2011

“Oly matematika” fanidandan amaliy mashg’ulotlar uchun USLUBIY KO’RSATMA

“Matemati analiz” bo`limi bo’yicha

Uslubiy ko’rsatmada “Oliy matematika” fanining “Matematik analiz” bo’limiga doir misollar va qisqa nazariy ma`lumot berilgan.

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Ushbu uslubiy ko’rsatma oliy texnika o‘quv yurtlarida barcha yo‘nalishlar bo‘yicha ta’lim olayotgan talabalarga mo‘ljallangan bo‘lib, unda “Oliy matematika”ning “Matematik analiz” bo’limi bo’yicha misollar va qisqa nazariy ma’lumotlar hamda mustaqil ishlash uchun misollar berilgan

Uslubiy ko`rsatma

«Oliy matematika» kafedrasining

yig‘ilishida muhokama qilingan

(bayonnoma №__________2011y.)

Uslubiy ko`rsatma iqtisod va menejment fakultetining uslubiy komissiyasi tomonidan nashrga tavsiya qilingan

(qaror № ___ “__” ________ 2011y.)

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Funksiya va uning berilish usullari

1. Funksia tushunchasi

Matematik analizda asosan sonli to‘plamni sonli to‘plamga akslantirishlar qaraladi. Bunday akslantirish *funksional bog‘lanish* deb ataladi. Funksional bog‘lanishlarning xususiy holi bo‘lgan *funksiya* tushunchasi matematikaning asosiy tushunchalaridan biri bo‘lib, uning ta’rifini keltiramiz:

Ta’rif. *Agar X sonli to‘plamning har bir x elementiga biror f qonunga binoan Y sonli to‘plamning faqat bitta y elementi mos qo‘yilgan bo‘lsa, y funksiya x esa uning argumenti deyiladi va* $y = f(x)$ *kabi yoziladi. Bu yerda X to‘plamni funksiyaning berilish yoki aniqlanish sohasi, Y to‘plam uning o‘zgarish sohasi yoki qiymatlar to‘plami deyiladi.*

1. *Misol :*

$y=(x-2)\sqrt{\dfrac{x+1}{x-1}}$. funksiyani aniqlanish sohasini toping.

Yechish:

$x-2$ ko’paytuvchi x ning barcha qiymatlarida ma`noga ega, shuning uchun bu ifodaning aniqlanish sohasi $\dfrac{x+1}{x-1}\geq 0$ tengsizlikni qanoatlantiruvchi batcha x lar dan iborat. Bu tengsizlikni yechib $-1\leq x\leq 1$ javobni olamiz.

Javob: $-1\leq x\leq 1$

2. Misol:

$y=\sqrt{\sin\sqrt{x}}$ funksiyani aniqlanish sohasini toping.

Yechish:

Bu ifoda $\sin\sqrt{x}\geq 0$ shartda ma`noga ega, yani

$$2k\pi\leq\sqrt{x}\leq\pi+2k\pi,\ \ (x\geq 0)\quad (k=0,1,2,3...)$$

Bu yerdan

$$4k^2\pi^2\leq x\leq\pi^2(1+2k)^2,\qquad (k=0,1,2,3...)$$ ga ega bo’lamiz.

Javob: $4k^2\pi^2\leq x\leq\pi^2(1+2k)^2,\qquad (k=0,1,2,3...)$

3- misol:

$y=\lg(\sin\dfrac{\pi}{x})$ funksiyani aniqlanish sohasini toping.

Yechish:

Aniqlanish sohasi $\sin\frac{\pi}{x} > 0$ bilan aniqlanadi. Bu tengsizlik uchun quyidagilar o'rinli:

$$2k\pi < \frac{\pi}{x} < \pi(2k+1) \qquad (k = 0, \pm 1, \pm 2, \ldots$$

$$agar \quad k = 0 \;\; bo'lsa \quad 0 < \frac{1}{x} < 1 \;\; va \;\; 1 < x < +\infty \;\; o'rinlidir;$$

$$agar \;\; k > 0 \;\; bo'lsa \quad \frac{1}{2k+1} < x < \frac{1}{2k} \qquad (k = 1, 2, ..)$$

$$agar \;\; k < 0 \quad bo'lsa \quad \frac{1}{2k} < x < \frac{1}{2k+1} \quad (k = -1, -2, \ldots)$$

bo'ladi.

Javob:

$$1 < x < +\infty, \quad \frac{1}{2k+1} < x < \frac{1}{2k} \quad (k = 1, 2, ..), \quad \frac{1}{2k} < x < \frac{1}{2k+1} \quad (k = -1, -2, \ldots)$$

2.Teskari funksiya tushunchasi

Faraz qilaylik, $y = f(x)$ funksiya biror X sohada aniqlangan bo'lib, uning o'zgarish sohasi Y to'plamdan iborat bo'lsin. Bu funksiya

$$f : X \Rightarrow Y$$

akslantirishdan iborat ekanligini e'tiborga olib, u teskarilanuvchi, ya'ni $\forall y \in Y$ uchun $f^{-1}(y) = (x : x \in X, f(x) = y)$ to'plam yagona elementga ega bo'lgan holni qarasak, qandaydir $x = \varphi(y)$ funksiyaga ega bo'lamiz va uni berilgan $y = f(x)$ funksiyaga *teskari funksiya* deb ataymiz.

$y = f(x)$ ga teskari bo'lgan $x = \varphi(y)$ funksiyada ham an'anaviy belgilashga o'tib, $y = \varphi(x)$ funksiyani olamiz. Odatda, $y = f(x)$ ga teskari funksiya deyilganda oxirgi $y = \varphi(x)$ funksiya tushuniladi va, ko'pincha, $y = f^{-1}(x)$ kabi belgilanadi. Kezi kelganda, bunday belgilashni teskari qiymat bilan adashtirib yubormaslik lozimligini eslatamiz. Undan tashqari, berilganga teskari funksiya mavjud bo'lsa, ular *o'zaro teskari funksiyalar* bo'lishi ravshandir.

1. Misol:

   $y = 2x + 3$ funksiyaga teskari bo'lgan funksiyani va hosil bo'lgan funksiyani aniqlanish sohasini toping.

Yechish:

x $-\infty$dan $+\infty$ gacha o'zgaradi, u $-\infty$ dan $+\infty$ ga monoton o'sadi. Shuning uchun yagona teskari funksiya mavjuddir, $x=\frac{1}{2}(y-3) \quad (-\infty<y<\infty)$

Javob: $x=\frac{1}{2}(y-3) \quad (-\infty<y<\infty)$

2- misol:

$y=\frac{1-x}{1+x} \quad (x\neq -1)$ uchun teskari funksiyani toping.

Yechish:

$+\infty<x<-1$intervalda funksiya -1 dan $-\infty$gacha monoton kamayuvchi, $-1<x<+\infty$ da $+\infty$dan -1 gacha monoton kamayuvchi bo'lganligi sababli y=-1 dan tashqari butun son o'qida yagona teskari funksiya mavjuddir. $x=\frac{1-y}{1+y} \quad (y\neq -1)$funksiya berilgan funksiyaga teskari bo'ladi.

Javob: $x=\frac{1-y}{1+y} \quad (y\neq -1)$

3.Davriy funksiyalar.

Agar $y=f(x)$ funksiya uchun shunday *T≠0* son mavjud bo'lib, bu funksiyaning aniqlanish sohasidan olingan ixtiyoriy *x* uchun *x+T* ham aniqlanish sohasiga tegishli va *f(x+T)=f(x)* tenglik bajarilsa, $y=f(x)$ *davriy funksiya, T esa uning davri* deyiladi.

1-misol: quyida berilgan funksiyalarni davriylikka tekshiring va eng kichik davrini toping.

a) $f(x)=A\cos\lambda x+B\sin\lambda x$

Yechish:

$$f(x+T)=A\cos\lambda(x+T)+B\sin\lambda(x+T)=A\cos\lambda x\cos\lambda T-A\sin\lambda x\sin\lambda T+$$
$$+B\sin\lambda x\cos\lambda T+B\sin\lambda T\cos\lambda x=A\cos\lambda x+B\sin\lambda x=f(x),$$
$$agar\ \ \cos\lambda T=1,\ \ ya'ni\ \ \lambda T=2\pi k,\ \ k=0,1,2..$$

Demak, berilgan funksiya uchun eng kichik davr $T=\frac{2\pi}{\lambda}$ ga teng.

b) $f(x)=\sin(x^2)$

Yechish:

Bu funksiya davriy emas, chunki uning ketma-ket nollari orasidagi masofa nolga intiladi, ya'ni

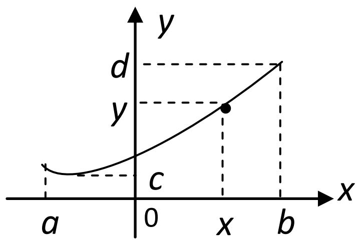
$$\sqrt{(2k+1)\pi}-\sqrt{k\pi}=\frac{\pi}{\sqrt{(k+1)\pi}+\sqrt{k\pi}}\to 0,\ k\to\infty$$

4.Funksiya grafigi tushunchasi

xOy koordinatalar tekisligidagi abssissasi $x\in D$, ordinatasi $f(x)$ dan iborat bo'lgan nuqtalar to'plami $y=f(x)$ *funksiyaning grafigi* deyiladi.

Agar koordinatalar tekisligidagi nuqtalar to'plami *Oy* o'qqa parallel to'g'ri chiziq bilan bittadan ortiq umumiy nuqtaga ega bo'lmasa, u biror funksiyaning grafigi bo'ladi.

Koordinatalar tekisligida shunday nuqtalar to'plami berilsa, uni *grafik usulda berilgan funksiya* deyiladi. Grafikdagi nuqtaning abssissalar o'qidagi proyeksiyasi argument qiymatiga, ordinatalar o'qidagi proyeksiyasi esa funksiya qiymatiga mos keladi



1. Misol

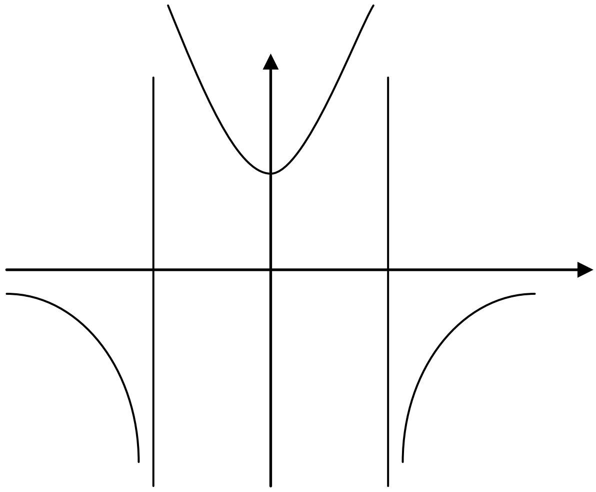
$y=\frac{1}{1-x^2}$ funksiya grafigini chizing.

Yechish:

Bu funksiyaning aniqlanish sohasi $-\infty<x<-1,\ -1<x<1,\ 1<x<+\infty$ bo'lib funksiya juftdir. Agar $|x|<1,\ bolsa\ y>0$, agar $-\infty<x<-1,\ 1<x<+\infty$ bo'lsa y<0.

Quyidagi jadval natijalaidan foydalanib grafikni chizamiz:

x	0	$\frac{1}{2}$	$x\to 1-0$	$x\to 1+0$	2	3	$x\to+\infty$
y	1	$\frac{4}{3}$	$y\to+\infty$	$y\to-\infty$	$-\frac{1}{3}$	$-\frac{1}{8}$	$y\to-0$



Auditoriyada ishlash uchun misollar

I. Quidagi funksiyalarnianiqlanish sohasini toping.

1.1. $y = \frac{\sqrt{x}}{\sin \pi x}$

1.2. $y = (x + |x|)\sqrt{x \sin^2 \pi x}$

1.3. $y = \arcsin \frac{2x}{1+x}$

1.4. $y = \lg[\cos(\lg x)]$

1.5. $y = (-1)^x$

II. Berilgan funksiyaga teskari funksiya va hosil bo'lgan funksiyaning aniqlanish sohasini toping.

2.1. $y = x^2$

2.2. $y = \sqrt{1 - x^2}$

2.3. $y = \frac{1}{2}(e^x - e^{-x})$

III. Berilgan funksiyani davriylikka tekshiring.

3.1. $f(x) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x$

3.2. $f(x) = 2tg\frac{x}{2} - 3tg\frac{x}{3}$

3.3. $f(x) = \sin^2 x$

3.4. $f(x) = \sqrt{tgx}$

3.5. $f(x) = tg\sqrt{x}$

IV. Funksiya grafigini quring.

4.1. $f(x) = x^2 - x^4$

4.2. $f(x) = (1 - x^2)(2 + x)$

4.3. $f(x) = e^{x^2}$

Uy vayifasi uchun misollar.

Quyidagi funksiyalarning aniqlanish sohasini toping.

5.1. $y = (2x)!$

5.2. $y = \lg(1 - 2\cos x)$

5.3. $y = \arccos(2\sin x)$

5.4. $y = \sqrt{\sin 2x} + \sqrt{\sin 3x}$

5.5. $y = \arccos\dfrac{2x}{1 + x^2}$

Quyidagi funksiyalarga teskari funksiyani toping.

5.6. $y = \lg x$

5.7. $y = 2\sin x$

5.8. $y = \arccos x$

Quyidagi funksiyalarni davrini toping.

