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TA'LIM VAZIRLIGI

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Fizika -matematika fakulteti

“Axborot texnologiyalari” kafedrasi

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“ Xususiy hosilali differensial tenglamalarni taqribiy yechish metodlari bo’yicha uslubiy qo’llanma”

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> Bu murakkab dunyoning azaliy va abadiy muammolari, shu bilan birga, har bir davrning dolzarb masalalariga har tomonlama asosli ilmiy javoblar topilgan taqdirdagina ma`naviyat olami yangi ma`no-mazmun bilan boyib boradi. Boshqacha aytganda, har bir ilmiy yangilik, yaratilgan kashfiyot – bu yangi davr va dunyoqarashga turtki beradi, ma`naviyatning shakllanishiga o`ziga xos ta`sir o`tkazadi.
>
> I.A. Karimov

Hozirgi kunda kompyuter texnologiyalari kirib bormagan soha deyarli uchramaydi. Kompyuter texnologiyalardan nafaqat hisoblash ishlarini olib borish uchun emas, balki, hayotga tadbiq qilinadigan elektron darsliklar, rasm va video tasmalarni qayta ishlovchi, katta hajmli ma’lumotlarni o’zida saqlovchi dasturlar yaratish uchun ham foydalaniladi.So’ngi yillarda kompyuter va uning dasturiy ta’minotiga bo’lgan talab va qiziqishlar ortib bormoqda. Bu esa o’z navbatida dasturchidan katta izlanish va mahoratni talab qiladi. Prezidentimiz I. A. Karimovning “ Yoshlarimiz bizdan ko’ra kuchli bilimli dono va albatta baxtli bo’lishlari kerak “,”O’zbekiston kelajagi yoshlar qo’lida” degan so’zlari va 2008 – yilning “Yoshlar yili” deb belgilaganlari, yoshlarga bo’lgan e’tiborni va katta ishonchni ko’rsatadi. Biz yoshlar o’z navbatida bunday ishonchni oqlashga va komillik sari intilishga harakat qilamiz.

Prezident I.A.Karimov tashabbusi bilan qabul qilingan “Ta`lim to`g`risida”gi qonun va kadrlar tayyorlash milliy dasturi ta`limning yangi milliy modelini yaratilishiga asos bo`ldi. Milliy modelning asosiy maqsadi ta`lim sohasini isloh qilish , mutaxassislar tayyorlashning yangi tizimini joriy etish va ta`lim sifatini hozirgi zamon talablari darajasiga ko`tarishdir. Ushbu maqsadni bitta qonun yoki farmonlar bilan amalga oshirib bo`lmaydi. U mashaqqatli mehnat va izlanishlarni talab qiladi.

Ta`lim sifatini oshirish bugungi kunning asosiy vazifalaridan biridir. Chunki bugungi kunda insoniyat rivojlanishi Shunday bosqichga yetib bordiki, bu bosqichda inson aql-zakovati, bilimi va uning mutaxassis sifatidagi malakasi hal qiluvchi ro’l o`ynaydigan bo`lib qoldi.

Ta`lim sifatini oshirish o`quv dargohlarini zamonaviy o`quv rejalari, dasturlar, o`quv jihozlari bilan ta`minlash, yangi darsliklar va qo`llanmalarni yaratish, ilg`or pedagogik texnologiyalarni joriy etishni taqazo etadi.

Albatta, jihozlar, darsliklar, texnologiyalar ham ushbu vazifani to`liq bajarilishiga olib kela olmaydi. Chunki ta`lim sifati ko`p jihatdan ta`lim beruvchining shaxsiga va malakasiga bog`liqdir.

Ta`lim jarayonini tashkil etish, uni yetarli materiallar bilan ta`minlash, o`quv jihozlaridan maqsadli foydalanish, o`quvchi fikrini darsga doimiy jalb etish o`qituvchidan katta mahorat, bilim va ko`nikma talab etadi.

Yangi axborot – kommunikatsion texnologiyalari hozirgi vaqtda eng dolzarb mavzulardan biri bo’lib kelmoqda, sababi har bir sohani o’rganish, izlanish va tajriba orttirish uchun turli usullardan foydalanish kerak bo’ladi. Shuning uchun yangi axborot – kommunikatsion texnologiyalardan foydalanish maqsadga muvoffiqdir.

Hozirgi zamon mutaxasislari, faoliyat doiralari qanday bo’lishidan qat’iy nazar informatika bo’yicha keng ko’lamdagi bilimlarga, zamonaviy hisoblash texnikasi, informatsion aloqa va kommunikatsiya tizimlari, orgtexnika vositalari va ulardan foydalanish borasida yetarli malakalarga ega bo’lishi, hamda yangi information texnika va texnologiya asoslarini uning ertangi kuni, rivoji to’g’risidagi bilimlarni o’zida mujassamlashtirgan bo’lishi kerak. Zamonviy hisoblash texnikasi va information texnologiyalarning kun sayin rivojlanib, jamiyatning esa tobora informatsiyalashib borishi sababli, uzluksiz ta’lim tizimining o’rta va yuqori bosqichlariga informatika, ishlab chiqarish va boshqarish jarayonlarini kompyuerlashtirish bo’yicha bir qator o’quv fanlari kiritilgan.

Mavzuning dolzarbligi: Xususiy hosilali differensial tenglamalar fan va texnikaning turli sohalarida uchraydi, ammo ularning yechimini oshkor ko’rinishda chekli formula shaklida topish kamdan-kam hollarda mumkin.Shu munosabat bilan matematik fizika masalalari deb ataluvchi har xil xususiy hosilalai differensial tenglamalarni, xususiy hosilali differensial tenglamalar sistemasi va integral tenglamalarni taqribiy yechish metodlari muhim ahamiyatga egadir.

Malakaviy bitiruv ishining maqsadi: Xususiy hosilali differensial tenglamalarni taqribiy yechish metodlari bo’yicha uslubiy qo’llanma tayyorlash.

Malakaviy bitiruv ishining vazifalari:

1. Xususiy hosilali differensial tenglamalar haqida tushunchaga ega bo’lish.
2. Mavzuga oid adabiyotlar bilan tanishish.
3. Xususiy hosilali differensial tenglamalarni taqribiy yechish usullarini o’rganish.
4. Elliptik, parabolik, giperbolik tenglamalarni to’r metodi bilan yechishni o’rganish.
6. Xususiy hosilali differensial tenglamalarni taqribiy yechish usullari algoritmini ishlab chiqish.

Mavzuning o’rganilganlik darajasi: Xususiy hosilali differensial tenglamalarni taqribiy yechish usullarining qanchasi o’rganildi. Bu usullarning barchasi uchun Mathcad muhitida natijalar olindi.

Malakaviy bitiruv ishimizning ob’ekti va predmeti: Xususiy hosilali differensial tenglamalar, to’r, ayirmali sxema, approksimassiya, Mathcad dasturi.

Malakaviy bitiruv ishining yangiligi: Xususiy hosilali differensial tenglamalarni taqribiy yechish bo'yicha uslubiy qo'llanma yaratish. Mathcad dasturida tuzilgan algoritm.

Malakaviy bitiriv ishi natijalarining nazariy va amaliy ahamiyati shundan iboratki, xususiy hosilali differensial tenglamalarni taqribiy yechish bo'yicha uslubiy qo'llanmadan fan va texnikaning turli masalalarini yechishda foydalanish.

Malakaviy bitiruv ishining metodologik asosi : Malakaviy bitiruv ishining usullari: Mavzuga oid adabiyotlarni o'rganish, tahlil qilish, aniqlikni solishtirish va tekshirish, tajriba sinovlar, olingan natijalarni algoritmlar asosida tekshirish va turg`unligini tekshirish.

Himoyaga olib chiqilayotgan asosiy holatlar:

- Xususiy hosilali differensial tenglamalar haqida ma'lumot;
- To'r metodi,turg'unlik,approksimatsiya yaqinlashish,
- Elliptik tenglamalarni to'r metodi bilan yechish.
- Chebishevning optimal oshkor iteratsion metodi va uning ayirmali elliptik tenglamalarga tadbiqi;
- Parabolitik tenglamalar uchun ayirmali sxemalar;
- Giperbolik tenglamalarni ayirmali metodlar bilan yechish;
- Xususiy hosilali differensial tenglamalarni taqribiy hisoblash algoritmini ishlab chiqish;
- Xususiy hosilali differensial tenglamalarni taqribiy yechish bo'yicha uslubiy qo'llanma.

Malakaviy bitiruv ishining tuzilishi va hajmi: Malakaviy bitiruv ishi kirish, ikki bob, xotima va adabiyotlar ro'yxatidan iborat bo'lib, unda 10 ta chizma, ishining hajmi 53 betda bayon qilingan.

I bob. Xususiy hosilali differensial tenglamalarni taqribiy yechish

Biz matematik fizika masalalarini taqribiy yechishning ayrim keng tarqalgan metodlarini ko’rib chiqamiz. Matematik fizika kurslarida o’zgaruvchilarning soni n va hosilalarning tartibi $m(\geq 2)$ bo’lgan tenglamalar qaraladi. Biz asosiy diqqatni ikki erkli o’zgaruvchili ikkinchi tartibli xususiy hosilali chiziqli differensial tenglamalarga qaratamiz.Bunday tenglamalar misolida qaraladigan metodlarning asosiy g’oyasi yaxshi tushunarli bo’lib, hisoblash sxemasi ham soddaroq bo’ladi.

Shuni ham ta’kidlash kerakki, bitta tenglama uchun qaraladigan metodlarni bir necha noma’lum funksiyalarni o’z ichiga olgan tenglamalar sistemasi uchun ham tadbiq qilish mumkin.

1.1.To’r metodi, turg’unlik, approksimatsiya yaqinlashish

To’r metodi (chekli-ayirmali metod) xususiy hosilali differensial tenglamalarni yechishning keng tarqalgan metodlaridandir.

To’r metodining g’oyasi. To’r metodining g’oyasi bilan

$$L(u) = a\frac{\partial^2 u}{\partial x_1^2} + 2b\frac{\partial^2 u}{\partial x_1 x_2} + c\frac{\partial^2 u}{\partial x_2^2} + d\frac{\partial u}{\partial x_1} + e\frac{\partial u}{\partial x_2} + gu = f \qquad (1.1.1)$$

Tenglama uchun Dirixle masalasini yechish misolida tanishamiz.Bunda a, b, c, d, e, g *koeffisientlar* va f *ozod had* chegarasi Γ dan iborat bo’lgan chekli D sohada aniqlangan ikki x_1 va x_2 o’zgaruvchilarning funksiyalaridir.Bu funksiyalar $\overline{G} = GU\Gamma$ yopiq sohada aniqlangan hamda $\overline{G}$ da $a > 0$, $\mathrm{c} > 0$ va $\mathrm{g} \leq 0$ shartlarni qanoatlantiradi, deb faraz qilamiz.

Faraz qilaylik, $(1.1.1)$ tenglamaning $\overline{G}$ da uzluksiz va Γda berilgan qiymatlarni qabul qiladigan, ya’ni

$$u|_{\Gamma} = \varphi \qquad (1.1.2)$$

Yechimini toppish talab qilinsin, bunda $\varphi = \varphi(x_1, x_2) \in \Gamma$ uzluksiz funksiyadir.

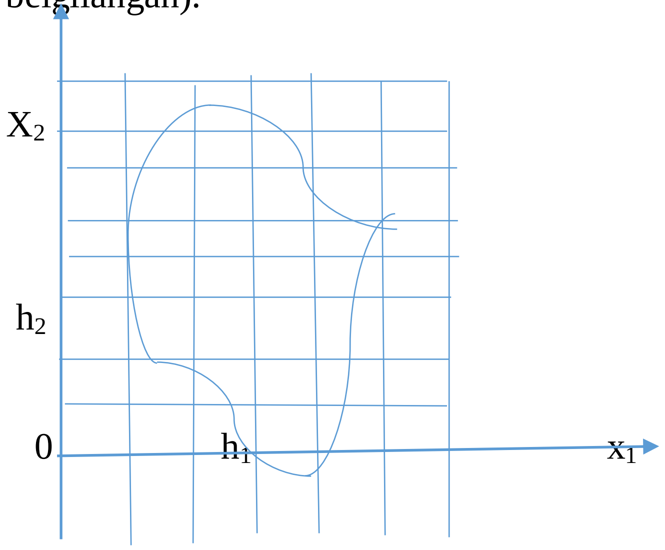
Taqribiy yechimning sonli qiymatlarini toppish uchun $0x_1x_2$ tekisligida

$$x_{1i} = x_{10} + ih_1,\, x_{2k} = x_{20} + kh_2, (i, k = 0, \pm 1, \pm 2, \ldots)$$

Parallel to'g'ri chiziqlarning ikkita oilasini o'tkazamiz.Bunda h_1 va h_2 mos ravishda absissa va ordinata yo'nalishlaridagi *qadamlar* deyiladi.Bu to'g'ri chiziqlarning kesishgan nuqtalari *tugunlar* deyiladi, tugunlar to'plami esa *to'rni* tashkil etadi.Odatda, h_1 va h_2 qadamlar bir-biriga bog'liq ravishda tanlanadi, masalan, $h_1 = h$. Ah^{α} (A va α qandaydir sonlar), xususiy holda $h_1 = h_2 = h$ Shuning uchun ham qaralayotgan to'r bitta h parametrga bog'liq bo'lib, qadam kichrayganda $h \to 0$,

Agar ikki tugun $0x_1$ o'qi yoki $0x_2$ o'qi bo'ylab to'rning shu yo'nalishi bo'yicha bir-biridan bir qadam uzoqlikda joylashgan bo'lsa, ularni *qo'shni tugunlar* deymiz.

Faqat G da yotgan tugunlar to'plamini qaraymiz.Agar biror tugunning to'rtala qo'shni tugunlari to'plamda yotsa , u hoilda bu tugunni *ichki tugun* deymiz.Ichki tugunlar to'plamini *to'r soha* deymiz va G_h orqali belgilaymiz.Agar tugunning hech bo'lmaganda birorta qo'shnisi G_h da yotmasa , u holda bu tugun *chegaraviy tugun*, ularning to'plamini esa *to'r sohaning chegarasi* deymiz va Γ orqali belgilaymiz(1.1-chizmada ichki tugunlar 0 bilan va chegaraviy tugunlar bilan belgilangan).



1.1-chizma. Ichki va chegaraviy tugunlar

Agar G_h to’r soha Γ_h chegarsi bilan birgalikda qaralsa, u holda u *yopiq to’r soha* deyiladi va $\overline{G_h} = C_h U \Gamma_h$ orqali belgilanadi.

Biz G_h to’r ustida aniqlangan $y(x_1, x_2)$ funksiya uchun $y_{ik} = y(x_{1i}, x_{2k})$ belgilash kiritamiz va har bir $(i,k) = (x_{1i}, x_{2k})$ tugun uchun $(1.1.1)$tenglamada qatnashadigan barcha hosilalarni bo’lingan ayirmalar bilan quyidagicha almashtiramiz:

$$\left(\frac{\partial u}{\partial x_1}\right)_{(i,k)} \approx \frac{y_{i+1,k} - y_{i-1,k}}{2h_1}, \left(\frac{\partial u}{\partial x_2}\right)_{(i,k)} \approx \frac{y_{i,k+1} - y_{i,k-1}}{2h_2} \qquad (1.1.3)$$

$$\left(\frac{\partial^2 u}{\partial x_1^2}\right)_{(i,k)} \approx \frac{y_{i+1,k} - 2y_{ik} + y_{i-1,k}}{h_1^2} \qquad (1.1.4)$$

$$\left(\frac{\partial^2 u}{\partial x_2^2}\right)_{(i,k)} \approx \frac{y_{i,k+1} - 2y_{ik} + y_{i,k-1}}{h_2^2} \qquad (1.1.5)$$

$$\left(\frac{\partial^2 u}{\partial x_1 \partial x_2}\right)_{(i,k)} \approx \frac{y_{i+1,k+1} - y_{i-1,k+1} - y_{i+1,k-1} + y_{i-1,k-1}}{4h_1 h_2} \qquad (1.1.6)$$

bunda y_{ik} miqdorlar $u(x_1, x_2)$ yechimning to’rning $(i,k) = (x_{1i}, x_{2i})$ tugunidagi taqribiy qiymatlaridir. Tenglama koeffisientlarining (i,k) tugundagi qiymatini $a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, g_{ik}, f_{ik}$, orqali belgilaymiz.Hosilalar o’rniga $(1.1.3)-(1.1.6)$taqribiy qiymatlarni qo’yib, natijada $(1.1.1)$ differensial tenglamaga mos keladigan quyidagi ayirmali tenglamaga ega bo’lamiz:

$$\begin{gathered} L_h y_{ik} \equiv \frac{1}{h_1^2} a_{ik}\left(y_{i+1,k} - 2y_{ik} + y_{i-1,k}\right) + \frac{b_{ik}}{4h_1 h_2}\left(y_{i+1,k+1} - y_{i-1,k+1} - y_{i+1,k} + y_{i-1,k-1}\right) + \\ + \frac{c_{ik}}{h_2^2}\left(y_{i,k+1} - 2y_{ik} + y_{i,k-1}\right) + \frac{d_{ik}}{2h_1}\left(y_{i+1,k} - y_{i-1,k}\right) + \frac{e_{ik}}{2h_2}\left(y_{i,k+1} - y_{i,k-1}\right) + g_{ik} y_{ik} = f_{ik} \end{gathered} \qquad (1.1.7)$$

Bunday tenglamani har bir ichki tugun uchun yozish mumkin.Agar (i,k) chegaraviy tugun bo’lsa, u holda y_{ik} ni bu tugunga yaqinroq bo’lgan φ ning Γ ustidagi qiymatiga teng deb olamiz (chegaraviy tugunlarda y_{ik} larning qiymatini boshqacha yo’l bilan topishni biz keyinroq ko’rib chiqamiz). Shunday qilib, yechimning ichki tugunlaridagi y_{ik} qiymatini topish uchun algebraik tenglamalar sistemasiga ega bo’lamiz. Bu sistemada tenglamalarning soni noma’lumlar soniga teng. Agar bu sistema yechimga ega bo’lsa , u holda uni yechib, ichki tugunlarda qidirilayotgan yechimning taqribiy qiymatiga ega bo’lamiz.

