O’ZBEKISTAN RESPUBLİKASI’ XALI’Q BİLİMLENDİRİW MİNİSTRLİGİ
A’JİNİYaZ ati’ndag’i’ NO’KİS MA’MLEKETLİK PEDAGOGİKALI’Q İNSTİTUTI’

A. Usakova, A. Jan’abergenova

Joqari’ matematika tiykarlari’ pa’ninen a’meliy ha’m baqlaw jumi’slari’

(«Su’wretlew wo’neri ha’m injenerlik grafikasi’» ta’lim bag’dari’ talabalari’ ushi’n)

NO’KİS- 2015

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Annotatsiya

Bul woqi'w-metodikali'q qollanba joqari' woqi'w wori'nlari' ushi'n arnalg'an boli'p, bunda talabalardi'n' a'meliy sabaqlari'n ha'm baqlaw jumi'slari'n wo'tkeriw boyi'nsha metodikali'q ko'rsetpeler berilgen. Qollanba talabalardi'n' pa'n boyi'nsha teoriyali'q bilimlerin teren'lestiriwge, lekcsiyali'q sabaqlarda berilgen tu'siniklerdin' mazmuni'n ken'nen bilip ali'wg'a ja'rdem beredi. Qollanbada temalar boyi'nsha testler ha'm baqlaw jumi'slari' variantlari' kiritilgen.

Qollanba 110000 – "Pedagogika" ha'm 5110800 – " Su'wretlew wo'neri ha'm injenerlik grafikasi'" bakalavriat ta'lim bag'dari' ushi'n mo'lsherlengen boli'p, ta'lim standartlari' ha'm oqi'w rejesi talaplari'na juwap beredi.

Woqi'w-metodikali'q qollanba No'kis ma'mleketlik pedagogikali'q instituti'ni'n' ilimiy-metodikali'q Ken'esinde talqi'lani'p, baspag'a usi'ni's etilgen(2014-ji'l, 27-dekabrdegi №3-sanli' Ken'es qarari').

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SO’Z BASI’

Joqari’ matematika tiykarlari’ pa’nin woqi’ti’wdi’n’ maqseti-talabalarda injenerlik grafikasi’ pa’nlerin jaqsi’ wo’zlestiriw ha’m wolardi’n’ ma’selelerin sheshiw ushi’n jeterli da’rejede matematikali’q bilim ha’m ko’nlikpelerin rawajlandi’ri’wdan ibarat.

Joqari’ matematika tiykarlari’ pa’ni su’wretlew wo’neri ha’m injenerlik grafikasi’ qa’nigeligi ushi’n za’ru’r bolg’an matematikani’n’: si’zi’qli’ algebra, vektorlar algebrasi’ elementleri, analitikali’q geometriya elementleri, matematikali’q analiz, tegisliktegi ha’m ken’isliktegi geometriyali’q jasawlar, su’wretlew metodlari’, differentsiyal ha’m integral esabi’ ha’m differentsiyal ten’lemeler boyi’nsha da’slepki tu’siniklerin wo’z ishine aladi’.

Bul metodikali’q qollanba «Su’wretlew wo’neri ha’m injenerlik grafikasi’» bakalavr ta’lim bag’dari’nda bilim ali’p ati’rg’an talabalarg’a mo’lsherlengen boli’p, ta’lim standartlari’ ha’m woqi’w rejesi talaplari’na juwap beredi.

Metodikali’q qollanba woqi’w rejedegi barli’q temalar boyi’nsha qi’sqasha tu’siniklerdi ha’m mi’sallardi’ wo’z ishine aladi’. Sonin’ menen birge qollanba talabalar bilimin reyting sistemasi’ tiykari’nda bahalawg’a arnalg’an boli’p, yeki arali’q bahalaw(AB) , to’rt ku’ndelik bahalaw (KB) ha’m bir juwmaqlawshi’ bahalaw (JB) sorawlari’n, test tapsi’rmalari’n wo’z ishine aladi’.

Qollanbada arali’q bahalaw ushi’n 400 test tapsi’rmalari’, ku’ndelik bahalawdi’n’ 160 mi’sal ha’m ma’seleleri ja’ne de juwmaqlawshi’ bahalawdi’n’ 116 sorawlari’ keltirilgen. Arali’q bahalaw ushi’n test tapsi’rmalari’ ha’m ku’ndelik bahalaw ushi’n mi’sallar talabalarg’a to’rt variantta tarqati’ladi’. Test tapsi’rmalari’ juwaplari’ menen berilgen.

I. HAQI'YQI'Y SANLAR KO'PLIGI. KOORDINATALAR METODI'. FUNKCIYA TU'SINIGI.

1. Haqi'yqi'y sanlar ko'pligi. Ko'plik tu'sinigi matematikani'n' tiykarg'i' tu'siniklerinin' biri boli'p, wol ani'qlamasi'z mi'sallar ja'rdeminde tu'sindiriledi. Ma'selen, auditoriyadag'i' talabalardi'n' ko'pligi, natural sanlar ko'pligi, Qaraqalpaqstan Respublikasi' boyi'nsha rayonlar ko'pligi, pu'tin sanlar ko'pligi ha'm tag'i' basqalar.

Ko'plikti payda yetiwshi' ob'ektler ko'pliktin' elementleri dep ataladi'.

Ko'plikler $A,B,C,\ldots$ lar menen, al woni'n' elementleri $a,b,c,\ldots$ lar menen belgilenedi. Ko'pliktin' elementi $a \in A$ ko'rinisinde jazi'ladi' ha'm «a element A ko'plikke tiyisli» dep ataladi'. Yeger tiyisli bolmasa, $a \notin A$ yamasa $a \overline{\in} A$ ko'rinisinde jazi'ladi'. Ma'selen, ha'mme natural sanlar ko'pligi H ha'm $4,\ 5,\ \frac{3}{4},\ \pi$ sanlari' ushi'n $4 \in N,\ 5 \in N,\ \frac{3}{4} \notin N,\ \pi \notin N$ mu'na'sebetleri wori'nli'.

Sanli' ko'plikler degende, ha'mme elementleri sanlardan ibarat bolg'an ha'r qanday ko'pliklerdi tu'siniwge boladi'. Bunda N-natural sanlar ko'pligi, Z-pu'tin sanlar ko'pligi, Q-ratsional sanlar ko'pligi, R-haqi'yqi'y sanlar ko'pligi.

Ko'plik wo'z elementlerinin' toli'q dizimin ko'rsetiw yaki sol ko'plikke tiyisli bolg'an elementlerge qanaatlandi'ratug'i'n sha'rtler sistemasi'n beriw menen toli'q ani'qlani'wi' mu'mkin. Ko'plikke tiyisli bolg'an elementler g'ana qanaatlandi'ratug'i'n sha'rtler sistemasi' sol ko'pliktin' xarakteristikali'q qa'siyeti dep ataladi'. Ha'mme x elementleri qanday da bir b qa'siyetke iye bolg'an ko'plik $X = \{x | b(x)\}$ ko'rinisinde jazi'ladi'. Ma'selen ratsional sanlar ko'pligin $Q = \left\{ r \middle| r = \frac{p}{q},\ p \in Z,\ q \in N \right\}$ ko'rinisinde, $ax^2 + bx + c = 0$ kvadrat ten'lemenin' korenler ko'pligin bolsa $X = \{x | ax^2 + bx + c = 0\}$ ko'rinisinde jazi'wg'a boladi'.

Elementlerinin' sani'na baylani'sli' halda ko'plikler shekli ha'm sheksiz ko'pliklerge aji'raladi'. Elementlerinin' sani' shekli bolg'an ko'plik **shekli ko'plik**, al elementlerinin' sani' sheksiz bolg'an ko'plik **sheksiz ko'plik** delinedi.

Hesh qanday elementke iye bolmag'an ko'plik **bos ko'plik** delinedi. Bos ko'plik Ø arqali' belgilenedi.

Yeger B ko'pliginin' ha'r bir elementi A ko'pliginin' de elementi bolsa, wonda B ko'plik A ko'pliginin' **u'les ko'pligi** delinedi. ha'm $B \subset A$ ko'rinisinde belgilenedi. Ma'selen. Yeger $A = \{3, 4, 5\}$, $B = \{x | x^2 - 7x + 12 = 0\}$ bolsa, $B \subset A$ boladi'.

A ha'm B ko'pliklerinin' yekewinde de bar bolg'an x elementke sol ko'pliklerdin' uli'wma elementi delinedi. A ha'm B ko'pliklerinin' **kesilispesi** dep, wolardi'n' ha'mme uli'wma elementlerinen du'zilgen ko'plikke ayti'ladi'. A ha'm B ko'pliklerinin' kesilispesi $A \cap B$ ko'rinisinde belgilenedi.

A ha'm B ko'pliklerinin' **birikpesi** dep, wolardi'n' keminde birewinde bar bolg'an ha'mme elementlerden du'zilgen ko'plikke ayti'ladi'. A ha'm B ko'pliklerinin' birikpesi $A \cup B$ ko'rinisinde belgilenedi, $A \cup B = \{x | x \in A \; yaki \; x \in B\}$

A ha'm B ko'pliklerinin' **ayi'rmasi'** dep, A ni'n' B da bar bolmag'an ha'mme elementlerinen du'zilgen ko'plikke ayti'ladi' ha'm wol A\B ko'rinisinde belgilenedi. A\B= $\{x | x \in A \; xam \; x \notin B\}$.

Ko'pliklerdin' kesilispesinin' qa'siyetleri:

1. $B \subset A$ bolsa, $A \cap B = B$ boladi'.
2. $A \cap B = B \cap A$ (kommutativlik qa'siyeti).
3. $A \cap (B \cap C) = (A \cap B) \cap C = A \cap B \cap C$ (assotsiativlik qa'siyeti).
4. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (kesilispenin' birikpege sali'sti'rg'anda distributivlik qa'siyeti).
5. $A \cap \varnothing = \varnothing$.
6. $A \cap A = A$

Ko'pliklerdin' birikpesinin' qa'siyetleri:

1. $B \subset A$ bolsa, $A \cup B = A$ boladi'.
2. $A \cup B = B \cup A$ (kommutativlik qa'siyeti).
3. $A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$ (assotsiativlik qa'siyeti).
4. $A \cup \varnothing = A$

5. $A \cup A = A$

6. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (kesilispenin' birikpege sali'sti'rg'anda distributivlik qa'siyeti).

Mi'sallar

1. $A = \{x | x \in N,\ x > 10\}$ ko'plik 10 nan u'lken bolg'an ha'mme natural sanlardan du'zilgen, yag'ni'y $A = \{11, 12, 13, 14, 15, 16, 17, \ldots\ldots\}$ ko'pligi sheksiz ko'plik boli'p yesaplanadi'.

2. $x^2 + 3x + 2 = 0$ ten'lemenin' korenleri $X = \{-2;\ -1\}$ shekli ko'plikti payda yetedi. $x^2 + 3x + 3 = 0$ ten'leme bolsa haqi'yqi'y korenlerge iye yemes, yag'ni'y woni'n' haqi'yqi'y sheshimler ko'pligi Ø boli'p yesaplanadi'.

3. $X = \{x | x \in N,\ x \le 2\}$ *ha'm* $Y = \{x | (x-1)(x-2) = 0\}$ ko'pliginin' ha'r biri tek g'ana 1, 2 sanlari'nan du'zilgen. Soni'n' ushi'n bul ko'plikler ten' boli'p yesaplanadi', yag'ni'y X=Y.

4. A-yeki xanali' sanlar ko'pligi, al B-yeki xanali' jup sanlar ko'pligi bolsi'n. Ha'r bir yeki xanali' jup sanlar A ko'pliginde de bar. Demek, $B \subset A$.

5. $A = \{4, 5, 9\}$, $B = \left\{\frac{16}{4}, \sqrt{25}, 3^2\right\}$ bolsa, wonda

$B = \left\{\frac{16}{4}, \sqrt{25}, 3^2\right\} = \{4,5,9\} = A$. Bunda $A \subset B$, $B \subset A$ boladi'.

6. $A = \{1;2;3\}$, $B = \{7;8\}$ ko'pliklerinin' dekart ko'beymesin tabi'n'

Sheshi'liwi: $A \times B = \{(1;7), (1;8), (2;7), (2;8), (3;7), (3;8)\}$

2. Koordinatalar metodi'.

1. Ko'sherdegi $A(x_1)$ ha'm $B(x_2)$ tochkalar arasi'ndag'i' arali'q:

$d = |x_2 - x_1| = \sqrt{(x_2 - x_1)^2}$ (1)

2. Ko'sherdegi bag'i'tlang'an AB kesindinin' shamasi'

$\vec{AB} = x_2 - x_1$ (2)

3. Tegisliktegi $A(x_1, y_1)$ ha'm $B(x_2, y_2)$ tochkalar arasi'ndag'i' arali'q:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad (3)$$

4. Tegisliktegi bag'i'tlang'an kesindinin', yamasa basi' $A(x_1, y_1)$ ha'm aqi'ri' $B(x_2, y_2)$ bolg'an $\vec{BA}$ vektordi'n' koordinata ko'sherlerindegi proektsiyalari':

$$np_x \vec{AB}=\mathrm{X}=x_2-x_1, \quad np_y \vec{AB}=Y=y_2-y_1 \qquad (4)$$

5. Kesindini berilgen qatnasta bo'li'w. $A(x_1, y_1)$ ha'm $B(x_2, y_2)$ tochkalar berilgen AB kesindini $AN:NB=\lambda$ qatnasi'nda bo'liwshi $N(x, y)$ tochkalardi'n' koordinatalari' to'mendegi:

$$x=\frac{x_1+x_2}{1+\lambda}, \quad y=\frac{y_1+y_2}{1+\lambda} \qquad (5)$$

formulalar menen ani'qlanadi'.

Dara jag'dayda kesindini ten' yekige, yag'ni'y $\lambda=1:1=1$ qatnasi'nda bo'lgende

$$x=\frac{x_1+x_2}{2}, \quad y=\frac{y_1+y_2}{2} \qquad (6)$$

6. To'beleri $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3), ..., F(x_n, y_n)$ tochkalarda bolg'an ko'pmu'yeshliktin' maydani':

$$S=\pm\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}+\begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix}+.....+\begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right] \qquad (7)$$

$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$ ko'rinisindegi an'latpa $x_1y_2-x_2y_1$ ge ten' boli'p, 2-ta'rtipli determinant dep ataladi'.

M i ' s a l l a r

1. $A(1;-4)$, $B(5;-1)$ tochkalar arasi'ndag'i' arali'qti' tabi'n'.

Sheshi'liwi: $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ formulasi' boyi'nsha bul tochkalar arasi'ndag'i' arali'q $AB=\sqrt{(5-1)^2+((-1)-(-4))^2}=\sqrt{16+9}=5$

2. ABC u'shmu'yeshliklerinin' to'belerinin' koordinatalari' berilgen. $A(10;-2)$, $B(-1;1)$, $C(8;4)$. AB ha'm BC ta'replerinin' ten'lemesin du'zin'.

Sheshi'liwi: $\frac{y+2}{3}=\frac{x-10}{-11}\Rightarrow 3(x-10)=-11(y+2)$ $\Rightarrow 3x-30=-11y-22$

$\Rightarrow 3x+11y-8=0$. Bul AB ta'repinin' ten'lemesi.

Yendi BC ta'repinin' ten'lemesin du'zeyik: $\frac{x+1}{8+1}=\frac{y-1}{4-1}\Rightarrow \frac{x+1}{9}=\frac{y-1}{3}$

$\Rightarrow x+1=3y-3 \Rightarrow x-3y+4=0$

3. Funkciya tu'sinigi.

Yeger wo'zgeriwshi x ti'n' ha'r bir ma'nisine bir san sa'ykes keltirilgen bolsa, wonda usi' sanlar ko'pligi menen ani'qlang'an y wo'zgeriwshi x ti'n' bir ma'nisli funkciyasi' dep ataladi'. Bunda wo'zgeriwshi x-argument, ma'nislerinin' berilgen ko'pligi bolsa funkciyani'n' ani'qlani'w oblasti' dep ataladi'. Y-x tin' funkciyasi' ekenligi $y=f(x),\ y=F(x),\ y=\varphi(x)$ ha'm tag'i' basqa ko'riniste jazi'ladi'.

M i ' s a l l a r

1. $f(x)=x^2-x-1$ funkciyasi' berilgen.

1) $f(0)$; 2) $f(-1)$; 3) $f(4)$; 4) $f(10)$; 5) $f\left(\frac{3}{5}\right)$; di yesaplan'.

Sheshi'liwi:

1) $f(0)=-1$

2) $f(-1)=(-1)^2-(-1)-1=1+1-1=1$

3) $f(4)=4^2-4-1=16-4-1=11$

4) $f(10)=10^2-10-1=100-10-1=89$

5) $f\left(\frac{3}{5}\right)=\left(\frac{3}{5}\right)^2-\frac{3}{5}-1=\frac{9}{25}-\frac{3}{5}-1=\frac{9-15-25}{25}=-\frac{31}{25}=-1\frac{6}{25}$

2. $y=\frac{7}{2-x}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'.

Sheshi'liwi: $y=\frac{7}{2-x}$ funkciyasi' $2-x=0$ yamasa $x=2$ den basqa x ti'n' qa'legen ma'nisinde ani'qlang'an. Funkciyani'n' ani'qlani'w oblasti': $(-\infty;2)\cup(2;+\infty)$.

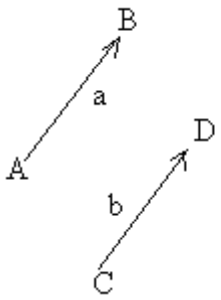
3. $y=\frac{3}{\sqrt{x-5}}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'.

Sheshi'liwi: $\sqrt{x-5}$ an'latpasi' $x-5\geq 0$ yamasa $x\geq 5$ bolg'anda haqi'yqi'y ma'niske iye. Biraq $x=5$ bolg'anda bo'lshektin' bo'limi nolge ten' boli'p, bo'lshek mag'anasi'z boli'p qaladi'. Demek $x=5$ ma'nisi funkciyani'n' ani'qlani'w oblasti'na kirmeydi. Demek funkciyani'n' ani'qlani'w oblasti' $(5;+\infty)$.

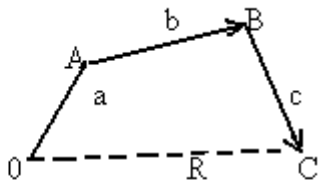
II. VEKTORLAR ALGEBRASI' HA'M SI'ZI'QLI' ALGEBRA ELEMENTLERI

1. Vektorlardi' qosi'w. Vektolardi' skalyarg'a ko'beytiw

Bag'i'tlang'an $\overrightarrow{AB}$ kesindi (1-su'wret) vektor delinedi. Bunda A tochka vektordi'n' basi', B tochka bolsa woni'n' aqi'ri' dep qaraladi'. Vektor basi' ha'm aqi'ri' ko'rsetilip to'besine strelkali' si'zi'qsha qoyi'lg'an $\overrightarrow{AB}$ ko'rinisinde yaki qandayda bir ha'rip, ma'selen **a** (baspada qalin' jazi'lg'an, jazi'wda bolsa to'besine strelkali' si'zi'qsha qoyi'lg'an) menen belgilenedi. Vektordi'n' moduli (uzi'nli'g'i') $|\overrightarrow{AB}|$ $|\mathbf{a}|$, yaki AB yaki **a** menen belgilenedi. Bir tuwri' si'zi'qqa parallel bolg'an vektorlar kollinear vektorlar delinedi. Bir tegislikke parallel bolg'an vektorlar komplanar vektorlar delinedi. Yeger **a** ha'm **b** (1-su'wret) vektorlar: 1) ten' modulge iye, 2) wo'z-ara collinear, 3) bir bag'i'tqa bag'i'tlang'an bolsa, wolar wo'z-ara ten' delinedi.



1-su'wret



2-su'wret

1) Vektorlardi' skalyarg'a ko'beytiw. **a** vektordi'n' qandayda bir m sang'a ko'beymesi dep, uzi'nli'g'i' a|m| g'a ten' bolg'an al bag'i'ti' bolsa berilgen vektor bag'i'ti'nday (m>0 bolg'anda) yaki wog'an qarama-qarsi' (m<0 bolg'anda) bolg'an jan'a vektorg'a ayti'ladi'.

2) Vektorlardi' qosiw. Bir neshshe vektorlardi'n' ji'yi'ndi'si' a+b+c dep sol vektorlardan du'zilgen (2-su'wret) OABC si'ni'q si'zi'qti'n' jabi'wshi'si'nan ibarat $\overrightarrow{OC} = R$ vektorg'a ayti'ladi'. Ma'selen, $\overrightarrow{OA} = a$ ha'm $\overrightarrow{OB} = b$ vektorlarda jasalg'an parallelogrammni'n' bir diagonal vektori' $\overrightarrow{OC}$ berilgen vektorlardi'n' ji'yi'ndi'si' a+b, yekinshi diagonal vektori' $\overrightarrow{BA}$ bolsa wolardi'n' ayi'rmasi' a-b dan ibarat.

3) Vektordi'n' ko'sherdegi proektsiyasi'. **A** vektor ox ko'sher menen φ mu'yesh payda yetsin. Wonda vektordi'n' bul ko'sherdegi proektsiyasi'

$$pr_x a = |a|cos\varphi = \text{acos}\,(\widehat{a,Ox})$$

formula menen ani'qlanadi'.

Bir neshshe vektor qosi'ndi'si'ni'n' ko'sherdegi proektsiyasi' qosi'li'wshi' vektorlar proektsiyalari'ni'n' qosi'ndi'si'na ten':

$$pr_x(a+b) = pr_x a + pr_x b$$

Mi'sallar:

1. $\vec{a} = \{1;-2;3\}; \vec{b} = \{2;-3;0\}$ vektorlari'ni'n' skalyar ko'beymesin tabi'n'

Sheshiliwi: $(\vec{a},\vec{b}) = 1\cdot 2 + (-2)\cdot(-3) + 3\cdot 0 = 8$

2. $\vec{a}$ vektori' wo'zinin' A ha'm B: $A(2,1,-4); B(1,3,2)$ ushl ari' menen berilgen. $\vec{a}$ vektori'i'ni'n' koordinata ko'sherine proektsiyasi'n ha'm woni'n' bag'i'tlawshi' kosinusi'n tabi'n'.

Sheshiliwi: $\vec{a}$ vektori'ni'n' koordinata ko'sherine proektsiyasi'

$a_x = x_2 - x_1; a_y = y_2 - y_1;\ a_z = z_2 - z_1$ formulasi' menen tabi'ladi'.

$a_x = -1; a_y = 2; a_z = 6$ $a = \sqrt{(-1)^2 + 2^2 + 6^2} = \sqrt{41}$ bag'i'tlawshi' kosinusi'

$$cos\alpha = \frac{a_x}{a} = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

$$cos\beta = \frac{a_y}{a} = \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

$$cos\gamma = \frac{a_z}{a} = \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

formulasi' menen tabi'ladi'

$$cos\alpha = -\frac{1}{\sqrt{41}}; cos\beta = \frac{2}{\sqrt{41}}; cos\gamma = \frac{6}{\sqrt{41}}$$

2. Determinantlar.

2-ta'rtipli determinant dep, $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ simvol menen belgileniwshi ha'm

$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$ (8) ten'lik penen ani'qlani'wshi' sang'a ayti'ladi'.

3-ta'rtipli determinant dep, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ simvol menen belgileniwshi ha'm

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} \qquad (9)$$

ten'lik penen ani'qlani'wshi' sang'a ayti'ladi'.

(9) ten'liktin' won' ta'repindegi 2-ta'rtipli determinanlardi'n' ha'r biri berilgen u'shinshi ta'rtipli determinantti'n' bir qatari'n ha'm bir bag'anasi'n wo'shiriwden payda boladi' ha'm wolar usi' determinantti'n' minorlari' delinedi. (9) formula bolsa 3-ta'rtipli determinantti' birinshi bag'anasi' elementleri boyinsha jayi'w formulasi' delinedi.

Determinantlardi'n' qa'siyetleri:

1. Determinantti'n' qatarlari' menen woni'n' bag'analari'n almasti'ri'wdan woni'n' ma'nisi wo'zgermeydi.

2. Determinantti'n' yeki parallel qatarlari'n wo'z-ara almasti'rg'anda determinant ma'nisinin' belgisi wo'zgeredi.

1 ha'm 2 qa'siyetlerinen, determinantti'n' qa'legen qatari'n birinshi qatar worni'na keltiriw mu'mkin, soni'n' ushi'n woni' qa'legen qatar elementleri boyinsha jayi'w mu'mkin.

3. Yeki parallel qatari' birdey bolg'an determinant nolge ten'

4. Bir qatar elementlerinin' uli'wma ko'beytiwshisin determinant belgisinen si'rtqa shi'g'ari'w mu'mkin.

5. Determinantti'n' qandayda bir qatari'ni'n' elementlerine wog'an parallel qatar elementlerin qa'legen birdey sang'a ko'beytip qosi'wdan determinant ma'nisi wozgermeydi. Ma'selen: $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + mc_1 & b_1 + nc_1 & c_1 \\ a_2 + mc_2 & b_2 + nc_2 & c_2 \\ a_3 + mc_3 & b_3 + nc_3 & c_3 \end{vmatrix}$

Bul qa'siyetke tiykarlani'p, 3-ta'rtipli determinantti'n' qa'legen qatari'nda yeki nol payda yetiw mu'mkin, buni'n' na'tiyjesinde determinantti'n' usi' qatar elementleri boyi'nsha jayi'lmasi' a'piwayi'lasadi'.

Ushlari' $A(x_1;y_1), B(x_2;y_2), C(x_3;y_3)$ tochkalarda bolg'an u'shmu'yeshlik maydani'

$$S = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad (10)$$

Mi'sallar

1. $\begin{vmatrix} 2 & 7 \\ 4 & 1 \end{vmatrix}$ yekinshi ta'rtipli determinantti' yesaplan'

Sheshiliwi: $\begin{vmatrix} 2 & 7 \\ 4 & 1 \end{vmatrix} = 2 \cdot 1 - 4 \cdot 7 = -26$

2. $\begin{vmatrix} 2 & 4 & -1 \\ 7 & 3 & 2 \\ 3 & 1 & -2 \end{vmatrix}$ u'shinshi ta'rtipli determinantti' yesaplan'

Sheshiliwi:

$$\begin{vmatrix} 2 & 4 & -1 \\ 7 & 3 & 2 \\ 3 & 1 & -2 \end{vmatrix} = 2 \cdot 3 \cdot (-2) + 7 \cdot 1 \cdot (-1) + 3 \cdot 4 \cdot 2 - 3 \cdot 3 \cdot (-1) - 7 \cdot 4 \cdot (-2) - 2 \cdot 1 \cdot 2 = -12 - 7 + 24 + 9 + 56 - 4 = 66$$

III. TEGISLIKTEGI ANALITIKALI'Q GEOMETRIYA.

1. Tuwri' si'zi'qti'n' mu'yesh koeffitsientli ten'lemesi

$$y = kx + b \qquad (11)$$

k parametr tuwri' si'zi'qti'n' o*x* ko'sherinin' won' bag'i'ti' menen jaylasqan mu'yeshi' ni'n' tangensine ten' boli'p $(k = tg\alpha)$, tuwri' si'zi'qti'n' mu'yesh koeffitsienti dep ataladi'. Parametr v-tuwri' si'zi'qti'n' o*y* ko'sheri menen kesilisiw noqati'ni'n' ordinatasi'.

2. Tuwri' si'zi'qti'n' uli'wma ten'lemesi: $Ax + By + C = 0$

Dara jag'daylari':

a) $C = 0$ bolsa, $y = -\frac{A}{B}x$ - tuwri' si'zi'q koordinatalar basi'nan wo'tedi;

b) $B = 0$ bolsa, $x = -\frac{C}{A} = a$ - tuwri' si'zi'q Oy ko'sherge parallel boladi';

c) $A = 0$ bolsa, $y = -\frac{C}{B} = b$ - tuwri' si'zi'q Ox ko'sherge parallel boladi';

d) $B = C = 0$ bolsa, $Ax = 0$ yamasa $x = 0$, tuwri' si'zi'q Oy ko'sherinen ibarat;

e) $A = C = 0$ bolsa, $By = 0$ yamasa $y = 0$, tuwri' si'zi'q Ox ko'sherinen ibarat;

3. Tuwri' si'zi'qti'n' ko'sherlerden aji'ratqan kesindiler boyi'nsha ten'lemesi

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad (12)$$

Bunda a ha'm в - sanlar tuwri' si'zi'qti'n' ko'sherlerden kesken noqatlari'ni'n' abstsissa ha'm wordinatalari'.

4. $y = k_1 x + b_1$ tuwri' si'zi'qtan $y = k_2 x + b_2$ tuwri' si'zi'qqa shekemgi, saat strelkasi'na qarsi' bag'i'tta yesaplani'wshi' φ mu'yesh

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 k_2} \qquad (13)$$

formulasi' menen ani'qlanadi'.

$A_1 x + B_1 y + C_1 = 0$ ha'm $A_2 x + B_2 y + C_2 = 0$ ten'lemeler menen berilgen tuwri' si'zi'qlar ushi'n (13) formula to'mendegi ko'riniske iye boladi':

$$tg\varphi = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}$$

yeki tuwri'ni'n' parallellik sha'rti: $k_1 = k_2$ yamasa $\frac{A_1}{A_2} = \frac{B_1}{B_2}$, al perpendikulyarli'q sha'rti: $k_2 = -\frac{1}{k_1}$ yamasa $A_1A_2 + B_1B_2 = 0$

5. Berilgen $A(x_1, y_1)$ tochkadan wo'tiwshi tuwri' si'zi'qlar da'stesinin' ten'lemesi to'mendegishe jazi'ladi':

$$y - y_1 = k(x - x_1) \qquad (14)$$

6. Beri'lgen yeki $A(x_1, y_1)$ ha'm $B(x_2, y_2)$ tochkalardan wo'tiwshi tuwri' si'zi'qti'n' ten'lemesi to'mendegishe jaziladi':

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \qquad (15)$$

7. Parallel bolmag'an yeki $A_1x + B_1y + C_1 = 0$ ha'm $A_2x + B_2y + C_2 = 0$ tuwri' si'zi'qlardi'n' kesilisiw tochkasi'n tabi'w ushi'n wolardi'n' ten'lemelerin birgelikte sheshi'w kerek. Bunday jag'dayda

$$x = \frac{\begin{vmatrix} -C_1 & B_1 \\ -C_2 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} A_1 & -C_1 \\ A_2 & -C_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}$$

kesilisiw tochkasi'ni'n' koordinatalari' boladi'.

8. Tuwri' si'zi'qti'n' normal ten'lemesi to'mendegishe jazi'ladi'.

$$x\cos\beta + y\sin\beta - \rho = 0 \qquad (16)$$

Bunda ρ - koordinatalar basi'nan tuwri' si'zi'qqa tu'sirilgen perpendikulyar (normal) uzi'nli'g'i', β bolsa usi' perpendikulyardi'n' Ox ko'sheri menen jasag'an mu'yeshi'. Tuwri' si'zi'qti'n' $Ax + By + C = 0$ uli'wma ten'lemesin normal ko'riniske keltiriw ushi'n woni'n' barli'q ag'zalari'n

$$M = \pm\frac{1}{\sqrt{A^2 + B^2}}$$

normallasti'ri'wshi' ko'beytiwshi'ge ko'beytiw kerek. *M* nin' belgisi ten'lemedegi saltan' ag'za *C* ni'n' belgisine keri yetip ali'nadi'.

9. $(x_0;\ y_0)$ tochkadan tuwri' si'zi'qqa shekemgi' bolg'an d arali'qti' tabi'w ushi'n tuwri' si'zi'qti'n' normal ten'lemesinin' shep ta'repindegi wo'zgeriwshi koordinatalardi'n' worni'na $(x_0;\ y_0)$ koordinatalari'n qoyi'p, payda bolg'an sanni'n' absolyut shamasi'n alami'z, yag'ni'y

$$d = |x_0 cos\beta + y_0 \sin\beta - \rho| \qquad (17)$$

yamasa

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \qquad (17')$$

10. $Ax + By + C = 0$ ha'm $A_1x + B_1y + C_1 = 0$ tuwri' si'zi'qlar arasi'ndag'i' mu'yeshler bissektrisalari'ni'n' ten'lemeleri:

$$\frac{Ax + By + C}{\sqrt{A^2 + B^2}} = \pm\frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}} \qquad (18)$$

11. Berilgen yeki tuwri' si'zi'qti'n' kesilisiw tochkasi'nan wo'tiwshi tuwri' si'zi'qlar da'stesinin' ten'lemesi:

$$\alpha(Ax + By + C) + \beta(A_1x + B_1y + C_1) = 0 \qquad (19)$$

$\alpha = 1$ dep ali'w mu'mkin.

Yekinshi ta'rtipli iymek si'zi'qlar.

1. Worayi' $C(a;b)$ tochkada ha'm radiusi' R bolg'an shen'berdin' ten'lemesi:

$$(x-a)^2 + (y-b)^2 = R^2 \quad (20)$$

2. Ellips dep ha'r bir tochkadan berilgen yeki F ha'm F_1 tochkag'a (fokuslarg'a) shekemgi arali'qlardi'n' qosi'ndi'si' FF_1 dep u'lken turaqli' $2a$ sha'mag'a ten' tochkalardi'n' geometriyali'q worni'na ayti'ladi'.

Ellipstin' kanonikali'q ten'lemesi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (21)$$

(21) ten'leme menen berilgen ellips koordinata ko'sherine sali'sti'rg'anda simmetriyali'.

a ha'm **b** parametri ellipstin' yari'm ko'sherleri dep ataladi'. a > b bolsi'n. Wonda F ha'm F_1 fokuslar Ox ko'sherde boli'p, woraydan $c=\sqrt{a^2-b^2}$ arali'qta boladi'. $\frac{c}{a}=e$ qatnasi' ellipstin' ekssentrisiteti dep ataladi'.

3. Giperbola dep, sonday tochkalardi'n' geometriyali'q worni'na ayti'ladi'. Wolardi'n' ha'r birinen berilgen yeki F ha'm F_1 tochkag'a (fokuslarg'a) shekemgi bolg'an arali'qlardi'n' ayi'rmasi'ni'n' absolyut shamasi' turaqli' $2a$ $0<2a<FF_1$ shamasi'nan ibarat.

Giperbolani'n' kanonikali'q ten'lemesi

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad (22)$$

(22) ten'leme menen berilgen giperbola koordinata ko'sherine sali'sti'rg'anda simmetriyali'.

4. Berilgen tochkadan (fokustan) ha'm berilgen tuwri' si'zi'qtan (direktrisadan) birdey qashi'qli'qta bolg'an tochkalardi'n' geometriyali'q worni' parabola dep ataladi'. Parabolani'n' kanonikali'q ten'lemesi to'mendegi yeki ko'riniske iye:

1) $y^2=px$ (21) Ox ko'sherge sali'sti'rg'anda simmetriyali' parabola.

2) $x^2=py$ (22) Oy ko'sherge sali'sti'rg'anda simmetriyali' parabola.

Ha'r yeki jag'dayda da parabolani'n' ushi' yag'ni'y simmetriya ko'sherinde jati'wshi' tochkasi', koordinatalar basi'nda boladi'.

Mi'sallar.

1. Tuwri' si'zi'qti'n' ten'lemesi $4x-4y-8=0$, woni'n' koordinata ko'sherleri menen kesilisiw tochkasi'n tabi'n'.

Sheshi'liwi: Kesilisken tochkalardi'n' koordinatalari'n tabi'w ushi'n, berilgen tuwri' si'zi'q ten'lemesin tuwri' si'zi'qti'n' koordinata ko'sherlerinen aji'ratqan kesindiler boyi'nsha ten'lemesin (12) formulag'a keltiremiz

$$\frac{x}{2}-\frac{y}{2}=1$$

Demek koordinata ko'sherleri menen kesilisiw tochkalari': $A(2;0)$ ha'm $A(0;-2)$.

