O’ZBEKISTON RESPU BLIKASI OLIY VA

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«OLIY MATEMATIKA» KAFEDRASI

«Oliy matematika» fanidan

REFERAT

Mavzu: Fazoda analitik geometriya. Tekislik va uning tenglamalari.

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Fazoda analitik geometriya. Tekislik va uning tenglamalari

Tayanch iboralar: berilgan nuqtadan o′tuvchi tekisliklar tenglamasi, uchta nuqtadan o′tuvchi tekislik tenglamasi, tekisliklar orasidagi burchak, tekisliklarni perpendikulyarlik va parallellik sharti, nuqtadan tekislikkacha masofa.

R e j a :

1. Berilgan nuqtadan o′tuvchi tekisliklar tenglamasi.
2. Berilgan uchta nuqtadan o′tuvchi tekislik tenglamasi.
3. Ikki tekislik orasidagi burchak.
4. Tekisliklarning perpendikulyarlik va parallellik sharti.
5. Uchta tekislikning kesishish nuqtasi.
6. Nuktadan tekislikkacha bo′lgan masofa.

Fazoda tekislik tenglamalari

Tekislikning umumiy tenglamasi. Aytaylik, fazoda ixtiyoriy $Oxyz$ dekart koordinatalar sistemasi berilgan bo'lsin, unda x, y, z o'zgaruvchili istalgan birinchi tartibli tenglama berilgan sistemaga nisbatan tekislikni ifodalaydi. Bunday tenglamaning ko'rinishi quyidagicha bo'lsin:

$$Ax + By + Cz + D = 0 \qquad (1)$$

Bu yerda A,B,C va D lar ixtiyoriy o'zgarmaslar bo'lib, A,B,C lar bir paytda nolga teng bo'lmaydi. (1)- tenglamani qanoatlantiradigan hech bo'lmaganda bitta $M_0(x_0, y_0, z_0)$ nuqtani belgilab olamiz, ya`ni

$$Ax_0 + By_0 + Cz_0 + D = 0 \qquad (2)$$

tenglik o'rinli bo'lsin. Agar (1)- dan (2)- ni ayirsak, tekislikning (1)-ga teng kuchli

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 \qquad (3)$$

ko'rinishidagi tenglamasini hosil qilamiz. Endi (3)- tenglama $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi, $\vec{n} = \{A, B, C\}$ vektorga perpendikulyar tekislikni ifodalashini ko'rsatamiz. $\vec{n}$ vektor nol vektor emas, chunki A,B,C lar bir paytda nolga teng bo'lolmaydi.

Agar haqiqattan ham, $M(x, y, z)$ nuqta tekshirilayotgan tekislikda yotsa, uning koordinatalari (4.28)- tenglamani qanoatlantirishi kerak. Demak, bu paytda shu tekislikda yotgan $\vec{r} = \overline{M_0}\vec{M} = \{x - x_0, y - y_0, z - z_0\}$ yo'nalgan kesma (radius-vektor) va $\vec{n} = \{A, B, C\}$ o'zaro perpendikulyar (ortogonal) bo'lishi, ularning skalyar ko'paytmasi esa

$$(\vec{n}\vec{r}) = \left(\vec{n} \cdot \overline{M_0}\vec{M}\right) = A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 \quad (4)$$

bo'lishi kerak, aks holda qaralayotgan vektorlar o'zaro perpendikulyar bo'lolmaydi. $\vec{n}$ vektor faqat tekislikdagina yotgan istalgan bir yo'nalgan kesmaga perpendikulyar bo'lishi mumkin. Shunday qilib, (3) va demak, (1)-tenlama fazoda tekislikni ifodalab, unga tekislikning umumiy tenglamasi deyiladi. $\vec{n} = \{A, B, C\}$ vektorga esa tekislikning normal vektori deyiladi.

Tekislikning umumiy tenglamasini tekshirish. Tekislikning umumiy ko'rinishidagi (1)-tenglamasida *A, B, C* va *D* lar noldan farqli bo'lsa, unda (1)-ga to'la tenglama deyiladi. Agar *A, B, C, D* lardan birortasi nolga teng bo'lsa, to'la bo'lmagan tenglama deyiladi. Quyida to'la bo'lmagan tenglamaning ba`zi xususiy hollari bilan tanishib o'tamiz.

1) $D = 0$; tenglama $Ax + By + Cz = 0$ ko'rinishini olib, kordinatalar boshidan o'tuvchi tekislikni ifodalaydi.

2) $A = 0$; tenglama $By + Cz + D = 0$ ko'rinishini olib, *Ox* o'qiga parallel tekislikni ifodalaydi. Chunki bunday tekislikning $\vec{n} = \{0, B, C\}$ normal vektor *Ox* o'qiga perpendikulyardir.

3) $B = 0$ tenglama $Ax + Cz + D = 0$ ko'rinishini olib, *Oy* o'qiga parallel tekislikni ifodalaydi. Chunki bunday tekislikning $\vec{n} = \{A, 0, C\}$ normal vektori *Oy* o'qiga perpendikulyardir.

4) $C = 0$; tenglama $Ax + By + D = 0$ ko'rinishini olib, *Oz* o'qiga parallel tekislikni ifodalaydi. Chunki bunday tekislikning $\vec{n} = \{A, B, 0\}$ normal vektori *Oz* o'qiga perpendikulyardir.

5) $B=0, C=0$ tenglama $Ax+D=0$ yoki $x=-\frac{D}{A}$ ko'rinishini oladi. Agar $a=-\frac{D}{A}$ deb olsak, $x=a$ tenglama Oyz koordinata tekisligiga parallel va Ox o'qidan a ga teng kesma ajratuvchi tekislikni ifodalaydi.

6) $A=0, B=0;$ tenglama $Cz+D=0$ yoki $z=-\frac{D}{C}$ ko'rinishni oladi. Agar $c=\frac{-D}{C}$ deb olsak, $z=c$ tenglama Oxy koordinata tekisligiga parallel va Oz o'qidan c ga teng kesma ajratuvchi tekislikni ifodalaydi.

7) $A=0, C=0;$ tenglama $By+D=0$ yoki $y=-\frac{D}{B}$ ko'rinishini oladi. Agar $b=-\frac{D}{B}$ deb olsak, $y=b$ tenglama Oxz kordinata tekisligiga parallel va Oy o'qidan b ga teng kesma ajratuvchi tekislikni ifodalaydi.

8) $B=0, C=0, D=0$ tenglama $Ax=0$ ko'rinishini olib, Oyz koordinata tekisligini ifodalaydi.

9) $A=0, C=0, D=0$ tenglama $By=0$ ko'rinishini olib, Oxz koordinata tekisligini ifodalaydi.