5.9. $y = \sin x + \sin \sqrt{2}x$

5.10. $\chi(x) = \begin{cases} 1, & agar \quad x \ \ ratsional \quad bo'lsa, \\ 0, & agar \quad x \ \ irrasional \quad bo'lsa \end{cases}$

Quyidagi funksiyalarni grafigini quring.

5.11. $y = \sin^2 x$

5.12. $y = \dfrac{1}{1 + x^2}$

”Ketma-ketlik va funksiya limiti.”

1. Sonly ketma-ketlik va uning limiti

Agar o‘sish tartibida olingan har bir n natural songa biror qoidaga binoan qandaydir x_n ifoda mos qo‘yilgan bo‘lsa, uni *ifodalar ketma-ketligi* deyiladi va

$$x_1, x_2, \ldots, x_n, \ldots \qquad (1)$$

kabi yoziladi. Bu yerda x_n *ketma-ketlikning n-hadi* deyiladi $(n \in N)$, ba’zan, uni *umumiy had* deb ham yuritiladi. Ketma-ketlik uchun $\{x_n\}$ yoki (x_n) kabi belgilashlar ham qo‘llaniladi.

Agar (1) ketma-ketlikning barcha hadlari sonlardan iborat bo‘lsa, uni *sonli ketma-ketlik* deb ataladi.

Ta’rif. *Agar berilgan $\{x_n\}$ ketma-ketlik uchun biror a son mavjud bo‘lib, ixtiyoriy olingan musbat ε son uchun shunday n_0 natural son topilsaki, $\forall n_0 < n \in N$ bo‘lganda $|x_n - a| < \varepsilon$ tengsizlik bajarilsa, a son $\{x_n\}$ ketma-ketlikning limiti* deyiladi va

$$x_n \to a \text{ yoki } \lim x_n = a \qquad \left(\lim_{n\to\infty} x_n = a\right)$$

kabi yoziladi.

Ta’rif. Limiti nolga teng bo‘lgan ketma-ketlik *cheksiz kichik miqdor* deyiladi. Agar $\{x_n\}$ cheksiz kichik miqdor bo‘lsa, yuqoridagi ta’rif asosida $\lim x_n = 0$ bo‘lib, $\forall \varepsilon > 0$ uchun shunday n_0 natural son mavjud bo‘ladiki, $\forall n_0 < n \in N$, $|x_n| < \varepsilon$ tengsizlik bajariladi.

Ta’rif. Agar $\{x_n\}$ ketma-ketlik berilgan bo‘lib, ixtiyoriy olingan musbat M son uchun shunday n_0 natural son mavjud bo‘lsaki, $\forall n_0 < n \in N$, $|x_n| > M$ o‘rinli bo‘lsa, *$\{x_n\}$ ketma-ketlik cheksiz katta miqdor* deyiladi va $x_n \to \infty$ yoki $\lim x_n = \infty$ kabi yoziladi.

Hossalari:

1. Agar $\{x_n\}, \{y_n\}, \ldots, \{z_n\}$ chekli sondagi ketma-ketliklarning har biri chekli limitga ega bo‘lsa, ularning algebraik yig‘indisi ham chekli limitga ega va

$$\lim(x_n \pm y_x \ldots \pm z_n) = \lim x_n \pm \lim y_n \pm \ldots \lim z_n$$

tenglik o‘rinli bo‘ladi.

2. Agar $\{x_n\},\{y_n\},\ldots,\{z_n\}$ chekli sondagi ketma-ketliklarning har biri chekli limitga ega bo‘lsa, ularning ko‘paytmasi ham chekli limitga ega bo‘lib,

$$\lim(x_n \cdot y_n \ldots \cdot z_n) = \lim x_n \cdot \lim y_n \cdot \ldots \cdot \lim z_n$$

tenglik o‘rinlidir.

3. Agar $\{x_n\}$ va $\{y_n\}$ ketma-ketliklarning chekli limitlari mavjud bo‘lib, $\lim y_n \neq 0$ bo‘lsa, $\left\{\frac{x_n}{y_n}\right\}$ ning limiti ham mavjud va

$$\lim \frac{x_n}{y_n} = \frac{\lim x_n}{\lim y_n}$$

tenglik o‘rinlidir.

1. Misol:

Yuqoridagi ta’rif bo‘yicha

$$\lim \frac{n-1}{n+1} = 1$$

ekanligini isbotlaylik. Buning uchun ihtiyoriy musbat ε son berilganda ta’rifda aytilgan n_0 natural son topilishini ko‘rsatish kifoya.

$$\left|\frac{n-1}{n+1} - 1\right| < \varepsilon \Rightarrow \left|\frac{2}{n+1}\right| < \varepsilon \Rightarrow \frac{2}{n+1} < \varepsilon \Rightarrow n > \frac{2}{\varepsilon} - 1$$

ε- ixtiyoriyligidan $\frac{2}{\varepsilon} - 1 > 0$ bo‘ladigan qilib olish mumkin ($\varepsilon < 2$ deb olinsa kifoya). Endi, $n_0 = \left[\frac{2}{\varepsilon} - 1\right] + 1$ deb olsak, yuqoridagi tengsizlik $\forall n_0 < n \in N$ uchun bajarilishi kelib chiqadi. Masalan, ε=0,001 desak, $n_0 = \left[\frac{2}{0{,}001} - 1\right] + 1 = [2000 - 1] + 1 = 2000.$

$[x]$ yozuv x sonining butun qismini anglatishini eslatamiz.

2. Misol:

Ihtiyoriy $\varepsilon > 0$ son uchun shartni $n > N$ qanoatlantiruvchi $N = N(\varepsilon)$ topilsaki, $|x_n| > 0$ bajarilganda

a) $x_n = \dfrac{(-1)^{n+1}}{n}$ $\qquad$ $b)\ x_n = \dfrac{2n}{n^3+1}$

Ketma-ketliklar cheksiz kichik miqdor ekanligini isbotlang. Bunda $\varepsilon > 0$ nimaga tengligini ko'rsating.

Isbot: a) Ihtiyoriy $\varepsilon > 0$ da va $n > \dfrac{1}{\varepsilon} = N(\varepsilon)$ uchun $\left|\dfrac{(-1)^{n+1}}{n}\right| = \dfrac{1}{n} < \varepsilon$ tengsizlik o'rinlidir. Ya'ni berilgan ketma-ketlik cheksiz kichikdir.

b) huddi shuningdek, $n > \sqrt{\dfrac{2}{\varepsilon}} = N(\varepsilon)$ $bo'\lg anda$ $\left|\dfrac{2n}{n^3+1}\right| < \dfrac{2n}{n^3} = \dfrac{2}{n^2} < \varepsilon$

ekanligi kelib chiqadi.

3. Misol:

$$\lim_{n\to\infty}\frac{1+a+a^2+...+a^n}{1+b+b^2+...+b^n} \quad (\,|a| < 1,\ \ |b| < 1\,)$$

Yechish:

Surat va mahrajning yig'indisini topib limitlar haqidagi hossalarga ko'ra, quyidagi natijaga erishamiz:

$$\lim_{n\to\infty}\frac{1+a+a^2+...+a^n}{1+b+b^2+...+b^n} = \lim_{n\to\infty}\frac{\dfrac{1-a^{n+1}}{1-a}}{\dfrac{1-b^{n+1}}{1-b}} = \frac{1-a}{1-b}\,\frac{\lim\limits_{n\to\infty} a^{n+1}}{\lim\limits_{n\to\infty} b^{n+1}} = \frac{1-a}{1-b}$$

Bu yerda

$\lim\limits_{n\to\infty} q^n = 0\ \ (|q| < 1)$ dan foydalandik. Haqiqattan ham

$$\frac{1}{|q|^n} = (1+\frac{1-|q|^n}{|q|})^n = 1+n\frac{1-|q|}{|q|}+...+(\frac{1-|q|}{|q|})^n > n\frac{1-|q|}{|q|}$$ dan

$|q|^n = |q^n| < \dfrac{|q|}{1-|q|}\cdot\dfrac{1}{n} < \varepsilon$ tengsizlik kelib chiqadi. Bu tengsizlik

$n > |q|(1-|q|)^{-1}\cdot\varepsilon^{-1}$ va $\forall\varepsilon > 0$ uchun o'rinlidir.

2. Funksiya limiti

ta'rif. Agar *f(x)* funksiya *a* quyuqlik (limitik) nuqtasiga ega bo'lgan *D* sohada aniqlangan (*a* ning o'zida shart emas) va biror *b* son mavjud bo'lib, $\forall\varepsilon$ *>0* son uchun shunday musbat δ son topilsaki, $0 < |\,x - a\,| < \delta$ munosabatni qanoatlantiruvchi $\forall x \in D$ uchun $|f(x)-b|<\varepsilon$ tengsizlik bajarilsa, *b* son *f(x)* funksiyaning $x \to a$ dagi (yoki *a* nuqtadagi) limiti deyiladi va

$$\lim_{x \to a} f(x) = b \qquad (1)$$

ko‘rinishda yoziladi.

Ta’rif. Agar *a* son *f(x)* funksiya aniqlanish sohasi bo‘lgan *D* to‘plamning limitik nuqtasi bo‘lib, $\forall \varepsilon > 0$ olinganda shunday $\delta > 0$ son topilsaki, $0<|x'-a|<\delta,\ 0<|x''-a|<\delta$ munosabatlarni qanoatlantiruvchi *D* ga tegishli barcha x'', x' nuqtalar uchun $|f(x'')-f(x')|<\varepsilon$ tengsizlik bajarilsa, *f(x)* funksiya *a* nuqtada *Koshi sharti*ni qanoatlantiradi deyiladi.

Bu ta’rif funksiya uchun *Koshi mezoni*dir.

Funksiya limitining asosiy xossalari:

1^0. Agar $f_i(x)$ $(i=1,2,\ldots,n)$ funksiyalar x_0 ning yetarlicha yaqin atrofida aniqlanish sohalari bir xil qismiy to‘plamga ega bo‘lgan holda bu nuqtada ularning chekli limitlari mavjud bo‘lsa,

$$\lim_{x \to x_0} \sum_{i=1}^{n} f_i(x) = \sum_{i=1}^{n} \lim_{x \to x_0} f_i(x),$$

$$\lim_{x \to x_0} \prod_{i=1}^{n} f_i(x) = \prod_{i=1}^{n} \lim_{x \to x_0} f_i(x)$$

o‘rinlidir.

2^0. Agar $\lim\limits_{x \to x_0} f(x) = b$ chekli limit mavjud va $b \neq 0$ bo‘lsa, $x = x_0$ nuqtaning shunday yaqin atrofi mavjud bo‘ladiki, argument qiymati undan va funksiya yaqinlanish sohasiga tegishli qilib olinganda *f(x)* ning ishorasi b ning ishorasi bilan bir xil bo‘ladi, xatto aytilgan yaqin atrofni toraytirish hisobiga $|f(x)| > \dfrac{|b|}{2}$ ni o‘rinli qilish mumkin.

3^0. Agar *f(x)* va $\varphi(x)$ funksiyalar x_0 nuqtaning yetarlicha yaqin atrofida bir xil qismiy aniqlanish sohaga ega va bu nuqtada ularning chekli limitlari mavjud bo‘lib, $\lim\limits_{x \to x_0} \varphi(x) \neq 0$ bo‘lsa,

$$\lim_{x \to x_0} \frac{f(x)}{\varphi(x)} = \frac{\lim\limits_{x \to x_0} f(x)}{\lim\limits_{x \to x_0} \varphi(x)}$$

o‘rinlidir.

4^0. Agar $\varphi(x)$ funksiya x_0 nuqta yaqin atrofida aniqlangan bo‘lib, $\lim\limits_{x\to x_0}\varphi(x)=z_0$ chekli limit mavjud va $f(z)$ funksiya z_0 nuqta yaqin atrofida aniqlangan bo‘lib, $\lim\limits_{z\to z_0} f(z)=b$ chekli limit mavjud bo‘lsa, $f[\varphi(x)]$ murakkab funksiyaning x_0 nuqtadagi chekli limiti mavjud bo‘lib, $\lim\limits_{x\to x_0} f\left[\varphi(x)\right]=b$ o‘rinli bo‘ladi, ya’ni $\lim\limits_{x\to x_0}\varphi(x)=z_0 \Rightarrow \lim\limits_{x\to x_0} f\left[\varphi(x)\right]=\lim\limits_{z\to z_0} f(z)$.

4. Misol : $\lim\limits_{x\to 1}\left(2x^2-1\right)=1$ ekanligini isbotlang

Eychish: $f(x)=2x^2-1$ ning aniqlanish sohasi R dir.

$\forall\varepsilon>0$ ni olaylik,

$$\left|(2x^2-1)-1\right|<\varepsilon \Longleftrightarrow 2\left|x^2-1\right|<\varepsilon \Rightarrow 2\,|x+1|\,|x-1|<\varepsilon\,. \qquad (2)$$

Bu yerda $x\to 1$, ya’ni $x=1$ nuqta yaqin atrofini qaralayotganligi sababli $x\in[0;1)\cup(1;2]$ desak bo‘ladi. U vaqtda, agar

$$2.\left(2+1\right)\cdot|x-1|<\varepsilon \Rightarrow |x-1|<\frac{\varepsilon}{6}$$

tengsizlik bajarilsa, (2) albatta bajariladi. Demak, $\delta=min\left\{1;\frac{\varepsilon}{6}\right\}$ deb olsak, $0<|x-1|<\delta$ bo‘lganda (2) bajariladi. Ya’ni $\lim\limits_{x\to 1}\left(2x^2-1\right)=1$ ekanligi kelib chiqadi.

