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A`JİNİYAZ ATINDAG`I NO`KİS MA`MLEKETLİK PEDAGOGİKALIQ İNSTİTUTI

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YEKİNSHI TA`RTİPLİ SIZIQLARDIN` KANONİKALIQ TEN`LEMELERİ

(matematika woqi`ti`w metodikasi`, informatika oqi`ti`w metodikasi`, fizika` woqi`ti`w metodikasi` ta`lim bag`dar`i`ndag`i` 1-kurs talabalari` ushi`n woqi`w-metodikali`q qollanba)

NO`KİS-2015

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Yekinshi ta`rtipli si`zi`qlardi`n` kanonikali`q ten`lemeleri

(matematika woqi`ti`w metodikasi`, informatika oqi`ti`w metodikasi`, fizika` woqi`ti`w metodikasi` ta`lim bag`dar`i`ndag`i` 1-kurs talabalari` ushi`n woqi`w-metodikali`q qollanba)

Woqi`w-metodikali`q qollanba matematika woqi`ti`w metodikasi` ta`lim bag`dari`ni`n da`stu`ri tiykari`nda jazi`lg`an boli`p, joqari' oqi'w ori'nlari'ni'n' matematika oqi`ti`w metodikasi`, informatika woqi`ti`w metodikasi`, fizika woqi`ti`w metodikasi` bag`dari`ndag`i` talabalarg`a usi`ni`ladi'. Woqi`w-metodikali`q qollanbada yekinshi ta`rtipli si`zi`qlar kanonikali`q ten`lemeleri ja`rdeminde u`yreniw qarali`p, og`an baylani`sli` mi`sallar, testler, o'z betinshe jumıs tapsi`rmalari` berilgen.

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KİRİSİW

Analitikali`q geometriya a`meliyatta ken` qollani`lg`anli`g`i` ushi`n matematikani`n` wo`z aldi`na bir bo`limine aylandi`.

Analitikali`q geometriyada birinshi h`am yekinshi da`rejeli ten`lemeler menen ani`qlanatug`i`n geometriyali`q figuralar u`yreniledi. Soni`n` ushi`n analitikali`q geometriya u`yrenetug`i`n ob`ektler klasi` u`lken yemes.

Biz geometriyali`q figuralardi` wolardi`n` ten`lemeleri ja`rdeminde u`yrengenimiz ushi`n analitikali`q geometriyani` u`yreniw metodi` algebrali`q metod yesaplanadi`.

Bazi` bir kosmosli`q ob`ektler (kometalar, asteroidlar) u`lken juldi`zlar a`tirapi`nan joqari` tezlikte wo`tip ati`rg`anda wolardi`n traektoriyalari` parabola yaki giperbola ko`rinisinde boladi`. Bizge ma`lim, jer joldaslari` jer a`tirapi`nda ellips boylap ha`reket yetedi.

Talabalarg`a usi`ni`li`p ati`rg`an bul woqi`w-metodikali`q qollanba pedagogikali`q institutlari`ni`n` "Matematika woqi`ti`w metodikasi`", "İnformatika woqi`ti`w metodikasi`" ha`m "Fizika woqi`ti`w metodikasi`" ta`lim bag`dari`ndag`i` talabalarg`a arnalg`an boli`p, wol yekinshi ta`rtipli si`zi`qlardi` u`yreniwge bag`i`shlanga`n.

Yekinshi ta`rtipli si`zi`qlarg`a bizge mektep kursi`nan tani`s bolg`an shen`ber, sonday-aq ellips, giperbola, parabolalar kiredi.

Bul oqi`w-metodikali`q qollanbada yekinshi ta`rtipli si`zi`qlar kanonikali`q ten`lemeler ja`rdeminde u`yrenilgen. Yekinshi ta`rtipli si`zi`qlardi`n` qa`siyetleri, uri`nba ten`lemeleri, diametr ten`lemeleri haqqi`nda mag`li`wmatlar keltirilgen. Soni`n` menen birge yekinshi ta`rtipli si`zi`qlardi`n` polyar koordinatalar sistemasi`ndag`i` ten`lemeleri keltirilip shi`g`ari`lg`an.

Ha`r bir bo`limde yekinshi ta`rtipli si`zi`qlarg`a baylani`sli` mi`sallar islep ko`rsetilgen ha`m ha`r bo`limnin` aqi`ri`nda talabalar wo`z u`stinde islew ushi`n wo`z betinshe jumi`s tapsi`rmalari` ha`m test sorawlari` keltirilgen.

1. ELLIPS

***1-ani`qlama***. Tegislikte yekinshi ta`rtipli si`zi`q ten`lemesin qanday da bir $Oxy$ dekart koordinatalar sistemasi`nda

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (1)$$

ko`rinisinde jazi`w mu`mkin bolsa, wol ***<u>ellips</u>*** dep ataladi`.

Bul jerde koefficientler $a \geq b > 0$ qatnaslardi`qanaatlandi`radi`. Yeger bul jerde a=b bolsa, onda ten'.

Bul ten`lemeni u`yreniw na`tiyjesinde ellipsti si`zami`z ha`m wolardi`n` qa`siyetlerin keltirip shi`g`arayi`q. Ten`lemeden ko`rinip turg`ani`nday x, y wo`zgeriwshiler $-a \leq x \leq a$, $-b \leq y \leq b$ ten`sizliklerdi qanaatlandi`radi`.

(1) ten`lemeni $x$ ha`m y ke qarata sheship, to`mendegilerge iye bolami`z:

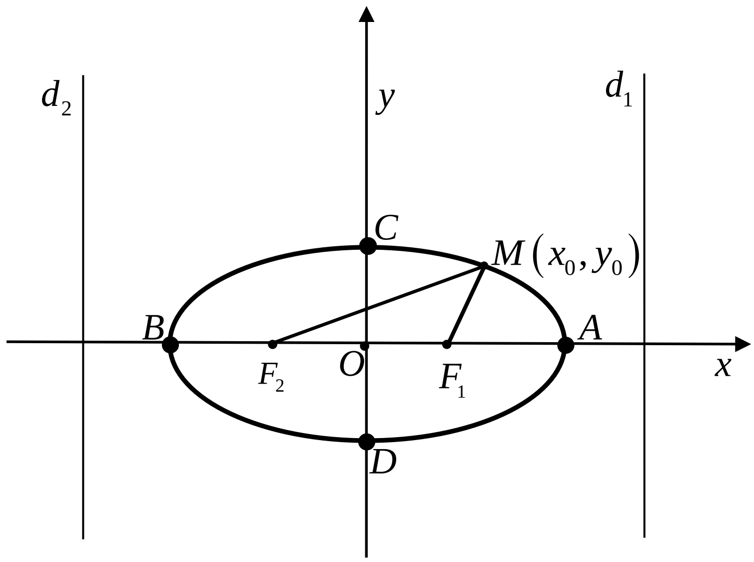
$$y = \pm\frac{b}{a}\sqrt{a^2-x^2} \quad (2)$$

$$x = \pm\frac{a}{b}\sqrt{b^2-y^2} \quad (3)$$

(2) ten`lemeden ellipstin` Ox ko`sherine sali`sti`rg`anda simmetriyali` yekenligi, al (3) ten`lemeden ellipstin` $Oy$ ko`sherine qarata simmetriyali` figura yekenligi kelip shi`g`adi`. Demek ellips koordinata ko`sherlerine qarata simmetriyali` jaylasqan boli`p, koordinata basi` wolardi`n` simmetriya worayi` boladi` (1-si`zi`lma).

(3) ten`lemede $y$ ti nolge ten`lesek, abscissalari` sa`ykes $x = \pm a$ bolg`an $Ox$ ko`sheri menen kesilisiwshi A ha`m $B$ noqatlarg`a iye bolami`z. (2) ten`lemede $x = 0$ dep alsaq, wonda ellipstin` ordinata ko`sherleri menen kesilisiwshi sa`ykes ordinatalari` $y = \pm b$ bolg`an $C$ ha`m D noqatlari`na iye bolami`z.

Bul $A(a,0)$, $B(-a,0)$, $C(0,b)$, $D(0,-b)$ noqatlari` ellipstin` to`beleri dep ataladi`. Bunda $|AB| = 2a, |CD| = 2b$ arali`qlari` ellipstin` ko`sherleri dep ataladi`. $2a$ arali`g`i` ***u`lken ko`sher*** dep, al wog`an vertikal ko`sher $2b$ arali`g`i` ***kishi ko`sher*** dep ataladi`, al $a$ ha`m b bolsa sa`ykes ***u`lken yari`m ko`sheri*** ha`m ***kishi yari`m ko`sheri*** dep ataladi`. Abscissa ko`sherinde jati`wshi` $F_1(c,\ 0)$, $F_2(-c,\ 0)$ noqatlar ellipstin` fokuslari`, $Oy$ ko`sherine parallel ha`m $x \pm \frac{a}{e} = 0$ ten`lemeler menen ani`qlani`wshi` $d_1$ ha`m d_2 tuwri` si`zi`qlar ellipstin` *direktrisalari`* dep ataladi`.



1-si`zi`lma

Bul jerde $c = \sqrt{a^2 - b^2}$, $e = \frac{c}{a}$ boli`p, $e$ sani` ellipstin` ekscentrisiteti dep ataladi`. Ellips ekscentrisiteti ellipstin` qi`si`li`w da`rejesin xarakterleydi. Ellipstin` ekscentrisiteti barli`q waqi`tta 0 menen 1 arali`g`i`nda jaylasadi`, yag`ni`y $0 \le e \le 1$, yeger ekscentrisitet qansha u`lken bolsa, ellips sonsha qi`si`lg`an boladi`. Yeger $e$ nolge ten` bolsa, wonda ellips shen`berge aylanadi`, sebebi fokuslar birigip ketip, ellips ko`sherleri wo`z ara ten` boli`p qaladi`, yag`ni`y $a = b$. Yeger $e$ birge ten` bolsa, wonda ellips qos yeki tuwri`si`zi`qqa aylanadi`, bul jerde ellips fokuslari` u`lken ko`sherdin` ushlari` menen u`stpe-u`st tu`sip qaladi`, al kishi ko`sher bolsa nolge aylanadi`.

Ellipstin` qa`siyetleri.

1^0. Ellipstin` qa`legen noqati`nan wolardi`n` fokuslari`na shekemgi arali`qlar qosi`ndi`si` wo`zgermes sang`a ten`.

Bul qa`siyet tuwri`dan-tuwri` $r_1 + r_2 = 2a$ ten`lemeni tekseriw ja`rdeminde da`lillenedi.

2^0. Ellipstin` qa`legen noqati`nan wolardi`n` fokuslari`na shekemgi arali`qlar sa`ykes direktrisalari`na shekemgi arali`qlardi`n`qatnasi` wo`zgermes ha`m $e$ sani`na ten`.

Bul qa`siyet bolsa, tuwri`dan tuwri` $\frac{r_1}{d_1}=\frac{r_2}{d_2}=e$ ten`lemeni tekseriw ja`rdeminde da`lillenedi.

$$r_1=\sqrt{(x+c)^2+y^2}=\sqrt{x^2+c^2+2xc+b^2-\frac{x^2b^2}{a^2}}=$$

$$\sqrt{x^2-\frac{x^2b^2}{a^2}+2aex+a^2}=\sqrt{x^2\frac{\left(a^2-b^2\right)}{a^2}+2aex+a^2}=|xe+a|$$

$$d_1=\left|-x-\frac{a}{e}\right|=\left|x+\frac{a}{e}\right|=\frac{|xe+a|}{e}\Rightarrow\quad\frac{r_1}{d_1}=e$$

Ellipstin` geometriyali`q ani`qlani`wi`

1. Tegislikte yeki noqat berilgen bolsa, bul noqatlarg`a shekemgi bolg`an arali`qlardi`n` qosi`ndi`si` wo`zgermes sang`a ten` bolatug`i`n noqatlardi`n` geometriyali`q worni` ellips boladi`.

***Da`lilleniwi.*** Tegislikte $F_{1,}F_2$ noqatlar berilgen bolsi`n. Biz tegisliktin` noqati`nan bul noqatlarg`a shekemgi bolg`an arali`qlardi` sa`ykes $r_1, r_2$ ko`riniste belgilep

$$r_1+r_2=2a$$

ten`lemeni qanaatlandi`ri`wshi` noqatlardi`n` geometriyali`q worni`n ani`qlawi`mi`z kerek. Bul arali`qlar ***ellipstin` fokal radiuslari`*** dep ataladi`. Berilgen noqatlar arasi`ndag`i` arali`qti` $2c$ menen belgilesek, ΔF_1MF_2 den $r_1+r_2>2a$ ten`sizlikten $a>c$ qatnas kelip shi`g`adi`. Tegislikte dekart koordinatalar sistemasi`n to`mendegishe kiritemiz. Berilgen F_1,F_2 noqatlardan wo`tiwshi tuwri` si`zi`qti` abscissa ko`sheri si`pati`nda alami`z, wonda won` bag`i`t F_1 noqatdan F_2 noqatqa qarap bag`i`tlang`an boladi`. Koordinata basi`n $F_1,F_2$ noqatlardi`n` wortasi`na jaylasti`ri`p, ordinata ko`sheri si`pati`nda abscissa ko`sherine perpendikulyar qa`legen ko`sherdi alami`z. Arali`qlar ushi`n

$$r_1=\sqrt{(x-c)^2+y^2}\ ,\ r_2=\sqrt{(x+c)^2+y^2}$$

an`latpalardi` joqari`dag`i` ten`likke qoyi`p

$$\sqrt{(x-c)^2+y^2}=2a-\sqrt{(x+c)^2+y^2}$$

ten`lemeni payda yetemiz. Bul ten`liktin` yeki ta`repinde kvadratqa ko`terip, ko`p ag`zali`lardi` a`piwayi`lasti`rami`z:

$$a\sqrt{(x+c)^2+y^2}=\left(cx+a^2\right).$$

Bul an`latpani` ja`ne bir ma`rte kvadratqa ko`terip, yesaplawlar wori`nlaymi`z:

$$a^2x^2+2a^2cx+a^2c^2+a^2y^2=c^2x^2+2a^2cx+a^4.$$

Bul an`latpani` x^2 ha`m $y^2$ qa qarata gruppalaymi`z:

$$\left(a^2-c^2\right)x^2+a^2y^2=a^2\left(a^2-c^2\right).$$

Buni` $a^2\left(a^2-c^2\right)$qa bo`li`p ha`m $a^2-c^2=b^2$ belgilew kiritsek

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

ten`lemeni payda yetemiz. Bul ten`leme *ellipstin` kanonikali`q ten`lemesi* dep ataladi`.