10) $A=0, B=0, D=0$ tenglama $Cz=0$ ko'rinishini olib, Oxy koordinata tekisligini ifolaydi.

Tekislikning kesmalar bo'yicha tenglamasi. Tekislikning (1)-ko'rinishidagi umumiy tenglamasini yozib olamiz:

$$Ax+By+Cz+D=0$$

Tenglamada D koeffitsientni tenglikning o'ng tomoniga o'tkazib, hamma hadlarni (-D) ga bo'lib chiqamiz:

$$\frac{A}{-D}x+\frac{B}{-D}y+\frac{C}{-D}z=1 \qquad (5)$$

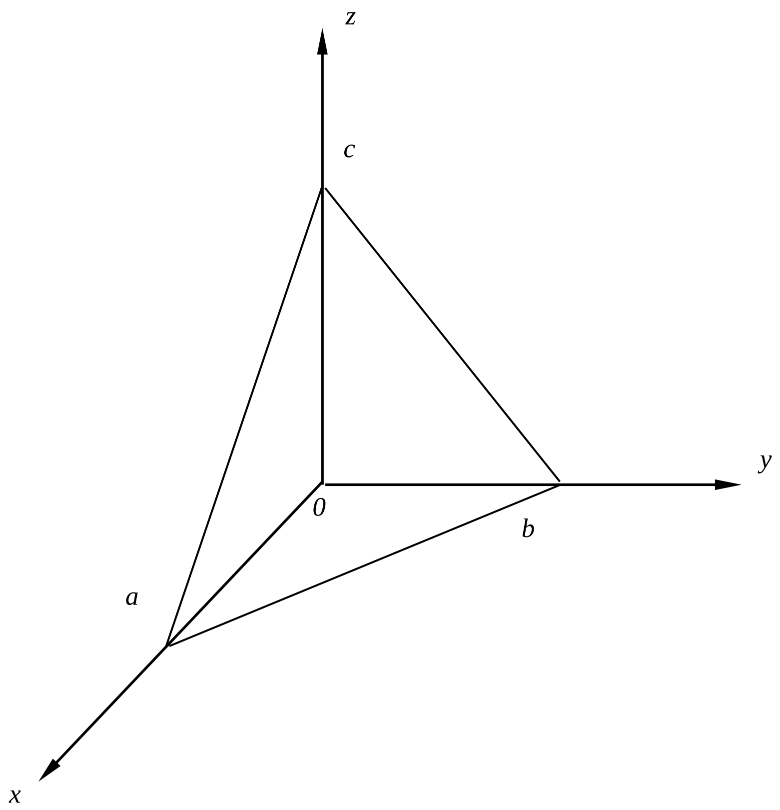
yoki

$$\frac{x}{-\frac{D}{A}}+\frac{y}{-\frac{D}{B}}+\frac{z}{-\frac{D}{C}}=1 \qquad (6)$$

(6)- tenglamada $a=-\frac{D}{A}, b=-\frac{D}{B}$ va $c=-\frac{D}{C}$ belgilashlarni kiritamiz, a,b,c lar tekislikning mos ravishda Ox, Oy, Oz o'qlardan ajratgan kesmalarini ko'rsatadi (1-chizma). Unda (6)- tenglama

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad (7)$$

ko'rinishini olib, tekislikning kesmalar bo'yicha tenglamasi deyiladi.



1-chizma.

Masala. Tekislikning umumiy tenglamasi $2x-y-4z+20=0$ ni uning kesmalar bo'yicha tenglamasiga keltiring.

Yechish. Tenglamaning ozod hadi 20 ni tenglikning o'ng tomoniga o'tkazib, hamma hadlarini -20 ga bo'lamiz:

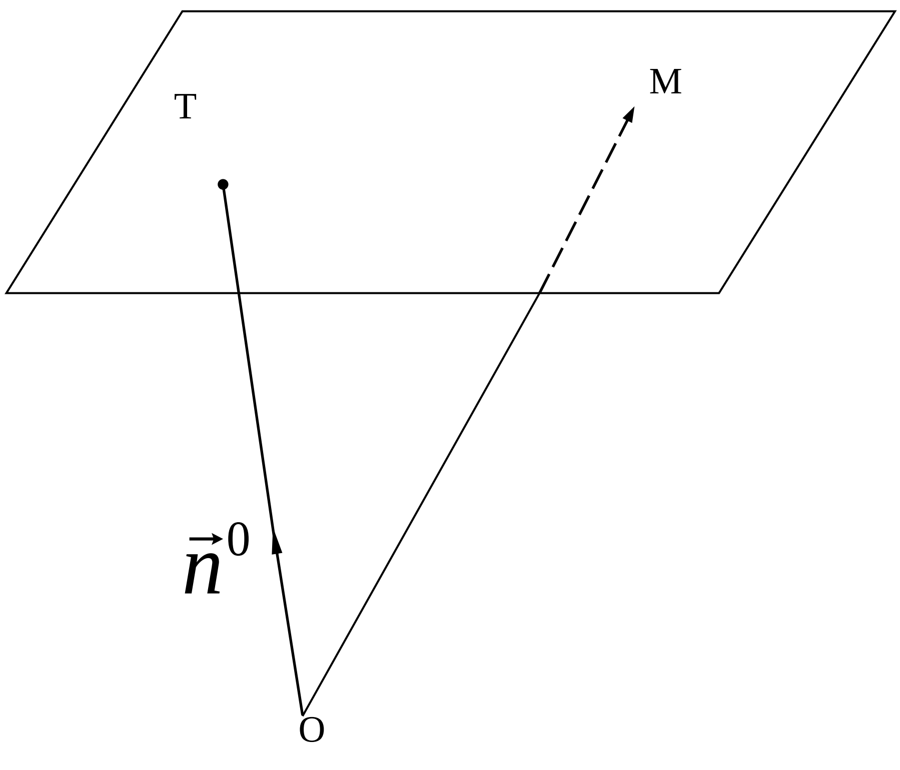
$$2x - y - 4z = -20, \qquad \frac{2}{-20}x - \frac{1}{-20}y - \frac{4}{-20}z = 1$$

yoki tekislikning

$$\frac{x}{-\frac{20}{2}} - \frac{y}{-\frac{20}{1}} - \frac{z}{-\frac{20}{4}} = 1 \quad \text{yoki} \quad \frac{x}{-10} + \frac{y}{20} + \frac{z}{5} = 1$$

ko’rinishidagi kesmalar bo’yicha tenglamasini hosil qilamiz. Demak, berilgan tekislik *Ox* o’qidan $a = -10$, *Oy* o’qidan $b = 20$ va *Oz* o’qidan $c = 5$ ga teng kesmalar ajratar ekan.

