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Diskret matematika va matematik mantiq fanidan misollar va masalalar to`plami

Uslubiy qo`llanma

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M u n d a r i j a

I bob. To`plamlar va binar munosabatlar.

1.1.To`plamlar ustida amallar.

$\in$ orqali *tegishlilik munosabat*ni belgilaymiz, ya`ni $a \in A$ belgi, $a$ elementni $A$ to`plamda yotishini bildiradi. Agar b element A to`plamga tegishli bo`lmasa, u holda bu $b \notin A$ deb yoziladi. Ikkita A va B to`plamlar *teng hisoblanadi*, agarda ular bir xil elementlardan tuzilgan bo`lsa. Agar A va B to`plamlar teng bo`lsa, u holda $A = B$ deb yozamiz, aks holda $A \neq B$ deb yozamiz. $\subseteq$ orqali to`plamni yotishi yoki *qismi munosabat*ni belgilaymiz, ya`ni $A \subseteq B$ belgi, A to`plamning har bir elementi $B$ to`plamning ham elementi ekanligini bildiradi. Bu holda A to`plam $B$ to`plamning *qism to`plami*, $B$ esa $A$ to`plam *ustidagi to`plam* deyiladi. Agar $A \subseteq B$ bo`lib, $A \neq B$ bo`lsa, u holda $A$ to`plam B to`plamning *xos qism to`plami* deyiladi, bu holda $A \subset B$ deb yozamiz. Hech bir elementi bo`lmagan to`plam *bo`sh to`plam* deyiladi va $\varnothing$ orqai belgilanadi.Berilgan A to`plamning barcha qism to`plamlari to`plami $P(A)$ orqali belgilanadi.

A va B to`plamlarning *birlashmasi* deb

$$A \cup B = \{x: \ x \in A \ \text{ yoki } \ x \in B\}$$

to`plamga aytiladi.

A_i (i $\in$ I) *to`plamlar oilasini birlashmasi* deb

$$\bigcup_{i \in I} A_i = \{x: \text{ shunday } i_0 \in I \ \text{ mavjudki, } x \in A_{i_0}\}$$

to`plamga aytiladi.

A va B to`plamlarning *kesishmasi* deb

$$A \cap B = \{x: \ x \in A \ \text{ va } \ x \in B\}$$

to`plamga aytiladi.

A_i (i $\in$ I) *to`plamlar oilasini kesishmasi* deb

$$\bigcap_{i \in I} A_i = \{x: \ x \in A_i \ \text{ barcha } i \in I \ \text{ lar uchun}\}$$

to`plamga aytiladi.

A va B to`plamlarning *ayirmasi* deb

$$A \setminus B = \{x: \ x \in A \ \text{ va } \ x \notin B\}$$

to`plamga aytiladi.

Biz ushbu mavzuda uchraydigan barcha to`plamlarni biror $U$ *universal to`plam*ning qism to`plami deb hisoblaymiz. $U \setminus A$ ayirma $A$ to`plamning to`ldiruvchisi deyiladi va $\bar{A}$ orqali belgilanadi.

A va B to`plamlarning *simmetrik ayirmasi* deb

$$A \Delta B = (A \setminus B) \cup (B \setminus A)$$

to`plamga aytiladi.

1.1. Quyidagilarni isbotlang:

a) $A \subseteq A$ (refleksivlik);

b) agar $A \subseteq B$ va $B \subseteq C$ bo`lsa, u holda $A \subseteq C$ bo`ladi (tranzitivlik);

c) $A \bigcap B \subseteq A \subseteq A \bigcup B$;

d) $A \bigcap B \subseteq B \subseteq A \bigcup B$;

e) $A \setminus B \subseteq A$.

1.2. Agar A ushbu

$$x^2 - 7x + 6 = 0$$

tenglamaning ildizlari to`plami va $B = \{1, 6\}$ bo`lsa, u holda $A = B$ ekanligini isbotlang.

1.3. $\varnothing \neq \{\varnothing\}$ ekanligini isbotlang.

Yechish. $\{\varnothing\}$ to`plamda bitta element $\varnothing$ bor, $\varnothing$ da esa biita ham element yo`q. Demak, ular teng emas.

1.4. $\{\{1,2\},\{2,3\}\}\neq\{1,2,3\}$ ekanligini isbotlang.

1.5. Ixtiyoriy A to`plam uchun quyidagilarni isbotlang:

a) $\varnothing\subseteq A\subseteq U$;

b) agar $A\subseteq\varnothing$ bo`lsa, u holda $A=\varnothing$; agar $U\subseteq A$ bo`lsa, u holda $U=A$;

c) $A\bigcup\varnothing=A,\quad A\bigcap\varnothing=\varnothing,\quad A\bigcup U=U,\quad A\bigcap U=A.$

1.6. Bitta ham elementi bo`lmagan faqat bitta to`plam mavjudligini isbotlang.

1.7. Quyidagi shartlarni qanoatlantiradigan A, B, C to`plamlar mavjudmi?

$A\bigcap B\neq\varnothing,\quad A\bigcap C=\varnothing,\quad \left(A\bigcap B\right)\setminus C=\varnothing.$

Yechish. Yo`q, bunday to`plamlar mavjud emas. $x\in A\cap B$ bo`lsin, u holda $x\notin C.$ Shunday qilib, $x\in(A\cap B)\setminus C.$

1.8 $\psi(x)=f(x)\cdot h(x)$ ko`phadning barcha ildizlari to`plami, $f(x)$ va $h(x)$ ko`phadlarning ildizlari to`plamlari birlashmasiga teng ekanligini isbotlang.

1.9. Haqiqiy koeffisientli $f(x)$ va $h(x)$ ko`phadlarning haqiqiy ildizlari to`plamlari kesishmasi $\psi(x)=f^2(x)+h^2(x)$ ko`phadning barcha haqiqiy ildizlari to`plami bilan ustma-ust tushishini isbotlang.

1.10. Quyidagini isbotlang.

$$A\subseteq B\Leftrightarrow A\bigcup B=B\Leftrightarrow A\bigcap B=A\Leftrightarrow A\setminus B=\varnothing\Leftrightarrow \bar{A}\bigcup B=U.$$

1.11. Quyidagi ayniyatlarni isbotlang:

a) $A\bigcup A=A\bigcap A=A$;

b) $A\bigcap B=B\bigcap A$;

c) $A\bigcup B=B\bigcup A$;

d) $A\bigcap\left(B\bigcap C\right)=\left(A\bigcap B\right)\bigcap C$;

e) $A\bigcup\left(B\bigcup C\right)=\left(A\bigcup B\right)\bigcup C$;

f) $A\bigcap\left(B\bigcup C\right)=\left(A\bigcap B\right)\bigcup\left(A\bigcap C\right)$;

g) $A\bigcup\left(B\bigcap C\right)=\left(A\bigcup B\right)\bigcap\left(A\bigcup C\right)$;

h) $\left(A\bigcap B\right)\bigcup\left(C\bigcap D\right)=\left(A\bigcup C\right)\bigcap\left(B\bigcup C\right)\bigcap\left(A\bigcup D\right)\bigcap\left(B\bigcup D\right).$

Yechish. f) $x\in A\cap(B\cup C)$ bo`lsin. U holda $x\in A$ va $x\in B\cup C$ . Agar $x\in B$ bo`lsa, u holda $x\in A\cap B$ va demak, $x\in(A\cap B)\cup(A\cap C).$ Agar $x\in C$ bo`lsa, u holda $x\in A\cap C.$ Demak, $x\in(A\bigcap B)\cup(A\cap C)$. Shunday qilib, $A\bigcap\left(B\bigcup C\right)\subseteq\left(A\bigcap B\right)\bigcup\left(A\bigcap C\right).$

$x\in(A\bigcap B)\cup(A\cap C)$ bo`lsin. Bundan agar $x\in A\cap B$ bo`lsa, u holda $x\in A$ va $x\in B$. Bu yerdan, $x\in A$ va $x\in B\cup C$ kelib chiqadi va demak, $x\in A\cap(B\cup C).$ Agar $x\in A\cap C$ bo`lsa, u holda $x\in A$ va $x\in C$ bo`ladi. Bundan esa $x\in A$ va $x\in B\cup C$ bo`ladi, demak, $x\in A\cap(B\cup C).$ Shunday qilib, $(A\bigcap B)\cup(A\cap C)\subseteq A\cap(B\cup C).$

1.12. Quyidagi ayniyatlarni isbotlang:

a) $\overline{\left(A\bigcap B\right)}=\bar{A}\bigcup\bar{B}$;

b) $\overline{\left(A\cup B\right)}=\bar{A}\cap\bar{B}$;

c) $A\setminus\left(B\bigcup C\right)=\left(A\setminus B\right)\bigcap\left(A\setminus C\right)$;

d) $A\setminus\left(B\bigcap C\right)=\left(A\setminus B\right)\bigcup\left(A\setminus C\right)$;

e) $A\setminus(A\setminus B)=\left(A\bigcap B\right)$;

f) $A\setminus B=A\setminus\left(A\bigcap B\right)$;

g) $A\bigcap(B\setminus C)=(A\bigcap B)\setminus(A\bigcap C)=(A\bigcap B)\setminus C;$

h) $(A\setminus B)\setminus C=(A\setminus C)\setminus(B\setminus C);$

i) $(A\bigcup B)=A\bigcup(B\setminus A);$

j) $\overline{\overline{A}}=A;$

k) $A\cup\overline{A}=U;$

l) $A\cap\overline{A}=\varnothing;$

m) $(A\cap B)\cup(A\cap\overline{B})=(A\cup B)\cap(A\cup\overline{B})=A;$

n) $(\overline{A}\cup B)\cap A=A\cap B;$

o) $A\cap(B\setminus A)=\varnothing;$

p) $(A\cup B)\setminus C=(A\setminus C)\cup(B\setminus C);$

q) $A\setminus(B\setminus C)=(A\setminus B)\cup(A\cap C);$

r) $A\setminus(B\cup C)=(A\setminus B)\setminus C.$

Yechish. a) $x\in\overline{(A\cap B)}$ bo`lsin. Bu $x\in U$ va $x\notin A\cap B$ ekanligini bildiradi.Bundan, $x\notin A$ yoki $x\notin B$ kelib chiqadi. Agar $x\notin A$ bo`lsa, u holda $x\in\overline{A}$ bo`ladi, demak, $x\in\overline{A}\cup\overline{B}.$ Agar $x\notin B$ bo`lsa, holda $x\in\overline{B}$ bo`ladi, demak, $x\in\overline{A}\cup\overline{B}.$ Shunday qilib, $\overline{(A\bigcap B)}\subseteq\overline{A}\bigcup\overline{B}.$

$x\in\overline{A}\cup\overline{B}$ bo`lsin. U holda $x\in\overline{A}$ yoki $x\in\overline{B}$. Agar $x\in\overline{A}$ bo`lsa, u holda $x\notin A$, demak, $x\notin A\cap B.$ Bundan esa $x\in\overline{(A\cap B)}$ kelib chiqadi. Agar $x\in\overline{B}$ bo`lsa, u holda $x\notin B$ va demak, $x\notin A\cap B$, bundan esa $x\in\overline{(A\cap B)}$.Shunday qilib, $\overline{A}\cup\overline{B}\subseteq\overline{(A\cap B)}.$

1.13. Quyidagilarni isbotlang:

a) $A\bigcup B\subseteq C\Leftrightarrow A\subseteq C$ va $B\subseteq C;$

b) $A\subseteq B\bigcap C\Leftrightarrow A\subseteq B$ va $A\subseteq C;$

c) $A\bigcap B\subseteq C\Leftrightarrow A\subseteq\overline{B}\bigcup C;$

d) $A\subseteq B\bigcup C\Leftrightarrow A\bigcap\overline{B}\subseteq C;$

e) $(A\setminus B)\bigcup B=A\Leftrightarrow B\subseteq A;$

f) $(A\bigcap B)\bigcup C=A\bigcap(B\bigcup C)\Leftrightarrow C\subseteq A;$

g) $A\subseteq B\Rightarrow A\bigcup C\subseteq B\bigcup C;$

h) $A\subseteq B\Rightarrow A\bigcap C\subseteq B\bigcap C;$

i) $A\subseteq B\Rightarrow(A\setminus C)\subseteq(B\setminus C);$

j) $A\subseteq B\Rightarrow(C\setminus B)\subseteq(C\setminus A);$

k) $A\subseteq B\Rightarrow\overline{B}\subseteq\overline{A};$

l) $A\bigcup B=A\bigcup B\Rightarrow A=B;$

m) $A=-B\Leftrightarrow A\bigcap B=\varnothing$ va $A\bigcup B=U.$

Yechish. c) $A\cap B\subseteq C$ va $x\in A$ bo`lsin. Ikkita holni qaraymiz: $x\in B$ yoki $x\in\overline{B}$. Agar $x\in B$ bo`lsa, u holda $x\in A\cap B\subseteq C,$ ya`ni $x\in\overline{B}\cup C.$

Agar $x\in\overline{B}$ bo`lsa, u holda $x\in\overline{B}\cup C.$

$A\subseteq\overline{B}\cup C$ va $x\in A\cap B$ bo`lsin. U holda $x\in A$ va $x\in B$ va demak, $x\in C$.

e) $(A\setminus B)\cup B=A$ va $x\in B$ bo`lsin. U holda tushunarliki, $x\in A$. $B\subseteq A$ bo`lsin. U holda $(A\setminus B)\cup B=(A\cap\overline{B})\cup B=(A\cup B)\cap(\overline{B}\cup B)=A.$

f) $(A\bigcap B)\bigcup C=A\bigcap(B\bigcup C)$ bo`lsin. U holda $C\subseteq A\cap(B\cup C),$ bundan esa $C\subseteq A.$

1.14. Quyidagi ayniyatlarni isbotlang:

a) $A\Delta B=B\Delta A;$

b) $A\Delta(B\Delta C)=(A\Delta B)\Delta C$;

c) $A\cap(B\Delta C)=(A\cap B)\Delta(A\cap C)$;

d) $A\Delta(A\Delta B)=B$;

e) $A\cup B=A\Delta B\Delta(A\cap B)$;

f) $A\setminus B=A\Delta(A\cap B)$;

g) $A\Delta\varnothing=A$;

h) $A\Delta A=\varnothing$;

i) $A\Delta U=\overline{A}$;

j) $A\cup B=(A\Delta B)\cup(A\cap B)$.

Yechish. c) $x\in A\cap(B\Delta C)$ bo`lsin. U holda $x\in A$ va $x\in B\Delta C$ bo`ladi. Bu yerdan, agar $x\in B$ bo`lsa, u holda $x\notin C$, demak, $x\in A\cap B,$ ammo $x\notin A\cap C.$ Agar $x\in C$ bo`lsa, u holda $x\notin B.$ Demak, $x\in A\cap C,$ ammo $x\notin A\cap B.$ Shunday qilib, $x\in(A\cap B)\Delta(A\cap C).$ Demak, $A\cap(B\Delta C)\subseteq(A\cap B)\Delta(A\cap C).$

$x\in(A\cap B)\Delta(A\cap C)$ bo`lsin. Agar $x\in A\cap B$ va $x\notin A\cap C,$ u holda $x\in A,\ x\in B,\ x\notin C$ bo`ladi.Demak, $x\in A\cap(B\Delta C).$ Agar $x\in A\cap C$ va $x\notin A\cap B$, u holda $x\in A,\ x\in C,\ x\notin B$. Demak, $x\in A\cap(B\Delta C).$ Shunday qilib, $(A\cap B)\Delta(A\cap C)\subseteq A\cap(B\Delta C).$

1.15. Isbotlang:

a) $(A_1\cup...\cup A_n)\Delta(B_1\cup...\cup B_n)\subseteq(A_1\Delta B_1)\cup...\cup(A_n\Delta B_n)$;

b) $(A_1\cap...\cap A_n)\Delta(B_1\cap...\cap B_n)\subseteq(A_1\Delta B_1)\cup...\cup(A_n\Delta B_n)$.

Yechish. a) $x\in(A_1\cup...\cup A_n)\Delta(B_1\cup...\cup B_n)$ bo`lsin. Agar shunday $i$ $(1\le i\le n)$ mavjud bo`lsaki, $x\in A_i$ bo`lsa, u holda barcha $j=1,2,...,n$ lar uchun $x\notin B_j$ bo`ladi. U holda $x\in A_i\Delta B_i$ va demak, $x\in(A_1\Delta B_1)\cup...\cup(A_n\Delta B_n).$ Agar shunday i $(1\le i\le n)$ mavjud bo`lsaki, $x\in B_i$ bo`lsa, u holda barcha $j=1,2,...,n$ lar uchun $x\notin A_j$ bo`ladi. U holda $x\in A_i\Delta B_i$ va demak, $x\in(A_1\Delta B_1)\cup...\cup(A_n\Delta B_n).$

1.16. Isbotlang.

a) $A\Delta B=\varnothing\Leftrightarrow A=B$;

b) $A\cap B=\varnothing\Rightarrow A\cup B=A\Delta B$;

c) $A\Delta B=C\Leftrightarrow B\Delta C=A\Leftrightarrow C\Delta A=B.$

Yechish. a) $A\Delta B=C$ bo`lsin.U holda $B\Delta C=B\Delta(A\Delta B)=B\Delta(B\Delta A)=A$ (14 masala a), d) ga qarang).

1.17. $\cup,\ \cap,\ \setminus$ amallarni quyidagi amallar orqali ifodalang:

a) $\Delta,\ \cap$;

b) $\Delta,\ \cup$;

c) $\setminus,\ \Delta$.

Yechish. a) $A\cup B=A\Delta B\Delta(A\cap B),\quad A\setminus B=A\Delta(A\cap B)$;

1.18. Quyidagi amallarni birini ikkinchisi orqali ifodalab bo`lmasligini isbotlang:

a) $\setminus$ ni $\cap$ va $\cup$ orqali;

b) $\cup$ ni $\cap$ va $\setminus$ orqali.

Yechish. a) A va B lardan $\cap$ va $\cup$ amallari yordamida faqat $A,\ B,\ A\cap B$ va $A\cup B$ to`plamlarnigina hosil qilish mumkin. Bu to`plamlarni barchasi $A\setminus B$ dan farq qiladi, masalan, $A=B\neq\varnothing$ bo`lgan holda.

1.19. To`plamlar to`plami halqa tashkil etadi, bunda Δ qo`shish amalini rolini, $\cap$ esa ko`paytirish amali rolini bajaradi. Bu halqada ayirish nima bo`ladi?

Yechish. Δ va $\cap$ amallarning zaruriy xossalari 11 b, d, 14 a, b, c, g, h, 16 c masalalarda keltirilgan. Qaralayotgan to`plamlar halqada ayirish bu 16 c masalaga ko`ra Δ bo`ladi.

1.20. Quyidagi to`plamlarning barcha qism to`plamlarini toping: $\varnothing,\ \{\varnothing\},\ \{x\},\ \{1,2\}$.

1.21. a) n ta elementli to`plamning $2^n$ ta qism to`plami bor ekanligini isbotlang.

b) n ta elementli to`plam nechta $k$ elementli qism to`plamlarga ega ($k \le n$)?

Yechish. a) $A=\{a_1,a_2,...,a_n\}$ va $B\subseteq A$ bo`lsin. Har bir $a_i$ element uchun ikkita holat bo`lishi mumkin: $a_i\in B$ yoki $a_i\notin B.$ Bundan A to`plamning barcha qism to`plamlari soni $\underbrace{2\cdot 2\cdot ...\cdot 2}_{n}=2^n$ teng ekanligi kelib chiqadi.

1.22. Quyidagilarni isbotlang:

a) $P(A\cap B)=P(A)\cap P(B);$

b) $P\left(\bigcap\limits_{i\in I}A_i\right)=\bigcap\limits_{i\in I}P(A_i);$

c) $P(A\cup B)=\{A_1\cup B_1: \mathrm{A}_1\in P(A),\ B_1\in P(B)\};$

d) $P\left(\bigcup\limits_{i\in I}A_i\right)=\left\{\bigcup\limits_{i\in I}B_i: B_i\in P(A_i)\right\}.$

Yechish. a) $C\in P(A\cap B)$ bo`lsin, ya`ni $C\subseteq A\cap B.$ U holda $C\subseteq A$ va $C\subseteq B,$ demak, $C\subseteq P(A)$ va $C\subseteq P(B)$. Shunday qilib, $P(A\cap B)\subseteq P(A)\cap P(B).$

$C\in P(A)\cap P(B)$ bo`lsin, ya`ni $C\in P(A)$ va $C\in P(B)$ bo`lsin. Bundan $C\subseteq A$ va $C\subseteq B$, demak, $C\subseteq A\cap B.$ Shunday qilib, $C\in P(A\cap B).$ Demak, $P(A)\cap P(B)\subseteq P(A\cap B).$

c) $C\in P(A\cup B)$ bo`lsin. U holda $C\subseteq A\cup B.$ $A_1=C\cap A$ va $B_1=C\cap B$ deb olamiz. U holda $C=A_1\cup B_1$ va $A_1\subseteq A$ va $B_1\subseteq B.$

Agar $A_1\in P(A)$ va $B_1\subseteq P(B)$ bo`lsa, u holda $A_1\subseteq A$ va $B_1\subseteq B.$ Bundan $A_1\cup B_1\subseteq A\cup B,$ ya`ni $A_1\cup B_1\in P(A\cup B).$

1.23. Ixtiyoriy a,b,c,d lar uchun quyidagi munosabatlar o`rinli ekanligini isbotlang:
$\{\{a\},\{a,b\}\}=\{\{c\},\{c,d\}\}\Leftrightarrow a=c,\ b=d.$

Yechish. Agar $\{\{a\},\{a,b\}\}=\{\{c\},\{c,d\}\}$ bo`lsa, u holda ikkinchi to`plam $\{a\}$ elementni o`z ichiga olishi lozim, ya`ni $\{a\}=\{c\}$ yoki $\{a\}=\{c,d\}$ bo`lishi lozim. Ikkala holda ham $a=c$. Endi $\{a,b\}=\{c,d\}$ tenglikdan $b=d$ kelib chiqishini isbotlash qoldi.Agar $a=b$ bo`lsa, u holda $a=d$ va demak, $b=d.$ Agar $a\neq b$ bo`lsa, u holda $a\neq d$ va demak, $b=d$. Teskarisi ravshan.

1.24. Barcha A,B,C lar uchun quyidagi tasdiqlardan qaysilari to`g`ri?

a) Agar $A\in B$ va $B\in C$ bo`lsa, u holda $A\in C;$

b) Agar $A\subseteq B$ va $B\in C$ bo`lsa, u holda $A\in C;$

c) Agar $A\cap B\subseteq \overline{C}$ va $A\cup C\subseteq B$ bo`lsa, u holda $A\cap C=\varnothing;$

d) Agar $A\neq B$ va $B\neq C$ bo`lsa, u holda $A\neq C;$

e) Agar $A\subseteq\overline{(B\cup C)}$ va $B\subseteq\overline{(A\cup C)}$ bo`lsa, u holda $B=\varnothing.$

Yechish. a) Noto`g`ri, masalan, $A=\varnothing,\ B=\{\varnothing\},\ C=\{\{\varnothing\}\}.$

b) Noto`g`ri, masalan, bu yerda ham a) dagi misolni keltirish mumkin.

c) To`g`ri. Teskarisini faraz qilish orqali isbotlaymiz. $x\in A\cap C$ bo`lsin, u holda shunday qilib, $A\cup C\subseteq B,$ u holda $x\in B.$ Ammo $x\in A\cap B$, demak, $x\in\overline{C}.$ Bu esa $x\in C$ ga zid. Demak, farazimiz noto`g`ri.

d) Noto`g`ri, masalan, $A=C\neq B$ deb olamiz.

e) Noto`g`ri. Masalan, uchta o`zaro kesishmaydigan, bo`sh bo`lmagan to`plamlar olamiz.

1.25. Ixtiyoriy $A_1,A_2,...,A_n$ lar uchun agar $A_1\subseteq A_2\subseteq...\subseteq A_n\subseteq A_1$ bo`lsa, u holda $A_1=A_2=...=A_n$ ekanligini isbotlang.

1.26. Har bir n musbat son uchun shunday n elementli A_n to`plam ko`rsatingki, agar $x, y \in A_n$ bo`lsa, u holda $x \in y$, yoki $y \in x$, yoki $x = y$.

Yechish. Masalan, $A_1 = \{\varnothing\}$, $A_{n+1} = A_n \cup \{A_n\}$

1.27. Tenglamalar sistemasini yeching:

$$\begin{cases} A \cap X = B, \\ A \cup X = C. \end{cases}$$

Bu yerda A, B va C – berilgan to`plamlar bo`lib, $B \subseteq A \subseteq C$.

Yechish. $X = (C \setminus A) \cup B$. Haqiqatan ham, $B \subseteq X \subseteq \overline{A} \cup B$ (birinchi tenglamadan 13 b), c) masalaga ko`ra) va $C \cap \overline{A} \subseteq X \subseteq C$ ( ikkinchi tenglamadan 13 a), d) masalaga ko`ra). Bu yerdan,

$$B \cup (C \cap \overline{A}) \subseteq X \subseteq (\overline{A} \cup B) \cap C = (\overline{A} \cap C) \cup (B \cap C) = (\overline{A} \cap C) \cup B.$$

Oson tekshirish mumkinki, $X = (C \setminus A) \cup B$ berilgan sistemani qanoatlantiradi.

1.28. Tenglamalar sistemasini yeching:

$$\begin{cases} A \setminus X = B, \\ X \setminus A = C. \end{cases}$$

Bu yerda A, B va C – berilgan to`plamlar bo`lib, $B \subseteq A$, $A \cap C = \varnothing$.

1.29. $\{A_i\}_{i \in I}$ va $\{B_i\}_{i \in I}$ to`plamlar sistemalari berilgan bo`lsin, bu yerda I – biror to`plam. Quyidagi tenglamalar sistemalarini yeching:

a) $A_i \cap X = B_i$, $i \in I$;

b) $A_i \cup X = B_i$, $i \in I$.

Qanday A_i va B_i larda bu sistemalar yechimga ega.

Yechish. a) Sistema yechimga ega faqat va faqat qachonki, ixtiyoriy $i, j \in I$ lar uchun $B_i \subseteq A_i$ va $B_i \subseteq \overline{A}_j \cup B_j$ bo`lsa. Bu shartlar bajarilsa, u holda $X$ $\bigcup_{i \in I} B_i \subseteq X \subseteq \bigcap_{i \in I} (\overline{A}_i \cup B_i)$ shartni qanoatlantiruvchi ixtiyoriy to`plam bo`ladi
(13 a), b), c) masalaga qarang).

1.30. Tenglamalar sistemasini yeching:

$$\begin{cases} A \setminus X = B, \\ A \cup X = C. \end{cases}$$

Bu yerda A, B va C – berilgan to`plamlar bo`lib, $B \subseteq A \subseteq C$.

