

O'zbekiston Respublikasi
Oliy va O'rta maxsus ta'lim vazirligi
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Matematika kafedrası

Oliy matematika asoslari fanidan Mehnat ta'limi yo'nalishi I
bosqich talabalari uchun tuzilgan

Mustaqil ishlar

I qism

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Matematika kafedrası yig' ilishida muhokama etilib, Fakultet uslubiy kengashi yig' ilishida qo' llashga tavsiya etilgan.

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uchun

Uslubiy qo'llanma

O'zbekiston davlat ta'lim standartiga asosan Mehnat ta'limi yo'nalishi bo'yicha bakalavriat bo'limida o'qiydigan talabalar Oliy matematika asoslari fanidan birinchi semestrda 52 soat hajmida Mustaqil ish bajaradilar. Bu mustaqil ishlar asosan uch qismga bo'linadi. Ular joriy, oraliq va yakuniy nazoratlardan iborat.

Mustaqil ishlarni bajarishda talabalarga yordam berishga mo'ljallab tuzilgan ushbu uslubiy qo'llanma asosan besh qismdan iborat. Qo'llanmaning birinchi qismida oraliq nazoratga mo'ljallangan mavzular qisqacha bayon etiladi va tegishli misollarning yechilish namunalari keltiriladi. Ikkinchi qismda har bir mavzu bo'yicha savollar tuziladi va ularga javob namunalari beriladi (0 variant). Uchinchi qismda talabalar mustaqil bajarishi lozim bo'lgan 10 ta variant beriladi. Variantlar quyidagi tartibda bajariladi: Talaba sinf jurnalida nechanchi o'rinda turgan bo'lsa, u shu o'ringa mos raqamni birlik xonasidagi variantni bajaradi. Masalan, 23-o'rindagi talaba 3-variantni, 20-o'rindagi talaba 10-variantni bajaradi. To'rtinchi qism yakuniy nazorat savollari. Beshinchi qism esa testlar va ularga mos kalitlardan iborat.

Mustaqil ishlarni bajarishda kelib chiqadigan qiyinchiliklar va kamchiliklarni oydinlashtirish uchun mazkur to'plam oxirida ko'rsatilgan adabiyotlardan va ma'ruza matnlaridan foydalanish mumkin.

Chiziqli algebra elementlari

I semestr

0 variant

1. Quyidagi determinantlarni hisoblang:

$$\text{a) } \begin{vmatrix} \sin^2 \alpha & \cos^2 \alpha \\ \sin^2 \beta & \cos^2 \beta \end{vmatrix}$$

$$\text{b) } \begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

Yechilishi: a) Ikkinchi tartibli $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ determinant quyidagicha hisoblanadi:

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1. \text{ Shunga ko'ra } \begin{vmatrix} \sin^2 \alpha & \cos^2 \alpha \\ \sin^2 \beta & \cos^2 \beta \end{vmatrix} = \sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta = \\ = (\sin \alpha \cos \beta - \cos \alpha \sin \beta)(\sin \alpha \cos \beta + \cos \alpha \sin \beta) = \sin(\alpha - \beta) \sin(\alpha + \beta).$$

b) Sarryus qoidasiga binoan:

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 2 \cdot (-2) \cdot 3 + 1 \cdot 3 \cdot 1 + 4 \cdot 5 \cdot 2 - (4 \cdot (-2) \cdot 1 + 3 \cdot 5 \cdot 3 + 2 \cdot 1 \cdot 2) = -12 + 3 + 40 - \\ -(-8 + 45 + 4) = 31 - 41 = -10.$$

Endi shu determinantni birorta satr (ustun) elementlari bo'yicha yoyib hisoblaylik. Determinantni birinchi ustun elementlari bo'yicha yoyaylik:

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 2 \cdot (-1)^{1+1} \begin{vmatrix} -2 & 1 \\ 2 & 3 \end{vmatrix} + 5 \cdot (-1)^{2+1} \begin{vmatrix} 3 & 4 \\ 3 & 3 \end{vmatrix} + 1 \cdot (-1)^{3+1} \begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} = \\ = 2 \cdot (-6 - 2) - 5 \cdot (9 - 8) + 1 \cdot (3 + 8) = -16 - 5 + 11 = -10.$$

2. Quyidagi chiziqli tenglamalar sistemalarini yeching:

$$\text{a) } \begin{cases} mx - ny = (m - n)^2 \\ 2x - y = n, \end{cases} \quad (m \neq 2n) \text{ bo'lganda.}$$

$$b) \begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 5x_1 + x_2 + 2x_3 = 29 \\ 3x_1 - x_2 + x_3 = 10 \end{cases}$$

Yechilishi: a) $\Delta = \begin{vmatrix} m & -n \\ 2 & -1 \end{vmatrix} = -m + 2n \neq 0$. Demak, berilgan sistema aniq sistema ekan. Bu sistemalarni Kramer formulalari yordamida yechish mumkin:

$$\Delta_1 = \Delta_x = \begin{vmatrix} (m-n)^2 & -n \\ n & -1 \end{vmatrix} = -(m-n)^2 + n^2 = -m^2 + 2mn - n^2 + n^2 = -m(m-2n),$$

$$\Delta_2 = \Delta_y = \begin{vmatrix} m & (m-n)^2 \\ 2 & n \end{vmatrix} = mn - 2(m-n)^2 = mn - 2m^2 + 4mn - 2n^2 = 5mn - 2m^2 - 2n^2.$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-m(m-2n)}{-m+2n} = \frac{m(-m+2n)}{-m+2n} = m,$$

$$y = \frac{\Delta_y}{\Delta} = \frac{5mn - 2m^2 - 2n^2}{-m+2n} = \frac{(-m+2n)(2m-n)}{-m+2n} = 2m-n.$$

b) Berilgan sistema uchun $\Delta, \Delta_{x_1}, \Delta_{x_2}, \Delta_{x_3}$ larni hisoblab olamiz:

$$\Delta = \begin{vmatrix} 1 & 2 & 4 \\ 5 & 1 & 2 \\ 3 & -1 & 1 \end{vmatrix} = (1+12-20) - (12+10-2) = -7-20 = -27,$$

$$\Delta_1 = \Delta_{x_1} = \begin{vmatrix} 31 & 2 & 4 \\ 29 & 1 & 2 \\ 10 & -1 & 1 \end{vmatrix} = (31+40-116) - (40+58-62) = -45-36 = -81,$$

$$\Delta_2 = \Delta_{x_2} = \begin{vmatrix} 1 & 31 & 4 \\ 5 & 29 & 2 \\ 3 & 10 & 1 \end{vmatrix} = (29+186+200) - (348+155+20) = 415-523 = -108,$$

$$\Delta_3 = \Delta_{x_3} = \begin{vmatrix} 1 & 2 & 31 \\ 5 & 1 & 29 \\ 3 & -1 & 10 \end{vmatrix} = (10+174-155) - (93+100-29) = 29-164 = -135.$$

Kramer formulalariga binoan, $x_1 = \frac{\Delta_{x_1}}{\Delta} = \frac{-81}{-27} = 3$, $x_2 = \frac{\Delta_{x_2}}{\Delta} = \frac{-108}{-27} = 4$ va

$x_3 = \frac{\Delta_{x_3}}{\Delta} = \frac{-135}{-27} = 5$. Shunday qilib $(3, 4, 5)$ vektor berilgan sistema yechimi ekan.

3. Quyidagi chiziqli tenglamalar sistemalarining birgalashgan, birgalashmagan, aniq yoki aniqmasekanligini aniqlang. Birgalashgan sistemalar uchun yechimlarini toping.