Turg’unlik, approksimatsiya va yaqinlashish.

Faraz qilaylik, chegarasi $\Gamma = \bigcup_{i=1}^{m} \Gamma_j$ bo’lgan sohada ushbu

$$L(u) = f \qquad (1.1.8)$$

$$R(u)\big|_j \equiv R_j(u) = \varphi_j, \quad j = 1,2,....,m \qquad (1.1.9)$$

chegaraviy masala berilgan bo’lsin.Bu yerda L-ixtiyoriy ikkinchi tartibli chiziqli differensial operator, R_j -birinchi tartibli differensial operator yoki chekli algebraik ifoda , xususiy holda $R_j u = u$ va $f, \varphi_1, \varphi_2, ...\varphi_m$-berilgan funksiyalar.

Endi $\vec{G}$ da yotuvchi qandaydir G_h to’r sohani quramiz,keyin U_h orqali G_h ning nuqtalarida (tugunlarida) aniqlangan u_h funksiyalarning fazosini belgilaymiz, L_h operator U_h dagi funksiyalarni biror $\overset{0}{G}_h \subset G_h$ to’r sohada aniqlangan funksiyalarga o’tkazsin; $\overset{0}{G}_h$ da aniqlangan funksiyalar to’plamini F_h orqali belgilaymiz. Chegaraviy shartlarni approksimatsiyalash uchun G sohaning Γ_j chegarasiga mos keladigan Γ_{jh} to’r chegarasini tanlab, Φ_{jh} orqali Γ_{jh} da aniqlangan funksiyalar to’plamini belgilaymiz.

1-ta’rif. Agar $X \subset Y$ bo’lib , ϑ funksiya Y da aniqlangan bo’lsa , u holda ϑ ning *to’plamdagi izi* deb shunday funksiyaga aytiladiki , y X to’plamda aniqlanga va bu yerda ϑ bilan ustma-ust tushadi.

Agar ϑ funksiya G_h ni o’z ichiga olgan to’plamda aniqlangan bo’lsa, u holda ϑ ning G_h dagi izini $[\vartheta]_h$ orqali belgilaymiz.

Faraz qilaylik , u $(1.1.8)$ va $(1.1.9)$ chegaraviy masala yechimlarining fazosi, Γ $(1.1.8)$ tenglamaning o’ng tomonidagi f funksiyalarning fazosi, Φ_j esa Γ_j da aniqlangan funksiyalarning fazosi bo’lsin.

2-ta’rif. Faraz qilaylik, $U, U_h, F, F_h, \Phi_j, \Phi_{jh}$ fazolarda

$$\|\bullet\|_U, \|\bullet\|_{U_h}, \|\bullet\|_F, \|\bullet\|_{F_{nh}}, \|\bullet\|_{\Phi_o}, \|\bullet\|_{\Phi_{op}}$$

Normalar aniqlangan bo’lsin. Bu normalar *moslangan* deyiladi, agar $h \to 0$ da har qanday yetarlicha silliq $u \in U,\ f \in F, \varphi_j \in \Phi_j$ funksiyalar uchun quyidagi

$$\|[u]\|_h\|_{U_h} \to \|u\|_U,$$

$$\|[f]\|_h\|_{F_h} \to \|f\|_F,$$

$$\|\varphi_{ij}\|_h\|_{\Phi_{jh}} \to \|\varphi_{ij}\|_{\Phi_j}$$

munosabatlar o’rinli bo’lsa.

3-ta’rif. Agar $h \to 0$ da

$$\left\|\left[u_h - [u]_h\right]\right\|_{U_h} \to 0$$

bo’lsa, u holda u_h to’r funksiyasi $(1.1.8)$, $(1.1.9)$ chegaraviy masalaning yechimiga *yaqinlashadi* deyiladi.

Agar h ga bog’liq bo’lmagan $C > 0$ va $\sigma > 0$ o’zgarmas sonlar uchun

$$\left\|\left[u_h - [u]_h\right]\right\|_{U_h} \le Ch^{\sigma}$$

tengsizlik bajarilsa , u holda *yaqinlashishning tartibi* h ga nisbatan σ ga teng deyiladi.

To’r ustida ushbu

$$L_h(u_h) = f_h \qquad (1.1.10)$$

$$R_{jh}(u_h) = \varphi_{jh} \qquad (j = 1,2,....,m) \qquad (1.1.11)$$

masalasini qaraymiz, bu yerda L_h va R_{jh} chi,ziqli operatorlar.

Endi quyidagi belgilashni kiritamiz:

$$W(h) = \|L_h([u]_h) - [L(u)]_h\|_{F_h} + \|f_h - [f]_h\|_{F_h} + \\ + \sum_{j=1}^{m}\left\{\left\|R_{jh}([u]_h) - [R_j(u)]_h\right\|_{\Phi_{jh}} + \left\|\varphi_{jh} - [\varphi_j]_h\right\|_{\Phi_{jh}}\right\} \qquad (1.1.12)$$

4-ta'rif. Agar ixtiyoriy silliq u, f, φ_j funksiyalar uchun $h \to 0$ da $W(h) \to 0$ bo'lsa , u holda $(1.1.8)$, $(1.1.9)$ chegaraviy masalani $(1.1.10)$, $(1.1.11)$ to'r ustidagi masala *approksimatsiya qiladi* deyiladi.

Agar $(1.1.10)$ tenglamaning o'ng tomonini

$$f_h\big|_{(i,k)} = f(x_{1i}, x_{2k})$$

deb olsak, u holda $W(h)$ ning ta'rifiga kirgan $\|f_h - [f]_h\|_{F_h}$ miqdor nolga teng bo'ladi. Ammo ayrim hollarda aniqlikni oshirish uchun $(1.1.8)$ tenglamaning o'ng tomoni (i,k) nuqtada $f(x_{1i}, x_{2k} + 0.5h_2)$ deb olinadi.

5-ta'rif. To'r ustidagi $(1.1.10),(1.1.11)$ masala *turg'un(korrekt)* deyiladi,agar $h \le h_0$ uchun h ga bog'liq bo'lmagan M_0 va M_j o'zgarmaslar topilib, ular uchun ushbu tengsizlik bajarilsa:

$$\|u_h\|_{U_h} \le M_0 \|L_n(u_h)\|_{F_n} + \sum_{j=1}^{m} M_j \|R_{jh}(u_h)\|_{\Phi_{jh}} \qquad (1.1.13)$$

Bu ta'rifdan ko'ramizki, chiziqli masala uchun turg'unlik f_h va φ_{jh} funksiyalar bog'liq emas.

Bu ta'rifning ma'nosini tushuntirishga harakat qilamiz. Chiziqli masala uchun $(1.1.10),(1.1.11)$ ayirmali sxema chiziqli algebraik tenglamalar sistemasidan iborat. Shuning uchun ham $(1.1.13)$ tengsizlikdan $f_h \equiv 0, \varphi_{jh} \equiv 0$ bo'lganda $(1.1.10)-(1.1.11)$ tenglamalar sistemasi faqat trivial yechimga ega. Bundan esa Kroneker-Kenelli teoremasiga ko'ra $(1.1.10),(1.1.11)$ masala o'ng tomonidagi ixtiyoriy f_h, φ_{jh} uchun yagona yechimga ega. Demak , chiziqli masala turg'unlik shartidan ayirmali tenglamalar sistemasining o'ng tomoni ixtiyoriy funksiyalar bo'lganda ham yagona yechikga egaligi kelib chiqadi.

Agar u_h^1, u_h^2 funksiyalar quyidagi

$$L_h u_h^1 = f_h^1, R_{jh} u_h^1 = \varphi_{jh}^1, j = 1,2,......,m;$$
$$L_h u_h^2 = f_h^2, R_{jh} u_h^2 = \varphi_{jh}^2, j = 1,2,......,m;$$

ayirmali masalalarning yechimi bo’lsa , u holda L_h va R_{jh} operatorlar chiziqli bo’lganda (1.1.13) tengsizlikka ko’ra quyidagi ega bo’lamiz:

$$\begin{aligned} \left\| u_h^1 - u_h^2 \right\| \le M_0 \left\| L_h u_h^1 - L_h u_u^2 \right\|_{Fh} + \sum_{j=1}^{m} M_j \left\| R_{jh} u_h^1 - R_{jh} u_h^2 \right\|_{\varPhi jh} = \\ = M_0 \left\| f_h^1 - f_h^2 \right\|_{Fh} + \sum_{j=1}^{m} M_j \left\| \varphi_{jh}^1 - \varphi_{jh}^2 \right\|_{\varPhi jh} \end{aligned} \qquad (1.1.14)$$

Shunday qilib , agar tenglama va chegaraviy shartlarning o’ng tomoni bir-biridan kam farq qilsa, u holda turg’unlik sharti bajarilganda to’rdagi masalaning yechimi bir-biridan kam farq qiladi.

Yuqorida keltirilgan yaqinlashish, approksimatsiya va turg’unlikning ta’rifidagi $U_h, F_h, \varPhi_{jh}$ fazolarda aniqlangan normalar muhim ahamiyatga ega. Shunday hollar bo’lishi mumkinki, (1.1.13) tengsizlik ayrim normalar uchun bajarilib, boshqalari uchun bajarilmaydi. Har gal (1.1.13) tengsizlik nima sabadan bajarilmasligini tekshirish kerak.

Agar normalar noqulay olinganligi sababli (1.1.13) tengsizlik bajarilmagan bo’lsa , u holda $U_h, F_h, \varPhi_{jh}$ fazolarda normalarni boshqacha tanlab, (1.1.13) tengsizlikning bajarilishini ta’minlash kerak. Agar (1.1.13) tengsizlik normaning hech biri uchun ham bajarilmasa , u holda bu ayirmali sxemaning *noturg’unligini* bildiradi. To’rdagi normalar moslangan bo’lishi kerak. Masalani tekshirishda ko’pincha $\|\bullet\|_{Uh}$ va $\|\bullet\|_{U}$ larning moslanish normalari sifatida quyidagilar olinadi:

$$\left.\begin{aligned} \|u_h\|_{U_h} &= \sup_{\substack{0 \le m \le M \\ 0 \le n \le N}} |u_{mn}| \\ \|u_h\|_{U} &= \sup_{\substack{a \le x \le b \\ 0 \le y \le T}} |u(x,y)| \end{aligned}\right\} \qquad (1.1.15)$$

yoki

$$\left.\begin{aligned}\|u_h\|_{U_h} &= \sup_{0\le n\le N}\sqrt{h\sum_{m=0}^{M}|u_{mn}|^2}\,,\\ \|u_h\|_{U} &= \sup_{0\le y\le T}\sqrt{\int_a^b|u(x,y)|^2\,dx}\,,\end{aligned}\right\} \qquad (1.1.16)$$

Bu normalar h=(b-a)/M (M-butun son) , $N=[T/h_2]$

Faraz qilaylik $u\in U$ bo'lsin. $r_h^0=L_h[u]_h-f_h$ miqdor *masalaning yechimidagi tenglama approksimatsiyasining xatoligi* deyiladi.

$r_h^j=R_{jh}[u]_h-\varphi_{jh}$ (j=1,2,…..,m) miqdorlar esa *masalaning yechimidagi chegaraviy shartlar approksimatsiyaning xatoligi* deyiladi. Ushbu

$$\rho_0(h)=\|L_h[u]_h-f_h\|_{Fh},\rho_j(h)=\|R_{jh}[u]_h-\varphi_{jh}\|_{\Phi jh}$$

belgilashni kiritamiz.

Agar u funksiya $(1.1.8),(1.1.9)$ masalaning yechimi bo'lsa , u holda

$$\rho(h)=\sum_{j=0}^{m}\rho_j(h)$$

miqdor $(1.1.8),(1.1.9)$ *differensial masalani* $(1.1.10),(1.1.11)$ *ayirmali sxema bilan approksimatsiyalashda yechimdagi xatoning o'lchovi* deyiladi. Agar $h\to 0$ da $\rho(h)$ ning tartibi *yechimdagi approksimatsiyaning tartibi* deyiladi.

Ayirmali sxemalarni qurish va ularni tekshirish to'g'risida ayrim mulohazalarni aytish mumkin:

1. Avvalo , to'rni tanlash, yani G soha va Γ konturni qandaydir to'r soha bilan almashtirish qoidasi ko'rsatiladi.
2. Keyin konkret ravishda bitta yoki bir nechta ayirmali sxema quriladi; approksimatsiya shartlarining bajarilishi tekshiriladi va approksimatsiyaning tartibi aniqlanadi.
3. Qurilgan ayirmali sxemaning turg'unligi tekshiriladi.Bu esa eng muhim va og'ir masala hisoblanadi. Agar ayirmali masala approksimatsiya va turg'unlikka ega bo'lsa, yuqoridagi teoremaga ko'ra u yaqinlashadi.

4. Ayirmali sxema tenglamalarini sonli yechish masalasi qaraladi. Odatda , tenglamalarning soni ko’p bo’lib, bunday sistemani yechish ko’p mehnat talab qiladi. Shuning uchun ham to’r metodida hosil bo’ladigan sistemalarni yechish uchun maxsus metodlar yaratilgan va yaratilmoqda.

1.2. Elliptik tenglamalarni to’r metodi bilan yechish

Quyidagi

$$L(u) \equiv a\frac{\partial^2 u}{\partial x_1^2} + c\frac{\partial^2 u}{\partial x_2^2} + d\frac{\partial u}{\partial x_1} + e\frac{\partial u}{\partial x_2} + gu = f \qquad (1.2.1)$$

Elliptik tenglamani $(1.1.7)$ ayirmali tenglama bilan almashtirganda hosil bo’ladigan xatolikni baholashni ko’rib chiqamiz. Bu yerda hisoblashlar soda bo’lishi uchun $\frac{\partial^2 u}{\partial x_1 \partial x_2}$ aralash hosilaning oldidagi koeffisientni $b(x_1, x_2) = 0$ deb oldik. $(1.2.1)$ differensial tenglamaning *u(x,y)* yechimini to’rtinchi tartibli xususiy hosilalarga ega deb faraz qilib va Teylor formulasidan foydalanib, $(1.1.3)-(1.1.6)$ taqribiy tengliklar o’rnida quyidagilarni hosil qilamiz:

$$\frac{y_{i+1,k} - y_{i-1,k}}{2h_1} = \left(\frac{\partial u}{\partial x_1}\right)_{(x_{1i}, x_{2k})} + \frac{h_1^2}{6}\left(\frac{\partial^3 u}{\partial x_1^3}\right)_{(\xi, x_{2k})} \qquad (1.2.2)$$

$$\left(x_{1,i-1} \le \xi \le x_{1,i+1}\right)$$

$$\frac{y_{i,k+1} - y_{i,k-1}}{2h_2} = \left(\frac{\partial u}{\partial x_2}\right)_{(x_{1j}, x_{2k})} + \frac{h_1^2}{6}\left(\frac{\partial^3 u}{\partial x_2^3}\right)_{(\xi_1, x_{2k})} \qquad (1.2.3)$$

$$\left(x_{2,k-1} \le \eta \le x_{2,k+1}\right)$$

$$\frac{y_{i,k+1} - 2y_{ik} + y_{i,k-1}}{h_1^2} = \left(\frac{\partial^2 u}{\partial x_1^2}\right)_{(x_{1j}, x_{2k})} + \frac{h_1^2}{12}\left(\frac{\partial^4 u}{\partial x_1^4}\right)_{(\xi_1, x_{2k})} \qquad (1.2.4)$$

$$\left(x_{1,i-1} \le \xi_1 \le x_{2,i+1}\right)$$

$$\frac{y_{i,k+1} - 2y_{ik} + y_{i,k-1}}{h_2^2} = \left(\frac{\partial^2 u}{\partial x_2^2}\right)_{(x_{1j}, x_{2k})} + \frac{h_2^2}{12}\left(\frac{\partial^4 u}{\partial x_2^4}\right)_{(x_{1j}, \eta_1)} \qquad (1.2.5)$$

$$\left(x_{2,k-1} \le \eta \le x_{2,k+1}\right)$$

Endi $(1.2.2)-(1.2.5)$ lardan foydalanib, $(1.1.7)$ dan quyidagiga ega bo’lamiz:

$$L_h y_{ik} \equiv \left\{ a_{ik} \frac{\partial^2 u}{\partial x_1^2} + c_{ik} \frac{\partial^2 u}{\partial x_2^2} + d_{ik} \frac{\partial u}{\partial x_1} + l_{ik} \frac{\partial u}{\partial x_2} + g_{ik} u \right\}_{(i,k)} +$$

$$+ \frac{h^2}{12} \left\{ a_{ik} \left(\frac{\partial^4 u}{\partial x_1^4} \right)_{(\xi_1, x_{2k})} + \alpha^2 c_{ik} \left(\frac{\partial^4 u}{\partial x_2^4} \right)_{(x_{1i}, \eta_1)} + 2 d_{ik} \left(\frac{\partial^3 u}{\partial x_1^3} \right)_{(\xi, x_{2k})} + 2\alpha l_{ik} \left(\frac{\partial^3 u}{\partial x_2^3} \right)_{(x_{1i}, \eta)} \right\} =$$

$$= [L(u)]_{(i,k)} + Ri,k,$$

Bunda $h = h_1, \alpha = \frac{h_2}{h_1}$ bo’lib, R_{ik} -qoldiq had. Agar ushbu

$$M_3 = \frac{\max}{G} \left\{ \left| \frac{\partial^3 u}{\partial x_1^3} \right|, \left| \frac{\partial^3 u}{\partial x_2^3} \right| \right\}, M_4 = \frac{\max}{G} \left\{ \left| \frac{\partial^4 u}{\partial x_1^4} \right|, \left| \frac{\partial^4 u}{\partial x_2^4} \right| \right\}$$

