O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

Abdulla Qodiriy nomidagi Jizzax Davlat pedagogika instituti

“Matematika o‘qitish metodikasi” kafedrasi

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Tasvirli masalalar

Annotasiya

Ushbu uslubiy qo‘llanmada umumta’lim maktablari, akademik litsey, kasb-hunar kollejlari geometriya, algebra va analiz fanlarida uchraydigan tasvirli masalalar klassifikatsiyalanib, har bir klassifikatsiyalangan turiga oid qisqacha nazariy ma’lumot, masalalarni yechish na’munalari, mustaqil yechish uchun masalalardan na’munalar keltirilgan. Fanlararo aloqadorlikka doir ham tasvirli masalalar berilgan. Ularda keltirilgan metodik tavsiyalar ushbu uslubiy qo‘llanmadan foydalanuvchilarga keng imkoniyatlar yaratadi.

Keltirilgan misol va masalalar nazariy bilimlarni chuqurlashtirish, tadbiqini kengaytirishga qaratilgan bo‘lib, talabalarni mustaqil, ijodiy izlanishga yo‘naltirilgan.

Uslubiy qo‘llanmadan umumta’lim maktablari, akademik litsey, kasb-hunar kollejlari o‘qituvchilari, o‘quvchilari hamda oliy o‘quv yurtiga kiruvchi abituriyentlar foydalanishi mumkin.

Tasvirli masalalarni tuzish va yechish usullari

KIRISH

Umumta’lim maktablari, akademik litsey va kasb-hunar kollejlarida o‘rgatiladigan aniq fanlardan geometriya fanining boshqa fanlardan tubdan farq qiladigan tomoni uning o‘quvchilarda fazoviy tasavvurini rivojlantirishidir. O‘quvchilarda fazoviy tasavvurning paydo bo‘lishi va rivojlanishi geometriyaning sodda tushunchalari va elementar shakllarni tasavvur etishdan boshlanadi. Sodda tushunchalar deb atalgan, aslida esa ilmiy jihatdan asosli bayon etilishi murakkab bo‘lgan, geometriya fanining boshlang‘ich tushunchalaridir. Bular “nuqta”, “to‘g‘ri chiziq” kabi asosiy tushunchalar.

Tabiiyki, geometriya fani o‘qitilishi boshlang‘ich davrida, ya’ni 7 – sinfda tushunchalarni ilmiy asoslashga va mantiqiy bexato tushuntirishga harakat qilinmaydi. Aksincha, ilmiylikdan ko‘ra bu tushunchalarni amaliy tomonga, ya’ni “mana bunday yasash mumkin”, “ko‘rinib turibdiki” kabi tushunchalariga tayanib o‘rgatiladi.

Geometriya fanini o‘rganishda shaklning alohida o‘rni bo‘lib, bu fanni amaliyotga va hayotiy masalalarni hal qilishdagi o‘rniga qaratilgan bo‘ladi. O‘quvchida geometrik tasavvur hosil qilishni eng sodda shakllar “nuqta”, “kesma”, “burchak”, “aylana”, “uchburchak” va ularning elementlarini chizishdan boshlab o‘rgatiladi. Albatta bu tushunchalar alohida – alohida emas, balki biror bir uyg‘unlikda bir – birlariga bog‘liq holda o‘rganiladi.

Matematika fanini o‘qitishda masalaning ahamiyati juda katta bo‘lib, bunda o‘quvchilarda matematikaga bo‘lgan qiziqishni orttirish, tayanch va fanga oid kompetensiyalarni shakllantirish uchun ta’lim jarayonida amaliy va nostandart xarakterdagi masalalardan foydalanish maqsadga muvofiq. Bunday masalalarni yechish o‘quvchilarda analiz, sintez, analogiya, umumlashtirish, deduksiya va induksiya kabi mantiqiy mushohada yuritish faoliyatini, intuitsiya, egiluvchanlik va moslashuvchanlik kabi fazilatlarni rivojlantirib, o‘quvchilarni olingan natijalar ustida tanqidiy fikrlashga o‘rgatadi. Ayrim hollarda amaliy va nostandart

xarakterdagi masalalarning yechimi darhol topilmasdan, bir necha bor urinishlar natijasidagina aniqlanilishi mumkinligi, bu maqsadga erishish uchun tirishqoq bo‘lishlikni, ya’ni shaxsning irodalilik kabi juda ahamiyatli sifatlarni tarkib topishiga imkon beradi. Eng asosiysi, bunday masalalarni yechilishi o‘quvchilarda natijaga erishilganlik, yechim yo‘lining go‘zalligi va noan’anaviyligi bilan bog‘liq bo‘lgan ruhiy ko‘tarinkilik bag‘ishlashi muhim ahamiyatga ega.

Geometrik masalalar ma’lum bir shartlar asosida qaralayotgan geometrik ob’ektga tegishli bo‘lgan geometrik kattaliklarni aniqlashga yo‘naltirilgan bo‘ladi. Geometriya fanini o‘rganish davrida o‘quvchilarga masalalar turli xil usullarda va har xil darajadagi murakkabliklarda beriladi. Shuningdek, masalani yechishning ham har xil usullari talab qilinadi.

Biz umumta’lim maktablari, akademik litsey va kasb – hunar kollejlari geometriya faniga oid masalalarni qo‘yilish usuliga qarab quyidagi uch tipga ajratdik.

1. **Mazmuniy masalalar.**

Bunda masalada qaralayotgan geometrik ob’ektlar mohiyati ham, talab qilinayotgan vazifa ham so‘zlarda bayon etiladi. O‘quvchidan esa bu so‘zlar mazmuniga tayangan holda, geometrik shaklni tasavvur qilishi, chizishi va vazifani hal qilishi talab etiladi.

2. **Mantiqiy masalalar.**

Bunda geometrik masalaga oid aniq mantiqiy fikrlar berilib, boshqa bir fikrning to‘g‘ri yoki noto‘g‘ri ekani mantiqan ko‘rsatiladi. Ba’zan bunday masalalarni hal qilishda o‘quvchi faqat tasavvur qilishining o‘zi yetarli bo‘lib, geometrik shakldan foydalanishga zarurat bo‘lmasligi mumkin.

3. **Tasvirli masalalar.**

Bunday masalalarda geometrik shakl masalaning asosiy qismi bo‘lib xizmat qiladi. O‘quvchi esa shaklga qarab talab qilingan vazifani hal qilishi zarur.

Keltirilgan bu uch holat geometriya fanida uchraydigan masalalarning koʻp qismini qamrab oladigan holatlardir. Bundan tashqari baʼzi masalalarda aytib oʻtilgan uch holatning oʻzaro uygʻunlashishini ham koʻrishimiz mumkin. Aytilgan hollardan birinchisida oʻquvchi qoʻyilgan masala mohiyatidan kelib chiqib, masalaga oid shaklni chizish talab etiladi. Albatta masalaning bu koʻrinishda berilishi oʻquvchida geometrik shakllar haqida tasavvurni hosil qilishga xizmat qiladi. Bunda shakl, yaʼni tasvir masalani yechish uchun vosita oʻrnida ishlatiladi. Tasvirni hosil qilish uchun esa, oʻquvchi masaladagi soʻzlar bilan ifoda etilgan geometrik obʼektlarni tasavvur etishi zarur. Masalaning bu holda berilishi geometriya fanining klassik usullaridan boʻlib qolgan.

Ammo soʻnggi 20 yillar ichida masalaning biz uchinchi tip shaklida ajratgan tasvirli usuli keng rivojlanib bormoqda. Masalaning tasvirli shaklda berilishi koʻpincha test materiallarida uchraydi, shuningdek oʻquv darsliklarida ham muvofaqiyatli qoʻllanib borilayotir.

Tasvirli masalalarni yechish oʻquvchida matematik savodxonlik, fan va texnika yangiliklaridan xabardor boʻlish hamda foydalanish kompitensiyasini rivojlantiradi.

Matematik savodxonlik, fan va texnika yangiliklaridan xabardor boʻlish hamda foydalanish kompitensiyasi – aniq hisob-kitoblarga asoslangan holda shaxsiy, oilaviy, kasbiy va iqtisodiy rejalarni tuza olish, kundalik faoliyatda turli diagramma, chizma va modellarni oʻqiy olish, inson mehnatini yengillashtiradigan, mehnat unumdorligini oshiradigan, qulay shart-sharoitlar olib keladigan fan va texnika yangiliklaridan foydalana olish layoqatini nazarda tutadi.

Matematik taʼlimga kompetensiyaviy yondashuv, oʻquvchilarda kasbiy, shaxsiy va kundalik hayotda uchraydigan holatlarda samarali harakat qilishga imkon beradigan amaliy koʻnikmalarni shakllantirish va rivojlantirishni hamda matematik taʼlimning amaliy, tatbiqiy yoʻnalishlarini kuchaytirishni nazarda tutadi.

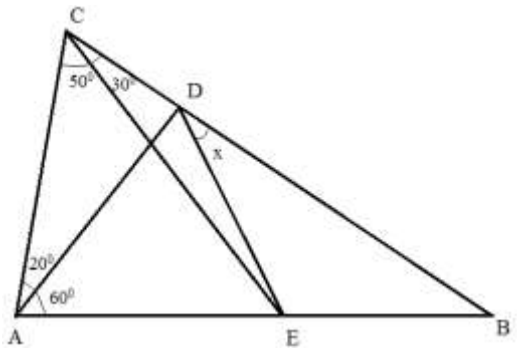
Biz ushbu uslubiy qo‘llanmada asosan uchinchi tipga tegishli masalalarni o‘rganishga e’tibor qaratamiz. Buning uchun tasvirli masalalarni geometriya fanining planometriya va stereometriya bo‘limiga oid masalalarini alohida – alohida beramiz. Shuningdek bu masalalarni murakkabligi va alohida mavzularga tegishliligi bo‘yicha guruhlarga ajratamiz.

Tasvirli masalalardan faqat geometriya fanini o‘qitish jarayonidagina emas, umumta’lim maktablari, akademik litsey va kasb – hunar kollejlaridagi boshqa fanlarda ham muvofaqiyatli qo‘llanib kelinmoqda. Shuning uchun bu tipdagi masalalarni algebra va analiz hamda fizika fanlarida qo‘llanishiga doir masalalarni ham keltirdik.

§1. Sodda tasvirli masalalar.

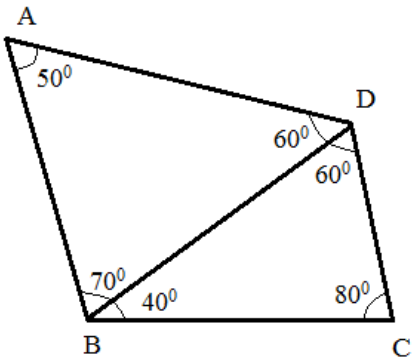
Planometriyaga oid eng sodda tasvirli masalalar, asosan berilgan tasvir(shakl)ga asoslanib, talab etilgan noma'lum kattaliklarni topishdan iborat bo'ladi. Bunda o'quvchi shakldagi geometrik tushunchalarning ta'rifini va elementar xossalarini bilishi yetarli bo'ladi. Shuningdek, eng sodda tasvirli masalalar lemma yoki sodda teoremalarni ifoda etadi. Bunday ko'rinishidagi tasvirli masalalarni yechishda o'quvchi o'rganilgan tushunchani bilishi, tushunishi va amalda qo'llay olishi mumkin, lekin tahlil, sintez, hulosa chiqara olmaydi.

1 – masala. Noma'lum *x* ni toping



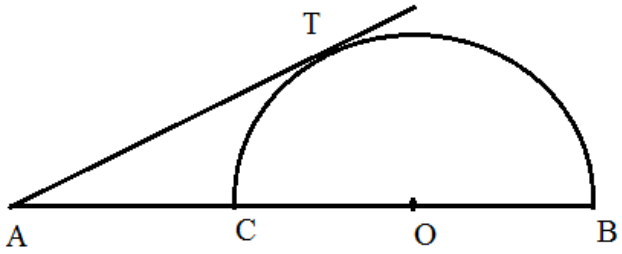
Masalaning yechilishi: Masala shartiga ko'ra, uchburchak ichki burchaklari yig'indisi 180^0 ekanligidan $\angle ADC = 80^0$ va $\angle AEC = 50^0$ ga teng. Bundan esa, $AC = AD = AE$ kelib chiqadi. Demak, ADE uchburchak teng tomonli va $\angle ADE = 60^0$. Yoyiq burchak ta'rifidan $x + 80° + 60° = 180°$, $x = 40°$ o'rinli.

2 – masala. Shakldagi eng katta tomonni toping.



Masalaning yechilishi: Uchburchakda katta tomon qarshisida katta burchak va aksincha katta burchak qarshisida katta tumon yotishini e'tiborga olsak, *BCD* uchburchakda katta tomon *BD* bo'ladi. Bu tomon esa *ABD* uchburchakning eng kichik tomoni. *ABD* uchburchakda eng katta burchak qarshisidagi *AD* tomon berilgan shakldagi eng katta tomon bo'ladi.

3 – masala. *AT* urinma, $AO = 2 \cdot OB$ va *AT* = 6 sm bo'lsa, *AC* = ?

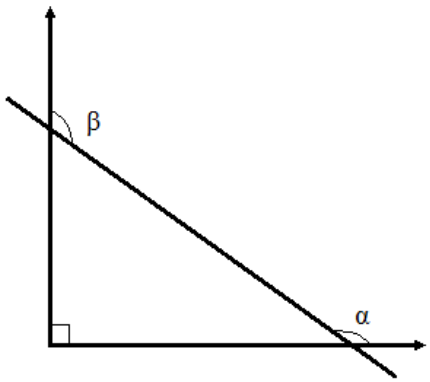


Masalaning yechilishi: *OB* = *R* deb olaylik, u holda *AO* = *2R* va *AC* = *R* ga teng bo'ladi. Aylanaga o'tkazilgan urinma va kesuvchi haqidagi teoremaga ko'ra,

$$AT^2 = AC \cdot AB \;\; => \;\; 36 = R \cdot 3R \;\; => AC = R = 2\sqrt{3}$$

ga teng bo'ladi.

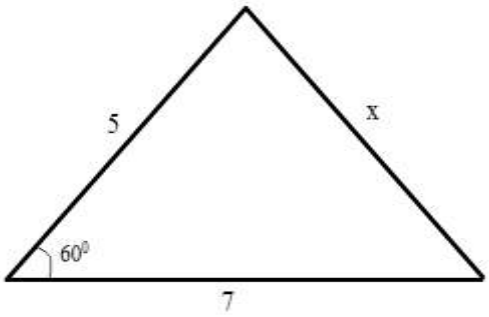
4 – masala. Shaklga ko'ra, $\alpha + \beta =$?



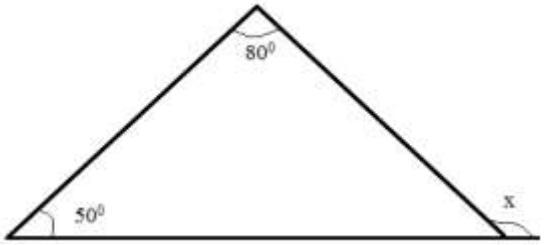
Masalaning yechilishi: Ko'pburchak tashqi burchaklari yig'indisi 360^0 ga tengligidan, $\alpha + \beta + 90° = 360°$, $\alpha + \beta = 270°$ kelib chiqadi.

Sodda tasvirli masalalarga namunalar.

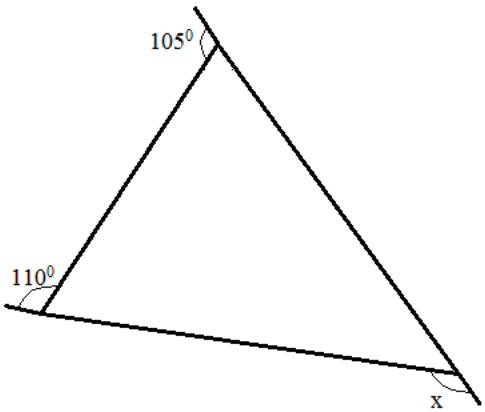
1. Berilgan shaklga ko‘ra, x = ?



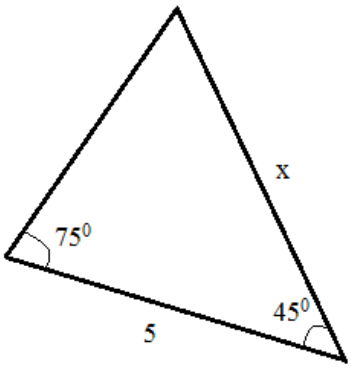
2.Berilgan shaklga ko‘ra, x = ?



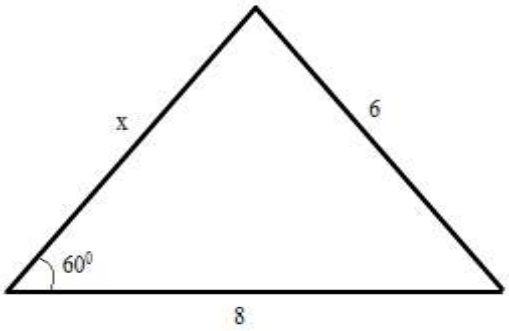
3.Berilgan shaklga ko‘ra, x = ?



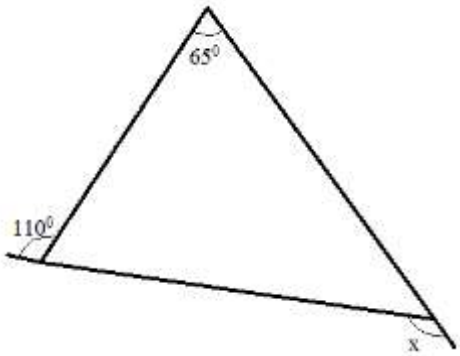
4. Berilgan shaklga ko‘ra, x = ?



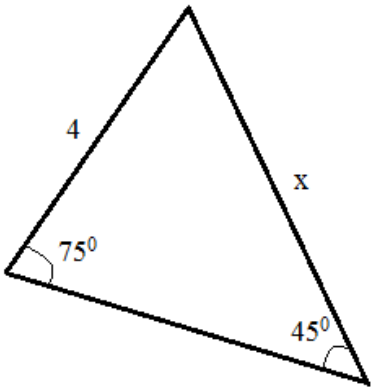
5. Berilgan shaklga ko‘ra, x = ?



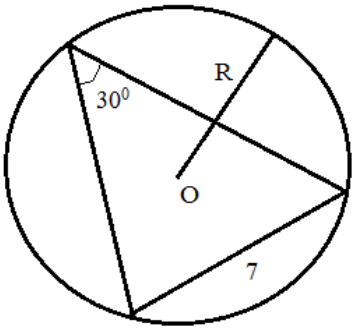
6. Berilgan shaklga ko‘ra, x = ?



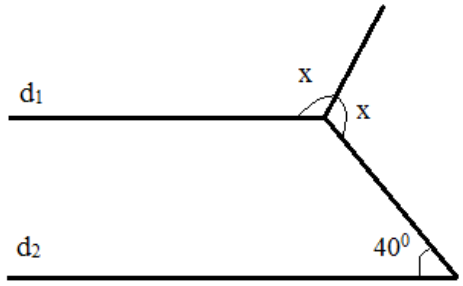
7. Berilgan shaklga ko‘ra, x = ?



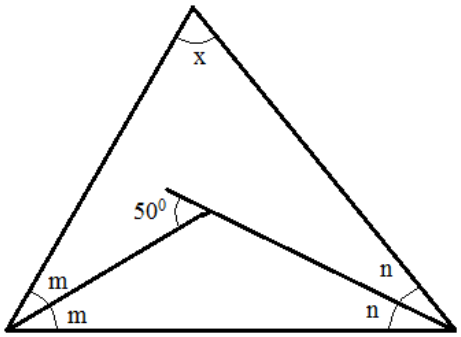
8. Berilgan shaklga ko‘ra, R = ?



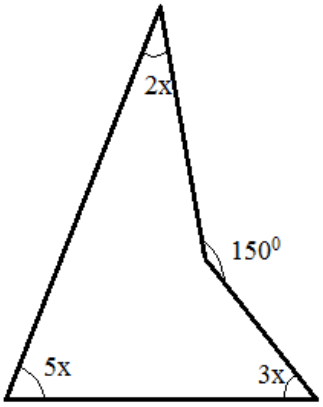
9. $d_1 // d_2$ bo‘lsa, berilgan shaklga ko‘ra x = ?



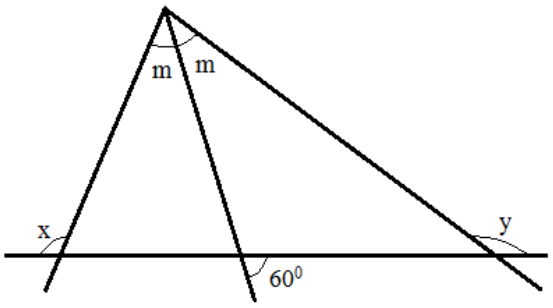
10. Shaklda berilganlarga ko‘ra x = ?



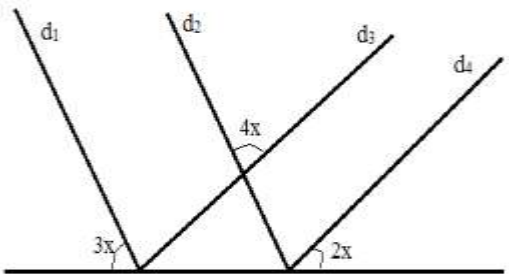
11. Shaklga ko‘ra x = ?



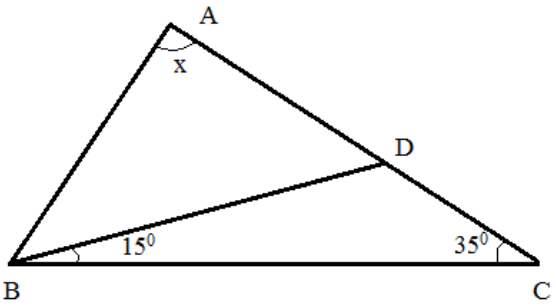
12. Shaklga ko‘ra $x + y$ = ?



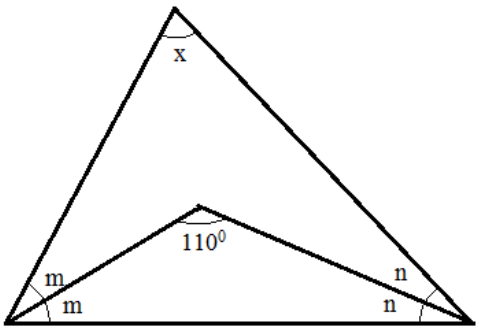
13. $d_1 // d_2$ va $d_3 // d_4$ bo‘lsa, shaklga ko‘ra x = ?



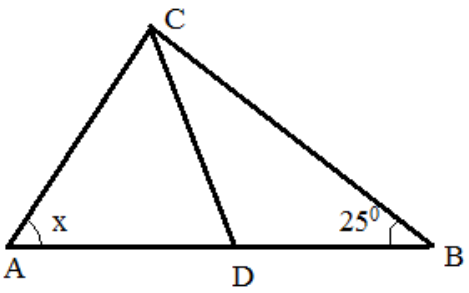
14. $AB = AD$ bo‘lsa, shaklga ko‘ra x = ?



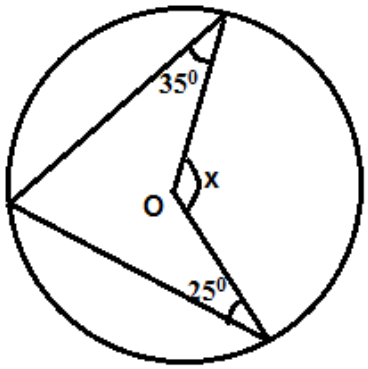
15. Shaklga ko‘ra x = ?



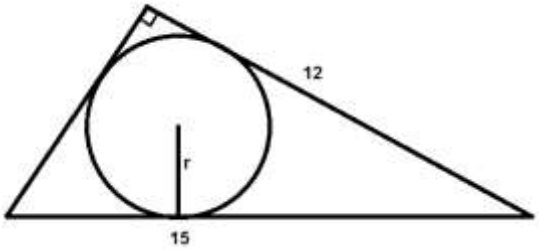
16. $AC = DC = DB$ bo‘lsa, shaklga ko‘ra x = ?