5.Misol: $\lim\limits_{x\to\infty}\frac{5x+3}{x}=5$ ni isbot qiling.

Isbot. $f(x)=\frac{5x+3}{x}$, $b=5$ va $f(x)$ funksiya $x\neq 0$ bo‘lgan barcha nuqtalarda aniqlangan. Ixtiyoriy musbat ε son olamiz va $f(x)-b$ ayirmaning absolut qiymatini qaraymiz:

$$|f(x)-b|=\left|\frac{5x+3}{x}-5\right|=\left|\frac{3}{x}\right|=\frac{3}{|x|}$$

Bu ayirma ε dan kichik bo‘lishi, ya’ni

$$\left|\frac{5x+3}{x}-5\right|=\frac{3}{|x|}<\varepsilon$$

tengsizlik bajarilishi uchun $|x|>\frac{3}{\varepsilon}$ bo‘lishi yetarli.

Bundan limitning 9.3.3- ta’rifida ko‘rsatilgan *M* son $M=\frac{3}{\varepsilon}>0$ bo‘lishini olamiz.

Shunday qilib, $\lim\limits_{x\to\infty}\frac{5x+3}{x}=5$ ekan.

6.Misol: $\lim\limits_{x\to 2}x^3$ topilsin.

Eychish: Ko‘paytmaning limiti haqidagi xossani qo‘llasak:

$$\lim_{x\to 2}x^3=\lim_{x\to 2}(x\cdot x\cdot x)=\lim_{x\to 2}x\cdot\lim_{x\to 2}x\cdot\lim_{x\to 2}x=2\cdot 2\cdot 2=8$$

7. Misol : $\lim\limits_{x\to 2}\frac{x^2-5x+6}{x-2}$ topilsin.

Yechish: Bu yerda maxrajning limiti nolga teng bo‘lgani uchun bo‘linmaning limiti haqidagi xossani qo‘llab bo‘lmaydi.

Bundan tashqari, kasr suratining limiti ham nolga teng bo‘lgani uchun berilgan ifoda $\frac{0}{0}$ ko‘rinishdagi aniqmaslikdir. Bu limit ostidagi kasrning surati

$$x^2-5x+6=(x-2)(x-3)$$

bo‘lgani uchun, $x\to 2$ da *x-2≠0* ekanligini e’tiborga olib,

$$\lim_{x\to 2}\frac{x^2-5x+6}{x-2}=\lim_{x\to 2}\frac{(x-2)(x-3)}{x-2}=\lim_{x\to 2}(x-3)=-1$$

ga ega bo‘lamiz.

8. Misol: $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}$

Bu misolni ishlashda $1+mx=t^3$ almashtirish kiritamiz, u holda $x=\frac{t^3-1}{m}$, $x\to 0$ *da* $t\to 1$ bo’lib, olingan natijalarni berilgan misolga qo’ysak,

$$\lim_{t\to 1}\frac{t-1}{\frac{t^3-1}{m}}=m\lim_{t\to 1}\frac{t-1}{(t-1)(t^2+t+1)}=m\lim_{t\to 1}\frac{1}{t^2+t+1}=\frac{m}{3}$$ hosil bo’ladi

9.Misol:

$$\lim_{x\to\infty}(\sqrt{x^2+3x}-x)=\lim_{x\to\infty}\frac{(\sqrt{x^2+3x}-x)(\sqrt{x^2+3x}+x)}{(\sqrt{x^2+3x}+x)}=\lim_{x\to\infty}\frac{x^2+3x-x^2}{(\sqrt{x^2+3x}+x)}=\lim_{x\to\infty}\frac{3x}{(\sqrt{x^2+3x}+x)}=$$

$$=\lim_{x\to\infty}\frac{3}{\sqrt{1+\frac{3}{x}}+1}=\frac{3}{2},\quad chunki\quad x\to\infty\ da\ \ \frac{3}{x}\to 0$$

Auditoriyada bajarish uchun misollar:

1.1. $x_n=(-1)^n\cdot n$ ketma-ketlik $n\to\infty$da cheksiz katta miqdor ekanligini isbotlang.

1.2. $x_n=2^{\sqrt{n}}$ ketma-ketlik $n\to\infty$da cheksiz katta miqdor ekanligini isbotlang.

1.3. $x_n=(-1)^n\cdot 0.999^n$ ketma-ketlik $n\to 0$da cheksiz kichik miqdor ekanligini isbotlang.

1.4. $\lim\limits_{x\to 1}\frac{a^n}{n!}=0$ ekanligini isbotlang

2.1. $\lim\limits_{x\to 1}\frac{x^2-3x+2}{x-1}$

2.2. $\lim\limits_{x\to -2}\frac{x^3+x^2-x+2}{x+2}$

2.3. $\lim\limits_{x\to 1}\frac{3x^2-4x^2+1}{2x^2-x-1}$

2.4. $\lim\limits_{x\to 5}\frac{x-5}{x^3-8x^2+15x}$

2.5. $\lim\limits_{x\to 0}\frac{x^4+3x^2}{x^5+x^3+2x^2}$

2.6. $\lim\limits_{x\to 1}\frac{x^2-1}{x^3-1}$

2.7.

2.8. $\lim\limits_{x\to a}\frac{x^6-a^6}{x^{10}-a^{10}}$

2.9. $\lim\limits_{x\to 1}\frac{x^m-1}{x^n-1}$ *m,n* – butun sonlar.

2.10. $\lim\limits_{x\to 0}\frac{x}{\sqrt{a+x}-\sqrt{a}}$

2.11. $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{x+3}-2}$

2.12. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{\sqrt{1+x}-\sqrt{1+x^2}}$

2.13. $\lim\limits_{x\to 5}\dfrac{x-5}{\sqrt{x-1}-2}$

2.14. $\lim\limits_{x\to\infty}\dfrac{15x^4-2x^3+3x^2-5x+1}{3x^4+4x^3+2x^2-4x-2}$

2.15. $\lim\limits_{x\to\infty}\dfrac{x^4-2x^3+3x}{4x^3+2x^2-4x-2}$

2.16. $\lim\limits_{x\to\infty}\dfrac{(x+2)(x+3)}{x^2+4x+4}$

2.17. $\lim\limits_{x\to\infty}\dfrac{1+2+3+...+n}{n^2-1}$

2.18. $\lim\limits_{x\to\infty}\left(\sqrt{x}-\sqrt{x+1}\right)$

2.19. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+3x+5}-\sqrt{x^2-3x-5}\right)$

2.20. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+3x+5}-\sqrt{x^2-3x-5}\right)$

2.21. $\lim\limits_{x\to\infty}\dfrac{1^2+3^2+...+(2n-1)^2}{2^2+4^2+...(2n)^2}$

2.22. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$

3. Uy vazifasi uchun misollar:

3.1. Umumiy hadi quyidagicha bo‘lgan ketma-ketlikni yozing. $x_n=\dfrac{1}{2^n}$

3.2. $\lim\limits_{n\to\infty}\left(2\cdot\left(3+\frac{1}{n}\right)\cdot\left(\frac{1}{n}-4\right)\right)$

3.3. $\lim\limits_{x\to 8}\frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$

3.4. $\lim\limits_{x\to -1}\frac{x^2-x-2}{1-x^2}$

3.5. $\lim\limits_{x\to 4}\frac{x^3-6x^2+8x}{x-4}$

3.6. $\lim\limits_{x\to 1}\frac{x^{-1}-2^{-1}}{x-2}$

3.7. $\lim\limits_{x\to o,5}\frac{4x^2-4x+1}{4x^2-1}$

3.8. $\lim\limits_{x\to 1}\frac{x^4+x-2}{x-1}$

3.9. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-1}{x}$

3.10. $\lim\limits_{x\to 7}\frac{\sqrt{x+2}-3}{x-7}$

3.11. $\lim\limits_{x\to\infty}x(\sqrt{x^2+2x}-2\sqrt{x^2+x}+x)$

3.12. $\lim\limits_{x\to\infty}(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x})$

Ajoyib limitlar.

1. 1- ajoyib limit.

Teorema: *x→0* da $\frac{\sin x}{x}$ ifoda 1 ga intiladi, ya`ni $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$

1. Misol: $\lim\limits_{x\to}\frac{\sin 3x}{x}$ limitni hisoblang.

   Yechish: $\lim\limits_{x\to 0}\frac{\sin 3x}{x}=\lim\limits_{x\to 0}\frac{\sin 3x}{3x}\cdot 3=3$

2. Misol: $\lim\limits_{x\to 0}\frac{tgx}{x}=\lim\limits_{x\to 0}\frac{\sin x}{x\cdot\cos x}=\lim\limits_{x\to 0}\frac{\sin x}{x}\cdot\lim\limits_{x\to 0}\frac{1}{\cos x}=1\cdot 1=1$

3. Misol: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\lim\limits_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}=\lim\limits_{x\to 0}2\left(\frac{\sin\frac{x}{2}}{2\cdot\frac{x}{2}}\right)^2=2\cdot\frac{1}{4}=\frac{1}{2}$

4. Misol:

$$\lim_{x\to 0}\frac{\sin 5x-\sin 3x}{\sin x}=\lim_{x\to 0}\frac{5\frac{\sin 5x}{5x}-3\frac{\sin 3x}{3x}}{\frac{\sin x}{x}}=\frac{5\lim\limits_{x\to 0}\frac{\sin 5x}{5x}-3\lim\limits_{x\to 0}\frac{\sin 3x}{3x}}{\lim\limits_{x\to 0}\frac{\sin x}{x}}=5-3=2$$

5. Misol:

$$\lim_{x\to 0}\frac{x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}=\lim_{x\to 0}\frac{x^2(\sqrt{1+x\sin x}+\sqrt{\cos x})}{(\sqrt{1+x\sin x}-\sqrt{\cos x})(\sqrt{1+x\sin x}+\sqrt{\cos x})}=$$

$$=\lim_{x\to 0}\frac{x^2(\sqrt{1+x\sin x}+\sqrt{\cos x})}{1+x\sin x-\cos x}=\lim_{x\to 0}\frac{\sqrt{1+x\sin x}+\sqrt{\cos x}}{\frac{1-\cos x}{x^2}-\frac{\sin x}{x}}=\frac{4}{3}$$

2. 2- ajoyib limit

Matematik analizda,

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e \quad \text{yoki} \quad \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$$

ikkinchi ajoyib (muhim) limit deb yuritiladi.

6. Misol: quyida berilgan misolni 2 hil usulda ishlash mumkin

1- usul: $\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^x=\lim\limits_{x\to\infty}\left(1+\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}\cdot 3}=e^3$

2-usul: $\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^x$ misolda $\frac{3}{x}=t$ almashtirish bajaramiz. U holda $x\to\infty$ *da* $\frac{3}{x}=t\to 0$ bo'lib biz izlayotgan limit $\lim\limits_{x\to\infty}((1+t)^{\frac{1}{t}})^3=e^3$

7. Misol: $\lim\limits_{x\to\infty}\left(\frac{x}{x-1}\right)^{2x+2}=\lim\limits_{x\to\infty}\left(\frac{x-1}{x}\right)^{-(2x+2)}=\lim\limits_{x\to\infty}\left(\left(1+\frac{1}{-x}\right)^{-x}\right)^{2}\cdot\left(1+\frac{1}{-x}\right)^{-2}=e^{2}$

8. Misol: $\lim\limits_{x\to0}(1+2x)^{\frac{1}{x}}=\lim\limits_{x\to0}\left((1+2x)^{\frac{1}{2x}}\right)^{2}=e^{2}$

9. Misol: $\lim\limits_{x\to0}\frac{\ln(1+x)}{x}=\lim\limits_{x\to0}\frac{1}{x}\ln(1+x)=\lim\limits_{x\to0}\ln(1+x)^{\frac{1}{x}}=\ln\lim\limits_{x\to0}(1+x)^{\frac{1}{x}}=\ln e=1.$

10. Misol: $\lim\limits_{x\to0}\left(\frac{1+tgx}{1+\sin x}\right)^{\frac{1}{\sin x}}$ bu misolni ishlshda quyidagi formuladan foydalanamiz: $\lim\limits_{x\to x_0}(u)^{v}=e^{\lim\limits_{x\to x_0}(u-1)v}$. Demak,

$$\lim_{x\to0}\left(\frac{1+tgx}{1+\sin x}\right)^{\frac{1}{\sin x}}=e^{\lim\limits_{x\to0}\frac{tgx-\sin x}{1+\sin x}\frac{1}{\sin x}}=e^{\lim\limits_{x\to0}\frac{2\sin^{2}\frac{x}{2}}{\cos x(1+\sin x)}}=e^{0}=1$$

Auditoriyada bajarish uchun misollar.