2. l **tuwri` si`zi`q ha`m wog`an tiyisli bolmag`an noqat** F **berilgen bolsa, tegislikte berilgen noqatqa shekemgi bolg`an arali`g`i`ni`n` berilgen tuwri` si`zi`qqa shekemgi arali`g`i`na qatnasi` wo`zgermes ha`m birden kishi** $e$ **sani`na ten` bolg`an noqatlardi`n` geometriyali`q worni` ellips boladi`.**

Bul faktti da`lillew ushi`n berilgen F noqattan tuwri` si`zi`qqa perpendikulyar tuwri` si`zi`q wo`tkizip, woni` abscissa ko`sheri si`pati`nda alami`z. Na`tiyjede abscissa ko`sherin F noqat yeki bo`lekke aji`ratadi`. Berilgen $F$ noqattan tuwri` si`zi`qqa shekemgi arali`qti` `n` e sani`na ko`beymesin p menen belgilep, to`mendegi ten`likler menen

$$a=\frac{p}{1-e^2}\ \text{ha`m}\ c=ea\,,\ b=\sqrt{a^2-c^2}$$

a, b c sanlardi` kiritemiz. Koordinata basi`n abscissa ko`sherinin` l tuwri` si`zi`qti` kespeytug`i`n bo`leginde $F$ noqattan $c$ birlik arali`qta jaylasti`rami`z. Na`tiyjede koordinata basi`nan l tuwri` si`zi`qqa shekemgi arali`q

$$p_1 + c = \frac{p}{e} + ea = \frac{a(1-e^2)}{e} + ea = \frac{a}{e}$$

shamalarg`a ten` boladi`. Bul jerde $p_1$ menen $F$ noqattan $l$ tuwri` si`zi`qqa shekemgi arali`q dep belgilengen. Demek $l$ tuwri` si`zi`qti`n` ten`lemesi

$$x - \frac{a}{e} = 0$$

ko`riniste boladi`. Yekinshi koordinata ko`sherin $l$ tuwri` si`zi`qqa parallel ju`rgizip, tegisliktin` $M(x,y)$ noqati`nan $F$ noqatqa shekemgi arali`qti' r menen, l tuwri` si`zi`qqa shekemgi arali`qti` d menen belgilesek,

$$r = ed$$

ten`likten

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ten`lemeni alami`z.

Biz joqari`da tegisliktin` noqati`nan berilgen noqatlarg`a shekemgi (ellips fokuslari`na shekemgi) bolg`an arali`qlardi` ***<u>ellipstin` fokal radiuslari`</u>*** dep keltirgen yedik. Yendi ellipstin` qa`legen $M(x,y)$ noqati`na ju`rgizilgen fokal radiuslari`

$$r_1 = a - ex, \; r_2 = a + ex \qquad (4)$$

formulalari` menen beriletug`i`ni`n ko`rsetemiz.

Bizge belgili $r_1 = MF_1$ ha`m $r_2 = MF_2$ arali`qlar sa`ykes

$$r_1 = \sqrt{(x+c)^2 + y^2} = \sqrt{x^2 + 2cx + c^2 + y^2},$$

$$r_2 = \sqrt{(x-c)^2 + y^2} = \sqrt{x^2 - 2cx + c^2 + y^2}$$

ten`lemeleri menen ani`qlanadi`. Ellips ten`lemesinen $y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$ ti` tawi`p, bul jerdegi y^2 ti`n` worni`na qoysaq, to`mendegige iye bolami`z:

$$r_1=\sqrt{x^2-2cx+c^2+b^2\left(1-\frac{x^2}{a^2}\right)}=\sqrt{\frac{a^2-b^2}{a^2}x^2-2cx+c^2+b^2}=$$

$$=\sqrt{\frac{c^2}{a^2}x^2-2cx+a^2}=\sqrt{\left(a-\frac{c}{a}x\right)^2}$$

Bul jerde $x\leq 0$ bolsa, $a-\frac{c}{a}x>0$ boladi\`, al $x>0$ bolsa, $x\leq a$ boladi\`. Demek $a-\frac{c}{a}x\geq a-\frac{c}{a}\cdot a=a-c>0$ boladi\`. Ha\`r yeki jag\`day ushi\`nda $a-\frac{c}{a}x>0$ boladi\` yeken. Sonli\`qtan,

$$r_1=\sqrt{\left(a-\frac{c}{a}x\right)^2}=a-\frac{c}{a}x \tag{5}$$

r_2 ushi\`nda joqari\`dag\`day tu\`rlendiriwlerdi wori\`nlasaq

$$r_1=\sqrt{\left(a+\frac{c}{a}x\right)^2}=a+\frac{c}{a}x \tag{6}$$

ten\`ligine iye bolami\`z. Yeger $\frac{c}{a}=e$ yekenligin yesapqa alsaq, wonda (5) ha\`m (6) ten\`liklerden (4) ten\`likke iye bolami\`z. Demek (4) ten\`leme ellipstin\` fokal radiuslari\`ni\`n\` ten\`lemeleri boladi\`.

***1-mi\`sal.*** Yari\`m ko\`sherleri sa\`ykes 5 ha\`m 4 ke ten\` bolg\`an ellipstin\` kanonikali\`q ten\`lemesin du\`zin\`.

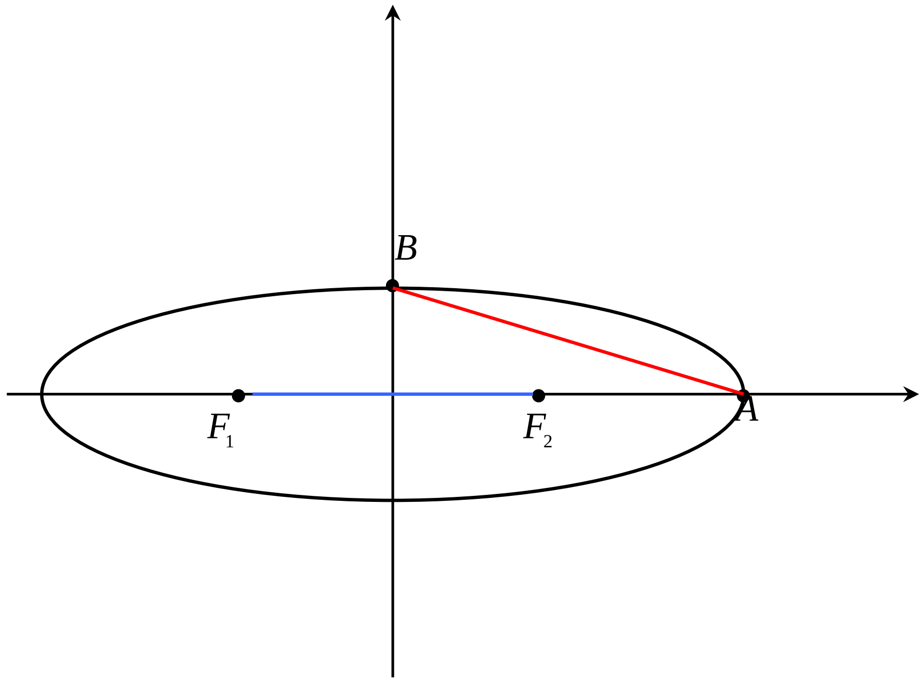
Sheshiliwi: Demek, ma\`sele sha\`rtine ko\`re $a=5,\quad b=4$, wonda ellips ten\`lemesi

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

ko\`riniste boladi\`.

***2-mi\`sal.*** Ha\`r qi\`yli\` to\`beleri arasi\`ndag\`i\` arali\`q fokuslari\` arasi\`ndag\`i\` arali\`qtan yeki yese u\`lken bolg\`an ellipstin\` ekscentrisitetin ani\`qlan\`.

Sheshiliwi: Ma\`sele sha\`rtine k\`ore $|F_1F_2|=2c$, $|AB|=4c$, $e=\frac{c}{a}-?$



$$\begin{cases} 16c^2 = a^2 + b^2 \\ c^2 = a^2 - b^2 \end{cases} \Rightarrow \frac{c^2}{a^2} = \frac{2}{17} \Rightarrow e = \frac{c}{a} = \sqrt{\frac{2}{17}}$$

***3-mi`sal.*** $x = \pm 8$ tuwri` si`zi`qlari` ellipstin` direktrisalari`, kishi ko`sheri 8 ge ten`. Ellipstin` ten`lemesin du`zin`.

S h e s h i l i w i : Demek $2b = 8 \Rightarrow$ $b = 4$, al $x = \pm \frac{a^2}{c} = \pm 8$.

$c^2 = a^2 - b^2 = a^2 - 16$ bul tabi`lg`andi` direktrisa ten`lemesine apari`p qoyami`z ha`m a`pi`wayi`lasti`ri`wlar wori`nlaymi`z:

$$\frac{a^2}{\sqrt{a^2 - 16}} = 8 \Rightarrow \quad a^2 = 8\sqrt{a^2 - 16}$$

$$a^4 - 64a^2 + 64 \cdot 16 = 0, \qquad a^2 = t$$

$t^2 - 64t + 64 \cdot 16 = 0 \Rightarrow t_1 = t_2 = 32$. Bunnan $a^2 = 32$.

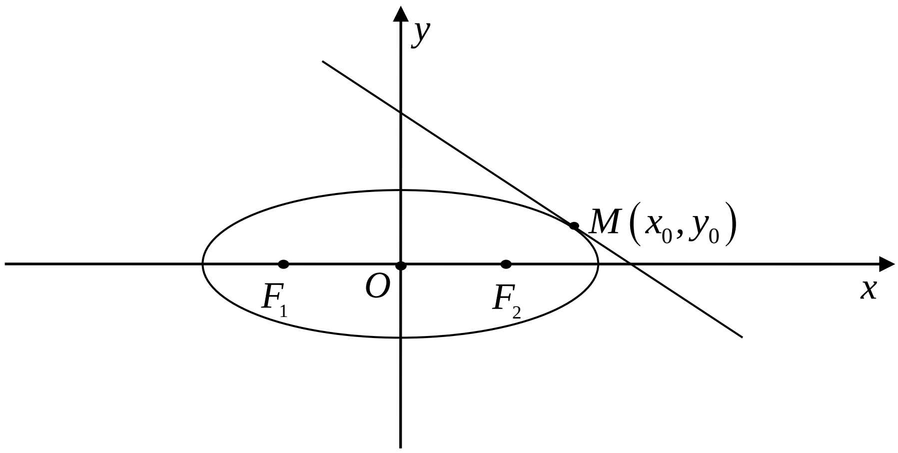
Wonda ellips ten`lemesinin' ko`rinisi $\frac{x^2}{32} + \frac{y^2}{16} = 1$ boladi`.

Ellipstin` uri`nbasi`

Yendi biz ellipstin` uri`nbasi`ni`n` ten`lemesin keltirip shi`g`arayi`q. Bizge

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

ellipsi berilgen ha`m woni`n` bazi`-bir $M(x_0, y_0)$ noqati`nan ju`rgizilgen uri`nbasi`ni`n` ten`lemesin du`ziwdi qarayi`q ***(2-si`zi`lma).***



2-si`zi`lma

Bizge mektep kursi`nan belgili uri`nbani`n` ten`lemesi

$$y-y_0=y'(x-x_0) \qquad (7)$$

menen beriledi. Bul ten`lemedegi $y'$ ti` tabi`w ushi`n ellipstin` kanonikali`q ali`q ten`lemesin, yag`ni`y (1) ten`lemeni $x$ boyi`nsha differenciallaymi`z, yag`ni`y $x$ boyi`nsha tuwi`ndi` alami`z:

$$\frac{2x}{a^2}+\frac{2yy'}{b^2}=0,$$

Bul jerde y quramali` funktsiya dep qaraladi`. Bunnan y' ti` tapsaq ha`m wo`zgeriwshi noqatti`n` koordinatalari`n berilgen $M(x_0, y_0)$ noqati`ni`n` koordinatalari` menen almasti`rsaq:

$$y'=-\frac{b^2x_0}{a^2y_0}$$

ten`ligin payda yetemiz.