Tekislikning normal tenglamasi. Bizga koordinatalar boshidan tekislikkacha bo’lgan masofa *p*, ya`ni *O* nuqtaning tekislikka o’tkazilgan *OT* perpendikulyarning uzunligi hamda *O* nuqtadan tekislikka yo’nalgan birlik normal vektor $\vec{n}_0$ berilgan bo’lsin (2-chizma).



2-chizma.

Shu berilgan kattaliklar yordamida tekislikning tenglamasini topish masalasini qo'yamiz. M ko'rilayotgan tekislikning biror nuqtasi bo'lsin. $\overrightarrow{OM} = \vec{r}$ deb, bu vektorning $\vec{n}^{\circ}$ vektor yo'nalishidagi proyektsiyasini olsak, u

$$ПР_{\vec{n}^{\circ}} OM = p \qquad (8)$$

bo'ladi, chunki shartga ko'ra $|\overrightarrow{OT}| = p$. Ikki vektorning skalyar ko'paytmasigi asosan

$$ПР_{\vec{n}^{\circ}} \overline{O}\vec{M} = \left(\overrightarrow{OM} \cdot \vec{n}^{\circ}\right)$$

yoki

$$ПР_{\vec{n}^{\circ}} \overline{O}\vec{M} = \left(\vec{r} \cdot \vec{n}^{\circ}\right) \qquad (9)$$

ni hosil qilamiz. Buni (4.33)- tenglikka qo'ysak,

$$\left(\vec{r}\vec{n}^{\circ}\right) - p = 0 \qquad (10)$$

bo'ladi. Tekislikning vektor shaklidagi normallashgan tenglamasi deyiladi.

$\vec{r}$ esa tekislikdagi ixtiyoriy M nuqtaning radius-vektori bo'lib, o'zgaruvchi kattalikdir.

(10)-tenglikni Dekart koordinatalar orqali yozish maqsadida $\vec{n}^{\circ}$ vektorni yo'naltiruvchi kosinuslar orqali $\vec{r}$ ni x, y, z koordinatalar orqali yozsak, ya`ni

$$\vec{n}^{0}\{\cos\alpha, \cos\beta, \cos\gamma\}, \vec{r}\{x, y, z\}$$

unda

$$\left(\vec{r} \cdot \vec{n}^{\circ}\right) = x\cos\alpha + y\cos\beta + z\cos\gamma \qquad (11)$$

bo'ladi.

Natijada (10)-ni (11)-yordamida quyidagi ko'rinishda yozish mumkin:

$$x\cos\alpha + y\cos\beta + z\cos\gamma - p = 0 \qquad (12)$$

(12)-tekislikning koordinatalar shaklidagi normal tenglamasidir.

Tekislikning umumiy tenglamasini normal ko'rinishiga keltirish. Yuqorida tekislikning (12)-ko'rinishidagi tenglamasini ikkita $\vec{n} = \{A, B, C\}$ va $\vec{r} = \overline{M}_0\vec{M}$ vektorlarning skalyar ko'paytmasi ko'rinishida yozish mumkinligini ko'rsatgan edik. Endi tekislikning (12)-ko'rinishidagi tenglamasining

$$Ax+By+Cz+D=0 \qquad (13)$$

ko'rinishini (13)- dan foydalanib, quyidagi $\vec{n}=\{A,B,C\}$ va $\vec{r}=\{x,y,z\}$ vektorlarning skalyar ko'paytmasi $(\vec{n}\vec{r}\}=Ax+By+Cz$ yordamida o'zgartirib yozamiz. Unda tekislikning vektor ko'rinishidagi tenglamasi

$$(\vec{n}\vec{r})+D=0 \qquad (14)$$

ko'rinishini oladi. Bu yerda $\vec{n}$ - tekislikning normal vektori, esa $\vec{r}$ tekislik nuqtasining radius-vektoridir. Agar normal vektorni

$$\vec{n}=\vec{n}^{\circ}\cdot|\vec{n}|=\vec{n}^{\circ}\cdot n \qquad (15)$$

ko'rinishida yozish mumkinligini e`tiborga olsak, (14)-ushbu ko'rinishga keladi.

$$(\vec{n}^{\circ}\vec{r})n+D=0 \qquad (16)$$

(16)-tenglikni $(\pm n)$ ga bo'lsak,

$$\pm(\vec{n}\vec{r})+\frac{D}{\pm n}=0 \qquad (17)$$

hosil bo'ladi. Quyidagi

$$\begin{aligned} D \succ 0 \quad da \quad \frac{D}{-n}=-p \\ D \prec 0 \quad da \quad \frac{D}{+n}=-p \end{aligned} \qquad (18)$$

belgilashlarni kiritib, tekislikning ushbu

$$\pm(\vec{n}^{\circ}\vec{r})-p=0$$

normal tenglamasini hosil qilamiz. Demak, tekislikning (13)-ko'rinishidagi tenglamasini normal ko'rinishga keltirish uchun tenglamaning hamma hadlarini $\pm n$ ga bo'lish yoki $M=\frac{1}{\pm n}$ ga ko'paytirish kerak. Agar

$n=\sqrt{A^2+B^2+C^2}$ ekanini e`tiborga olsak,

$$M=\pm\frac{1}{\sqrt{A^2+B^2+C^2}} \qquad (19)$$

ga ega bo'lib, unga normallovchi ko'paytuvchi deyiladi. Agar $D>0$ bo'lsa, M manfiy ishora bilan, agar $D \prec 0$ bo'lsa, M musbat ishora bilan olinadi.

Shunday qilib, (4.38)- tenglamani M ga ko'paytirsh namunasiga (12)- ko'rinishidagi normal tenglama ko'rinishiga keladi.

$$MAx + MBy + MCz + MD = 0 \qquad (20)$$

(20)- va (8)- ni solishtirib, quyidagi tengliklarga ega bo'lamiz.

$$\cos\alpha = \pm\frac{A}{\sqrt{A^2+B^2+C^2}}, \quad \cos\beta = \pm\frac{B}{\sqrt{A^2+B^2+C^2}},$$

$$cos\gamma = \pm\frac{C}{\sqrt{A^2+B^2+C^2}}, \quad p = \pm\frac{D}{\sqrt{A^2+B^2+C^2}}$$

Misol. Tekislikning $6x + 7y + 6z - 34 = 0$ ko'rinishidagi umumiy tenglamasi berilgan. Bu tekislikning normal tenglamasini tuzing va normalining yo'naltiruvchi kosinuslarini toping.