1.1. $\lim\limits_{x\to0}\frac{\sin x-x}{\sin x}$

1.2. $\lim\limits_{x\to0}\frac{tg3x}{x}$

1.3. $\lim\limits_{x\to0}\frac{x^{3}}{tgx-\sin x}$

1.4. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos 2x}$

1.5. $\lim\limits_{x\to0}\frac{\sqrt{1-\cos 2x}}{x}$

1.6. $\lim\limits_{x\to0}\frac{\sin 4x}{\sqrt{x+1}-1}$

1.7. $\lim\limits_{x\to0}\frac{1-\cos 2x+tg^{2}x}{x\sin x}$

1.8. $\lim\limits_{h\to0}\frac{\cos(x+h)-\cos(x-h)}{h}$

1.9. $\lim\limits_{x\to2}\left[\frac{\sin(x-2)}{x^{2}-4}+2^{-\frac{1}{(x-2)^{2}}}\right]$

1.10. $\lim\limits_{x\to-2}\frac{\arcsin(x+2)}{x^{2}+2x}$

1.11. $\lim\limits_{x\to\infty}\left(\frac{n}{n+1}\right)^{n}$

1.12. $\lim\limits_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{3x+2}$

1.13. $\lim\limits_{x\to\infty}\left(\frac{3x+4}{3x+7}\right)^{2x-5}$

1.14. $\lim\limits_{x\to\infty} n[\ln(n+3)-\ln n]$

1.15. $\lim\limits_{x\to 0}(\cos x)^{ctg^2 x}$

1.16. $\lim\limits_{x\to 0}(1+3tg^2 x)^{ctg^2 x}$

1.17. $\lim\limits_{x\to 0}\frac{e^{-x}-1}{x}$

1.18. $\lim\limits_{x\to 0}\frac{\ln(1+x)}{x}$

1.19. $\lim\limits_{x\to 0}\frac{a^{2x}-1}{x}$

1.20. $\lim\limits_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{2x}$

3. Uy vazifasi uchun misollar.

3.1. $\lim\limits_{x\to 0}\frac{x}{\sin 3x}$

3.2. $\lim\limits_{x\to 0}\frac{1-\cos mx}{x^2}$

3.3. $\lim\limits_{x\to 0}\frac{\sqrt{x+4}-2}{\sin 5x}$

3.4. $\lim\limits_{x\to 0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}$

3.5. $\lim\limits_{x\to 1}\frac{\sin(1-x)}{\sqrt{x}-1}$

3.6. $\lim\limits_{n\to+\infty}\left(1+\frac{2}{n}\right)^{3n}$

3.7. $\lim\limits_{x\to+\infty}\left(\frac{x-3}{x}\right)^{\frac{x}{2}}$

3.8. $\lim\limits_{x\to 0}(1+2x)^{\frac{1}{x}}$

3.9. $\lim\limits_{n\to\infty} n[\ln n-\ln(x+2)]$

3.10. $\lim\limits_{x\to\frac{\pi}{4}}(\sin 2x)^{tg^2 2x}$

Funksiya uzluksizligi.

1. Funksiyaning uzluksizligi

Ta`rif: *Agar* $f(x)$ *funksiya* x_0 *nuqtaning atrofida aniqlangan bo'lib,* $\forall\varepsilon>0$ *olinganda shunday* $\delta>0$ *son mavjud bo'lsaki, argument x ning* $|x-x_0|<\delta$ *tengsizlikni qanoatlantiruvchi barcha qiymatlari uchun* $|f(x)-f(x_0)|<\varepsilon$ *tengsizlik o'rinli bo'lsa, u* x_0 *nuqtada uzluksiz deyiladi.*

Buni *funksiya uzluksizlgining «*$\varepsilon-\delta$*»tilidagi ta'rifi* deb yuritiladi.

Ta'rif. *Agar* $f(x)$ *funksiya* x_0 *nuqtada va uning o'ng (chap) yaqin atrofida aniqlangan bo'lib,*

$$f(x_0+0)=f(x_0)\big(f(x_0-0)=f(x_0)\big)$$

o'rinli bo'lsa, funksiya x_0 *nuqtada o'ngdan (chapdan) uzluksiz* deyiladi.

Funksiya uzilishini ikki turga ajratiladi. Agar x_0 $f(x)$ funksiyaning uzilish nuqtasi bo'lib, bu nuqtadagi funksiyaning o'ng va chap chekli limitlari mavjud bo'lsa, bu uzilish nuqtasi *birinchi jins (tur) uzilish nuqtasi*; agar o'ng va chap chekli limitlardan aqalli bittasi mavjud bo'lmasa, *ikkkinchi jins (tur) uzilish nuqtasi* deyiladi.

1. Misol. $y=\frac{x}{|x|}$ funksiya $x=0$ nuqtada aniqlanmagan, lekin, uning ixtiyoriy yaqin atrofida aniqlangan bo'lib,

$$f(-0)=\lim_{x\to-0}\frac{x}{|x|}=\lim(-1)=-1$$

$$f(+0)=\lim_{x\to+0}\frac{x}{|x|}=\lim 1=1$$

O'ng va chap limitlar mavjud. Demak, $x=0$ funksiyaning birinchi jins uzilish nuqtasi bo'lib, uning bu nuqtadagi sakrashi

$$f(+0)-f(-0)=1-(-1)=2$$

bo'ladi.

2. Misol:. $f(x)=\frac{\sin x}{x}$, $x=0$ da aniqlanmagan, ammo,

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

ekanligidan $f(+0)=f(-0)=1$

Endi,

$$f(x)=\begin{cases}\dfrac{\sin x}{x}, & x\neq 0,\\ 1\quad , & x=0\end{cases}$$

deb olsak, uzilish chetlantiriladi.

3. Misol: $f(x)=2^{\frac{1}{x}}$ funksiya $x=0$ nuqtada aniqlanmagan, lekin, uning ixtiyoriy yaqin atrofida mavjuddir. Demak, $x=0$ uzilish nuqtsidir.

$f(+0)=\lim\limits_{x\to+0}2^{\frac{1}{x}}=\lim\limits_{y\to+\infty}2^{y}=+\infty$ - o‘ng limit cheksiz,

$f(-0)=\lim\limits_{x\to-0}2^{\frac{1}{x}}=\lim\limits_{z\to-\infty}2^{z}=0$ - chekli chap limit mavjud.

O‘ng chekli limit mavjud emas, demak, uzilish ikkinchi jinsdir.

4. Misol: $f(x)=\sin\dfrac{1}{x}$; $x=0$ nuqtada funksiya aniqlanmagan, lekin, uning ixtiyoriy yaqin atrofida aniqlangan.

$$\lim_{x\to\pm 0}\sin\frac{1}{x}$$

ikkalasi ham mavjud emas. Demak, uzilish ikkinchi jins.

5. Misol: $f(x)=arctg\dfrac{1}{x}$;

$\lim\limits_{x\to+0}arctg\dfrac{1}{x}=\lim\limits_{x\to+\infty}arctgt=\dfrac{\pi}{2},\ chunki\ \ t>tg(\dfrac{\pi}{2}-\varepsilon)\ bo'lg\,anda\,arctgt>\dfrac{\pi}{2}-\varepsilon$

huddi shunga o’hshash

$\lim\limits_{x-+0}arctg\dfrac{1}{x}=-\dfrac{\pi}{2},$

Ga teng bo’ladi. Demak, x=0 birinchi jins uzilish nuqtasi ekan.

Ta’rif. *Agar* $f(x)$ *funksiya* $(a;b)$ *oraliqda aniqlangan va uning har bir nuqtasida uzluksiz bo‘lsa, bu funksiya* $(a;b)$ *oraliqda uzluksiz deyiladi; agar*

$[a;b]$ kesmada aniqlangan bo'lib, $(a;b)$ oraliqda uzluksiz, a nuqtada o'ngdan b nuqtada esa chapdan uzluksiz bo'lsa, funksiya $[a;b]$ kesmada uzluksiz deyiladi.

Ta`rif. *(Tekis uzluksizlik tushunchasi)*. Agar $f(x)$ funksiya biror *D* sohada aniqlangan bo'lib, ixtiyoriy olingan musbat ε uchun shunday musbat δ son topilsaki, $|x_1 - x_2| < \delta$ tengsizlikni qanoatlantiruvchi *D* sohadan olingan ixtiyoriy x_1 va x_2 nuqtalar uchun

$$|f(x_1) - f(x_2)| < \varepsilon$$

tengsizlik o'rinli bo'lsa, $f(x)$ funksiya *D* sohada *tekis uzluksiz* deyiladi.

Kantor teoremasi. *Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz bo'lsa, u bu kesmada tekis uzluksizdir.*

6. Misol: Berilgan sohada funksiyani tekis uzluksizlikka tekshiring.

   $a)\ \ f(x) = \dfrac{x}{4 - x^2} \quad (-1 \le x \le 1)$ Kantor teoremasiga ko'ra berilgan funksiya ko'rsatilgan sohada tekis uzluksizdir.

   b) $f(x) = \ln x \quad (0 < x < 1)$ tekis uzluksiz emas, chunki agar $x_n' = e^{-n},\ x_n'' = e^{-n-1}$ bo'lganda $n \to \infty\ da\ \left|x_n' - x_n''\right| = \dfrac{e-1}{e^{n+1}} \to 0$ bo'lib $\forall \varepsilon > 0\ uchun \left|f(x_n') - f(x_n'')\right| = |-n + n + 1| = 1 > \varepsilon$ bo'ladi.

Auditoriyada bajarish uchun misollar.

1.1. Quyidagi funksiyalarni uzluksiz ekanligini isbotlang.

$a)\ \ ax + b \quad b)\ \ x^2 \quad c)\ \ x^3 \quad d)\ \sqrt{x} \quad e)\ \ \sin x \quad j)\ \ \cos x \quad z)\ \ arctgx$

1.2. Quyidagi funksiyalarni uzluksizlikka tekshiring va uzulish nuqtalarini toping, qaysi turga tegishli ekanligini aniqlang

$a)\quad y = x\sin\frac{1}{x}$

$b)\quad y = x\ln x^2$

$c)\quad y = \frac{x}{(1+x)^2}$

$d)\quad y = \sqrt{x}\,arctg\,\frac{1}{x}$

$e)\quad y = e^{x+\frac{1}{x}}$

1.3. Berilgan funksiyalarni ko'rsatilgan sohada tekis uzluksizlikka tekshiring;

$a)\quad y = x\sin x\quad (0 \le x < +\infty)$

$b)\quad y = arctgx\quad (-\infty < x < +\infty)$

$c)\quad y = \sqrt{x}\qquad (1 \le x < +\infty)$

$d)\quad y = e^x\cos\frac{1}{x}\quad (0 < x < 1)$

$e)\quad y = \frac{\sin x}{x}\quad (0 < x < \pi)$

Uy vazifasi uchun misollar

1.1. Quyidagi funksiyalarni uzluksizlikka tekshiring va uzulish nuqtalarini toping, qaysi turga tegishli ekanligini aniqlang.

$a)\quad y = arctg\,\frac{1}{1-x^2}$

$b)\quad y = \arcsin(\sin x)$

$c)\quad y = \ln arctg\,\frac{1}{x}$

$d)\quad y = tg\,\frac{1}{x}$

$e)\quad y = \frac{e^x - 1}{x}$

1.2. Berilgan sohada funksiyalarni tekis yaqinlashishga tekshiring;

$a)\quad y = \sqrt{x^2+1}\quad (-\infty < x < +\infty)$

$b)\quad y = \sqrt{x}\ln x\quad (1 \le x < +\infty)$

$c)\quad y = x^\alpha\qquad (1 \le x < +\infty)$

$d)\quad y = \frac{x^2}{x+1}\quad (0 \le x < +\infty)$

Funksiya hosilasi.

Aytaylik, $y=f(x)$ funksiya x_0 nuqtaning atrofida aniqlangan bo‘lsin. Bu nuqtadagi argument va funksiya orttirmalari mos ravishda $\Delta x=x-x_0$ va $\Delta y=f(x)-f(x_0)$ bo‘lishi ma’lumdir.

Ta’rif. Agar $y=f(x)$ funksiya x_0 nuqta atrofida aniqlangan bo‘lib, shu nuqtadagi funksiya orttirmasi Δy ning argument orttirmasi Δx ga nisbatining argument orttirmasi nolga intilgandagi $(\Delta x\to 0)$ chekli limiti mavjud bo‘lsa, bu limit *funksiyaning x_0 nuqtadagi hosilasi* deyiladi va $y',\ f'(x_0), \frac{dy}{dx}, \frac{df(x_0)}{dx}$ lardan biri bilan belgilanadi.

1-misol. $y=x^2$ funksiyaning ixtiyoriy x_0 nuqtadagi hosilasi topilsin.

Yechish.

1) $\Delta x=x-x_0 \Rightarrow x=x_0+\Delta x;$

2) $\Delta y=f(x)-f(x_0)=x^2-x_0^2=(x+x_0)(x-x_0)=(2x_0+\Delta x)\cdot\Delta x;$

3) $\frac{\Delta y}{\Delta x}=\frac{(2x_0+\Delta x)\cdot\Delta x}{\Delta x}=2x_0+\Delta x;$

4) $\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}(2x_0+\Delta x)=2x_0 \Rightarrow y'=2x_0$

2-misol. $y=\sin x$ ning ixtiyoriy nuqtadagi hosilasi topilsin.

Yechish.

$$\Delta y=\sin(x+\Delta x)-\sin x=2\sin\frac{x+\Delta x-x}{2}\cdot\cos\frac{x+\Delta x+x}{2}=2\sin\frac{\Delta x}{2}\cdot\cos\left(x+\frac{\Delta x}{2}\right)$$

$$\frac{\Delta y}{\Delta x}=\frac{2\sin\frac{\Delta x}{2}}{\Delta x}\cdot\cos\left(x+\frac{\Delta x}{2}\right)=\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}\cdot\cos\left(x+\frac{\Delta x}{2}\right)$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=1\cdot\cos x$$

Demak, $(\sin x)'=\cos x$

Differensiallashning asosiy qoidalari

1. O‘zgarmasni differensiallash. $y=c$, bu yerda c- o‘zgarmas.

$\Delta y=0$ ekanligidan $y'=0$, ya’ni $c'=0$

kelib chiqadi. Demak, *o‘zgarmasning hosilasi nolga teng* ekan.