Belgilashlarni kritsak, qoldiq had uchun

$$|R_{ik}| \le \frac{h^2}{12} \left\{ \left(|a_{ik}| + \alpha^2 |b_{ik}| \right) M_4 + 2\left(|d_{ik}| + \alpha |l_{ik}| \right) M_3 \right\} \qquad (1.2.6)$$

Baho o’rinli bo’ladi. Demak ,

$$L_h y_{ik} - f_{ik} = \{L(u) - f\}_{(i,k)} + R_{ik} = R_{ik}$$

Bundan ko’ramizki, $(1.2.1)$ differensial tenglamani $(1.1.7)$ ayirmali tenglama bilan almashtirganda R_{ik} xatolik hosil bo’lib , uning h qadamga nisbatan tartibi h^2 dir. Agar R_{ik} qoldiq hadni tashlasak, to’r ustidagi y_{ik} funksiya uchun

$$L_h y_{ik} = f_{ik} \qquad (1.2.7)$$

Tenglamalar sistemasiga ega bo’lamiz. Xususiy holda ushbu

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = f(x_1, x_2) \qquad (1.2.8)$$

Puasson tenglamasi uchun $h_1 = h_2 = h$ kvadrat to’rni qarasak , u holda $(1.2.7)$ tenglamalar sistemasi

$$y_{i+1,k} + y_{i-1,k} + y_{i,k+1} + y_{i,k-1} - 4y_{ik} = h^2 f_{ik} \qquad (1.2.9)$$

Ko’rinishga ega bo’lib, $(1.2.6)$ dan qoldiq had uchun

$$|R_{ik}| \le \frac{h^2}{6} M_4 \qquad (1.2.10)$$

bahoga ega bo’lamiz.

Ayirmali tenglama hosil qilish uchun aniqmas koeffisientlar metodi.

Yuqoridagi differensial tenglamani (i,k) nuqtada ayirmali tenglama bilan almashtirganda har bir xususiy hosilali alohida-alohida bo'lingan ayirmalar bilan almashtirgan edik. Differensial tenglamani to'laligicha ayirmali tenglama bilan almashtirish mumkin. Hozir qaraladigan metodda to'r soha to'g'ri to'rtburchakdan iborat bo'lishi shart emas, to'r uchburchaklar , parallelogramlardan iborat yoki umuman notekis bo'lishi ham mumkin. Differensial tenglamani (i,k) tugunda ayirmali sxema bilan almashtirish uchun (i,k) tugun atrofini ma'lum tartibda joylashgan P ta tugunni qaraymiz. Qulay bo'lishi uchun (i,k) tugunni 0 orqali belgilab , qolgan tugunlarni $1,2,.......,P$ orqali belgilaymiz. Endi c_i aniqmas koeffisientlar bilan ushbu

$$\sum_{j=0}^{P} c_j u_j \qquad (1.2.11)$$

chiziqli kombinatsiyani tuzamiz, bunda u_j miqdor u ning j tugundagi qiymati. Faraz qilaylik , u funksiya $(n+1)$ tartibli hosilalarga ega bo'lsin, u holda u_j larni 0 tugun atrofida Teylor qatoriga yoyamiz:

$$u_j = u(x_{1j}, x_{2j}) = \sum_{k_1+k_2=n} \frac{(x_{1j}-x_{10})^{k_1}}{k_1!} - \frac{(x_{2j}-x_{20})^{k_2}}{k_2!} - \left(\frac{\partial^{k_1+k_2} u}{\partial x_1^{k_1} \partial x_2^{k_2}}\right)_0 + R(j), \qquad (1.2.12)$$

$j = 1,2,.....,...$

Bu ifodalarni $(1.2.11)$ ga qo'yib , u funksoyaning bir xil hosilalari oldidagi koeffisientlarni qo'shib chiqamiz, natijada

$$\sum_{j=0}^{P} c_j u_j = \sum_{0 \le i+k \le n} \alpha_{ik} \left(\frac{\partial^{i+k} u}{\partial x_1^{l} \partial x_2^{k}}\right)_0 + \sum_{j=0}^{P} c_j R(j) \qquad (1.2.13)$$

Bu yerda α_{ik} koeffisientlar c_j lar orqali chiziqli ravishda ifodalanadi. Qoldiq had esa $\theta h^{n+1} K M_{n+1}$ ko'rinishga ega bo'ladi, bunda $|\theta| \le 1,$ K qandaydir son bo'lib, h bog'liq emas; h ning o'zi esa 0 tugun va $j(j = 1,2,.....,P)$ tugunlar koordinatalari ayirmalarining moduli bo'yicha eng kichigi hamda

$$M_{n-1}=\frac{\max}{i+k=n+1}\frac{\max}{G}\left|\frac{\partial^{i+k}u}{\partial x_1^i\partial x_2^k}\right|$$

Endi G sohada $(n+1)$ tartibli uzluksiz hosilaga ega bo'lgan har qanday $u(x_1,x_2)$ funksiya uchun

$$\sum_{i\le i+k\le n}\alpha_{ik}\left(\frac{\partial^{i+k}u}{\partial x_1^l\partial x_2^k}\right)_0=[L(u)]_0 \qquad (1.2.14)$$

tenglikning bajarilishini talab qilamiz. Buning uchun c_j koeffisientlarni shunday tanlashimiz kerakki , $0\le i+k\le n$ shartni qanoatlantiruvchi barcha i va k uchun $(1.2.14)$ tenglikning chap va o'ng tomonlarida $\left(\frac{\partial^{i+k}u}{\partial x_1^i\partial x_2^k}\right)_0$ oldidagi koeffisientlar ustma-ust tushsin. Bu esa $c_1,c_2,...,c_p$ noma'lum koeffisientlarga nisbatan quyidagi chiziqli algebraik tenglamalar sistemasiga olib keladi:

$$\begin{aligned}
&\alpha_{00}=g_0(i+k=0),\\
&\alpha_{10}=d_0,\alpha_{01}=l_0(i+k=1),\\
&\alpha_{20}=\alpha_0,\alpha_{02}=c_0,\alpha_{11}=0(i+k=2),\\
&\alpha_{30}=\alpha_{21}=\alpha_{12}=\alpha_{03}=0(i+k=3),\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
&\alpha_{n0}=\alpha_{n-1,1}=..\alpha_{1,n-1}=\alpha_{0n}=0(i+k=n).
\end{aligned}$$

Agar bu sistema yechimga ega bo'lib, yechim c_j (j=0,1,….,P) bo'lsa, u holda

$$\sum_{j=0}^{P}c_ju_j=[L(u)]_0+\theta Kh^{n+1}M_{n+1} \qquad (1.2.15)$$

Endi qoldiq hadni tashlab yuborib, u_j ning to'r ustidagi taqribiy qiymati y_j uchun ushbu

$$\sum_{j=0}^{P}c_jy_j=f_0 \qquad (1.2.16)$$

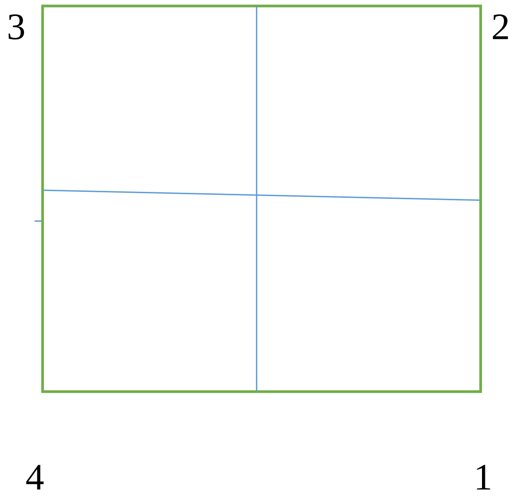
Ayirmali tenglamaga ega bo'lamiz. Bu tenglama $(1.2.1)$ differensial tenglamani 0 tugunda $O(h^{n+1})$ aniqlikda almashtiradi.

Chegaradan uzoqroq ichki tugunlar uchun ayirmali tenglamani tuzishda qatnashadigan tugunlarining joylanishini $(1.2.16)$ dagidek saqlash maqsadga muvofiq bo'ladi. Chegaraga yaqin tugunlar uchun bu holatni saqlash har doim ham mumkin bo'lavermaydi. Ammo qaralayotga metodda tugunlarni biroz boshqacha joylashtirib , differensial tenglamani kerakli aniqlikda ayirmali tenglama bilan almashtirish mumkin. Bu metod chegaraviy shartlarni approksimatsiya qilish uchun yaxshi natujaga olib keladi.

Puasson tenglamasi uchun aniqmas koeffisientlar metodi asosida ayirmali sxema qurish. Faraz qilaylik , G soha kvadrat bo'lib , shu sohada ushbu

$$\Delta u \equiv \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = f(x_1, x_2) \qquad (1.2.17)$$

Puasson tenglamasi uchun ayirmali sxema qurish talab qilinsin.



1.2-chizma. Tugun nuqtalar.

Bunday sxemani ikki xil to'r ustida bajaramiz. Avvalo, qadami h ga teng bo'lgan kvadrat to'rni qaraymiz, 1.2-chizmada ko'rsatilganidek, 0 tugun atrofida $1,2,3,4$ bilan belgilangan tugunlarni olamiz. Bu yerda x_1 va x_2 teng huquqli bo'lganligi hamda tugunlar simmetrik ravishda joylashganligi sababli ayirmali approksimatsiyani quyidagi ko'rinishda izlash mumkin:

$$L_h u_0 = c_0 u_0 + c_1 (u_1 + u_2 + u_3 + u_4). \qquad (1.2.18)$$

Qaralayotgan sohada $(1.2.17)$ tenglamaning yechimi to’rtinchi tartibli uzluksiz hosilalarga ega deb faraz qilib $(1.2.18)$ ifoda uchun quyidagiga ega bo’lamiz:

$$L_h u_0 = c_0 u_0 + 4c_1 u_0 + c_1 \left\{ \begin{array}{l} \left[h\left(\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}\right)u + \frac{h^2}{2!}\left(\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}\right)^2 u - \frac{h^3}{3!}\left(\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}\right)^3 u \right]_0 + \\ + \frac{h^4}{4!}\left[\left(\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}\right)^4 u\right]_{(\xi_1,\eta_1)} + \\ \left[h\left(\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right)u + \frac{h^2}{2!}\left(\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right)^2 u + \frac{h^3}{3!}\left(\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right)^3 u \right]_0 + \\ + \frac{h^4}{4!}\left[\left(\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right)^4 u\right]_{(\xi_2,\eta_2)} + \\ + \left[h\left(-\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}\right)u + \frac{h^2}{2!}\left(-\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}\right)^2 u + \frac{h^3}{3!}\left(-\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}\right)^3 u \right]_0 \end{array} \right\}$$

Bunda

$$x_{10} - h \le \xi_j \le x_{10} + h, x_{20} - h \le \eta_j \le x_{20} + h, \; j = 1,2,3,4.$$

Bu ifodani soddalashtirib quyidagini olamiz:

$$L_h u_0 = (c_0 + 4c_1)u_0 + 4c_1 \frac{h^2}{2}\left(\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2}\right)_0 + R(h), \qquad (1.2.19)$$

Bunda $R(h)$- qoldiq had. $(1.2.19)$ ifoda $(1.2.17)$ tenglamani approksimatsiya qilishi uchun

$$c_0 + 4c_1 = 0, 2c_1 h^2 = 1$$

shartlar bajarilishi kerak. Bulardan esa

$$c_0 = -\frac{2}{h^2}, c_1 = \frac{1}{2h^2}$$

kelib chiqadi. Shunday qilib, natijada quyidagiga ega bo’ldik:

$$L_h u_0 = \frac{1}{2h^2}(u_1 + u_2 + u_3 + u_4 - 4u_0) = (\Delta u)_0 + R(h) \qquad (1.2.20)$$

Agar M_4 orqali to’rtinchi hosilalarning G dagi maksimumi modulini belgilasak , u holda $R(h)$ qoldiq had uchun ushbu bahoga ega bo’lamiz:

$$|R(h)| \le 4|c_1| \frac{h^4}{4!} 2^4 M_4 = \frac{4h^2}{3} M_4 \qquad (1.2.21)$$

Yuqoridagi $(1.2.20)$ ifodada $(\Delta u)_0$ ni f_0 orqali almashtirib , $R(h)$ qoldiq hadni tashlab yuborsak , natijada u_j ning to'rdagi taqribiy qiymati y_j uchun ushbu

$$y_1 + y_2 + y_3 + y_4 - 4y_0 = 2h^2 f_0$$

ayirmali tenglamaga ega bo'lamiz. Bundan approksimatsiyaning xatoligi $\frac{4}{3}h^2 M_4$ dan oshmaydi.

Izoh. $(1.2.10)$ va $(1.2.21)$ baholarni solishtirish shuni ko'rsatadiki , $(1.2.10)$ baho $(1.2.21)$ ga nisbatan 8 marta kichik.

Ayirmali sxemaning turg'unligi. Biz yozuvni qisqaroq qilish maqsadida ayirmali sxemaning turg'unligini Puasson tenglamasi uchun Dirixle masalasini yechish misolida ko'rib chiqamiz.

Faraz qilaylik, G soha to'g'ri burchakli to'rtburchak $G\ \{0 < x_1 < a, 0 < x_2 < b\}$ bo'lib , Γ uning chegarasi bo'lsin. Shunday $u(x_1, x_2)$ funksiyani topish kerakki , u G da

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = f(x_1, x_2) \qquad (1.2.22)$$

Tenglamani qanoatlantirib, Γ chegarada Dirixle shartini qanoatlantirsin:

$$u|_\Gamma = \varphi(x_1, x_2) \qquad (1.2.23)$$

Bunda $\varphi(x_1, x_2)$ ma'lum funksiya. Faraz qilaylik, $(1.2.22)-(1.2.23)$ chegaraviy masala $\vec{G} = GU\Gamma$ sohada yagona yechimga ega va bu yechim G da $\frac{\partial^4 u}{\partial x_1^4}$ va $\frac{\partial^4 u}{\partial x_2^4}$ uzluksiz hosilalarga ega bo'lsin.

Biz quyidagi to'g'ri burchakli to'rtburchaklardan iborat bo'lgan to'rni qaraymiz:

$$\left.\begin{aligned} x_{1i} &= ih_1, i = 0,1,.....,M, Mh_1 = a; \\ x_{2k} &= kh_2, k = 0,1,....,N, Nh_2 = b. \end{aligned}\right\} \qquad (1.2.24)$$

Endi G da yotuvchi barcha tugunlarni G_h^0 deb olib chegaraviy nuqtalar Γ_h sifatida Γ da yotuvchi tugunlarni olamiz. Keyin

$$\Delta u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2}$$

Laplas operatorini G_h^0 ga tegishli nuqtalarda 3-chizmadagi besh nuqtali andaza yordamida

$$\Delta_h y_{ik} \equiv \frac{y_{i+1,k} - 2y_{ik} + y_{i-1,k}}{h_1^2} + \frac{y_{i,k+1} - 2y_{ik} + y_{i,k-1}}{h_2^2} = f_{ik} \qquad (1.2.25)$$
$$i = 1,2,....,M-1, k = 1,2,......,N-1$$

Ayirmali sxema bilan approksimatsiya qilamiz. Agar $h_1 = h, h_2 = \alpha h, (\alpha = const)$ bo'lsa, u holda $(1.2.26)$ approksimatsiyaning xatoligi

$$|R_{ik}(h)| \le \frac{h^2}{12}(1+\alpha^2)M_4$$

dan iborat. $(1.2.23)$ shartni quyidagilarga almashtiramiz:

$$y_{ik}\big|_{\Gamma h} = \varphi(ih_1, kh_2), (ih_1, kh_2) \in \Gamma_h \qquad (1.2.26)$$

Qaralayotgan soha to'g'ri burchakli to'rtburchak bo'lganligi tufayli $(1.2.26)$approksimatsiyaning xatoligi nolga teng. Chegaradagi $(1.2.26)$ qiymatlar ma'lum bo'lganligi uchunularni $(1.2.25)$ tenglamaga qo'yib, keyin ma'lum hadlarni o'ng tomonga o'tqazib, quyidagi chiziqli algebraik tenglamalar sistemasiga ega bo'lamiz:

$$L_h y_{ik} = f_{ik}, i = 1,2,....,M-1; k = 1,2,.....,N-1. \qquad (1.2.27)$$

Ravshanki , $(1.2.27)$ tenglamalar faqat chegara yaqinidagi tugunlarda $(1.2.25)$ tenglamalardan farq qiladi. Masalan , $(i,1)$ ko'rinishdagi tugunlarda $(1.2.27)$ tenglama quyidagi ko'rinishga ega bo'ladi:

$$\frac{y_{i+1,1} - 2y_{i1} + y_{i-1,1}}{h_1^2} + \frac{y_{i,2} - 2y_{i1}}{h_2^2} = f_{i1} - \frac{\varphi(ih_1, 0)}{h_2^2} \equiv \psi_{i1}$$

$(1.2.27)$ sistemada tenglamalarning soni noma'lumlarning soniga teng. Shuning uchun ham $(1.2.25)$ sistemaning matritsasini G_h to'r ustidagi funksiyani o'ziga akslantiradigan chiziqli operatordek qarash mumkin.