Uch noma'lumli tenglamalar sistemasining umumiy ko'rinishi

$$\left. \begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned} \right\} \quad (1)$$

dan iborat bo'lib, bu yerda $a_i, b_i, c_i (i=1,2,3)$ koeffitsiyentlar, $d_i (i=1,2,3)$ lar esa ozod hadlar deb yuritiladi.

Agar d_1, d_2, d_3 larning birortasi noldan farqli bo'lsa, (1) sistema bir jinslimas chiziqli tenglamalar sistemasi, barcha ozod hadlar nolga teng bo'lganda esa (1) sistema bir jinsli chiziqli tenglamalar sistemasi deyiladi.

(1) sistema uchun quyidagi teorema o'rinli:

Teorema (Kroneker- Kopelli):

(1) sistema birlashgan bo'lishi (yechimga ega bo'lishi) uchun uning asosiy matritsasi

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

rangi bilan kengaytirilgan matritsasi

$$B = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix}$$

ning ranglari o'zaro teng bo'lishi zarur va yetarli ("Oliy algebra" darslilariga qarag), ya'ni A matritsaning rangi (chiziqli bog'lanmagan satrlarning maksimal soni) ni r bilan, B matritsaning rangini esa \bar{r} desak, ya'ni $r(A)=r$, $r(B)=\bar{r}$ desak, $r=1,2$ va $\bar{r}=1,2,3$ larga teng bo'lishi mumkin.

Kroneker- Kopelli teoremasiga asosan quyidagi holler yuz berishi mumkin:

1) $r=1, \bar{r}=1$ 2) $r=2, \bar{r}=2$ 3) $r=3, \bar{r}=3$ 4) $r=1, \bar{r}=2$.

Agar 1), 2) yoki 3) holler o'rinli bo'lsa, berilgan sistema birgalashgan (yechimga ega) bo'ladi. 3) holda, ya'ni $r=3, \bar{r}=3$ bo'lsa berilgan sistema aniq sistema (yagona yechimga ega) bo'ladi. Berilgan sistema uch noma'lumli bo'lib, $r=1, \bar{r}=2$ bo'lsa bu sistema birgalashmagan (yechimga ega emas) bo'ladi.

1- misol:
$$\begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 5x_1 + x_2 + 2x_3 = 29 \\ 3x_1 - x_2 + x_3 = 10 \end{cases}$$
 sistemani tekshiring. Agar sistema birgalashgan

bo'lsa uni yeching.

Yechish: Asosiy va kengaytirilgan matritsalarini quyidagi ko'rinishda yozib

olamiz:
$$A/B = \left(\begin{array}{ccc|c} 1 & 2 & 4 & 31 \\ 5 & 1 & 2 & 29 \\ 3 & -1 & 1 & 10 \end{array} \right).$$

Bu yerda vertikal chiziqqacha bo'igan matritsa asosiy matritsani, vertical chiziqqacha olinmasa A/B matritsa kengaytirilgan B matritsani bildiradi.

Endi quyidagi elementar almashtirishlarni bajaramiz:

1) A/B matritsaning birinchi yo'lini ketma- ket -5 va -3 ga ko'paytirib hosil bo'lgan sonlarni ikkinchi va uchinchi satrlarga qo'shamiz. U holda A/B matritsa

$$\left(\begin{array}{ccc|c} 1 & 2 & 4 & 31 \\ 0 & -9 & -18 & -126 \\ 0 & -7 & -11 & -83 \end{array} \right)$$
 ga o'tadi. Endi hosil bo'lgan matritsaning ikkinchi satrini $-\frac{7}{9}$ ga ko'paytirib, natijalarni uchinchi satrning mos elementlariga qo'shamiz.

Natijada
$$\left(\begin{array}{ccc|c} 1 & 2 & 4 & 31 \\ 0 & -9 & -18 & -126 \\ 0 & 0 & 3 & 15 \end{array} \right)$$
 hosil bo'ladi.

Oxirgi matritsalarining har birida uchtadan nolmas satrlar bor. Ular 1, 2 va 3-satrlardir. Shuning uchun asosiy matritsa $A = \begin{pmatrix} 1 & 2 & 4 \\ 5 & 1 & 2 \\ 3 & -1 & 1 \end{pmatrix}$ va kengaytirilgan

matritsa $B = \begin{pmatrix} 1 & 2 & 4 & 31 \\ 5 & 1 & 2 & 29 \\ 3 & -1 & 1 & 10 \end{pmatrix}$ larning har birining rangi 3 ga teng. Demak,

berilgan sistema yagona yechimga ega ekan. Bu yechimni topish uchun
$$\left(\begin{array}{ccc|c} 1 & 2 & 4 & 31 \\ 0 & -9 & -18 & -126 \\ 0 & 0 & 3 & 15 \end{array} \right)$$
 matritsadan tenglamalar sistemasiga qaytamiz. Bu

sistemaning ko'rinishi quyidagicha bo'ladi:
$$\begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 9x_2 + 18x_3 = 126 \\ 3x_3 = 15 \end{cases}$$

Bu sistemadan $x_3 = \frac{15}{3} = 5$, $9x_2 + 18 \cdot 5 = 126 \Rightarrow x_2 = \frac{126 - 18 \cdot 5}{9} = 4$, $x_1 + 2 \cdot 4 + 4 \cdot 5 = 31 \Rightarrow x_1 = 31 - 2 \cdot 4 - 4 \cdot 5 = 3$. Shunday qilib yechim (3, 4, 5) dan iborat ekan.

2- misol:
$$\begin{cases} x - 2y + z = 4 \\ 2x + 3y - z = 3 \\ 4x - y + z = 11 \end{cases}$$
 sistemaning birgalashgan yoki birgalashmagan

ekanligini aniqlang. Sistema birgalashgan bo'lsa uni yeching.

Yechish: Kroneker- Kopelli teoremasidan foydalanamiz. Buning uchun vertikal chiziqcha orqali asosiy va kengaytirilgan matritsalarini yozib olamiz:

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 2 & 3 & -1 & 3 \\ 4 & -1 & 1 & 11 \end{array} \right)$$
. Bu matritsaning birinchi satr elementlarini -2 ga ko'paytirib

ikkinchi satrning mos elementlariga, so'ngra birinchi satr elementlarini -4 ga ko'paytirib, uchinchi satrning mos elementlariga qo'shamiz. Natijada

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 7 & -3 & -5 \\ 0 & 7 & -3 & -5 \end{array} \right)$$
 matritsa hosil bo'ladi. Endi ikkinchi satr elementlarini -1 ga

ko'paytirib uchinchi satr elementlariga qo'shamiz. Natijada $\left(\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 7 & -3 & -5 \\ 0 & 0 & 0 & 0 \end{array} \right)$

matritsa hosil bo'ladi. Oxirga matritsada asosiy va kengaytirilgan matritsalar ranglari o'zaro teng (ya'ni bu rang 2 ga teng).

Endi oxirgi matritsaga mos keluvchi chiziqli tenglamalar sistemasini yozamiz: $\begin{cases} x - 2y + z = 4 \\ 7y - 3z = -5 \end{cases}$.

$7y - 3z = -5$ tenglamadan $y = -\frac{5}{7} + \frac{3}{7}z$ ni, $x - 2y + z = 4$ tenglamadan esa $x = \frac{18}{7} - \frac{1}{7}z$ ni topib olamiz. Bu yerda z noma'lum parametrni bildiradi. Natijada, berilgan sistemaning umumiy yechimi $\left(\frac{18}{7} - \frac{z}{7}, -\frac{5}{7} + \frac{3z}{7}, z \right)$ dan iborat bo'ladi.

Parametrning aniq qiymatlarida xususiy yechimlar hosil bo'ladi. Masalan, $z = 0$ yoki $z = 7$ larda $\left(-\frac{18}{7}, -\frac{5}{7}, 0 \right)$ va $\left(\frac{11}{7}, \frac{16}{7}, 7 \right)$ xususiy yechimlar hosil bo'ladi.