2. Yigindini differensiallash. Agar u va υ funksiyalar differensiallanuvchi bo‘lsa, ularning algebraik yig‘indisi ham differensiallanuvchi va

$$(u\pm\upsilon)'=u'\pm\upsilon'$$

o‘rinlidir.

Haqiqatdan ham, $\Delta(u\pm\upsilon)=\Delta u\pm\Delta\upsilon$ ekanligidan yuqoridagini keltirib chiqarish qiyin emas.

3. Ko‘paytmani differensiallash. Agar u va υ funksiyalar differensiallanuvchi bo‘lsa,

$$(u\upsilon)'=u'\upsilon+u\upsilon'$$

o‘rinlidir.

4. O‘zgarmas ko‘paytuvchili ko‘paytmani differensiallash. Agar u differensiallanuvchi funksiya bo‘lib, c o‘zgarmas bo‘lsa,

$$(cu)'=cu'$$

o‘rinlidir, ya’ni *o‘zgarmas ko‘paytuvchini hosila belgisi tashqarisiga chiqarish mumkin.*

5. Bo‘linmani differensiallash. Agar u va υ funksiyalar differensiallanuvchi bo‘lib, $\upsilon\neq 0$ bo‘lsa,

$$\left(\frac{u}{\vartheta}\right)' = \frac{u'\vartheta - u\vartheta'}{\vartheta^2}$$

o‘rinlidir.

Hosila jadvali

1. $\left(x^{\alpha}\right)' = \alpha x^{\alpha-1} \qquad (\alpha \neq 0, \quad \alpha \in R).$

2. $\left(a^{x}\right)' = a^{x}\ln a \qquad \left(a \in R^{+},\ a \neq 1\right).$ $(e^x)' = e^x$

3. $\left(\log_a x\right)' = \dfrac{1}{x\ln a} = \dfrac{\log_a e}{x} \qquad \left(a \in R^{+}, \quad a \neq 1\right). \left(\ln x\right)' = \dfrac{1}{x}$

4. $(\sin x)' = \cos x.$

5. $(\cos x)' = -\sin x.$

6. $\left(tg\, x\right)' = \dfrac{1}{\cos^2 x}.$

7. $\left(ctg\, x\right)' = -\dfrac{1}{\sin^2 x}.$

8. $\left(\arcsin x\right)' = \dfrac{1}{\sqrt{1-x^2}}.$

9. $\left(\arccos x\right)' = -\dfrac{1}{\sqrt{1-x^2}}.$

10. $\left(arctg\, x\right)' = \dfrac{1}{1+x^2}.$

11. $\left(arcctg x\right)' = -\dfrac{1}{1+x^2}.$

3. Misol: $y = \dfrac{x^3}{3} + 2x^2 + 4x - 5 \quad y' = ?$

$$y' = \left(\frac{x^3}{3}\right)' + (2x^2)' + (4x)' - (5)' = \frac{3x^2}{3} + 2\cdot 2x + 4x^0 - 0 = x^2 + 4x + 4$$

Bu misolni ishlashda quyidagi formuladan foydalandik:

$$\left(x^{\alpha}\right)' = \alpha x^{\alpha-1} \qquad (\alpha \neq 0, \quad \alpha \in R).$$

4. Misol: $y = x + \dfrac{1}{x^2} - \dfrac{1}{5x^5} \quad y' = ?$

$$y' = x' + (x^{-2})' - \frac{1}{5}(x^{-5})' = 1 - 2x^{-3} - \frac{1}{5}\cdot(-5)x^{-6} = 1 - \frac{2}{x^3} + \frac{1}{x^6}$$

5. Misol: $y = x^2 \cos x \quad y' = ?$

Bu misolni ishlashda ko'paytmani hosilasini topish formulasidan foydalanamiz: *(uυ)'=u'υ+uυ'*

$$y' = (x^2 \cos x)' = (x^2)' \cos x + x^2 (\cos x)' = 2x \cos x - x^2 \sin x$$

6. Misol: $y = \sqrt{x} \ln x \quad y' = ?$

$$y' = (\sqrt{x}\ln x)' = (\sqrt{x})' \ln x + \sqrt{x}(\ln x)' = \frac{1}{2\sqrt{x}} \ln x + \sqrt{x}\cdot\frac{1}{x} = \frac{1}{2\sqrt{x}} \ln x + \frac{1}{\sqrt{x}} = \frac{\ln x + 2}{2\sqrt{x}}$$

7. Misol:

$$y = \frac{\cos x}{1 - \sin x} \quad y' = ?$$

$$y' = \frac{(\cos x)'(1-\sin x) - \cos x(1-\sin x)'}{(1-\sin x)^2} = \frac{-\sin x(1-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2} =$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2} == \frac{1-\sin x}{(1-\sin x)^2} = \frac{1}{1-\sin x}$$

Bu misolfa ishlashda boälinmaning hosilasini topish formulasidan fozdalandik $\left(\frac{u}{\vartheta}\right)' = \frac{u'\vartheta - u\vartheta'}{\vartheta^2}$

8. Misol: $y = \frac{x^2 - 1}{x^2 + 1} \quad y' = ?$

$$y' = \frac{(x^2-1)'(x^2+1) - (x^2-1)(x^2+1)'}{(x^2+1)^2} = \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2} =$$

$$= \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

Hosilaning fizik va geometrik ma'nolari

Fizik ma’nosi:

Moddiy nuqta to‘g‘ri chiziqli harakatda bo‘lsa, bosib o‘tayotgan yo‘lning funksiyasidan olingan hosila uning qaralayotgan paytdagi tezligidan iborat:

$$\vartheta = \lim_{\Delta t \to 0} \overline{\vartheta} = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = S'(t)$$

Geometric ma’nosi:

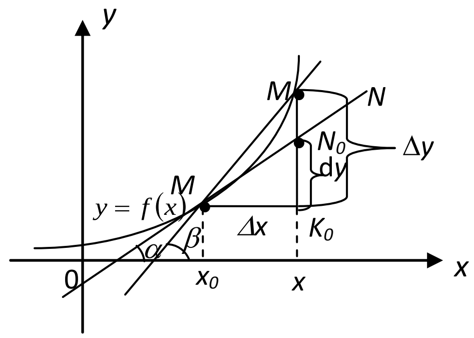
Funksiya grafigiga o‘tkazilgan urinma tenglamasi quyidagi ko’rinishda bo‘ladi:

$$y - f(x_0) = f'(x_0)\,(x - x_0).$$

Qaralayotgan nuqtadagi normal (urinmaga perpendikulyar to‘g‘ri chiziq) tenglamasi

$$y - f(x_0) = -\frac{1}{f'(x_0)}(x - x_0)$$

bo‘ladi(bu yerda $f'(x_0) \neq 0$ deb faraz qilindi).



9. Misol: $y = 4 - x^2$ parabolaning O_x o’qi bilan kesishish nuqtasiga o’tkazilgan urinma va normal tenglamasini tuzing.
   Yechish:
   O_x *o'qi bilan* $x = -2$ *va* $x = 2$ *nuqtada kesishadi, demak urinma chiziq tenglamasi:*
   $y' = -2x \quad y'_{x=-2} = -2 \cdot (-2) = 4$ *bo'lib* $y - 0 = 4(x+2) \Rightarrow y = 4x + 8$
   $y'_{x=2} = -2 \cdot 2 = -4$ *bo'lib* $y - 0 = -4(x-2) \Rightarrow y = 8 - 4x$
   normal tenglama: $y - 0 = -\frac{1}{4}(x+2) \Rightarrow y = -\frac{1}{4}x - \frac{1}{2} \quad x = -2$ *da*
   $y - 0 = \frac{1}{4}(x-2) \Rightarrow y = \frac{1}{4}x - \frac{1}{2} \quad x = 2$ *da*

Auditoriyada bajariladigan misollar.

1.1. $a)\ y=\frac{x^4}{4}-\frac{x^3}{3}+\frac{x^2}{2}+4x-5$ $\qquad b)\ y=\left(1-\frac{x^2}{2}\right)^2$

1.2. $a)\ y=\frac{10}{x^3}$ $\qquad b)\ y=x+\frac{1}{x^2}+\frac{1}{5x^5}$

1.3. $a)\ y=x+2\sqrt{x}$ $\qquad b)\ y=3x-6\sqrt{x}$

1.4. $a)\ y=6\sqrt[3]{x}-4\sqrt[4]{x}$ $\qquad b)\ y=\frac{8}{\sqrt[4]{x}}-\frac{6}{\sqrt[6]{x}}$

1.5. $a)\ y=x\cos x$ $\qquad b)\ y=x^2 ctgx$

1.6. $a)\ y=\sqrt{x}\sin x$ $\qquad b)\ y=x^3 tgx$

1.7. $a)\ y=\frac{\cos x}{1-\sin x}$ $\qquad b)\ y=\frac{x^2}{x^2+1}$

1.8. $a)\ y=\frac{\cos x}{1+2\sin x}$ $\qquad b)\ y=\frac{x}{1-4x}$

1.9. $f(x)=\frac{x^3}{3}-x^2+1$ *bo'lsa* $f'(0),\ f'(1)$ *ni hisoblang*

1.10. $f(x)=\frac{x}{2x-1}$ *bo'lsa,* $f'(0),\ f'(-2)$ *ni hisoblang*

Quyidagi egri chiziqlarga o'tkazilgan ueinma chiziq va normal tenglamasini tuzing:

2.1. $y=\frac{x^3}{3}$ $\quad x=-1$ *nuqtada o'tkazilgan bo'lsa*

2.2. $y^2=x^3$ $\quad x_1=0$ *va* $x_2=1$ *nuqtalarda o'tkazilgan bo'lsa*

2.3. $y=\frac{8}{4+x^2}$ $\quad x=2$ *nuqtada o'tkazilgan bo'lsa*

2.4. $y=\sin x$ $\quad x=\pi$ *nuqtada o'tkazilgan bo'lsa*

3.1. Jism $x=\frac{t^3}{3}-2t^2+3t$ qonunga qonunga asosan O_x to'g'ri chiziq bo`yicha harakat qiladi. Harakat tezligi va tezlanish aniqlansin.

3.2. Radiusi a gat eng g'ildirak to`g`ri chiziq bo'yicha yumalaydi. G'ildirakning t sekunddagi burilish burchagi $\varphi=t+\frac{t^2}{2}$ tenglama bilan aniqlanadi. G'ildirak markazining harakat tezligi va tezlanishi aniqlansin.

3.Uy vazifasi uchun misollar.

3.1. $a)\ \ y=(1+\sqrt{x})^2$ $\qquad b)\ \ y=\frac{1}{x^3}-\frac{1}{x^2}+1$

3.2. $a)\ \ y=\frac{3}{\sqrt[3]{x}}-\frac{2}{\sqrt{x}}$ $\qquad b)\ \ y=x+ctgx$

3.3. $a)\ \ y=x^2e^x$ $\qquad b)\ \ y=\frac{1}{x}\ln x$

3.4. $a)\ \ y=(x^3-1)\sin x$ $\qquad b)\ \ y=\sqrt[4]{x}\cos x$

3.5. $a)\ \ y=\frac{x^2-1}{x^2-3}$ $\qquad b)\ \ y=\frac{\sin x}{e^x}$

3.6. $a)\ \ y=\frac{tgx-2}{ctgx+2}$ $\qquad b)\ \ y=\frac{e^x}{x-1}$

Quyidagi egri chiziqlarga berilgan nuqtada o'tkazilgan urinma chiziq va normal tenglamasini tuzing:

3.7. $y=4x-x^2 \qquad x=0 \ \ va \ \ x=4 \ \ nuqtalarda$

3.8. $xy=4 \qquad x=1 \quad x=-4 \ \ nuqtalarda$

3.9. $y^2=4-x \quad x=4 \ \ nuqtada$

3.10.Burchak tezligi $\frac{d\varphi}{dt}=\omega$ burchak tezlanishi esa $\frac{d\omega}{dt}=\varepsilon$ bo'lsin. $\frac{d(\omega^2)}{d\mu}=2\varepsilon$ Ekanligi isbotlansin

3.11. tormoz bilan to'htatiladigan aylanuvchi mahovik t sekundda $\varphi=a=bt-ct^2$ burchakka buriladi. Bundagi a, b va c lar musbat o'zgarmas miqdorlar. Tezlik va tezlanish aniqlansin. G'ildirak qachon to'htaydi?

Murakkab funksiya hosilasi

Ta'rif: Agar $z=\varphi(x)$ biror D(z) oraliqda, $y=f(z)$ esa biror D_0(z) oraliqda aniqlangan funksiyalar bo'lib, $\forall\times\in D(z)\Rightarrow\varphi(x)\in D_0(z)$ hamda $\varphi(x)$ funksiya x_0 nuqtada, $f(z)$ esa $z_0=\varphi(x_0)$ nuqtada differensiallanuvchi bo'lsa, $y=f[\varphi(x)]$ murakkab funksiya x_0 nuqtada differensiallanuvchi va

$$u'=f'[\varphi(x_0)]\cdot\varphi'(x_0)$$

o‘rinli bo‘ladi.

