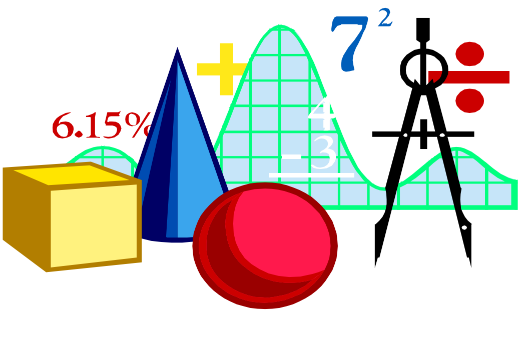
Namangan Viloyat Xalq ta'limi boshqarmasi
Viloyat metodika markazi

Matematika

fanidan
test ishlanmalari to`plami

Uslubiy qo`llanma



Namangan-2005

Ushbu test ishlanmalari to'plamidan matematika fani o'qituvchlari, akademik litseylar va kasb-hunar kollejlari hamda umumiy o'rta ta'lim maktablari o'quvchilari foydalanishlari mumkin.

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Mazkur qo'llanma viloyat metodika markazi huzuridagi o'quv-metodika kengashining 2005 yil "___" ___________ dagi № ________ sonli qarori bilan foydalanishga tavsiya etilgan.

So'z boshi

Mazkur to'plamga DTM "Axborotnoma"sining 1999-2003 yillardagi sonlarida e'lon qilingan test topshiriqlari orasidan tanlab olingan 100 ta qiyin va o'rtacha qiyinlikdagi masalalarning yechimlari kiritilgan. Kitobda masalalarning to'liq yechimlari keltirilgan bo'lib, masalalarni yechishdagi bu usullar eng qulay usul bo'lmasligi mumkin. Chunki, o'quvchilarga tushunarli bo'lishi uchun, yechish usullari iloji boricha maktab matematika dasturi doirasida tanlab olindi.

Bu to'plam o'rta umumiy ta'lim maktablarining yuqori sinf o'quvchilariga mo'ljallangan bo'lib, undan oliy o'quv yurtlariga kirish uchun mustaqil tayyorgarlik ko'rayotganlar va o'rta maktab matematika o'qituvchilari ham foydalanishlari mumkin.

To'plam haqida o'z fikr va mulohazalarini bildirgan kishilarga oldindan minnadtorchilik bildiraman.

Tuzuvchi-mualliflar.

Eslatma: To'plamda masalalrning yechimlari "##" belgisi bilan ajratib ko'rsatilgan.

Matematika fanidan test ishlanmari.

1.(2002-1.6). $2\cos\frac{x}{20} = 2^x + 2^{-x}$ tenglama nechta ildizga ega?

A) 1   B) 2   C) cheksiz ko'p   D) $\varnothing$ E) 5

Bu tenglamada $2\cos\frac{x}{20} \le 2$ va $2^x + 2^{-x} \ge 2$ bo'lgani uchun bu tenglama ko'pi bilan bitta ildizga ega bo'ladi. Bu ildiz $\begin{cases} 2^x + 2^{-x} \ge 2 \\ 2\cos\frac{x}{20} \le 2 \end{cases}$ sistema ildizlaridan iborat.

Ko'rinib turibdiki, bu sistema yagona $x=0$ yechimga ega.

(to'g'ri javob: A).

2. $\sqrt{x-4} < 6 - x$ tengsizlikni yeching.

Tengsizlikni $x + \sqrt{x-4} < 6$ ko'rinishda qayta yozamiz va $f(x) = x + \sqrt{x-4}$ funksiyani qaraymiz. Bu funksiya [4; +∞) oraliqda aniqlangan va o'sadi. Demak, $x + \sqrt{x-4} = 6$ tenglama yagona $x=5$ ildizga ega. Shunday qilib, berilgan tengsizlikning yechimlari to'plami [4;5) yarim intervaldan iborat.

3. $5^{2x} + 16x = 9$ tenglamani yeching.

Tenglamani $5^{2x} + 4^{2x} = 9$ ko'rinishda qayta yozamiz va $f(x) = 5^{2x} + 4^{2x}$ funksiyani qaraymiz. Bu funksiya barcha sonlar to'plamida aniqlangan va o'sadi. Uning qiymatlari sohasi R_+ dan iborat. Demak, $5^{2x} + 4^{2x} = 9$ tenglama yagona ildizga ega. Ko'rinib turibdika, bu ildiz $x=0,5$.

4. $|2x-5| = 5 - 2x$ tenglamani yeching.

Tenglamani $|2x-5| = -(2x-5)$ ko'rinishda qayta yozamiz. Modulning ta'rifiga ko'ra $2x-5 \le 0$ bo'lgandagina bajariladi. Bu tengsizlikni yechub, (-∞; 2,5] ga ega bo'lamiz.

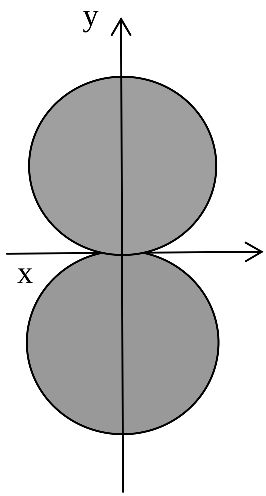
5.(2001-8.15). Koordinatalar tekisligida $x^2 + y^2 \le 4|y|$ tengsizlik bilan berilgan shaklning yuzini toping.

A) 4π B) 6,5π C) 12π D) 8π E) 16π

Ikki holni qarab chiqamiz:

1-hol. $y>0$ da berilgan tengsizlikni quyidagicha qayta yozamiz: $x^2+y^2\le 4y$, bundan $x^2+y^2-4y\le 0$, nihoyat, $x^2+(y-2)^2\le 2^2$ ga ega bo`lamiz.

2-hol. $y<0$ da berilgan tengsizlikni quyidagicha qayta yozamiz $x^2+y^2+4y\le 0$, nihoyat, $x^2+(y+2)^2\le 2^2$ ga ega bo`lamiz.



Bu ikki doira tengdosh bo`lib, ularning har biri 4π yuzaga ega.

Shunday qilib, yuqoridagi tengsizlik bilan berilgan shaklning yuzi S$=8\pi$ ga teng.
To'g'ri javob: D

6.(1999-4.39). Qavariq ko'pburchak ichi burchaklarining va bitta tashqi burchagining yig'indisi $\frac{23\pi}{2}$ ga teng. Ko'pburchakning nechta tomoni bor?

A) 10 B) 11 C) 13 D) 15 E) 16

Ma'lumki, qavariq n burchak ichki burchaklarining yig'indisi $\pi(n-2)$ ga teng. Bundan $\frac{23\pi}{2}=11\pi+\frac{\pi}{2}$ ning o'g tomonidagi 1-qo'shiluvchi ichki burchaklar yig'indisi ekanligi kelib chiqadi:

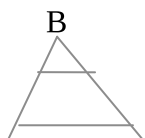
$$\pi(n-2)=11\pi$$

bu tenglamani n ga nisbatan yechib, kerakli javobni topamiz: n=13.
To'g'ri javob: C.

7.(1999-4.44). Uchburchakning yon tomoni uchidan boshlab hisoblaganda 2:3:4 kabi nisbatda bo'lindi va bo'linish nuqtalaridan asosiga parallel to'g'ri chiziqlar o'tkazildi. Hosil bo'lgan figuralar yuzlarining nisbatini toping.

A) 4:9:16 B) 2:5:9 C) 4:25:49 D) 4:21:56 E) 4:25:81

BD=2x, DN=3x, NA=4x bo'lsin.



U holda BN=2x+3x=5x va
AB=2x+3x+4x=9x bo’ladi.
Bundan BD:BN:BA=2:5:9 ekani
Kelib chiqadi. ΔDBE, ΔNBM, ΔABC-
Lar o’xshash. Shuning uchun:

D E
N M
A C

$S_{DBE}:S_{NBM}:S_{ABS}=(2:5:9)^2=4:25:81$. bundan foydalanib quyidagi belgilash-larni kiritamiz:

$$S_{DBE}=4k,\ \ S_{NBM}=25k,\ \ S_{ABC}=81k.$$

Budan: $S_{NDEM}=25k-4k=21k$, $S_{ANMC}=81k-25k=56k$.
Shunday qilib, $S_{DBE}:S_{NDEM}:S_{ANMC}=4:21:56$. To’gri javob: D.

8. (1999-5.1) $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+...+\frac{1}{182}$ ni hisoblang.

A) 11/42 B) 10/33 C) 1/4 D) 12/35 E) 15/56

$\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+...+\frac{1}{182}=\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+...+\frac{1}{13\cdot 14}=$

$\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+...+\frac{1}{13}-\frac{1}{14}=\frac{1}{3}-\frac{1}{14}=\frac{11}{42}$.

To’g’ri javob: A.

9.(1999-5.6). $\overline{abc}+\overline{dec}=\overline{fkmc}$ bo’lsa, $f^{a+d}+(b+d)^c$ ni hisoblang.

A) aniqlab bo’lmaydi. B) 1 C) 2 D) 3 E) 4

Ikkita uch xonali sonning yig’indisi 2000 dan kichik bo’ladi, shuning uchun $f=1$ bo’ladi. $c+c=c$ dan $c=0$ ekani kelib chiqadi.

Bulardan $f^{a+d}+(b+d)^c=1+1=2$ ekanligi kelib chiqadi. To’gri javob C.

10.(1999-5.27). $5\sin 2x+8\cos x=13$ tenglama $[-\pi;2\pi]$ kesmada nechta ildizga ega?

A) ∅ B) 1 C) 2 D) 3 E) 4

Bu tengalamani yechishda sinus va kosinus funksiyalarining chegaralanganlik xossasidan foydalanamiz. $\sin 2x\le 1$ va $\cos x\le 1$ e’tiborga olsak, bu tenglik $\begin{cases}\sin 2x=1\\ \cos x=1\end{cases}$ bo’lgandagina bajariladi.

Bu sistemani yechsak, $\begin{cases} x=\dfrac{\pi}{4}+\pi n \\ x=2\pi n \end{cases}$ ko'rinib turibdiki, sistema yechimga ega emas. To'g'ri javob: A.

11.(1999-5.57). [-10;10] oraliqdagi nechta butun son

$y=2^{\cos x}\sqrt{x^3\sin^2(\frac{\pi x}{3})e^{-x}}$ funksiyaning aniqlanish sohasiga tegishli?

