yib a_{11} ni aniqlaymiz

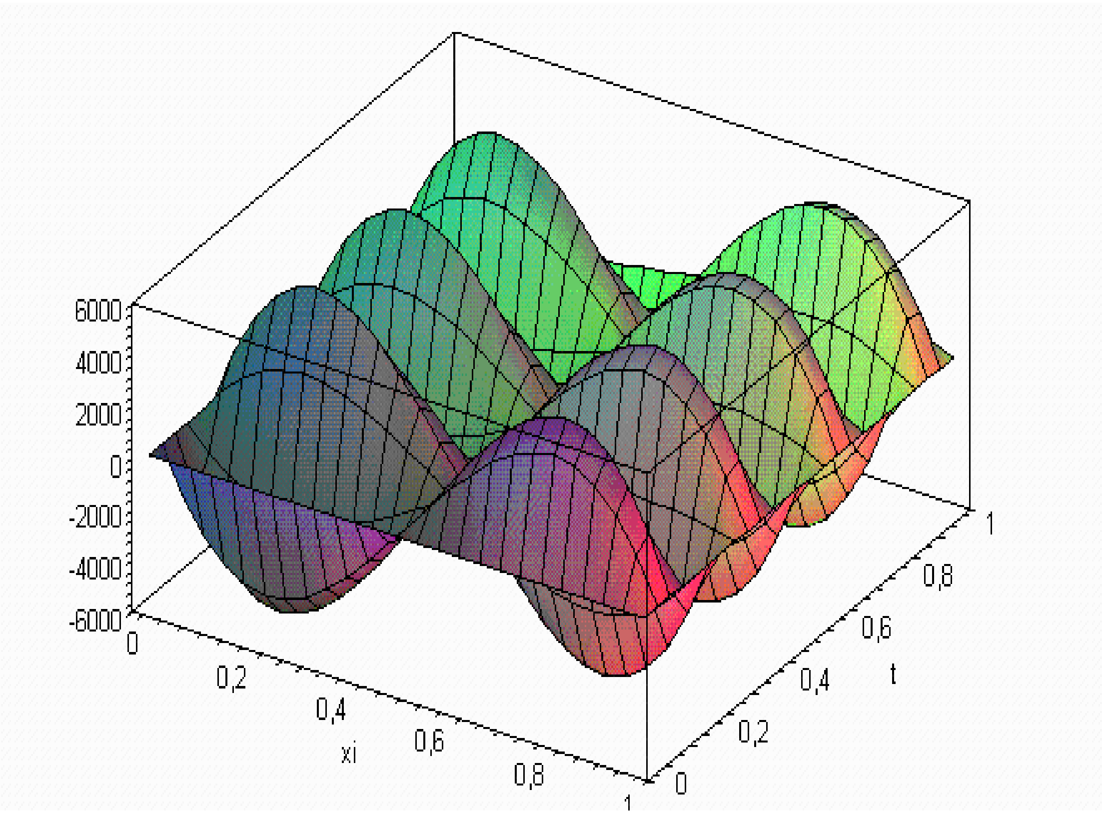
$$a_{11}=60k^2\upsilon\,. \tag{3.2.13}$$

Yuqoridagi (3.2.7) ni birinchi tenglamasidan a_{12} topamiz.

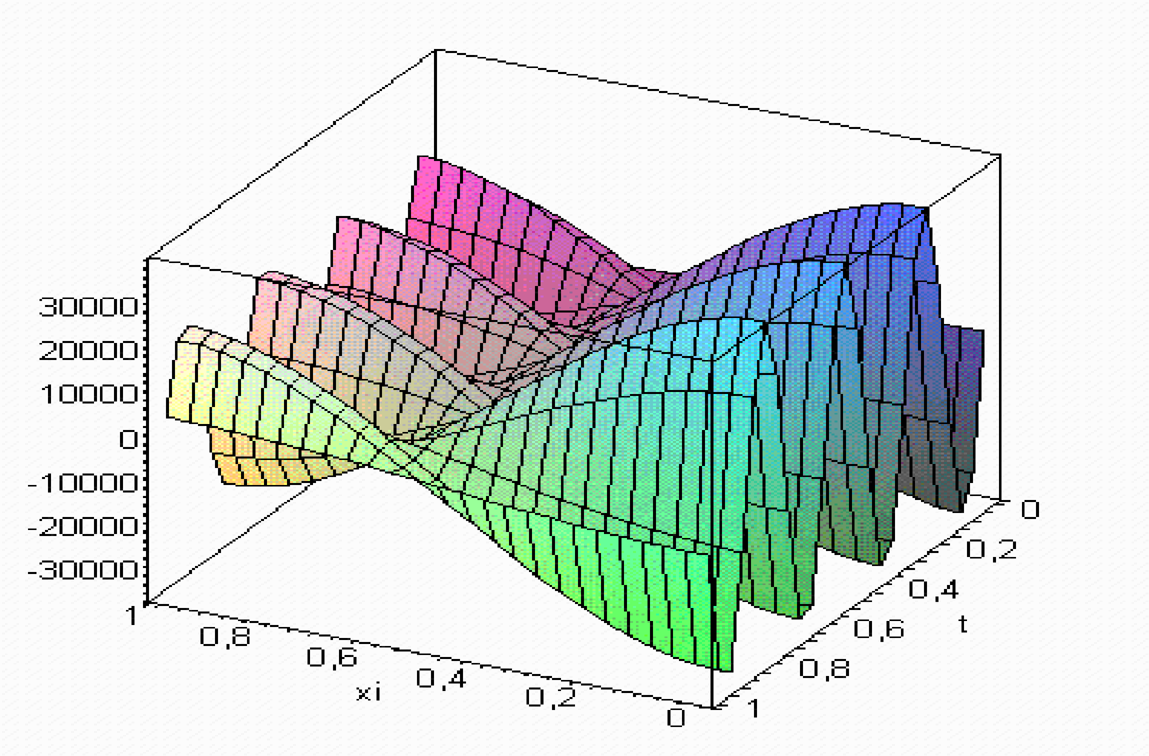
$$a_{12}=-30k\upsilon-\frac{1}{2} \tag{3.2.14}$$