2. Ox ko'sheri menen 45^0 mu'yesh jasap $A(-4;7)$ tochka arqali' wo'tiwshi tuwri' si'zi'q ten'lemesin du'zin'.

Sheshiliwi: İzlenip ati'rg'an tuwri' si'zi'qti'n' mu'yeshlik koeffitsienti $k = tg\alpha = tg45^0 = 1$ge ten'. (14) ten'lemege $x_1 = -4$; $y_1 = 7$ ma'nislerin qoyi'p to'mendegi ten'lemege iye bolami'z: $y - 7 = x + 4$ yamasa $x - y + 11 = 0$.

3. $3x - 5y + 8 = 0$ tuwri' si'zi'qti'n' normal vektori'n ko'rsetin'.

Sheshiliwi: Normal vektor $\vec{N} = \{A;B\}$ ko'riniste berilgen tuwri' si'zi'q ten'lemesinde $A = 3$; $B = -5$. Soni'n' ushi'n $\vec{N}(3;-5)$.

4. Worayi' $C(-4;5)$ tochkada ha'm radiusi' 5 g'a ten' bolg'an shen'berdin' ten'lemesin jazi'n'.

Sheshiliwi: (20) ten'leme boyi'nsha $a = -4, b = 5, R = 5$ bolg'ani' ushi'n $(x+4)^2 + (y-5)^2 = 25$ yamasa $x^2 + y^2 + 8x - 10y + 16 = 0$.

5. Giperbolani'n' $F_1(20;0), F_2(-20;0)$ fokuslari'n ha'm wog'an tiyisli $A(24;6\sqrt{5})$ noqati'n bilgen halda woni'n' ten'lemesin du'zin'.

Sheshiliwi: Giperbolani'n' fokal radiuslari' formulasi'nan paydalanami'z, yag'ni'y

$$r_1 = \sqrt{(x-c)^2 + y^2}; r_2 = \sqrt{(x+c)^2 + y^2}$$
$$r_1 = \sqrt{4^2 + 36 \cdot 5} = \sqrt{196} = 14$$
$$r_2 = \sqrt{44^2 + 36 \cdot 5} = \sqrt{2116} = 46$$

Giperbolani'n' ani'qlamasi'na go're:

$$|r_1 - r_2| = 2a \text{ yaki } |46 - 14| = 2a$$
$$2a = 32 \Rightarrow a = 16$$

Giperbola ushi'n:

$$b^2 = c^2 - a^2 = 20^2 - 16^2 = 144$$

Demek,

$$\frac{x^2}{256}-\frac{y^2}{144}=1$$

giperbola ten'lemesine iye bolami'z.

6. $xy=4$ giperbola ten'lemesin kanonik ko'riniske keltirin'.

Sheshiliwi: Koordinatalar basi'n qozg'altpag'an halda koordinata ko'sherlerin $\alpha=+45^0$ mu'yeshke burami'z, yag'ni'y usi' formuladan paydalanami'z:

$$\begin{cases} x=x'\cos 45^0-y'\sin 45^0=\dfrac{\sqrt{2}}{2}(x'-y') \\ y=x'\sin 45^0+y'\cos 45^0=\dfrac{\sqrt{2}}{2}(x'+y') \end{cases}$$

x,y tin' bul ma'nislerin berilgen ten'lemege qoyami'z:

$$\frac{\sqrt{2}}{2}(x'-y')\cdot\frac{\sqrt{2}}{2}(x'+y')=4$$

Bul ten'lemeni a'piwayi'lasti'rsaq,

$$x'^2-y'^2=8$$

ko'rinistegi ten' ta'repli giperbolani'n' kanonik ten'lemesine iye bolami'z.

7. Direktrisalari' $x=\pm 4\sqrt{2}$ ten'lemeler menen berilgen ha'm asimptotalari' arasi'ndag'i' mu'yesh 90^0 bolg'an giperbolani'n' ten'lemesin du'zin'.

Sheshiliwi: Ma'sele sha'rtinde, yag'ni'y asimptotalardi'n' wo'z-ara perpendikulyarli'g'i'nan giperbola ten' ta'repli ekenligi kelip shi'g'adi', wol $x^2-y^2=a^2$ ten'leme menen an'lati'ladi'. Bunnan $a=b$. Giperbolani'n' direktrisalari' $x=\pm\dfrac{a}{\varepsilon}$ ten'lemeler menen an'lati'ladi'. Ma'selenin' sha'rtine go're $\dfrac{a}{\varepsilon}=4\sqrt{2},\varepsilon=\dfrac{c}{a}$ ni' esapqa alsaq, $\dfrac{a^2}{c^2}=4\sqrt{2}\Rightarrow a^2=4\sqrt{2}c^2, b^2=c^2-a^2$ ten'likten $a^2=c^2-a^2\Rightarrow 2a^2=c^2\Rightarrow a^2=\dfrac{1}{2}c^2$ bolg'ani' ushi'n $a^2=4\sqrt{2}c\Rightarrow\dfrac{1}{2}c^2=4\sqrt{2}c\Rightarrow c=8\sqrt{2}$ ge iye bolami'z. Wonda $a^2=4\sqrt{2}c=4\sqrt{2}\cdot 8\sqrt{2}=64$. Demek geperbolani'n' ten'lemesi:

$$x^2-y^2=64$$

8. Asimptotalari' $2x-y=0;\ 2x+y=0$ ten'lemeler menen berilgen ha'm fokuslari' woraydan 5 birlik qashi'qli'qta bolg'an giperbolani'n' kanonik ten'lemesin du'zin'.

Sheshiliwi: Berilgen ten'lemelerdi $y=2x, y=-2x$ ko'riniste jazi'p alsaq ha'mde $y=\frac{b}{a}x;\ \ y=-\frac{b}{a}x$ ten'lemeler menen sali'sti'rsaq, $\frac{b}{a}=2;$ yaki $b=2a$ boladi'. Fokuslar woraydan 5 birlik qashi'qli'qta bolg'ani' ushi'n $c=5$ boli'p, $b^2=c^2-a^2$ ten'likten paydalansaq, $4a^2=25-a^2$, bunnan $a^2=5; a=\sqrt{5}$, wonda $b=2\sqrt{5}$. Usi'larg'a tiykarlani'p giperbolani'n' izlep ati'rg'an ten'lemesi:

$$\frac{x^2}{5}-\frac{y^2}{20}=1$$

9. Parabola ox ko'sherine sali'sti'rg'anda simmetriyali' ha'm A(5;-2) tochkasi' arqali' wo'tedi, al to'besi koordinata basi'nda jatadi'. Parabolani'n' ten'lemesin du'zin'.

Sheshi'liwi. Parabola A(5;-2) tochkasi'nan wo'tedi, won' abtsissa ko'sherine simmetriyali' bolsa, wonda parabolani'n' ten'lemesin $y^2=2px$ tu'rinde izlewimiz kerek. Bul ten'lemege A tochkani'n' koordinatalari'n qoysaq: $4=10p,\ \ p=\frac{2}{5},\ 2p=\frac{4}{5}$.

Demek, izlenip ati'rg'an ten'lememiz: $y^2=\frac{4}{5}x$.

IV. KEN'ISLIKTEGI ANALITIKALI'Q GEOMETRIYA.

1. Tegisliktin' uli'wma ten'lemesi to'mendegishe:

$$Ax + By + Cz + D = 0 \quad (24)$$

$N(A,B,C)$ vektor (23) ha'm (24) tegislikke normal vektor dep ataladi'.

2. $Ax + By + Cz + D = 0$ ten'lemesinin' dara jag'daylari':

1) $D = 0$ bolg'anda $Ax + By + Cz = 0$ - tegislik koordinatalar basi'nan wo'tedi.

2) $C = 0$ bolg'anda $Ax + By + D = 0$ - tegislik Oz ko'sherine parallel.

3) $C = D = 0$ bolg'anda $Ax + By = 0$ - tegislik Oz ko'sherden wo'tedi.

4) $B = C = 0$ bolg'anda $Ax + D = 0$ - tegislislik yOC tegislikke parallel.

5) Koordinata tegisliginin' ten'lemeleri: $x = 0$, $y = 0$ ha'm $z = 0$

3. Tegisliktin' koordinata ko'sherinen aji'ratqan kesindiler boyi'nsha ten'lemesi:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad (25)$$

4. Yeki tegislik arasi'ndag'i' mu'yesh:

$$\cos\varphi = \pm\frac{N \cdot N_1}{NN_1} = \pm\frac{AA_1 + BB_1 + CC_1}{NN_1}$$

formulasi' menen tabi'ladi'. Bunda N ha'm N_1 sa'ykes tu'rde $Ax + By + Cz + D = 0$ ha'm $A_1x + B_1y + C_1z + D_1 = 0$ tegisliklerge normal vektorlar.

Paralellik sha'rti

$$\frac{A}{A_1} = \frac{B}{B_1} = \frac{C}{C_1} \neq \frac{D}{D_1} \quad (26)$$

Perpendikulyarli'q sha'rti

$$AA_1 + BB_1 + CC_1 = 0 \quad (27)$$

5. $M(x_0, y_0, z_0)$ tochkadan $Ax + By + Cz + D = 0$ tegisligine shekemgi arali'q:

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{N} \quad (28)$$

6. Berilgen yeki tegisliktin' kesilisken si'zi'g'i'nan wo'tiwshi tegislikler da'stesinin' ten'lemesi:

$$\alpha(Ax + By + Cz + D) + \beta(A_1x + B_1y + C_1z + D_1) = 0 \quad (29)$$

$\alpha = 1$ dep ali'w mu'mkin, wonda (12) da'steden berilgen tegisliklerden yekinshi'sin shi'g'ari'p taslag'an bolami'z.

7. $A(a;b;c)$ tochkadan wo'tiwshi ha'm $P(m;n;p)$ vektorg'a parallel bolg'an tuwri' si'zi'q ten'lemeleri $N(x,y,z)$- tuwri' si'zi'qti'n' qa'legen tochkasi' bolsi'n. Wonda $\overline{AN} \parallel P$ ha'm yeki vektordi'n' parallellik sha'rtinen:

$$\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p} \qquad (30)$$

(30) ten'leme tuwri' si'zi'qti'n' kanonikalq ten'lemeleri dep ataladi'. $P(m;n;p)$ vektor tuwri' si'zi'qti'n' bag'i'tlang'an vektori' dep ataladi'.

8. (30) ten'lemedegi ha'r bir qatnasti' t dep belgilep, tuwri' si'zi'qti'n'

$$\begin{cases} x = mt + a \\ y = nt + b \\ z = pt + c \end{cases} \quad (31)$$

ko'rinistegi parametrlik ten'lemesine iye bolami'z.

9. Yeki tochkadan wo'tiwshi tuwri' si'zi'qti'n' ten'lemesi

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \qquad (32)$$

10. Tuwri' si'zi'qti'n' uli'wma ten'lemesi:

$$\begin{cases} Ax + By + Cz + D = 0 \\ A_1x + B_1y + C_1z + D_1 = 0 \end{cases} \quad (33)$$

11. (33) ten'likten bir ma'rte y ti, 2 ma'rte x ti jog'alti'p, tuwri' si'zi'qti'n' proektsiyalari' boyi'nsha jazi'lg'an ten'lemelerge iye bolami'z:

$$\begin{cases} x = mz + a \\ y = nz + b \end{cases} \qquad (34)$$

12. $\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p}$ tuwri' si'zi'q penen $Ax + By + Cz + D = 0$ tegislik arasi'ndag'i' mu'yesh:

$$\sin\alpha = \frac{|N \cdot P|}{NP} = \frac{|Am + Bn + Cp|}{NP} \quad (35)$$

Olardi'n' paralellik sha'rti: $Am+Bn+Cp=0$ (36).

Olardi'n' perpendikulyarli'q sha'rti: $\frac{A}{m}+\frac{B}{n}=\frac{C}{p}$ (36)

Mi'sallar

1. Ox ko'sherine perpendikulyar ha'm $A(2;-4;-2)$ tochkasi' arqali' wo'tiwshi tegisliktin' ten'lemesin du'zin'.

Sheshi'liwi: Tegislik ox ko'sherine perpendikulyar bolsa, wonda wol yOz tegisligine parallel'. Demek, woni'n' ten'lemesi $Ax+D=0$ tu'rine iye. Bul ten'lemege A tochkasi'ni'n' koordinatalari'n qoysaq, $D=-2A$ g'a iye bolami'z. D ni'n' bul ma'nisin $Ax+D=0$ ten'lemesine qoyi'p, A g'a qi'sqartsaq $x-2=0$ ge iye bolami'z.

2. $3x+4y-12z+48=0$ tegisliginin' ten'lemesin (25) tu'rindegi tegisliktin' koordinata ko'sherinen aji'ratqan kesindiler boyi'nsha ten'lemesi tu'rine keltirin':

Sheshi'liwi: Saltan' ag'za 48 di ten'lemenin' won' jag'i'na shi'g'arami'z:

$3x+4y-12z=-48$. Ten'lemenin' yeki jag'i'n da –48 ke bo'lsek: $\frac{x}{-16}+\frac{y}{-12}+\frac{z}{4}=1$. '

V. Jasawg'a baylani'sli' ma'seleler

Geometriyada ha'r dayi'm si'zba instrumentlerinin' ja'rdemi menen geometriyali'q figuralar jasawg'a mi'sallar sheshiwge tuwra keledi. Geybir jag'daylarda mi'sallar jasaw waqti'nda qanday si'zba instrumentlerin qollani'wg'a bolatug'i'nli'g'i' aytilg'an. Al bul ayti'lmag'an jag'daylarda jasaw ushi'n kerekli bolg'an instrumentlerdi wo'zimiz tan'lap aliwg'a da boladi'. Mektep geometriya kursi'nda biz si'zg'i'sh (bir ta'repli) ha'm tsirkul' instrumentlerinen paydalanami'z. I''nstrumenttin' tan'lap ali'wi'na baylani'sli' jasawg'a mi'sallardi'n' ha'r tu'rli tipleri ushi'rasadi'. Tsirkul' ha'm si'zg'i'sh penen jasawlar tek bir tsirkuldin' ja'rdeminde jasawlar (Mor-Markeroni jasawlari') yeger tegislikte shen'ber ha'm woni'n' worayi' si'zi'lg'an bolsa, tek bir si'zg'i'shti'n' ja'rdeminde jasawlar (Shteyner jasawlari') boladi'. Geometriyada ha'r dayi'm si'zba instrumentlerinin' ja'rdemi menen geometriyali'q figuralar jasawg'a mi'sallar sheshiwge tuwra keledi. Si'zg'i'sh ja'rdeminde (kesindi tu'rinde) : a) erikli tuwri'ni'n', b) berilgen noqat arqali' wo'tetug'i'n tuwri'ni'n', v) berilgen yeki noqat arqali' wo'tetug'i'n tuwri'ni'n' su'wretleniwin si'zi'wg'a boladi'. Si'zg'i'shti'n' ja'rdemi

menen, ja'ne de yeger wonda bo'linbeler bar bolsa da kesindilerdi wo'lshep tu'siriwge bolmaydi', si'zg'i'shti'n' yeki jiyeginen paydalani'wg'a bolmaydi. Tsirkuldin' ja'rdeminde: a) worayi' berilgen noqatta bolg'an berilgen radiustag'i' shen'ber jasawg'a boladi', b) berilgen kesindini berilgen tuwri'ni'n' u'stine berilgen noqattan baslap wo'lshep tu'siriwge boladi'.

Jasawg'a arnalg'an mi'sallardi' sheshiwdegi basqi'shlar.

Geometriyada geometriyali'q denelerdi "jasaw" degende woni'n' barli'q elementlerin tabi'wdi' tu'sinemiz. Geometriyani'n' jasawg'a arnalg'an tiykarg'i' talaplari tiyisli aksiomalar arqali' aniqlanadi'. Geometriya mi'sallari'n qa'legen qurallar ja'rdeminde sheshiwde to'mendegi aksiomalar wori'nli' dep qabi'l yetiledi.

1. Berilgen $F_1, F_2, \ldots, F_k$ denelerdin' ha'r biri jasalg'an. Bul jerde "berilgen dene" ha'm "ani'qlang'an dene" tu'siniklerin shatasti'ri'p almaw kerek. Yeger geybir "dene berildi" dep ayti'lsa, bul dene su'wretlengen, si'zi'lg'an, yamasa jasalg'an dep tu'siniw kerek. Yeger geybir "dene ani'qlang'an" dep ayti'lsa, bul an'latpa arqali' denenin' wo'zi berilmegen boli'p, tek g'ana denenin' jag'dayin ani'qlaytug'i'n elementler berilgen degen ma'nisti tu'siniw kerek. Mi'sali', tuwri' si'zi'qti'n' yeki noqati' berilgen bolsa, bul noqatlardi' tutasti'ratug'i'n jalg'i'z tuwri' si'zi'q bar, yamasa bul tuwri' si'zi'q wo'zinin' yeki noqati' menen ani'qlang'an, biraq bul tuwri' si'zi'q jasalmag'an (si'zi'lmag'an) , woni' jasaw kerek.

2. Yeki dene jasalg'an bolsa, wol jag'dayda bul denelerdin' birikpesi de jasalg'an boladi.

3. Yeki F_1 *ha'm* F_2 dene jasalg'an boli'p, wolardi'n' kesispesi bos bolmasa, wolardi'n' $F_1 \cap F_2$ kesispesi jasalg'an boladi'.

4. Yeger F_1, F_2 deneler jasalg'an ha'm $F_2 \subset F_1, F_1 \neq F_2$ bolsa, $F_1 \setminus F_2$ dene jasalg'an yesaplanadi'.

5. Yeger F dene jasalg'an bolsa, bul denege tiyisli noqatti' jasaw mu'mkin.

Biz Evklid tegisligine tiyisli jasawg'a arnalg'an mi'sallar menen g'ana shug'i'llanami'z. Tegislikte jasawg'a arnalg'an mi'sallardi' sheshiwde jasaw qurallari'nan negizinde si'zg'i'sh ha'm tsirkul' jumsaladi'. Jasawg'a arnalg'an mi'sallardi' si'zg'i'sh ha'm tsirkul' ja'rdeminde sheshiwde si'zba praktikasi'nda

qollanatug'i'n si'zg'i'sh ha'm tsirkullar yemes, abstrakt si'zg'i'sh ha'm tsirkul' na'zerge ali'nadi'.

Bu'l qurallardin' jag'daylari to'mendegi yeki aksioma arqali an'lati'ladi'.

6. Yeger *A* ha'm B noqatlar *(A ≠B)* belgilengen bolsa, *AB* sa'wlesin jasaw mu'mkin.

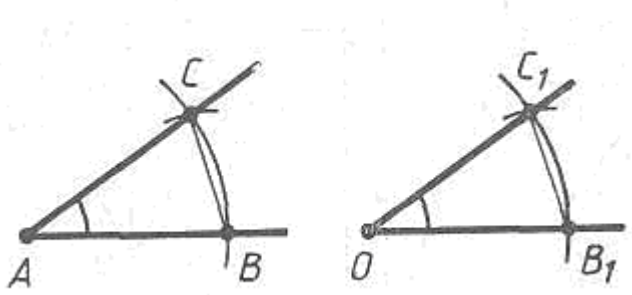
7. Yeger wo noqat ha'm *AB* kesindi jasalg'an bolsa, worayi' wo noqatta ha'm *R=AB* bolg'an shen'berdi si'zi'w mu'mkin.

Jasawg'a arnalg'an misallardi' sheshiw degende wolardi' shekli ret jasaw joli' menen wori'nlang'an en' a'piwayi' ma'selelerge keltiriwdi tu'sinemiz.

Mi'sallar

l. Berilgen mu'yeshke ten' mu'yesh jasan'.

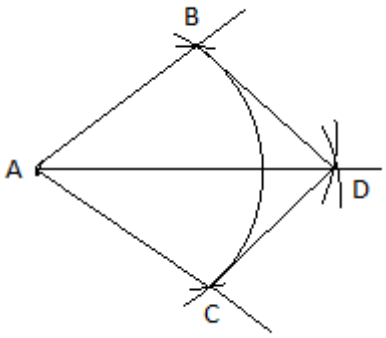
Qa'legen yari'm tuwri'dan berilgen yari'm tegislikke, berilgen mu'yesh wo'lshemi 180^0 tan kishi bolg'an bir ha'm tek bir g'ana mu'yesh wo'lshep tu'siriwge boladi'. Buni' si'zg'i'shti'n' ha'm tsirkuldin' ja'rdeminde qalay islewge boladi.



3-su'wret

3-su'wrette jasaw boyi'nsha wori'nlang'an ∠A-berilgen mu'yesh, woB_1-berilgen yari'm tuwri', birdey erikli radiusta woraylari' A ha'm wo noqatlari'nda bolg'an yeki shen'ber ha'm BC radiusi'nda worayi' B_1 noqatta bolg'an bir shen'ber ju'rgizilgen. U'shmu'yeshliktin' ten'liginin' u'shinshi belgisi boyinsha yag'niy yeger bir u'shmu'yeshliktin' u'sh ta'repi yekinshi u'shmu'yeshliktin' sa'ykes u'sh ta'repine ten' bolsa, wonda bunday u'shmu'yeshlikler ten' boladi'. $\Delta BAC=\Delta B_1OC_1$ ekenligi ko'rinip tur, bunnan $\angle A=\angle O$.

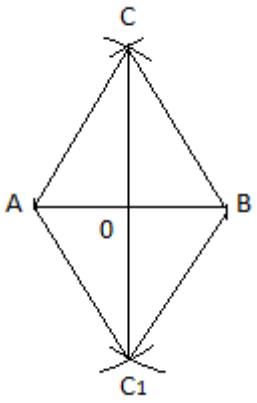
2. Berilgen mu'yeshtin' bissektrisasi'n jasan'.



4-su’wret

4-su’wrette berilgen BAC mu’eyshtin’ AD bissektrisasi’n jasaw bi’layi’nsha worinlang’an. Bir erikli radius penen woraylari’ A, B ha’m C noqatlari’nda bolg’an u’sh shen’ber ju’rgizilgen. Woraylari’ B ha’m C noqatlari’nda bolg’an shen’berlerdin’ kesilisiw noqati’ D noqati’n A noqati’ menen tutasti’rami’z. Bul fakttin’ da’liylleniwi u’shmu’yeshliktin’ ten’liginin’ u’shinshi belgisi boyi’nsha yag’ni’y yeger bir u’shmu’yeshliktin’ u’sh ta’repi yekinshi u’shmu’yeshliktin’ sa’ykes u’sh ta’repine ten’ bolsa, wonda bunday u’shmu’yeshlikler ten’ boladi’. Usi boyi’nsha ABD ha’m ACD u’shmu’yeshliktin’ ten’ligine tiykarlang’an.

3. Kesindini ten’ wortadan bo’lin’.



5-su’wret

5-su’wrette AB kesindisinin’ wortasi’n jasaw to’mendegishe wori’nlang’an. AB radiusi menen woraylari’ A ha’m B noqatlari’nda bolg’an yeki shen’ber jasalg’an. C ha’m C_1 noqatlari’ ha’r tu’rli yari’m tegislikte jati’r, sonli’qtan CC_1 kesindisi AB ni wo noqatta AB kesindisinin’ wortasi’nda kesip wo’tedi.

Da’liyllew: ten’ u’shmu’yeshliklerdi qarasti’ri’wg’a tiykarlang’an yag’niy $\Delta CAC_1=\Delta CBC_1$ u’shmu’yeshliktin’ ten’liginin’ u’shinshi belgisi boyi’nsha yag’niy yeger bir u’shmu’yeshliktin’ u’sh ta’repi yekinshi u’shmu’yeshliktin’ sa’ykes u’sh ta’repine ten’ bolsa, wonda bunday u’shmu’yeshlikler ten’ boladi’.

Ja’ne $\Delta ACO=\Delta BCO$ yeger bir u’shmu’yeshliktin’ yeki ta’repine wolardi’n’ arasi’ndag’i’ mu’yesh yekinshi u’shmu’yeshliktin’ sa’ykes yeki ta’repine ha’m wolardi’n’ arasi’ndag’i’ mu’yeshke ten’ bolsa, wonda bunday u’shmu’yeshlikler ten’ boladi’.

VI. LIMITLER TEORIYASI'.

1. Sanli' izbe-izlikler.

O'zgeriwshi

$$x, x_1, x_2, \ldots x_n, \ldots \quad (37)$$

ma'nislerin izbe-iz qabi'l yetsin. Bunday nomerlengen sanlar ko'pligi izbe-izlik dep ataladi'. (37) izbe-izliktin' du'ziliwi n-ag'za formulasi' menen beriledi.

Ma'selen: $x_n = n + (-1)^n$ bolsi'n; $n = 1,2,3,\ldots$ dep alsaq,

$$0,3,2,5,4,7,\ldots \quad (38)$$

izbe-izlik payda boladi'.

2. Sheksiz kishi wo'zgeriwshi.

Yeger ha'r qanday won' *e* san wo'zegriwshinin' sonday α_0 ma'nisi bar bolsa, α ni'n' wonnan son'gi' ha'r bir ma'nisinin' absolyut shamasi' *e* den kishi bolsa, α wo'zgeriwshi sheksiz kishi dep ataladi'.

yeger α sheksiz kishi bolsa, wol nolge umti'ladi' dep ataladi' ha'm $\alpha \to 0$ ko'rinisinde jazi'ladi'.

3. Sheksiz u'lken wo'zgeriwshi.

Yeger ha'r qanday won' *s* sani' ushi'n wo'zgeriwshinin' sonday x_0 ma'nisi bar bolsa, x ti'n' wonnan son'g'i' ha'r bir ma'nisinin' absolyut shamasi' S dan u'lken bolsa, wonda x wo'zgeriwshi sheksiz u'lken dep ataladi'. Bul $x \to \infty$ ko'rinisinde jazi'ladi'.

Soni'n' menen birge, yeger x tin' x_0 dan keyingi ma'nisleri wo'z belgilerin saqlasa, wonda $x \to +\infty$ (yamasa $x \to -\infty$) dep jazi'ladi'.

4. Wo'zgeriwshinin' limiti.

Yeger A ha'm wo'zgeriwshi x arasi'ndag'i' ayi'rma sheksiz kishi sha'm a, yag'ni'y yeger $x = a + \alpha$ bolsa, turaqli' a wo'zgeriwshi x ti'n' limiti dep ataladi' ha'm $\lim x = a$ tu'rinde jazi'ladi'.

5. Funkciyani'n' limiti.

Yeger x ti'n' **a** g'a ten' bolmastan wog'an umti'li'wi'nan ha'r dayi'm $f(x)$ ti'n' **b** g'a umti'li'wi' kelip shi'qsa, **b** san $f(x)$ funkciyani'n' x ti'n' a g'a umti'lg'andag'i' limiti dep ataladi'.

Buni' $\lim\limits_{x\to a} f(x)=b$ ko'rinisinde jazadi'.

6. Limitlerdin' qa'siyetleri:

1) Turaqli' shamani'n' limiti wo'zine ten'.

2) lim $(u+v)=\lim u+\lim v$

3) $(u\bullet v)=\lim u\bullet\lim v$

4) Yeger $\lim u$ *ha`m* $\lim v$ bar boli'p, $\lim v\neq 0$ bolsa, wonda $\lim\dfrac{u}{v}=\dfrac{\lim u}{\lim v}$

5) Yeger a tochkani'n' qandayda bir a'tirapi'ndag'i' x ti'n', balki tek $x=a$ dan basqa barli'q ma'nislerinde $f(x)$ ha'm $\varphi(x)$ funkciyalar bir-birine ten' bolsa ha'm wolardi'n' birewi $x\to a$ da limitke iye bolsa, ekinshiside usi' limitke iye boladi'.

7. A'jayi'p limitler.

1. $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1;\ \lim\limits_{x\to 0}\dfrac{x}{\sin x}=1.$

2.. $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n=\lim\limits_{n\to-\infty}\left(1+\dfrac{1}{n}\right)^n=\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=e$

8. **e** sani' irratsional san boli'p, $e\approx 2{,}71828...$ Tiykari' ***e*** ge ten' bolg'an logorifmler natural logarifmler dep ataladi' ha'm $\log_e x=\ln x$ ko'rinisinde belgilenedi.

Onli'q logorifm $\lg x=M\ln x$, bunda $M=0{,}43429...$

Mi'sal

1. $\lim\limits_{n\to\infty}\dfrac{4n+9}{2n-3}$ yesaplan'.

Sheshi'liwi. Bo'lshektin' ali'mi'da bo'limide shegaralabang'an izbe-izlikler bolg'ani' ushi'n bo'lshektin' limiti haqqi'ndag'i' teoremani' qollana almaymi'z. Usi' sebepli bo'lshektin' ali'mi'nda bo'liminde **n** ge bo'li'p son' bo'lshektin' limiti haqqi'ndag'i' teoremadan paydalanami'z.

$$\lim_{n\to\infty}\frac{4n+9}{2n-3}=\lim_{n\to\infty}\frac{\frac{4n+9}{n}}{\frac{2n-3}{n}}=\frac{\lim\limits_{n\to\infty}\left(\frac{4n+9}{n}\right)}{\lim\limits_{n\to\infty}\left(\frac{2n-3}{n}\right)}=\frac{\lim\limits_{n\to\infty}\left(4+\frac{9}{n}\right)}{\lim\limits_{n\to\infty}\left(2-\frac{3}{n}\right)}=\frac{4+0}{2-0}=2$$

2. $\lim\limits_{x\to 1}\dfrac{2x^3+3x^2-x-4}{3x^4-x^2+2x-4}$ ti yesaplan'.

Sheshi'liwi: **yeger** berilgen bo'lshekke x ti'n' worni'na 1 di qoysaq, wonda $\dfrac{0}{0}$ tu'rindegi ani'q yemeslik kelip shi'g'adi'. Lopital' qa'desin paydalani'p to'mendegige iye bolami'z.

$$\lim_{x\to 1}\frac{2x^3+3x^2-x-4}{3x^4-x^2+2x-4}=\lim_{x\to 1}\frac{6x^2+6x-1}{12x^3-2x+2}=\frac{11}{12}.$$

3. $\lim\limits_{x\to 1}\dfrac{x^2-1}{2x^2-x-1}$ ti yesaplan'.

Sheshiliwi:

$$\lim_{x\to 1}\frac{x^2-1}{2x^2-x-1}=\lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)(2x+1)}=\lim_{x\to 1}\frac{x+1}{2x+1}=\frac{2}{3}$$

4. $\lim\limits_{x\to 2}\dfrac{x^3-2x^2-4x+8}{x^4-8x^2+16}$ ti yesaplan'.

Sheshiliwi:

$$\lim_{x\to 2}\frac{x^3-2x^2-4x+8}{x^4-8x^2+16}=\lim_{x\to 2}\frac{x^2(x-2)-4(x-2)}{(x^2-4)^2}=\lim_{x\to 2}\frac{(x^2-4)(x-2)}{(x^2-4)^2}=$$
$$=\lim_{x\to 2}\frac{x-2}{x^2-4}=\lim_{x\to 2}\frac{x-2}{(x-2)(x+2)}=\lim_{x\to 2}\frac{1}{x+2}=\frac{1}{4}$$

5. $\lim\limits_{x\to 5}\dfrac{\sqrt{x^2-9}-4}{(x-5)}$

Sheshiliwi:

$$\lim_{x\to 5}\frac{\sqrt{x^2-9}-4}{(x-5)}=\lim_{x\to 5}\frac{\left(\sqrt{x^2-9}-4\right)\left(\sqrt{x^2-9}+4\right)}{(x-5)\left(\sqrt{x^2-9}+4\right)}=$$
$$=\lim_{x\to 5}\frac{x^2-9-16}{(x-5)\left(\sqrt{x^2-9}+4\right)}=\lim_{x\to 5}\frac{x^2-25}{(x-5)\left(\sqrt{x^2-9}+4\right)}=$$
$$=\lim_{x\to 5}\frac{(x-5)(x+5)}{(x-5)\left(\sqrt{x^2-9}+4\right)}=\lim_{x\to 5}\frac{x+5}{\sqrt{x^2-9}+4}=\frac{10}{8}=1\frac{1}{4}$$

VII. FUNKCIYANI'N' U'ZLIKSIZLIGI

1. Yeger $f(x)$ funkciya a ni'n' qandayda bir do'gereginde ani'qlang'an ha'm

$$\lim_{x\to a} f(x) = f(a)$$

bolsa, wol $x = a$ bolg'anda u'zliksiz delinedi. Bul ani'qlama to'mendegi to'rt u'zliksizlik sha'rtin wo'z ishine aladi':

1) $f(x)$ funkciya a ni'n' qandayda bir do'gereginde ani'qlang'an boli'wi' kerek;

2) shekli $\lim\limits_{x\to a-0} f(x)$ ha'm $\lim\limits_{x\to a+0} f(x)$ limitler bar boli'wi' kerek;

3) bul (shep ha'm won') limitler birdey boli'wi' kerek;

4) Bul limitler $f(a)$ g'a ten' boli'wi' kerek;

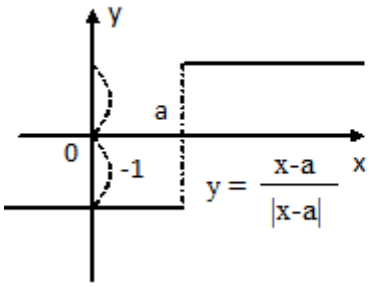
Yeger funkciya $[x_1; x_2]$ segmenttin' ha'r bir ishki noqati'nda u'zliksiz bolsa ha'm woni'n' shegaralari'nda bolsa $\lim\limits_{x\to x_1+0} f(x) = f(x_1)$ ha'm $\lim\limits_{x\to x_2-0} f(x) = f(x_2)$ bolsa, wol sol segmentte u'zliksiz delinedi.

2. Funkciyani'n' u'zilisleri. Yeger funkciya a dan won'da ha'm shepte ani'qlang'an bolsa, biraq a noqatta u'zliksizliktin' to'rt sha'rtinen hesh bolmag'anda birewi wori'nlanbasa, $f(x)$ funkciya $x = a$ bolg'anda u'ziliske iye boladi'. U'zilislerdi tiykarg'i' yeki tu'rge aji'rati'wg'a boladi'.

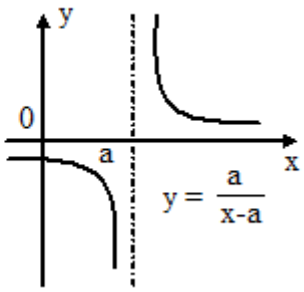
1) Birinshi tu'r u'zilis-shekli $\lim\limits_{x\to a-0} f(x)$ ha'm $\lim\limits_{x\to a+0} f(x)$ limitler bar boli'p, yag'niy u'zliksizlik shartlerinen ekinshisi wori'nlanadi' ha'm qalg'anlari' (yaki wolardan hesh bolmag'anda birewi) wori'lanbaydi'.

Ma'selen, $x < a$ bolg'anda -1 ge ten' $x > a$ bolg'anda +1 ge ten' bolg'an $y = \dfrac{x-a}{|x-a|}$ funkciya $x = a$ da birinshi tu'r u'ziliske iye (6-su'wret), sebebi $\lim\limits_{x\to a-0} y = -1$ ha'm $\lim\limits_{x\to a+0} y = +1$ limitler bar, biraq bul limitler wo'z-ara ten' yemes.

2) yekinshi tu'r u'zilis-$\lim\limits_{x\to a} f(x)$ won'nan yaki shepten $\pm\infty$ ke ten'. Ma'selen, $y = f(x) = \dfrac{a}{x-a}$ funkciya (7-su'wret) $x = a$ bolg'anda yekinshi tu'r u'ziliske iye. $x = a$ bolg'anda bo'limi 0 (nol) ge ten' boli'p, ali'mi' 0 (nol) ge ten' bolmag'an ba'rshe bo'lshek funkciyalar $x = a$ bolg'anda yekinshi tu'r u'ziliske iye boladi'.