Yechish. Berilgan tenglamada: $A = 6, B = 7, C = 6, D = -34$ (20)-formulaga asosan normallovchi ko'paytuvchini topamiz.

$$M = \frac{1}{+\sqrt{6^2+7^2+6^2}} + \frac{1}{+\sqrt{36+49+36}} = \frac{1}{11}$$

Berilgan umumiy ko'rinishdagi tenglamani $\frac{1}{11}$ ga ko'paytirib, tekislikning

$$\frac{6}{11}x + \frac{7}{11}y + \frac{6}{11}z - \frac{34}{11} = 0$$

ko'rinishidagi normal tenglamasini hosil qilamiz. Bu yerdan esa yo'naltiruvchi kosinuslarini topamiz:

$$\cos\alpha = \frac{6}{11}, \quad \cos\beta = \frac{7}{11}, \quad \cos\gamma = \frac{6}{11}.$$

Berilgan nuqtadan tekislikkacha bo'lgan masofa. Berilgan $M(x_1, y_1, z_1)$ nuqtadan tekislikkacha bo'lgan masofasini topish masalasini qaraymiz. Bu chetlanishni δ deb, M nuqtadan tekislikkacha bo'lgan masofani d deb olsak, ular orasida quyidagi munosabat mavjud: agar M nuqta va koordinatalar boshi tekislikning har tomonida bo'lsa, $d = \delta$ aks holda, ya`ni M nuqta va koordinatalar boshi tekislikning bir tomonida bo'lsa, $d = -\delta$ bo'ladi. Biz birinchi hol bilan chegaralanamiz. 4.19-chizmadan ko'rinib turibdiki,

$$\delta = |PQ| = |OQ| - |OP| \qquad (21)$$

shu bilan birga, $\overrightarrow{OM}$ vektorning $\vec{n}^\circ$ birlik normal vektorga proyektsiyasi ikki vektorning skalyar ko'paytmasiga teng:

$$|OQ| = \text{ÏÐ}_{\vec{n}^\circ}\overrightarrow{\text{ÎÌ}} = \left(\vec{n}^0 \cdot \overrightarrow{\text{ÎÌ}}\right)$$

Agar $\vec{n}^\circ = \{\cos\alpha;\ \ \cos\beta;\ \ \cos\gamma\}$, $\overrightarrow{OM} = \{x_1; y_1; z_1\}$ ekanligini nazarda tutsak,

$$|OQ| = x_1\cos\alpha + y_1\cos\beta + z_1\cos\gamma \qquad (22)$$

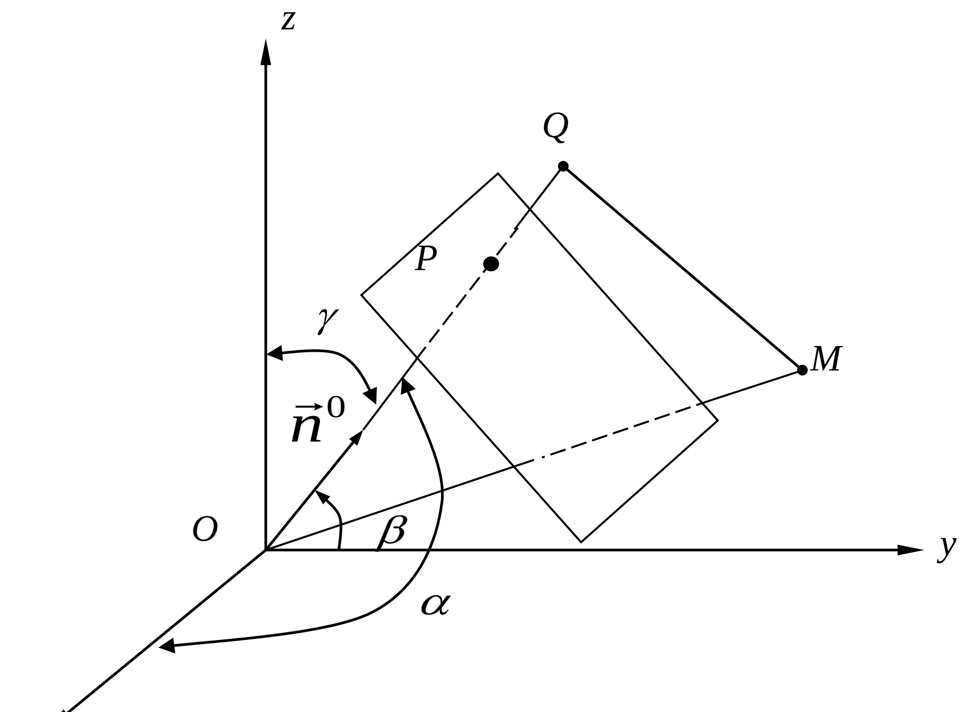
ni topamiz. $|OP| = p$ ligini (I punktga qarang) e`tiborga olib, (21) ni (22) yordamida qayta yozamiz:

$$\delta = x_1\cos\alpha + y_1\cos\beta + z_1\cos\gamma - p \qquad (23)$$

Bu nuqtaning tekislikdan chetlanishidir. Nuqtadan tekislikkacha bo'lgan masofa quyidagi formula yordamida topiladi:

$$d = \pm\left|x_1\cos\alpha + y_1\cos\beta + z_1\cos\gamma - p\right| \qquad (24)$$

Masala. $M(2, 1, 2)$ nuqtadan $6x + 7y + 6z - 34 = 0$ tekislikkacha bo'lgan masofani toping.



3-chizma.

Yechish. Yuqorida bu tekislik tenglamasi normal ko'rinishiga keltirilgan edi:

$$\frac{6}{11}x+\frac{7}{11}y+\frac{6}{11}z-\frac{34}{11}=0$$

formulaga asosan, $x_1=2,\ y_1=1$ va $z_1=2$ ligini e`tiborga olib, d ni topamiz:

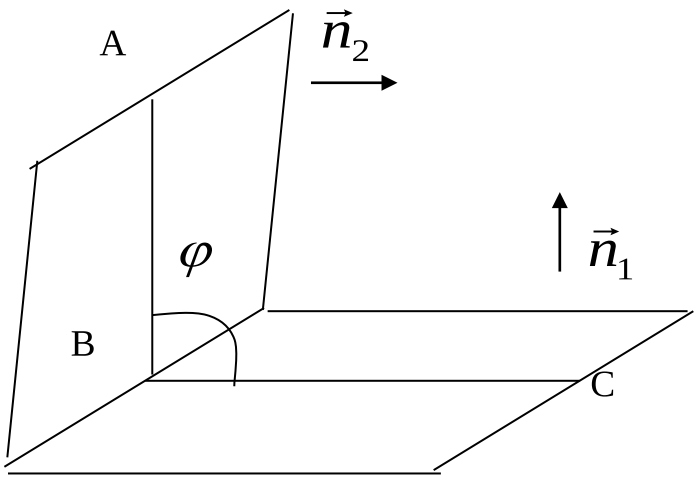
$$d=\left|\frac{6}{11}\cdot 2+\frac{7}{11}\cdot 1+\frac{6}{11}\cdot 2-\frac{34}{11}\right|=\left|\frac{12+7+12-34}{11}\right|=\left|\frac{-3}{11}\right|=\frac{3}{11}$$

Demak, $d=\frac{3}{11}$.