Endi $(1.2.25),(1.2.26)$ tenglamalar sistemasining yagona yechimi mavjudligini ko'rsatamiz.

1-lemma. Faraz qilaylik , $\vartheta^{(h)}=\{\vartheta_{ik}\}$ miqdorlar $\overline{G_h}=G_h^0 U \Gamma_h$ to'r ustida aniqlangan qandaydir funksiya bo'lsin. Agar G_h^0 sohaning tugunlarida $\Delta_h \vartheta^{(h)} \ge 0$ shart bajarilsa , u holda $\vartheta^{(h)}$ o'zining eng katta qiymatini $\overline{G_h}$ ning chegarasida , yani Γ_h da qabul qiladi.

Isboti. Teskarisini faraz qilamiz. Aytaylik , $\vartheta^{(h)}$ o'zining eng katta qiymatini ichki nuqtada qabul qilsin. Umuman aytganda , shunday nuqtalar ko'p bo'lishi mumkin. Ular orasida shunday $(i,k)\in G_h^0$ tugunni tanlaymizki, $\vartheta_{i+1,k}, \vartheta_{i-1,k}, \vartheta_{i,k+1}, \vartheta_{i,k-1}$ qiymatlarning birortasida ϑ_{ik} dan qat'iyan kichik , masalan, $\vartheta_{i+1,k}<\vartheta_{ik}$ bo'lsin. U holda (i,k) tugunda quyidagiga ega bo'lamiz:

$$\Delta_h \vartheta +_{ik} = \frac{1}{h_1^2}\left(\vartheta_{i-1,k}-2\vartheta_{ik}+\vartheta_{i+1,k}\right)+\frac{1}{h_2^2}\left(\vartheta_{i,k+1}-2\vartheta_{ik}+\vartheta_{i,k-1}\right)=$$

$$=\frac{1}{h_1^2}\left[\left(\vartheta_{i-1,k}-\vartheta_{ik}\right)+\left(\vartheta_{i+1,k}\vartheta_{ik}\right)\right]+ \qquad (1.2.28)$$

$$+\frac{1}{h_2^2}\left[\left(\vartheta_{i,k+1}-\vartheta_{ik}\right)+\left(\vartheta_{i,k-1}-\vartheta_{ik}\right)\right]<0,$$

chunki $\vartheta_{i+1,k}-\vartheta_{ik}<0$ bo'lib, qolgan kichik qavslar ichidagi ifoda musbat emas, $(1.2.28)$ tengsizlik esa lemma shartiga ziddir. Demak , bizning farazimiz noto'g'ri ekan. Shu bilan lemma isbotlandi.

2-lemma. Faraz qilaylik, $\vartheta^{(h)}$ miqdorlar $\overline{G_h}$ to'r ustida aniqlangan qandaydir funksiya bo'lsin. Agar G_h^0 ning tugunlarida $\Delta_h \vartheta^h \le 0$ shart bajarilsa , u holda $\vartheta^{(h)}$ o'zining eng kichik qiymatini $\overline{G_h}$ ning chegarasida , yani Γ_h da qabul qiladi.

Teorema.(maksimum prinsipi). *Faraz qilaylik* , $\vartheta^{(h)}=\{\vartheta_{ik}\}$ *miqdorlar* $\overline{G_h}$ *da aniqlangan bo'lib,* G_h^0 *tugunlarda*

$$\Delta_h \vartheta_{ik}=0, i=1,2,....,M-1; k=1,2,...,N-1$$

Tenglamalarni qanoatlantirsin. U holda $\vartheta^{(h)}$ *o'zining modul bo'yicha eng katta qiymatini* Γ_h *chegarada qabul qiladi.*

Teoremaning isboti 1- va 2- lemmalardan kelib chiqadi.

Boshqa chegaraviy shartlarda $(1.2.1)$ tenglama uchun to'r metodining turg'unlik masalasini [3,8,9] da ko'rish mumkin.

1.3. Chebishevning optimal oshkor iteratsion metodi va uning ayirmali elliptik tenglamalarga tadbiqi

Biz bu yerda Chebishev ko'phadi ildizlarining xossalaridan foydalanib, oshkor iteratsion metodning yaqinlashishini tezlashtirish masalasini ko'ramiz va uni elliptik tipdagi tenglamalarni approksimatsiyalashda hosil bo'ladigan ayirmali sistemani yechishda qo'llaymiz.

Bu yerda

$$T_n(x) = 2^{1-n}\cos(n\arccos x) \qquad (1.3.1)$$

Chebishev ko'phadlari. Bu ko'phadning bosh koeffisienti 1 ga teng bo'lib, u $[-1,1]$ kesmada eng kam og'uvchi ko'phaddir. Ixtiyoriy $[a,b]$ kesma uchun

$$t = \frac{2x-a-b}{b-a}, \quad -1 \le t \le 1, \ a \le x \le b$$

almashtirish vositasida $(1.3.1)$ ko'phad quyidagi ko'rinishga ega bo'ladi:

$$T_n^{[a,b]}(x) = \frac{(b-a)^n}{2^{2n-1}}\cos\left(n\arccos\frac{2x-a-b}{b-a}\right) \qquad (1.3.2)$$

Bu ko'phadning maksimal og'ishi

$$\max_{a\le x\le b}\left|T_n^{[a,b]}(x)\right| = \frac{(b-a)^n}{2^{2n-1}}$$

Bo'lib, uning ildizlari quyidagilardan iborat:

$$x_k = \frac{a+b}{2} + \frac{b-a}{2}\cos\frac{(2k+1)\pi}{2}, \quad k = 0,1,.....,n-1 \qquad (1.3.3)$$

Endi quyidagi masalani yechamiz: $x=0$ nuqtada 1 qiymatni qabul qiladigan ko'phadlar orasida $[a,b]$ kesmada noldan eng kam og'adigan n- darajali

$P_n(x)$ ko'phad topilsin. Ravshanki , izlanayotgan ko'phad $(1.3.2)$ ko'phaddan o'zgarmas ko'payuvchi bilan farq qilishi kerak, ya'ni

$$P_n(x)=\frac{T_n^{[a,b]}(x)}{T_n^{[a,b]}(0)} \qquad (1.3.4)$$

Biz keyinchalik $T_n^{[a,b]}(0)\neq 0$ deb qaraymiz.

Agar $T_n^{[a,b]}(0)=0$ bo'lsa , u holda qaralayotgan masala darajasi aniq n bo'lgan ko'phadlar sinfida yechimga ega emas. Masalan , birinchi darajali ko'phad $P_1(x)=ax+1$ uchun

$$\max_{-1\le x\le 1}|P_1(x)|=|a|+1$$

va u $a=0$ bo'lganda minimumga erishadi. Ammo bu holda $P_1(x)$ birinchi darajali ko'phad bo'lmay qoladi.

Agar $b>a>0$ bo'lsa $(1.3.2)$ va $(1.3.4)$ lardan quyidagini hosil qilamiz:

$$P_n(x)=P_n\cos\left(n\arccos\frac{2x-a-b}{b-a}\right), \qquad (1.3.5)$$

bunda

$$P_n=\left(\cos\left(n\arccos\frac{a+b}{a-b}\right)\right)^{-1} \qquad (1.3.6)$$

Endi

$$\xi=\frac{a}{b},\quad \rho_0=\frac{1-\xi}{1+\xi} \qquad (1.3.7)$$

belgilash kiritsak,

$$P_n=\left(\cos\left(n\arccos\left(-\frac{1}{\rho_0}\right)\right)\right)^{-1} \qquad (1.3.8)$$

hosil bo'ladi.

Quyidagi

$$\begin{aligned}&\cos(n\arccos(-z))=(-1)^n\cos(n\arccos z)=\\&=(-1)^n 0.5\left(\left(z+\sqrt{z^2-1}\right)^n+\left(z-\sqrt{z^2-1}\right)^n\right)\end{aligned} \qquad (1.3.9)$$

Ayniyatlar ixtiyoriy haqiqiy yoki kompleks z sonlar uchun o’rinli ekanligi ma’lum.

Agar $(1.3.9)$ da $z = {}^{1}\!/_{\rho_0}$ deb olsak, unda

$$z-\sqrt{z^2-1}=\frac{1}{\rho_0}-\sqrt{\frac{1}{\rho_0^2}-1}=\frac{1-\sqrt{1-p_0^2}}{\rho_0}$$

bo’lib, bunda ρ_0 ning $(1.3.7)$ dagi qiymatini qo’ysak,

$$z-\sqrt{z^2-1}=\frac{1-\sqrt{1-(1-\xi)^2(1+\xi)^{-2}}}{(1-\xi)/(1+\xi)}=\frac{1+\xi-2\sqrt{\xi}}{1-\xi}=\frac{1-\sqrt{\xi}}{1+\sqrt{\xi}},$$

$$z+\sqrt{z^2-1}=\frac{1+\sqrt{\xi}}{1-\sqrt{\xi}}$$

larni hosil qilamiz. Bu ifodalarni $(1.3.9)$ ga qo’ysak , $(1.3.8)$ quyidagi ko’rinishga ega bo’ladi:

$$P_n=2(-1)^n\left(\rho^n+\rho^{-n}\right)^{-1}=(-1)^n\frac{2\rho S^n}{1+\rho^{2n}}$$

Bunda $\rho = {}^{\left(1-\sqrt{\xi}\right)}\!\Big/_{\left(1+\sqrt{\xi}\right)}$. Shunday qilib, $x=0$ nuqtada 1 qiymatni qabul qiladigan ko’phadlar orasida $[a,b]$ kesmada noldan eng kam og’adigan n- darajali ko’phad quyidagi ko’rinishga ega:

$$P_n(x)=(-1)^n q_n\cos\left(n\arccos\frac{2x-a-b}{b-a}\right) \qquad (1.3.10)$$

bunda

$$q_n=\frac{2\rho^n}{1+\rho^{2n}},\ \rho=\frac{1\text{-}\sqrt{\xi}}{1+\sqrt{\xi}},\ \xi=\frac{\mathrm{a}}{\mathrm{b}}\ (\mathrm{b}>\mathrm{a}>0) \qquad (1.3.11)$$

Bu ko’phadning ildizlari $(1.3.3)$ formula bilan topiladi.

Endi ikkinchi masalaga o’tamiz. Ushbu

$$F_n(\lambda)=(1-\tau_1\lambda)(1-\tau_2\lambda).........(1-\tau_n\lambda) \qquad (1.3.12)$$

Ko’phad uchun $\tau_1,\tau_2,.....,\tau_n$ parametrlarni shunday tanlash kerakki,

$$0<\gamma_1\le\overset{\max}{\lambda}\le\gamma_2\left|F_n(x)\right|$$

miqdor o’zining minimal qiymatiga erishsin.

Qaralayotgan ko’phad $F_n(0)=1$ shartni qanoatlantiradi. Shuning uchun ham qaralayotgan bu masala Chebishevning $(1.3.10)$ ko’phadi yordamida yechiladi. $(1.3.12)$ ning ildizlari

$$\lambda_k = \tau_k^{-1}, \quad \mathrm{k}=1,2,.....,\mathrm{n}$$

ushbu

$$P_n(\lambda)=(-1)^n q_n \cos\left(n \arccos\frac{2\lambda-\gamma_1-\gamma_2}{\gamma_2-\gamma_1}\right) \tag{1.3.13}$$

ko’phadning

$$\bar{\lambda}_k = \frac{\gamma_1+\gamma_2}{2}+\frac{\gamma_2-\gamma_1}{2}\cos\frac{(2k-1)\pi}{2n}, \quad \mathrm{k}=1,2,....,\mathrm{n} \tag{1.3.14}$$

ildizlar bilan ustma-ust tushishi kerak, bunda

$$q_n = \frac{2\rho^n}{1+\rho^{2n}}, \quad \rho=\frac{1\text{-}\sqrt{\xi}}{1+\sqrt{\xi}}, \quad \xi=\frac{\gamma_1}{\gamma_2} \tag{1.3.15}$$

Demak,

$$\tau_k^{-1} = \frac{\gamma_1+\gamma_2}{2}+\frac{\gamma_2-\gamma_1}{2}\cos\frac{(2k-1)\pi}{2n}, \quad \mathrm{k}=1,2,....,\mathrm{n} \tag{1.3.16}$$

deb olsak, u holda $F_n(\lambda)$ ning noldan og’ishi minimal bo’lib,

$$\max_{\gamma_1\le\lambda\le\gamma_2}|F_n(\lambda)| = q_n$$

bo’ladi, bunda q_n miqdor $(1.3.15)$ tengliklar bilan aniqlanadi.

Parametrlarning $(1.3.16)$ tengliklar bilan aniqlanadigan $\{\tau_k^{-1}\}_{k=1}^n$ to’plamini optimal deyish tabiiydir.

Shuni ham ta’kidlash kerakki, $\{\tau_k\}_{k=1}^n$ parametrlarning to’plami optimal bo’lishi uchun τ_k ni $(1.3.16)$ tengliklarda ko’rsatilgan tartibda olish shart emas. Buning uchun $\{\tau_k^{-1}\}_{k=1}^n$ to’plam Chebishev ko’phadlari ildizlarining $\{\bar{\lambda}_k\}_{k=1}^n$ to’plami bilan ustma-ust tushishi yetarlidir.

1.4. Parabolitik tenglamalar uchun ayirmali sxemalar

$G=\{0<x<l,\ \ 0<\mathrm{t}<\mathrm{T}\}$ sohada ushbu

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+f(x,t) \tag{1.4.1}$$

Parabolitik tenglamaning (issiqlik o'tkazuvchanlik tenglamasining)

$$u(x,0)=u_0(x) \tag{1.4.2}$$

Dastlabki shart va

$$u(0,t)=\mu_1(t),u(l)=\mu_2(t) \tag{1.4.3}$$

Chegaraviy shartlarni qanoatlantiradigan $u(x,t)$ yechimini topish talab qilinsin. Bu yerda $u_0(x),\mu_1(t),\mu_2(t)$ -berilgan funksiyalar. Ma'lumki $(1.4.1)$-$(1.4.3)$ masalaning yechimi mavjud va yagona. $u(x,t)$ barcha kerakli hosilalarga ega deb faraz qilamiz.