6-su’wret



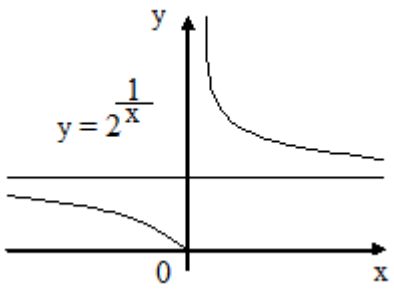
7-su’wret

Mi’sallar

1. $y = -\frac{5}{x^2-x-6}$ funkciyani’n’ u’zliksizlik arali’qlari’n tabi’n’.

Sheshiliwi: Berilgen ratsyonal funkciyani’n’ ani’qlani’w oblasti’ $(-\infty;2) \cup (-2;3) \cup (3;+\infty)$ arali’qlar birikpesinen ibarat. Demek, bul funkciya usi’ arali’qlardi’n’ barli’q tochkalari’nda u’zliksiz boli’p, $x = -2$ ha’m $x = 3$ tochkalarda u’ziliske iye.

2. $f(x) = 2^{\frac{1}{x}}$ funkciya (8-su’wret) ha’m $x = 0$ bolg’anda yekinshi tu’r u’ziliske iye, sebebi $\lim\limits_{x\to-0} f(x) = 0$, biraq $\lim\limits_{x\to+0} f(x) = \infty$.



8-su’wret

VIII. BIR WO'ZGERIWSHILI FUNKCIYANI'N' DIFFERENTSIYAL ESABI'.

1. $(Cf(x))' = Cf'(x)$ S-turaqli' san. 2. $\left(f(x)+g(x)\right)' = f'(x)+g'(x)$

3. $\left(f^n(x)\right)' = nf^{n-1}f'(x)\ \ d\left(f^n(x)\right) = nf^{n-1}f'(x)dx$

4. $\left(x^n\right)' = nx^{n-1}\ \ d\left(x^n\right) = nx^{n-1}dx$

5. $(f\cdot g)' = f'\cdot g + f\cdot g'\ \ d(f\cdot g) = f\cdot dg + g\cdot df$

6. $\left(\frac{1}{f(x)}\right) = \frac{f'(x)}{f^2(x)}$ 7. $\left(\frac{f}{g}\right) = \frac{f'\cdot g - f\cdot g'}{g^2}$

8. $(\sin x)' = \cos x\ \ d(\sin x) = \cos x dx$

9. $(\cos x)' = \sin x\ \ d(\cos x) = \sin x dx$

10. $(tgx)' = \frac{1}{\cos^2 x}\ \ d(tgx)' = \frac{dx}{\cos^2 x}$

11. $(ctgx)' = -\frac{1}{\sin^2 x}\ \ d(ctgx) = -\frac{dx}{\sin^2 x}$

12. $\left(e^x\right)' = e^x$ 13. $\left(a^x\right)' = a^x \ln a$

14. $(\log_a x)' = \frac{1}{x\ln a}$ 15. $(\ln x)' = \frac{1}{x}$

16. Yeger y wo'zgeriwshi u di'n' funkciyasi' boli'p, yag'ni'y $y = f(u)$, al u bolsa wo'z gezeginde x argumenttin' funkciyasi' bolsa, yag'ni'y $u = \varphi(x)$ bolsa, wonda y wo'zgeriwshi x qa arali'q argument u arqali' baylani'si'p, x tin' quramali' funkciyasi' dep ataladi' ha'm $y = f(\varphi(x))$ tu'rinde jazi'ladi'.

Yeger $y = f(u)$ ha'm $u = \varphi(x)$ funkciyalar differentsiyallani'wshi' funkciyalar bolsa, wonda quramali' $y = f(\varphi(x))$ funkciyasi'ni'n' erikli wo'zgeriwshi x boyi'nsha tuwi'ndi'si' bul funkciyani'n' arali'q argumenti boyi'nsha tuwi'ndi'si'ni'n' arali'q argumenttin' erikli wo'zgeriwshi x boyi'nsha tuwi'ndi'si'na ko'beymesine ten', yag'ni'y $y`_x = y`_u \cdot u`_x$.

17. Yeger bazi' bi'r x tochkada differentsiyalani'wshi' ha'm nol'den wo'zgeshe tuwi'ndi'g'a iye $y=f(x)$ funkciyasi'ni'n' $x=\varphi(y)$ keri funkciyasi' bar bolsa, wonda bul keri funkciya da usi' tochkada differentsiyallani'wshi' boladi' ha'm woni'n' tuwi'ndi'si' $\varphi'(y)=\frac{1}{f'(x)}$ g'a ten' boladi'

18. Ferma teoremasi'.

$f(x)$ funkciya $(a;b)$ intervalda berilgen boli'p, wol usi' intervaldan bazi'bi'r c tochkasi'nda wo'zinin' yen' u'lken (yen' kishi) ma'nisinde erissin. Yeger funkciya c tochkada shekli tuwi'ndi'g'a iye bolsa, wonda $f'(c)=0$ boladi'.

19. Roll teoremasi'.

Sheksiz u'lken wo'zgeriwshi.

$f(x)$ funkciya $[a;b]$ segmentte ani'qlang'an ha'm u'zliksiz boli'p, $f(a)=f(b)$ bolsi'n. Yeger funkciya $(a;b)$ intervalda shekli tuwi'ndi'g'a iye bolsa, wonda sonday bir c tochka $(c\in(a;b))$ tabi'li'p, $f'(c)=0$ boladi'.

20. Lagranj teoremasi'.

$f(x)$ funkciya $[a;b]$ segmentte ani'qlang'an ha'm u'zliksiz bolsi'n. Yeger funkciya $(a;b)$ da shekli tuwi'ndi'g'a iye bolsa, wonda sonday bir c tochka $(c\in(a;b))$ tabi'li'p, $\frac{f(b)-f(a)}{b-a}=f'(c)$ boladi'.

21. Koshi' teoremasi'.

$f(x)$ ha'm $g(x)$ funkciyalar $[a;b]$ segmentte ani'qlang'an ha'm u'zliksiz bolsi'n. Yeger bul funkciyalar $(a;b)$intervalda shekli tuwi'ndi'g'a iye boli'p, $\forall x\in(a;b)$ ushi'n $g'(x)\neq 0$ bolsa, wonda sonday bir c tochka $(c\in(a;b))$, $\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$ boladi'.

22. $\frac{0}{0}$ <u>ko'rinisindegi ani'q yemeslik. Lopital'din' birinshi qa'desi.</u>

Yeger $\lim\limits_{x\to a}f(x)=\lim\limits_{x\to a}\varphi(x)=0$ ha'm $\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ bar bolsa, wonda

$\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)}=\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ boladi'.

23. $\frac{\infty}{\infty}$ ko'rinisindegi ani'q yemeslik. Lopital'din' yekinshi qa'desi.

Yeger $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a}\varphi(x)=\infty$ ha'm $\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ bar bolsa, wonda

$\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)}=\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ boladi'.

24. $0\cdot\infty$, $\infty-\infty$, 1^{∞} ha'm 0^0 ko'rinisindegi ani'q yemeslikler algebrali'q almasti'ri'wlar ja'rdeminde $\frac{0}{0}$ ha'm $\frac{\infty}{\infty}$ ko'rinisindegi ani'q yemesliklerge keltiriledi.

Mi'sallar

1. $y=x+\sqrt{x}+\sqrt[3]{x}$ funkciya tuwi'ndi'si'n tabi'n'

Sheshiliwi:

$$y'=(x)'+\left(x^{\frac{1}{2}}\right)'+\left(x^{\frac{1}{3}}\right)'=1+\frac{1}{2\sqrt{x}}+\frac{1}{3\sqrt[3]{x^2}}\,(x>0)$$

2. $y=\frac{2x}{1-x^2}$ funkciya tuwi'ndi'si'n tabi'n'

Sheshiliwi:

$$y'=\frac{(2x)'(1-x^2)-2x(1-x^2)'}{(1-x^2)^2}=\frac{2(1-x^2)-2x(-2x)}{(1-x^2)^2}=$$

$$=\frac{2-2x^2+4x^2}{(1-x^2)^2}=\frac{2+2x^2}{(1-x^2)^2}=\frac{2(1+x^2)}{(1-x^2)^2}\,(|x|\neq 1)$$

3. $y=\frac{1+x-x^2}{1-x+x^2}$ funkciya tuwi'ndi'si'n tabi'n'

Sheshiliwi:

$$y'=\frac{(1+x-x^2)'(1-x+x^2)-(1+x-x^2)(1-x+x^2)'}{(1-x+x^2)^2}=$$

$$=\frac{(1-2x)(1-x+x^2)-(1+x-x^2)(-1+2x)}{(1-x+x^2)^2}=$$

$$=\frac{1-x+x^2-2x+2x^2-2x^3+1-2x+x-2x^2-x^2+2x^3}{(1-x+x^2)^2}=$$

$$=\frac{2-4x}{(1-x+x^2)^2}=\frac{2(1-2x)}{(1-x+x^2)^2}$$

4. $\left(\sin\left(x^5 - x\ -8\right)\right)'$ funkciyasi'ni'n' tuwi'ndi'si'n tabi'n'.

Seshiliwi: $\left(\sin\left(x^5 - x\ -8\right)\right)' = (5x^4 - 1)\cos\left(x^5 - x - 8\right)$

5. $\left(\log_3 10x\right)'$ funkciyasi'ni'n' tuwi'ndi'si'n tabi'n'.

$$\left(\log_3 10x\right)' = \frac{10}{10x\ln 3} = \frac{1}{x\ln 3}$$

6. $y = \ln\cos^2 12x$ funkciyani'n' tuwi'ndi'si'n tabi'n'.

Sheshi'liwi: $y' = (\ln\cos^2 12x)' = \frac{(\cos^2 12x)'}{\cos^2 12x} = -\frac{24\cos 12x \cdot \sin 12x}{\cos^2 12x} = -24tg12x$

Joqari' ta'rtipli tuwi'ndi'lar

Biz $y = f(x)$ funkciyanin' $y' = f'(x)$ tuwi'ndi'si'n tapti'q dep ko'z aldi'mi'zg'a keltireyik. Bul tuwi'ndi'ni'n' tuwi'ndi'si' $f(x)$ funkciyani'n' yekinshi ta'rtipli tuwi'ndi'lari' delinedi ha'm y'' yaki $f''(x)$ yaki $\frac{d^2y}{dx^2}$ lar menen belgilenedi. Sog'an uqsas, joqari' ta'rtipli tuwi'ndi'lar ani'qlanadi' ha'm 3-ta'rtipli tuwi'ndi' $y''' = f'''(x) = \frac{d^3y}{dx^3}$ penen ha'm uli'wma tu'rde n - ta'rtipli tuwi'ndi' $y^{(n)} = f^{(n)}(x) = \frac{d^ny}{dx^n}$ penen belgilenedi.

Mi'sal

$y = 4x^3 - 7x^2 + 5$ funkciyani'n' yekinshi ta'rtipli tuwi'ndi'si'n tabi'n'

Sheshi'liwi:

$y' = 12x^2 - 14x$

$y'' = (y')' = (12x^2 - 14x)' = 24x - 14$

IX. BIR WO'ZGERIWSHILI FUNKCIYANI'N' INTEGRAL ESABI'

1. Bazi' bir X arali'qtagi' barli'q x lar ushi'n $F'(x)=f(x)$ wori'nli' bolsa, $F(x)$ funkciya usi' arali'qta $f(x)$ funkciyani'n daslepki funkciyasi' dep ataladi'.

1. $\int f(x)dx = F(x)+C$ **C** – turaqli' san (39)

2. $\int F'(x)dx = F(x)+C$ (40)

3. $\int(\varphi(x)+\psi(x))dx = \int\varphi(x)dx+\psi(x)dx$ (41)

4. $\int k\cdot f(x)dx = \int k\cdot f(x)dx, \quad k-turaqli\ \ san$ (42)

5. $\alpha\neq -1\ \ ushin\ \int x^{\alpha}dx = \frac{x^{\alpha+1}}{\alpha+1}+C$ (43)

6. İntegrallaw formulalari':

1	$\int f(x)dx = F(x)+C$	$F'(x)=f(x)$
2	$\int x^{\alpha}dx = \frac{x^{\alpha+1}}{\alpha+1}+C,\ (\alpha\neq -1)$	$\left(\frac{x^{\alpha+1}}{\alpha+1}+C\right)=\frac{\alpha+1}{\alpha+1}x^{\alpha}=x^{\alpha},\ \ \alpha\neq -1$
3	$\int\cos xdx = \sin x+C$	$(\sin x+C)' = \cos x$
4	$\int\sin xdx = \cos x+C$	$(-\cos x+C)' = -(\sin x)+0=\sin x$
5	$\int\frac{dx}{\cos^2 x} = tgx+C$	$(tgx+C)' = \frac{1}{\cos^2 x},\quad x\neq\frac{\pi}{2}+2k\pi,\quad k\in Z$
6	$\int\frac{dx}{\sin^2 x} = ctgx+C$	$(-ctgx+C)' = -\left(-\frac{1}{\sin^2 x}\right)=\frac{1}{\sin^2 x},$ $x\neq\pi k,\quad k\in Z$
7	$\int\frac{dx}{\sqrt{1-x^2}} = \arcsin x+C$	$(\arcsin x+C)' = \frac{1}{\sqrt{1-x^2}},\quad \lvert x\rvert<1$
8	$\int\frac{dx}{1+x^2} = arctgx+C$	$(arctgx+C)' = \frac{1}{1+x^2}$
9	$\int\sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2}+$ $+\frac{a^2}{2}\arcsin\frac{x}{a}+C$	$\left(\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}+C\right)'=$ $=\sqrt{a^2-x^2},\ \ \lvert a\rvert\geq\lvert x\rvert$

10 $\int e^x dx = e^x + C$ $\left(e^x + C\right)' = e^x$

11 $\int a^x dx = \frac{a^x}{Ina} + C$ $\left(a^x + C\right)' = a^x Ina$

12 $\int \frac{dx}{x} In|x| + C$ $x > 0$ da $\int \frac{dx}{x} Inx + C$, $\left(Inx + C\right)' = \frac{1}{x}$;

$x > 0$ da $\int \frac{dx}{x} In(-x) + C$

$\left(Inx + C\right)' = -\frac{1}{x} \cdot (-1) = \frac{1}{x}$

7. Wo'zgeriwshini almasti'ri'w usi'li'.

$F(\varphi(t))$ funkciya berilgen ha'm $x = \varphi(t)$ almasti'ri'li'wi' kiritilgen bolsi'n.

$(F(\varphi(t)))' = F'(x)\varphi'(t) = f(x)\varphi'(t) = f(\varphi(t))\varphi'(t)$ ha'm

$\int f(\varphi(t))\varphi'(t) = F(\varphi(t)) + C$ (44)

yamasa $\int f(\varphi(t))d\varphi(t) = F(\varphi(t)) + C$ (45)

Demek, $F(\varphi(t))$ funkciya $f(\varphi(t))\varphi'(t)$ funkciyani'n daslepki funkciyasi' $\varphi(t) = kt + b$ bolsi'n. Wonda $\varphi'(t) = k$ yamasa $d\varphi(t) = kdt$ ha'm (u) boyi'nsha

$\int f(kt+b) \cdot kdt = F(kt+b) + C$ yamasa $\int f(kt+b)dt = \frac{1}{k} F(kt+b) + C$ (46)

8. Bo'leklep integrallaw.

$u = u(x)$, $v = v(x)$ funkciyalar differentsiyallani'wshi' bolsi'n. Bizge ma'lim

$$d(uv) = udv + vdu \quad (47)$$

(9) ni' integrallap, $uv = \int udv + \int vdu$ yamasa

$$\int udv = uv - \int vdu \quad (48)$$

g'a iye bolami'z. (48) formula bo'leklep integrallaw formulasi' dep ataladi'.

9. Ratsional bolsheklerdi integrallaw.

a) $\int \frac{A}{x-a} dx = A \cdot \ln|x-a| + C$

b) $\int \frac{A}{(x-a)^n}dx = A\int (x-a)^{-n}dx = \frac{A}{(1-n)(x-a)^{n-1}} + C$

$$\int \frac{Bx+C}{x^2+\rho^x+q}dx = \int \frac{Bx+C}{\left(x+\frac{\rho}{2}\right)^2+k^2}dx = \frac{B}{2}\ln\left(x^2+\rho x+q\right)+$$

c)

$$+\left(C-\frac{B\rho}{2}\right)\frac{1}{\sqrt{q-\frac{\rho^2}{4}}}arctg\frac{x+\frac{\rho}{2}}{\sqrt{q-\frac{\rho^2}{4}}}+C$$

M i ' s a l l a r

1. $\int (7x^5+11x^3+6x^2-15)dx$ İntegrali'n yesaplan'.

$$\int (7x^5+11x^3+6x^2-15)dx = 7\int x^5dx+11\int x^3dx+6\int x^2dx-15\int dx =$$

$$= 7\cdot\frac{x^6}{6}+11\cdot\frac{x^4}{4}+6\frac{x^3}{3}-15x+C = \frac{7}{6}x^6+\frac{11}{4}x^4+2x^3-15x+C$$

2. $\int \frac{dx}{x-25}(x\neq 25)$ integrali'n yesaplan'.

$$\int \frac{dx}{x-25} = \int \frac{d(x-25)}{x-25} = \ln(x-25)+C$$

3. İntegraldi' wo'zgeriwshini almasti'ri'w usi'li' menen yesaplan'.

$$\int \cos^2 2x\cdot\sin 2xdx$$

Sheshi'w: $(\cos 2x)' = -2\sin 2x$ bolg'anli'qtan $\cos 2x = t$ dep almasti'rami'z. Wonda

$$\int \cos^2 2x\cdot\sin 2xdx = \frac{1}{2}\int t^2\cdot(-dt) = -\frac{t^3}{6}+C = -\frac{\cos^3 2x}{6}+C$$

4. $\int x\cdot e^{2x}dx$ integrali'n bo'leklep integrallaw usi'li' menen sheshi'n'.

Sheshi'w: $u = x,\quad dv = e^{2x}\ du = dx,\quad v = \frac{1}{2}e^{2x}$

(48) formula boyi'nsha

$$\int x\cdot e^{2x}dx = \frac{x}{2}e^{2x}-\frac{1}{2}\int e^{2x}dx = \frac{x}{2}e^{2x}-\frac{1}{4}e^{2x}+C$$

5. Ani'q yemes integraldi' yesaplan' : $\int (3-x^2)^3 dx$

Sheshiliwi:

$$\int (3-x^2)^3 dx = \int \left(27 - 27x^2 + 9x^4 - x^6\right)dx = \int 27dx - \int 27x^2 dx +$$
$$+ \int 9x^4 dx - \int x^6 dx = 27\int dx - 27\int x^2 dx + 9\int x^4 dx - \int x^6 dx =$$
$$= 27x - 27\frac{x^3}{3} + 9\frac{x^5}{5} - \frac{x^7}{7} = 27x - 9x^3 + \frac{9}{5}x^5 - \frac{1}{7}x^7$$

6. Ani'q yemes integraldi' yesaplan': $\int (1-x)(1-2x)(1-3x)dx$

Sheshiliwi:

$$\int (1-x)(1-2x)(1-3x)dx = \int (1-6x+11x^2-6x^3)dx = \int dx -$$
$$-\int 6xdx + \int 11x^2 dx - \int 6x^3 dx = \int dx - 6\int xdx + 11\int x^2 dx - 6\int x^3 dx =$$
$$= x - \frac{6}{2}x^2 + \frac{11}{3}x^3 - \frac{6}{4}x^4 = x - 3x^2 + \frac{11}{3}x^3 - \frac{3}{2}x^4$$

7. Ani'q yemes integraldi' yesaplan': $\int \frac{dx}{1+e^x}$

Sheshiliwi: $1+e^x = t$ belgilep ali'p, bunnan $e^x = t-1$, $x = \ln(t-1), dx = \frac{dt}{t-1}$.

Onda

$$\int \frac{dx}{1+e^x} = \int \frac{dt}{t(t-1)}$$

$\frac{1}{t(t-1)} = \frac{1}{t-1} - \frac{1}{t}$ ten'likti itibarg'a ali'p,

$$\int \frac{dt}{t(t-1)} = \int \frac{dt}{t-1} - \int \frac{dt}{t} = \ln|t-1| - \ln|t| + C$$

ti payda yetemiz. Aldi'ng'i' belgilewge qaytami'z:

$$\int \frac{dx}{1+e^x} = \ln e^x - \ln(1+e^x) + C = x - \ln(1+e^x) + C$$

Ani'q integral

1. $\int_a^b f(x)dx = F(x)\Big|_a^b$ (49)

N'yuton-Leybnits formulasi' dep ataladi'. (a ha'm b-integrallaw shegaralari').

2. $\int_a^b (f_1(x)+ f_2(x))dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx$ (50)

3. $\int_a^b Af(x)dx = A\int_a^b f(x)dx$ (51) **A –** turaqli’ san

4. $\int_a^b f(x)dx = -\int_b^a f(x)dx$ (52)

5. $\int_a^a f(x)dx = 0$ (53)

6. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (54)

7. [a~b] arali’qta $u = u(x),\ v = v(x)$ funkciyalar u’zliksiz tuwi’ndi’larg’a iye bolsa, tomendegi bo’leklep integrallaw formulasi’ wori’nli’ boladi’

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu \qquad (55)$$

8. $\int_a^b f(x)dx$ integraldi’ esaplaw kerek bolsi’n.

$x = \varphi(t),\ \alpha \le t \le \beta$ almasti’ri’wi’n qollanami’z.

Yeger $[\alpha;\ \beta]$ arali’qta $x = \varphi(t),\ \varphi'(t),\ f(\varphi(t))$ funkciyalar u’zliksiz ha’m $\varphi(\alpha) = a,\ \varphi(\beta) = b$ bolsa, to’mendegi

$$\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\varphi'(t)dt \qquad (56)$$

formulasi’ wori’nli’ boladi’.

Ayi’ri’m jag’daylarda $x = \varphi(t)$ almasti’ri’w worni’na $t = \varphi(x)$ turindegi almasti’ri’wdan paydalani’ladi’. Bul jag’dayda $t = \varphi(x)$ funkciyag’a keri funkciya bar boli’p, bul funkciya joqari’dag’i’ sha’rtlerdi qanaatlandi’ri’wi’ kerek.

M i ’ s a l l a r

To’mendegi integrallardi’ yesaplan’.

1. Ani’q integraldi’ yesaplan’ $\int_0^{\pi} \cos^2 x dx$

Sheshiliwi:

$$\int_0^{\pi} \cos^2 x dx = \int_0^{\pi} \frac{1+\cos 2x}{2} dx = \frac{1}{2}\left(\int_0^{\pi} dx + \int_0^{\pi} \cos 2x dx \right) =$$

$$= \frac{1}{2}\left(x\Big|_0^{\pi} + \frac{1}{2}\sin 2x\Big|_0^{\pi} \right) = \frac{1}{2}\left(\pi - 0 + \frac{1}{2}\cdot 0 \right) = \frac{\pi}{2}$$

2. Ani’q integraldi’ yesaplan’ $\int_0^4 \left(x - 4\sqrt{x} \right) dx$

Sheshiliwi:

$$\int_0^4 \left(x - 4\sqrt{x} \right) dx = \left(\frac{x^2}{2} - 4\frac{x^{3/2}}{\frac{3}{2}} \right)\Bigg|_0^4 = \left(\frac{x^2}{2} - \frac{8}{3}x\sqrt{x} \right)\Big|_0^4 =$$

$$= 8 - \frac{8}{3}\cdot 4\cdot 2 - 0 = 8 - \frac{64}{3} = -13\frac{1}{3}$$

3. Ani’q integraldi’ yesaplan’ $\int_0^{\frac{\pi}{4}} \left(\cos^2 x - \sin^2 x \right) dx$

Sheshiliwi:

$$\int_0^{\frac{\pi}{4}} \left(\cos^2 x - \sin^2 x \right) dx = \int_0^{\frac{\pi}{4}} \cos 2x dx = \frac{1}{2}(\sin 2x)\Big|_0^{\frac{\pi}{4}} =$$

$$= \frac{1}{2}\left(\sin 2\cdot\frac{\pi}{4} - \sin 2\cdot 0 \right) = \frac{1}{2}(1-0) = \frac{1}{2}$$

4. Ani’q integraldi’ yesaplan’ $\int_0^{7/3} \frac{x+1}{\sqrt[3]{3x+1}} dx$

$$\int_0^{7/3}\frac{x+1}{\sqrt[3]{3x+1}}dx=\int_0^{7/3}\frac{3(x+1)}{3\sqrt[3]{3x+1}}dx=\frac{1}{3}\int_0^{7/3}\frac{x+1}{\sqrt[3]{3x+1}}dx+$$

$$+\frac{1}{3}\int_0^{7/3}\frac{2}{\sqrt[3]{3x+1}}dx=\frac{1}{3}\left(\int_0^{7/3}(3x+1)^{2/3}dx+2\int_0^{7/3}(3x+1)^{-1/3}dx\right)=$$

Sheshiliwi:

$$=\frac{1}{3}\left(\frac{1}{3}\frac{(3x+1)^{5/3}}{\frac{5}{3}}\Bigg|_0^{7/3}+\frac{2}{3}\frac{(3x+1)^{2/3}}{\frac{2}{3}}\Bigg|_0^{7/3}\right)=$$

$$=\frac{1}{3}\left(\frac{1}{5}\left(8^{5/3}-1\right)+\left(8^{2/3}-1\right)\right)=\frac{46}{15}=3\frac{1}{15}$$

5. $\int_1^6\frac{2dx}{\sqrt{9x-5}}$

Sheshi'liwi': $9x-5=t^2$ almasti'ri'wi'n qollanami'z. Bunnan

$9x=t^2+5,\quad 9dx=2t\ dt$

jan'a wo'zgeriwshinin' shegaralari'n ani'qlaymi'z. $x=1$ bolg'anda $t=2,\quad x=6$ bolg'anda $t=7$ wonda $\int_1^6\frac{2dx}{\sqrt{9x-5}}=\int_2^7\frac{4tdt}{9t}=\frac{4}{9}\int_2^7dt=\frac{4t}{9}\Bigg|_2^7=\frac{4}{9}(7-2)=\frac{20}{9}$

6. $\int_0^{\pi}x\cos xdx$

Sheshiliwi: $u=x$, $dv=\cos xdx$ dep alsaq, $du=dx,\quad v=\sin x$ bo'leklep integrallaw formulasi'na tiykarlani'p

$$I=\int_0^{\pi}x\cos xdx=x\sin x\Big|_0^{\pi}-\int_0^{\pi}\sin xdx=0+\cos x\Big|_0^{\pi}=\cos\pi-\cos 0=-1-1=-2$$

X. KO'P WO'ZGERIWSHILI FUNKCIYANI'N' DIFFERENTSIYAL ESABI'

$\frac{\Delta xU}{\Delta x}$ qatnasi'ni'n' Δx nolge umti'lg'andag'i' shegi $u = f(x, y, z)$ funkciyasi'ni'n' x g'a'rezsiz wo'zgeriwshisiz boyi'nsha dara tuwi'ndi'si' dep ataladi' ha'm wol to'mendegi simvollardi'n' biri menen belgilenedi:

$\frac{\partial U}{\partial x}; \frac{\partial f}{\partial x}; U'_x; f'_x$

Solay yetip, $\frac{\partial U}{\partial x} = \lim\limits_{\Delta x \to 0} \frac{\Delta_x U}{\Delta x}$

Sol si'yaqli' $u = f(x, y, z)$ funkciyasi'ni'n' y g'a'rezsiz wo'zgeriwshisi boyi'nsha dara tuwi'ndi'si' $\frac{\partial U}{\partial y}; \frac{\partial f}{\partial y}; U'_y; f'_y$ simvollari'ni'n' biri menen ha'm z g'a'rezsiz wo'zgeriwshisi boyi'nsha dara tuwi'ndi'si' $\frac{\partial U}{\partial z}; \frac{\partial f}{\partial z}; U'_z; f'_z$ simvollari'ni'n' biri menen belgilenedi

Solay yetip,

$\frac{\partial U}{\partial y} = \lim\limits_{\Delta y \to 0} \frac{\Delta_y U}{\Delta y}; \ \frac{\partial U}{\partial z} = \lim\limits_{\Delta z \to 0} \frac{\Delta_z U}{\Delta z}$

$U = f(x, y, z)$ funkciyasi'ni'n' toli'q differentsiyali' dU arqali' belgilenedi ha'm to'mendegi formula boyi'nsha yesaplanadi': $dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} dz$

Yeger $z = f(x, u, v)$ boli'p, $U = \varphi(x), v = \psi(x)$ bolsa, wonda z x argumentinin' quramali' funkciyasi' delinedi. Bul jag'dayda z funtsiyasi'ni'n' toli'q tuwi'ndi'si'ni'n' to'mendegi formula menen yesaplanadi': $\frac{dz}{dx} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial U} \frac{dU}{dx} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dx}$

bul jerde $\frac{\partial z}{\partial x}$ dara tuwi'ndi'si'n esaplag'anda, U ha'm v shamalari'n turaqli' dep alami'z.

$z = f(x, y)$ funkciyasi'ni'n' yekinshi ta'rtipli differentsiyali' dep, birinshi ta'rtipli differentsiyaldan ali'ng'an differentsiyalg'a aytami'z ha'm wol $d^2 z$ tu'rinde belgilenedi. Demek, $d^2 z = d(dz)$. Yekinshi ta'rtipli differentsiyal to'mendegi formula menen yesaplanadi':

$d^2 z = \frac{\partial^2 z}{\partial x^2} dx^2 + 2 \frac{\partial^2 z}{\partial x \partial y} dxdy + \frac{\partial^2 z}{\partial y^2} dy^2.$

Uli'wma alg'anda, $z = f(x, y)$ funkciyasi'ni'n' n-shi ta'rtipli differentsiyali'

$d^2 z = \left(\frac{\partial}{\partial x} dx + \frac{\partial}{\partial y} dy \right)^n \cdot z$ formulasi' menen yesaplanadi'.

Yeki argumentli funkciyalar ha'm wolardi'n' geometriyali'q ko'rinisi

1. Ani'qlama. Yeger x ha'm y wo'zgeriwshilerdin' qandayda bir wo'zgeriw oblasti'ndag'i' ha'r bir ma'nisleri jubi'na wo'zgeriwshi z tin' ani'q bir ma'nisi sa'ykes keltirilse, z wo'zgeriwshi x ha'm y wo'zgeriwshilerinin' bir ma'nisli funkciyasi' delinedi z tin' x ha'm y funktsional baylani'sli' boli'wi'

$$z = F(x, y) \qquad (57)$$

ko'riniste jazi'ladi'.

2. Geometriyali'q ko'rinisi. (57) ten'leme geometriyali'q ko'z qarastan qandayda bir betlikti ani'qlaydi', x ha'm y ti'n' ma'nisleri jubi' xOy tegislikte $P(x, y)$ noqatti' ani'qlaydi', $z = F(x, y)$ bolsa betliktegi wog'an sa'ykes $M(x, y, z)$ noqatti'n' applikatsiyasi'n ani'qlaydi'. Sol sebepli z wo'zgeriwshi $P(x, y)$ noqatti'n' funkciyasi' delinedi, ha'm $z = F(P)$ dep jazi'ladi'.

3. Funkciyani'n' limiti. Yeger ha'rekettegi P noqat ha'r qanday usi'l menen P_0 noqatqa jaqi'nlasqanda (ma'selen, qa'legen si'zi'q boylap) , yag'ni'y $\rho = P_0 P$ nolge umti'lg'anda $(\rho = P_0 P \to 0) F(P) - A$ ayi'rma sheksiz kishi bolsa, $\lim\limits_{P \to P_0} F(P) = A$ delinedi.

4. Funkciyani'n' u'zliksizligi. Yeger $\lim\limits_{P \to P_0} F(P) = F(P_0)$ bolsa $F(x, y)$ funkciya P_0 noqatta u'zliksiz delinedi. Basqasha aytqanda, yeger $\lim\limits_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} (x + \Delta x, y + \Delta y) = F(x, y)$ bolsa $F(x, y)$ funkciya (x, y) noqatta u'zliksiz delinedi.

Birinshi ta'rtipli dara tuwi'ndi'lar

$z = F(x, y)$ funkciyada y turaqli' dep qarap, wonnan x boyi'nsha ali'ng'an tuwi'ndi' z tin' x boyi'nsha dara tuwi'ndi'si' delinedi ha'm wol $\frac{\partial z}{\partial x}$ yaki $F_x'(x, y)$ ko'riniste belgilenedi. z tin' y boyi'nsha dara tuwi'ndi'si' da sog'an uqsas ani'qlanadi' ha'm belgilenedi:

$$\frac{\partial z}{\partial y} = F_y'(x, y)$$

Birinshi ta'rtipli toli'q differentsiyal

Yeger $z = F(x, y)$ funkciya (x, y) noqatta u'zliksiz dara tuwi'ndi'larg'a iye bolsa, woni'n' toli'q artti'rmasi'

$$\Delta z = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y + \varepsilon \cdot \rho \qquad (58)$$

ko'riniste jazi'ladi', bunda $\rho = \sqrt{|\Delta x|^2 + |\Delta y|^2}$ nolge umti'lg'anda $(\rho \to 0)\varepsilon \to 0$. Wonda $\frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$ an'latpa toli'q artti'rma Δz tin' bas bo'legi boladi'; wol funkciyani'n' toli'q differentsiyali' delinedi ha'm ∂z arqali' belgilenedi:

$$dz = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y \qquad (59)$$

Yeger (59) da z ti: 1) x qa 2) y ke ten' dep alsaq, $dx = \Delta x, dy = \Delta y$ boladi'. Soni'n' ushi'n

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \qquad (60)$$

(58) den

$$\Delta z \approx dz \qquad (61)$$

ekenligi kelip shi'g'adi', yag'niy Δx ha'm Δy jeterli kishi bolg'an halda funkciyanin' toli'q artti'rmasi' shama menen toli'q differentsiyalg'a ten' boladi'.

Yeger $F(x,y)$ funkciyanin' (x,y) noqatta toli'q differentsiyali' bar bolsa, funkciya bul noqatta differentsiyallani'wshi' delinedi.

Quramali' funkciyani'n' tuwi'ndi'lari'

1. Yeger $z = F(x,y)$ boli'p, $x = f(t), y = \varphi(t)$ bolsa, z,t ni'n' quramali' funkciyasi' delinedi. Wonda, yeger F, f ha'm φ differensiallani'wshi' funkciyalar bolsa,

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} \qquad (62)$$

boladi'.

2. Yeger $z = F(x,y)$ boli'p, $x = f(u,v), y = \varphi(u,v)$ ha'm F, f, φ differensiallani'wshi' funkciyalar bolsa,

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} \qquad (63)$$

boladi'.

Mi'sallar

1. $U = \sqrt{4 - x^2 - y^2}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'.

Sheshimi: Funkciya x ha'm y tin' $x^2 + y^2 \le 4$ ten'sizlikti qanaatlandi'ratug'i'n barli'q ma'nislerinde ani'qlang'an. Geometriyali'q jaqtan, bul $x^2 + y^2 = 4$ shen'berinin' ishinde ha'm shegarasi'nda jaylasqan barli'q tochkalardi' bildiredi.

2. $U = e^{\frac{x}{y}}$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'.