A) 10 B) 11 C) 12 D) 13 E) 14

Shartga ko'ra ildiz ostidagi ifoda nomanfiy bo'lishi kerak. Buning uchun $x\geq 0$ yoki $\sin\frac{\pi x}{3}=0$ bo'lishi yetarli.

Bundan *x*=-9;-6;-3;0;1;2;3;4;5;6;7;8;9;10. To'gri javob: E.

12.(1999-6.40). $a^2+\frac{9}{a^2}=22$ bo'lsa, $a-\frac{3}{a}$ nimaga teng?

A) 3 B) -3 C) 2 D) ± 4 E) 1

$(a-\frac{3}{a})^2=a^2-6+\frac{9}{a^2}=(a^2+\frac{9}{a^2})-6=22-6=16$

$a-\frac{3}{a}=\pm 4$. To'g'ri javob: D.

13.(1999-6.53). $\cos\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{5\pi}{7}$ ni hisoblang.

A) -1/8 B) 1/4 C) 1/2 D) 1 E) 1/8

$\cos\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{5\pi}{7}=\cos\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot(-\cos\frac{2\pi}{7})=$

$$=-\frac{2\sin\frac{\pi}{7}\cdot\cos\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{2\pi}{7}}{2\sin\frac{\pi}{7}}=-\frac{\sin\frac{2\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}}{2\sin\frac{\pi}{7}}=$$

$$=-\frac{2\sin\frac{2\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}}{2\cdot 2\sin\frac{\pi}{7}}=-\frac{\sin\frac{4\pi}{7}\cdot\cos\frac{4\pi}{7}}{4\sin\frac{\pi}{7}}=-\frac{2\sin\frac{4\pi}{7}\cdot\cos\frac{4\pi}{7}}{2\cdot 4\sin\frac{\pi}{7}}$$

$$=-\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}=-\frac{\sin(\pi+\frac{\pi}{7})}{8\sin\frac{\pi}{7}}=-\frac{-\sin\frac{\pi}{7}}{8\sin\frac{\pi}{7}}=\frac{1}{8}.$$ To’g’ri javob: E.

14.(1999-8.4). $|x+1|+|x-4|>7$ tengsizlikni qanoatlantiruvchi x ning eng kichik natural qiymatini toping.

A) 1 B) 3 C) 6 D) 5 E) 2

Modullarning nollarini topamiz: -1; 4.

Berilgan tengsizlikni quyidagi oraliqlarda yechamiz:

$$x<-1;\ -1\le x<4;\ x\ge 4.$$

a). $x<-1$ oraliqda $-(x+1)-(x-4)>7$

$$-x-1-x+4)>7$$
$$-2x+3>7$$
$$-2x>4$$
$$x<-2.$$

Bu $x<-1$ shartga zid emas. Shuning uchun $x<-2$ oraliq tengsizlikning yechimi bo’ladi.

b) $-1\le x<4$ oraliqda $(x+1)-(x-4)>7$

$$x+1-x+4>7$$
$$0\cdot x>2$$

bu tengsizlik yechimga ega emas

c) $x\ge 4$ oraliqda $(x+1)+(x-4)>7$

$$x+1+x-4>7$$
$$2x-3>7$$
$$2x>10$$
$$x>5$$

Bu $x\ge 4$ shartga zid emas. Shuning uchun $x>5$ oraliq tengsizlikning yechimi bo’ladi.

Shunday qilib, dastlabki tengsizlikning yechimlari to’plami: $(-\infty;2]\cup(5;\infty)$.

Eng kichik natural yechimi esa: 6. To’g’ri javob: C

15.(1999-9.10). $\frac{x^2+2x+8}{x^2+2x+3}$ ifodaning eng katta qiymatini toping.

A) 3,5   B) 2,6   C) 2,4   D) 2,8   E) 3

Berilgan ifodani quyidagicha qayta yozamiz:

$$\frac{(x^2+2x+3)+5}{x^2+2x+3}=1+\frac{5}{(x+1)^2+2}$$

ko'rinib turibdiki, bu ifodaning eng katta qiymati 3,5 ga teng.

To'g'ri javob: A.

16.(1999-9.22). $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{9}+\sqrt{8}}$ ni hisoblang.

A) 2   B) 3   C) 4   D) 1   E) 5

Har bir kasrning surati va mahrajini shu kasr mahrajining qo'shmasiga ko'paytiramiz:

$$\frac{\sqrt{2}-1}{2-1}+\frac{\sqrt{3}-\sqrt{2}}{3-2}+\frac{\sqrt{4}-\sqrt{3}}{4-3}+\ldots+\frac{\sqrt{9}-\sqrt{8}}{9-8}=$$

$$=\frac{\sqrt{2}-1}{1}+\frac{\sqrt{3}-\sqrt{2}}{1}+\frac{\sqrt{4}-\sqrt{3}}{1}+\ldots+\frac{\sqrt{9}-\sqrt{8}}{1}=$$

$$=\sqrt{2}-1+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+\ldots+\sqrt{9}-\sqrt{8}=-1+3=2$$

To'g'ri javob: A.

17.(1999-9.34). $tgx-tg\frac{\pi}{3}-tgxtg\frac{\pi}{3}=1$ tenglamani yeching.

A) $\frac{7\pi}{6}+\pi k, k\in Z$ B) $\frac{5\pi}{6}+2\pi k, k\in Z$ C) $\frac{7\pi}{12}+2\pi k, k\in Z$

D) $\frac{7\pi}{12}+\pi k, k\in Z$ E) $\frac{5\pi}{6}+\pi k, k\in Z$

Tenglamani quyidagicha qayta yozamiz:

$$tgx-tg\frac{\pi}{3}=1+tgxtg\frac{\pi}{3}$$

$$\frac{tgx-tg\frac{\pi}{3}}{1+tgxtg\frac{\pi}{3}}=1$$

$$tg(x-\frac{\pi}{3})=1$$

bu tenglamani yechib, $x=\frac{7\pi}{12}+\pi k, k\in Z$ ga ega bo'lamiz.

To'g'ri javob: D.

18.(2000-1.17). $2x^2+2xy+2y^2+2x-2y+3$ ko'phad eng kichik qiymatga erishganda, xy ning qiymati qanday bo'ladi?

A) 1 B) -2 C) 2 D) 1,5 E) -1

Ifodani quyidagicha qayta yozamiz:

$(x^2+2xy+y^2)+(x^2+2x+1)+(y^2-2y+1)+1=(x+y)^2+(x+1)^2+(y-1)^2+1$

ifodaning qiymati eng kichik bo'lishi uchun $x+y=0$; $x+1=0$; $y-1=0$ bo'lishi kerak. Bundan $x=-1$; $y=1$ ekani kelib chiqadi.

Shunday qilib, $xy=-1$. To'g'ri javob: E.

19.(2000-1.33). $2(arc^2cosx)+\pi^2=3\pi \arccos x$ tenglamaning ildizlari yig'indisini toping.

A) $\sqrt{2}/2$ B) -1 C) 1 D) $-\sqrt{2}/2$ E) -1/2

Quyidagicha belgilash kiritamiz: $\arccos x=t$.

Bundan $2t^2+\pi^2=3\pi t$

$$2t^2-3\pi t+\pi^2=0$$

Bundan, $t_1=\pi$; $t_2=\frac{\pi}{2}$ kelib chiqadi. Bularni t ning o'rniga qo'yib,

$x_1=-1$; $x_2=0$. $x_1+x_2=-1$ ni topamiz.

To'g'ri javob: B.

20.(2000-2.9). $1\cdot 2\cdot 3\cdot\ldots\cdot 50$ ko'paytma nechta nol bilan tugaydi?

A) 8 B) 10 C) 9 D) 14 E) 12

Ko'paytmani tub ko'paytuvchilarga ajratamiz. Hosil bo'lgan yoyilmada nechta 5 raqami qatnashgan bo'lsa, ko'paytma shuncha 0 raqami bilan tugaydi. 5; 10; 15; 20; 30; 35; 40; 45 larning tub ko'paytmalarga yoyilmasida 1 tadan 8 ta, 25 va 50 ning tub ko'paytuvchilarga yoyilmasida esa 2 tadan 4 ta, jami 12 ta 5 raqami bo'lganligi uchun, ko'paytma 12 ta 0 raqami bilan tugaydi. To'g'ri javob: E.

21.(2000-3.26). $(x^2+5x+4)(x^2+5x+6)=120$ tenglamaning haqiqiy ildizlari yig'indisini toping.

A) 3 B) -3 C) 2 D) -5 E) -4

Quyidagicha belgilash kiritib olamiz: $x^2+5x+5=t$

bunda berilgan tenglama quyidagi ko'rinishga keladi:

$$(t-1)(t+1)=120$$
$$t^2-1=120$$
$$t^2=121$$
$$t_{1,2}=\pm 11$$
a) $x^2+5x+5=11;\quad x^2+5x-6=0;\quad x_1=-6;\ \ x_2=1$

b) $x^2+5x+5=-11;\quad x^2+5x+16=0$ yechimi yoq

Yuqoridagilardan: $x_1+x_2=-6+1=-5$ To`g`ri javob: D.

22.(2000-3.57) y=6sin2x+8cos2x funksiyaning qiymatlari to'plamini toping.

A) [-10;10] B) [-14;14] C) (-∞;∞) D) [0;6] E) [0;8]

max(y) $=\sqrt{36+64}=10$ va min(y) $=-\sqrt{36+64}=$ -10 bo'lgani uchun, berilgan funksiyaning qiymatlari to'plami [-10;10]. To'g'ri javob: A.

23.(2000-3.68) $\int_{-3}^{6} x|x|dx$ ni hisoblang.

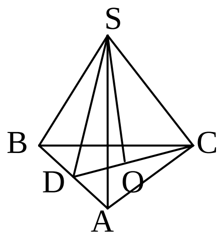
A) 81 B) 63 C) 60 D) 84 E) 80

Modulning ta'rifidan foydalanib topamiz:

$\int_{-3}^{6} x|x|dx=-\int_{-3}^{0} x^2dx+\int_{0}^{6} x^2dx=-\frac{x^3}{3}\Big|_{-3}^{0}+\frac{x^3}{3}\Big|_{0}^{6}=$ -9+72=63. To`g`ri javob: B.