Bul tabi`lg`andi` (7) ten`lemege qoyami`z.

$$y - y_0 = -\frac{b^2 x_0}{a^2 y_0}\left(x - x_0\right).$$

yamasa

$$a^2 y_0\left(y - y_0\right) + b^2 x_0\left(x - x_0\right) = 0 \qquad (8)$$

Tiyisli a`piwayi`lasti`ri`wlardi` wori`nlasaq, to`mendegi ten`likke iye bolami`z:

$$b^2 x x_0 + a^2 y y_0 = b^2 x_0^2 + a^2 y_0^2.$$

Bul ten`liktin` yeki ta`repin $a^2 b^2$ qa bo`lsek:

$$\frac{x x_0}{a^2} + \frac{y y_0}{b^2} = \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2}. \qquad (9)$$

Bul jerde $M\left(x_0, y_0\right)$ noqati` ellipske tiyisli bolg`anli`qtan, woni`n` koordinatalari` ellipstin` ten`lemesin qanaatlandi`wi` tiyis, yag`ni`y:

$$\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1.$$

Sonli`qtan (9) ten`likti to`mendegi ko`riniste jaza alami`z:

$$\frac{x x_0}{a^2} + \frac{y y_0}{b^2} = 1 \qquad (10)$$

Bul (10) ten`leme ellipstin` $M\left(x_0, y_0\right)$ noqati` arqali` ju`rgizilgen uri`nbasi`ni`n` ten`lemesi boladi`.

***4-mi`sal.*** $\frac{x^2}{32} + \frac{y^2}{18} = 1$ ellipske $M(4,3)$ noqatta ju`rgizilgen uri`nbasi`ni`n` ten`lemesin du`zin` .

Sheshiliwi: Berilgen mag`li`wmatlardi` ellips uri`nbasi` ten`lemesindegi worni`na apari`p qoyami`z:

$$\frac{4x}{32} + \frac{3y}{18} = 1,$$

a`piwayi`lasti`ri`wlar wori`nlaymi`z ha`m $3x + 4y - 24 = 0$ ten`lemege iye bolami`z. Demek soralg`an uri`nba ten`lemesi $3x + 4y - 24 = 0$.

***5-mi`sal.*** $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipske $N(10;4)$ noqattan o`tiwshi uri`nba ten`lemelerin jazi`n`.

Sheshiliwi: Bul mi`sal sha`rtine ko`re 4-mi`salg`a uqsap ketedi. Bul mi`saldi` talabalarg`a bergenimizde da`rhal 4-mi`sal sheshiliwindey etip isleydi. Lekin bul naduri`s. Sebebi ma`sele sha`rtinde berilip turg`an noqat ellipske tiyisli emes, demek da`slep bul noqatti`n` ellipske qarata awhali`n ani`qlaymi`z.

$\frac{10^2}{25}+\frac{4^2}{16}=1 \Rightarrow 2000>1$. Demek noqat ellipstin` si`rti`nda jaylasqan, onda ellipske bul noqattan o`tiwshi eki uri`nba ten`lemesin jazi`wi`mi`z mu`mkin.

Ellipstin` uri`nba ten`lemesin jazi`w ushi`n ellipske tiyisli noqatti`da ani`qlawi`mi`z kerek. Ol ushi`n ellipske tiyisli bol`gan noqatti` to`mendegi ten`lemeler sistemasi`nan ani`qlaymi`z :

$$\begin{cases} \frac{{x_0}^2}{25}+\frac{{y_0}^2}{16}=1 \\ \frac{10x_0}{25}+\frac{4y_0}{16}=1 \end{cases}$$

Bul jerden x_0, y_0 lerdin` ma`nisleri tabi`ladi`, soni`nan (10) ten`leme ja`rdeminde $N(10;4)$ ha`m $M(x_0, y_0)$ noqatlari`nan o`tiwshi uri`nba ten`lemeleri jazi`ladi`. ***Juwap*:** $y=4$ ha`m $16x-15y-100=0$.

Ellipstin` diametrleri

Mektep kursi`nda shen`berdin` diametri degen tu`sinikke iye bolg`an yedik. Wol jerde shen`ber diametri dep shen`ber worayi`nan wo`tiwshi yen` u`lken xorda dep keltiriler edi. Biraq bul ani`qlama yekinshi ta`rtipli si`zi`qlar ushi`n toli`q ani`qlama bola almaydi`. Sonli`qtan yekinshi ta`rtipli si`zi`qlardi`n` diametrleri haqqi`nda so`z bolg`anda, ha`r qanday yekinshi ta`rtipli si`zi`qlarg`a uli`wma bolg`an diametrge ani`qlama beriwimiz kerek.

***2-ani`qlama*.** Yekinshi ta`rtipli si`zi`qlardi`n` wo`z-ara parallel bolg`an xordalari`ni`n` wortalari`n tutasti`ri`wshi` tuwri` si`zi`q, woni`n` diametri dep ataladi`.

Usi` ani`qlamadan ellipstin` diametrinin` ten`lemesin keltirip shi`g`arami`z:

Bizge

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

ellips ha`m usi` ellipstin` $\overline{a}$ vektor bag`i`ti`na parallel bolg`an

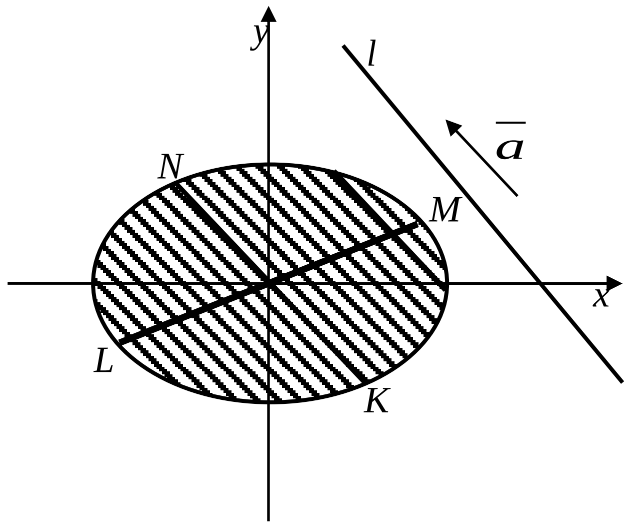
$$y=kx+m$$

ten`lemesi menen ani`qlani`wshi` xordasi` berilgen bolsi`n.

Berilgen ellips ha`m xordani`n` kesilisiw noqatlari`n ani`qlaymi`z. Buni`n` ushi`n

$$\begin{cases}\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1\\ y=kx+m\end{cases} \qquad (11)$$

ten`lemeler sistemasi`n sheshiwdi qarasti`rami`z. Sebebi kesilisiw noqatlari` ellipskede, xordag`ada tiyisli boli`wi` kerek. To`mendegi tu`rlendiriwlerdi wori`nlaymi`z:



3-si`zi`lma

$$\frac{x^2}{a^2}+\frac{(kx+m)^2}{b^2}=1,$$

$$b^2x^2+a^2(kx+m)^2=a^2b^2,$$

$$b^2x^2+a^2k^2x^2+2a^2kmx+a^2m^2-a^2b^2=0,$$

$$\left(b^2+a^2k^2\right)x^2+2a^2kmx+a^2\left(m^2-b^2\right)=0.$$

Keyingi ten`liktin` yeki ta`repinde $b^2+a^2k^2$ qa bo`lemiz ha`m:

$$x^2+2\frac{a^2km}{b^2+a^2k^2}x+\frac{a^2\left(m^2-b^2\right)}{b^2+a^2k^2}=0$$

ten`lemesine iye bolami`z.

Bul kvadrat ten`lemenin` korenleri ellips ham xordani`n` kesilisiw noqati`ni`n` abscissalari`n beredi. Xordani` ten` yekige bo`liwshi noqatti`n` koordinatalari`n sa`ykes $x$ ha`m y penen belgilesek, kesindini ten` wortadan bo`liw formulasi`nan

$$x=\frac{x_1+x_2}{2} \qquad (12)$$

ten`lemege iye bolami`z. Kvadrat ten`leme ushi`n Viet teoremasi`nan ha`m (12) ten`lemesinen to`mendegini jaza alami`z:

$$x=-\frac{ka^2m}{b^2+a^2k^2} \qquad (13)$$

Bul tabi`lg`an x ti` (11) sistemani`n` yekinshi ten`lemesine qoyi`p, $y$ tin` ma`nisin tabami`z:

$$y=m-k\frac{ka^2m}{b^2+a^2k^2}$$

Bul ten`leme ushi`n a`piwayi`lasti`ri`wlar wori`nlasaq, ten`leme

$$y=\frac{b^2m}{b^2+a^2k^2} \qquad (14)$$

ko`riniske keledi.

Yeger (14) ten`likti (13) ten`likke bo`lsek

$$\frac{y}{x}=-\frac{b^2}{a^2k} \qquad (15)$$

ten`ligine iye bolami`z. Bul ten`likte $m$ qatnaspaydi`, demek bul qatnas berilgen bag`i`tqa parallel bolg`an barli`q xordalardi`n` ten` wortasi`ndag`i` noqatlar ushi`nda wori`nli`. Bul noqatlardi`n` ko`pligi, yag`ni`y bul noqatlardan wo`tiwshi tuwri` si`zi`q diametrden ibarat. Demek (15) ten`leme diametrdin` ten`lemesi boli`p tabi`ladi`. (15) ten`lemeni a`piwayi`lasti`ri`p

$$\frac{x}{a^2}+k\frac{y}{b^2}=0 \qquad (16)$$

ko`riniske keltiremiz. (16) ten`leme ellipstin` diametrinin` ten`lemesi boladi`.

E l l i p s t i n ` d i a m e t r i n i n ` q a ` s i y e t l e r i

1^0. Ellipstin` diametri ellips worayi` arqali` wo`tiwshi tuwri` si`zi`q boli`p tabi`ladi`;

2^0. Ellipstin` diametrinin` mu`yeshlik koefficienti $k_1$ bolsa, wonda bul diametrge tu`yinles diametrdin` mu`yeshlik koefficienti k ha`m ellips ko`sherleri ja`rdeminde to`mendegishe tabi`ladi`:

$$k_1 = -\frac{b^2}{a^2 k}$$

3^0. Diametr menen wog`an tu`yinles xordalardi`n` mu`yeshlik koefficientlerinin` belgileri qarama-qarsi` boladi`, yag`ni`y diametr Ox ko`sheri menen su`yir mu`yesh jasasa, wog`an tu`yinles xordalar dog`al mu`yesh jasaydi` ha`m kerisinshe.

Bir-birine parallel xordalardi` ten` wortadan bo`letug`i`n yeki diametr wo`z-ara tu`yinles diametr dep ataladi`. Mi`sali` 3-si`zi`lmada LM ha`m $KN$ wo`z-ara tu`yinles diametrler boladi`.

***6-mi`sal.*** $\frac{x^2}{100} + \frac{y^2}{64} = 1$ ellipstin` $2x - y + 7 = 0,\ 2x - y - 1 = 0$ xordalari`ni`n` wortalari` arqali` wo`tiwshi tuwri` si`zi`q ten`lemesin du`zin`.

S h e s h i l i w i : Bizge belgili ellips xordalari`ni`n` wortalari`nan ibarat tuwri` si`zi`q bul ellipstin` diametri boli`p tabi`ladi`. Demek diametr ten`lemesin duziwimiz kerek. Diametr ten`lemesinde $k$ koefficient bul ellipstin` parallel xordalari`ni`n` mu`yeshlik koefficienti edi, ma`sele shartine ko`re $k = 2$.

Wol jag`dayda soralg`an diametr ten`lemesi $\frac{x}{100} + 2 \cdot \frac{y}{64} = 0$ ko`riniske keledi ha`m bunnan $8x + 25y = 0$ ten`lemege iye bolami`z. Demek diametr ten`lemesi $8x + 25y = 0$.

***7-mi`sal.*** $\frac{x^2}{25} + \frac{y^2}{16} = 1$ ellipstin` $M(2,1)$ noqatta ten` yekige bo`liniwshi xorda ten`lemesin du`zin`.

S h e s h i l i w i : Xorda ten`lemesin $y = kx + b$ ko`rinisinde izleymiz. Bul jerde k to`mendegishe izlenedi:

$M(2,1)$ noqat diametrgede tiyisli. Sonli`qtan

$$\frac{x}{25}+k\frac{y}{16}=0,\ \frac{2}{25}+k\frac{1}{16}=0 \Rightarrow k=-\frac{32}{25}.$$

$M(2,1)$ noqat xordag`ada tiyisli, demek $1=-\frac{32}{25}\cdot 2+b \ \Rightarrow b=\frac{89}{25}$

Demek xorda ten`lemesi $32x+25y-89=0$.

2. GIPERBOLA

***3-ani`qlama.*** Yekinshi ta`rtipli si`zi`q ten`lemesin qanday da bir dekart koordinatalar sistemasi`nda

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad (1)$$

ko`riniste an`lati`w mu`mkin bolsa, bul si`zi`q giperbola dep ataladi`. Bul jerde koefficientler $a \geq b > 0$ qatnaslardi` qanaatlandi`radi`.