Topilgan no'malumlarni bitta sistema qilib quyidagicha yozamiz

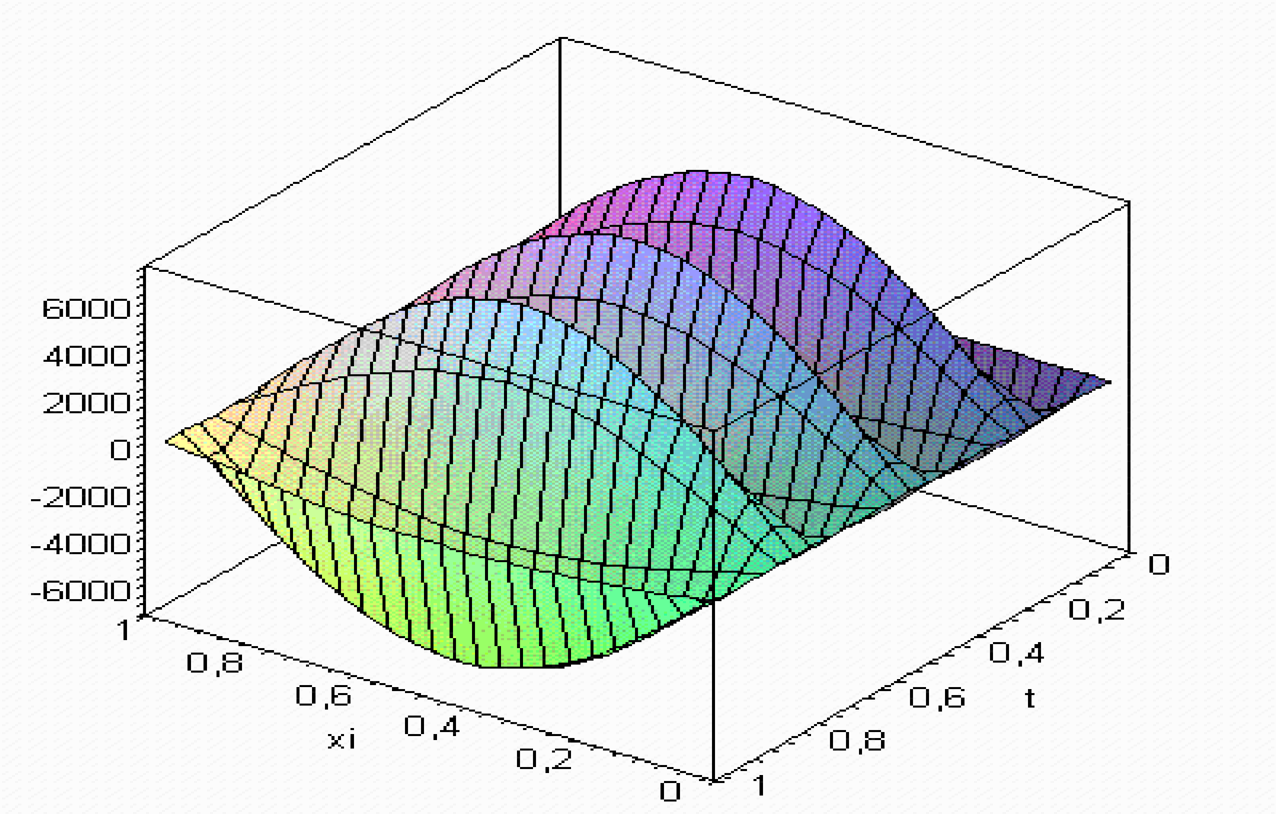
$$\begin{cases} a_{11}=60k^2\upsilon; \\ a_{12}=-30k\upsilon-\dfrac{1}{2}; \\ a_{14}=0; \\ a_{13}=30k\upsilon; \\ a_{15}=\dfrac{5}{2}; \\ a_{16}=-3. \end{cases}$$



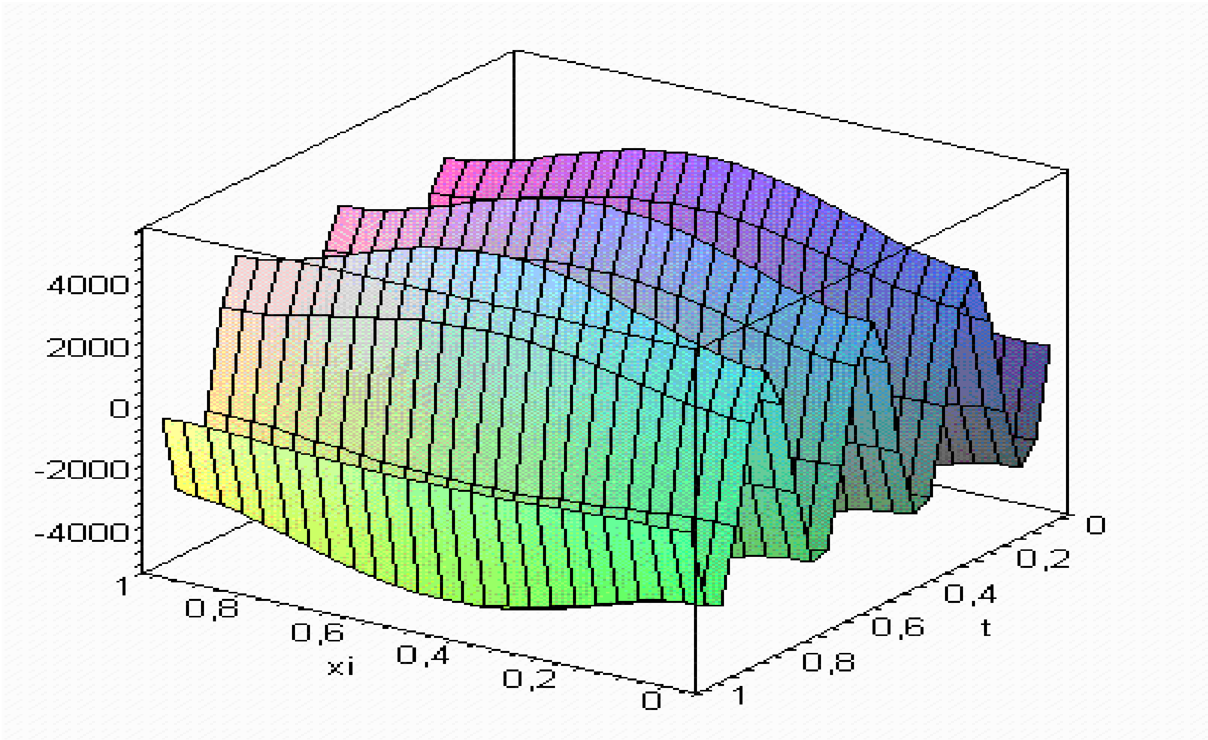
3.1-rasm to`ldiruvchi qatlamda qirquvchi kuchning koordinata va vaqtdan bog`liq o`zgarishi



3.2-rasm sterjen kesimlaridagi qirquvchi kuchning koordinata va vaqtdan bog`liq o`zgarishi



3.3-rasm sterjen kesimlarida eguvchi momentning koordinata va vaqtdan bog`liq o`zgarishi



3.4-rasm sterjen kesimlarida to`ldiruvchi qatlamdagi eguvchi momentning koordinata va vaqtdan bog`liq o`zgarishi

§ 3.3. Bir uchi qistirib mahkamlangan, ikkinchi uchi erkin uch qatlamli sterjenning xususiy tebranishi.