Bu ta’rifni hosila jadvaliga tatbiq etsak, quyidagilarga ega bo’lamiz:

1. $\left(u^n\right)' = nu^{n-1}\cdot u'$
2. $(\sin u)' = \cos u\cdot u'$
3. $(\cos u)' = -\sin u\cdot u'$
4) $\left(\sqrt{u}\right)' = \dfrac{u'}{2\sqrt{x}}$
5. $(tgu)' = \dfrac{u'}{\cos^2 u}$
6. $(ctgu)' = -\dfrac{u'}{\sin^2 u}$
7. $(\ln u)' = \dfrac{u'}{u}$
8. $\left(e^u\right)' = e^u u'$
9. $(a^u)' = a^u u'\ln a$
10. $(\arcsin u)' = \dfrac{u'}{\sqrt{1-u^2}}$
11. $(\arccos u)' = -\dfrac{u'}{\sqrt{1-u^2}}$
12. $(arctgu)' = \dfrac{u'}{1+u^2}$
13. $(arcctgu)' = -\dfrac{u'}{1+u^2}$

1. Misol: $y=(1-5x)^4 \qquad y'=?$

$$y'=4(1-5x)^3\cdot(1-5x)'=4(1-5x)^3\cdot(-5)=-20(1-5x)^3$$

2. Misol: $y=\sin\dfrac{x}{2} \qquad y'=?$

$$y'=\cos\frac{x}{2}\cdot\left(\frac{x}{2}\right)'=\frac{1}{2}\cos\frac{x}{2}$$

3. Misol:

$$y=\sqrt{\cos 2x} \qquad y'=?$$

$$y'=\frac{(\cos 2x)'}{2\sqrt{\cos 2x}}=\frac{-2\sin 2x}{2\sqrt{\cos 2x}}=-\frac{\sin 2x}{\sqrt{\cos 2x}}$$

4. Misol:

$$y=\sin^2 x \qquad y'=?$$

$$y'=((\sin x)^2)'=2\sin x\cos x=\sin 2x$$

5. Misol: $y=\ln(x^2+2x) \qquad y'=?$

$$y'=\frac{2x+2}{x^2+2x}$$

6. Misol: $y=\ln\sqrt{\dfrac{1+2x}{1-2x}} \qquad y'=?$

$$y' = \frac{1}{\sqrt{\frac{1+2x}{1-2x}}} \cdot \frac{1}{\sqrt{\frac{1+2x}{1-2x}}} \cdot \frac{2(1-2x)+2(1+2x)}{(1-2x)^2} = \frac{1-2x}{1+2x} \cdot \frac{4}{(1-2x)^2} = \frac{4}{1-4x^2}$$

7. Misol: $y = \sqrt{x}e^{\sqrt{x}} \qquad y' = ?$

$$y' = \frac{1}{2\sqrt{x}} e^{\sqrt{x}} + \sqrt{x}e^{\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{1+\sqrt{x}}{2\sqrt{x}} e^{\sqrt{x}}$$

8. Misol: $y = \arcsin\sqrt{1-2x} \qquad y' = ?$

$$y' = \frac{1}{\sqrt{1-(\sqrt{1-2x})^2}} \cdot \frac{-2}{2\sqrt{1-2x}} = \frac{-1}{\sqrt{2x(1-2x)}}$$

9. Misol: $y = arctg\,\frac{1+x}{1-x} \qquad y' = ?$

$$y' = \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \cdot \frac{1-x+1+x}{(1-x)^2} = \frac{(1-x)^2}{1-2x+x^2+1+2x+x^2} \cdot \frac{2}{(1-x)^2} = \frac{2}{2(1+x^2)} = \frac{1}{1+x^2}$$

10. Misol: $y = \sqrt{4x-1} + arcctg\sqrt{4x-1} \qquad y' = ?$

$$y' = \frac{4}{2\sqrt{4x-1}} - \frac{1}{\sqrt{1-4x+1}} \cdot \frac{4}{2\sqrt{4x-1}} = \frac{2}{\sqrt{4x-1}} - \frac{1}{\sqrt{x(4x-1)}} = \frac{2\sqrt{x}-1}{\sqrt{x(4x-1)}}$$

Auditoriyada bajariladigan misollar

2.1. $а)\ y = \sqrt{x} + 2\sin 2x - 3e^{2x} + \frac{1}{x}$ $\qquad б)\ y = \ln x + \sqrt[3]{x} - \frac{x+1}{x^2+2}$

2.2. $а)\ y = \sin 5x + e^{3x} - \frac{\sin x}{x}$ $\qquad б)\ y = \frac{e^{2x}}{x^2+1} + x^2\cos x$

2.3. $а)\ y = \sin^2 x - \sqrt{x} + x^2\cos x$ $\qquad б)\ y = \frac{\cos x}{e^x} - \ln\left(x^2+1\right)$

2.4. $а)\ y = \sqrt[3]{x+1}\cos\frac{x}{2} + \frac{\sin 2x}{e^x}$ $\qquad б)\ y = x^2e^x - \ln\left(x^2+1\right)$

2.5. $а)\ y = \left(2x+1\right)^3 - x^2\sin x$ $\qquad б)\ y = \frac{\cos x}{e^{2x}} + \sqrt{x^3+1}$

2.6. $а)\ y = \sqrt{4x+\sin 4x}$ $\qquad б)\ y = tgx + \frac{2}{3}tg^3x + \frac{1}{5}tg^5x$

2.7. $а)\ y = \sin^2 x^3$ $\qquad б)\ y = \frac{1+\sin 3x}{1-\sin 3x}$

2.8. $a)\ y = x\sqrt{x^2+1}$ $\quad б)\ y=\sqrt{1+\sin 2x}-\sqrt{1-\sin 2x}$

2.9. $a)\ y=\ln(\sqrt{x}-\sqrt{x+1})$ $\quad б)\ y=\ln\dfrac{1+\sqrt{x^2+1}}{x}$

2.10. $a)\ y=\ln(\sin x+\sqrt{1+\sin^2 x})$ $\quad б)\ y=\ln\sqrt{\dfrac{\sin 2x}{1-\sin 2x}}$

2.11. $a)\ y=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$ $\quad б)\ y=\ln\sqrt{\dfrac{e^{4x}}{e^{4x}+1}}$

2.12. $a)\ y=x^{\frac{1}{x}}$ $\quad б)\ y=arctgx+\ln\sqrt{\dfrac{1-x}{1+x}}$

2.13. $a)\ y=xarctg\dfrac{x}{a}-\dfrac{a}{2}\ln(x^2+a^2)$ $\quad б)\ y=e^x\sqrt{1-e^{2x}}+\arcsin e^x$

2.14. $a)\ y=\arccos\sqrt{1-2x}+\sqrt{2x-4x^2}$ $\quad б)\ y=\sin^4 x+\cos^4 x$

2.15. $a)\ y=\arcsin\dfrac{x-1}{x}$ $\quad б)\ s=\dfrac{1}{4}tg^4t-\dfrac{1}{2}tg^2t-\ln(\cos t)$

3. Uy vazifasi uchun misollar

3.1. $a)\ y=6\cos^3 x$ $\quad б)\ y=\sqrt[4]{(2+3x)^3}$

3.2. $a)\ y=\dfrac{1}{(1-x^3)^4}$ $\quad б)\ y=\sin^4 x$

3.3. $a)\ y=\dfrac{1}{(1+\cos 4x)^5}$ $\quad б)\ y=ctg^3x$

3.4. $a)\ y=\ln(x^3-3x)$ $\quad б)\ y=\ln\sin x-\dfrac{1}{2}\sin^2 x$

3.5. $a)\ y=\ln tg(\dfrac{\pi}{4}+\dfrac{x}{2})$ $\quad б)\ y=\ln\dfrac{x}{\sqrt{1-x^2}}$

3.6. $a)\ y=\ln(e^{2x}+\sqrt{e^{4x}+1})$ $\quad б)\ s=\ln\dfrac{2+tgt}{2-tgt}$

3.7. $a)\ y=\arcsin(e^{3x})$ $\quad б)\ y=arctg\sqrt{5x-1}$

3.8. $a)\ y=\dfrac{\sqrt{x^2-1}}{x}+\arcsin\dfrac{1}{x}$ $\quad$ $б)\ y=\dfrac{tg^2x}{2}+\ln\cos x$

3.9. $a)\ y=\sqrt{\sin x^2}$ $\quad$ $б)\ y=e^{-\frac{x}{a}}\cos\dfrac{x}{a}$

3.10. $a)\ x=\ln(e^{2t}+1)-2arctg(e^t)$ $\quad$ $б)\ s=\sqrt{4t-t^2}+4\arcsin\dfrac{\sqrt{t}}{2}$

Yuqori tartibli hosilalar va funksiya differensiali.

1. Yuqori tartibli hosilalar

Yuqorida funksiyaning hosilasi argumentning ixtiyoriy qiymatida (aniqlanish sohasiga tegishli) mavjud boʻlsa, u ham funksiyadan iborat ekanligini koʻrdik.

Agar funksiya hosilasi ham hosilaga ega boʻlsa, *hosiladan olingan hosilani ikkinchi tartibli hosila* deb yuritiladi.

Funksiyaning hosilasini uning *birinchi tartibli hosilasi* deb qabul qilsak, umumiy holda quyidagi ta’rifni berish mumkin.

T**a’rif.** *Agar funksiyaning (n-1) tartibli hosilasi differensialanuvchi boʻlsa, uning hosilasini funksiyaning n-tartibli hosilasi* deyiladi va $y^{(n)}$, $\dfrac{d^n y}{dx^n}$, $f^{(n)}(x)$, $\dfrac{d^n f(x)}{dx^n}$ kabi belgilanadi. Bu holda *funksiya n marta differensiallanuvchi* deyiladi.

1. Misol. $y=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n$ boʻlsa,

$y'=na_0x^{n-1}+(n-1)a_nx^{n-2}+\ldots+a_{n-1}$,

- - - - - - - - - - - - - - - - - - -

$y^{(n)}=n\cdot(n-1)\cdot\ldots\cdot2\cdot1\cdot a_0=a_0\,n!$,

$y^{(n+1)}=y^{(n+2)}=\ldots=0$.

Demak, n – darajali koʻphadning n – tartibli hosilasi oʻzgarmas son boʻlib, $(n+1)$- tartibli hosilasidan boshlab yuqori tartibli hosilalarining barchasi nolga teng boʻlar ekan.

2. Misol. $f(x)=e^{kx}$, k – oʻzgarmas $(k\neq 0)$.

$$f'(x)=e^{kx}(kx)'=ke^{kx};$$

$$f''(x)=(f'(x))'=(ke^{kx})'=k(e^{kx})'=k\cdot ke^{kx}=k^2e^{kx}$$

va hokazo,

$$f^{(n)}(x)=k^ne^{kx}$$

ni olamiz. Demak,

$$(e^{kx})^{(n)}=k^ne^{kx},\ \ n\in N$$

3. Misol. $f(x)=\sin x$.

$$f'(x)=\cos x=\sin(x+\frac{\pi}{2}),$$

$$f''(x)=(f'(x))'=(\sin(x+\frac{\pi}{2}))'=\cos(x+\frac{\pi}{2})\cdot 1=\sin(x+\pi),$$

- -

$$f^{(n)}(x)=\sin(x+n\cdot\frac{\pi}{2}),$$

ya’ni $\quad (\sin x)^{(n)}=\sin(x+n\cdot\frac{\pi}{2}),\ \ n\in N$

4. Misol. $f(x)=\cos x$.

Yuqoridagiga oʻxshash,

$$(\cos x)^{(n)}=\cos(x+n\cdot\frac{\pi}{2}),\ \ n\in N$$

ni olish mumkin.

5. Misol. $f(x)=U\cdot V$, bu yerda U va V lar ixtiyoriy tartibli hosilalari mavjud funksiyalardir.

$$(U\cdot V)'=U'V+UV'$$

(UV)′′=(U′V+UV′)′=U′′V+U′V′+U′V′+UV′′=U′′V+ 2U′V′+UV′′

va hokazo.

$$(U\cdot V)^{(n)}=\sum_{k=0}^{n}C_n^K\,U^{(n-k)}\cdot V^{(k)}$$

ni olish mumkin. Bu *Leybnis formulasi* deb yuritiladi. Bu yerda nolinchi tartibli hosila funksiyaning o‘zi ekanligini eslash lozim.

6.**Misol**: $y=x^2e^{2x}$ $\qquad y^{(20)}=?$

Leybnitsning formulasidan foydalanamiz: $u=x^2 \quad v=e^{2x}$ desak

$u'=2x \quad u''=2, \quad u'''=.....=u^{(20)}=0, \quad v'=2e^{2x}, \quad v''=4e^{2x},...., \; v^{(20)}=2^{20}e^{2x}$

ni hisoblab, formulaga qo’ysak,

$$(x^2e^{2x})^{(20)}=u^{(0)}v^{(20)}+C_{20}^1u'v^{(19)}+C_{20}^2u''v^{(18)}=x^2\cdot 2^{20}e^{2x}+20\cdot 2x\cdot 2^{19}e^{2x}+\frac{20\cdot 19}{2}\cdot 2\cdot 2^{18}\cdot e^{2x}=$$

$$=2^{20}(x^2+20x+95)e^{2x}$$

7. Misol: $y=\dfrac{e^x}{x} \qquad y^{(10)}=?$

$$u=\frac{1}{x} \qquad v=e^x \text{ desak,}$$

$$(\frac{1}{x}e^x)^{(10)}=\frac{e^x}{x}-C_{10}^1\frac{e^x}{x^2}+2C_{10}^2\frac{e^x}{x^3}-2\cdot 3C_{10}^3\frac{e^x}{x^4}+2\cdot 3\cdot 4C_{10}^4\frac{e^x}{x^5}-5!C_{10}^5\frac{e^x}{x^6}+6!C_{10}^6\frac{e^x}{x^7}-7!C_{10}^7\frac{e^x}{x^8}+$$

$$+8!C_{10}^8\frac{e^x}{x^9}-9!C_{10}^9\frac{e^x}{x^{10}}+10!\frac{e^x}{x^{11}}=e^x\sum_{n=0}^{10}(-1)^n\,C_{10}^n\,\frac{n!}{x^{n+1}}$$

2. Funksiya differensiali

Aytaylik, *y=f(x)* funksiya x_0 nuqtaning qandaydir atrofida aniqlangan bo‘lsin.