3- misol: $\begin{cases} x + y - 3z = -1 \\ 2x + y - 2z = 1 \\ x + y + z = 3 \\ x + 2y - 3z = 1 \end{cases}$ sistemaning birgalashgan yoki birgalashmagan

ekanligini tekshiring.

Yechish: Sistemaga mos keluvchi asosiy va kaengaytirilgan matritsalarini

yozib olamiz: $\left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 2 & 1 & -2 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 1 \end{array} \right)$.

Matritsaning birinchi satrini ketma- ket -2, -1 va -1 larga ko'paytirib 2, 3 va

4- satrlarga qo'shamiz. Natijada $\left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -1 & 4 & 3 \\ 0 & 0 & 4 & 4 \\ 0 & 1 & 0 & 2 \end{array} \right)$ hosil bo'ladi. Bu

matritsaning ikkinchi satrini 1 ga ko'paytirib to'rtinchi satrga qo'shamiz. U holda

$\left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -1 & 4 & 3 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 4 & 5 \end{array} \right)$ hosil bo'ladi. Endi oxirgi matritsaning uchinchi satrini -1 ga

ko'paytirib, natijani to'rtinchi satrga qo'shamiz.

Nihoyat $\left(\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -1 & 4 & 3 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 1 \end{array} \right)$ matritsa hosil bo'ladi. Bu yerda asosiy

matritsaning rangi 3 ga, kengaytirilgan matritsa rangi esa 4 ga teng, ya'ni $r = 3, \bar{r} = 4$. $3 \neq 4$ bo'lgani uchun berilgan sistema yechimga ega emas (ya'ni sistema birlashmagan).

4. Quyidagi funksiyalarning aniqlanish sohaslarini toping:

a) $y = \frac{3x}{\sqrt{x^2 + 3x + 2}}$

Yechish: Berilgan funksiya mavjud bo'lishi uchun $x^2 + 3x + 2 > 0$ bo'lishi kerak. Bu tengsizlikni qanoatlantiradigan x ning qiymatlarini topish uchun, kvadrat uchhadni

ko'paytuvchilarga ajratamiz: $x^2 + 3x + 2 = 0, \quad x_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2},$

$x_1 = -1, \quad x_2 = -2.$

Shunday qilib, $x^2 + 3x + 2 = (x+1)(x+2) > 0$

Tengsizlik o'rinli bo'lishi uchun

$\begin{cases} x+1 > 0 \\ x+2 > 0 \end{cases}$ yoki $\begin{cases} x+1 < 0 \\ x+2 < 0 \end{cases}$ bo'lishi kerak.

Hosil bo'lgan tengsizliklar sistemalarini yechsak $x > -1$ va $x < -2$ hosil bo'ladi.

Demak, berilgan funksiyaning aniqlanish sohasi $(-\infty; -2) \cup (-1; +\infty)$ intervaldir.

b) $y = \lg \sqrt{x-3} + \frac{1}{x-2}$

Yechish: Ushbu funksiya mavjud bo'lishi uchun (asos musbat bo'lganda faqat musbat sonning logarifmi mavjud bo'lganligi uchun sistemadagi birinchi tengsizlik, maxraji 0 ga teng bo'lganda kasr mavjud bo'lmaganligi uchun ikkinchi tengsizlik olinadi):

$\begin{cases} x-3 > 0 \\ x-2 \neq 0 \end{cases}$ bo'lishi kerak.

Bu tengsizlikdan:

$\begin{cases} x > 3 \\ x \neq 2 \end{cases}$ kelib chiqadi.

Demak, berilgan funksiyaning aniqlanish sohasi $(3, +\infty)$ bo'ladi.

5. Quyidagi limitlar hisoblansin:

a) $\lim_{x \rightarrow \infty} \frac{x^3 - 2x^2 + 1}{2x^3 - 3x^2 + 2x}$.

Yechish: $x \rightarrow \infty$ da limit ishorasi ostidagi ifoda $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslik bo'lib, uni ochish uchun kasrning surat va maxrajini x^3 ga bo'lamiz:

$$\lim_{x \rightarrow \infty} \frac{x^3 - 2x^2 + 1}{2x^3 - 3x^2 + 2x} = \lim_{x \rightarrow \infty} \frac{1 - \frac{2}{x} + \frac{1}{x^3}}{2 - \frac{3}{x} + \frac{2}{x^2}};$$

$x \rightarrow \infty$ da $\frac{2}{x}$, $\frac{1}{x^2}$, $\frac{3}{x}$ va $\frac{2}{x^2}$ ifodalarning har biri 0 ga intiladi. Demak,

$$\lim_{x \rightarrow \infty} \frac{x^3 - 2x^2 + 1}{2x^3 - 3x^2 + 2x} = \frac{1}{2}.$$

b) $\lim_{x \rightarrow -1} \frac{2x^2 + x - 1}{x^2 - x - 2}$

Yechish: $x \rightarrow -1$ bo'lgani uchun limit ishorasi ostida $\frac{0}{0}$

Ko'rinishdagi aniqmaslik hosil bo'ladi, buni ochish uchun esa, ifodaning surat va maxrajini ko'paytuvchilarga ajratib hisoblaymiz:

$$\lim_{x \rightarrow -1} \frac{2x^2 + x - 1}{x^2 - x - 2} = \lim_{x \rightarrow -1} \frac{2(x+1)\left(x - \frac{1}{2}\right)}{(x+1)(x-2)} = 2 \lim_{x \rightarrow -1} \frac{x - \frac{1}{2}}{x - 2} = 1.$$

c) $\lim_{x \rightarrow 6} \frac{\sqrt{x+3} - 3}{x - 6}$

Yechish: Ushbu limitda $x \rightarrow 6$ bo'lgani uchun $\frac{0}{0}$ ko'rinishdagi aniqmaslik kelib chiqadi. Bu aniqmaslikni ochish uchun kasrning surat va maxrajini suratning qo'shmasiga ko'paytiramiz va hisoblaymiz:

$$\begin{aligned} \lim_{x \rightarrow 6} \frac{\sqrt{x+3} - 3}{x - 6} &= \lim_{x \rightarrow 6} \frac{(\sqrt{x+3} - 3)(\sqrt{x+3} + 3)}{(x-6)(\sqrt{x+3} + 3)} = \lim_{x \rightarrow 6} \frac{x+3-9}{(x-6)(\sqrt{x+3} + 3)} = \lim_{x \rightarrow 6} \frac{x-6}{(x-6)(\sqrt{x+3} + 3)} = \\ &= \lim_{x \rightarrow 6} \frac{1}{\sqrt{x+3} + 3} = \frac{1}{6}. \end{aligned}$$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan. Quyidagilar talab qilinadi:

1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishini aniqlang;

2) Sxematik chizma chizish.

$$f(x) = 2^{\frac{1}{x-2}}, \quad x_1 = 2, \quad x_2 = 4$$

Yechish: Ma'lumki, $y = f(x)$ funksiya $x = a$ nuqtada uzluksiz bo'lishi uchun $f(a-0) = f(a+0) = f(a)$ tengliklar bajarilishi kerak, aks holda funksiya $x = a$ nuqtada uzlukli bo'ladi. Shunga asosan $x_1 = 2$ nuqtada:

$$f(2-0) = \lim_{x \rightarrow 2-0} f(x) = \lim_{x \rightarrow 2-0} 2^{\frac{1}{x-2}} = 0,$$

$$f(2+0) = \lim_{x \rightarrow 2+0} f(x) = \lim_{x \rightarrow 2+0} 2^{\frac{1}{x-2}} = \infty.$$

$f(2)$ aniqlanmagan. Demak, $x_1 = 2$ nuqtada funksiya ikkinchi tur uzilishga ega.