O'zgaruvchan koeffisisentli issiqlik o'tkazuvchanlik tenglamasini yechish. Koeffisientlari o'zgaruvchan bo'lgan quyidagi issiqlik o'tkazuvchanlik tenglamasi uchun birinchi chegaraviy masalani qaraylik:

$$\begin{gathered}\rho(x,t)\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(p(x,t)\frac{\partial u}{\partial x}\right)+f(x,t),\ \ 0<\mathrm{x}<\mathrm{l},\ \ 0<\mathrm{t}<\mathrm{T},\\ \mathrm{u}(\mathrm{x},0)=u_0(x),\ u(0,t)=\mu_1(t),\ \mathrm{u}(\mathrm{l},\mathrm{t})=\mu_2(t)\end{gathered} \tag{1.4.4}$$

Bunda $\rho(x,t),\ \ \mathrm{p}(\mathrm{x},\mathrm{t}),\ \ \mathrm{f}(\mathrm{x},\mathrm{t})$ yetarlicha silliq funksiyalar bo'lib,

$$C_1\geq p(x,t)\geq C_2>0,\ \ \rho(\mathrm{x},\mathrm{t})\geq C_3>0 \tag{1.4.5}$$

Shartlarni qanoatlantirsin. Har bir belgilangan t uchun (x_i,t) nuqtada $Lu=\frac{\partial}{\partial x}\left(p(x,t)\frac{\partial u}{\partial x}\right)$ differensial ifodani

$$\wedge_1(t)y_i=\frac{1}{h}\left[a(x_{i+1},t)\frac{y_{i+1}-y_i}{h}-a(x_i,t)\frac{y_i-y_{i-1}}{h}\right] \tag{1.4.6}$$

Ayirmali nisbat bilan approksimatsiya qilamiz. Bunda $a(x_i,t)$ koeffisient balans metodidagidek ikkinchi tartibli approksimatsiya shartlarini qanoatlantirishi kerak:

$$\left.\begin{aligned} \frac{a(x_{i+1},t)+a(x_i,t)}{2} &= p(x_i,t)+o(h^2),\\ \frac{a(x_{i+1},t)+a(x_i,t)}{2} &= p^{'}(x_i,t)+o(h^2) \end{aligned}\right\} \qquad (1.4.7)$$

Balans metodida ko'rganimizdek, $a(x_i,t)$ ni quyidagi

$$a(x_i,t)=\frac{p(x_i,t)+p(x_{i-1},t)}{2}, \quad \mathrm{a}(\mathrm{x}_\mathrm{i},t)=p\left(x_i-\frac{h}{2},t\right),$$

$$a(x_i,t)=\frac{2p(x_{i-1},t)p(x_i,t)}{p(x_{i-1},t)+p(x_i,t)}$$

Formulalarning birortasi bilan hisoblasak, $(1.4.7)$ munosabatlar o'rinli bo'ladi. Shunday qilib, $(1.4.4)$ differensial tenglamaga ushbu vazniy ayirmali masala mos keladi:

$$\begin{aligned} &\rho(x_i,t)\frac{y_i^{k+1}-y_i^k}{\tau}=\Delta(t)\left(\sigma y_i^{k+1}+(1-\sigma)y_i^k\right)+f(x_i,t),\\ &i=1,2,\ldots\ldots.,M-1, \qquad\qquad (1.4.8)\\ &y_i^0=u_0(x_i),\ \mathrm{y}_\mathrm{o}^\mathrm{k}=\mu_1(t_k),\quad \mathrm{y}_\mathrm{M}^\mathrm{k}=\mu_2(t_k) \end{aligned}$$

Bunda $t=t_k+0.5\tau$ va $\sigma=0.5$ bo'lsa , u holda $(1.4.8)$ sxema approksimatsiyaning xatoligi $r=0(\tau^2+h^2)$ bo'lib, $\sigma\neq 0.5$ bo'lganda $r=0(\tau+h^2)$ bo'ladi. Shunday qilib , biz oshkormas sxemaga ega bo'ldik. Bu sistemani yechish uchun haydash metodini qo'llash mumkin. Ayirmali sxemaning turg'unligini tekshirishda , oldingi bandlarda qaraganlarimizdan tashqari, *koeffisientlarni muzlatish prinsipi* ham ishlatiladi. Bu prinsip o'zgaruvchan koeffisientli masalani o'zgarmas koeffisientli masalaga keltiradi. Misol uchun $(1.4.8)$ sxemada $\partial=0$ va $\mathrm{f}(\mathrm{x}_\mathrm{i},t)=0$ deb olib, quyidagi oshkor sxemani qaraymiz:

$$\rho(x_i,t)\frac{y_i^{k+1}-y_i^k}{\tau}=\frac{1}{h}\left[a(x_{i+1},t)\frac{y_{i+1}^k-y_i^k}{h}-a(x_i,t)\frac{y_i^k-y_{i-1}^k}{h}\right] \qquad (1.4.9)$$

Faraz qilaylik , $\rho(x_i,t)$, $\mathrm{a}(\mathrm{x}_\mathrm{i},t)$ koeffisientlar o'zgarmas bo'lsin, yani $\rho(x_i,t)=\rho=\mathrm{const},\ \mathrm{a}(\mathrm{x}_\mathrm{i},t)=a=const$. U holda $(1.4.9)$ tenglamani quyidagicha yozish mumkin:

$$\rho\frac{y_i^{k+1}-y_i^k}{\tau}=\frac{a}{h^2}\left(y_{i+1}^k-2y_i^k+y_{i-1}^k\right)$$

Yoki

$$\frac{y_i^{k+1}-y_i^k}{\tau_i}=\frac{y_{i+1}^k-2y_i^k+y_{i-1}^k}{h^2}=,\quad \tau_1=\frac{\tau a}{\rho}$$

Ma’lumki, bu oshkor sxema $\tau_1 \le \frac{1}{2}h^2$ bo’lganda, yani

$$\frac{\tau a}{\rho}\le\frac{h^2}{2} \tag{1.4.10}$$

Bo’lganda turg’un bo’ladi.

Koeffisientlarni muzlatish prinsipi shuni tasdiqlaydiki, agar barcha x_i va $\mathrm{t}=\mathrm{t}_{\mathrm{k}}+0.5\tau$ lar uchun

$$\frac{\tau a(x_i,t)}{\rho(x_i,t)}\le\frac{h^2}{2} \tag{1.4.13}$$

Tengsizlik bajarilsa, u holda $(1.4.9)$ sxema turg’un bo’ladi. Agar $C_1\ge a(x_i,t)\ge C_2>0,\ \rho(\mathrm{x}_i,t)>C_3>0$ munosabatlar ma’lum bo’lsa, u holda

$$\frac{\tau}{h^2}\le\frac{C_3}{2C_1}$$

Bajarilganda $(1.4.13)$ tengsizlik o’rinli bo’ladi. $(1.4.9)$ sxemaning turg’unligini qat’iy ravishda asoslashni $[47]$ dan qarash mumkin.

Agar $\sigma\ge 0{,}5$ bo’lsa u holda koeffisientlarni muzlatish prinsipidan $(1.4.8)$ sxemaning absolyut turg’unligi kelib chiqadi.

Chiziqli bo’lmagan issiqlik o’tkazuvchanlik tenglamasini yechish. Quyidagi chegaraviy masalani qaraymiz:

$$\begin{aligned}&\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(p(u)\frac{\partial u}{\partial x}\right)+f(u),\quad 0<\mathrm{x}<\mathrm{l},0<\mathrm{t}<\mathrm{T},\\&\mathrm{u}(\mathrm{x},0)=u_0(x),u(0,t)=\mu_1(t),u(1,t)=\mu_2(t)\end{aligned} \tag{1.4.14}$$

Odatda , chiziqli bo’lmagan tenglamalarda $p(u)$ funksiyaning o’zgarish sohasi oldindan ma’lum bo’lmasa, oshkor sxemalar ishlatilmaydi.

Sof oshkormas sxema $y_i^{k+1}\left(i=\overline{1,M-1}\right)$ noma’lumlarga nisbatan chiziqli sistemani ham , chiziqli bo’lmagan sistemani ham tashkilk etish mumkin. Ushbu sxema

$$\frac{y_i^{k+1}-y_i^k}{\tau}=\frac{1}{h}\left[a_{i+1}\cdot\frac{y_{i+1}^{k+1}-y_i^{k+1}}{h}-a_i\cdot\frac{y_i^{k+1}-y_{i-1}^{k+1}}{h}\right]+f\left(y_i^k\right) \qquad (1.4.15)$$

Da $a_i=\frac{1}{2}\left[p\left(y_i^k\right)+p\left(y_{i-1}^k\right)\right]$ deb olsak, u holda $y_i^{k+1}\left(i=\overline{1,M-1}\right)$ noma’lumlarga nisbatan chiziqli, absolyut turg’un bo’lib, approksimatsiya xatoligi $r=0\left(\tau+h^2\right)$ bo’ladi. Bu sistemaning yechimi haydash metodi bilan topiladi.

Ko’pincha $(1.4.4)$ tenglama uchun ushbu

$$\begin{gathered}\frac{y_i^{k+1}-y_i^k}{\tau}=\frac{1}{h}\left[a\left(y_{i+1}^{k+1}\right)\frac{y_{i+1}^{k+1}-y_i^{k+1}}{h}-a\left(y_i^{k+1}\right)\frac{y_i^{k+1}-y_{i-1}^{k+1}}{h}\right]+f\left(y_i^{k+1}\right),\\ a\left(y_i^{k+1}\right)=\frac{p\left(y_i^{k+1}\right)+p\left(y_{i-1}^{k+1}\right)}{2}\end{gathered} \qquad (1.4.16)$$

sof oshkormas sxema ishlatiladi. Bu sxemani qo’llash uchun u yoki bu iteratsion metod qo’llaniladi. Masalan, iteratsion jarayonni quyidagicha olib borish mumkin:

$$\begin{gathered}\frac{y_i^{(S+1)}-y_i^k}{\tau}=\frac{1}{h}\left[a\left(y_{i+1}^{(S)}\right)\frac{y_{i+1}^{(S+1)}-y_i^{(S+1)}}{h}-a\left(y_i^{(S)}\right)\frac{y_i^{(S+1)}-y_{i-1}^{(S+1)}}{h}\right]+f\left(y_i^{(S)}\right),\\ S=0,1,.......,L-1,\ \ \mathrm{y}_{\mathrm{i}}^{(0)}=y_i^k,\ \mathrm{y}_{\mathrm{i}}^{(\mathrm{L})}=y_i^{k+1},\end{gathered} \qquad (1.4.17)$$

Bu yerda S-iteratsiya nomeri. Bu iteratsion jarayondan ko’ramizki, chiziqli bo’lmagan koeffisientlar oldingi iteratsiyada , yani y_i^{k+1} da hisoblanadi, y_i^{k+1} ning dastlabki yaqinlashishi sifatida y_i^k olinadi. Agar τ qadam qancha kichik bo’lsa , bu dastlabki yaqinlashish shuncha yaxshi bo’ladi. Agar koeffisientlar silliq bo’lib, $p(u)\geq C_2>0$ shart bajarilsa, odtda, ikki-ucgta iteratsiya qoniqarli natijaga olib keladi. Har bir yangi iteratsiyada $y_i^{(s+1)}$ ning qiymatlari $(1.4.17)$ sistemadan haydash metodi bilan aniqlanadi. Shuningdek, $(1.4.17)$sistemani yechish uchun ikkinchi tartibli aniqlikka ega bo’lgan prediktor-korrektor sxemasi ham ishlatiladi. Bunda k-qatlamdan $(k+1)$-qatlamga o’tish ikki bosqichda bajariladi. Birinchi bosqichda haydash metodi bilan oshkormas chiziqli sistema

$$\frac{y_i^{k+\frac{1}{2}}-y_i^k}{0.5\tau}=\frac{1}{2}\left[a\left(y_{i+1}^k\right)\frac{y_{i+1}^{k+\frac{1}{2}}-y_i^{k+\frac{1}{2}}}{h}-a\left(y_i^k\right)\frac{y_i^{k+\frac{1}{2}}-y_{i-1}^{k+\frac{1}{2}}}{h}\right]+$$

$$+f\left(y_i^k\right),\ \mathrm{i}=1,2,.....,\mathrm{M}-1,$$

$$\mathrm{y}_0^{\mathrm{k}+\frac{1}{2}}=\mu_1\left(t_k+0.5\tau\right),\ \mathrm{y}_{\mathrm{M}}^{\mathrm{k}+\frac{1}{2}}=y_2\left(t_k+0.5\tau\right)$$

Yechilib, oradagi $y_i^{k+\frac{1}{2}}$ $(\mathrm{i}=0,1,2,....,\mathrm{M})$ qiymatlar topiladi. Ikkinchi bosqichda esa $a(y),\ \mathrm{f}(\mathrm{y})$ chiziqli bo'lmagan koeffisientlar $y=y_i^{k+\frac{1}{2}}$ hisoblanib, y_i^{k+1} larni topish quyidagi olti nuqtali simmetrik sxema

$$\frac{y_i^{k+1}-y_i^k}{2}=\frac{1}{2h}\left[a\left(y_{i+1}^{k+\frac{1}{2}}\right)\frac{y_{i+1}^{k+1}-y_i^{k+1}}{h}-a\left(y_i^{k+\frac{1}{2}}\right)\frac{y_{i+1}^k-y_i^k}{h}\right]+$$

$$+f\left(y_i^{k+\frac{1}{2}}\right),\ \mathrm{i}=1,2,....,\mathrm{M}-1,\ \mathrm{y}_0^{\mathrm{k}+1}=\mu_1\left(t_{k+1}\right),\ \mathrm{y}_{\mathrm{M}}^{\mathrm{k}+1}=\mu_2\left(t_{k+1}\right)$$

asosida olib boriladi.

1.5. Giperbolik tenglamalarni ayirmali metodlar bilan yechish

Bir jinsli giperbolik tenglama uchun Koshi masalasini va birinchi chegaraviy masalani ko'rib chiqamiz.

Koshi masalasini yechish. Ma'lumki , Koshi masalasi quyidagicha qo'yiladi: $G=\{t>0,-\infty<x<\infty\}$ sohada ikki marta uzluksiz differensiallanuvchi shunday $u(x,t)$ funksiyani topish kerakki, bu sohada u

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2} \tag{1.5.1}$$

differensial tenglamani qanoatlantirib, $t=0$ to'g'ri chiziqda

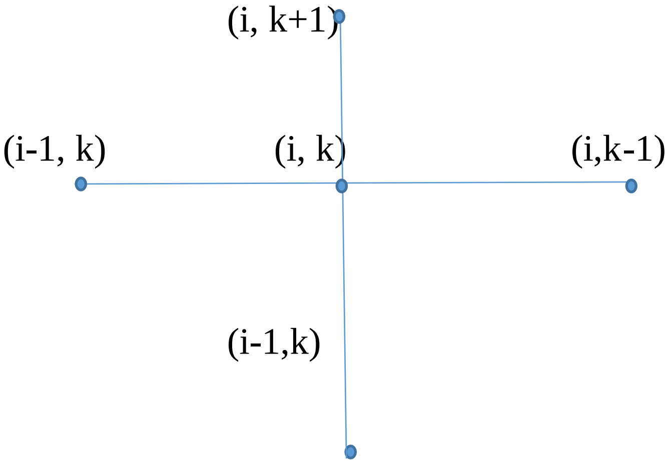
$$u(x,0)=\varphi(x),\ \left.\frac{\partial \mathrm{u}}{\partial \mathrm{t}}\right|_{t=0}=\psi(x) \tag{1.5.2}$$

dastlabki shartlarni qanoatlantirsin, bunda $\varphi(\mathrm{x})$ va $\psi(\mathrm{x})$ berilgan funksiyalar.

Differensial tenglamani ayirmali tenglama bilan almashtirish uchun $G_{h\tau}=\omega_h\omega_\tau$ to'rni kiritamiz, bunda

$$\omega_h = \{x_i = ih, i = 0,\pm1,\pm2,......,h > 0\},$$
$$\omega_1 = \{t_k = kt, k = 0,1,2,.....,t > 0\},$$

Keyin (1.3)-chizmadagidek besh nuqtali andazadan foydalanamiz. Bu andaza asosida qurilgan sxema *uch qatlamli sxema* deyiladi. Bu andazadan quyidagi ayirmali sxema kelib chiqadi:



(1.3)- chizma. Besh nuqtali andaza.

$$\frac{y_i^{k+1} - 2y_i^k + y_i^{k-1}}{\tau^2} = \frac{y_{i+1}^k - 2y_i^k + y_{i-1}^k}{h^2} \qquad (1.5.3)$$
$$i = 0,\pm1,\pm2,......,k = 1,2,........$$

Biz bilamizki, bu sxema $(1.5.1)$ differensial tenglamani $0(\tau^2 + h^2)$ aniqlikda approksimatsiya qiladi. Chegaraviy shartning ikkinchisini

$$\frac{y_i^1 - y_i^0}{\tau} = \varphi(x_1) \qquad (1.5.4)$$

Bilan almashtirsak, u holda approksimatsiya tartibi $0(\tau)$ bo'ladi. Ammo chegaraviy shartni ham $0(\tau^2)$ aniqlikda approksimatsiya qilish mumkin. Haqiqatan ham,

$$\frac{u(x,\tau) - u(x,0)}{\tau} = \frac{\partial u(x,0)}{\partial t} + \frac{\tau}{2}\frac{\partial^2 u(x,0)}{\partial t^2} + 0(\tau^2)$$

yoyilmadan hamda $(1.5.1)$ differensial tenglamadan hosil bo'ladigan

$$\frac{\partial^2 u(x,0)}{\partial t^2}=\frac{\partial^2 u(x,0)}{\partial x^2}=\varphi''(x)$$

munosabatdan foydalanib, quyidagiga ega bo'lamiz:

$$\frac{\partial u(x,0)}{\partial t}=\frac{u(x,\tau)-u(x,0)}{\tau}-\frac{\tau}{2}\varphi''(x)+0(\tau^2)$$

bundan esa

$$\frac{y_i^1-y_i^0}{\tau}=\psi(x_i)+\frac{\tau}{2}\varphi''(x_i) \qquad (1.5.5)$$

ga ega bo'lamiz. Agar $\varphi(x)$ ning analitik ifodasi berilgan bo'lmasa , u holda $\varphi''(x_i)$ ni $0(h^2)$ aniqlikda

$$\Delta_2\varphi_i=\frac{1}{h^2}(\varphi(x_{i+1})-2\varphi(x_i)+\varphi(x_{i-1}))$$

bilan almashtirish mumkin, natijada

$$\frac{y_i^1-y_i^0}{\tau}=\psi(x_i)+\frac{\tau}{2}\Delta_2\varphi_i \qquad (1.5.6)$$

ga ega bo'lamiz.