Bir uchi qistirib mahkamlangan, ikkinchi uchi erkin uch qatlamli sterjenni qaraymiz. Bu holda masalaning chegaraviy shartlari, [5] ilmiy tadqiqot ishi natijalari asosida, sterjenning uchlarida quyidagi ko`rinishda bo`ladi:

1) $x=0$ uchida

$$\begin{cases} \left(1-\frac{h^2}{\beta}\frac{d^2}{dx^2}\right)\chi=0, \\ \frac{d\chi}{dx}=0, \\ \frac{d^3\chi}{dx^3}=0. \end{cases} \qquad (3.3.1)$$

2) $x=l$ uchida

$$\begin{cases} \frac{d^2\chi}{dx^2}=0, \\ \frac{d^4\chi}{dx^4}=0, \\ \left(1-\frac{\upsilon h^2}{\beta}\frac{d^2}{dx^2}\right)\frac{d^3\chi}{dx^3}=0. \end{cases} \qquad (3.3.2)$$

Olingan (3.3.1), (3.3.2) chegaraviy shartlarda (3.1.3) formulalar yordamida o’lchamsiz patametrlarga o’tib

1) $\xi=0$ bo`lganda

$$\begin{cases} \chi(\xi)-k\chi''(\xi)=0, \\ \chi'(\xi)=0, \\ \chi'''(\xi)=0, \end{cases} \qquad (3.3.3)$$

2) $\xi=1$ bo`lganda

$$\begin{cases} \chi''(\xi)=0, \\ \chi^{IV}(\xi)=0, \\ \chi'''(\xi)-k\upsilon\chi^{V}(\xi)=0, \end{cases}$$

(3.3.4)

chegaraviy shartlarga ega bo`lamiz. Hosil qilingan (3.2.5) ni (3.3.3) va (3.3.4) chegaraviy shartlarga qo`yib noma’lum koeffitsientlar $a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}$ larni aniqlaymiz. Olingan (3.2.5) ni ketma ket hosilalarni aniqlaymiz.

$$\begin{cases} \chi_1^{I}(\xi)=a_{12}+2a_{13}\xi+3a_{14}\xi^2+4a_{15}\xi^3+5a_{16}\xi^4+6\xi^5; \\ \chi_1^{II}(\xi)=2a_{13}+6a_{14}\xi+12a_{15}\xi^2+20a_{16}\xi^3+30\xi^4; \\ \chi_1^{III}(\xi)=6a_{14}+24a_{15}\xi+60a_{16}\xi^2+120\xi^3; \\ \chi_1^{IV}(\xi)=24a_{15}+120a_{16}\xi+360\xi^2; \\ \chi_1^{V}(\xi)=120a_{16}+720\xi; \\ \chi_1^{VI}(\xi)=720; \end{cases}$$

Topilgan hosilalarni (3.3.1) tenglamaga $\xi=0$ bo`lganga qo`yamiz. U holda

$$\begin{cases} a_{11} - 2ka_{13} = 0 \\ a_{12} = 0 \\ a_{14} = 0 \end{cases} \qquad (3.3.5)$$

hosil bo`ladi.

Endi $\xi = 1$ bo'lganda (3.3.4) tenglamani yechamiz.

$$\begin{cases} 2a_{13} + 6a_{14} + 12a_{15} + 20a_{16} + 30 = 0 \\ 24a_{15} + 120a_{16} + 360 = 0 \\ 6a_{14} + 24a_{15} + 60a_{16} + 120 - k\upsilon(120a_{16} + 720) = 0 \end{cases} \qquad (3.3.6)$$

(3.3.6) tenglamaga topilgan (3.3.5) tenglamaning qiymatlarini qo`yib hisoblaymiz.

$$\begin{cases} 2a_{13} + 12a_{15} + 20a_{16} + 30 = 0 \\ 24a_{15} + 120a_{16} + 360 = 0 \\ 24a_{15} + 60a_{16}(1 - 2k\upsilon) + 120 - 720k\upsilon = 0 \end{cases} \qquad (3.3.7)$$

(3.3.7) tenglamalar sistemasini ikkinchi tenglamasidan uchunchi tenglamasini ayirib a_{16} ni aniqlaymiz

$$a_{16} = \frac{-3 - 12k\upsilon}{1 + 2k\upsilon}; \qquad (3.3.8)$$

(3.3.8) ni (3.3.7) ning ikkinchi tenglamasiga qo`yib a_{15} ni topamiz

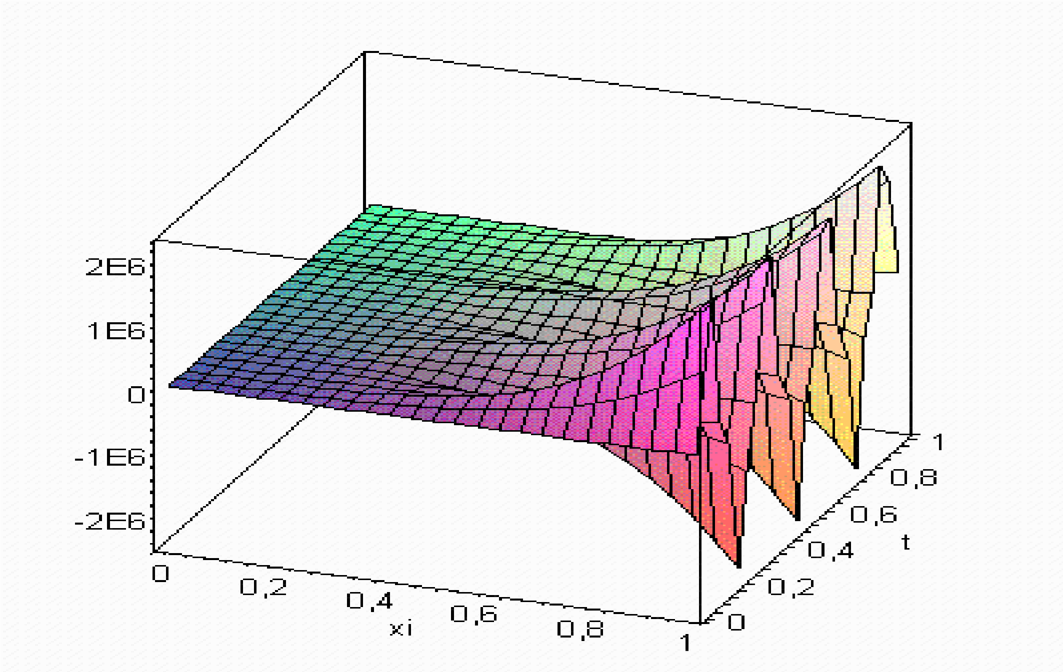
$$a_{15} = \frac{90k\upsilon}{1 + 2k\upsilon}; \qquad (3.3.9)$$