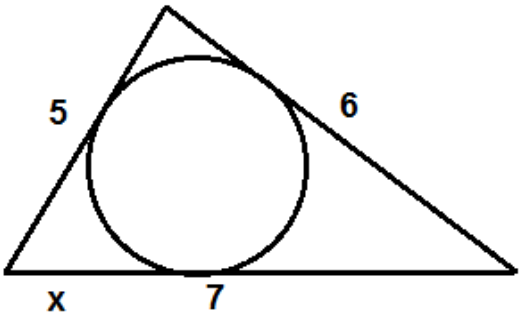
17. Shaklga ko‘ra, x = ?



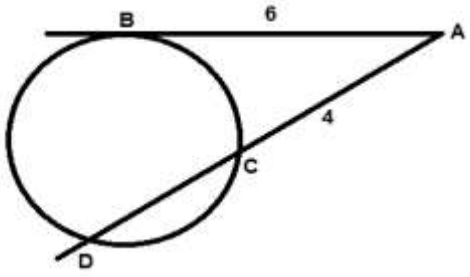
18. Shaklga ko‘ra, r = ?



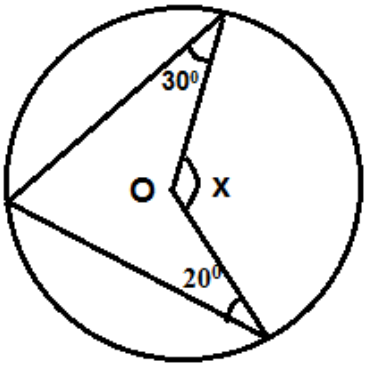
19. Shaklga ko‘ra, x = ?



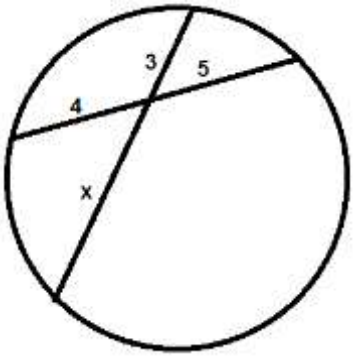
20. AB urunma. Shaklga ko‘ra, AD = ?



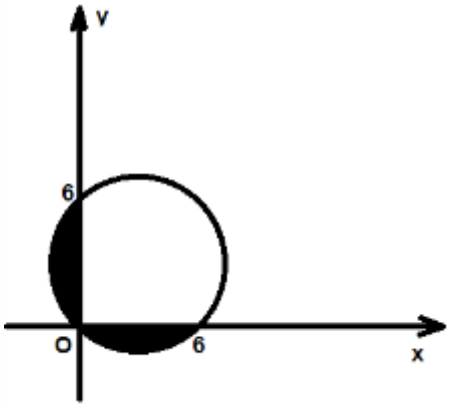
21. Shaklga ko‘ra, x = ?



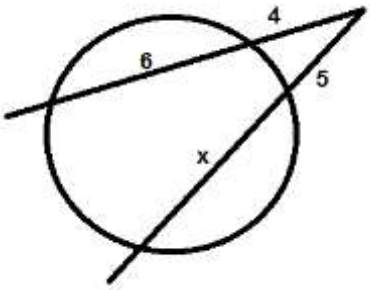
22. Shaklga ko‘ra, x = ?



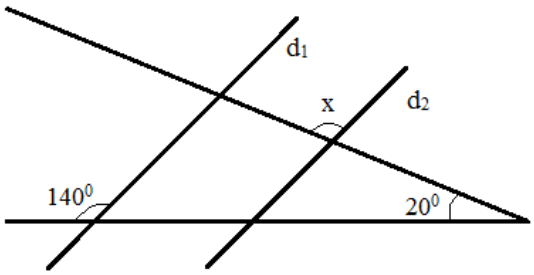
23. Shaklga ko‘ra shtrixlangan yuza qancha?



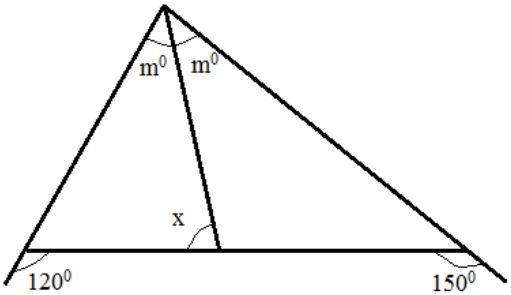
24. Shaklga ko‘ra, x = ?



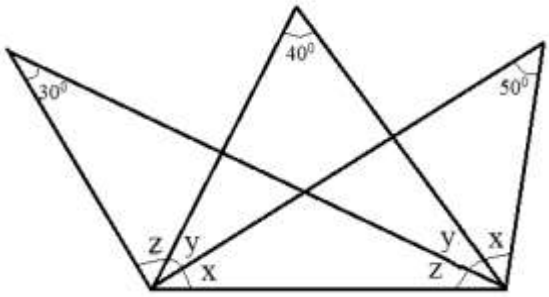
25. $d_1 // d_2$ bo‘lsa, shaklga ko‘ra, x = ?



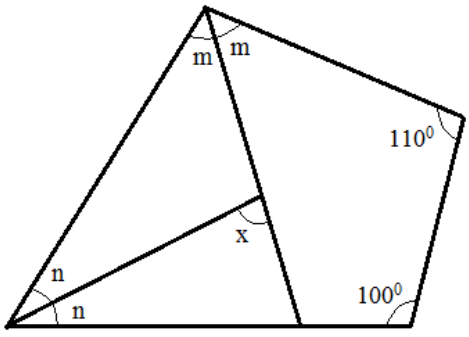
26. Shaklda berilganlarga ko‘ra, x = ?



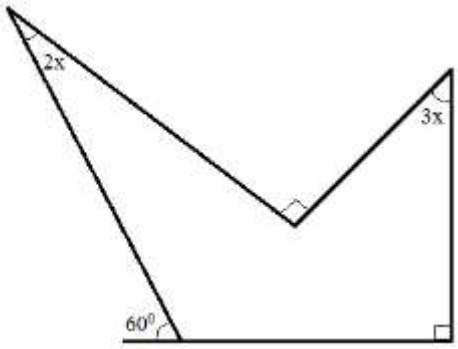
27. Berilgan shaklga ko‘ra, $x+y+z$ = ?



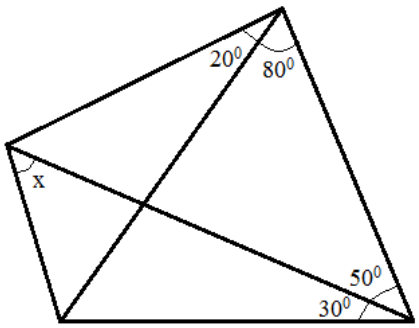
28. Shaklda berilganlarga ko‘ra x = ?



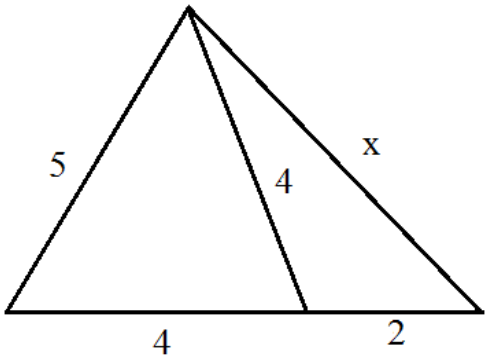
29. Shaklda berilganlarga ko‘ra, x = ?



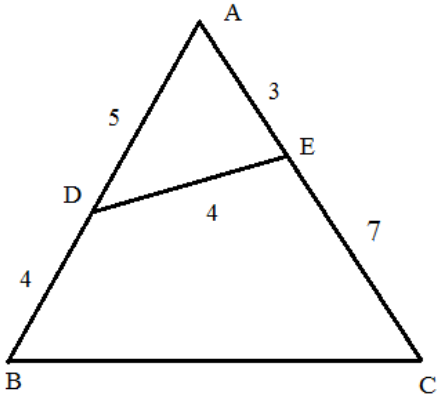
30. Shaklda berilganlarga ko‘ra, x = ?



31. Berilgan shaklga ko‘ra x = ?



32. Berilgan shaklga ko‘ra ABC uchburchak yuzini toping.

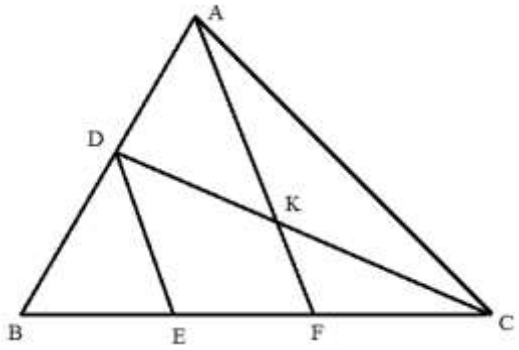


§2. Tasvirdagi shakl yetarli bo‘lgan masalalar.

Bunday tasvirli masalalarda berilgan shakl masalani yechish uchun zarur va yetarli bo‘lgan tushunchalarni o‘z ichiga olgan bo‘lib, o‘quvchi tasvirga o‘zgartirish kiritishiga zarurat bo‘lmaydi. Bu kabi masalalar tasvirli masalalarning asosiy qismini tashkil etadi. O‘quvchidan esa masalani mantiqiy hal qilinishi talab etiladi. Bunday ko‘rinishidagi masalalarni tuzish ancha murakkab, chunki bunday masalalar o‘quvchidan o‘rganilgan bilimlarni bilishi, tushunishi, amalda qo‘llay olishi, tahlil, sintez va hulosa chiqarishni talab qiladi.

Ushbu ko‘rinishidagi tasvirli masalalarni yechishga na’munalar keltiramiz.

1 – masala. Berilgan tasvirda *AD = DB*, *BE = EF = FC* va *DEFK* trapetsiyaning yuzi 12 sm^2 bo‘lsa, *DAK* uchburchakning yuzini toping.



Masalaning yechilishi: Masala shartiga ko‘ra *DEFK* trapetsiya va *EF = FC* ekanligidan *KF* kesma *DEC* uchburchakning o‘rta chizig‘i. Demak, *KFC* va *DEC* uchburchaklar o‘xshash. Uchburchaklar o‘xshashligiga ko‘ra

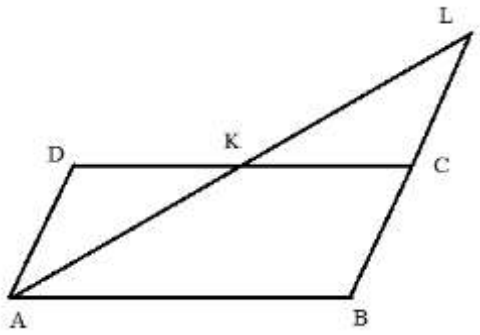
$$\frac{S_{KFC}}{S_{DEC}} = \left(\frac{FC}{EC}\right)^2 \Rightarrow \frac{S_{KFC}}{12 + S_{KFC}} = \frac{1}{4} \Rightarrow S_{KFC} = 4\ sm^2$$

kelib chiqadi. Quyidagi nisbatni qaraymiz

$$\frac{S_{BED}}{S_{BCD}} = \frac{\frac{BE \cdot BD}{2} \cdot sin\angle B}{\frac{BC \cdot BD}{2} \cdot sin\angle B} \Rightarrow \frac{S_{BED}}{16 + S_{BED}} = \frac{1}{3} \Rightarrow S_{BED} = 8\ sm^2,$$

D va *E* mos tomonlarning o‘rtasi ekanligidan *DE* kesma *ABF* uchburchakning o‘rta chizig‘i. Shuning uchun *DBE* va *ABF* uchburchaklar o‘xshash. Yuqoridagiga ko‘ra $S_{DEFK} = 24\ sm^2$ ga tengligi kelib chiqadi. Demak, $S_{DAK} = 12\ sm^2$.

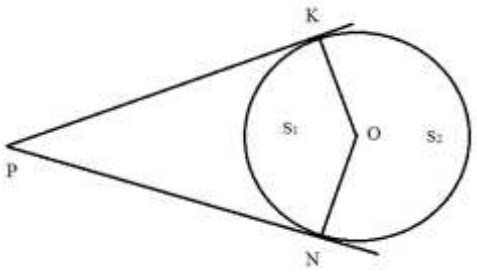
2 – masala. Tasvirda *ABCD* parallelogramm. *AL* bissektrisa va $AB = 3 \cdot AD$ bo‘lsa, $\frac{S_{ABCK}}{S_{KCL}} = ?$



Masalaning yechilishi: *ABCD* parallelogramm va *AL* bissektrisa ekanligidan ∠*KAB* =∠*DAK* = ∠*AKD* tenglik o‘rinli. Bundan esa *AD* = *DK* va $AB = \frac{3}{2} \cdot KC$ ekanligi kelib chiqadi. *L* burchak umumiy va ∠*LAB*=∠*LKC* dan *LAB* va *LKC* uchburchaklar o‘xshash. Demak,

$$\frac{S_{LKC}}{S_{LAB}} = \left(\frac{KC}{AB}\right)^2 \Rightarrow \frac{S_{LKC}}{S_{LKC}+S_{ABCK}} = \frac{4}{9} \Rightarrow \frac{S_{ABCK}}{S_{KCL}} = \frac{5}{4}.$$

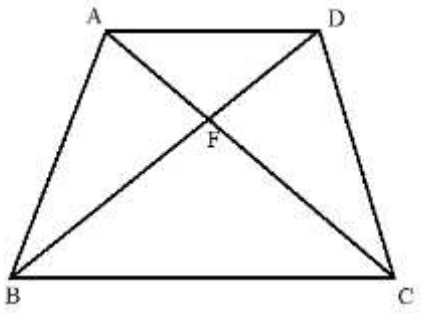
3 – masala. Tasvirda *PK* va *PO* doiraga urinmalar. $\angle P = 30°$ bo‘lsa, $\frac{S_2}{S_1} = ?$



Masalaning yechilishi: Berilgan doira radiusi R bo‘lsin. PK va PN urinmalar va $\angle P = 30°$ ekanligidan S_1 sektor $\frac{5\pi}{6}$, S_2 esa $\frac{7\pi}{6}$ radian markaziy burchakka tiralganligi aniqlanadi. Demak,

$$\frac{S_2}{S_1} = \frac{\frac{1}{2}\cdot R^2 \cdot \frac{7\pi}{6}}{\frac{1}{2}\cdot R^2 \cdot \frac{5\pi}{6}} = \frac{7}{5}.$$

4 – masala. $ABCD$ trapetsiya. $3 \cdot BF = 5 \cdot FD$, $S_{BFC} - S_{AFD} = 64\ sm^2$ bo‘lsa, $S_{ABC} = ?$



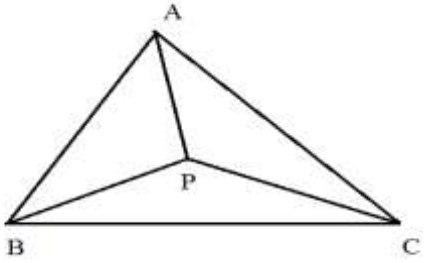
Masalaning yechilishi: $ABCD$ trapetsiyada $\angle FBC = \angle FDA$, $\angle FCB = \angle DAF$ ekanligidan BFC va AFD uchburchaklar o‘xshash. Demak,

$$\frac{S_{BFC}}{S_{AFD}} = \left(\frac{BF}{FD}\right)^2 = \frac{25}{9}$$

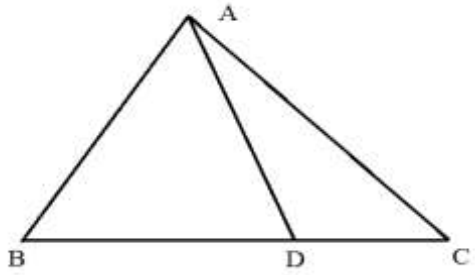
ga teng va $S_{BFC} - S_{AFD} = 64\ sm^2$ dan $S_{BFC} = 100\ sm^2$, $S_{AFD} = 36\ sm^2$ kelib chiqadi. Trapetsiyaning xossasiga ko‘ra, $S_{ABF} = \sqrt{S_{BFC} \cdot S_{AFD}} = 60\ sm^2$. U holda $S_{ABC} = S_{ABF} + S_{BFC} = 160\ sm^2$ natijaga ega bo‘lamiz.

Tasvirdagi shakl yetarli bo‘lgan masalalarga namunalar

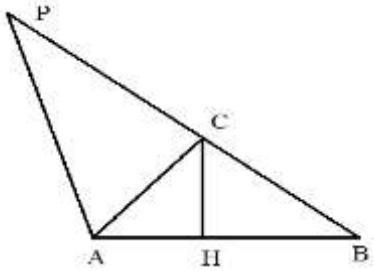
1. *AB=8 sm, AC=6 sm, BC=x sm, BP=5 sm, AP=PC=4 sm* va *P* uchburchak ichidagi bir nuqta bo‘lsa, *x* ning olishi mumkin bo‘lgan eng katta butun qiymati qancha?



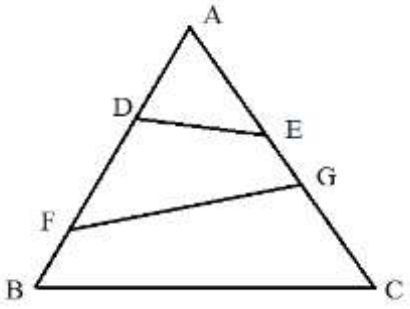
2. $S_{ABC} = 15\ sm^2$, *AB = AD,* $BD = 2 \cdot DC = 4\ sm$ bo‘lsa, *AD* necha sm?



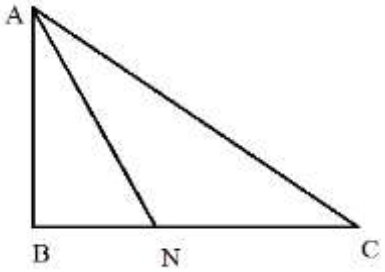
3. *AB⊥CH,* $BH = 2 \cdot AH = 4\ sm$, $PC = 3 \cdot CB$ va *CH = 3 sm* bo‘lsa, S_{ABP} =?



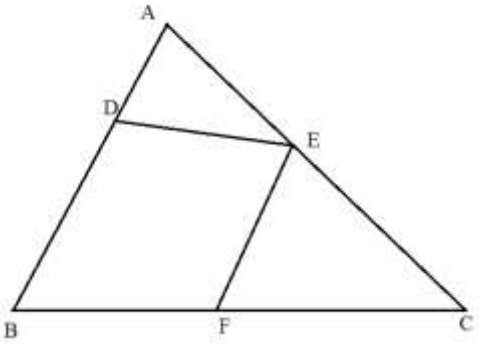
4. $AD = DF = 2 \cdot BF$,
$AE = 2 \cdot GC = 4 \cdot EG$ bo‘lsa, $\frac{S_{DFGE}}{S_{ABC}}$ =?



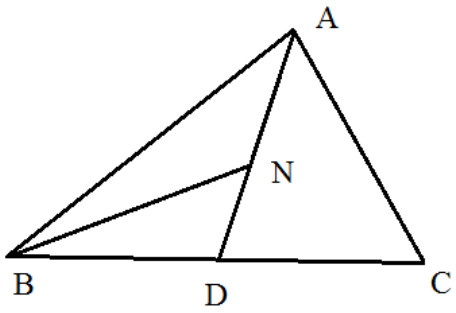
5. *ABC* teng yonli to‘g‘ri burchakli uchburchak va *AN* bissektrisa bo‘lsa, $\frac{BN}{NC}$ =?



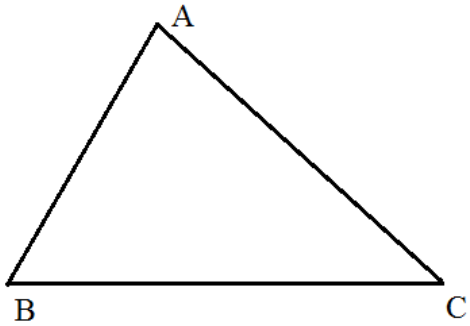
6. *AB⊥DE, AB//EF,* $AE = 2 \cdot EC$, $AD = 4\ sm$ va $EF = 3\ sm$ bo‘lsa, BD =?



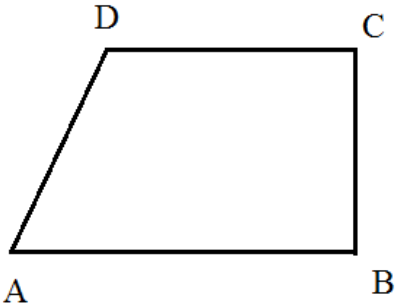
7. *AB*⊥*AC*, *AD* mediana, *BN* bissektrisa, *AB = 8 sm*, *AC = 6 sm* bo‘lsa, *AN* = ?



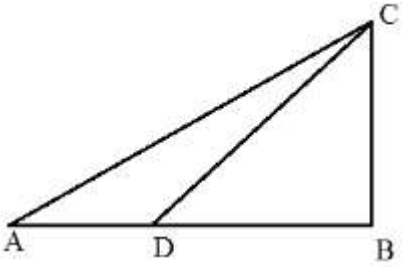
8. $\angle A = 105^\circ, \angle B = 60^\circ$ va *AB=2 sm* bo‘lsa, *BC* necha sm bo‘ladi?



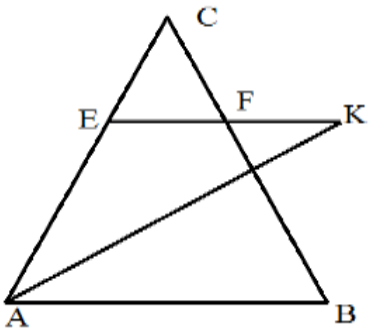
9. *ABCD* to‘g‘ri burchakli trapetsiya. *BC = CD = 3 sm* va $\angle A = 60^\circ$ bo‘lsa, *AB* necha *sm*?



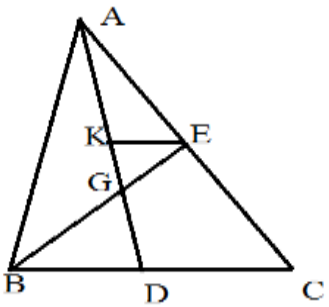
10. *AB*⊥*BC*, $\angle A = 30^\circ, \angle CDB = 45^\circ$ bo‘lsa, $\frac{CA}{CD} = ?$



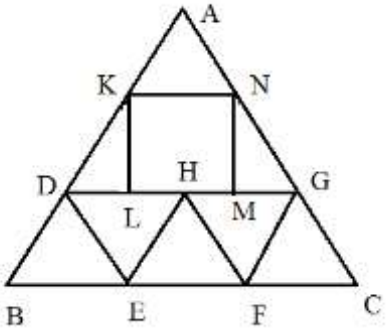
11. *EK // AB*, *AK* bissektrisa, *EF = EC* va $AC = 3 \cdot CE$ bo‘lsa, $\frac{AB+AC}{FK} = ?$



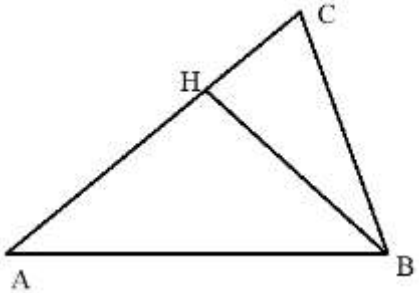
12. *G*, *ABC* uchburchakning og‘irlik markazi. $AK = 3 \cdot GK$ bo‘lsa, $\frac{S_{GEK}}{S_{GBD}} = ?$



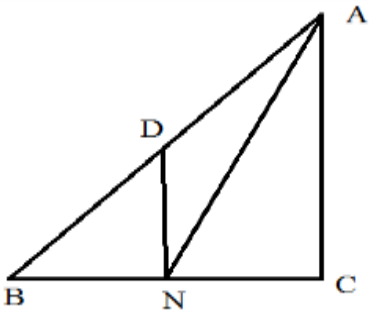
13. *ABC*, *DEH* va *HFG* muntazam uchburchaklar, *AN = a*, *NG = b*, *GC= =c* va *KLMN* kvadrat bo‘lsa, *a*,*b*,*c* larni taqqoslang.