Ikki tekislik orasidagi burchak. Ikki tekislikning parallellik va perpendikulyarlik shartlari. Ikki tekislik o'zining (23)-ko'rinishidagi tenglamalari bilan berilgan bo'lsin:

$$(\vec{r}\vec{n}_1)+D_1=0,\quad (\vec{r}\vec{n}_2)+D_2=0$$

bu yerda $\vec{n}_1$ va $\bar{n}_2$ -tekisliklarning normal vektorlaridir. Bu ikki tekislik orasidagi burchak ikki yoqli burchak bilan aniqlanib, u esa o'z navbatida o'zining chiziqli burchagi $A\hat{B}C=\varphi$ bilan o'lchanadi (4.20-chizma).



4 –chizma.

Shu bilan birga ikki tekislik orasidagi burchak ularning normal vektorlari orasidagi burchakka tengdir. Shuning uchun ham berilgan ikki tekislik orasidagi burchakni ularning normallari, ya`ni ikki vektor orasidagi burchak sifatida aniqlaymiz.

Ma`lumki, $\cos\varphi = \frac{(\vec{n}_1 \cdot \vec{n}_2)}{|n_1||n_2|}$. Agar berilgan tekisliklar bir-biriga parallel bo'lsa, ularning normal vektorlari kollinear bo'ladi va shuning uchun tekisliklar parallelligining zarur va yetarli sharti

$$\vec{n}_1 = \lambda \vec{n}_2$$

tenglik bilan ifodalanadi, bu yerda λ biror o'zgarmas son. Shunga o'xshash berilgan tekisliklarning perpendikulyarlik sharti, ularning normallarining perpendikulyarlik shartiga teng bo'lib, bu shart normal vektorlari skalyar ko'paytmasining nolga teng bo'lishi bilan aniqlanadi.

$$(\vec{n}_1 \cdot \vec{n}_2) = 0$$

Endi tekisliklar umumiy tenglamalari bilan berilganda, yuqoridagi shartlar qanday ko'rinishda bo'lishini aniqlaylik. Ushbu

$$A_1x + B_1y + C_1z + D_1 = 0$$

va

$$A_2x + B_2y + C_2z + D_2 = 0$$

tekisliklar berilgan bo'lsin. Bu tekisliklarning normal vektorlari $n_1 = \{A_1, B_1, C_1\}$ va $n_2 = \{A_2, B_2, C_2\}$ bo'ldi. Unda ikki vektorning skalyar ko'paytmasi ta`rifidan foydalanib, ikki tekislik orasidagi burchakni (ularning normallari orasidagi burchak) hisoblash formulasini quyidagi ko'rinishda olamiz:

$$\cos\varphi = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}} \qquad (25)$$

Ikki tekislikning parallellik sharti (normallarining parallelligi)

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \qquad (26)$$

ko'rinishni oladi.

Va nihoyat ikki tekislikning perpendikulyarlik sharti (normallarining perpendikulyarligi)

$$A_1A_2 + B_1B_2 + C_1C_2 = 0 \qquad (27)$$

ko'rinishini oladi.

Masala. $2x+y-3z+3=0$ va $x-y+2z-4=0$ tekisliklar orasidagi burchakni toping.

Yechish. Bu yerda

$$A_1=2,\quad B_1=1,\quad C=-3$$
$$A_2=1,\quad B_2=-1,\quad C_2=2$$

Unda (27)- formulaga asosan

$$\cos\varphi=\frac{2\cdot1+1(-1)+(-3)\cdot2}{\sqrt{2^2+1^2+(-3)}\,\sqrt{1^2+(-1)^2+2^2}}=\frac{-5}{\sqrt{14}\,\sqrt{6}}=-\frac{5}{2\sqrt{21}}$$

$$\varphi=\arccos\left(-\frac{5}{2\sqrt{21}}\right)=\pi-\arccos\frac{5}{2\sqrt{21}}$$

Bir to'g'ri chiziqda yotmagan uch nuqtadan o'tuvchi tekislikning tenglamasi. Fazoda bir to'g'ri chiziqda yotmaydigan uchta $M_1(x_1,\, y_1,\, z_1)$, $M_2(x_2,\, y_2,\, z_2)$ va $M_3(x_3,\, y_3,\, z_3)$ nuqta berilgan bo'lsin. Shu nuqtadan o'tuvchi tekislikning tenglamasini topamiz. Shartga ko'ra nuqtalar bir to'g'ri chiziqda yotmagani uchun, $\overrightarrow{M_1M_2}=\{x_2-x_1,\, y_2-y_1,\, z_2-z_1\}$ va $\overrightarrow{M_1M_3}=\{x_3-x_1,\, y_3-y_1,\, z_3-z_1\}$ vektorlar kolleniar bo'lolmaydi, ya`ni ular parallel yoki bir to'g'ri chiziqda yotmaydi. Shuning uchun ham ixtiyoriy $M(x,y,z)$ nuqta M_1, M_2 va M_3 nuqtalar bilan bir tekislikda yotishi uchun $\overrightarrow{M_1M_2}, \overrightarrow{M_1M_3}$ va $\overrightarrow{M_1M}=\{x-x_1, y-y_1, z-z_1\}$ vektorlar komplanar va shu sababli ularning aralash ko'paytmasi nolga teng bo'lishi shart. Shunday qilib, $\overrightarrow{M_1M_2}, \overrightarrow{M_1M_3}$ va $\overrightarrow{M_1M}$ vektorlarning komplanarlik sharti yoki M, M_1, M_2 va M_3 nuqtalarning bir tekislikda yotish sharti quyidagidan iborat ekan:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0.$$

Bu esa bir to'g'ri chiziqda yotmagan uch nuqtadan o'tuvchi tekislikning tenglamasidir.

Masala. $A(4;2;5)$, $B(0;7;2)$ va $C(0;3;7)$ nuqtalardan o'tuvchi tekislikning tenglamasini tuzing.