24.(2000-2.45). Uchburchakli piramida asosining tomonlari 9; 10 va 17 ga teng. Piramidaning barcha yon yoqlari asos tekisligi bilan 45° li burchak tashkil etsa, uning hajmini toping.

A) 24 B) 36 C) 32 D) 21 E) 33



P_{ABC}=(9+10+17)/2=18.

$S_{ABC}=\sqrt{18\cdot 9\cdot 8\cdot 1}=36$.

r=2S/(a+b+c) =2·36/36=2

ΔSOD dan SO=OD=2

V= (36·2)/3=24. To'g'ri javob: A

25.(2000-4.11). $x^2-3|x|-40=0$ tenglamaning ildizlari ko'paytmasini toping.

A) -40 B) 40 C) -32 D) -64 E) -56

Tenglamani quyidagicha qayta yozamiz: $|x|^2-3|x|-40=0$

$|x|=t$ belgilashni kiritamiz:

$$t^2-3t-40=0$$
$$t_1=8;\ \ t_2=-5$$

O'rniga qo'yamiz: a) $|x|=8;\ \ x_{12}=\pm 8$.

b) $|x|=-5$; yechimi yoq.

$x_1 \cdot x_2=-64$. To'g'ri javob: D.

26.(2000-6.19). Agar $a+b=7$ va $ab=2$ bo'lsa, $a^2b^4+a^4b^2$ ning qiymatini toping.

A) 196 B) 180 C) 112 D) 98 E) To'g'ri javob keltirilmagan.

$a^2+2ab+b^2=49$; $a^2+2\cdot 2+b^2=49$; $a^2+b^2=45$; $a^2b^2=4$;

$(a^2+b^2)\cdot a^2b^2=a^2b^4+a^4b^2=45\cdot 4=180$. To'g'ri javob: B.

27.(2000-8.21). $x^2+y^2-4x-6y-12\le 0$ tengsizlik bilan berilgan shaklning yuzini toping.

A) 25π B) 36π C) 20π D) 16π E) 40π

Tengsizlikni quyidagicha qayta yozamiz:

$$x^2-4x+4+y^2-6y+9\le 12+4+9;$$
$$(x-2)^2+(y-3)^2\le 25.$$
$$(x-2)^2+(y-3)^2\le 5^2.$$

Demak, bu tengsizlik bilan radiusi 5 ga teng bo'lgan doira berilgan bo'lib, uning yuzi 25π teng bo'ladi. To'g'ri javob: A

28.(2000-8.65). $a_1,a_2,a_3,...,a_n$ sonlar arifmetik progressiya tashkil etsa,

$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\frac{1}{a_3a_4}+\ldots+\frac{1}{a_{n-1}a_n}$ yig`indini toping.

A) a_1 B) a_1a_{n+1} C) $1/a_1$ D) n/a_1 E) $(n-1)/a_1a_n$

$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\frac{1}{a_3a_4}+\ldots+\frac{1}{a_{n-1}a_n}=$

$$=\frac{1}{d}\cdot\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_3}-\frac{1}{a_4}+\ldots+\frac{1}{a_{n-1}}-\frac{1}{a_n}\right)=\frac{1}{d}\cdot\left(\frac{1}{a_1}-\frac{1}{a_n}\right)=\frac{1}{d}\cdot\left(\frac{a_n-a_1}{a_1a_n}\right)=$$

$$=\frac{1}{d}\cdot\left(\frac{a_1+(n-1)d-a_1}{a_1a_n}\right)=\frac{1}{d}\cdot\frac{(n-1)d}{a_1a_n}=\frac{(n-1)}{a_1a_n}$$. To'g'ri javob: E.

29.(2000-9.17). Lagerda dam olayotga o'g'il bolalar va qizlarning soni teng òåíã. 13 yoshgacha bo'lgan bolalar soni 13 yoshdan katta bolalar sonidan 2 marta ko'p. Agar 4 raqamining o'ng va chap tomoniga bir xil raqam yozilsa, lagerdagi bolalar soni hosil bo'ladi. Bu qanday raqam?

A) 2 B) 3 C) 4 D) 6 E) 8

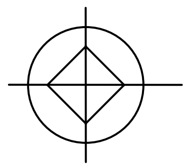
a). “qizlar soni”=”o`g`il bolalar soni”=*x* ta desak, hamma bolalar soni 2*x* ta bo’ladi.
b). 13 yoshdan katta bolalar sonini y ta desak, 13 yoshdan kichik bolalar soni 2y ta boladi. Hamma bolalar 3y ta bo’ladi.

Yuqoridagilarga asoslanib, biz shunday raqamni tanlashimiz kerak-ki, hosli bo’gan son 3 ga karrali va juft son bo’lishi kerak. Demak, 4 raqamini olamiz. To’g’ri javob: C.

30. (2000-10.53). Agar $16\le x\le y\le z\le t\le 100$ bo’lsa, $x/y+z/t$ ifodaning eng kichik qiymatini toping.
A) 0,9 B) 200 C) 0,8 D) 0,2 E) topib bo’lmaydi.
Shartga ko’ra $x/y\ge 16/z$; $z/t\ge z/100$ deb yozish mumkin. Bu tengsizliklarni hadlab qo’shamiz va Koshi tengsizligidan foydalanamiz:
$x/y+z/t\ge 16/z+z/100\ge 2\sqrt{(16/z)(z/100)}=2\cdot(4/10)=0{,}8.$
To’g’ri javob: C.

31.(2001-8.20). Agar $x>0$ bo`lsa, $x+81/x$ ning eng kichik qiymatini toping.
A) 30 B) 24 C) 6 D) 12 E) 18
Koshi tengsizligidan foydalanib topamiz:
$x+81/x\ge 2\sqrt{x(81/x)}=2\cdot 9=18.$ To’g’ri javob: E.

32.(2001-7.20) $\begin{cases} |x|+|y|=1 \\ x^2+y^2=4 \end{cases}$ tenglamalar sistemasi nechta yechimga ega?
A) 1 B) 2 C) 4 D) ∅ E) to’g’ri javob keltirilmagan.
Sistemaning birinchi tenglamasi bilan markazi koordinatalar boshida va diagonali 2 ga teng bo’lgan kvadrat, ikkinchi tenglama bilan esa, markazi koordinatalar boshida va diametri 4 ga teng bo’lgan aylana berilgan. Bu ikki shakl kesishmaydi, shuning uchun berilgan sistema yechimga ega emas. To’g’ri javob: D.



33.(2001-7.21). $x^2+5x+\sqrt{x^2+5x-5}=17$ tenglamaning ildizlari ko’paytmasini toping.
A) 5 B) -5 C) 8 D) -8 E) -14
Tenglamani quyidagicha qayta yozamiz:
$x^2+5x-5+\sqrt{x^2+5x-5}-12=0$. So’ngra $\sqrt{x^2+5x-5}=t$ belgilsh kiritsak, $t^2+t-12=0$ hosil bo’ladi. Bundan $t_1=-4$; $t_2=3$ larni topamiz.

O'rniga qo'ysak:

a) $\sqrt{x^2+5x-5}=-4$ *b)* $\sqrt{x^2+5x-5}=3$

Yechimi yo'q. $x^2+5x-5=9$

$x^2+5x-14=0$

Viyet teoremasiga ko'ra: $x_1\cdot x_2=-14$

To'g'ri javob: E.

34.(2001-10.8). Nechta butun *x va y* sonlar jufti $x^2-y^2=31$ tenglikni qanoatlantiradi?

A) ∅ B) 1 C) 2 D) 3 E) 4

$(x-y)(x+y)=31\cdot 1=-31\cdot(-1)=-1\cdot(-31)=1\cdot 31$

Bundan kelib chiqib, 4 ta holni qaraymiz:

1). $\begin{cases} x-y=31 \\ x+y=1 \end{cases}$ 2). $\begin{cases} x-y=-31 \\ x+y=-1 \end{cases}$ 3). $\begin{cases} x-y=-1 \\ x+y=-31 \end{cases}$ 4). $\begin{cases} x-y=1 \\ x+y=31 \end{cases}$

(16;-15) (-16;15) (-16;-15) (16;15)

Demak, 4 ta juftlik. To`g`ri javob: E.

35.(2001-9.37). Yo`lovchi metroning harakatlanayotgan eskalatorida to`xtab turib 56 sekundda, yurib esa 24 sekundda pastga tushadi. Yo`lovchi to`xtab turgan eskalatorda xuddi shunday tezlik bilan yursa, necha sekundda pastga tushadi.

A) 40 B) 42 C) 41 D) 44 E) 43

1 sekundda eskalator butun masofaning 1/56 qismini, harakatlanayotgan eskalatorda yurayotgan kishi esa 1/24 qismini bosib o'tadi. Bundan to'xtab turgan eskalatorda yurayotgan kishi 1 sekundda butun masofaning 1/24-1/56=1/42 qismini bosib o'tishi kelib chiqadi. Demak, bu kishi to'xtab turgan eskalatorda 42 sekundda pastga tushadi. To'g'ri javob: B.

36.(2001-12.24). $(x^2-2)^2=5x^3+7x$ tenglamaning nechta manfiy ildizi bor?

A) 1 B) 2 C) 3 D) 4 E) manfiy ildizlari yo'q.

x ning istalgan manfiy qiymatida tenglamaning chap qismi nomanfiy< o'ng qismi esa manfiy son bo'ladi. Demak tenglamaning manfiy ildizlari mavjud emas: To'g'ri javob E.

37.(1999-4.20). $\left(\sqrt{7}+\sqrt{2}-1\right)\left(\sqrt{7}+1-\sqrt{2}\right)$ ni soddalashtiring.

A) $4+2\sqrt{2}$ B) $2-\sqrt{2}$ C) $4-\sqrt{2}$ D) $6+2\sqrt{2}$ E) $3\sqrt{2}+2\sqrt{7}$

$\left(\sqrt{7}+\sqrt{2}-1\right)\left(\sqrt{7}+1-\sqrt{2}\right)=\left(\sqrt{7}+(\sqrt{2}-1)\right)\left(\sqrt{7}-(\sqrt{2}-1)\right)=$

$=7-\left(\sqrt{2}-1\right)^2=7-(2-2\sqrt{2}+1)=4+2\sqrt{2}$. To’g’ri javob: A.

38.(1999-5.30). $\dfrac{8\cos 2\alpha-5\cos 3\beta}{7+2\cos 4\gamma}$ ifodaning eng katta qiymatini toping.