Sheshimi: Berilgen funkciya eni g'a'rezsiz x ha'm y wo'zgeriwshilerinin' funkciyasi'. Bulardi'n' biri menen differentsiyallag'ani'mi'zda, yekinshi wo'zgeriwshi turaqli' shama si'pati'nda qaraladi'. Demek, $\frac{\partial U}{\partial x} = \frac{1}{y} e^{\frac{x}{y}}; \frac{\partial U}{\partial y} = -\frac{x}{y^2} e^{\frac{x}{y}}$. Sebebi

$$\left(\frac{x}{y}\right)'_x = \frac{1}{y}; \left(\frac{x}{y}\right)'_y = -\frac{x}{y^2}$$

3. Yeger $z = \sin(3u + 2v - 4w)$, boli'p $u = 2x^3; v = 3x^2; w = x^4$ bolsa, $\frac{dz}{dx}$ ti tabi'n'.

Sheshimi:

$$\frac{\partial z}{\partial u} = 3\cos(3u + 2v - 4w); \frac{\partial z}{\partial v} = 2\cos(3u + 2v - 4w); \frac{\partial z}{\partial w} = -4\cos(3u + 2v - 4w)$$
$$\frac{du}{dx} = 6x^2; \frac{dv}{dx} = 6x; \frac{dw}{dx} = 4x^3$$

Demek,

$$\frac{dz}{dx} = \left[3\cos(3u + 2v - 4w)\right] \cdot 6x^2 + \left[2\cos(3u + 2v - 4w)\right] \cdot 6x - \left[4\cos(3u + 2v - 4w)\right] \cdot 4x^3 =$$
$$= (18x^2 + 12x - 16x^3) \cdot \cos(6x^3 + 6x^2 - 4x^4)$$

4. Yeger $z = f(x,u,v)$ boli'p, $u = \frac{1}{x}; v = \ln x$ bolsa, $\frac{dz}{dx}$ ti tabi'n'.

Sheshimi: $\frac{dz}{dx} = \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \cdot \frac{1}{x^2} + \frac{\partial z}{\partial v} \cdot \frac{1}{x}$

5. $z = x^2 y^2$ funkciyasi'ni'n' yekinshi ta'rtipli tuwi'ndi'si'n tabi'n'.

Sheshimi: $\frac{\partial z}{\partial x} = 2xy^2; \frac{\partial z}{\partial y} = 2x^2 y; \frac{\partial^2 z}{\partial x^2} = 2y^2; \frac{\partial^2 z}{\partial x \partial y} = 4xy; \frac{\partial^2 z}{\partial y^2} = 2x^2$

Bunnan, $d^2 z = 2y^2 dx^2 + 8xy \cdot dxdy + 2x^2 \cdot dy^2$

XI. DIFFERENTSIYAL TEN'LEMELER

1. n-ta'rtipli a'piwayi' differentsiyal ten'leme dep

$$F\left(x, y, y', y'', \dots, y^{(n)}\right) = 0 \quad (64)$$

ko'rinistegi ten'lemege ayti'ladi'.

Ten'lemedegi y worni'na qoyg'anda woni' birdeylikke aylandi'ri'wshi' $\varphi(x)$ funkciya ten'lemenin' sheshimi delinedi. Sol funkciyani' ani'qlawshi' $y = \varphi(x)$ yamasa $\Phi(x, y) = 0$ ten'leme differentsiyal ten'lemenin' integrali' delinedi. Ha'r bir integral xOy tegislikte differentsiyal ten'lemenin' integral si'zi'g'i' dep atali'wshi' iymer si'zi'qti' ani'qlaydi'.

Yeger x, y ha'm n qa'legen $C_1, C_2, \dots, C_n$ turaqli'lari'n wo'z ishine alg'an

$$\Phi(x, y, C_1, C_2, \dots, C_n) = 0 \quad (65)$$

ten'lemedegi qa'legen turaqli'larg'a ha'r qi'yli' ma'nisler bergende (64) ten'leme sheshimlerinin' bar ha'm birden birlik oblasti'nan wo'tiwshi ha'mme integral si'zi'qlar ha'm tek sol si'zi'qlar g'ana payda bolsa, (65) ten'leme (64) differentsiyal ten'lemenin' sol oblasti'ndag'i' uliwma integrali' delinedi.

Qa'legen turaqli'larg'a ani'q ma'nisler berip, uli'wma integraldan payda yetilgen integral dara integral delinedi.

Uli'wma integral (65) ti n ma'rte x boyi'nsha differentsiyallap, payda bolg'an n ten'lemeden ha'm (65) ten'lemeden n qa'legen turaqli'ni' jog'altsaq, berilgen (64) differentsiyal ten'lemege iye bolami'z.

2. Birinshi ta'rtipli differentsiyal ten'leme.

$$F\left(x, y, \frac{dy}{dx}\right) = 0 \qquad (66)$$

ko'riniske iye. (66) ten'lemeni $\frac{dy}{dx}$ ke sali'sti'rg'anda sheshiw mu'mkin bolsa, woni' sheship

$$\frac{dy}{dx} = f(x, y) \qquad (67)$$

ten'lemege iye bolami'z.

(67) ten'leme, integral si'zi'qti'n' (x, y) noqattag'i' $r = tg\alpha = \frac{dy}{dx} = f(x, y)$ qi'yali'g'i'n ani'qlaydi', yag'ni'y wol integral si'zi'qlar bag'i'tlari'ni'n' maydani'n ani'qlaydi'.

Yeger de qandayda bir oblastta $f(x,y)$ funkciya u'zliksiz bolsa ha'm shegaralang'an $f'_y(x,y)$ dara tuwi'ndi'g'a iye bolsa, wonda sol oblastti'n' ha'r bir ishki $(x_0;y_0)$ noqati'nan birden-bir integral si'zi'q wo'tetug'i'n eken.

Bunday oblastta (67) ten'leme $y=\varphi(x)$ yaki $\Phi(x,y,c)=0$ uli'wma integralg'a iye; bul uli'wma sheshimnen $x=x_0$ bolg'anda $y=y_0$ bolatug'i'n da'slepki sha'rtlerdi qanaatlandi'ri'wshi' birden-bir dara integral tabi'w mu'mkin.

O'zgeriwshileri aji'ralatug'i'n 1-ta'rtipli differentsiyal ten'leme. Ortogonal traektoriyalar

1-ta'rtipli

$$Pdx+Qdy=0 \qquad (68)$$

differentsiyal ten'lemedegi differentsiyallar aldi'ndag'i' P ha'm Q koeffitsientler tek x yaki tek y tin' funkciyalari'nan ibarat bolg'an ko'beytiwshilerge aji'ralsa, yag'ni'y yeger de ten'leme

$$f(x)\varphi(y)dx+f_1(x)\varphi_1(y)dy=0 \qquad (69)$$

ko'riniste bolsa, (68) ten'leme wo'zgeriwshileri aji'ralatug'i'n ten'leme delinedi.

(69) ten'lemenin' yeki ag'zasi'n $\varphi(y)\,f_1(x)$ ko'beymege bo'lip,

$$\frac{f(x)dx}{f_1(x)}+\frac{\varphi_1(y)dy}{\varphi(y)}=0 \qquad (70)$$

ten'lemeni payda yetemiz.

(68) ten'lemenin', demek, (69) ten'lemenin' uli'wma integrali'

$$\int\frac{f(x)dx}{f_1(x)}+\int\frac{\varphi_1(y)dy}{\varphi(y)}=C \quad (71)$$

dan ibarat boladi'.

Berilgen $F(x,y,a)=0$ si'zi'qlar da'stesinin' ha'r bir si'zi'g'i'n tuwri' mu'yesh asti'nda kesiwshi si'zi'qlar usi' si'zi'qlar da'stesinin' worthogonal traektoriyalari' delinedi.

$F(x,y,a)=0$ ten'lemeni x boyi'nsha differentsiyallap, payda bolg'an ha'm $F(x,y,a)=0$ ten'lemelerden a jog'alti'lsa, berilgen si'zi'qlar da'stesinin' $y'=f(x,y)$ differentsiyal ten'lemesine iye bolami'z. Wonda worthogonal traektoriyalardi'n' differentsiyal ten'lemesi $y'=-\frac{1}{f(x,y)}$ ten ibarat boladi'.

1-ta'rtipli: 1) bir tekli, 2) si'zi'qli' differentsiyal ten'lemeler ha'm 3) Bernulli differentsiyal ten'lemesi

Bir tekli ten'leme. $Pdx + Qdy = 0$ ko'rinistegi ten'leme P ha'm Q x ha'm y tin' birdey wo'lshemli bir tekli funkciyalari' bolg'anda, bir tekli ten'leme delinedi. Bul ten'leme $\frac{dy}{dx} = \varphi\left(\frac{y}{x}\right)$ ko'riniske keltirilip, $\frac{y}{x} = u$ yaki $y = ux$ almasti'ri'w menen sheshiledi.

Si'zi'qli' ten'leme. I''zleniwshi y ha'm woni'n' ba'rshe tuwi'ndi'lari'na sali'sti'rg'anda birinshi da'rejeli bolg'an ten'leme si'zi'qli' delinedi. 1-ta'rtipli si'zi'qli' ten'leme $y^1 + Py = Q$ ko'riniske iye. $y = uv$ almasti'ri'w menen bul ten'leme wo'zgeriwshileri aji'ralatug'i'n yeki ten'lemege keltiriledi. 1-ta'rtipli si'zi'qli' ten'lemenin' sheshiw jollari'nan ekinshisi (qa'legen turaqli'ni' variatsiyalaw) to'mendegiden ibarat: aldi'n $y' + Py = 0$ ten'lemeni sheship, $y = Ae^{-\int Pdx}$ sheshimin tabami'z. Bundag'i' A ni' x tin' funkciyasi'n esaplap, berilgen ten'lemege qoyami'z. Bunnan A' ha'm A ni' tabami'z.

Bernulli ten'lemesi. $y' + Py = Qy^n$ si'zi'qli' ten'lemege uqsas $y = ux$ almasti'ri'w menen yaki qa'legen turaqli'ni' variatsiyalaw menen sheshiledi. Bernulli ten'lemesi $z = y^{1-n}$ almasti'ri'wlar na'tiyjesinde si'zi'qli' ten'lemege keltiriledi.

Tuwi'ndi'g'a sali'sti'rg'anda sheshilmegen 1-ta'rtipli differentsiyal ten'lemeler. Lagranj ha'm Klero ten'lemeleri

Yeger $F(x, y, y') = 0$ ten'leme y' qa sali'sti'rg'anda 2-da'rejeli bolsa, bul ten'leme y' qa sali'sti'rg'anda yeki qandayda bir oblastta x ha'm y qa sali'sti'rg'anda u'zliksiz $y' = f_1(x, y)$ ha'm $y' = f_2(x, y)$ sheshimge iye. Geometriyali'q pikirlewlerden bul ten'leme sol oblastti'n' qa'legen (x_0, y_0) noqati'nda yeki integral si'zi'g'i'ni'n' bag'i'tlari'n ani'qlaydi'.

Bunday $F(x, y, y') = 0$ differentsiyal ten'lemeler, $\Phi(x, y, C) = 0$ uli'wma ha'm dara integrallardan ti'sqari' qa'legen turaqli'ni' wo'z ishine almag'an ha'm uli'wma integraldan qa'legen turaqli'g'a qandayda bir ma'nis beriwden payda bolmaytug'i'n arnawli' integralg'a iye boladi'.

Arnawli' integral bar bolsa, woni' $F(x, y, p) = 0$ ha'm $F'_p(x, y, p) = 0$ ten'lemelerden $y' = p$ ni' joq yetip yaki uli'wma integral $\Phi(x, y, C) = 0$ menen

$\Phi'_c(x, y, C) = 0$ dan C ni' joq yetip tabi'w mu'mkin. Geometriyali'q ko'z qarastan arnawli' integral si'zi'qlar da'stesinin' worami'n ani'qlaydi'.

Lagranj ten'lemesi:

$$y = xf(p) + \varphi(p) \qquad (72)$$

ko'riniste jazi'ladi', bunda $p = y'$. Bul ten'leme to'mendegishe integrallanadi'.

(72) ni x boyi'nsha differentsiyallap,

$$p = f(x) + [xf'(p) + \varphi'(p)]\frac{dp}{dx}$$

ten'lemeni tabami'z.

Bul ten'leme x ha'm $\frac{dx}{dp}$ qa sali'sti'rg'anda si'zi'qli'. Woni' sheship

$$x = CA(p) + B(p) \quad (73)$$

g'a iye bolami'z.

(72) ha'm (73) ten'lemeler uli'wma integraldi' parameter arqali' ani'qlaydi'. Bul ten'lemelerden (mu'mkin bolsa) p ni' joq yetip, $\Phi(x, y, C) = 0$ ko'rinistegi uli'wma integralg'a iye bolami'z.

Klero ten'lemesi

$$y = px + \varphi(p) \quad (74)$$

Lagranj ten'lemesinin' dara hali' boli'p tabi'ladi'. Bul ten'leme $y = Cx + \varphi(C)$ uli'wma integralg'a ha'm $y = px + \varphi(p)$ ha'm $x = -\varphi'(p)$ ten'lemelerden p parametrdi joq yetiwden payda bolatug'i'n arnawli' sheshimge iye.

Mi'sallar

1. $y' = x + 2$ differentsiyal ten'lemeni sheshin'

Sheshiliwi: Tuwi'ndi'si' $x+2$ ge ten' bolg'an $y(x)$ funkciyani', yag'ni'y $x+2$ funkciyani'n' da'slepki funkciyasi'n tabami'z ha'm to'mendegige iye bolami'z: $y = \frac{x^2}{2} + 2x + C$, bul jerde C – turaqli' shama.

2. $y = \cos x$ differentsiyal ten'lemenin' $y(0) = 2$ sha'rtti qanaatlandi'ri'wshi' sheshimin tabi'n'

Sheshiliwi: Bul ten'lemenin' barli'q sheshimleri $y = \sin x + C$ ko'rinisinde jazi'ladi'. $y(0) = 2$ sha'rtinen $\sin 0 + C = 2$ ge iye bolami'z, bunnan $C = 2$ ni tabami'z. Demek ten'lemenin' sheshimi: $y = \sin x + 2$

3. $y = \frac{e^x}{1+e^x} \cdot \frac{1}{y}$ differentsiyal ten'lemenin' $y(0) = \sqrt{2}$ da'slepki sha'rtin qanaatlandi'ri'wshi' dara sheshimin tabi'n'.

Sheshiliwi: $y' = \frac{dy}{dx}$ dep, ten'lemenin' wo'zgeriwshilerin aji'ratami'z. $ydy = \frac{e^x}{1+e^x}dx$. Buni' integrallap uli'wma sheshimin tabami'z: $\frac{y^2}{2} = \ln\left(1+e^x\right) + C$, bunda $2C = \ln C$ dep alsaq, $y = \sqrt{2\ln C \cdot (1+e^x)}$ qa iye bolami'z. Bul uli'wma sheshimge $x = 0; y = \sqrt{2}$ ni qoyi'p, $C = \frac{e}{2}$ ni tabami'z. Demek, izlenip ati'rg'an dara sheshim: $y = \sqrt{2\ln\frac{e}{2}\left(1+e^x\right)}$ ko'rinisinde boladi'.

BAQLAW JUMI''SLARI''

1-Kundelik baqlaw tapsi'rmalari'

I-variant

1. $A=\{2;3;5;7;9;10\}$ $B=\{3;5;7;8\}$ $C=\{4;7;11\}$ ko'plikleri berilgen bolsa, $A\cap(B\cup C)$ ko'pliginin' elementlerin ko'rsetin'.

2. To'beleri $A(3;1), B(4;6), C(6;3)$ ha'm $D(5;-2)$ tochkalarda bolg'an to'rtmu'yeshliktin' maydani'n yesaplan'.

3. $A(-3;1)$ ha'm $B(4;2)$ tochkalari' jasalsi'n ha'm AB kesindini $AN:NB=3:2$ qatnasi'nda bo'liwshi $N(x;y)$ tochka tabi'lsi'n.

4. Worayi' $C(3;4)$ tochkada, radiusi' $R=5$ bolg'an shen'berdin' ten'lemesi jazi'lsi'n. $A(-1;1), B(2;3), O(0;0)$ ha'm $D(4;1)$ tochkalar usi' shen'berde jatama?

5. $x=4$ bolg'anda $\left|\dfrac{x-8}{5-x^2}\right|$ an'latpasi'ni'n' san ma'nisin ani'qlan'.

6. $y=\dfrac{5-x}{\sqrt[3]{x-5}}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n':

7. $\begin{cases} x_1+2x_2-x_3=-3 \\ 2x_1+3x_2+x_3=-1 \\ x_1-x_2-x_3=3 \end{cases}$ ten'lemeler sistemasi'n Kramer formulasi' menen sheshin'.

8. $a=2m-n$ vektor berilgen boli'p, bunda m ha'm n-aralari'ndag'i' mu'yesh $120°$ qa ten' bolg'an birlik vektorlar $\cos(a;m)$ ha'm $\cos(a;n)$ lerdi tabi'n':

9. To'mendegi tuwri' si'zi'qlar arasi'ndag'i' mu'yeshti ani'qlan'. $\begin{cases} y=2x-3 \\ y=\dfrac{1}{2}x+1 \end{cases}$

10. $5x-12y+26=0$ tuwri' ten'lemesin normal tu'rge keltirin'.

II-variant

1. $A=\{2;3;5;7;9;10\}$ $B=\{3;5;7;8\}$ $C=\{4;7;11\}$ ko'plikleri berilgen bolsa, $A\cup(B\cap C)$ ko'pliginin' elementlerin ko'rsetin'.

2. To'beleri $A(-2;1), B(2;3), C(2;-4)$ ha'm $D(-2;-3)$ tochkalarda bolg'an to'rtmu'yeshliktin' maydani'n yesaplan'.

3. $A(-4;-3)$ ha'm $B(5;3)$ tochkalari' jasalsi'n ha'm AB kesindini ten' wortadan bo'liwshi $N(x;y)$ tochka tabi'lsi'n.

4. $y = x^2 - 4x + 3$ si'zi'qti'n' koordinata ko'sherleri menen kesilisken tochkalari' ani'qlansi'n ha'm si'zi'q jasalsi'n.

5. $x = 0$ bolg'anda $\left|\frac{2x^2 - 4}{5 - x}\right|$ an'latpasi'ni'n' san ma'nisin ani'qlan'.

6. $y = \arcsin\left(\frac{x}{2} - 1\right)$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n':

7. $\begin{cases} 3x + y + z = -2 \\ 5x - y - z = 10 \\ x - y + 5z = -12 \end{cases}$ ten'lemeler sistemasi'n Kramer formulasi' menen sheshin'.

8. $\overrightarrow{OA} = a$ ha'm $\overrightarrow{OB} = b$ vektorlar berilgen. $a = 2, b = 4$ ha'm $(a; b) = 60°$, woAB u'shmu'yeshliginin' $\overrightarrow{OM}$ medianasi' menen $\overrightarrow{OA}$ ta'repi arasi'ndag'i' mu'yesh ani'qlansi'n.

9. To'mendegi tuwri' si'zi'qlar arasi'ndag'i' mu'yeshti ani'qlan'. $\begin{cases} 5x - y + 7 = 0 \\ 2x - 3y + 1 = 0 \end{cases}$

10. $7x + y - 3 = 0$ tuwri' ten'lemesin normal tu'rge keltirin'.

III-variant

1. $A = \{2;3;5;7;9;10\}$ $B = \{3;5;7;8\}$ $C = \{4;7;11\}$ ko'plikleri berilgen bolsa, $A \cap (B \cap C)$ ko'pliginin' elementlerin ko'rsetin'.

2. To'beleri $A(-4;2), B(-1;3), C(2;1)$ ha'm $D(-2;-2)$ tochkalarda bolg'an to'rtmu'yeshliktin' maydani'n yesaplan'.

3. $A(-2;4)$ ha'm $B(2;-4)$ tochkalari' jasalsi'n ha'm AB kesindini $AN : NB = 3 : 2$ qatnasi'nda bo'liwshi $N(x; y)$ tochka tabi'lsi'n.

4. $3x - 2y = 12$ si'zi'qti'n' koordinata ko'sherleri menen kesilisken tochkalari' ani'qlansi'n ha'm si'zi'q jasalsi'n.

5. $x = 2$ bolg'anda $\left|\frac{2x + 5}{7 - 2x^2}\right|$ an'latpasi'ni'n' san ma'nisin ani'qlan'.

6. $y = \arccos(3x - 5)$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n':

7. $\begin{cases} 2x_1 + x_2 - x_3 = 0 \\ x_1 - x_2 - 3x_3 = 13 \\ 3x_1 - 2x_2 + 4x_3 = -15 \end{cases}$ ten'lemeler sistemasi'n Kramer formulasi' menen sheshin'.

8. Ta'repleri 6 ha'm 4sm bolg'an tuwri' to'rtmu'yeshliktin' to'besinen qarsi' ta'replerin ten' yekige bo'liwshi tuwri' si'zi'qlar ju'rgizilgen. Usi' tuwri' si'zi'qlar arasi'ndag'i' φ mu'yesh tabi'lsi'n.

9. To'mendegi tuwri' si'zi'qlar arasi'ndag'i' mu'yeshti ani'qlan'. $\begin{cases} 2x + y = 0 \\ y = 3x - 4 \end{cases}$

10. $6x - 8y - 15 = 0$ tuwri' ten'lemesin normal tu'rge keltirin'.

IV-variant

1. $A = \{2;3;5;7;9;10\}$ $B = \{3;5;7;8\}$ $C = \{4;7;11\}$ ko'plikleri berilgen bolsa, $B \cap (A \cup C)$ ko'pliginin' elementlerin ko'rsetin'.

2. To'beleri $A(1;), B(3;3), C(3;-2)$ ha'm $D(1;-2)$ tochkalarda bolg'an to'rtmu'yeshliktin' maydani'n yesaplan'.

3. $A(3;3)$ ha'm $B(3;-5)$ tochkalari' jasalsi'n ha'm AB kesindini ten' wortadan bo'liwshi $N(x;y)$ tochka tabi'lsi'n.

4. $y = x^2 + 4x$ si'zi'qti'n' koordinata ko'sherleri menen kesilisken tochkalari' ani'qlansi'n. Usi' si'zi'q jasalsi'n.

5. $x = -3$ bolg'anda $\left|\frac{5 - x^2}{1 - x}\right|$ an'latpasi'ni'n' san ma'nisin ani'qlan'.

6. $y = \log_2(x-1) + x^2$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n':

7. $\begin{cases} x_1 + 3x_2 - 3x_3 = 13 \\ 2x_1 - 3x_2 + 3x_3 = -10 \\ x_1 + x_3 = 0 \end{cases}$ ten'lemeler sistemasi'n Kramer formulasi' menen sheshin'.

8. $A(3;3;-2), B(0;-3;4), C(0;-3;0)$ ha'm $D(0;2;-4)$ tochkalar berilgen. $\overrightarrow{AB} = a$ ha'm $\overrightarrow{CD} = b$ vektorlar jasalsi'n ha'm $ïð_a b$ tabi'lsi'n.

9. To’mendegi tuwri’ si’zi’qlar arasi’ndag’i’ mu’yeshti ani’qlan’. $\begin{cases} 3x+2y=0 \\ 6x+4y+9=0 \end{cases}$

10. $x+3y-4=0$ tuwri’ ten’lemesin normal tu’rge keltirin’.

2-Kundelik baqlaw tapsi’rmalari’

I-variant

1. $x^2+y^2+4x-6y-3=0$ din’ shen’ber ten’lemesi ekenligin ko’rsetin’. Woni’n’ worayi’ ha’m radiusi’n tabi’n’:

2. $5x-2y+3z-10=0$ tegisligin jasan’.

3. $x-2y+2z-8=0$ ha’m $x+z-6=0$ tegislikler arasi’ndag’i’ mu’yesh tabi’lsi’n.

4. $A(-1;2;3)$ ha’m $B(2;6;-2)$ tochkalardan wo’tiwshi tuwri’ si’zi’q ten’lemeleri jazi’lsi’n ha’m woni’n’ bag’i’tlawshi’ kosinuslari’ tabi’lsi’n.

5. $\frac{x^2}{a^2}+\frac{y^2}{c^2}=1; y=0$ ellipstin’ woz ko’sheri do’gereginde aylani’wi’nan payda bolg’an bet ten’lemesin jazi’n’.

6. To’rtmu’yeshli duri’s prizmani’n’ ultani’ni’n’ maydani’ $144\ \text{sm}^2$, biyikligi 14 sm. Usi’ prizmani’n’ diagonali’n tabi’n’:

7. Tsilindrdin’ toli’q beti $50\ \text{sm}^2$, qaptal beti $30\ \text{sm}^2$ bolsa, tsilindrdin’ radiusi’n tabi’n’:

8. Limitti yesaplan’. $\lim\limits_{n\to\infty}\frac{n^2-1}{n^2+n+1}$

9. Limitti yesaplan’. $\lim\limits_{x\to 2} 4^{\frac{2x}{x+1}}$

10. $f(x)=\frac{1}{x}$ funkciyasi’n u’zliksizlikke tekserin’.

II-variant

1. Ellipstin’ fokuslari’ arasi’ndag’i’ arali’q $2c=10$, al u’lken ko’sheri $2a=16$ beligili bolsa, ellipstin’ a’piwayi’ ten’lemesin du’zin’.

2. $3x+2y-z=0$ tegisligin jasan’.

3. $x+2z-6=0$ ha’m $x+2y-4=0$ tegislikler arasi’ndag’i’ mu’yesh tabi’lsi’n.

4. $A(2;-1;3)$ ha'm $B(2;3;3)$ tochkalardan wo'tiwshi tuwri' si'zi'q jasalsi'n ha'm woni'n' ten'lemeleri jazi'lsi'n.

5. $\frac{x^2}{16}+\frac{y^2}{4}-\frac{z^2}{36}=1$ giperbaloid jasalsi'n ha'm woni'n' $(4;1;-3)$ tochkadan wo'tiwshi jasawshi'lari' tabi'lsi'n.

6. Duri's parallelepipedtin' ultani'ni'n' ta'repleri 8 ha'm 4 ke ten' boli'p, wolar $60°$ li' mu'yesh payda yetedi. Parallelepipedtin' kishi diagonali' $8\sqrt{3}$ ke ten' bolsa, usi' diagonaldi'n' ultan tegisligi menen payda yetken mu'yeshin tabi'n':

7. Jasawshi'si' l boli'p, ultan tegisligi menen $60°$ li' mu'yesh payda yetiwshi konusti'n' ultani'ni'n' maydani'n tabi'n':

8. Limitti yesaplan'. $\lim\limits_{n\to\infty}\frac{n^2-n-1}{2n^2+n-1}$

9. Limitti yesaplan'. $\lim\limits_{x\to\infty}2^{\frac{3x}{x+2}}$

10. $f(x)=x^2+1$ funkciyasi'n u'zliksizlikke tekserin'.

III-variant

1. Giperbola $\left(3;\frac{2\sqrt{15}}{5}\right)$ ha'm $(-2\sqrt{5};3)$ tochkalari' arqali' wo'tedi. Giperbolani'n' ten'lemesin tabi'n':

2. $3x+2z=6$ tegisligin jasan'.

3. $(2;2;-2)$ tochkadan wo'tiwshi ha'm $x-2y-3z=0$ tegislikke parallel' tegislik tabi'lsi'n.

4. $A(3;-1;4)$ ha'm $B(1;1;2)$ tochkalardan wo'tiwshi tuwri' si'zi'qti'n' ten'lemeleri jazi'lsi'n.

5. $\frac{x^2}{16}-\frac{y^2}{9}=2z$ giperbolali'q parabaloidti'n' $(4;3;0)$ tochkadan wo'tiwshi tuwri' si'zi'qli' jasawshi'lari'ni'n' ten'lemeleri jazi'lsi'n.

6. Duri's to'rtmu'yeshli piramidani'n' biyikligi 6 sm, apofemasi' 6,5 sm, Piramidani'n' ultani'ni'n' perimetrin tabi'n':

7. Betinin' maydani' $16\ \pi$ ge ten' bolg'an shardi'n' ko'lemin tabi'n':

8. Limitti yesaplan'. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n}}{3\sqrt{n}+2}$

9. Limitti yesaplan'. $\lim\limits_{x\to 2} a^{\frac{\sqrt{2+x}-2}{x-2}}\ (a>0)$

10. $f(x)=\dfrac{3x^2+x+5}{x^2-6x+8}$ funkciyasi' x qanday ma'nisinde u'zliksiz.

IV-variant

1. Parabola wox ko'sherine qarata simmetriyali' ha'm wol $A(4;-1)$ tochkasi' arqali' wo'tedi, al to'besi koordinata basi'nda jatadi'. Usi' parabolani'n' ten'lemesin du'zin.

2. $2z-7=0$ tegisligin jasan'.

3. $(-1;-1;2)$ tochkadan wo'tiwshi ha'm $x-2y+z-4=0$ ja'ne $x+2y-2z+4=0$ tegisliklerge perpendikulyar tegisliktin' ten'lemesi jazi'lsi'n.

4. $\begin{cases} y=2 \\ z=x+1 \end{cases}$ tuwri' si'zi'qlar jasalsi'n ha'm wolardi'n' bag'i'tlawshi' vektorlari' ani'qlansi'n.

5. $\dfrac{x^2}{169}+\dfrac{y^2}{25}+\dfrac{z^2}{9}=1$ ellipsoidti'n' en' u'lken do'n'gelek kesimi tabi'lsi'n.

6. Duri's to'rtmu'yeshli kesik piramidani'n' diagonallari' wo'z-ara perpendikulyar ha'm wolardi'n' ha'r biri 8 ge ten'. Piramidani'n' biyikligin tabi'n':

7. Ko'lemi $432\ \pi$ ge ten' bolg'an tsilindrge ishley si'zi'lg'an shar betinin' maydani'n tabi'n':

8. Limitti yesaplan'. $\lim\limits_{n\to\infty}\dfrac{2^n-1}{2^n}$

9. Limitti yesaplan'. $\lim\limits_{x\to 0}(1+x)^{\frac{1}{3x}}$

10. $f(x)=\dfrac{3x^2-x-4}{3x^2+x+5}$ funkciyasi' x tin' qanday ma'nisinde u'zliksiz.

3-Kundelik baqlaw tapsi'rmalari'

I-variant

1. Funkciyani'n' tuwi'ndi'si'n tabi'n': $y = x^2 \cdot tgx$

2. Funkciyani'n' tuwi'ndi'si'n tabi'n': $y = \ln\left(e^{-x} + xe^{-x}\right)$

3. Funkciyani'n' 3-ta'rtipli tuwi'ndi'si'n tabi'n': $y = x\ln x$

4. Funkciya differentsiyali'n tabi'n': $d(\sin^2 t)$

5. $f(x) = x^3 - 3x - 2$ funkciyani'n' wo'siw ha'm kemeyiw arali'qlari'n tabi'n': woni'n' grafiklerin si'zi'n'?

6. $y = (x-1)^3 \cdot (x+1)^2$ funkciyani'n' ekstremumi'n tabi'n':

7. İntegraldi' tabi'n': $\int\left(x^2 + 2x + \frac{1}{x}\right)dx$

8. İntegraldi' tabi'n': $\int \sin^2 x \cdot \cos^2 x dx$

9. Bo'leklep integrallaw formulasi'nan paydalani'p, integraldi' tabi'n': $\int e^x \sin x dx$

10. İntegraldi' tabi'n': $\int \frac{\ln(x+1)dx}{x^2}$

II-variant

1. Funkciyani'n' tuwi'ndi'si'n tabi'n': $y = \frac{\cos x}{1 + 2\sin x}$

2. Funkciyani'n' tuwi'ndi'si'n tabi'n': $y = \ln\frac{e^x}{x^2 + 1}$

3. Funkciyani'n' 3-ta'rtipli tuwi'ndi'si'n tabi'n': $y = arctg\,\frac{x}{a}$

4. Funkciya differentsiyali'n tabi'n': $d\left(\frac{a}{x} + arctg\,\frac{x}{a}\right)$

5. $y = \frac{2x}{1 + x^2}$ funkciyani'n' wo'siw ha'm kemeyiw arali'qlari'n tabi'n': woni'n' grafiklerin si'zi'n'?

6. $y = 2x + 3\sqrt[3]{(2-x)^2}$ funkciyani'n' ekstremumi'n tabi'n':

7. İntegraldi' tabi'n': $\int \frac{10x^8 + 3}{x^4} dx$

8. İntegraldi’ tabi’n’: $\int \frac{dx}{x^2+4x+29}$

9. Bo’leklep integrallaw formulasi’nan paydalani’p, integraldi’ tabi’n’: $\int x^2 \cdot \cos x dx$

10. İntegraldi’ tabi’n’: $\int \frac{\ln(\cos x)}{\sin^2 x} dx$

III-variant

1. Funkciyani’n’ tuwi’ndi’si’n tabi’n’: $y = \left(1+\frac{1}{\sqrt[3]{x}}\right)^2$

2. Funkciyani’n’ tuwi’ndi’si’n tabi’n’: $y = \ln\cos x - \frac{1}{2}\cos^2 x$

3. Funkciyani’n’ 3-ta’rtipli tuwi’ndi’si’n tabi’n’: $y = x \cdot e^{\frac{x}{a}}$

4. Funkciya differentsiyali’n tabi’n’: $d\left(\arcsin\frac{1}{x}\right)$

5. $f(x) = \frac{2x^2}{1-x^2}$ funkciyani’n’ wo’siw ha’m kemeyiw arali’qlari’n tabi’n’: woni’n’ grafiklerin si’zi’n’?

6. $y = 4x^3 - 21x^2 + 18x + 20$ funkciyani’n’ ekstremumi’n tabi’n’.

7. İntegraldi’ tabi’n’: $\int \left(\sqrt{x} + \sqrt[3]{x}\right) dx$

8. İntegraldi’ tabi’n’: $\int \frac{dx}{\sqrt{4x^2+4x+3}}$

9. Bo’leklep integrallaw formulasi’nan paydalani’p, integraldi’ tabi’n’: $\int x \cdot e^{2x} dx$

10. İntegraldi’ tabi’n’: $\int \frac{e^{2x}dx}{e^{2x}-1}$

IV-variant

1. Funkciyani’n’ tuwi’ndi’si’n tabi’n’: $y = \sqrt{x} \cdot \cos x$

2. Funkciyani’n’ tuwi’ndi’si’n tabi’n’: $y = \ln\left(e^{2x} + \sqrt{e^{4x}+1}\right)$

3. Funkciyani'n' 3-ta'rtipli tuwi'ndi'si'n tabi'n': $y = e^x \cdot \cos x$

4. Funkciya differentsiyali'n tabi'n': $d(1-\cos x)$

5. $y = 4x^3 - 21x^2 + 18x + 20$ funkciyani'n' wo'siw ha'm kemeyiw arali'qlari'n tabi'n': woni'n' grafiklerin si'zi'n'.

6. $y = x^4 - \frac{20}{3}x^3 + 8x^2$ funkciyani'n' ekstremumi'n tabi'n'.

7. İntegraldi' tabi'n': $\int\left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt[3]{x^2}}\right)dx$

8. İntegraldi' tabi'n': $\int \sin 3x \cdot \cos x dx$

9. Bo'leklep integrallaw formulasi'nan paydalani'p, integraldi' tabi'n': $\int \frac{\ln x dx}{x^2}$

10. İntegraldi' tabi'n': $\int \frac{1+\cos x}{\sin^2 x} dx$

4-Kundelik baqlaw tapsi'rmalari'

I-variant

1. Ani'q integraldi' yesaplan': $\int_0^{\frac{\pi}{4}} \sin 4x dx$

2. To'mendegi si'zi'qlar menen shegaralang'an maydan esaplansi'n: $y = 4 - x^2;\ \ y = 0$

3. İntegraldi' yesaplan': $\int_0^{\frac{\pi}{2}} \sin^2 x dx$

4. $z = x^2 + y^2$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n';

5. $u = ax + by + cz$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n';

6. $z = \frac{ay - bx}{by - ax}$ funkciyasi'ni'n' toli'q differentsiyali'n tabi'n';

7. $z = x^3y^2(12 - x - y)$ funkciyani'n' ekstremumi'n tabi'n';

8. Differentsiyal ten'lemeni sheshin': $xy + y^2 = (2x^2 + xy)y'$;

9. $y = xy' - y'^2$ Klero ten'lemesinin' uli'wma ha'm arnawli' integrali'n tabi'n';

10. $x = t, y = t^2, z = t^3$ qa'legen tochkasi'nda ha'm $t = 1$ bolg'anda, berilgen iymek si'zi'qlardi'n' uri'nba si'zi'g'i'ni'n' ha'm normal tegisliginin' ten'lemesin jazi'n' ;

(Ko'rsetpe: Ha'r bir ten'lemenin' won' ha'm shep ta'replerinen differentsiyal ali'p, son' $dx : dy : dz$ qatnaslar tabi'lsi'n.).