Ta’rif. Agar *y=f(x)* funksiyaning x_0 nuqtadagi orttirmasi Δy argument orttirmasi Δx ga nisbatan chiziqli bosh qismga ega bo‘lsa, bu chiziqli bosh qism funksiyaning x_0 nuqtadagi *differensiali* deyiladi va *dy* yoki $df(x_0)$ bilan belgilanadi.

Demak, ta’rif bo‘yicha x_0 nuqtada *y=f(x)* funksiya differensiali mavjud bo‘lsa,

$$\Delta y=(A+\alpha)\Delta x=A\Delta x+\alpha\Delta x$$

kabi yozish mumkin bo‘lib, bu yerda A-o‘zgarmas, α esa $\Delta x \to 0$ da cheksiz kichik miqdordir.

Bu vaqtda,

$$dy = A\Delta x$$

bo‘ladi.

Agar x_0 nuqtada $y=f(x)$ funksiya chekli hosilaga ega bo‘lsa, yuqoridagi formulada $A=f'(x_0)$ bo‘ladi. Buning aksinchasini ham ko‘rsatish qiyin emas. Demak, funksiyaning differensiali mavjud bo‘lishi uchun qaralayotgan nuqtada u differensiallanuvchi bo‘lishi zarur va yetarli, ya’ni

$$dy = f'(x_0)\Delta x$$

o‘rinli ekan.

Endi, $y=x$ funksiyani olsak, yuqoridagi formula asosida

$dy=\Delta x$ bo‘lishini yoki $dx = \Delta x$ ekanligini ko‘ramiz. Shunday qilib, argument (erkli o‘zgaruvchi) differensialini orttirmasiga teng deb olsak, x_0 o‘rniga ixtiyoriy x nuqta deb olsak funksiya differensiali uchun

$$dy = f'(x)\, dx$$

ni olamiz.

Bu yerda differensiallashning asosiy qoidalaridagi barcha formulalar funksiyaning differensiali uchun ham o‘rinli ekanligini eslatamiz:

1. $dC = 0$ *(C-constant);*
2. $d(U \pm V) = dU \pm dV$;
3. $d(UV) = VdU + VdU$;
4. $d(CU) = CdU$ *(C-constant);*
5. $d\left(\dfrac{U}{V}\right) = \dfrac{VdU - VdU}{V^2}$.

7. Misol: $y = x^n$ $\quad dy = ?$

   $y' = nx^{n-1}$ $\quad dy = nx^{n-1}dx$

8. Misol:

$$y = \sqrt{1+x^2} \qquad dy = ?$$

$$y' = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}} \qquad dy = \frac{xdx}{\sqrt{1+x^2}}$$

9. Misol: $d(1-\cos u) = ?$

$d(1-\cos u) = \sin u du,\ chunki \quad (1-\cos u)' = \sin u$

10. Misol: $d(\arcsin\frac{1}{x}) = ?$

$$d(\arcsin\frac{1}{x}) = -\frac{1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}} = -\frac{1}{x\sqrt{x^2-1}}dx$$

2. 1. Yuqori tartibli differensial

Endi, yuqori tartibli differensial tushunchasini kiritamiz. Buning uchun funksiya differensialini uning birinchi tartibli differensiali argument orttirmasini oʻzgarmas deb qabul qilgan holda (*n–1*) – tartibli differensialning differensialini *n-tartibli differensial deb ataymiz* va uning uchun $d^n y$, $d^n f(x)$ kabi belgilashlarni qoʻllaymiz.

Demak, taʼrif boʻyicha $d^n y=d(d^{n-1}y)$ ekan. Oxirgi formula asosida

$$d^2y=d(dy)=d[f'(x)dx]=(f''(x)dx)dx=f''(x)dx^2$$

va hokazo,

$$d^n y=f^{(n)}(x)dx^n$$

formulani olamiz.

11. Misol: $y = a^x \quad d^n y = ?$

$d^n y = (a^x)^{(n)} dx = a^x \ln^n a dx^n$

12. Misol: $y = \sin^2 x \quad d^n y = ?$

$d^n y = -2^{n-1}\cos(2x + n\frac{\pi}{2})$

Auditoriyada bajarish uchun misollar

1.1. $y = x\ln x \quad y''' = ?$

1.2. $y = x\sin x \quad y''' = ?$

1.3 $y = arctg\frac{x^2}{2} \quad y'' = ?$

1.4. $y = \arcsin\frac{1}{x} \quad y''(2) = ?$

2.1. $y = e^{-\frac{x}{a}}$ $y^{(n)} = ?$

2.2. $y = \sqrt{x}$ $y^{(n)} = ?$

2.3. $y = \cos^2 x$ $y^{(n)} = ?$

2.4. $y = \dfrac{1}{1+2x}$ $y^{(n)} = ?$

3.1. $y = x^3 e^x$ $y^{(5)} = ?$

3.2. $y = x^3 \sin\dfrac{x}{a}$ $y^{(3)} = ?$

3.3. $y = x^4 \ln x$ $y^{(4)} = ?$

3.4. $y = x^2 \sin x$ $y^{(2)} = ?$

4.1. $y = x^3 + 2x^2 - 4x - 4$ $dy = ?$

4.2. $y = \ln\cos x$ $dy = ?$

4.3. $y = arctg\sqrt{4x-1}$ $dy = ?$

4.4. $y = e^{-2x}$ $dy = ?$

5.1. $y = \sqrt{x+1}$ $d^{(5)}y = ?$

5.2. $y = e^{-x^2}$ $d^{(n)} = ?$

5.3. $y = \ln\dfrac{1}{x}$ $d^{(6)}y = ?$

5.4. $d^{(n)}(\sqrt{1+2x}) = ?$

Uy vazifasi uchun misollar

6.1. $y = \dfrac{1}{(x-1)(x-2)}$ $y'' = ?$

6.2. $y = \dfrac{x}{\sqrt{x^2+1}}$ $y'' = ?$

6.3. $y = \sin^2 x$ $y^{(n)} = ?$

6.4. $y = \cos 2x$ $y^{(n)} = ?$

6.5. $y = e^x \cos x$ $y^{(4)} = ?$

6.6. $y = x^2 \sin\alpha x$ $y^{(n)} = ?$

6.7. $y = (arctgx)^2$ $dy = ?$

6.8. $y = \dfrac{1}{\sqrt{x^3 - x^2 - 1}}$ $dy = ?$

6.9. $y = 2^x$ $d^n y = ?$

6.10. $y = \arcsin x$ $d^n y = ?$

Differensiallanuvchi funksiyalar haqidagi teoremalar. Lopital qoidasi.

1-teorema (Ferma). Agar *f(x)* funksiya (*a;b*) oraliqda aniqlangan bo‘lib, $x_0 \in (a;b)$ nuqtada eng kichik yoki eng katta qiymatga erishsa va shu nuqtada differensiallanuvchi bo‘lsa, $f'(x_0)=0$ bo‘ladi.

2–teorema(Roll). Agar *f(x)* funksiya [*a;b*] kesmada aniqlangan, uzluksiz va (*a;b*) oraliqda differensiallanuvchi bo‘lib, kesmaning chetki nuqtalarida teng *(f(a)=f(b))* qiymatlar qabul qilsa, (*a;b*) oraliqda shunday *c* nuqta topiladiki, bu nuqtada funksiya hosilasi nolga teng $(f'(c)=0)$ bo‘ladi.

3–teorema(Lagranj). Agar *f(x)* funksiya [*a;b*] kesmada aniqlangan, uzluksiz va (*a;b*) oraliqda differensiallanuvchi bo‘lsa, (*a;b*) oraliqda shunday *c* nuqta topiladiki, $\frac{f(b)-f(a)}{b-a}=f'(c)$ o‘rinli bo‘ladi.

4 - teorema (Koshi). Agar *f(x)* va *g(x)* funksiyalar [*a;b*] kesmada aniqlangan, uzluksiz va (*a;b*) oraliqda differensiallanuvchi bo‘lib, $g'(x)\neq 0$ bo‘lsa, shunday $c\in(a;b)$ topiladiki,

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$$

o‘rinli bo‘ladi.

1. Misol: berilgan funksiya uchun Roll teoremasi shartlari bajarilishini tekshiring $f(x)=(x-1)(x-2)(x-3)$

Yechish: berilgan funksiya [1;3] kesmada differensiallanuvchi va x=1, x=2 x=3 nuqtalarda nolga tengdir. Bundan [1;2] va [2;3] kesmalarda f(x) funksiya uchun Roll teoremasining barcha shartlari bajarilishi kelib chiqadi. (1;3) kesmaning kamida ikkita nuqtasida berilgan funksiyaning hosilasi nolga teng bo’ladi:

$$f'(x)=0$$

Berilgan funksiyani differensiallab va nolga tenglab quyidagi kvadrat tenglamaga ega bo’lamiz:

$$3x^2-12x+11=0$$

Bundan

$$c_1=2-\frac{1}{\sqrt{3}}\quad c_2=2+\frac{1}{\sqrt{3}}\qquad 1<c_1<2,\quad 2<c_2<3$$

2. Misol: $f(x)=x^2$ *va* $g(x)=x^3$, $x\in[-1;1]$ funksiya uchun Koshi teoremasi o’inlimi?

Yechish: Bunda Koshining shartlari buzilgan $[f'(x)]^2+[g'(x)]^2\neq 0$ $x\in[-1;1]$ da. X=0 da esa $[f'(0)]^2+[g'(0)]^2=0$

Lopital qoidalari

Agar $\frac{f(x)}{g(x)}$ nisbatdan iborat funksiyada surat va maxrajdagi funksiyalarning ikkalasi ham x_0 nuqtaning biror yaqin atrofida aniqlangan va $x\to x_0$ da cheksiz kichik yoki cheksiz katta bo‘lsa, $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko‘rinishdagi aniqmasliklarga ega bo‘lamiz. Bu aniqmasliklarni ochishda oddiy usullar bilan bir qatorda *Lopital qoidalari* deb ataluvchi hosila yordamida ochish usuli ham mavjud.

Teorema. Agar *f(x)* va *g(x)* funksiyalar x_0 nuqtaning qandaydir yaqin atrofida aniqlangan uzluksiz va differensiallanuvchi hamda $g'(x)\neq 0$, $\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0} g(x)=0$ bo‘lib, $\lim\limits_{x\to x_0}\frac{f'(x)}{g'(x)}$ limit (chekli yoki cheksiz) mavjud bo‘lsa,

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)} \qquad (1)$$

Agar teorema shartlari bajarilsa, (1) o‘rinli bo‘lib, uni $x\to x_0$ da $\frac{0}{0}$ aniqmaslikni ochishda *Lopital qoidasi* deb yuritiladi. Xuddi shunga o‘xshash, $x\to\infty$ da ham $\frac{0}{0}$ aniqmaslikni

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=\lim_{x\to\infty}\frac{f'(x)}{g'(x)}$$

tenglik yordamida oxirgi limit (chekli yoki cheksiz) mavjud bo‘lganda ochish mumkin.

3. Misol**:** $\lim\limits_{x\to 0}\frac{1-cosx}{x^2}$ limit hisoblansin.

Bu yerda $\frac{0}{0}$ aniqmaslikka egamiz. Surat va maxrajdagi funksiyalar 0 nuqtaning qisqa atrofida yuqoridagi teorema shartlarini qanoatlantiradi.

$f(x)=1-cosx\Rightarrow f'(x)=sinx,\ g(x)=x^2\Rightarrow g'(x)=2x,\ g'(x)\neq 0(x\neq 0);$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}(1-cosx)=1-1=0,\qquad \lim_{x\to 0} g(x)=\lim_{x\to 0} x^2=0;$$

$$\lim_{x\to 0}\frac{f'(x)}{g'(x)}=\lim_{x\to 0}\frac{\sin x}{2x}=\frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x}=\frac{1}{2}\cdot 1=\frac{1}{2}.$$

Demak, $\lim\limits_{x\to 0}\frac{1-cosx}{x^2}=\frac{1}{2}.$

4. Misol: $\lim\limits_{x\to1}\frac{\ln x}{1-x}$ ni hisoblang.

$$f(x)=\ln x, \qquad g(x)=1-x; \qquad \lim_{x\to1}\ln x=\lim_{x\to1}(1-x)=0$$

$$f'(x)=\frac{1}{x}, \qquad g'(x)=-1; \qquad \lim_{x\to1}\frac{f'(x)}{g'(x)}=\lim_{x\to1}\frac{\frac{1}{x}}{-1}=-\lim_{x\to1}\frac{1}{x}=-1$$

$$\lim_{x\to1}\frac{\ln x}{1-x}=-1.$$

5. Misol: $\lim\limits_{x\to+0}x\ln x$ ni hisoblang.

$x\to+0\Rightarrow x\cdot\ln x\to0\cdot\infty$ - aniqmaslikka egamiz.