A) 2,2 B) 2,3 C) 2,4 D) 2,5 E) 2,6

α, β, γ - lar o’zaro bog’liq bo’lmagani uchun $\cos 2\alpha=1$; $\cos 3\beta=-1$; $\cos 4\gamma=-1$ deb olishimiz mumkin. Bunda ifoda eng katta 2,6 qiymatga erishadi. To’g’ri javob: E.

39.(1999-8.29). Agar $x^2+\left(\dfrac{x}{x-1}\right)^2=8$ bo`lsa, $\dfrac{x^2}{x-1}$ ifodaning eng katta qiymatini toping.

A) 4 B) 8 C) 2 D) 16 E) 1/4

Koshi tengsizligidan foydalanamiz:

$$x^2+\left(\frac{x}{x-1}\right)^2\geq 2\sqrt{x^2\cdot\left(\frac{x}{x-1}\right)^2}=\frac{2x^2}{x-1}.$$

Bundan $\dfrac{2x^2}{x-1}\leq 8$; $\dfrac{x^2}{x-1}\leq 4$. To’g’ri javob: A.

40.(2002-6.29). $f(x)=\dfrac{x^2-4x+8}{x^2-4x+5}$ funksiyaning qiymatlari sohasini toping.

A) [1,6;5] B) [1,6;4] C) [1;4] D) (1;4] E) (0;5]

Fuksiya formulasini quyidagicha qayta yozamiz:

$$f(x)=\frac{x^2-4x+8}{x^2-4x+5}=\frac{x^2-4x+5+3}{x^2-4x+5}=1+\frac{3}{(x-2)^2+1}$$

Ko’rinib turibdiki, x=2 ikkinchi qo’shiluvchi eng katta 4 qiymatga erishadi. Ikkinchi tomondan bu kasr x ning istalgan qiymatida musbat. Shunday qilib, bu funksiyaning qiymatlari sohasi (1; 4] dan iborat.
To’g’ri javob: D.

41.(2002-3.16). $\dfrac{x}{3}+\dfrac{x}{15}+\dfrac{x}{35}+\dfrac{x}{63}+\dfrac{x}{99}+\dfrac{x}{143}=12$ tenglamani yeching.

A) 26 B) 13 C) 18 D) 16 E) 24

Tenglamani quyidagicha qayta yozamiz:

$$\left(\frac{1}{3}+\frac{2}{3}-\frac{3}{5}+\frac{3}{5}-\frac{4}{7}+\frac{4}{7}-\frac{5}{9}+\frac{5}{9}-\frac{6}{11}+\frac{6}{11}-\frac{7}{13}\right)\cdot x=12$$

$$(1-\frac{7}{13})\cdot x=12$$

x=26. To'g'ri javob: A.

42. (2002-7.41). *(x+1)(x+2)(x+4)(x+5)=40* Tenglamaning haqiqiy ildizlari yig'indisini toping.

A) -6 B) 0 C) -5 D) 6 E) 7

\## Quyidagi belgilashni kiritamiz: $x+3=t$

Berilgan tenglama quyidagi ko`rinishni oladi:

$$(t-2)(t-1)(t+1)(t+2)=40$$
$$(t^2-1)(t^2-4)=40$$
$$t^4-5t^2-36=0$$

Bikvadrat tenglamani yechib: a) $t^2=9;$ b) $t^2=-4$

$t_{12}=\pm 3$. yechimi yo'q.

O'rniga qo'ysak: a) $x+3=-3$ b) $x+3=3$

$x_1=-6$ $x_2=0$

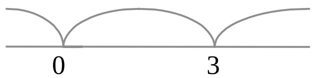
Bulardan $x_1+x_2=-6$. To'g'ri javob: A.

43.(2002-1.59). $\log_2^3 x-3\log_2^2 x\geq 0$ tengsizlikni yeching.

A) [16;∞) B) {1}∪[16;∞) C) [8; ∞) D) {1}∪[9;∞) E) {1}∪[8;∞)

\## Quyidagi belgilshni kiritamiz: $\log_2 x=t$

$t^3-3t^2\geq 0$



$t^2(t-3)\geq 0$

$t=0,\ t\geq 3$

o'rniga qo'ysak,

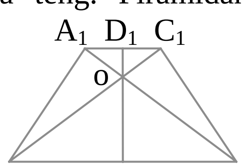
a) $\log_2 x=0,$ b) $\log_2 x\geq 3.$

$x=1$ $x\geq 8$

javob: {1}∪[8;∞)

To'g'ri javob: E.

44.(2002-2.44). Muntazam to'rtburchakli kesik piramidaning diagonallari o'zaro perpendikulyar va ularning har biri 8 ga teng. Piramidaning balandligini toping.



A) $4\sqrt{2}$ B) $2\sqrt{2}$ C) 4 D) 6 E) $3\sqrt{2}$

\## Piramidaning diagonal kesimini qaraymiz.

ΔA_1OC_1 teng yonli to'g'ri burchakli uchburchak bo'lgani uchun ΔC_1OD_1 ham teng yonli to'g'ri burchakli bo'ladi. Shu kabi ΔCOD ham teng yonli to'g'ri burchakli uchburchak bo'ladi.

$C_1D_1=D_1O=x$, $CD=DO=y$ deb belgilasak, $H=x+y$ bo'ladi.

AA_1C_1C trapetsiyaning yuzini ikki usul bilan topib tengalymiz:

$(2x+2y)(x+y)/2=8\cdot 8/2$; $(x+y)^2=32$; $x+y=4\sqrt{2}$

Bundan $H=4\sqrt{2}$. To'g'ri javob: A

45.(2002-2.60). $y=\cos^4x-2\sin^2x+7$ funksiyaning eng kichik qiymatini toping.

A) 5 B) 3 C) 2 D) 1 E) -5

Formulani quyidagicha almashtiramiz:

$y=\cos^4x-2\sin^2x+7=\cos^4x-2(1-\cos^2x)+7=\cos^4x-2+2\cos^2x+7=$
$=(\cos^2x+1)^2+4\geq 5$. To'g'ri javob: A.

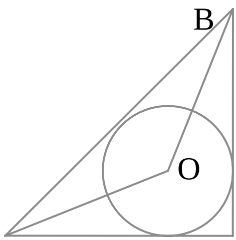
46.(2002-3.47). A(2;5) nuqtadan $4x-3y+1=0$ to'g'ri chiziqqacha bo'lgan masofani aniqlang.

A) 1,2 B) 1 C) 1,4 D) 1,3 E) 0,8

$(x_o;y_o)$ nuqtadan $ax+by+c=0$ to'g'ri chiziqqacha bo'lgan masofa

$d=\frac{|ax_o+by_o+c|}{\sqrt{a^2+b^2}}$ formula bilan hisoblanadi.

$d=\frac{|4\cdot 2-3\cdot 5+1|}{\sqrt{4^2+(-3)^2}}=\frac{6}{5}=1{,}2$. To'g'ri javob: A.



47.(2002-3.56). To'g'ri burchakli uchburchakka ichki chizilgan aylananing markazidan gipotenuzaning uchlarigacha bo'lgan masofalar $\sqrt{5}$ va $\sqrt{10}$ ga teng.
Gipotenuzaning uzunligini toping.

A) 5 B) $0{,}5\sqrt{50}$ C) $\sqrt{50}$ D) 6 E) 5,2

Shakldan $\angle AOB=180^o-(0{,}5\angle A+0{,}5\angle B)=180^o-0{,}5(\angle A+\angle B)= =180^o-0{,}5\cdot 90^o=135^o$.

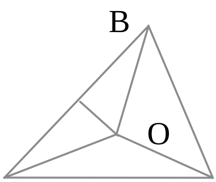
ΔAOB ga kosinuslar teoremasni qo'llab topamiz:

$AB=\sqrt{10+5-2\sqrt{10}\cdot\sqrt{5}\cdot(-1/\sqrt{2})}=5$. To'g'ri javob: A.

48.(2002-3.58). ABC uchburchakda medianalar kesishgan nuqtadan AB tomonigacha bo'lgan masofa 1 ga teng. Agar AB=8 bo'lsa, ABC uchburchakning yuzini toping.

A) 12 B) 16 C) 9 D) 13 E) 10

Uchburchakda medianalar kesishgan nuqtani- uchburchak uchlari bilan tutashtirilsa, uchta tengdosh uchburchak hosil bo'ladi.
Berilgan uchburchakning yuzi shu



uchburchaklardan istalgan biri yuzining uch baravariga teng bo’ladi. A C

$S_{ABC}=3S_{AOB}=3\cdot(0,5\cdot8\cdot1)=12$. To’g’ri javob: A.

49.(2002-7.42) $(x^2+3x+1)(x^2+3x-3)\geq5$ tengsizlikni yeching.

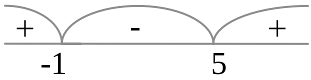
A)$(-\infty;-4]\cup[-2;-1]\cup[1;\infty)$ B) $(-\infty;-4]\cup[1;\infty)$

C) $(-4;-2]\cup[-1;\infty)$ D) $(-2;-1]\cup[1;\infty)$

E) $(-\infty;-4]\cup[-2;-1]$

Quyidagi belgilashni kiritamiz: $x^2+3x+1=t$.

$t\cdot(t-4)\geq5$; $t^2-4t-5\geq0$; $(t-5)(t+1)\geq0$

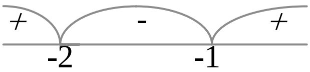


$t\leq-1$; $t\geq5$.

a) $x^2+3x+1\leq-1$

$x^2+3x+2\leq0$

$(x+1)(x+2)\leq0$

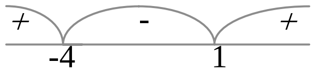


$[-2;-1]$

b) $x^2+3x+1\geq5$

$x^2+3x-4\geq0$

$(x+4)(x-1)\geq0$



$(-\infty;-4]\cup[1;\infty)$

Umumiy yechim: $(-\infty;-4]\cup[-2;-1]\cup[1;\infty)$

To’g’ri javob: A.

50.(2002-10.28). $y=x^x$ funksiyaning hosilasini toping.

A) $x^x(1+\ln x)$ B) $x^{x-1}(1+\ln x)/\ln x$ C) x^x D) $x^x\ln x$ E) x^{x-1}

Formulani asosiy logarifmik ayniyatdan foydalanib, quyidagicha almashtiramiz: $y=x^x=(e^{\ln x})^x=e^{x\ln x}$.