Shunday qilib, dastlabki shart, $(1.5.3)$ va $(1.5.6)$ dan quyidagilarni hosil qilamiz:

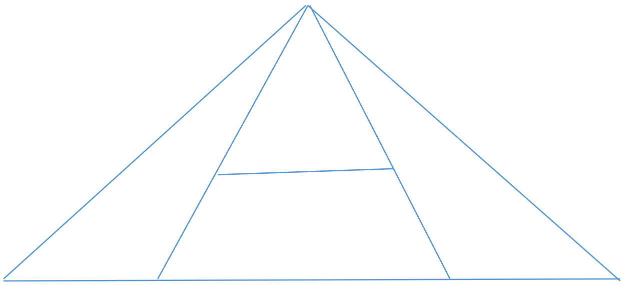
$$y_i^0=\varphi(x_i),\ \mathrm{y}_\mathrm{i}^{'}=\varphi(x_i)+\tau\psi(x_i)+\frac{\tau^2}{2}\Delta_2\varphi_i, \qquad (1.5.7)$$

$$y_i^{k+1}=2y_i^{k+1}+\tau^2\Delta_2 y_i^k-y_i^{k-1},\ \ \mathrm{i}=0,\pm1,\pm2,.....\mathrm{k}=1,2,..... \qquad (1.5.8)$$

Bunda ko'ramizki, y_i^0 va $y_i^{'}$ $(\mathrm{i}=0,\pm1,\pm2,......)$ qiymatlar $(1.5.7)$ dan ma'lum. $(1.5.8)$dan barcha $k=1,2,....$ uchun ketma-ket avval y_i^1 $(\mathrm{i}=0,\pm1,\pm2,...)$, keyin y_i^3 $(\mathrm{i}=0,\pm1,\pm2,.....)$ va boshqalarni topib olamiz.

Parabolik tenglamada sxemaning turg'unligi uchun qadamalar orasida $\tau\le\frac{1}{2}h^2$ shartning bajarilishi kerakligini ko'rgan edik. Endi giperbolik tenglama $\gamma=\frac{\tau}{h}$ uchun qanday shartni bajarish kerakligini tekshiramiz.

Faraz qilaylik, ixtiyoriy i va $j \geq 2$ uchun $M(x_i, t_j)$ tugunda y_i^j ning qiymatini $(1.5.8)$ formula bilan topish kerak bo'lsin. Buning uchun $(1.5.8)$ da $k = j-1$ deb olib, ko'ramizki, y_i^j ning qiymati y_{i+1}^{j-1}, y_i^{j-1}, y_{i+1}^{j-1} va y_i^{j-2} lar orqali ifodalanadi. Agar $j > 3$ bo'lsa, o'z navbatida, y_{i+1}^{j-1}, y_i^{j-1}, y_{i-1}^{j-1}, y_i^{j-2} larning qiymatlari past qatlamlardagi y_{i+2}^{j-2}, y_{i+1}^{j-2}, y_i^{j-2}, y_{i-1}^{j-2}, y_{i-2}^{j-2}, y_{i+1}^{j-3}, y_i^{j-3}, y_{i-1}^{j-3}, y_i^{j-4} lar orqali ifodalanadi. Bu jarayonni davom ettirib, oxirgi natijada y_i^j ni $y_m^0 (m = i+s, s = 0, \pm 1, \ldots, \pm j-2)$ va $y_m^1 (m = i+s, s = 0, \pm 1, \ldots, \pm j-1)$ orqali ifodalaymiz. Bu qiymatlarning barchasi teng yonli ΔMCD uchburchak ichida yotadi (1.4-chizma). Bu uchburchakning uchi $M(x_i, t_j)$ nuqtada bo'lib, bir tomoni Ox o'qida, qolgan ikki tomoni MC va MD dan iborat. Ular Ox o'qi bilan $\pm arctg\gamma$, $\gamma = \tau / h = const$ burchakni tashkil etadi. MCD uchburchak $(1.5.8)$ *ayirmali sxemaning aniqlanganlik uchburchagi* deyiladi.



1.4-chizma. To'r nuqtalari.

Shunday qilib, y_i^j ning qiymati M nuqtada $(1.5.8)$ tenglamaning CD hamda EF kesmalarda yotuvchi y_m^0 va y_m^1 dastlabki qiymatlari orqali aniqlanadi. Matematik fizikadan ma'lumki, $u(x,t)$ yechimning $M(x_i, t_j)$ nuqtadagi qiymati $(1.5.1)$ tenglama hamda $M(x_i, t_j)$ nuqtadan o'tuvchi

$$t - t_j = x - x_i, \quad t - t_j = -x + x_i \qquad (1.5.9)$$

xarakteristikalar $t = 0$ to'g'ri chiziqda ajratadigan kesmadagi shartlar bilan , ya'ni AB kesmadagi boshlang'ich shartlar bilan bir qiymatli ravishda aniqlanadi. $(1.5.1)$ tenglamaning $(1.5.9)$ xarakteristikalari o'zaro perpendikulyar bo'lib, Ox o'qi bilan $\frac{\pi}{4}$ va $\frac{3\pi}{4}$ burchaklarni tashkil etadi; MAB uchburchak (6.1) *differensial tenglamaning aniqlanganlik uchburchagi* deyiladi.

Faraz qilaylik, to'rning τ qadami h dan katta bo'lsin (14-chizma). Bu holda $\angle MAB < \angle MCD$ va $\mathrm{tg}(\angle \mathrm{MCD}) = \gamma > 1$ bo'lib, ayirmali tenglamaning aniqlik burchagi ichida yotadi. Shuning uchun ham *CD* kesmada beriladigan dastlabki shartlar *M* nuqtada yechimni aniqlash uchun yetarli emas. Agar biz AC va *DB* kesmalarda boshlang'ich shartlarni o'zgartirsak, $(1.5.1),(1.5.2)$ masalaning yechimi butun *G* sohada jumladan, *M* nuqtada o'zgarishi kerak. Ammo y_i^j ning to'rdagi qiymati *M* nuqtada bunday o'zgarishlarga bog'liq bo'lmasdan, o'zgarmay qoladi. Demak, $\gamma > 1$ bo'lganda $(1.5.7),(1.5.8)$ ayirmali masalaning yechimi $h \to 0$ da $(1.5.1),(1.5.2)$ Koshi masalasining yechimiga yaqinlashmaydi, $(1.5.7),(1.5.8)$ayirmali masala $(1.5.1),(1.5.2)$ differensial masalani approksimatsiya qilganligi sababli u turg'un bo'la olmaydi, chunki approksimatsiya va turg'unlikdan yaqinlashish kelib chiqishi kerak. Bundan biz shunday xulosaga kelamiz: $\gamma = \tau/h = const$ bo'lganda to'r metodi bilan topilgan taqribiy yechimlar ketma-ketligi $h \to 0$ da yaqinlashishi uchun $\gamma \le 1$ shartning bajarilishi zarurdir, ya'ni differensial tenglamaning aniqlanganlik uchburchagi ayirmali tenglamaning aniqlanganlik uchburchagi bilan ustma-ust tushishi yoki uning ichida yotishi mumkin. Umumiy holda differensial tenglamaning aniqlanganlik uchburchagi egri chiziqli uchburchak bo'ladi, ammo bu holda ham differensial tenglamaning aniqlanganlik uchburchagi ayirmali sxemaning aniqlanganlik uchburchagi ichida yotishi lozim. Bu shartning bajarilishi uchun to'r qadamlari ma'lum munosabatda olinishi, yani to'rning maxsus tanlanishi talab etiladi. Differensial tenglamaning koeffisientlaridan va boshlang'ich shartlaridan ma'lum silliqlik talab qilinganda taqribiy yechimlar ketma-ketligining Koshi masalasi yechimiga yaqinlashishi uchun yuqoridagi shart yetarli bo'ladi. [7]

Birinchi chegaraviy masalani yechish. Biz endi tebranish tenglamasi uchun $G=\{0<x<1, 0<t<T\}$ sohada ushbu birinchi chegaraviy masalani ko'rib chiqamiz. Yani G sohada ikki marta uzluksiz differensiallanuvchi $u(x,t)$ funksiyani topish kerakki , bu sohada u

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2} \qquad (1.5.10)$$

tenglamani qanoatlantirib, $t=0$ to'g'ri chiziqda

$$u(x,0)=\varphi(x), \frac{\partial u}{\partial t}(x,0)=\psi(x) \qquad (1.5.11)$$

dastlabki shartlarni va

$$u(0,t)=\mu_1(t), \ u(1,t)=\mu_2(t), \ 0\le \mathrm{t} \le \mathrm{T} \qquad (1.5.12)$$

Chegaraviy shartlarni qanoatlantirsin. Bu maslani to'r metodi bilan yechish uchun ushbu

$$G_{h\tau}=\{x_i=ih, i=\overline{0,M}, hM=1; t_k=k\tau, k=\overline{0,N}, N\tau=T\}$$

To'rni kiritamiz va 1.3-chizmadagidek uch qatlamli andaza bo'yicha (1.5.1) differensial tenglamani (1.5.3) dagi ayirmali sxema bilan almashtiramiz, bu yerda i va k quyidagi qiymatlarni qabul qiladi:

$$i=1,2,......,M-1; k=1,2,.....,N-1$$

Dastlabki shartlar uchun (1.5.7) formuladan foydalanamiz. Chegaraviy shartlar quyidagicha yoziladi:

$$y_0^{k+1}=\mu_1(t_{k+1}), y_M^{k+1}=\mu_2(t_{k+1}), k=0,1,....,N-1$$

Bularning hammasini birlashtirib, ayirmali sxemaning quyidagi hisoblash algoritmiga ega bo'lamiz:

$$y_i^0=\varphi(x_i), y_i^1=\varphi(x_i)+\tau\psi(x_i)+\frac{\tau^2}{2}\Delta_2\varphi_i, \qquad (1.5.13)$$

$$y_i^{k+1}=2y_i^k+\tau^2\Delta_2 y_i^k-y_i^{k-1}, i=1,2,.....,M-1, \qquad (1.5.14)$$

$$y_0^{k+1}=\mu_1(t_{k+1}), y_M^{k+1}=\mu_2(t_{k+1}), k=0,1,......,N-1 \qquad (1.5.15)$$

Yuqorida ko'rdikki, bu sxema $(1.5.1),(1.5.3)$ chegaraviy masalani $0(\tau^2+h^2)$ aniqlikda approksimatsiya qiladi. Ko'rsatish mumkinki, agar ixtiyoriy $\varepsilon>0$ uchun τ va h qadamlar quyidagi

$$\frac{\tau^2}{h^2}\le\frac{1}{1+\varepsilon} \qquad (1.5.16)$$

Shartni qanoatlantirsa , $(1.5.7),(1.5.10),(1.5.11)$ sxema turg'un bo'ladi. Biz buning isbotiga to'xtalib o'tirmaymiz.

Misol. To'r metodi bilan $G=\{0<x<1,\ 0<t<T\}$ sohada

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$$

To'r tenglamasining

$$u(0,t)=u(1,t)=0(0\le t\le 1),$$
$$u(x,0)=\sin\pi x,\frac{\partial u}{\partial t}(x,0)=0$$

Chegaraviy va dastlabki shartlarni qanoatlantiradigan taqribiy yechimi topilsin.

Yechish. Bu yerda $h=0{,}1$ va $\tau=0.08$ deb olamiz. Keyin $\varphi(x)=\sin\tau x,\ \varphi''(x)=-\tau^2\sin\pi x$ hamda $\psi(x)=0$ ligini hisobga olib, (6.5) formulani quyidagicha yozamiz:

$$y_i^1=y_i^0+\frac{\tau^2}{2}\varphi''(x_i)=(1-0.0032\pi^2)\sin\pi x_i$$

Endi Δ_2 operatorning ko'rinishini e'tiborga olsak, hisoblash uchun quyidagi algoritm hosil bo'ladi:

$$y_i^0=\sin\pi x_i, y_i^1=(1-0.0032\pi^2)\sin\pi x_i, i=1,2,......,M-1,$$
$$y_0^{k+1}=y_M^{k+1}=0, k=0,1,........,N-1,$$
$$y_i^{k+1}=0.64y_{i+1}^k+0.72y_i^k+0.64y_{i-1}^k-y_i^{k-1}$$

Hisoblashni faqat $0\le x\le\frac{1}{2}$ uchun bajarsa yetarli bo'ladi, chunki $u=u(x,t)$ yechimning grafigi $x=\frac{1}{2}$ tekislikka nisbatan simmetrik ravishda joylashgan.

I bobning qisqacha xulosasi

I bobda xususiy hosilali differensial tenglamalarni taqribiy yechish metodlari haqida gapirilgan.

II bob. Xususiy hosilali differensial tenglamalarni Mathcad muhitida taqribiy yechish

2.1. Elliptik tenglamalarni Mathcad dasturi yordamida taqribiy yechish.

Faraz qilamiz:

$$u(A,y) = u(y)_1 \qquad u(B,y) = u(y)_2$$

$$u(x,C) = u(x)_3 \qquad u(x,D) = u(x)_4$$

chegaraviy shartlar bilan

$$u_{x,x} + u_{y,y} = f(x,y) \qquad A < x < B \quad C < y < D$$

tenglamani sonli yechish talab qilingan bo'lsin.

$$\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = y_{i,j}$$

$$i = 1,..N-1 \quad j = 1,..M-1$$

ayirmali sxema bilan almashtiramiz.

Bu sistemani yechib: $y_{i,j} \quad i = 1,..N \qquad j := 0,1..M$

taqribiy yechimini topamiz .

Konkret bir masalani qaraymiz.

$$\frac{d^2}{dx_1^2}u(x_1,x_2) + \frac{d^2}{dx_2^2}u(x_1,x_2) = 2x_2$$

$$f(x_1,x_2) := -2(x_1 + x_2)$$

$$u(0,x_2) = 0 \quad \mu \text{ chap- } \mu_{left}(x_2) := 0$$

$$u(x_1,0) = 0 \quad \mu \text{ quyi- } \mu_{bottom}(x_1) := 0$$

$$u(1,x_2) = x_2 + (x_1)^2 \quad \mu_{right}(x_2) := x_2 + x_2^2$$

$$u(x_1,1) = (x_1^2) + x_1 \qquad \mu_{top}(x_1) := x_1^{\ 2} + x_1$$

tenglamaning aniq yechimi:

$$u(x_1,x_2) := x_1^{\ 2}x_2 + x_1x_2^{\ 2}$$

$$G = \{(x_1, x_2) : 0 \le x_1 \le 1, 0 \le x_2 \le 1\}$$

ekanligi ma’lum.

$$N := 5 \quad M := 5 \quad a := 0 \quad b := 1 \quad c := 0 \quad d := 1 \quad h := \frac{b-a}{N} \quad l := \frac{d-c}{M}$$

$$h = 0.2 \quad l = 0.2 \quad i := 0...N \quad j := 0..M \quad x_{1,,i} := ih \quad x_{2,j} := jl \quad y_{j,0} := \mu_{left}(x_{2,j})$$

$$y_{j,N} := \mu_{right}(x_{2,j}) \quad y_{0,i} := \mu_{bottom}(x_{1,i}) \quad y_{M,i} := \mu_{top}(x_{1,i}) \quad j := 0..M-2 \quad i := 0...N$$

$$F_{j,i} := f(x_{1,i}, x_{2,j+1}) \quad F^{<1>} = \begin{pmatrix} -0.8 \\ -1.2 \\ -1.6 \\ -2 \end{pmatrix} \quad F = \begin{bmatrix} -0.4 & -0.8 & -1.2 & -1.6 & -2 & -2.4 \\ -0.8 & -1.2 & -1.6 & -2 & -2.4 & -2.8 \\ -1.2 & -1.6 & -2 & -2.4 & -2.8 & -3.2 \\ -1.6 & -2 & -2.4 & -2.8 & -3.2 & -3.6 \end{bmatrix}$$

$$F_{0,2} := F_{0,i} + \frac{y_{0,i}}{l^2} \quad F_{M-2,i} := F_{M-2,i} + \frac{y_{0,i}}{l^2} \quad F = \begin{bmatrix} -0.4 & -0.8 & -1.2 & -1.6 & -2 & -2.4 \\ -0.8 & -1.2 & -1.6 & -2 & -2.4 & -2.8 \\ -1.2 & -1.6 & -2 & -2.4 & -2.8 & -3.2 \\ -1.6 & -2 & -2.4 & -2.8 & -3.2 & -3.6 \end{bmatrix}$$

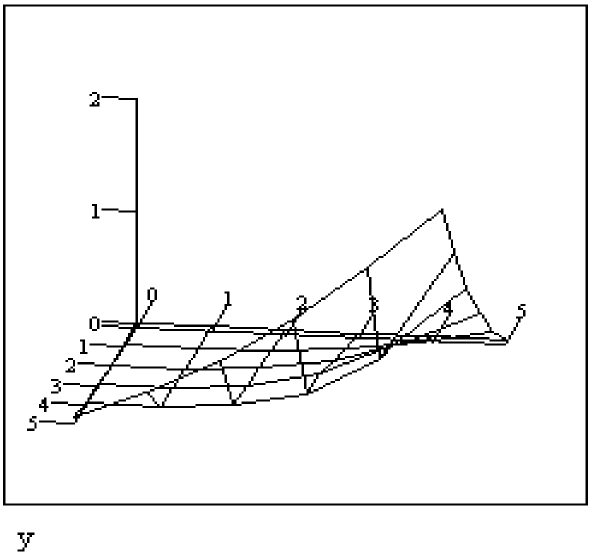
$$A := \frac{1}{h^2} identity(M-1) \quad B := A \quad m := 0,..M-2 \quad n := 0,..N-2$$

$$\Lambda_{m,n} := if\,(m = n-1) + (m-1 = n), 1, 0 \quad C := \frac{2}{h^2} identity(M-1) - \frac{1}{l^2}\Lambda \quad \alpha_1 := C^{-1}B \quad \beta_1 := C^{-1}F^{<0>}$$