Giperbola ten`lemesin tekseriw na`tiyjesinde to`mendegilerdi alami`z: Da`slep (1) ten`lemeni x ha`m $y$ ke qarata sheship to`mendegi yeki ten`likke iye bolami`z:

$$x_{1,2}=\pm\frac{a}{b}\sqrt{b^2+y^2} \qquad (2)$$

$$y_{1,2}=\pm\frac{b}{a}\sqrt{x^2-a^2} \qquad (3)$$

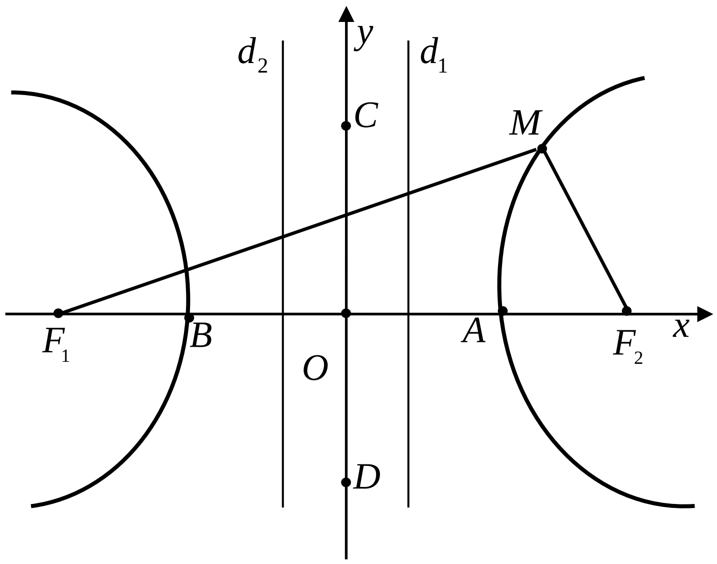
Bul ten`lemelerden giperbolani`n` koordinata ko`sherleri menen kesilisiw noqatlari`n ani`qlaymi`z.

a) Yeger (2) ten`lemede $y=0$ dep alsaq, wonda giperbolani`n` $Ox$ ko`sheri menen kesilisiw noqatlari`nan, abscissalari` sa`ykes $x=a$ ha`m $x=-a$ bolg`an $A(a,0)$ ha`m $B(-a,0)$ noqatlari`na iye bolami`z. Bul yeki noqat giperbolani`n` to`beleri dep ataladi`, al bul yeki noqat arasi`ndag`i` arali`q, $|AB|=2a$ arali`g`i` giperbolani`n` *haqi`yqi`y ko`sheri* dep ataladi`.

b) Giperbolani`n` Oy ko`sheri menen kesilisiw noqatlari`n ani`qlaw ushi`n (3) ten`lemede $x=0$ dep alami`z, wonda $y=\pm bi$ g`a iye bolami`z. Bul bolsa giperbola Oy ko`sheri menen kesilisiw noqati`na iye yemes yekenligin ko`rsetedi. Sonli`qtan, $2b=2\sqrt{c^2-a^2}$ arali`g`i`n giperbolani`n` *jori`ma ko`sheri* dep ataladi` (1-si`zi`lma).

(2) ha`m (3) ten`liklerden x, y wo`zgeriwshiler $|x| \geq a$, $-\infty < y < \infty$ ten`sizliklerdi qanaatlandi`radi`, yag`ni`y $|x|$ hesh waqi`tta $a$ dan kishi yemes, $x$ ti`n` $a$ dan $+\infty$ ke shekem wo`zgergende y tin` absolyut ma`niside 0 den $+\infty$ ke shekem wo`zgeretug`i`ni`n ko`remiz. (1) ten`lemede x, y wo`zgeriwshilerdin` tek g`ana yekinshi da`rejeleri qatnasqanli`g`i` ushi`n giperbola koordinata ko`sherlerine qarata simmetriyali` jaylasqan boladi`. Bunnan basqa koordinata basi` giperbolani`n` simmetriya worayi` boladi`.

Abscissa ko`sherindegi $F_1(-c, 0)$, $F_2(c, 0)$ noqatlar giperbolani`n` *fokuslari`*, $x \pm \frac{a}{e} = 0$ ten`lemeler menen ani`qlani`wshi` l_1, l_2 tuwri` si`zi`qlar giperbolani`n` *direktrisalari`* dep ataladi`. Bul jerde $c = \sqrt{a^2 + b^2}$, $e = \frac{c}{a} > 1$ boli`p, e sani` giperbolani`n` *ekscentrisiteti* dep ataladi`. Ellipstegi si`yaqli` giperboladada ekscentrisitititeti woni`n` qi`sili`n`qi` yaki jazi`li`n`qi` boli`p keliwin, yag`ni`y giperbolani`n` formasi`na ta`sir jasaydi`. Yeger e sani` birge umti`lsa, giperbolani`n` shaqalari` qi`si`li`n`qi` boli`p keledi, yag`ni`y wolar $Ox$ ko`sheri menen betlesiwge umti`ladi`, yeger e sani` sheksiz wo`sse giperbolani`n` shaqalari` jazi`li`n`qi` boli`p keledi, yag`ni`y wolar Oy ko`sherine parallel bolg`an d_1, d_2 tuwri` si`zi`qlari` menen u`stpe-u`st tu`siwge umti`ladi`.



1-si`zi`lma

Giperbola qa`siyetleri

1[0]. Giperbolani`n` qa`legen noqati`nan wolardi`n` fokuslari`na shekemgi bolg`an arali`qlar ayi`rmasi`ni`n` moduli wo`zgermes ha`m $2a$ ga ten`.

2[0]. Giperbolani`n` qa`legen noqati`nan wolardi`n` fokuslari`na shekemgi arali`qlar sa`ykes direktrisalarg`a shekemgi arali`qqa qatnasi` wo`zgermes ha`m e sani`na ten`.

Bul qa`siyet tuwri`dan-tuwri` $\frac{r_1}{d_1} = \frac{r_2}{d_2} = e$ ten`lemeni tekseriw ja`rdeminde da`lillenedi. Giperbolani`n` $M(x, y)$ noqati`nan fokuslarg`a shekemgi arali`qlar ushi`n

$$r_1 = \sqrt{(ex + a)^2}, r_2 = \sqrt{(ex - a)^2} \tag{4}$$

ten`likler wori`nli`.

Bul arali`qlar giperbolani`n` *fokal radiuslari`* dep ataladi`. (4) ten`lemelerden korenden shi`g`ari`w a`melin wori`nlasaq, yeger $x > 0$ bolsa, yag`ni`y $M(x, y)$ noqat giperbolani`n` won` shaqasi`nda jaylasqan bolsa,

$$r_1 = a + ex, \ r_2 = -a + ex \tag{5}$$

yeger $x < 0$ bolsa, yag`ni`y $M(x, y)$ noqat giperbolani`n` shep shaqasi`nda jaylasqan bolsa,

$$r_1 = -a - ex, \ r_2 = a - ex \tag{6}$$

ten`liklerdi payda yetemiz. (5) ha`m (6) ten`lemeler giperbolani`n` fokal radiuslari`ni`n` ten`lemeleri boladi`. Na`tiyjede yeger $x > 0$ bolsa $r_1 - r_2 = 2a$ , yeger $x < 0$ bolsa $r_1 - r_2 = -2a$ ten`lik wori`nli` boladi`. Demek qa`legen x ushi`n

$$|r_1 - r_2| = 2a$$

ten`lik wori`nli` boladi`.

G i p e r b o l a n i ` n ` g e o m e t r i y a l i ` q  a n i ` q l a n i ` w i `

1. Tegislikte yeki noqat berilgen bolsa, bul noqatlarg`a shekemgi arali`qlar ayi`rmasi`ni`n` moduli wo`zgermes sang`a ten` bolatug`i`n noqatlardi`n` geometriyali`q worni` giperbola boladi`.

Tegislikte $F_{1,}F_2$ noqatlar berilgen. Biz tegisliktin` noqati`nan bul noqatlarg`a shekemgi arali`qlardi` sa`ykes r_1, r_2 ko`riniste belgilep

$$|r_1 - r_2| = 2a$$

ten`lemeni qanaatlandi`ri`wshi` noqatlar ko`pligi giperbola yekenligin da`lilleymiz. Berilgen noqatlar arasi`ndag`i` arali`qti` $2c$ menen belgileymiz ha`m tegislikte dekart koordinatalar sistemasi`n to`mendegishe kiritemiz. Berilgen F_1, F_2 noqatlardan wo`tiwshi tuwri` si`zi`qti` abscissa ko`sheri si`pati`nda alami`z, wonda won` bag`i`t F_1 noqatdan F_2 noqatqa qarap bag`i`tlang`an. Koordinata basi`n $F_{1,}F_2$ noqatlardi`n` wortasi`na jaylasti`ri`p, ordinata ko`sheri si`pati`nda abscissa ko`sherine perpendikulyar qa`legen ko`sherdi alami`z. Arali`qlar ushi`n

$$r_1 = \sqrt{(ex+a)^2}\,, r_2 = \sqrt{(ex-a)^2}$$

an`latpalardi` joqari`dag`i` ten`likke qoyi`p

$$\left|\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2}\right| = 2a$$

ten`lemeni payda yetemiz. Bul jerde ellipste wori`nlang`anday a`piwayi`lasti`ri`wlar jasasaq

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

qatnaslardi` alami`z. Bul jerde $b^2 = c^2 - a^2$ belgilew kiritilgen.

2. **Bizge** l **tuwri` si`zi`q ha`m wog`an tiyisli bolmag`an noqat** F **berilgen bolsa, tegislikte berilgen noqatqa shekemgi arali`qti` n` berilgen tuwri` si`zi`qqa shekemgi arali`qqa qatnasi` wo`zgermes birden u`lken** e **sani`na ten` bolg`an noqatlardi`n` geometriyali`q worni` giperbola boladi`.**

Biz joqari`da $e<1$ bolg`anda ellips payda boli`wi`n ko`rsetken yedik. Bul jerde $p$ sani` ellipstegi si`yaqli`, giperbolani`n` u`lken ha`m kishi yari`m ko`sherleri

$$a=\frac{p}{e^2-1}\ ,\ b=\sqrt{c^2-a^2}$$

ten`likleri menen ani`qlanadi`. Bul jerde $c$ sani` $c=ea$ ten`lik penen ani`qlanadi`.

***8-mi`sal.*** Haqi`yqi`y ko`sheri 48ge ha`m ekscentrisiteti $e=\frac{13}{12}$ ge ten` bolg`an giperbola ten`lemesin du`zin`.

Sheshiliwi: Ma`sele sha`rtine ko`re $2a=48\Rightarrow\ \ a=24$

$e=\frac{c}{a}=\frac{13}{12}$ yekenligi ma`lim. Bul jerde $c=\sqrt{a^2+b^2}$. Bunnan:

$$\frac{576+b^2}{576}=\frac{169}{144}\ \Rightarrow b^2=100.$$

Demek giperbola ten`lemesi

$$\frac{x^2}{576}-\frac{y^2}{100}=1.$$

Giperbolani`n` uri`nbasi`

Bizge giperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

ten`lemesi menen berilgen bolsi`n. Giperbolani`n` bazi`-bir $M(x_0,y_0)$ noqati`nan wo`tkizilgen uri`nbasi`ni`n` ten`lemesin du`ziwdi qarayi`q (2-si`zi`lma).

Uri`nbani`n` ten`lemesin du`ziw ushi`n, biz joqari`da ellipstin` uri`nbasi`ni`n` ten`lemesin du`zgendegidey (1) ten`lemeden $y'$ tabi`w ha`m oni`

$$y-y_0=y'(x-x_0) \tag{7}$$

ten`lemege qoyi`y jetkilikli . (1) ten`lemeden tuwi`ndi` alami`z:

$$\frac{2x}{a^2}-\frac{2yy'}{b^2}=0,$$

bunnan $$y' = \frac{b^2 x_0}{a^2 y_0}.$$

Yendi y' tin` bul ma`nisin (7) ten`lemedegi worni`na qoyami`z:

$$y - y_0 = \frac{b^2 x_0}{a^2 y_0}\left(x - x_0\right).$$

Bul ten`likke tu`rlendiriwler jasap, to`mendegi ko`riniske keltiremiz:

$$b^2 x x_0 - a^2 y y_0 = b^2 x_0^2 - a^2 y_0^2.$$

Bul ten`liktin` yeki ta`repinde $a^2 b^2$ qa bo`lemiz:

$$\frac{x x_0}{a^2} - \frac{y y_0}{b^2} = \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2}. \qquad (8)$$

ten`lemesine iye bolami`z.

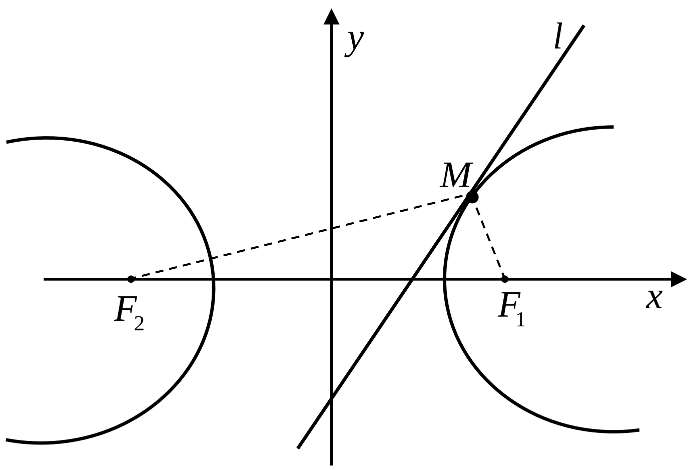
$M\left(x_0, y_0\right)$ noqati` giperbolag`a tiyisli, sonli`qtan:

$$\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1.$$

Usi`g`an tiykarlani`p, (8) ten`likti to`mendegi ko`riniste jaza alami`z:

$$\frac{x x_0}{a^2} - \frac{y y_0}{b^2} = 1 \qquad (9)$$

Bul (9) ten`leme giperbolani`n` $M\left(x_0, y_0\right)$ noqati` arqali` ju`rgizilgen uri`nbasi`ni`n` ten`lemesi boladi`.