Murakkab funksiya hosilasini topish formulasini qo’llab topamiz:

$y'=e^{x\ln x}\cdot(\ln x+x\cdot(1/x))=x^x(\ln x+1)$. To’g’ri javob: A.

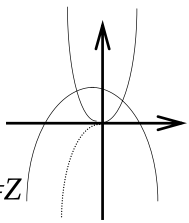
51(2002-1.57). $1+x-x^2=|x^3|$ tenglama nechta haqiqiy ildizga ega?

A) 1 B) 2 C) 3 D) 4 E) Yechimi yo’q.

$y=-x^2+x+1$ va $y=|x^3|$ funksiyalarning grafiklarini qaraymiz. Ular ikkita nuqtada kesishadi.

Demak, berilgan tenglama 2 ta ildizga ega.

To’g’ri javob: B.



52(2002-1.62). cos(sinx)<0 tengsizlikni yechig.

A) $(\frac{\pi}{2}+2\pi n;\frac{3\pi}{2}+2\pi n)$, $n\in Z$ B) $(\frac{\pi}{2}+\pi n;\frac{3\pi}{2}+\pi n)$, $n\in Z$

C) $(0;\frac{3\pi}{2}+2\pi n)$, $n\in Z$ D) $(0;\frac{3\pi}{2})$, $n\in Z$ E) yechimi yo'q

$sinx=t$ deb belgilasak, $cost<0$ bo'ladi, bundan $\frac{\pi}{2}<t<\frac{3\pi}{2}$ yoki $\frac{\pi}{2}<\sin x<\frac{3\pi}{2}$. Bu tengsizlikning esa yechimi yo'q.

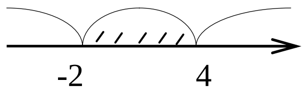
To'g'ri javob: E.

53(2002-2.35). $\int_0^a xdx \le a+4$ tengsizlikni qanoatlantiruvchi a ning qiymatlari oralig'i uzunligini toping.

A) 6 B) 5 C) 4 D) 8 E) 7

$\int_0^a xdx=\frac{x^2}{2}\Big|_0^a=\frac{a^2}{2}$ dan $\frac{a^2}{2}\le a+4$; a^2-2a-8≤0; (a+2)(a-4)≤0

[-2;4] oraliqning uzunligi 6 birlik.



To'g'ri javob: A.

54(2002-2.51). $2sin6x(cos^4 3x-sin^4 3x)=sinkx$ tenglik hamma vaqt o'rinli bo'lsa, k ni toping.

A) 12 B) 24 C) 6 D) 18 E) 4

$2sin6x(cos^4 3x-sin^4 3x)$q $2sin6x(cos^2 3x-sin^2 3x)(cos^2 3x+sin^2 3x)=$
$=2sin6x(cos^2 3x-sin^2 3x)=2sin6xcos6x=sin12x.$
sin12x=sinkx dan x=12. to'g'ri javob: A.

55(2002-3.9) Agar $a(x-1)^2+b(x-1)+c=2x^2-3x+5$ bo'lsa, $a+b+c$ yig'indi nechaga teng bo'ladi?

A) 7 B) 8 C) 6 D) 4 E) 5

$a(x-1)^2+b(x-1)+c=ax^2-2ax+a+bx-b=ax^2-(2a-b)x+a-b+c.$

Bundan $\begin{cases} a=2 \\ 2a-b=3 \\ a-b+c=5 \end{cases}$ Demak , b=1; c=4. $a+b+c$=7.

To'g'ri javob: A.

56(2002-3.78). $\cos^2 x+6\sin x=4a^2-2$ tenglama a ning qanday qiymatlarida yechimga ega?

A) $[-\sqrt{2};\sqrt{2}]$ B) $[0;\sqrt{2}]$ C) [0;2] D) (-2;2) E) [1;0)

max(cos2x+6sinx)=6 va min(cos2x+6sinx)=-6 bo'lgani uchun.

$$\begin{cases}4a^2-2\ge-6\\4a^2-2\le 6\end{cases}\Rightarrow\begin{cases}4a^2\ge-4\\4a^2\le 8\end{cases}\Rightarrow\begin{cases}a^2\ge-1\\a^2\le 2\end{cases}\Rightarrow a^2\le 2;\quad |a|\le\sqrt{2};$$

$-\sqrt{2}\le a\le\sqrt{2}$. To'g'ri javob: A.

57(2002-3.80). $(8x-1)(x+2)\mathrm{ctg}\pi x=0$ tenglama [-2;2] kesmada nechta ildizga ega?

A) 5 B) 4 C) 6 D) 7 E) 3

a) 8x-1=0; $x_1=1/8$. bu ildiz [-2;2] kesmaga tegishli.

b) x+2=0; x=-2. bu chet ildiz, chunki ctg2π aniqlanmagan.

c) ctgπx=0; πx=π/2+πn; x=1/2+n;

bu ko'rinishdagi sonlardan -3/2; -1/2; 1/2; 3/2 lar [-2;2] kesmaga tegishli.

Shunday qilib, tenglamaning [-2;2] kesmadagi ildizlari 5 ta.

To'g'ri javob: A.

58(2002-4.17). Geometrik progressiyaning mahraji 1/2 ga teng.

$b_1(b_2)^{-1}b_3(b_4)^{-1}\cdot\ldots\cdot b_{13}(b_{14})^{-1}$ ning qiymatini toping.

A) 64 B) 32 C) 16 D) 128 E) 256

$b_1(b_2)^{-1}b_3(b_4)^{-1}\cdot\ldots\cdot b_{13}(b_{14})^{-1}=\frac{b_1}{b_2}\cdot\frac{b_3}{b_4}\cdot\ldots\cdot\frac{b_{13}}{b_{14}}=\left(\frac{1}{q}\right)^7=2^7=128.$

To'g'ri javob: D.

59(2002-4.19). Arifmetik progressiya hadlari uchun

$a_1+a_3+\ldots+a_{21}=a_2+a_4+\ldots+a_{20}+15$ tenglik o'rinli bo'lsa, a_{11} ni toping.

A) 11 B) 13 C) 15 D) 17 E) 19

$a_1+(a_3-a_2)+(a_5-a_4)+\ldots+(a_{21}-a_{20})=15;\quad a_1+10d=15;\quad a_{11}=15.$

To'g'ri javob: C.

60(2002-4.41). $f(x)=\sqrt{5^x-1/25}+\sqrt{-x}$ funksiyaning aniqlanish sohasiga tegishli barcha butun sonlarning o'rta arifmetigini toping.

A) -2 B) -1 C) 0 D) 1 E) 2

$$\begin{cases}5^x-\frac{1}{25}\ge 0\\-x\ge 0\end{cases}\Rightarrow\begin{cases}5^x\ge 5^{-2}\\x\le 0\end{cases}\Rightarrow\begin{cases}x\ge-2\\x\le 0\end{cases}\Rightarrow -2\le x\le 0.$$

$\frac{-2+(-1)+0}{3}=-1$. To'g'ri javob: B.

61(2002-4.44). $\sqrt{1+x}\le\arccos(x+2)$ tengsizlikning eng katta butun yechimini toping.

A) -2   B) -1   C) 0   D) 1   E) 2

$\begin{cases} 1+x\ge 0 \\ -1\le x+2\le 1 \end{cases} \Rightarrow \begin{cases} x\ge -1 \\ -3\le x\le -1 \end{cases} \Rightarrow x = -1.$

To'g'ri javob: B.

62(2002-5.14). $\sqrt{x+3-\sqrt{x+14}}+\sqrt{x+3+\sqrt{x+14}}=4$ bo'lsa, $x/(x+1)$ ning qiymatini toping.

A) 2/3   B) -2/3   C) 3   D) 3/2   E) -3/2

$\sqrt{x+14}=t$ deb belgilasak, $x=t^2-14$ bo'ladi. Bundan

$\sqrt{t^2-11-t}+\sqrt{t^2-11+t}=4$.   Bu tenglamani yechsak: $t_{1,2}=\pm 4$.

a) $\sqrt{x+14}=-4$ tenglama yechimga ega emas.

b) $\sqrt{x+14}=4$; $x+14=16$; $x=2$.   To'g'ri javob: A.

63(2002-6.14). Nechta natural $(x;y)$ sonlar jufti $x^2-y^2=53$ tenglikni qanoatlantiradi?

A) $\varnothing$ B) 1   C) 2   D) 3   E) 4

$x^2-y^2=1\cdot 53$; $(x-y)(x+y)=1\cdot 53$

$\begin{cases} x-y=1 \\ x+y=53 \end{cases} \Rightarrow x=27;\ y=26.$ (27;26) Bitta juftlik.

To'g'ri javob: B.

64(2002-6.20). $\log_{128}\left((0{,}25)^{\log_6(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...)}\right)$ ni hisoblang.

A) 2/7   B) 3/8   C) 1/14   D) 2/5   E) 1/12

$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...$ - birinchi hadi $\frac{1}{3}$ va mahraji $\frac{1}{3}$ ga teng bo'lgan cheksiz kamayuvchi geometrik progressiyaning yig'indisi. $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...=\frac{1}{2}$.

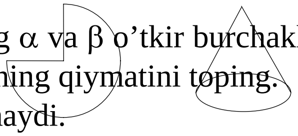
Buni berilgan ifodaga qo'yamiz:

$$\log_{128}\left((0{,}25)^{\log_{16}(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...)}\right)=\log_{128}(1/4)^{\log_{16}(1/2)}=\frac{1}{7}\log_2(4^{\log_4 2})^{1/2}=$$

$$=\frac{1}{7}\log_2 2^{1/2}=\frac{1}{7}\cdot\frac{1}{2}=\frac{1}{14}.$$ To'g'ri javob: C.

65(2002-2.46). Doiradan markaziy burchagi 90° bo'lgan sektor qirqib olingach, uning qolgan qismi o'ralib konus shakliga keltirildi. Bu konus diametrining yasovchisiga nisbatini toping.

A) 3/2 B) 2 C) 5/4 D) 2/3 E) 4/5
Konus asosining uzunligi
sektor yoyining uzunligiga, konusning
yasovchisi esa sektorning radiusiga teng.
$C_{konus}=1,5\pi R_{sektor}$; $d_{konus}=1,5R_{sektor}$;
$L_{konus}= R_{sektor}$
$D_{konus} : l_{konus}$=1,5R:R=1,5=3/2.