Endi $x_2 = 4$ nuqtada tekshiramiz:

$$f(4-0) = \lim_{x \rightarrow 4-0} f(x) = \lim_{x \rightarrow 4-0} 2^{\frac{1}{x-2}} = 2^{\frac{1}{2}} = \sqrt{2},$$

$$f(4+0) = \lim_{x \rightarrow 4+0} f(x) = \lim_{x \rightarrow 4+0} 2^{\frac{1}{x-2}} = 2^{\frac{1}{2}} = \sqrt{2},$$

$$f(4) = 2^{\frac{1}{4-2}} = 2^{\frac{1}{2}} = \sqrt{2}.$$

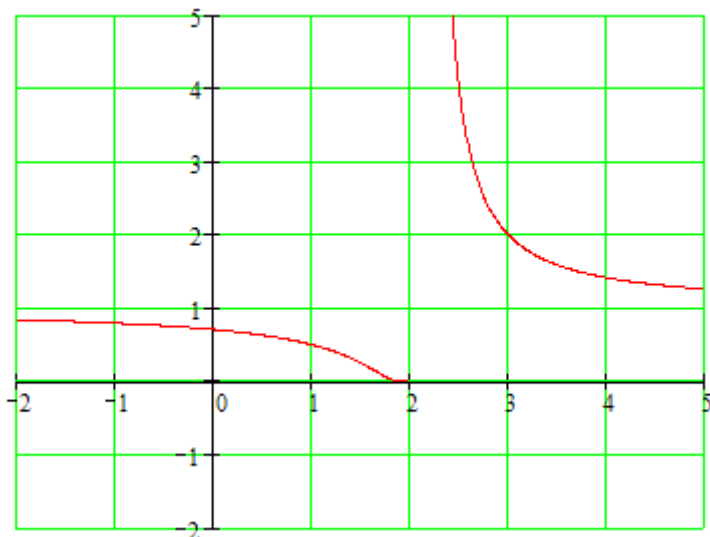
Demak, $f(4-0) = f(4+0) = f(4) = \sqrt{2}$ bo'lganligi uchun berilgan funksiya $x_2 = 4$ nuqtada uzluksiz ekan.

Sxematik chizma chizish uchun qo'shimcha ravishda $x \rightarrow -\infty$ va $x \rightarrow +\infty$ da funksiya limitlarini izlaymiz.

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} 2^{\frac{1}{x-2}} = 1,$$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} 2^{\frac{1}{x-2}} = 1.$$

Sxematik chizma chizamiz:



Variantlar

1- variant

1. Determinantlarni hisoblang:

$$\text{a) } \begin{vmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{vmatrix}$$

$$\text{b) } \begin{vmatrix} a+b & b+d \\ a+c & c+d \end{vmatrix}$$

$$\text{c) } \begin{vmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{vmatrix}$$

$$\text{d) } \begin{vmatrix} \sin 3\alpha & \cos 3\alpha & 1 \\ \sin 2\alpha & \cos 2\alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \end{vmatrix}$$

2. Chiziqli tenglamalar sistemalarini yeching:

$$\text{a) } \begin{cases} 3x + 2y = 7 \\ 4x - 5y = 40 \end{cases}$$

$$\text{b) } \begin{cases} 2x - 3y + z - 2 = 0 \\ x + 5y - 4z + 5 = 0 \\ 4x + y - 3z + 4 = 0 \end{cases}$$

3. Matritsalarini ko'paytiring:

$$\text{a) } \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\text{b) } \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 0 & 2 \end{pmatrix}$$

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

$$\text{a) } y = \frac{x}{x^2 + 3x + 2}$$

$$\text{b) } y = \lg(x-1) + \frac{1}{x-3}$$

5. Berilgan limitlarni hisoblang:

$$\text{a) } \lim_{x \rightarrow \infty} \frac{3x^5 - 4x^2 - 1}{2x^5 + 3x^2 - x}$$

$$\text{b) } \lim_{x \rightarrow 1} \frac{2x^2 + x - 3}{x^2 + x - 2}$$

$$\text{c) } \lim_{x \rightarrow 3} \frac{\sqrt{5x+1} - 4}{x-3}$$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan.

Quyidagilar talab qilinadi:

1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;

2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;

3) sxematik chizma chizish.

$$f(x) = 4^{\frac{1}{x-5}}, \quad x_1 = 5, \quad x_2 = 7$$

2- variant

1. Determinantlarni hisoblang:

$$\text{a) } \begin{vmatrix} 3 & -2 \\ -4 & 5 \end{vmatrix}$$

$$\text{b) } \begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix}$$

$$\text{c) } \begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$$

$$\text{d) } \begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$$

2. Chiziqli tenglamalar sistemalarini yeching:

$$\text{a) } \begin{cases} 2x + 3y = 5 \\ 6x - 10y = -4 \end{cases} \quad \text{b) } \begin{cases} 2x - 4y + 3z = 1 \\ x - 2y + 4z = 3 \\ 3x - y + 5z = 2 \end{cases}$$

3. Matritsalarini ko'paytiring:

$$\text{a) } \begin{pmatrix} 3 & 5 \\ 6 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix} \quad \text{b) } \begin{pmatrix} 3 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

$$\text{a) } y = \frac{2x-1}{\sqrt{x^2-5x+6}} \quad \text{b) } y = \lg(x^2+1) - \frac{3}{2x-3}$$

5. Berilgan limitlarni hisoblang:

$$\text{a) } \lim_{x \rightarrow \infty} \frac{4x^3 + 3x^2 - 1}{2x^3 - 3x + 1} \quad \text{b) } \lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{x^2 - 3x + 2}$$
$$\text{c) } \lim_{x \rightarrow 0} \frac{\sqrt{4+3x} - \sqrt{4-3x}}{7x}$$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan. Quyidagilar talab qilinadi:

- 1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;
- 2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;
- 3) sxematik chizma chizish.

$$f(x) = 3^{\frac{1}{x-2}}, \quad x_1 = 2, \quad x_2 = 3.$$

3-variant

1. Determinantlarni hisoblang:

$$\text{a) } \begin{vmatrix} a & c+di \\ c-di & b \end{vmatrix} \quad \text{b) } \begin{vmatrix} x-1 & 1 \\ x^3 & x^2+x+1 \end{vmatrix}$$
$$\text{c) } \begin{vmatrix} a & a & a \\ -a & a & x \\ -a & -a & x \end{vmatrix} \quad \text{d) } \begin{vmatrix} m+a & m-a & a \\ n+a & 2n-a & a \\ a & -a & a \end{vmatrix}$$

2. Chiziqli tenglamalar sistemalarini yeching:

$$\text{a) } \begin{cases} 2x + y = 4 \\ 7x + 6y = -1 \end{cases} \quad \text{b) } \begin{cases} 2x - y + z = 2 \\ 3x + 2y - 2z = -2 \\ x - 2y + z = 1 \end{cases}$$

3. Matritsalarini ko'paytiring:

$$\text{a) } \begin{pmatrix} 3 & 4 \\ 5 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix} \quad \text{b) } \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

$$\text{a) } y = \frac{x^2}{\sqrt{x^2 - 9}} \quad \text{b) } y = \lg \sqrt{x^2 - 1} + \frac{1}{x - 1}$$

5. Berilgan limitlarni hisoblang:

$$\text{a) } \lim_{x \rightarrow \infty} \frac{x^6 - 3x^2 - 2}{2x^6 + 4x^3 + 5x} \quad \text{b) } \lim_{x \rightarrow 3} \frac{3x^2 - 10x + 3}{x^2 - 2x - 3}$$

$$\text{c) } \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{\sqrt{2x + 1} - 3}$$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan. Quyidagilar talab qilinadi:

- 1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;
- 2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;
- 3) sxematik chizma chizish.

$$f(x) = 7^{\frac{1}{x-4}}, \quad x_1 = 4, \quad x_2 = 5$$

4-variant

1. Determinantlarni hisoblang:

$$\text{a) } \begin{vmatrix} b & a + bi \\ a - bi & b \end{vmatrix} \quad \text{b) } \begin{vmatrix} 4 & -5 \\ 5 & 4 \end{vmatrix}$$

$$\text{c) } \begin{vmatrix} 1 & 2 & 4 \\ 5 & 1 & 2 \\ 3 & -1 & 1 \end{vmatrix} \quad \text{d) } \begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$$

2. Chiziqli tenglamalar sistemalarini yeching:

$$\text{a) } \begin{cases} 3x + y + 5 = 0 \\ 7x + 6y + 8 = 0 \end{cases} \quad \text{b) } \begin{cases} x + 2y + 3z = 5 \\ 2x - y - z = 1 \\ x + 3y + 4z = 6 \end{cases}$$

3. Matritsalarini ko'paytiring:

$$\text{a) } \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix} \quad \text{b) } \begin{pmatrix} a & b & c \\ c & b & a \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & a & c \\ 1 & b & b \\ 1 & c & a \end{pmatrix}$$

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

$$a) y = \frac{5x}{\sqrt{x^2 - 6x + 8}}$$

$$b) y = \sqrt{x-2} + \lg(x+2)$$

5. Berilgan limitlarni hisoblang:

$$a) \lim_{x \rightarrow \infty} \frac{6x^5 - 3x^3 + 4}{3x^5 - 2x^4 + 3}$$

$$b) \lim_{x \rightarrow 5} \frac{3x^2 - 14x - 5}{x^2 - 6x + 5}$$

$$c) \lim_{x \rightarrow 1} \frac{\sqrt{5x+4} - 3}{\sqrt{2x-1} - 1}$$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan. Quyidagilar talab qilinadi:

1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;

2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;

3) sxematik chizma chizish.

$$f(x) = 8^{\frac{1}{x-3}}, \quad x_1 = 3, \quad x_2 = 6$$

5-variant

1. Amallarni bajaring:

$$a) 3(1, -1, 0, 3) + 2(-1, 2, 3, 1) - (1, 1, 6, 11) \quad b) (1, 2, 1, -1) + (3, 2, -1, 2)$$

2. Chiziqli tenglamalar sistemalarini yeching:

$$a) \begin{cases} x + y + 4z = -6 \\ 5x + y + 2z = 2 \\ 7x + 3y - 8z = -6 \end{cases} \quad b) \begin{cases} 7x - 3y = 44 \\ 8x + 7y = 19 \end{cases}$$

3. Matritsalarini ko'paytiring:

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 & 1 \\ -1 & 1 & 0 \\ 1 & 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{pmatrix}$$

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

$$a) y = \frac{x^2 + 1}{x^2 - 16}$$

$$b) y = \lg(x^2 + 5) + \frac{1}{\sqrt{x-3}}$$

5. Berilgan limitlarni hisoblang:

$$a) \lim_{x \rightarrow \infty} \frac{x^3 - 4x^2 + 5}{3x^3 - 2x^2 - x}$$

$$b) \lim_{x \rightarrow 7} \frac{2x^2 - 13x - 7}{x^2 - 9x + 14}$$

$$c) \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x+1} - \sqrt{3}}$$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan. Quyidagilar talab qilinadi:

- 1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;
- 2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;
- 3) sxematik chizma chizish.

$$f(x) = 9^{\frac{1}{x-7}}, \quad x_1 = 7, \quad x_2 = 9$$

6- variant

1. Amallarni bajaring:

$$\text{a) } \begin{pmatrix} 3 & 2 \\ -4 & -2 \end{pmatrix}^5 \qquad \text{b) } \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}^n$$

2. Chiziqli tenglamalar sistemalarini yeching:

$$\text{a) } \begin{cases} 6x + 9y = 6 \\ 2x + 3y = -2 \end{cases} \qquad \text{b) } \begin{cases} 3x + 2y + z = 5 \\ 2x + 3y + z = 1 \\ 2x + y + 3z = 11 \end{cases}$$

3. Teskari matritsalarini hisoblang:

$$\text{a) } \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{b) } \begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$$

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

$$\text{a) } y = \frac{x^3}{x^2 - 4x + 3} \qquad \text{b) } y = \sqrt{x} - \lg(2x - 3)$$

5. Berilgan limitlarni hisoblang:

$$\text{a) } \lim_{x \rightarrow \infty} \frac{x^4 - 3x^2 + 2}{5x^4 - 3x - 2} \qquad \text{b) } \lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^2 - 5x + 6}$$

$$\text{c) } \lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{\sqrt{2x-1} - 3}$$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan.

Quyidagilar talab qilinadi:

- 1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;
- 2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;
- 3) sxematik chizma chizish.

$$f(x) = 16^{\frac{1}{x-2}}, \quad x_1 = 2, \quad x_2 = 6$$

7- variant

1. Uchlari A(2; 3), B(4; -1) va C(6; 8) nuqtalarda bo'lgan uchburchakning yuzini hisoblang.

2. $\bar{a}_1 = (3; 2; 1; 2)$, $\bar{a}_2 = (-1; 3; 2; 1)$, $\bar{a}_3 = (4; 3; -1; 2)$, $\bar{a}_4 = (18; 20; 4; 14)$ vektorlar sistemasining chiziqli bog'langan yoki chiziqli erkli ekanligi aniqlansin.

3.
$$\begin{cases} x + 2y + 3z = 4 \\ 2x + 4y + 6z = 3 \\ 3x + y - z = 1 \end{cases}$$
 sistemaning birgalashgan yoki birgalashmagan ekanligini aniqlang.

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

a) $y = \frac{2x-1}{\sqrt{x^2-7x+10}}$ b) $y = \lg(x-5) + \frac{1}{2x-7}$

5. Berilgan limitlarni hisoblang:

a) $\lim_{x \rightarrow \infty} \frac{9x^5 - 4x^4 + 12}{3x^5 - 2x - 1}$ b) $\lim_{x \rightarrow 5} \frac{2x^2 - 11x + 5}{x^2 - 7x + 10}$

c) $\lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{x}$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan.

Quyidagilar talab qilinadi:

1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;

2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;

3) sxematik chizma chizish.

$f(x) = 3^{\frac{1}{x-4}}$, $x_1 = 4$, $x_2 = 6$

8-variant

1. Uchlari A(3; 4), B(5; 0) va C(7; 6) nuqtalarda bo'lgan uchburchakning yuzini hisoblang.

2. $\bar{a}_1 = (1; 1; -1; 4)$, $\bar{a}_2 = (2; 4; 1; 9)$, $\bar{a}_3 = (1; -1; 1; -2)$, $\bar{a}_4 = (2; 5; -3; 15)$ vektorlar sistemasining chiziqli bog'langan yoki chiziqli erkli ekanligi aniqlansin.

3.
$$\begin{cases} x_1 + x_2 - 3x_3 = -1 \\ 2x_1 + x_2 - 2x_3 = 1 \\ x_1 + x_2 + x_3 = 3 \\ x_1 + 2x_2 - 3x_3 = 1 \end{cases}$$
 sistemaning birgalashgan yoki birgalashmagan ekanligini aniqlang.