(3.3.8) va (3.3.9) ni (3.3.7) tenglamaning birinchi tenglamasiga qo`yib a_{13} ni topamiz

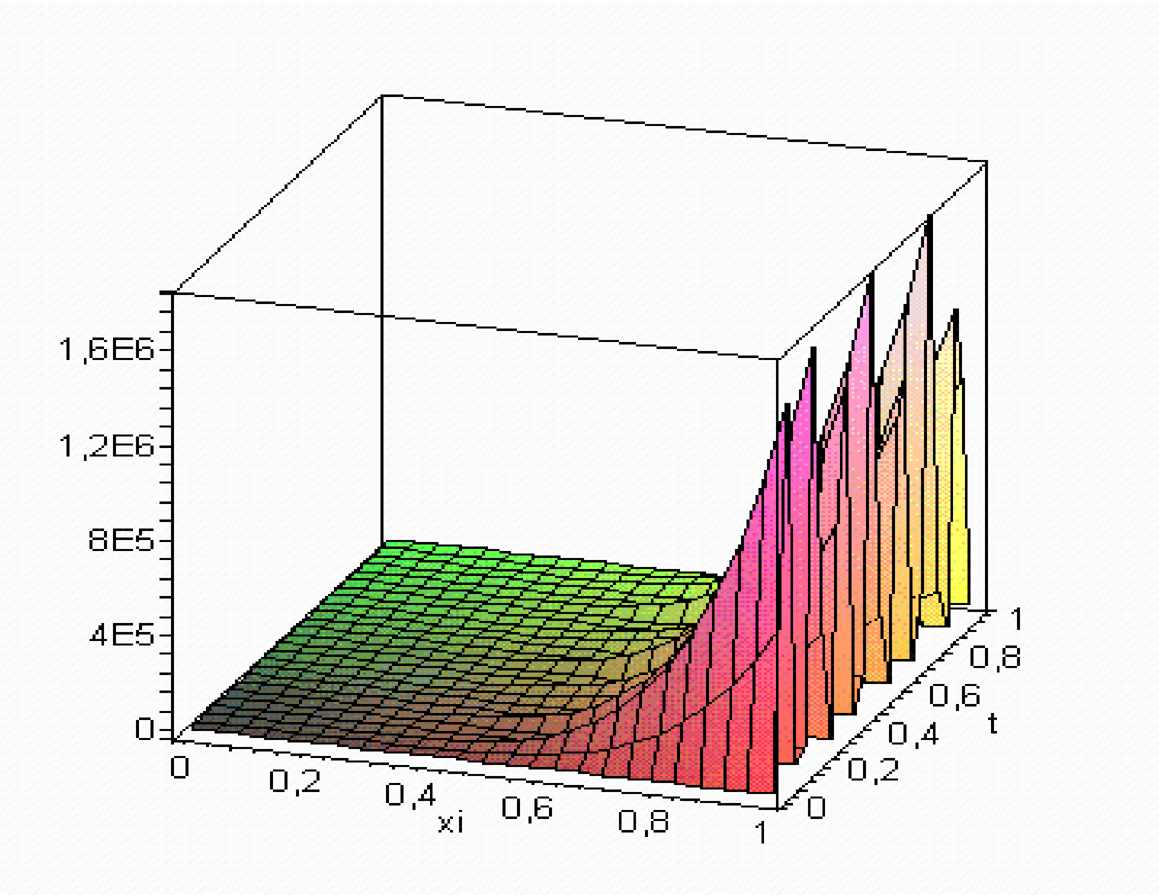
$$a_{13} = \frac{-530k\upsilon + 15}{1 + 2k\upsilon}; \qquad (3.3.10)$$

Topilgan no'malumlarni bitta sistema qilib quyidagicha yozamiz

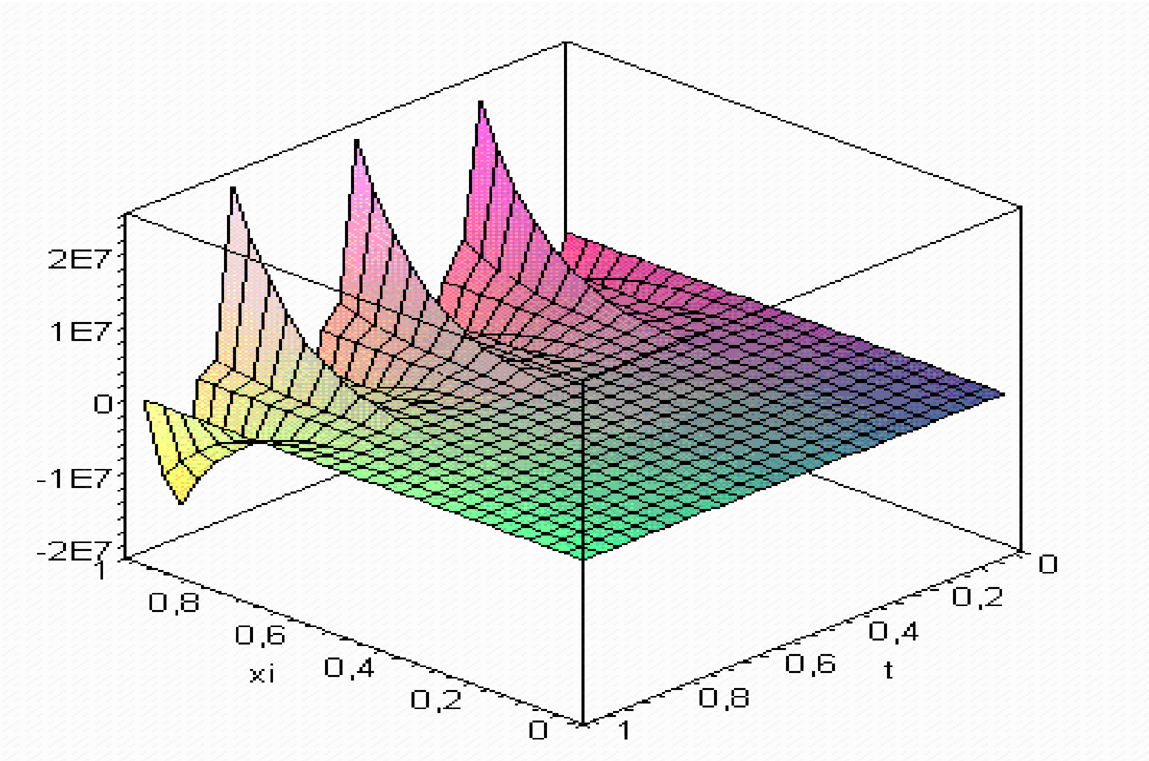
$$a_{11} - 2ka_{13} = 0;\ a_{12} = 0;\ a_{14} = 0;\ a_{13} = \frac{-530k\upsilon + 15}{1 + 2k\upsilon};\ a_{15} = \frac{90k\upsilon}{1 + 2k\upsilon};\ a_{16} = \frac{-3 - 12k\upsilon}{1 + 2k\upsilon}.$$