66(2002-3.57). To'g'ri burchakli uchburchakning α va β o'tkir burchaklari uchun cosα+sin(α-β)=1 tenglik o'rinli bo'lsa, β ning qiymatini toping.
A) 30° B) 45° C) 60° D) 75° E) aniqlab bo'lmaydi.
$\alpha+\beta=90°$ dan $\alpha=90°-\beta$ bo'ladi.
$\cos\alpha+\sin(\alpha-\beta)=\cos(90°-\beta)+\sin(90°-2\beta)=\sin\beta+\cos2\beta=\sin\beta+1-2\sin^2\beta$.
$\sin\beta+1-2\sin^2\beta=1$; $2\sin^2\beta-\sin\beta=0$; $\sin\beta(2\sin\beta-1)=0$;
$\sin\beta=0$ bo'la olmaydi, shuning uchun $2\sin\beta-1=0$; $\beta=30°$.
To'g'ri javob: A.
67(2002-4.49). Uchburchakning burchaklari 1:2:3 kabi nisbatda. Uchburchak katta tomonining kichik tomoniga nisbatini toping.
A) 1 B) 2 C) 3 D) 4 E) 5
$\angle A:\angle B:\angle C=1:2:3$ dan $\angle A=30°$; $\angle B=60°$; $\angle C=90°$.
$c=a/\sin A=a/\sin30°=a/0,5=2a$. $c:a=2a:a=2$. To'g'ri javob: B
68(2002-4.50). Uchburchakning tomonlar 10, 13 va 17 ga teng. Bu uchburchakka tashqi chizilgan aylananing markazi qayerda bo'lishini aniqlang.
A) uchburchak ichida B) uchburchakning kichik tomonida
C) uchburchak tashqarisida D) aniqlab bo'lmaydi
E) uchburchakning katta tomonida
$10^2+13^2<17^2$ bo'lgani uchun bu uchburchak o'tmas burchakli. O'tmas burchakli uchburchakka tashqi chizilgan aylananing markazi uchburchak tashqarisida joylashadi.
To'g'ri javob: C.
69(2002-4.52). Piramidaning asosi gipotenuzasining uzunligi 2 bo'lgan to'g'ri burchakli uchburchakdan iborat. Piramidaning qirralari asos tekisligi bilan α burchak tashkil qiladi. Agar uning balandligi 5 ga teng bo'lsa, $tg\alpha$ ning qiymatini toping.
A) 1 B) 2 C) 3 D) 4 E) 5
Piramidaning barcha yon qirralari asos tekisligiga bir xil og'ishganda piramida balandligining asosi, piramida asosiga tashqi chizilgan aylana

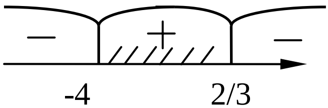
markazi bilan ustma-ust tushadi. Piramida asosi to'g'ri burchakli uchburchak bo'lgani uchun balandlikning asosi gipotenuzaning o'rtasida yo'tadi. Shunday qilib, tgα=5/1=5. To'g'ri javob: E.

70(2003-1.8). $\sqrt{\dfrac{2-3x}{x+4}} > -2$ tengsizlikning eng kichik butun yechimini toping.

A) 0 B) -1 C) -2 D) -3 E) -5

Arifmetik kvadrat ildizning qiymati doimo nomanfiy son bo'lgani uchun, bu tengsizlik $\dfrac{2-3x}{x+4} \geq 0$ tengsizlikka teng kuchli. Bundan

(2-3x)(x+4)≥0



(-4; 2/3]

Eng kichik butun yechimi: -3. To'g'ri javob: D.

71(2003-1.52). $\int_1^2 \dfrac{x}{x+1}dx$ ni hisoblang.

A) 2+ln(1/2) B) 1+ln(2/3) C) 3-ln(2/3) D) 1-ln(2/3) E) 2-ln(2/3)

$\int_1^2 \dfrac{x}{x+1}dx = \int_1^2 \dfrac{(x+1)-1}{x+1}dx = \int_1^2 (1-\dfrac{1}{x+1})dx =$ (x-ln(x+1)) $\Big|_1^2 =$

=(2-ln(2+1))-(1-ln(1+1))=2-ln3-1+ln2=1+ln(2/3). To'g'ri javob: B.

72(2003-6.5). Agar $\dfrac{29}{31}+\dfrac{38}{41}+\dfrac{47}{51}=a$ bo'lsa, $\dfrac{2}{31}+\dfrac{3}{41}+\dfrac{4}{51}$ quyidagilarning qaysi biriga teng?

A) 3-a B) 4-a C) 5-a D) 3-a/2 E) 4-a/2

$\dfrac{2}{31}+\dfrac{3}{41}+\dfrac{4}{51}=\dfrac{31-29}{31}+\dfrac{41-38}{41}+\dfrac{51-47}{51}=1-\dfrac{29}{31}+1-\dfrac{38}{41}+1-\dfrac{47}{51}=$

3-($\dfrac{29}{31}+\dfrac{38}{41}+\dfrac{47}{51}$)=3-a. To'g'ri javob: A.

73(2003-6.15). $\sqrt[3]{x+\sqrt[3]{x+\sqrt[3]{x+...}}} = 4$ tenglamani yeching.

A) 56 B) 48 C) 60 D) 54 E) 64

Har ikki qismini kubga ko'taramiz:

$$x+\sqrt[3]{x+\sqrt[3]{x+\sqrt[3]{x+...}}}=64.$$

$\sqrt[3]{x+\sqrt[3]{x+\sqrt[3]{x+...}}}=4$ ni e'tiborga olsak, *x*+4=64 bo'ladi.
Bundan *x*=60. To'g'ri javob: C

74(2003-6.26). Agar *sin*37°=*a* bo'lsa, *sin*16° ni *a* orqali ifodalang.
A) a^2 B) *a*-1 C) $2a^2-1$ D) $1-2a^2$ E) aniqlab bo'lmaydi.

sin37°=sin(45°-8°)=sin45°cos8°-cos45°sin8°=$\frac{\sqrt{2}}{2}$cos8°-$\frac{\sqrt{2}}{2}$sin8°=

=$\frac{\sqrt{2}}{2}$(cos8°-sin8°). Demak: $\frac{\sqrt{2}}{2}$(cos8°-sin8°)=*a;* cos8°-sin8°=$\sqrt{2}$ *a;*

$(\cos8°-\sin8°)^2=(\sqrt{2}\,a)^2$; $\cos^2 8°-2\cos8°\sin8°+\sin^2 8°=2a^2$.
Bundan $\sin16°=1-2a^2$. To'g'ri javob: D.

75(2003-7.30). Agar $f(x)=\frac{7x^2+ax+b}{x}$ funksiya grafigi (2;0) nuqtada absissalar o'qiga urinib o'tsa, *a*+*b* nimaga teng?

A) 0 B) 20 C) -21 D) 28 E) -56

Shartga ko'ra f(2)=0 bo'lishi kerak, shuning uchun:

$\frac{7\cdot2^2+a\cdot2+b}{2}=0$. Bundan *b= -2a-28.*

Ikkinchi tomondan urinma OX o`qiga parallel (aniqrog'i OX o`qining o'zi) bo'lgani uchun *f′(2)=0* bo'lishi kerak. Shuning uchun

$$\frac{(14x+a)x-7x^2-ax-b}{x^2}=0$$

$$14x^2+ax-7x^2-ax-b=0$$

$$a=-28$$

$$b=-28-2\cdot(-28)=28$$

$$a+b=-28+28=0.$$

To'g'ri javob: A.

76(2003-8.34). Agar $\begin{cases} x+y-\sqrt{xy}=7 \\ x^2+y^2+xy=133 \end{cases}$ bo'lsa, xy ning qiymatini toping.

A) 36 B) 42 C) 25 D) 81 E) 16

$$\begin{cases} x+y=7+\sqrt{xy} \\ (x+y)^2=133+xy \end{cases} \Rightarrow (7+\sqrt{xy}) = 133 + xy \Rightarrow$$

$$49+14\sqrt{xy}+xy=133+xy$$

$$\sqrt{xy}=6$$

xy=36. To`g`ri javob: A

77(2003-8.40). $\begin{cases} \dfrac{xy}{x+y}=\dfrac{10}{7} \\ \dfrac{yz}{y+z}=\dfrac{40}{13} \\ \dfrac{xz}{x+z}=\dfrac{5}{8} \end{cases}$ tenglamalar sistemasidan *x* ni toping.

A) 80/79 B) 5/7 C) 7/13 D) 79/80 E) 7/5

Tenglamalar sistemasini quyidagicha qayta yozamiz:

$$\begin{cases} \dfrac{x+y}{xy}=\dfrac{7}{10} \\ \dfrac{y+z}{yz}=\dfrac{13}{40} \\ \dfrac{x+z}{xz}=\dfrac{8}{5} \end{cases} \Rightarrow \begin{cases} \dfrac{1}{y}+\dfrac{1}{x}=\dfrac{7}{10} \\ \dfrac{1}{z}+\dfrac{1}{y}=\dfrac{13}{40} \\ \dfrac{1}{z}+\dfrac{1}{x}=\dfrac{8}{5} \end{cases}$$

quyidagicha belgilashlar kiritamiz: $\dfrac{1}{x}=a$, $\dfrac{1}{y}=b$, $\dfrac{1}{z}=c$.

$\begin{cases} b+a=\dfrac{7}{10} \\ c+b=\dfrac{13}{40} \\ c+a=\dfrac{8}{5} \end{cases}$ bu sistemani yechsak, *a*=79/80. Bundan x=80/79.

To’g’ri javob: A

78(2003-9.48). $f(x)=x-1-ctg^2x$ funksiyaning boshlang'ich funksiyasini toping.

A) $\frac{x^2}{2}-ctgx+C$ B) $\frac{x^2}{2}+ctgx+C$ C) $\frac{x^2}{2}-tgx+C$ D) $\frac{x^2}{2}+tgx+C$

E) $x^2+ctgx+C$

Funksiya formulasini quyidagicha qayta yozamiz:

$f(x)=x-1-ctg^2x=x-(1+ctg^2x)=x-\frac{1}{\sin^2 x}$. Bu funksiyaning boshlang'ich funksiyasi: $F(x)=x^2/2+ctgx+C$. To'g'ri javob: B.