Yechish. Berilishiga ko'ra

$$x_1 = 4, \quad y_1 = 2, \quad z_1 = 5$$
$$x_2 = 0, \quad y_2 = 7, \quad z_2 = 2$$
$$x_3 = 0, \quad y_3 = 3, \quad z_3 = 7$$

Bu qiymatlardan foydalanib tekislikning tenglamasini tuzamiz:

$$\begin{vmatrix} x-4 & y-2 & z-5 \\ 0-4 & 7-2 & 2-5 \\ 0-4 & 3-2 & 7-5 \end{vmatrix} = 0, \qquad \begin{vmatrix} x-4 & y-2 & z-5 \\ -4 & 5 & -3 \\ -4 & 1 & 2 \end{vmatrix} = 0$$

$$(x-4)\begin{vmatrix} 5 & -3 \\ 1 & 2 \end{vmatrix} - (y-2)\begin{vmatrix} -4 & -3 \\ -4 & 2 \end{vmatrix} + (z-5)\begin{vmatrix} -4 & 5 \\ -4 & 1 \end{vmatrix} = 0$$

$$(x-4)[5\cdot 2 - 1(-3)] - (y-2)[(-4)\cdot 2 - (-3)(-4)] + (z-5)[(-4)\cdot 1 - 5(-4)] = 0$$

$$13(x-4) + 20(y-2) + 16(z-5) = 0$$

Demak, tekislik tenglamasi

$$13x - 52 + 20y - 40 + 16z - 80 = 0$$
$$13x + 20y + 16z - 172 = 0$$

ga teng.

Tekislik tenglamasiga doir masala.

Berilgan nuqtadan o′tuvchi tekisliklar dastasi tenglamasi masalasi.

Aytaylik, tekislik berilgan $M_1(x_1;\ y_1;\ z_1)$ nuqtadan o’tsin va uning tenglamasini topish talab etilsin. Izlanayotgan tekislikning umumiy tenglamasini qaraymiz:

$$Ax + By + Cz + D = 0$$

M_1 nuqta tekislikda yotgani uchun uning koordinatalari bu tenglamani qanoatlantirishi kerak:

$$Ax_1 + By_1 + Cz_1 + D = 0$$

Hosil bo′lgan bu tenglikni yuqoridagi tenglamadan ayirib, izlangan

$$A(x-x_1) + B(y-y_1) + C(z-z_1) = 0,$$

ya'ni berilgan M_1 nuqtadan o′tuvchi tekisliklar tenglamasini hosil qilamiz. Undagi koeffitsientlarga turli qiymatlar berib, M_1 nuqtadan o′tuvchi tekisliklar dastasini olamiz.

Tekislik va uning tenglamalari

1. Tekislikning umumiy tenglamasi qayerda to'liq va to'g'ri ifodalangan?
*A) Ax+By+Cz+D=0. B) Ax+By+CDz=0. C) Ax+By+(C+D)z=0.
D) Ax^{-1}+By^{-1}+Cz^{-1}+D=0. E) Ax^2+By^2+Cz^2+D=0.

2. Umumiy tenglamasi $2x-5y+4z+9=0$ bo'lgan tekislikka tegishli va koordinatalarining yig'indisi 15 bo'lgan M(x,y,5) nuqtaning abssisasini toping.
A) 4. B) −7. *C) 3. D) 0. E) −1.

3. Umumiy tenglamasi $2x-5y+4z-9=0$ bo'lgan tekislikka tegishli, OZ o'qda yotuvchi va koordinatalarining yig'indisi birga teng bo'lgan nuqtaning ordinatasini toping.
A) 4. B) −7. C) 3. D) 0. *E) −1.

4. Quyidagilardan qaysi biri Ax+By+Cz+D=0 tenglamali tekislikning ***n*** normal vektori bo'ladi ?
A) ***n***=(B,C,D). B) ***n***=(A,C,D). *C) ***n***=(A,B,C).
D) ***n*** =(A,B,D). E) ***n***=(C,A,B).

5. ***Tasdiqni yakunlang:*** Umumiy tenglamasi Ax+By+Cz+D=0 bo'lgan tekislikning ***n***=(A,B,C) normal vektori shu ··· .
A) tekislikda yotadi . B) tekislikka parallel bo'ladi .
*C) tekislikka perpendikulyar bo'ladi . D) tekisliikka og'ma bo'ladi.
E) to'g'ri javob keltirilmagan .

6. $3x+4y+7z-81=0$ tenglama bilan berilgan tekislik normalini aniqlang.
A) ***n*** =(3,4,–81). B) ***n*** =(3,–4,–81). C) ***n*** =(4,7,–81).
D) ***n*** =(–3,–4,81). *E) ***n*** =(3,4,7).

7. Quyidagi tenglamalardan qaysi biri koordinatalar boshidan o'tuvchi tekislikni ifodalaydi ?
A) Ax+By+D=0. *B) Ax+By + Cz=0. C) By+Cz+D=0.
D) Ax+By+Cz+D=0. E) Ax+Cz+D=0.

8. $x+y-z=0$ tenglamali *P* tekislik to'g'risidagi quyidagi tasdiqlardan qaysi biri o'rinli ?
*A) *P* koordinatalar boshidan o'tadi. B) *P* OXY tekisligiga parallel.
C) *P* OXZ tekisligiga parallel . D) *P* OYZ tekisligiga parallel.
E) *P* tekislik OZ koordinata o'qiga perpendikulyar.

9. Ax+By+Cz+D=0 tenglama A=D=0 holda qanday *P* tekislikni ifodalaydi ?
A) *P* OX o'qiga parallel. B) *P* OX o'qiga perpendikulyar.
*****C) *P* OX o'qi orqali o'tadi. D) *P* OY o'qiga perpendikulyar.

E) P OY o‘qiga parallel .

10.Quyidagi tenglamalardan qaysi biri OZ koordinata o‘qidan o‘tuvchi tekislikni ifodalaydi ?

A) Ax+By+Cz=0. B) Ax+Cz+D=0. *C) Ax+By=0.
D) By+Cz+D=0. E) By +D=0.

11.Quyidagi tenglamalardan qaysi biri OY koordinata o‘qiga parallel tekislikni ifodalaydi ?

A) Ax+By+Cz=0. *B) Ax+Cz+D=0. C) Ax+By+D=0.
D) By+Cz+D=0. E) Ax +D=0.

12.Quyidagi tenglamalardan qaysi biri XOZ koordinata tekisligiga parallel tekislikni ifodalaydi ?

A) Ax+By+Cz=0. B) Ax+Cz+D=0. C) Ax+By=0.
D) By+Cz+D=0. *E) By +D=0.

13.Quyidagi tenglamalardan qaysi biri YOZ koordinata tekisligini ifodalaydi ?