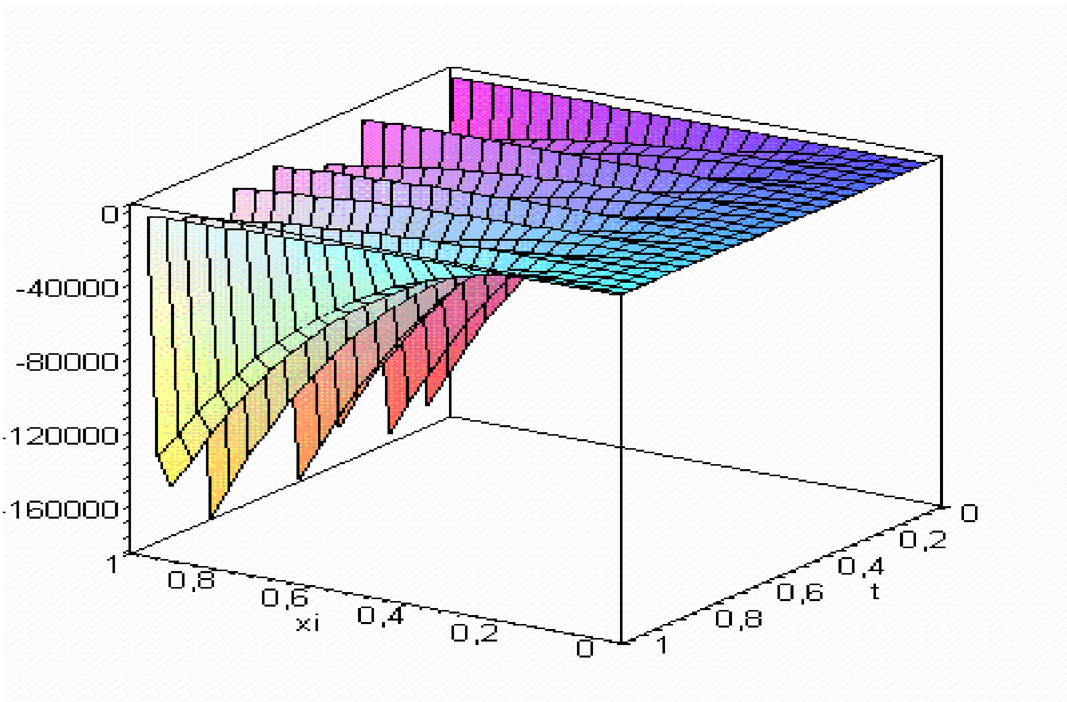
3.5-rasm to`ldiruvchi qatlamda qirquvchi kuchning koordinata va vqatdan bog`liq o`zgarishi



3.6-rasm sterjen kesimlarida eguvchi momentning koordinata va vqatdan bog`liq o`zgarishi



3.7-rasm sterjen kesimlaridagi qirquvchi kuchning koordinata va vaqtdan bog`liq o`zgarishi



3.8-rasm sterjen kesimlarida to`ldiruvchi qatlamdagi eguvchi momentning koordinata va vaqtdan bog`liq o`zgarishi

§ 3.4. Bir uchi qistirib mahkamlangan, ikkinchi erkin uchida cheksiz katta diafragma bor uch qatlamli sterjenning xususiy tebranishi.

Bir uchi qistirib mahkamlangan, ikkinchi erkin uchida cheksiz katta diafragma bor uch qatlamli sterjenni qaraymiz. Bu holda masalaning chegaraviy shartlari, [5] ilmiy tadqiqot ishi natijalari asosida, sterjenning uchlarida quyidagi ko`rinishda bo`ladi:

1) $x = 0$ uchida

$$\begin{cases} \left(1-\frac{h^2}{\beta}\frac{d^2}{dx^2}\right)\chi = 0, \\ \frac{d\chi}{dx} = 0, \\ \frac{d^3\chi}{dx^3} = 0. \end{cases} \qquad (3.4.1)$$

2) $x = l$ uchida

$$\begin{cases} \frac{d^2\chi}{dx^2} = 0, \\ \frac{d^4\chi}{dx^4} = 0, \\ \left(1-\frac{\upsilon h^2}{\beta}\frac{d^2}{dx^2}\right)\frac{d^3\chi}{dx^3} = 0. \end{cases} \qquad (3.4.2)$$

Olingan (3.4.1), (3.4.2) chegaraviy shartlarda (3.1.3) formulalar yordamida o`lchamsiz patametrlarga o`tib

1) $\xi = 0$ bo`lganda

$$\begin{cases} \chi(\xi) - k\chi''(\xi) = 0, \\ \chi'(\xi) = 0, \\ \chi'''(\xi) = 0, \end{cases} \qquad (3.4.3)$$

2) $\xi = 1$ bo`lganda

$$\begin{cases} \chi''(\xi) - k\upsilon\chi^{IV}(\xi) = 0, \\ \chi'''(\xi) = 0, \\ \chi^{V}(\xi) = 0, \end{cases} \qquad (3.4.4)$$

chegaraviy shartlarga ega bo`lamiz. Hosil qilingan (3.2.5) ni (3.4.3) va (3.4.4) chegaraviy shartlarga qo`yib noma'lum koeffitsientlar $a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}$ larni aniqlaymiz. Olingan (3.2.5) ni ketma ket hosilalarni aniqlaymiz.