1.31. Quyidagilarni isbotlang:

a) $A = B \Leftrightarrow (A \setminus B) \cup (B \setminus A) = \varnothing$;

b) O`ng tomonida $\varnothing$ to`plam turgan X to`plamga nisbatan ixtiyoriy tenglama

$$(A \cap X) \cup (B \cap \overline{X}) = \varnothing$$

tenglamaga teng kuchli, bu yerda A va B – berilgan to`plamlar bo`lib, yozuvida X belgi qatnashmaydi;

c) $$\begin{cases} A \cap X = \varnothing, \\ B \cap \overline{X} = \varnothing, \end{cases}$$

tenglamalar sistemasi yechimga ega bo`ladi, faqat va faqat qachonki $B \subseteq \overline{A}$ bo`lsa. Ushbu shartda sistemaning yechimi X quyidagi shartni qanoatlantiradigan ixtiyoriy to`plam bo`ladi:

$$B \subseteq X \subseteq \overline{A}.$$

d) Bitta no`malumli, tenglamalar sistemasini yechish usulini yozing.

Ko`rsatma. b) 11 va 12 masalalardan foydalaning.

d) a) banddan foydalanib, sistemani har bir tenglamasini o`ng tomonida $\varnothing$ to`plam turgan tenglamalar bilan almashtiring. Hosil qilingan $A_1=\varnothing,...,A_n=\varnothing$ sistemani bitta $A_1\cup A_2\cup...\cup A_n=\varnothing$ tenglama bilan almashtiring.

So`ngra b) banddan foydalanib, hosil bo`lgan tenglamani $(A\cap X)\cup(B\cap\overline{X})=\varnothing$ ko`rinishdagi tenglamaga olib keling. Natijada hosil bo`lgan tenglamani

$$\begin{cases} A\cap X=\varnothing, \\ B\cap\overline{X}=\varnothing, \end{cases}$$

tenglamalar sistemasi bilan almashtiring.

Nihoyat, c) banddan foydalanib, yechimni mavjudlik shartini yozing va yechimni toping.

1.32. 1.31. masaladagi usuldan foydalanib, quyidagi sistemalarni yeching:

a) $\begin{cases} A\cup X=B\cap X, \\ A\cap X=C\cup X; \end{cases}$

b) $\begin{cases} A\setminus X=X\setminus B, \\ X\setminus A=C\setminus X; \end{cases}$

c) $\begin{cases} A\cap X=B\setminus X, \\ C\cup X=X\setminus A. \end{cases}$

Qanday A,B va C larda bu sistemalar yechimga ega bo`ladi?

Javoblar:

a) Agar $C\subseteq A\subseteq B$ bo`lsa, u holda $X=A$.

b) Agar $C\subseteq A\subseteq B$ bo`lsa, u holda $X=A$.

c) Agar $B\cup C\subseteq\overline{A}$ bo`lsa, u holda $B\cup C\subseteq X\subseteq\overline{A}$.

1.33. Har qanday to`plam quyidagi xossalarga ega ekanligini isbotlang:

a) O`zining barcha qism to`plamlari birlashmasiga teng;

b) O`zining barcha chekli qism to`plamlari birlashmasiga teng;

c) o`zining barcha bir elementli qism to`plamlari birlashmasiga teng.

1.2.Munosabat va funksiya

$A_1,A_2,...,A_n$ to`pamlarning to`g`ri (dekart) ko`paytmasi deb $A_1\times A_2\times...\times A_n=\{(a_1,a_2,...,a_n):a_1\in A_1,a_2\in A_2,...,a_n\in A_n\}$ to`plamga aytiladi. Agar $A_1=A_2=...=A_n=A$ bo`lsa, u holda $A_1\times A_2\times...\times A_n$ to`plam $A$ to`plamni to`g`ri darajasi deyiladi va A^n orqali belgilanadi.

A va B to`plamlar elementlari orasida binar munosabat deb $A\times B$ to`plamning ixtiyoriy R qism to`plamiga aytiladi. Agar $A=B$ bo`lsa, u holda R munosabat A to`plamda binar munosabat deyiladi. $(x,y)\in R$ o`rniga ko`p hollarda xRy deb yoziladi.

R binar munosabatning aniqlanish sohasi deb $\delta_R=\{x:$ shunday y mavjudki, $(x,y)\in R\}$ to`plamga aytiladi.

R binar munosabatning qiymatlar sohasi deb $\rho_R=\{x:$ shunday y mavjudki, $(y,x)\in R\}$ to`plamga aytiladi.

Binar munosabatlar uchun odatdagidek nazariy-to`plamiy amallar birlashma, kesishma va h.k. amallar aniqlangan.

A va B to`plamlar elementlari orasidagi $R$ binar munosabatning to`ldiruvchisi deb

$$\overline{R}=(A\times B)\setminus R$$

to`plamga aytiladi.

R binar munosabatga teskari munosabat deb quyidagi to`plamga aytiladi:

$$R^{-1}=\{(x,y):(y,x)\in R\}.$$

R binar munosabatga nisbatan X to`plamning obrazi deb
$R(X)=\{y:$ shunday $x\in X$ mavjudki, $(x,y)\in R\}$ to`plamga aytiladi.
R binar munosabatga nisbatan X to`plamning proobrazi deb $R^{-1}(X)$ to`plamga aytiladi.
$R_1\subseteq A\times B$ va $R_2\subseteq B\times C$ binar munosabatlarning ko`paytmasi deb
$R_1\circ R_2=\{(x,y):$ shunday z mavjudki, $(x,z)\in R_1$ va $(z,y)\in R_2\}$
munosabatga aytiladi.
f munosabat A dan B ga (A dan B da) funksiya deyiladi, agarda $\delta_f=A,\ \rho_f\subseteq B$ (mos ravishda $\rho_f=B$) va barcha x,y_1,y_2 lar uchun $(x,y_1)\in f$ va $(x,y_2)\in f$ ekanligidan $y_1=y_2$ kelib chiqsa. A dan B ga f funksiya $f:A\to B$ orqali belgilanadi. Agar $f-$ funksiya bo`lsa, $(x,y)\in f$ o`rniga $y=f(x)$ deb yozamiz va y ni x argumenti qiymatidagi f funksiyani qiymati deyiladi. Ixtiyoriy A to`plam uchun $i_A:A\to A$ ni quyidagicha aniqlaymiz:

$$i_A(x)=x.$$

f funksiyani $1-1$-funksiya deymiz, agarda barcha x_1,x_2,y lar uchun $y=f(x_1)$ va $y=f(x_2)$ ekanligidan $x_1=x_2$ kelib chiqsa.
$f:A\to B$ funksiya A va B orasida bir qiymatli munosabatni ifodalaydi deymiz, agarda $\delta_f=A,\ \rho_f=B$ va $1-1$-funksiya bo`lsa. O`zaro bir qiymatli $f:A\to A$ moslik A to`plamni o`rniga qo`yish deyiladi.
A dan B ga barcha funksiyalar to`plami B^A orqali belgilanadi.
$A_i\ (i\in I)$ to`plamlar oilasining dekart ko`paytmasi deb

$$\prod_{i\in I}A_i=\{f:\quad f:I\to\bigcup_{i\in I}A_i \text{ va barcha } i\in I \text{ lar uchun } f(i)\in A_i\}$$

to`plamga aytiladi.
A to`plamda $n$-o`rinli munosabat deb A^n to`plamning ixtiyoriy qism to`plamiga aytiladi.
$f:A^n\to B$ funksiya A dan B ga n-o`rinli funksiya deymiz va $y=f((x_1,...,x_n))$ o`rniga $y=f(x_1,...,x_n)$ deb yozamiz hamda y ni $x_1,...,x_n$ argumentlarning qiymatlarida f funksiyani qiymati deb ataymiz.

1.34. Quyidagi munosabatlarni qanoatlantiruvchi $A,\ B$ va C to`plamlar mavjudligini isbotlang:
a) $A\times B\neq B\times A;$
b) $A\times(B\times C)=(A\times B)\times C.$

1.35. Quyidagi to`plamlarning geometrik interpretatsiyasini toping:
a) $[a,b]\times[c,d],$ bu yerda $[a,b],\ [c,d]-D$ haqiqiy to`g`ri chiziqning kesmalari;
b) $[a,b]^2;$
c) $[a,b]^3;$
d) $D^n.$

1.36. Agar $A,\ B,\ C$ va D lar bo`sh bo`lmagan to`plamlar bo`lsa, u holda quyidagilarni isbotlang:
a) $A\subseteq B$ va $C\subseteq D\ \Leftrightarrow\ A\times C\subseteq B\times D;$
b) $A=B$ va $C=D\ \Leftrightarrow\ A\times C=B\times D.$

1.37. Quyidagilarni isbotlang:
a) $(A\cap B)\times(C\cap D)=(A\times C)\cap(B\times D);$
b) $\bigcap_{t\in T}A_t\times\bigcap_{t\in T}B_t=\bigcap_{t\in T}(A_t\times B_t).$

1.38. Quyidagi munosabatni isbotlang:

$(A\times B)\cup(C\times D)\subseteq(A\cup C)\times(B\cup D).$

Qanday A, B, C va D larda quyidagi tenglik o`rinli bo`ladi?

Yechish. $x\in(A\times B)\cup(C\times D)$ bo`lsin. U holda $x=(y,z)$ va $y\in A$ va $z\in B$ yoki $y\in C,\ z\in D.$ Bu yerdan $y\in A\cup C,\ z\in B\cup D$ va $x=(y,z)\in(A\cup C)\times(B\cup D).$ Shunday qilib, $(A\times B)\cup(C\times D)\subseteq(A\cup C)\times(B\cup D).$" ($C\subseteq A$ va $D\subseteq B$) yoki ($A\subseteq C$ va $B\subseteq D$) " shartlar tenglik bajarilishi uchun zarur va yetarlidir.

1.39. Quyidagilarni isbotlang:

a) $(A\bigcup B)\times C=(A\times C)\bigcup(B\times C);$

b) $A\times(B\bigcup C)=(A\times B)\bigcup(A\times C);$

c) $(A\bigcup B)\times(C\bigcup D)=(A\times C)\bigcup(B\times C)\bigcup(A\times D)\bigcup(B\times D);$

d) $(A\setminus B)\times C=(A\times C)\setminus(B\times C);$

e) $A\times(B\setminus C)=(A\times B)\setminus(A\times C);$

f) $(A\times B)=(A\times D)\bigcap(C\times B)$, bu yerda $A\subseteq C$ va $B\subseteq D;$

g) $U^2\setminus(A\times B)=\left[(U\setminus A)\times U\right]\bigcup\left[U\times(U\setminus B)\right];$

h) $\bigcup\limits_{k\in K}A_k\times\bigcup\limits_{t\in T}B_t=\bigcup\limits_{(k,t)\in K\times T}(A_k\times B_t);$

i) $\bigcap\limits_{k\in k}A_k\times\bigcap\limits_{t\in T}B_t=\bigcap\limits_{(k,t)\in K\times T}(A_k\times B_t).$

Yechish. a) $x\in(A\cup B)\times C$ bo`lsin. U holda $x=(y,z)$ va $y\in A\cup B,\ z\in C.$ Bu yerdan $y\in A$ yoki $y\in B.$ Demak, $(y,z)\in A\times C$ yoki $(y,z)\in B\times C$. Shunday qilib, $(A\bigcup B)\times C\subseteq(A\times C)\bigcup(B\times C).$

$x\in(A\times C)\bigcup(B\times C)$ bo`lsin. U holda $x\in A\times C$ yoki $x\in B\times C.$ Bu esa $x=(y,z)$ va birinchi holda $y\in A,\ z\in C$, ikkinchi holda esa $y\in B,\ z\in C$ ekanligini bildiradi.Demak, $y\in A\cup B$ hamda $x=(y,z)\in(A\cup B)\times C$. Shunday qilib, $(A\times C)\bigcup(B\times C)\subseteq(A\bigcup B)\times C.$

1.40. Agar $A, B\neq\varnothing$ va $(A\times B)\cup(B\times A)=C\times D$ bo`lsa, u holda $A=B=C=D$ ekanligini isbotlang.

Yechish. $a\in A,\ b\in B$ bo`lsin. U holda $(a,b)\in A\times B$,demak, $(a,b)\in C\times D$, ya`ni $a\in C,\ b\in D.$ Ikkinchi tomondan, $(b,a)\in B\times A,$ demak, $(b,a)\in C\times D,$ ya`ni $b\in C,\ a\in D.$ U holda $(a,a)\in C\times D,$ demak, $a\in B.$ Shunga o`xshash, $(b,b)\in C\times D,$ demak, $b\in A.$ Shunday qilib, Bundan $A\times B=C\times D$ va 36 b) masalaga ko`ra $A=C,\ B=D.$

N kengaytirilgan natural sonlar to`plami bo`lib, u barcha natural sonlar va 0 sonidan tuzilgan.

1.41. Quyidagi munosabatlar uchun $\delta_R, \rho_R, R\circ R,\ R\circ R^{-1},\ R^{-1}\circ R$ larni toping :

a) $R=\{(x,y): x,y\in N$ va x y ni bo`luvchisi$\}$;

b) $R=\{(x,y): x,y\in N$ va y x ni bo`luvchisi$\}$;

c) $R=\{(x,y): x,y\in D$ va $x+y\leq 0\}$;

d) $R=\{(x,y): x,y\in D$ va $2x\geq 3y\}$;

e) $R=\left\{(x,y): x,y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \text{ va } y\geq\sin x\right\}.$ $A=B.$

Yechish. 0 soni 0 sonini bo`luvchisi deb hisoblaymiz.

a) $\delta_R=\rho_R=N,$ chunki $(x,x)\in R.$ $R^{-1}=\{(x,y):\ x,y\in N$ va y x ni bo`luvchisi$\}$. chunki $(x,y)\in R\circ R^{-1}\Leftrightarrow$ shunday $z$ mavjudki, $x$ $z$ ni bo`luvchisi va y z ni bo`luvchisi. Ammo har doim ixtiyoriy $x$ va $y$ uchun bunday $z$ ni oson topish mumkin. Masalan, $z = x \cdot y$ deb olish mumkin. $R^{-1} \circ R = N^2$, chunki, $(x, y) \in R^{-1} \circ R \Leftrightarrow$ shunday $z$ mavjudki, $z$ $x$ ni bo`luvchisi va z y ni bo`luvchisi. Ixtiyoriy x va y uchun $z = 1$ olish lozim.

b) ham a) ga o`xshash bajariladi.

c) $\delta_R = \rho_R = D,\ \ R^{-1} = R,\ R \circ R = R \circ R^{-1} = R^{-1} \circ R = D^2.\ R \circ R = R,\ R \circ R^{-1} = N^2,$

d) $\delta_R = \rho_R = D,\ R^{-1} = \{(x, y):\ x, y \in D$ va $2y \geq 3x\},\ R \circ R = \{(x, y): x, y \in D$ va $2x \geq 9y\},$ $R \circ R^{-1} = R^{-1} \circ R = D.$

e) $\delta_R = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right],\ \rho_R = \left[-1, \frac{\pi}{2}\right],\ \ R^{-1} = \{(x, y): x, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ va $x \geq \sin y\},$

$R \circ R = \{(x, y):\ \sin\sin x \leq y\},\ R \circ R^{-1} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]^2,\ R \circ R^{-1} = \{(x, y): x, y \in \left[-1, \frac{\pi}{2}\right].$

1.42. Quyidagilarni isbotlang:

a) $\delta_R = \varnothing \Leftrightarrow R = \varnothing \Leftrightarrow \rho_R = \varnothing;$

b) $\delta_{R^{-1}} = \rho_R,\ \ \rho_{R^{-1}} = \delta_R;$

c) $\delta_{R_1 \circ R_2} = R_1^{-1}(\rho_{R_1} \cap \delta_{R_2});$

d) $\rho_{R_1 \circ R_2} = R_2(\rho_{R_1} \cap \delta_{R_2}).$

Yechish. c) $x \in \delta_{R_1 \circ R_2} \Leftrightarrow$ shunday y mavjudki, $(x, y) \in R_1 \circ R_2 \Leftrightarrow$ shunday y va z mavjudki, $(x, z) \in R_1$ va $(z, x) \in R_2 \Leftrightarrow$ shunday z mavjudki, $(x, z) \in R_1$ va $z \in \rho_{R_1} \Leftrightarrow$ shunday z mavjudki, $(z, x) \in R_1^{-1},\ z \in \rho_{R_1}$ va $z \in \delta_{R_2} \Leftrightarrow \ \Leftrightarrow x \in R_1^{-1}(\rho_{R_1} \cap \delta_{R_2}).$

d) $x \in \rho_{R_1 \circ R_2} \Leftrightarrow$ shunday y mavjudki, $(y, x) \in R_1 \circ R_2 \Leftrightarrow$ shunday y va z mavjudki, $(y, z) \in R_1$ va $(z, y) \in R_2 \Leftrightarrow$ shunday z mavjudki, $(z, x) \in R_1$ va $z \in \delta_{R_2} \Leftrightarrow$ shunday z mavjudki, $z \in \rho_{R_1}$ va $z \in \delta_{R_2}$ va $(z, x) \in R_2 \Leftrightarrow x \in R_2(\rho_{R_1} \cap \delta_{R_2}).$

1.43. Isbotlang:

a) agar $B \neq \varnothing$ bo`lsa, u holda $\delta_{A \times B} = A;$

b) agar $A \neq \varnothing$ bo`lsa, u holda $\rho_{A \times B} = B.$

1.44. $R - A$ dagi binar munosabat bo`lsin. $R = i_A$ bo`ladi faqat va faqat qachonki, $R \circ R_1 = R_1 \circ R = R_1$ bo`lsa, bu yerda $R_1 -\ A$ dagi ixtiyoriy munosabat.

Yechish. Agar $R = i_A$ bo`lsa, u holda $A$ dagi ixtiyoriy $R_1$ munosabat uchun $(x, y) \in R \circ R_1 \Leftrightarrow$ shunday $z$ mavjudki, $(x, z) \in R$ va $(z, y) \in R_1$, ammo $(x, z) \in R$ faqat $x = z$ da. Shunday qilib, $R \circ R_1 = R_1$. Xuddi shunday $R_1 \circ R = R_1$ ekanligini ham ko`rsatish mumkin. Aksincha, $R_1 = i_A$ deb olamiz. U holda $R \circ R_1 = R_1$ ekanligidan $R_1 = i_A$.

1.45. Ixtiyoriy binar munosabat uchun quyidagilar o`rinli ekanligini isbotlang:

a) $R \cup R = R \cap R = R;$

b) $\left(R^{-1}\right)^{-1} = R;$

c) $(R_1 \cup R_2)^{-1} = R_1^{-1} \cup R_2^{-1};$

d) $(R_1 \cap R_2)^{-1} = R_1^{-1} \cap R_2^{-1};$

e) $\overline{R^{-1}} = \overline{R}^{-1};$

f) $\left(\bigcup_{i \in I} R_i\right)^{-1} = \bigcup_{i \in I} R_i^{-1};$

g) $\left(\bigcap_{i\in I} R_i\right)^{-1} = \bigcap_{i\in I} R_i^{-1}.$

Yechish. c) $(x,y)\in(R_1\cup R_2)^{-1} \Leftrightarrow (y,x)\in(R_1\cup R_2) \Leftrightarrow (y,x)\in R_1$ yoki $(y,x)\in R_2 \Leftrightarrow (x,y)\in R_1^{-1}$ yoki $(x,y)\in R_2^{-1} \Leftrightarrow (x,y)\in R_1^{-1}\cup R_2^{-1}.$

e) $R - A$ va B to`plamlar orasidagi binar munosabat bo`lsin.

$(x,y)\in\overline{(R^{-1})} \Leftrightarrow (x,y)\in(B\times A)\setminus R^{-1} \Leftrightarrow x\in B,\ y\in A$ va $(x,y)\notin R^{-1} \Leftrightarrow x\in B,\ y\in A$ va $(y,x)\notin R \Leftrightarrow (y,x)\in(A\times B)\setminus R \Leftrightarrow (y,x)\in\bar{R} \Leftrightarrow (x,y)\in\bar{R}^{-1}.$

1.46. Qanday R binar munosabatlar uchun $R^{-1}=\bar{R}$ tenglik o`rinli bo`ladi?

Yechish. Agar $A\neq\varnothing$ va $B\neq\varnothing$ bo`lsa, u holda bunday R munosabat mavjud emas.

$x\in A\cap B$ bo`lsin. U holda $(x,x)\in R \Leftrightarrow (x,x)\in R^{-1} \Leftrightarrow (x,x)\in\bar{R}.$ Qarama-qarshilik hosil qildik.

$A\cap B=\varnothing$ bo`lsin. Ma`lumki, $R^{-1}\subseteq B\times A,\ \bar{R}\subseteq A\times B$, u holda $R^{-1}=\bar{R}=\varnothing.$ Bu yerdan $R=\varnothing$ va $R=A\times B$. Qarama-qarshilik hosil qildik.

1.47. A va B lar mos ravishda n va m ta elementlardan iborat chekli to`plamlar bo`lsin.

a) A va B to`plam elementlari orasida nechta har xil binar munosabatlar mavjud?

b) A dan B ga nechta har xil funksiyalar bor?

c) A dan B ga nechta har xil $1-1$-funksiyalar bor?

c) Qanday n va m larda A va B to`plamlar elementlari orasida o`zaro bir qiymatli moslik mavjud?

Yechish. c) Agar $n\geq m$ bo`lsa, u holda bu son $A_n^m -$ $n$ ta elementdan $m$ tadan o`rinlashtirishlar soniga teng, agar $n<m$ bo`lsa, u holda bunday funksiyalar mavjud emas.

1.48. Ixtiyoriy binar munosabatlar uchun quyidagi xossalarni isbotlang:

a) $R_1\circ(R_2\circ R_3)=(R_1\circ R_2)\circ R_3;$

b) $(R_1\circ R_2)^{-1}=R_2^{-1}\circ R_1^{-1};$

c) $\left(\bigcup_{i\in I} R_i\right)\circ Q=\bigcup_{i\in I}(R_i\circ Q);$

d) $Q\circ\left(\bigcup_{i\in I} R_i\right)=\bigcup_{i\in I}(Q\circ R_i).$

Yechish. a) $(x,y)\in R_1\circ(R_2\circ R_3) \Leftrightarrow$ shunday z mavjudki, $(x,z)\in R_1$ va $(z,y)\in R_2\circ R_3 \Leftrightarrow$ shunday z, u lar mavjudki, $(x,z)\in R_1, (z,u)\in R_2$ va $(u,y)\in R_3 \Leftrightarrow$ shunday u mavjudki, $(x,u)\in R_1\circ R_2$ va $(u,y)\in R_3 \Leftrightarrow (x,y)\in(R_1\circ R_2)\circ R_3.$

b) $(x,y)\in(R_1\circ R_2)^{-1} \Leftrightarrow (y,x)\in R_1\circ R_2 \Leftrightarrow$ shunday z mavjudki, $(y,z)\in R_1$ va $(z,x)\in R_2 \Leftrightarrow$ shunday z mavjudki, $(x,z)\in R_2^{-1}$ va $(z,y)\in R_1^{-1} \Leftrightarrow (x,y)\in R_2^{-1}\circ R_1^{-1}.$

c) $(x,y)\in\left(\bigcup_{i\in I} R_i\right)\circ Q \Leftrightarrow$ shunday z mavjudki, $(x,z)\in\bigcup_{i\in I} R_i$ va $(z,y)\in Q \Leftrightarrow$

$\Leftrightarrow$ shunday z va $i\in I$ mavjudki, $(x,z)\in R_i$ va $(z,y)\in Q \Leftrightarrow$ shunday $i\in I$ mavjudki, $(x,y)\in R_i\circ Q \Leftrightarrow (x,y)\in\bigcup_{i\in I}(R_i\circ Q).$

d) c) ga o`xshash isbotlanadi.

1.49. Quyidagilarni isbotlang:

a) $Q\circ\left(\bigcap_{i\in I} R_i\right)\subseteq\bigcap_{i\in I}(Q\circ R_i);$

b) $\left(\bigcap_{i\in I} R_i\right)\circ Q\subseteq\bigcap_{i\in I}(R_i\circ Q);$

c) a) va b) bandlardagi qism belgisini tenglik bilan almashtirish mumkin emas.

Yechish. a) $(x,y)\in Q\circ\left(\bigcap_{i\in I}R_i\right)\Leftrightarrow$ shunday z mavjudki, $(x,z)\in Q$ va $(z,y)\in\bigcap_{i\in I}R_i\Leftrightarrow$ shunday z mavjudki, $(x,z)\in Q$ va barcha $i\in I$ lar uchun $(z,y)\in R_i\Leftrightarrow$ barcha $i\in I$ lar uchun $(x,y)\in Q\circ R_i\Leftrightarrow(x,y)\in\bigcap_{i\in I}(Q\circ R_i)$.

b) a) ga o`xshash isbotlanadi.

c) Masalan, a) uchun : $R_1=\{(1,1)\}$, $R_2=\{(0,1)\}$, $Q=\{(1,0),(1,1)\}$.

1.50. Binar munosabatlar $\circ$ va $^{-1}$ amallarga nisbatan gruppa tashkil etadimi?

1.51. Agar $R_1\subseteq R_2$ bo`lsa, u holda quyidagiarni isbotlang:

a) $Q\circ R_1\subseteq Q\circ R_2$;

b) $R_1\circ Q\subseteq R_2\circ Q$;

c) $R_1^{-1}\subseteq R_2^{-1}$.

1.52. Quyidagilarni isbotlang:

a) Agar $B\neq\varnothing$ bo`lsa, u holda $B^A\neq\varnothing$;

b) $B^A\subseteq P(A\times B)$.

Yechish. a) $b\in B$ bo`lsin. U holda $B^A$ barcha $x\in A$ uchun $f(x)=b$ kabi aniqlangan $f:A\to B$ funksiyani o`z ichiga oladi.

1.53. $l=\{1,2,...,n\}$ lar uchun A^n va A^l orasida bir qiymatli munosabat o`rnating.

Yechish. A^n dagi $(a_1,a_2,...,a_n)$ elementga $f(i)=a_i$ deb aniqlangan $f:\ I\to A$ funksiyani mos qo`yamiz.

1.54. Ta`rifdan foydalanib, $D^{\left(D^D\right)}$ to`plamni ifodalang, bu yerda $D-$haqiqiy sonlar to`plami.

1.55. Agar f A dan B ga va g B dan C ga funksiya bo`lsa, u hoda $f\circ g$ $A$ dan $C$ ga funksiya bo`lishini isbotlang.

1.56. f va g funksiyalar bo`lsin. Qanday shartlarda quyidagi mulohazalar o`rinli?

a) f^{-1} funksiya bo`ladi;

b) $f\circ g$ 1 – 1-funksiya bo`ladi.