II-variant

1. Ani'q integraldi' yesaplan': $\int_0^3 e^{\frac{x}{3}} dx$

2. To'mendegi si'zi'qlar menen shegaralang'an maydan esaplansi'n: $y = 3 - 2x - x^2;\ \ y = 0$

3. İntegraldi' yesaplan': $\int_0^{\frac{\pi}{4}} \sin^4 x dx$

4. $z = \sqrt{4 - x^2 - y^2}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n';

5. $u = y \sin x + \sin y$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n';

6. $z = arctg \frac{y}{x}$ funkciyasi'ni'n' toli'q differentsiyali'n tabi'n';

7. $z = xy(xy(x + y - 1))$ funkciyani'n' ekstremumi'n tabi'n';

8. Differentsiyal ten'lemeni sheshin': $(a^2 + x^2)y' + xy = 1$;

9. $y = xy'^2 + y'^2$ Logranj ten'lemesinin' uli'wma ha'm arnawli' integrali'n tabi'n';

10. $y = x^2, z^2 = x$ qa'legen tochkasi'nda $(x \geq 0)$ ha'm $x = 4$ bolg'anda, berilgen iymek si'zi'qlardi'n' uri'nba si'zi'g'i'ni'n' ha'm normal tegisliginin' ten'lemesin jazi'n';

(Ko'rsetpe: Ha'r bir ten'lemenin' won' ha'm shep ta'replerinen differentsiyal ali'p, son' $dx : dy : dz$ qatnaslar tabi'lsi'n.).

III-variant

1. Ani'q integraldi' yesaplan': $\int_0^{\frac{\pi}{2}} \sin x \cdot \cos^2 x dx$

2. To'mendegi si'zi'qlar menen shegaralang'an maydan esaplansi'n: $xy = 4;\ \ x = 1$, $x = 4\ \ y = 0$

3. İntegraldi’ yesaplan’: $\int_0^{\frac{\pi}{2}} \cos^2 x dx$;

4. $z = \ln(x - y)$ funkciyasi’ni’n’ ani’qlani’w oblasti’n tabi’n’;

5. $u = x^{\sin y} (x > 0)$ funkciyasi’ni’n’ dara tuwi’ndi’si’n tabi’n’;

6. $z = x \sin y + y \sin x$ funkciyasi’ni’n’ toli’q differentsiyali’n tabi’n’;

7. $z = x^3 + y^2 - 6xy - 39x + 18y + 20$ funkciyani’n’ ekstremumi’n tabi’n’;

8. Differentsiyal ten’lemeni sheshin’: $xy' + 2\sqrt{xy} = y$;

9. $y = xy' + \frac{1}{2y'^2}$ Klero ten’lemesinin’ uli’wma ha’m arnawli’ integrali’n tabi’n’;

10. $\begin{cases} x^2 + y^2 = 10 \\ y^2 + z^2 = 25 \end{cases}$ (1;3;4) tochkasi’nda berilgen iymek si’zi’qlardi’n’ uri’nba si’zi’g’i’ni’n’ ha’m normal tegisliginin’ ten’lemesin jazi’n’;

(Ko’rsetpe: Ha’r bir ten’lemenin’ won’ ha’m shep ta’replerinen differentsiyal ali’p, son’ $dx : dy : dz$ qatnaslar tabi’lsi’n.).

IV-variant

1. Ani’q integraldi’ yesaplan’: $\int_0^{\sqrt{3}} \frac{x dx}{\sqrt{4 - x^2}}$

2. To’mendegi si’zi’qlar menen shegaralang’an maydan esaplansi’n: $y = x^2 + 4x, \quad y = x + 4$

3. İntegraldi’ yesaplan’: $\int_0^{\frac{\pi}{2}} \cos^4 x dx$

4. $z = xy$ funkciyasi’ni’n’ ani’qlani’w oblasti’n tabi’n’;

5. $u = z^{xy} (z > 0)$ funkciyasi’ni’n’ dara tuwi’ndi’si’n tabi’n’;

6. $z = x + y \cdot e^{\frac{x}{y}}$ funkciyasi’ni’n’ toli’q differentsiyali’n tabi’n’;

7. $u = x^2 + y^2 + z^2 - xy + x - 2z$ funkciyani’n’ ekstremumi’n tabi’n’;

8. Differentsiyal ten’lemeni sheshin’: $(2x + 1)y' + y = x$;

9. $y = 2xy' + \frac{1}{y'^2}$ Logranj ten'lemesinin' uli'wma ha'm arnawli' integrali'n tabi'n';

10. $x = e^t, y = e^{-t}, z = t$ iymek si'zi'qti'n' $t = 0$ tochkasi'nda ju'rgizilgen bas normal ha'm binormaldi'n' ten'lemelerin jazi'n'.

1-arali'q baqlaw ushi'n test tapsi'rmalari'

I-variant

(Duri's juwabi'n ko'rsetin')

1. A={1, 2, 3, 5, 7, 8} ha'm B={2, 3, 4, 5, 9} ko'pliklerinin' kesilispesin tabi'n'.

a) $A \cap B$={2, 3, 5}, b) $A \cap B$={1, 2, 7, 8}, c) $A \cap B$={4, 9}, d) $A \cap B$={1, 7, 8}

2. A={6, 7, 8, 9, 10} ha'm B={2, 3, 5, 6, 9, 10}ko'pliklerinin' birikpesin tabi'n'.

a) $A \cup B$={6, 7, 8, 10} b) $A \cup B$={2, 5, 3, 10, 8}

c) $A \cup B$={2, 3, 5} d) $A \cup B$={2, 3, 5, 6, 7, 8, 9, 10}

3. A={3, 5, 6, 12} ha'm B={1, 2, 3, 5}ko'plikleri berilgen. A\B ni' tabi'n'.

a) A\B={3, 5, 6} b) A\B={3, 5} c) A\B={6, 12} d) A\B={1, 2}

4. A={5, 6, 8} ha'm B={4, 5, 7} ko'pliklerinin' Dekart ko'beymesin tabi'n'.

a) $A \times B$={(5, 5) ;(5, 3) ;(5, 4) ;(5, 7) ;}

b) $A \times B$={(5, 4) ;(5, 5) ;(5, 7) ;(6, 4) ;(6, 5) ;(6, 7) ;(8, 4) ;(8, 5) ;(8, 7) }

c) $A \times B$={(5, 4) ;(5, 5) ;(6, 8) ;(4, 6) }

d) $A \times B$={(4, 5) ;(5, 5) ;(7, 6) ;(6, 6) ;(5, 1) ;(5, 7) }

5. Yeger A=(a|aϵN, 17≤a≤23) ha'm B=(b|bϵN, 8≤b≤21) bolsa, ko'plikler kesispesin tabi'n'

a) (8, 9, 10, 11, 12, 13, 14, 15, 16) , b) (17, 18, 19, 20, 21) , c) (22, 23) , d) (12, 13, 14, 15)

6. $M_0(5,2)$ tochka arqali' wo'tiwshi ha'm bag'i'tlawshi' vektor $\vec{a} = \{2;-1\}$ bolg'an tuwri' si'zi'q ten'lemesin du'zin'.

a) $x = 5 + 2t, y = 2 - t$, b) $x = 5 - 2t, y = 2 - t$, c) $x = 5 - 2t, y = 2 + t$, d) $x = 5 + 2t, y = 2 + t$

7. $x-4y+3=0$ ha'm $2x-y+5=0$ tuwri' si'zi'qlardi'n' tegislikte jaylasi'wi'n tekserin'.

a) $\left(\frac{17}{7};\frac{1}{7}\right)$ b) $\left(-\frac{17}{7};\frac{1}{7}\right)$ c) $\left(\frac{1}{5};\frac{1}{3}\right)$ d) $\left(\frac{1}{5};-\frac{1}{3}\right)$

8. $2x-3y+4=0$ ha'm $3x+2y-6=0$ tuwri' si'zi'qlardi'n' tegislikte wo'z-ara jaylasi'wi'n korsetin'.

a) $x=\frac{7}{2};y=-\frac{5}{2}$, b) $x=\frac{1}{2};y=\frac{5}{2}$, c) $x=1;y=2$, d) $x=4;y=2$

9. $M_1(3;2)$ ha'm $M_2(4;3)$ tochkalar arqali' wo'tiwshi tuwri' si'zi'qti'n' mu'yeshlik koeffitsientin tabi'n.

a) 60^0 b) 45^0 c) 75^0 d) 40^0

10. Tuwri' si'zi'qlar $x+y+10=0$ ha'm $2x-3y-5=0$ ten'lemeler menen berilgen. Sol tuwri' si'zi'qlar da'stesine tiyisli ha'm $M(1,2)$ tochka arqali' wo'tiwshi tuwri' si'zi'q ten'lemesin du'zin'.

a) $7x-6y+5=0$, b) $x-y-1=0$, c) $x-3y-5=0$, d) $-9x-y-7=0$

11. Abtsissa ko'sherine sali'sti'rg'anda (1;-3) tochkasi'na simmetriyali' tochkani' tabi'n'.

a) (1;3) , b) (-1;-3) , c) (-1;3) , d) (1;-3)

12. (1;-3) tochkasi'na wordinata ko'sherine sali'sti'rg'anda simmetriyali' tochkani' tabi'n'.

a) (1;3) , b) (-1;-3) , c) (-1;3) , d) (1;-3)

13. (1;-3) tochkasi'na koordinata basi'na sali'sti'rg'anda simmetriyali' tochkani' tabi'n'.

a) (1;3) , b) (-1;-3) , c) (-1;3) , d) (1;-3)

14. A(-7;0) ha'm B(-4;4) tochkalari' arasi'ndag'i' arali'qti' tabi'n'

a) 3, b) 4, c) 12, d) 5

15. $f(z)=4^{\frac{1}{z-1}}$ funkciyasi' berilgen $f(1)$ di tabi'n'.

a) 1, b) 4, c) 64, d) 16

16. $y=\frac{7}{1-x}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'

a) $(1;\infty)$, b) $(-\infty;1)\cup(1;+\infty)$, c) $(-\infty;-1)$, d) $(-1;1)$

17. $f(x)=\frac{\sin x}{x}$ bolsa, $f(-x)$ ti' tabi'n'

a) $f(-x)=-f(x)$, b) $f(-x)=0$, c) $f(-x)=f(x)$, d) $f(-x)=1$

18. Determinantti' yesaplan' $\begin{vmatrix} 2 & -3 \\ -4 & -5 \end{vmatrix}$

a) -2, b) -23, c) 2, d) -22

19. Diterminanatti' yesaplan' $\begin{vmatrix} 1 & 3 & 5 \\ 0 & 2 & 1 \\ 4 & 1 & 2 \end{vmatrix}$

a) 57, b) 25, c) -24, d) -25

20. $Ax=B$ ten'lemesin sheshin', bunada $A=\begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix}$, $B=\begin{vmatrix} 2 & 3 \\ -2 & 2 \end{vmatrix}$

a) -2, b) 2, c) 1, d) -1

21. Determinantti' yesaplan' $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}$

a) 54, b) 12, c) -12, d) 1

22. Ten'lemeler sistemasi'n sheshin' $\begin{cases} x-y=2 \\ x+y=6 \end{cases}$

a) (3;1) , b) (2;4) , c) (6;4) , d) (4;2)

23. Ten'lemeler sistemasi'n sheshin' $\begin{cases} x-y=3 \\ 3x-2y=1 \end{cases}$

a) (-5;-8) , b) (-5;8) , c) (5;-8) , d) (5;8)

24. Vektorlardi'n' qosi'ndi'si'n tabi'n' $\vec{a}=2\vec{i}+3\vec{j}-4\vec{k}$, $\vec{b}=3\vec{i}-4\vec{j}+6\vec{k}$

a) $\vec{a}+\vec{b}=5\vec{i}+7\vec{j}+10\vec{k}$, b) $\vec{a}+\vec{b}=5\vec{i}-\vec{j}+2\vec{k}$,

c) $\vec{a}+\vec{b}=2\vec{i}+3\vec{j}-4\vec{k}$, d) $\vec{a}+\vec{b}=3\vec{i}-4\vec{j}+6\vec{k}$

25. $\vec{a}(7;2;-1)$, $\vec{b}(1;2;-3)$ vektorlari'ni'n' skalyar ko'beymesin tabi'n'

a) 8, b) -20, c) 6, d) 14

26. $\vec{a}$ vektori' wo'zinin' $A(1;-2;3)$ ha'm $B(2;-1;-4)$ ushlari' menen berilgen. AB kesindisinin' wortasi'ni'n' koordinatalari'n tabi'n'

a) $x=3; y=-3; z=-1$, b) $x=-1; y=-1; z=7$

c) $x=1,5; y=-1,5; z=-0,5$, d) $x=1; y=1; z=-7$

27. ABC u'shmu'yeshliginin' to'beleri $A(-5;3), B(-3;3), C(7;-1)$ berilgen. Woni'n' ta'replerinin' wortasi'ni'n' koordinatalari'n tabi'n'. (AB ta'replerinin' wortasi' E, AC ta'repinin' F, BC ta'repinin' wortasi' K menen belgilensin).

a) $E(-8;6), F(1;1), K(2;1)$, b) $E(-4;3), F(1;1), K(2;1)$

c) $E(-4;3), F(2;2), K(2;1)$, d) $E(-4;3), F(1;1), K(4;2)$

28. $A(1;2)$ ha'm $B(3;4)$ tochkalari'n tutasti'ri'wshi' AB kesindisin $\lambda=\frac{1}{2}$ qatnasi'nda bo'lin'.

a) $\left(\frac{7}{3};\frac{8}{3}\right)$, b) $(7;8)$, c) $\left(\frac{7}{3};\frac{8}{3}\right)$, d) $(2;3)$

29. $3x-6y=0$ tuwri'si' koordinata basi'nan wo'tedi. Bul tuwri' to'mendegi tochkalardi'n' qaysi' birinen wo'tedi?

a) (0;0) ha'm (2;2) , b) (0;0) ha'm (2;0) ,

c) (0;0) ha'm (3;1) , d) (0;0) ha'm (0;6)

30. $2x-6y+12=0$ tuwri' si'zi'g'i'ni'n' mu'yeshlik koeffitsientin ko'rsetin'.

a) $k=2$, b) $k=3$, c) $k=\frac{1}{6}$, d) $k=\frac{1}{3}$

31. $2x-6y+12=0$ tuwri' si'zi'g'i'ni'n' ko'sherlerden aji'ratqan kesindiler boyi'nsha ten'lemesin ko'rsetin'.

a) $\frac{x}{6}-\frac{y}{2}=1$, b) $\frac{x}{-6}+\frac{y}{2}=1$, c) $\frac{x}{12}-\frac{y}{12}=1$, d) $\frac{x}{-12}+\frac{y}{6}=1$

32. (1;2) ha'm (2;1) tochkalari'nan wo'tiwshi tuwri'ni'n' ten'lemesin du'zin'.

a) $y-x-3=0$, b) $x+y+3=0$, c) $x-y+3=0$, d) $x+y-3=0$

33. Kubti'n' wo'z-ara parallel' neshe jup qabi'rg'alari' bar?

a) 18, b) 16, c) 12, d) 9

34. Trapetsiyani'n' 3 ke ten' bolg'an duri's u'shmu'yeshliktin' to'belerinin' ha'r birinen 2 birlik qashi'qli'qta jaylasqan tochkalardan u'shmu'yeshlik tegisligine shekemgi bolg'an qashi'qli'qti' tabi'n'?

a) 1, b) $\sqrt{2}$, c) $\sqrt{6}$, d) $\sqrt{3}$

35. AB kesindinin' A ushi'nan tegislik ju'rgizilgen. Usi' kesindinin' B ushi'nan ha'm C tochkasi'nan tegislikti B_1 ha'm C_1 tochkalarda kesiwshi parallel' tuwri' si'zi'qlar

ju'rgizilgen. Yeger $|AB|=8$ ha'm $|CC_1|:|AC|=3:4$ bolsa, $|BB_1|$ kesindinin' uzi'nli'g'i'n tabi'n'.

a) 3 b) 5 c) 4 d) 6

36. $N(0;y;0)$ tochka $A(0;2;0)$ ha'm $B(3;1;0)$ tochkalardan ten' uzaqli'kta ekenligi ma'lim bolsa, y ti tabi'n'.

a) 1 b) 1, 5 c) -1, 5 d) -3

37. To'beleri $A(4;5;1)$ $B(2;3;0)$ ha'm $C(2;1;-1)$ tochkalarda jaylasqan u'shmu'yeshliktin' BD medianasi'ni'n' uzi'nli'g'i'n tabi'n'?

a) 1 b) $\sqrt{2}$ c) 3 d) 2

38. Kubti'n' barli'q qabi'rg'alari'ni'n' uzi'nli'qlari'ni'n' qosi'ndi'si' 48 ge ten'. Kub betinin' maydani'n tabi'n'.

a) 96 b) 24 c) 36 d) 48

39. Duri's to'rtmu'yeshli prizmani'n' ultani'ni'n' ta'repi 4 ke, biyikligi $4\sqrt{6}$ g'a ten'. Prizmani'n' diagonali' ultan tegisligi menen qanday mu'yesh payda yetedi?

a) 30^0 b) 45^0 c) 75^0 d) 60^0

40. Tuwri' parallelepiped ultani'ni'n' ta'repleri 8 ha'm 4 ke ten' boli'p, wolar 60º li' mu'yesh jasaydi'. Parallelepipedtin' kishi diagonali' $8\sqrt{3}$ ke ten' bolsa, usi' diagonaldi'n' ultan tegisligi menen payda yetken mu'yeshin tabi'n'.

a) 60º b) 30º c) arctg2 d) 45º

41. Piramidani'n' ultani'ni'n' ta'repleri 6 ha'm 8 ge ten' bolg'an tuwri' to'rtmu'yeshlikten ibarat. Piramidani'n' har bir qaptal qabi'rg'asi' $5\sqrt{5}$ ke ten' bolsa, biyikligin tabi'n'.

a) 5 b) 10 c) 100 d) 25

42. Duri's tetraedrdin' ushraspaytug'i'n (ayqasi'wshi') qabi'rg'alari' arasi'ndag'i' mu'yeshti tabi'n'.

a) 60º b) 90º c) 45º d) 120º

43. Shen'berdin' $\frac{1}{3}$ bo'legi neshe gradusli' bo'lek boladi'.

a) 60º b) 90º c) 120º d) 180º

44. Tsilindrdin' biyikligi b g'a, ko'sherlik kesindinin' diagonali' d g'a ten'. Ultani'ni'n' radiusi'n tabi'n'.

a) $\sqrt{b^2+d^2}$ b) $\sqrt{b^2-d^2}$ c) $\frac{1}{2}\sqrt{d^2-b^2}$ d) $\sqrt{d^2-b^2}$

45. Ko'sherlik kesimi ta'repi 6sm ge ten'. Ten'ta'repli u'shmu'yeshlik bolg'an konusti'n' radiusi'n tabi'n'.

a) 3 b) 4 c) 5 d) 2

46. Ultanlari'ni'n' radiusi' 2 ha'm 7 ge, ko'sherlik kesiminin' diagonali' 15 ke ten' bolg'an kesik konusti'n', jasawshi'si'n tabi'n'.

a) 6 b) 13 c) 4 d) 5

47. Radiusi' 13 ke ten' bolg'an shar tegislik penen kesilisken. Yeger shar worayi'nan kesimge shekemgi arali'q 10g'a ten' bolsa, kesimnin' maydani'n tabi'n'.

a) 69π b) $3\sqrt{6}\pi$ c) 100 π d) 9 π

48. Ko'lemi 432π ge ten' bolg'an tsilindrge ishley si'zi'lg'an shar betinin' maydani'n tabi'n'.

a) 120 π b) 130 π c) 144 π d) 150 π

49. Piramidani'n' toli'q beti 60qa ten', wog'an ishley si'zi'lg'an shardi'n' radiusi' 5 ke ten'. Piramidani'n' ko'lemin tabi'n'.

a) 100 b) 80 c) 90 d) 120

50. U'shmu'yeshli piramida ultani'ni'n' ta'repleri 6;8 ha'm 10g'a ten'. Piramidani'n' qaptal qabi'rg'alari' ultan tegisligi menen birdey mu'yesh payda yetedi. Yeger piramidani'n' biyikligi 4 ke ten' bolsa, qaptal qabi'rg'asi' neshege ten' boladi'.

a) $\sqrt{41}$ b) 3 c) 4 d) 5

II-variant

(Duri's juwabi'n ko'rsetin')

1. A={5, 6, 8, 10, 11} ha'm B={3, 4, 5, 6, 8} ko'pliklerinin' kesilispesin tabi'n'.

a) A∩B={3, 4, 5, 6, 8} b) A∩B={5, 6, 8} c) A∩B={10, 11} d) A∩B={3, 4, 11}

2. A={3, 5, 6, 12} ha'm B={1, 2, 3, 5}ko'pliklerinin' birikpesin tabi'n'.

a) A∪B={1, 2, 3, 3, 5, 6} b) A∪B={1, 2, 12, 3}

c) A∪B={1, 2, 3, 5, 6, 12} d) A∪B={3, 5, 6, 12, 3}

3. A={1, 2, 3, 5, 7, 8} ha'm B={2, 3, 4, 5, 9} ko'plikleri berilgen. A\B ni' tabi'n'.

a) A\B={1, 7, 8} b) A\B={4, 9} c) A\B={1, 2, 3} d) A\B={4, 5, 6}

4. A={7, 8, 9} ha'm B={6, 9, 10}ko'pliklerinin' Dekart ko'beymesin tabi'n'.

a) $A\times B$={(7, 6) ;(7, 9) ;(7, 10) ;(8, 6) ;(8, 9) }

b) $A\times B$={(7, 6) ;(7, 9) ;(7, 10) ; (9, 6) ;(9, 9) ;(9, 10) }

c) $A\times B$={(8, 6) ;(8, 9) ;(8, 10) ;(9, 6) ;(9, 9) ;(9, 10) }

d) $A\times B$={(7, 6) ;(7, 9) ;(7, 10) ;(8, 6) ;(8, 9) ;(8, 10) ;(9, 6) ;(9, 9) ;(9, 10) }

5. Yeger A=[-1, 2] ha'm B=(4, 5, 6) bolsa, A ha'm B ni'n' kesilispesin tabi'n'

a) (4, 5, 6 b) (-1, 0, 1, 2, 4, 5, 6) c) (-1, 0, 1, 2, 3, 4) d) (Ø)

6. ABC u'shmu'yeshlik ushlari'ni'n' koordinatalari' berilgen: A(-1;4); B(11;-5); C(15;17). AB ha'm BC ta'replerinin' ten'lemesin du'zin'

a) 5y+3x+13=0 b) 4y+3x-13=0 c) 11y-3x-13=0 d) 17y-3x+13=0

7. 2x+y-4=0 ha'm x-y+1=0 tuwri' si'zi'qlardi'n' tegislikte jaylasi'wi'n tekserin'.

a) (-1;-2) b) (2;4) c) (1;2) d) (1;-4)

8. $x-y-2=0$ ha'm $x+y-6=0$ tuwri' si'zi'qlardi'n' tegislikte wo'z-ara jaylasi'wi'n korsetin'.

a) $x=\frac{7}{2}; y=-\frac{5}{2}$ b) $x=\frac{1}{2}; y=\frac{5}{2}$ c) $x=1; y=2$ d) $x=4; y=2$

9. Tuwri' si'zi'qlar 3x-4y-3=0 ha'm x+2y-1=0 ten'lemeler menen berilgen. Sol tuwri' si'zi'qlar ha'm M(2;1) tochka arqali' wo'tiwshi tuwri' si'zi'q ten'lemesin du'zin'.

a) 7x-6y+5=0 b) x-y-1=0 c) x-3y-5=0 d) -9x-y-7=0

10. M_1(2;3) ha'm M_2(4;3) tochkalar arqali' wo'tiwshi tuwri' si'zi'qti'n' mu'yesh koeffitsientin tabi'n'.

a) 45^0 b) 60^0 c) 135^0 d) 120^0

11. (5;-2) tochkasi'na abstsissa ko'sherine sali'sti'rg'anda simmetriyali' tochkani' tabi'n'.

a) (5;2) b) (-5;-2) c) (-5;2) d) (5;-2)

12. (5;-2) tochkasi'na wordinata ko'sherine sali'sti'rg'anda simmetriyali' tochkani' tabi'n'.

a) (5;2) b) (-5;-2) c) (-5;2) d) (5;-2)

13. (5;-2) tochkasi'na koordinata basi'na sali'sti'rg'anda simmetriyali' tochkani' tabi'n'.

a) (5;2) b) (-5;-2) c) (-5;2) d) (5;-2)

14. A(-3;-7) ha'm B(3;1) tochkalari' arasi'ndag'i' arali'qti' tabi'n'

a) 90 b) 8 c) 6 d) 10

15. $f(z)=4^{\frac{1}{z}-1}$ funkciyasi' berilgen $f\left(\frac{1}{2}\right)$ di tabi'n'.

a) 4 b) 1 c) 64 d) 16

16. $y=\frac{7}{x-2}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'

a) $(2;+\infty)$ b) $(-\infty;2)\cup(2;+\infty)$ c) $(-\infty;-2)$ d) $(-2;2)$

17. $f(x)=\frac{\sin x}{x}$ bolsa, $f(\pi)$ ti' tabi'n'

a) $f(\pi)=f(x)$ b) $f(\pi)=-f(x)$ c) $f(\pi)=0$ d) $f(\pi)=1$

18. Ditermenanatti' yesaplan' $\begin{vmatrix}2 & -3\\ 3 & -5\end{vmatrix}$

a) -19 b) 1 c) 19 d) -1

19. Ditermenanatti' yesaplan' $\begin{vmatrix}2 & 4 & -1\\ 7 & 3 & 2\\ 3 & 1 & -2\end{vmatrix}$

a) -74 b) 74 c) -66 d) 66

20. $Ax=B$ ten'lemesin sheshin', bunda $A=\begin{vmatrix}1 & 0\\ 2 & 3\end{vmatrix}$, $B=\begin{vmatrix}3 & 7\\ 0 & 3\end{vmatrix}$

a) 3 b) -3 c) $\frac{1}{3}$ d) $-\frac{1}{3}$

21. Determenantti' yesaplan' $\begin{vmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ -1 & -3 & -2\end{vmatrix}$

a) 54 b) 12 c) -12 d) 1

22. Ten'lemeler sistemasi'n sheshin' $\begin{cases}2x-3y=1\\ x+y=3\end{cases}$

a) (2;1) b) (1;2) c) (0;3) d) (3;0)

23. Ten'lemeler sistemasi'n sheshin' $\begin{cases}x+3y=-2\\ 3x-y=7\end{cases}$

a) (1;-1) b) (1, 9;1, 3) c) (1, 9;-1, 3) d) (-1, 9;1, 3)

24. Vektorlardi'n' ayi'rmasi'n tabi'n' $\vec{a}=2\vec{i}+3\vec{j}-4\vec{k},\ \vec{b}=3\vec{i}-4\vec{j}+6\vec{k}$

a) $\vec{a}-\vec{b}=-\vec{i}+7\vec{j}-10\vec{k}$ b) $\vec{a}-\vec{b}=-\vec{i}-\vec{j}+2\vec{k}$

c) $\vec{a}-\vec{b}=-\vec{i}-7\vec{j}+10\vec{k}$ d) $\vec{a}-\vec{b}=5\vec{i}-\vec{j}+2\vec{k}$

25. $\vec{a}(2;4;-3),\ \vec{b}(6;-4;2)$ vektorlari'ni'n' skalyar ko'beymesin tabi'n'

a) -10 b) 2 c) -2 d) 34

26. $\vec{a}$ vektori' wo'zinin' $A(0;1;2)$ ha'm $B(0;-1;-2)$ ushlari' menen berilgen. AB kesindisinin' wortasi'ni'n' koordinatalari'n tabi'n'

a) $x=0;y=0;z=0$ b) $x=0;y=2;z=4$

c) $x=0;y=1;z=2$ d) $x=0;y=4;z=8$

27. ABC u'shmu'yeshliginin' to'beleri A(1;3) , B(2;2) , C(-2;-1) berilgen. Woni'n' ta'replerinin' wortasi'ni'n' koordinatalari'n tabi'n'. (AB ta'replerinin' wortasi' E, AC ta'repinin' F, BC ta'repinin' wortasi' K menen belgilensin).

a) E(1, 5;2, 5) , F(-0, 5;1) , K(0;1) b) E(1, 5;2, 5) , F(-1;2) , K(0;0, 5)

c) E(1, 5;2, 5) , F(-0, 5;1) , K(0;0, 5) d) E(3;5) , F(-0, 5;1) , K(0;0, 5)

28. A(0;2) ha'm B(2;4) tochkalari'n tutasti'ri'wshi' AB kesindisin $\lambda=3$ qatnasi'nda bo'lin'.

a) $\left(\frac{3}{2};\frac{7}{2}\right)$ b) $(1;3)$ c) $(-1;-1)$ d) $(1;1)$

29. $4x-8y=0$ tuwri'si' koordinata basi'nan wo'tedi. Bul tuwri' to'mendegi tochkalardi'n' qaysi' birinen wo'tedi?

a) (0;0) ha'm (2;0) b) (0;0) ha'm (2;1)

c) (0;0) ha'm (1;2) d) (0;0) ha'm (0;4)

30. $3x-3y+6=0$ tuwri' si'zi'g'i'ni'n' mu'yeshlik koeffitsientin ko'rsetin'.

a) $k=3$ b) $k=1$ c) $k=0$ d) $k=6$

31. $3x-3y+6=0$ tuwri' si'zi'g'i'ni'n' ko'sherlerden aji'ratqan kesindiler boyi'nsha ten'lemesin ko'rsetin'.

a) $\frac{x}{-2}+\frac{y}{2}=1$ b) $\frac{x}{2}-\frac{y}{2}=1$ c) $\frac{x}{6}+\frac{y}{6}=1$ d) $\frac{x}{-6}-\frac{y}{6}=1$

32. (2;3) ha'm (3;4) tochkalari'nan wo'tiwshi tuwri'ni'n' ten'lemesin du'zin'.

a) $y-x+1=0$ b) $x-y+1=0$ c) $x-y-1=0$ d) $y-x-1=0$

33. Kubti'n' wo'z-ara parallel' neshe jup jaqlari' bar

a) 6 b) 4 c) 3 d) 8

34. Tegislikke ju'rgizilgen perpendikulyar ha'm qi'ya arasi'ndag'i' mu'yesh 30º, perpendikulyardi'n' uzi'nli'g'i' 10g'a ten'. Qi'yani'n' uzi'nli'g'i'n tabi'n'?

a) 20 b) $10\sqrt{3}$ c) $20\sqrt{3}$ d) $\frac{20}{\sqrt{3}}$

35. Duri's ABC u'shmu'yeshliktin' AC ta'repi arqali' tegislik ju'rgizilgen. U'shmu'yeshliktin' BD medianasi' tegislik penen 60º li' mu'yesh jasaydi'. AB tuwri' si'zi'q penen tegislik arasi'ndag'i' mu'yeshtin' sinusi'n tabi'n'.