$$\begin{cases} \chi_1^{I}(\xi) = a_{12} + 2a_{13}\xi + 3a_{14}\xi^2 + 4a_{15}\xi^3 + 5a_{16}\xi^4 + 6\xi^5; \\ \chi_1^{II}(\xi) = 2a_{13} + 6a_{14}\xi + 12a_{15}\xi^2 + 20a_{16}\xi^3 + 30\xi^4; \\ \chi_1^{III}(\xi) = 6a_{14} + 24a_{15}\xi + 60a_{16}\xi^2 + 120\xi^3; \\ \chi_1^{IV}(\xi) = 24a_{15} + 120a_{16}\xi + 360\xi^2; \\ \chi_1^{V}(\xi) = 120a_{16} + 720\xi; \\ \chi_1^{VI}(\xi) = 720; \end{cases}$$

Topilgan hosilalarni (3.4.3) tenglamaga $\xi = 0$ bo`lganga qo`yamiz. U holda

$$\begin{cases} a_{11} - 2ka_{13} = 0 \\ a_{12} = 0 \\ a_{14} = 0 \end{cases} \qquad (3.4.5)$$

hosil bo`ladi.

Endi $\xi = 1$ bo'lganda (3.4.4) tenglamani yechamiz.

$$\begin{cases} 2a_{13} + 6a_{14} + 12a_{15} + 20a_{16} + 30 - k\upsilon(24a_{15} + 120a_{16} + 360) = 0 \\ 6a_{14} + 24a_{15} + 60a_{16} + 120 = 0 \\ 120a_{16} + 720 = 0 \end{cases} \qquad (3.4.6)$$

(3.4.6) tenglamaga topilgan (3.4.5) tenglamaning qiymatlarini qo`yib hisoblaymiz.

$$\begin{cases} 2a_{13} + 12a_{15} + 20a_{16} + 30 - k\upsilon(24a_{15} + 120a_{16} + 360) = 0 \\ 24a_{15} + 60a_{16} + 120 = 0 \\ 120a_{16} + 720 = 0 \end{cases} \qquad (3.4.7)$$

(3.4.7) tenglamalar sistemasini uchinchi tenglamasidan a_{16} ni aniqlaymiz

$$a_{16} = -6; \qquad (3.4.8)$$

(3.4.8) ni (3.4.7) ning ikkinchi tenglamasiga qo`yib a_{15} ni topamiz

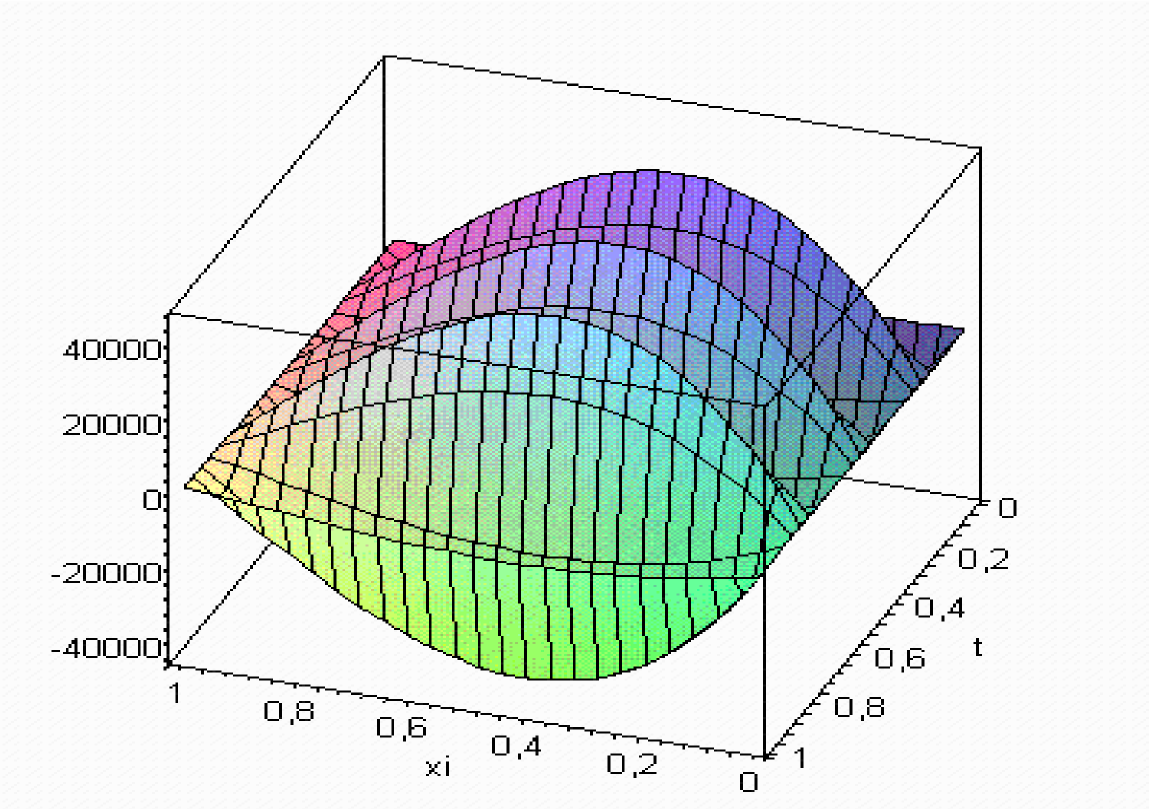
$$a_{15} = 10; \qquad (3.4.9)$$

(3.4.8) va (3.4.9) ni (3.4.7) tenglamaning birinchi tenglamasiga qo`yib a_{13} ni topamiz