2-si`zi`lma

***9-mi`sal.*** $\frac{x^2}{5}-\frac{y^2}{4}=1$ giperbolag`a $M(5;-4)$ noqatta uri`ni`wshi` uri`nba ten`lemesin jazi`n`.

Sheshiliwi: $\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1$ ten`leme giperbolani`n` uri`nba ten`lemesi ekenligin bilgen halda to`mendegilerdi jaza alami`z:

$$\frac{5x}{5}-\frac{4y}{4}=1 \Rightarrow \quad x+y-1=0.$$

Giperbolani`n` asimptotalari`

Bizge (1) ten`leme ja`rdeminde giperbola berilgen bolsi`n. Usi`giperbolani`n` koordinata basi`nan wo`tetug`i`n

$$y=kx \qquad (10)$$

tuwri` si`zi`g`i` menen kesilisiw ma`selesin qarayi`q .

Bul yeki si`zi`qti`n` wo`z-ara kesilisiw noqatlari`n tabi`w ushi`n, wolardi`n` ten`lemelerin sistema si`pati`nda qarap, (11) ten`lemeni (1) ten`lemedegi $y$ tin` worni`na qoyami`z.

$$\frac{x^2}{a^2}-\frac{k^2y^2}{b^2}=1$$

Bunnan

$$b^2x^2-a^2k^2x^2=a^2b^2,$$

yaki

$$x^2\left(b^2-a^2k^2\right)=a^2b^2$$

Buni` $x$ qa qarata sheship, to`mendegi ten`likke iye bolami`z:

$$x=\pm\frac{ab}{\sqrt{b^2-a^2k^2}} \qquad (11)$$

Buni` (10) ten`likke qoyi`p, $y$ ushi`n to`mendegi an`latpani` payda yetemiz:

$$y=\pm\frac{abk}{\sqrt{b^2-a^2k^2}} \qquad (12)$$

Yendi x ha`m $y$ ushi`n tabi`lg`an bul an`latpalardi` izertlep ko`remiz:

a) yeger radikal asti`ndag`i` an`latpa $\delta=b^2-a^2k^2>0$ bolsa, yag`ni`y $k^2<\frac{b^2}{a^2}$, yaki $|k|<\frac{b}{a}$ bolsa, wonda (11) ha`m (12) formulada $x$ ha`m y tin` ha`r qaysi`si` yeki haqi`yqi`y ma`niske iye boladi`, demek qarali`p ati`rg`an tuwri` si`zi`q penen giperbola yeki noqatda kesilisedi.

b) Yeger $\delta<0$ bolsa, radikal asti`ndag`i` an`latpa teris san boladi`, ten`lemenin` yeki korenide jori`ma boladi`, bul tuwri` si`zi`q penen giperbola kesilispeytug`i`ni`n bildiredi.

c) $\delta=0$ bolg`an jag`daydi` qarasaq, $b^2=a^2k^2$ yaki $k^2=\frac{b^2}{a^2}$, bunnan $k=\pm\frac{b}{a}$ boladi`. Bul jag`dayda $y=\pm\frac{b}{a}x$ tuwri` si`zi`qlari` giperbolag`a qarata jaylasi`wi`n qarap ko`reyik. Buni`n` ushi`n usi` tuwri` si`zi`qlardi`n` giperbola menen kesilisiw noqatlari`ni`n` koordinatalari` qalay wo`zgeretug`i`ni`n tekseriw jetkilikli.

$\delta=0$ bolg`anli`qtan (11) ha`m (12) ten`lemelerdegi an`latpalardi`n` yekewinin`de bo`limleri nolge umti`ladi`, yag`ni`y $x$ ha`m y sheksizlikke umti`ladi`. Bul jag`dayda giperbola ha`m tuwri` si`zi`q kesilisedi, yaki kesilispeydi dep ayta almaymi`z. $\delta=0$ jag`day ayri`qsha jag`day boli`p, $y=\pm\frac{b}{a}x$ tuwri` si`zi`qlari` berilgen giperbola menen kesilisetug`i`n barli`q tuwri`lardi`n` ji`ynag`i`n, giperbola menen kesilispeytug`i`n barli`q tuwri` si`zi`qlardi`n` ji`ynag`i`nan aji`rati`p turi`wshi` bazi`-bir tuwri` si`zi`q wazi`ypasi`n atqaradi`. Bul tuwri`lardi`n` yekewide giperbolag`a qarata ayri`qsha wori`ndi` iyeleydi.

$$y=\frac{b}{a}x$$

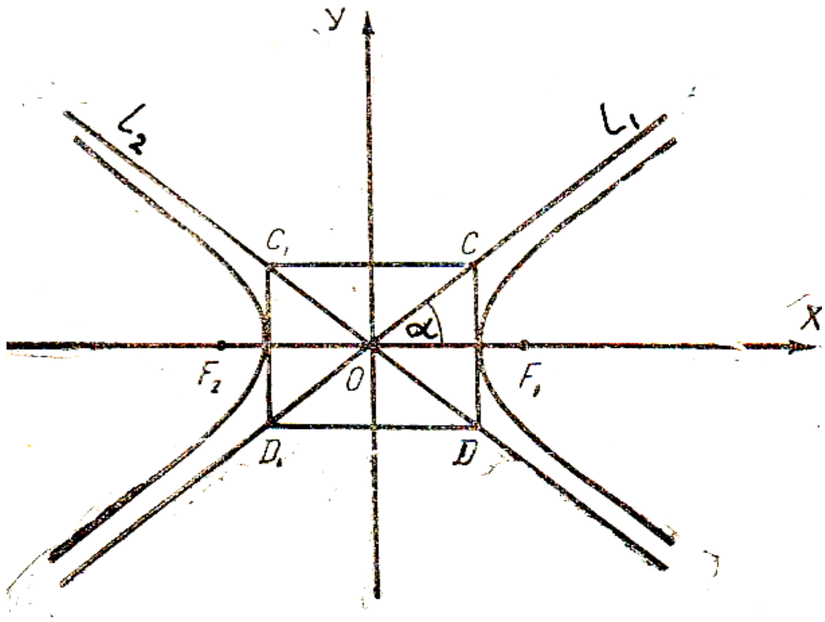
$$y=-\frac{b}{a}x$$

ten`lemeleri menen ani`qlani`wshi` tuwri` si`zi`qlar ***giperbolani`n` asimptotalari`*** dep ataladi`.

Yendi giperbolani`n` asimptotalari`n jasaw ma`selesin qarayi`q . Giperbolani`n` ko`sherleri u`stinen $CDD_1C_1$ tuwri`mu`yeshligin jasaymi`z, son` woni`n` diagonallari`n ju`rgizemiz. Bul diagonallardi` dawam ettirsek, payda bolg`an $l_1$ ha`m l_2 tuwri` si`zi`qlar bizge giperbolani`n` asimptotalari`n beredi. Sebebi jasaw boyi`nsha bul tuwri` si`zi`qlardi`n` mu`yesh koefficientleri

$$k_1 = tg\alpha = \frac{b}{a} \text{ ha`m } k_2 = tg\left(180^0 - \alpha\right) = -\frac{b}{a}$$

Sonli`qtanda, $y = \frac{b}{a}x$ ten`leme l_1 tuwri` si`zi`qti`n` ten`lemesi, al $y = -\frac{b}{a}x$ ten`lemesi $l_2$ tuwri` si`zi`qti`n` ten`lemesi boli`p tabi`ladi` (3-si`zi`lma).



3-si`zi`lma

Yeger giperbolani`n` asimptotalari`, woni`n` ko`sherleri si`pati`nda ali`nsa, wonda giperbola ten`lemesi $xy = c$ ko`riniste boladi`, bul jerde $c \neq 0$.

***10-mi`sal.*** Giperbolani`n` asimptotalari`ni`n` ten`lemeleri $y = \pm\frac{5}{12}x$ ha`m giperbolada jati`wshi` $M(24{,}5)$ noqat berilgen. Giperbola ten`lemesin du`zin`.

Sheshiliwi: Ma`sele sha`rtine ko`re $\frac{b}{a} = \frac{5}{12}$. To`mendegi ten`lemeler sistemasi`n sheshiwdi qarasti`rami`z:

$$\begin{cases} \frac{b}{a} = \frac{5}{12} \\ \frac{24^2}{a^2} - \frac{25}{b^2} = 1 \end{cases}$$

ha`m bul jerden $a^2$, $b^2$ lardi` ani`qlaymi`z, wolar sa`ykes $a^2 = 432,\ b^2 = 75$ ke ten`. Sonli`qtan izlengen giperbola ten`lemesi

$\frac{x^2}{432} - \frac{y^2}{75} = 1$ ko`riniske iye boladi`.

Giperbolanin` diametrleri`

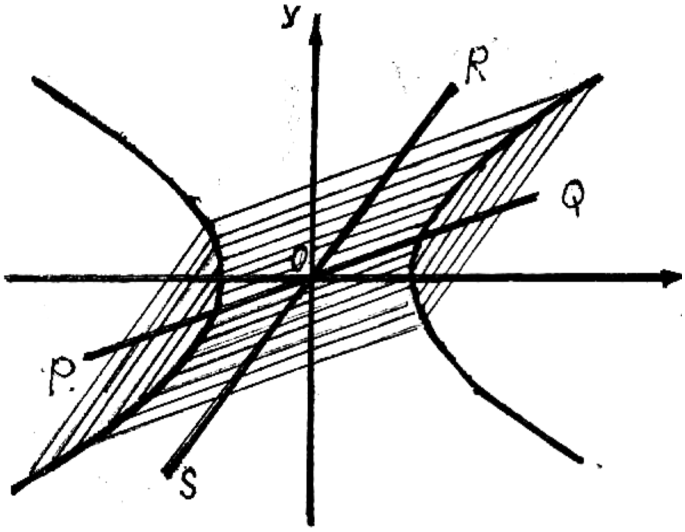
***4-ani`qlama.*** *Giperbolani`n` wo`z ara parallel bolg`an xordalari`ni`n` wortalari`ni`n` geometriyali`q worni` giperbolani`n` diametri dep ataladi`.*

Biz joqari`da ellips ushi`n wori`nlang`an talqi`lawlardan paydalani`p, woni`n` mu`yeshlik koefficienti $k$ menen ani`qlanatug`i`n berilgen bazi`-bir bag`i`tqa parallel xordalari`na tu`yinles bolg`an diametrinin` ten`lemesin payda yete alami`z:

$$\frac{x}{a^2} - k\frac{y}{b^2} = 0 \qquad (13)$$

(13) ten`leme giperbola diametrinin` ten`lemesi boli`p tabi`ladi`. Ellipstegi si`yaqli` giperbola diametri, giperbola worayi`nan o`tedi (4-si`zi`lma).

Giperbolada da ha`r bir diametrdin` wo`zine tu`yinles diametri bar boladi`. 4-si`zi`lmada $PQ$ ha`m RS tu`yinles diametrler boladi`. Tu`yinles diametrdin` birewi giperbolani` kesip wo`tse (PQ-haqi`yqi`y diametr), al yekinshi diametr (RS-jori`ma diametr) giperbolani` kesip wo`tpeydi. Giperbolani`n` ko`sherleri woni`n` tu`yinles diametrleri boli`p tabi`ladi`, sebebi wolardi`n` ha`r qaysi`si` wo`zlerine perpendikulyar bolg`an xordalardi` ten` wortadan bo`ledi. Wolar giperbolani`n` *bas diametrleri* dep ataladi`.



4-si`zi`lma

***11-mi`sal.*** $\frac{x^2}{9} - \frac{y^2}{4} = 1$ giperbolani`n` $M(5{,}1)$ noqatta ten` yekige bo`liniwshi xordasi`ni`n` ten`lemesin du`zin`.

Sheshiliwi: Giperbolani`n` $\frac{x}{a^2}-k\frac{y}{b^2}=0$ diametr ten`lemesinen paydalani`p $k=\frac{20}{9}$ ni` tabami`z, soni`nan $y=kx+b$ ten`lemeden b ni` ani`qlaymi`z; $1=\frac{20}{9}\cdot 5+b\Rightarrow b=-\frac{91}{9}$. Izlengen xorda ten`lemesi bolsa $20x-9y-91=0$ ko`riniste boladi`.

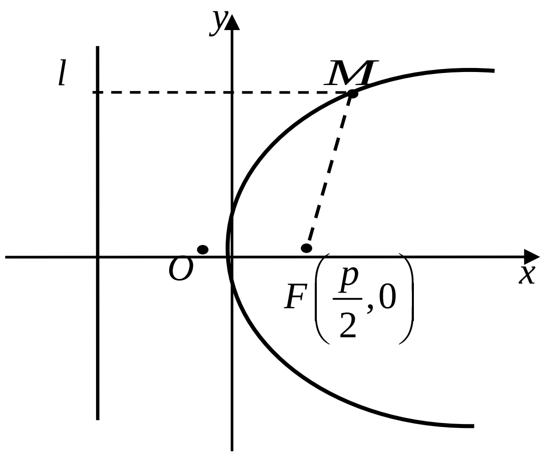
3. PARABOLA

***Ani`qlama-5.*** *Yekinshi ta`rtipli si`zi`qti`n` ten`lemesin qanday da bir dekart koordinatalar sistemasi`nda*

$$y^2=2px \tag{1}$$

ko`riniste jazi`w mu`mkin bolsa, wol parabola dep ataladi`.