A) By=0. B) Cz=0. *C) Ax=0. D) By+Cz=0. E) Ax +D=0.

14.Tekislikning kesmalardagi tenglamasi qayerda to‘gr’ri yozilgan ?

A) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1.$ B) $ax+by+cz=1.$ C) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0.$
*D) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$ E) $\frac{x}{a}\cdot\frac{y}{b}\cdot\frac{z}{c}=1.$

15.***Tasdiqni yakunlang:*** Koordinata boshidan o’tmaydigan, koordinata o’qlariga parallel bo’lmagan va $Ax+By+Cz+D$=0 umumiy tenglama bilan berilgani tekislikning kesmalardagi tenglamasiga o‘tish uchun umumiy tenglama ··· soniga bo‘linadi .

A) $–C$. B) $–A$. C) $–B$. *D) $–D$. E) $-(A^2+B^2+C^2)$.

16.3x–4y+12z–24=0 tekislikning kesmalardagi tenglamasini toping.

A) $\frac{x}{8}+\frac{y}{6}+\frac{z}{12}=1.$ B) $\frac{x}{4}+\frac{y}{-6}+\frac{z}{1}=1.$ C) $\frac{x}{3}-\frac{y}{2}-\frac{z}{4}=1.$
D) $\frac{x}{8}+\frac{y}{3}-\frac{z}{4}=1.$ *E) $\frac{x}{8}+\frac{y}{-6}+\frac{z}{2}=1.$

17.Tekislikning normal tenglamasi qayerda to‘g’ri yozilgan ?

A) $x\cos^{-1}\alpha+y\cos^{-1}\beta+z\cos^{-1}\gamma-p=0.$ B) $x^{-1}\cos\alpha+y^{-1}\cos\beta+z^{-1}\cos\gamma-p=0$.
*C) $x\cos\alpha+y\cos\beta+z\cos\gamma-p=0$. D) $x^2\cos\alpha+y^2\cos\beta+z^2\cos\gamma-p=0$.
E) $x\cos\alpha-y\cos\beta-z\cos\gamma+p=0$.

18.***Tasdiqni to‘ldiring:*** Tekislikning umumiy $Ax+By+Cz+D=0$ tenglamasidan uning normal tenglamasiga o‘tish uchun bu tenglama ··· ifodaga bo‘linadi .

*A) $\pm\sqrt{A^2+B^2+C^2}$. B) $\pm\sqrt{A^2+B^2+D^2}$. C) $\pm\sqrt{B^2+C^2+D^2}$.

D) $\pm\sqrt{A^2+B^2+C^2+D^2}$. E) $\pm\sqrt{|A|+|B|+|C|+|D|}$.

19. Umumiy tenglamasi $2x+2y+z-18=0$ bo‘lgan tekislikning normal tenglamasini toping .

*A) $\frac{2}{\sqrt{5}}x+\frac{2}{\sqrt{5}}y+\frac{1}{\sqrt{5}}z-\frac{18}{\sqrt{5}}=0$. B) $\frac{2}{\sqrt{3}}x+\frac{2}{\sqrt{3}}y+\frac{1}{\sqrt{3}}z-\frac{18}{\sqrt{3}}=0$.

C) $\frac{2}{3}x+\frac{2}{3}y+\frac{1}{3}z-6=0$. D) $\frac{2}{5}x+\frac{2}{5}y+\frac{1}{5}z-\frac{18}{5}=0$.

E) $x\cdot\cos60^0+y\cdot\cos30^0+z\cdot\cos45^0-9=0$.

20.Quyidagilardan qaysi biri tekislikning normal tenglamasi bo’ladi?

A) $\frac{2}{13}x+\frac{5}{13}y+\frac{7}{13}z-5=0$. B) $\frac{4}{13}x-\frac{3}{13}y-\frac{6}{13}z-13=0$.

C) $-\frac{5}{13}x+\frac{1}{13}y+\frac{8}{13}z-9=0$. *D) $\frac{3}{13}x-\frac{4}{13}y+\frac{12}{13}z-2=0$.

E) $x\cdot\cos60^0+y\cdot\cos30^0+z\cdot\cos45^0-9=0$.

Tekislikka doir asosiy masalalar

1. Berilgan $M_0(x_0,y_0,z_0)$ nuqtadan o‘tuvchi tekisliklar dastasining tenglamasi qayerda to’g’ri ifodalangan?

A) $A(x+x_0)+B(y+y_0)+C(z+z_0)=0$. B) $Axx_0+Byy_0+Czz_0=0$.

C) $\frac{x-x_0}{A}+\frac{y-y_0}{B}+\frac{z-z_0}{C}=0$. D) $\frac{x+x_0}{A}+\frac{y+y_0}{B}+\frac{z+z_0}{C}=0$.

*E) $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$.

2. Fazoning $M(1,2,-3)$ nuqtasidan o‘tuvchi tekisliklar dastasi tenglamasini ko‘rsating.

A) $Ax+2By-3Cz=0$. *B) $A(x-1)+B(y-2)+C(z+3)=0$.

C) $A(x+1)+B(y+2)+C(z-3)=0$. D) $Ax+2By-3Cz+D=0$.

E) $A(x-1)+B(y-2)+C(z-3)+D=0$.

3. $M_1(3,2,-1)$, $M_2(0,3,1)$ va $M_3(4,5,0)$ nuqtalardan o‘tuvchi tekislik tenglamasini ko‘rsating.

A) $2x-y+z-3=0$. B) $x-2y+z+2=0$. *C) $x-y+2z+1=0$.

D) $x-y+z=0$. E) $x-2y+2z+3=0$.

4. $M_1(3,2,-1)$, $M_2(0,3,1)$ va $M_3(4,5,0)$ nuqtalardan o‘tuvchi tekislikda yotuvchi $M_0(x,-4, 7)$ nuqtaning abssissasini toping.

A) $x_0=-3$. B) $x_0=5$. C) $x_0=-1{,}5$. *D) $x_0=9$. E) $x_0=0$.

5. Berilgan $M_0(x_0,y_0,z_0)$ nuqtadan o‘tuvchi va $\boldsymbol{n}=(A,B,C)$ vektorga perpendikulyar tekislik tenglamasini ko‘rsating.

A) $A(x+x_0)+B(y+y_0)+C(z+z_0)=0$. B) $Axx_0+Byy_0+Czz_0=0$.

C) $\frac{x-x_0}{A}+\frac{y-y_0}{B}+\frac{z-z_0}{C}=0$. D) $\frac{x+x_0}{A}+\frac{y+y_0}{B}+\frac{z+z_0}{C}=0$.

*E) $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$.