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

a) $y = \frac{x-3}{x^2-81}$ b) $y = \sqrt{x-3} + \frac{1}{\lg(x-1)}$

5. Berilgan limitlarni hisoblang:

a) $\lim_{x \rightarrow \infty} \frac{3x^3 - 4x + 1}{x^3 - 2x^2 + 1}$ b) $\lim_{x \rightarrow 6} \frac{2x^2 - 9x - 18}{x^2 - 7x + 6}$

c) $\lim_{x \rightarrow 2} \frac{\sqrt{3x+10} - 4}{x-2}$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan.

Quyidagilar talab qilinadi:

1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;

2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;

3) sxematik chizma chizish.

$$f(x) = 9^{\frac{1}{x-6}}, \quad x_1 = 6, \quad x_2 = 8$$

9-variant

1. Uchlari A(5; 6), B(7; 2) va C(9; 8) nuqtalarda bo'lgan uchburchakning yuzini hisoblang.

2. $\vec{a}_1 = (-2; 3; -4; 5)$, $\vec{a}_2 = (4; -3; 2; -1)$, $\vec{a}_3 = (-2; -5; 3; 1)$, $\vec{a}_4 = (6; 5; -7; 7)$ vektorlar sistemasining chiziqli bog'langan yoki chiziqli erkli ekanligi aniqlansin.

$$3. \begin{cases} x_1 + x_2 - 3x_3 = -1 \\ 2x_1 + x_2 - 2x_3 = 1 \\ x_1 + x_2 + x_3 = 3 \\ x_1 + 2x_2 - 3x_3 = 1 \end{cases} \text{ sistemaning birgalashgan yoki birgalashmagan ekanligini}$$

aniqlang.

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

a) $y = \frac{x+1}{x^2 - 10x + 16}$

b) $y = \lg(x^2 - 16) - \frac{1}{x-4}$

5. Berilgan limitlarni hisoblang:

a) $\lim_{x \rightarrow \infty} \frac{4x^5 - 3x^2 + 1}{2x^5 - 2x^3 + 3}$

b) $\lim_{x \rightarrow 7} \frac{3x^2 - 17x - 28}{x^2 - 9x + 14}$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan.

Quyidagilar talab qilinadi:

1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;

2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;

3) sxematik chizma chizish.

$$f(x) = 7^{\frac{1}{x-3}}, \quad x_1 = 3, \quad x_2 = 5$$

10-variant

1. A(1; 3), B(2; 4) va C(3; 5) nuqtalar bir to'g'ri chiziqda yotadimi?

2. Agar $\vec{a}_1 = (1; 0; 3; -2)$, $\vec{a}_2 = (-1; 1; 4; 3)$, $\vec{a}_3 = (-5; 3; 5; 3)$ bo'lsa $\vec{a} = 2\vec{a}_1 - 3\vec{a}_2 + \vec{a}_3$ vector topilsin.

$$3. \begin{cases} x_1 - 2x_2 + x_3 + x_4 = 1 \\ x_1 - 2x_2 + x_3 - x_4 = -1 \\ x_1 - 2x_2 + x_3 - 5x_4 = 5 \end{cases} \text{ sistemaning aniq yoki aniqmas ekanligini aniqlang.}$$

4. Berilgan funksiyalarning aniqlanish sohalarini toping:

$$a) y = \frac{3x+1}{\sqrt{x^2-36}}$$

$$b) y = \sqrt{x^2+1} - \lg(x^2-25)$$

5. Berilgan limitlarni hisoblang:

$$a) \lim_{x \rightarrow \infty} \frac{9x^3 - 4x^2 + 1}{6x^3 + 3x + 2}$$

$$b) \lim_{x \rightarrow 3} \frac{3x^2 - 8x - 3}{x^2 - x - 6}$$

$$c) \lim_{x \rightarrow 7} \frac{x-7}{\sqrt{2x+11}-5}$$

6. $y = f(x)$ funksiya va argumentning ikkita x_1 va x_2 qiymatlari berilgan. Quyidagilar talab qilinadi:

- 1) x ning berilgan qiymatlarning har biri uchun berilgan funksiya uzluksiz yoki uzlukli bo'lishi aniqlansin;
- 2) funksiya uzilishga ega bo'lgan nuqtaga x chapdan va o'ngdan yaqinlashganda funksiyaning limiti topilsin;
- 3) sxematik chizma chizish.

$$f(x) = 8^{\frac{1}{x-2}}, \quad x_1 = 2, \quad x_2 = 5$$

Test

1. $-\frac{8}{9} \cdot 18,75 \cdot 1\frac{1}{8} - (-4,25)$ ni hisoblang.

- A) -14,5 B) -8 C) 14,5 D) -10

2. $6\frac{2}{3} \cdot 2\frac{1}{4} \cdot \left(-\frac{1}{2}\right) \cdot \frac{2}{5}$ ni hisoblang.

- A) -3 B) 3 C) -2,5 D) 2,5

3. 36455472363 ni 2, 4, 5, 9, 10 va 25 ga bo'lganda hosil bo'lgan qoldiqlar yig'indisini toping.

- A) 16 B) 26 C) 14 D) 15

4. Quyidagi oddiy kasr ko'rinishida qaysilarini chekli o'nli kasr ko'rinishiga keltirib bo'lmaydi :

- 1) $\frac{7}{32}$ 2) $\frac{10}{55}$ 3) $\frac{11}{160}$ 4) $\frac{20}{35}$

- A) 3; 4 B) 2; 3 C) 2; 4 D) 4; 1

5. $\frac{0,202 - 0,004}{\frac{8}{9} \cdot 81 \cdot 0,125}$ ni hisoblang.

- A) 0,99 B) 0,099 C) 0,022 D) 0,0099

6. 55 dan katta bo'lmagan barcha natural sonlarning ko'paytmasi nechta nol bilan tugaydi?

- A) 12 B) 14 C) 11 D) 13

7. $\frac{n^2 - 24}{n}$ ifoda natural son bo'ladigan n ning barcha natural qiymatlari yig'indisini toping.

- A) 54 B) 44 C) 48 D) 50

8. $\frac{2^8 \cdot 10^{10} \cdot 50^5}{(80 + 20)^{10}}$ ni hisoblang.

- A) 16 B) $\frac{1}{32}$ C) $\frac{1}{64}$ D) 8

9. $2^{10} + 3^{12}$ yig'indining oxirgi raqamini toping.

- A) 9 B) 5 C) 1 D) 4

10. $a(b + c - bc) - b(c + a - ac) - c(b + a)$ ni soddalashtiring.

- A) $2ac - 2bc$ B) $-2abc$ C) $ab - ac$ D) $-2bc$

11. Agar $a = 6 + \sqrt{3}$ va $b = 6 - \sqrt{3}$ bo'lsa $\frac{a^3 - b^3}{a^2 - b^2} : \frac{a^2 + ab + b^2}{a^3 + 3a^2b + 3ab^2 + b^3}$ ni hisoblang.

- A) 198 B) 144 C) 169 D) 196

12. $\left(\frac{1}{a(a+1)} + \frac{1}{(a+1)(a+2)}\right) \cdot \frac{a^2 + 2a}{8}$ ni soddallashtiring.

- A) $\frac{1}{8}$ B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) $\frac{3}{4}$

13. Amallarni bajaring: $\frac{9}{5 - \sqrt{7}} - \frac{22}{7 - \sqrt{5}} + \frac{1}{\sqrt{7} + \sqrt{5}}$.

- A) 6 B) $\sqrt{7} - 1$ C) 5 D) $\sqrt{7} - \sqrt{5}$

14. $a^3 - 9a^2 + 27a - 19$ ni ko'paytuvchilarga ajrating.

- A) $(a+1)(a^2 + 8a - 19)$ B) $(a-1)(a^2 - 8a + 19)$
C) $(a-1)(a^2 + 8a - 19)$ D) $(a+1)(a^2 + 8a + 19)$

15. $\left(\frac{3}{7}\right)^{-1} + 0,3^{-3} + (-0,5)^{-2} \cdot \frac{3}{4} - 10\frac{19}{27}$ ni hisoblang.