Biz (1) ten`lemeni tekseriw ja`rdeminde parabolani`n` qa`siyetlerin u`yrenemiz ha`m woni` si`zami`z. Ten`lemeden ko`rinip turg`ani`nday, yeger(x,y) koordinatali` noqat parabolag`a tiyisli bolsa, $(x,-y)$ noqat ham parabolaga tiyisli boladi`. Demek parabola $Ox$ ko`sherine qarata simmetriyali` jaylasqan. Bunnan basqa koordinata basi` parabolag`a tiyisli, $x$ teris ma`nislerdi qabi`l qi`lmag`anli`g`i` ushi`n parabola $Oy$ ko`sherinin` wo`n ta`repinde jaylasqan. Bul talqi`lawlardan paydalani`p biz si`zi`lmada parabolani` to`mendegi ko`riniste su`wretlewimiz mu`mkin.



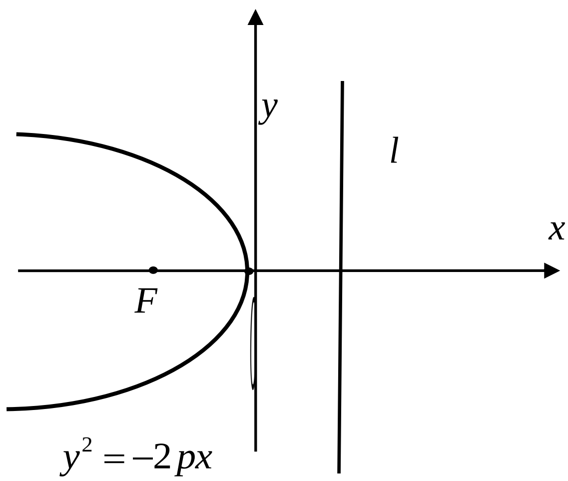
1-si`zi`lma

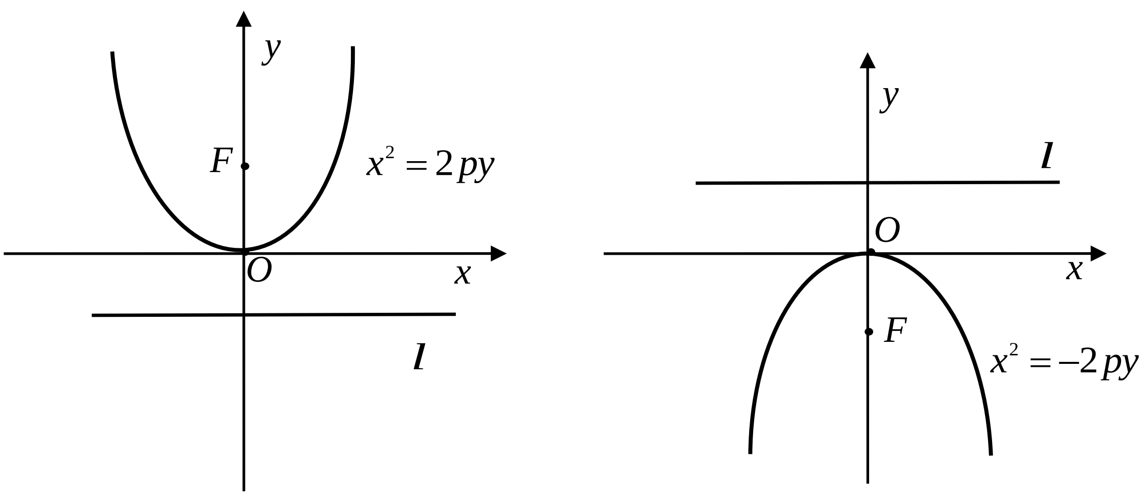
[Tegislikte

$$x+\frac{p}{2}=0 \tag{2}$$

ten`leme menen berilgen $\boldsymbol{l}$ tuwri` si`zi`q *parabolani`n` direktrisasi`*], $F\left(\frac{p}{2},0\right)$ noqat bolsa woni`n` *fokusi`* dep ataladi`. Parabolani`n` $Ox$ ko`sheri menen kesilisiw noqati` woni`n` to`besi dep ataladi`. (1) ten`lemede p sani` parabola *parametri* ha`m parabola fokusi`nan, woni`n` direktrisasi`na shekemgi arali`qti` bildiredi.

Parabola $y^2 = 2px$ ten`lemesinen basqa $y^2 = -px$, $x^2 = 2py$, $x^2 = -2py$ ten`lemeleri ko`rinisindede beriliwi mu`mkin, bul jag`daylarda parabolani`n` koordinata ko`sherlerine qarata jaylasi`wi` wo`zgeredi (2-si`zi`lma).





2-si`zi`lma

Parabola qa`siyetleri

1^0. Parabolani`n` qa`legen noqati`nan direktrisag`a shekemgi arali`q fokusqa shekemgi bolg`an arali`qqa ten`.

Parabola noqati`nan $F\left(\frac{p}{2},0\right)$ noqatqa shekemgi arali`qti` $r$ menen, direktrisag`a shekemgi arali`qti` d menen belgilep, $r=d$ yekenligin da`lilleymiz.

$$r=\sqrt{\left(x-\frac{p}{2}\right)^2+y^2}=\sqrt{x^2-px+\frac{p^2}{4}+y^2}$$

An`latpada $y^2=2px$ ten`likten paydalansaq, ha`m $x\geq 0$ qatnaslardi` yesapqa
alsaq

$$r=\sqrt{\left(x+\frac{p}{2}\right)^2}=x+\frac{p}{2}$$

formulani` payda yetemiz.

Parabolani`n` qa`legen noqati`nan direktrisag`a shekemgi arali`qti` yesaplaw ushi`n noqatdan tuwri` si`zi`qqa shekemgi bolg`an arali`qti tabi`w formulasi`nan paydalani`p

$$d=\left|-x-\frac{p}{2}\right|=x+\frac{p}{2}=r$$

ten`lemeni payda yetemiz.

Parabolani`n` geometriyali`q ani`qlani`wi`

1. Berilgen tuwri` si`zi`q ha`m wonda jatpaytug`i`n noqattan birdey uzaqli`qta jaylasqan noqatlar ko`pligi parabola boli`p tabi`ladi`.

Tegislikte ℓ tuwri` si`zi`q ha`m wog`an tiyisli bolmag`an F noqat berilgen bolsi'n. Berilgen F noqatdan ℓ tuwri` si`zi`qqa shekemgi bolg`an arali`qti` p menen belgilep ha`m $F$ noqatdan $\ell$ tuwri` si`zi`qqa perpendikulyar wo`tiwshi tuwri` si`zi`qti` abscissa ko`sheri si`pati`nda ali`p koordinatalar sistemasi`n kiritemiz. Abscissa ko`sherinin` won` bag`i`ti` ℓ tuwri` si`zi`qtan $F$ noqat ta`repke bag`i`tlang`an, al koordinata basi`n

ℓ tuwri` si`zi`q ha`m F noqat wortasi`na jaylasti`rami`z. Ordinata ko`sheri bolsa ℓ tuwri` si`zi`qqa parallel. Na`tiyjede ℓ tuwri` si`zi`q: $x+\frac{p}{2}=0$ ten`lemege, F noqat bolsa $\left(\frac{p}{2},0\right)$ koordinatalarg`a iye boladi`. Tegisliktin` $M(x,y)$ noqati`nan ℓ tuwri` si`zi`qqa shekemgi arali`qti` n` sol noqatdan F noqatqa shekemgi bolg`an arali`qqa ten`liginen

$$y^2=2px$$

ten`lemeni payda yetemiz.

***12-mi`sal.*** $y^2=4x$ parabolani`n` fokusi`nin` koordinatalari`n ani`qlan`.

Sheshiliwi: Ma`sele sha`rtine ko`re $\left(\frac{p}{2},0\right)$di tabi`wi`mi`z kerek. Berilgen ten`lemeden $p$ ni ani`qlaymi`z ha`m wol $p=2$ ge ten`. Sonli`qtan parabola fokusi`ni`n` koordinatalari` $F(1,0)$ boladi`.

***13-mi`sal.*** Parabola $Ox$ ko`sherine simmetriyali` jaylasqan boli`p, koordinatalar basi`nan ha`m $M(1,-4)$ noqattan wo`tedi. Parabolani`n` ten`lemesin du`zin`.

Sheshiliwi: Parabola ten`lemesin $y^2=2px$ ko`riniste izleymiz. $M(1,-4)$ noqattan wo`tkenligi ushi`n $16=2p$, bunnan $p=8$. Demek parabola ten`lemesi $y^2=16x$ boladi`.

Parabolani`n` uri`nbasi`

Bizge parabola (1) ten`lemesi menen berilgen bolsi`n. Parabolani`n` $M(x_0, y_0)$ noqati`nan ju`rgizilgen uri`nbani`n` ten`lemesin du`zeyik (3-si`zi`lma). Buni`n` ushi`n da`slep (1) ten`lemeden y' ti` tabami`z. Sonda $2yy' = 2p$.

Bunnan $y' = \frac{p}{y}$.

Yendi $M(x_0, y_0)$ noqati`nan wo`tiwshi da`stenin`

$$y - y_0 = k(x - x_0) \quad (3)$$

ten`lemesindegi $k$ ni`n` worni`na qoyami`z ha`m to`mendegi ten`lemeni payda yetemiz:

$$y - y_0 = \frac{p}{y_0}(x - x_0) \quad (4)$$

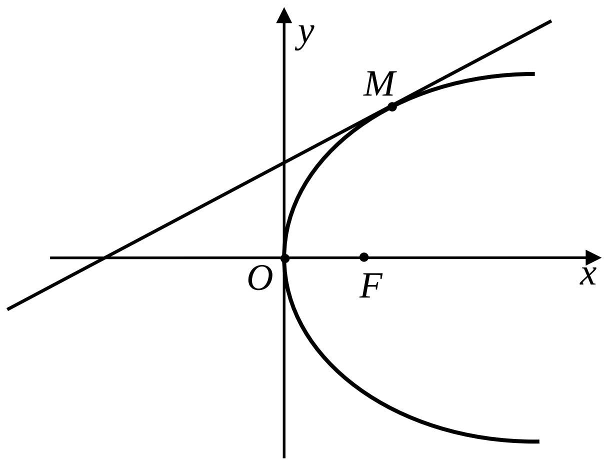
yaki

$$yy_0 - y_0^2 = px - px_0 \quad (5)$$

Bunda $y_0^2 = 2px_0$, woni` (5) ten`likke qoysaq, to`mendegige iye bolami`z:

$$yy_0 = p(x + x_0) \quad (6)$$

Bul (6) ten`leme parabolani`n` $M(x_0, y_0)$ noqati`na ju`rgizilgen uri`nbani`n` ten`lemesi boli`p tabi`ladi`.



3-si`zi`lma

***14-mi`sal.*** $y^2 = 4x$ parabolani`n` $M(9,6)$ noqati`nda ju`rgizilgen uri`nbasi`ni`n` ten`lemesin du`zin`.

Sheshiliwi: $yy_0 = p(x + x_0)$ ten`lemeden paydalanami`z: $p = 2$ ge ten` yekenligin bilgen halda $6y = 2(x+9)$ ten`lemeni jaza alami`z, apiwayi`lasti`rami`z: $x - 3y + 9 = 0$. Demek uri`nba ten`lemesi $x - 3y + 9 = 0$.