6. M(1,2,3) nuqtadan o‘tuvchi va $\boldsymbol{n}=(3,2,1)$ normal vektorga ega tekislik tenglamasini yozing.

*A) $3x+2y+z-10=0$. B) $x+2y+3z-14=0$. C) $2x+3y+z-11=0$. D) $x+3y+2z-13=0$. E) $3x+y+2z-11=0$.

7. Berilgan $M_0(3,-4,0)$ nuqtadan o‘tuvchi va $\boldsymbol{n}=(1,2,-3)$ normal vektorga ega bo’lgan tekislikda yotuvchi $N(-3,8,z)$ nuqtaning aplikatasini toping.

A) $z=0$. B) $z=5$. C) $z=-1$. *D) $z=6$. E) $z=-3{,}5$.

8. $M_0(x_0,y_0,z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikkacha bo‘lgan masofani topish formulasini ko’rsating.

A) $d=|Ax_0+By_0+Cz_0+D|$. B) $d=\sqrt{|Ax_0+By_0+Cz_0+D|}$.

*C) $d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$. D) $d=\frac{|A+x_0+B+y_0+C+z_0+D|}{\sqrt{A^2+B^2+C^2}}$.

E) $d=\sqrt{\frac{Ax_0+By_0+Cz_0+D}{A^2+B^2+C^2}}$.

9. $3x+4y-2\sqrt{6}z+14=0$ tekislikdan koordinata boshigacha bo‘lgan masofani toping.

A) 14. B) 7. *C) 2. D) 1. E) $2\sqrt{6}$.

10. Ushbu $4x+3y-5z-8=0$ va $4x+3y-5z-12=0$ parallel tekisliklar orasidagi masofani toping.

A) 4 . B) 20 . C) $\sqrt{2}$. *D) $\frac{2\sqrt{2}}{5}$. E) $2\sqrt{2}$.

11. $x+y-z-1=0$ va $2x-2y-2z+1=0$ tekisliklar orasidagi burchak kosinusini toping.

A) 0. B) 1. C) $\sqrt{3}/4$. *D) 1/3. E) 3/4.

12. $x+y-18=0$ va $y+z-72=0$ tenglamalar bilan berilgan tekisliklar orasidagi burchak topilsin.

A) 30^0 . B) $\arccos\frac{\sqrt{3}}{4}$. C) 45^0 . D) $\arccos\frac{\sqrt{3}}{5}$. *E) 60^0 .

13.Normal vektorlari $\boldsymbol{n}_1=(-1, -1,0)$ va $\boldsymbol{n}_2=(0, -1, -1)$ bo‘lgan tekisliklar orasidagi burchak topilsin.

A) 45^0 . B) 30^0 . C) $\arccos\frac{\sqrt{2}}{3}$. *D) 60^0 . E) 90^0.

14. $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tekisliklarning parallellik sharti qayerda to‘gr’ri ko‘rsatilgan ?

A) $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{D_1}{D_2}$. B) $\frac{A_1}{A_2}=\frac{C_1}{C_2}=\frac{D_1}{D_2}$. *C) $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$.

D) $\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{D_1}{D_2}$. E) $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{D_1}{D_2}$.

15. $kx–2y+5z+10=0$ va $6x– (1+k)y+10z–2=0$ tekisliklar k parametrning qanday qiymatida parallel bo‘ladi ?

*A) $k=3$. B) $k= – 4$. C) $k=2$. D) $k= –5$. E) $k=\pm1$.

16.Umumiy tenglamalari $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ bilan berilgan tekisliklarning perpendikulyarlik shartini ko‘rsating.

A) $A_1A_2+B_1B_2+C_1C_2+D_1D_2=0$. B) $A_1A_2+B_1B_2+D_1D_2=0$.

*C) $A_1A_2+B_1B_2+C_1C_2=0$. D) $B_1B_2+C_1C_2+D_1D_2=0$.

E) $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$.

17. $kx–2y-5z+10=0$ va $6x–(1+k)y+10z–2=0$ tekisliklar k parametrning qanday qiymatida parallel bo‘ladi ?

A) $k=3$. B) $k= – 4$. *C) $k=6$. D) $k= –5$. E) $k=\pm1$.

18. $x–y–1=0$, $y+z=0$ va $x–z–1=0$ tekisliklarning kesishish nuqtasi koordinatalarining yig’indisini toping.

A) 0 . *B) 1 . C) −1 . D) 4 . E) −4 .

19. $2x–3y+4z–12=0$ tenglama bilan berilgan tekislikning koordinata o‘qlari bilan kesishgan nuqtalarining koordinatalari topilsin.

A) $(-1,0,0)$, $(0, -5,0)$, $(0,0,4)$. B) $(-6,0,0)$, $(0, 3,0)$, $(0,0,4)$.

C) $(5,0,0)$, $(0, -4,0)$, $(0,0,-3)$. *D) $(6,0,0)$, $(0, -4,0)$, $(0,0,3)$.

E) $(1,0,0)$, $(0, 3,0)$, $(0,0,5)$.

20. M(2, −1,1) nuqtadan o‘tib, $3x+2y-z+4=0$ tekislikka parallel bo‘lgan tekislikning koordinata o‘qlari bilan kesishish nuqtalarining koordinatalarini toping.

A) (−1,0,0), (0, −5,0), (0,0,4). B) (−6,0,0), (0, 3,0), (0,0,4).
C) (5,0,0), (0, −4,0), (0,0,−3). D) (6,0,0), (0, −4,0), (0,0,3).
*E) (1,0,0), (0, 3/2,0), (0,0,−3).

21. Kesmalardagi tenglamasi

$$\frac{x}{5}+\frac{y}{-2}+\frac{z}{9}=1$$

bo‘lgan P tekislik va koordinata tekisliklari bilan chegaralangan piramida hajmini toping.

A) 90 B) 60. C) 45. D) 30. *E) 15.

22. Kesmalardagi tenglamasi

$$\frac{x}{a}+\frac{y}{4}+\frac{z}{-9}=1$$

bo‘lgan P tekislik va koordinata tekisliklari bilan chegaralangan piramida hajmi 72 kub birlikka teng . Noma’lum a parametr qiymatini toping.

A) 2. B) −2. C) ±2. *D) ±12. E) 6.

23. M(3, −2,1) nuqtadan o‘tib, $\boldsymbol{a}_1=(0,1,-2)$ va $\boldsymbol{a}_2=(5,0,2)$ vektorlarga parallel bo’lgan tekislik tenglamasini toping.

A) $3x+2y-z-4=0$. B) $3x-5y+2z-21=0$. *C) $2x-10y-5z-21=0$.
D) $2x+5y-z+5=0$. E) $5x-12y-3z-36=0$.

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