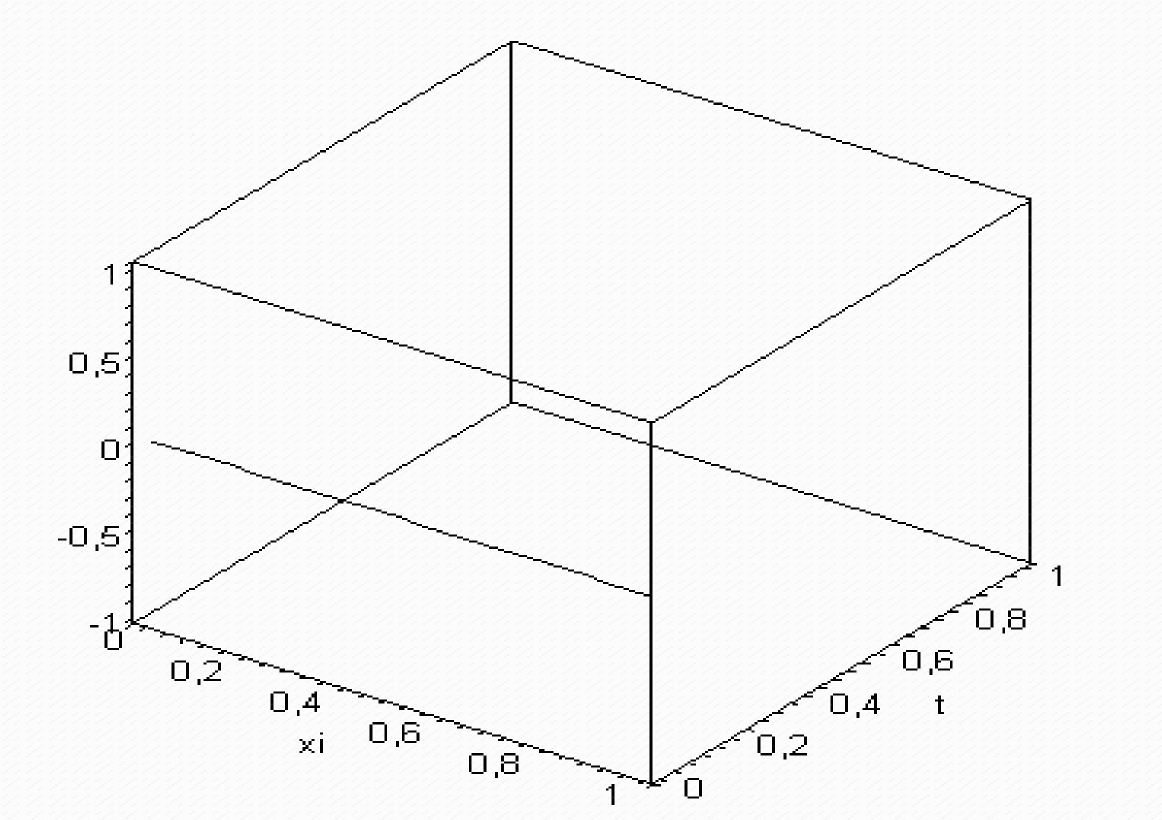
$$a_{13} = -15 - 60k\upsilon; ; \qquad (3.4.10)$$

Topilgan no'malumlarni bitta sistema qilib quyidagicha yozamiz

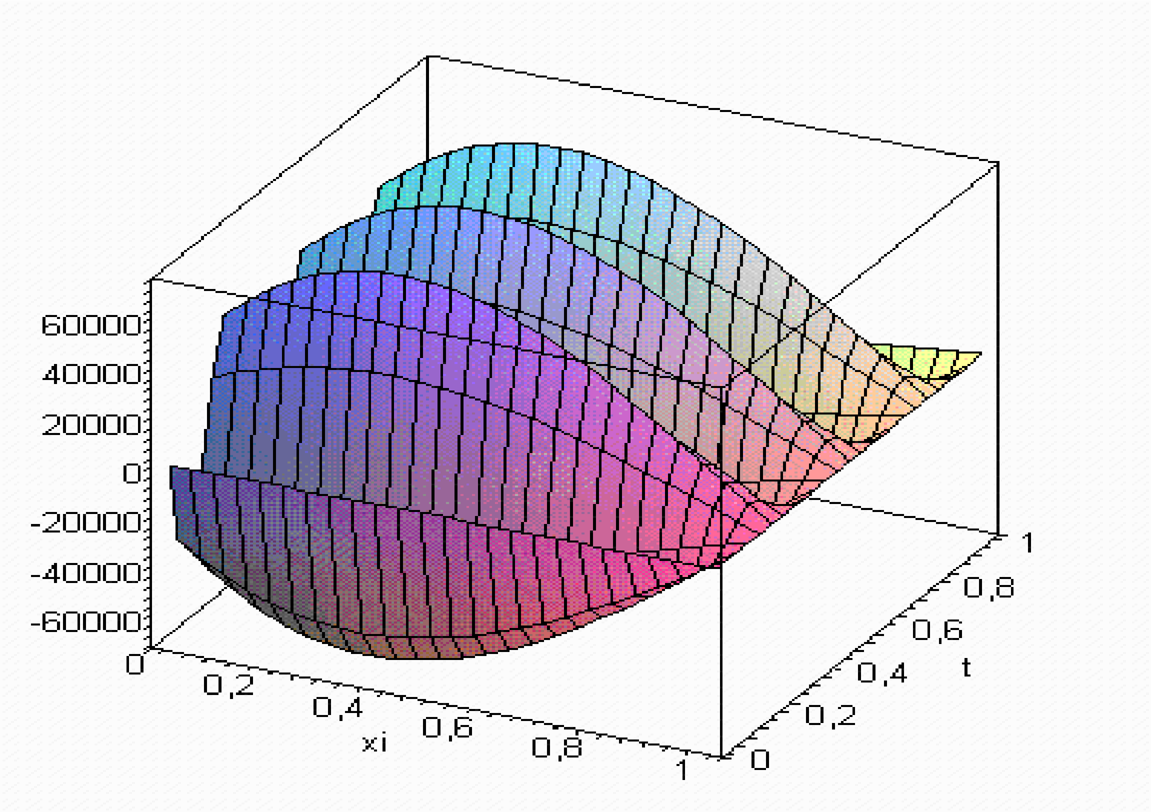
$$\begin{cases} a_{11} = 2k(-15 - 60k\upsilon); \\ a_{12} = 0; \\ a_{14} = 0; \\ a_{13} = -15 - 60k\upsilon; \\ a_{15} = 10; \\ a_{16} = -6. \end{cases}$$