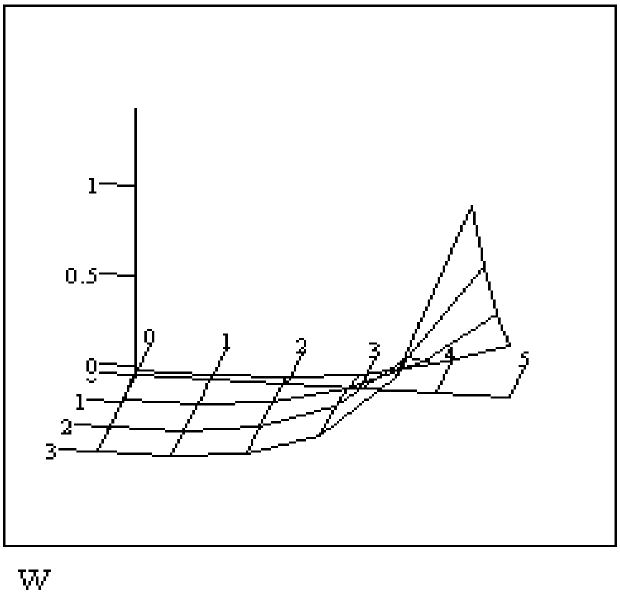
$$i := 1,..N \quad \alpha_i := (C - A\alpha_i)^{-1}B \quad \left|C^{-1}A\right| + \left|C^{-1}B\right| = 0.01 \quad \left|C^{-1}B\right| = 0.005 \quad \alpha_{i+1} := (C - A\alpha_i)^{-1}B$$

$$\beta_{i+1} := (C - A\alpha_i)^{-1}(A\beta_i + F^{<i>}) \quad i := 0,..N \quad j := 0,..M-2 \; W_{j,i} := y_{j+1,i} \quad i := N-1,...0$$

$$W^{<i>} := \alpha_{i+1}W^{<i+1>} + \beta_{i+1} \quad i := 0,..N \quad j := 0,...M-2 \quad y_{j+1,i} := W_{j,i}$$



2.1 chima. Taqribiy yechim grafigi



2.2. chizma. To’rni ichki nuqtalaridagi yechim grafigi

2.2. Chebishev parametrlar majmuasi bilan oshkor iteratsion metodi Mathcad dasturida qo’llash.

Ax=f chiziqli algebraik tenglamalar sistemasini yechish talab qilingan bo'lsin. Bu yerda A>0 -nxn o'lchovli simmetrik kvadrat matritsa ; f - n o'lchovli berilgan vektor; x - topilishi lozim bo'lgan n- o'lchovli noma'lum vektor;

Yechimni $\varepsilon := 10^{-2}$ aniqlik bilan topish talab qilingan bo'lsin. Soddalik uchun A matrirsa va f vektor sifatida

$$A := \begin{pmatrix} 6 & 1 & -1 \\ 1 & 7 & 2 \\ -1 & 2 & 8 \end{pmatrix} \qquad f := \begin{pmatrix} 6 \\ 10 \\ 9 \end{pmatrix} \qquad E := \mathrm{identity}(\mathrm{rows}(A))$$

berilgan bo'lsin. Chebishev parametrlar majmuasini hisoblaymiz.

$$\Lambda = \begin{pmatrix} 7.143 \\ 9.571 \\ 4.286 \end{pmatrix}$$

$\Lambda := \mathrm{eigenvals}(A)$ A matritsaning xos sonlari vektori ;

$\lambda max := \max(\Lambda)$ A matritsaning eng katta xos soni ;

$\lambda min := \min(\Lambda)$ A matritsaning eng kichik xos soni ;

$$\tau 0 := \frac{2}{\lambda max + \lambda min} \qquad \xi := \frac{\lambda min}{\lambda max} \qquad \rho 0 := \frac{1-\xi}{1+\xi} \qquad \rho 1 := \frac{1-\sqrt{\xi}}{1+\sqrt{\xi}}$$

ε aniqlikkacha yechimni topish uchun lozim bo'ladigan iteratsiyalar sonini esa

$$n0 := \mathrm{ceil}\left(\frac{\ln\left(\frac{1}{\varepsilon}\right)}{\ln\left(\frac{1}{\rho 1}\right)}\right)$$

formula yordamida hisoblaymiz. Bu yerda ceil(r) - r sonining r - dan kichik bo'lmagan eng kichik butun son. Hisoblash natijalariga ko'ra

$$n0 = 3 \qquad \rho 1 = 0.198 \qquad k := 1..n0 \qquad t_k := \cos\left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n0}\right] \qquad \tau_k := \frac{\tau 0}{1 + \rho 0 \cdot t_k}$$

Chebishev parametrlar majmuasi; Boshlang'ich yaqinlashish vektori sifatida

$$x^{\langle 0 \rangle} := \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}$$

vektorni olamiz. Navbatdagi yaqinlashishlarni topish uchun iteratsiya usuli

$$x^{\langle k \rangle} := \left(E - \tau_k \cdot A\right) \cdot x^{\langle k-1 \rangle} + \tau_k \cdot f$$

rekurrrent formulalarda bajarib

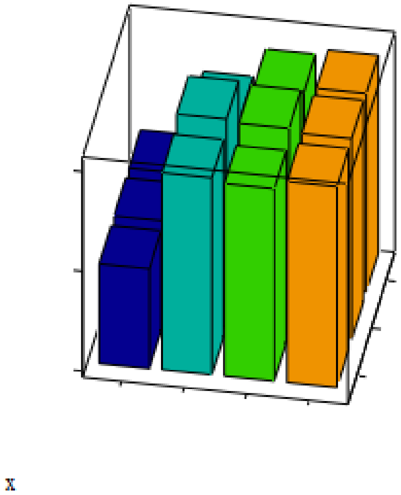
$$x^{\langle n0 \rangle} = \begin{pmatrix} 0.995406379986161 \\ 1.00776398941987 \\ 1.00694138362392 \end{pmatrix}$$

yechimni

$$x^{\langle n0 \rangle} - x^{\langle n0-1 \rangle} = \begin{pmatrix} 0.027 \\ -0.02 \\ 0.043 \end{pmatrix}$$

aniqlikda topadi.

$$x = \begin{pmatrix} 0.5 & 0.825 & 0.969 & 0.995 \\ 0.5 & 1.042 & 1.028 & 1.008 \\ 0.5 & 0.988 & 0.964 & 1.007 \end{pmatrix}$$



2.3 chizma. Ildizlarning yaqinlashish grafigi.

x-absissa o'qi - iteratsiyalar tartibi;

y-ordinata o'qi - yaqinlashish qiymati;

har bir ildiz alohida rangda berilgan.

2.3. Parabolik tenglamalarni Mathcad dasturi yordamida taqribiy yechish.

Issiqlik o’tkazuvchanlik tenglamasini sonli yechish.

Faraz qilamiz :

$$\frac{du}{dt} = \frac{d^2}{dx^2}u + f(x,t) \qquad 0 < x < L \quad o < t < T$$

issiqlik o’tkazuvchanlik tenglamasi berilgan bo’lib

$u(x,0) = u_0(x)$ boshlang’ich shart va

$u(0,t) = \mu_1(t)$ $\quad u(L,t) = \mu_2(t)$ chegaraviy shartlarni qanoatlantiruvchi yechimni topish talab qilingan bo’lsin.

Bu masalani sonli yechishning ko’pgina usullari mavjud.

Bu erda biz bu masalani sof oshkormas ayirmali sxema bilan yechishni ko’rib chiqamiz.

Oshkormas sxema quyidagi ko'rinishda yoziladi :

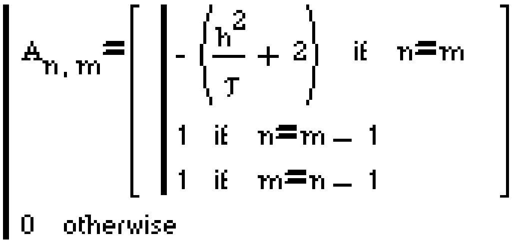
$$\frac{(y_n)^{m+1}-(y_n)^m}{\tau}=\frac{(y_{n+1})^{m+1}-2(y_n)^{m+1}(y_{n-1})^{m+1}}{h^2}+f(x_n,t_n) \qquad (2.3.1)$$

munosobat o'rinli ekanligi ma'lum .

(2.3.1) sistema *m* ning har bir qiymati uchun uch diagonalli sistemalardan iborat. Bu sistema uchun progonka usulini qo'llash mumkin .

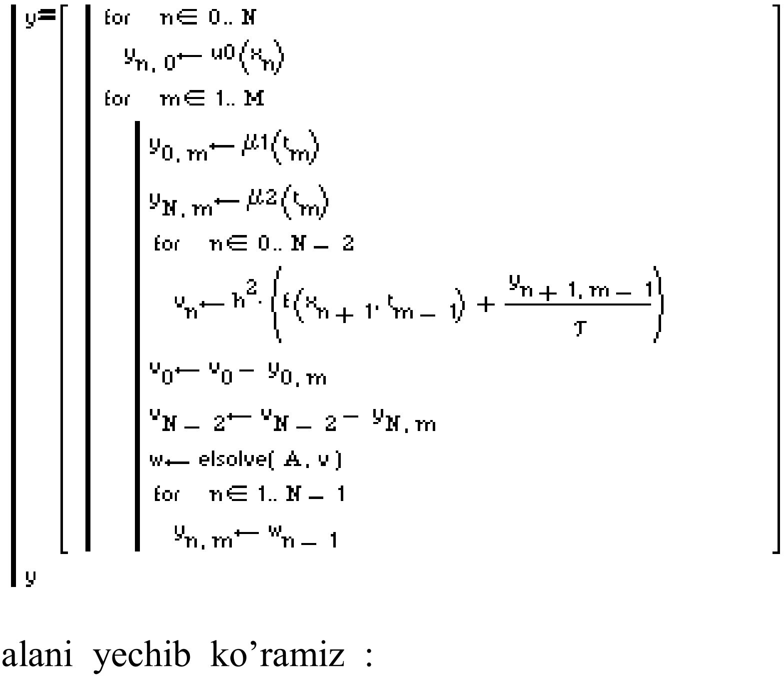
Barcha qatlamlar uchun (2.3.1) sitsemaning matristasi bir xil bo'ladi .

Bu matristani A bilan belgilaymiz . A matristani yaratish quyidagicha programma yordamida amalga oshiriladi .



```
A_{n,m} = [ | -(h^2/τ + 2)  if  n=m
            | 1  if  n=m - 1
            | 1  if  m=n - 1 ]
0  otherwise
```

Barcha qatlamlardagi yechim qiymatini topish programmasi quyidagidan iborat :



```
y=[ | for  n∈ 0.. N
    |   y_{n,0} ← u0(x_n)
    | for  m∈ 1.. M
    |   | y_{0,m} ← μ1(t_m)
    |   | y_{N,m} ← μ2(t_m)
    |   | for  n∈ 0.. N - 2
    |   |   v_n ← h^2·( f(x_{n+1}, t_{m-1}) + y_{n+1,m-1}/τ )
    |   | v_0 ← v_0 - y_{0,m}
    |   | v_{N-2} ← v_{N-2} - y_{N,m}
    |   | w ← elsolve( A, v )
    |   | for  n∈ 1.. N - 1
    |   |   y_{n,m} ← w_{n-1} ]
y
```

Endi bitta masalani yechib ko'ramiz :

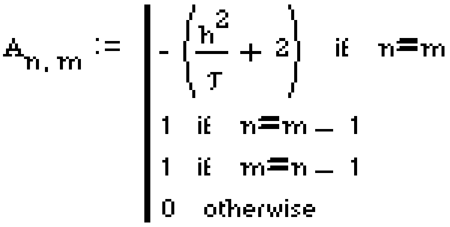
$$L:=1 \quad T:=1 \quad f(x,t):=2(t-1)+(x-t^2)e^{tx} \; N:=50 \quad M:=50 \quad u_0(x):=x^2+1$$

$n := 0...N \quad m := 0..M \quad \mu_1(t) := t^2 + 1 \quad \mu_2(t) := t^2 + e^t + 1 \; \tau := \frac{T}{M} \quad h := \frac{L}{N}$

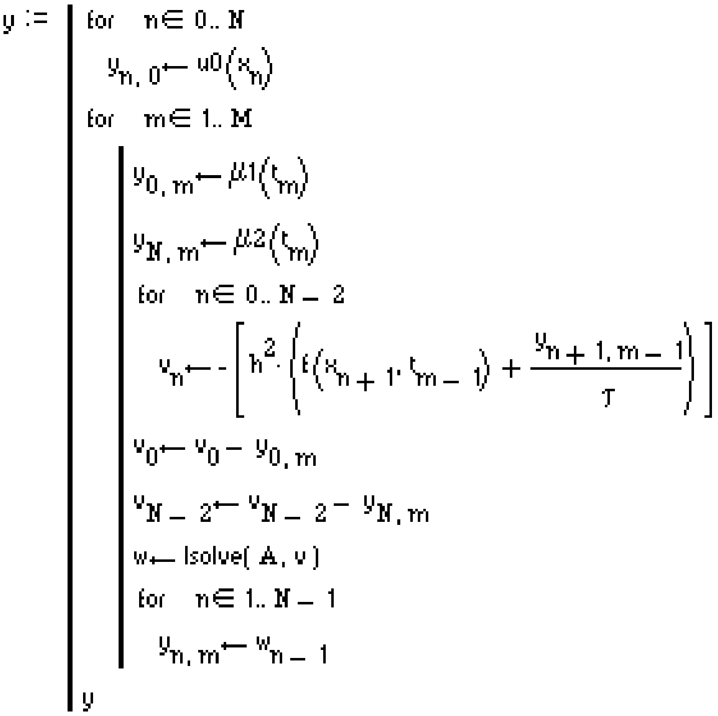
$\tau := 0.02 \quad h := 0.02 \; y_{n,m} = (y_n)^m = y(x_n, t_n)$ kabi belgilab $x_n := nh \quad t_m := \tau m$

to'rni aniqlaymiz :

$n := 0..N-2 \quad m := 0..M-2$



```
A_{n,m} := | $-\left(\frac{h^2}{\tau} + 2\right)$ if $n = m$
           | 1 if $n = m - 1$
           | 1 if $m = n - 1$
           | 0 otherwise
```



```
y := | for $n \in 0..N$
     |   $y_{n,0} \leftarrow u0(x_n)$
     | for $m \in 1..M$
     |   | $y_{0,m} \leftarrow \mu1(t_m)$
     |   | $y_{N,m} \leftarrow \mu2(t_m)$
     |   | for $n \in 0..N-2$
     |   |   $v_n \leftarrow -\left[h^2 \cdot \left(f(x_{n+1}, t_{m-1}) + \frac{y_{n+1,m-1}}{\tau}\right)\right]$
     |   | $v_0 \leftarrow v_0 - y_{0,m}$
     |   | $v_{N-2} \leftarrow v_{N-2} - y_{N,m}$
     |   | $w \leftarrow \mathrm{lsolve}(A, v)$
     |   | for $n \in 1..N-1$
     |   |   $y_{n,m} \leftarrow w_{n-1}$
     | y
```

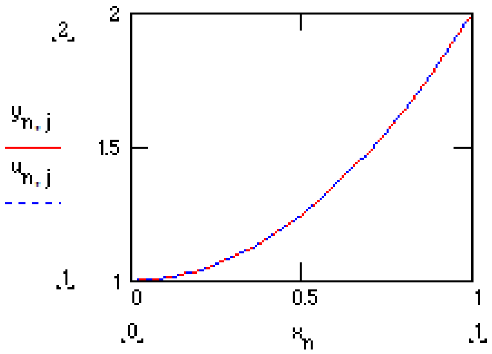
$M := 50 \quad N := 50$

$i := 0..N \qquad j := 0..M$

$g(x,t) := t^2 + x^2 + e^{xt} \qquad u_{i,j} := g(x_i, t_j)$

$n := 0,1..N$

$j := FRAME$



2.4 chizma. Qatlamlardagi yechim

2.4. Giperbolik tenglamalarni Mathcad dasturi yordamida taqribiy yechish.

Faraz qilamiz:

$$u(x,0)=\mu_1(x) \qquad (2.4.1)$$

$$\frac{du(x,0)}{dt}=\mu_2(x)$$

boshlang’ich shartlar va

$$u(0,t)=u_0(t)$$

$$(2.4.2)$$

$$u(L,t)=u_L(t)$$

chegaraviy shartlar bilan

$$\frac{d^2}{dt^2}u=\frac{d^2}{dt^2}u+f(x,t) \quad t>0, 0<x<L \qquad (2.4.3)$$

tenglamani yechish talab qilingan bo’lsin.