79(2003-10.13). $\sqrt{x-2}+\sqrt{1-x}=2$ tenglamani yeching.

A) $\varnothing$ B) 2 C) 1,2 D) 0,4 E) 0,9

Bu tenglamani yechishdan oldin uning aniqlanish sohasini topish maqsadga muvofiq:

$\begin{cases} x-2\geq 0 \\ 1-x\geq 0 \end{cases} \Rightarrow \begin{cases} x\geq 2 \\ x\leq 1 \end{cases} \Rightarrow$ sistema yechimga ega emas.

Demak, tenglamaning aniqlanish sohasi, shuningdek, yechimlar to'plami ham bo'sh to'plam - $\varnothing$. To'g'ri javob: A.

80(2003-11.9). S_n arifmetik progressiyaning dastlabki n ta hadining yig'indisi bo'lsa, $S_5-3S_4+3S_3-S_2$ ning qiymatini toping.

A) 0 B) $-2a_1$ C) $2a_1$ D) $3a_1$ E) $-3a_1$

$S_5-3S_4+3S_3-S_2=S_5-S_4-2(S_4-S_3)+S_3-S_2=a_5-2a_4+a_3=$

$=a_1+4d-2(a_1+3d)+a_1+2d=0.$

To'g'ri javob: A.

81(2003-11.15). $y=\sin(\sin x)$ funksiyaning eng katta qiymatini aniqlang.

A) sin1 B) 1 C) 1/2 D) aricsin1 E) $\pi/2$

t=sinx deb olsak, $-1 \leq t \leq 1$ bo'ladi. Bu oraliqda y=sin(sinx)=sint funksiya o'suvchi. Shuning uchun y_{max}= sin1. To'g'ri javob: A.

82(2003-12.58). $5^{\sqrt{\log_5 a}}-a^{\sqrt{\log_a 5}}$ ifodani soddalashtiring.

A) a B) a^2 C) $5a$ D) 1 E) 0

$5^{\sqrt{\log_5 a}}-a^{\sqrt{\log_a 5}}=a^{\log_a 5\cdot\sqrt{\frac{1}{\log_a 5}}}-a^{\sqrt{\log_a 5}}=a^{\sqrt{\log_a 5}}-a^{\sqrt{\log_a 5}}=0$

To'g'ri javob: E.

83(2003-12.77). $\left(\left(tg^2\frac{7\pi}{24}-tg^2\frac{\pi}{24}\right):\left(1-tg^2\frac{7\pi}{24}\cdot tg^2\frac{\pi}{24}\right)\right)^2$ ni hisoblang.

A) 1/9 B) 9 C) 1/3 D) 1 E) 3

$\left(\left(tg^2\frac{7\pi}{24}-tg^2\frac{\pi}{24}\right):\left(1-tg^2\frac{7\pi}{24}\cdot tg^2\frac{\pi}{24}\right)\right)^2=$

$$=\frac{\left(tg^2\frac{7\pi}{24}-tg^2\frac{\pi}{24}\right)^2}{\left(1-tg^2\frac{7\pi}{24}\cdot tg^2\frac{\pi}{24}\right)^2}=\frac{\left(\left(tg\frac{7\pi}{24}-tg\frac{\pi}{24}\right)\left(tg\frac{7\pi}{24}+tg\frac{\pi}{24}\right)\right)^2}{\left(\left(1-tg\frac{7\pi}{24}\cdot tg\frac{\pi}{24}\right)\left(1+tg\frac{7\pi}{24}\cdot tg\frac{\pi}{24}\right)\right)^2}=$$

$$=\left(\frac{tg\frac{7\pi}{24}+tg\frac{\pi}{24}}{1-tg\frac{7\pi}{24}tg\frac{\pi}{24}}\cdot\frac{tg\frac{7\pi}{24}-tg\frac{\pi}{24}}{1+tg\frac{7\pi}{24}tg\frac{\pi}{24}}\right)^2=\left(tg\frac{\pi}{3}tg\frac{\pi}{4}\right)^2=3.$$

To’g’ri javob: E.

84(2003-7.61). Uchburchakning ikki tomoni 7 va 11 ga teng, uchinchi tomoniga o’tkazilgan medianasi 6 ga teng. Uchburchakning uchinchi tomonini toping.

A) 12 B) 8 C) 14 D) 10 E) 13

Berilgan: ABC uchburchakda AB=7, AC=11, AO=6 (AO-mediana). BC=?

Yechilishi: ABDC parallelogrammga to’ldiramiz.

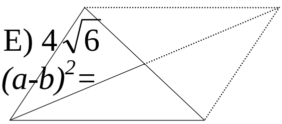
$AD^2+BC^2=2(AB^2+AC^2)$;

$(2\cdot6)^2+BC^2=2(7^2+11^2)$

Bundan BC=14. To’g’ri javob: C.

85(2003-9.36). Agar $|\vec{a}|=7, |\vec{b}|=17$ va $|\vec{a}-\vec{b}|=3\sqrt{35}$ bo’lsa, $|\vec{a}+\vec{b}|$ ning qiymatini toping.

A) 19 B) 20 C) $8\sqrt{3}$ D) $9\sqrt{2}$ E) $4\sqrt{6}$

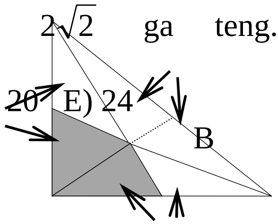


$(a+b)^2+(a-b)^2=2(a^2+b^2)$, $(a+b)^2=2(a^2+b^2)-(a-b)^2=$
$=2(49+289)-315=676-315=361$. $a+b=19$

To’g’ri javob: A.

86(2003-9.52). To’g’ri burchakli uchburchakka ichki chizilgan aylananing urinish nuqtasi gipotenuzani 2:3 nisbatda bo’ladi. To’g’ri burchak uchidan aylana markazigacha bo’lgan masofa $2\sqrt{2}$ ga teng.Berilgan uchburchakning yuzini toping..

A) 12 B) 16 C) 18 D) 20 E) 24



AF=AD=2x, BE=BD=3x.

$(3x+2)^2+(2x+2)^2=(5x)^2$; $3x^2-5x-2=0$

$x_1=2$; $x_2=-1/3$.

AC=2·2+2=6; BC=3·2+2=8.

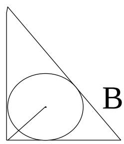
$S_{ABC}=24$.

87(2003-10.48). To’g’ri burchakli uchburchakning katetlaridan biri ikkinchisidan ikki marta katta.Shu uchburchakning gipotenuzasiga tushirilgan balandligi 12 ga teng. Uchburchakning yuzini toping.

A) 180 B) 84 C) 120 D) 90 E) 108

AC=x; BC=2x deb olsak,

$AB=\sqrt{x^2+(2x)^2}=x\sqrt{5}$



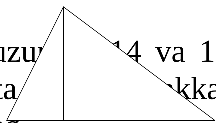
$S=\frac{1}{2}AC\cdot BC=\frac{1}{2}AB\cdot CD.$ $AC\cdot BC=AB\cdot CD$

$x\cdot 2x=x\sqrt{5}\cdot 12;$

$x=6\sqrt{5}.$

$S_\Delta=\frac{1}{2}AC\cdot BC=\frac{1}{2}6\sqrt{5}\cdot 12\sqrt{5}=180.$ To’g’ri javob: A

88(2003-10.55). To’g’ri burchakli uchburchakning uzunligi 14 va 18 ga teng katetlariga tushirilgan medianalari uni uchta uchburchakka va to’rtburchakka ajratadi. To’rtburchakning yuzini toping.



A) 56 B) 64 C) 48 D) 72 E) 42

Berilgan uchburchakning yuzi S bo’lsin.

$S=(18\cdot 14):2=126.$

Shakldan $\begin{cases} S_1+2S_2=S/2 \\ 2S_1+S_2=S/2 \end{cases}$

S_1 S_2 S_3

$S_1=S_2$;

$3S_1=S/2$

$S_1=S/6.$

$S_{shakl}=\frac{S}{2}-S_1=\frac{S}{2}-\frac{S}{6}=\frac{2S}{6}=\frac{S}{3}=42$

To’g’ri javob: E.

89(2003-10.61). To’rtburchakli piramidaning barcha yon qirralari asos tekisligi bilan 60° li burchak hosil qiladi. Uning asosi teng yonli trapetsiyadan iborat. Trapetsiyaning diagonallari uning o’tkir burchaklarining bissektrisalaridir. Piramidaning balandligi $4\sqrt{3}$ ga teng. Trapetsiyaning katta asosini toping.

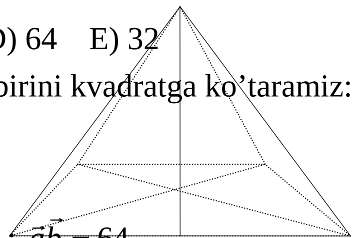
A) $4\sqrt{3}$ B) 8 C) $8\sqrt{3}$ D) 12 E) $3\sqrt{6}$

Masala shartidan ∠ACD=90° ekani kelib chiqadi. Piramidaning yon qirralari asos tekisligiga bir xil og’ishganligi uchun, piramida balandligining asosi ΔACD ning gipotenuzasi AD ning o’rtasida yotadi.

ΔASE dan AE=SE·ctg60°=

$=4\sqrt{3}\cdot\frac{1}{\sqrt{3}}=4$; AD=2AE=8.

To’g’ri javob: B



90(2003-10.63). Ikki vektor yig’indisining uzunligi 20 ga, shu vektorlar ayirmasining uzunligi 12 ga teng. Shu vektorlarning skalyar ko’paytmasini toping.

A) 16 B) 48 C) 24 D) 64 E) 32

$\left|\vec{a}+\vec{b}\right|=20$; $\left|\vec{a}-\vec{b}\right|=12$ larning har birini kvadratga ko’taramiz:

$$\vec{a}^2+2\vec{a}\vec{b}+\vec{b}^2=400 \quad (1)$$
$$\vec{a}^2-2\vec{a}\vec{b}+\vec{b}^2=144 \quad (2)$$

(1) dan (2) ni hadlab ayiramiz: $4\vec{a}\vec{b}=256$; $\vec{a}\vec{b}=64$

To’g’ri javob: D.

91(2003-12.31). Parallelogrammning diagonali $8\sqrt{2}$ ga teng. Shu parallelogrammga ichki va tashqi aylanalar chizish mumkin bo’lsa, parallelogrammning yuzini toping.