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) $\frac{3}{2}$

36. Yeger kesindinin' bir ushi' $A(1;-5;4)$ wortasi' $C(4;-2;3)$ tochkada bolsa, yekinshi ushi'ni'n' koordinatalari'n tabi'n'.

a) (6;5;3) b) (7;-1;2) c) (7;1;2) d) (5;4;6)

37. A(2;-1;0) ha'm B(-2;3;2) tochkalar berilgen. Koordinata basi'nan AB kesindinin' wortasi'na shekemgi bolg'an arali'qti' tabi'n'?

a) $\sqrt{2}$ b) $-\sqrt{2}$ c) $2\sqrt{2}$ d) 1

38. Diagonali' $\sqrt{3}$ ke ten' bolg'an kub betinin' maydani'n tabi'n'.

a) 6 b) 3 c) 9 d) 2

39. U'shmu'yeshli tuwri' prizmani'n' ultani'ni'n' ta'repleri 36, 29 ha'm 25 ke, toli'q beti 1620 g'a ten'. Prizmani'n' biyikligin tabi'n'.

a) 20 b) 12, 6 c) 10 d) 18

40. Tuwri' parallelepiped ultani'ni'n' ta'repleri $2\sqrt{2}$ ha'm 5 sm boli'p, wo'z-ara 45^0 li' mu'yesh jasaydi'. Parallelepipedtin' kishi diagonali' 7 sm. Woni'n' ko'lemi qansha?

a) $60sm^3$ b) $120sm^3$ c) $80sm^3$ d) $90sm^3$

41. Piramidani'n' ultani' tuwri' mu'yeshli u'shmu'yeshlik boli'p, gipotenuzasi'ni'n' uzi'nli'g'i' 10 g'a ten'. Piramidani'n' qaptal qabi'rg'alari' 13 ke ten' bolsa, biyikligin tabi'n'.

a) 11 b) 12 c) 10 d) 13

42. Ko'lemi $8\sqrt{3}$ ke ten' bolg'an tetraedrdin' biyikligin tabi'n'.

a) 3 b) 4 c) $2\sqrt{3}$ d) $3\sqrt{3}$

43. Saatti'n' minut tili 20 minutta neshe gradusli' bo'lek «si'zadi'»?

a) 150º b) 120º c) 90º d) 60º

44. Tsilindrdin' ultani'ni'n' radiusi' 3ese artti'ri'lsa, woni'n' ko'lemi neshe ese artadi'?

a) 3 b) 4 c) 6 d) 9

45. l jasawshi'si' ultan diametrine ten' bolg'an konusti'n' biyikligin tabi'n'?

a) $\frac{l\sqrt{2}}{2}$ b) $\frac{l\sqrt{3}}{2}$ c) $\frac{l}{2}$ d) $\frac{l}{4}$

46. Ultanlari'ni'n' radiusi' 2 ha'm 7 ge, ko'sherlik kesiminin' diagonali' 15 ke ten' bolg'an kesik konus qaptal betinin' maydani'n tabi'n'.

a) 112π b) 115π c) 117π d) 120π

47. Betinin' maydani' 16π ge ten' bolg'an shardi'n' ko'lemin tabi'n'?

a) $\frac{32\pi}{3}$ b) 69π c) 33π d) 100π

48. Kubqa ishley ha'm si'rtlay si'zi'lg'an sferalar maydanlari'ni'n' qatnasi'n tabi'n'?

a) 1:2 b) 1:3 c) 2:3 d) 3:4

49. Duri's u'shmu'yeshli piramidag'a konus ishley si'zi'lg'an. Yeger Piramidani'n' qaptal betleri menen ultani' 60º li' mu'yesh payda yetip, piramidani'n' ultani'na 16 g'a ten' bolsa, konusti'n' qaptal betin tabi'n'.

a) 512π b) 524π c) 518π d) 514π

50. Duri's to'rtmu'yeshli piramidani'n' biyikligi 8 ge, ultani'ni'n' ta'repi 12ge ten'. Piramidani'n' qaptal jag'i'na parallel' yetip ultani'ni'n' worayi' arqali' wo'tkerilgen kesimnin' maydani'n yesaplan'.

a) 45 b) 30 c) 50 d) 60

III-variant

(Duri's juwabi'n ko'rsetin')

1. A={3, 5, 6, 12} ha'm B={1, 2, 3, 5} ko'pliklerinin' kesilispesin tabi'n'.

a) $A \cap B=\{6, 12\}$ b) $A \cap B=\{1, 2, 3\}$

c) $A \cap B=\{3, 5\}$ d) $A \cap B=\{1, 2, 5, 12\}$

2. A={5, 6, 8, 10, 11} ha'm B={3, 4, 5, 6, 8} ko'pliklerinin' birikpesin tabi'n'.

a) $A \cup B=\{3, 4, 6, 11\}$ b) $A \cup B=\{3, 4, 5, 6, 8, 10, 11\}$

c) $A \cup B=\{5, 10, 11\}$ d) $A \cup B=\{1, 2, 5, 6\}$

3. A={6, 7, 8, 9, 10} ha'm B={2, 3, 5, 6, 9, 10}ko'plikleri berilgen. A\B ni' tabi'n'.

a) A\B={6, 7, 8} b) A\B={9, 10}

c) A\B={2, 5, 9} d) A\B={7, 8}

4. A={1, 2, 3} ha'm B={2, 3, 4} ko'pliklerinin' Dekart ko'beymesin tabi'n'.

a) $A \times B=\{(1;2) , (1;3) , (1;4) , (2;2) , (2;3) , (2;4) , (3;2) , (3;3) , (3;4) \}$

b) $A \times B=\{(1;3) , (1;5) , (2;3) , (2;4) , (3;2) , (3;3) , (3;4) \}$

c) $A \times B=\{(4;3) , (4;2) , (4;1) , (4;0) , (3;1) , (3;2) , (3;3) , (3;0) , (2;2) \}$

d) $A \times B=\{(1;2) , (1;3) , (1;4) , (1;0) , (2;2) , (2;3) , (2;4) , (3;2) , (3;6) \}$

5. Yeger A=[-2, 1] ha'm B=(2, 3, 4) bolsa, A ha'm B ni'n' dekart ko'beymesin tabi'n' (Aє Z)

a) ((-1;2) , (-1;3) , (-1;4) , (0;2) , (0;3) , (0;4) , (1;2) , (1;3) , (1;4))

b) ((-2;2) , (-2;3) , (-2;4) , (-1;2) , (-1;3) , (-1;4))

c) ((-2;2) , (-2;3) , (-2;4) , (-1;2) , (-1;3) , (-1;4) , (0;2) , (0;3) , (0;4) , (1;2) , (1;3) , (1;4))

d) ((-1;4) , (0;2) , (0;3) , (0;4) , (1;2) , (1;3) , (1;4))

6. M1(3;1) tochka arqali' wo'tiwshi ha'm $\vec{N}=\{-1;1\}$ vektorlarg'a perpendikulyar bolg'an tuwri' si'zi'q ten'lemesin du'zin'.

a) 3x-y+2=0 b) x+y+2=0 c) x-y-2=0 d) 3x-y-2=0

7. 6x-2y-6=0 ha'm 5x-y-17=0 tuwri' si'zi'qlardi'n' tegislikte jaylasi'wi'n tekserin'.

a) (-7;18) b) (6;-6) c) (7;-18) d) (7;18)

8. $x-y-6=0$ ha'm $x+y-1=0$ tuwri' si'zi'qlardi'n' tegislikte wo'z-ara jaylasi'wi'n korsetin'.

a) $x=\frac{7}{2}; y=-\frac{5}{2}$ b) $x=\frac{1}{2}; y=\frac{5}{2}$ c) $x=1; y=2$ d) $x=4; y=2$

9. Tuwri' si'zi'qlar 2x-y-5=0 ha'm 3x+2y-4=0 ten'lemeler menen berilgen. Sol tuwri' si'zi'qlar ha'm M(-1;-2) tochka arqali' wo'tiwshi tuwri' si'zi'q ten'lemesin du'zin'.

a) 7x-6y+5=0 b) x-y-1=0 c) x-3y-5=0 d) -9x-y-7=0

10. M_1(3;3) ha'm M_2(4;4) tochkalar arqali' wo'tiwshi tuwri' si'zi'qti'n' mu'yesh koeffitsientin tabi'n.

a) 90^0 b) 45^0 c) 60^0 d) 0^0

11. (-7;4) tochkasi'na abstsissa ko'sherine qarata simmetriyali' tochkani' tabi'n'.

a) (-7;-4) b) (7;-4) c) (-7;4) d) (7;4)

12. (-7;4) tochkasi'na wordinata ko'sherine sali'sti'rg'anda simmetriyali' tochkani' tabi'n'.

a) (7;-4) b) (7;4) c) (-7;-4) d) (-7;4)

13. (-7;4) tochkasi'na koordinata basi'na sali'sti'rg'anda simmetriyali' tochkani' tabi'n'.

a) (7;4) b) (-7;-4) c) (7;-4) d) (-7;4)

14. A(2;0) ha'm B(14;-5) tochkalari' arasi'ndag'i' arali'qti' tabi'n'

a) 281 b) 5 c) 12 d) 13

15. $f(z)=4^{\frac{1}{z}-1}$ funkciyasi' berilgen $f\left(\frac{1}{3}\right)$ di tabi'n'.

a) 16 b) 1 c) 64 d) 4

16. $y=\frac{5}{2x-8}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'

a) $(4;+\infty)$ b) $(-\infty;4)\cup(4;+\infty)$ c) $(-\infty;4)$ d) $(-4;4)$

17. $f(x)=\frac{\cos x}{x}$ bolsa, $f(-x)$ ti' tabi'n'

a) $f(-x)=f(x)$ b) $f(-x)=0$ c) $f(-x)=-f(x)$ d) $f(-x)=1$

18. Ditermenanatti' yesaplan' $\begin{vmatrix}-3 & -5\\ 5 & 3\end{vmatrix}$

a) 16 b) -34 c) -16 d) 34

19. Ditermenanatti' yesaplan' $\begin{vmatrix}2 & 1 & 1\\ 3 & 2 & 1\\ 1 & 4 & -3\end{vmatrix}$

a) 2 b) -2 c) -20 d) 0

20. $Ax=B$ ten'lemesin sheshin', bunada $A=\begin{vmatrix}2 & -3\\ 0 & 2\end{vmatrix}$, $B=\begin{vmatrix}4 & 0\\ -2 & 8\end{vmatrix}$

a) 2 b) 0 c) -2 d) 4

21. Determenantti’ yesaplan’ $\begin{vmatrix} 1 & 2 & 3 \\ -3 & -2 & -1 \\ 1 & 3 & 2 \end{vmatrix}$

a) 54 b) 12 c) -12 d) 1

22. Ten’lemeler sistemasi’n sheshin’ $\begin{cases} 7x - y = 3 \\ 2x + y = 6 \end{cases}$

a) (2;11) b) (4;2) c) (1;4) d) (4;1)

23. Ten’lemeler sistemasi’n sheshin’ $\begin{cases} x + 4y = 12 \\ 3x - 2y = -6 \end{cases}$

a) (0;-3) b) (0;3) c) (3;0) d) (-3;0)

24. Vektorlardi’n’ qosi’ndi’si’n tabi’n’ $\vec{a} = 2\vec{i} + 3\vec{j} - 4\vec{k}$, $\vec{b} = -3\vec{i} + 2\vec{j} + 5\vec{k}$

a) $\vec{a} + \vec{b} = 5\vec{i} + 5\vec{j} + 9\vec{k}$ b) $\vec{a} + \vec{b} = -\vec{i} + 5\vec{j} + \vec{k}$

c) $\vec{a} + \vec{b} = -\vec{i} + \vec{j} - 9\vec{k}$ d) $\vec{a} + \vec{b} = 5\vec{i} + 5\vec{j} + \vec{k}$

25. $\vec{a}(-1;2;4)$, $\vec{b}(2;-1;-4)$ vektorlari’ni’n’ skalyar ko’beymesin tabi’n’

a) 12 b) -20 c) -12 d) 20

26. $\vec{a}$ vektori’ wo’zinin’ $A(-3;0;3)$ ha’m $B(1;2;4)$ ushlari’ menen berilgen. AB kesindisinin’ wortasi’ni’n’ koordinatalari’n tabi’n’

a) $x = -1; y = 2; z = 7$ b) $x = -1; y = 1; z = 3{,}5$

c) $x = -2; y = 1; z = 7$ d) $x = -2; y = 2; z = 3{,}5$

27. ABC u’shmu’yeshliginin’ to’beleri A(-1;1) , B(-2;2) , C(3;-3) berilgen. Woni’n’ ta’replerinin’ wortasi’ni’n’ koordinatalari’n tabi’n’. (AB ta’replerinin’ wortasi’ E, AC ta’repinin’ F, BC ta’repinin’ wortasi’ K menen belgilensin).

a) E(-1, 5;1, 5) , F(1;-1) , K(0, 5;-0, 5) b) E(-1, 5;1, 5) , F(2;-2) , K(0, 5;-0, 5)

c) E(-1, 5;1, 5) , F(1;-1) , K(1;-1) d) E(-3;3) , F(1;-1) , K(0, 5;-0, 5)

28. A(-1;1) ha’m B(-2;2) tochkalari’n tutasti’ri’wshi’ AB kesindisin $\lambda = \frac{1}{4}$ qatnasi’nda bo’lin’.

a) $\left(-\frac{3}{2};\frac{3}{2}\right)$ b) $\left(-\frac{6}{5};\frac{6}{5}\right)$ c) $\left(\frac{1}{2};-\frac{1}{2}\right)$ d) $\left(-\frac{1}{2};\frac{1}{2}\right)$

29. $2x-8y=0$ tuwri'si' koordinata basi'nan wo'tedi. Bul tuwri' to'mendegi tochkalardi'n' qaysi' birinen wo'tedi?

a) (0;0) ha'm (4;0) b) (0;0) ha'm (4;1)

c) (0;0) ha'm (0;4) d) (0;0) ha'm (2;0)

30. $4x-8y+16=0$ tuwri' si'zi'g'i'ni'n' mu'yeshlik koeffitsientin ko'rsetin'.

a) $k=2$ b) $k=4$ c) $k=\frac{1}{2}$ d) $k=\frac{1}{4}$

31. $4x-8y+16=0$ tuwri' si'zi'g'i'ni'n' ko'sherlerden aji'ratqan kesindiler boyi'nsha ten'lemesin ko'rsetin'.

a) $\frac{x}{16}-\frac{y}{2}=1$ b) $\frac{x}{4}-\frac{y}{2}=1$ c) $\frac{x}{-4}+\frac{y}{2}=1$ d) $\frac{x}{-2}+\frac{y}{2}=1$

32. (-2;2) ha'm (2;5) tochkalari'nan wo'tiwshi tuwri'ni'n' ten'lemesin du'zin'.

a) $3x-4y+14=0$ b) $-3x-4y-14=0$

c) $3x-4y-14=0$ d) $-3x-4y+14=0$

33. Kubti'n' wo'z-ara ayqasi'wshi' tuwri' si'zi'qlari'n ani'qlawshi' qabi'rg'alari' neshe jupli'qtan ibarat?

a) 24 b) 20 c) 16 d) 12

34. Tegislikke ju'rgizilgen perpendikulyar ha'm qi'ya arasi'ndag'i' mu'yesh 60º ha'm qi'yani'n' uzi'nli'g'i' $20\sqrt{3}$. Perpendikulyardi'n' uzi'nli'g'i'n tabi'n'.

a) 10 b) 40 c) $10\sqrt{3}$ d) $5\sqrt{3}$

35. Tochkadan tegislikke uzi'nli'qlari' 10 ha'm 15 bolg'an qi'yalar ju'rgizilgen. Birinshi qi'yani'n' tegisliktegi proektsiyasi' 7 bolsa, yekinshi qi'yani'n' proektsiyasi'n tabi'n'?

a) $\sqrt{170}$ b) $\sqrt{171}$ c) $\sqrt{172}$ d) $\sqrt{174}$

36. Koordinatalar basi'nan sali'sti'rg'anda (1;2;3) tochkag'a simmetriyali' bolg'an tochkani' tabi'n'?

a) (-1;2;3) b) (-1;-2;3) c) (-1;-2;-3) d) (1;2;-3)

37. $A(-3;8;3\sqrt{33})$ tochkadan wox ko'sherge shekemgi bolg'an arali'qti' tabi'n'?

a) 17 b) 18 c) 19 d) 21

38. To’rtmu’yeshli duri’s prizmani’n’ ultani’ni’n’ maydani’ 144sm^2, biyikligi 14sm. Usi’ prizmani’n’ diagonali’n tabi’n’.

a) 18 b) 22 c) 16 d) $14\sqrt{2}$

39. Kubti’n’ barli’q qabi’rg’alari’ni’n’ qosi’ndi’si’ 96 g’a ten’. Kubti’n’ ko’lemin tabi’n’?

a) 256 b) 216 c) 64 d) 512

40. U’shmu’yeshli tuwri’ prizmani’n’ barli’q qabi’rg’alari’ birdey uzi’nli’qqa iye, toli’q beti $8+16\sqrt{3}$ ke ten’. Prizmani’n’ ultani’ni’n’ maydani’n tabi’n’.

a) 4 b) $2\sqrt{6}$ c) $2\sqrt{3}$ d) 3

41. Piramidani’n’ ultani’ gipotenuzasi’ni’n’ uzi’nli’g’i’ 2 ge ten’ bolg’an tuwri’ mu’yeshli u’shmu’yeshlikten ibarat. Piramidani’n’ qaptal qabi’rg’alari’ ultan tegisligi menen α mu’yesh jasaydi’. Yeger woni’n’ biyikligi 5 ke ten’ bolsa, tgα ni’n’ ma’nisin tabi’n’.

a) 1 b) 2 c) 4 d) 5

42. Woktaedrdin’ qabi’rg’asi’ a g’a ten’. Woni’n’ toli’q betin yesaplan’.

a) $2a^2\sqrt{3}$ b) $a^2\sqrt{3}$ c) $4a^2\sqrt{3}$ d) $3a^2\sqrt{3}$

43. Radiuslari’ni’n’ uzi’nli’qlari’ni’n’ qatnasi’ 1:3 tu’rinde bolg’an shen’berler uzi’nli’qlari’ni’n’ qatnasi’n tabi’n’.

a) 1:3 b) 2:3 c) 1:4 d) π:25

44. Tsilindrdin’ qaptal betinin’ jayi’lmasi’ ta’repi a g’a ten’ bolg’an kvadrattan ibarat. Tsilindrdin’ ko’lemin tabi’n’.

a) $\frac{a^3}{4\pi}$ b) πa^3 c) $4\pi a^3$ d) $\frac{2\pi a^3}{3}$

45. Konusti’n’ biyikligi woni’n’ ultani’ni’n’ diametrinen yeki ese kishi bolsa, ko’sherlik kesiminin’ to’besindegi mu’yeshin tabi’n’.

a) 45º b) 60º c) 90º d) 30º

46. Kesik konus ultanlari’ni’n’ radiusi’ R ha’m r. Jasawshi’si’ ultan tegisligi menen 45º mu’yesh payda yetedi. Kesik konusti’n’ ko’lemin tabi’n’.

a) $\frac{1}{3}\pi\sqrt{R^2-r^2}$ b) $\frac{1}{3}\pi\left(R^3-r^3\right)$ c) $\frac{1}{3}\pi(R-r)^3$ d) $\frac{1}{3}\pi(R+r)^3$

47. Radiuslari' 15 ke ha'm 20g'a ten' bolg'an yeki shar woraylari' arasi'ndag'i' arali'q 25ke ten'. Shar betleri kesiliwinen payda bolg'an shen'berdin' uzi'nli'g'i'n tabi'n'.

a) 24π b) 20π c) 25π d) 15π

48. Yeger shar sektori' konusi'ni'n' radiusi' 60sm ge, shar radiusi' bolsa 75sm ga ten' bolsa, shar sektori'ni'n' ko'lemin tabi'n'.

a) 112000π b) 12500 π c) 112500π d) 121500π

49. Duri's to'rtmu'yeshli piramida ultani'ni'n' ta'repi 12 ge, wog'an ishley si'zi'lg'an shardi'n' radiusi' 3 ke ten'. Piramidani'n' qaptal betin tabi'n'.

a) 480 b) 360 c) 280 d) 240

50. Piramidani'n' ultani' tuwri'mu'yeshli u'shmu'yeshlikten ibarat. U'shmu'yeshliktin' katetleri 3 ha'm 4 ke ten'. Piramidani'n' qaptal jaqlari' ultan tegisligi menen 60º'li' mu'yeshler payda yetedi. Piramidani'n' toli'q betin tabi'n'.

a) 15 b) 18 c) 20 d) 24

IV-variant

(Duri's juwabi'n ko'rsetin')

1. A={6, 7, 8, 9, 10} ha'm B={2, 3, 5, 6, 9, 10} ko'pliklerinin' kesilispesin tabi'n'.

a) $A \cap B=\{6, 7, 8\}$ b) $A \cap B=\{2, 3, 5\}$

c) $A \cap B=\{2, 3, 5, 7, 9\}$ d) $A \cap B=\{6, 9, 10\}$

2. A={1, 2, 3, 5, 7, 8} ha'm B={2, 3, 4, 5, 9} ko'pliklerinin' birikpesin tabi'n'.

a) $A \cup B=\{1, 2, 3, 4, 5, 7, 8, 9\}$ b) $A \cup B=\{2, 3, 4\}$

c) $A \cup B=\{5, 7, 8\}$ d) $A \cup B=\{9\}$

3. A={5, 6, 8, 10, 11} ha'm B={3, 4, 5, 6, 8}ko'plikleri berilgen A\B ni' tabi'n'.

a) A\B={3, 4} b) A\B={10, 11} c) A\B={6, 8} d) A\B={1, 5}

4. A={3, 5, 12} ha'm B={2, 3, 5}ko'pliklerinin' Dekart ko'beymesin tabi'n'.

a) $A \times B=\{(3, 2) ;(3, 3) ;(3, 5) ;(5, 2) ;(5, 3) ;(5, 5) \}$

b) $A \times B=\{(5, 2) ;(5, 12) ;(5, 3) ;(5, 2) \}$

c) $A \times B=\{(3, 2) ;(3, 3) ;(3, 5) ;(5, 2) ;(5, 3) ;(5, 5) ;(12, 2) ;(12, 3) ;(12, 5) \}$

d) $A \times B=\{(2, 2) ;(2, 3) ;(2, 5) ;(2, 12) ;(3, 3) ;(3, 5) ;(3, 12) \}$

5. Yeger A=[-1, 2] ha'm B=(4, 5, 6) bolsa A ha'm B ni'n' birikpesin tabi'n' ($A \epsilon Z$).

a) (4, 5, 6) b) (-1, 0, 1, 2, 4, 5, 6)

c) (-1, 0, 1, 2, 3, 4) d) (-2, -1, 0, 1, 2, 3)

6. 2x+y-4=0 ha'm x-y+1=0 tuwri' si'zi'qlardi' kesisken tochkasi' arqali' wo'tip, x+y-5=0 tuwri' si'zi'qqa perpendikulyar bolg'an tuwri' ten'lemesin du'zin'.

a) x-y-3=0 b) 4x-y-2=0 c) 2x-y-2=0 d) x+y-3=0

7. x-4y+3=0 ha'm 2x-y+5=0 tuwri' si'zi'qlardi'n' tegislikte jaylasi'wi'n tekserin'.

a) (5;-2) b) (-2;-5) c) (4;3) d) (2;-5)

8. $x+y-3=0$ ha'm $x-y-2=0$ tuwri' si'zi'qlardi'n' tegislikte wo'z-ara jaylasi'wi'n korsetin'.

a) $x=\frac{7}{2}; y=-\frac{5}{2}$ b) $x=\frac{1}{2}; y=\frac{5}{2}$ c) $x=1; y=2$ d) $x=4; y=2$

9. Tuwri' si'zi'qlar 3x-4y-1=0 ha'm 4x-y+2=0 ten'lemeler menen berilgen. Sol tuwri' si'zi'qlar ha'm M(-1;2) tochka arqali' wo'tiwshi tuwri' si'zi'q ten'lemesin du'zin'.

a) 7x-6y+5=0 b) x-y-1=0 c) x-3y-5=0 d) -9x-y-7=0

10. $M_1(5;4)$ ha'm $M_2(5;4)$ tochkalar arqali' wo'tiwshi tuwri' si'zi'qti'n' mu'yesh koeffitsientin tabi'n.

a) 45^0 b) 90^0 c) 120^0 d) 180^0

11. (6;9) tochkasi'na abstsissa ko'sherine qarata simmetriyali' tochkani' tabi'n'.

a) (6;-9) b) (-6;-9) c) (-6;9) d) (6;9)

12. (6;9) tochkasi'na wordinata ko'sherine sali'sti'rg'anda simmetriyali' tochkani' tabi'n'?

a) (-6;-9) b) (-6;9) c) (-6;9) d) (6;-9)

13. (6;9) tochkasi'na koordinata basi'na sali'sti'rg'anda simmetriyali' tochkani' tabi'n'.

a) (-6;9) b) (6;-9) c) (-6;-9) d) (6;9)

14. A(-2;3) ha'm B(6;-3) tochkalari' arasi'ndag'i' arali'qti' tabi'n'

a) 4 b) 6 c) 8 d) 10

15. $f(z)=4^{\frac{1}{z}-1}$ funkciyasi' berilgen $f\left(\frac{1}{4}\right)$ di tabi'n'.

a) 64 b) 1 c) 16 d) 4

16. $y=\frac{10}{x^2+1}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'

a) $(-1;1)$ b) $(-\infty;+\infty)$ c) $(-\infty;1)\cup(1;+\infty)$ d) $(1;+\infty)$

17. $f(x)=\dfrac{\cos x}{x}$ bolsa, $f\left(\dfrac{\pi}{2}\right)$ ti' tabi'n'

a) $f\left(\dfrac{\pi}{2}\right)=1$ b) $f\left(\dfrac{\pi}{2}\right)=f(x)$ c) $f\left(\dfrac{\pi}{2}\right)=0$ d) $f\left(\dfrac{\pi}{2}\right)=-f(x)$

18. Ditermenanatti' yesaplan' $\begin{vmatrix}3 & 9\\ 2 & 10\end{vmatrix}$

a) 12 b) -12 c) 48 d) -48

19. Ditermenanatti' yesaplan' $\begin{vmatrix}2 & 4 & -1\\ -4 & 2 & 1\\ 3 & 1 & 5\end{vmatrix}$

a) -120 b) 20 c) -20 d) 120

20. $Ax=B$ ten'lemesin sheshin', bunada $A=\begin{vmatrix}2 & 3\\ 1 & 2\end{vmatrix}$, $B=\begin{vmatrix}1 & 0\\ 2 & 8\end{vmatrix}$

a) 8 b) $\dfrac{1}{8}$ c) -8 d) $-\dfrac{1}{8}$

21. Determenantti' yesaplan' $\begin{vmatrix}-1 & -2 & -3\\ 3 & 2 & 1\\ 1 & 3 & 2\end{vmatrix}$

a) 54 b) 12 c) -12 d) 1

22. Ten'lemeler sistemasi'n sheshin' $\begin{cases}-3x+4y=-5\\ 3x+2y=11\end{cases}$

a) (-3;1) b) (1;3) c) (-1;3) d) (3;1)

23. Ten'lemeler sistemasi'n sheshin' $\begin{cases}13x-12y=-9\\ 2x+3y=18\end{cases}$

a) (3;4) b) (3;-4) c) (-3;4) d) (-3;-4)

24. Vektorlardi'n' ayi'rmasi'n tabi'n' $\vec{a}=2\vec{i}+3\vec{j}-4\vec{k}$, $\vec{b}=-3\vec{i}+2\vec{j}+5\vec{k}$

a) $\vec{a}-\vec{b}=5\vec{i}+\vec{j}-9\vec{k}$ b) $\vec{a}-\vec{b}=-\vec{i}+5\vec{j}+\vec{k}$

c) $\vec{a}-\vec{b}=-\vec{i}+\vec{j}+\vec{k}$ d) $\vec{a}-\vec{b}=5\vec{i}+5\vec{j}+5\vec{k}$

25. $\vec{a}(-3;-7;-7)$, $\vec{b}(-6;2;3)$ vektorlari'ni'n' skalyar ko'beymesin tabi'n'

a) -11 b) 11 c) 17 d) -17

26. $\vec{a}$ vektori' wo'zinin' $A(5;3;2)$ ha'm $B(-3;3;4)$ ushlari' menen berilgen. AB kesindisinin' wortasi'ni'n' koordinatalari'n tabi'n'

a) $x=2; y=6; z=6$ b) $x=2; y=3; z=3$

c) $x=1; y=6; z=6$ d) $x=1; y=3; z=3$

27. ABC u'shmu'yeshliginin' to'beleri A(0;5) , B(-1;2) , C(1;3) berilgen. Woni'n' ta'replerinin' wortasi'ni'n' koordinatalari'n tabi'n'. (AB ta'replerinin' wortasi' E, AC ta'repinin' F, BC ta'repinin' wortasi' K menen belgilensin).

a) E(-0, 5;3, 5) , F(0, 5;4) , K(-1;-0, 5) b) E(-0, 5;3, 5) , F(-0, 5;1) , K(0;2, 5)

c) E(0, 5;1, 5) , F(0, 5;4) , K(0;2, 5) d) E(-0, 5;3, 5) , F(0, 5;4) , K(0;2, 5)

28. $A\left(\frac{1}{2};\frac{1}{2}\right)$ ha'm $B\left(1;\frac{3}{2}\right)$ tochkalari'n tutasti'ri'wshi' AB kesindisin $\lambda=\frac{1}{2}$ qatnasi'nda bo'lin'.

a) $\left(\frac{3}{4};1\right)$ b) $\left(-\frac{1}{4};-\frac{1}{2}\right)$ c) $\left(\frac{1}{4};\frac{1}{2}\right)$ d) $\left(\frac{2}{3};\frac{5}{6}\right)$

29. $2x-2y=0$ tuwri'si' koordinata basi'nan wo'tedi. Bul tuwri' to'mendegi tochkalardi'n' qaysi' birinen wo'tedi?

a) (0;0) ha'm (2;2) b) (0;0) ha'm $\left(0;\frac{1}{2}\right)$

c) (0;0) ha'm (1;1) d) (0;0) ha'm $\left(\frac{1}{2};2\right)$

30. $6x-8y+18=0$ tuwri' si'zi'g'i'ni'n' mu'yeshlik koeffitsientin ko'rsetin'.

a) $k=\frac{9}{4}$ b) $k=6$ c) $k=18$ d) $k=\frac{3}{4}$

31. $6x-9y-18=0$ tuwri' si'zi'g'i'ni'n' ko'sherlerden aji'ratqan kesindiler boyi'nsha ten'lemesin ko'rsetin'.

a) $\frac{x}{18}-\frac{y}{18}=1$ b) $\frac{x}{-3}+\frac{y}{2}=1$ c) $\frac{x}{2}-\frac{y}{3}=1$ d) $\frac{x}{3}-\frac{y}{2}=1$

32. (-3;0) ha'm (3;4) tochkalari'nan wo'tiwshi tuwri'ni'n' ten'lemesin du'zin'.

a) $-4x-6y+12=0$ b) $4x-6y+12=0$

c) $4x-6y-12=0$ d) $4x+6y+12=0$

33. Ken'isliktegi to'rtmu'yeshlik ta'replerinin' wortalari' qanday figurani'n' ushlari' boladi'?

a) duri’s to’rtmu’yeshlik b) romb

c) trapetsiya d) parallelogramm

34. Bir tochkadan tegislikke qi’ya ha’m perpendikular ju’rgizilgen. Qi’yani’n’ uzi’nli’g’i’ 10, perpendikulyardi’n’ uzi’nli’g’i’ 6. Qi’yani’n’ tegisliktegi proektsiyasi’ni’n’ uzi’nli’g’i’n tabi’n’.

a) 4 b) 2 c) 8 d) 5

35. Tochkadan tegislikke yeki qi’ya ju’rgizilgen. Qi’yalar 3:5 ke ten’ qatnasta boli’p, wolardi’n’ proektsiyalari’ 33 ha’m 17 ge ten’. Qi’yalardi’n’ uzi’nli’g’i’n tabi’n’.

a) $10\sqrt{2};5\sqrt{2}$ b) $25\sqrt{2};10\sqrt{2}$ c) $15\sqrt{2};5\sqrt{2}$ d) $25\sqrt{2};5\sqrt{2}$

36. oxy tegislikke sali’sti’rg’anda (a;b;c) tochkag’a simmetriyali’ bolg’an tochkani’ tabi’n’.

a) (-a;b;c) b) (-a;-b;-c) c) (a;b;-c) d) (a;-b;c)

37. AB kesindinin’ wortasi’ ox ko’sherinde jatadi’. Yeger A(-3;m;5) ; B(2;-2;n) bolsa, m ha’m n di tabi’n’?

a) m=2; n=-5 b) m=-5; n=2 c) m=-2; n=4 d) m=1; n=4

38. Tuwri’ prizmani’n’ biyikligi 50ge, ultani’ni’n’ ta’repleri 13, 37 ha’m 40 qa ten’. Prizmani’n’ toli’q betin tabi’n’.

a) 2730 b) 3900 c) 4500 d) 4980

39. Kubti’n’ jag’i’ni’n’ maydani’ 2 ma’rte artti’ri’lsa, woni’n’ ko’lemi neshe ma’rte artadi’?

a) 2 b) 8 c) 4 d) 6

40. Qi’ya prizmani’n’ qaptal qabi’rg’asi’ 20g’a ten’ ha’m ultan tegisligi menen 30º li’ mu’yesh payda yetedi. Prizmani’n’ biyikligin tabi’n’.

a) 12 b) $10\sqrt{3}$ c) 10 d) $10\sqrt{2}$

41. Duri’s to’rtmu’yeshli piramidani’n’ biyikligi 6 sm, apofemasi’ 6, 5sm. Piramida ultani’ni’n’ perimetrin tabi’n’

a) 10 b) 12 c) 24 d) 20

42. Duri’s tetraedrdin’ qabi’rg’asi’ 1 ge ten’. Woni’n’ ultani’na si’rtlay si’zi’lg’an shen’berdin’ worayi’nan woni’n’ qaptal jag’ani’ shekemgi bolg’an arali’qti’ tabi’n’

a) $\frac{2\sqrt{3}}{6}$ b) $\frac{\sqrt{6}}{9}$ c) $\frac{2\sqrt{2}}{5}$ d) $\frac{3\sqrt{2}}{8}$

43. Shen'berdin' uzi'nli'g'i' radiusi' 4 ke ha'm worayli'q mu'yesh 120º qa ten' bo'lek uzi'nli'g'i'na ten'. Shen'berdin' radiusi' tabi'n'.

a) $2\frac{2}{3}$ b) $1\frac{1}{3}$ c) 2 d) $\frac{2\sqrt{2}}{3}$

44. Tsilindrdin' qaptal beti jayi'lmasi'ni'n' diagonali' ultan tegisligi menen 45º li' mu'yesh jasaydi'. Tsilindrdin' qaptal beti $144\pi^2$ qa ten' bolsa, radiusi'n tabi'n'.

a) 12 b) $\sqrt{12}$ c) 6 d) 36

45. Jasawshi'si' l boli'p, ultan tegisligi menen 60º li' mu'yesh jasawshi' konusti'n' ultani'ni'n' maydani'n tabi'n'.

a) $\frac{\pi l^2}{2}$ b) $\frac{\pi l^2}{3}$ c) πl^2 d) $\frac{\pi l^2}{4}$

46. Kesik konus ko'sherlik kesiminin' maydani' ultanlari' maydanlari'ni'n' ayi'rmasi'na ten'. Ultanlari'ni'n' raduislari' R ha'm r bolsa, kesik konusti'n' ko'lemin tabi'n'.

a) $\frac{1}{3}\pi\left(R^3-r^3\right)$ b) $\frac{1}{3}\pi\left(R^3+r^3\right)$ c) $\frac{1}{3}\pi(R+r)^3$ d) $\frac{1}{3}\pi(R-r)^3$

47. Tsilindrge shar ishley si'zi'lg'an. Tsilindrdin' ko'lemi 16π ge ten' bolsa, shardi'n' ko'lemin tabi'n'.

a) $\frac{32\pi}{3}$ b) 30π c) 32π d) 30

48. Biyikligi shar diametrinin' 0, 1 bo'legine ten' bolg'an shar sigmentinin' ko'lemi shar ko'leminin' qanday bo'legin payda yetedi.

a) 0, 028 b) 0, 28 c) 2, 8 d) 0, 8

49. Sharg'a ishley si'zi'lg'an konusti'n' biyikligi 3 ke, ultani'ni'n' radiusi' $3\sqrt{3}$ ke ten'. Shardi'n' radiusi'n tabi'n'.

a) $5\sqrt{2}$ b) $4\sqrt{3}$ c) 6 d) 5

50. U'shmu'yeshli piramidani'n' ultani'ni'n' ta'repleri 4; 4 ha'm 2 ge ten' bolg'an u'shmu'yeshlikten ibarat. Piramidani'n' barli'q qaptal jaqlari' ultan tegisligi menen 60º li' mu'yesh payda yetedi. Piramidani'n' ko'lemin tabi'n'.

a) 6 b) $2\sqrt{3}$ c) 3 d) $\sqrt{3}$

II-arali’q baqlaw ushi’n test tapsi’rmalari’

I-variant

(Duri’s juwabi’n ko’rsetin’)

1. Yesaplan’ $\lim\limits_{x\to 2}\left(x^3-3x^2+7x+5\right)$

a) 39 b) 15 c) 20 d) 25

2. Yesaplan’ $\lim\limits_{x\to 3}\dfrac{x^2-9}{x-3}$

a) 6 b) 0 c) -6 d) 3

3. Yesaplan’ $\lim\limits_{x\to 0}\dfrac{4x^3+5x^2}{x^4-3x^3+x^2}$

a) -9 b) 0 c) 5 d) ∞

4. Yesaplan’ $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 4x}$

a) 0 b) 2 c) $-\dfrac{1}{2}$ d) $\dfrac{1}{2}$

5. Yesaplan’ $\lim\limits_{x\to 0}2^{\frac{3x}{x+2}}$

a) 1 b) 2 c) 0 d) -1

6. Yesaplan’ $\lim\limits_{x\to 9}\lg(x+1)$

a) 1 b) 0 c) 10 d) e

7. $y=\dfrac{5}{x-10}$ funkciyasi’ni’n’ u’zilis tochkasi’n ko’rsetin’.

a) $x=5$ b) $x=-10$ c) $x=0$ d) $x=10$

8. $f(x)=\dfrac{x^2}{9-x^2}$ funkciyasi’ x barli’q ma’nislerinde u’zliksiz be?

a) $x_1=-3$ ha’m $x_2=3$ ma’nislerinen basqa barli’q ma’nislerinde u’zliksiz.

b) x tin’ barli’q ma’nislerinde u’zliksiz.

c) $x_1=-9$ ha’m $x_2=9$ ma’nislerinen basqa barli’q ma’nislerinde u’zliksiz.

d) $x=0$ ma’nisinen basqa barli’q ma’nislerinde u’zliksiz.