(2.4.3) tenglamani

$$\omega_{h,\tau}=\left[(ih,j,\tau),i=0...N,j=0,1..M\right]$$

to’rda ayirmali sxema bilan apraksimastiyalaymiz :

$$\frac{y_{i,j-1}-2y_{i,j}+y_{i,j+1}}{h^2}-\frac{y_{i-1,j}-2y_{i,j}+y_{i+1,j}}{\tau^2}=f(x_i,t_j) \qquad (2.4.4)$$

$$i=1...N-1, j=1..M-1$$

$$y_{i,j}=y(x_i,t_j)=u(x_i,t_j)+o(h_1^2+h_2^2)$$

ekanligi ma’lum.

$$h=\frac{L}{N}\qquad \tau=\frac{T}{M}$$

(2.4.4) tenglama besh nuqtali ayirmali sxemadan iborat bo’lib bu sxemani yechish uchun yechimni qatlamma-qatlam usulini qo’llash lozim. Xar bir tenglamada uchta qatlamdan noma’lum qiymatlar qatnashadi. Shu sababli, agar biz ikki oldingi qatlamdagi qiymatlarni bilsak keyingi qatlamdagi yechim qiymatlarini topamiz. Nolinchi va birinchi qatlamlardagi yechim qiymatlarini boshlang’ich shartlardan foydalanib topamiz:

(2.4.1) shartlardan

$$y_{i,0}=\mu_1(x_i)\quad i=0...N$$

va

$$\frac{y_{i,1}-y_{i,0}}{\tau}=\mu_2(x_i)\quad i=0...N\qquad y_{i,1}=y_{i,0}+\tau\mu_2(x_i)\quad i=0...N$$

Bu tengliklardan nolinchi va birinchi qatlamlardagi yechim qiymatini aniqlanadi. Chap va o’ng, (2.4.2) chegaraviy shartlardan

$$y_{0,j}=u_0(t_j)\qquad j=0..M$$

$$y_{N,j}=u_L(t_j)\qquad j=0..M$$

Qiymatlarni aniqlanadi. (2.4.4) sistemadan

$$y_{i,j+1}=2y_{i,j}-y_{i,j-1}+\left(\frac{\tau}{h}\right)^2\left(y_{i-1,j}-2y_{i,j}+y_{i+1,j}\right)+\tau^2 f(x_i,t_i)$$

$$i=1..N-1\qquad j=1..M-1$$

tengliklarni hosil qilamiz. Bu tengliklardan qolgan qatlamlardagi yechim qiymatlari topiladilar.

Endi (2.4.1)- (2.4.3) masalaning konkret holda yechish tartibini qarab chiqamiz.

$$\frac{d^2}{dt^2}u(x,t)-\frac{d^2}{dt^2}u(x,t)=0\qquad o<x<L\qquad 0<t<\pi$$

Tenglamani $\mu_1(x)=\sin(x)\qquad \mu_2(x)=\cos(x)$ boshlang’ich shartlar va

$u_0(t)=\sin(t)\quad u_L(t)=\cos(t)$ chegaraviy shartlar bilan echamiz.

Mathcad dagi programma quyidagicha bo’ladi:

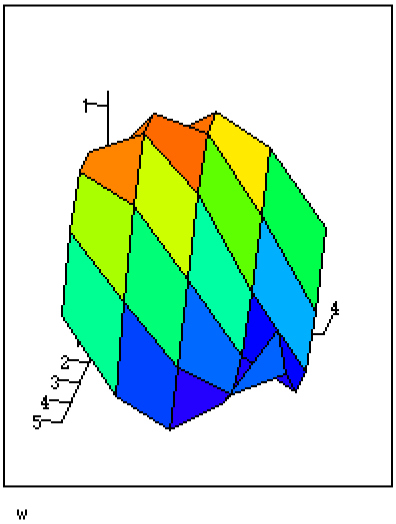
$$L:=\pi\quad T:=\pi\quad N:=4\quad M:=5\qquad u(x,t):=\sin(x+t)\ h:=\frac{L}{N}\quad \tau:=\frac{T}{M}\quad i:=0..N$$

$$j:=0..M\ x_i:=ih\quad t_j:=j\tau\ \ f(x,t):=0\ \mu_1(x):=\sin(x)\quad \mu_2(x):=\cos(x)\quad u_0(t):=\sin(t)$$

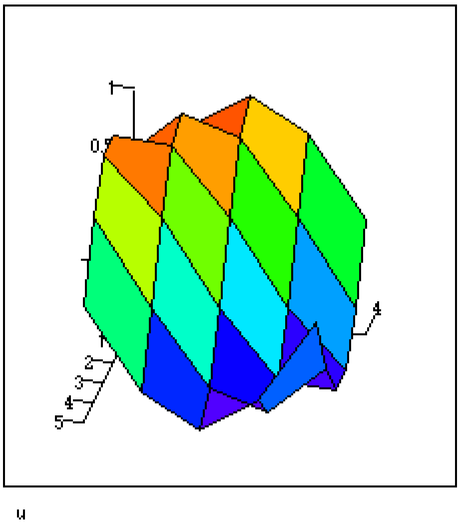
$$u_L(t):=\cos(t)\ \ y_{0,i}:=\mu_1(x_i)\ \ y_{1,i}:=y_{0,i}+\tau\mu_2(x_i)\ y_{j,0}:=u_0(t_j)\quad y_{j,N}:=u_L(t_j)$$

```
solve(y) := | for  j ∈ 1.. M − 1
            |   for  i ∈ 1.. N − 1
            |     $y_{j+1,i} \leftarrow 2\cdot y_{j,i} - y_{j-1,i} + \left(\frac{\tau}{h}\right)^2\cdot\left(y_{j,i-1} - 2\cdot y_{j,i} + y_{j,i+1}\right) + \tau^2\cdot f\left(x_i, t_j\right)$
            | y
```

$$w:=solve(y)\quad u_{j,i}:=u(x_i,t_j)$$



2.5 chizma. Taqribiy yechim grafigi.



2.6 chizma. Aniq yechim grafigi.

Taqribiy va aniq yechimlar jadvallari quyidagicha:

$$w=\begin{bmatrix}0 & 0.707 & 1 & 0.707 & 0\\ 0.588 & 1.151 & 1 & 0.263 & -0.588\\ 0.951 & 1.138 & 0.625 & -0.254 & -0.951\\ 0.951 & 0.677 & 0.016 & -0.654 & -0.951\\ 0.588 & -0.032 & -0.599 & -0.816 & -0.588\\ 0 & -0.707 & -0.99 & -0.693 & 0\end{bmatrix}\qquad u=\begin{bmatrix}0 & 0.707 & 1 & 0.707 & 0\\ 0.588 & 0.988 & 0.809 & 0.156 & -0.588\\ 0.951 & 0.891 & 0.309 & -0.454 & -0.951\\ 0.951 & 0.454 & -0.309 & -0.891 & -0.951\\ 0.588 & -0.156 & -0.809 & -0.988 & -0.588\\ 0 & -0.707 & -1 & -0.707 & 0\end{bmatrix}$$

2.5. Topshiriqlar

Chegaraviy masalalarning $a \le x_k \le b, k = 0,...10$ nuqtalardagi yechim qiymatlari chekli – ayirmali usullardan birini qo’llab 1×10^{-3} aniqlikda yechilsin.

№1. $y'' + xy' + y = x + 1,\ y(0.5) + 2y'(0.5) = 1,\ y'(0.8) = 0.2$

№2. $y'' + 0.5xy' + \left(1 + 2p^2 \cdot x^2\right)y = 4x,\ y(0) = 1,\ y(1) = 1.367$

№3. $y'' + (x-1)y' + 3.125y = 4x,\ y(0) = 1,\ y(1) = 1.367$

№4. $y'' + 2xy' + 2y = \dfrac{2(5-2x)}{(2-x)^3},\ y(0) = 1,\ y(1) = 1.367$

№5. $y'' + \left(1 + x^3\right)y' + \left(1 - x^2\right)y = e^{1-2.5x^2},\ y(0) = 0,\ y(1) = 0$

№6. $y'' + x^2 \cdot y' + (1-x)y = \dfrac{x}{x^2 + 2.5},\ y(0) = 0,\ y(1) = 0$

№7. $\left(x^2 + 1\right)\cdot y'' - 2xy' + 2y = 0,\ y(0) = -1,\ y(1) = 1$

№8. $y'' + \sin x \cdot y' + y = \dfrac{1}{2.5 + \sin^2 x},\ y(0) = 0,\ y(1) = 0$

№9. $x^2 \cdot (x+1)\cdot y'' - y = 0,\ y(1) = 2,\ y(2) = 1.5$

№10. $y'' + \dfrac{1}{\sqrt{x^2 + 2.5}} \cdot y' + y = x,\ y(0) = 0,\ y(1) = 0$

№11. $y'' - y \cdot 2\left(1 - tg^2 x\right) = 0,\ y(0) = 0,\ y(0.785) = 3.4938$

№12. $\left(x^2 + 1\right)\cdot y'' + 5x \cdot y' + 4y = \dfrac{1}{2.5 + \sin^2 x},\ y(0) = 0,\ y(1) = 0.3534$

№13. $y'' + x^2 \cdot y' + (1.4 - x)y = \dfrac{x}{x^5 + 3},\ y(0) = 0,\ y(1) = 0$

№14. $y'' - x \cdot y' + x \cdot y = 0,\ y(0) = 0,\ y(1) = 1.083$

№15. $\left[\left(x^2 + 1\right)\cdot y\right]'' + 5x \cdot y' + 3y = 0,\ y(0) = 0,\ y(1) = 0.3534$

№16. $xy'' + y' - xy = 0,\ y(1) = 1.266059,\ y(2) = 2.277778$

№17. $xy'' + 2 \cdot y' + xy = 0,\ y(1) = 0.841471,\ \ y(2) = 0.841471$

№18. $2x^2 \cdot y'' + \left(3x - 2x^2\right) \cdot y' - (x+1) \cdot y = 0,\ y(1) = 2.718182,\ \ y(2) = 3.69453$

№19. $9x^2 \cdot y'' - \left(x^2 - 2\right) \cdot y = 0,\ y(0) = 0,\ \ y(1) = 1.033586$

Erkin tebranishlar tenglamasi $y'' + ay' + by = 0$

№20. $a = 2,\ b = 0.5,\ y(0) = 2,\ y(2) = 1.602618$

№21. $a = 2,\ b = 1,\ y(-1) = 0,\ y(0) = 1$

№22. $a = 0,\ b = \frac{\pi^2}{4},\ y(0) = 1,\ y(1) = 1$

№23. $3x^2 \cdot y'' - x^2 \cdot y' + (x-2) \cdot y = 0,\ y(1) = 2.5,\ \ y(2) = 2.5$

№24. $y'' + \left(x^4 + 1\right) \cdot y = 0,\ y(1) = 0.3149857,\ \ y(2) = -0.4446632$

№25. $\left(x^2 + 1\right) \cdot y'' - y = 0,\ y(0) = 0,\ \ y(1) = 1$

№26. $y'' - y' \cdot \cos x + y \cdot \sin x = \cos x,\ y(-\pi) = 2,\ \ y(\pi) = 2$

№27. $y'' + x^2 \cdot y' - xy = e^x,\ y(0) = 0,\ \ y(1) = 0$

№28. $y'' - y' \cdot \cos x + y \cdot \sin x = \sin x,\ y(-\pi) = 2,\ \ y(\pi) = 2$

№29. $y'' - y' \cdot \cos x + y \cdot \sin x = \cos 2x,\ y(-\pi) = 2,\ \ y(\pi) = 2$

№30. $y'' - y' \cdot \cos x + y \cdot \sin x = \sin 2x,\ y(-\pi) = 2,\ \ y(\pi) = 2$

№31. $y'' + x^2 \cdot y' - xy = \cos x,\ y(0) = 0,\ \ y(1) = 0$

№32. $y'' - 2x \cdot y' + 2y = 3x^2 + x - 1,\ y(0) = 0,\ \ y'(1) = 1$

№33. $y'' + y' - \frac{y}{x} = x^2 - \frac{3}{4}x + \frac{1}{8},\ y(0) = 0,\ \ y'(1) = 1$

№34. $y'' - 2x \cdot y' + 2y = 5x^3,\ y(0) = 0,\ \ y'(1) = 1$

№35. $y''+x^2\cdot y'-xy=tgx,\ y(0)=0,\ \ y(1)=0$

№36. $y''-x\cdot y'-2y=1,\ y(0)=1,\ \ y'(1)=1.297443$

№37. $y''-\frac{2x}{x^2+1}\cdot y'+\frac{2y}{x^2+1}=0,\ y(0)=-1,\ \ y'(1)=3$

№38. $x^2\cdot y''-x\cdot y'+y=4x^3,\ y(1)=2,\ \ y(e)=25.5221$

№39. $(1+x^2)\cdot y''+2x\cdot y'-2y=4x^2+2,\ y(1)=1,\ \ y(2)=4$

№40. $y''-(1+x^2)\cdot y=0,\ y(0)=-2,\ \ y'(1)=\frac{7}{12}$

№41. $y''-2y=4x^2\cdot\left(e^{(x^2)}\right),\ y(0)=3,\ \ y(1)=7.07465$

№42. $y''+a^2\cdot y=ctg(ax),\ y\left(\frac{\pi}{4}\right)=0.5328401,\ \ y\left(\frac{\pi}{2}\right)=1,\ a=1$

№43. $y''+a^2\cdot y=ctg(ax),\ y(0.6)=0.6728232,\ \ y(0.8)=-0.15481,\ a=\pi$

Tenglamalarning $[0;1]$ oraliqda $y(0)=y(1)=0$ chegaraviy shartlarni qanoatlantiruvchi yechim qiymatlari topilsin. $(x_i=0.1\cdot i,\ i=1..9,\ \alpha=1+0.4\cdot k,\ k=0..3,\ \beta=2.5+0.5\cdot n,\ n=0..5)$:

№44. $y''+(\alpha+x^3)\cdot y'+(1-x^2)\cdot y=e^{1-\beta}\cdot x^2$

№45. $y''+x^2\cdot y'+(\alpha-x)\cdot y=\frac{x}{x^2+\beta}$

№46. $y''+\sin(\alpha x)\cdot y'+y=\frac{1}{\beta+\sin^2(\alpha x)}$

№47. $y''+\frac{1}{\sqrt{x^2+\beta}}\cdot y'+\alpha y=x$

Chegaraviy qiymatlari bilan berilgan ikkinchi tartibli bo’lmagan differenstial tenglamalarni $[a;b]$ oraliqning x_k nuqtalardagi yechim qiymatlari iterastiya usuli qo’llanib topilsin.

№48. $x\cdot y''+x\cdot(y')^2-y'=0,\ y(2)=2,\ \ y(4)=3.386294,\ h=0.2,\ \varepsilon=1\cdot10^{-4}$

№49. $2y'' = 3y^2,\ y(-2) = 1,\ \ y(0) = 0.25,\ h = 0.2,\ \varepsilon = 1\cdot 10^{-3}$

№50. $8y'' + (9y')^4 = 0,\ y(0) = -1,\ \ y(7) = 2,\ h = 1,\ \varepsilon = 1\cdot 10^{-3}$

Chegaraviy masalalarning x_k nuqtalardagi yechim qiymatlari chekli ayirmalar usullari qo’llanilib topilsin.

№51. $y'' + y' - \frac{1}{x}\cdot y = \frac{x+1}{x},\ y(0.5) = -0.5\cdot \ln 2,\ \ y(1) = 0,\ \ \varepsilon = 1\cdot 10^{-2},\ x_k = 0.1k,\ k = 5..10$

№52. $4y'' + 16y' + 15y = 4e^{-1.5k},\ y(0) = 3,\ \ y(1) = 0.89252,\ \ \varepsilon = 1\cdot 10^{-3},\ x_k = 0.2k,\ k = 1..4$

№53. $y'' - \frac{2}{x}\cdot y' - \frac{4}{x^2+2}\cdot y = 8,\ y'(0.5) = 0.5,\ \ y(1) + y'(1) = 1,\ \ \varepsilon = 1\cdot 10^{-2},\ x_k = 0.1k,\ k = 5..10$

II bobning qisqacha xulosasi.

II bobda xususiy hosilali differensial tenglamalarni Mathcad muhitida sonli yechish metodlari ko’rsatilgan.

Xotima

Xususiy hosilali differensial tenglamalar fan va texnikaning turli sohalarida uchraydi, ammo ularning yechimini oshkor ko’rinishda chekli formula shaklida toppish kamdan-kam hollarda mumkin.Shu munosabat bilan matematik fizika masalalari deb ataluvchi har xil xususiy hosilalai differensial tenglamalarni, xususiy hosilali differensial tenglamalar sistemasi va integral tenglamalarni taqribiy yechish metodlari muhim ahamiyatga egadir.

Ushbu BMIda xususiy hosilali differensial tenglamalarni taqribiy yechish metodlari bo’yicha uslubiy qo’llanma tayyorlandi. Xususiy hosilali differensial tenglamalarni taqribiy yechishning bir qancha usullari o’rganildi. To’r metodi, turg’unlik, approksimatsiya yaqinlashish , elliptik tenglamalarni to’r metodi bilan yechish, Chebishevning optimal oshkor iteratsion metodi va uning ayirmali elliptik tenglamalarga tadbiqi, parabolitik tenglamalar uchun ayirmali sxemalar, giperbolik tenglamalarni ayirmali metodlar bilan yechish o’rganildi. Bu usullarning barchasi uchun Mathcad muhitida natijalar olindi.

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