- A) $42\frac{4}{9}$ B) $31\frac{2}{3}$ C) $48\frac{10}{27}$ D) $34\frac{2}{3}$

16. $4\sqrt{7\frac{1}{2}} - \frac{2\sqrt{10}}{2\sqrt{3} - \sqrt{10}} + 8 + 3\sqrt{10}$ ni hisoblang.

- A) 10 B) $2 - 3\sqrt{10}$ C) -10 D) $3\sqrt{10} - 2$

17. $\frac{4^{\frac{2}{3}} \cdot 40^{\frac{1}{3}}}{10^{\frac{2}{3}}}$ ni hisoblang.

- A) 20 B) 15 C) 40 D) 60

18. $\frac{\left(\frac{1}{343}\right)^{\frac{1}{3}} + \left(\frac{1}{8}\right)^{\frac{1}{3}}}{\sqrt[3]{18\sqrt{144}}}$ ni hisoblang.

- A) $\frac{5}{16}$ B) $\frac{3}{2}$ C) $\frac{4}{7}$ D) $\frac{2}{3}$

19. Agar $a^2 - 6a + 10 + b^2 + 2b = 0$ bo'lsa, $(a+b)^3$ ni hisoblang.

- A) 27 B) 64 C) 25 D) 8

20. $\frac{\sqrt{3+2\sqrt{2}} + \sqrt{3-2\sqrt{2}} + \sqrt{2}}{4\sqrt{2}}$ ni hisoblang.

- A) 0,5 B) $\frac{\sqrt{2}}{4}$ C) 0,75 D) $\frac{\sqrt{2}}{2}$

21. $\frac{5x+6}{x^2-4} - \frac{x}{x^2-4} : \frac{x}{x-2} + 1$ ifodani soddalashtiring.

- A) -1 B) 1 C) $\frac{x+2}{x-2}$ D) $\frac{x-2}{x+2}$

22. $\frac{1-x^2}{1+x^2} \cdot \left(\frac{1}{(x-1)^2} - \frac{x}{1-x^2} \right)$ ifodani soddalashtiring.

- A) -1 B) $\frac{1}{1-x}$ C) $\frac{1}{x-1}$ D) $\frac{x+1}{1-x}$

23. Birinchi kuni ish normasining $\frac{2}{5}$ qismi bajarildi. Ikkinchi kuni birinchi kunda bajarilgan ishning $\frac{1}{6}$ qismicha ko'p ish bajarildi. Shu ikki kunda qancha ish normasi bajarildi?

- A) $\frac{7}{15}$ B) $\frac{13}{15}$ C) $\frac{11}{15}$ D) $\frac{4}{5}$

24. $\frac{2,72^4 - 0,72^4}{3,44^2 - 2,72 \cdot 1,44}$ ni hisoblang.

- A) 6,88 B) 5,68 C) 6,84 D) 5,28

25. $\sqrt{17-12\sqrt{2}} \cdot (9+6\sqrt{2})$ ning qiymatini hisoblang.

- A) 3 B) $2\sqrt{2}$ C) 2 D) $\sqrt{3+\sqrt{8}}$

26. $\frac{n^2-8n+7}{n^2-1}$ ni qisqartiring.

- A) $\frac{n-7}{n+1}$ B) $\frac{n+7}{n-1}$ C) $\frac{n-7}{n-1}$ D) $\frac{n+7}{n+1}$

27. $(a-3b)^2 - (3a+b)^2$ ni soddalashtiring.

- A) $-8a^2 + 12ab - 8b^2$ B) $8a^2 + 12ab - 8b^2$
C) $-8a^2 - 12ab + 8b^2$ D) $8a^2 - 12ab + 8b^2$

28. Agar $f(x) = (2x+3)\left(\frac{3}{x}-3\right)$ bo'lsa, $f(-1)$ ni toping.

- A) 6 B) 0 C) -3 D) -6

29. $0,34 \cdot 0,00025$ ko'paytma quyidagi sonlardan qaysi biriga teng emas?

- A) $850 \cdot 10^{-7}$ B) $8,5 \cdot 10^{-5}$ C) $8,5 \cdot 10^{-6}$ D) $85 \cdot 10^{-6}$

30. Bir son ikkinchi sondan 6 ta ortiq, ularning o'rta arifmetiga 23 ga teng. Shu sonlardan kattasini toping.

- A) 27 B) 23 C) 26 D) 33

31. $3\frac{3}{5} : 2\frac{7}{10} = 3\frac{3}{4} : x$ proporsiyaning noma'lum hadini toping.

- A) $2\frac{3}{10}$ B) $2\frac{13}{16}$ C) $1\frac{15}{16}$ D) $3\frac{1}{3}$

32. $\sqrt{a} - \sqrt{b} = 3$ va $a - b = 24$ bo'lsa, $\sqrt{a} + \sqrt{b}$ nimaga teng?

- A) 4 B) 6 C) 8 D) 5

33. a ning qanday qiymatlarida $ax - 3 = a + 4x$ tenglama yechimga ega emas ?

- A) 2 B) 4 C) -2 D) -1

34. $2x^2 - 26x + 32 = 0$ tenglama ildizlarining o'rta proporsionalini toping.

- A) 5 B) 4 C) 6 D) 7

35. $2,5(ax - 5,2) = 2a - 5x - 9$ tenglama a ning qanday qiymatlarida yagona yechimga ega?

- A) $-\frac{1}{2}$ B) $\left(-\infty; -\frac{1}{2}\right) \cup \left(-\frac{1}{2}; \infty\right)$ C) $(-\infty; -2) \cup (-2; \infty)$ D) $\frac{1}{5}$

36. a ning qanday qiymatlarida $ax^2 - 3x + 3 = 0$ tenglama bitta ildizga ega bo'ladi?

- A) 0 va 1 B) $\frac{1}{3}$ C) $\frac{1}{3}$ va 0 D) $\frac{3}{4}$ va 0

37. $x^2 + px - 12 = 0$ tenglamaning ildizlaridan biri 3 ga teng. Shu tenglamaning barcha koeffitsiyentlari yig'indisini toping.

- A) -10 B) -13 C) -11 D) -12

38. $\frac{x^3 - 8}{x - 2} = 9 - 2x$ tenglamaning ildizlari yig'indisini toping.

- A) 4 B) 6 C) 3 D) -4

39. $x^2 + 5x - \sqrt{x^2 + 5x + 25} = 17$ tenglamaning ildizlari ko'paytmasini toping.

- A) 12 B) -24 C) -8 D) -16

40. $\begin{cases} x + 4 = 2 \\ xy^2 = 4 \end{cases}$ tenglamalar sistemasini yeching.

- A) $(-1; -2)$ B) $(1; -2)$ C) $(-1; -2); (1; -2)$ D) \emptyset

41. $\begin{cases} x^3 + y^3 = 126 \\ x^2y + xy^2 = 30 \end{cases}$ tenglamalar sistemasining haqiqiy yechimlaridan iborat barcha x va y larning yig'indisini toping.

- A) 2 B) 12 C) 10 D) 30

42. $x^2 - 3|x| - 28 = 0$ tenglamaning ildizlari ko'paytmasini toping.

- A) -36 B) -49 C) -64 D) -32

43. Agar $\begin{cases} (x-2)^2 + |y-1| = 4 \\ |x-2| + |y-1| = 2 \end{cases}$ bo'lsa, $x - y$ ning qiymatini toping.

- A) 0 yoki 4 B) 3 yoki -1 C) 1 yoki 5 D) -2 yoki 4

44. Quyidagilardan qaysi biri $(x-4)\sqrt{x^2 + x - 2} \leq 0$ tengsizlikning yechimi?

- A) $(-\infty; -2] \cup [1; 4]$ B) $(-\infty; 4]$ C) $[-1; 2] \cup [4; \infty)$ D) $[-2; 4]$

45. t ning qanday qiymatlarida $3x + 2 = 2(x - t)$ tenglama musbat ildizga ega?