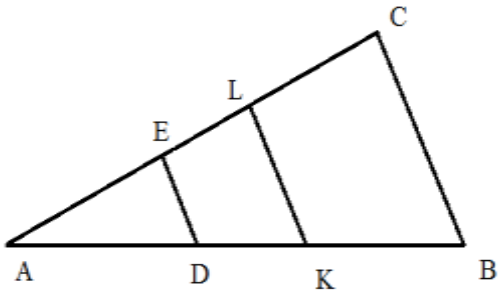
14. *AC*⊥*BH*, *AH = 12 sm*, *HC = 5 sm*, *BC = 13 sm* bo‘lsa, *AB* necha *sm*?



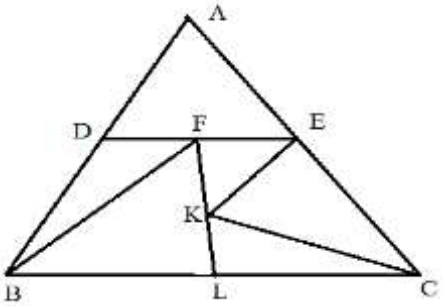
15. *AC⊥BC*, *AN* bissektrisa, *AD= 2 sm* va $\frac{S_{ANC}}{S_{AND}} = 3$ bo‘lsa, *AC* necha *sm*?



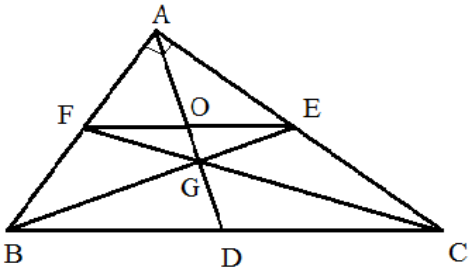
16. *BC//KL//DE*, $KB = 3 \cdot KD$, $AD = 2 \cdot KD$, $S_{KBCL} - S_{ADE} = 92\ sm^2$ bo‘lsa, $S_{DKLE} = ?$



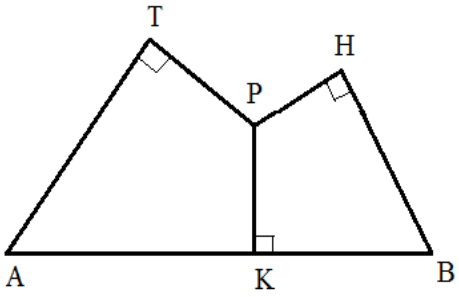
17. *D*, *E*, *F*, *L*, *K* nuqtalar mos tomonlarning o‘rtalari, *FEK* tomoni 1 *sm* bo‘lgan muntazam uchburchak bo‘lsa, *ABC* uchburchakning perimetri necha sm?



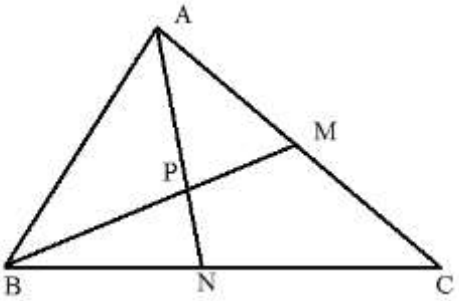
18. *ABC* to‘g‘ri burchakli uchburchak, *D*, *E*, *F* mos tomonlarning o‘rta nuqtalari bo‘lsa, $\frac{BC}{OG+OE} = ?$



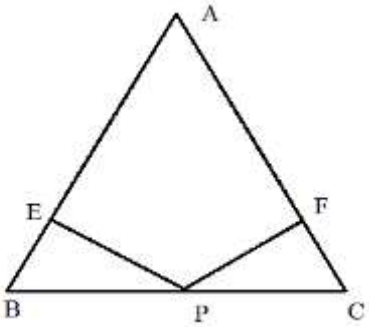
19. $PT + PK + PH = 12\ sm$, $\angle A = \angle B = 60^\circ$ bo‘lsa, *AB* necha *sm*?



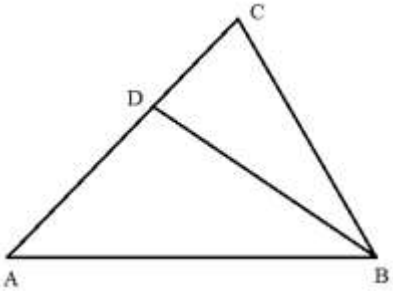
20. *AB⊥AC*, *AN* va *BM* bissektrisalar. *AB=6 sm* va *BC=10 sm* bo‘lsa, *PM*=?



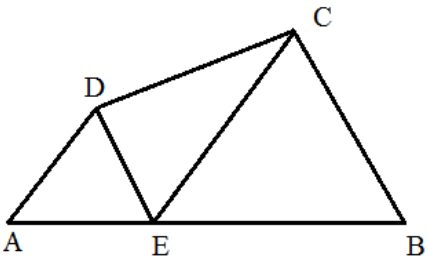
21. *ABC* muntazam uchburchak, *EP⊥AB*, *PF⊥AC*, *EP = PF* va *AC = 4 sm* bo‘lsa, $S_{BEP} = ?$



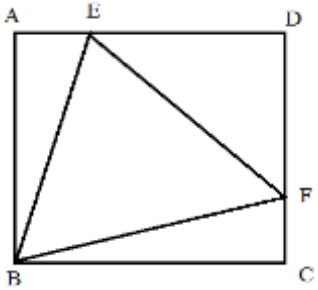
22. $\angle CAB = \angle DBC$ va $AD = 2 \cdot DC = 6\ sm$ bo‘lsa, *BC* necha *sm*?



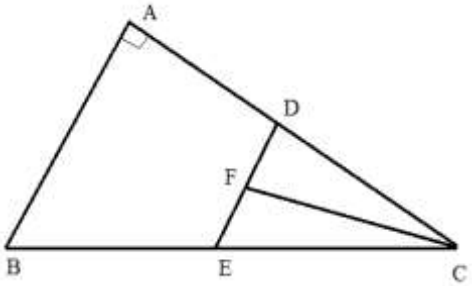
23. *AED* va *BCE* muntazam uchburchaklar, $DE \perp DC$ va $DC = 6\sqrt{3}$ bo‘lsa, *AB* necha *sm*?



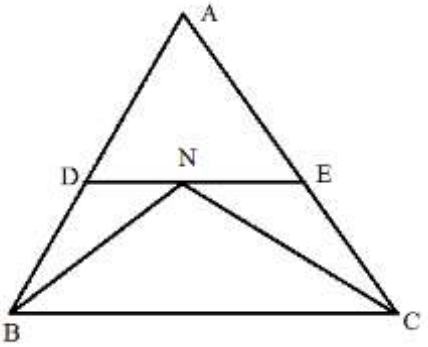
24. *ABCD* kvadrat va *BEF* muntazam uchburchak bo‘lsa, $\frac{S_{BEF}}{S_{ABCD}} = ?$



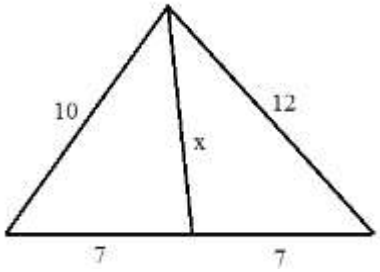
25. *ABC* to‘g‘ri burchakli uchburchakda *D* va *E* mos tomonlarning o‘rtalari. *FD=FE*, *AC = 8 sm* va $S_{EFC} = 6\ sm^2$ bo‘lsa, *BE* necha *sm*?



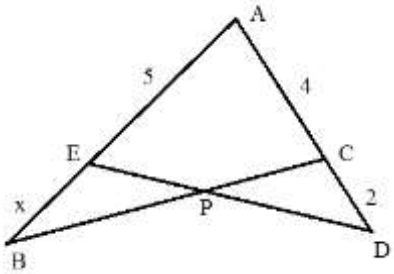
26. *N* bissektrisalar kesishish nuqtasi, *DE//BC* va *DB + EC = 8 sm* bo‘lsa, *DE* necha *sm*?



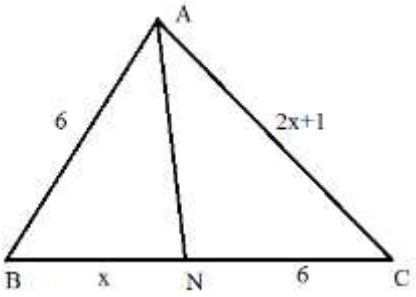
27. Shaklga ko‘ra *x* qancha?



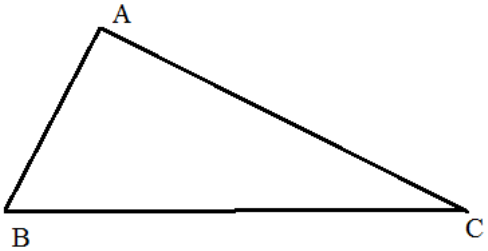
28.Shaklda berilganlarga ko‘ra $\frac{S_{ABC}}{A_{AED}} = \frac{6}{5}$ bo‘lsa, *x* = ?



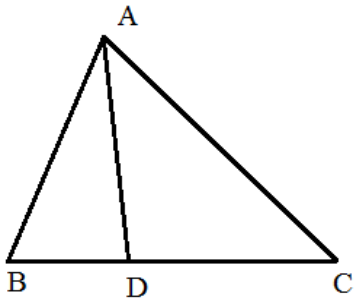
29. *AN* bissektrisa bo‘lsa, shaklga ko‘ra *x* = ?



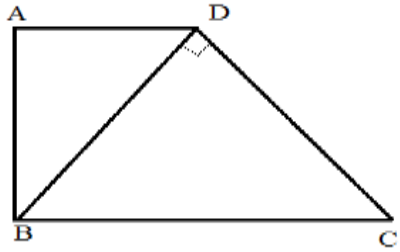
30. $\angle B = 67{,}5°$, $\angle C = 22{,}5°$ va *AB=2 sm* bo‘lsa, *ABC* uchburchakning yuzini toping.



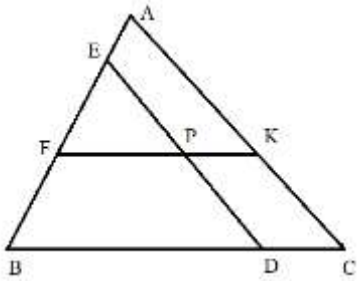
31. *AC = 7 sm*, *AD = 5 sm*, *DC=4 sm* va *BD = 2 sm* bo‘lsa, *ABC* uchburchakning yuzini toping.



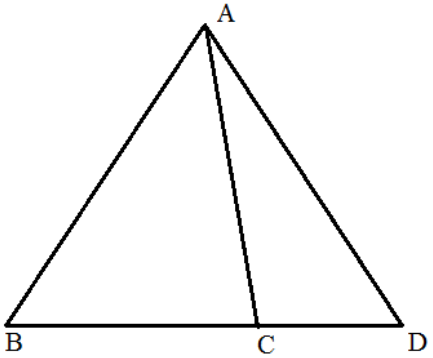
32. *ABCD* to‘g‘ri burchakli trapetsiya, *BD⊥DC*, *AD = 12 sm* va *BC = 25 sm* bo‘lsa, *DC* necha *sm*?



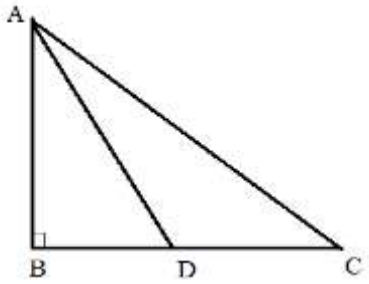
33. *FK//BC*, *AC//ED*, $BF = FE = 2 \cdot EA$, $BD = 4 \cdot DC$ bo‘lsa, $\frac{S_{ABC}}{S_{FEP}} = ?$



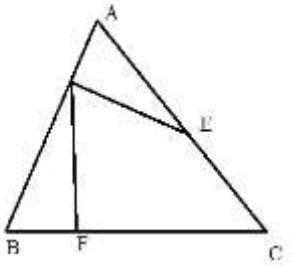
34. Muntazam *ABC* uchburchakda $BC = 2 \cdot CD = 2\ sm$ bo‘lsa, *AD* = ?



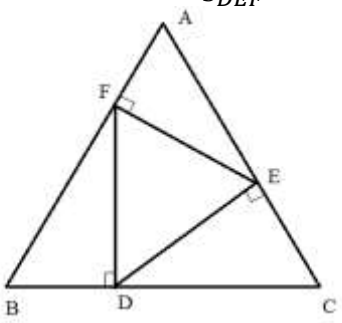
35. *BD = DC*, *AD = 5 sm*, $AC = 2\sqrt{13}$ sm bo‘lsa, *AB* = ?



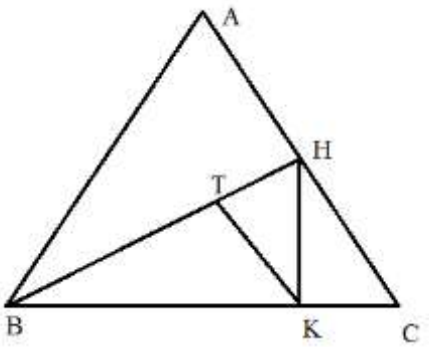
36. $BD = 3 \cdot AD$, $CF = 4 \cdot FB$, $AE = EC$ va $S_{ABC} = 80\ sm^2$ bo‘lsa, $S_{DECF} = ?$



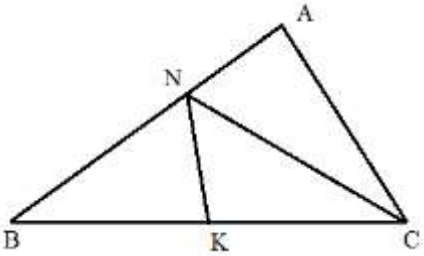
37. *ABC* va *DEF* muntazam uchburchaklar bo‘lsa, $\frac{S_{ABC}}{S_{DEF}} = ?$



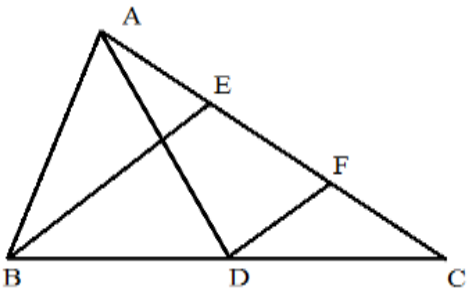
38. Muntazam *ABC* uchburchakning tomoni 16 *sm*. *BH⊥AC*, *BH⊥TK*, *HK⊥BC* bo‘lsa, S_{THK} =?



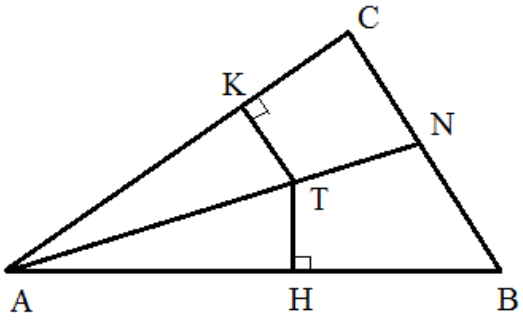
39. *CN* bissektrisa, $AC = 2 \cdot AN = 6sm$, $BK = KC = 4\ sm$ bo‘lsa, *NK* necha *sm*?



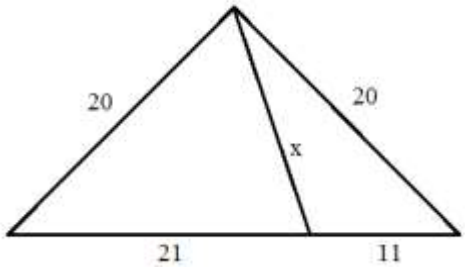
40. *BE//DF*, *AB = AD*, *DF = FC*, *AD=5sm*, *EF = 3 sm*, bo‘lsa, *AE* = ?



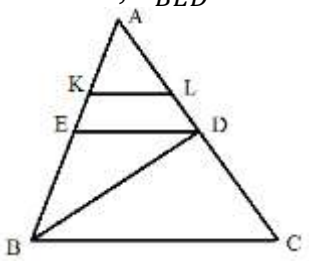
41. *AN* bissektrisa, *AH = x+1*, *HB=2x-1*, *NB = x+2*, *NC = x*, *KC=x-1* bo‘lsa, *x* = ?



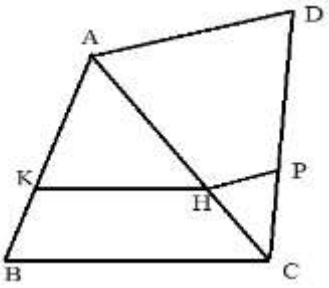
42. Shaklda berilganlarga ko‘ra *x* = ?



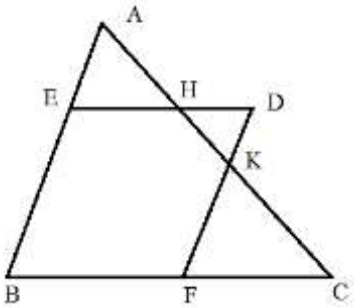
43. *BD* bissektrisa, *AD = DC*, *KL//BC*, $3 \cdot KL = 2 \cdot ED$ va *KEDL* trapetsiyaning yuzi 15 sm^2 bo‘lsa, $S_{BED} = ?$



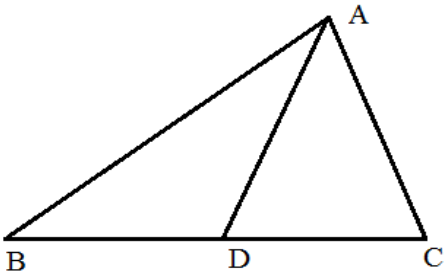
44. $\frac{AK}{KB} = \frac{3}{2}$, *KH//BC*, *AD//HP* bo‘lsa, $\frac{S_{ADPH}}{S_{HPC}} = ?$



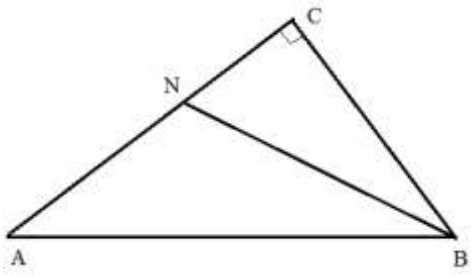
45. *BEDF* parallelogramm $BE = 2 \cdot AE$, *BC = 9 sm* va *HD = 2 sm* bo‘lsa, $\frac{S_{AEH}}{S_{KFC}} = ?$



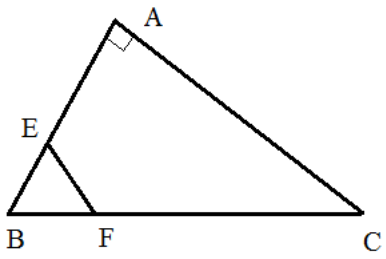
46. $\angle BDA = 120°$, *BD = DC = 4 sm*, *AD = 6 sm* bo‘lsa, $S_{ABC} = ?$



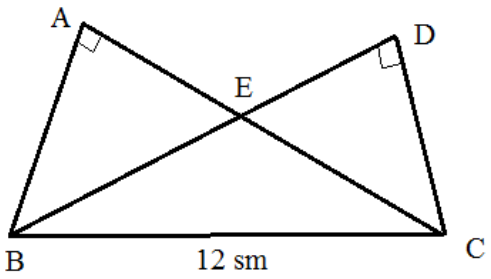
47. *BN* bissektrisa, *AN = 6 sm*, *NC = 4 sm* bo‘lsa, *AB* = ?



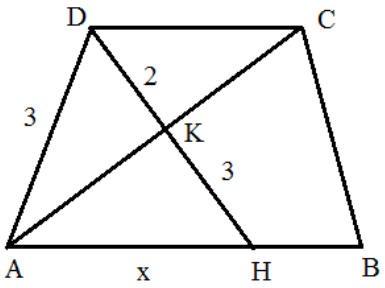
48. *ABC* to‘g‘ri burchakli va *BEF* muntazam uchburchak. $AE = 2 \cdot EF$ va *FC = 10 sm* bo‘lsa, *BF* = ?



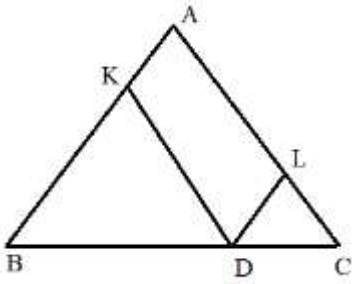
49. $\angle DBC = \angle ACD = 30°$ bo‘lsa, shaklga ko‘ra, $S_{ABE} = ?$



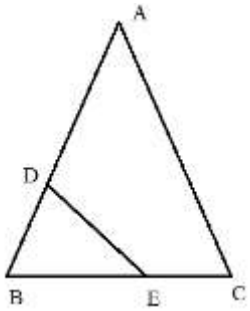
50. *ABCD* trapetsiya va *AC* bissektrisa. Shaklga ko‘ra *x* qancha?



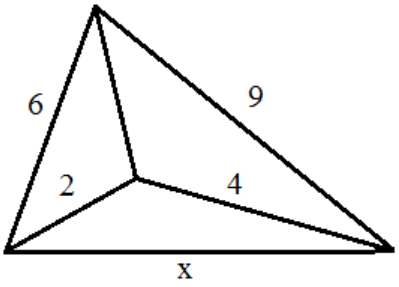
51. *AKDL* parallelogramm. *AB = AC*, *AB = 8 sm* bo‘lsa, *KD + DL* = ?



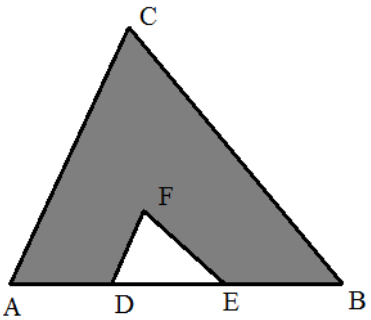
52. $\angle BAC = \angle BED = 40°$, $\frac{S_{ADEC}}{S_{BED}} = 8$, $BE = 2 \cdot BD = 2\ sm$ bo‘lsa, *AD* = ?



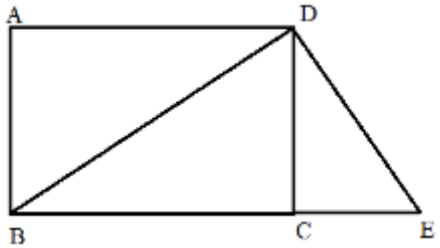
53. Shaklga ko‘ra *x* qanday oraliqda bo‘ladi?



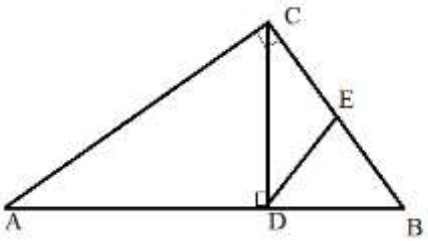
54. *AC//DF*, *BC//EF*, *AD=DE=EB* bo‘lsa, bo‘yalgan yuza *DEF* uchburchak yuzidan necha marta katta?



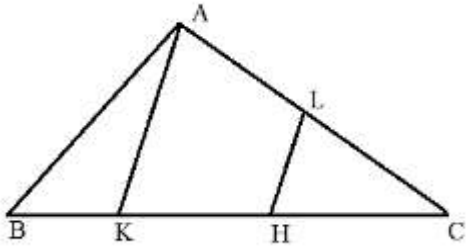
55. *ABCD* to‘g‘ri to‘rtburchak. *BD⊥DE*, $BC = 3 \cdot CE = 6\ sm$ bo‘lsa, *ABD* uchburchak yuzini toping.