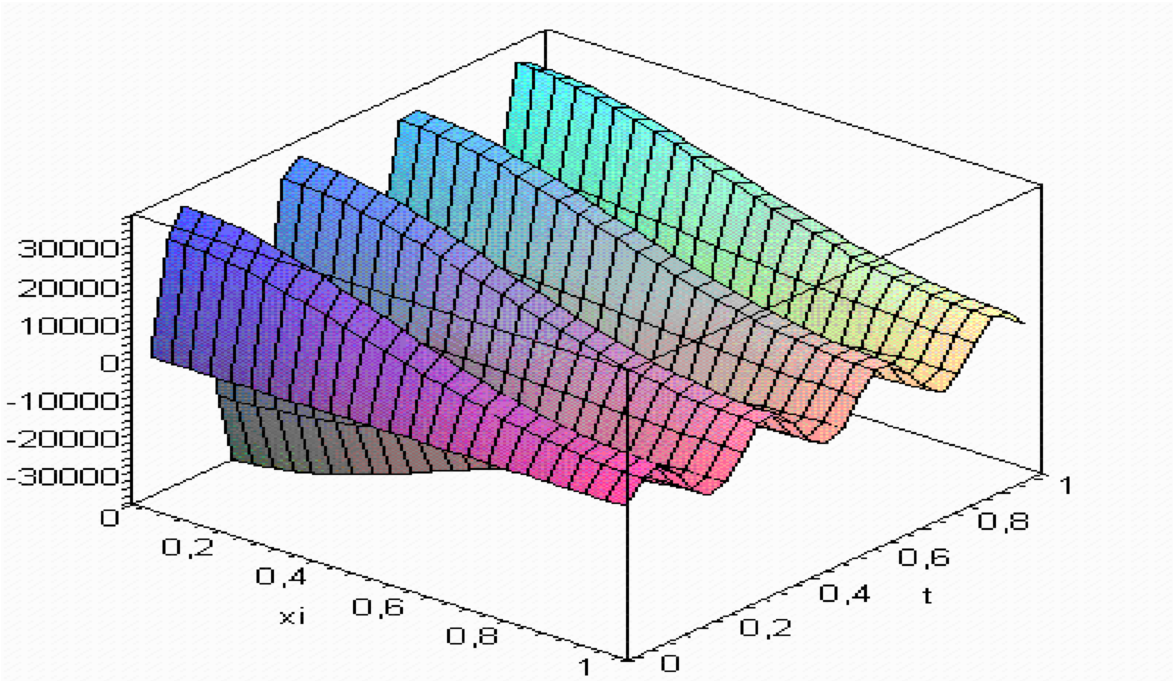
3.9-rasm to`ldiruvchi qatlamda qirquvchi kuchning koordinata va vaqtdan bog`liq o`zgarishi



3.10-rasm sterjen kesimlarida eguvchi momentning koordinata va vaqtdan bog`liq o`zgarishi



3.11-rasm sterjen kesimlaridagi qirquvchi kuchning koordinata va vaqtdan bog`liq o`zgarishi



3.12-rasm sterjen kesimlarida to`ldiruvchi qatlamdagi eguvchi momentning koordinata va vaqtdan bog`liq o`zgarishi

XULOSA

Shunday qilib magistirlik dissertatsiya ishi doirasida yechilgan masalalar va olingan natijalar quyidagi xulosalarni chiqarishga imkon beradi.

a) Ishning birinchi bobida:

-uch qatlamli konstruktiv elementlar tebranishlari bo`yicha ba’zi tadqiqot ishlari bilan tanishib chiqildi.

-elastiklik nazariyasining asosiy tenglamalari va masalalari yanada chuqurroq o`rganildi va qisqacha bayon etiladi;

-uch qatlamli sterjenning tebranish tenglamasi (Grigolyuk-Chulkov) hamda statik yuk ta’siridagi tenglamasi keltirildi. Bunda statik yuk ta’siridagi muvozanat tenglamasining umumiy yechimi va keltirib chiqarish algoritmi bayon etildi;

b) Ishning ikkinchi bobida:

-uchlari cheksiz katta bikrlikka ega va erkin tayangan uch qatlamli sterjenning egilishi haqidagi masala yechildi. Olingan natijalar shuni ko`rsatadiki, kesimlarda vujudga keladigan eguvchi moment va qirquvchi kuch hamda to`ldiruvchi qatlamdagi eguvchi moment va qirquvchi kuchlarning koordinatadan bog`liq o`zgarishlari qonuniyatlari aniqlanadi. (2.1-2.4 rasmlar)

- bir uchi qistirib mahkamlangan, ikkinchi uchi erkin uch qatlamli sterjenning (konsol balkaning) statik yuk ta’sirida egilishi haqidagi masalaning analitik yechimi olindi va sonli natijalar quyidagi hollar uchun grafiklar ko`rinishida keltirildi.

a) sterjen kesimlaridagi eguvchi moment (2.5-rasm);
b) sterjen kesimlaridagi qirquvchi kuch (2.6-rasm);
c) to`ldiruvchi qatlamdagi eguvchi moment (2.7-rasm);
d) to`ldiruvchi qatlamdagi qirquvchi kuch (2.8-rasm):

-bir uchi qistirib mahkamlangan, ikkinchi uchi erkin cheksiz katta bikrlikka ega diafragmali uch qatlamli sterjenning statik yuk ta’sirdagi egilishi haqidagi masala analitik usul bilan yechildi va sonli tadqiqb etildi. Olingan natijalar sterjenning har uchchala qatlami bo`ylab kesimlardagi, hamda to`ldiruvchi qatlam kesimlardagi eguvchi moment (2.9, 2.10- rasmlar) hamda qirquvchi kuch (2.11, 2.12-rasmlar) larning koordinata bo`yicha o`zgarish qonuniyatlari grafik ko`rinishda tasvirlanadi;

c) Ishning uchinchi bobida

-uch qatlamli sterjenlar uchun tebranishlar tenglamasi, uning xususiy tebranishlar holidagi ko`rinishi va yechimi Bubnov-Galerkin usulida tadqiq etildi. Xususiy tebranish chastotasi aniqlandi va uning analitik ifoldasi topildi.

-uchlari diafragmali va erkin tayangan uch qatlamli sterjenning xususiy tebranishlari tadqiq etildi. Buning uchun ko`chish funksiyasining vaqt bo`yicha o`zgarishi garmonik ya’ni sinusoidal xarakterga ega deb hisoblanadi. Sterjen kesimlaridagi va to`ldiruvchi qatlamdagi eguvchi moment bilan qirquvchi kuchlarning (3.1-3.4-rasmlar) vaqt va koordinatadan bog`liq o`zgarishlari uch o`lchovli to`g`ri burchakli koordinatalar sistemalarida keltirildi.

-uch qatlamli konsal balkaning xususiy tebranishlari tadqiq etildi. Balka kesimlari va qatlamlaridagi zo`riqishlar aniqlandi va grafik ko`rinishda taqdim etildi (3.5-3.8-rasmlar);

-erkin uchiga cheksiz katta bikrlikka ega bo`lgan diafragma keydirilgan konsol balkaning xususiy tebranishlari haqidagi masalaning analitik yechimi, sonli natijalar va ular asosidagi grafiklar (3.9-3.12-rasmlar) dissertatsiyaning yakunlovchi qismida bayon etildi.

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