9. $y=-\dfrac{5}{x}$ funkciyani’n’ tuwi’ndi’si’n tabi’n’?

a) -5 b) 5 c) $\frac{5}{x}$ d) $\frac{5}{x^2}$

10. $f(x)=4\sqrt[3]{x}-\sqrt[3]{4}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{4}{3\sqrt[3]{x^2}}$ b) $\frac{4\sqrt[3]{x^2}}{3}$ c) $\left(\frac{1}{\sqrt[3]{x^2}}-1\right)$ d) 4

11. $f(x)=(x+2)(4x-3)$ bolsa, $f'(2)$ ni tabi'n'?

a) 20 b) 21 c) 8 d) 9

12. $f(x)=\frac{x^2+7}{x^2-5}$ funkciyasi'ni'n' tuwi'ndi'si'n tabi'n'?

a) 1 b) $\frac{2x}{(x^2-5)^2}$ c) $-\frac{24}{(x^2-5)^2}$ d) $\frac{2x}{x^2-5}$

13. $y=\sqrt[3]{\frac{1}{1+x}}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $-\frac{1}{3\cdot\sqrt[3]{(1+x)^4}}$ b) $\frac{1}{3\cdot\sqrt[3]{(1+x)^4}}$ c) 1 d) $-\frac{1}{3\sqrt[3]{(1+x)^4}}$

14. $y=\sin^2 x+\cos^2 x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) 1 b) 0 c) $\sin 2x$ d) $2\sin 2x$

15. $y=\frac{1}{3}tg^3 x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{\sin^2 x}{\cos^4 x}$ b) $tg^2 x$ c) $3tg^2 x$ d) $\frac{\sin^2 x}{\cos^3 x}$

16 $y=\sin 2x$ funkciyani'n' tuwi'ndi'si'ni'n' $x=\frac{\pi}{2}$ tochkadag'i' ma'nisin tabi'n'?

a) 2 b) -2 c) 0 d) $\frac{\pi}{2}$

17. $y=\arcsin x^2$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{2x}{\sqrt{1-x^2}}$ b) $-\frac{2x}{\sqrt{1-x^2}}$ c) $\frac{2x}{\sqrt{1-x^4}}$ d) $-\frac{2x}{\sqrt{1-x^4}}$

18. $y=x\cdot\arcsin x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\arcsin x$ b) $\frac{x}{\sqrt{1-x^2}}$ c) $-\frac{2x}{\sqrt{1-x^4}}$ d) $\arcsin x+\frac{x}{\sqrt{1-x^2}}$

19. Tuwi'ndi'si'n tabi'n'. $y=\ln^5 x$

a) $5\ln^4 x$ b) $\frac{5}{x}\ln^4 x$ c) $\frac{\ln x}{x}$ d) $\ln x$

20. $y=\sin^2 x$ funkciyani'n' 4-ta'rtipli tuwi'ndi'si'n tabi'n'.

a) $\cos 2x$ b) $2\sin x$ c) $-8\cos 2x$ d) $-2\cos x$

21. $y=e^{4x}$ funkciyani'n' 5-ta'rtipli tuwi'ndi'si'n tabi'n'.

a) $1024e^{4x}$ b) e^{4x} c) $4e^{4x}$ d) $256e^{4x}$

22. $\int\left(5x^4+3x^2\right)dx$ ti tabi'n'.

a) x^4+x^3+C b) x^5+x^3+C c) $\frac{x^5}{5}+\frac{x^3}{3}+C$ d) $\frac{x^4}{20}+\frac{x^3}{6}+C$

23. $\int\frac{x^2dx}{8+x^3}$ integrali'n tabi'n'.

a) $\frac{x^3}{3}+\ln(8+x^3)+C$ b) $\frac{x^3}{3}+\frac{x^4}{4}+8x+C$

c) $\frac{1}{3}\ln\left(8+x^3\right)+C$ d) $x^2\cdot\ln\left(8+x^3\right)+C$

24. $\int(x-5)\sin x dx$ integrali'n tabi'n'.

a) $(5-x)\cos x+\sin x+C$ b) $-x\cos x-\frac{5\sin^2 x}{2}+C$

c) $(5-x)\cos x-\sin x+C$ d) $-\frac{(x-5)^2}{2}\cos x+C$

25. $\int_1^2 6x^5dx$ ti tabi'n.

a) 31 b) 63 c) 1 d) 4800

26. $\int_0^{e^2-1}\frac{dx}{x+1}$ ti tabi'n.

a) 3 b) -3 c) 2 d) -2

27. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\sin^3 x+1}{\sin^2 x}dx$ ti tabi'n.

a) $\frac{1}{2}-\frac{\sqrt{3}}{3}+\frac{\sqrt{2}}{2}$ b) $\frac{1}{2}-\frac{\sqrt{3}}{3}$ c) $-\frac{1}{2}-\frac{1}{\sqrt{3}}$ d) 1

28. $\int_0^3 \frac{dx}{1+x^2}$ ti tabi'n.

a) $arctg3$ b) $arctg2$

c) $arctg3 - \pi n; n = 0,1,2,3,..$ d) $arctg3 - \frac{\pi n}{2}; n = 0,1,2,3,..$

29. $\int_0^3 (x^2 - 2x + 2)dx$ ti tabi'n.

a) 24 b) 6 c) 8 d) 12

30. $\int_1^5 x \ln x dx$ ti tabi'n.

a) $\frac{25}{2}\ln 5 - 6$ b) $\frac{25}{2}\ln 5 + 6$ c) $\frac{25}{2}\ln 5$ d) $\frac{25}{2}\ln 5 - \frac{26}{4}$

31. $u = x^2 + 3xy + 4y^2$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'.

a) $\frac{\partial u}{\partial x} = 2x + 3y + 8y; \frac{\partial u}{\partial y} = 2x + 3x + 8y$ b) $\frac{\partial u}{\partial x} = 2x + 3xy + 8y; \frac{\partial u}{\partial y} = 2x + 3xy$

c) $\frac{\partial u}{\partial x} = 2x + 3y; \frac{\partial u}{\partial y} = 3x + 8y$ d) $\frac{\partial u}{\partial x} = 2x + 8y; \frac{\partial u}{\partial y} = 3xy + 8y$

32. $z = x^n + y^n$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'.

a) $\frac{\partial z}{\partial x} = nx^{n-1}; \frac{\partial z}{\partial y} = ny^{n-1}$ b) $\frac{\partial z}{\partial x} = nx^{n-1} + ny^{n-1}; \frac{\partial z}{\partial y} = ny^{n-1} - ny^{n-1}$

c) $\frac{\partial z}{\partial x} = ny^{n-1}; \frac{\partial z}{\partial y} = nx^{n-1}$ d) $\frac{\partial z}{\partial x} = 1; \frac{\partial z}{\partial y} = 1$

33. $z = \frac{ay - bx}{by - ax}$ funkciyasi'ni'n' toli'q differentsiyali'n tabi'n'.

a) $dz = \frac{(a^2 - b^2)(ydx + xdy)}{(by - ax)^2}$ b) $dz = \frac{(a^2 - b^2)ydx}{(by - ax)^2}$

c) $dz = \frac{(a^2 - b^2)(ydx - xdy)}{(by - ax)^2}$ d) $dz = \frac{(a^2 - b^2)xdy}{(by - ax)^2}$

34. $u = \ln \frac{x^2 + y^2}{x^2 - y^2}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'

a) $|y| < |x|$ b) $|y| \le |x|$ c) $|y| > |x|$ d) $|y| \ge |x|$

35. $u = x^2 + 3xy + 4y^2$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'

a) $\frac{\partial u}{\partial x}=2x+11y;\frac{\partial u}{\partial y}=5x+8y$ b) $\frac{\partial u}{\partial x}=2x+8y;\frac{\partial u}{\partial y}=3x+4y$

c) $\frac{\partial u}{\partial x}=2x+8y;\frac{\partial u}{\partial y}=2x+4y^2$ d) $\frac{\partial u}{\partial x}=2x+3y;\frac{\partial u}{\partial y}=3x+8y$

36. $z=\ln\left(x^2+y^2\right)$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'

a) $\frac{\partial u}{\partial x}=\frac{2(x+y)}{\left(x^2+y^2\right)};\frac{\partial u}{\partial y}=\frac{2(x+y)}{\left(x^2+y^2\right)}$ b) $\frac{\partial u}{\partial x}=\frac{2x}{\left(x^2+y^2\right)};\frac{\partial u}{\partial y}=\frac{2y}{\left(x^2+y^2\right)}$

c) $\frac{\partial u}{\partial x}=x^2+y^2;\frac{\partial u}{\partial y}=x^2-y^2$ d) $\frac{\partial u}{\partial x}=\frac{x^2}{\left(x^2+y^2\right)};\frac{\partial u}{\partial y}=\frac{y^2}{\left(x^2+y^2\right)}$

37. $u=\frac{1}{\sqrt{x^2+y^2+z^2}}$ funkciyani'n' toli'q differentsiyali'n tabi'n'

a) $du=\frac{xdx+ydy+zdz}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$ b) $du=\frac{xdx+ydy+zdz}{x^2+y^2+z^2}$

c) $du=-\frac{xdx+ydy+zdz}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$ d) $du=-\frac{xdx+ydy+zdz}{x^2+y^2+z^2}$

38. $z=\sin(3u+2v)$ bolsa, $\frac{dz}{dx}$ ti tabi'n'. (bul jerde $u=2x^3,v=3x^2$)

a) $\frac{dz}{dx}=\left(18x^2+12x\right)\cdot\cos(3u+2v)$ b) $\frac{dz}{dx}=\cos(3u+2v)$

c) $\frac{dz}{dx}=(6x^2+6x)\cdot\cos(3u+2v)$ d) $\frac{dz}{dx}=(6x^2-6x)\cdot\cos(3u+2v)$

39. $z=\sqrt{x^2+y^2}$ funkciyasi' berilgen (bunda $y=\sin^2 x$). $\frac{dz}{dx}$ ti tabi'n'.

a) $\frac{dz}{dx}=\frac{x}{\sqrt{x^2+y^2}}+\frac{\sin 2x}{\sqrt{x^2+y^2}}$ b) $\frac{dz}{dx}=\frac{x}{\sqrt{x^2+y^2}}+\frac{y}{\sqrt{x^2+y^2}}\cdot\sin 2x$

c) $\frac{dz}{dx}=\frac{x}{\sqrt{x^2+y^2}}+\frac{y}{\sqrt{x^2+y^2}}\cdot 2\cos x$ d) $\frac{dz}{dx}=\frac{x}{\sqrt{x^2+y^2}}-\frac{y}{\sqrt{x^2+y^2}}\cdot\sin 2x$

40. Differentsiyal ten'lemeni sheshin' $3y-xy'=0$

a) $y=cx^2$ b) $y=cx^3$ c) $y=-x^3$ d) $y=-x^2$

41. $xy'-y=0$ differentsiyal ten'lemenin' uli'wma integrali'n tabi'n'

a) $y=e^x$ b) $y=\ln x$ c) $y=e^{cx}$ d) $y=cx$

42. $xy'-y=0$ differentsiyal ten'lemenin' $x=-2$ bolg'anda $y=4$ da'slepki sha'rtleri boyi'nsha dara integral tabi'lsi'n.

a) $y=-2x$ b) $y=4-\frac{1}{e^2}$ c) $y=e^{-2x}$ d) $y=2x$

43. $f(x,y)=y^x+x^{y-1}$ funkciyasi' berilgen. $f(1,1)$ di tabi'n

a) 1 b) 2 c) 0 d) -1

44. $f(x,y)=e^{\sin(x+y)}$ funkciyasi' berilgen. $f\left(\frac{\pi}{2};\frac{\pi}{2}\right)$ di tabi'n

a) 1 b) e c) $\sqrt{e}$ d) $\frac{1}{e}$

45. Yeger betlik $z=x^2+y^2$ ten'lemesi menen berilgen bolsa, wonda woni'n' $(1;1;2)$ tochkadag'i' uri'nba tegislik ten'lemesin tabi'n'

a) $z=2x+2y+2$ b) $z=2x+2y-2$

c) $z=-2x-2y+2$ d) $z=-2x-2y-2$

46. Yeger betlik $z=x^2+y^2$ ten'lemesi menen berilgen bolsa, wonda woni'n' $(1;1;2)$ tochkadag'i' normaldi'n' ten'lemesin tabi'n'

a) $\frac{x-1}{-2}=\frac{y-1}{-2}=\frac{z-1}{2}$ b) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{-2}$

c) $\frac{x-1}{-2}=\frac{y-1}{-2}=\frac{z-2}{1}$ d) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-1}{1}$

47. $z=2xy$ funkciyani'n' $(3;2)$ tochkadag'i' gradientin tabi'n'

a) $4\vec{i}+6\vec{j}$ b) $4\vec{i}-6\vec{j}$ c) $4\vec{i}+2\vec{j}$ d) $4\vec{i}-2\vec{j}$

48. $f(x,y)=x^3+3x^2y+12xy^3$ berilgen. $f''_{x^2}(0;1)$ di tabi'n'

a) 0 b) 1 c) 6 d) -1

49. $f(x;y)=x^3+y^3+x^3y^3$ berilgen. $f''_{xy}(1;2)$ ti tabi'n'

a) 36 b) 18 c) 9 d) 1

50. $f(x;y)=e^{xy}$ berilgen. f''_{xy} ti tabi'n'

a) y^2e^{xy} b) $e^{xy}(1+xy)$ c) x^2e^{xy} d) 1

II-variant

(Duri's juwabi'n ko'rsetin')

1. Yesaplan' $\lim\limits_{x\to 0}\left(x^3-4x^2+6x-5\right)$

a) -5 b) 5 c) 0 d) ∞

2. Yesaplan' $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}$

a) 0 b) -12 c) 8 d) 12

3. Yesaplan' $\lim\limits_{x\to 0}\dfrac{3x^2-4x}{5x^3-7x^2+10x}$

a) -0, 4 b) 0, 4 c) 0 d) ∞

4. Yesaplan' $\lim\limits_{x\to 0}\dfrac{\sin 6x}{\sin 3x}$

a) 6 b) 2 c) -2 d) 3

5. Yesaplan' $\lim\limits_{x\to 0} 4^{\frac{2x}{x-3}}$

a) 4 b) 1 c) 0 d) -1

6. Yesaplan' $\lim\limits_{x\to 0}\ln\dfrac{2x+1}{x+1}$

a) 1 b) -1 c) 2 d) 0

7. $y=\dfrac{1}{x}$ funkciyasi'ni'n' u'zilis tochkasi'n ko'rsetin'.

a) $x=1$ b) $x=0$ c) $x=-1$ d) $x=2$

8. $f(x)=\dfrac{\sin x}{1-x^3}$ funkciyasi' x barli'q ma'nislerinde u'zliksiz be?

a) $x=-1$ ma'nisinen basqa barli'q ma'nislerinde u'zliksiz.

b) $x=1$ ma'nisinen basqa barli'q ma'nislerinde u'zliksiz.

c) $x=0$ ma'nisinen basqa barli'q ma'nislerinde u'zliksiz.

d) x tin' barli'q ma'nislerinde u'zliksiz.

9. $y=\dfrac{10}{x}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) 10 b) -10 c) $-\dfrac{10}{x^2}$ d) $-\dfrac{10}{x}$

10. $f(x)=\dfrac{x}{4}+\sqrt[4]{4x}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\dfrac{1}{4}\left(1+\dfrac{1}{2\sqrt[4]{4x^3}}\right)$ b) $\dfrac{1}{4}+4x$ c) $1+\dfrac{1}{16x^2}$ d) $\dfrac{1}{4}+\dfrac{1}{16x^2}$

11. $f(x)=(x-5)(7x-4)$ bolsa $f'(1)$ di tabi'n'

a) 25 b) -53 c) -25 d) 53

12. $f(x)=\frac{x^3-1}{x^3+7}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{24x}{\left(x^3+7\right)^2}$ b) $\frac{3x}{\left(x^3+7\right)^2}$ c) $-\frac{24x}{\left(x^3+7\right)^2}$ d) 1

13. $y=\sqrt[3]{\frac{2}{1-x}}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $-\frac{2}{3\cdot\sqrt[3]{4(1-x)^4}}$ b) $\frac{2}{3\cdot\sqrt[3]{4(1-x)^4}}$

c) 1 d) $\frac{2}{3\cdot\sqrt[3]{4(1-x)^2}}$

14. $y=\sin^2 x-\cos^2 x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) 1 b) 0 c) $\sin 2x$ d) $2\sin 2x$

15. $y=\frac{1}{2}tg^2 x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{\sin^2 x}{\cos^4 x}$ b) $\frac{\sin x}{\cos^3 x}$ c) $-\frac{\sin x}{\cos^3 x}$ d) $-\frac{\sin^2 x}{\cos^4 x}$

16. $y=\cos 2x$ funkciya tuwi'ndi'si'ni'n' $x=\frac{\pi}{2}$ tochkasi'ndag'i' ma'nisin tabi'n'

a) 2 b) -2 c) 0 d) $\frac{\pi}{2}$

17. $y=\arccos x^2$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{2x}{\sqrt{1-x^2}}$ b) $-\frac{2x}{\sqrt{1-x^2}}$ c) $\frac{2x}{\sqrt{1-x^4}}$ d) $-\frac{2x}{\sqrt{1-x^4}}$

18. $y=x\cdot\arccos x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\arccos x-\frac{x}{\sqrt{1-x^2}}$ b) $\arccos x+\frac{x}{\sqrt{1-x^2}}$

c) $\arccos x$ d) $-\frac{x}{\sqrt{1-x^2}}$

19. Tuwi'ndi'si'n tabi'n'. $y=\ln\sin x$

a) $ctgx$ b) $\frac{1}{\sin x}$ c) $\ln\cos x$ d) $\frac{1}{\cos x}$

20. $y=\cos^2 x$ funkciyani'n' 4-ta'rtipli tuwi'ndi'si'n tabi'n'.

a) $-8\cos 2x$ b) $8\sin 2x$ c) $-8\sin 2x$ d) $8\cos 2x$

21. $y=2e^{2x}$ funkciyani'n' 5-ta'rtipli tuwi'ndi'si'n tabi'n'.

a) $2e^{2x}$ b) $64e^{2x}$ c) 1 d) $4e^{2x}$

22. $\int\left(4\sqrt[3]{x}-6\sqrt{x}\right)dx$ ti tabi'n'.

a) $12x\sqrt[3]{x}-12x\sqrt{x}+C$ b) $16x\sqrt[3]{x+}-x^3\sqrt{x}+C$

c) $3x\sqrt[3]{x}-4x\sqrt{x}+C$ d) $3\sqrt[3]{x^2}-4\sqrt{x}+C$

23. $\int\frac{dx}{x\ln x}$ integrali'n tabi'n'.

a) $\ln x+\frac{1}{\ln x}+C$ b) $-\frac{1}{x^2}+\frac{1}{\ln x}+C$

c) $\frac{\ln x}{x}+C$ d) $\ln(\ln x)+C$

24. $\int x^2\cdot e^{2x}dx$ integrali'n tabi'n'.

a) $\frac{x^3}{3}e^{2x}+C$ b) $\frac{x^3e^{2x}}{6}+C$

c) $\frac{1}{2}e^{2x}\left(x^2-x+\frac{1}{4}\right)+C$ d) $\frac{1}{4}e^{2x}\cdot(2x^2-2x+1)+C$

25. $\int_0^3\left(3x-e^{\frac{x}{3}}\right)$ ti tabi'n.

a) $16{,}5-3e$ b) $10{,}5-3e$ c) $13{,}5-3e$ d) $16{,}5-\frac{e}{3}$

26. $\int_{-1}^{3}(3-x)dx$ ti tabi'n.

a) 2 b) 4 c) 6 d) 8

27. $\int_0^{\frac{\pi}{3}}(\sin 2x-\cos x)dx$ ti tabi'n.

a) $\frac{3-2\sqrt{3}}{4}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{3-2\sqrt{3}}{2}$ d) $-\frac{\sqrt{3}}{2}$

28. $\int_2^3\frac{dx}{\sqrt{1-x^2}}$ ti tabi'n.

a) $\arcsin 3$ b) $\arcsin 2$ c) $\arcsin 3+\arcsin 2$ d) $\arcsin 3-\arcsin 2$

29. $\int_{-1}^{1}\left(x^2-2x+2\right)dx$ ti tabi'n.

a) -2 b) 2 c) $\frac{14}{3}$ d) $-\frac{14}{3}$

30. $\int_{1}^{4} x\ln x dx$ ti tabi'n.

a) $8\ln 4$ b) $8\ln 4-\frac{15}{4}$ c) $8\ln 4+\frac{15}{4}$ d) $8\ln 4-\frac{3}{4}$

31. $u=\sin(3x+5y-4z)$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'.

a) $\frac{\partial u}{\partial x}=\cos(3x+5y-4z);\frac{\partial u}{\partial y}=\cos(3x+5y-4z);\frac{\partial u}{\partial z}=\cos(3x+5y-4z);$

b) $\frac{\partial u}{\partial x}=60\cos(3x+5y-4z);\frac{\partial u}{\partial y}=12\cos(3x+5y-4z);\frac{\partial u}{\partial z}=20\cos(3x+5y-4z);$

c) $\frac{\partial u}{\partial x}=3\sin(3x+5y-4z);\frac{\partial u}{\partial y}=5\sin(3x+5y-4z);\frac{\partial u}{\partial z}=-4\sin(3x+5y-4z);$

d) $\frac{\partial u}{\partial x}=3\cos(3x+5y-4z);\frac{\partial u}{\partial y}=5\cos(3x+5y-4z);\frac{\partial u}{\partial z}=-4\cos(3x+5y-4z);$

32. $z=\cos(ax+by)$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'.

a) $\frac{\partial z}{\partial x}=-\sin(ax+by);\frac{\partial z}{\partial y}=-\sin(ax+by);$

b) $\frac{\partial z}{\partial x}=-a\sin(ax+by);\frac{\partial z}{\partial y}=-b\sin(ax+by);$

c) $\frac{\partial z}{\partial x}=a\sin(ax+by);\frac{\partial z}{\partial y}=b\sin(ax+by);$

d) $\frac{\partial z}{\partial x}=a\cos(ax+by);\frac{\partial z}{\partial y}=b\cos(ax+by);$

33. $z=arctg\frac{y}{x}$ funkciyasi'ni'n' toli'q differentsiyali'n tabi'n'.

a) $dz=\frac{ydx+xdy}{x^2+y^2}$ b) $dz=\frac{xdx-ydy}{x^2+y^2}$

c) $dz=\frac{dx-dy}{x^2+y^2}$ d) $dz=\frac{ydx-xdy}{x^2+y^2}$

34. $u=\ln x+\ln y$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'

a) $x=0; y=0$ b) $x>0; y>0$

c) $x\geq 0; y\geq 0$ d) $x<0; y<0$

35. $u=\sin(3x+5y)$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'

a) $\frac{\partial u}{\partial x}=\frac{1}{3}\cos(3x+5y); \frac{\partial u}{\partial y}=\frac{1}{5}\cos(3x+5y)$

b) $\frac{\partial u}{\partial x}=3\sin(3x+5y); \frac{\partial u}{\partial y}=5\sin(3x+5y)$

c) $\frac{\partial u}{\partial x}=3\cos(3x+5y); \frac{\partial u}{\partial y}=5\cos(3x+5y)$

d) $\frac{\partial u}{\partial x}=\cos(3x-5y); \frac{\partial u}{\partial y}=\cos(3x+5y)$

36. $z=x^3+3x^2y-y^2$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'

a) $\frac{\partial u}{\partial x}=3x^2+6xy; \frac{\partial u}{\partial y}=3x^2-2y$ b) $\frac{\partial u}{\partial x}=3x^2+3y; \frac{\partial u}{\partial y}=3x^2-y^2$

c) $\frac{\partial u}{\partial x}=x^3+3x^2; \frac{\partial u}{\partial y}=3y^2-y^2$ d) $\frac{\partial u}{\partial x}=3x^2-2y; \frac{\partial u}{\partial y}=x^3-2y$

37. $u=\frac{y}{1+x^2}$ funkciyani'n' toli'q differentsiyali'n tabi'n'

a) $du=\frac{(1+x^2)dy-2xydx}{\left(1+x^2\right)^2}$ b) $du=\frac{2xydx-(1+x^2)dy}{\left(1+x^2\right)^2}$

c) $du=\frac{-2xydx-dy}{1+x^2}$ d) $du=\frac{(1+x)^2dy-2xydx}{1+x^2}$

38. $z=\cos(3u+2v)$ bolsa, $\frac{dz}{dx}$ ti tabi'n'. (bul jerde $u=2x^3, v=3x^2$)

a) $\frac{dz}{dx}=\left(18x^2+12x\right)\cdot\sin\left(3u+2v\right)$ b) $\frac{dz}{dx}=-\left(18x^2+12x\right)\cdot\sin\left(3u+2v\right)$

c) $\frac{dz}{dx}=-3\sin\left(3u+2v\right)$ d) $\frac{dz}{dx}=-2\sin\left(3u+2v\right)$

39. $z=u^v$ funkciyasi' berilgen (bunda $y=\sin x; v=\cos x$). $\frac{dz}{dx}$ ti tabi'n'.

a) $\frac{dz}{dx}=(\sin x)^{\cos x}\cdot\left(\frac{\cos^2 x}{\sin x}-\sin x\ln\sin x\right)$

b) $\frac{dz}{dx}=(\sin x)^{\cos x}\cdot\left(\frac{\cos^2 x}{\sin x}+\sin x\ln\sin x\right)$

c) $\frac{dz}{dx}=\frac{\cos^2 x}{\sin x}-\sin x\ln\sin x$

d) $\frac{dz}{dx}=\frac{\cos^2 x}{\sin x}+\sin x\ln\sin x$

40. Differentsiyal ten'lemeni sheshin' $5y-xy'=0$

a) $y=-x^5$ b) $y=cx^4$ c) $y=-x^4$ d) $y=cx^5$

41. $xy'+y=0$ differentsiyal ten'lemenin' uli'wma integrali'n tabi'n'

a) $y=\frac{c}{x}$ b) $y=cx$ c) $y=\ln x$ d) $y=e^{cx}$

42. $xy'+y=0$ differentsiyal ten'lemenin' $x=-2$ bolg'anda $y=4$ da'slepki sha'rtleri boyi'nsha dara integral tabi'lsi'n.

a) $y=-2x$ b) $y=-\frac{8}{x}$ c) $y=\frac{8}{x}$ d) $y=2x$

43. $f(x,y)=y^x+x^{y-1}$ funkciyasi' berilgen. $f(1,2)$ di tabi'n

a) 1 b) 2 c) 3 d) 4

44. $f(x,y)=e^{\cos(x+y)}$ funkciyasi' berilgen. $f\left(\frac{\pi}{2};\frac{\pi}{2}\right)$ di tabi'n

a) 1 b) e c) $\sqrt{e}$ d) $\frac{1}{e}$

45. Yeger betlik $z=x^2-y^2$ ten'lemesi menen berilgen bolsa, wonda woni'n' $(1;2;-1)$ tochkadag'i' uri'nba tegislik ten'lemesin tabi'n'

a) $z=2x-4y+5$ b) $z=2x-4y-5$

c) $z=-2x-4y+5$ d) $z=2x+4y-5$

46. Yeger betlik $z=x^2-y^2$ ten'lemesi menen berilgen bolsa, wonda woni'n' $(1;2;-1)$ tochkadag'i' normaldi'n' ten'lemesin tabi'n'

a) $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z-1}{1}$ b) $\frac{x-1}{-2}=\frac{y+2}{-4}=\frac{z-1}{1}$

c) $\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-1}{1}$ d) $\frac{x-1}{-2}=\frac{y-2}{4}=\frac{z+1}{1}$

47. $z=x^2+y^2$ funkciyani'n' $(3;2)$ tochkadag'i' gradientin tabi'n'

a) $6\vec{i}-4\vec{j}$ b) $6\vec{i}+4\vec{j}$ c) $3\vec{i}+2\vec{j}$ d) $3\vec{i}-2\vec{j}$

48. $f(x,y)=x^3+3x^2y+12xy^3$ berilgen. $f''_{xy}(-1;1)$ di tabi'n'

a) -1 b) 1 c) -10 d) 30

49. $f(x;y)=x^3+y^3+x^3y^3$ berilgen. $f''_{x^2}(1;1)$ ti tabi'n'

a) 12 b) 3 c) 6 d) 9

50. $f(x;y)=e^{xy}$ berilgen. f''_{yx} ti tabi'n'

a) y^2e^{xy} b) $e^{xy}(1+xy)$ c) x^2e^{xy} d) 1

III-variant

(Duri's juwabi'n ko'rsetin')

1. Yesaplan' $\lim\limits_{x\to -1}\left(x^3-x^2-3x+7\right)$

a) 10 b) 12 c) -8 d) 8

2. Yesaplan' $\lim\limits_{x\to 0}\dfrac{3x^3+2x^2+5x}{x}$

a) 0 b) 5 c) -5 d) 10

3. Yesaplan' $\lim\limits_{x\to 0}\dfrac{5x^3-6x^2}{3x^4+10x^3-2x^2}$

a) 0 b) ∞ c) 3 d) -3

4. Yesaplan' $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\sin 5x}$

a) 0, 6 b) 0 c) ∞ d) $\dfrac{5}{3}$

5. Yesaplan' $\lim\limits_{x\to 0}3^{\frac{3x-2}{x-2}}$

a) 1 b) 0 c) -1 d) 3

6. Yesaplan' $\lim\limits_{x\to 8}\lg(x+2)$

a) 1 b) 2 c) 10 d) e

7. $y=\dfrac{1}{x-5}$ funkciyasi'ni'n' u'zilis tochkasi'n ko'rsetin'.

a) $x=5$ b) $x=0$ c) $x=1$ d) $x=-5$

8. $f(x)=\dfrac{x^2+x+1}{\sin x}$ funkciyasi' x barli'q ma'nislerinde u'zliksiz be?

a) $x=n\pi$ (bul jerde n-qa'legen pu'tin san) ma'nisinen basqa barli'q ma'nislerinde u'zliksiz.

b) x tin' barli'q ma'nislerinde u'zliksiz.

c) $x=0$ ma'nisinen basqa barli'q ma'nislerinde u'zliksiz.

d) $x=1$ma'nisinen basqa barli'q ma'nislerinde u'zliksiz.

9. $y=-\frac{2}{x^2}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{4}{x^3}$ b) $\frac{2}{x^4}$ c) $-\frac{1}{x}$ d) -2

10. $f(x)=\frac{2}{x^2}-\sqrt{x}+\sqrt[3]{15}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $-\frac{4}{x^3}-\frac{1}{2\sqrt{x}}$ b) $-\frac{4}{x^3}-\frac{1}{2\sqrt{x}}+\frac{1}{\sqrt[3]{225}}$

c) $\frac{1}{x}-\frac{1}{2\sqrt{x}}$ d) $\frac{1}{x}-\frac{1}{2\sqrt{x}}+\frac{1}{\sqrt[3]{15}}$

11. $f(x)=(2x-4)(3x+2)$ bolsa, $f'(-1)$ di tabi'n'?

a) 4 b) -4 c) 20 d) -20

12. $f(x)=\frac{x^2-9}{x^3+3}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{-x^4+27x^2+6x}{\left(x^3+3\right)^2}$ b) $\frac{2}{3x}$

c) 1 d) $\frac{5x^4+27x^2+6x}{(x^3+3)^2}$

13. $y=\sqrt[3]{\frac{2}{1+x}}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $-\frac{2}{3\cdot\sqrt[3]{4(1+x)^4}}$ b) $\frac{2}{3\cdot\sqrt[3]{4(1+x)^4}}$

c) 1 d) $\frac{2}{3\cdot\sqrt[3]{4(1+x)^2}}$

14. $y=\cos^2 x-\sin^2 x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) 1 b) $-2\sin 2x$ c) $2\sin 2x$ d) $\sin 2x$

15. $y=\frac{1}{3}ctg^3 x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{\cos^2 x}{\sin^4 x}$ b) $-\frac{\cos^2 x}{\sin^4 x}$ c) $\frac{\cos x}{\sin^3 x}$ d) $-\frac{\cos x}{\sin^3 x}$

16. $y=tg2x$ funkciya tuwi'ndi'si'ni'n' $x=\frac{\pi}{2}$ tochkadag'i' ma'nisin tabi'n'

a) 2 b) -2 c) 0 d) $\frac{\pi}{2}$

17. $y = arctgx^2$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{2x}{1+x^4}$ b) $-\frac{2x}{1+x^4}$ c) $\frac{2x}{1+x^2}$ d) $-\frac{2x}{1+x^2}$

18. $y = x \cdot arctgx$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $arctgx - \frac{x}{1+x^2}$ b) $\frac{x}{1+x^2}$ c) $arctgx + \frac{x}{1+x^2}$ d) $-\frac{x}{1+x^2}$

19. Tuwi'ndi'si'n tabi'n'. $y = x \cdot \ln x$

a) $\ln x$ b) 1 c) 0 d) $\ln x + 1$

20. $y = 5x^5$ funkciyani'n' 4-ta'rtipli tuwi'ndi'si'n tabi'n'.

a) 600 b) $600x$ c) $100x^3$ d) $25x^4$

21. $y = e^{3x}$ funkciyani'n' 5-ta'rtipli tuwi'ndi'si'n tabi'n'.

a) $243e^{3x}$ b) e^{3x} c) $81e^{3x}$ d) 1

22. $\int\left(e^{\frac{x}{5}} + \cos 2x\right)dx$ ti tabi'n'.

a) $5e^{\frac{x}{5}} + \frac{1}{2}\sin 2x + C$ b) $e^{\frac{x}{5}} + \sin 2x + C$

c) $\frac{1}{5}e^{\frac{x}{5}} + \frac{1}{2}\sin x + C$ d) $5e^{\frac{x}{5}} + \frac{1}{2}\cos 2x + C$

23. $\int x\sqrt{x^2-7}dx$ integrali'n tabi'n'.

a) $\frac{2\sqrt{(x^2-7)^3}}{3} + C$ b) $\frac{\sqrt{(x^2-7)^3}}{3} + C$

c) $\frac{x^2 \cdot \sqrt{(x^2-7)^3}}{6} + C$ d) $\frac{x^2}{2} - \frac{2\sqrt{(x^2-7)^3}}{3} + C$

24. $\int e^x \cos x dx$ integrali'n tabi'n'.

a) $\frac{e^x}{2}(\sin x + \cos x) + C$ b) $e^x + \sin x + C$

c) $e^x \sin x + C$ d) $\frac{e^x}{2}(\sin x - \cos x) + C$

25. $\int_0^{\frac{\pi}{3}} \frac{\sin 2x}{\cos x} dx$ ti tabi'n.

a) $\frac{1}{4}$ b) -1 c) 1 d) $-\frac{1}{2}$

26. $\int_0^{2\pi} sixdx$ ti tabi'n.

a) 0 b) 2 c) 1 d) 3

27. $\int_0^{\frac{\pi}{3}} tgxdx$ ti tabi'n.

a) $-\ln 2$ b) $\ln 2$ c) $\ln(\cos x)$ d) $2\ln 2$

28. $\int_1^3 \frac{dx}{(x+3)^2}$ ti tabi'n.

a) $\frac{1}{2}$ b) $\frac{1}{12}$ c) $-\frac{1}{12}$ d) $-\frac{1}{2}$

29. $\int_{-1}^{2} (x^2 - 2x + 2)dx$ ti tabi'n.

a) 4 b) $-\frac{10}{3}$ c) $\frac{10}{3}$ d) $\frac{22}{3}$

30. $\int_1^3 x \ln x dx$ ti tabi'n.

a) $\frac{9}{2}\ln 3$ b) $\frac{9}{2}\ln 2$ c) $\frac{9}{2}\ln 3 - 2$ d) $\frac{9}{2}\ln 3 + 2$

31. $u = e^{\frac{x}{y}}$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'.

a) $\frac{\partial u}{\partial x} = \frac{1}{y} e^{\frac{x}{y}}; \frac{\partial u}{\partial y} = -\frac{x}{y^2} e^{\frac{x}{y}}$ b) $\frac{\partial u}{\partial x} = e^{\frac{x}{y}}; \frac{\partial u}{\partial y} = e^{\frac{x}{y}}$

c) $\frac{\partial u}{\partial x} = \frac{x}{y} e^{\frac{x}{y}}; \frac{\partial u}{\partial y} = \frac{x}{y} e^{\frac{x}{y}-1}$ d) $\frac{\partial u}{\partial x} = \frac{1}{y} e^{\frac{x}{y}-1}; \frac{\partial u}{\partial y} = -\frac{x}{y^2} e^{\frac{x}{y}-1}$

32. $z = \sqrt{x^2 + y^2}$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'.

a) $\frac{\partial z}{\partial x} = \frac{2x}{\sqrt{x^2+y^2}}; \frac{\partial z}{\partial y} = \frac{2y}{\sqrt{x^2+y^2}}$ b) $\frac{\partial z}{\partial x} = \sqrt{2x}; \frac{\partial z}{\partial y} = \sqrt{2y}$

c) $\frac{\partial z}{\partial x}=\frac{x}{\sqrt{x^2+y^2}};\frac{\partial z}{\partial y}=\frac{y}{\sqrt{x^2+y^2}}$ d) $\frac{\partial z}{\partial x}=x;\frac{\partial z}{\partial y}=y$