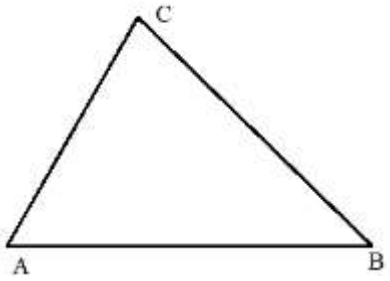
56. *ABC* va *BCD* lar to‘g‘ri burchakli, *BED* muntazam uchburchaklar. *BE = EC = 2 sm* bo‘lsa, *ABC* uchburchak yuzini toping.



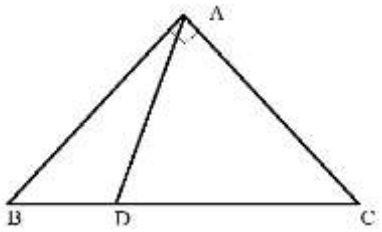
57. *AL = LC, BK = HC, AKHL* trapetsiyaning yuzi 12 sm^2 bo‘lsa, *ABK* uchburchak yuzini toping.



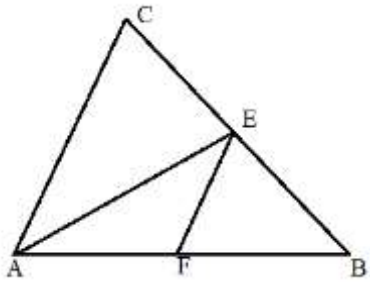
58. *AB = 5 sm, BC = 12 sm. ABC* uchburchakning yuzasi eng katta bo‘lganda *AB* = ?



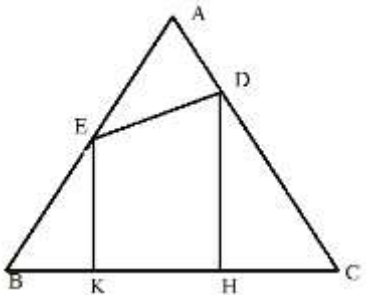
59. *ABC* teng yonli to‘g‘ri burchakli uchburchak. $BD = \sqrt{5}\ sm$, $DC = 3\sqrt{5}\ sm$ bo‘lsa, *AD* = ?



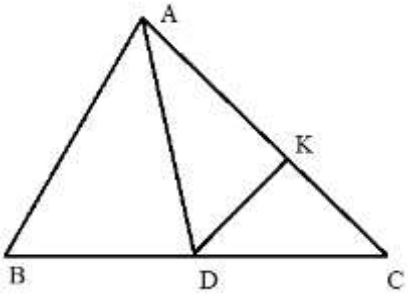
60. *AE* bissektrisa, *E* va *F* mos tomonlarning o‘rtasi. $\frac{AC}{2} = \frac{BC}{3} = 4\ sm$ bo‘lsa, *ABC* uchburchakning perimetrini toping.



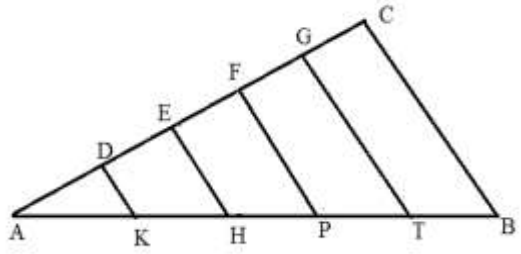
61. *ABC* muntazam uchburchak. *EK = ED, EK⊥BC, DH⊥BC*, $\angle DEK = 120°$, $DC = 4\sqrt{3}\ sm$ bo‘lsa, *AD* necha *sm*?



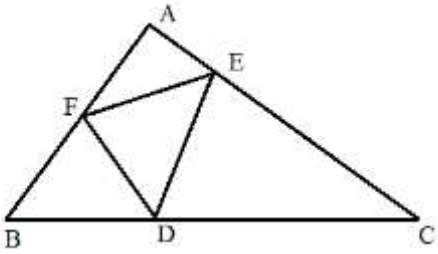
62. *AD* bissektrisa, *DC = 3 sm*, $AK = BD = 2 \cdot KC$ bo‘lsa, $\frac{S_{ABDK}}{S_{DCK}} = ?$



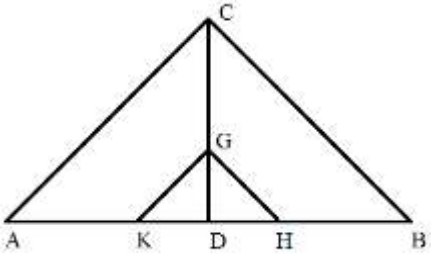
63. *DE//EH//FP//GT//CB, AK = KH = = HP = PT = TB,* va $S_{EFPH} = 10\ sm^2$ bo‘lsa, $S_{EFPH} = ?$



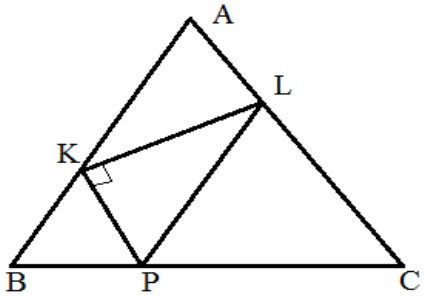
64. $AF = FB$, $EC = 3 \cdot EA$, $DC = 2 \cdot DB$ va $S_{DEF} = 10\ sm^2$ bo‘lsa, $S_{ABC} = ?$



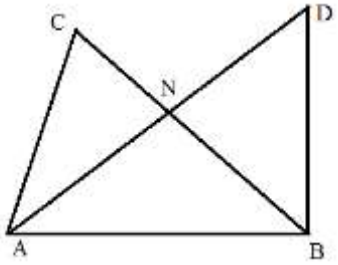
65. *AC⊥CB, AB⊥CD, G* og‘irlik markazi, *AC//KG, BC//HG* va *GD=4sm* bo‘lsa, $S_{KGH} = ?$



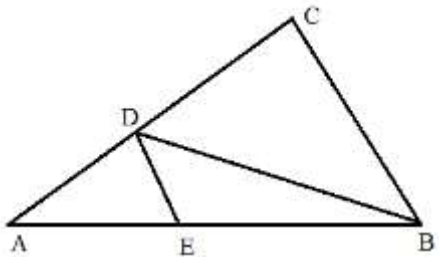
66. *AB = AC, AKPL* parallelogramm. $4 \cdot AK = 5 \cdot AL$ va *AK = 5 sm* bo‘lsa, $\frac{AC}{KL} = ?$



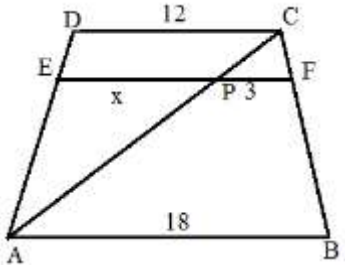
67. *AD* bissektrisa, *AB⊥BD, NA = ND, AC = 5 sm, AB = 8 sm* va $S_{ABD} = 24\ sm^2$ bo‘lsa, $S_{ACN} = ?$



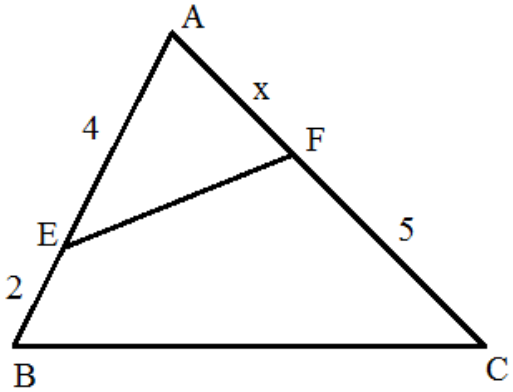
68. *AC⊥CB, AC⊥DE, DE = 3 sm, DC = 8 sm* va *CB = 9 sm* bo‘lsa, $S_{DEB} = ?$



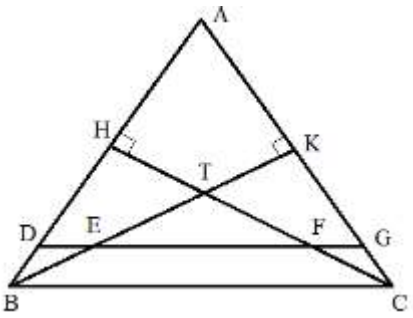
69. *ABCD* trapetsiya. *EF//DC* bo‘lsa, shaklga ko‘ra *x* topilsin.



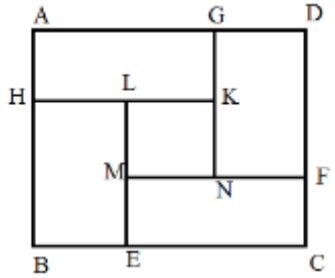
70. Shaklga ko‘ra $\frac{S_{AEF}}{S_{BEFC}} = \frac{1}{3}$ bo‘lsa, *x* = ?



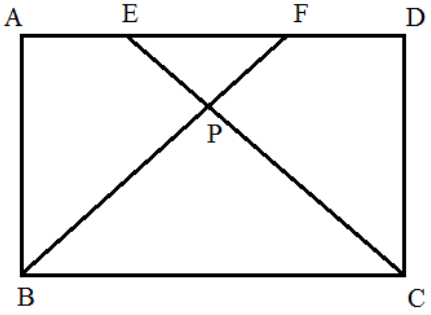
71. *ABC* muntazam uchburchak bo‘lib, $AD = 11 \cdot DB$ va *BC//DG* bo‘lsa, $\frac{S_{TBC}}{S_{TEF}} = ?$



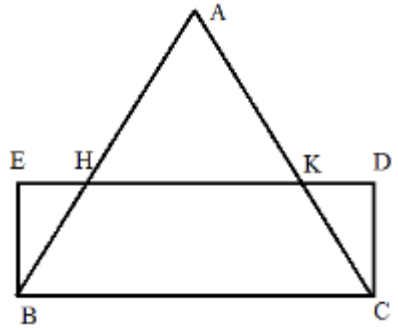
72. *ABCD* va *KLMN* kvadratlar *HK = MF = EL, AH = a, AG = b* bo‘lsa, S_{KLMN} ning yuzini toping.



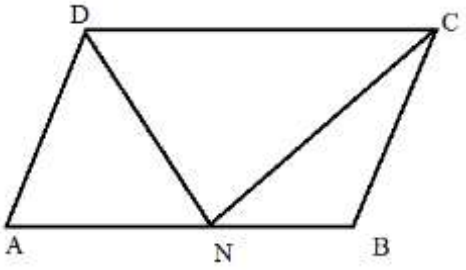
73. *ABCD* to‘g‘ri to‘rtburchak. *P* nuqta *BF* va *CE* bissektrisalar kesishish nuqtasi, *AB = 6 sm, BC = 8 sm* bo‘lsa, $S_{PEF} = ?$



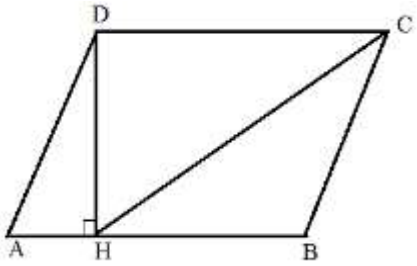
74. *ABC* muntazam uchburchak va *BCDE* to‘g‘ri to‘rtburchak. $AC = 3 \cdot CK$ bo‘lsa, $\frac{S_{HAK}}{S_{BCDE}} = ?$



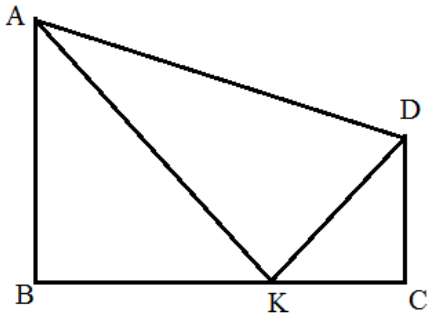
75. *ABCD* parallelogramm, *DN* va *CN* bissektrisa va *AN = 1 sm* bo‘lsa, parallelogrammning perimetrini toping.



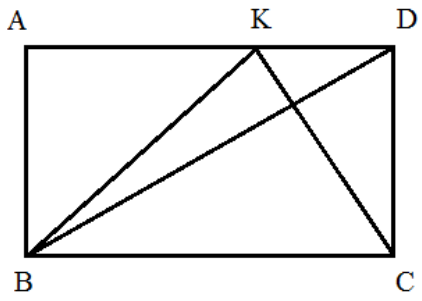
76. *ABCD* parallelogramm va *CH* bissektrisa. $\angle A = 60°$ va *AB = 12 sm* bo‘lsa, *AD* = ?



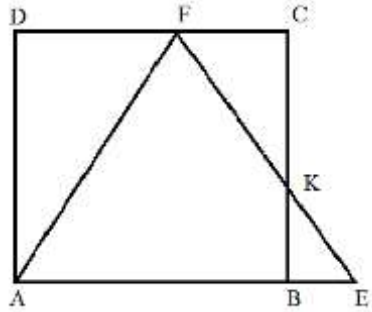
77. *AB⊥BC, BC⊥CD, AK⊥KD, AB = 8 sm, BK = 6 sm, KC = 4 sm* bo‘lsa, $S_{AKD} = ?$



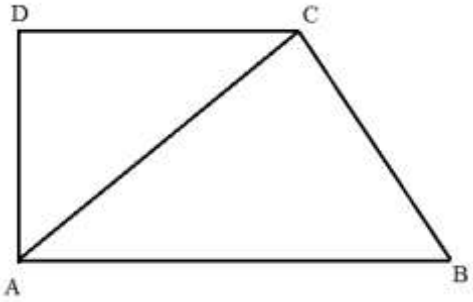
78. *ABCD* to‘g‘ri to‘rtburchak. $AK = 2 \cdot KD$ va $S_{ABK} = 6\ sm^2$ bo‘lsa, $S_{ABCD} = ?$



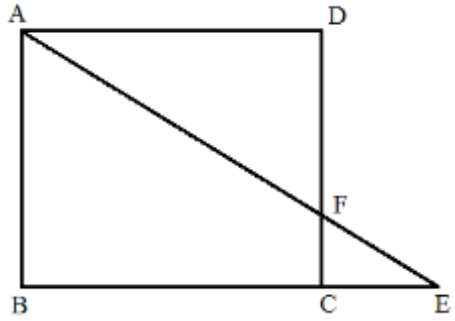
79. *ABCD* kvadrat, *AEF* muntazam uchburchak. $\frac{S_{KCF}}{S_{KBE}} = ?$



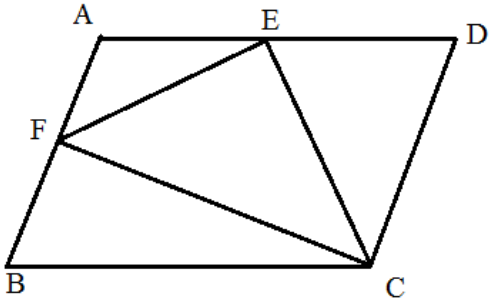
80. *ABCD* to‘g‘ri burchakli trapetsiya. *DC = 4 sm* va *DA = 3 sm*, *BC⊥AC* bo‘lsa, *AB* necha *sm*?



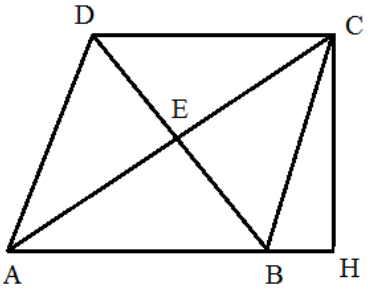
81. *ABCD* kvadrat. *BE = 4 sm*, *DF=3sm* bo‘lsa, *AB* = ?



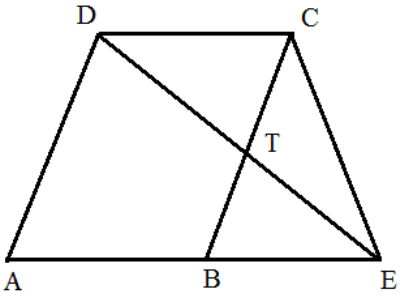
82. *ABCD* parallelogramm. *AE = ED*, *AF = FB* va $S_{ABCD} = 48\ sm^2$ bo‘lsa, $S_{FEC} = ?$



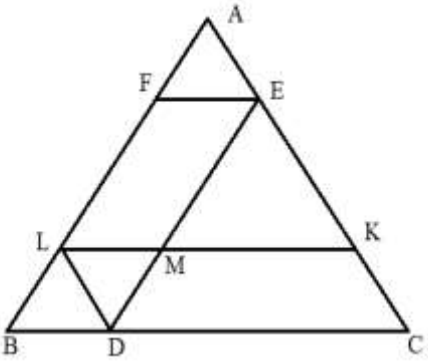
83. *ABCD* romb. *AE = 8 sm*, *EB = 6 sm* bo‘lsa, *BH* = ?



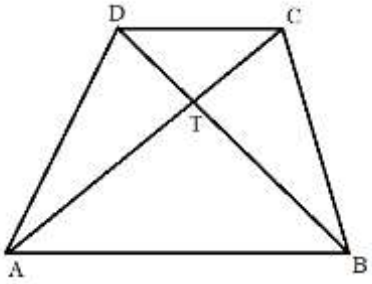
84. *ABCD* parallelogramm. $AB = 2BE$ va $S_{DEC} = 12\ sm^2$ bo‘lsa, $S_{ABTD} = ?$



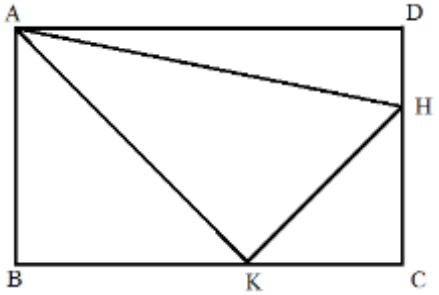
85. *ABC* tomoni 4 *sm*, *BDL* tomoni 1 *sm* bo‘lgan muntazam uchburchaklar. *BDEF* va *DCKL* parallelogrammlar bo‘lsa, $S_{DCKM} = ?$



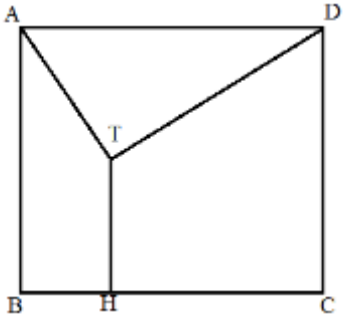
86. *ABCD* trapetsiya. $S_{DTC} = 9\ sm^2$ va $S_{TAB} = 16\ sm^2$ bo‘lsa, $S_{TAD} = ?$



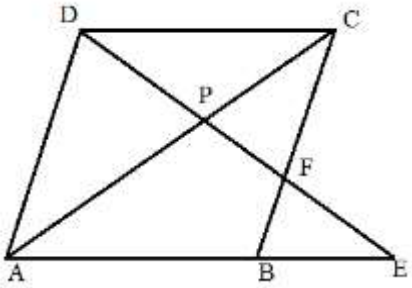
87. *ABCD* to‘g‘ri to‘rtburchak. *AB=BK, KC=CH,* $AD = 7 \cdot DH$, *DH=1 sm* bo‘lsa, $S_{AHK} = ?$



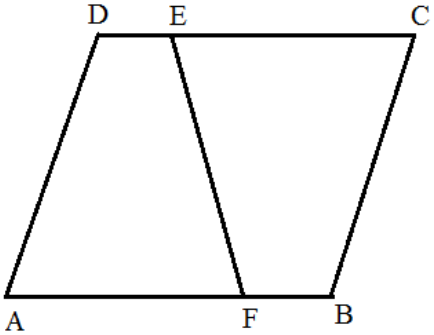
88. *ABCD* tomoni 6 *sm* bo‘lgan kvadrat. *BC⊥TH, AT⊥TD* va *TH=3sm* bo‘lsa, *TD* = ?



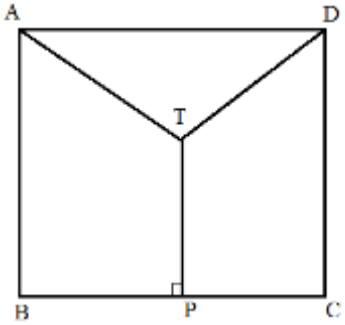
89. *ABCD* parallelogramm. $AB = 2 \cdot BE,\ \ 3 \cdot PF = 2 \cdot PD$ bo‘lsa, $\frac{S_{PFC}}{S_{BFE}} = ?$



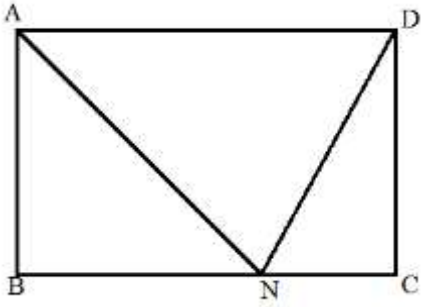
90. *ABCD* tomoni 6 *sm* bo‘lgan romb. *DE = FB = 2 sm* va $\angle A = 60°$ bo‘lsa, *EF* necha *sm*?



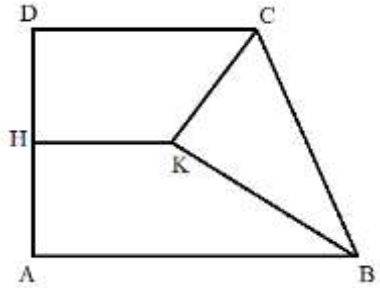
91. *ABCD* tomoni 2 *sm* bo‘lgan kvadrat. *AT = DT = PT* bo‘lsa, *PT* = ?



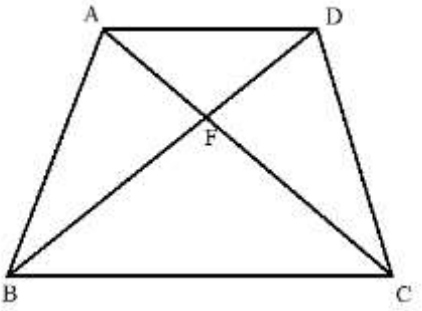
92. *ABCD* to‘g‘ri to‘rtburchak, *AN* bissektrisa, $3 \cdot AB = 2 \cdot BC$ bo‘lsa, $\frac{S_{BNA}}{S_{NCD}} = ?$



93. *ABCD* to‘g‘ri burchakli trapetsiya. *AD⊥HK, CK⊥KB, DH = HA = 4 sm, DC = 5 sm* va *AB = 11 sm* bo‘lsa, *HK* necha *sm*?



94. *ABCD* trapetsiya. $3 \cdot BF = 5 \cdot FD$, $S_{BFC} - S_{AFD} = 64\ sm^2$ bo‘lsa, $S_{ABC} = ?$

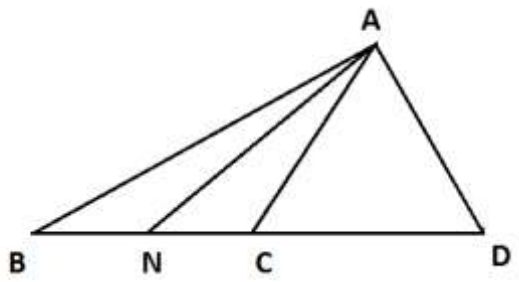


§3. Tasvirdagi shaklni to‘ldirish yordamida yechiladigan masalalar.

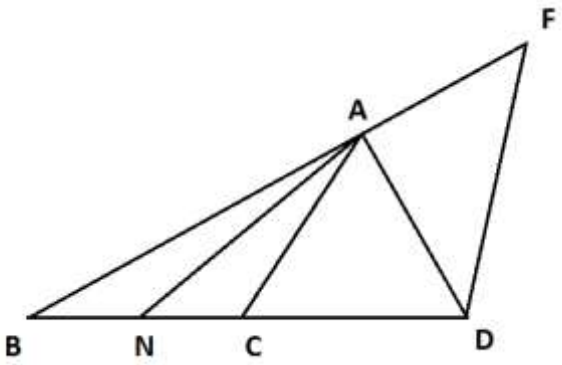
Ba’zan tasvirli masala shunday shaklda beriladiki, masalani hal qilish uchun berilgan chizma yordamida masalani hal qilish juda murakkab bo‘ladi. Unga qo‘shimcha element kiritilib masalani hal qilish ancha sodda bo‘lishi mumkin.

Bunday masala o‘quvchidan o‘rganilgan bilimlarni bilishi, tushunishi, amalda qo‘llay olishi, tahlil, sintez va hulosa chiqara olishidan tashqari ijodiy fikrlashini ham talab qiladi. Bu bo‘limda biz ana shunday masalalarning tipik namunalarini keltiramiz va ulardan ba’zilarining yechilish usullarini keltiramiz.

1 – masala. *AN* ichki va *AD* tashqi bissektrisalar. *BN=6 sm, NC=x sm, CD=x+1 sm* bo‘lsa, x = ?



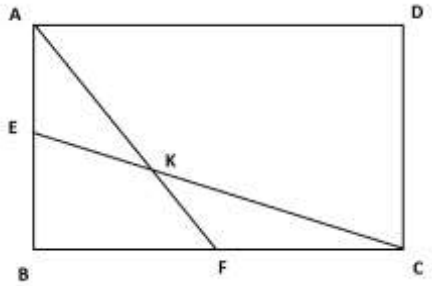
Masalaning yechilishi: Bu tasvirli masalani yechish uchun, *BA* kesmaning davomida uzunligi *AC* kesmaga teng bo‘lgan *AF* kesmani olamiz. U holda berilgan tasvirli masala quyidagicha to‘ldirilgan ko‘rinishni oladi.



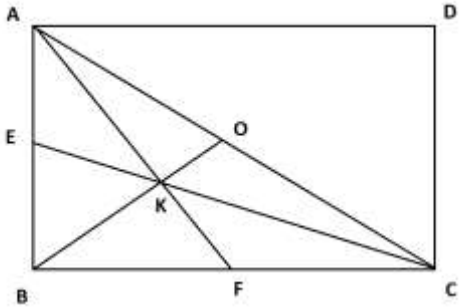
AN bissektrisa ekanligidan, uchburchak bissektrisasi ajratgan kesmaga ko‘ra, $\frac{AB}{AC} = \frac{BN}{NC} = \frac{6}{x}$ o‘rinli. Bunda *AD* bissektrisa, $\angle CAD = \angle DAB$, *AD* tomon umumiy va *AC=AF*. Uchburchaklar tengligi alomatidan, *ACD* va *ADF* uchburchaklar teng. Demak, $\angle ADC = \angle ADF$ va *AD* kesma *BDF* uchburchakning bissektrisasi bo‘ladi. Shuning uchun,

$$\frac{BD}{DF} = \frac{AB}{AF} = \frac{BN}{NC}, \qquad \frac{7+2x}{x+1} = \frac{6}{x}, \quad x = 1{,}5\ sm.$$

2 – masala. *ABCD* to‘g‘ri to‘rtburchak. *E* va *F* mos tomonlarning o‘rta nuqtalari bo‘lsa, $\frac{S_{AKCD}}{S_{EBFK}} = ?$



Masalaning yechilishi: Berilgan tasvir masalani yechish uchun yetarli emas, shuning uchun berilgan tasvirlga qo‘shimcha elementlar kiritib, tasvirli masalani osonlashtirish uchun chizmaga qo‘shimcha qilib, *AC* diagonal va *AC* diogonalning o‘rtasi va *B* nuqtani tutashtiruvchi kesma o‘tkazamiz va quyidagi tasvirli masalaga ega bo‘lamiz. $S_{ABCD} = S$ belgilash olaylik.

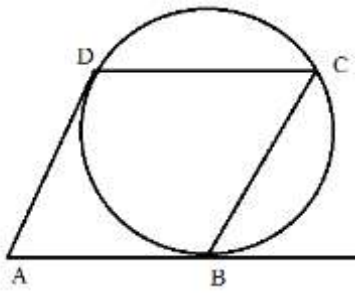


Bunda *E, F, O* nuqtalar mos tomonlarning o‘rtasi ekanligidan *K* nuqta *ABC* uchburchakning medianalari kesishish nuqtasi bo‘ladi. $S_{ABC} = \frac{S}{2}$ va *AF, BO, CE* lar mediana ekanligidan,

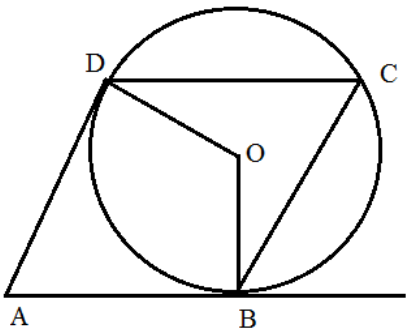
$$S_{AEK} = S_{EKB} = S_{BKF} = S_{KCF} = S_{KCO} = S_{AKO} = \frac{S}{12}$$

kelib chiqadi. Demak, $\frac{S_{AKCD}}{S_{EBFK}} = \frac{\frac{S}{2}+2\cdot\frac{S}{12}}{2\cdot\frac{S}{12}} = 4.$

3 – masala. *AB* va *AD* urinmalar . *ABCD* parallelogramm bo‘lsa, $\angle ABC$ = ?



Masalaning yechilishi: Aylana markazini *O* bilan belgilaylik. *O* nuqtani *B* va *D* nuqtalar bilan tutashtiramiz.



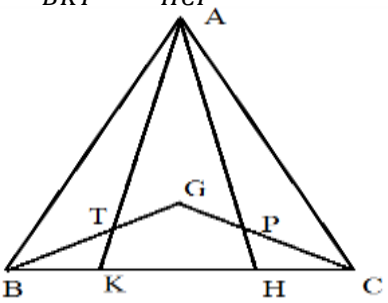
Masala shrtiga ko‘ra, *AD* va *AB* urinmalar. Bundan esa $\angle ADO = \angle ABO = 90°$ o‘rinli. Ichki chizilgan va markaziy burchakning xossasiga ko‘ra $\angle DOB = 2 \cdot \angle DCB$. *ABOD* to‘rtburchak va *ABCD* parallelogramm ekanligidan,

$$\angle A + 90° + 2 \cdot \angle A + 90° = 360°, \quad \angle A = 60°$$

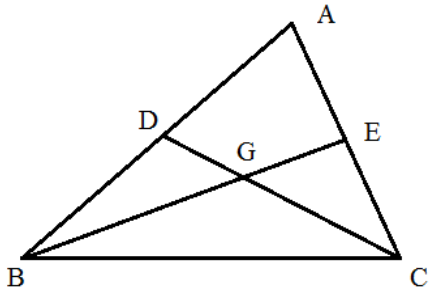
kelib chiqadi. Bundan esa, $\angle ABC = 120°$ ga teng.

Tasvirdagi shaklni to‘ldirish yordamida yechiladigan masalalarga namunalar.

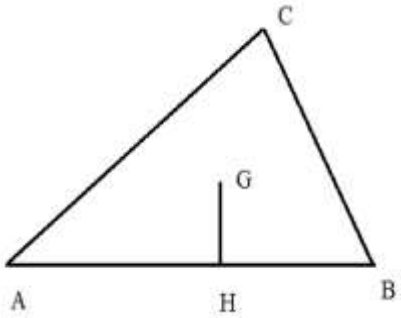
1. *G*, *ABC* va *AKH* uchburchaklarning og‘irlik markazi. $BK = CH$, $BC = 2 \cdot KH$ va $S_{ABC} = 48\ sm^2$ bo‘lsa, $S_{ATGP} - S_{BKT} - S_{HCP}$ necha sm^2 ?



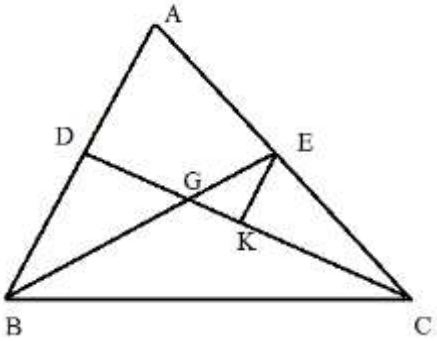
2. *AB = BC, BG = 6 sm,* $S_{DBG} = 6\ sm^2$ va *G* og‘irlik markazi bo‘lsa, *DG* = ?



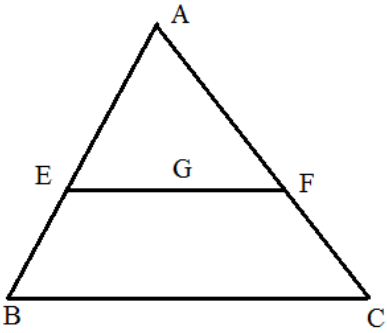
3. *G* og‘irlik markazi, *AB⊥GH, AB=AC, AH = 8 sm, GH = 6 sm* bo‘lsa, *HB* necha *sm*?



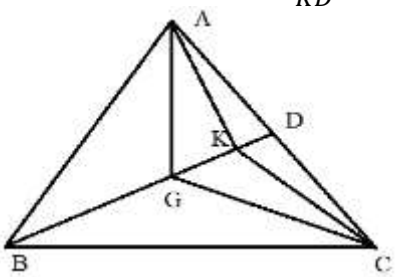
4. *G* og‘irlik markazi, *AB//KE* bo‘lsa, $\frac{S_{KEG}}{S_{ABC}} = ?$



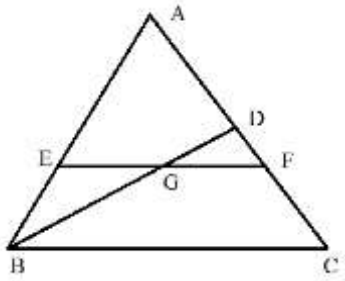
5. *G* og‘irlik markazi. *EF//BC*, *BC=9 sm* bo‘lsa, *EG* necha *sm*?



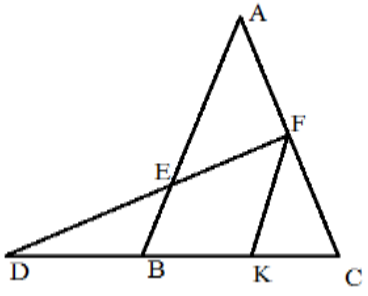
6. *ABC* uchburchagining og‘irlik markazi *G* va *AGD* uchburchagining og‘irlik markazi *K* bo‘lsa, $\frac{BK}{KD} = ?$



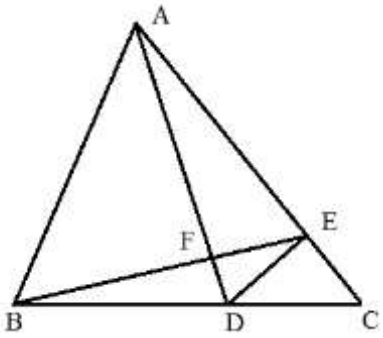
7. *G* og‘irlik markazi. *EF//BC* bo‘lsa, $\frac{S_{AEGD}}{S_{BGFC}} = ?$



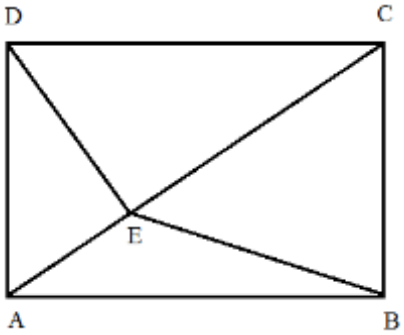
8. *ABC* teng yonli uchburchak. $\angle D = 30°$, $EF = 3\sqrt{3}$ *sm*, *AB//FK*, *BD = 3sm* bo‘lsa, *KC* = ?



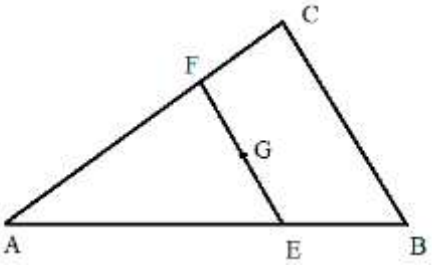
9. $\frac{BC}{DC}=\frac{7}{2}$, $\frac{AE}{AC}=\frac{4}{5}$, $AF=3\cdot FD$ bo‘lsa, $\frac{S_{ABC}}{S_{DEF}}=?$



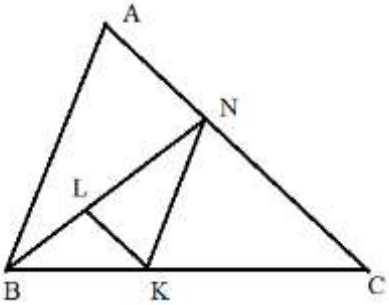
10. *ABCD* to‘g‘rito‘rtburchakda *ED⊥AC, AD = 6 sm, DC = 8 sm* bo‘lsa, $S_{ABED}=?$



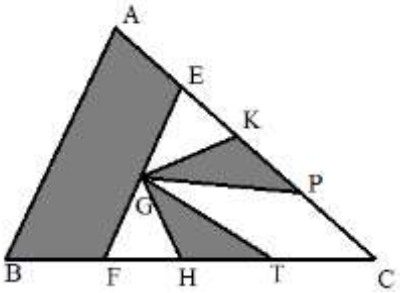
11. *G* og‘irlik markazi, *FE//CB* va $S_{BEFC}=10\ sm^2$ bo‘lsa, $S_{AEF}=?$



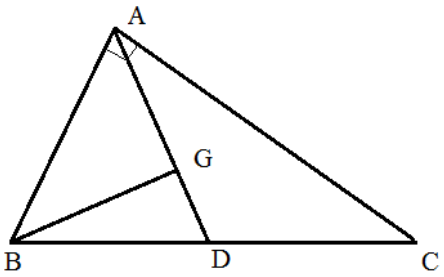
12. *BN* bissektrisa, *AC//LK*, $\frac{S_{ABN}}{S_{KLN}}=9$ va $KC=2\cdot BK$ bo‘lsa, $\frac{S_{KNC}}{S_{KLN}}=?$



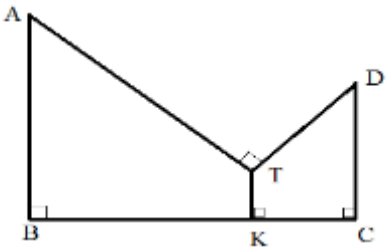
13. *G* og‘irlik markazi, *AB//EF*, *K* va *H* mos tomonlarning o‘rtalari, *KP = PC*, *HT = TC* bo‘lsa, *ABC* uchburchak yuzining bo‘yalgan sohalar yuzlari yig‘indisiga nisbatini toping.



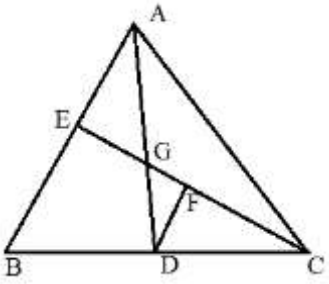
14. *G*, *ABC* to‘g‘ri burchakli uchburchakning og‘irlik markazi. *BG⊥AD* va *BC = 12 sm* bo‘lsa, *BG*=?



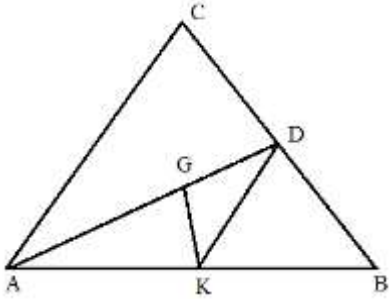
15. Shaklda *AB = 9 sm, DC = 4 sm, BC = 10 sm* va *TK = 1 sm* bo‘lsa, *AT + TD* = ?



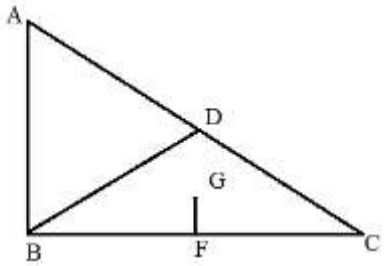
16. *G* og‘irlik markazi. *AB//FD*, *E* va *D* mos tomonlarning o‘rtalari bo‘lsa, $\frac{S_{AGC}}{S_{GDF}}=?$



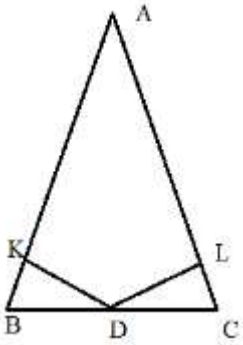
17. G o‘girlik markazi. $AK = KB$ va $S_{ABC} = 48\ sm^2$ bo‘lsa, $S_{GKD} = ?$



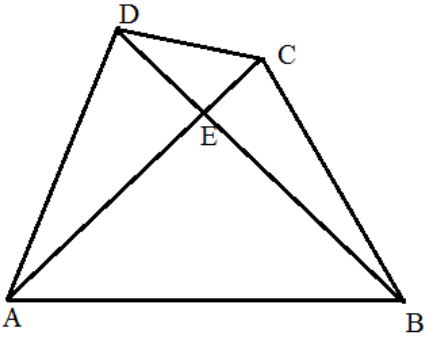
18. $AB \perp BC$, $AD = DC$, $BF = FC$, $\angle DBC = 30°$ va $GF = 1\ sm$. G nuqta BCD uchburchakning og‘irlik nuqtasi bo‘lsa, $S_{ABD} = ?$



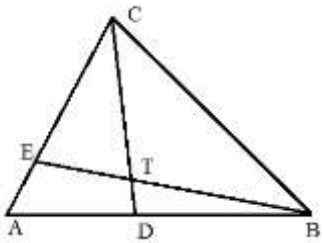
19. $AB = AC$, $DK \perp AB$, $DL \perp AC$, $\angle A = 30°$ va $AB = 12\ sm$ bo‘lsa, $DK+DL = ?$



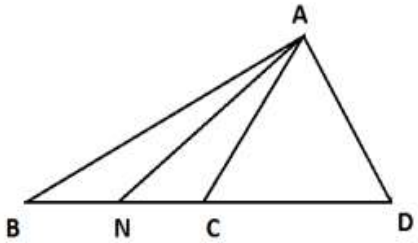
20. $\frac{AE}{EC} = \frac{5}{2}$ bo‘lsa, $\frac{S_{ABD}}{S_{BCD}} = ?$



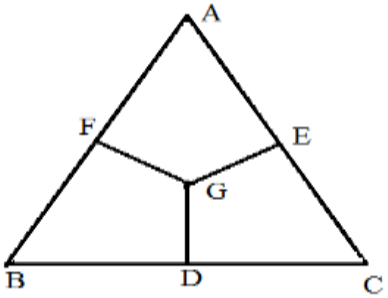
21. $AC = 4 \cdot AE$, $\frac{AD}{DB} = \frac{2}{3}$ bo‘lsa, $\frac{ET}{TB} = ?$



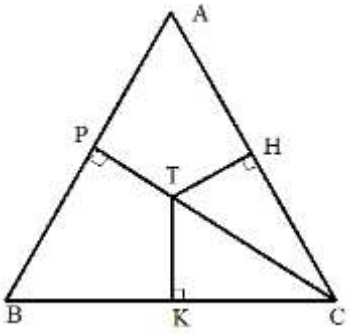
22. AN ichki va AD tashqi bissektrisalar. $BN=6\ sm$, $NC=x\ sm$, $CD=x+1\ sm$ bo‘lsa, $x = ?$



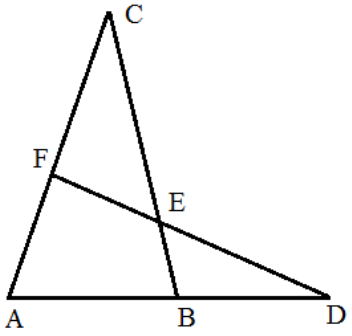
23. G og‘irlik markazi, D, E, F mos tomonlarning o‘rtalari. $EG = 8\ sm$ va $DG = FG = 5\ sm$ bo‘lsa, ABC uchburchakning perimetrini toping.



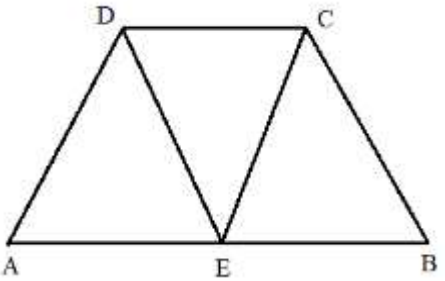
24. ABC uchburchakning medianalari kesishish nuqtasi T bo‘lib, $AP = 4\ sm$, $PT = 3\ sm$ bo‘lsa, $TC = ?$



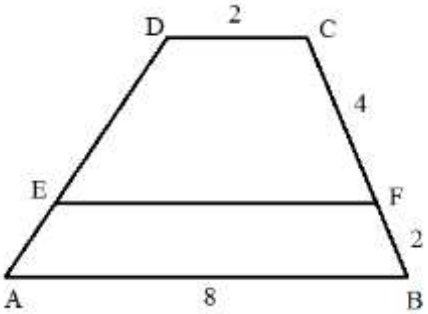
25. *AF = BD = 4 sm, FC = 6 sm* va $S_{FEC} = S_{BED}$ bo‘lsa, *AB* = ?



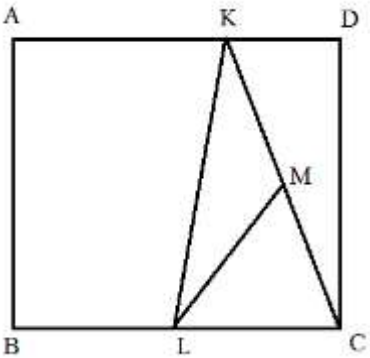
26. *AECD* romb, *ABCD* teng yonli trapetsiya. *DC = 2 sm* bo‘lsa, $S_{ABCD} = ?$



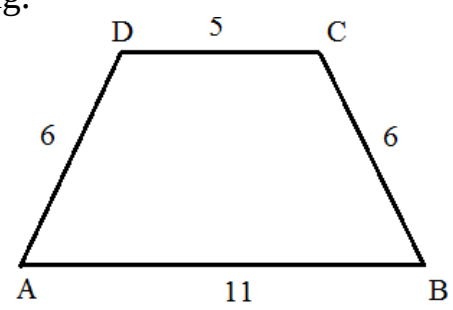
27. *ABCD* trapetsiya. *EF//AB* bo‘lsa, shaklga ko‘ra *EF* = ?



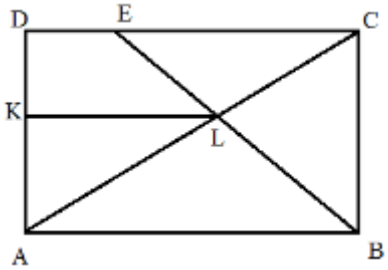
28. *ABCD* kvadrat. $AK = 2 \cdot KD$, *KM = MC, BL = LC* bo‘lsa, $\frac{S_{ABCD}}{S_{KLM}} = ?$



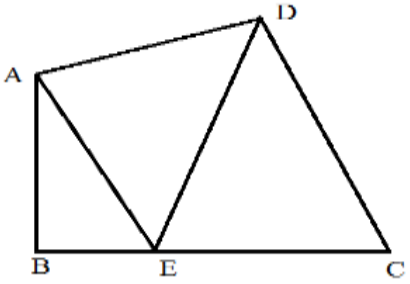
29. *ABCD* trapetsiya bo‘lsa, shaklda berilganlarga ko‘ra *ADC* burchakni toping.



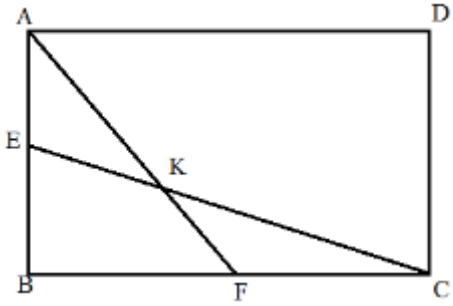
30. *ABCD* to‘g‘ri to‘rtburchak. *AD⊥KL*, *AB = 4 sm, DE = 1 sm* bo‘lsa, $\frac{S_{DELK}}{S_{KLBA}} = ?$



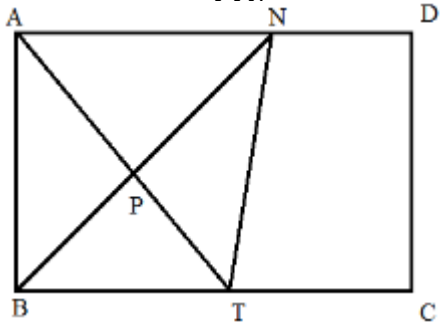
31. *AB⊥BC, AE//DC, AB = 4 sm, EC = 6 sm* bo‘lsa, $S_{AED} = ?$



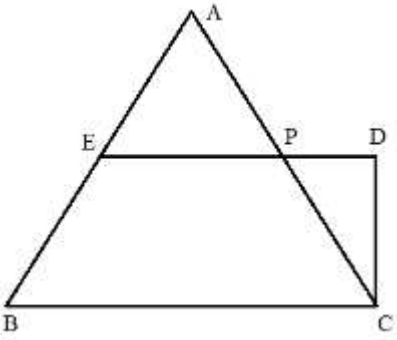
32. *ABCD* to‘g‘ri to‘rtburchak. *E* va *F* mos tomonlarning o‘rtalari bo‘lsa, $\frac{S_{AKCD}}{S_{EBFK}} = ?$



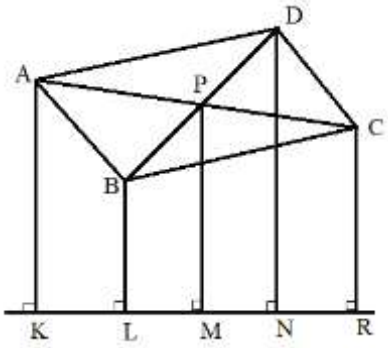
32. *ABCD* to‘g‘ri to‘rtburchak. *BN* bissektrisa, *BT = TC = 6 sm* va *AB = 8 sm* bo‘lsa, $S_{PTN} = ?$



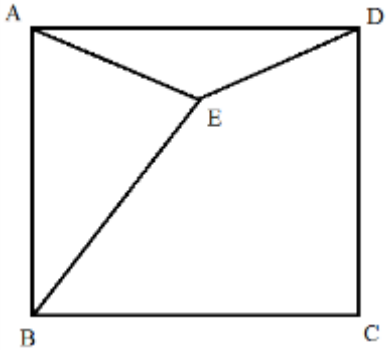
34. *ABC* muntazam uchburchak va *BCDE* to‘g‘ri burchakli trapetsiya. *BE = EA = 4 sm* bo‘lsa, *PD* = ?



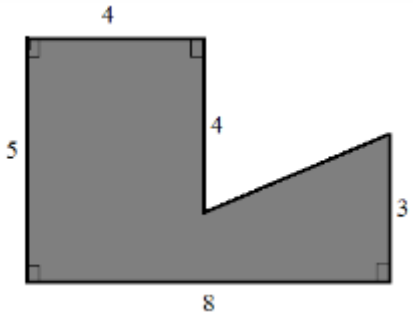
35. *ABCD* parallelogramm. *AK = 8 sm, BL = 4 sm, CR = 6 sm* bo‘lsa, *DN – PM* necha *sm*?



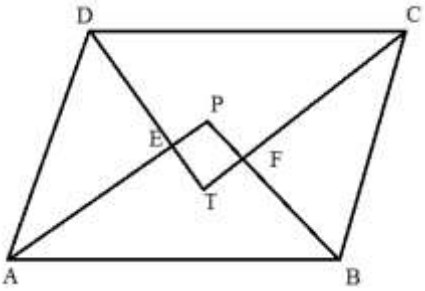
36. *ABCD* tomoni *4 sm* bo‘lgan kvadrat. *AE = ED* va *AB = BE* bo‘lsa, $S_{BEDC} = ?$



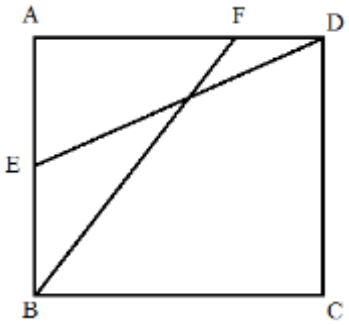
37. Shaklga ko‘ra bo‘yalgan sohaning yuzini toping.



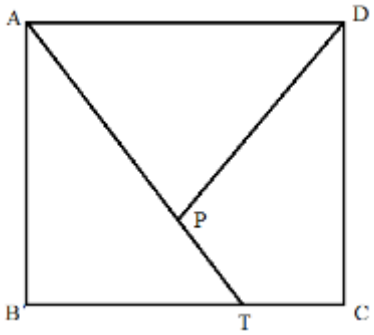
38. *ABCD* parallelogramm. *AP, BP, DT* va *CT* bissektrisalar. *EP = 3 sm, PF=4sm* va *EA = 6 sm* bo‘lsa, *AD* = ?



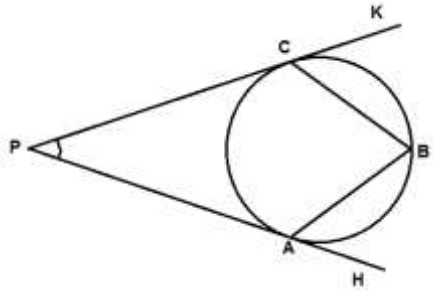
39. *ABCD* kvadrat. *AE = EB*, $AF = 2 \cdot FD$ bo‘lsa, $\frac{S_{ABKD}}{S_{BCDK}} = ?$



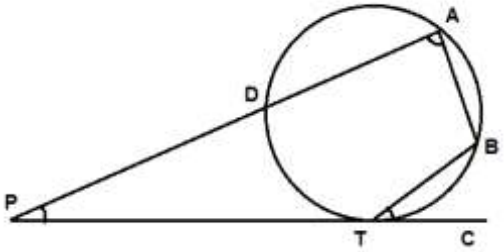
40. *ABCD* kvadrat. $BT = 2 \cdot TC$, $AP = 2 \cdot PT$ va $S_{DPTC} = 20\ sm^2$ bo‘lsa, $S_{ABCD} = ?$



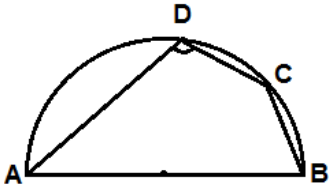
41. PK va PH aylanaga urunmalar $\angle KCB=60^0$ va $\angle BAH=50^0$ bo‘lsa, $\angle P$=?



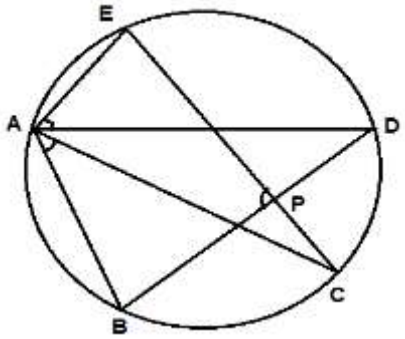
42. PT urunma $\angle PAB=80^0$, $\angle BTC=40^0$ va $\overset{\smile}{DA}=120^0$ bo‘lsa, $\angle APC$=?



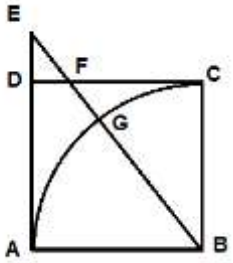
43. $\overset{\smile}{DB}=100^0$ va $DC = CB$ bo‘lsa, $\angle ADC$=?



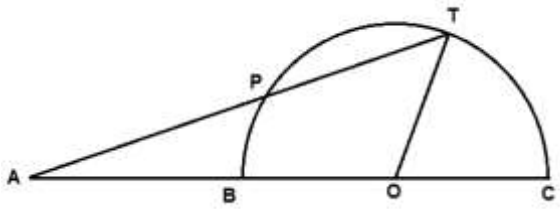
44. $\angle EAD=50^0$ va $\angle BAC=40^0$ bo‘lsa, $\angle BPE$=?



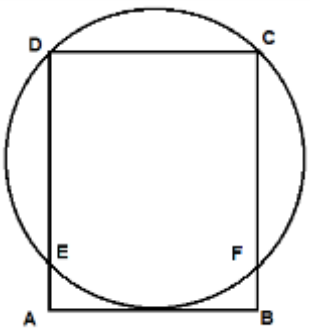
45. $AB=3sm$, $DE=3$ sm G nuqtasi EB kesma va B markazli aylananing kesishish nuqtasi. Bunga ko‘ra FG necha sm?



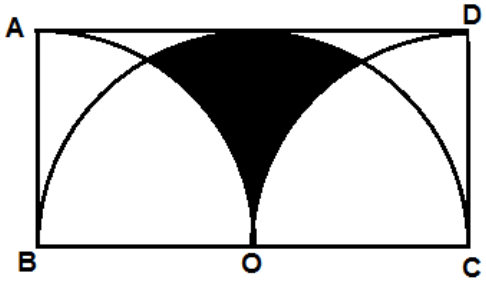
46. $\frac{AP}{AB}=\frac{3}{2}$, $PT=3$ sm va $OT=4$ sm bo‘lsa, AB necha sm?



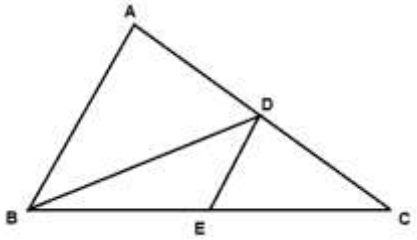
47. $ABCD$ to‘g‘ri to‘rtburchak. $\overset{\smile}{DC}=\overset{\smile}{CF}$ va $DC=8$ sm bo‘lsa, EA necha sm?



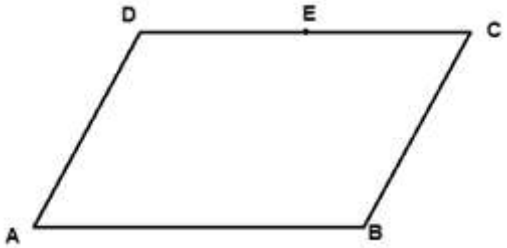
48. $ABCD$ to‘g‘ri to‘rtburchak. O markazli yarim aylana bilan B va C markazli chorak aylanalarning radiuslari 6 sm bo‘lsa, shtrixlangan yuza necha sm^2 ?



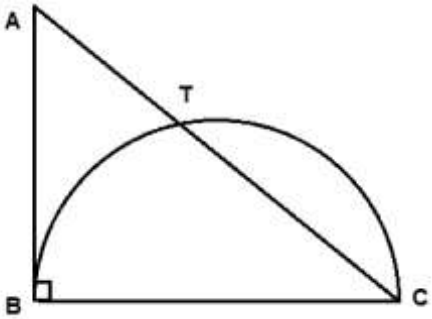
49. *AD = DC, BE = EC* bo‘lsa, $\overrightarrow{DC}+(\overrightarrow{BA}+\overrightarrow{AD})$=?



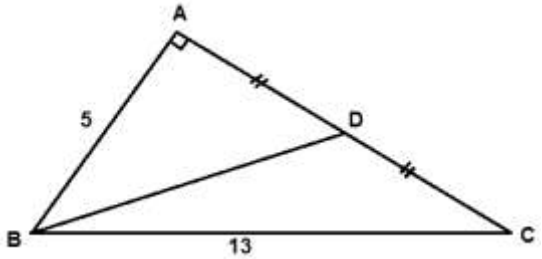
50. *ABCD* parallelogramm. *DE = EC* bo‘lsa, $(\overrightarrow{BC}+\overrightarrow{CB})-(\overrightarrow{DC}+\overrightarrow{DE})$ =?



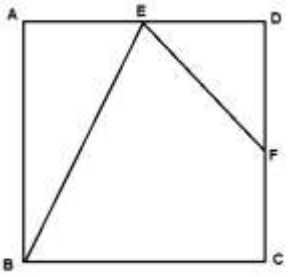
51. *ABC* teng yonli to‘g‘ri burchakli uchburchak. *BC* diametr va *BC=2 sm* bo‘lsa, $\overrightarrow{BC}\cdot\overrightarrow{CT}$ =?



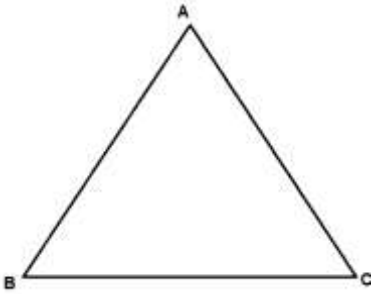
52. *AB⊥AC , AD = DC, AB=5, BC=13* bo‘lsa, $\overrightarrow{BD}\cdot\overrightarrow{CD}$ =?



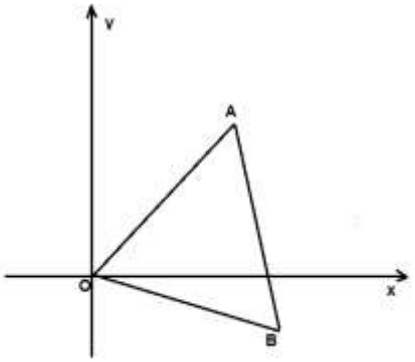
53. *ABCD* tomoni 2 birlik bo‘lgan kvadrat. *AE = ED* va *DF = FC* bo‘lsa, $\overrightarrow{EF}\cdot(\overrightarrow{EA}+\overrightarrow{AB})$ =?



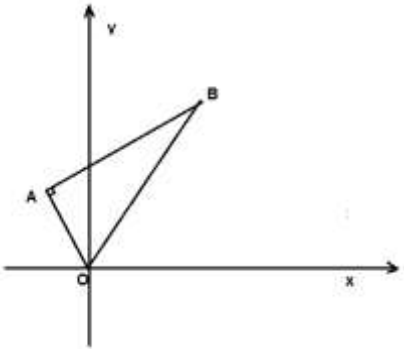
54. *ABC* teng tomonli uchburchak. $\overrightarrow{BA}-\overrightarrow{CA}$ =?



55. *AOB* teng tomonli uchburchak. *A*(12, 5) bo‘lsa, $\overrightarrow{OA}\cdot\overrightarrow{OB}$ =?



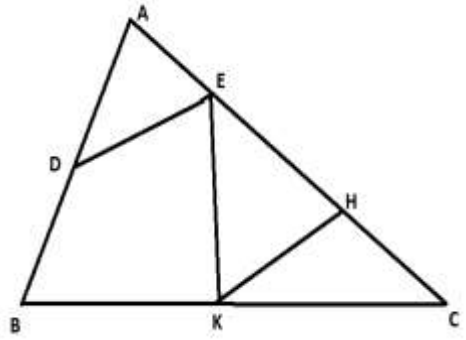
56. *AOB* to‘g‘ri burchakli uchburchak. $\overrightarrow{OB}=2\cdot\overrightarrow{OA}$ va *A*(-6 , 8) bo‘lsa, $\overrightarrow{AO}\cdot\overrightarrow{OB}$ =?



§4. Nuqtaning vaziyatiga bog‘liq tasvirli masalalar.

Tasvirli masalalarda shunday masalalarni uchratish mumkinki, ularda boshlang‘ich berilgan shakl masalani yechish uchun noqulay bo‘ladi. Shaklning umumiy elementlarini saqlagan holda tasvirni o‘zgartirish yo‘li bilan masalani yechishga qulay bo‘ladigan holatga olib kelish imkoni bo‘ladi. O‘quvchi tasvirda qatnashayotgan elementlarning shartlarini o‘zgartirmagan holda, ya’ni masala mohiyatini, qo‘yilgan talablarni saqlagan holda tasvirdagi shaklni o‘zgartirishi mumkin. Bunday masala o‘quvchidan o‘rganilgan bilimlarni bilishi, tushunishi, amalda qo‘llay olishi, tahlil, sintez va hulosa chiqara olishidan tashqari ijodiy fikrlashini ham talab qiladi. Bu masalalar tasvirli masalalarni tuzishning uchinchi ko‘rinishiga qaraganda ham murakkabroq ko‘rinishida beriladi.Bunday ko‘rinishdagi tasvirli masalalarga misollar keltiramiz.

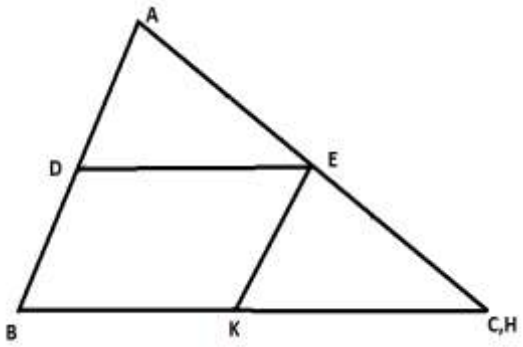
1-masala. Tasviri quyida berilgan shaklda *D* va *K* mos tomonlarning o‘rtasi, *DE//KH* bo‘lsa, $\frac{S_{KEH}}{S_{DEKB}} = ?$



Izoh: Masala shartiga ko‘ra *D* va *K* nuqtalar yagona mumkin bo‘lgan aniq nuqtalar. Ammo *DE//KH* shartni qanoatlantiruvchi *E* va *H* nuqtalar *AC* kesmada ixtiyoriy joylashishi mumkin. Masalada berilgan tasvir, masala shartini qanoatlantiruvchi yagona tasvir emas. O‘quvchilarda bunday masalalarni yechish ko‘nikmasini hosil qilish uchun o‘qituvchi ushbu tushunchalarni o‘rgatish zarur. Tekislikda ikki nuqtadan bir-biriga parallel istalgancha to‘g‘ri chiziqlar o‘tkazish

mumkin ekanligini tushuntirish. Berilgan *DK* kesma o‘rta chiziq bo‘lgani uchun, *DEHK* – parallelogramm. Demak, *EH=DK*. Shuningdek, xususiy xollarda *E* nuqta *A* bilan yoki *H* nuqta *C* nuqta bilan ustma-ust tushishi mumkin.

Masalaning yechilish: Masalaning shartiga ko‘ra *D* va *K* nuqtalarning o‘rni aniq, ular mos ravishda *AB* va *BC* tomonlarning o‘rtasi. Ammo *DE//KH* shartidan *E* va *H* nuqtaning o‘rni muhim emas. Agar *DE//KH* ni e’tiborga olgan holda *E* nuqtani *AC* tomonning o‘rtasiga ko‘chiramiz, u holda *H* nuqta *C* nuqta bilan ustma-ust tushadi. Sababi *DE* kesma uchburchakning o‘rta chizig‘i bo‘lib qoladi. Demak *DE//BC*, *DE//KH* va *KH=KC*. Bunday holda masalaning sharti hech qanday o‘zgarmaydi. Berilgan masalaga mos chizma quyidagicha o‘zgartiriladi.



Bu tasvir masalani hal qilishni ancha osonlashtiradi. $S_{ABC} = S$ deb belgilash olaylik, uchburchak *ADE* va uchburchak *ABC* o‘xshash ekanligidan,

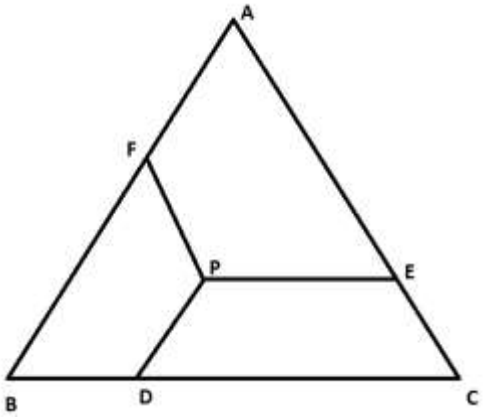
$$\frac{S_{ADE}}{S_{ABC}} = \left(\frac{DE}{BC}\right)^2 = \frac{1}{4}$$

Demak, $S_{ADE} = \frac{S}{4}$, huddi shuningdek $S_{KEH} = \frac{S}{4}$ kelib chiqadi. U holda,

$$\frac{S_{KEH}}{S_{DEKH}} = \frac{\frac{S}{4}}{\frac{S}{2}} = \frac{1}{2}.$$

Bunda albatta *H* nuqtani *AC* tomonning o‘rtasiga olib borib joylashtirgan holni ham qarash kerak. Bunday ko‘rinishida ham masalaning yechimi aynan bir xil usulda bo‘ladi.

2-masala. *AB//PD, BC//PE, AC//FP*. *ABC* tomoni 6 *sm* ga teng bo‘lgan muntazam uchburchak bo‘lsa, *PD+PE+PF*=?



Izoh: Bu masala ham *PD, PE, PF* kesmalarning mos tomonlarga parallelligini saqlagan holda, *P* nuqtani uchburchak ichidagi yoki tomonlaridagi ixtiyoriy nuqtaga ko‘chirganda masala yechimi o‘zgarmasligiga asoslanadi.

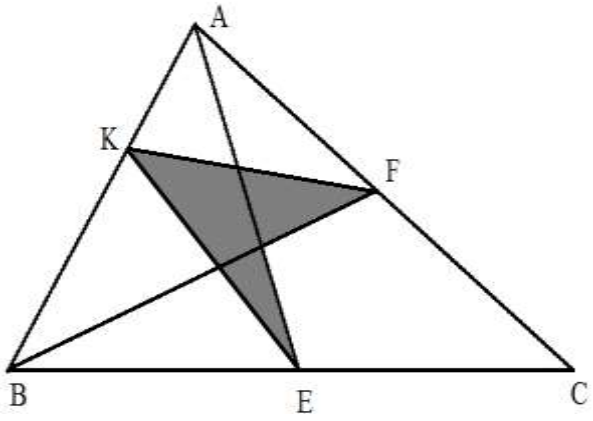
Masalaning yechimi: Masala shartiga ko‘ra, *P* nuqta ixtiyoriy nuqta(*P* nuqtaning aniq o‘rni ko‘rsatilmagan). Bunda *AB//PD, BC//PE, AC//FP* shartlarni e’tiborga olgan holda *P* nuqtaning o‘rnini *ABC* uchburchak ichida hohlagancha o‘zgartira olamiz. Agar *P* nuqtani uchburchakning *B* uchiga olib kelsak, masala sharti o‘zgarmaydi, ammo *PF=0, PD=0, PE=BC* va *BC//PC* kelib chiqadi. Bu esa masalani hal qilishni osonlashtiradi. Demak,

$$PD+PE+PF= 6\ sm.$$

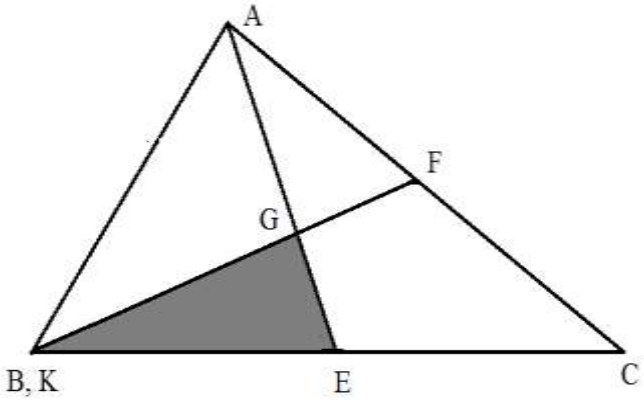
Bu yechim masalaning sodda yechimlaridan biri. Masalani *P* nuqtani uchburchakning *A* yoki *C* uchlariga olib borib joylashtirganimizda ham masala shartini o‘zgartirmagan holda huddi shu yechimni hosil qilamiz. *ABC* muntazam uchburchak ekanligidan topilgan yechimning barcha hollarda ham bir xil bo‘ladi.

Demak berilgan nuqtani almashtirishlar faqat *ABC* uchburchak muntazam ekanligidagina o‘rinli bo‘lar ekan.

3 – masala. *K* nuqta *AB* ustidagi va *E*, *F* mos tomonlarning o‘rtasi. Agar $S_{ABC} = 48\ sm^2$ bo‘lsa, shtrixlangan sohaning yuzini toping.



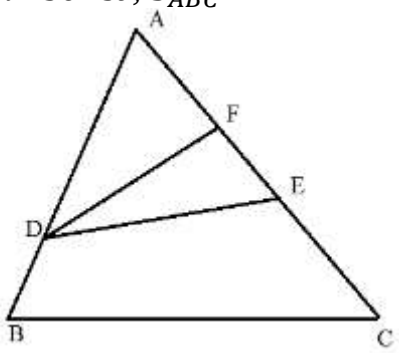
Masalaning yechilishi: Bu masalada *K* nuqtaning aniq vaziyati ko‘rsatilmagan. Shuning uchun *K* nuqtaning vaziyatini masala shartlarini o‘zgartirmagan holda, *B* nuqta bilan ustma – ust qo‘yamiz va masalani yechish uchun qulay ko‘rinishga keltiramiz. U holda,



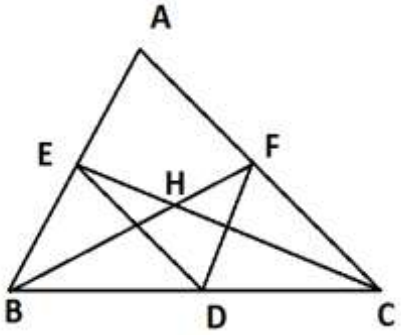
ko‘rinishidagi tasvir hosil bo‘ladi. *BGE* uchburchakning yuzi $S_{BGE} = \frac{S_{ABC}}{6} = 8\ sm^2$ ga teng. *K* nuqtani *A* nuqtaning ustiga olib borib joylashtirgan holda ham huddi shu natijaga ega bo‘lamiz.

Nuqtaning vaziyatiga bog‘liq tasvirli masalalarga namunalar.

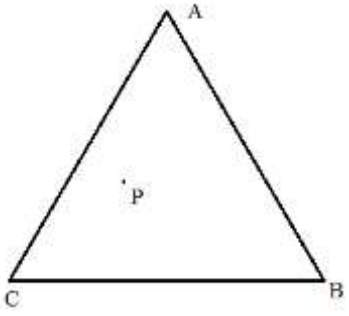
1. $AD = 3 \cdot DB, EF = \frac{1}{4} \cdot AC,\ S_{DEF} = 6\ sm^2$ bo‘lsa, $S_{ABC} = ?$



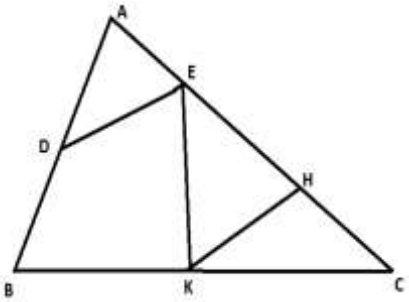
2. *E* va *F* tegishli tomonlarning o‘rtalari. *D* nuqta *BC* kesma ustida bo‘lsa, $\frac{S_{ABC}}{S_{DEHF}} = ?$



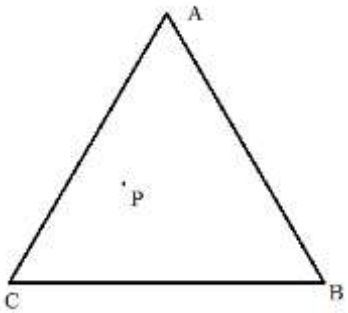
3. Muntazam *ABC* uchburchakning tomoni 12 *sm*. *P* nuqtadan tomonlargacha masofalar yig‘indisini toping.



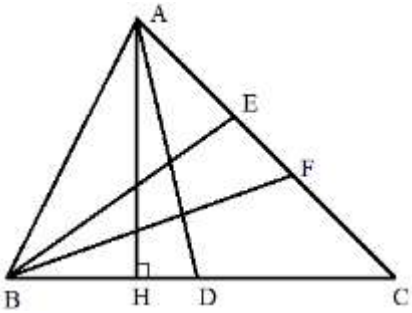
4. *D* va *K* mos tomonlarning o‘rtalari. *DE//KH* bo‘lsa, $\frac{S_{KEH}}{S_{DEKB}} = ?$



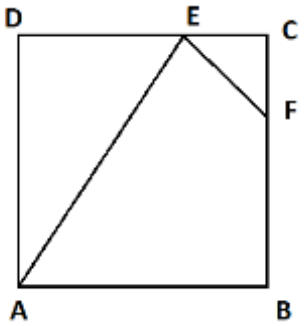
5. Rasmda *ABC* muntazam uchburchak. *D* nuqtadan tomonlargacha masofalar yig‘indisi 10 *sm* bo‘lsa, *ABC* uchburchak perimetrini toping.



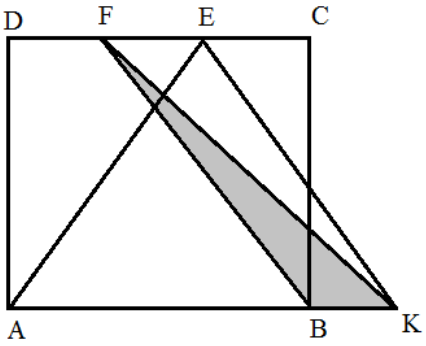
6. *AH⊥BC*, *BD = DC*, $AC = 5 \cdot EF$, *AC = 8 sm*, *AB = 6 sm*, *BC = 7 sm* bo‘lsa, $\frac{S_{AHD}}{S_{BEF}} = ?$



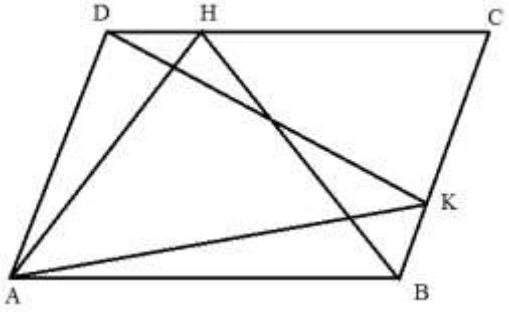
7. *ABCD* tomoni 12 *sm* bo‘lgan kvadrat, *E* nuqta *DC* kesma ustida, *F* nuqta *BC* kesma ustida va *EC=FC* bo‘lsa, S_{ABFE} ning eng katta va eng kichik qiymatlari orasidagi farqni toping.



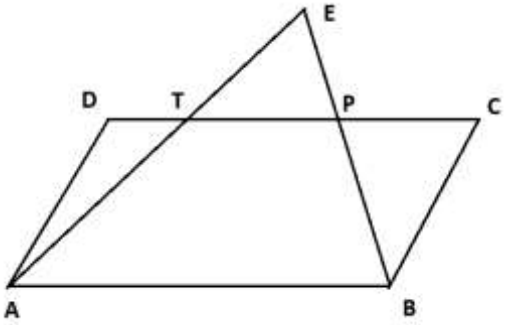
8. *ABCD* tomoni 1 *sm* bo‘lgan kvadrat. *AKE* muntazam uchburchak bo‘lsa, $S_{FBK} = ?$



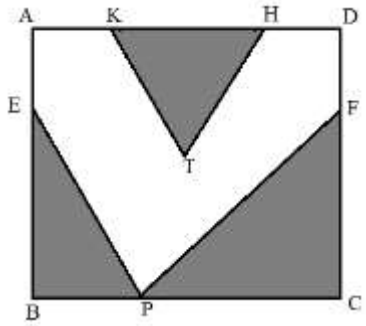
9. *H*, *DC* kesma ustidagi va *K*, *BC* kesma ustidagi ixtiyoriy ikki nuqta. *ABCD* parallelogramm bo‘lsa, $\frac{S_{DKC}+S_{ABK}}{S_{ABH}} = ?$



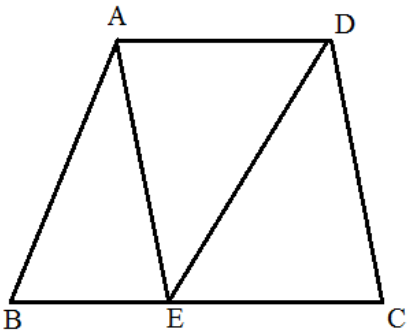
10. *ABCD* parallelogramm, *T* va *P* nuqtalar *DC* kesma ustida yotuvchi ikki nuqta va $S_{TAD} + S_{PBC} = S_{TEP}$ bo‘lsa, $\frac{S_{ATPB}}{S_{TEP}} = ?$



11. *P*, *BC* kesma ustidagi ixtiyoriy nuqta va *ABCD* tomoni 6 *sm* bo‘lgan kvadrat. *T* nuqtaning *BC* tomondan uzoqligi 3 *sm*. *AE=AK=HD=DF*, *KH=2AK* bo‘lsa, shaklga ko‘ra bo‘yalgan yuzalar yig‘indisini toping.



12. *ABCD* trapetsiya. $BC = 2 \cdot AD$ va *E* nuqta *BC* ustida bo‘lsa, $\frac{S_{ABE}+S_{DEC}}{S_{AED}} = ?$

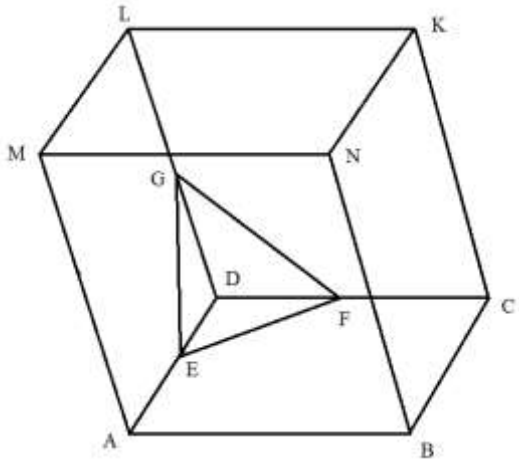


§5. Streometriyaga oid tasvirli masalalar.

Avvalgi paragraflarda biz asosan planometriyaga oid tasvirli masalalar va ularning asosiy hollari bilan tanishdik. Bu paragrafda beriladigan tasvirli masalalar geometriya fanining stereometriya bo‘limiga doir. Stereometriyaga oid tasvirli masalalarni ham berilish va yechilish usullariga qarab sinflarga ajratsa bo‘ladi. Ammo biz bu usullarni planometriya qismida berganmiz va ular stereometriyaning asosini tashkil etishini hisobga olgan holda masalalarni guruhlarga ajratmadik.

Stereometriya bo‘limiga oid tipik masalalardan ba’zilarini yechish na’munasini keltiramiz.

1 – masala. Shaklda *E, F, G* nuqtalar parallelopipedning mos tomonlarining o‘rtalari bo‘lsa, parallelepiped hajmining *DEFG* piramida hajmiganisbatini toping.

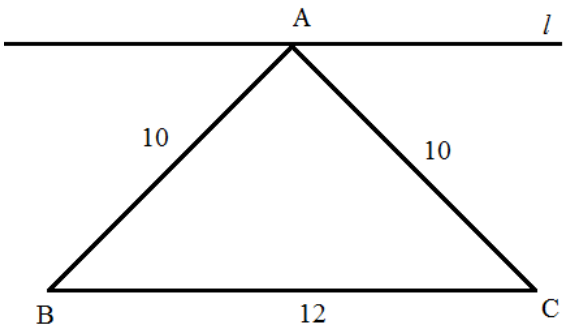


Masalaning yechilishi: Parallelepipedning balandligi *H* bo‘lsin. Parallelepiped asosiga *AC* diagonal o‘tkazamiz. *E, F, G* mos tomonlarning o‘rtasi ekanligidan, $S_{DEF} = \frac{S_{ACD}}{4} = \frac{S_{ABCD}}{8}$, va *DEFG* piramida balandligi $H_1 = \frac{H}{2}$ ga teng ekanligi kelib chiqadi. U holda,

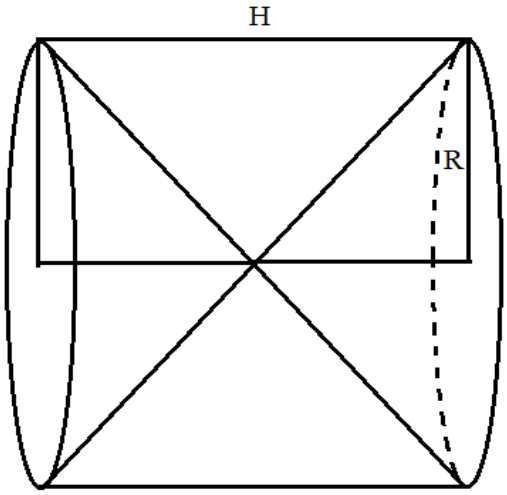
$$\frac{V_{par}}{V_{pir}} = \frac{S_{ABCD} \cdot H}{\frac{1}{3} \cdot S_{DEF} \cdot H_1} = 48$$

ga teng natijaga ega bo‘lamiz.

2 – masala. Shaklda l o‘q BC tomonga parallel. Berilgan uchburchakni l o‘qi atrofida aylantirishdan hosil bo‘lgan aylanma jismning hajmini toping.



Masalaning yechilishi: Masala shartiga ko‘ra, ABC uchburchakni l oq atrofida aylantirib, quyidagi aylanma jismni hosil qilamiz.

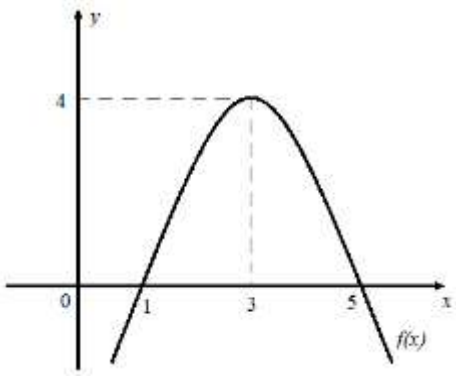


Ko‘rinib turibdiki, aylanma jismning hajmi radiusi R va balandligi H bo‘lgan silindr hajmidan ikkita radiusi R va balandligi $\frac{H}{2}$ ga teng konus hajmining ayirmasiga teng. Masaladan $R = 8$ va $H = 12$ ekanligi kelib chiqadi. U holda,

$$V = V_{sil} - 2 \cdot V_{kon} = \pi \cdot R^2 \cdot H - 2 \cdot \frac{1}{3} \cdot \pi \cdot R^2 \cdot \frac{H}{2} = 512\pi$$

ga teng natijaga ega bo‘lamiz.

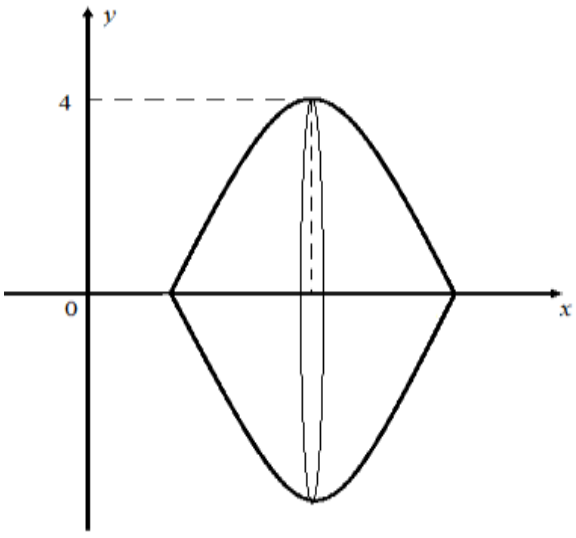
3 – masala. Rasmda berilgan parabola bilan Ox o‘qi chegaralagan sohani Ox o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini toping.



Masalaning yechilishi: Dastlab biz rasmda berilgan parabola tenglamasini tuzamiz. Faraz qilaylik parabola temglamasi $f(x) = ax^2 + bx + c$ ko‘rinishida bo‘lsin. Bu parabola (1;0), (3;4), (5;0) nuqtalardan o‘tadi. Shuning uchun, quyidagi sistemani tuzamiz va uning yechimlarini topib,

$$\begin{cases} a + b + c = 0 \\ 9a + 3b + c = 0 \\ 25a + 5b + c = 0 \end{cases} => \begin{cases} a = -1 \\ b = 6 \\ c = -5 \end{cases}$$

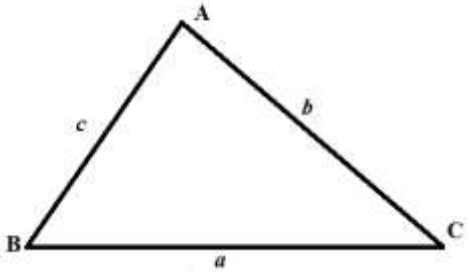
parabola tenglamasi $f(x) = -x^2 + 6x - 5$ ko‘rinishida ekanligini aniqlaymiz. Bu parabola va *Ox* oqi bilan chegaralangan sohani *Ox* oqi atrofida aylantirib, hosil bo‘lgan aylanma jismning hajmini topamiz;



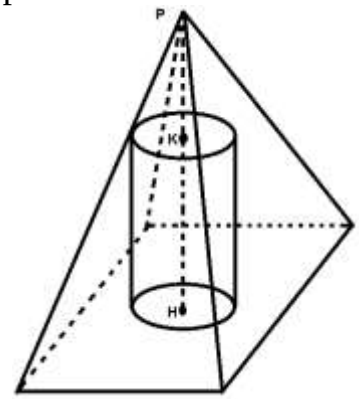
$$V = \pi \cdot \int_1^5 f^2(x)dx = \pi \cdot \int_1^5 (-x^2 + 6x - 5)^2 dx = \frac{512\pi}{15}\ br^3.$$

Streometriyaga oid tasvirli masalalarga namunalar.

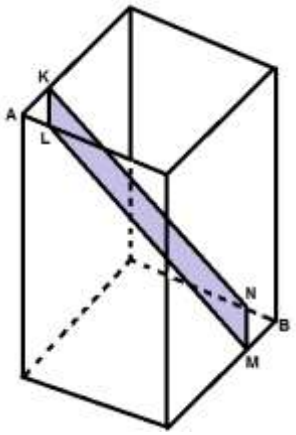
1. *BC=6 sm* va $h_a = 8$ *sm* bo‘lsa, *ABC* uchburchakning *BC* atrofida 360^0 aylantirilishi bilan hosil bo‘lgan jismning hajmi necha sm^3?



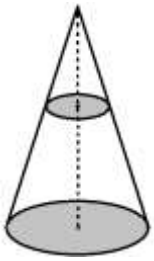
2. Shaklda, asosi kvadrat piramida berilgan. Asosi kvadratda bo‘lgan va ustki asosi piramidaning yon yoqlariga to‘rtta nuqtada urunadigan bir silindr uchun *PK=KH* bo‘lsa, piramidaning asos yuzasini silindrning asos yuzasiga nisbati topilsin.



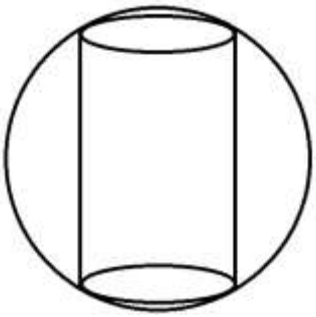
3. Shakldagi kubning bir tomoni 5 sm. *AK=AL=1 sm* va *BN=BM=1 sm* bo‘lsa, *KLMN* to‘g‘ri to‘rtburchakning yuzasini toping.



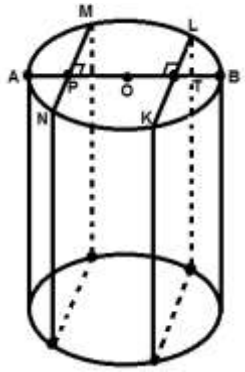
4. Shakldagi to‘g‘ri konsning asosiga parallel tekislik bilan kesilib hosil qilingan doiradan konus uchigacha va asosigacha masofa teng bo‘lsa, asos yuzasi bu yuzadan necha marta katta?



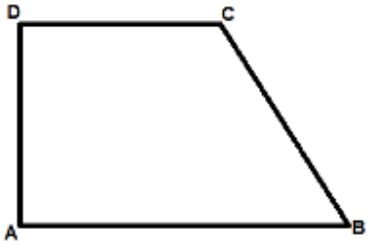
5. Shar ichiga joylashtirilgan to‘g‘ri silindrning balandligi asos radiusidan 2 marta katta bo‘lsa, silindr hajmining shar hajmiga nisbati topilsin.



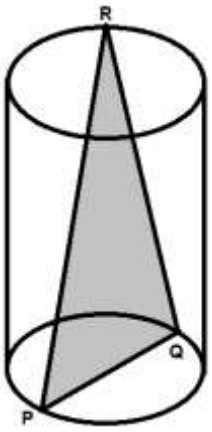
6. Shakldagi to‘g‘ri silindrda ko‘rsatilgan to‘g‘ri to‘rtburchaklar parallel. *KL⊥PB, AP=BT=OT=2 sm, h=3 sm* va ustki asosi *MNKL* figura, pastki asosi *MNKL* ning tik proeksiyasi bo‘lgan jismning hajmi $a(2\pi + 3\sqrt{3})\ sm^3$ bo‘lsa, *a* = ?



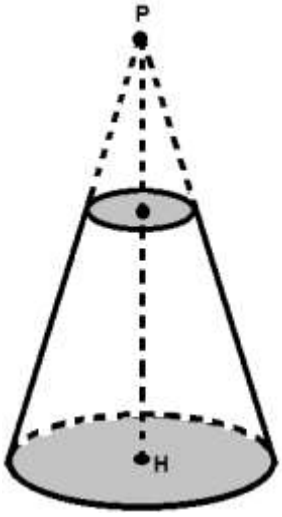
7. *ABCD* to‘g‘ri burchakli trapetsiyada *AD=DC=6 sm* va *AB=10 sm* bo‘lsa, *ABCD* trapetsiyani *AB* tomon atrofida aylantirilishi bilan hosil bo‘lgan jismning hajmi necha sm^3?



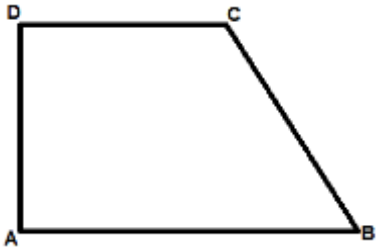
8. Shakldagi silindrning radiusi 5 *sm*, balandligi $\sqrt{17}$ *sm*, *PQ=8 sm* va *PQR* teng yonli uchburchak bo‘lsa, *PQR* uchburchakning yuzasi necha sm^2?



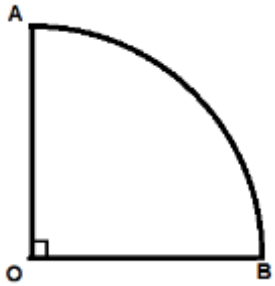
9. Shakldagi kesik konus, bir to‘g‘ri konusni asosiga parallel va uchi bilan asosdan bir xil masofada bo‘lgan tekislik bilan kesilishidan paydo bo‘lgan. Asos yuzining radiusi 4 *sm* va *PH=10 sm* bo‘lsa, kesik konusning hajmi necha sm^3?



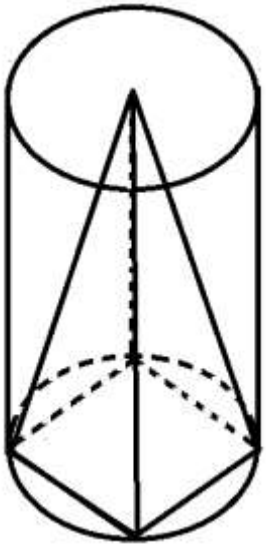
10. *ABCD* to‘g‘ri to‘rtburchakli trapetsiyada *AD=DC=6 sm* va *AB=10 sm* bo‘lsa, *ABCD* trapetsiyani *AD* tomon atrofida aylantirilishi bilan hosil bo‘lgan jismning hajmi necha sm^3?



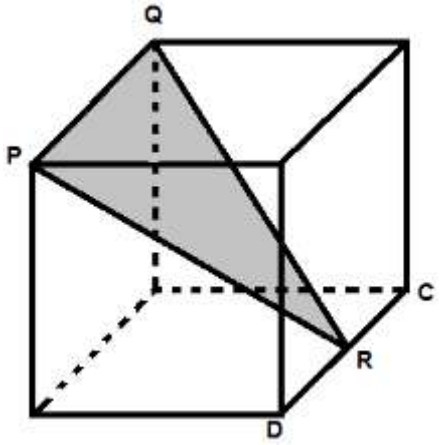
11. *OA=4 sm* bo‘lsa, *AOB* sektorni *AO* o‘qi atrofida 90^0 aylantirilishi natijasida hosil bo‘lgan jismning hajmi necha sm^3?



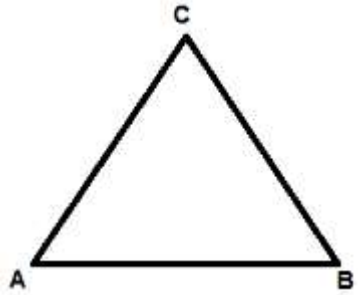
12. Piramidaning asosi silindrning asosida va uchi silindrning yuqori asosida. Piramida asosining uchlari silindr yon yog‘ida bo‘lsa, silindr hajmining kvadrat asosli piramida hajmiga bo‘lgan nisbatni toping.