A) berilganlar yetarli emas B) 32 C) 64 D) 128 E) 256

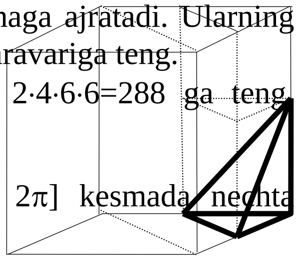
Tashqi chizilgan to’rtburchakning qarama-qarshi tomonlari yig’indisi teng. Agar u parallelogramm bo’lsa, bu parallelogramm romb bo’ladi. Ichki chizilgan to’rtburchak qarama-qarshi burchaklari yig’indisi 180° ga teng bo’ladi. Agar bu to’rtburchak romb bo’lsa, u holda bu romb kvadrat

bo’ladi. Diagonali $8\sqrt{2}$ ga teng bo’lgan kvadratning tomoni 8 ga, yuzi esa 64 ga teng bo’ladi. To’g’ri javob: C.

92(2003-12.41). Parallelepipedning bir uchidan chiquvchi uchta qirrasining o’rtalari orqali o’tkazilgan tekislik undan hajmi 6 ga teng piramida kesib ajratadi. Parallelepipedning hajmini toping.

A) 120 B) 144 C) 180 D) 288 E) 276

Hosil bo’lgan piramidani uchburchakli prizmaga to’ldiramiz. Bu prizma hajmi piramida hajmining 3 baravariga teng. Asosi piramida asosiga teng va Balandligi parallelepipedning Balandligiga teng uchburchakli prizma hajmi piramida hajmining 6 baravariga tengligi shakldan ko’rinib turibdi. Parallelepiped diagonali orqali o’tgan tekislik uni ikkita uchburchakli prizmaga ajratadi. Ularning har birining hajmi avvalgi prizma hajmining 2^2=4 baravariga teng. Shunday qilib, berilgan parallelepipedning hajmi 2·4·6·6=288 ga teng. To’g’ri javob: D.



93(2003-5.43). $\frac{|\cos x|}{\cos x} = \cos 2x - 1$ tenglama [π; 2π] kesmada nechta ildizga ega?

A) 1 B) 2 C) 3 D) 4 E) ∅

cosx ning qiymati [π; 1,5π) oraliqda manfiy bo’lgani uchun, bu oraliqda berilgan tenglama -1=cos2x-1 ko’rinishda bo’ladi. Bundan cos2x=0; x=π/4+πn/2 (Bulardan 5π/4 berilgan oraliqqa tegishli)

cosx ninq qiymati (1,5π; 2π] oraliqda musbat bo’lgani uchun, bu oraliqda berilgan tenglama 1=cos2x-1 ko’rinishda bo’ladi. Bundan cos2x=2. bu tenglama yechimga ega emas. To’g’ri javob: A.

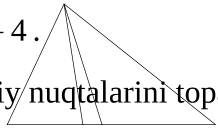
94(2003-9.44). A(1;4) nuqtadan y=-2-2/x funksiya grafigiga ikkita urinma o’tkazilgan. Urinish nuqtalari abssissalarining yg’indisini toping.

A) –1 B) 1 C) 1/3 D) 2/3 E) –2/3

A(1;4) nuqtadan o’tadigan to’g’ri chiziq tenglamasi y=kx+b ko`rinishda bo’lsin. Bunga A nuqtaning koordinatalarini qo’yib, topamiz: 4=k·1+b; b=4-k. bu to’g’ri chiziq urinma bo’lgani uchun k=y′ bo’lishi kerak. $y'=2/x^2$.

b va y ning ifodalarini to'g'ri chiziq tenglamasiga qo'yamiz:

$$y=\frac{2}{x^2}\cdot x+4-\frac{2}{x^2}=-\frac{2}{x^2}+\frac{2}{x}+4.$$



Bu chiziq bilan berilgan funksiya grafigining umumiy nuqtalarini topamiz:

$$\begin{cases} y=-2-\frac{2}{x} \\ y=-\frac{2}{x^2}+\frac{2}{x}+4 \end{cases}$$ bundan $-\frac{2}{x^2}+\frac{2}{x}+4=-2-\frac{2}{x}$. Bu tenglamani yechib, $x_1=-1$ va $x_2=1/3$ ga ega bo'lamiz. $x_1+x_2=-2/3$.

To'g'ri javob: E.

95(2003-11.20) y=3-|x-3| funksiya grafigi va OX o'qi bilan chegaralangan shaklning yuzini toping.

A) 9 B) 8 C) 12 D) 6 E) 10

Modulning ta'rifidan foydalanib, bu funksiyani quyidagicha yozish mumkin: y=$\begin{cases} x; x\le 3 \\ -x+6; x\ge 3 \end{cases}$

shunga ko'ra, grafikni x≤3 va x≥3 oraliqlarda chizamiz.

3

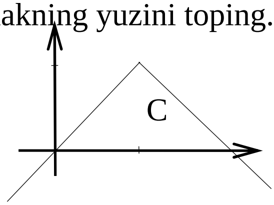
S=$\frac{1}{2}\cdot 6\cdot 3=9$

3 6

To'g'ri javob: A.

96(2003-1.40). To'g'ri burchakli ACB uchburchakning katetlari 8 ga va 10 ga teng. Shu uchburchakning C –to'g'ri burchagi uchidan CE-mediana va CD-bissektrisa o'tkazilgan. CDE uchburchakning yuzini toping.

A) $2\frac{2}{9}$ B) $2\frac{2}{7}$ C) $2\frac{3}{8}$ D) $3\frac{2}{5}$ E) $3\frac{2}{3}$



$S_{ACE}=S_{\Delta}/2=(8\cdot10/2)/2=20$

AB=$\sqrt{8^2+10^2}$ =2$\sqrt{41}$

h_c=80/2$\sqrt{41}$ =40/$\sqrt{41}$

Bissektrisa xossasidan AD:DB=AC:BC

AD:BD=8:10=4:5

AD=4x; BD=5x; 4x+5x=2$\sqrt{41}$; x=2$\sqrt{41}$/9; AD=8$\sqrt{41}$/9;

S_{ACD}=(8$\sqrt{41}$/9)(40/$\sqrt{41}$)/2=160/9.

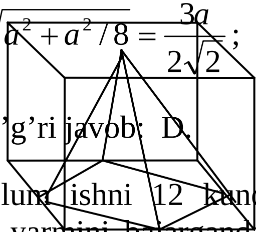
S_{DCE}=S_{ADE}-S_{ACD}=20-160/9=20/9=$2\frac{2}{9}$. To'g'ri javob: A.

97(2003-7.83). Kubning ostki asosidagi tomonlarining o'rtalarini ketma-ket tutashtirildi. Hosil bo'lgan to'rtburchakning uchlari kub ustki asosining markazi bilan tutashtirildi.
Agar kubning qirrasi *a* ga teng bo'lsa, hosil bo'lgan piramidaning to'la sirtini toping.

A) $2a^2/3$ B) $3a^2$ C) $1{,}5a^2$ D) $2a^2$ E) $\frac{2a^2\sqrt{3}}{3}$

Hosil bo'lgan piramidaning asosi kvadrat bo'lib, uning diagonali kub qirrasiga teng.
Shuning uchun piramida asosining yuzi $S_{asos}=a^2/2$.

Asos tomoni $b=a/\sqrt{2}$. Piramidaning apofemasi $h_a=\sqrt{a^2+a^2/8}=\frac{3a}{2\sqrt{2}}$;



$S_{yon}=4\cdot\frac{1}{2}\cdot\frac{a}{\sqrt{2}}\cdot\frac{3a}{2\sqrt{2}}=\frac{3a^2}{2}$ $S_t=\frac{a^2}{2}+\frac{3a^2}{2}=2a^2$. To'g'ri javob: D.

98(2003-9.7). Ikkita ishchi birgalikda ishlab, ma'lum ishni 12 kunda tamomlaydi. Agar ishchilarning bittasi shu ishning yarmini bajargandan keyin, ikkinchi ishchi qolgan yarmini bajarsa, shu ishni 25 kunda tamomlashlari mumkin. Ishchilardan biri boshqasiga qaraganda necha marta tez ishlaydi?

A) 1,2 B) 1,5 C) 1,6 D) 1,8 E) 2

1-ishchi yolg'iz o'zi butun ishni *x* kunda, 2-si esa *y* kunda tamomlaydi.

$$\begin{cases}\frac{1}{x}+\frac{1}{y}=\frac{1}{12}\\ 0{,}5x+0{,}5y=25\end{cases}$$ bu sistemani yechsak, *x*=20; *y*=30

30/20=1,5 marta tezroq ishlaydi. To'g'ri javob: B.

99(2003-11.4). $\frac{x-1}{x}+\frac{x-2}{x}+\frac{x-3}{x}+\ldots+\frac{1}{x}=4$ tenglamaning ildizi 10 dan nechta kam?

A) 1 B) 2 C) 3 D) 4 E) 5

Kasrlarning surati birinchi hadi x-1 ga, ayirmasi -1 ga teng bo’lgan arifmetik progressiya tashkil etadi.

S= $\frac{(x-1)+1}{2}\cdot(x-1)=\frac{x(x-1)}{2}$ bo’lgani uchun, berilgan tenglamani $\frac{x(x-1)}{2x}=4$ ko’rinishda yozish mumkin.

Bu tenglamani yechib, x=9 ga ega bo’lamiz. U 10 dan 1 ta kam.

To’g’ri javob: A.

100(2003-9.17). $\frac{2\cdot 7^x}{7^{2x}-1}\geq\frac{7^x}{7^x-1}-\frac{7^x}{7^x+1}$ tengsizlikni yeching.

A) (0;∞) B) (-∞;0) C) (-∞;0] D) (-1; 1) E) (1; ∞)

$$\frac{2\cdot 7^x}{7^{2x}-1}-\frac{7^x}{7^x-1}+\frac{7^x}{7^x+1}\geq 0$$

$$\frac{2\cdot 7^x-7^x(7^x+1)+7^x(7^x-1)}{7^{2x}-1}\geq 0$$

$$\frac{1}{7^{2x}-1}\geq 0$$

$$7^{2x}-1>0$$

$$7^{2x}>1$$

$$2x>0$$

$$x>0$$

To’g’ri javob: A.