33. $z=x+ye^{\frac{x}{y}}$ funkciyasi'ni'n' toli'q differentsiyali'n tabi'n'.

a) $dz=e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dx+\left(1+e^{\frac{x}{y}}\right)dy$ b) $dz=e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dx-\left(1+e^{\frac{x}{y}}\right)dy$

c) $dz=\left(1+e^{\frac{x}{y}}\right)dx-e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy$ d) $dz=\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy$

34. $u=\frac{z}{x+y}$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'

a) $x+y\neq 0$ b) $x+y=0$ c) $x>y$ d) $x<y$

35. $u=e^{\frac{x}{y}}$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'

a) $\frac{\partial u}{\partial x}=ye^{\frac{x}{y}};\frac{\partial u}{\partial y}=xe^{\frac{x}{y}}$ b) $\frac{\partial u}{\partial x}=\frac{1}{y}e^{\frac{x}{y}};\frac{\partial u}{\partial y}=-\frac{x}{y^2}e^{\frac{x}{y}}$

c) $\frac{\partial u}{\partial x}=e^{\frac{x}{y}};\frac{\partial u}{\partial y}=ye^{\frac{x}{y}}$ d) $\frac{\partial u}{\partial x}=\frac{1}{y}e^{\frac{x}{y}};\frac{\partial u}{\partial y}=xe^{\frac{x}{y}}$

36. $z=\ln\left(x^2-y^2\right)$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'

a) $\frac{\partial u}{\partial x}=\frac{2x}{x^2-y^2};\frac{\partial u}{\partial y}=\frac{2y}{x^2-y^2}$ b) $\frac{\partial u}{\partial x}=\frac{1}{x^2-y^2};\frac{\partial u}{\partial y}=\frac{1}{x^2-y^2}$

c) $\frac{\partial u}{\partial x}=\frac{x}{x^2-y^2};\frac{\partial u}{\partial y}=\frac{-y}{x^2-y^2}$ d) $\frac{\partial u}{\partial x}=\frac{2x}{x^2-y^2};\frac{\partial u}{\partial y}=\frac{-2y}{x^2-y^2}$

37. $u=e^{\frac{x}{y}}$ funkciyani'n' toli'q differentsiyali'n tabi'n'

a) $du=e^{\frac{x}{y}}dx+e^{\frac{x}{y}}dy$ b) $du=e^{\frac{x}{y}}dx-e^{\frac{x}{y}}dy$

c) $du=\frac{e^{\frac{x}{y}}}{x}dx-\frac{x}{x^2}e^{\frac{y}{x}}dy$ d) $du=\frac{e^{\frac{y}{x}}}{y}dx-\frac{x}{y^2}e^{\frac{y}{x}}dy$

38. $z=\sin(u+4v)$ bolsa, $\frac{dz}{dx}$ ti tabi'n'. (bul jerde $u=x^3,v=x^2$)

a) $\frac{dz}{dx}=\cos(u+4v)$ b) $\frac{dz}{dx}=4\cos(u+4v)$

c) $\frac{dz}{dx}=\left(3x^2+8x\right)\cdot\cos(u+4v)$ d) $\frac{dz}{dx}=-\left(3x^2+8x\right)\cdot\cos(u+4v)$

39. $z = u \cdot v$ funkciyasi' berilgen (bunda $u = \sin x; v = \ln x$). $\frac{dz}{dx}$ ti tabi'n'.

a) $\frac{dz}{dx} = \sin x \cdot \ln x + \frac{\sin x}{x}$ b) $\frac{dz}{dx} = \cos x \cdot \ln x + \frac{\sin x}{x}$

c) $\frac{dz}{dx} = \cos x \cdot \ln x - \frac{\sin x}{x}$ d) $\frac{dz}{dx} = \sin x \cdot \ln x - \frac{\sin x}{x}$

40. Differentsiyal ten'lemeni sheshin' $7y - xy' = 0$

a) $y = cx^7$ b) $y = -x^7$ c) $y = cx^6$ d) $y = -x^6$

41. $yy' - x = 0$ differentsiyal ten'lemenin' uli'wma integrali'n tabi'n'

a) $y = x^2 + c$ b) $y = \sqrt{x^2 + c}$ c) $y = cx^2$ d) $y = x + c$

42. $yy' - x = 0$ differentsiyal ten'lemenin' $x = -2$ bolg'anda $y = 4$ da'slepki sha'rtleri boyi'nsha dara integral tabi'lsi'n.

a) $y = x^2 + \sqrt{12}$ b) $y = 12x^2$ c) $y = \sqrt{x^2 + 12}$ d) $y = x + 12$

43. $f(x, y) = y^x + x^{y-1}$ funkciyasi' berilgen. $f(2,2)$ di tabi'n

a) 2 b) 4 c) 1 d) 6

44. $f(x, y) = e^{\sin(x+y)}$ funkciyasi' berilgen. $f(0; \pi)$ di tabi'n

a) 0 b) 1 c) e d) $\frac{1}{e}$

45. Yeger betlik $z = x^3 + y^3$ ten'lemesi menen berilgen bolsa, wonda woni'n' $(1;0;1)$ tochkadag'i' uri'nba tegislik ten'lemesin tabi'n'

a) $z = 3x - 2$ b) $z = 3x + 2$ c) $z = 3x + y - 2$ d) $z = 3x - y + 2$

46. Yeger betlik $z = x^3 + y^3$ ten'lemesi menen berilgen bolsa, wonda woni'n' $(1;0;1)$ tochkadag'i' normaldi'n' ten'lemesin tabi'n'

a) $\frac{x-1}{-3} = \frac{y-1}{1} = \frac{z-1}{1}$ b) $\frac{x-1}{-3} = \frac{y}{-3} = \frac{z-1}{2}$

c) $\frac{x-1}{-3} = \frac{y}{0} = \frac{z-1}{1}$ d) $\frac{x-1}{-3} = \frac{y}{1} = \frac{z-1}{1}$

47. $z = \sin\frac{x}{y}$ funkciyani'n' $(x_0; y_0)$ tochkadag'i' gradientin tabi'n'

a) $\frac{1}{y_0}\cos\frac{x_0}{y_0}\left(\vec{i} + x_0\vec{j}\right)$ b) $\frac{1}{y_0}\cos\frac{x_0}{y_0}\left(\vec{i}\, x 0\, \vec{j}\right)$

c) $\frac{1}{y_0}\sin\frac{x_0}{y_0}\left(\vec{i} + x_0\vec{j}\right)$ d) $\frac{1}{y_0}\sin\frac{x_0}{y_0}\left(\vec{i} - x_0\vec{j}\right)$

48. $f(x,y)=x^3+3x^2y+12xy^3$ berilgen. $f''_{y2}(2;0)$ di tabi'n'

a) 2 b) 0 c) -20 d) 1

49. $f(x;y)=x^3+y^3+x^3y^3$ berilgen. $f''_{yx}(1;1)$ ti tabi'n'

a) 9 b) 18 c) 3 d) 36

50. $f(x;y)=e^{xy}$ berilgen. f''_{x^2} ti tabi'n'

a) y^2e^{xy} b) $e^{xy}(1+xy)$ c) x^2e^{xy} d) 1

IV-variant

(Duri's juwabi'n ko'rsetin')

1. Yesaplan' $\lim\limits_{x\to-2}\left(x^3+x^2+8\right)$

a) 4 b) 20 c) -4 d) -20

2. Yesaplan' $\lim\limits_{x\to-3}\dfrac{x^2-9}{x+3}$

a) 0 b) 6 c) -9 d) -6

3. Yesaplan' $\lim\limits_{x\to0}\dfrac{3x^3-2x^2}{5x^4-2x^2}$

a) -1 b) ∞ c) 1 d) 0

4. Yesaplan' $\lim\limits_{x\to0}\dfrac{\sin 6x}{\sin 4x}$

a) 0 b) ∞ c) 6 d) 1, 5

5. Yesaplan' $\lim\limits_{x\to0}7^{\frac{x-3}{2x-3}}$

a) 0 b) 1 c) 7 d) -1

6. Yesaplan' $\lim\limits_{x\to0}\ln\dfrac{2}{x+2}$

a) e b) 2 c) 1 d) 0

7. $y=\dfrac{2}{x^2-1}$ funkciyasi'ni'n' u'zilis tochkasi'n ko'rsetin'.

a) $x=2$ b) $x=0$

c) $x=-1$ ha'm $x=+1$ d) $x=-2$ ha'm $x=+2$

8. $f(x)=x^2+2x+1$ funkciyasi' x barli'q ma'nislerinde u'zliksiz be?

a) $x=-1$ ma'nisinen basqa barli'q ma'nislerinde u'zliksiz.

b) x tin' barli'q ma'nislerinde u'zliksiz.

c) $x=0$ ma'nisinen basqa barli'q ma'nislerinde u'zliksiz.

d) $x=1$ ma'nisinen basqa barli'q ma'nislerinde u'zliksiz.

9. $y=\frac{3}{x^3}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{1}{x^2}$ b) $-\frac{1}{x^2}$ c) $-\frac{9}{x^4}$ d) $\frac{1}{x^3}$

10. $f(x)=\sqrt{x}+\frac{1}{x}-\sqrt[4]{8}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{1}{2\sqrt{x}}-\frac{1}{x^2}-\frac{1}{4\sqrt[4]{8^3}}$ b) $\frac{1}{2\sqrt{x}}-\frac{1}{x^2}$

c) $\frac{1}{\sqrt{x}}+\frac{1}{x^2}$ d) $\frac{1}{2\sqrt{x}}+\frac{1}{x^2}$

11. $f(x)=(9x-1)(3x+2)$ bolsa, $f'(0)$ di tabi'n'?

a) 69 b) -39 c) 0 d) 15

12. $f(x)=\frac{x-8}{x^2-4}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{1}{2x}$ b) 1 c) $\frac{-x^2+16x-4}{\left(x^2-4\right)^2}$ d) $\frac{3x^2+16x-4}{\left(x^2-4\right)^2}$

13. $y=\sqrt[3]{\frac{1}{1-x}}$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $-\frac{1}{3\cdot\sqrt[3]{4(1-x)^4}}$ b) $\frac{1}{3\cdot\sqrt[3]{4(1-x)^4}}$ c) 1 d) $\frac{1}{3\cdot\sqrt[3]{4(1-x)^2}}$

14. $y=\cos^2 x+\sin^2 x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) 1 b) 0 c) $\sin 2x$ d) $2\sin 2x$

15. $y=\frac{1}{2}ctg^2 x$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{\cos^2 x}{\sin^4 x}$ b) $-\frac{\cos^2 x}{\sin^4 x}$ c) $\frac{\cos x}{\sin^3 x}$ d) $-\frac{\cos x}{\sin^3 x}$

16. $y=ctgx$ funkciya tuwi'ndi'si'ni'n' $x=\frac{\pi}{2}$ tochkadag'i' tuwi'ndi'si'n tabi'n'?

a) 1 b) -1 c) 0 d) $\frac{\pi}{2}$

17. $y = arcctgx^2$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $\frac{2x}{1+x^4}$ b) $-\frac{2x}{1+x^4}$ c) $\frac{2x}{1+x^2}$ d) $-\frac{2x}{1+x^4}$

18. $y = x \cdot arcctgx$ funkciyani'n' tuwi'ndi'si'n tabi'n'?

a) $arcctgx + \frac{x}{1+x^2}$ b) $\frac{x}{1+x^2}$ c) $arcctgx - \frac{x}{1+x^2}$ d) $-\frac{x}{1+x^2}$

19. Tuwi'ndi'si'n tabi'n'. $y = \ln(\ln x)$

a) $\frac{1}{x \cdot \ln x}$ b) $\frac{1}{\ln x}$ c) $\frac{1}{x}$ d) 1

20. $y = (x-2)^{\frac{1}{2}}$ funkciyani'n' 4-ta'rtipli tuwi'ndi'si'n tabi'n'.

a) $(x-2)$ b) $\frac{1}{2\sqrt{x-2}}$ c) $-\frac{15}{16}(x-2)^{-\frac{7}{2}}$ d) $\frac{x-2}{2}$

21. $y = e^{5x}$ funkciyani'n' 5-ta'rtipli tuwi'ndi'si'n tabi'n'.

a) e^{5x} b) $3125e^{5x}$ c) $5e^{5x}$ d) 1

22. $\int 2\sin(2x-1)dx$ ti tabi'n'.

a) $\cos(2x-1)+C$ b) $2\cos(2x-1)+C$

c) $-2\cos(2x-1)+C$ d) $-\cos(2x-1)+C$

23. $\int \sin x \cdot \cos^2 x dx$ integrali'n tabi'n'.

a) $-\cos x + \frac{\cos^3 x}{3} + C$ b) $\frac{\sin^3 x}{3} + C$

c) $-\frac{\cos^3 x}{3} + C$ d) $-\frac{\sin^3 x}{3} + C$

24. $\int x\ln(2x)dx$ integrali'n tabi'n'.

a) $\frac{x^2}{2}\ln(2x) + 2x + C$ b) $\frac{x^2}{2}\ln(2x) - \frac{x}{2} + C$

c) $\frac{x^2}{2} + \ln(2x)$ d) $\frac{x^2}{2}\ln(2x) - \frac{x}{4} + C$

25. $\int_{\sqrt{e}}^{e} x\ln x dx$ ti tabi'n.

a) $\frac{e^2 - e}{2}$ b) $\frac{e^2}{4}$ c) $\frac{e^2}{2}$ d) $e^2 - \frac{e}{4}$

26. $\int_{-2}^{2}(x+1)dx$ ti tabi'n.

a) 4 b) 3 c) -4 d) -3

27. $\int_{0}^{\frac{\pi}{3}}\frac{dx}{\cos^2 x}$ ti tabi'n.

a) $\sqrt{3}-1$ b) $\sqrt{3}$ c) $\sqrt{3}+1$ d) 3

28. $\int_{2}^{3}\frac{dx}{(x-5)^2}$ ti tabi'n.

a) $-\frac{1}{6}$ b) $\frac{1}{3}$ c) $-\frac{1}{3}$ d) $\frac{1}{6}$

29. $\int_{0}^{2}(x^2-2x+2)dx$ ti tabi'n.

a) $\frac{8}{3}$ b) $\frac{4}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

30. $\int_{1}^{2}x\ln xdx$ ti tabi'n.

a) $\ln 2-\frac{5}{2}$ b) $\ln 4-\frac{3}{4}$ c) $\ln 4+\frac{5}{4}$ d) $\ln 4$

31. $u=\frac{x}{y}+\frac{y}{x}$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'.

a) $\frac{\partial u}{\partial x}=-\frac{x}{y^2}+\frac{1}{x};\frac{\partial u}{\partial y}=\frac{1}{y}-\frac{y}{x^2}$ b) $\frac{\partial u}{\partial x}=\frac{1}{y}-\frac{y}{x^2};\frac{\partial u}{\partial y}=-\frac{x}{y^2}+\frac{1}{x}$

c) $\frac{\partial u}{\partial x}=\frac{x}{y}-\frac{y}{x^2};\frac{\partial u}{\partial y}=\frac{x}{y}+\frac{1}{x}$ d) $\frac{\partial u}{\partial x}=\frac{1}{y}+\frac{1}{x};\frac{\partial u}{\partial y}=-\frac{x}{y^2}-\frac{y}{x^2}$

32. $z=arctg\frac{x}{y}$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'.

a) $\frac{\partial z}{\partial x}=\frac{y}{x^2+y^2};\frac{\partial z}{\partial y}=-\frac{x}{x^2+y^2}$ b) $\frac{\partial z}{\partial x}=-\frac{y}{x^2+y^2};\frac{\partial z}{\partial y}=\frac{x}{x^2+y^2}$

c) $\frac{\partial z}{\partial x}=-\frac{y}{x^2+y^2};\frac{\partial z}{\partial y}=\frac{y}{x^2+y^2}$ d) $\frac{\partial z}{\partial x}=\frac{x}{x^2+y^2};\frac{\partial z}{\partial y}=-\frac{y}{x^2+y^2}$

33. $z=x\sin y+y\sin x$ funkciyasi'ni'n' toli'q differentsiyali'n tabi'n'.

a) $dz=(\sin y+y)dx+\cos ydy$

b) $dz=(\sin y+y\cos x)dx-(x\cos y+\sin x)dx$

c) $dz = y\cos x dx + x\cos y dy$

d) $dz = (\sin y + y\cos x)dx + (x\cos y + \sin x)dx$

34. $z = \arcsin(3 - x^2 - y^2)$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n'

a) $-2 \le x^2 + y^2 \le -4$ b) $2 \le x^2 - y^2 \le 4$

c) $-4 \le x^2 + y^2 \le -2$ d) $2 \le x^2 + y^2 \le 4$

35. $z = \cos(ax + by)$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'

a) $\frac{\partial z}{\partial x} = a \cdot \cos(ax + by); \frac{\partial z}{\partial y} = b \cdot \cos(ax + by)$

b) $\frac{\partial z}{\partial x} = -a \cdot \sin(ax + by); \frac{\partial z}{\partial y} = -b \cdot \sin(ax + by)$

c) $\frac{\partial z}{\partial x} = -a \cdot \cos(ax + by); \frac{\partial z}{\partial y} = -b \cdot \cos(ax + by)$

d) $\frac{\partial z}{\partial x} = -b \cdot \sin(ax + by); \frac{\partial z}{\partial y} = -a \cdot \sin(ax + by)$

36. $z = \frac{y}{x}$ funkciyasi'ni'n' dara tuwi'ndi'si'n tabi'n'

a) $\frac{\partial z}{\partial x} = \frac{y}{x^2}; \frac{\partial z}{\partial y} = -\frac{1}{x}$ b) $\frac{\partial z}{\partial x} = \frac{1}{x}; \frac{\partial z}{\partial y} = y$

c) $\frac{\partial z}{\partial x} = -\frac{y}{x^2}; \frac{\partial z}{\partial y} = \frac{1}{x}$ d) $\frac{\partial z}{\partial x} = \frac{1}{x}; \frac{\partial z}{\partial y} = -\frac{y}{x^2}$

37. $u = e^{\frac{y}{x}}$ funkciyani'n' toli'q differentsiyali'n tabi'n'

a) $du = \frac{e^{\frac{y}{x}}}{x}dy - \frac{y}{x^2}e^{\frac{y}{x}}dx$ b) $du = \frac{y}{x}e^{\frac{y}{x}}dx - \frac{e^{\frac{y}{x}}}{x^2}dy$

c) $du = e^{\frac{y}{x}}dx - e^{\frac{y}{x}}dy$ d) $du = e^{\frac{y}{x}}dx + e^{\frac{y}{x}}dy$

38. $z = \cos(u + 4v)$ bolsa, $\frac{dz}{dx}$ ti tabi'n'. (bul jerde $u = x^3, v = x^2$)

a) $\frac{dz}{dx} = -\sin(u + 4v)$ b) $\frac{dz}{dx} = (3x^2 + 8x) \cdot \sin(u + 4v)$

c) $\frac{dz}{dx} = -(3x^2 + 8x) \cdot \sin(u + 4v)$ d) $\frac{dz}{dx} = -4\sin(u + 4v)$

39. $z = e^{\frac{x}{y}}$ funkciyasi' berilgen (bunda $y = \sin^3 x;$). $\frac{dz}{dx}$ ti tabi'n'.

a) $\frac{dz}{dx}=\frac{1}{y}e^{\frac{x}{y}}\cdot\left(1+\frac{x}{y}\sin 2x\right)$ b) $\frac{dz}{dx}=\frac{1}{y}e^{\frac{x}{y}}\cdot\left(1+\frac{3x}{y}\sin^2 x\cdot\cos x\right)$

c) $\frac{dz}{dx}=\frac{1}{y}e^{\frac{x}{y}}\cdot\left(1-\frac{x}{y}\sin 2x\right)$ d) $\frac{dz}{dx}=\frac{1}{y}e^{\frac{x}{y}}\cdot\left(1-\frac{3x}{y}\sin^2 x\cdot\cos x\right)$

40. Differentsiyal ten'lemeni sheshin' $10y-xy'=0$

a) $y=-x^{10}$ b) $y=cx^{10}$ c) $y=-x^9$ d) $y=cx^9$

41. $y'=y$ differentsiyal ten'lemenin' uli'wma integrali'n tabi'n'

a) $y=e^{x+c}$ b) $y=\ln x$ c) $y=\frac{1}{e^x}+c$ d) $y=-\ln x$

42. $y'=y$ differentsiyal ten'lemenin' $x=-2$ bolg'anda $y=4$ da'slepki sha'rtleri boyi'nsha dara integral tabi'lsi'n.

a) $y=e^{x+2}$ b) $y=e^{x+\ln 4+2}$ c) $y=e^{x+\ln 4}$ d) $y=e^{x+1}$

43. $f(x,y)=y^x+x^{y-1}$ funkciyasi' berilgen. $f(3,3)$ di tabi'n

a) 27 b) 9 c) 3 d) 36

44. $f(x,y)=e^{\cos(x+y)}$ funkciyasi' berilgen. $f(0;\pi)$ di tabi'n

a) e b) 1 c) $\frac{1}{e}$ d) 0

45. Yeger betlik $z=x^3-y^3$ ten'lemesi menen berilgen bolsa, wonda woni'n' $(1;2;1)$ tochkadag'i' uri'nba tegislik ten'lemesin tabi'n'

a) $z=3x-3y+4$ b) $z=3x-12y+22$ c) $z=3x-12y+4$ d) $z=3x-3y+22$

46. Yeger betlik $z=x^3-y^3$ ten'lemesi menen berilgen bolsa, wonda woni'n' $(1;2;1)$ tochkadag'i' normaldi'n' ten'lemesin tabi'n'

a) $\frac{x-1}{3}=\frac{y-2}{-12}=\frac{z-1}{-1}$ b) $\frac{x-1}{3}=\frac{y-2}{3}=\frac{z-1}{-1}$

c) $\frac{x-1}{-3}=\frac{y-2}{12}=\frac{z-1}{1}$ d) $\frac{x-1}{-3}=\frac{y-2}{3}=\frac{z-1}{-22}$

47. $z=\frac{x}{y}$ funkciyani'n' $(2;1)$ tochkadag'i' gradientin tabi'n'

a) $2\vec{i}+\vec{j}$ b) $2\vec{i}-\vec{j}$ c) $\vec{i}+2\vec{j}$ d) $\vec{i}-2\vec{j}$

48. $f(x,y)=x^3+3x^2y+12xy^3$ berilgen. $f''_{yx}(0;1)$ di tabi'n'

a) 0 b) 1 c) 36 d) -36

49. $f(x;y)=x^3+y^3+x^3y^3$ berilgen. $f''_{y^2}(1;2)$ ti tabi'n'

a) 24 b) 17 c) 12 d) 36

50. $f(x;y)=e^{xy}$ berilgen. f''_{y^2} ti tabi'n'

a) y^2e^{xy} b) $e^{xy}(1+xy)$ c) x^2e^{xy} d) 1

1-arali'q baqlaw juwaplari'

I-variant

Sorawlar	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Juwaplar	a	d	c	b	b	a	b	c	B	a	a	b	c	D	a	b	c	d	d	A

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
B	d	a	b	d	c	B	a	A	d	b	d	a	A	d	d	a	a	d	C

41	42	43	44	45	46	47	48	49	50
b	B	c	C	a	b	a	c	a	a

II-variant

Sorawlar	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Juwaplar	b	c	a	d	d	b	c	d	B	c	a	b	c	d	a	b	c	d	d	A

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
C	a	c	a	a	a	C	a	b	b	a	b	c	d	c	c	d	a	c	A

41	42	43	44	45	46	47	48	49	50
b	B	b	d	b	c	a	b	a	a

III-variant

Sorawlar	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
juwaplar	c	b	d	a	c	c	d	a	c	d	a	b	c	d	a	b	c	a	d	A

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	c	b	b	b	c	a	b	b	c	c	a	a	c	d	c	c	b	d	A

41	42	43	44	45	46	47	48	49	50
d	a	a	a	c	b	a	c	d	b

IV-variant

Sorawlar	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
juwaplar	d	a	b	c	b	d	a	b	d	a	a	b	c	d	a	b	c	a	d	A

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	d	a	a	d	d	d	d	c	d	d	d	d	c	d	c	a	d	b	C

41	42	43	44	45	46	47	48	49	50
d	b	b	c	d	a	a	b	c	d

2-arali'q baqlaw juwaplari'

I-variant

Sorawlar	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Juwaplar	b	a	c	d	a	a	d	a	d	a	b	c	a	b	a	b	c	d	b	C

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	b	c	a	b	c	a	c	b	a	c	a	c	a	d	b	c	a	b	B

41	42	43	44	45	46	47	48	49	50
d	a	b	a	b	c	a	c	a	b

II-variant

Sorawlar	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Juwaplar	a	d	a	b	b	d	b	b	C	a	c	a	b	d	b	c	d	a	a	D

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

B	c	d	d	a	d	a	d	C	b	d	b	d	b	c	a	a	b	a	D

41	42	43	44	45	46	47	48	49	50
a	b	c	D	a	d	b	d	a	b

III-variant

Sorawlar	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Juwaplar	d	b	c	a	d	a	a	a	a	a	d	a	a	b	b	a	a	c	d	B

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	a	b	a	c	b	a	b	D	c	a	c	d	a	b	d	d	c	b	A

41	42	43	44	45	46	47	48	49	50
b	c	d	B	a	c	b	b	a	b

IV-variant

Sorawlar	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
juwaplar	a	d	c	d	c	d	c	b	c	b	d	c	b	b	d	b	b	c	a	C

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	d	c	d	b	a	b	d	a	b	b	a	d	d	b	c	a	c	d	B

41	42	43	44	45	46	47	48	49	50
a	b	d	c	b	c	d	c	a	c

Juwmaqlawshi’ baqlaw sorawlari’

1. Ko’plik tu’sinigi. Ko’plikler u’stinde a’meller
2. Haqi’yqi’y sanlar ko’pligi. Haqi’yqi’y sanni’n’ absolyut shamasi’, qa’siyetleri
3. Tegisliktegi ha’m ken’isliktegi tuwri’ mu’yeshli koordinatalar sistemasi’
4. Koordinatalardi’ almasti’ri’w, parallel’ ko’shiriw ha’m buri’w
5. Yeki tochka arasi’ndag’i’ arali’q
6. Kesindini berilgen qatnasta bo’liw
7. Funkciya ha’m woni’n’ beriliw usi’llari’
8. Monoton, keri, periodli’ funkciyalar
9. Quramali’ funkciya
10. Funkciyalar u’stinde arifmetikali’q a’meller
11. Determinantlar haqqi’nda tu’sinik
12. Si’zi’qli’ ten’lemeler sistemasi’. Kramer formulasi’
13. Vektorlar ha’m wolar u’stinde si’zi’qli’ a’meller
14. Yeki vektor arasi’ndag’i’ mu’yesh
15. Vektordi’n’ ko’sherge proektsiyasi’
16. Vektordi’n’ si’zi’qli’ baylani’sli’li’g’i’
17. Tegislik ha’m ken’isliktegi bazis
18. Vektordi’n’ bag’i’tlawshi’ kosinuslari’
19. Yeki vektordi’n’ kollenearli’q sha’rti
20. Skalyar, vektor ha’m aralas ko’beymeler.
21. Tuwri’ si’zi’qti’n’ uli’wma ten’lemesi
22. Tuwri’ si’zi’qti’n’ normal ten’lemesi
23. Berilgen tochkadan wo’tiwshi, berilgen vektorg’a perpendikulyar tuwri’ si’zi’q ten’lemesi
24. Yeki tuwri’ si’zi’qti’n’ kesilisiw tochkasi’
25. Tuwri’ si’zi’qti’ woni’n’ ten’lemesi boyi’nsha jasaw
26. Tuwri’ si’zi’qti’n’ bag’i’tlawshi’ vektori’
27. Tuwri’ si’zi’qti’n’ kanonikali’q ten’lemesi

28. Berilgen tochkadan berilgen bag'i't boyi'nsha wo'tiwshi tuwri' si'zi'q ten'lemesi

29. Tuwri' si'zi'qlar da'stesi

30. Tuwri' si'zi'qti'n' mu'yeshlik koeffitsientli ten'lemesi

31. Berilgen yeki tochka arqali' wo'tiwshi tuwri' si'zi'q ten'lemesi

32. Yeki tuwri' si'zi'q arasi'ndag'i' mu'yeshti esaplaw

33. Yeki tuwri' si'zi'qti'n' parallellik ha'm perpendikulyarli'q sha'rti

34. Yekinshi ta'rtipli iymek si'zi'qti'n' ani'qlamasi'

35. Shen'ber, ellips, giperbola, parabola

36. Tegisliktin' normal vektori'

37. Berilgen tochkadan wo'tiwshi, berilgen vektorg'a perpendikulyar tegislik ten'lemesi

38. Tegisliktin' uli'wma ten'lemesi

39. Tegislikti woni'n' ten'lemesi boyi'nsha jasaw

40. Tegislikler arasi'ndag'i' mu'yesh

41. Yeki tegisliktin' parallellik ha'm perpendikulyarli'q sha'rtleri

42. U'sh tegisliktin' kesilisiw sha'rtleri

43. Tochkadan tegislikke shekemgi arali'q

44. Ken'isliktegi tuwri' si'zi'q ten'lemesi

45. Tuwri' si'zi'qti'n' vektor, parametrlik ha'm kanonikali'q ten'lemeleri

46. Yeki tochka arqali' wo'tiwshi tuwri' si'zi'q ten'lemesi

47. Yekinshi ta'rtipli betliktin' ani'qlamasi'

48. Sfera, ellipsoid, giperbaloid, parabaloid

49. Tsirkul' ha'm si'zg'i'sh ja'deminde jasawg'a baylani'sli' en' a'piwayi' ma'seleler

50. Jasawg'a baylani'sli' ma'selelerdi sheshiwdegi algebrali'q metod

51. Tsirkul' ha'm si'zg'i'sh ja'deminde sheshiletug'i'n klassikali'q ma'seleler

52. Do'n'es ko'pmu'yeshlikler ha'm ko'pjaqli'lar

53. Duri's ko'pjaqli'lar

54. Su'wretlew metodlari'

55. Orayli'q ha'm parallel' proektsiyalaw

56. Proektiv ken’islik tu’sinigi

57. Proektiv geometriyani’n’ tiykarg’i’ faktleri

58. Topologiya elementleri

59. Evklid ken’islikte si’zi’qlar ha’m betlikler

60. Sanli’ izbe-izlik ha’m woni’n’ limiti

61. Limitler haqqi’nda teoremalar, wolar u’stinde a’meller

62. Monoton izbe-izlik limiti, e sani’.

63. Funkciyani’n’ limiti

64. Quramali’ funkciyani’n’ limiti

65. Birinshi ha’m yekinshi a’jayi’p limitler

66. Funkciyani’n’ u’zliksizligi

67. Quramali’ ha’m keri funkciyani’n’ u’zliksizligi

68. Tiykarg’i’ elementar funkciyalardi’n’ u’zliksizligi, wolardi’n’ tiykarg’i’ qa’siyetleri

69. Bir wo’zgeriwshili funkciyani’n’ differentsiyal esabi’

70. Funkciyani’n’ tuwi’ndi’si’, woni’n’ geometriyali’q ha’m mexanikali’q mag’anasi’

71. Tuwi’ndi’ haqqi’ndag’i’ teoremalar

72. Uri’nba ha’m normal ten’lemeler

73. Quramali’ funkciyani’n’ tuwi’ndi’si’

74. Tiykarg’i’ elementar funkciyalardi’n’ tuwi’ndi’lari’. Tuwi’ndi’lar tablitsasi’.

75. Funkciyani’n’ differentsiyali’, woni’n’ geometriyali’q mag’anasi’

76. Parametrik ha’m keri funkciyalardi’n’ tuwi’ndi’lari’

77. Joqari’ ta’rtipli tuwi’ndi’ ha’m differentsiyallar

78. Differentsiyallani’wshi’ funkciyalar haqqi’nda tiykarg’i’ teoremalar. Lopital’ qa’desi

79. Teylor formulasi’

80. Funkciyani’n’ wo’siw ha’m kemeyiwi, ekstremumlari’

81. Funkciyani’n’ kesindidegi en’ u’lken ha’m en’ kishi ma’nisleri

82. Funkciya grafiginin’ do’n’esligi ha’m woyi’sli’g’i’, buri’li’w tochkalari’, asimptotalari’

83. Funkciyani' toli'q tekseriw
84. Ani'q yemes integral, woni'n' qa'siyetleri
85. İntegrallawdi'n' wo'zgeriwshini almasti'ri'w usi'li'
86. Bo'leklep integrallaw usi'li'
87. Ratsional' funkciyalardi' integrallaw
88. Ayi'ri'm irrotsional funkciyalardi' integrallaw
89. Trigonometriyali'q an'latpalardi' integrallaw
90. Ani'q integral. N'yuton-Leybnits formulasi'
91. Ko'p wo'zgeriwshili funkciyalardi'n' differentsiyal esabi'.
92. Ko'p wo'zgeriwshili funkciyalardi'n' limiti ha'm u'zliksizligi
93. Dara tuwi'ndi'lar
94. Toli'q differentsiyal
95. Yeki wo'zgeriwshili funkciya ushi'n Teylor formulasi'
96. Ko'p wo'zgeriwshili funkciyalardi'n' ekstremumlari'
97. Differentsiyal ten'lemeler
98. Birinshi ta'rtipli differentsiyal ten'lemeler
99. Wo'zgeriwshileri aji'ralg'an ha'm aji'ralatugi'n differentsiyal ten'lemeler
100. Bir tekli ha'm wog'an keltiriletug'i'n differentsiyal ten'lemeler
101. Bernulli ten'lemesi
102. Toli'q differentsiyal ten'lemeler
103. Klero ha'm Logranj ten'lemeleri
104. Ortogonal ha'm izogonal traektoriyalar
105. Ta'rtibi pa'seyetug'i'n joqari' ta'rtipli ten'lemeler
106. Birinshi ta'rtipli differentsiyal ten'lemenin' arnawli' sheshimi
107. Bir tekli joqari' ta'rtipli ten'lemeler, tiykarg'i' tu'sinikler ha'm teoremalar
108. Variatsiya metodi'. Si'zi'qli' ten'lemenin' bir belgili sheshimi boyi'nsha uli'wma sheshimin tabi'w.
109. Qatar ja'rdeminde differentsiyal ten'lemelerdi sheshiw
110. Mexanikali'q terbelislerdin' differentsiyal ten'lemeleri. Rezonans
111. Vektor funkciya ha'm woni'n' tuwi'ndi'si'
112. Si'zi'qti'n' uri'nbasi' ha'm normal tegisligi

113. Si’zi’qti’n’ iymekligi
114. Si’zi’qti’n’ buri’li’wi’
115. Frene formulalari’
116. Vint si’zi’qlar

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(«Su'wretlew wo'neri ha'm injenerlik grafikasi'» ta'lim bag'dari' talabalari' ushi'n)

Woqi'w-metodikali'q qollanba A`jiniyaz ati'ndag`i' No`kis ma`mleketlik pedagogikali'q instituti' ilimiy-metodikali'q ken`esinin` 27-dekabr 2014-ji'lg`i' № 3 protokoli' menen baspadan shi'g`ari'wg`a usi'ni's etilgen.

Bas redaktor ***K. M. Koshanov***
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A`jiniyaz ati'ndag`i' NMPİ redaktsiya-baspa bo`limi
A`jiniyaz ati'ndag`i' NMPİ baspaxanasi'nda basi'lg`an. 2015-j.
Buyi'rtpa № 0128. Nusqasi' 100 dana. Formati' 60x84. Ko`lemi 7,5 b.t.
230105, No`kis qalasi', A. Dosnazarov ko`shesi-104. Reestr № 11-3084.