- A) $t < 2$ B) $t > -2$ C) $t < -1$ D) $t \leq 1$

46. Arifmetik progressiyaning 6-hadi 10 ga, dastlabki 16 ta hadining yig'indisi 200 ga teng. Bu progressiyaning 9-hadini toping.

- A) 14 B) 16 C) 13 D) 18

47. Tomoni 1000 dm gat eng bo'lgan kvadrat tomoni 5 sm ga teng bo'lgan kvadratchalarga ajratildi. Shu kvadratchalar kengligi 10 sm bo'lgan tasma shaklida joylashtirilsa, uning uzunligi qancha bo'ladi?

- A) 200 km B) 100 km C) 1 km D) 20 km

48. Ikkinchi hadi 6 ga teng, birinchi uchta hadining yig'indisi 26 ga teng o'suvchi geometrik progressiyaning 4- va 2-hadlari ayirmasini toping.

- A) 16 B) 32 C) 48 D) 36

49. Geometrik progressiyaning maxraji 3 ga, dastlabki 4 ta hadining yig'indisi 120 ga teng. Birinchi hadining qiymatini toping.

- A) 2 B) 1 C) 4 D) 3

50. Mahsulotning bahosi 30% ga oshirildi. Ma'lum vaqtdan keyin 20% ga arzonlashtirildi, shundan so'ng uning narxi 8944 so'm bo'ldi. Mahsulotning dastlabki bahosi necha so'm bo'lgan?

- A) 8400 B) 8600 C) 9300 D) 8500

51. 15 kg eritmaning 40 foizi tuzdan iborat. Tuzning miqdori 25 foiz bo'lishi uchun eritmaga necha kg chuchuk suv qo'shish kerak?

- A) 6 B) 9 C) 8 D) 10

52. $\operatorname{tg}\left(\frac{1}{2}\arcsin\frac{24}{25}\right)$ ni hisoblang.

- A) $\frac{1}{2}$ B) $\frac{1}{7}$ C) $\frac{3}{4}$ D) $\frac{1}{4}$

53. $\frac{\sin\alpha+2}{5+\cos\beta} + \frac{3}{\operatorname{tg}^2\gamma + \operatorname{ctg}^2\gamma}$ ifodaning eng katta qiymatini toping.

- A) 4,75 B) 6,25 C) 2,75 D) 3,45

54. $4\cos 5x = 6 + 3\cos\left(\frac{\pi}{2} + 5x\right)$ tenglama $[-\pi; 2\pi]$ oraliqda nechta ildizga ega?

- A) 1 B) \emptyset C) 3 D) 2

55. k ning quyida ko'rsatilgan qiymatlarining qaysi birida $\sin kx \cdot \cos x - \sin x \cdot \cos kx = 0$ tenglamaning ildizlari $\frac{\pi n}{7}$, ($n \in \mathbb{Z}$) bo'ladi?

- A) 8 B) 5 C) 7 D) 6

56. $\frac{1}{\sqrt{2} + \sqrt{3} + \sqrt{5}}$ kasrning maxrajini irratsionallikdan qutqaring.

- A) $\frac{2\sqrt{3} - 3\sqrt{2} + \sqrt{30}}{12}$ B) $\frac{2\sqrt{3} - 3\sqrt{2} - \sqrt{30}}{12}$
C) $\frac{3\sqrt{2} - 2\sqrt{3} + \sqrt{30}}{12}$ D) $\frac{3\sqrt{2} - 2\sqrt{3} - \sqrt{30}}{12}$

57. $a = 2\log_2 5$, $b = 4\log_{\frac{1}{4}} \frac{5}{26}$, $c = 3\log_{\frac{1}{8}} \frac{1}{23}$ sonlarni o'sish tartibida joylashtiring.

- A) $a < b < c$ B) $b < a < c$ C) $c < a < b$ D) $b < c < a$

58. $2\log_8 x - \log_8(x-1) > \frac{2}{3}$ tengsizlikni yeching.

- A) $(2; \infty)$ B) $(3; 5)$ C) $(3; \infty)$ D) $(1; 2) \cup (2; \infty)$

59. $y = \sqrt{\lg^2|2x-7| \cdot (5x-6-x^2)}$ funksiyaning aniqlanish sohasiga tegishli butun sonlarning yig'indisini toping.

- A) 5 B) 14 C) 12 D) 9

60. $y = x^2 - 2x - 2,75$ dagi qanday nuqtada o'tkazilgan urinma $y = -4(x+1)$ to'g'ri chiziqqa parallel bo'ladi?

- A) $(-1; 4)$ B) $\left(-1; \frac{1}{4}\right)$ C) $(1; 4)$ D) $\left(1; \frac{1}{4}\right)$

61. $y = e^{2-3x}$ funksiyaning boshlang'ich funksiyasini toping.

A) $e^{2-3x} + C$ B) $\frac{1}{3}e^{2-3x} + C$ C) $-\frac{1}{3}e^{2-3x} + C$ D) $-3e^{2-3x} + C$

62. $\frac{(-x^2 + x - 1)(x^2 - 3x + 2)}{(x^2 - 7x + 12)} \geq 0$ tengsizlikning butun sonlardan iborat yechimlari nechta?

A) 1 B) 4 C) 3 D) 2

63. Teng yonli ABC uchburchakning ($AB=AC$) A uchidan uchburchak tekisligiga uzunligi 32 ga teng bo'lgan AD perpendikulyar o'tkazildi. D nuqtadan BC tomongacha bo'lgan masofa 40 ga teng. ABC uchburchakning BC tomoniga o'tkazilgan balandligi qanchaga teng?

A) 12 B) 24 C) 20 D) 14

64. Radiusi 15 ga teng bo'lgan sharga ichki chizilgan konusning balandligi 12 ga teng. Konusning hajmini toping.

A) 486π B) 756π C) 864π D) 672π

65. $x^{\log_2 x + 2} < 8$ tengsizlikni yeching.

A) $(2^{-2}; 2)$ B) $(2^{-5}; 2)$ C) $(2^{-4}; 2)$ D) $(2^{-3}; 2)$

66. Rombning yuzi 120 ga teng. Diagonallaridan biri 24 ga teng. Uning tomonini toping.

A) 13 B) 10 C) 14 D) 8

67. ABCD parallelogramm C uchining koordinatalari (5; 8), O(3; 6) esa parallelogramm diagonallarining kesishish nuqtasi. Parallelogramm A uchining koordinatalarini toping.

A) (3; 2) B) (2; 3) C) (4; 1) D) (1; 4)

68. Ikki tomoni yig'indisi 1,8 ga va ular orasidagi burchagi 150° gat eng bo'lgan uchburchaklar ichida yuzasi eng katta bo'lgan uchburchakning yuzini toping.

A) $\frac{4}{25}$ B) $\frac{9}{10}$ C) $\frac{81}{400}$ D) $\frac{81}{100}$

69. $\sin^4 \frac{17\pi}{8} - \cos^4 \frac{15\pi}{8}$ ni hisoblang.

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $-\frac{\sqrt{2}}{2}$ D) $-\frac{\sqrt{3}}{2}$

70. Muntazam uchburchakli piramidaning yon qirrasini 20 ga, asosining tomoni $16\sqrt{3}$ ga teng. Piramidaning balandligini toping.

A) $8\sqrt{3}$ B) 12 C) 8 D) 16

Adabiyotlar:

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