Parabolani`n` diametri

Yendi biz (1) ten`leme menen berilgen parabolani`n` diametrinin` ten`lemesin du`ziwdi qarasti`rayi`q. Deylik berilgen bag`i`tqa parallel bolg`an xordalardi`n` birewi

$$y = kx + m \tag{7}$$

ten`lemesi menen berilgen bolsi`n. Bul xordani`n` parabola menen kesilisiw noqatlari`n ani`qlaymi`z. Demek

$$\begin{cases} y^2 = 2px \\ y = kx + m \end{cases} \tag{8}$$

Ten`lemeler sistemasi`n sheshiwdi qaraymi`z. Sistemani`n` yekinshi ten`lemesinen x ti tabami`z:

$$x = \frac{y - m}{k}$$

Buni` birinshi ten`lemege qoyami`z:

$$y^2 = 2p\frac{(y-m)}{k} \tag{9}$$

Bul ten`lemeni

$$y^2 - 2\frac{p}{k}y + \frac{2pm}{k} = 0$$

ko`riniske keltiremiz. Usi`keltirilgen kvadrat ten`lemenin` korenleri parabola menen xordani`n` kesilisiw noqati`ni`n` ordinatalari`n beredi. Xordani`n` wortasi`ni`n` ordinatasi`n

$$y = \frac{y_1 + y_2}{2} = \frac{p}{k}$$

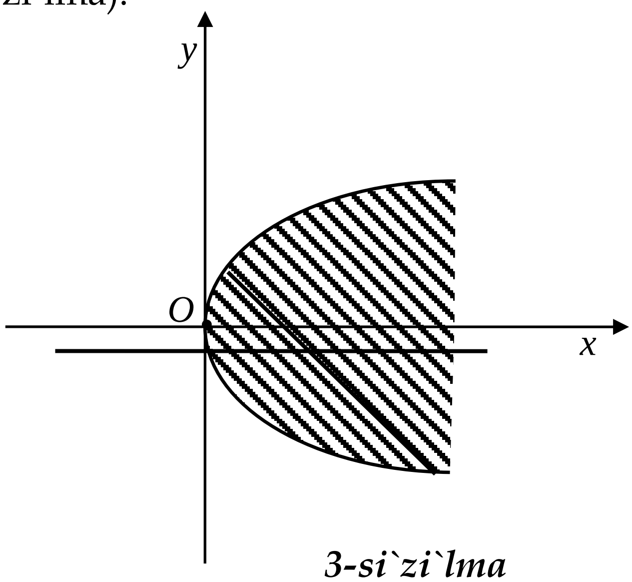
formulasi` menen tabami`z. Bunnan parabolani`n` wo`z ara parallel xordalari`ni`n` wortalari`ni`n` ordinatalari` tek wolardi`n` mu`yeshlik koefficientine ha`m parabola parametrine baylani`sli` bolatug`i`ni`n, yag`ni`y berilgen bag`i`tqa parallel bolg`an xordalardi`n` barli`g`i` ushi`n birdey ha`m

$$y = \frac{p}{k} \tag{10}$$

ten`lemesi menen beriliwin ko`remiz. Bul ten`leme biz izlegen parabola diametrinin` ten`lemesi boli`p tabi`ladi`.

Yeger biz xordalardi` $Oy$ ko`sherine parallel yetip alsaq, wonda xordalar Ox ko`sherinin` noqatlari` menen ten` wortadan bo`liniwin ko`riwge boladi`. Bul jag`dayda Ox ko`sherinin` wo`zi diametr boladi`. Sonli`qtan wol bas diametr dep ataladi`.

Yeger (10) ten`lemede $k > 0$ bolsa, yag`ni`y berilgen bag`i`tqa parallel xordalar $Ox$ ko`sheri menen su`yir mu`yesh jasasa, wonda parabolani`n` diametri Ox ko`sherinin` u`stinde jaylasadi`. Yeger $k < 0$ bolsa, parallel xordalar Ox ko`sheri menen dog`al mu`yesh jasasa, wonda diametr $Ox$ ko`sheri asti`nda jaylasqan boladi`. Yeger $k = \infty$ bolsa, wonda diametr Ox ko`sheri menen betlesedi. Bul jag`dayda xordalar Ox ko`sherine perpendikulyar boladi`. Parabolada ellips ha`m giperboladag`i` si`yaqli` tu`yinles diametrler bolmaydi`, sebebi parabolada barli`q diametrleri wo`z ara parallel boladi` (3-si`zi`lma).



3-si`zi`lma

Ellips, parabola ha`m giperbolani`n` ten`lemelerinin` uli`wma ko`rinistegi ten`lemeleri

1. Koordinata basi` yekinshi ta`rtipli si`zi`qti`n` ushi`nda bolg`an jag`day:

a) Ellips kanonikali`q ko`rinistegi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{1}$$

ten`leme menen berilgen bolsa,

$$x'=x+a, \quad y'=y \tag{2}$$

tu`rlendiriw wori`nlasaq, taza $O'x'y'$ koordinatalar sistemasi`ni`n` basi` ellipstin` shep $(-a,0)$ to`besinde jaylasadi` ha`m (1) ten`leme

$$\frac{(x'-a)^2}{a^2}+\frac{y'^2}{b^2}=1 \tag{3}$$

ko`riniske keledi. Bul ten`lemeni

$$y'^2=2px'+qx'^2 \tag{4}$$

ko`riniste jazip alami`z. Bul jerde $p=\frac{b^2}{a}$, $q=-\frac{b^2}{a^2}=e^2-1$ boli`p, $-1\le q<0$ qatnas wori`nlanadi`.

b)Yeger giperbolani`n`

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \tag{5}$$

ten`lemesinde

$$x'=x-a, y'=y \tag{6}$$

tu`rlendiriw wori`nlasaq ten`leme

$$y'^2=2px'+qx'^2 \tag{7}$$

ko`riniste boli`p, koeeffisientler ushi`n

$$q=\frac{b^2}{a^2}=e^2-1>0, \quad p=\frac{b^2}{a}$$

qatnaslar wori`nli` boladi`.

c) Yeger (7) ten`elemede $q=0$ bolsa, onda bul ten`leme $y'^2=2px'$ ko`riniske keledi, bul bolsa parabola ten`lemesi.

Demek giperbola, ellips ha`m parabola ten`lemelerin (7) ko`riniste jazi`w mu`mkin.

Ellips, parabola ha`m giperbolani`n` polyar koordinatalar sistemasi`ndag`i` ten`lemeleri

Ellips. Ellipstin` polyar koordinatalar sistemasi`ndag`i` ten`lemesin keltirip shi`g`arayi`q. Buni`n` ushi`n polyusti` ellipstin` shep fokusi`na jaylasti`ri`p, abscissa ko`sherin polyar ko`sheri si`pati`nda alami`z (1-si`zi`lma).

Ellipstin`

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

kanonikali`q ten`lemesin polyar koordinatalar sistemasi`na wo`tkiziw ushi`n

$$\begin{cases} x'=x+c \\ y'=y \end{cases}$$

tu`rlendiriwler ja`rdeminde taza $O'x'y'$ dekart koordinatlar sistemasi`n kiritemiz. Bul koordinatalar sistemasi` ha`m polyar koordinatalar arasi`ndag`i` baylani`s

$$x'=r\cos\varphi, \quad y'=r\sin\varphi$$

formulalar ja`rdeminde beriledi. Ellipstin` M noqati` ushi`n shep fokal radius woni`n` polyar radiusi`na ten`liginen paydalani`p

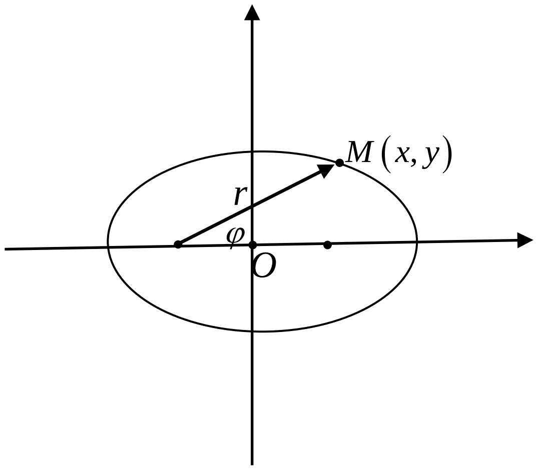
$$MF_1=r=ex+a$$

ten`lemeni jazami`z. Bul jerde $r=ex+a$ ha`m $x+c=r\cos\varphi$ ten`lemelerden x wo`zgeriwshini shi`g`ari`p taslasaq, wonda

$$r=\frac{p}{1-e\cos\varphi}$$

ten`lemeni payda yetemiz. Bul jerde

$$p=\frac{b^2}{a}=a-ec$$ ten`likten paydalandi`q.



1-si`zi`lma

Giperbola. Giperbolani`n` shep shaqasi`ni`n` ten`lemesin polyar koordinatalar sistemasi`nda jazi`wdi` qaraymi`z. Bul ushi`n polyusti` shep fokusqa jaylasti`rami`z ha`m abscissa ko`sherin qarama-qarsi` bag`i`t penen polyar ko`sheri si`pati`nda alami`z (2-si`zi`lma).

Yeger

$$x' = -x - c$$
$$y' = y$$

formulalar menen taza dekart koordinatalar sistemasi`n kiritsek, wolar ushi`n

$$x' = r\cos\varphi$$
$$y' = r\sin\varphi$$

formulalar wori`nli` boladi`. Bul jerde polyar radius shep fokal radiusqa ten` bolg`anli`g`i` ushi`n

$$r = -ex - a$$

ten`lik wori`nli` boladi`. Bul ten`liktegi $r$ din` an`lati`li`wi`n joqari`dag`i` formulalardan keltirip shi`g`aratug`i`n bolsaq

$$-x - c = r\cos\varphi$$

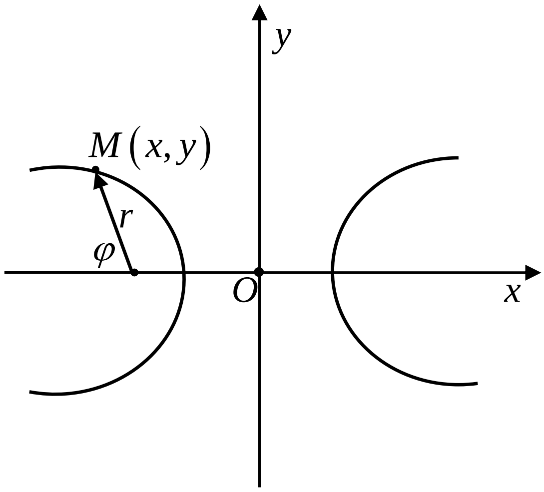
ten`likke qoyi`p

$$r = \frac{p}{1 - e\cos\varphi}$$

ten`lemeni payda yetemiz. Bul jerdede

$$p = \frac{b^2}{a} = \frac{c^2 - a^2}{a} = ec - a$$

ten`lik wori`nli`.



2-si`zi`lma

Parabola. Yendi parabolani`n` polyar koordinatalar sistemasi`ndag`i` ten`lemesin keltirip shi`g`arayi`q. Bizge parabola

$$y^2 = 2px$$

kanonikali`q ko`rinistegi ten`lemesi menen berilgen bolsi`n. Polyusti` parabola fokusi`na jaylasti`ri`p, polyar ko`sheri si`pati`nda abscissa ko`sherin alami`z (3-si`zi`lma).

Yeger biz

$$x' = x - \frac{p}{2}, \qquad y' = y$$

tu`rlendiriwler wori`nlasaq

$$x' = r\cos\varphi, \qquad y' = r\sin\varphi$$

ten`likler wori`nli` boladi`. Bul jerde r, φ ler noqatti`n` polyar koordinatalari` boli`p, yeger noqat parabolag`a tiyisli bolsa, $r$ wolardi`n` fokal radiusi`na ten` boladi`.

Biz

$$x - \frac{p}{2} = r\cos\varphi$$

ten`likte $r$ din` noqattan direktrisag`a shekemgi bolg`an arali`qqa ten`ligin yesapqa ali`p $r = x + \frac{p}{2}$ an`latpani` joqari`dag`i` ten`likke qoysaq

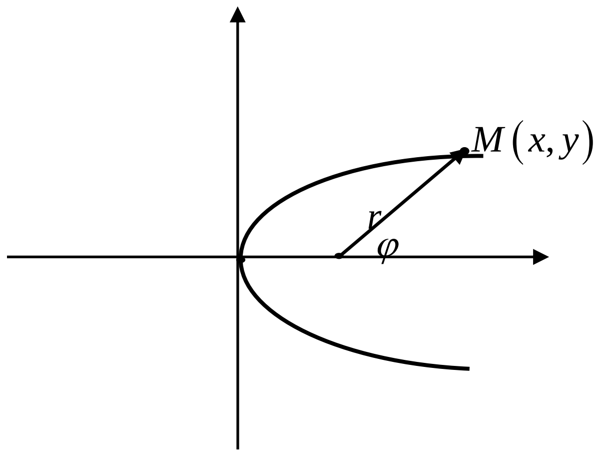
$$r = \frac{p}{1 - \cos\varphi}$$

qatnaslardi` payda yetemiz. Bul qatnas parabolani`n` polyar koordinatalar sistemasi`ndag`i ten`lemesi boli`p tabi`ladi`.

Demek, polyar koordinatalar sistemasi`nda yekinshi tartipli si`zi`qti`n` ten`lemesin

$$r = \frac{p}{1 - e\cos\varphi}$$

ko`riniste jazi`w mu`mkin yeken. Bul ten`lemede yeger $e < 1$ bolsa ellips, $e > 1$ bolg`anda giperbola, $e = 1$ bolsa parabolani`n` ten`lemesi boladi` yeken.



3-si`zi`lma

***15-mi`sal.*** Giperbolani`n` polyar koordinatalar sistemasi`ndag`i` $p = \frac{9}{4 - 5\cos\varphi}$ ten`lemesi berilgen. Woni`n` dekart koordinatalar sistemasi`ndag`i` ten`lemesin du`zin`.

Sheshiliwi: $p = \frac{b^2}{a} = 9$ g`a al $\frac{c}{a} = \frac{5}{4}$ ke ten`.

To`mendegilerdi ani`qlaymi`z:

$$a^2 + b^2 = c^2, \; a^2 + \frac{9}{4}a = \frac{25}{16}a^2 \Rightarrow \; \frac{9}{4}a = a^2\left(\frac{25}{16} - 1\right)$$

$$\Rightarrow \frac{1}{4} = \frac{a}{16} \; \Rightarrow a = 4.$$

$$b^2 = \frac{9}{4} \cdot a = 9 \Rightarrow b^2 = 9.$$

Demek giperbola ten`lemesi $\frac{x^2}{16} - \frac{y^2}{9} = 1.$

***16-mi`sal.*** Giperbolani`n` dekart koordinatalar sistemasi`nda ten`lemesi $\frac{x^2}{144}-\frac{y^2}{25}=1$ bolsa, woni`n` polyar koordinatalar sistemasi`ndag`i` ten`lemesin du`zin`.