$\lim\limits_{x\to+0}x\ln x=\lim\limits_{x\to+0}\frac{\ln x}{\frac{1}{x}}$ ko‘rinishga keltirsak,

$$f(x)=\ln x,\ g(x)=\frac{1}{x}\Rightarrow\lim_{x\to+0}f(x)=\infty, \qquad \lim_{x\to+0}\frac{1}{x}=\infty;$$

ya’ni, $\frac{\infty}{\infty}$ aniqmaslikni olamiz.

$$f'(x)=(\ln x)'=\frac{1}{x}, \qquad g'(x)=\left(\frac{1}{x}\right)'=-\frac{1}{x^2};$$

$$\lim_{x\to+0}\frac{f'(x)}{g'(x)}=\lim_{x\to+0}\frac{\frac{1}{x}}{-\frac{1}{x^2}}=-\lim_{x\to+0}x=0\,.$$

Demak , $\lim\limits_{x\to+0}x\ln x=0.$

6. Misol: $\lim\limits_{x\to+\infty}\frac{e^x}{x^2}$ ni hisoblang.

$$f(x)=e^x, \qquad g(x)=x^2; \qquad \lim_{x\to+\infty}e^x=\infty, \qquad \lim_{x\to+\infty}x^2=\infty;$$

$$f'(x)=e^x, \qquad g'(x)=2x; \qquad \lim_{x\to+\infty}\frac{f'(x)}{g'(x)}=\lim_{x\to+\infty}\frac{e^x}{2x}.$$

Yana $\frac{\infty}{\infty}$ aniqmaslikni oldik.

$$f''(x)=e^x, \qquad g''(x)=2; \qquad \lim_{x\to+\infty}\frac{f''(x)}{g''(x)}=\lim_{x\to+\infty}\frac{e^x}{2}=\infty .$$

$$\lim_{x\to+\infty}\frac{f(x)}{g(x)}=\lim_{x\to+\infty}\frac{f'(x)}{g'(x)}=\lim_{x\to+\infty}\frac{f''(x)}{g''(x)}=\infty .$$

Demak, $\lim\limits_{x\to+\infty}\frac{e^x}{x^2}=\infty$. o‘rinli bo‘ladi.

Agar $\lim\limits_{x\to x_0}[f(x)]^{\varphi(x)}$ limitni hisoblashda

$\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0}\varphi(x)=0$ bo‘lsa, 0^0;

$\lim\limits_{x\to x_0} f(x)=1,\quad \lim\limits_{x\to x_0}\varphi(x)=\infty$ bo‘lsa, 1^∞;

$\lim\limits_{x\to x_0} f(x)=\infty,\quad \lim\limits_{x\to x_0}\varphi(x)=0$ bo‘lganda esa, ∞^0 ko‘rinishdagi aniqmasliklarga ega bo‘lamiz. Agar *f(x)>0* bo‘lsa, $[f(x)]^{\varphi(x)}=e^{\varphi(x)\ln f(x)}$ ayniyat yordamida ularni $0\cdot\infty$ ko‘rinishdagi aniqmaslikka keltirish mumkin.

7. Misol: $\lim\limits_{x\to+0} x^x$ hisoblansin.

Bu yerda 0^0 aniqmaslikka egamiz. $x^x=e^{x\ln x}$.

$$\lim_{x\to+0}(x\ln x)=\lim_{x\to+0}\frac{\ln x}{\frac{1}{x}}=\lim_{x\to+0}\frac{\frac{1}{x}}{-\frac{1}{x^2}}=\lim_{x\to+0}(-x)=0 .$$

$\lim\limits_{x\to+0} x^x = e^0 = 1.$

Auditoriyada bajariladigan misollar.

1.1. $f(x)=x^2-4x+3$funksiyaning ildizlari orasida funksiyaning hosilasi ildizi joylashishini tekshiring. Grafik yordamida izohlang.

1.2. Poll teoremasi $f(x)=1-\sqrt[3]{x^2}$ funksiyasi uchun [-1;1] kesmada o’rinlimi? Grafik asoslang.

1.3. $f(x)=x^2$ funksiya uchun [a;b] kesmada Lagranj formulasini yozing va c ni toping.

1.4. [-1,2] kesmada Lagranj formulasini $\frac{4}{x}$ *va* $1-\sqrt[3]{x^2}$ funksiyalar uchun o’rinli emasligini ko’rsating.

1.5. $f(x)=x^3$ *va* $\varphi(x)=x^2$ funksiyalar uchun Koshi formulasini yozing va c ni toping.

1.6. $f(x)=arctgx$ funksiya uchun [0;1] kesmada Lagranj formulasini yozing.

1.7. $f(x)=\sin x$ funksiya uchun [0;π/2] kesmada Koshi formulasini yozing va c ni toping.

Lopital qoidasini qo'llab aniqmasliklarni oching:

2.1. $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sin 2x}$

2.2. $\lim\limits_{x\to 0}\dfrac{x-arctgx}{x^3}$

2.3. $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{x-\sin x}$

2.4. $\lim\limits_{x\to 0}\dfrac{1-\cos ax}{1-\cos bx}$

2.5. $\lim\limits_{x\to 0}\dfrac{a^x-l^x}{tgx}$

2.6. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{1-2\sin x}{\cos 3x}$

2.7. $\lim\limits_{x\to\infty}\dfrac{e^x}{x^3}$

2.8. $\lim\limits_{x\to\infty}\dfrac{x-\sin x}{x+\sin x}$

2.9. $\lim\limits_{x\to\infty}\dfrac{1+x+\sin x\cos x}{(x+\sin x\cos x)e^{\sin x}}$

2.10. $\lim\limits_{x\to\pi}(\pi-x)tg\dfrac{x}{2}$

2.11. $\lim\limits_{x\to 0}(\sin x)^{tgx}$

2.12. $\lim\limits_{x\to 1}x^{\frac{1}{1-x}}$

Uy vazifasi uchun misollar.

1.1. [1;4] kesmada $f(x)=\sqrt{x}$ funksiya uchun Lagranj formulasini yozing va c ni toping.

1.2. [0;1] da $f(x)=\arcsin x$ funksiya uchun Lagranj formulasini yozing.

1.3. [0;$\frac{\pi}{2}$] kesmada $f(x)=\cos x$ funksiya uchun Koshi formulasini yozing.

2.1. $\lim\limits_{x\to 0}\dfrac{e^{ax}-e^{bx}}{\sin x}$

2.2. $\lim\limits_{x\to\frac{\pi}{2a}} \frac{1-\sin ax}{(2ax-\pi)^2}$

2.3. $\lim\limits_{x\to 0} \frac{e^{2x}-1}{\ln(2x+1)}$

2.4. $\lim\limits_{x\to 0}(1-e^{2x})ctgx$

2.5. $\lim\limits_{x\to 0}(e^{2x}+x)^{\frac{1}{x}}$

2.6. $\lim\limits_{x\to 0}(\frac{1}{x\sin x}-\frac{1}{x^2})$

2.7. $\lim\limits_{x\to 0}(\frac{arctgx}{x})^{\frac{1}{x^2}}$

Teylor va Makloren formulalari.

Teylor formulasi

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\ldots+\frac{f^{(n)}(a)}{n!}(x-a)^n+R_n(x)$$

ni olamiz. Bu *f(x) ning a nuqta atrofidagi Teylor formulasi* deb ataladi, $R_n(x)$ esa uning *qoldiq hadi* deyiladi.

Bu qoldiq had uchun turli koʻrinishlar mavjud boʻlib, bu yerda ulardan ikkitasini keltiramiz:

1. $R_n(x)=\frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$ - Lagarnj shaklidagi (koʻrinishidagi) qoldiq had deyilib, bu yerda c qiymat *a* va *x* lar orasida boʻlib, *a* nuqtaning qandaydir atrofida *f*(*x*) funksiya (*n+1*) marta differensiallanuvchi deb talab qilinadi.

2. $R_n(x)=\frac{f^{(n+1)}(a)+\alpha}{(n+1)!}(x-a)^{n+1}$ - Peano shaklidagi (koʻrinishidagi) qoldiq had deyilib, bu yerda α miqdor $x\to a$ da cheksiz kichik miqdor boʻlib, *a* nuqtada *f*(*x*) funksiyaning chekli (*n+1*)- tartibli hosilasi mavjud deb talab qilinadi.

Makloren formulasi

Agar Teylor formulasini 0 nuqta atrofida yozilsa,

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\ldots+\frac{f^{(n)}(0)}{n!}x^n+R_n(x)$$

ga ega bo‘lamiz. Uni *Makloren formulasi* deb ham yuritiladi.

Makloren formulasidan quyidagi beshta muhim yoyilmalarga ega bo’lamiz:

1. $e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + o(x^n)$
2. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + (-1)^{n+1}\frac{x^{2n-1}}{(2n-1)!} + o(x^{2n})$
3. $cos\, x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$
4. $(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \ldots + \frac{m(m-1)\ldots(m-n+1)}{n!}x^n + o(x^n)$
5. $\ln(1+x) = x - \frac{x^2}{2} + \ldots + (-1)^{n-1}\frac{x^{2n}}{n} + o(x^n)$

Misol 1: e^{-x} ni *x* ning darajalari bo’yicha yoying.

Yeching: $e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + o(x^n)$ formulada x ning o`rniga –x almashtirish kiritamiz. $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \ldots + \frac{(-1)^n x^n}{n!} + o(x^n)$

Misol 2: $cosx$ ni $(x - \frac{\pi}{4})$ darajalari bo’yicha yoying.

Yechish:

$$f(\frac{\pi}{4}) = \frac{1}{\sqrt{2}} \qquad f'(\frac{\pi}{4}) = -\sin(\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}, \qquad f''(\frac{\pi}{4}) = -\cos(\frac{\pi}{4}) = -\frac{1}{\sqrt{2}},$$
$$f'''(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$$

Topilganlarni Teylor formulasiga qo`ysak,

$$\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}(x - \frac{\pi}{4}) - \frac{1}{2\sqrt{2}}(x - \frac{\pi}{4})^2 + \frac{1}{6\sqrt{2}}(x - \frac{\pi}{4})^3 + \ldots$$

Misol 3 $\ln x$ ni (x-1) ning darajalari bo`yicha yoying

Yechish:

$$f(1) = \ln 1 = 0 \qquad f'(1) = \frac{1}{1} = 1, \qquad f''(1) = -1, \qquad f'''(\frac{\pi}{4}) = 2$$

Topilganlarni Teylor formulasiga qo’ysak,

$$(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\frac{1}{4}(x-1)^4+....$$

Misol 4 $\sin 10^0$ ni 10^{-5} gacha aniqlik bilan hisoblang.

Yechish: 10^0 radian hisobida $\frac{\pi}{18}\approx 0.174533$

bo'lganligi uchun, Makloren formulasinidan foydalabsak,

$$\sin 10^0=\frac{\pi}{18}-\frac{1}{3!}(\frac{\pi}{18})^3+\frac{1}{5!}(\frac{\pi}{18})^5-\frac{1}{7!}(\frac{\pi}{18})^7+..$$

Birinchi ikkita had bilan chegaralanib, ushbu taqribiy tenglikni hosil qilamiz

$$\sin\frac{\pi}{18}\approx\frac{\pi}{18}-\frac{1}{6}(\frac{\pi}{18})^3$$

Bunda biz absolyut qiymat jihatdan tashlab yuborilgan hadlarning birinchisidan kichik bo'lgan δ hatoga yol qoyamiz.

$$\delta<\frac{1}{5!}\left(\frac{\pi}{18}\right)^5<\frac{1}{120}(0.2)^5<4\cdot 10^{-6}$$

Agar $\sin\frac{\pi}{18}$ uchun hosil qilingan ifodadagi har bir qo'shiluvchini oltita raqam bilan hisoblasak,

$$\sin 10^0=0.173647$$

hosil bo'ladi.

Auditoriyada bajariladigan misollar

1. e^x *ni* $(x-2)$ *ning darajalari bo'yicha yoying*.

2. x^3-2x^2+5x-7 *ni* $(x-1)$ *ning darajalari bo'yicha yoying*.

. $\cos^2 x$ *ni* $(x+2)$ *ning darajalari bo'yicha yoying*.

4. $\cos(x+a)$ *ni* x *ningdarajalari bo'yicha yoying*.

5. $\sin kx$ *ni* x *ning darajalari bo'yicha yoying*.

6. $\sin^2 x$ *ni* x *ning darajalari bo'yicha yoying*.

7. $\dfrac{1}{1+x^2}$ *ni* x *ning darajalari bo'yicha qatorga yoying*.

8. *arctgx ni x ning darajalari bo'yicha qatorga yoying*

9. $\cos 10^0$ *ni* 0.0001 *gacha aniqlik bilanhisoblang*

10. $\sin 1^0$ *ni* 0.0001 *gacha aniqlik bilan hisoblang*

11. $\ln 5$ *ni* 0.001 *gacha aniqlik bilan hisoblang.*

12. $\sqrt{e}$ *ni* 0.0001 *gacha aniqlik bilan hisoblang.*

Uy vazifasi uchun misollar:

1. e^x *ni* $(x+2)$ *ning darajalari bo'yicha yoying*.

2. $x^{10}-2x^9-3x^7-6x^6+3x^4+6x^3-x-2$ *ni* $(x-1)$ *ning darajalari bo'yicha yoying*.

3. *tgx ni x ningdarajalari bo'yicha yoying*.

4. $e^{\sin x}$ *ni* x *ning darajalari bo'yicha yoying*.

5. $\sqrt[3]{30}$ *ni hisoblang*

6. $\lg e$ *ni* 0.00001 *aniqlik bilan hisoblang*

Mundarija: