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darajasini olish uchun

Salimova Nodira Namoz qizining

“Integro-differensial tenglamalar va ularni yechish usullari’’

mavzusida

BITIRUV MALAKAVIY ISHI

“Ish ko’rildi va himoyaga ruxsat berildi’’ Ilmiy rahbar___ Durdiyev

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«Himoya qilishga ruxsat berildi »

Fakultet dekani ______prof. Sh.M. Mirzayev

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Mundarija:

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Kirish.

Har qanday millatning ravnaqi,

umumbashariyat tarixida tutgan

o’rni, mavqei va shuhrati bevosita

o’z farzandlarining aqliy va jismoniy

yetukligiga bo’q’liqdir.

I.A.Karimov

Bugun jamiyatimizning taraqqiyotini malakali kadr, salohiyatli va yetuk mutaxassislarsiz tasavvur etib bo’lmaydi. Shu bois prezidentimiz Islom Karimov tashabbusi bilan mamlakatimizda barkamol avlodni tarbiyalash eng ustivor vazifalardan biriga aylandi. Zamonaviy bilim va malakaga ega kadrlar tayyorlashga davlat siyosati darajasidagi vazifa sifatida qaralmoqda “Ta’lim to’g’risida”gi Qonun, Kadrlar tayyorlash milliy dasturi bu boradagi ishlarni butunlay yangi bosqichga olib chiqdi. Mamlakatimizda sog’lom va barkamol avlodni tarbiyalash, yoshlarning o’z ijodi va intelektual salohiyatini ro’yobga chiqarish, mamlakatimiz yigit-qizlarini XXI asr talabalariga to’liq javob beradigan har tomonlama rivojlangan shaxslar etib voyaga yetkazish uchun zarur shart-sharoitlar va imkoniyatlarni yaratish bo’yicha keng ko’lamli aniq yo’naltirilgan chora tadbirlarni amalga oshirish maqsadida keng ko’lamli ishlar olib borilmoqda.

Tayyorlanayotgan mutaxassislarga real iqtisodiyot tarmoqlari va sohalardagi mavjud talabga alohida e’tibor qaratilgan holda o’sib kelayotgan yosh avlodga ta’lim va tarbiya berish sohasidagi moddiy texnika bazani yanada mustahkamlash undan oqilona va samarali foydalanishni ta’minlash, davlat ta’lim standartlari, o’quv dasturlari va o’quv-uslubiy adabiyotlarni takomillashtirish;

ta’lim jarayoniga yangi axbarot-kammunikatsiya va pedagogik texnalogiyalarni,elektron darsliklar,multimediya vositalarni keng joriy etish orqali mamlakatimiz maktablarida,kasb-huhar kollejlarida,litseylari va oily o’quv yurtlarda o’qish sifatini tubdan yaxshilash,talim muassasalarning o’quv labaratorya bazasini zamonaviy turdagi o’quv va labaratorya uskunalari,kompyuter texnikasi bilan mustahkamlash, shuningdek, o’qituvchilar va murabbiylar mehnatini moddiy hamda ma’naviy rag’batlantirish bo’yicha samarali tizimni yanada rivojlantirish;

zamonaviy axborot va kammunikatsiya texnologiyalari raqamli va keng farmatli telekammunikatsiya aloqa vositalari hamda internet tizimini yanada rivojlantirish ,ularni har bir oila hayotida joriy etish va keng o’zlashtirish;

yosh avlodni jismonan barkamol etib tarbiyalash,bolalar sportni rivojlantirish sohasida yoshlarni, ayniqsa qishloq qizlarni sport bilan muntazam shug’ullanishga keng jalb qilish yangi sport majmualarni stadion va inshootlarni qurish,ularni zamonaviy sport anjomlari va jihozlari bilan ta’minlash, yuqori malakali ustoz va murabbiylar bilan mustahkamlash bo’yicha amalga oshirilayotgan ishlarni izchil kuchaytirish;

iqtisodiyotni tarkibiy o’zgartirishning muhim yo’nalishi , aholi va o’rta sinf mulkdorlari daromadlarini shakllantirish asosi bo’lgan kichik biznes hamda xususiy tadbirkorlikni rivojlantirishni yanada rag’batlantirish,bu sohadagi mavjud muammolarni hal etish, yoshlar avvalambor, kasb hunar kollejlari va oliy ta’lim muassasalari bitiruvchilarni ayniqsa, qishloq joylarda tadbirkorlik faoliyatiga keng jalb etish uchun sharoit yaratish;

ilm-fanni yanada rivojlantirish, iqtidorli va qobilyatli yoshlarni ilmiy faoliyatiga keng jalb etish ularning o’z ijodiy va intelektual salohiyatini ro’yobga chiqarish uchun sharoit yaratishga doir kompleks chora tadbirlarni ishlab chiqish;

Yosh avlodga g'amxo'rlik qilish ishlarini kuchaytirish,ularni huquqiy va ijtimoiy muhofaza qilishni ta'minlash jismonan va har tamonlama rivojlangan barkamol avlodni milliy va umuminsoniy qadriyatlar hamda vatanga muhabbat ruhida tarbiyalash borasida jamiyatning muhim bo'g'ini bo'lgan sog'lom va mustahkam oilani shakllantirish uchun zarur shart-sharoitlarni yaratish;

yoshlar o'rtasida sog'lom turmush tarzini qaror toptirish, ularni ichkilik va giyohvandlik illatlardan,boshqa turli halokatli tahlidlar hamda biz uchun yod bo'lgan diniy va ekstrimik tasirlardan,tuban “ommaviy madaniyat” xurujlardan himoya qilishga doir kompleks chora tadbirlarni amalga oshirish.

Hozirgi kunda insoniyatning oldida ko'plab og'riqli muammolar turibdi. Bunga misol sifatida eng – dolzarb muammo – 2008-yilgi moliyaviy inqiroz tufayli yuzaga kelgan vaziyatni keltirish mumkin g'arbning rivojlangan mamlakatlarida necha yillardan beri faxrlanib kelgan iqtisodiy barkamollik bir necha kunda inqirozga yuz tutdi. Buning natijasida yer yuzida ko'plab kishilar ishsiz qoldi. Katta-katta kompaniyalar esa o'z faoliyatini tugatishga yoki qisqartirishga majbur bo'lmoqda xo'sh, bu muammoning yechimi qanday topiladi? Albatta, bunday paytda birinchi tayanch nuqtasi iqtisodchi olimlarga qaratiladi.Chunki ularning tog'ri ishlab chiqqan siyosati vaziyatni to'g'rilashga yordam beradi.Iqtisodchilar esa to'g'ri qarorlarni qabul qilish uchun o'z navbatida matematik qonuniyatlar va modellarga suyanadilar. Bu birgina misol matematika fanining naqadar muhimligini anglashimizga kifoya qiladi. Bunday misollarni minglab keltirish mumkin.Nafaqat iqtisodiyot balki boshqa barcha sohalarda ham matematika katta ahamiyatga ega. Galiley tabiri bilan aytadigan bo'lsak “ Tabiatning buyuk kitobi matematika tilida yozilgan” .Bu fikrni tasdiqlash uchun uzoqqa bormaylik, tanangizdagi hujayralarni faoliyatini o'rganishni ham matematik hisoblashlarsiz amalga oshirib bo'lmaydi. Hujayra faoliyatining bir me'yorda amalga oshishi va undagi himoyaviy jarayonlar xuddi matematik tenglama kabi doim muvozanatda . Agar shu muvozanat buzilsa, demak tanangiz biror kasallikda duchor bo’gan . Kasallikga to’g’ri diagnoz qo’yish uchun esa, o’xshatish bilan aytadigan bo’lsak, tenglamadagi muozanatni buzgan x noma’lumni topish kerak bo’ladi. Ba’zi kasalliklarning shifosini topish ham global, ya’ni butun insoniyatni o’ylantiradigan muammo hisoblanadi.

Insoniyat oldida yechimi topilishini kutub turgan yana ko’plab muammolar mavjud . Bularga ekologik muammolar, terrorizm, davlatlar o’rtasidagi kelishmovchiliklar va ko’plab boshqa muammolar kiradi. Bu ham yetmagandek insoniyat koinotni o’rganishga ham jon-jahti bilan kirishgan. Har yili kosmik tadqiqotlarga milliardlab dollorlar sarflanmoqda xo’sh, yuqoridagi muammolarni bartaraf qilish va insoniyat o’z oldiga qo’ygan maqsadlarini amalga oshirishni matematik, fanisiz tasavur qilish mumkinmi? Yo’q albatta!

Endi bevosita davlatlarning rivoji uchun matematika faninig ahamiyatini baholashga harakat qilib ko’raylik. Biz millat sifatida o’z oldimizga eng buyuk va eng ezgu maqsadni qo’ydik.O’zbek davlati nafaqat rivojlanib dunyoda o’z o’rnini topish, balki dunyodagi ma’naviyat bilan bezangan hayot qanday ekanligini ko’rsatib qoymoq lozim, chunki boy davlatga aylanish unchalik qiyin ish emas. Boy bo’lib turib, ma’naviy boylikga erishish juda mushkul vazifa . Bunga erishish uchun esa , kelajak uchun har tomonlama mukammal rejalarni ishlab chiqadigan, ma’naviy jihatdan yetuk mutaxassislar kerak. Mukammal rejalar, barchaga ma’lumki, matematik hisoblashlar orqali ishlab chiqiladi.

Bitiruv malakaviy ishning dolzarbligi: Bitiruv malakaviy ishim matematik- fizika fanida muhim o’rin tutgan bo’lib, differensial va integral tenglamalar va integro- differensial tenglamalarni yechish usullariga bag’ishlangan. Talabalar bunday masalalarni yechishda qator qiyinchiliklarga duch keladilar. Tekshirishlar shuni ko’rsatadiki, integral tenglamani yechishga nisbatan, unga mos differensial tenglamani yechish, umuman olganda ancha murakkab. Ammo masalani tahlil qilish vaqtida ba’zi savollarni oydinlashtirish uchun berilgan integral tenglamani differensial tenglamaga aylantirish zarur bo’lib qolishi ehtimoldan holi emas.

Tadqiqot obyekti: Integro- differensial tenglamalarni yechish usullarini o’rganish.

Tadqiqot predmeti: Differensial tenglamalar, Integral tenglamalar, Koshi formulasi, bir argumentli va ikki argumentli funksiyalar uchun integro- differensial tenglamalarni ketma- ket yaqinlashish usuli bilan yechish.

Bitiruv malakaviy ishning asosiy maqsadi: Integro- differensial tenglamalarni yechish usullarini o’rganish.

Bitiruv malakaviy ishning asosiy vazifalari: Bir argumentli va ikki argumentli funksiyalar uchun integro-differensial tenglamalarni yechish usullarini o’rganish.

Tadqiqot usuli va uslubiyati: Bir argumentli va ikki argumentli integro- differensial tenglamalarni ketma- ket yaqinlashish usuli bilan yechish.

Olingan asosiy natijalar: Bir argumentli va ikki argumentli funksiyalarni yechish usullari o’rganildi.

Natijalarning ilmiy yangiligi va amaliy ahamiyati: Fizikaning ko’pgina masalalari integro- differensial tenglamaga keltiriladi. Shu jihatdan bu mavzu analiz ahamiyatga ega.

Tadbiq etish darajasi va iqtisodiy samaradorligi: Matematika va fizika fanini o’rganishda , fizikaviy masalalarni o’rganishda qo’llaniladi.

Ishning hajmi va tuzilishi: Bitiruv malakaviy ishim: kirish qism, ikkita bob, har bir bobda ikkitadan to’rtta paragraf, har bir bobning xulosasi, xotima va foydalanilgan adabiyotlar ro’yxati. Ishning hajmi 61 betdan iborat.

Metodologik asosi: Integro- differensial tenglamalarni yechish usullari.

I.Oddiy differensial tenglamalar va integral tenglamalar.

I. 1. Differensial tenglamalar haqida umumiy tushunchalar.

1.1.1-ta'rif. OddiyDifferensial tenglama deb, o'zgaruvchi x, noma'lum $y = f(x)$ funksiya va uning $y', y'', \dots, y^{(n)}$ hosilalari orasidagi bog'lanishni ifodalaydigan tenglamaga aytiladi. Differensial tenglama umumiy holda quyidagicha yoziladi:

$$F(x, y, y', y'', \dots, y^{(n)}) = 0$$

yoki

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots, \frac{d^ny}{dx^n}\right) = 0.$$

izlanayotgan funksiya $y = f(x)$ bitta erkli o'zgaruvchining funksiyasi bo'lgani sababli differensial tenglama $oddiy\ differensial\ tenglama$ deyiladi.

Umuman, noma'lum funksiya ko'p argumentli bo'lgan hollar ham tez-tez uchraydi. Bunday holda differensial tenglama $xususiy\ hosilali\ differensial\ tenglama$ deb ataladi.

1.1.2-ta'rif. Differensial tenglamaning tartibi deb, tenglamada qatnashgan hasilaning eng yuqori tartibiga aytiladi.

Masalan, $(y')^2 + 2y' + xy^3 = 0$ tenglama birinchi tartibli differensial tenglamadir.

Mana bu $\left(y''\right)^2 + ay' + by + \cos x = 0$ tenglama esa ikkinchi tartibli differensial tenglama.

1.1.3-ta'rif. Differensial tenglamaning yechimi yoki integrali deb, differensial tenglamaga qo'yganda uni ayniyatga aylantiradigan har qanday $y = f(x)$ funksiyaga aytiladi.

Biz birinchi tartibli hosilaga nisbatan yechilmagan oddiy differensial tenglamalarni ko'ramiz:

$$F(x, y, y') = 0 \qquad \textbf{(1.1.1)}$$

Bunda x-erkli o'zgaruvchi, y-uning noma'lum funksiyasi $, y' = \frac{dy}{dx}$ esa noma'lum funksiyaning hosilasi.

(1.1.1) tenglamaning muhim xususiy hosilasiga to'xtalamiz.

$$\frac{dy}{dx} = f(x, y) \qquad \textbf{(1.1.1')}$$

bu tenglamaga hosilaga nisbatan yechilgan oddiy diferensial tenglama deyiladi. (1.1.1')tenglama(1.1.1')tenglamani y` ga nisbatan yechish natijasida hosil bo'lgan deb qaramasdan, balki (1.1.1')ga f(x,y) funksiya Γ sohada berilgan deb qaraymiz.

1.1.1-izoh. Soha deyilganda faqat yopiq yoki faqat ochiq bog'langan to'plamni olamiz. Agar berilgan Γ to'plamning ixtiyoriy ikki nuqtasini tutashtiruvchi va shu to'plamga tegishli biror chiziq mavjud bo'lsa,u holda Γ to'plam bog'langan bo'ladi.

1.1.2-izoh. Agar I intervalda yopiq bo'lsa u holda uning chap uchiga o'ng hosila, o'ng uchiga esa chap hosila nazarda tutiladi.

1.1.4-ta'rif .(1.1.1) tenglama berilgan bo'lib, unda f(x,y) funktsiya R^2 tekislikning Γ sohasida aniqlangan bo'lsin. Agar I (ochiq,yopiq yoki yarim ochiq) intervalda aniqlangan $\varphi(x)$ funksiya uchun quyidagi uch shart

$$\left.\begin{aligned} &1^\circ \;\; (\mathrm{x}, \; \varphi(\mathrm{x})) \in \Gamma, \Gamma \subset R^2, x \in I \\ &2^\circ \;\; \varphi(\mathrm{x}) \in C^1(I) \\ &3^\circ \;\; \frac{\alpha\varphi(\mathrm{x})}{\alpha x} = f(x, \varphi(x)), x \in I \end{aligned}\right\} \qquad \textbf{(1.1.2)}$$

bajarilsa, u holda bu funksiya I intervalda (1.1.1')differensial tenglamaning yechimi deyiladi. (1.1.1)differensial tenglamaning har bir $y = \varphi(x)$ yechimga mos kelgan egri chiziq (ya'ni $y = \varphi(x)$ funksiyaning grafigi) shu tenglamaning integral egri chizig'i deyiladi. (1.1.1')Tenglamaning yechimi ba'zi hollarda oshkormas

F(x,y)=0 ko'rinishda bo'lsa, ba'zi hollarda parametrik $x = x(t),\ \ y = y(t),\ \ t_\circ < t < t_1\ \ x'(t) \neq 0$ ko'rinishda bo'lishi mumkin.

Koshi masalasining qo'yilishi.

(1.1.$1'$)tenglama berilgan bo'lib unda f(x,y) funksiya R^2 tekislikning Γsohasida aniqlangan, uzluksiz va I interval x o'qidagi interval bo'lsin, x_0 ni o'z ichiga oladigan I intervalni va shu I intervalda aniqlangan uzluksiz differensiallanuvchi hamda ushbu

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$$\left.\begin{array}{l} 1^\circ\ \ (x,\ \varphi(x)) \in \Gamma, \qquad x \in I \\ 2^\circ\ \ \varphi'(x) = f(x, \varphi(x)) \qquad (\in I) \\ 3^\circ\ \ \varphi(x_\circ) = y_\circ\,,\ (x_\circ, y_\circ) \in \Gamma \end{array}\right\} \qquad \textbf{(1.1.3)}$$

shartlarni qanoatlantiruvchi $y = \varphi(x)$ funksiyani topish talab etiladi.

Bu masala qisqacha $y' = f(x, y),\ \ y(x_\circ) = y_\circ$ kabi yoziladi va(1.1.$1'$)tenglama uchun Koshi masalasi (yoki boshlang'ich masala) deyiladi.

3-shartni qanoatlantiruvchi $y = \varphi(x)$ funksiya I intervalda (k) Koshi masalasini yechimi deyiladi. Endi Γ sohaning (k) masalasi yagona yechimga ega bo'ladigan (x,y) nuqtalaridan tuzilgan kesmini $D_2^* \subset \Gamma\ (D_2 \equiv \Gamma)$ deb belgilaylik. Shunga ko'ra D_2^* to'plamning har bir (x,y) nuqtasida (1.1.$1'$)tenglamaning yagona integral chizig'i o'tadi.

1.1.5-ta'rif.(1.1.$1'$)differensial tenglama x,c o'zgaruvchilarning biror o'zgarish sohasida aniqlangan hamda x bo'yicha uzluksiz differensiallanuvchi

$$y = \varphi(x, c) \qquad \textbf{(1.1.4)}$$

funksiya berilgan bo'lsin. Agar $\forall\ (x, y) \in D_2^*$ nuqta uchun (1.1.4) munosabat c ning $c = \psi(x, y)$ (1.1.4`) qiymatini bir qiymatli aniqlasa va bu qiymatni ushbu $\frac{dy}{dx} = \varphi_x^`(x, c)$ (1.1.4) tenglikka qo'yish natijasida (1.1.$1'$)tenglama hosil bo'lsa, u holda (1.1.4) funksiya (1.1.$1'$)tenglamaning D_2 to'plamda aniqlangan umumiy yechimi deyiladi.

1.1.6-ta'rif.(1.1.$1'$)tenglama va (1.1.4) chiziqlar oilasi berilgan bo'lsin.

Agar

1) f(x,c) funksiya I intervalda x bo'yicha uzluksiz hosilaga ega bo'lsa;

2) Har bir $(x, y) \in D_2^*$ nuqta uchun (1.1.4) munosabat c ning (1.1.4`) qiymatini bir qiymatli aniqlasa;

3) $y = \varphi(x, \psi(x, y))$funksiya(1.1.$1'$)tenglamaning yechimi bo'lsa, uholda (1.1.4) funksiya (1.1.$1'$)tenglamaning umumiy yechimi deyiladi.

Har bir nuqtasida Koshi masalasi yagona yechimga ega bo'ladigan yechim xususiy yechim deyiladi, (1.1.$1'$) tenglamaning barcha yechimlarini topish asosiy masala hisoblanadi.Barcha yechimlarini topish jarayoni differensial tenglamani integrallash deyiladi.Agar (1.1.$1'$) chi tenglamaning yechimini elementar funksiyalar va ularning integrallari yordamida yozish mumkin bo'lsa, u holda differensial tenglama kvadraturalarda integrallanadi deyiladi.

$D = D_2 / D_2^*$ to'plamning har bir (x,y) nuqtasidan o'tadigan integral chiziqlar yagona emasligi kelib chiqadi. Har bir nuqtasidan yechimning yagonaligi buziladigan yechimlar maxsus yechimlar deyiladi.Umumiy yechish formulasi (1.1.4) maxsus yechimlarni o'z ichiga olmaydi. Agar Φ(x,y,c)=0 munosabat D_2^* to'plamda $y = \varphi(x, c)$ umumiy yechimni aniqlasa, u holda (1.1.4```) ni (1.1.$1'$)differensial tenglamaning umumiy integrali deyiladi.

Masalan ; $y = ce^x$ chiziqlar oilasi berilgan bo'lsin. U holda $y' = ce^x$ izlangan differensial tenglama y`=y bo'ladi. Ravshanki, bu tenglamaning umumiy yechimi: $y = ce^x$

Agar umumiy yechim ma'lum bo'lmasa, Koshi masalasini yechish qiyinlashadi.Bunda differensial tenglama taqribiy integrallash metodlari yordamida yechiladi.

HOSILAGA NISBATAN YECHILMAGAN TENGLAMALAR.

Hosilaga nisbatan yechilmagan 1- tartibli oddiy differensial tenglamalar ushbu

$$F(x, y, y') = 0 \qquad \textbf{(1.1.5)}$$

ko'rinishda yoziladi. Bu yerda F uch argumentli funksiya bo'lib, uch o'lchovli fazoning ochiq D_3 to'plamida (D_3 sohaga) aniqlangan. Agar bu to'plamni R^2tekisligiga ortogonal proeksiyalasak, R^2 ga biror ochiq $\Gamma$to'plam (Γ soha) hosil bo'ladi.

1.1.7-ta'rif.(1.1.5) differensial tenglama berilgan bo'lib, $F(x, y, y') = 0$ funksiya R^3 fazoning D_3 sohasida aniqlangan bo'lsin.

Agar I (ochiq,yopiq va yoki yarim ochiq) intervalda aniqlangan (x) funksiya uchun quyidagi uchta shart

$$\left.\begin{array}{l} 1^\circ \ (\mathrm{x}, \varphi(\mathrm{x})) \in \Gamma, \mathrm{x} \in \mathrm{I}, (\mathrm{x}, \varphi(\mathrm{x})), \varphi`(x) \in D_3, \Gamma \subset \mathrm{R}^2 \ \ D_3 \subset \mathrm{R}^3 \\ 2^\circ \ \varphi(x) \in \mathrm{C}^1(\mathrm{I}) \\ 3^\circ \ \mathrm{F}(\mathrm{x}, \varphi(x)), \ \varphi`(x) \equiv 0, \mathrm{x} \in \mathrm{I} \end{array}\right\} \qquad \textbf{(1.1.6)}$$

bajarilsa, bu funksiya I intervalda (1.1.5) differensial tenglamaning yechimi deyiladi. (1.1.5) tenglamaning yechimiga mos egri chiziq, uning integral egri chizig'i deyiladi.

Agar parametrik ko'rinishda berilgan $x = x(t),\ \mathrm{y} = y(t),\ \ \mathrm{t} \in \mathrm{I_t}$ ($\mathrm{I_t}$ parametr t ning o'zgarish sohasi yopiq, ochiq, yarim ochiq intervaldan iborat) funksiya uchun $x'(t) \neq 0,\ \ t \in I_t$ bo'lib, quyidagi uchta shart

1° (x(t),y(t))∈ Γ, (x(t),y(t)), $\frac{y'(t)}{x'(t)}$ ∈D_3, t∈I_t;

2°y(t)∈$C^1(I_t)$, (x(t)∈$C^1(I_t)$);

3° F(x(t),y(t), $\frac{y'(t)}{x'(t)}$)=0, t∈I_t

bajarilsa u holda x=x(t), y=y(t) funksiya I_t intervalda (1.1.5) differensial tenglamaning yechimi deyiladi. Ba'zi hollarda yechimni shu ko'rinishida yozish yoki izlash qulay bo'ladi.

(1.1.5) differensial tenglama uchun ham (1.1.1′)differensial tenglama uchun aytilganidek yechim uch : $x = x(t),\ y = y(t),\ (t \in I_t);\ y = f(x);\ \Phi(x, y) = 0$ ko'rinishdan bittasi orqali izlanadi.

(1.1.5) differensial tenglama ochiq Г to'plamning har bir (x,y) nuqtasida y` ning bitta yoki bir necha qiymatlarini aniqlasin deylik. Har bir (x,y) nuqtada y` dan foydalanib, bitta yoki bir necha birlik vektor chizamiz. Natijada yo'nalishlar maydoni hosil bo'ladi.

Umumiy yechim tushunchasini kiritishdan avval (1.1.5) tenglama uchun Koshi masalasini qo'llaymiz.

Koshi masalasi.

(1.1.5) differensial tenglamaning $y(x_0) = y_0,\ (x_0, y_0) \in \Gamma$ boshlang'ich shartni qanoatlantiruvchi yechimi topilsin yoki geometrik nuqtai nazardan (1.1.5) differensial tenglamaning $(x_0, y_0) \in \Gamma$ nuqtadan o'tuvchi integral chizig'i ko'rsatilsin. (1.1.5) differensial tenglama y` ga nisbatan yechilishi mumkin deylik.U holda $(x_0, y_0)$ nuqtaning biror atrofida y` uchun bir necha haqiqiy qiymatlarni topamiz.

$$y` = f_k(x, y),\ k = 1,2...,m \qquad \textbf{(1.1.7)}$$

Agar har bir $f_k(x, y),\ k = 1,2...,m$ funksiya biror mavjudlik va yagonalik teoremasining shartlarini qanoatlantirsa, u holda (x_0, y_0) nuqtadan (1.1.5) differensial tenglamaning m ta integral chizig'i o'tadi. Ba'zan $f_{k_1}, f_{k_2}, ..., f_{k_{2n}}$ $(f_{k_{2n}} \le M)$funksiyalar kompleks bo'lsa , u holda biz faqat $f_{k_{2n+1}}, ..., f_n$ holda (x_0, y_0) nuqtadan tegishli differensial tenglamaning m-k_{2n} ta integral chizig'i o'tadi.

Agar (1.1.5) differensial tenglamaning haqiqiy $f_1(x,y),..,f_k(x,y)$ $(\mathrm{k} \le \mathrm{m})$ funksiyalarga mos kelgan va (x_0, y_0) nuqtada uning integral chiziqlariga o'tkazilgan urunmalar turli burchak koeffisientlariga ega bo'lsa,u holda Koshi masalasi yagona yechimga ega deyiladi.

1.1.8-ta'rif.(1.1.5) differensial tenglama (x_0, y_0)nuqtaning biror atrofida y` ga nisbatan yechilishi mumkin , ya'ni (1.1.7) tenglamalarga ajraladi deylik.

Agar har bir (1.1.7) tenglama

$$y = f_k(x,c), \ \mathrm{k} = 1,2...,\mathrm{m} \qquad \textbf{(1.1.8)}$$

umumiy yechimga yoki $\Phi_k(x,y) = c, \ \mathrm{k} = 1,2...,\mathrm{m}$ c- ixtiyoriy o'zgarmas (1.1.8) umumiy integralga ega bo'lsa, u holda (1.1.7) umumiy yechimlar to'plami berilgan (1.1.5) differensial tenglamaning umumiy yechimi deyiladi.

1.1.6-ta'rif. Agar (1.1.5) tenglamaning biror I intervalda aniqlangan $y = \varphi(x)$ yechimning har bir nuqtasida Koshi masalasi yechimga ega bo'lsa, u holda $y = \varphi(x)\ x \in I$ yechim berilgan tenglamaning xususiy yechimi deyiladi.

Yuqoridagi ta'riflar munosabati bilan maxsus yechim tushunchasini kiritish lozim bo'ladi.

1.1.7-ta'rif. Agar $y = \varphi(x)$ funksiya biror I intervalda (1.1.5) differensial tenglamaning yechimi bo'lib, uning har bir nuqtasida yagona yechimga ega (yagonalik xossaga ega) bo'lmasa, yani uning har bir nuqtasidan bir xil yo'nalishda kamida ikkita integral chiziq o'tsa, u holda $y = \varphi(x)$ funksiya (1.1.5) tenglamaning o'sha intervalda aniqlangan maxsus yechimi deyiladi.

Misol

.$(y`)^3 = y^2, D_3 = \{(x,y,y`): -\infty < x < +\infty, \ -\infty < y < +\infty, \ 0 \le y` < +\infty\}$differensial

tenglamani $y` = y^{2/3}$ ko'rinishda yozish mumkin.

Ma'lumki, absissa o'qi (ya'ni y=0 chiziq) va $y = \frac{(x+1)^3}{27}$ kubik parabolalar bu tenglama uchun integral chiziq bo'lib xizmat qiladi. Ammo y=0 chiziqning har bir nuqtasidan kamida ikkita integral chiziq o’tadi. Shuning uchun y=0 maxsus yechimdir.

Masalaning qo’yilishi.

(1.1.1′)differensial tenglama uchun Koshi masalasi ((1.1.1′),(1.1.3)) ning yechimi bormi yoki yo’qmi? Agar bunday yechim bor bo’lsa, ular nechta?Qachon Koshi masalasi yechimga ega emas?

Bu savolga javob beradigan teoremalar mavjudlik va yagonalik teoremalari deb ataladi.

1.1.1 –teorema. (Koshi teoremasi) Agar f(x,y) funksiya Γ sohada aniqlangan va uzluksiz bo’lib, uning y bo’yicha xususiy hosilasi $\frac{\partial f(x,y)}{\partial y}$ biror $Q(Q \subset \Gamma)$ sohada aniqlangan va uzluksiz bo’lsa , u holda.

1^0(1.1.1′)tenglamaning x_0 o’z ichiga oladigan biror intervalda aniqlangan va har bir berilgan $(x_0, y_0) \in Q$ nuqta uchun $y(x_0)= y_0$ boshlang’ich shartni qanoatlantiruvchi yechimi mavjud.

2^0Agar(1.1.1′)tenglamaning ikkita $y = \varphi(x)$ va $y = \psi(x)$yechimlari x_0 ga ustma – ust tushsa ya’ni $\varphi(x_0) = \psi(x_0) = y_0$ bo’lsa, u holda bu $y = \varphi(x),\ y = \psi(x)$ yechimlar aniqlanish sohalarining umumiy qismiga ustma – ust tushadi.

1.1.8-ta’rif. Agar f(x,y) funksiya Γsohada aniqlangan bo’lib, shu funksiya uchun shunday musbat L son mavjud bo’lsaki, $\forall (x, y_1) \in \Gamma, (x, y_2) \in \Gamma$ nuqtalar uchun $|f(x, y_1) - f(x, y_2)| \le L|y_1 - y_2|$ (L) tengsizlik bajarilsa, u holda f(x,y) funksiya Γsohada y bo’yicha Lipshist shartini qanoatlantiradi deyiladi, L esa Lipshis o’zgarmasi deyiladi.

1.1.2-teorema. (Koshi – Pikar – Lindelef teoremasi).

Agar f(x,y) funksiya Γ sohada x va y bo’yicha aniqlangan va uzluksiz bo’lib, Γ sohada y bo’yicha Lipshist shartini qanoatlantirsa, u holda shunday o’zgarmas h>0 son topiladiki, natijada (1.1.1′)tenglamaning $(x_0, y_0) \in \Gamma$ bo’lgan (1.1.3) boshlang'ich shartni qanoatlantiradigan va $I=\{x:|x-x_0|\le h\}$ yopiq intervalda aniqlangan yagona yechim mavjud bo'ladi.

1.1.3-teorema. (Peano teoremasi)

Agar f(x,y) funksiya Γ sohada aniqlangan va uzluksiz bo'lsa, u holda Γ sohaning berilgan $(x_0,y_0)\in\Gamma$ nuqtasidan (1.1.1′)tenglamaning kamida bitta integral chizig'i o'tadi.Yuqoridagi teoremalarning qo'llanilishiga doir misol ko'raylik.

Misol. $\begin{cases} y` = y^{2/3} \\ y(-2)=1 \end{cases}$

Koshi masalasida

$$\Gamma=R^2,\ \frac{\partial f(x,y)}{\partial y}=\frac{2}{3}y^{-1/3}$$

ga ko'ra $Q=Q_1\cup Q_2,\ Q_1=R^1\times R^1,\ Q_2=R^1\times R^1,\ Q\subset\Gamma$ ekani kelib chiqadi.

$\Gamma=Q\cup\{(x,y),y=0\}$ **va** $(-2,1)\in Q\subset\Gamma$ umumiy yechim $y=\left(\frac{x+c}{3}\right)^2$ $x=-2, y=1, c=5.$

Koshi masalasi yechimi $y=\left(\frac{x+c}{3}\right)^2$ bo'lib, bu yechim Q_2 ga yagonadir.

$$\begin{cases} y` = y^{2/3} \\ y(-2)=0 \quad \Gamma=R^2,\ (-2,0)\in\Gamma \quad \frac{\partial f}{\partial y}=\frac{2}{3}y^{-1/3} \end{cases}$$

(-2;0) nuqtada uzluksiz emas. Shuning uchun (-2;0) nuqtadan cheksiz ko'p integral chiziqlar o'tadi, (-2;0) nuqtadan $y=\left(\frac{x+2}{3}\right)^3$ y=0 integral chiziqlar o'tadi. Shuning uchun

$$(x)=\begin{cases} y=\left(\frac{x+2}{3}\right)^3, & x<-2 \\ 0, & x=2 \\ y=\left(\frac{x+k}{3}\right)^3, & x\geq -k,\ k>-2 \end{cases}$$

Funksiya berilgan tenglamaning R^2 ga aniqlangan yechimi bo'ladi.

Agar $y=\varphi(x),\ I_r=\{x: r_1<x<r_2\}$ ga $\psi(x)$ funksiya $I_S=\{x: S_1<x<S_2\}$ aniqlangan $x_0\in I_r\bigcap I_S$ uchun $\varphi(x_0)=\psi(x_0)$ bo'lsa, u holda $\varphi(x)\equiv\psi(x),\ x\in I_r\bigcap I_s$ bunda $I_r\neq I_s$. Agar $I_r\supset I_s$ bo'lsa, $\varphi(x),\ \psi(x)$ ning davomi deyiladi.

PIKAR TEOREMASI

Agar

$$\frac{dy}{dx}=f(x,y) \qquad \textbf{(1.1.9)}$$

tenglamada $f(x,y)$ funksiya

1^0 $D=\{(x,y)\ x_0-a\leq x\leq x_0+a,\ \ y_0-b\leq y\leq y_0+b\}$ to'g'ri to'rburchakda uzluksiz (demak unda chegaralangan, ya'ni $|f(x,y)|\leq M,\ M>0$) bo'lsa.

2^0 y bo'yicha Lipshist shartlarini qanoatlantirsa, u holda (9) tenglama

$$y(x_0)=y_0 \qquad \textbf{(1.1.10)}$$

shartni qanoatlantiradigan va $|x-x_0|\leq h,\ h=\min\left(a,\frac{b}{m}\right)$ intervalda aniqlangan yagona yechimga ega. Agar D to'plamning $\forall$ ikki (x, y_1) va (x, y_2) nuqtasi ushbu

$$|f(x,y_1)-f(x,y_2)|\leq L|y_1-y_2| \qquad \textbf{(1.1.11)}$$

tengsizlik o'rinli bo'lsa f(x,y) funksiya D da y bo'yicha Lipshist shartini qanoatlantiradi deyiladi, L esa Lipshist o'zgarmasi deyiladi.

Pikart teoremasining isbotini keltirishdan avval zarur ikki tasdiqni keltiramiz.

Ekvivalentlik lemmasi

Agar $y=\varphi(x)$ funksiya x_0 nuqtani o'z ichiga olgan biror I intervalda aniqlangan bo'lib, (1.1.9) – (1.1.10) Koshi masalasining yechimi bo'lsa, u holda $y=\varphi(x)$ funksiya I intervalda

$$y(x)=y_0+\int_{x_0}^{x} f(\tau)y(\tau)d\tau \qquad \mathbf{(1.1.12)}$$

integral tenglamaning yechimi bo'ladi, aksincha agar $y=\varphi(x)$ funksiya I intervalda uzluksiz bo'lsa, u holda $y=\varphi(x)$ funksiya (9)-(10) Koshi masalasining ham yechimi bo'ladi.

Gronuoll lemmasi

Agar u(x) funksiya $[x_0, x_0+h]$ intervalda manfiymas, uzluksiz bo'lib, shu intervalda ushbu

$$u(x)\le A+B\int_{x_0}^{x} u(\tau)d\tau,\ \mathrm{A}\ge 0,\ \mathrm{B}\ge 0 \qquad \mathbf{(1.1.13)}$$

integral tengsizlikka qanoatlantirsa, shu u(x) funksiya uchun quyidagi

$$u(x)\le Ae^{B(x-x_0)},\quad \mathrm{x}_0\in[x_0, x_0+h] \qquad \mathbf{(1.1.14)}$$

tengsizlik o'rinli bo'ladi.

Pikar teoremasining isboti. Mavjudligi. Ekvivalentlik lemmasiga ko'ra Koshi masalasi (1.1.9)-(1.1.10) o'rniga ushbu

$$y=y_0+\int_{x_0}^{x} f(\tau,y)d\tau \qquad \mathbf{(\ 1.1.15)}$$

integral tenglamani yechish masalasini ko'ramiz. Bu tenglamaning yechimini Pikarning ketma-ket yaqinlashish metodi bilan izlaymiz. $|x - x_0| \le h$ intervalda yaqinlashgan funksiyalar ketma-ketligini quyidagicha ko'ramiz;

$$y_0(x) = y_0 \quad (\text{nolinchi yaqinlashish})$$

$$y_1(x) = y_0 + \int_{x_0}^{x} f(\tau, y_0) d\tau \quad (1\text{-yaqinlashish})$$

$$y_2(x) = y_0 + \int_{x_0}^{x} f(\tau, y(\tau)) d\tau \quad (2\text{-yaqinlashish})$$

– –

$$y_n(x) = y_0 + \int_{x_0}^{x} f(\tau, y_{n-1}(\tau)) d\tau \quad (\text{n-yaqinlashish})$$

shu funksiyalarning grafigi $|x - x_0| < h,$ intervalda $D_h = \{(x, y) : |x - x_0| \le h, |y - y_0| \le b,\}$ to'g'ri to'rtburchakdan chiqib ketmaydi, ya'ni $(x, y_n(x)) \in D_h$, $\text{n} = 0,1,2,3...$ haqiqatdan.

$$(x_0, y_0) \in D_k$$

$$|y_1(x) - y_0| = \left| \int_{x_0}^{x} f(\tau, y_0) d\tau \right| \le \left| \int_{x_0}^{x} |f(\tau, y_0(\tau))| d\tau \right| \le M|x - x_0| \le Mh \le b_1$$

$$|y_2(x) - y_0| = \left| \int_{x_0}^{x} f(\tau, y_1(\tau)) d\tau \right| \le \left| \int_{x_0}^{x} |f(\tau, y_1(\tau))| d\tau \right| \le M|x - x_0| \le Mh \le b$$

– –

$$|y_n(x) - y_0| = \left| \int_{x_0}^{x} f(\tau, y_{n-1}(\tau)) d\tau \right| \le \left| \int_{x_0}^{x} |f(\tau, y_{n-1}(\tau))| d\tau \right| \le M|x - x_0| \le Mh \le b$$

tasdiqlab o'tamizki, $\{y_n(x)\}$ ketma-ketlikning hadlari ko'rilayotgan $|x - x_0| \le h,$ intervalda uzluksiz, hatto differensiallanuvchidir.

Endi qurilgan $\{y_n(x)\}$ ketma-ketlik $|x - x_0| \le h,$ intervalda tekis yaqinlashuvchi ekanligini intervalda tekis yaqinlashuvchi ekanligini isbotlaymiz. Ushbu

$$y_0 + [y_1(x) - y_0] + [y_2(x) - y_1(x)] + \ldots + [y_n(x) - y_{n-1}(x)] + \ldots \quad \textbf{(1.1.16)}$$

Funksional qatorni ko'ramiz.Uning **n-** xususiyyig'indisi $S_n(x) = y_n(x)$ bundan $\lim\limits_{n\to\infty} S_n(x) = \lim\limits_{n\to\infty} y_n(x)$. Shuning uchun (1.1.16) qatorning tekis yaqinlashuvchi ekanligi isbot qilish yetarli, (1.1.16) qatorning har bir hadini baholaymiz , (1.1.11) tengsizlikni hisobga olgan holda

$$|y_1(x) - y_0| = \left|\int_{x_0}^{x} f(\tau, y_0) d\tau\right| \le M|x - x_0|$$

$$|y_2(x) - y_1(x)| = \left|\int_{x_0}^{x} [f(\tau, y_1(\tau)) - f(\tau, y_0)] d\tau\right| \le \left|\int_{x_0}^{x} [f(\tau, y(\tau)) - f(\tau, y_0)] d\tau\right| \le L\left|\int_{x_0}^{x} |y_1(\tau) - y_0| d\tau\right| \le$$

$$\le LM\left|\int_{x_0}^{x} |\tau - x_0| d\tau\right| = LM\,\frac{(x - x_0)^2}{2!}$$

Induksiya usuli bilan:

$$|y_n(x) - y_{n-1}(x)| \le L^{n-1} M\,\frac{(x - x_0)^n}{n!} \qquad \textbf{(1.1.17)}$$

Tengliksiz o'rinli bo'lsa, shu qonun **n** dan **n+1** ga o'tganda ham o'rinli ekanligini isbotlash mumkin:

$$|y_{n+1}(x) - y_n(x)| = \left|\int_{x_0}^{x} [f(\tau) y_n(\tau) - f(\tau, y_{n-1}(\tau))] d\tau\right| \le \int_{x_0}^{x} [f(\tau) y_n(\tau) - f(\tau, y_{n-1}(\tau))] d\tau \le$$

$$\le L\int_{x_0}^{x} [y_n(\tau) - y_{n-1}(\tau)] d\tau \le \frac{L^n M}{n!}\left|\int_{x_0}^{x} |\tau - x_0|^n d\tau\right| = \frac{L^n M}{(n+1)!}|x - x_0|^{n+1}$$

Shundayqilib (1.1.12) tengsizlikixtiyoriynaturalnlaruchunto'g'ri. Haqiqatdan (1.1.12) ga ko'ra

$$|y_n(x) - y_{n-1}(x)| \le L^{n-1} M\,\frac{h^n}{n!} \quad \text{va} \quad |\mathrm{y}_0| + \sum_{n=1}^{\infty} L^{n-1} M\,\frac{h^n}{n!}$$

sonli qator yaqinlashuvchi, shunki Dalamber alomatiga ko'ra

$$\lim_{n\to\infty}\frac{L^n M\frac{h^{n+1}}{(n+1)!}}{L^{n-1}M\frac{h^n}{n!}}=\lim_{n\to\infty}\frac{h}{n+1}=0<1$$

Shunday qilib, matematik analiz kursidan ma'lum bo'lgan Veyershtrass teoremasiga ko'ra $\{S_n(x)=y_n(x)\}$ ketma – ketlik uzluksiz $\varphi(x)$ funksiyaga tekis yaqinlashadi $\varphi(x)$ funksiyaning uzluksizligi har bir $y_n(x)-y_{n-1}(x)$ ayirma yuqori limiti o'zgaruvchi bo'lgan integraldan iboratligidan ko'rinadi. Ma'lumki, bunday integral yuqori limitining uzluksiz funksiyasidan iboratdir.

Enditopilganshuy=$\varphi(x)$limitfunksiya (1.1.9) –(1.1.10) masalasiningyechimiekanliginiisbotqilamiz,

buninguchun $n\to\infty$ da $y_{n+1}(x)=y_0+\int_{x_0}^{x} f(\tau,y_n(\tau))d\tau$ tenglikdan

$$\varphi(x)=y_0+\int_{x_0}^{x} f(\tau,\varphi(\tau))d\tau \qquad \textbf{(1.1.18)}$$

Tenglik kelib chiqishini isbotlash lozim, haqiqatdan ravshanki

$$\left|\int_{x_0}^{x} f(\tau,\varphi(\tau))d\tau-\int_{x_0}^{x} f(\tau,y_n(\tau))d\tau\right|\le\left|\int_{x_0}^{x}\left|f(\tau,\varphi(\tau))-f(\tau,y_n(\tau))\right|d\tau\right|\le L\left|\int_{x_0}^{x}\left|\varphi(\tau)-y_n(\tau)\right|d\tau\right|$$

$\{y_n\}$ketma –ketlikning $\varphi(x)$ funksiyaga tekis yaqinlashuvidan $\forall\varepsilon>0$ uchun shunday N nomer topiladiki, n>N bo'lganda $|\varphi(x)-y_n(x)|\le\frac{\varepsilon}{Lh}$ tengsizlik o'rinli bo'ladi shuning uchun

$L\left|\int_{x_0}^{x}\left|\varphi(\tau)-y_n(\tau)\right|d\tau\right|\le L\frac{\varepsilon}{Lh}\left|\int_{x_0}^{x}d\tau\right|=\frac{\varepsilon}{h}|x-x_0|\le\frac{\varepsilon}{h}\cdot h=\varepsilon$ bo'ladi. Bunda

$\lim_{n\to\infty}\int_{x_0}^{x} f(\tau,y_n(\tau))d\tau=\int_{x_0}^{x} f\left(\tau,\lim_{n\to\infty}y_n(\tau)\right)d\tau=\int_{x_0}^{x} f(\tau,\varphi(\tau))d\tau$ shunday qilib

$\lim_{n\to\infty} y_{n+1}=\lim_{n\to\infty}\left(y_0+\int_{x_0}^{x} f(\tau,\varphi(\tau))d\tau\right)$ dan (18) ning o'rinli ekanligini kelib chiqadi.

Yagonaligi

(1.1.9) tenglamaning (1.1.10) shartni qanoatlantiradigan yana bitta $y=\psi(x)$ yechim bo'lsin. Uning aniqlanish intervali $|x-x_0|\le h$ bo'lib $\varphi(x)$ va $\psi(x)$ funksiyalaning aniqlanish intervallarining umumiy qismi $|x-x_0|\le h_0$ dan iborat bo'lsin. U holda $|x-x_0|\le h_0$ da $\varphi(x)\equiv\psi(x)$ ekanligini isbotlaymiz.

Shartga ko'ra $\varphi(x)=y_o+\int_{x_o}^{x} f(\tau,\varphi(\tau))d\tau,\quad \psi(x)=y_0+\int_{x_0}^{x} f(\tau,\psi(\tau))d\tau$)

ayniyatlarga egamiz.

Bundan $[x_0,x_0+h]$ uchun

$|\varphi(x)-\psi(x)|=\left|\int_{x_o}^{x} f(\tau,\varphi(\tau))d\tau-\int_{x_0}^{x} f(\tau,\psi(\tau))d\tau\right|\le L\int_{x_0}^{x}|\varphi(\tau)-\psi(\tau)|d\tau$, yani

$|\varphi(x)-\psi(x)|\le L\int_{x_0}^{x}|\varphi(\tau)-\psi(\tau)|d\tau$ ga egamiz. Bu yerdan Gronuall lemmasining natijasiga ko'ra $\varphi(x)\equiv\psi(x)$, $x\in[x_0,x_0+h]$ kelib chiqadi $x\in[x_0-h,\ x_0]$ uchun ham mulohazalar shunga o'xshashdir. Yagonaligi isbot etiladi.Pikar teoremasi isbotlanadi

Hosilaga nisbatan yechilmagan tenglama uchun yechimning mavjudligi va yagonaligi

1.1.4-teorema. Agar

$$F(x,y,y`)=0 \qquad \textbf{(1.1.19)}$$

Differensial tenglamada $F(x,y,y`)$ funksiya uchun ushbu ikki shart.

1^0 $F(x,y_0,y`)=0$ **(1.1.20)**

Tenglamaning haqiqiy ildizi , $y_0^`$ uchun $(x,y_0,y_0^`)\in D_3((x_0,y_0)\in \Gamma)$ nuqtaning biror yopiq D_3 atrofida $F(x,y,y`)$ funksiya uzluksiz va birinchi tartibli xususiy hosilalarga ega.

$2^0\ F_y^{`}(x_0, y_0, y_0^{`}) \neq 0$ bajardi u holda, shunday h>0 mavjud bo'ladiki, (1.1.19) tenglamaning $|x - x_0| \leq h$ intervalda aniqlangan $y(x_0) = y,\ y^{`}(x) = y_0^{`}$ shartlarini qanoatlantiruvchi yagona y=y(x) yechimi mavjud.

Isbot. Oshkormas funksiyalar haqidagi ma'lum teoremaga ko'ra (1.1.19) tenglama y` ni bir qiymatli funksuiya sifatida aniqlaydi, ya'ni

$$\frac{dy}{dx} = f(x, y) \qquad \textbf{(1.1.21)}$$

Bunda f(x,y) funksiya yopiq $\overline{\Gamma}_0(\overline{\Gamma}_0 \subset \Gamma)$ to'plamda uzluksiz, 1-tartibli uzluksiz hosilaga ega va $f(x_0, y_0) = y_0^{`};\quad (x_0, y_0) \in \overline{\Gamma}_0$ shuning uchun f(x,y) funksiya yopiq $\overline{\Gamma}_0$ to'plamda y bo'yicha Lipshist shartini qanoatlantiradi demak (1.1.21) differensial tenglama Pikar teoremasiga asosan $|x - x_0| \leq h$ intervalda aniqlangan va yagona y=y(x) yechimga ega bo'lib , $y(x_0) = y_0$ bo'ladi. Xuddi shu yechimga (1.1.19) tenglama ham ega.Endi $y^{`}(x_0) = y_0^{`}$ ekanini ko'rsataylik. Haqiqatdan (1.1.21) tenglama y=y(x) uchun ayniyatga aylanadi:

$\frac{dy(x)}{dx} = f(x, y(x)),\quad |x - x_0| \leq h$ agar x=x_0 bo'lsa $y^{`}(x_0) = f(x_0, y(x_0)) = f(x_0, y_0) = y^{`}$

Natija 1.1.1teorema shartiga ko'ra $(x_0, y_0 y_i^{`})$ nuqtaning $\overline{D}_3^0$ atrofida $\frac{\partial F(x,y,y^{`})}{\partial y^{`}} \neq 0,\ \left|\frac{\partial F(x,y,y^{`})}{\partial y^{`}}\right| \leq A,\ 0 < A = \mathrm{const}$

Natija1.1.2 agar (1.1.20) tenglama bir necha haqiqiy ildizga ega bo'lsa, har bir $(x_0, y_0 y_i^{`})$ nuqtaning yopiq $\overline{D}_3^0$ atrofida (1.1.19)tenglama y` ni bir qiymatli aniqlaydi, y`=f_i(x,y). shu bilan birga har bir $i(i = \overline{1,m})$ uchun tegishli differensial tenlama $(x_0, y_0) \in \overline{\Gamma}_0$ nuqtadan o'yuvchi yagona integralchiziqqa ega. Boshqacha aytganda, (x_0,y_0) nuqtadan m ta yo'nalish bo'yicha faqat m ta integral chiziq o'tadi.

Agar (x_0,y_0) nuqtada Koshi masalasi yagona yechimga ega bo'lsa u nuqtaga oddiy nuqta deyiladi bu nuqtaga mos yechim oddiy yechim, integral chiziqni oddiy integral chiziq deyiladi.

Agar (x_0,y_0) nuqtada Koshi masalasi uchun yagonalik o'rinli bo'lmasa, u holda bu nuqta (1.1.19) tenglamaning maxsus nuqtasi deyiladi hamda uning grafigi maxsus integral chiziq deyiladi. Demak, $(x_0, y_0 y_0^{`})$ nuqtaning yetarli kichik yopiq atrofida teoremaning biror sharti buzilganda maxsus nuqtaga ega bo'lishimiz mumkin. Bu teorema faqat yetarli shartni belgilagani uchun $(x_0, y_0 y_0^{`})$nuqta aytilgan holda maxsus bo'lishi ham bo'lmasli ham mumkin .

1.2.Integral tenglamalar haqida umumiy tushunchalar.

1.2.1-ta'rif. Agar tenglamada noma'lum funksiya shu funksiyaning argumenti bo'yicha olinadigan integral belgisi ostida albatta qatnashsa, bunday tenglama **integral tenglama** deb ataladi.

1.2.2-ta'rif. Agar funksional tenglamada noma'lum funksiya integral ishorasi ostida qatnashsa, u holda bu tenglama **integral tenglama** deb ataladi.

1.2.3-ta'rif. Agar integral tenglamada integral belgisi ostidagi funksiya va tenglamaning boshqa qismlari (hadlari) noma'lum funksiyaga nisbatan chiziqli bo'lsa, u holda tegishli tenglama **chiziqli integral tenglama** deyiladi.

1.2.4-ta'rif. Integral tenglamadagi ifoda noma'lum funksiyaga nisbatan chiziq bo'lsa, u holda tenglama **chiziqli integral tenglama** deyiladi.

1.2.5-ta'rif. Noma'lum funksiya integral ishorasi ostida chiziqsiz ishtirok etsa, bunday tenglamalar ***chiziqli bo'lmagan integral tenglamalar*** deyiladi.

1.2.6-ta'rif. Ushbu

$$f(t) = \lambda \int_a^t K(t,s)\varphi(s)ds \qquad \textbf{(1.2.1)}$$

ko'rinishdagi tenglama Vol'terraning *birinchi tur chiziqli integral tenglamasi* deyiladi. Unda integral ostidagi funksiya noma'lum $\varphi(t)$ funksiyaga nisbatan chiziqli. (1.2.1)tenglamada $f(t)$ funksiya $I(a \le t \le b)$ segmentda, $K(t,s)$ funksiya esa $R(a \le t \le b, a \le s \le t)$ sohada berilgan hamda o'z argumentlari bo'yicha uzluksiz funksiyalardir. Undan tashqari, $\lambda -$ o'zgarmas son (parametr), $a,b -$ berilgan haqiqiy o'zgarmas sonlar, $K(t,s)$ funksiya (1.2.1)tenglamaning *yadrosi*, $f(t)$ esa *ozod hadi* deyiladi.

1.2.7-ta'rif. Ushbu

$$\varphi(t) = f(t) + \lambda \int_a^t K(t,s)\varphi(s)ds \qquad (1.2.2)$$

ko'rinishdagi tenglama Vol'terraning *ikkinchi tur chiziqli integral tenglamasi* deb ataladi. Bunda $\varphi(t)$ noma'lum funksiya integral belgisidan tashqarida ham alohida va chiziqli bo'lib qatnashmoqda.

Agar (1.2.2) tenglamada ixtiyoriy $t \in I$ uchun $f(t) \equiv 0$ bo'lsa, (1.2.2) dan

$$\varphi(t) = \lambda \int_a^t K(t,s)\varphi(s)ds \qquad (1.2.3)$$

tenglama hosil bo'ladi va u Vol'terraning *bir jinsli chiziqli integral tenglamasi* deb ataladi.

1.2.8-ta'rif. Ushbu

$$\psi(t)\varphi(t) = f(t) + \lambda \int_a^t K(t,s)\varphi(s)ds \qquad (1.2.4)$$

ko'rinishdagi tenglama Vol'terraning *uchinchi tur chiziqli integral tenglamasi* deb ataladi.

Agar ixtiyoriy $t \in I$ uchun $\psi(t) \equiv 0$ bo'lsa, (1.2.4)dan (1.2.1) tenglama, $\psi(t) = 1$ bo'lsa, (1.2.4)dan (1.2.2) tenglama kelib chiqadi.

Ba'zi hollarda $K(t,s)$ yadro xususan $K(t-s)$, ya'ni $t-s$ ayirmaning funksiyasi ko'rinishiga ega bo'lishi mumkin. Bu holda, jumladan, ushbu

$$\varphi(t) = f(t) + \lambda\int_a^t K(t-s)\varphi(s)ds \qquad (1.2.5)$$

tenglama Vol'terraning *o'ram (yig'ma) tipidagi integral tenglamasi* deb yuritiladi.

1.2.9-ta'rif. Agar Vol'terra tenglamalarida $a\to-\infty$ yoki $b\to\infty$ bo'lsa, ushbu

$$\varphi(t) = f(t) + \lambda\int_t^\infty K(t,s)\varphi(s)ds \qquad (1.2.6)$$

$$\varphi(t) = f(t) + \lambda\int_{-\infty}^t K(t,s)\varphi(s)ds \qquad (1.2.7)$$

tenglamalarga, agar $K(t,s)$ yadro qaralayotgan sohaning bir yoki bir nechta nuqtasida cheksizlikka aylansa, masalan ushbu

$$\varphi(t) = f(t) + \lambda\int_a^t \frac{\varphi(s)}{s-a}ds, \qquad (1.2.8)$$

$$\varphi(t) = f(t) + \lambda\int_a^t \frac{H(t,s)}{(s-a)^\alpha}\varphi(s)ds,\ 0<\alpha<1 \qquad (1.2.9)$$

tenglamalarga ega bo'lamiz. Ularga Vol'terraning *maxsus integral tenglamalari* deyiladi.

Vol'terra tenglamalarida integrallash chegaralaridan biri yoki ikkalasi ham funksiyadan iborat bo'lishi mumkin. Ushbu

$$\varphi(t) = f(t) + \int_a^{g(t)} K(t,s)\varphi(s)ds, \qquad (1.2.10)$$

$$\varphi(t) = f(t) + \int_{t-a}^t K(t,s)\varphi(s)ds, \alpha>0, \qquad (1.2.11)$$

$$\varphi(t) = f(t) + \int_{pt}^{t} K(t,s)\varphi(s)ds, 0 < p < 1 \qquad (1.2.12)$$

tenglamalardir.

Integral tenglamalarni yechish usullari.

Integral tenglamalar nazariyasi matematikaning eng rivojlangan sohalaridan biri bo'lib, fan va texnikaning rivojlanishida katta rol o'ynaydi. Bunda Vol'terra tenglamalari muhum o'rin egallaydi. Biz bu yerda Vol'terra tenglamalariga oid dastlabki ma'lumotlardan ba'zilarini keltiramiz.

Ketma-ket o'rniga qo'yish usuli.

Quyidagi

$$\varphi(t) = f(t) + \lambda \int_{a}^{t} K(t,s)\varphi(s)ds \qquad (1.2.13)$$

ikkinchi tur Vol'terra tenglamasini qaraymiz. Bu yerda va kelgusida uchraydigan barcha Vol'terra tenglamalarining ozod hadi va yadrosi haqiqiy o'zgaruvchili noldan farqli funksiyalardan, λ parametr esa haqiqiy sondan iborat deb faraz qilamiz.

Teorema. Agar (1.2.13) tenglamada $f(t)$ funksiya $I(a \le t \le b)$ segmentda, $K(t,s)$ funksiya esa $R(a \le t \le b, a \le s \le t)$ sohada uzluksiz bo'lib, $f(t) \ne 0,\ K(t,s) \ne 0,\ \lambda = const$ bo'lsa, u holda (1.2.13) tenglama I segmentda yagona uzluksiz yechimga ega bo'ladi.

Isbot. Bu teoremani isbotlash usuli ketma-ket o'rniga qo'yish usuli deb yuritiladi. Bu usul bilan (1.2.13) tenglamaning yagona uzluksiz yechimi quriladi. Izlanuvchi $\varphi(t)$ funksiyaning (1.2.13)tenglamadagi ifodasini shu tenglamaning o'ng tomoniga qo'yib, quyidagini hosil qilamiz:

$$\varphi(t) = f(t) + \lambda \int_{a}^{t} K(t,s)\left[f(s) + \lambda \int_{a}^{s} K(s,s_1)ds_1 \right]ds = f(t) + \lambda \int_{a}^{t} K(t,s) f(s)ds +$$

$$+\lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1)\varphi(s_1)ds_1ds.$$

Bu yerdagi $\varphi(s_1)$ o'rniga yana uning (1.2.13) tenglamadagi ifodasini qo'ysak, quyidagiga ega bo'lamiz

$$\varphi(t)=f(t)+\lambda\int_a^t K(t,s)f(s)ds+\lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1)\left[f(s_1)+\lambda\int_a^{s_1}K(s_1,s_2)\varphi(s_2)ds_2\right]ds_1ds=$$

$$=f(t)+\lambda\int_a^t K(t,s)f(s)ds+\lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1)f(s_1)ds_1ds+$$

$$+\lambda^3\int_a^t K(t,s)\int_a^s K(s,s_1)\int_a^{s_1}K(s_1,s_2)u(s_1)ds_2ds_1ds.$$

Bu jarayonni davom ettirib, n marta o'rniga qo'yishni bajarsak, quyidagiga ega bo'lamiz:

$$\varphi(t)=f(t)+\lambda\int_a^t K(t,s)f(s)ds+\lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1)f(s_1)ds_1ds+\ldots+$$

$$+\lambda^n\int_a^t K(t,s)\int_a^s K(s,s_1)\ldots\ldots\int_a^{s_{n-2}}K(s_{n-2},s_{n-1})f(s_{n-1})ds_{n-1}\ldots ds+R_{n+1}(t) \qquad (1.2.14)$$

bu yerda

$$R_{n+1}(t)=\lambda^{n+1}\int_a^t K(t,s)\int_a^s K(s,s_1)\ldots\int_a^{s_{n-1}}K(s_{n-1},s_n)\varphi(s_n)ds_n\ldots ds.$$

Bunga asosan quyidagi qatorni qaraymiz:

$$f(t)+\lambda\int_a^t K(t,s)f(s)ds+\ldots+\lambda^n\int_a^t K(t,s)\int_a^s K(s,s_1)\ldots\int_a^{s_{n-2}}K(s_{n-2},s_{n-1})f(s_{n-1})ds_{n-1}\ldots ds+\ldots$$

(1.2.15)

Teoremaning shartlariga asosan, bu qatorning har bir hadi t ning I segmentda uzluksiz funksiyasidan iborat.Demak, agar bu qator I segmentda tekis yaqinlashuvchi bo'lsa, uning yig'indisi ham shu segmentda aniqlangan uzluksiz funksiyadan iborat bo'ladi.

$K(t,s)$ va $f(t)$ funksiyalar mos ravishda yopiq R va I sohalarda uzluksiz ekanidan quyidagini yozish mumkin:

$$\max_{(t,s)\in R}|K(t,s)| = M, (yoki\, |K(t,s)| < M);$$

$$\max_{t\in I}|f(t)| = N, (yoki\, |f(t)| < N).$$

Bularga asosan (1.2.15) qatorning $n-$hadi ($f(t)$ 0-had)

$$V_n(t) = \lambda^n \int_a^t K(t,s)\int_a^s K(s,s_1)\ldots \int_a^{s_{n-2}} K(s_{n-2},s_{n-1})f(s_{n-1})ds_{n-1}\ldots ds \qquad (1.2.16)$$

quyidagicha baholanadi:

$$|V_n(t)| \le |\lambda^n| NM\frac{(t-a)^n}{n!} \le |\lambda|^n N\frac{[M(b-a)]^n}{n!}, t\in I.$$

Bundan ko'rinadiki, umumiy hadi ($n-$hadi) ushbu

$$|\lambda|^n N\frac{[M(b-a)]^n}{n!}$$

ko'rinishda bo'lgan musbat hadli sonli qator λ, M, N va $b-a$ larning har qanday musbat chekli qiymatlarida yaqinlashuvchi ekani ravshan (bunga Dalamber alomatini bevosita qo'llanish yordamida ishonch hosil qilish mumkin). Shuning uchun Veyershtrass teoremasiga asosan umumiy hadi (1.2.16) dan iborat bo'lgan (1.2.15)qator I da absolyut va tekis yaqinlashuvchi bo'ladi.

Agar (1.2.13) tenglama uzluksiz yechimga ega bo'lsa, bu yechim (1.2.14)tenglamaning ham yechimi bo'lishi kerak, ya'ni bu yechim qoldiq hadi $R_{n+1}(t)$ dan iborat bo'lgan (1.2.14) qator bilan ifodalanishi kerak.Agar (1.2.13) tenglama uzluksiz yechimga ega bo'lsa, o'sha yechim (1.2.14) ko'rinishda yozilishi mumkin. $\varphi(t)$ ning I da uzluksizligi esa ushbu munosabatni yozishga asos bo'ladi: $\max_{t\in I}|\varphi(t)| = Q$. Shuning uchun ushbuga egamiz:

$$|R_{n+1}(t)| \le |\lambda^{n+1}| QM^{n+1} \frac{(t-a)^{n+1}}{(n+1)!} \le |\lambda|^{n+1} Q \frac{[M(b-a)]^{n+1}}{(n+1)!}.$$

Demak, $\lim_{n\to\infty} R_{n+1}(t) = 0.$

Shunday qilib, ixtiyoriy n uchun (1.2.13) tenglamaning yechimi (bu yechim mavjud bo'lganda) (1.2.15) qatorga yoyildi.

Endi (1.2.15) qatorning yig'indisidan iborat uzluksiz $\varphi(t)$ funksiya (1.2.13) tenglamaning yechimi ekanini ko'rsatamiz. Bunga ushbu

$$\varphi(t) = f(t) + \lambda\int_a^t K(t,s) f(s)ds + \lambda^2 \int_a^t K(t,s)\int_a^s K(s,s_1) f(s_1)ds_1 ds + \dots \tag{1.2.17}$$

qatorni (1.2.13) tenglamadagi $\varphi(t)$ ning o'rniga bevosit qo'yish natijasida ishonch hosil qilish mumkin. Haqiqatan, yuqoridagi (1.2.17) tenglikning har ikkala tomonini $\lambda K(t,s)$ ga ko'paytirib, uning har ikkala tomonini a dan t gacha integrallaymiz. U holda quyidagiga ega bo'lamiz:

$$\lambda\int_a^t K(t,s)\varphi(s)ds = \lambda\int_a^t K(t,s)\left[f(s) + \lambda\int_a^s K(s,s_1) f(s_1)ds_1 + \dots \right]ds =$$

$$= \lambda\int_a^t K(t,s) f(s)ds + \lambda^2 \int_a^t K(t,s)\int_a^s K(s,s_1) f(s_1)ds_1 ds + \dots = \varphi(t) - f(t).$$

Bu esa (1.2.17) tenglik bilan aniqlanuvchi $\varphi(t)$ funksiya (1.2.13) tenglikning yechimi ekanini ko'rsatadi.

Ketma-ket yaqinlashish usuli

Vol'terra tenglamasi yechimining mavjudligi va yagonaligi ketma-ket yaqinlashish usuli yordamida ham ko'rsatiladi.

Ushbu

$$\varphi(t) = f(t) + \lambda \int_a^t K(t,s)\varphi(s)ds \qquad (1.2.18)$$

Vol'terraning chiziqli tenglamasini qaraymiz.

Teorema. Agar $f(t)$ funksiya $I(a \le t \le b)$ segmentda, $K(t,s)$ funksiya (yadro) esa $R(a \le t \le b, a \le s \le t)$ sohada uzluksiz bo'lib, $f(t) \ne 0,\ K(t,s) \ne 0,\ \lambda = const$ bo'lsa, u holda (1.2.18) tenglama I segmentda yagona uzluksiz yechimga ega bo'ladi va bu yechim $\varphi_0(t)$ ixtiyoriy uzluksiz funksiya bo'lganda ushbu

$$\varphi_n(t) = f(t) + \lambda \int_a^t K(t,s)\varphi_{n-1}(s)ds \ (n = 1, 2, 3, \ldots)$$

rekurrent formula yordamida aniqlanuvchi $\{\varphi_n(t)\}$ ketma-ketlikning $n \to \infty$ dagi limitidan iborat bo'ladi.

Isbot. Teoremaning yechimi mavjudligi haqidagi qismi yangi tasdiq emas.Biz uning ikkinchi qismini isbotlashimiz lozim. I segmentda uzluksiz bo'lgan ixtiyoriy $\varphi_0(t)$ funksiyani tanlaymiz. Bu funksiyani (1.2.13) tenglamaning o'ng tomonini $\varphi(s)$ ning o'rniga qo'yib,

$$\varphi_1(t) = f(t) + \lambda \int_a^t K(t,s)\varphi_0(s)ds$$

tenglikni hosil qilamiz. Shu tarzda topilgan $\varphi_1(t)$ funksiya I segmentda uzluksiz funksiyadan iborat.

Endi topilgan $\varphi_1(t)$ funksiyani yana(1.2.18) tenglamaning o'ng tomonidagi $\varphi(s)$ ning o'rniga qo'yib,

$$\varphi_2(t) = f(t) + \lambda \int_a^t K(t,s)\varphi_1(s)ds$$

tenglikni hosil qilamiz. Ko'rinib turibdiki, $\varphi_2(t)$ funksiya ham I segmentda uzluksizdir.

Bu jarayonni davom ettirib,

$$\varphi_0(t),\ \varphi_1(t),\ \varphi_2(t),\ \ldots,\ \varphi_n(t),\ldots$$

funksiyalar ketma-ketligini hosil qilamiz. Bu yerda $\varphi_i(t), i=1,2,\ldots$ funksiyalar quyidagi tengliklarni qanoatlantiradi:

$$\varphi_1(t)=f(t)+\lambda\int_a^t K(t,s)\varphi_0(s)ds,$$

$$\varphi_2(t)=f(t)+\lambda\int_a^t K(t,s)\varphi_1(s)ds, \qquad (1.2.19)$$

$$\varphi_n(t)=f(t)+\lambda\int_a^t K(t,s)\varphi_{n-1}(s)ds.$$

Yuqorida $f(t)$ va $K(t,s)$ funksiyalar uchun qo'yilgan shartlarga asoslanib, n cheksizlikka intilganda $\{\varphi_n(t)\}$ ketma-ketlik (1.2.19) tenglamaning $\varphi(t)$ yechimiga yaqinlashishini ko'rsatamiz. Shu maqsadda $\varphi_i(t)$ lar uchun yozilgan (1.2.19) ifodalarni yuqoridan boshlab birin-ketin o'zidan keyingisiga qo'yib chiqamiz. Natijada quyidagi ifodalar hosil bo'ladi:

$$\varphi_1(t)=f(t)+\lambda\int_a^t K(t,s)\varphi_0(s)ds,$$

$$\varphi_2(t)=f(t)+\lambda\int_a^t K(t,s)f(s)ds+\lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1)\varphi_0(s_1)ds_1ds$$

…………………...

$$\varphi_n(t)=f(t)+\lambda\int_a^t K(t,s)f(s)ds+\lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1)\varphi_0(s_1)ds_1ds+\ldots+$$

$$+\lambda^{n-1}\int_a^t K(t,s)\int_a^s K(s,s_1)\ldots\int_a^{s_{n-3}} K(s_{n-3},s_{n-2})f(s_{n-2})ds_{n-2}\ldots ds_1ds+R_n(t), \qquad (1.2.20)$$

bu yerda

$$R_n(t)=\lambda^n\int_a^t K(t,s)\int_a^s K(s,s_1)\ldots\int_a^{s_{n-3}} K(s_{n-2},s_{n-1})\varphi_0(s_{n-1})ds_{n-1}\ldots ds_1ds.$$

Shartga ko'ra $K(t,s)$ va $\varphi_0(t)$ funksiyalar mos ravishda R va I larda uzluksiz, demak,

$$|K(t,s)|\le M, |\mu_0(t)|\le Q.$$

Bularga asosan quyidagiga ega bo'lamiz:

$$|R_n(t)|\le QM^n|\lambda|^n\int_a^t ds\int_a^s ds_1\ldots\int_a^{s_{n-1}} ds_{n-2}=QM^n|\lambda|^n\frac{(t-a)^n}{n!},$$

ya'ni

$$|R_n(t)|\le QM^n|\lambda|^n\frac{(b-a)^n}{n!}.$$

Shuning uchun ravshanki, $\lim\limits_{n\to\infty}R_n(t)=0.$

Buni nazarda tutib, (1.2.21) tenglikning har ikkala tomonida $n\to\infty$ da limitga o'tsak, $\varphi_n(t)$ funksiyalarning limiti

$$f(t)+\lambda\int_a^t K(t,s)f(s)ds+\ldots+\lambda^n\int_a^t K(t,s)\int_a^s K(s,s_1)\ldots\int_a^{s_{n-1}} K(s_{n-2},s_{n-1})f(s_{n-1})ds_{n-1}\ldots ds+\ldots \qquad (1.2.21)$$

qatorning yig'indisiga teng ekanligi va uning (1.2.18) tenglamaning yechimi ekaniga ishonch hosil qilamiz. Demak,

$$\lim_{n\to\infty}\varphi_n(t)=\varphi(t).$$

Ko’rinib turibdiki, topilgan $\varphi(t)$ yechim I segmentda uzluksiz funksiyadir.

Yuqorida bayon etilgan usul **Vol’terraning chiziqli integral tenglamasini yechishning ketma-ket yaqinlashish usuli** deyiladi.

Integral tenglamalar deb, noma’lum funksiya integral ishorasi ostida bo’lgan tenglamalarga aytiladi.

Mexanika , matematik fizika va texnikaning juda ko’p masalalari ushbu

$$\varphi(x) - \lambda \int_a^b K(x,y)\varphi(y)dy = f(x) \qquad (1.2.22)$$

ko’rinishdagi integral tenglamalarni tekshirishga olib kelinadi, bu yerda $\varphi(x)$-noma’lum funksiya, $K(x,y)$ va $f(x)$ funksiyalar mos ravishda $a \le x \le b,\ \ a \le y \le b,$ va $a \le x \le b$ $(a, b - o'zgarmas\ \ sonlar)$yopiq sohalarda berilgan uzluksiz haqiqiy funksiyalardir. $f(x)$ funksiya (1.2.22) integral tenglamaning ozod hadi, $K(x,y)$ uning yadrosi , sonli λ ko’paytma tenglamaning parametri deyiladi. Bunday parametrni kiritish shart emas, agar $\lambda K(x,y)$ ko’paytmani $K_1(x,y)$ bilan belgilab, $K_1(x,y)$ ni yangi yadro deb qarasak, parametr 1 ga teng bo’lib qoladi. Lekin , keyinchalik biz bunga ishonch hosil qilamiz, bunday parametrni kiritish integral tenglamalarni o’rganishda foydali bo’ladi. Biz (1.2.22) tenglama ikkinchi tur chiziqli integral tenglama yoki bunday tenglamalarni birinchi bo’lib o’rgangan matematik nomi bilan Fredgolm integral tenglamasi deyiladi.

Fredgolm birinchi tur tenglamasi deb,

$$\int_a^b K(x,y)\varphi(y)dy = f(x)$$

ko’rinishdagi integral tenglamaga aytiladi. Bunda $\varphi(y) -$ noma’lum funksiya, $f(x) -$ ozod had va $K(x,y)$ tenglamaning yadrosi ma’lum funksiyalar, integrallash chegaralari a va b berilgan haqiqiy o'zgarmas sonlardir.

Agar (1.2.22) tenglamada $f(x) \equiv 0$ bo'lsa, ya'ni

$$\varphi(x) - \lambda \int_a^b K(x,y)\varphi(y)\,dy = 0$$

tenglama (1.2.22) ga mos bo'lgan bir jinsli integral tenglama deyiladi.

Xulosa.

Yuqorida differensial va integral tenglamalarga bog'liq asosiy tushunchalar ya'ni differensial tenglama turlari, differensial tenglama yechimi, differensial tenglamaga qo'yilgan Koshi masalasi, integral tenglama turlari, integral tenglamalarni yechish kabi masalalar ko'rib o'tildi.

II. Integro-differensial tenglamalar.

2.1. Bir argumentli funksiyalar uchun integro-differensial tenglamalarni yechish.

Agar tenglamadagi noma'lum funksiya bir vaqtda ham integral ishorasi ostida qatnashsa, ham uning hosilalari qatnashsa, bunday tenglama integro-differensial tenglama deyiladi. Biz bir argumentli va ikki argumentli noma'lum funksiyalar uchun yozilgan eng sodda integro-differensial tenglamalarni yechish bilan chegaralanamiz.

Bir argumentli funksiya uchun

Bir argumentli noma'lum funksiyaning integro-differensial tenglamalari bilan shug'ullanamiz. Bunday tenglamalarning yechimini ushbu

$$u(x) = u_0(x) + \lambda u_1(x) + \lambda^2 u_2(x) + \cdots + \lambda^n u_n(x) + \cdots \qquad (2.1.1)$$

funksional qator shaklida izlaymiz, ya'ni ularni ketma-ket yaqinlashish usuli bilan yechamiz.

2.1.1-misol. Ushbu tenglama

$$u'(x) = \lambda \int_{px}^{x} xtu(t)dt \qquad (2.1.2)$$

berilgan, $0 < p < 1, u -$noma'lum funksiya.

Izlanayot yechimni (2.1.1)*qator ko'rinishida olamiz va uni* (2.1.2) *tenglamaga* qo'yamiz. Natijada quyidagi ayniyat hosil bo'ladi:

$$u_0'(x) + \lambda u_1'(x) + \lambda^2 u_2'(x) + \cdots + \lambda^n u_n'(x) + \ \ldots \equiv$$
$$\equiv \lambda \int_{px}^{x} xt[u_0(t) + \lambda u_1(t) + \lambda^2 u_2(t) + \cdots]\, dt.$$

Bu tenglikning ikki tomonidagi teng darajali $\lambda^m (m = 0,1,2, \ldots)$ larning koeffisientlarini tenglab, ketma-ket $u_0, u_1, u_2, \ldots$ larni topamiz:

$u_0'(x) = 0,$ bundan $u_0(x) = c_0.$

$c_0 -$ixtiyoriy o'zgarmas son.

$$u_1'(x) = \int_{px}^{x} xtu_0(t)dt$$

$u_0(t) - c_0$ ga teng.

$$u_1'(x) = \int_{px}^{x} xtu_0(t)dt = \int_{px}^{x} xtc_0 dt = c_0 x \int_{px}^{x} t\,dt = \frac{1}{2}c_0(1-p^2)x^3,$$

bundan x bo'yicha integral olinsa,

$$u_1(x) = c_0\frac{x^4}{2\cdot 4}(1-p^2) + c_1,$$

$$A_1 = \frac{1}{2\cdot 4}(1-p^2);$$

$$u_1(x) = c_1 + c_0A_1x^4,$$

Shuningdek,

$$u_2'(x) = \int_{px}^{x} xtu_1(t)dt,$$

$$u_1(t) = c_1 + c_0A_1t^4,$$

$$u_2'(x) = \int_{px}^{x} xt(c_1 + c_0A_1t^4)dt = xc_1\int_{px}^{x} t\,dt + c_0A_1x\int_{px}^{x} t^5dt =$$

$$= xc_1\left(\frac{x^2}{2} - \frac{p^2x^2}{2}\right) + c_0A_1x\left(\frac{x^6}{6} - \frac{p^6x^6}{6}\right) =$$

$$= c_1\frac{x^3}{2}(1-p^2) + c_0A_1\frac{x^7}{6}(1-p^6)$$

bundan x bo'yicha integral olinsa,

$$u_2(x) = c_1\frac{x^4}{2\cdot 4}(1-p^2) + c_0A_1\frac{x^8}{6\cdot 8}(1-p^6) + c_2,$$

$$A_2 = \frac{1}{6\cdot 8}(1-p^6),$$

$$u_2(x) = c_2 + c_1A_1x^4 + c_0A_1A_2x^8,$$

Shu yo'sinda $u_3(x)$ ni topish mumkin:

$$u_3(x) = c_3 + c_2A_1x^4 + c_1A_1A_2x^8 + c_0A_1A_2A_3x^{12},$$

$$A_3 = \frac{1}{10 \cdot 12}(1 - p^{10})$$

$$u_4(x) = c_4 + c_3 A_1 x^4 + c_2 A_1 A_2 x^8 + c_1 A_1 A_2 A_3 x^{12} + c_0 A_1 A_2 A_3 A_4 x^{16},$$

$$A_4 = \frac{1}{14 \cdot 16}(1 - p^{14})$$

va hokazo, umuman,

$$u_n(x) = c_n + c_{n-1} A_1 x^4 + c_{n-2} A_1 A_2 x^8 + \cdots + C_0 A_1 A_2 \cdot \ldots \cdot A_n x^{4n},$$

$$A_n = \frac{1}{(4n-2) \cdot 4n}(1 - p^{4n-2})$$

$$A_{n+1} = \frac{1}{8(2n+1)(n+1)}\left(1 - p^{2(2n+1)}\right); \qquad n = 0,1,2,3, \ldots \ldots .$$

Mana shu u_n larning ifodalarini (2.1.1) qatorga qo'yib va o'xshash hadlarni ixchamlash natijasida berilgan tenglamaning umumiy yechimi kelib chiqadi:

$$u(x) = c[1 + A_1(\lambda x^4) + A_1 A_2 (\lambda x^4)^2 + \cdots + A_1 A_2 \ldots A_n (\lambda x^4)^n + \cdots] \qquad (2.1.3)$$

Bunda $c = c_0 + \lambda c_1 + \lambda^2 c_2 + \cdots$

Xususiy holda, agar $p = 0$ bo'lsa, (2.1.2) tenglama ushbu

$$u'(x) = \lambda \int_0^x xtu(t)dt$$

ko'rinishga ega bo'lib (2.1.3) yechimdagi koeffisientlar esa

$$A_{n+1} = \frac{1}{8(2n+1)(n+1)}, \qquad n = 0,1,2,3, \ldots$$

bo'ladi.

2.1.2-misol. Ushbu tenglama yechilsin:

$$u'(x) = e^x + \lambda \int_{x-\sigma}^{x} e^{x-t} u(t)dt,$$

$$\sigma > 0, \quad |\lambda| = \frac{1}{\sigma}$$

Bu tenglamadagi u o'rniga ham (2.1.1) qatorni qo'yib, $u_0, u_1, u_2 \ldots$ larni quyidagicha topamiz:

$$u_0'(x)+\lambda u_1'(x)+\lambda^2 u_2'(x)+\cdots+\lambda^n u_n'(x)+\ \ldots\equiv$$
$$\equiv e^x+\lambda\int_{x-\sigma}^{x} e^{x-t}\,[u_0(t)+\lambda u_1(t)+\lambda^2 u_2(t)+\cdots+\lambda^n u_n(t)]dt$$

Bu tenglikning ikki tomonidagi teng darajali $\lambda^m(m=0,1,2,3,\ldots)$ larning koeffisientlarini tenglab, ketma-ket $u_0,u_1,u_2\ldots$ larni topamiz.

$$u_0'(x)=e^x, \qquad \text{bundan} \qquad u_0(x)=e^x+c_0,$$

c_0 –ixtiyoriy o'zgarmas son; hisoblash ishlarini qisqartish maqsadida c_0 ni va bundan keyingi ixtiyoriy o'zgarmas $c_1,c_2,\ldots$ larni nolga teng deb hisoblaymiz . U holda biz tenglamaning bitta xususiy yechimini topgan bo'lamiz. Shu sababli

bundan, x bo'yicha integral olinsa

$$u_1(x)=\sigma e^x+c_1;\quad c_1=0,\ bo'lsa\ \ u_1(x)=\sigma e^x.$$

$$u_2'(x)=\int_{x-\sigma}^{x} e^{x-t}u_1(t)dt=\int_{x-\sigma}^{x} e^{x-t}\sigma e^t dt=\sigma e^x\int_{x-\sigma}^{x} dt=\sigma^2 e^x$$

bundan, x bo'yicha integral olinsa,

$$u_2(x)=\sigma^2 e^x+c_1x+c_2,\quad c_1=0,\quad c_2=0.$$

$$u_2(x)=\sigma^2 e^x.$$

$$u_3'(x)=\int_{x-\sigma}^{x} e^{x-t}u_2(t)dt=\int_{x-\sigma}^{x} e^{x-t}\sigma^2 e^t dt=\sigma^2 e^x\int_{x-\sigma}^{x} dt=\sigma^3 e^x$$

bundan, x bo'yicha integral olinsa,

$$u_3(x)=\sigma^3 e^x.$$

Buning u_0 dan farqi σ ko'paytuvchidan iborat. Shuning uchun

$$u_n(x)=\sigma^n e^x,\qquad n=0,1,2,3,\ldots$$

deb yozish mumkin.

Endi u_n larning ifodalarini (2.1.1) qatorga qo'ysak, ushbu yechim hosil bo'ladi:

$$u(x)=\frac{e^x}{1-\lambda\sigma}$$

hosil bo'ladi.

2.1.3-misol. Ushbu tenglama yechilsin:

$$u''(x) = e^{-x} + \lambda \int_{x-\sigma}^{x} e^{-(x-t)} u(t)\, dt, \qquad \sigma > 0, \quad |\lambda| < \frac{1}{\sigma}.$$

Yuqoridagi usul bilan ketma-ket $u_n(x)$ larni topamiz:

$$u_0''(x) + \lambda u_1''(x) + \lambda^2 u_2''(x) + \cdots + \lambda^n u_n''(x) + \cdots \equiv$$
$$\equiv e^{-x} + \lambda \int_{x-\sigma}^{x} e^{-(x-t)} [u_0(t) + \lambda u_1(t) + \lambda^2 u_2(t) + \cdots]\, dt.$$

Bu tenglikning ikki tomonidagi teng darajali $\lambda^m (m = 0,1,2,3,\dots)$ larning koeffisientlarini tenglab, ketma-ket $u_0, u_1, u_2, \dots$ larni topamiz:

$$u_0''(x) = e^{-x}, \qquad u_0'(x) = -e^{-x} + C_0, \qquad u_0(x) = D_0 + xC_0 + e^{-x}.$$

Hisoblash ishlarini qisqartish maqsadida $C_0 = D_0 = 0$ deb hisoblaymiz. U holda

$$u_0(x) = e^{-x},$$

shu sababli

$$u_1''(x) = \int_{x-\sigma}^{x} e^{-(x-t)} u_0(t) dt = \int_{x-\sigma}^{x} e^{-(x-t)} e^{-t} dt = e^{-x} \int_{x-\sigma}^{x} dt = \sigma e^{-x}.$$

Bundan x bo'yicha integral olinsa,

$$u_1'(x) = -\sigma e^{-x} + C_1, \qquad u_1(x) = D_1 + C_1 x + \sigma e^{-x};$$

bu yerda ham yuqoridagidek, $D_1 = C_1 = 0$ deb belgilaymiz. U holda

$$u_1(x) = \sigma e^{-x};$$

shu sababli

$$u_2'(x) = \int_{x-\sigma}^{x} e^{-(x-t)} u_0(t) dt = \int_{x-\sigma}^{x} e^{-(x-t)} \sigma e^{-t} dt = \sigma e^{-x} \int_{x-\sigma}^{x} dt = \sigma^2 e^{-x}.$$

Bundan x bo'yicha integral olinsa,

$$u_2'(x) = -\sigma^2 e^{-x} + C_2, \qquad u_2(x) = D_2 + C_2 x + \sigma^2 e^{-x};$$

bu yerda ham yuqoridagidek, $D_2 = C_2 = 0$ deb belgilaymiz. U holda

$$u_2(x) = \sigma^2 e^{-x};$$

umuman,

$$u_n(x) = \sigma^n e^{-x} \qquad n = 0,1,2,\dots$$

deb yozish mumkin. Bularni (2.1.1) qatorga qo'yib soddalashtirilgandan so'ng quyidagi xususiy yechim hosil bo'ladi:

$$u(x) = \frac{e^{-x}}{1 - \lambda\sigma}.$$

Mashqlar

Quyidagi integro-differensial tenglamalar ketma-ket yaqinlashish usuli bilan yechilsin:

2.1.1-misol. $u'(x) = \lambda \int_{px}^{x} (x-t)u(t)dt, \qquad 0 < p < 1.$

Izlanayotgan yechimni (1) qator ko'rinishida olamiz va uni tenglamaga qo'yamiz. Natijada quyidagi ayniyat hosil bo'ladi:

$$u_0'(x) + \lambda u_1'(x) + \lambda^2 u_2'(x) + \cdots + \lambda^n u_n'(x) + \cdots \equiv$$

$$\equiv \lambda \int_{px}^{x} (x-t)[u_0(t) + \lambda u_1(t) + \lambda^2 u_2(t) + \cdots]dt$$

Bu tenglikning ikki tomonidagi teng darajali λ^m $(m = 0,1,2,\dots)$ larning koeffisientlarini tenglab, ketma-ket $u_o, u_1, u_2, \dots$ larni topamiz:

$$u_0'(x) = 0, \qquad bundan \qquad u_0(x) = C_0$$

C_0 −ixtiyoriy o'zgarmas son.

$$u_1'(x) = \int_{px}^{x} (x-t)u_0(t)dt = \int_{px}^{x} (x-t)C_0 dt = C_0 x \int_{px}^{x} dt - C_0 \int_{px}^{x} t dt =$$

$$= C_0 x(x - px) - C_0\left(\frac{x^2}{2} - \frac{p^2x^2}{2}\right) = C_0 x^2(1-p) - C_0 x^2\left(\frac{1}{2} - \frac{p^2}{2}\right)$$

$$= C_0 \frac{x^2}{2}(1-p)^2,$$

bundan x bo'yicha integral olinsa,

$$u_1(x) = C_0 \frac{x^3}{2 \cdot 3}(1-p)^2 + C_1, \qquad A_1 = \frac{1}{2 \cdot 3}(1-p)^2,$$

$$u_1(x) = C_1 + C_0 A_1 x^3.$$

Shuningdek,

$$u_2'(x) = \int_{px}^{x} (x-t)u_1(t)dt = \int_{px}^{x} (x-t)(C_1 + C_0A_1t^3)dt =$$

$$= C_1x\int_{px}^{x} dt + C_0A_1x\int_{px}^{x} t^3dt - C_1\int_{px}^{x} tdt - C_0A_1\int_{px}^{x} t^4dt =$$

$$C_1x(x-px) + C_0A_1x\left(\frac{x^4}{4} - \frac{p^4x^4}{4}\right) - C_1\left(\frac{x^2}{2} - \frac{p^2x^2}{2}\right) - C_0A_1\left(\frac{x^5}{5} - \frac{p^5x^5}{5}\right) =$$

$$= C_1x^2(1-p) + C_0A_1x^5\left(\frac{1}{4} - \frac{p^4}{4}\right) - C_1x^2\left(\frac{1}{2} - \frac{p^2}{2}\right) - C_0A_1x^5\left(\frac{1}{5} - \frac{p^5}{5}\right) =$$

$$= C_1\frac{x^2}{2}(1-p)^2 + C_1A_1\frac{x^5}{4}(1-p^4) - C_0A_1\frac{x^5}{5}(1-p^5),$$

bundan x bo'yicha integral olinsa,

$$u_2(x) = C_1\frac{x^3}{2\cdot 3}(1-p)^2 + C_0A_1\frac{x^6}{4\cdot 6}(1-p^4) - C_0A_1\frac{x^6}{5\cdot 6}(1-p^5) =$$

$$= C_1\frac{x^3}{2\cdot 3}(1-p)^2 + C_0A_1x^6\left(\frac{1}{4\cdot 6}(1-p^4) - \frac{1}{5\cdot 6}(1-p^5)\right) + C_2$$

$$A_2 = \frac{1}{4\cdot 6}(1-p^4) - \frac{1}{5\cdot 6}(1-p^5);$$

$$u_2(x) = C_2 + C_1A_1x^3 + C_0A_1A_2x^6,$$

shu yo'sinda $u_3(x)$ ni topish mumkin:

$$u_3(x) = C_3 + C_2A_1x^3 + C_1A_1A_2x^6 + C_0A_1A_2A_3x^9$$

$$A_3 = \frac{1}{7\cdot 9}(1-p^7) - \frac{1}{8\cdot 9}(1-p^8)$$

va hokazo, umuman,

$$u_n(x) = C_n + C_{n-1}A_1x^3 + C_{n-2}A_1A_2x^6 + \dots + C_0A_1A_2\dots A_nx^{3n}$$

$$A_{n+1} = \frac{1}{(3n+1)(3n+3)}(1-p^{3n+1}) - \frac{1}{(3n+2)(3n+3)}(1-p^{3n+2})$$

$n = 1,2,3,\dots$

Mana shu u_n larning ifodalarini (2.1.1) qatorga qo'yib va o'shash hadlarni ixchamlash natijasida berilgan tenglamaning umumiy yechimi kelib chiqadi:

$$u(x) = C[1 + A_1(\lambda x^3) + A_1A_2(\lambda x^3)^2 + \cdots + A_1A_2 \ldots A_n(\lambda x^3)^n + \cdots]$$

bunda $C = C_0 + \lambda C_1 + \lambda^2 C_2 + \cdots$

2.1.2-misol. $u'(x) = ax + \lambda \int_{px}^{x} \frac{u(t)}{t} dt, \qquad 0 < p < 1.$

Izlanayotgan yechimni (2.1.1) qator ko'rinishida olamiz va uni tenglamaga qo'yamiz. Natijada quyidagi ayniyat hosil bo'ladi.

$$u_0'(x) + \lambda u_1'(x) + \lambda^2 u_2'(x) + \cdots + \lambda^n u_n'(x) + \cdots \equiv$$

$$\equiv ax + \lambda \int_{px}^{x} \frac{1}{t}[u_0(t) + \lambda u_1(t) + \lambda^2 u_2(t) + \cdots]\, dt.$$

Bu tenglikning ikki tomonidagi teng darajali $\lambda^m (m = 0,1,2,\ldots)$ larning koeffisientlarini tenglab, ketma-ket $u_0, u_1, u_2, \ldots$ larni topamiz:

$u_0'(x) = ax,$ bundan $u_0(x) = a\frac{x^2}{2} + C_0.$

$C_0 -$ ixtiyoriy o'zgarmas son; hisoblash ishlarini qisqartish maqsadida C_0 ni va bundan keyingi ixtiyoriy o'zgarmas $C_1, C_2, \ldots$ larni nolga teng deb hisoblaymiz. U holda biz tenglamaning bitta xususiy yechimini topgan bo'lamiz. Shu sababli

$$u_1'(x) = \int_{px}^{x} \frac{1}{t} u_0(t) dt\, u_0(t) = a\frac{t^2}{2};$$

$$u_1'(x) = \int_{px}^{x} \frac{1}{t} \cdot \frac{at^2}{2} dt = \frac{a}{2}\int_{px}^{x} t dt = \frac{a}{2}\left(\frac{x^2}{2} - \frac{p^2x^2}{2}\right) = \frac{a}{2} \cdot \frac{x^2}{2}(1 - p^2),$$

bundan x bo'yicha integral olinsa,

$$u_1(x) = \frac{a}{2} \cdot \frac{x^3}{2 \cdot 3}(1 - p^2) + C_1; \qquad C_1 = 0, \qquad A_1 = \frac{1}{2 \cdot 3}(1 - p^2),$$

$$u_1(x) = \frac{a}{2} A_1 x^3.$$

shuningdek,

$$u_2'(x) = \int_{px}^{x} \frac{1}{t} u_1(t)dt =$$

$$= \frac{a}{2} A_1 \int_{px}^{x} \frac{1}{t} \cdot t^3 dt = \frac{a}{2} A_1 \int_{px}^{x} t^2 dt = \frac{a}{2} A_1 \left(\frac{x^3}{3} - \frac{p^3 x^3}{3} \right) =$$

$$= \frac{a}{2} A_1 \frac{x^3}{3} (1 - p^3),$$

bundan x bo'yicha integral olinsa,

$$u_2(x) = \frac{a}{2} A_1 \frac{x^4}{3 \cdot 4} (1 - p^3) + C_2, \qquad C_2 = 0, \qquad A_2 = \frac{1}{3 \cdot 4} (1 - p^3),$$

$$u_2(x) = \frac{a}{2} A_1 A_2 x^4.$$

Shu yo'sinda $u_3(x)$ ni topish mumkin:

$$u_3(x) = \frac{a}{2} A_1 A_2 A_3 x^5, \qquad A_3 = \frac{1}{4 \cdot 5} (1 - p^4)$$

va hokazo, umuman,

$$u_n(x) = \frac{a}{2} A_1 A_2 \dots A_n x^{n+2},$$

$$A_n = \frac{1}{(n+1)(n+2)} (1 - p^{n+1}) \ ; \qquad n = 0,1,2,\dots$$

Mana shu u_n larning ifodalarini (2.1.1) qatorga qo'yib va o'xshash hadlarni ixchamlash natijasida berilgan tenglamaning umumiy yechimi kelib chiqadi:

$$u(x) = C + \frac{a}{2} x^2 [1 + A_1(\lambda x) + A_1 A_2 (\lambda x)^2 + \dots + A_1 A_2 \dots A_n (\lambda x)^n + \dots]$$

2.1.3-misol.

$$u'(x) = \lambda \int_{x-\sigma}^{x} e^{it} u(t)\, dt, \qquad \sigma > 0, \qquad i = \sqrt{-1}\,, \qquad i^2 = -1.$$

Izlanayotgan yechimni (2.1.1) qator ko'rinishida olamiz va uni tenglamaga qo'yamiz. Natijada quyidagi ayniyat hosil bo'ladi:

$$u_0'(x) + \lambda u_1'(x) + \lambda^2 u_2'(x) + \dots + \lambda^n u_n'(x) + \dots \equiv$$

$$\equiv \lambda \int_{x-\sigma}^{x} e^{it} [u_0(t) + \lambda u_1(t) + \lambda^2 u_2(t) + \dots] dt.$$

Bu tenglikning ikki tomonidagi teng darajali $\lambda^m (m = 0,1,2,\dots)$ larning koeffisientlarini tenglab, ketma-ket $u_0, u_1, u_2, \dots$ larni topamiz:

$u_0'(x) = 0,$ bundan $u_0(x) = c_{0.}$

$C_0 -$ ixtiyoriy o'zgarmas son.

$$u_1'(x) = \int_{x-\sigma}^{x} e^{it} u_0(t) dt = C_0 \int_{x-\sigma}^{x} e^{it} dt =$$

$$= C_0 \frac{1}{i}\left(e^{ix} - e^{ix-i\sigma}\right) = C_0 \frac{1}{i} e^{ix}\left(e^{-i\sigma} - 1\right),$$

bundan x bo'yicha integral olinsa,

$$u_1(x) = C_1 + C_0 A_1 e^{ix}, \qquad A_1 = \frac{1}{i^2}\left(1 - e^{-i\sigma}\right), \quad i = \sqrt{-1}.$$

$A_1 = (e^{-i\sigma} - 1).$

Shuningdek

$$u_2'(x) = \int_{x-\sigma}^{x} e^{it} u_1(t)\, dt = \int_{x-\sigma}^{x} e^{it}\left(C_1 + C_0 A_1 e^{it}\right) dt =$$

$$= C_1 \int_{x-\sigma}^{x} e^{it} dt + C_0 A_1 \int_{x-\sigma}^{x} e^{2it} dt =$$

$$= C_1 \frac{1}{i}\left(e^{ix} - e^{ix-i\sigma}\right) + C_0 A_1 \frac{1}{2i}\left(e^{2ix} - e^{2ix-2i\sigma}\right) =$$

$$= C_1 \tfrac{1}{i} e^{ix}\left(1 - e^{-i\sigma}\right) + C_0 A_1 \tfrac{1}{2i} e^{2ix}(1 - e^{-2i\sigma}),$$

bundan x bo'yicha integral olinsa,

$$u_2(x) = C_2 + C_1 A_1 e^{ix} + C_0 A_1 A_2 e^{2ix}, \qquad A_2 = \frac{1}{2i^2}\left(1 - e^{-2i\sigma}\right),$$

$A_2 = \frac{1}{2^2}(e^{-2i\sigma} - 1).$

Shu yo'sinda $u_3(x)$ ni topish mumkin:

$$u_3(x) = C_3 + C_2 A_1 e^{ix} + C_1 A_1 A_2 e^{2ix} + C_0 A_1 A_2 A_3 e^{3ix},$$

$$A_3 = \frac{1}{3^2}(e^{-3i\sigma} - 1)$$

va hokazo, umuman,

$$u_n(x) = C_n + C_{n-1}A_1e^{ix} + C_{n-2}A_1A_2e^{2ix} + \cdots + C_0A_1A_2 \dots A_ne^{ixn},$$

$$A_n = \frac{1}{n^2}\left(e^{-ni\sigma} - 1\right); \qquad n = 1,2,3,\dots$$

Mana shu u_n larning ifodalarini (2.1.1) qatorga qo'yib va o'shash hadlarni ixchamlash natijasida berilgan tenglamaning umumiy yechimi kelib chiqadi:

$$u(x) = C\left[1 + A_1\left(\lambda e^{ix}\right) + A_1A_2(\lambda e^{ix})^2 + \cdots + A_1A_2 \dots A_n(\lambda e^{ix})^n + \cdots\right],$$

bunda $C = C_0 + \lambda C_1 + \lambda^2 C_2 \dots$

2.2. Ikki argumentli funksiyalar uchun Integro-differensial tenglamalarni yechish.

Agar integro-differensial tenglamadagi noma'lum funksiya ikki argumentli bo'lsa, tenglamada uning xususiy hosilalari qatnashadi. Shuning uchun bunday tenglamalar xususiy hosilali integro-differensial tenglamalar deyiladi. Ularning yechimini ushbu

$$u(x,y) = u_0(x,y) + \lambda u_1(x,y) + \lambda^2 u_2(x,y) + \cdots + \lambda^n u_n(x,y) + \cdots \qquad (2.2.1)$$

funksional qator shaklida izlaymiz , ya'ni ketma-ket yaqinlashish usulida yechamiz.

2.2.1-misol.

$$\frac{\partial u(x,y)}{\partial x} = \lambda \int_{px}^{x}\int_{qy}^{y} xyt_1t_2u(t_1,t_2)dt_1,dt_2, \qquad (2.2.2)$$

bunda $0 < p < 1, \qquad 0 < q < 1.$

Yechim (2.2.1) qator ko'rinishida yozilgan deb faraz qilib, uni tenglamaga qo'yamiz. Natijada ushbu ayniyat hosil bo'ladi:

$$\frac{\partial u_0(x,y)}{\partial x} + \lambda\frac{\partial u_1(x,y)}{\partial x} + \lambda^2\frac{\partial u_2(x,y)}{\partial x} + \cdots + \lambda^n\frac{\partial u_n(x,y)}{\partial x} + \cdots \equiv$$

$$\equiv \lambda \int_{px}^{x}\int_{qy}^{y} xyt_1t_2[u_0(t_1,t_2) + \lambda u_1(t_1,t_2) + \lambda^2 u_2(t_1,t_2) + \cdots]dt_1dt_2.$$

Tenglikning ikki tomonidagi bir xil darajali $\lambda^m(m = 0,1,2,\dots)$ larning koeffisientlarini tenglab, ketma-ket $u_0, u_1, u_2, \dots$ larni topib olamiz:

$$\frac{\partial u_0(x,y)}{\partial x} = 0,$$

buni x ga nisbatan integrallasak,

$$u_0(x,y) = C_0(y)$$

hosil bo'ladi. $C_0(y)$- o'zgaruvchi , y ning ixtiyoriy funksiyasi bo'lib, x ga esa bo'liq emas. Endi $u_1(x,y)$ ni aniqlaymiz:

$$\frac{\partial u_{1(x,y)}}{\partial x} = \int_{px}^{x}\int_{qy}^{y} xyt_1t_2u_0(t_1,t_2)dt_1dt_2 = \int_{px}^{x}\int_{qy}^{y} xy\, t_1t_2C_0(t_2)dt_1dt_2 =$$

$$= xy\int_{px}^{x} t_1dt_1\int_{qy}^{y} t_2C_0(t_2)dt_2 = xy\left(\frac{x^2}{2} - \frac{p^2x^2}{2}\right)\varphi_0(y),$$

bu yerda $\varphi_0(y)=\int_{qy}^{y} t_2C_0(t_2)dt_2;$

bundan x bo'yicha integral olsak,

$$u_1(x,y) = \frac{x^4}{2\cdot 4}(1-p^2)y\varphi_0(y) + C_1(y)A_1 = \frac{1}{2\cdot 4}(1-p^2):$$

$$u_1(x,y) = C_1(y) + A_1x^4y\varphi_0(y).$$

Endi $u_2(x,y)$ ni aniqlaymiz:

$$\frac{\partial u_2(x,y)}{\partial x} = \int_{px}^{x}\int_{qy}^{y} xyt_1t_2u_1(t_1t_2)dt_1dt_2 =$$

$$= \int_{px}^{x}\int_{qy}^{y} xyt_1t_2\big(C_1(t_2) + A_1t_1^4t_2\varphi_0(t_2)\big)dt_1dt_2 =$$

$$= xy\int_{px}^{x} t_1dt_1\int_{qy}^{y} t_2c_1(t_2)dt_2 + A_1\int_{px}^{x} t_1^5dt_1\int_{qy}^{y} t_2^2\varphi_0(t_2)dt_2 =$$

$$= \frac{x^3}{2}(1-p^2)y\varphi_1(y) + A_1y\frac{x^7}{6}(1-p^6)\psi_0(y),$$

bu yerda $\varphi_1(y) = \int_{qy}^{y} t_2C_1(t_2)dt_2;$ $\psi_0(y) = \int_{qy}^{y} t_2^2\varphi_0(t_2);$

bundan x bo'yicha integral olsak,

$$u_2(x,y) = \frac{x^4}{2\cdot 4}(1-p^2)y\varphi_1(y) + A_1 y\frac{x^8}{6\cdot 8}(1-p^8)\psi_0(y) + C_2(y).$$

$$A_2 = \frac{1}{6\cdot 8}(1-p^8),$$

$$u_2(x,y) = C_2(y) + A_1x^4y\varphi_1(y) + A_1A_2x^8y\psi_0(y).$$

Shuningdek,

$$u_3(x,y) = C_3(y) + A_1x^4y\varphi_2(y) + A_1A_2x^8y\psi_1(y) + A_1A_2A_3x^{12}y\omega_0(y),$$

bunda

$$A_3 = \frac{1}{10\cdot 12}(1-p^{10}), \varphi_2(y) = \int_{qy}^{y} t_2C_2(t_2)\,dt_2,$$

$$\psi_1(y) = \int_{qy}^{y} t_2^2\varphi_1(t_2)\,dt_2, \qquad \omega_0(y) = \int_{qy}^{y} t_2^2\psi_0(t_2).$$

va hokazo. Umumiy qonuniyat ma'lum bo'lib qoldi. Endi $u_0, u_1, u_2, \dots$ larning shu topilgan ifodalarini (2.2.1) qatorga qo'yib ixchamlashtirsak, ushbu yechim hosil bo'ladi:

$$u(x,y) = C_0(y) + \lambda[C_1(y) + A_1x^4y\varphi_0(y)] +$$

$$+\lambda^2[C_2(y) + A_1x^4y\varphi_1(y) + A_1A_2x^8y\psi_0(y)] +$$

$$+\lambda^3[C_3(y) + A_1x^4y\varphi_2(y) + A_1A_2x^8y\psi_1(y) + A_1A_2A_3x^{12}y\omega_0(y)] + \cdots =$$

$$= [C_o(y) + \lambda C_1(y) + \lambda^2C_2(y) + \cdots + \lambda^nC_n(y) + \cdots] +$$

$$+\lambda A_1x^4y[\varphi_0(y) + \lambda\varphi_1(y) + \lambda^2\varphi_2(y) + \cdots] +$$

$$+\lambda^2A_1A_2x^8y[\psi_0(y) + \lambda\psi_1(y) + \lambda^2\psi_2(y) + \cdots] +$$

$$+\lambda^3A_1A_2A_3x^{12}y[\omega_0(y) + \lambda\omega_1(y) + \lambda^2\omega_2(y) + \cdots] + \cdots$$

Qisqacha bu yechimni quyidagicha belgilaymiz:

$$u(x) = C(y) + \lambda A_1x^4yP_1(y) + \lambda^2A_1A_2x^8yP_2(y) + \cdots +$$

$$+\lambda^nA_1A_2\dots A_nx^{4n}yP_n(y) + \cdots, \qquad (2.2.3)$$

bunda

$$A_n = \frac{1}{(4n-2)\cdot 4n}(1-p^{4n-2}), \qquad n = 1,2,\dots \qquad (2.2.4)$$

va

$$C(y) = C_0(y) + \lambda C_1(y) + \lambda^2C_2(y) + \cdots$$

$C(y)$ funksiya y ning ixtiyoriy funksiyasidir.

Endi $P_i(y)$ funksiyalarni aniqlash maqsadida (2.2.3) qatorni berilgan (2.2.2) tenglamaga qo'yib , ushbu ayniyatni hosil qilamiz:

$$4\lambda A_1 x^3 y P_1(y) + 8\lambda^2 A_1 A_2 x^7 y P_2(y) + 12\lambda^3 A_1 A_2 A_3 x^{11} y P_3(y) + \cdots \equiv$$

$$\equiv \lambda \int_{px}^{x} \int_{qy}^{y} x y t_1 t_2 [C(t_2) + \lambda A_1 t_1^4 t_2 P_1(t_2) + \lambda^2 A_1 A_2 t_1^8 t_2 P_2(t_2) + \cdots] dt_1 dt_2,$$

buning ikki tomonidagi bir xil darajali $\lambda, \lambda^2, \lambda^3, \dots$ larning koeffisientlarini tenglab olsak, quyidagi munosabatlar hasil bo'ladi:

$$P_1(y) = \int_{qy}^{y} t_2 C(t_2) dt_2,$$

$$P_2(y) = \int_{qy}^{y} t_2^2 P_1(t_2) dt_2,$$

$$P_3(y) = \int_{qy}^{y} t_2^2 P_2(t_2) dt_2, \qquad (2.2.5)$$

$$P_n(y) = \int_{qy}^{y} t_2^2 P_n(t_2) dt_2,$$

. . . .

Demak, barcha $P_i(y)$ lar ixtiyoriy va integrallanuvchi $C(y)$ funksiyaga bog'liq ekan.

Xususiy holda , agar $C(y) \equiv 1$ deb faraz qilsak, (2.2.5) dan

$$P_1(y) = \int_{qy}^{y} t_2 dt_2 = \left(\frac{y^2}{2} - \frac{q^2 y^2}{2}\right) = \frac{y^2}{2}(1 - q^2), \qquad B_1 = \frac{1}{2}(1 - q^2),$$

$P_1(y) = B_1 y^2$.

shuningdek,

$$P_2(y) = \int_{qy}^{y} t_2^2 P_1(t_2) dt_2 = \int_{qy}^{y} B_1 t_2^4 dt_2 = B_1 \left(\frac{y^5}{5} - \frac{q^5 y^5}{5}\right) = B_1 \frac{y^5}{5}(1 - q^5),$$

$$B_2 = \frac{1}{5}(1 - q^5),$$

$$P_2(y) = B_1 B_2 y^5.$$

shu yo'sinda $P_n(y)$ ni topish mumkin:

$$P_n(y) = B_1 B_2 B_3 \dots B_n y^{3n-1}, \qquad B_n = \frac{1}{3n-1}(1 - q^{3n-1}),$$

$$n = 1,2,3,\dots \tag{2.2.6}$$

bo'ladi. U holda berilgan (2.2.2) tenglamaning ushbu xususiy yechimi kelib chiqadi.

$$u(x,y) = 1 + A_1 B_1(\lambda x^4 y^3) + A_1 A_2 B_1 B_2 (\lambda x^4 y^3)^2 +$$

$$+A_1 A_2 A_3 B_1 B_2 B_3 (\lambda x^4 y^3)^3 + \dots + A_1 A_2 \dots A_n B_1 B_2 \dots B_n (\lambda x^4 y^3)^n + \dots \tag{2.2.7}$$

Tekshirish shuni ko'rsatadiki, bu qator λ, x, y larning barcha chekli qiymatlarida absolyut va tekis yaqinlashuvchi bo'ladi.

2.2.2-misol. Ushbu tenglama yechilsin:

$$\frac{\partial u(x,y)}{\partial x} = e^{x+y} + \lambda \int_{x-\sigma}^{x} \int_{y-\omega}^{y} e^{x+y+t_1+t_2} u(t_1,t_2) dt_1 dt_2, \tag{2.2.8}$$

bunda $\sigma > 0$, $\omega > 0$ o'zgarmas haqiqiy sonlardir.

Bu tenglamadagi noma'lum funksiya o'rniga (2.2.1) qatorni qo'yamiz. Hosil bo'lgan ayniyatning ikki tomonidagi bir xil darajali $\lambda^m (m = 0,1,\dots)$ larning koeffisientlarini o'zaro tenglab birin- ketin $u_0, u_1, u_2, \dots$ larni topamiz:

$$\frac{\partial u_0(x,y)}{\partial x} + \lambda \frac{\partial u_1(x,y)}{\partial x} + \lambda^2 \frac{\partial u_2(x,y)}{\partial x} + \dots + \lambda^n \frac{\partial u_n(x,y)}{\partial x} + \dots \equiv$$

$$\equiv e^{x+y} + \lambda \int_{x-\sigma}^{x} \int_{y-\sigma}^{y} e^{x+y+t_1+t_2} [u_0(t_1,t_2) + \lambda u_1(t_1,t_2) + \dots] dt_1 dt_2.$$

$$\frac{\partial u_0(x,y)}{\partial x} = e^{x+y} = e^x e^y;$$

buni x ga nisbatan integrallasak,

$$u_0(x,y) = e^y \int e^x dx + C_0(y) = e^y e^x + C_0(y),$$

ya'ni

$$u_0(x,y) = C_0(y) + e^{x+y}.$$

Bu yerda $C_0(y)-$ ixtiyoriy funksiya bo'lib, x ga bog'liq emas. Endi $u_1(x,y)$ ni aniqlaymiz:

$$\frac{\partial u_1(x,y)}{\partial x} = \int_{x-\sigma}^{x}\int_{y-\omega}^{y} e^{x+y} e^{t_1+t_2} u_0(t_1,t_2) dt_1 dt_2 =$$

$$= \int_{x-\sigma}^{x}\int_{y-\omega}^{y} e^{x+y} e^{t_1+t_2} [C_0(t_2) + e^{t_1+t_2}]\, dt_1 dt_2 =$$

$$= e^{x+y}\int_{x-\sigma}^{x}\int_{y-\omega}^{y} C_0(t_2) e^{t_1+t_2} dt_1 dt_2 + e^{x+y}\int_{x-\sigma}^{x} e^{2t_1} dt_1 \int_{y-\omega}^{y} e^{2t_2} dt_2,$$

bundan

$$\varphi_1(x,y) = e^{x+y}\int_{x-\sigma}^{x}\int_{y-\omega}^{y} C_0(t_2) e^{t_1+t_2} dd_1 dt_2,$$

$$\frac{\partial u_1(x,y)}{\partial x} = \varphi_1(x,y) + \frac{1}{2^2}(1-e^{-2\sigma})(1-e^{-2\omega})e^{3(x+y)}.$$

Bu yerda birinchi integralni $\varphi_1(x,y)$ orqali belgiladik, endi tenglikning ikkala tomonini x bo'yicha integrallash natijasida

$$u_1(x,y) = C_1(y) + \psi_1(x,y) + \frac{1}{3}B_1 e^{3(x+y)}$$

kelib chiqadi, bunda

$$B_1 = \frac{1}{2^2}(1-e^{-2\sigma})(1-e^{-2\omega}), \qquad \psi_1(x,y) = \int \varphi_1(x,y) dx.$$

shuningdek,

$$\frac{\partial u_2(x,y)}{\partial x} = \int_{x-\sigma}^{x}\int_{y-\omega}^{y} e^{x+y} e^{t_1+t_2} u_1(t_1,t_2) dt_1 dt_2 =$$

$$= \int_{x-\sigma}^{x}\int_{y-\omega}^{y} e^{x+y} e^{t_1+t_2}\left[C_1(t_2) + \psi_1(t_1,t_2) + \frac{1}{3}B_1 e^{3(t_1+t_2)}\right] dt_1 dt_2 =$$

$$= e^{x+y}\int_{x-\sigma}^{x}\int_{y-\omega}^{y} C_1(t_2)e^{t_1+t_2}dt_1dt_2 +$$

$$+e^{x+y}\int_{x-\sigma}^{x}\int_{y-\omega}^{y} \psi_1(t_1,t_2)e^{t_1+t_2}dt_1dt_1 +$$

$$+\frac{1}{3}B_1e^{x+y}\int_{x-\sigma}^{x} e^{4t_1}dt_1\int_{y-\omega}^{y} e^{4t_2}dt_2,$$

$$\frac{\partial u_2(x,y)}{\partial x} = \varphi_2(x,y) + \phi_2(x,y) + \frac{1}{3}B_1\frac{1}{4^2}(1-e^{-4\sigma})(1-e^{-4\omega})e^{5(x+y)}.$$

bunda

$$\varphi_2(x,y) = e^{x+y}\int_{x-\sigma}^{x}\int_{y-\omega}^{y} C_1(t_2)e^{t_1+t_2}dt_1dt_2,$$

$$\phi_2(x,y) = e^{x+y}\int_{x-\sigma}^{x}\int_{y-\omega}^{y} \psi_1(t_1,t_2)e^{t_1+t_2}dt_1dt_2.$$

Bu yerda birinchi va ikkinchi integralni $\varphi_2(x,y)$ va $\phi_2(x,y)$ orqali belgiladik, endi tenglikning ikkala tomonini x bo'yicha integrallash natijasida

$$u_2(x,y) = C_2(y) + \psi_2(x,y) + \omega_2(x,y) + \frac{1}{3\cdot 5}B_1B_2e^{5(x+y)}$$

kelib chiqadi, bunda

$$B_2 = \frac{1}{4^2}(1-e^{-4\sigma})(1-e^{-4\omega}), \qquad \psi_2(x,y) = \int \varphi_2(x,y)dx,$$

$$\omega_2(x,y) = \int \phi_2(x,y)dx.$$

va hokazo. Umumiy qonuniyat ko'rinib qolgani uchun ixtiyoriy u_n ning ifodasini yozish mumkin. Endi $u_0, u_1, u_2, \ldots$ larning ifodalarini (2.2.1) qatorga qo'yamiz, u holda

$$u(x,y) = C_0(y) + e^{x+y} + \lambda\left[C_1(y) + \psi_1(x,y) + \frac{1}{3}B_1e^{3(x+y)}\right] +$$

$$+\lambda^2\left[C_2(y) + \psi_2(x,y) + \omega_2(x,y) + \frac{1}{3\cdot 5}B_1B_2e^{5(x+y)}\right] + \cdots =$$

$$= [C_0(y) + \lambda C_1(y) + \lambda^2 C_2(y) + \cdots + \lambda^n C_n(y) + \cdots] +$$

$$+\lambda[\psi_1(x,y)+\lambda\psi_2(x,y)+\lambda^2\psi_3(x,y)+\cdots]+$$

$$+\lambda^2[\omega_2(x,y)+\lambda\omega_3(x,y)+\lambda^2\omega_4(x,y)+\cdots]+\cdots+$$

$$+e^{x+y}\left[1+\frac{\lambda}{3}B_1e^{2(x+y)}+\frac{\lambda^2}{3\cdot 5}B_1B_2e^{4(x+y)}+\cdots\right]$$

hosil bo'ladi. Buni qisqacha quyidagicha yozish qulay:

$$u(x,y)=C(y)+e^{x+y}\left[1+\lambda A_1e^{2(x+y)}+\lambda^2A_2e^{4(x+y)}+\lambda^3A_3e^{6(x+y)}+\cdots\right]+$$

$$+\lambda[P_1(x,y)+\lambda P_2(x,y)+\lambda^2P_3(x,y)+\cdots+\lambda^{n-1}P_n(x,y)+\cdots] \quad (2.2.9)$$

Mana shu yechimdagi $P_i(x,y)$ hamda A_i larni topish maqsadida (2.2.9) yechimni berilgan (2.2.8) tenglamaga qo'yib ayniyat hosil qilamiz. So'ngra uning ikki tomonidagi bir xil darajali λ^m larning koeffisientlarini o'zaro tenglab olamiz. U holda qator tengliklar hosil bo'lib, ulardan birinchisi quyidagicha yoziladi:

$$3A_1e^{3(x+y)}+\frac{\partial P_1(x,y)}{\partial x}=e^{x+y}\int_{x-\sigma}^{x}e^{2t_1}\,dt_1\int_{y-\omega}^{y}e^{2t_2}dt_2+$$

$$+e^{x+y}\int_{x-\sigma}^{x}e^{t_1}dt_1\int_{y-\omega}^{y}C(t_2)e^{t_2}dt_2.$$

Agar ikkala tomondagi birinchi hadlarning bir-biriga o'shashligidan foydalanib, ularni o'zaro tenglab olsak, u holda ikkinchi hadlar ham o'zaro teng bo'lib qoladi. Demak,

$$3A_1e^{3(x+y)}=e^{x+y}\int_{x-\sigma}^{x}e^{2t_1}dt_1\int_{y-\omega}^{y}e^{2t_2}dt_2=$$

$$=\frac{1}{2^2}(1-e^{-2\sigma})(1-e^{-2\omega})e^{3(x+y)},$$

bundan

$$A_1=\frac{1}{3}\cdot\frac{1}{2^2}(1-e^{-2\sigma})(1-e^{-2\omega}).$$

Xuddi shu usulda A_2 ni topish mumkin:

$$5A_2e^{5(x+y)}=e^{x+y}A_1\int_{x-\sigma}^{x}e^{4t_1}dt_1\int_{y-\omega}^{y}e^{4t_2}dt_2=$$

$$=\frac{1}{4^2}(1-e^{-4\sigma})(1-e^{-4\omega})e^{5(x+y)},$$

bundan

$$A_2=\frac{1}{5}\cdot\frac{1}{4^2}A_1(1-e^{-4\sigma})(1-e^{-4\omega})$$

Xuddi shu usulda A_3 ni topish mumkin:

$$A_3=\frac{1}{7}\cdot\frac{1}{6^2}A_1A_2(1-e^{-6\sigma})(1-e^{-6\omega})$$

Tekshirishlar shuni ko'rsatadiki, umuman,

$$A_n=\frac{1}{2n+1}\cdot\frac{1}{(2n)^2}A_1A_2\ldots A_n(1-e^{-2n\sigma})(1-e^{-2n\omega}),n=1,2,3,\ldots \tag{2.2.10}$$

So'ngra

$$\frac{\partial P_1(x,y)}{\partial x}=e^{x+y}\int_{x-\sigma}^{x}e^{t_1}dt_1\int_{y-\omega}^{y}e^{t_2}C(t_2)dt_2 \tag{2.2.11}$$

Agar $C(t)$ ma'lum bo'lsa, (2.2.11) dan $P_1(x,y)$ ni topa olamiz.

$C(y)\equiv 1$ bo'lsin, u holda (2.2.11) dan

$$\frac{\partial P_1(x,y)}{\partial x}=e^{x+y}\int_{x-\sigma}^{x}e^{t_1}dt_1\int_{y-\omega}^{y}e^{t_2}dt_2=$$

$$=e^{x+y}(e^x-e^{x-\sigma})(e^y-e^{y-\omega})=e^{2x}e^{2y}(1-e^{-\sigma})(1-e^{-\omega}),$$

bundan x bo'yicha integral olinsa,

$$P_1(x,y)=\frac{1}{2}q_1e^{2(x+y)},\qquad q_1=(1-e^{-\sigma})(1-e^{-\omega})$$

kelib chiqadi;

$$\frac{\partial P_2(x,y)}{\partial x}=e^{x+y}\int_{x-\sigma}^{x}e^{t_1}dt_1\int_{y-\omega}^{y}e^{t_2}P_1(t_1,t_2)dt_2.$$

Bu yerda P_1 o'rniga uning yuqoridagi aniqlangan ifodasini qo'yib, P_1 ni topish mumkin.

$$P_2(x,y)=\frac{1}{2\cdot 4}q_2e^{4(x+y)},$$

$$q_2=\frac{1}{3^2}q_1(1-e^{-3\sigma})(1-e^{-3\omega})=$$

$$=\frac{1}{1^2\cdot 3^2}(1-e^{-\sigma})(1-e^{-\omega})(1-e^{-3\sigma})(1-e^{-3\omega})$$

kelib chiqadi. Shuningdek,

$$P_3(x,y) = \frac{1}{2\cdot 4\cdot 6} q_3 e^{6(x+y)}, \qquad q_3 = \frac{1}{5^2} q_2 (1-e^{-5\sigma})(1-e^{-5\omega})$$

va hokazo.

$$P_n(x,y) = \frac{1}{2\cdot 4\cdot 6 \ldots (2n)} q_n e^{2n(x+y)},$$

$$q_n = \frac{1}{(2n-1)^2} q_n \left(1-e^{-(2n-1)\sigma}\right)\left(1-e^{-(2n-1)\omega}\right), \qquad n = 1,2,3,\ldots$$

Natijada yechimning ko'rinishi quyidagicha bo'lib qoladi:

$$u(x,y) = 1 + e^{x+y}\left[1 + \lambda A_1 e^{2(x+y)} + \lambda^2 A_2 e^{4(x+y)} + \lambda^3 A_3 e^{6(x+y)} + \cdots\right] +$$

$$+\lambda e^{2(x+y)}\left[\frac{q_1}{2} + \frac{\lambda q_2}{2\cdot 4} e^{2(x+y)} + \frac{\lambda^2 q_3}{2\cdot 4\cdot 6} e^{4(x+y)} + \cdots\right]$$

Mashqlar.

Quyidagi xususiy hosilali integro-differensial tenglamalar ketma-ket yaqinlashish usuli bilan yechilsin:

$$2.2.1 - misol. \qquad \frac{\partial u(x,y)}{\partial y} = \lambda \int_{px}^{x}\int_{qy}^{y} (xy - t_1 t_2) u(t_1,t_2)\,dt_1 dt_2,$$

bunda $0 < p < 1$ va $0 < q < 1$.

Yechim (1) qator ko'rinishida yozilgan deb faraz qilib, uni tenglamaga qo'yamiz. Natijada ushbu ayniyat hosil bo'ladi:

$$\frac{\partial u_0(x,y)}{\partial y} + \lambda \frac{\partial u_1(x,y)}{\partial y} + \lambda^2 \frac{\partial u_2(x,y)}{\partial y} + \cdots + \lambda^n \frac{\partial u_n(x,y)}{\partial y} + \cdots \equiv$$

$$\equiv \lambda \int_{px}^{x}\int_{qy}^{y} (xy - t_1 t_2)[u_0(t_1,t_2) + \lambda u_1(t_1,t_2) + \lambda^2 u_2(t_1,t_2) + \cdots]\,dt_1 dt_2.$$

Tenglikning ikki tomonidagi bir xil darajali λ^m $(m = 0,1,2,\ldots)$ larning koeffisientlarini tenglab birin-ketin $u_0, u_1, u_2, \ldots$ larni topib olamiz:

$$\frac{\partial u_0(x,y)}{\partial y} = 0,$$

buni y ga nisbatan integrallasak,

$u_0(x,y) = C_0(x)$

hosil bo'ladi. $C_0(x)-$ o'zgaruvchi, x ning ixtiyoriy funksiyasi bo'lib, y ga esa bog'liq emas. Endi $u_1(x,y)$ ni aniqlaymiz:

$$\frac{\partial u_1(x,y)}{\partial y} = \int_{px}^{x}\int_{qy}^{y}(xy - t_1t_2)u_0(t_1,t_2)dt_1dt_2 =$$

$$= xy\int_{px}^{x} C_0(t_1)dt_1 - \int_{px}^{x} t_1C_0(t_1)dt_1\int_{qy}^{y} t_2dt_2 =$$

$$= \left(\frac{y^2}{2} - \frac{q^2x^2}{2}\right)\varphi_0(x) = \frac{y^2}{2}(1-q^2)\varphi_0(x),$$

bu yerda $\varphi_0(x) = \int_{px}^{x} t_1C_1(t_1)dt_1;$

bundan y bo'yicha integral olsak,

$u_1(x,y) = C_1(x) + A_1y^3\varphi_0(x)$

bo'ladi , bunda

$$A_1 = \frac{1}{2\cdot 3}(1-q^2);$$

xuddi shu usulda topish mumkinki,

$u_2(x,y) = C_2(x) + A_1y^3\varphi_1(x) + A_1A_2y^6\psi_0(x),$

bunda

$$\varphi_1(x) = \int_{px}^{x} t_1C_1(t_1)dt_1, \qquad \psi_0(x) = \int_{px}^{x} t_1^2\varphi_0(t_1)dt_1, A_2 = \frac{1}{5\cdot 6}(1-q^5).$$

Shuningdek,

$u_3(x,y) = u_3(x) + A_1y^3\varphi_2(x) + A_1A_2y^6\psi_1(x) + A_1A_2A_3y^9\omega_0(x),$

bunda

$$A_3 = \frac{1}{8\cdot 9}(1-q^8), \qquad \varphi_2(x) = \int_{px}^{x} t_1C_2(t_1)dt_1,$$

$$\psi_1(x) = \int_{px}^{x} t_1^2\varphi_1(t_1)dt_1, \omega_0(x) = \int_{px}^{x} t_1^2\psi_0(t_1)dt_1$$

va hokazo. Umumiy qonuniyat ma'lum bo'lib qoldi. Endi $u_0, u_1, u_2, \dots$ larning shu topilgan ifodalarini (2.2.1) qatorga qo'yib ixchamlashtirsak, ushbu yechim hosil bo'ladi:

$$u(x,y) = C_0(x) + \lambda[C_1(x) + A_1x^3\varphi_0(x)] +$$
$$+\lambda^2[C_2(x) + A_1y^3\varphi_1(x) + A_1A_2y^6\psi_0(x)] +$$
$$+\lambda^3[C_3(x) + A_1y^3\varphi_2(x) + A_1A_2y^6\psi_1(x) + A_1A_2A_3y^9\omega_0(x)] + \cdots =$$
$$= [C_0(x) + \lambda C_1(x) + \lambda^2 C_2(x) + \cdots] +$$
$$+\lambda A_1y^3[\varphi_0(x) + \lambda\varphi_1(x) + \lambda^2\varphi_2(x) + \cdots] +$$
$$+\lambda^2A_1A_2y^6[\psi_0(x) + \lambda\psi_1(x) + \lambda^2\psi_2(x) + \cdots] +$$
$$+\lambda^3A_1A_2A_3y^9[\omega_0(x) + \lambda\omega_1(x) + \lambda^2\omega_2(x) + \cdots] + \cdots$$

Qisqacha bu yechimni quyidagicha belgilaymiz:

$$u(x,y) = C(x) + \lambda A_1y^3P_1(x) + \lambda^2A_1A_2y^6P_2(x) + \lambda^3A_1A_2A_3y^9P_3(x) +$$
$$+ \cdots + \lambda^nA_1A_2\dots A_ny^{3n}P_n(x) + \cdots \qquad (2.2.12)$$

bunda

$$A_n = \frac{1}{(3n-1)\cdot 3n}\left(1 - q^{(3n-1)}\right), \qquad n = 1,2,3,\dots$$

va

$$C(x) = C_0(x) + \lambda C_1(x) + \lambda^2C_2(x) + \cdots$$

$C(x)$ funksiya y ning ixtiyoriy funksiyasidir.

Endi $P_i(x)$ funksiyalarni aniqlash madsadida (2.2.12) qatorni berilgan (2.2.11) tenglamaga qo'yib, ushbu ayniyatni hosil qilamiz:

$$3\lambda A_1y^2P_1(x) + 6\lambda^2A_1A_2y^5P_2(x) + 9\lambda^3A_1A_2A_3y^8P_3(x) + \cdots \equiv$$
$$\equiv \lambda\int_{px}^{x}\int_{qy}^{y}(xy - t_1t_2)[C(t_1) + \lambda A_1t_1^3P_1(t_1) + \lambda^2A_1A_2t_1^6P_2(t_1) + \cdots]dt_1dt_2.$$

buning ikki tomonidagi bir xil darajali $\lambda, \lambda^2, \lambda^3, \dots$ larning koeffisientlarini tenglab olsak, quyidagi munosabatlar hosil bo'ladi:

$$3y^2P_1(x) = \int_{px}^{x}\int_{qy}^{y}(xy - t_1t_2)C(t_1)dt_1dt_2,$$

Agar $C(x) \equiv 1$ deb faraz qilinsa.

$$3y^2P_1(x) = \int_{px}^{x}\int_{qy}^{y}(xy - t_1t_2)dt_1dt_2 =$$

$$= x\int_{px}^{x}dt_1 y\int_{qy}^{y}dt_2 - \int_{px}^{x}t_1dt_1\int_{qy}^{y}t_2dt_2 =$$

$$= x^2y^2(1-p)(1-q) - \frac{x^2}{2}\cdot\frac{y^2}{2}(1-p^2)(1-q^2) =$$

$$= x^2y^2\left[(1-p)(1-q) - \frac{1}{4}(1-p^2)(1-q^2)\right],$$

$$P_1(x) = \frac{1}{3}x^2(1-p)(1-q) - \frac{1}{3\cdot 4}(1-p^2)(1-q^2).$$

$$P_1(x) = \frac{1}{3}x^2B_1.$$

$$B_1 = (1-p)(1-q) - \frac{1}{2\cdot 2}(1-p^2)(1-q^2)$$

va hokazo.

$$P_n(x) = \frac{A_1A_2...A_n}{3\cdot 6\cdot 9\dots(3n)}x^{2n}$$

$$B_n = \frac{1}{(2n-1)(3n-2)}(1-p^{2n-1})(1-q^{3n-2}) -$$

$$-\frac{1}{2n(3n-1)}(1-p^{2n})(1-q^{3n-1}),$$

bo'ladi. U holda berilgan tenglamaning ushbu xususiy yechimi kelib chiqadi.

$$u(x,y) = 1 + \frac{\lambda}{3}A_1B_1x^2y^3 + \frac{\lambda^2}{6}A_1A_2B_1B_2x^4y^6 + \dots +$$

$$+\frac{\lambda^n}{3n}A_1A_2\dots A_nB_2B_1\dots B_nx^{2n}y^{3n} + \cdots$$

Tekshirish shuni ko'rsatadiki, bu qator λ, x, y larning barcha chekli qiymatlarida absolyut va tekis yaqinlashuvchi bo'ladi.

Xulosa.

Ushbu bobda integro- differensial tenglamalarni yechish usullari. Ya'ni bir argumentli va ikki argumentli noma'lum funksiyalar uchun yozilgan integro- differensial tenglamalarni ketma- ket yaqinlashish usulida yechish kabi masalalar ko'rib o'tildi.

Xotima.

Bitiruv malakaviy ish ikki bobdan iborat bo'lib, ular differensial va integral tenglamalar haqida umumiy ma'lumotlar va integro- differensial tenglamalarni yechish usullaridir.

Differensial va integral tenglamalar haqida umumiy ma'lumot bobi ikki qismdan iborat. Differensial tenglamalar haqida boshlang'ich tushunchalar, integral tenglamalar haqida asosiy tushunchalar.

Differensial tenglamalar haqida boshlang'ich tushunchalar qismida differensial tenglama ta'rifi, differensial tenglamaga qo'yilgan Koshi masalasi, differensial tenglamalar yechishga oid misollar keltirilgan.

Integral tenglamalar haqida asosiy tushunchalar qismida integral tenglama turlari, integral tenglamalarni yechish, Fedgolm integral tenglamalari, Volterra integral tenglamalari kabi masalalar keltirilgan.

Integro- differensial tenglamalar bobi ikki qismdan iborat. Bir argumentli funksiyalar uchun integro- differensial tenglamalarni yechish, ikki argumentli funksiyalar uchun integro- differensial tenglamalarni yechish.

Bir argumentli funksiyalar uchun va ikki argumentli funksiyalar uchun integro- differensial tenglamalarni yechish qismida ketma- ket yaqinlashish usuli bilan yechish o'rganildi

Bitiruv malakaviy ishim refarativ xarakterga ega bo'lib, mazkur bitiruv malakaviy ishdan oliy o'quv yurtlaridagi talabalar mustaqil ta'limdan foydalanishlari mumkin.

Foydalanigan adabiyotlar ro'yxati.

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