Sheshiliwi: $r=\frac{p}{1-e\cos\varphi}=\frac{\frac{b^2}{a}}{1-\frac{c}{a}\cos\varphi}=\frac{\frac{25}{12}}{1-\frac{13}{12}\cos\varphi}=\frac{25}{12-13\cos\varphi}$,

bul jerde $c=\sqrt{144+25}=\sqrt{169}=13$.

Demek juwap: $r=\frac{25}{12-13\cos\varphi}$.

T E S T L E R

1. $\frac{x^2}{16}+\frac{y^2}{9}=1$ ***ellipstin` kishi ha`m u`lken ko`sherin ani`qlan`.***

A. $a=4, b=3$

B. $a=3, b=4$

C. $a=16, b=9$

D. $a=9, b=16$

2. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ***ellipstin` fokusi`ni`n` koordinatalari`n ani`qlan`.***

A. $(\pm 3{,}0)$

B. $(5{,}4)$

C. $(9{,}0)$

D. $(-9{,}0)$

3. $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1$ ***ten`leme ellipstin` ... ten`lemesi***

A. Uri`nba

B. Diametr

C. Direktrisa

D. Duri`s juwap joq

4. $Ax+By+C=0$ ***tuwri` si`zi`q qanday sha`rt wori`nlang`anda*** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ***ellipske uri`ni`p wo`tedi?***

A. $A^2a^2+B^2b^2=C^2$

B. $A^2a^2+B^2b^2=0$

C. $A^2a+B^2b=C^2$

D. $Aa^2+B\,b^2=C$

5. $\frac{x^2}{32}+\frac{y^2}{18}=1$ ***ellipstin` $M(4,3)$ noqattan wo`tiwshi uri`nba ten`lemesin du`zin`.***

A. $3x+4y-24=0$

B. $3x-4y-24=0$

C. $3x+4y+24=0$

D. $3x+4y-18=0$

6. *Ellipstin` diametri ten`lemesin ko`rsetin`*

A. $\frac{x}{a^2}+k\frac{y}{b^2}=0$

B. $\frac{x}{a^2}+k\frac{y}{b^2}=1$

C. $\frac{x}{a}+k\frac{y}{b}=0$

D. $\frac{x}{a}+k\frac{y}{b}=1$

7. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ***ellipstin` ekscentrisitetin ani`qlan`.***

A. $e=\frac{3}{5}$

B. $e=\frac{5}{3}$

C. $e=-\frac{3}{5}$

D. $e=-\frac{5}{3}$

8. $\frac{x^2}{9}+\frac{y^2}{25}=1$ ***ellipske qarata*** $\left(\sqrt{3},5\sqrt{\frac{2}{3}}\right)$ ***noqatti`n` jaylasi`wi`n ani`qlan`.***

A. Ellipske tiyisli

B. Ellipske tiyisli yemes

C. Ellips si`rti`nda jaylasqan

D. Ellips ishinde jaylasqan

9. U`lken ko`sheri 6 g`a, kishi ko`sheri 4 ke ten` bolg`an ellipstin` ten`lemesin du`zin`.

A. $\frac{x^2}{36}+\frac{y^2}{16}=1$

B. $\frac{x^2}{6}+\frac{y^2}{4}=1$

C. $\frac{x^2}{16}+\frac{y^2}{36}=1$

D. $\frac{x^2}{36}+\frac{y^2}{16}=0$

10. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ***ellipstin` direktrisa ten`lemesin ani`qlan`***

A. $x\pm\frac{25}{3}=0$

B. $x\pm\frac{3}{25}=0$

C. $x\pm\frac{20}{3}=0$

D. $x\pm\frac{15}{3}=0$

11. $x\pm\frac{a}{e}=0$ ***ten`leme giperbolani`n` ... ten`lemesi***

A. Uri`nba

B. Diametr

C. Direktrisa

D. Asimptota

12. Giperbolani`n` (x_0, y_0) ***noqattag`i` uri`nba ten`lemesin ko`rsetin`.***

A. $\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1$

B. $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1$

C. $\frac{xx_0}{a}+\frac{yy_0}{b}=1$

D. $\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=0$

13. Giperbolani`n` kanonikali`q ten`lemesin ko`rsetin`.

A. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

B. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

C. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

D. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$

14. Giperbolani`n` asimptotalari` ten`lemesin ko`rsetin`.

A. $y = \pm\frac{b}{a}x$

B. $y = \pm\frac{a}{b}x$

C. $y = abx$

D. $y = -abx$

15. Haqi`yqi`y ko`sheri 48 ha`m jori`ma ko`sheri 10 g`a ten` bolg`an giperbola ten`lemesin du`zin`.

A. $\frac{x^2}{576} - \frac{y^2}{100} = 1$

B. $\frac{x^2}{576} - \frac{y^2}{100} = 0$

C $\frac{x^2}{100} - \frac{y^2}{576} = 1$

D. $\frac{x^2}{100} - \frac{y^2}{576} = 0$

16. $\frac{x}{a^2} - k\frac{y}{b^2} = 0$ ***ten`leme giperbolani`n` ... ten`lemesi***

A. Uri`nba

B. Diametr

C. Direktrisa

D. Asimptota

17. $\frac{x^2}{25}-\frac{y^2}{144}=1$ ***giperbolani`n` fokusi` koordinatalari`n ani`qlan`.***

A. $F_1(-13{,}0),\ F_2(13{,}0)$

B. $F_1(-9{,}0),\ F_2(9{,}0)$

C. $F_1(-5{,}0),\ F_2(5{,}0)$

D. $F_1(-12{,}0),\ F_2(12{,}0)$

18. $\frac{x^2}{9}-\frac{y^2}{16}=1$ ***giperbolani`n` ekscentrisitetin tabi`n`.***

A. $\frac{5}{3}$

B. $\frac{3}{5}$

C. 5

D. 3

19. Asimptotalar ten`lemeleri*** $y=\pm\frac{5}{3}x$ ***ha`m $N(6{,}9)$ ***noqattan wo`tiwshi giperbolani`n` yari`m ko`sherlerin ani`qlan`.***

A. $a=\frac{3\sqrt{19}}{5},\ b=\sqrt{19}$

B. $a=5,\ b=3$

C. $a=\frac{\sqrt{19}}{5},\ b=\sqrt{19}$

D. $a=\frac{3\sqrt{19}}{3},\ b=\sqrt{19}$

20. $\frac{x^2}{5}-\frac{y^2}{4}=1$ ***giperbolani`n`*** $M(5,-4)$ ***noqattan wo`tiwshi uri`nbasi`ni`n` ten`lemesin du`zin`.***

A. $x+y-1=0$

B. $x-y-1=0$

C. $x+y+1=0$

D. $x-y+1=0$

21. Parabolani`n` kanonikali`q ten`lemesin ko`rsetin`.

A. $y^2 = 2px$

B. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

C. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

D. $\frac{x^2}{a} - \frac{y^2}{b} = 1$

22. $y^2 = 4px$ ***parabolani`n` fokusi`ni`n` koordinatalari`n ani`qlan`.***

A. $F(1{,}0)$

B. $F(-1{,}0)$

C. $F(2{,}0)$

D. $F(-2{,}0)$

23. Yeger parabola to`besinen fokusi`na shekemgi arali`q 3 ke ten` bolsa, parabolani`n` kanonikali`q ten`lemesi`n du`zin`.

A. $y^2 = 12x$

B. $y^2 = 3x$

C. $x^2 = 12y$

D. $x^2 = 3y$

24. $y^2 = 4x$ ***parabolani`n`*** $M(3{,}1)$ ***noqatta ten` yekige bo`liniwshi xordasi`ni`n` ten`lemesin tabi`n`.***

A. $y = 2x - 5$

B. $y = 5x - 2$

C. $y = x - 5$

D. $y = x - 3$

25. Parabolani`n` uri`nbasi` ten`lemesin ko`rsetin`.

A. $yy_0 = p(x + x_0)$

B. $y = p(x + x_0)$

C. $xx_0 = p(y + y_0)$

D. $y = p(y_0 + x_0)$

26. $x=-\frac{p}{2}$ ***ten`leme parabolani`n` ... ten`lemesi.***

A. Direktrisa

B. Uri`nba

C. Diametr

D. Duri`s juwap joq.

27. $y=\frac{p}{k}$ ***ten`leme parabolani`n ... ten`lemesi.***

A. Diametr B. Duri`s juwap joq. C. Direktrisa D. Uri`nba

28. $y^2=4x$ ***parabolani`n`*** $M(9,6)$ ***noqati`nda ju`rgizilgen uri`nbani`n` ten`lemesin du`zin`.***

A. $x-3y+9=0$

B. $x-3y-9=0$

C. $x+3y+9=0$

D. $3x+y+9=0$

29. Uri`nba ten`lemesi $x-3y+9=0$ ***bolg`an parabolani`n` ten`lemesin tabi`n`.***

A. $y^2=4x$

B. $y^2=x$

C. $y^2=-4x$

D. $y^2=14x$

30. $y^2=12x$ ***parabolani`n` direktrisa ten`lemesin du`zin`.***

A. $x=-3$

B. $y=-3$

C. $x=3$

D. $y=3$

WO`Z BETİNSHE JUMIS USHIN TAPSIRMALAR

1. $\frac{x^2}{100}+\frac{y^2}{36}=1$ ellipste won` fokusqa shekemgi arali`q shep fokusqa shekemgi arali`qqa qarata 4 ma`rte u`lken bolg`an noqat tabi`lsi`n.

2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipstin` $F(c,0)$ fokusi` arqali` u`lken ko`sherine perpendikulyar bolg`an xorda wo`tkizilgen. Bul xorda uzi`nli`g`i`n tabi`n`.

3. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipske ishki si`zi`lg`an kvadrat ta`replerinin` uzi`nli`qlari` tabi`lsi`n.

4. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipstin` $N(10,4)$ noqat arqali` wo`tiwshi uri`nbalari`ni`n` ten`lemelerin du`zin`.

5. To`mendegi ellipslerge uli`wma bolg`an uri`nba ten`lemelerin du`zin`:

$\frac{x^2}{4}+\frac{y^2}{5}=1$ ha`m $\frac{x^2}{5}+\frac{y^2}{4}=1$

6. Tas gorizont penen su`yir muyesh payda yetken bag`i`tta i`laqti`ri`ldi` ha`m parabola dog`asi` boyi`nsha ha`reket yetip, baslang`i`sh halati`nan 16 metr uzaqli`qta jerge tu`sti. Wolardi`n` yen` joqarg`i halati` 12 metr biyiklikte bolsa, parabolani`n` parametrin tabi`n`.

7. Asimptotasi` $y=\pm\frac{1}{2}x$ bolg`an ha`m $(12;3\sqrt{3})$ noqattan wo`tiwshi giperbola ten`lemesin jazi`n`.

8. Berilgen $\frac{x^2}{9}-\frac{y^2}{36}=1$ giperbolaga:

1) $3x-y-17=0$ tuwri` si`zi`qqa parallel;

2) $2x+5y+11=0$ tuwri` si`zi`qqa perpendikulyar yetip ju`rgizilgen uri`nbalardi`n` ten`lemeleri du`zilsin.

9. Giperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ten`leme menen berilgen bolsi`n. Wolardi`n` parallel diametrlerinin` wortalari`ni`n` geometriyali`q worni`n tabi`n`.

10. $Ax+By+C=0$ tuwri` si`zi`qti`n` $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolag`a uri`ni`w sha`rtin jazi`n`.

11. Parabolani`n` fokusi`nan direktrisag`a shekemgi arali`q 2 ge ten`. Parabolani`n` kanonikali`q ten`lemesin du`zin`.

12. Parabolani`n` fokusi` $F(3{,}0)$ noqatta ha`m $x=-1$ direktrisasi`ni`n` ten`lemesi bolsa, parabolani`n` ten`lemesin du`zin`.

13. $y=x^2-4x+5$ parabolani`n` fokusi`n ani`qlan`.

14. $4x+3y+46=0$ tuwri` si`zi`qtan $y^2=64x$ parabolag`a shekemgi bolg`an yen` qi`sqa arali`qti` tabi`n`.

15. Ellips $\rho^2=\frac{288}{16-7\cos\varphi}$ ten`leme menen berilgen bolsa, dekart koordinatalar sistemasi`ndag`i` ten`lemesin du`zin`.

16. Giperbola $\rho=\frac{2}{1-\sqrt{2}\cos\varphi}$ ten`leme menen berilgen bolsa, wolardi`n` asimptotalari` ha`m direktrisalari` ten`lemesin du`zin`.

12. To`mendegi si`zi`qlardi`n` dekart koordinatalar sistemasi`ndag`i ten`lemesin jazi`n`.

a). $\rho=\frac{2}{13-12\cos\varphi}$

b). $\rho=\frac{2}{3-3\cos\varphi}$

c). $\rho=\frac{2}{4-5\cos\varphi}$

d). $\rho=\frac{2}{\sqrt{5}-3\cos\varphi}$.

PAYDALANILG`AN A`DEBİYATLAR.

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M A Z M U N I

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