Yechish. a) f 1 – 1-funksiya bo`lsa.

b) $f\cap(\delta_f\times(\rho_f\cap\delta_g)$ va $g\cap((\rho_f\cap\delta_g)\times\rho_g)$ lar 1 – 1-funksiyalar bo`lishi lozim.

1.57. $A,B,A_1,B_1-$to`plamlar shunday to`plamlar bo`lsinki, $A$ to`plam A_1 to`plam bilan o`zaro bir qiymatli moslikda hamda B to`plam $B_1$ to`plam bilan o`zaro bir qiymatli moslikda bo`lsin. U holda quyidagi bir qiymatli mosliklarni o`rnatish mumkinligini ko`rsating:

a) $A\times B$ va $A_1\times B_1$ to`plamlar orasida;

b) A^B va $A_1^{B_1}$ to`plamlar orasida;

c) Agar $A\cap B=\varnothing$ va $A_1\cap B_1=\varnothing$ bo`lsa, u holda $A\cup B$ va $A_1\cup B_1$ to`plamlar orasida.

Yechish. $f_1:A\to A_1$ va $f_2:B\to B_1$ funksiyalar mos ravishda A va A_1 hamda B va B_1 to`plamlar orasidagi o`zaro bir qiymatli moslikni ifodalasin.

a) $F((a,b))=(f_1(a),f_2(b))$ deb aniqlangan $F:A\times B\to A_1\times B_1$ funksiya $A\times B$ va $A_1\times B_1$ to`plamlar orasidagi bir qiymatli moslikni ifodalaydi.

b) $h\in A^B$ bo`lsin. U holda $F(h)=f_2^{-1}\circ h\circ f_1$ funksiya $A^B$ va $A_1^{B_1}$ to`plamlar orasida bir qiymatli moslikni ifodalaydi.

1.58. Quyidagi to`plamlar orasida o`zaro bir qiymatli moslik o`rnatish mumkinligini isbotlang:

a) $A\times B$ va $B\times A$;

b) $A\times(B\times C)$ va $(A\times B)\times C$;

c) $(A\times B)^C$ va $A^C\times B^C$;

d) $(A^B)^C$ va $A^{B\times C}$;

e) Agarda $B\cap C=\varnothing$ bo`lsa, u holda $A^{B\cup C}$ va $A^B\times A^C$;

f) $\prod\limits_{i\in I}A_i$ va $\prod\limits_{i\in I}A_{\varphi(i)}$, bu yerda $\varphi - I$ to`plamni o`rniga qo`yishi;

g) $\prod\limits_{i\in I}A_i$ va $\prod\limits_{k\in K}\left(\prod\limits_{j\in T_k}A_j\right)$, bu yerda $\bigcup\limits_{k\in K}T_k=I$ va barcha T_k o`zaro kesishmaydi;

h) A^I va $\prod\limits_{k\in K}A^{T_k}$, bu yerda $\bigcup\limits_{k\in K}T_k=I$ va barcha T_k o`zaro kesishmaydi.

1.59. $\varphi: A\to A - A$ toplamni o`rniga qo`yishi bo`lsin. U holda $\varphi^{-1}-A$ to`plamni o`rniga qo`yishi ekanligini isbotlang.

1.60. A to`plamni o`rniga qo`yishlari to`plami gruppa tashkil etishini isbotlang.

1.61. $\varphi: A\to B-$ o`zaro bir qiymatli moslik bo`lsin. U holda quyidagilarni isbotlang:

a) $\varphi^{-1}-B$ va A orasida o`zaro bir qiymatli moslik;

b) $\varphi^{-1}\circ\varphi=i_B$;

c) $\varphi\circ\varphi^{-1}=i_A$.

1.62. $R\subseteq A\times B$ munosabat A va B to`plamlar orasidagi o`zaro bir qiymatli munosabat bo`lishi uchun $R^{-1}\circ R=i_B$ va $R\circ R^{-1}=i_A$ bo`lishi zarur va yetarli ekanligini isbotlang.

*Ko`rsatma.* Agar $R-A$ va $B$ to`plamlar orasida o`zaro bir qiymatli moslik bo`lsa, u holda bu tasdiq 61 masaladan kelib chiqadi.

Aksincha, $\delta_R=A$, $R\circ R^{-1}=i_A$, $R^{-1}\circ R=i_B$. Agar $(x,y)\in R$ va $(x,z)\in R$ bo`lsa, u holda $(y,z)\in R^{-1}\circ R$, demak, $y=z$. Agar $(y,x)\in R$ va $(z,x)\in R$ bo`lsa, u holda $(y,z)\in R\circ R^{-1}$, demak, $y=z$.

1.63. A dan B ga ikkita f_1 va f_2 funksiyalarning birlashmasi (kesishmasi) A dan B ga funksiya bo`lishi uchun $f_1=f_2$ bo`lishi zarur va yetarli ekanligini isbotlang.

1.64. Ixtiyoriy f funksiya uchun quyidagilarni o`rinli ekanligini isbotlang:

a) $f(A\cup B)=f(A)\cup f(B)$;

b) $f\left(\bigcup\limits_{i\in I}A_i\right)=\bigcup\limits_{i\in I}f(A_i)$.

1.65. Ixtiyoriy f funksiya uchun quyidagilarning o`rinli ekanligini, qism belgisini tenglik bilan almashtirish mumkin emasligini isbotlang:

a) $f(A\cap B)\subseteq f(A)\cap f(B)$;

b) $f\left(\bigcap\limits_{i\in I}A_i\right)\subseteq\bigcap\limits_{i\in I}f(A_i)$.

1.66. Ixtiyoriy A va B lar uchun f funksiya

$$f(A\cap B)=f(A)\cap f(B)$$

tenglikni qanoatlantiradi faqat va faqat qachonki, u 1 – 1- funksiya bo`lsa, tasdig`ini isbotlang.

Yechish. f 1 – 1-funksiya bo`lmasin. U holda $a,b\in\delta_f$ lar mavjudki, $a\neq b$ va $f(a)=f(b)$. $A=\{a\}$ va $B=\{b\}$ deb olamiz. Teskarisi ravshan.

1.67. Ixtiyoriy f funksiya uchun $f(A)\setminus f(B)\subseteq f(A\setminus B)$ ekanligini isbotlang.

Yechish. Agar $x\in f(A)\setminus f(B)$ bo`lsa, u holda shunday $y\in A$ mavjudki, $f(y)=x$ va $y\notin B$. Shunday qilib, $x\in f(A\setminus B)$.

1.68. Oldingi misolda agar f 1 – 1-funksiya bo`lsa, tenglik bo`lishini isbotlang.

Yechish. Agar $x \in f(A \setminus B)$ bo`lsa, u holda u holda shunday $y \in A$ va $y \in \overline{B}$ mavjudki, $f(y) = x$. Shunday qilib, $x \in f(A)$. Tushunarliki, $x \notin f(B)$, shunday qilib, f 1 – 1-funksiya.

1.69. Ixtiyoriy f funksiya uchun $A \subseteq B$ ekanligidan $f(A) \subseteq f(B)$ ekanligini isbotlang.

1.70. Ixtiyoriy f funksiya uchun quiyidagi munosabat o`rinli ekanligini isbotlang:

$$f(A) = \varnothing \Leftrightarrow A \cap \delta_f = \varnothing.$$

1.71. Ixtiyoriy f funksiya uchun quyidagi ayniyatlar o`rinli ekanligini isbotlang:

a) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B);$

b) $f^{-1}\left(\bigcup_{i \in I} A_i\right) = \bigcup_{i \in I} f^{-1}(A_i);$

c) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B);$

d) $f^{-1}\left(\bigcap_{i \in I} A_i\right) = \bigcap_{i \in I} f^{-1}(A_i);$

e) $f^{-1}(A \setminus B) = f^{-1}(A) \setminus f^{-1}(B).$

Yechish. a) $x \in f^{-1}(A \cup B)$ bo`lsin. Bu esa $f(x) \in A \cup B$ ekanligini bildiradi. Agar $f(x) \in A$ bo`lsa, u holda $x \in f^{-1}(A)$. Agar $f(x) \in B$ bo`lsa, u holda $x \in f^{-1}(B)$. Shunday qilib, $f^{-1}(A \cup B) \subseteq f^{-1}(A) \cup f^{-1}(B)$.

Endi $x \in f^{-1}(A) \cup f^{-1}(B)$ bo`lsin. Agar $x \in f^{-1}(A)$ bo`lsa, u holda $f(x) \in A \subseteq A \cup B$, ya`ni $x \in f^{-1}(A \cup B)$. Agar $x \in f^{-1}(B)$ bo`lsa, u holda $f(x) \in B \subseteq A \cup B$, ya`ni $x \in f^{-1}(A \cup B)$. Shunday qilib, $f^{-1}(A) \cup f^{-1}(B) \subseteq f^{-1}(A \cup B)$.

1.72. Ixtiyoriy f funksiya uchun, agar $A \subseteq B$ bo`lsa, u holda $f^{-1}(A) \subseteq f^{-1}(B)$ ekanligini isbotlang.

1.73. Ixtiyoriy f funksiya uchun qiyidagi munosabat o`rinli ekanligini isbotlang:

$$f^{-1}(A) = \varnothing \Leftrightarrow A \cap \rho_f = \varnothing.$$

1.74. Agar $A \subseteq \delta_f$ va $B \subseteq \rho_f$ bo`lsa, u holda quyidagilarni isbotlang:

a) $A \subseteq f^{-1}(f(A));$

b) $f(f^{-1}(B)) = B;$

c) $f(A) \cap B = f(A \cap f^{-1}(B));$

d) $f(A) \cap B = \varnothing \Leftrightarrow A \cap f^{-1}(B) = \varnothing;$

e) $f(A) \subseteq B \Leftrightarrow A \subseteq f^{-1}(B).$

1.75. $f : A \to B$ bo`lsin. $f_* : P(A) \to P(B)$ va $f^* : P(B) \to P(A)$ quyidagicha aniqlaymiz: $f_*(X) = \{f(x) : x \in X\}$ va $f^*(Y) = \{x : f(x) \subseteq Y\}$. Qanday shartda $f^* \circ f_* = i_{P(B)}$ bo`ladi? Qanday shartda $f_* \circ f^* = i_{P(A)}$ bo`ladi?

Javob. Birinchi holda, $\rho_f = B.$ Ikkinchi holda esa f 1 – 1-funksiya bo`lishi lozim.

1.76. Oldingi masala belgilashlarda quyidagilarni isbotlang:

a) $f^*(X \cap Y) = f^*(X) \cap f^*(Y);$

b) $(f \circ g)^*(X) = f^*(g^*(X)).$

1.3.Maxsus binar munosabatlar

Bu mavzuda aniqlanish sohasi bo`sh to`plam bo`lmagan binar munosabatlar qaraladi. $R$ binar munosabat $A$ to`plamda **refleksiv** deyiladi, agarda barcha $x \in A$ lar uchun $< x, x > \in R$ bo`lsa.

R binar munosabat A to`plamda **irrefleksiv** deyiladi, agarda barcha $x \in A$ lar uchun $< x, x > \notin R$ bo`lsa.

R binar munosabat **simmetrik** deyiladi, agarda $< x, y > \in R \Rightarrow < y, x > \in R$ bo`lsa va **antisimmetrik** deyiladi, agarda $< x, y > \in R$ va $< y, x > \in R \Rightarrow x = y$ bo`lsa.

R binar munosabat **tranzitiv** deyiladi, agarda $< x, y > \in R$ va $< y, z > \in R$, $< x, z > \in R$ bo`lsa.

A to`plamda refleksiv, simmetrik va tranzitiv bo`lgan munosabat **ekvivaentlik** munosabati deyiladi. $x \in A$ elementni R ekvivalentlik bo`yicha **ekvivalentlik sinfi ( qo`shni sinfi**) deb

$$[x]_R = x / R = \{y : < x, y > \in R\}$$

to`plamga aytiladi. $A$ to`plam elementlarini R ekvivalentlik bo`yicha ekvivalentlik sinflari to`plami R **bo`yicha** $A$ **to`plamni factor-to`plami** deyiladi va A / R deb belgilanadi.

A to`plamdagi binar munosabat $A$ da **old tartib (предпорядком**) deyiladi, agarda u refleksiv va tranzitiv bo`lsa.

A to`plamda refleksiv, tranzitiv va antisimmetrik bo`lgan munosabat A da **qisman tartib** deyiladi.Qisman tartib ko`p hollarda $\leq$ belgi bilan belgilanadi. $\leq^{-1}$ tartib esa $\leq$ tartibga **qo`shma (ikki tomonlama)** deyiladi va $\geq$ orqali belgilanadi. $x < y$ deb yozamiz agarda $x \leq y$ va $x \neq y$ bo`lsa. $\leq$ qisman tartib $A$ to`plamda **chiziqli** deyiladi, agarda A dagi ixtiyoriy ikkita element $\leq$ bo`yicha taqqoslansa, ya`ni ixtiyoriy $x, y \in A$ lar uchun $x \leq y$ yoki $y \leq x$ bo`lsa.

A to`plam unda berilgan qisman (chiziqli) tartib $\leq$ bilan **qisman (chiziqli) tartiblangan** deyiladi.

1.77. Agar R_1 va R_2 munosabatlar refleksiv bo`lsa, u holda $R_1 \cup R_2$, $R_1 \cap R_2$, R_1^{-1} va $R_1 \circ R_2$ munosabatlar ham refleksiv ekanligini isbotlang.

*Ko`rsatma.* $R - A$ to`plamdagi refleksivlik $\Leftrightarrow i_A \subseteq R$.

1.78. Agar R_1 va R_2 munosabatlar irrefleksiv bo`lsa, u holda $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1^{-1}$ munosabatlarni irrefleksiv ekanligini isbotlang. Irrefleksiv munosabatlarning $R_1 \circ R_2$ ko`paytmasi irrefleksiv bo`lmasligi mumkin ekanligini ko`rsating.

Yechish. $R - A$ to`plamdagi irrefleksivlik $\Leftrightarrow R \cap i_A = \varnothing$. Masalan, $R_1 = \{(x, y) : x, y \in N, x < y\}, R_2 = R_1^{-1}$ bo`lsin u holda $R_1 \circ R_2$ refleksiv.

1.79. Agar R_1 va R_2 munosabatlar simmetrik bo`lsa, u holda $R_1 \cup R_2$, $R_1 \cap R_2$, R_1^{-1} va $R_1 \circ R_1^{-1}$ munosabatlar ham simmetrik ekanligini isbotlang.

Ko`rsatma. $R -$ simmetrik $\Leftrightarrow R = R^{-1}$.

1.80. R_1 va R_2 simmetrik munosabatlarning $R_1 \circ R_2$ ko`paytmasi simmetrik bo`ladi, faqat va faqat qachonki, $R_1 \circ R_2 = R_2 \circ R_1$ bo`lsa tasdig`ini isbotlang.

Yechish. $R_1 \circ R_2$ simmetrik $\Rightarrow R_1 \circ R_2 = (R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1} = R_2 \circ R_1$, $\quad R_1 \circ R_2 = R_2 \circ R_1 \Rightarrow$ $R_1 \circ R_2 = (R_1 \circ R_2)^{-1} = (R_2 \circ R_1)^{-1} = R_1^{-1} \circ R_2^{-1} = R_1 \circ R_2$.

1.81. Quyidagilarni isbotlang:

a) Agar R_1 va R_2 munosabatlar antisimmetrik bo`lsa, u holda $R_1 \cap R_2$ va R_1^{-1} munosabatlar ham antisimmetrik;

b) R_1 va R_2 antisimmetrik munosabatlarning $R_1 \cup R_2$ birlashmasi antisimmetrik bo`ladi, faqat va faqat qachonki, $R_1 \circ R_2^{-1} \subseteq i_A$ bo`lsa tasdig`ini isbotlang.

Yechish. a) R antisimmetrik $\Leftrightarrow R \cap R^{-1} \subseteq i_A$.

b) $(R_1 \cup R_2) \cap (R_1 \cup R_2)^{-1} = (R_1 \cup R_2) \cap (R_2^{-1} \cup R_1^{-1})$; $\quad R_1^{-1} \cap R_2 = (R_1 \cap R_2^{-1})^{-1}$.

1.82. Quyidagi xossalarga ega bo`lgan binar munosabatlar quring:

a) refleksiv, simmetrik ammo tranzitiv bo`lmagan;

b) refleksiv, antisimmetrik ammo tranzitiv bo`lmagan;

c) refleksiv, tranzitiv ammo simmetrik bo`lmagan;
d) antisimmetrik, tranzitiv ammo refleksiv bo`lmagan.
Yechish. a) Masalan, $\{(x,y): x,y\in D, |x-y|\le 1\}$;
b) $\{(x,y): x,y\in Z,\ x\le y\le x^2\}$;
c) $\{(x,y): x,y\in D,\ x\le y\}$;
d) $\{(x,y): x,y\in D,\ x=y=0\}$.
1.83. a) simmetrik, tranzitiv ammo refleksiv bo`lmagan binar munosabat quring,
b) Agar A to`plamda $R$ munosabat tranzitiv va simmetrik va $\delta_R \cup \rho_R = A$ bo`lsa, u holda R ekvivalentlik munosabati ekanligini isbotlang.
Yechish. a) Masalan, $\{(x,y): x,y\in D,\ x,y>0\}$;
b) $x\in A \Rightarrow$ biror y uchun $(x,y)\in R$ yoki $(y,x)\in R$ $\Rightarrow$ $(x,y)\in R$ va $(y,x)\in R$ $\Rightarrow (x,x)\in R$.
1.84. Bir paytda simmetrik va antisimmetrik bo`lgan ixtiyoriy $R$ munosabat tranzitiv ham bo`lishini isbotlang.
Ko`rsatma. $R\subseteq i_A$.
1.85. A to`plamda aniqlangan $R$ munosabat bir vaqtda ekvivalentlik va qisman tarib munosabati bo`ladi, faqat va faqat qachonki, $R=i_A$ bo`lsa tasdig`ini isbotlang.
1.86. N va $N\times N$ to`plamlarda aniqlangan R_m, Q, S munosabatlarni quyidagicha aniqlaymiz:
a) $(a,b)\in R_m \Leftrightarrow (a-b)$ m ga bo`linadi $(m>0)$;
b) $((a,b),(c,d))\in Q \Leftrightarrow a+d=b+c$;
c) $((a,b),(c,d))\in S \Leftrightarrow [((a\cdot d=b\cdot c)$ va $b\ne 0$ va $d\ne 0)$ yoki $(a=c,\ b=0,\ d=0)]$.
R_m, Q va S lar ekvivalentlik munosabati ekanliklarini isbotlang.
1.87. $A-$ tekislikdagi barcha to`g`ri chiziqlar to`plami bo`lsin. U holda quyidagilar ekvivalentlik munosabatlari bo`ladimi?
a) to`g`ri chiziqlarning parallellik munosabati;
b) to`g`ri chiziqlarning perpendikulyarlik munosabati.
1.88. $D-$haqiqiy sonlar to`plamida R munosabatlarni quyidagicha aniqlaymiz:
$$\alpha R\beta \Leftrightarrow (\alpha-\beta)- \text{ ratsional son.}$$
R ekvivalentlik munosabati ekanligini isbotlang.
1.89. Agar $R-$ekvivalentlik munosabati bo`lsa, quyidagilarni isbotlang:
a) $x\in[x]_R$;
b) $(x,y)\in R \Leftrightarrow [x]_R=[y]_R$.
1.90. Agar R ekvivalentlik munosabati bo`lsa, u holda R^{-1} ham ekvivalentlik munosabati ekanligini isbotlang.
Yechish. $R^{-1}=R$.
1.91. $R\subseteq A^2$ bo`lsin. U holda
$$R \text{ ekvivalentlik } \Leftrightarrow (R\circ R^{-1})\cup i_A = R$$
o`rinli ekanligini isbotlang.
Yechish. $R-$ ekvivalentlik $\Rightarrow R^{-1}=R,\ R\circ R\subseteq R,\ i_A\subseteq R$. Aksincha, ixtiyoriy R uchun $R\circ R^{-1}$ simmetrik. Shu sababli R simmetrik va $R\circ R = R\circ R^{-1}\subseteq R$.
1.92. Agar R_1 va $R_2 - A$ dagi ekvivalentlik munosabati bo`lsa, u holda quyidagilarni isbotlang:
a) $R_1\circ R_2 = A^2 \Leftrightarrow R_1 = A^2$;
b) $R_1\circ R_2 = A^2 \Leftrightarrow R_2\circ R_1 = A^2$.
Yechish. a) $R_1 = R_1\circ R_1$.
b) $A^2=(A^2)^{-1}=(R_1\circ R_2)^{-1}=R_2^{-1}\circ R_1^{-1}=R_2\circ R_1$.

1.93. A to`plamni kesishmaydigan bo`sh bo`lmagan qism to`plamlariga barcha ajratmalari (yoyilmasi) sinflari bilan A to`plamni barcha ekvivalentlik munosabatlari oilalari orasida o`zaro bir qiymatli moslik mavjud ekanligini isbotlang. (Agar $\bigcup_{i\in I} A_i = A$ va A_i to`plamlar o`zaro kesishmasa, u holda $\{A_i\}_{i\in I}$ to`plamlar oilasi $A$ ning bo`linmasi deyiladi.)

Yechish. $\{A_i\}_{i\in I}$ bo`linmaga $R=\{(x,y):$ shunday $i\in I$ mavjudki, $x,y\in A_i\}$ ekvivalentlikni mos qo`yamiz.

1.94. R A to`plamda ekvivalentlik munosabati bo`ladi, faqat va faqat qachonki, shunday o`zaro kesishmaydigan $P$ to`plamlar sistemasi mavjud bo`lib, uning uchun

$$R=\bigcup_{C\in P} C\times C \text{ va } \bigcup_{C\in P} C = A$$

bo`lsa tasdig`ini isbotlang.

*Ko`rsatma*. Agar $R-$ ekvivalentlik bo`lsa, u holda $P=A/R$ bo`ladi (89 masalaga qarang).

1.95. $f:A\to B-$ ixtiyoriy funksiya bo`lsin.

$$Q=\{(x,y):\ f(x)=f(y)\}$$

deb olamiz. Q A dagi ekvivalentlik munosabati ekanligini va f akslantirish uchun

$$f=\varepsilon\cdot f'$$

yoyilma mavjudligini isbotlang, bu yerda $\varepsilon - A$ ni $A/Q=\{[x]_Q : x\in A\}$ ga tabiiy akslantirish, ya`ni $\varepsilon(x)=[x]_Q$, $f' - A/Q$ va $f(A)$ to`plamlar orasidagi o`zaro bir qiymatli moslik.

Yechish. $f'\left([x]_Q\right)=f(x)$ deb olamiz. U holda ravshanki, $[x]_Q=[y]_Q \Leftrightarrow f(x)=f(y)$. Shu sababli ham $f' - A/Q$ va $f(A)$ to`plamlar orasidagi o`zaro bir qiymatli moslik.

$(\varepsilon\circ f')(x)=f'(\varepsilon(x))=f'\left([x]_Q\right)=f(x).$

1.96. A to`plamdagi ixtiyoriy ekvivalentlik sistemalarining kesishmasi yana $A$ da ekvivalentlik bo`lishini isbotlang.

1.97. R_1 va R_2 ekvivalentlik munosabatlarining $R_1\cup R_2$ birlashmasi ekvivalentlik munosabati bo`ladi faqat va faqat qachonki, $R_1\cup R_2 = R_1\circ R_2$ bo`lsa tasdig`ini isbotlang.

Yechish. $R_1\cup R_2 -$ ekvivalentlik $\Rightarrow R_1\circ R_2 \subseteq (R_1\cup R_2)\circ(R_1\cup R_2)\subseteq R_1\cup R_2$,

$R_1\cup R_2 = (R_1\circ i_A)\cup(i_A\circ R_2)\subseteq R_1\circ R_2.$

Aksincha, $R_1\cup R_2 = R_1\circ R_2$ bo`lsin. U holda

$R_1\circ R_2 = R_2^{-1}\circ R_1^{-1} = (R_1\circ R_2)^{-1}=(R_1\cup R_2)^{-1}= R_1\cup R_2.$

$(R_1\cup R_2)\circ(R_1\cup R_2)=(R_1\circ R_2)\cup(R_1\circ R_2)\cup(R_1\circ R_2)\cup(R_1\circ R_2)\subseteq(R_1\cup R_2)$, ya`ni $R_1\cup R_2$ tranzitiv. $R_1\cup R_2$ ni simmetrik va refleksiv ekanligi ravshan.

1.98. R_1 va R_2 ekvivalentlik munosabatlarining $R_1\circ R_2$ ko`paytmasi ekvivalentlik munosabati bo`ladi faqat va faqat qachonki, $R_1\circ R_2 = R_2\circ R_1$ bo`lsa tasdig`ini isbotlang.

Yechish. $R_1\cup R_2 -$ ekvivalentlik $\Rightarrow R_1\circ R_2=(R_1\cup R_2)^{-1}=R_2^{-1}\circ R_1^{-1}=R_2\circ R_1$. $R_1\circ R_2 = R_2\circ R_1$ bo`lsin. U holda $(R_1\circ R_2)^{-1}=(R_2\circ R_1)^{-1}=R_1^{-1}\circ R_2^{-1}=R_1\circ R_2$, ya`ni $R_1\circ R_2$ simmetrik, $(R_1\cup R_2)\circ(R_1\cup R_2)=R_1\circ(R_2\circ R_1)\circ R_2 = R_1\circ(R_1\circ R_2)\circ R_2\subseteq R_1\circ R_2$, ya`ni $R_1\circ R_2$ tranzitiv, refleksiv ekanligi ravshan.

1.99. Agar R_1 va R_2 ekvivalentlik munosabatlari va $R_1\circ R_2 = R_2\circ R_1$ bo`lsa, u holda $R_1+R_2=R_1\circ R_2$ ekanligini ko`rsating, bu yerda $R_1+R_2-R_1\cup R_2$ birlashmaga kiruvchi eng kichik ekvivalentlik munosabati.

Yechish. $R_1 \circ R_2$ ekvivalentlik (98 masalaga qarang). Ravshanki, $R_1 \cup R_2 \subseteq R_2 \circ R_1$. Endi biror Q ekvivalentlik munosabati uchun $R_1 \cup R_2 \subseteq Q$ bo`lsin. U holda
$R_1 \circ R_2 \subseteq (R_1 \cap R_2) \circ (R_1 \cup R_2) \subseteq Q \circ Q \subseteq Q.$

1.100. Har qanday $\{R_i\}_{i\in I}$ ekvivalentlik oilasi uchun $\bigcup_{i\in I} R_i \subseteq Q$ shartni qanoatlantiradigan Q ekvivalentlik mavjudligini va agar ixtiyoriy R ekvivalentlik munosabati uchun $\bigcup_{i\in I} R_i \subseteq R$ bo`lsa, u holda $Q \subseteq R$ bo`lishini isbotlang.

Yechish. Q $R_{i_1} \circ R_{i_2} \circ \ldots \circ R_{i_k}$ $(k \geq 1,\ i_1, i_2, \ldots, i_k \in I)$ ko`rinishdagi barcha mumkin bo`lgan ko`paytmalarning birlashmasiga teng.

1.101. $p_{n+1} = \sum_{i=0}^{n} C_n^i p_i \quad (p_0 = 1)$ ekanligini isbotlang. Bu yerda $p_n - n$ ta elementli to`plamdagi ekvivalentlik munosabatlar soni.

Yechish. $A - n+1$ ta elementdan tuzilgan to`plam, $a \in A$, $B \subseteq A \setminus \{a\}$ to`plam i ta elementni o`z ichiga olsin. A dagi $[a]_R = B \cup \{a\}$ shartni qanoatlantiradigan ekvivalentliklar soni p_{n-1} ga teng.

II bob. Mulohazalar algebrasi asosiy tushunchalari

2.1.Mulohazalar va ular ustida amallar.

*Ta`rif.* Rost yoki yo`lg`onligini bir qiymatli aniqlash mumkin bo`lgan darak gap mulohaza deb ataladi.

Masalan, “Qarshi shahri – Qashqadaryo viloyati markazi” ,”Oy Marsning yo`ldoshi” Kislorod –gaz” , “5 > 4”, “Temir qo`rg`oshindan og`ir” kabi gaplar mulohazalarga misol bo`ladi. Ammo har qanday gap ham mulohaza bo`la olmaydi, masalan, “ Qarshi davlat universiteti talabasi”, “ “Sen qaysi oliy o`quv yurtida o`qiysan?”,”Yashasin O`zbekiston yoshlari” kabi gaplar mulohaza emas, chunki bu gaplar darak gap emas va rost yoki yo`lgon ekanligini aniqlash mumkin emas. Shunday qilib, biror bir gap mulohaza bo`lishi uchun , u albatta darak gap bo`lishi va rost va yo`lg`on ekanligini bir qiymatli aniqlanishi lozim.

Mulohazalarni lotin alifbosining bosma indeksli yoki indekssiz bosh harflari bilan belgilaymiz: $A, B, C, ..., X, Y, Z, A_1, A_2, ..., A_n, ..., Z_1, Z_2, ..., Z_n$.

Barcha mulohazalar to`plamida {0,1} to`plamdan qiymatlar qabul qiluvchi rostlik funksiyasi quyidagicha aniqlanadi:

$$\lambda(A) = \begin{cases} 1, \text{ agarda } A \text{ mulohaza rost bo`lsa,} \\ 0, \text{ agarda } A \text{ mulohaza yo`lgon bo`lsa.} \end{cases}$$

$\lambda(A)$ qiymat A mulohazaning mantiqiy qiymati yoki rostlik (chinlik) qiymati deyiladi. Keyingi yozivlarimizni ixchamlashtirish maqsadida $\lambda(A)$ o`rniga A deb yozamiz.

Mulohazalar ustida quyidagi asosiy amallar (mantiqiy bog`lovchilar) aniqlanadi. Ular yordamida bor mulohazalardan yangi mulohazalar qurish mumkin bo`ladi:

1) Inkor: $\bar{A}$ (“ A emas” deb o`qiladi);
2) Kon`yunsiya: $A \wedge B$ (“ $A$ va $B$ “ deb o`qiladi, hamda $A \& B$ belgilash ham ishlatiladi);
3) Diz`yunksiya: $A \vee B$ ( “ $A$ yoki $B$ “ deb o`qiladi);
4) Implikatsiya: $A \rightarrow B$ (“ Agar A bo`lsa, u holda $B$ “, yoki “ $A$ dan $B$ kelib chiqadi” yoki “ $B$ uchun $A$ yetarli” yoki “ $A$ uchun $B$ zarur” deb o`qiladi);
5) Ekvivalentlik: $A \leftrightarrow B$ (“ A B ga teng kuchli” yoki “ A faqat va faqat qachonki, B bo`lsa” yoki “ $A$ $B$ uchun zarur va yetarli” deb o`qiladi).

Bu amallarning mantiqiy qiymati boshlang`ich mulohazalar bilan qanday bog`langanligi quyidagi jadvalda (mos amallarning rostlik (chinlik) jadvalida) berilgan:

A	B	$\bar{A}$	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$
0	0	1	0	0	1	1
0	1	1	0	1	1	0
1	0	0	0	1	0	0
1	1	0	1	1	1	1

Bu amallarni har birini 0 va 1 belgilar ustidagi amallar kabi tushunish lozim. Masalan, kon`yunksiya va implikatsiya amallari quyidagi qoida bo`yicha bajariladi: $0 \wedge 0 = 0$; $0 \wedge 1 = 0$; $1 \wedge 0 = 0$; $1 \wedge 1 = 1$; $0 \rightarrow 0 = 1$; $0 \rightarrow 1 = 1$; $1 \rightarrow 0 = 0$; $1 \rightarrow 1 = 1$.

2.1. Quyidagi gaplardan qaysilari mulohaza bo`ladi:

a) Toshkent – O`zbekiston poytaxti;
b) Universitetni mexanika-matematika fakul`teti talabasi;
c) ABC uchburchak $A'B'C'$ uchburchakka o`xshash;

d) Yer Marsning yo`ldoshi;
e) $2+3-6$;
f) Vodorod – gaz;
g) Palov –shirin ta`om;
h) Matematika – qiziq predmet;
i) Pikassoning rasmlari juda abstract;
j) Temir qo`rg`oshindan og`ir;
k) "Yashasin mustaqillik";
l) Uchburchak teng tomonli deyiladi, agarda uning barcha tomonlari teng bo`lsa;
m) Agar uchburchakning barcha burchaklari teng bo`lsa, u teng tomonli;
n) Bugun yomon ob-havo;
o) A.Qahhorning " O`tgan kunlar" romanida 1 233 564 ta harf bor;
p) Angar daryosi Baykal ko`liga quyiladi.

*Yechish.*b) Ushbu gap mulohaza bo`lmaydi, chunki u talaba haqida hech qanday narsani tasdiqlamaydi.

c) Bu gap ham mulohaza bo`lmaydi, chunki, biz uni rost yoki yolg`on ekanligini bilmaymiz, ya`ni aynan qanday uchburchak to`g`risida gap ketayotganligini. Bu yerda $ABC$ biror o`zgaruvchi, uning o`rniga aniq quymat (uchburchaklar) qo`yish mumkin.

g) Bu gap ham mulohaza bo`lmaydi, chunki "shirin taom" juda aniq bo`lmagan tushuncha.

o) Bu gap mulohaza bo`ladi, ammo uni rost yoki yog`on ekanligini aniqlash uchun ko`p vaqt talab etiladi.

2.2. Yuqoridagi masaladagi mulozalardan qaysilari rost, qaysilari yo`lg`on ekanligini ko`rsating:

2.3. Quyidagi mulohazalarni teskarini toping; berilgan mulohazalarning va inkorlarini rostlik qiymatini ko`rsating:

a) Volga daryosi Kasbiy dengiziga quyiladi;
b) 28 soni 7 soniga bo`linadi;
c) $6>3$;
d) $4\le 5$;
e) Barcha tub sonlar toq sonlar;
f) $\sqrt{2}-$ratsional son;
g) $5+3=8$;
h) Afrika – orollar;
i) Barcha so`zlarni bo`g`inlarga ajratish mumkin;
j) Ba`zi qo`ziqorinlar zaharli.

2.4. Quyidagi qaysi juft mulohazalar biri ikkinchisining inkori bo`ladi (nima uchun ekanligini, tushuntiring):

a) "$4<5$", " $5<4$ ";
b) "$6<9$ ", " $6\ge 9$ ";
c) " ABC uchburchak to`g`ri burchakli", " ABC uchburchak o`tmas burchakli";
d) "n natural son juft", "n natural son toq";
e) " f funksiya toq", " f funksiya juft";
f) " Barcha tub sonlar toq sonlar", " Barcha tub sonlar juft sonlar";
g) " Barcha tub sonlar toq sonlar", " Tub juft son mavjud";
h) "Insonlarga yer yuzida yashovchi barcha jonzodlar turlari ma`lum" , " Yer yuzida yashovchi shunday jonzod turi mavjudki, u insonlarga ma`lum emas"
i) " Irratsional sonlar mavjud" , " Barcha sonlar – ratsional sonlar.";
j) " Agar n 3ga bo`linsa, u holda $n$ 9ga bo`linadi", "Agar n 3ga bo`linmasa, u holda $n$ 9ga bo`linmaydi";
k) "$2<0$", " $2>0$".

Yechish. k) "$2<0$" mulohaza "$2>0$" mulohazaning inkori bo`lmadi, ya`ni inkori emas, chunki, 0 dan kichik bo`lmaslik sharti ikkita imkoniyatni qoldiradi: $0$ ga teng bo`lishni va 0 dan katta bo`lishni. Shunday qilib, "$2<0$" mulohazaning inkori "$2\geq 0$" mulohaza bo`ladi.

2.5. Quyidagi mulohazalarning rostlik qiymatini toping:

a) Sankt-Peterburg Neva daryosi bo`yida joylashgan va $2+3=5$;
b) $7-$ tub son va $9-$ tub son;
c) $7-$ tub son yoki $9-$ tub son;
d) 2 soni juft yoki yoki bu son tub;
e) $2\leq 3,\ 2\geq 3,\ 2\cdot 2=4,\ 2\cdot 2\geq 4$;
f) $2\cdot 2=4$ yoki oq ayiqlar Afrikada yashaydi;
g) $2\cdot 2=4$ va $2\cdot 2\leq 5$ va $2\cdot 2\geq 4$;
h) $2-$ratsional son yoki $5-$irratsional son;
i) Fobus va Oy - Martsning yo`ldoshlari;
j) Teng tomonli uchburchakda ikkita yoki uchta burchagi o`zari teng;
k) $3\cdot 3=9$ va $4+7=11$.

Yechish. k) Ikkala sodda mulohazalarga kon`yunksiya amali qo`llanilayapti, shu sababli bu amalni ta`rifiga ko`ra ularni kon`yunksiyasi rost mulohaza bo`ladi.

2.6. Agarda quyidagi a) – e) mulohazalar rost , f) – k) mulohazalar yo`lg`on bo`lsa, A,B,C,D,E,F,G,H,I,J,K mulohazalarning rostlik qiymatini toping:

a) $A\wedge(2\cdot 2=4)$; e) $\bar{E}\vee(2\cdot 2=5)$; i) $\bar{I}\wedge(2\cdot 2=4)$;
b) $B\vee(2\cdot 2=5)$; f) $F\wedge(2\cdot 2=4)$; j) $\bar{J}\vee(2\cdot 2=5)$;
c) $C\vee(2\cdot 2=4)$; g) $G\vee(2\cdot 2=5)$; k) $K\wedge(2\cdot 2=4)$.
d) $\bar{D}\wedge(2\cdot 2=4)$; h) $H\wedge(2\cdot 2=5)$;

Yechish. k) Mulohazalarning kon`yunksiyasi yolg`on bo`ladi, agarda kon`yunksiyaga kiruvchu mulohazalardan (kon`yunksiya hadlaridan) kamida bittasi yolg`on bo`lsa. Bizning holimizda ikkinchi "$2\cdot 2=4$" mulohaza rost , ikkita mulohazaning kon`yunksiyasi esa yolg`on. Shu sababli birinchi $K$ mulohaza yolg`on bo`lishi lozim.

2.7. Quyidagi har bir gapni rostlik shartini kon`yunksiya va diz`yunksiya ko`rinishda yo`zing ($a,b-$haqiqiy sonlar):

a) $a\cdot b\neq 0$; d) $a\cdot b>0$; g) $|a|>3$; j) $a\cdot b<0$;
b) $a\cdot b=0$; e) $|a|=3$; h) $a^2+b^2\neq 0$; k) $a/b=0$.
c) $a^2+b^2=0$; f) $|a|<3$; i) $a/b\neq 0$;

Yechish. k) Kasr faqat surati nol bo`lgan, maxraji esa noldan farqli bo`lgan holda nolga teng bo`ladi, ya`ni $(a=0)\wedge(b\neq 0)$.

2.8. Agarda quyidagi a) – e) mulohazalar yolg`on , f) – k) mulohazalar rost bo`lsa, A,B,C,D,E,F,G,H,I,J,K mulohazalarning rostlik qiymatini toping:

a) Agar $4-$juft son bo`lsa, u holda A;
b) Agar B bo`lsa, u holda $6-$juft son;
c) Agar $2\cdot 2=4$ bo`lsa, u holda $\bar{C}$;
d) agar $\bar{D}$ bo`lsa, u holda $2\cdot 2=5$;
e) Agar $6-$juft son bo`lsa, u holda $\bar{E}$;
f) Agar F bo`lsa, u holda $4-$toq son;
g) Agar $3\cdot 2=6$ bo`lsa, u holda $\bar{G}$;
h) Agar $\bar{H}$ bo`lsa, u holda $2\cdot 2=5$;
i) Agar $2-$juft son bo`lsa, u holda I;
j) Agar $3-$juft son bo`lsa, u holda J;
k) Agar $4-$juft son bo`lsa, u holda K;

Yechish. k) Ikkita mulohazaning implikatsiyasi yolg`on bo`ladi faqat yagona holda, qachonki, jo`natma rost, xulosa esa yolg`on bo`lsa.Bu berilgan holda jo`natma "$4-$ juft son" rost mulohaza va shartga ko`ra butun mulohazaning o`zi ham rost mulohaza. Shu sababli K mulohaza yolg`on bo`lishi mumkin emas, yani K mulohaza rost.

2.9. Quyidagi mulohazalarning rostlik qiymatini qaniqlang:

a) Agar $9\ 3$ga bo`linsa, u holda $4\ 2$ga bo`linadi;

b) Agar $11\ 6$ga bo`linsa, u holda $11\ 3$ga bo`linadi;

c) Agar $15\ 6$ga bo`linsa, u holda $15\ 3$ga bo`linadi;

d) Agar $15\ 3$ga bo`linsa, u holda $15\ 6$ga bo`linadi;

e) Agar Saratov Neva bo`yida joylashgan bo`lsa, u holda fillar – hashoratlar;

f) $12\ 6$ga bo`linadi faqat va faqat qachonki, $12\ 3$ ga bo`linsa;

g) $4>5$ faqat va faqat qachonki, $-4>-5$ bo`lsa;

h) $15\ 6$ga bo`linadi faqat va faqat, qachonki $15\ 3$ga bo`linsa;

i) $15\ 5$ga bo`linadi faqat va faqat, qachonki $15\ 4$ga bo`linsa;

j) Agar $12\ 6$ga bo`linsa, u holda $12\ 3$ga bo`linadi;

k) $11\ 6$ga bo`linadi faqat va faqat, qachonki $11\ 3$ga bo`linsa.

Yechish. j) Shunday qilib, mulohazada "$12\ 6$ga bo`linadi" jo`natma rost va mulohazada xulosa "$12\ 3$ga bo`linadi" rost, u holda butun mulohaza ta`rifga ko`ra rost.

k) $P\leftrightarrow Q$ ko`rinishdagi mulohaza ekvivalentlik ta`rifiga ko`ra, rost bo`ladi, agar P va Q mulohazalarning mantiqiy qiymatlari ustma- ust tushsa, aks holda yolg`on. Berilgan musolda ekvivalentlik bilan bog`langan ikkala mulohaza ham yolg`on. Shu sababli butun murakkab mulohaza rost.

2.10. Agarda quyidagi a) – e) mulohazalar rost , f) – j) mulohazalar yo`lg`on bo`lsa, $A, B, C, D, E, F, G, H, I, J$ mulohazalarning rostlik qiymatini toping:

$a)\ A\leftrightarrow(2<3);$ e) $(2\cdot 2=4)\leftrightarrow E;$ $h)\ (6\le 7)\leftrightarrow\overline{H}$

$b)\ B\leftrightarrow(2>3);$ $f)\ F\leftrightarrow(2<3);$ $i)\ (6\ge 7)\leftrightarrow\overline{I};$

c) $(6\le 7)\leftrightarrow\overline{G};$ g) $G\leftrightarrow(2>3);$ j) $(2\cdot 2=4)\leftrightarrow\overline{J}.$

d) $(6\ge 7)\leftrightarrow\overline{D};$

2.11. A orqali "$9\ 3$ga bo`linadi" mulohazani, $B$ orqali esa "$8\ 3$ga bo`linadi" mulohazani belgilangan bo`lsa. Quyidagi mulohazalarning rostlik qiymatini aniqlang:

$a)\ A\to B;$ d) $\overline{B}\to A;$ g) $B\to\overline{A};$ j) $\overline{A}\leftrightarrow B;$

$b)\ B\to A;$ e) $\overline{A}\to\overline{B};$ h) $A\leftrightarrow B;$ k) $A\leftrightarrow\overline{B};$

$c)\ \overline{A}\to B;$ f) $A\to\overline{B};$ i) $\overline{A}\to\overline{B};$ l) $\overline{B}\to\overline{A}.$

Yechish. k) Berilishiga ko`ra $A=1,\ B=0.$ Shu sababli $A\leftrightarrow\overline{B}=1\leftrightarrow\overline{0}=1\leftrightarrow 1=1.$

l) Berilishiga ko`ra $A=1,\ B=0.$ Shu sababli $\overline{B}\to\overline{A}=\overline{0}\to\overline{1}=1\to 0=0.$

2.12. A orqali "Bu uchburchak teng yonli" mulohazani, B orqali esa "Bu uchburchak teng tomonli" mulohazani belgilangan bo`lsa. Quyidagi mulohazalarni o`qing:

$a)\ \overline{A}\wedge\overline{B};$ $e)\ \overline{(A\wedge B)}\leftrightarrow\overline{B};$ $i)\ \overline{(A\wedge B)}\vee B;$

$b)\ \overline{(A\vee B)};$ f) $B\to(\overline{A}\vee B);$ j) $\overline{A}\wedge\overline{(A\vee\overline{B})};$

$c)\ \overline{A}\to\overline{B};$ g) $(\overline{A}\vee\overline{B})\to\overline{(A\wedge B)};$ k) $(A\wedge\overline{B})\to\overline{\overline{A}}.$

$d)\ (A\vee B)\leftrightarrow A;$ h) $\overline{(A\wedge B)}\to\overline{A};$

Yechish. k) Agar uchburchak teng yonli va teng tomonli bo`lmasa, u holda u teng yonli emas ekanligi yo`lg`on.

2.13. A orqali " Bu son – butun", B orqali " Bu son musbat" , C orqali " Bu son tub", D orqali " Bu son 3ga bo`linadi" mulohazalar belgilangan bo`lsa. Quyidagi mulohazalarni o`qing.

$a)\ (A\vee B)\to\bar{C};$ e) $D\leftrightarrow(\bar{C}\wedge A);$ i) $\bar{A}\vee\bar{D};$

$b)\ (A\wedge B)\to D;$ f) $(A\wedge C)\to D;$ j) $(A\wedge B\wedge C)\vee D;$

$c)\ (A\vee\bar{A})\to(B\wedge C);$ g) $(A\wedge D)\to\bar{C};$ k) $(A\wedge C)\vee(B\wedge D).$

d) $(B\wedge\bar{B})\leftrightarrow(C\vee A);$ h) $(A\vee B)\wedge(C\vee D);$

Yechish. k) Bu son yoki butun va tub, yoki musbat va 3 ga bo`linuvchi.

2.14. Quyidagi murakkab mulohazalarni sodda(elementar) mulohazalarga ajrating, elementar va murakkab mulohazalar uchun belgilashlar kiritib, bu belgilashlar orqali ularni yozing:

a) Agar son 2 ga bo`linsa va 3 ga bo`linmasa, u holda u 6 ga bo`linmaydi.

b) Uchta sonning ko`paytmasi nolga teng faqat va faqat qachonki, ulardan bittasi nolga teng bo`lsa.

c) Agar funksiyaning hosilasi nuqtada nolga teng va bu funksiyaning shu nuqtada ikkinchi hosilasi manfiy bo`lsa, u holda bu berilgan nuqta funksiyaning local maksimum nuqtasi bo`ladi.

d) Agar to`g`ri chiziq ikkita kesishuvchi tekisliklarning har biriga parallel bo`lsa, u holda ularning kesishish chizig`iga ham parallel bo`ladi.

e) Agar l tog`ri chiziq $\pi$ tekislikda yotuvchi ikkita $a$ va $b$ tog`ri chiziqlarga perpendikulyar bo`lsa ( tasdiq $A$), va $a$ va $b$ tog`ri chiziqlar parallel bo`lmasa $a \nparallel b$ (tasdiq $B$) , u holda $l$ to`g`ri chiziq $\pi$ tekislikda yotgan ixtiyoriy $c$ to`g`ri chiziqqa perpendikulyar bo`ladi (tasdiq C).

f) Agar l tog`ri chiziq $\pi$ tekislikda yotuvchi ikkita $a$ va $b$ tog`ri chiziqlarga perpendikulyar bo`lsa ( tasdiq $A$), va $\pi$ tekislikda yotuvchi biror $c$ tog`ri chiziqqa perpendikulyar bo`lmasa ( tasdiq $\bar{C}$ ), u holda $a$ va $b$ to`g`ri chiziqlar parallel ($a \parallel b$ - tasdiq $\bar{B}$).

g) Agar π tekislikda yotuvchi a va b to`g`ri chiziqlar parallel bo`lmasa $a \nparallel b$ (tasdiq $B$ ) va $l$ to`g`ri chiziq $\pi$ tekislikda yotuvchi $c$ to`g`ri chiziqqa perpendikulyar bo`lmasa (tasdiq $\bar{C}$), u holda l to`g`ri chiziq a yoki b to`g`ri chiziqqa perpendikulyar emas. (tasdiq $\bar{A}$)

h) Agar uchta $\vec{a},\vec{b},\vec{c}$ vektorlardan biror ikkitasi kolleniar bo`lsa, u holda ularning aralash ko`paytmasi nolga teng bo`ladi $\left([\vec{a},\vec{b}],\vec{c}\right)=0$.

i) Biror musbat sonning logarifmi musbat bo`ladi, agarda logarifm asosi va logarifmlanadigan son 1 dan katta bo`lsa yoki logarifm asosi va logarifmlanadigan son 0 va 1 oralig`ida yotsa.

j) Agar parallelogramda barcha burchaklari to`g`ri bo`lmasa yoki barcha tomonlari teng bo`lmasa, u holda bu parallelogram to`g`ri to`rt burchak yoki romb emas.

k) Agar uchburchakning ixtiyoriy medianasi balandlik va bissektrisa bo`lmasa, u holda bu uchburchak teng yonli va teng tomonli emas.

Yechish. k) Murakkab mulohazani elementar mulohazalarga ajratamiz va quyidagicha belgilaymiz:

A: Uchburchakning biror medianasi balandligi ham bo`ladi”;

B: “ Uchburchakning biror medianasi bissektrisasi ham bo`ladi”;

C: “ Bu uchburchak teng yonli”;

D: “ Bu uchburchak teng tomonli”.

U holda berilgan mulohaza simvolik quyidagicha yoziladi:

$(\bar{A}\wedge\bar{B})\to(\bar{C}\wedge\bar{D}).$

2.15. Berilgan uchta A,B,C mulohazalardan quyidagi shartlarni qanoatlantiradigan mulohazalar tuzing:

a) Rost faqat va faqat qachonki, barcha berilgan mulohazalar rost bo`lsa;

b) Yolg`on faqat va faqat qachonki, barcha berilgan mulohazalar yolg`on bo`lsa;

c) Rost faqat va faqat qachonki, barcha berilgan mulohazalar yo`lg`on bo`lsa;

d) Yolg`on faqat va faqat qachonki, barcha berilgan mulohazalar rost bo`lsa;

e) Rost faqat va faqat qachonki, A va B mulohazalar rost bo`lsa;

f) Rost faqat va faqat qachonki, A va B mulohazalar yolg`on bo`lsa;

g) Yolg`on faqat va faqat qachonki, $A$ va $B$ mulohazalar rost bo`lsa;

h) Yolg`on faqat va faqat qachonki, $A$ va $B$ mulohazalar yolg`on bo`lsa;

i) Rost faqat va faqat qachonki, barcha mulohazalar rost yoki barcha mulohazalar yolg`on bo`lsa;
j) yolg`on faqat va faqat qachonki, barcha mulohazalar rost yoki barcha mulohazalar yolg`on bo`lsa;
k) yolg`on faqat va faqat qachonki, faqat $C$ mulohaza yolg`on bo`lsa.
Yechish. k) Qidirilayotgan mulohaza yolg`on bo`lishi lozim faqat bitta holda, qachonki, C yolg`on, $A$ va $B$ mulohazalarning ikkialasi ham rost bo`lsa. Shunday qilib, bu qidirilayotgan mulohaza $M \to C$ ko`rinishdagi mulohaza bo`lishi mumkin, bu yerda A va B mulohazalardan tuzilgan M mulohaza rost bo`lishi lozim. Agar $A$ va $B$ mulohazalardan birortasi yolg`on bo`lsa, u holda $M$ mulohaza ham yolg`on bo`ladi. Tushunarliki, $M$ mulohaza sifatida kon`yunksiya $A \wedge B$ olish kerak bo`ladi. Shunday qilib, qidirilayotgan mulohaza quyidagi ko`rinishda bo`ladi: $(A \wedge B) \to C$.

2.16. Barcha oldingi mulohazalarning mantiqiy qiymatidan kelib chiqqan holda, oxirgi mulohazaning mantiqiy qiymatini aniqlang.

$a)$ $A \to B = 1,\ A \leftrightarrow B = 0,\ \ B \to A =$;

$b)$ $A \to B = 1,\ (\bar{A} \wedge B) \to (\bar{A} \vee B) =$;

c) $A \leftrightarrow B = 0,\ \ \bar{B} \to A =$;

$d)$ $A \wedge B = 0,\ A \to B = 1,\ B \to \bar{A} =$;

$e)$ $A \leftrightarrow B = 0,\ A \to B = 1,\ (\bar{A} \to B) \leftrightarrow A =$;

f) $A \vee B = 1,\ A \to B = 1,\ \bar{B} \to A =$;

$g)$ $A \wedge B = 0,\ A \leftrightarrow B = 0,\ A \to B = 1,\ A =$;

$h)$ $A \wedge B = 0,\ A \leftrightarrow B = 0,\ A \to B = 1,\ B =$;

i) $A \wedge B = 0,\ A \vee B = 1,\ A \to B = 1,\ B \to A =$;

j) $A \to (B \leftrightarrow A) = 0,\ A \to B =$;

k) $(A \vee B) \to A = 1,\ A \to B = 1,\ \bar{A} \leftrightarrow \bar{B} =$;

l) $A \leftrightarrow B = 1,\ (A \to B) \wedge (\bar{A} \to \bar{B}) =$.

Yechish. k) Birinchi $((A \vee B) \to A) = 1$ shartdan $(A \vee B) = 1$ va $A = 0$ bo`lishi mumkin emas ekanligi kelib chiqadi, ya`ni $A = 0$ bo`lsa, u holda $B = 1$ bo`lishi lozim. Ikkinchi $A \to B = 1$ shartdan $A = 1$ va $B = 0$ holni bo`lishi mumkin emas ekanligi kelib chiqadi. Shunday qilib, $A$ va $B$ mulohazalar bir xil mantiqiy qiymatga ega. Demak, ularning inkorlari $\bar{A}$ va $\bar{B}$ lar ham bir xil mantiqiy qiymat qabul qiladi. Bu holda $\bar{A} \leftrightarrow \bar{B}$ mulohaza rost bo`ladi.
l) $A \leftrightarrow B$ shartdan A va B mulohazalar bir xil mantiqiy qiymatga ega ekanligi kelib chiqadi. U holda ularning inkorlari $\bar{A}$ va $\bar{B}$ lar ham bir xil mantiqiy qiymat qabul qiladi.Demak, ikkala implikatsiya $A \to B$ va $\bar{A} \to \bar{B}$ rost qiymat qabul qiladi. Shunday qilib, oxirgi ikkita mulohazaning konyunksiyasi rost qiymat qabul qiladi.

2.17. Quyida joylashgan har bir muohaza uchun mantiqiy qiymatini bir qiymatli aniqlashda keltirilgan ma`lumotlar yetarli ekanligini aniqlang ( agar yetarli bo`lsa, bu qiymatni ko`rsating, agar yetarli bo`lmasa, har xil qiymatlar qabul qilishini misollarda ko`rsating):

$a)$ $A \wedge (B \to C),\ B \to C = 0$;

$b)$ $A \vee (B \to C),\ B = 0$;

$c)$ $\overline{(A \vee B)} \leftrightarrow (\bar{B} \wedge \bar{A}),\ A = 1$;

$d)$ $(A \to B) \to (\bar{A} \to \bar{B}),\ B = 1$;

$e)$ $(A \wedge B) \to (A \vee C),\ A = 1$;

$f)$ $\overline{(B \to A)} \leftrightarrow \overline{(A \vee C)},\ A = 1$;

g) $(A \leftrightarrow B) \vee (A \wedge C),\ A = 0$;

$h)$ $\overline{(A \to B)} \to (A \wedge \bar{B}) \vee C, A \to B = 0$;

i) $(A \wedge \bar{C}) \leftrightarrow (\bar{A} \vee \bar{C}) \to (B \wedge D),\ A \vee B = 0$;

j) $A \to (B \leftrightarrow C),\ B = 1$;

k) $A \to (B \vee \bar{C}) \leftrightarrow C,\ C = 0$;

$l)$ $(A \wedge B) \to ((\bar{A} \leftrightarrow C) \wedge (B \vee C)),\ A \wedge B = 1$.

Yechish. k) $C=0$ ekanligidan, $\bar{C}=1$ kelib chiqadi va demak, $B\vee\bar{C}=1$ bo`lib, bu qiymat $B$ ning qiymatiga bog`liq emas. Bundan esa $A\to(B\vee\bar{C})=1$ kelib chiqadi, bu qiymat A ning qiymatiga bog`liq emas. Nihoyat, $(A\to(B\vee\bar{C}))\leftrightarrow C=1\leftrightarrow 0=0$ ni topamiz.Shunday qilib, agar $C-$ yolg`on mulohaza bo`lsa, u holda berilgan butun mulohaza ham yolg`on qiymat qabul qiladi, ya`ni berilgan ma`lumotlar berilgan murakkab mulohazaning qiymatini qaniqlash uchun yetarli.

l) $A\wedge B=1$ ekanligidan, $A=1$ va $B=1$ ekanligi kelib chiqadi. U holda $B\vee C=1$ bo`lib, bu qiymat $C$ ning qiymatiga bog`liq emas.Ikkinchi tomondan, $\bar{A}=0$ ekanligidan, $\bar{A}\leftrightarrow C$ mulohazaning qiymati haqida aniq bir fikr ayta olmaymiz. U $C=0$ da rost va $C=1$ da yolg`on qiymat qabul qilishi mumkin. Birinchi holda , ya`ni $C=0$ da berilgan mulohaza rost , ikkinchi holda, ya`ni $C=1$ da yolg`on. Shunday qilib, berilgan mulohazaning qiymatini bir qiymatli aniqlash uchun keltirilgan ma`lumot etarli emas.

2.18. Quyidagi sharlarni bir vaqtda bajaradigan uchta A,B,C mulohazalar mavjudmi:

$a)\ A\wedge B=1,\ A\wedge C=0,\ A\wedge B\wedge\bar{C}=0;$

$b)\ B\to A=1,\ A\vee C=0,\ A\leftrightarrow(B\wedge\bar{C})=0;$

$c)\ A\vee B=0,\ \bar{B}\wedge C=1,\ (A\vee\bar{C})\leftrightarrow(\bar{B}\to\bar{C})=1;$

$d)\ A\wedge\bar{B}=1,\ B\vee C=1,\ \overline{(B\to A)}\vee C=0;$

$e)\ \bar{A}\wedge B=0,\ B\vee C=0,\ (A\vee B)\vee\bar{C}=1;$

$f)\ A\vee B=0,\ B\vee C=1,\ (\mathrm{C}\to A)\vee(\mathrm{C}\to\mathrm{B})=1;$

g) $\mathrm{A}\to\mathrm{B}=0,\ \mathrm{A}\to\mathrm{C}=1,\ (\mathrm{C}\to A)\vee(\mathrm{C}\to\mathrm{B})=1;$

$h)\ A\vee C=1, A\vee B=0,\ C\to(A\vee B)=1;$

$i)\ B\vee C=0,\ \bar{C}\to A=0,\ A\to B=0;$

$j)\ A\wedge C=1,\ C\leftrightarrow\bar{B}=0,\ A\to B=1;$

$k)\ A\vee\bar{B}=0,\ B\to(A\vee C)=0,\ \bar{C}\to\bar{B}=1.$

Yechish. k) Birinchi shartdan diz`yunksiya ta`rifiga ko`ra $A=0$ va $\bar{B}=0$, ya`ni $B=1$. U holda ikkinchi berilgan shartdan, implikatsiya ta`rifiga ko`ra $A\vee C=0$ kelib chiqadi. Bu yerdan $C=0$. Shunday qilib, $\bar{C}\to\bar{B}=\bar{0}\to\bar{1}=1\to 0=0$, bu esa berilgan uchinchi shartga zid. Demak, berilgan shartlarni qanoatlantiruvchi uchta A,B,C mulohazalar mavjud emas.

2.19. Quyidagi mulohazalarning qaysilari uchun , ularning qiymatlari A mulohazaning mantiqiy qiymatiga bog`liq emas:

$a)\ A\wedge 0;$ d) $\mathrm{A}\vee\bar{\mathrm{A}};$ g) $\mathrm{A}\to\bar{\mathrm{A}};$ j) $\bar{\mathrm{A}}\to\mathrm{A};$

$b)\ A\to 1;$ e) $\mathrm{A}\vee 0;$ h) $\mathrm{A}\wedge 1;$ k) $\mathrm{A}\vee 1;$

$c)\ A\to A;$ f) $0\to\mathrm{A};$ i) $\mathrm{A}\leftrightarrow\mathrm{A};$ l) $\mathrm{A}\leftrightarrow\bar{\mathrm{A}}.$

Yechish. k) Ta`rifga ko`ra ikkita mulohazalarning diz`yunksiyasi rost bo`ladi faqat va faqat qachonki, ulardan kamida bittasi rost bo`lsa. Demak, $A\vee 1$ mulohaza $A$ mulohazaning mantiqiy qiymatiga bog`liq bo`lmagan holda rost bo`ladi.

l) Tushunarliki, A va $\bar{A}$ mulohazalar qarama-qarshi mantiqiy qiymatlarni qabul qiladi (ya`ni 0 va 1 yoki 1 va 0 ). Ta`rifga ko`ra ikkita mulohazalar ekvivalentligi rost bo`ladi faqat va faqat qachonki, bu mulohazalar bir xil mantiqiy qiymat qabul qilsa. U holda $A\leftrightarrow\bar{A}$ mulohaza yolg`on va $A$ mulohazaning qanday mantiqiy qiymatiga bog`liq emas.

2.20. Ikkita implikatsiyalar sistemasining teskarilanishi haqidagi teoremani isbotlang: Agar $A_1\to B_1, A_2\to B_2, A_1\vee A_2,\ \overline{(B_1\wedge B_2)}$ mulohazalar rost bo`lsa, u holda $B_1\to A_1, B_2\to A_2$ mulohazalar ham rost bo`ladi.

2.21. m ta implikatsiyalar sistemasining teskarilanishi haqidagi teoremani isbotlang (to`la diz`yunksiya prinsipi):

Agar $A_1 \to B_1, A_2 \to B_2, \ldots, A_m \to B_m,\ A_1 \vee A_2 \vee \ldots \vee A_m,\ \overline{(B_1 \wedge B_2 \wedge \ldots \wedge B_m)}$ mulohazalar rost bo`lsa, u holda $B_1 \to A_1, B_2 \to A_2, \ldots B_m \to A_m$ mulohazalar ham rost bo`ladi.

2.2.Mulohazalar algebrasi formulalari.

Propositsional o`zgaruvchlar deb shunday o`zgaruvchilarni aytamizki, ular o`rniga aniq (konkret) mulohazalarni qo`yish mumkin bo`lsa. Bu o`zgaruvchilarni $P, Q, R, S, \ldots,\ P_1, P_2, P_3, \ldots,$ $X, Y, Z, X_1, X_2, X_3, \ldots$ lar orqali belgilaymiz.

Mulohazalar algebrasi formulasi quyidagicha (induktiv tarzda) aniqlanadi:

a) Har bir propozitsional o`zgaruvchi formula;
b) Agar A va B formula bo`lsa, u holda $\bar{A}$, $(A \wedge B)$, $(A \vee B)$, $(A \to B)$, $(A \leftrightarrow B)$ ifodalar ham formulalardir;
c) Ikkita oldingi bandlardagi qoidalar bo`yicha qurilgan formulalardan tashqari formulalar yo`q.

Odatda, formulalarda tashqi qavsni yozmaslikka kelishiladi.

Qism formula deb formulaning shunday ixtiyoriy bo`lagiga(qismiga) aytiladiki, uning o`zi ham formula bo`ladi.

Agar $F(X_1, X_2, \ldots, X_n) - X_1, X_2, \ldots, X_n$ propozitsinal o`zgaruvchilarni o`z ichiga olgan mulohazalar algebrasi formulasi va $A_1, A_2, \ldots, A_n -$ biror aniq mulohazalar bo`lsa, u holda oxirgilarni berilgan formulaning mos propozitsional o`zgaruvchilar o`rinlariga qo`yib murakkab $F(A_1, A_2, \ldots, A_n)$ mulohazani hosil qilamiz. $F(A_1, A_2, \ldots, A_n)$ bu mulohazaning mantiqiy qiymatini aniqlash uchun $A_1, A_2, \ldots, A_n$ mulohazalarning mantiqiy qiymatlarini (0 yoki 1) ular o`rniga qo`yib, so`ngra bu belgilar ustida ketma-ket formuladagi amallarni bajaramiz. Masalan, agar $A_1 = 1,\ A_2 = 0,\ A_3 = 1$ bo`lsa, u holda $((A_1 \to A_2) \wedge \bar{A}_3)$ murakkab mulohazaning mantiqiy qiymati $((1 \to 0) \wedge \bar{1}) = 0 \wedge 0 = 0$ bo`ladi. Bu holda $((X_1 \to X_2) \wedge \bar{X}_3)$ formula unga kiruvchi $X_1, X_2, X_3$ o`zgaruvchilarning mos ravishda $1, 0, 1$ qiymatida 0 qiymat qabul qiladi.

Ko`p hollarda, mulohazalar algebrasi mulohazasining tuzilishi bilan umuman qiziqmaydi, ularning faqat rost yoki yolg`on bo`lish xossalari nuqtai nazaridan o`rganadi. Shu sababli har bir yolg`on mulohazani 0 element kabi, rost mulohazani esa 1 element kabi qarash mumkin.

$F(X_1, X_2, \ldots, X_n)$ formula bajariluvchi (rad etuvchi) deyiladi, agarda shunday $A_1, A_2, \ldots, A_n$ qiymatlar satri mavjud bo`lsaki, $F(A_1, A_2, \ldots, A_n)$ mulohaza rost(yolg`on) bo`lsa.

Formula aynan rost yoki tavtologiya (aynan yolg`on, yoki qarama-qarshilikli) deyiladi,agarda o`zgaruvchilarning barcha qiymatlarida rost(yolg`on) qiymat qabul qilsa.Tavtologiya $\models F(X_1, X_2, \ldots, X_n)$ ko`rinishda belgilanadi.

2.22. Quyidagi belgilar ketma-ketligi formula bo`lishini aniqlang:

a) (PQ);
b) $((P \leftrightarrow Q) \wedge R) \to (P \vee R)$;
c) $((\bar{P} \to Q) \to (R \wedge (Q \vee S)))$;
d) $((P \vee \bar{Q}) \to (R\bar{S}))$;
e) $(P \to (Q \wedge R \to \bar{P}))$;
f) $\overline{((\bar{P} \wedge \bar{Q}) \to (P \vee (R \wedge \bar{S})))}$;
g) $((P \vee \bar{Q}) \to (\bar{P} \wedge \bar{R} \wedge (Q \leftrightarrow R)))$;
h) $P \to Q \to R$;
i) $\bar{\bar{P}} \to P$;
j) $(P \to Q) \vee (Q \to P)$;
k) $((P \wedge Q) R) \to \bar{S}$;
l) $((P \wedge (\bar{Q} \to R)) \vee ((\bar{P} \leftrightarrow R) \wedge \bar{Q}))$.

Yechish. k) Berilgan ketma-ketlik formula bo`lmaydi. Haqiqatan ham, formulaning a) bandiga ko`ra P, Q va R propozitsional o`zgaruvchilar formulalar bo`ladi. Formula ta`rifining b) bandiga ko`ra $((P \wedge Q)R)$ ketma-ketlik formula bo`lmaydi, chunki unga kirgan $(P \wedge Q)$ va $R$ formulalar hech bir ma`lum belgilar: $\wedge, \vee, \rightarrow$ yoki $\leftrightarrow$ bilan bog`lanmagan. Shu sababli berilgan ketma-ketlik formula bo`lmaydi.

l) Formulaning ta`rifidagi a) va b) bandlarga ko`ra P, Q, R propzitsional o`zgaruvchilar va $\bar{P}, \bar{Q}, (\bar{Q} \rightarrow R), (\bar{P} \leftrightarrow R)$ ifodalar formula bo`ladi. Keyin esa $(P \wedge (\bar{Q} \rightarrow R)), ((\bar{P} \leftrightarrow R) \wedge \bar{Q})$ ifodalar ham formulalar bo`ladi.

Nihoyat, berilgan ketma-ketlik bo`lgan $((P \wedge (\bar{Q} \rightarrow R)) \vee ((\bar{P} \leftrightarrow R) \wedge \bar{Q}))$ ifoda ham formula bo`ladi.

2.23. Quyidagi belgilar ketma-ketligiga barcha mumkin bo`lgan usullar bilan qavslarni shunday qo`yingki, natijada formula hosil bo`lsin:

$a)\ P \rightarrow Q \wedge \bar{R} \vee S;$ g) $\mathrm{P} \leftrightarrow \mathrm{Q} \wedge \bar{\mathrm{R}} \rightarrow \mathrm{S};$

$b)\ P \rightarrow \bar{Q} \vee R \rightarrow \bar{P} \rightarrow \bar{R};$ h) $\mathrm{P} \vee \mathrm{Q} \wedge \bar{\mathrm{R}} \wedge \mathrm{P} \vee \mathrm{R};$

$c)\ \bar{P} \wedge Q \rightarrow R;$ i) $\bar{\mathrm{P}} \wedge \mathrm{Q} \vee \mathrm{R} \rightarrow \mathrm{Q};$

$d)\ P \vee \bar{Q} \rightarrow \bar{R} \wedge Q;$ j) $\bar{\mathrm{P}} \wedge \mathrm{Q} \rightarrow \bar{\mathrm{P}} \vee \mathrm{R};$

$e)\ \bar{P} \vee R \wedge Q \vee R;$ k) $\bar{\mathrm{P}} \leftrightarrow \bar{\mathrm{Q}} \vee \mathrm{R} \wedge \mathrm{Q}.$

$f)\ \bar{P} \vee R \vee P \rightarrow Q;$

Yechish. k) So`ralgan formulalar (tashqi qavs tashlab yozilgan):

$(\bar{P} \leftrightarrow \bar{Q}) \vee (R \wedge Q);$ $\overline{(P \leftrightarrow \overline{((Q \vee R) \wedge Q)})};$ $\bar{P} \leftrightarrow (\overline{(Q \vee R)} \wedge Q);$

$(\bar{P} \leftrightarrow (\bar{Q} \vee R)) \wedge Q;$ $\overline{((P \leftrightarrow \bar{Q}) \vee (R \wedge Q))};$ $\overline{(P \leftrightarrow ((\bar{Q} \vee R) \wedge Q))};$

$(\bar{P} \leftrightarrow \overline{(Q \vee R)}) \wedge Q;$ $\overline{((P \leftrightarrow \overline{(Q \vee R)}) \wedge Q)};$ $\overline{(P \leftrightarrow (\bar{Q} \vee (R \wedge Q)))};$

$\overline{(P \leftrightarrow (\bar{Q} \vee R))} \wedge Q;$ $\overline{(P \leftrightarrow (\overline{(Q \vee R)} \wedge Q))};$ $\overline{(P \leftrightarrow \overline{(Q \vee R)})} \wedge Q;$

$\overline{((P \leftrightarrow \bar{Q}) \vee R)} \wedge Q;$ $\overline{(P \leftrightarrow \bar{Q})} \vee (R \wedge Q);$ $\overline{(P \leftrightarrow \overline{(Q \vee (R \wedge Q))})}.$

$\overline{((P \leftrightarrow (\bar{Q} \vee R)) \wedge Q)};$ $\bar{P} \leftrightarrow ((\bar{Q} \vee R) \wedge Q);$

2.24. Quyidagi har bir formulaning barcha mumkin bo`lgan qism formulalarini yozing (formulalarda tashqi qavs tashlab yozilgan):

$a)\ ((P \vee Q) \vee \bar{R}) \wedge (\bar{P} \vee (\bar{Q} \vee R));$

$b)\ (P \rightarrow Q) \rightarrow ((P \rightarrow \bar{Q}) \rightarrow (P \wedge Q));$

$c)\ (P \wedge (Q \vee \bar{P})) \wedge ((\bar{Q} \rightarrow P) \vee Q);$

$d)\ \overline{(P \vee R)} \wedge (P \rightarrow Q);$

$e)\ (P \leftrightarrow Q) \wedge \overline{(R \rightarrow S)};$

$f)\ (P \vee (Q \wedge \bar{R})) \wedge (P \vee R);$

$g)\ ((P \vee Q) \rightarrow (R \rightarrow \bar{P})) \rightarrow (\bar{R} \rightarrow \bar{Q});$

$h)\ (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow \bar{R}) \rightarrow (P \rightarrow \bar{Q}));$

$i)\ P \vee (Q \rightarrow (R \leftrightarrow (P \wedge Q)));$

$j)\ (P \wedge ((Q \wedge R) \vee S)) \vee \bar{S};$

$k)\ ((P \leftrightarrow (Q \wedge \bar{R})) \vee (P \wedge Q)) \rightarrow (P \vee (\bar{Q} \wedge R).$

Yechish. k) Qism formula – bu berilgan formulaning shunday bo`lagiki, uning o`zi ham formula bo`ladi. Birinchidan, berilgan formulaga kiruvchi barcha propozitsional o`zgaruvchilar qism formula bo`ladi ( bu qism formulalar nol sondagi mantiqiy bog`lovchilarga ega): P, Q, R. Keyin esa bitta mantiqiy bog`lovchisi bo`lgan qism formulalar: $\bar{R},\ P \wedge Q,\ \bar{Q}$. Ikkita mantiqiy bog`lovchisi bo`lgan qism formulalar: $Q \wedge \bar{R},\ \bar{Q} \wedge R$ Uchta mantiqiy bog`lovchisi bo`lgan qism formulalar: $P\leftrightarrow(Q\wedge\bar{R})$, $\text{P}\vee(\bar{\text{Q}}\wedge\text{R})$. Berilgan formulada to`rtta mantiqiy bog`lovchisi bo`lgan qism formulalar yo`q. Beshta bog`lovchisi bo`lgan bitta qism formula bor: $(P\leftrightarrow(Q\wedge\bar{R}))\vee(\text{P}\wedge\text{Q})$.Nihoyat, yana bitta qism formula bu berilgan formulaning o`zi bilan ustma-ust tushadi. Shunday qilib, berilgan formuada 12 ta qism formula bor.

2.25. Quyidagi formulalar uchun rostlik jadvalini tuzing va bu formulalardan qaysilari bajariluvchi,qaysilari aynan rost (tavtologiya), qaysilari aynan yolg`on ekanligini ko`rsating:

$a)\ (P\to Q)\to((P\to\bar{Q})\to\bar{P});$

$b)\ ((P\to Q)\to P)\to Q;$

$c)\ (P\wedge(Q\vee\bar{P}))\wedge((\bar{Q}\to P)\vee Q);$

$d)\ ((P\wedge\bar{Q})\to Q)\to(P\to Q);$

$e)\ P\wedge(Q\wedge(\bar{P}\vee\bar{Q});$

$f)\ (((P\to Q)\to Q)\to Q)\to Q;$

$g)\ (((P\vee\bar{Q})\wedge(Q\vee R))\vee\bar{R})\vee Q;$

h) $(\text{P}\wedge(\text{Q}\vee\text{R}))\to((\text{R}\to(\text{P}\to\text{Q}))\leftrightarrow(\text{Q}\to(\text{R}\to\text{P})));$

i) $(((\text{P}\leftrightarrow\text{Q})\leftrightarrow(\text{P}\leftrightarrow\text{R}))\leftrightarrow(\text{Q}\leftrightarrow\text{R}))\leftrightarrow\text{P};$

j) $\overline{((\bar{R}\to\overline{(P\to\overline{(Q\to R)})})\to\overline{(P\to\bar{Q})})};$

$k)\ ((P\vee\bar{Q})\to Q)\wedge(\bar{P}\vee Q).$

Yechish. k) Mantiqiy bog`lovchilar (mulohazalar ustida amallar) ta`rifiga ko`ra berilgan formula uchun rostlik jadvalini tuzamiz ( bu formulaning mantiqiy qiymati jadvalning oxirgi ustunuda yoziladi, bu yerda $F(P,Q)$ orqali berilgan formulaning o`zi belgilangan):

P	Q	$\bar{Q}$	$P\vee\bar{Q}$	$(P\vee\bar{Q})\to Q$	$\bar{P}$	$\bar{P}\vee Q$	$F(P,Q)$
0	0	1	1	0	1	1	0
0	1	0	0	1	1	1	1
1	0	1	1	0	0	0	0
1	1	0	1	1	0	1	1

Tuzilgan rostlik jadvalidan ko`rinadiki, berilgan formula bajariluvchi. Shunday qilib, agar formulada $P$ propozitsional o`zgaruvchi o`rniga yolg`on mulohaza, Q propozitsinal o`zgaruvchi o`rniga rost mulohaza qo`ysak, bu butun mulohaza rost mulohazaga aylanadi. Agar masalan, formulada $P$ propozitsional o`zgaruvchi o`rniga rost mulohaza, $Q$ propozitsinal o`zgaruvchi o`rniga yolg`on mulohaza qo`ysak, bu butun mulohaza yolg`on mulohazaga aylanadi. Demak, berilgan formula aynan rost ham emas, aynan yolg`on ham emas.

2.26. Quyidagi formulalarni ularning rostlik jadvalini tuzmasdan, unga kiruvchi propozitsional o`zgaruvchilarga biror qiymatlar berib, ular rost mulohazalarga aylanishini ko`rsatib, bajariluvchi ekanliklarini isbotlang:

$a)\ \overline{(P \to \overline{P})};$

$b)\ (P \to Q) \to (Q \to P);$

$c)\ (Q \to (P \wedge R)) \wedge \overline{((P \vee R) \to Q)};$

$d)\ \overline{((P \leftrightarrow \overline{Q}) \vee R)} \wedge Q;$

$e)\ (((P \to Q) \to (R \to Q)) \to (R \to P)) \to (P \to Q);$

f) $((Q \to \overline{P}) \to P) \to (P \to (\overline{P} \to Q));$

g) $(P \to ((Q \to R) \to R)) \to ((P \to (Q \to R)) \to (P \to R));$

h) $((P \to Q) \wedge (Q \to R)) \to (\overline{R} \to \overline{P});$

i) $((P \leftrightarrow Q) \wedge (Q \leftrightarrow R)) \to (P \vee R);$

j) $((P \wedge \overline{Q}) \vee (\overline{P} \wedge Q)) \leftrightarrow (P \leftrightarrow Q);$

k) $(P \wedge Q) \to ((R \vee Q) \to (Q \wedge \overline{Q})).$

Yechish. k) Ikkinchi implikatsiyaning xulosasi,ravshanki, aynan yolg`on formula. Shu sababli agar $R \vee Q$ shart ikkinchi implikatsiyada biror o`rniga qo`yishda yolg`on mulohazaga aylansa, u holda bu mulohaza rost mulohazaga aylanadi va demak, butun berilgan mulohaza $P \wedge Q$ shart qanday mulohazaga bog`liqmas holda rost mulohazaga aylanadi. $R \vee Q$ shart ikkinchi implikatsiyada biror o`rniga qo`yishda yolg`on mulohazaga aylanadi, agarda R va Q o`zgaruvchilar o`rniga yolg`on mulohaza qo`ysak. Shunday qilib, berilgan mulohaza bajariluvchi, chunki, agar R va Q o`zgaruvchilar o`rniga yolg`on mulohaza, $P$ ixtiyoriy mulohaza qo`ysak, u rost mulohazaga aylanadi (bu holdagi uning rost ekanligi butun mulohazaning rostlik qiymatiga ta`sir eta olmaydi).

2.27. Quyidagi formulalarni ularning rostlik jadvalini tuzmasdan, unga kiruvchi propozitsional o`zgaruvchilarga biror qiymatlar berib, ular rost mulohazalarga aylanishini ko`rsatib, bajariluvchi ekanliklarini isbotlang:

$a)\ ((X \to (Y \wedge Z)) \to (\overline{Y} \to \overline{X})) \to \overline{Y};$

$b)\ ((X \vee Y) \vee Z) \to ((X \vee Y) \wedge (X \vee Z));$

$c)\ ((X \vee Y) \wedge ((Y \vee Z) \wedge (Z \vee X))) \to ((X \wedge Y) \wedge Z);$

$d)\ (X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z) \vee (U \wedge V) \vee (U \wedge W) \vee (V \wedge W) \wedge (\overline{X} \wedge \overline{U});$

$e)\ ((\overline{P} \to P) \to P) \to \overline{((\overline{Q} \to \overline{P}) \to ((\overline{Q} \to P) \to Q))};$

$f)\ (((P \to Q) \to (R \to Q)) \to (R \to P)) \to (P \to Q);$

$g)\ ((Q \to \overline{P}) \to P) \to (P \to (P \to \overline{Q}));$

$h)\ (P \to ((Q \to R) \to R)) \to ((P \to (Q \to R)) \to (P \to \overline{R}));$

$i)\ ((P \to Q) \wedge (R \to \overline{Q}) \wedge (P \vee R)) \to R;$

$j)\ ((P \wedge \overline{Q}) \wedge (Q \to R) \wedge (R \vee \overline{S})) \to (S \wedge Q);$

$k)\ (X \vee Y) \to ((\overline{X} \wedge Y) \vee (X \wedge \overline{Y})).$

Yechish. k) Implikatsiya yolg`on bo`ladi faqat qachonki, uning shart rost, xulosasi esa yolg`on bo`lsa. Bizning implikatsiyamiz xulosasi diz`yunksiya bo`lib, u faqat va faqat ikkala qo`shiluvchisi ham yolg`on b`olgandagina yo`lg`on bo`ladi. Shunday qilib, bizning formulamiz yolg`on mulohazaga aylanadi, agarda shunday $A$ va $B$ mulohazalar topilsaki, $A \vee B$ rost, ikkala $\overline{A} \wedge B$ va $A \wedge \overline{B}$ mulohazalar yolg`on b`olsa. Agar $A$ va $B$ mulohazalar har xil rostlik qiymatiga ega bo`lsa, u holda $\overline{A} \wedge B$ va $A \wedge \overline{B}$ mulohazalarning ikkalasi ham yolg`on mulohaza bo`lishi mumkin emas (nima uchun?). Shu sababli A va B mulohazalarning har ikkalasi rost yoki har ikkalasi yolg`on. Ammo agar $A$ va $B$ mulohazalarning har ikkalasi yolg`on bo`lsa, u holda $A \vee B$ mulohaza yolg`on bo`ladi, bu bizni qanoatlantirmaydi. Demak, $A$ va $B$ mulohazalar rost bo`lishi lozim. Shunday qilib, biz berilgan formula faqat va faqat X va Y o`zgaruvchilar o`rniga rost qiymat qo`yganda yolg`on mulohazaga aylanishini isbotladik.

2.3.Mulohazalar algebrasi tavtologiyalari.

2.28. Quyidagi formulalar uchun rostlik jadvalini tuzib, ularni tavtologiya ekanligini isbotlang:

a) $P \vee \bar{P}$ (uchinchini chiqarish qonuni);

b) $\overline{(P \wedge \bar{P})}$ (qarama-qarshilikni inkor qonuni);

c) $\bar{\bar{P}} \to P$ (ikki marta inkor qonuni);

d) $P \to P$ (ayniylik qonuni);

e) $(P \to Q) \leftrightarrow (\bar{Q} \to \bar{P})$ (kontrpozitsiya qonuni);

f) $((P \to Q) \wedge (Q \to R)) \to (\mathrm{P} \to \mathrm{R})$ (zanjirli xulosa qonuni);

g) $(P \leftrightarrow Q) \leftrightarrow (\bar{Q} \leftrightarrow \bar{P})$ (qarama-qarshilik qonuni);

h) $(P \wedge Q) \leftrightarrow (Q \wedge P)$ (kon`yunksiyaning kommutativligi);

i) $(P \vee Q) \leftrightarrow (Q \vee P)$ (diz`yunksiyaning kommutativligi);

j) $((P \wedge Q) \wedge \mathrm{R}) \leftrightarrow (P \wedge (Q \wedge \mathrm{R}))$ (kon`yunksiyaning assosiativligi);

k) $((P \vee Q) \vee \mathrm{R}) \leftrightarrow (P \vee (Q \vee \mathrm{R}))$ (diz`yunksiyaning assosiativligi);

l) $((P \wedge (Q \vee \mathrm{R})) \leftrightarrow ((P \wedge Q) \vee (P \wedge \mathrm{R}))$ (kon`yunksiyaning diz`yunksiyaga nisbatan distributivligi);

m) $((P \vee (Q \wedge \mathrm{R})) \leftrightarrow ((P \vee Q) \wedge (P \vee \mathrm{R}))$ (diz`yunksiyaning kon`yunksiyaga nisbatan distributivligi);

n) $(P \wedge P) \leftrightarrow P$ (kon`yunksiyaning idempotentligi);

o) $(P \vee P) \leftrightarrow P$ (diz`yunksiyaning idempotentligi)

p) $(P \leftrightarrow Q) \leftrightarrow ((P \to Q) \wedge (Q \to P))$;

q) $(P \to Q) \leftrightarrow (\bar{P} \vee Q)$;

r) $(P \wedge (Q \vee P) \leftrightarrow P$ (birinchi yutish qonuni);

s) $(P \vee (Q \wedge P) \leftrightarrow P$ (ikkinchi yutish qonuni);

t) $\overline{(P \wedge Q)} \leftrightarrow (\bar{P} \vee \bar{Q})$ (birinchi de Morgan qonuni);

u) $\overline{(P \vee Q)} \leftrightarrow (\bar{P} \wedge \bar{Q})$ (ikkinchi de Morgan qonuni);

v) $(P \vee Q) \leftrightarrow (\bar{P} \to Q)$.

Yechish. e) Berilgan formula uchun rostlik jadvalini tuzamiz:

P	Q	$P \to Q$	$\bar{Q}$	$\bar{P}$	$\bar{Q} \to \bar{P}$	$(P \to Q) \leftrightarrow (\bar{Q} \to \bar{P})$
0	0	1	1	1	1	1
0	1	1	0	1	1	1
1	0	0	1	0	0	1
1	1	1	0	0	1	1

Javvaldan ko`rinadiki, propozitsional o`zgaruvchilar P va Q lar o`rniga qanday rostlik qiymatlarini qo`ymaylik berilgan formula rost mulohazaga aylanadi. Demak, bu formula tavtologiya.

2.29. Quyidagi formulalarga mos rostlik jadvalini tuzib, ularni tavtologiya ekanligini isbotlang:

a) $P \to (Q \to P)$;

b) $(P \to Q) \to ((P \to (Q \to R)) \to (\mathrm{P} \to \mathrm{R}))$;

c) $P \to (Q \to (P \wedge Q))$;

d) $P \to (P \vee Q)$;
e) $(P \wedge Q) \to P$;
f) $(P \to (Q \wedge R)) \leftrightarrow ((P \to Q) \wedge (P \to R))$;
g) $((P \to Q) \wedge (P \to \bar{Q})) \to \bar{P}$
h) $(P \to R) \to ((Q \to R) \to ((P \vee Q) \to R))$;
i) $(P \to Q) \to ((P \to \bar{Q}) \to \bar{P})$;
j) $\bar{\bar{P}} \to P$;
k) $(P \to Q) \vee (Q \to P)$;
l) $(P \to Q) \to ((Q \to P) \to (P \leftrightarrow Q))$;
m) $((P \to Q) \to P) \to P$;
n) $(P \leftrightarrow Q) \to (P \to Q)$;
o) $(P \to R) \to ((P \vee Q) \to (R \vee Q))$;
p) $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$;
q) $(F_1 \to (F_2 \to G)) \leftrightarrow ((F_1 \wedge F_2) \to G)$;
r) $(F_1 \to (F_2 \to \ldots \to (F_m \to G)\ldots)) \leftrightarrow ((F_1 \wedge F_2 \ldots \wedge \mathrm{F}_m) \to G)$;
s) $(P \to (Q \to R)) \leftrightarrow (Q \to (P \to R))$;
t) $(\bar{P} \to P) \to P$;
u) $((P \wedge Q) \to R) \to (P \wedge (Q \to R))$;
v) $(P \to (Q \to R)) \leftrightarrow (Q \to (P \to R))$.

2.30. Quyidagi formulalarning tavtologiya bo`lishini isbotlang.

$a)$ $(\bar{P} \to (Q \wedge \bar{Q})) \to P$;
$b)$ $((\bar{P} \to Q) \wedge (\bar{P} \to \bar{Q})) \to P$;
$c)$ $((P \wedge \bar{Q}) \to (R \wedge \bar{R})) \to (P \to Q)$;
$d)$ $(P \wedge (P \to Q)) \to Q$;
$e)$ $((P \to Q) \wedge \bar{Q}) \to \bar{P}$;
$f)$ $((P \to Q) \wedge (Q \to P)) \to (P \to R)$;
$g)$ $((P \to Q) \wedge (R \to S) \wedge \overline{(Q \vee S)}) \to \overline{(P \vee R)}$;
$h)$ $((P \to Q) \wedge (R \to S)) \to ((P \wedge R) \to (Q \wedge S))$;
$i)$ $((P \to Q) \vee R) \leftrightarrow (P \to (Q \vee R))$;
$j)$ $P \to (Q \to ((P \vee Q) \to (P \wedge Q)))$.

2.31. Agar F va $F \to G$ formulalar tavtologiya bo`lsa, u holda $G$ formula ham tavtologiya bo`lishini isbotlang, ya`ni agar $\models F$ va $\models F \to G$ bo`lsa, u holda $\models G$ bo`ladi. (Bu keltirib chiqarish qoidasi "modus ponens" – MP deb ataladi)

2.32. Isbotlang:

a) agar $\models F \wedge G$, $\models F \leftrightarrow G$ bo`lsa, u holda $\models G \to H$;
b) agar $\models F \vee G$, $\models G \to H$ bo`lsa, u holda $\models F \vee H$;
c) agar $\models \bar{F} \to G$, $\models \bar{G} \vee \bar{H}$ bo`lsa, u holda $\models H \to F$;
d) agar $\models \bar{G} \wedge \bar{H}$, $\models F \vee G$ bo`lsa, u holda $\models \bar{F} \to H$;
e) agar $\models F \vee G$, $\models F \leftrightarrow G$ bo`lsa, u holda $\models$ G;
f) agar $\models F$, $\models F \leftrightarrow G$, $\models F \leftrightarrow H$ bo`lsa, u holda $\models G \wedge H$;
g) agar $\models \bar{F} \to G$, $\models \bar{G} \wedge \bar{H}$ bo`lsa, u holda $\models F \vee H$;
h) agar $\models F \leftrightarrow G$, $\models G \leftrightarrow H$ bo`lsa, u holda $\models F \leftrightarrow H$;

i) agar $\models F$, $\models G$, $\models H$ bo`lsa, u holda $\models F \to (G \to H)$;

j) agar $\models F \wedge G$, $\models G \to \bar{H}$ bo`lsa, u holda $\models F \wedge \bar{H}$;

k) agar $\models \bar{F} \vee G$, $\models \bar{G} \vee \bar{H}$ bo`lsa, u holda $\models F \to \bar{H}$;

l) agar $\models G \to F$, $\models (\bar{F} \wedge H) \leftrightarrow G$, $\models H$ bo`lsa, u holda $\models \bar{G} \wedge H$.

Yechish. k) $F(X_1,...,X_n), G(X_1,...,X_n), H(X_1,...,X_n)-$ formulalar masaladagi formulalar bo`lsin. Faraz qilaylik, $F \to \bar{H}$ formula tavtologiya bo`lmasin. Bu rost bo`lishini, $\overline{H(A_1,...,A_n)}$ esa yolg`on bo`lishini bildiradi. U holda $\overline{F(A_1,...,A_n)}$ yolg`on bo`ladi. Berilishiga ko`ra $\bar{F} \vee G$ formulaning tavtologiya ekanligidan, $G(A_1,...,A_n)$ mulohaza rost bo`ladi. Ikkinchi tomondan $\bar{G} \vee \bar{H}-$ tavtologiya, bundan $\overline{G(A_1,...,A_n)}$ rost ekanligi kelib chiqadi. Qarama-qarshilik hosil bo`ldi. Demak, $F \to \bar{H}$ formula tavtologiya.

l) faraz qilaylik, berilgan tasdiqni jo`natmasi to`g`ri, xulosasi esa noto`g`ri bo`lsin, ya`ni $G \to F$, $(\bar{F} \wedge H) \leftrightarrow G$ va $H$ formulalar tavtologiyalar bo`lsin, $\bar{G} \wedge H$ formula esa tavtologiya bo`lmasin. Oxirgi farazimiz shuni anglatadiki, shunday aniq $A_1,...,A_n$ mulohazalar topiladiki, $\overline{G(A_1,...,A_n)} \wedge H(A_1,...,A_n)$ yolg`on bo`ladi. Bu o`z navbatida $\overline{G(A_1,...,A_n)}$ va $H(A_1,...,A_n)$ mulohazalardan kamida bittasi yolg`on bo`lishini bildiradi. $H(A_1,...,A_n)$ yolg`on bo`lishi mumkin emas, chunki bu $H(A_1,...,A_n)$ ni aynan rost bo`lishiga zid. Demak, $\overline{G(A_1,...,A_n)}$ yolg`on mulohaza, bu holda $G(A_1,...,A_n)$ rost mulohaza. Agar shunday bo`ladigan bo`lsa, $G(A_1,...,A_n) \to F(A_1,...,A_n)$ mulohazaning rost ekanligidan $F(A_1,...,A_n)$ mulohazning rost ekanligi kelib chiqadi.

Endi $(\overline{F(A_1,...,A_n)} \wedge H(A_1,...,A_n)) \leftrightarrow G(A_1,...,A_n)$ rost mulohazaga qaraymiz, chunki, berilishiga ko`ra $(\bar{F} \wedge H) \leftrightarrow G$ formula tavtologiya. $F(A_1,...,A_n)$ formulaning rost ekanligidan ekvivalentlikning chap tomoni yolg`on mulohaza. Demak, bu ekvivalentlikning o`ng tomoni ya`ni $G(A_1,...,A_n)$ mulohaza ham yolg`on. Ammo bu yuqorida hosil qilingan rostlikka zid.

Shunday qilib, qilgan farazimiz qarama-qarshilikka olib keldi. Demak, farazimiz noto`g`ri, isbotlanayotgan tasdig`imiz o`rinli.

2.33. Aniqlashtiring, quyidagi tasdiqlar o`rinlimi (agar tasdiq o`rinli bo`lmasa, u holda aniqlashga harakat qilingki, ikkala tomonda yoki bir tomonida "faqat" va "faqat qachonki" bajarilmaydi):

a) $\models F \leftrightarrow G$ bo`ladi, faqat va faqat qachonki, $\models (F \to G) \wedge (G \to F)$ bo`lsa;

b) $\models F \vee G$ bo`ladi, faqat va faqat qachonki, $\models F$ yoki $\models G$ bo`lsa;

c) $\models F \leftrightarrow G$ bo`ladi, faqat va faqat qachonki, $\models (F \to G)$ va $\models G \to F$ bo`lsa;

d) $\models F \vee G$ bo`ladi, faqat va faqat qachonki, $\models F$ va $\models G$ bo`lsa;

e) $\models F \to G$ bo`ladi, faqat va faqat qachonki, $\models F$ bo`lsa;

f) $\models F \to G$ bo`ladi, faqat va faqat qachonki, $\models G$ bo`lsa;

g) $\models F \to G$ bo`ladi, faqat va faqat qachonki, $\models \bar{F}$ yoki $\models G$ bo`lsa;

h) $\models F \wedge G$ bo`ladi, faqat va faqat qachonki, $\models F$ va $\models G$ bo`lsa;

i) $\models F \leftrightarrow G$ bo`ladi, faqat va faqat qachonki, $\models F$ va $\models G$ bo`lsa;

j) $\models F \vee G$ bo`ladi, faqat va faqat qachonki, $\models F$ va $\models G$ bo`lsa;

k) $\models F \to G$ bo`ladi, faqat va faqat qachonki, $\models F$ va $\models G$ bo`lsa;

l) $\models \overline{F \vee G}$ bo`ladi, faqat va faqat qachonki, $\models \bar{F}$ va $\models \bar{G}$ bo`lsa;

Yechish. k) Berilgan tasdiq to`liq hajmda noto`g`ri: uning "qachonki"(zaruruy) qismi noto`g`ri. Buni tasdiqlash uchun shunday aniq $F$ va $G$ formulalar ko`rsatish lozimki, $F \to G$ tavtologiya bo`lsa ham ulardan kamida bittasi tavtologiya bo`lmasin. Bunday formulalarga misol: $F \equiv P$,

$G \equiv Q \to P$. Bu formulalarning hech qaysisi tavtologiya bo`lmaydi, ammo $P \to (Q \to P)$ formula tavtologiya. Yana bir misol: $F \equiv P \to (Q \to R), G \equiv (P \wedge Q) \to R$ Bu misol haqiqatan ham zaruruy shartni inkor qilishini tekshirib ko`rish mumkin. Mustaqil shunga o`xshash misollar keltiring.

Endi berilgan tasdiqni " faqat qachonki" qismini (yetarliligini) qaraymiz.U qismi to`g`ri ekan. Haqiqatan ham, faraz qilaylik $\models F$ va $\models G$ bo`lsin. Bu ixtiyoriy $A_1,...,A_n$ mulohazalar uchun $F(A_1,...,A_n)$ va $G(A_1,...,A_n)$ mulohazalar rost bo`lishini bildiradi. Demak, ixtiyoriy $A_1,...,A_n$ mulohazalar uchun $F(A_1,...,A_n) \to G(A_1,...,A_n)$ mulohazalar rost bo`ladi.

Bu esa $F \to G$ formulani tavtologiya ekanligini bildiradi, ya`ni $\models F \to G$.

l) Bu tasdiqni to`g`ri ekanligini ko`rsatamiz. Zarurligi. $\models \overline{(F \vee G)}$ bo`lsin. Demak, $F \vee G$ formula aynan yolg`on. Ammo, u holda diz`yunksiya ta`rifiga ko`ra ikkala F va G mulohazalar ham aynan yolg`on. Shunday qilib, bu formulalarning inkorlari $\bar{F}$ va $\bar{G}$ formulalar har doim rost qiymat qabul qiladi, ya`ni $\models \bar{F}$ va $\models \bar{G}$ .Yetarliligini mustaqil isbotlang, zaruruyligini isbotlashga qilingan har bir qadamni teskari tartibda bajarib isbotlash mumkin ekanligiga ishonch hosil qiling.

` **2.4.Formulaning mantiqiy xulosasi.**

$G(X_1,...,X_n)$ formula $F_1(X_1,...,X_n),...,F_m(X_1,...,X_n)$ formulalarning mantiqiy xulosasi deyiladi, agarda $F_1(X_1,...,X_n),...,F_m(X_1,...,X_n)$ formulalar rost qiymat qabul qiladigan o`zgaruvchilarning barcha qiymatlari satrida (naborida) u ham rost qiymat qabul qilsa. Bu $F_1,...,F_m \models G$ deb belgilanadi.

1.34. Uchta o`zgaruvchili $F_i(P,Q,R)$ va $G_j(P,Q,R)$ $(i=1,2,3;\ j=1,2,...,11)$ formulalar quyidagi rostlik jadvali orqali berilgan:

P	Q	R	F_1	F_2	F_3	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	G_{11}
0	0	0	0	1	0	1	1	1	1	0	1	0	1	1	1	0
0	0	1	1	0	1	1	0	0	0	1	1	0	1	1	1	1
0	1	0	1	1	1	0	1	1	1	1	1	1	0	1	0	1
0	1	1	0	1	0	1	1	1	1	0	1	1	0	0	1	0
1	0	0	0	1	1	0	1	0	0	1	1	0	0	1	1	1
1	0	1	1	1	1	1	1	0	1	0	1	1	0	1	0	0
1	1	0	1	0	0	0	0	1	1	0	1	1	1	1	0	1
1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1

$G_j(P,Q,R)$ ($j=1,2,...,11$) formulalarning qaysilari uchta F_1,F_2,F_3 formulalarning mantiqiy xulosasi bo`lishini aniqlang.

Yechish. 11) G_{11} formula F_1,F_2,F_3 formulalarning mantiqiy natijasi bo`lmaydi, chunki $P=1, Q=0, R=1$ larda barcha $F_1,F_2,F_3$ formulalar 1 qiymat qabul qiladi (rost mulohazalarga aylanadilar), $G_{11}$ formula esa o`zining bu qiymatlarida 0 qiymat qabul qiladi, ya`ni yolg`on mulohazaga aylanadi (jadvalning $6-$satriga qarang).

2.35. Mantiqiy xulosa ta`rifidan foydalanib, quyidagi mantiqiy xulosalarni to`g`ri ekanligini isbotlang, teskari xulosa o`rinliligini, ya`ni chapdagi formula o`ngdagi formulaning mantiqiy xulosasi bo`lishi yoki bo`lmasligini aniqlang.

a) $P \leftrightarrow Q \models P \to Q$;

b) $P \leftrightarrow \bar{Q} \models P \vee Q$;

c) $P \wedge Q \models P \vee Q$;

d) $((P \wedge Q) \to (P \vee Q)) \to P \quad \models \; P \vee Q$;

e) $(P \vee Q) \to (P \wedge Q) \quad \models \; P \to Q$;

f) $P \wedge \bar{Q} \quad \models \; (\bar{P} \vee Q) \to \bar{Q}$;

g) $(P \to Q) \to \bar{Q} \quad \models \; (\bar{Q} \to P) \to P$;

h) $(\bar{Q} \to P) \to P \quad \models \; \overline{(Q \to P)} \to (P \leftrightarrow Q)$;

i) $(P \to Q) \wedge (\bar{P} \to Q) \quad \models \; Q$;

j) $\overline{(P \wedge Q)} \vee P \quad \models \; \bar{Q}$;

k) $\overline{(P \vee Q)} \quad \models \; \bar{P} \vee \bar{Q}$;

l) $\bar{P} \wedge \bar{Q} \quad \models \; \overline{(P \wedge Q)}$.

Yechish. l) Avvalo $\models$ belgisidan chapda turgan $\bar{P} \wedge \bar{Q}$ formula va bu belgidan o`ngda turgan $\overline{(P \wedge Q)}$ formula uchun rostlik jadvalini tuzamiz:

P	Q	$\bar{P}$	$\bar{Q}$	$\bar{P} \wedge \bar{Q}$	$P \wedge Q$	$\overline{(P \wedge Q)}$
0	0	1	1	1	0	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	0	0	0	1	0
				(*)		(**)

Shunday qilib, berilgan $\bar{P} \wedge \bar{Q}$ va $\overline{(P \wedge Q)}$ formulalarning mantiqiy qiymatlari turgan ustunlar mos ravishda (*) va (**) belgilar bilan belgilangan.Endi bu ustunlarni mantiqiy xulosa ta`rifi asosida taqqoslaymiz. Bu formulalarga kiruvchi $P$ va $Q$ propozitsional o`zgaruvchilarning qiymatlari uchun ustunlarni satrma-satr taqqoslash lozim. (*) ustunning birinchi satrida 1 turibdi, Demak, (mantiqiy xulosaning ta`rifiga asosan) (**) ustunning birinchi satrida ham 1 turishi lozim, ya`ni $\bar{P} \wedge \bar{Q}$ formula propozitsional o`zgaruvchilar $P$ va $Q$ larning bu qiymatlarida ( bu satrda $P=0, Q=0$ ) 1 qiymat qabul qilganda $\overline{(P \wedge Q)}$ formula ham 1 qiymat qabul qilishi lozim edi. (**) ustinning birinchi satrini ko`rib, bu holda haqiqatan ham shunday ekanligiga ishonch hosil qilamiz.

Ikkinchi satrga o`tamiz.Bu satrda (*) ustunda 0 turibdi.Mantiqiy xulosa tarifiga ko`ra bu holda ikkinchi formula $\overline{(P \wedge Q)}$ ga hech qanday talab qo`yilmaydi, bu satrda uning qiymati ixtiyoriy bo`lishi mumkin.Shu sababli (**) ustunning ikkinchi satridagi qiymatiga umuman qaramasligimiz mumkin.

Uchinchi satrga o`tamiz. (*) ustunda 0 turganini ko`ramiz.Shu sababli oxirgi to`rtinchi satrga o`tamiz. Bu (*) ustunda yana 0 turibdi.

Shunday qilib, jadvalning barcha ustunlarni qarab chiqdik.Biz ko`rdikki, propozitsional o`zgaruvchilarning barcha mumkin bo`lgan qiymatlari satrida $\bar{P} \wedge \bar{Q}$ formula 1 qiymat qabul qilsa ( bizning hosirgi holimizda faqat birinchi satrda: $P=0, Q=0$ da), bu satrda ikkinchi $\overline{(P \wedge Q)}$ formula ham 1 qiymat qabul qildi. Mantiqiy xulosa ta`rifiga ko`ra bu $\overline{(P \wedge Q)}$ formula $\bar{P} \wedge \bar{Q}$ formulaning mantiqiy xulosasi bo`ladi.

Endi ikkinchi savolga javob beramiz, ya`ni $\bar{P} \wedge \bar{Q}$ formula $\overline{(P \wedge Q)}$ formulaning mantiqiy xulosasi bo`lishi yoki bo`lmasligiga. Masalan, ikkinch satrni qaraylik, bu satrda (**) ustunda 1 turibdi, (*) ustunda esa 0 turibdi. Bu esa $\overline{(P \wedge Q)}$ formula o`zgaruvchilarning $P=0, Q=1$ qiymatida 1 qiymat qabul qilishini, shu vaqtda (holda) $\bar{P} \wedge \bar{Q}$ formula 0 qiymat qabul qilishini bildiradi. Boshqa satrlarga qaramasdan mantiqiy xulosa ta`rifiga ko`ra , shunday xulosaga kelamiz: $\bar{P}\wedge\bar{Q}$ formula $\overline{(P\wedge Q)}$ formulaning mantiqiy xulosasi emas, chunki ikkinchi satrda mantiqiy xulosa ta`rifi "bajarilmaydi".

2.36. Mantiqiy xulosa ta`ifidan foydalanib, quyidagi tasdiqlar o`rinli ekanligini isbotlang:

a) $(P\vee\bar{R})\to Q \models (P\to Q)\wedge R$;

b) $(P\to Q)\to R \models P\to(Q\to R)$;

c) $(P\vee Q)\to R \models (P\wedge\bar{Q})\vee R$;

d) $(P\to Q)\to R \models (P\wedge Q)\to R$;

e) $(P\wedge Q)\to R \models P\to(Q\to R)$;

f) $(P\leftrightarrow Q)\vee R \models (\bar{P}\to\bar{Q})\vee \mathrm{R}$;

g) $(P\vee R)\leftrightarrow Q \models (P\vee R)\leftrightarrow R$;

h) $\overline{(P\vee Q)} \models \bar{P}\vee R$;

i) $(P\vee Q)\to R \models (P\to Q)\vee(\mathrm{P}\leftrightarrow \mathrm{R})$;

j) $P\wedge(Q\vee R) \models (P\vee Q)\wedge(P\vee R)$;

k) $(P\wedge Q)\vee R \models P\vee(Q\to R)$.

Teskari xulosa o`rinli ekanligini ham aniqlang, ya`ni chapda turgan formula o`ngda turgan formulaning mantiqiy xulosasi bo`ladimi?

Yechish. k) mantiqiy xulosa munosabatiga qatnashadigan formulalar $(P\vee Q)\vee R$ va $P\vee(Q\to R)$ formulalar uchun rostlik jadvalini tuzamiz:

P	Q	R	$P\wedge Q$	$(P\wedge Q)\vee R$	$Q\to R$	$P\vee(Q\to R)$
0	0	0	0	0	1	1
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	1	1	1
1	0	0	0	0	1	1
1	0	1	0	1	1	1
1	1	0	1	1	0	1
1	1	1	1	1	1	1
				(*)		(**)

(*) va (**) ustunlarni satrlar bo`ylab ketma-ket qarash bilan shunga ishonch hosil qilish mumkinki, agar biror-bir satrda (*) ustunda 1 bo`lsa, u holda ushbu satrda (**) ustunda ham 1 turibdi. Demak, mantiqiy xulosa talabi bajariladi.

Teskari xulosa o`rinli emas, chunki masalan, birinchi satrda ( ya`ni $P=0,\ Q=0,\ R=0$ da) $P\vee(Q\to R)$ formula ((**) ustun) 1 qiymat qabul qiladi, $P\vee(Q\to R)$ formula ((*) ustun) esa 0 qiymat qabul qiladi.

2.37. Quyidagi formulalardan qaysi biri ikkinchisining mantiqiy xulosasi bo`lishini aniqlang:

a) $(P\to Q)\to R,\ P\vee Q\vee R;$

b) $P\to(Q\to R),\ (P\to Q)\to R;$

c) $R\to(Q\vee\bar{P}),\ P\to(Q\wedge R);$

d) $P\to(Q\wedge R),\ (P\wedge Q)\to R;$

e) $(P\vee R)\leftrightarrow Q,\ (P\vee Q)\leftrightarrow R;$

f) $P\vee(Q\to R)\vee Q,\ (P\vee Q)\leftrightarrow R;$

g) $(P\leftrightarrow Q)\to(Q\leftrightarrow R),\ P\to(Q\to R);$

h) $P\to Q,\ \ (P\to R)\vee Q;$

i) $(P\wedge Q)\to R,\ (P\to Q)\vee R;$

j) $(P\to Q)\to(P\to R),\ (P\to Q)\to R;$

k) $(P\wedge Q)\to R,\ (P\vee Q)\to R;$

l) $(P\wedge Q)\to R,\ P\vee(Q\to R).$

Yechish. k) Berilgan formulalar uchun rostlik jadvalini tuzamiz:

P	Q	R	$P\wedge Q$	$(P\wedge Q)\to R$	$P\vee Q$	$(P\vee Q)\to R$
0	0	0	0	1	0	1
0	0	1	0	1	0	1
0	1	0	0	1	1	0
0	1	1	0	1	1	1
1	0	0	0	1	1	0
1	0	1	0	1	1	1
1	1	0	1	0	1	0
1	1	1	1	1	1	1
				(*)		(**)

(*) va (**) ustunlarni satrlar bo`ylab solishtirib, agar (**) ustunning qaysi satrida 1 tursa, (*) ustunning ham o`sha satrida 1 turganligini, ammo teskarisi o`rinli emas ekanligini ( masalan, uchinchi satr) ko`ramiz. Bu esa berilgan birinchi formula ikkinchisining mantiqiy xulosasi ekanligini, ammo ikkinchi formula birinchisining mantiqiy xulosasi emas ekanligini ko`rsatadi.

l) Berilgan formulalar uchun rostlik jadvalini tuzamiz:

P	Q	R	$P\wedge Q$	$(P\wedge Q)\to R$	$Q\to R$	$P\vee(Q\to R)$
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	0	1	0	0
0	1	1	0	1	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1
				(*)		(**)

Berilgan formulalarning qiymatlari ustunlarini solishtirib, ko`ramizki, birinchi formula uchinchi satrda 1 qiymat qabul qiladi, ikkinchisi esa $0$, yettinchi satrda esa ikkinchi formula 1 qiymat qabul qiladi, birinchisi esa $0$. Demak, berilgan formulalardan hech biri ikkinchisining mantiqiy xulosasi bo`lmaydi.

2.38. Mantiqiy xulosa ta`rifidan foydalab, quyidagi mantiqiy xulosalar o`rinli yoki yo`qligini aniqlang:

a) $P\to Q,\ P\to\overline{Q}\ \models\ \overline{P};$

b) $P\to Q,\ Q\to\overline{P}\ \models\ \overline{P};$

c) $\overline{P}\to\overline{Q},\ P\ \models\ Q;$

d) $\overline{Q}\to\overline{P},\ P\ \models\ Q;$

e) $P\to Q,\ \overline{P}\to Q\ \models\ Q;$

f) $(P\to Q)\to Q,\ \overline{P},\ \overline{Q}\ \models\ R;$

g) $P\to Q,\ P\vee R\ \models\ (P\vee R)\to(P\wedge Q);$

h) $P\vee R,\ R\to Q\ \models\ P\vee Q;$

i) $(P\wedge Q)\to R,\ \overline{Q}\ \models\ \overline{R};$

j) $P,\ R\to\overline{(P\vee Q)}\ \models\ \overline{R};$

k) $P \wedge Q,\ \bar{R} \to \bar{Q} \quad \models\ R$;

l) $(P \wedge Q) \to R,\ \bar{R} \quad \models\ \bar{Q}$.

Yechish. k) Berilgan $P \wedge Q,\ \bar{R} \to \bar{Q}$ va R formulalar uchun rostlik jadvalini tuzamiz:

P	Q	R	$P \wedge Q$	$\bar{R}$	$\bar{Q}$	$\bar{R} \to \bar{Q}$
0	0	0	0	1	1	1
0	0	1	0	0	1	1
0	1	0	0	1	0	0
0	1	1	0	0	0	1
1	0	0	0	1	1	1
1	0	1	0	0	1	1
1	1	0	1	1	0	0
1	1	1	1	0	0	1
		(***)	(*)			(**)

Ushbu jadvaldagi $P \wedge Q,\ \bar{R} \to \bar{Q}$ va R formulalarning qiymatlari joylashgan ustunlarni mos ravishda (*), (**), (***) belgilar orqali belgilaymiz. Berilgan formulalar uchun mantiqiy xulosa ta`rifini bajarilishini tekshirish uchun jadvalning barcha shunday satrlarini topish lozimki, bu satrar bilan ikkala (*) va (**) ustunlarning kesishish joylarida birlar tursin hamda bu satrlarning (***) ustuni bilan kesishish joylarida ham birlar tursin.
Demak, isbotlanayotgan mantiqiy xulosa o`rinli ( (*) va (**) ustunlarning ikkalsida ham bir turmaydigan satrlarda mantiqiy xulosa sharti avtomatik tarzda bajariladi: ular uchun bu shartla implikatsiyaning sharti yolg`on, demak, implikatsiyaning o`zi rost).
l) Berilgan barcha uchta formulalar uchun rostlik jadvalini tuzamiz:

P	Q	R	$P \wedge Q$	$(P \wedge Q) \to R$	$\bar{R}$	$\bar{Q}$
0	0	0	0	1	1	1
0	0	1	0	1	0	1
0	1	0	0	1	1	0
0	1	1	0	1	0	0
1	0	0	0	1	1	1
1	0	1	0	1	0	1
1	1	0	1	0	1	0
1	1	1	1	1	0	0

Ushbu jadvaldan shunday satrlarni topamizki, ikkala $(P \wedge Q) \to R$ va $\bar{R}$ shartlar 1 qiymat qabul qilsin.Bu birinchi, uchinchi va beshinchi satrlar. Bu holda $\bar{Q}$ formula ham birinchi va beshinchi satrlarda 1 qiymat qabul qiladi, ammo uchinchi satrda esa $\bar{Q}$ 0 qiymat qabul qiladi.Aynan bu yerda mantiqiy xulosa ta`rifi "buziladi" (bajarilmaydi), demak, $\bar{Q}$ formula $(P \wedge Q) \to R$ va $\bar{R}$ formulalarning mantiqiy xulosasi emas.

2.39. Quyidagi formulalarni shunday joylashtiringki, har bir keyingisi barcha oldingilarining mantiqiy xulosasi bo`lsin:

a) $P \vee Q,\ \overline{(P \to (Q \to P))},\ \overline{(\bar{P} \wedge \bar{Q})},\ \bar{P} \leftrightarrow Q,\ \bar{P} \wedge Q$;

b) $P \to Q,\ \bar{P} \wedge \bar{Q},\ P \to (Q \to (P \wedge Q)),\ Q \vee \bar{P},\ P \leftrightarrow Q$;

c) $(P \to Q) \vee P,\ \overline{(P \to Q)} \wedge \overline{(Q \to P)},\ \overline{(P \leftrightarrow Q)},\ \overline{(P \wedge Q)},\ \bar{P} \wedge Q$;

d) $P \leftrightarrow Q,\ \overline{(P \vee Q)},\ \overline{(P \to (\bar{P} \to Q))},\ \bar{P} \to (P \to Q),\ Q \to (P \vee \bar{Q})$;

e) $(\bar{P} \wedge Q) \to P,\ P \vee \bar{Q},\ (P \vee Q) \wedge (P \wedge Q),\ P \leftrightarrow Q,\ (P \to Q) \vee (Q \to P)$;

f) $(P\vee Q)\leftrightarrow P,\ \bar{Q}\vee P,\ (\bar{P}\to Q)\leftrightarrow(Q\vee P),\ \ P\wedge Q,\ Q\to(Q\to P);$

g) $\overline{(\bar{Q}\wedge P)}\wedge(Q\to P),\ Q\vee\bar{P},\ (\bar{P}\to\bar{Q})\to(\mathrm{Q}\to\mathrm{P}),\ \bar{Q}\to\bar{P},\ \ P\to Q;$

h) $(P\wedge Q)\to Q,\ \bar{P}\to Q,\ \overline{((P\leftrightarrow\bar{Q})\leftrightarrow Q)},\ \ P\wedge Q,\ (\bar{P}\to Q)\wedge P;$

i) $\overline{(Q\wedge(\bar{P}\vee\bar{Q}))}\vee(\bar{P}\to Q),\ \ \bar{\bar{P}},\ \overline{(P\to Q)},\ \ \mathrm{Q}\leftrightarrow(\mathrm{P}\wedge\mathrm{Q}),\ Q\wedge\overline{(P\to Q)};$

j) $\bar{P}\wedge Q,\ \bar{Q}\to(P\vee\bar{P}),\ Q\leftrightarrow\bar{P},\ \ (P\wedge Q)\to(P\leftrightarrow Q),\ P\to\bar{Q};$

k) $(\bar{Q}\to\bar{P})\vee\bar{P},\ P\to Q,\ (\bar{P}\to Q)\vee\bar{P},\ \ \bar{P}\leftrightarrow\bar{Q},\ P\wedge Q.$

Yechish. k) Berilgan formulalarni shartda qanday tartibda yozilgan bo`lsa, o`sha tartibda nomerlab chiqamiz va ular uchun rostlik jadvalini tuzamiz:

P	Q	(1)	(2)	(3)	(4)	(5)
0	0	1	1	1	1	0
0	1	1	1	1	0	0
1	0	0	0	1	0	0
1	1	1	1	1	1	1

Jadvaldan ko`rinadiki, (5) $\models$(4) va (4) $\models$(2) . Bundan tashqari (1) va (2) formulalar o`zaro teng kuchli. Nihoyat (1) $\models$ (3). Shunday qilib, berilgan formulalarning talab qilingan joylashishi quyidagicha bo`ladi: (5) , (4) , (2), (1) , (3) .

2.40. Teskari faraz qilish usulidan foydalanib, quyidagi mantiqiy xulosalar o`rinli yoki yo`qligini aniqlang:

a) $F\to G,\ K\to L,\ F\vee K\ \ \models\ G\vee L;$

b) $F\to G,\ ((\mathrm{F}\vee\mathrm{L})\wedge\mathrm{H})\to M,\ L\to H\ \ \models\ ((F\vee L)\wedge G)\to\bar{M};$

c) $(F\wedge G)\to\bar{R},\ (F\wedge H)\to K,\ F\to\bar{K},\ (F\wedge\bar{G})\to H\ \ \models\ F\to\bar{R};$

d) $F\to G,\ \bar{K}\to\bar{L},\ S\to H,\ \bar{F}\to\bar{K},\ H\to L\ \ \models\ S\to G;$

e) $(F\wedge G)\to H,\ (H\wedge K)\to L,\ \bar{M}\to(K\wedge L)\ \ \models\ (F\wedge G)\to M;$

f) $F\to(G\to H),\ (H\wedge K)\to L,\ \bar{M}\to(K\wedge\bar{L})\ \ \models\ F\to(G\to M);$

g) $(F\vee G)\to(H\wedge K),\ (K\vee L)\to\mathrm{M}\ \ \models\ F\to M;$

h) $F\to(G\wedge H),\ \bar{G}\vee K,\ (L\to\bar{M})\to\bar{K},\ G\to(F\wedge\bar{L})\ \ \models\ G\to L;$

i) $(F\to G)\wedge(H\to K),\ (G\to L)\wedge(\mathrm{K}\to M),\ \overline{(L\wedge M)},\ F\to H\ \ \models\ \bar{\mathrm{F}};$

j) $F\to G,\ G\to H,\ \bar{H}\ \ \models\ \bar{\mathrm{F}};$

k) $F\to G,\ K\to\bar{H},\ H\vee\bar{G}\ \ \models\ F\to\bar{K}.$

Yechish. k) Faraz qilaylik, berilgan mantiqiy xulosa bajarilmasin, ya`ni shunday aniq mulohazalar borki, barcha shart-formulalar rost mulohazaga aylanadi, xulosa-formula $F\to\bar{K}$ esa yolg`on mulohaza bo`ladi. U holda $F\to\bar{K}=0$ dan $F=1$ va $\bar{K}=0$, ya`ni $K=1$. $F\to G=1$ va $F=1$ ekanligidan $G=1$ kelib chiqadi. Bundan tashqari, $H\vee\bar{G}=1$ va $G=1$ ekanligidan $H=1$ degan xulosaga kelamiz, yani $\bar{H}=0$. Nihoyat, $K\to\bar{H}=1$ va $\bar{H}=0$ dan $K=0$ ni hosil qilamiz. Biz qarama-qarshilikka keldik. Demak, agar $F\to G,\ K\to\bar{H},\ H\vee\bar{G}$ formulalar rost mulohazaga aylansa, u holda $F\to\bar{K}$ formula yolg`on mulohaza bo`lishi mumkin emas. Bu qaralayotgan mantiriy xulosani to`g`ri ekanligini bildiradi.

2.41. Quyidagi mantiqiy xulosalarning bajarishi yoki bajarilmasligini aniqlang:

a) $P\to Q,\ R\to S,\ P\wedge R\ \ \models\ Q\wedge S;$

b) $P\to Q,\ R\to S,\ \bar{P}\vee\bar{R}\ \ \models\ \bar{Q}\vee\bar{S};$

c) $P\to Q,\ R\to S,\ \bar{Q}\wedge\bar{S}\ \ \models\ \bar{P}\wedge\bar{Q};$

d) $(P\vee Q)\to(R\wedge S),\ P,\ \ \models\ R\wedge S;$

e) $P\to Q,\ R\to S,\ P\wedge R\ \ \models\ Q\wedge S;$

f) $(P\to Q)\to(\mathrm{R}\wedge S),\ \bar{R}\ \ \models\ P\wedge\bar{Q};$

g) $(P\vee Q)\to(R\to S),\ \bar{P}\wedge\bar{Q}\ \models\ R\wedge\bar{S}$;
h) $P\to Q,\ R\to S,\ P\vee S\ \models\ Q\vee R$;
i) $P\to Q,\ R\to S,\ P\vee R\ \models\ Q\wedge S$;
j) $(P\vee Q)\to(R\wedge S),\ (S\vee K)\to L\ \models\ \mathrm{P}\to \mathrm{L}$;
2.42. $F\ \models G$ faqat va faqat qachonki, $\models F\to G$ bo`lsa, tasdigi`ni isbotlang.
2.43. " $F\ \models G$ " tasdiq, " Agar $\models F$, u holda $\models G$ " tasdiqdan kuchli ekanligini ko`rsating. Boshqacha so`z bilan aytganda, birinchi tasdiqdan ikkinchi tasdiq kelib chiqishini ko`rsating.
2.44. " Agar $\models F$, u holda $\models G$ " tasdiqdan har doim ham " $F\ \models G$ " tasdiq kelib chiqavermasligini ko`rsating. Aytilganni tasdiqlash uchun aniq F va G formulalarga misol keltiring.
*Ko`rsatma.* Masalan, $F(P,Q,R)=(P\to Q)\wedge(P\vee R)$ va $G(P,Q,R)=(P\to R)$ formulalarni qarang. Avvalo " Agar $\models F$, u holda $\models G$ " implikatsiya rost, uning jo`natmasi $\models F$ esa yolg`on, so`ngra esa " $F\ \models G$ " tasdiqning yolg`on bo`lishini ko`rsating.
2.45. Agar F va $\bar{G}$ formulalardan mantiqiy xulosa sifatida aynan yolg`on formulani chiqarish mumkin bo`lsa (masalan, $P\wedge\bar{P}$), u holda $F\ \models G$ ekanligini ko`rsating. Boshqacha so`z bilan aytganda, agar $F,\bar{G}\ \models P\wedge\bar{P}$ u holda $F\ \models G$.
2.46. $F\models G\to H$ faqat va faqat qachonki, $F,\bar{G}\ \models H$ bo`lishini isbotlang yoki inkor qiling.
2.47. Agar $F\ \models G$ va $F\ \models H$ bo`lsa, u holda $F\ \models G\wedge H$ va $F\ \models G\vee H$ ekanligini isbotlang.
2.48. Agar $G-$tavtologiya bo`lsa, u holda $F_1,\mathrm{F}_2,...,F_m\models G$ ekanligini isbotlang bu yerda $F_1,F_2,...,F_m-$ixtiyoriy formulalar.
2.49. Agar $F-$aynan yolg`on formula bo`lsa, u holda uning mantiqiy xulosasi ixtiyoriy formula bo`ladi: $F\ \models G$ tasdig`ini isbotlang.
2.50. Agar $F,G\ \models H$ va $\models F$ bo`lsa, u holda $G\models H$ ekanligini isbotlang.
2.51. Agar $F\ \models G$ va $F\ \models\bar{G}$ bo`lsa, u holda $\models\bar{F}$ ekanligini isbotlang.
2.52. Agar $F,G\ \models H$, u holda $F,G,E\ \models H$ bo`lishini isbotlang. Bu yerda $E-$ixtiyoriy formula.
2.53. $F(P,Q,R)=(P\wedge Q\wedge R)\vee(\bar{P}\wedge Q\wedge R)\vee(P\wedge\bar{Q}\wedge\bar{R})\vee(P\wedge\bar{Q}\wedge R)\vee(\bar{P}\wedge\bar{Q}\wedge\bar{R})$;
$\mathrm{G}(P,Q,R)=(\bar{P}\wedge\bar{Q}\wedge\bar{R})\vee(\bar{P}\wedge\bar{Q}\wedge R)\vee(\bar{P}\wedge Q\wedge R)\vee(P\wedge\bar{Q}\wedge R)$;
$H(P,Q,R)=(\bar{P}\wedge\bar{Q}\wedge\bar{R})\vee(\bar{P}\wedge Q\wedge R)\vee(P\wedge\bar{Q}\wedge R)$.
Mulohazalar algebrasining ixtiyoriy E formulasi uchun $F,G\models\ E$ faqat va faqat qachonli, $H\ \models E$ bo`lsa tasdiqni isbotlang.

2.5.Formulalar tengkuchliligi.

F va G formulalar tengkuchli yoki ekvivalent deyiladi ($F\cong G$ yoki $F=G$ deb belgilanadi), agar o`zgaruvchilarning ixtiyoriy qiymatlarida $F$ va $G$ formulalardan hosil bo`ladigan mulohazalarning mantiqiy qiymatlari ustma-ust tushsa.
2.54. F va G formulalar teng kuchli bo`ladi faqat va faqat qachonki, $F\leftrightarrow G$ formula tavtologiya bo`lsa tasdig`ini isbotlang, ya`ni $F=G\Leftrightarrow\ \models\ F\leftrightarrow G$.
2.55. Oldingi masaladagi tasdiqdan va 28 masaladagi tavtologiyalardan foydalanib, quyidagi tengkuchliliklarni isbotlang:
a) $P\vee\bar{P}=1,\ P\wedge\bar{P}=0$;
b) $P\vee 0=P,\ P\vee 1=1$;
c) $P\wedge 0=0,\ P\wedge 1=P$;
d) $\bar{\bar{P}}=P$;
e) $P\wedge Q=Q\wedge P$;

f) $P \vee Q = Q \vee P$;

g) $(P \wedge Q) \wedge \mathrm{R} = P \wedge (Q \wedge R)$;

h) $(P \vee Q) \vee \mathrm{R} = P \vee (Q \vee R)$;

i) $P \wedge (Q \vee \mathrm{R}) = (P \wedge Q) \vee (P \wedge R)$;

j) $P \vee (Q \wedge \mathrm{R}) = (P \vee Q) \wedge (P \vee R)$;

k) $P \wedge P = P$;

l) $P \vee P = P$;

m) $P \wedge (Q \vee P) = P$;

n) $P \vee (Q \wedge P) = P$;

o) $P \to Q = \bar{P} \vee Q$;

p) $P \leftrightarrow Q = (P \to Q) \wedge (Q \to P)$;

q) $\overline{(P \wedge Q)} = \bar{P} \vee \bar{Q}$;

r) $\overline{(P \vee Q)} = \bar{P} \wedge \bar{Q}$;

s) $P \vee Q = \bar{P} \to Q$.

2.56. Quyidagi formulalarni shunday tengkuchli formulalarga almashtiringki, ularda faqat $\neg, \vee, \wedge$ amallari qatnashsin:

a) $((X \to Y) \wedge (Y \to X)) \to (X \vee Y)$;

b) $((X \to Y) \wedge (Y \to \bar{X})) \to (\mathrm{Z} \to \mathrm{X})$;

c) $((X \to Y) \wedge (\bar{\mathrm{X}} \to \bar{Y})) \to ((X \vee Y) \wedge (\bar{\mathrm{X}} \vee \bar{Y}))$;

d) $((\mathrm{X} \leftrightarrow \bar{Y}) \to \mathrm{Z}) \to (\mathrm{X} \leftrightarrow \bar{\mathrm{Z}})$;

e) $(X \to (Y \leftrightarrow Z)) \leftrightarrow ((X \to Y) \leftrightarrow Z)$;

f) $(X \to Y) \to ((X \to Y) \to \bar{X})$;

g) $((X \wedge \bar{Y}) \to Y) \to (X \to \bar{Y})$;

h) $((X \to Y) \to Y) \to Y$;

i) $(X \to Y) \to ((X \to \bar{Y}) \to (X \wedge Y))$;

j) $(X \to Z) \to ((X \vee Y) \to (\bar{Z} \vee Y))$;

k) $X \to \overline{(Y \leftrightarrow Z)}$.

Yechish. k) Lozim bo`lgan teng kuchli almashtirishlarni bajaramiz:

$$X \to \overline{(Y \leftrightarrow Z)} = \bar{X} \vee \overline{(Y \leftrightarrow Z)} = \bar{X} \vee \overline{(Y \to Z) \wedge (Z \to Y)} = \bar{X} \vee \overline{(\bar{Y} \vee Z) \wedge (\bar{Z} \vee Y)}$$

2.57. Quyidagi formulalarni shunday tengkuchli formulalarga almashtiringki, ularda faqat inkor amali propozitsional o`zgaruvchilar ustida qolsin, ya`ni qavslar oldida qolmasin:

a) $\overline{((X \wedge (\bar{Y} \vee \bar{Z})) \vee Z)}$;

b) $\overline{((X \wedge Y) \vee \bar{Z})} \to \overline{(X \wedge Z)}$;

c) $\overline{(U \to \overline{(Z \wedge \overline{(Y \wedge \bar{X})})})}$;

d) $\overline{(\overline{(\overline{(X \wedge Y)} \to Y)} \to (\bar{X} \wedge Z))}$;

e) $\overline{(\overline{(X \vee (\bar{\mathrm{Y}} \wedge \mathrm{Z}) \vee \bar{\mathrm{Z}})} \vee (Y \wedge Z))}$;

f) $\overline{(\bar{X} \wedge \bar{Y})} \to (X \vee (Z \wedge \bar{T}))$;

g) $\overline{((X \leftrightarrow (\bar{Y} \vee Z)) \wedge \mathrm{Y})}$;

h) $\overline{((\bar{X} \leftrightarrow \bar{Y}) \vee Z)} \wedge Y$;

i) $\overline{((X \to Y) \to X)} \to Y$;

j) $\overline{(((X \vee \bar{Y}) \to Y) \wedge (\bar{X} \vee Y))}$;

k) $(X \to Y) \to \overline{(X \leftrightarrow \bar{Z})}$.

Yechish. k) Lozim bo`lgan tengkuchli almashtirishlarni bajaramiz:

$$(X \to Y) \to \overline{(X \leftrightarrow \bar{Z})} = (\bar{X} \vee Y) \to \overline{(X \leftrightarrow \bar{Z})} = \overline{(\bar{X} \vee Y)} \vee \overline{(X \to \bar{Z}) \wedge (\bar{Z} \to X)} =$$

$$= (X \wedge \bar{Y}) \vee \overline{(\bar{X} \vee \bar{Z}) \wedge (Z \vee X)} = (X \wedge \bar{Y}) \vee (X \wedge Z) \vee (\bar{X} \wedge \bar{Z}).$$

2.58. Quyidagi formulalarni shunday tengkuchli formulalarga almashtiringki, ularda faqat $\neg$ va $\wedge$ amallari qatnashsin:

a) $(X \vee Y) \to (\bar{X} \to Z)$;

b) $(\bar{X} \to Y) \vee \overline{(X \to Y)}$;

c) $((X \vee Y \vee Z) \to X) \vee Z$;

d) $((X \to Y) \to Z) \to \bar{X}$;

e) $(X \vee (Y \to Z)) \to X$;

f) $(X \to Y) \to (Y \wedge Z)$;

g) $(\bar{X} \wedge \bar{Y}) \to (X \wedge Y)$;

h) $((\bar{X} \wedge \bar{Y}) \vee Z) \to (Z \wedge \bar{Y})$;

i) $(((X \to (Y \wedge Z)) \to (\bar{Y} \to \bar{X})) \to \bar{Y}$;

j) $((X \to Y) \wedge (Y \to Z)) \to (X \to Z)$;

k) $(\bar{X} \leftrightarrow Y) \to Z$.

Yechish. k) Teng kuchli almashtirishlar bajaramiz:

$$(\bar{X} \leftrightarrow Y) \to Z = \overline{(\bar{X} \leftrightarrow Y)} \vee Z = \overline{((\bar{X} \to Y) \wedge (Y \to \bar{X}))} \vee Z =$$

$$= \overline{((\bar{\bar{X}} \vee Y) \wedge (\bar{Y} \vee \bar{X})} \vee Z = \overline{(X \vee Y)} \vee \overline{(\bar{X} \vee \bar{Y})} \vee Z = (\bar{X} \wedge \bar{Y}) \vee (\bar{\bar{X}} \wedge \bar{\bar{Y}}) \vee Z =$$

$$= (\bar{X} \wedge \bar{Y}) \vee (X \wedge Y) \vee Z = \overline{(\overline{(\bar{X} \wedge \bar{Y})} \wedge \overline{(X \wedge Y)} \wedge Z)}.$$

2.59. Yuqoridagi masaladagi formulalarni shunday tengkuchli formulalarga almashtiringki, ularda faqat $\neg$ va $\vee$ amallari qatnashsin:

Yechish. k) Berilgan formula uchun yuqoridagi masalada qilingan teng kuchli almashtirishlardan foydalanamiz va berilgan masalani yechish uchun almashtirishlarni davom ettiramiz:

$$(\bar{X} \leftrightarrow Y) \to Z = (\bar{X} \wedge \bar{Y}) \vee (X \wedge Y) \vee Z = \overline{(\bar{\bar{X}} \vee \bar{\bar{Y}})} \vee \overline{(\bar{X} \wedge \bar{Y})} \vee Z =$$

$$= \overline{(X \vee Y)} \vee \overline{(\bar{X} \wedge \bar{Y})} \vee Z.$$

2.60. 29 masaladagi barcha formulalar ustida teng kuchli almashtirishlar qilib hamda 55 masaladagi tengkuchliliklardan foydalanib, ularni tavtologiya bo`lishligini isbotlang.

Yechish. f) Chap tomondagi ekvivalentlik o`ng tomonga teng kuchli ekanligini isbotlaymiz. Buning uchun chap tomonda teng kuchli almashtirishlar bajaramiz

(2.55 masaladagi p, j teng kuchliliklardan foydalanamiz):

$P \to (Q \wedge R) = \bar{P} \vee (Q \wedge R) = (\bar{P} \vee Q) \wedge (\bar{P} \vee R) = (P \to Q) \wedge (P \to R)$. Bu holda 2.54 masaladagi tasdiqqa ko`ra berilgan formula haqiqatan ham tavtologiya ekan.

m) Bu formula 1 ga (rost mulohazaga) ekvivalent ekanligini ko`rsatamiz:

$$((P \to Q) \to P) \to P = \overline{(\overline{(\bar{P} \vee Q)} \vee P)} \vee P = \overline{((\bar{\bar{P}} \wedge \bar{Q}) \vee P)} \vee P =$$

$$= \overline{((P \wedge \bar{Q}) \vee P)} \vee P = (\overline{(P \wedge \bar{Q})} \wedge \bar{P}) \vee P = ((\bar{P} \vee Q) \wedge \bar{P}) \vee P = \bar{P} \vee P = 1.$$

Almashtirishlar davomida 1.55 masaladagi tengkuchliliklardan foydalanildi.

2.61. Teng kuchli almashtirishlar yordamida 30 masaladagi formulalarni aynan rost ekanligini ko`rsating.

2.62. Tengkuchli almashtirishlarni qo`llab, quyidagi formulalarni imkon boricha sodda ko`rinishga keltiring:

a) $\overline{(\bar{P}\vee Q)}\to((P\vee Q)\to P)$;

b) $\overline{(\bar{P}\wedge\bar{Q})}\vee((P\to Q)\wedge P)$;

c) $(P\to Q)\wedge(Q\to P)\wedge(P\vee Q)$;

d) $(P\to Q)\wedge(Q\to\bar{P})\wedge(\mathrm{R}\to P)$;

e) $(P\wedge R)\vee(\mathrm{P}\wedge\bar{\mathrm{R}})\vee(\mathrm{Q}\wedge\mathrm{R})\vee(\bar{P}\wedge Q\wedge R)$;

f) $(P\to Q)\to((\mathrm{P}\to\bar{Q})\to\bar{P})$;

g) $\overline{((P\leftrightarrow\bar{Q})\vee R)}\wedge Q$;

h) $(P\leftrightarrow Q)\to(\bar{P}\to Q)$;

i) $(P\to\bar{Q})\wedge((P\to Q)\vee(R\to P))$;

j) $\overline{((P\to Q)\wedge P)}\wedge(\bar{P}\vee\bar{Q})$;

k) $(P\leftrightarrow Q)\wedge(P\vee Q)$.

Yechish. k) Zarur bo`lgan teng kuchli almashtirishlar qilamiz (har bir qadamda qo`llanilgan teng kuchliliklarni mustaqil tahlil qiling):

$$(P\leftrightarrow Q)\wedge(P\vee Q)=(P\to Q)\wedge(Q\to P)\wedge(P\vee Q)=(\bar{P}\vee Q)\wedge(\bar{Q}\vee P)\wedge$$
$$\wedge(P\vee Q)=(\bar{P}\vee Q)\wedge((\bar{Q}\vee P)\wedge(P\vee Q))=(\bar{P}\vee Q)\wedge(P\vee(Q\wedge\bar{Q}))=$$
$$=(\bar{P}\vee Q)\wedge(P\vee 0)=(\bar{P}\vee Q)\wedge P=(\bar{P}\wedge P)\vee(Q\wedge P)=P\wedge Q.$$

2.63. Tengkuchli almashtirishlar yordamida quyidagi formulalar aynan yolg`on ekanligini isbotlang:

a) $((P\to Q)\to P)\wedge\bar{P}$;

b) $(\overline{((X\vee Y)\wedge\overline{(X\to Y)})}\vee\overline{(Z\wedge Y)})\to\overline{((Z\to\bar{Z})\vee Z)}$;

c) $\overline{(((X\to Y)\wedge(Y\to Z))\to(X\to Z))}$;

d) $((X\to\bar{Y})\to\overline{(X\to Z)})\wedge\overline{(Z\to Y)}$;

e) $(Z\to\overline{(X\wedge\bar{Z})})\to(\overline{(X\vee Z)}\wedge X\wedge Y)$;

f) $((\bar{P}\to\bar{Q})\to((\bar{P}\to Q)\to P))\to\overline{((\bar{\mathrm{P}}\to P)\to P)}$;

g) $\bar{Q}\wedge P\wedge(P\to Q)$;

h) $(P\vee Q)\leftrightarrow(\bar{P}\wedge(Q\to\bar{Q}))$;

i) $(P\to(Q\to R))\wedge(P\to Q)\wedge P\wedge\bar{R}$;

j) $((X\wedge\bar{Y})\vee(X\wedge\bar{Z}))\leftrightarrow((X\to Y)\wedge(X\to Z))$;

k) $((P\to\bar{Q})\to((\bar{R}\to\bar{S})\to(P\wedge Q)))\wedge\overline{(R\to P)}$.

Yechish. k) Tengkuchli almashtirishlar bajaramiz:

$$((P\to\bar{Q})\to((\bar{R}\to\bar{S})\to(P\wedge Q)))\wedge\overline{(R\to P)}=(\overline{(\bar{P}\vee\bar{Q})}\vee\overline{(R\vee\bar{S})}\vee$$
$$\vee(P\wedge Q)))\wedge\overline{(\bar{R}\vee P)}=((P\wedge Q)\vee(\bar{R}\wedge S)\vee(P\wedge Q))\wedge(\bar{\mathrm{P}}\wedge\mathrm{R})=$$
$$=((P\wedge Q)\vee(\bar{R}\wedge S))\wedge(\bar{\mathrm{P}}\wedge\mathrm{R})=(P\wedge Q\wedge\bar{P}\wedge R)\vee(\bar{R}\wedge S\wedge\bar{P}\wedge R)=0\vee 0=0.$$

2.64. Teng kuchli almashtirishlar yordamida quyidagi teng kuchliliklardan qaysilari haqiqatan ham bajarilishini aniqlang:

a) $P\to(Q\vee R)=(P\to Q)\vee(P\to R)$;

b) $P\to(Q\wedge R)=(P\to Q)\wedge(P\to R)$;

c) $P\to(Q\leftrightarrow R)=(P\to Q)\leftrightarrow(P\to R)$;

d) $P\wedge(Q\leftrightarrow R)=(P\wedge Q)\leftrightarrow(P\wedge R)$;

e) $P \vee (Q \leftrightarrow R) = (P \vee Q) \leftrightarrow (P \vee R)$;
f) $P \wedge (Q \to R) = (P \wedge Q) \to (P \wedge R)$;
g) $P \vee (Q \to R) = (P \vee Q) \to (P \vee R)$;
h) $(P \to Q) \wedge R = (P \wedge R) \to (\mathrm{Q} \wedge R)$;
i) $(P \to Q) \vee R = (P \vee R) \to (\mathrm{Q} \vee R)$;
j) $P \to (P \leftrightarrow Q) = P \to Q$;
k) $P \to (P \wedge Q) = P \to Q$;
l) $P \to (P \vee Q) = P \to Q$.

Yechish. k)
$P \to (P \wedge Q) = \bar{P} \vee (P \wedge Q) = (\bar{P} \vee P) \wedge (\bar{P} \vee Q) = 1 \wedge (\bar{P} \vee Q) = P \to Q$.
l) Berilgan tengkuchlilik bajarilmaydi, chunki chapdagi formula uchun
$P \to (P \vee Q) = \bar{P} \vee (P \vee Q) = (\bar{P} \vee P) \vee Q = 1 \vee Q = 1$, ya`ni $P \to (P \vee Q)$ formula tavtologiya bo`ladi, o`ngdagi $P \to Q$ formula esa tavtologiya bo`lmaydi.

2.65. Quyidagi ifodalar (formulalar) nimaga teng kuchli:

$a)\ P \to 0$; $d)\ 1 \to P$; g) $0 \to \overline{\mathrm{P}}$; j) $0 \leftrightarrow \overline{\mathrm{P}}$;

$b)\ P \to 1$; $e)\ P \leftrightarrow 1$; $h)\ \bar{P} \to 0$; k) $1 \to \overline{\mathrm{P}}$;

$c)\ 0 \to P$; $f)\ \bar{P} \to 1$; i) $1 \leftrightarrow \overline{\mathrm{P}}$; l) $\mathrm{P} \leftrightarrow 0$.

Yechish. k) Implikatsiyaning ta`rifiga ko`ra $1 \to 0 = 0, 1 \to 1 = 1$, ya`ni agar jo`natma 1 ga teng bo`lsa, implikatsiyaning butun qiymati xulosaning qiymati bilan ustma-ust tushadi. Demak, $1 \to \bar{P} = \bar{P}$.
l) Ekvivalentlik uchun $0 \leftrightarrow 0 = 1, 1 \leftrightarrow 0 = 0$ larga egamiz, ya`ni ikkita mulohazaning ekvivalentligida bir mulohaza yolg`on mantiqiy qiymat qabul qilsa, u holda u ikkinchisining teskari mantiqiy qiymatiga teng bo`ladi. Demak, $P \leftrightarrow 0 = \bar{P}$.

2.66. Propozisional o`zgaruvchilar va $\neg, \vee, \wedge$ mantiqiy bog`lovchilardan tuzilgan ixtuyoriy formulaning inkorini topish uchun $\wedge$ belgisini $\vee$ belgisiga, $\vee$ belgisini $\wedge$ belgisiga, formulaga kiruvchi barcha inkori bo`lmagan o`zgaruvchilarni, inkor belgisi bilan, barcha inkorlari bor o`zgaruvchilarni esa inkorsiz olish lozim.

2.67. Quyidagi formulalarning inkorini toping:
a) $(X \wedge (Y \vee \bar{Z})) \vee (\bar{X} \wedge Y)$;
b) $((\bar{X} \wedge \bar{Y} \wedge \bar{Z}) \vee \mathrm{R}) \wedge \overline{\mathrm{U}} \wedge \overline{\mathrm{V}} \wedge \bar{W}$;
c) $(((\bar{X} \wedge (\bar{Y} \vee Z)) \vee P) \wedge \bar{Q}) \vee (\bar{R} \wedge (S \vee \bar{T}))$;
d) $\overline{(X \vee \bar{Y})} \vee ((\bar{X} \vee \bar{Y} \vee Z) \wedge X \wedge Y \wedge \bar{Z})$;
e) $(X \vee \overline{(\bar{X} \vee Y)} \vee (\bar{Y} \wedge \bar{Z})) \wedge \overline{(X \wedge Z)}$;
f) $X \wedge ((X \wedge \bar{Y}) \vee \overline{(\bar{X} \vee \bar{Y} \vee Z)})$;
g) $(((\bar{X} \wedge Y \wedge \bar{Z}) \vee (X \wedge \bar{Y} \wedge Z)) \wedge (Y \vee Z)) \vee (X \wedge Y \wedge Z)$;
h) $\overline{(X \vee \bar{Y})} \wedge (\bar{X} \vee Y \vee \bar{Z}) \wedge (\bar{Y} \vee \bar{Z})$;
i) $(\bar{X} \wedge (Y \vee Z)) \vee (X \wedge \bar{Z})$;
j) $X \vee \overline{((\bar{X} \vee \bar{Y} \vee \bar{Z}) \wedge (X \vee Y \vee Z))}$;
k) $((X \wedge (\bar{Y} \vee (\bar{Z} \wedge P))) \vee \bar{Q}) \wedge R$.

Yechish. k) 66 masalada ta`riflangan qoidaga ko`ra berilgan formulaning inkorini yozamiz: $((\bar{X} \vee (Y \wedge (Z \vee \bar{P}))) \wedge Q) \vee \bar{R}$. Shuni qayd etish mumkinki, ba`zida berilgan formulani soddalashtirish uchun avvalo uni inkorini topish va uni soddalashtirish, so`ngra olingan natijaning inkorini olish qo`layroq bo`ladi.

2.68. $G(P,Q,R)$ formula bilan tengkuchli bo`lmagan uch o`zgaruvchili berilgan formulaning mantiqiy xulosasi bo`lgan nechta formula mavjud:

a) $(P \to Q) \to R$; g) $(P \leftrightarrow Q) \wedge \bar{R}$;
b) $P \vee Q \vee R$; h) $P \wedge Q \wedge R$;
c) $(P \vee R) \to (P \wedge Q)$; i) $(\bar{P} \wedge Q) \leftrightarrow \bar{R}$;
d) $(P \wedge Q) \to \bar{R}$; j) $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$;
e) $P \leftrightarrow (Q \to R)$; k) $\bar{P} \vee (Q \wedge R)$.
f) $(P \to Q) \vee (P \to R)$;

Yechish. k) Berilgan formula uchun rostlik jadvalini tuzamiz:

P	Q	R	$\bar{P}$	$Q \wedge R$	$\bar{P} \vee (Q \wedge R)$	$G(P,Q,R)$
0	0	0	1	0	1	1
0	0	1	1	0	1	1
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	*
1	0	1	0	0	0	*
1	1	0	0	0	0	*
1	1	1	0	1	1	1

Mantiqiy xulosaning ta`rifini eslab, berilgan $\bar{P} \vee (Q \wedge R)$ formulaning mantiqiy xulosasi bo`lgan $G(P,Q,R)$ formulaning qiymatlari ustuni qanday ko`rinishini tushunishga harakat qilamiz. Berilgan formula 1 qiymat qabul qiladigan satrlarida ( bizda bu satrlar 1, 2, 3, 4 va 8 ) uning mantiqiy xulosasi bo`lgan G formula ham 1 qiymat qabul qilishi mumkin. Berilgan formula 0 qiymat qabul qiladigan satrlarida (bizda bu satrlar 5, 6 va 7) uning mantiqiy xulosasi bo`lgan $G$ formula ixtiyoriy qiymat qabul qilishi mumkin ( jadvaldagi $G$ ustunda bu o`rinlar * orqali belgilangan.

Shunday qilib, berilgan masala, 1,2,3,4 va 8 satrlarda 1 turgan, qolgan satrlarda esa ixtiyoriy qiymatlar turgan rostlik qiymatlari ustuniga ega bo`lgan formulalar sonini aniqlashga keldi. Bu son esa ravshanki, uchta vakant o`rinlarda (* bilan belgilangan o`rinlarda) 0 va 1 larni joylashtirish usullari (kombinatsiyalari ) soniga, ya`ni 0 va 1 lardan tuzilgan uch uzunlikdagi naborlar soniga teng. Oxirgisi, ma`lumki, $2^3$ ga teng.Ravshanki, barcha bunday formulalar o`zaro teng kuchli emas, chunki, ular har xil qiymatli ustunlarga ega.

2.69. Yuqoridagi masalada berilgan formula mantiqiy xulosasi bo`lgan tengkuchli bo`lmagan uch o`zgaruvchili nechta $F(P,Q,R)$ formula mavjud:

Yechish. k) Berilgan $\bar{P} \vee (Q \wedge R)$ formula uchun oldingi masalada tuzilgan rostlik jadvalidan foydalanamiz:

P	Q	R	$\bar{P}$	$Q \wedge R$	$\bar{P} \vee (Q \wedge R)$	$F(P,Q,R)$
0	0	0	1	0	1	*
0	0	1	1	0	1	*
0	1	0	1	0	1	*
0	1	1	1	1	1	*
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	0	0
1	1	1	0	1	1	*

Mantiqiy xulosaning ta`rifini eslab, berilgan $\overline{P}\vee(Q\wedge R)$ formula mantiqiy xulosasi bo`lgan F(P,Q,R) formulaning qiymatlari ustuni qanday ko`rinishini tushunishga harakat qilamiz. Berilgan formula 0 qiymat qabul qiladigan satrlarida ( bizda bu satrlar 5, 6 va 7 ) u mantiqiy xulosasi bo`lgan F formula ham 0 qiymat qabul qilishi mumkin. Berilgan formula 1 qiymat qabul qiladigan satrlarida (bizda bu satrlar 1,2,3,4 va 8) u mantiqiy xulosasi bo`lgan $F$ formula ixtiyoriy qiymat qabul qilishi mumkin ( jadvaldagi $F$ ustunda bu o`rinlar * orqali belgilangan). Shunday qilib, qidirilayotgan formulalar soni besh uzunlikdagi 0 va 1 lardan tuzilgan ikkilik naborlar soniga teng. Bizga ma`lumki, bu son $2^5$ ga teng.Har bir aytilgan ikkilik naborga biror formula mos keladi ( uni MDNSh yoki MKNSh yordamida topish mumkin), har xil ustunga mos kelgan formulalar o`zaro tengkuchli bo`lmaydi.

2.6.Mulohazalar sistemasini soddalashtirish.

2.70. Berilgan rostlik mulohazalar sistemasini soddalashtiring, ya`ni bu sistemaga ekvivalent bo`lgan, murakkab mulohazalarga nisbatan kam sondagi mulohazalardan tuzilgan sistemani toping:

a) $C\to(A\vee B),\ (B\wedge C)\to A,\ (A\wedge B)\to C;$

b) $A\to(B\vee C),\ B\to(A\vee C),\ (A\wedge B)\to C;$

c) $A\to B,\ A\to(B\vee C),\ B\to C;$

d) $P\to(Q\vee R),\ W\to(S\vee T),\ R\to(Q\vee\overline{P}),\ (W\wedge T)\to\overline{S};$

e) $W\to(M\vee S),\ R\to T,\ \overline{Q}\to T,\ M\to(S\vee W),\ P\to(T\vee R);$

f) $\overline{A}\to(B\vee C),\ B\to\overline{(A\wedge C)},\ C\to(A\vee\overline{B}),\ A\to(B\vee C),$
$(A\wedge C)\to B,\ (\overline{A}\wedge\overline{B})\to C;$

g) $P\to Q,\ P\vee Q,\ Q\to P;$

i) $A\vee C,\ B\to C,\ (B\wedge C)\to A;$

j) $A\wedge B,\ B\to C,\ C\to(A\vee B);$

k) $A\to B,\ C\to B,\ (B\wedge C)\to A.$

Yechish. k) Mulohazalar to`plamini soddalashtirish, berilgan mulohazalar to`plamidagi har bir mulohaza rost bo`ladi faqat va faqat qachonki, ularning kon`yunksiyasi rost bo`lsa degan tasdiqqa asoslangan. Shu sababli berilgan mulohazalarning kon`yunksiyasini olib, uning ustida tengkuchli almashtirishlar bajarib, sodda ko`rinishdagi kon`yuksiyalar olib kelamiz. Ravshanki, bunday yo`l bilan hosil qilingan sistema berilgan sistemaga ekvivalent bo`ladi. Bizning holimizda quyidagi kon`yunksiyaga egamiz va uni ketma-ket soddalashtiramiz:

$$(A\to B)\wedge(C\to B)\wedge((B\wedge C)\to A)=(\overline{A}\vee B)\wedge(\overline{C}\vee B)\wedge(\overline{B\wedge C}\vee A)=$$
$$=(\overline{A}\vee B)\wedge(\overline{C}\vee B)\wedge(\overline{B}\vee\overline{C}\vee A)=(\overline{A}\vee B)\wedge((A\wedge\overline{A})\vee B\vee\overline{C})\wedge(A\vee\overline{B}\vee\overline{C})=$$
$$=(\overline{A}\vee B)\wedge(A\vee B\vee\overline{C})\wedge(\overline{A}\vee B\vee\overline{C})\wedge(A\vee\overline{B}\vee\overline{C})=$$
$$=(\overline{A}\vee B)\wedge(A\vee(B\wedge\overline{B})\vee\overline{C})=(\overline{A}\vee B)\wedge(A\vee\overline{C})=(A\to B)\wedge(C\to A).$$

Demak, berilgan sistemaning barcha mulohazalari rost faqat va faqat qachonki, $A\to B$ va $C\to A$ mulohazalar rost bo`lsa. Shu sababli uchta berilgan mulohazalar sistemasi ikkita sodda $A\to B$ va $C\to A$ mulohazalardan tuzilgan sistemaga mantiqiy ekvivalent bo`ladi.

2.71. Agar berilgan sistemadagi kamida bitta mulohazasi rost bo`lsa, quyidagi har bir sistema uchun unga mantiqiy ekvivalent, ammo sodda bo`lgan sistemalarni toping:

a) $\overline{(A\to B)},\ \overline{B}\wedge A,\ \overline{D}\wedge C,\ \overline{(C\to A)};$

b) $A\wedge B\wedge C,\ \overline{A}\wedge\overline{B}\wedge\overline{C},\ A\wedge B\wedge\overline{C},\ \overline{(A\vee B\vee\overline{C})};$

c) $P\to Q,\ \overline{(Q\to P)},\ P\wedge Q;$

d) $A\wedge\bar{B}\wedge C,\ \ A\wedge\overline{(B\to\bar{C})},\ A\wedge\overline{(B\vee C)};$

e) $A\wedge\bar{B}\wedge C,\ \overline{(A\to C)}\wedge\bar{B},\ \bar{A}\wedge\overline{(C\to B)};$

f) $\bar{A}\vee B,\ A\leftrightarrow B;$

g) $A\to B,\ B\vee C,\ A\wedge C;$

h) $A\to(B\vee C),\ B\vee(A\to C),\ C\vee(A\to B),\ B\vee C;$

i) $A\wedge B\wedge\bar{C},\ \overline{(A\to C)}\wedge\bar{A},\ \ A\wedge(B\vee C);$

j) $\bar{A}\wedge C,\ A\to B,\ B\vee(A\to C);$

k) $A\wedge B\wedge\bar{C},\ \overline{(A\to B)}\wedge\bar{C},\ \ \bar{A}\wedge\overline{(B\to C)}.$

Yechish. k) Berilgan mulohazalar to`plamidagi kamida bitta mulohaza rost bo`ladi faqat va faqat qachonki, ularning diz`yunksiyasi rost bo`lsa. Shu sababli berilgan mulohazalardan diz`yunksiya tuzib, uni tengkuchli almashtirishlar yordamida sodda bo`lgan diz`yunksiyaga olib kelamiz. Bizning holimizda quyidagi diz`yunksiyaga ega bo`lamiz, so`ngra uni soddalashtiramiz:

$$(A\wedge B\wedge\bar{C})\vee(\overline{(A\to B)}\wedge\bar{C})\vee(\bar{A}\wedge\overline{(B\to C)})=(\mathrm{A}\wedge\mathrm{B}\wedge\bar{\mathrm{C}})\vee$$

$$\vee(\overline{(\bar{A}\vee B)}\wedge\bar{C})\vee(\bar{A}\wedge\overline{(\bar{B}\vee C)})=(A\wedge B\wedge\bar{C})\vee(A\wedge\bar{B}\wedge\bar{C})\vee$$

$$\vee(\bar{A}\wedge B\wedge\bar{C})=((A\wedge B\wedge\bar{C})\vee(A\wedge\bar{B}\wedge\bar{C}))\vee((A\wedge B\wedge\bar{C})\vee(\bar{A}\wedge B\wedge\bar{C}))=$$

$$(\mathrm{A}\wedge(\mathrm{B}\vee\bar{\mathrm{B}})\wedge\bar{\mathrm{C}})\vee((\mathrm{A}\vee\bar{\mathrm{A}})\wedge\mathrm{B}\wedge\bar{\mathrm{C}})=(A\wedge\bar{C})\vee(B\wedge\bar{C}).$$

Demak, berilgan sistemadan kamida bitta mulohaza rost bo`ladi faqat va faqat qachonki, $A\wedge\bar{C}$ yoki $B\wedge\bar{C}$ mulohazalardan kamida bittasi rost bo`lsa.Shu sababli berilgan uchta mulohazalar sistemasi nisbatan sodda bo`lgan ikkita $A\wedge\bar{C}$, $B\wedge\bar{C}$ mulohazalar sistemasiga mantiqiy ekvivalent.

Foydalanilgan adabiyotlar ro`yxati.

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