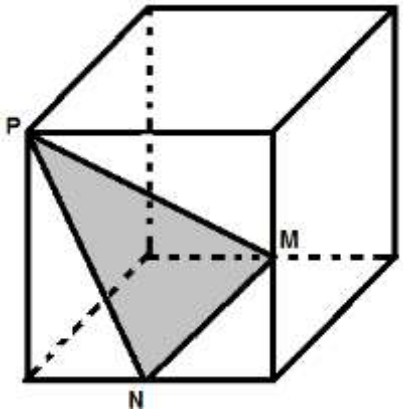
13. Shakldagi kubning tomoni *6sm. R, DC* ustidagi ihtiyoriy bir nuqta bo‘lsa, *PQR* uchburchakning yuzasi necha sm^2?



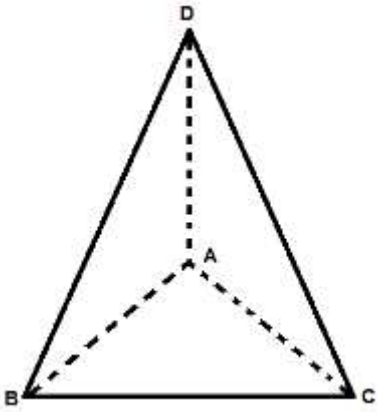
14. *ABC* teng tomonli uchburchk. *AB=6 sm* bo‘lsa, *ABC* uchburchakning *AB* atrofida aylantirilishi natijasida hosil bo‘lgan jismning hajmi necha sm^3?



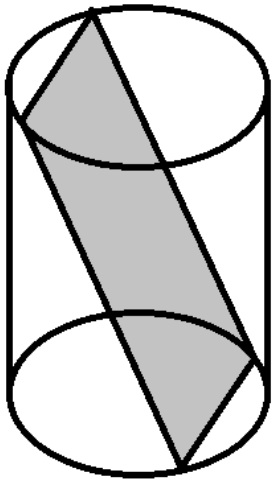
15. *M* va *N* kub tomonlarining o‘rta nuqtalari va $S_{PNM}=\sqrt{5}\ sm^2$ bo‘lsa, kubning hajmi necha sm^3?



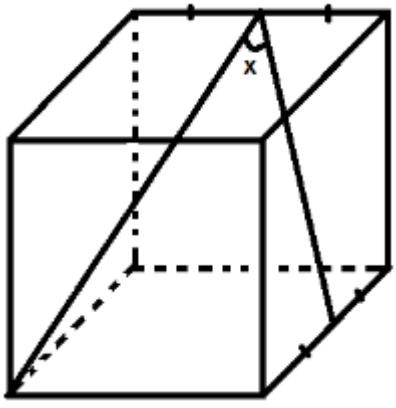
16. *ABCD* to‘g‘ri piramidaning yoqlari *A* nuqtasida bir-biriga perpendikulyar. *AD=AB=AC=4 sm* bo‘lsa, piramidaning hajmi necha sm^3?



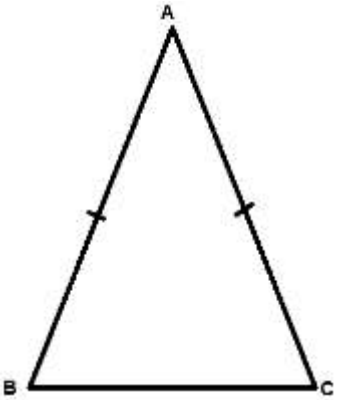
17. Shakldagi to‘g‘ri silindr asosining radiusi 5 *sm*. Kichik tomoni 6 *sm* va uchlari pastki va ustki aylanada bo‘lgan to‘g‘ri to‘rtburchak bilan silindr asoslari orasidagi burchak 60^0 bo‘lsa, to‘g‘ri to‘rtburchak yuzasi necha sm^2?



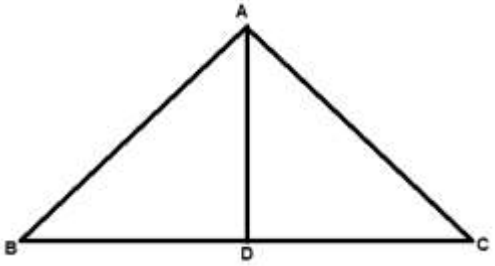
18. Quyidagi berilgan kub uchun *cosx* = ?



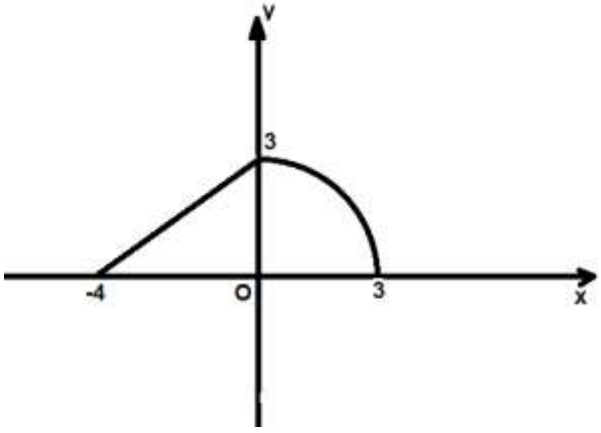
19. *ABC* teng yonli uchburchak berilgan. *AB=AC=8 sm* va *BC=6 sm* bo‘lsa, *BC* tomoni atrofida aylantirishdan hosil bo‘lgan jismning hajmi necha sm^3?



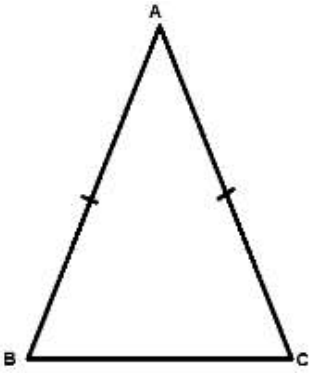
20. *ABC* teng yonli uchburchak berilgan. *AB=AC=5 sm* va *BC=8 sm* bo‘lsa, *AD* medianasi atrofida aylantirishdan hosil bo‘lgan jismning hajmi necha sm^3?



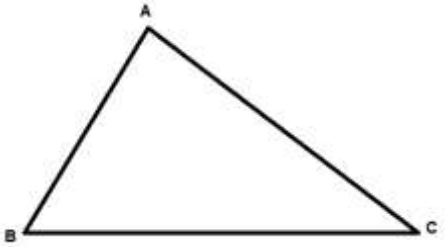
21. Quyidagi shaklni *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini toping.



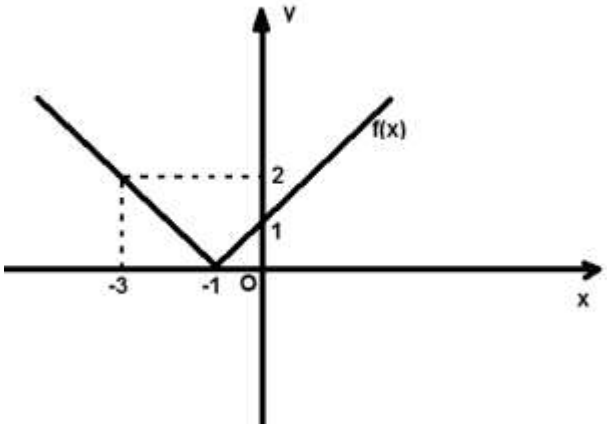
22. *ABC* teng yonli uchburchak berilgan. *AB=AC=10 sm* va *BC=8 sm* bo‘lsa, *AC* tomoni atrofida aylantirishdan hosil bo‘lgan jismning hajmi necha sm^3?



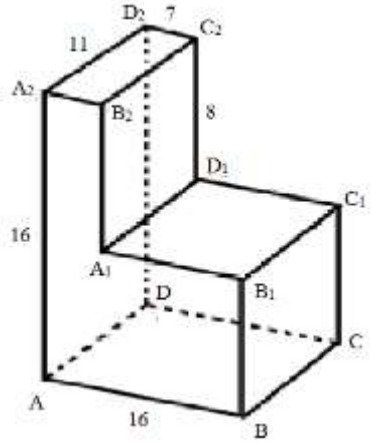
23. *ABC* uchburchak berilgan. *AB=5 sm*, *BC=9 sm* va $AC=2\sqrt{13}$ *sm* bo‘lsa, *BC* tomoniga parallel va *A* uchidan o‘tuvchi to‘g‘ri chiziq atrofida aylantirishdan xosil bo‘lgan jismning hajmi necha sm^3?



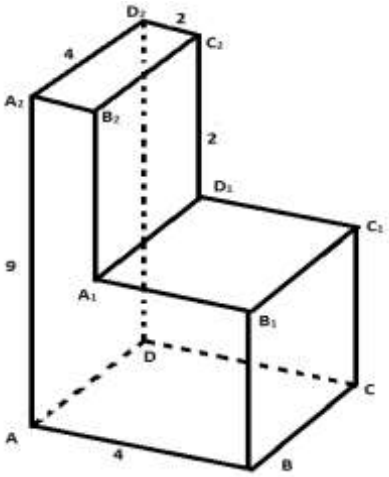
24. Quyidagi shaklga ko‘ra *f(x)* funksiya, *x=-3, x=0* va *Ox* o‘qibilan chegaralangan soxani *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmi necha sm^3?



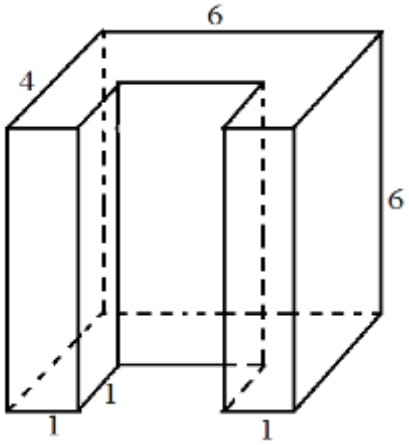
25. Rasmda tasvirlangan ko‘pyoqning D va B_1 uchlari orasidagi masofani toping. Bunda ko‘pyoqning barcha ikkiyoqli burchaklari to‘g‘ri burchakli



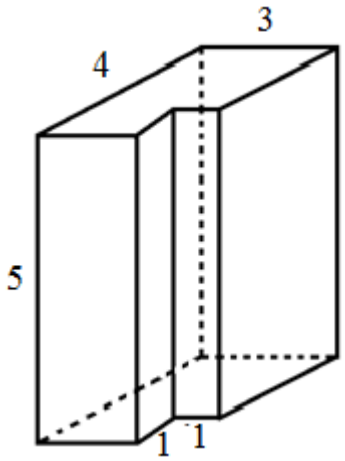
26. Rasmda tasvirlangan ko‘pyoqning D_2 va B_1 uchlari orasidagi masofani toping. Bunda ko‘pyoqning barcha ikkiyoqli burchaklari to‘g‘ri burchakli



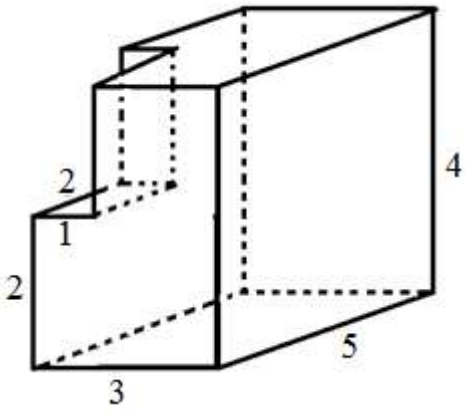
27. Chizmada tasvirlangan ko‘pyoq sirtining yuzini toping. Bunda barcha ikkiyoqli burchaklar to‘g‘ri.



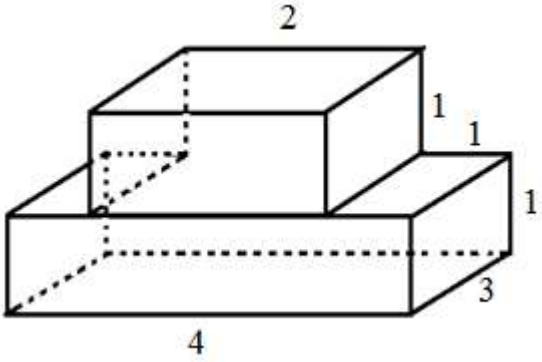
28. Chizmadatasvirlangan ko‘pyoqning hajmini toping. Bunda barcha ikkiyoqli burchaklar to‘g‘ri.



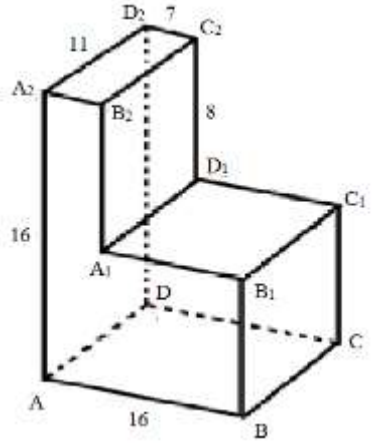
29. Chizmada tasvirlangan ko‘pyoq sirtining yuzini toping. Bunda barcha ikkiyoqli burchaklar to‘g‘ri.



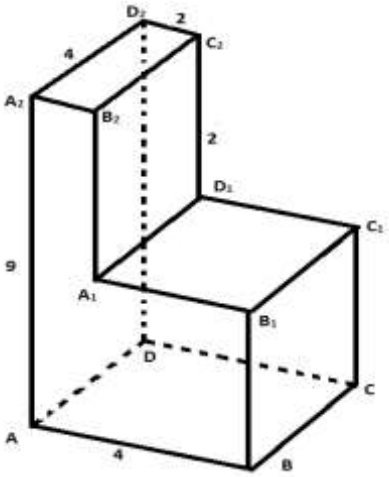
30. Chizmada tasvirlangan ko‘pyoq sirtining yuzini toping. Bunda barcha ikkiyoqli burchaklar to‘g‘ri.



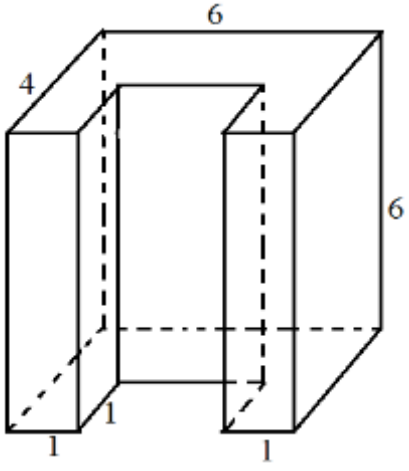
31. Rasmda tasvirlangan ko‘pyoqning B va D_2 uchlari orasidagi masofani toping. Bunda ko‘pyoqning barcha ikkiyoqli burchaklari to‘g‘ri burchakli



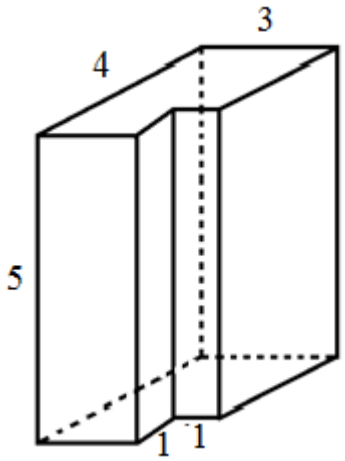
32. Rasmda tasvirlangan ko‘pyoqning B_2 va C_1 uchlari orasidagi masofani toping. Bunda ko‘pyoqning barcha ikkiyoqli burchaklari to‘g‘ri burchakli



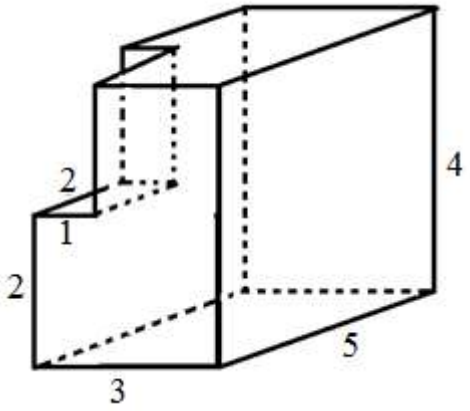
33. Chizmada tasvirlangan ko‘pyoq hajmini toping. Bunda barcha ikkiyoqli burchaklar to‘g‘ri.



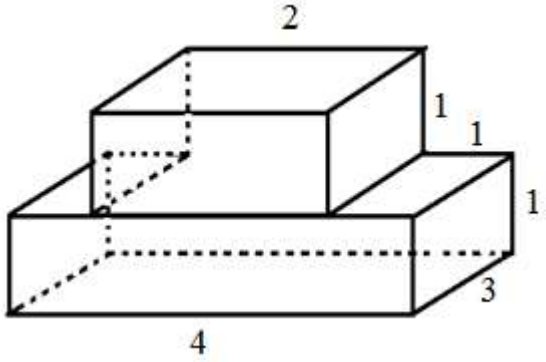
34. Chizmadatasvirlangan ko‘pyoqning sirtini toping. Bunda barcha ikkiyoqli burchaklar to‘g‘ri.



35. Chizmada tasvirlangan ko‘pyoq hajmini toping. Bunda barcha ikkiyoqli burchaklar to‘g‘ri.



36. Chizmada tasvirlangan ko‘pyoq hajmni toping. Bunda barcha ikkiyoqli burchaklar to‘g‘ri.

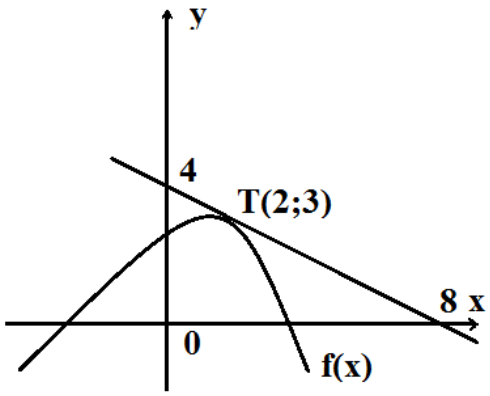


§6. Algebra va analizga oid tasvirli masalalar.

Analizning asosiy mavzularidan biri funksiya grafigini tekshirishdan iborat. Bunda analitik ifoda hossalariga ko‘ra funksiya grafigi yasaladi. Funksiya grafigi geometrik shakl namunasidir. Albatta bu geometrik shaklni hosil qilishda funksiya xossalari va uning qiymatlaridan foydalaniladi. O‘quvchilar geometrik shakldan foydalanib, analitik ifodalar va ularning xossalari haqida fikr yuritish ko‘nikmalarini shakllantirishda tasvirli masalalarni qo‘llash yaxshi natija beradi.

Biz quyida matematikaning analiz va algebra bo‘limi tushunchalarini o‘rgatishda tasvirli masalalardan foydalanishga doir masalalardan namunalar keltiramiz.

1 – masala. Quyida berilgan tasvirli masalada $g(x) = [f(x)]^4$ bo‘lsa, $g'(2) = ?$



Izoh: Bunda o‘quvchi funksiyaning berilgan nuqtadagi hosilasi uning shu nuqtasiga o‘tkazilgan urinmasining burchak koeffisiyentiga teng ekanligidan foydalaniladi. Yana uchburchak tashqi burchagi tushunchasini eslashiga hamda to‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmasi bo‘yicha tenglamasi tushunchalaridan foydalanishi kerak.

Masalaning yechilishi: Berilgan funksiyaga o‘tkazilgan urinma tenglamasini tuzamiz. Berilgan tasvirdan urinma *Ox* o‘qidan *x=8* va *Oy* o‘qidan *y=4* kesma ajratadi. To‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalariga ko‘ra urinma tenglamasi

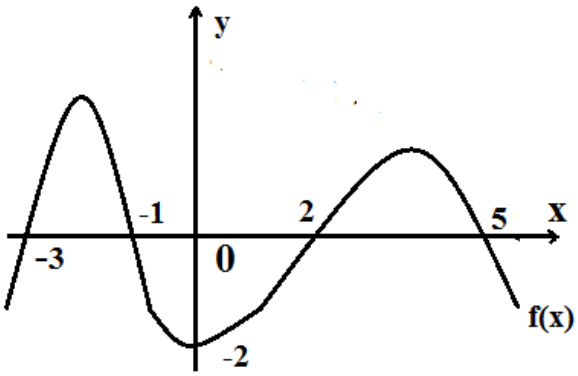
$$\frac{x}{8}+\frac{y}{4}=1,\ \ y=\ -\frac{1}{2}x+4$$

hosil bo‘ladi. Hosilaning geometrik ma’nosiga ko‘ra $f'(2)=-\frac{1}{2}$ ga teng. Berilgan $g(x)$ funksiyaning hosilasini va masala shartiga ko‘ra hosilaning $x_0=2$ nuqtadagi qiymatini topamiz

$$g'(x)=4\cdot[f(x)]^3\cdot f'(x),\ \ g'(2)=4\cdot 3^3\cdot\left(-\frac{1}{2}\right)=-54.$$

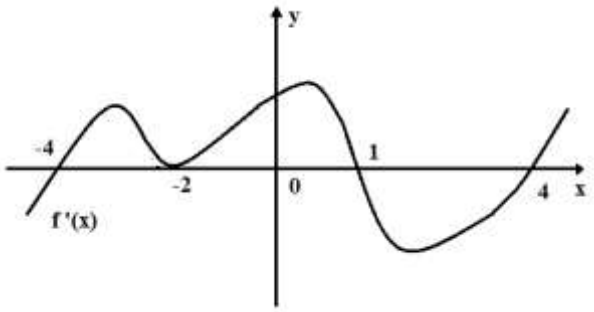
Biz masala shartiga ko‘ra kerakli yechimni aniqladik.

2 – masala. $f(x)$ funksiyaning grafigi quyida berilgan. $f\big(f(k)\big)=-2$ tenglikni [-3;5] oraliqdagi qanoatlantiradigan k ning qiymatlarini toping.



Masalaning yechilishi: Masalada o‘quvchi funksiyaning (-2) qiymati ordinatalar o‘qi bilan kesishgan nuqtasi ekani va bu nuqta abssisasi 0 ga teng ekanligini anglash zarur. Demak, masalaning yechimi grafikdagi absissalar o‘qining kesishish nuqtalari($f(x)=0$)ga mos keladi. Bu nuqtalar grafikdan ko‘rinib turibdiki, $x_1=-3,\ \ x_2=-1,\ \ x_3=2,\ \ x_4=5$ lardan iborat.

3 – masala. Hosilasining grafigi quyida berilgan funksiya *x* ning qanday qiymatlarida ekstremumga ega bo‘ladi.

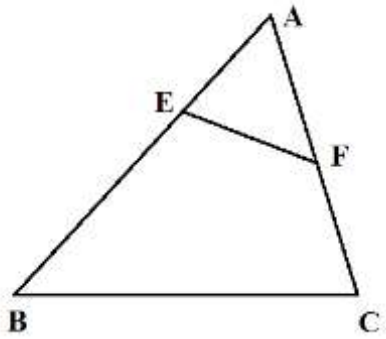


Izoh: Funksiya ekstremumlarini aniqlashga doir masalalarda, funksiya grafigi bilan birga, funksiya hosilasining grafigi tushunchalaridan ham foydalanish mumkin. Garchan funksiya hosilasining grafigi o‘quvchilar uchun ancha murakkab tushuncha bo‘lishiga qaramay uni o‘zlashtirish uncha murakkab emas. Faqat o‘quvchiga funksiya hosilasi ham yangi bir funksiya bo‘lishi haqidagi tushunchani berish lozim. Bunday masalalar o‘quvchilarga hosilani o‘zgarish qonuniyati ham oddiy funksiya kabi ekani haqida ma’lumot beradi. O‘quvchi e’tiborini ordinata o‘qida funksiya hosilasi qiymatlari berilganligiga qaratish zarur.

Masalaning yechilishi: Chizmadan ko‘rinadiki funksiya hosilasi argumentning -4, -2, 1, 4 nuqtalarda nolga teng. Bu nuqtalar kritik nuqtalarni tashkil etadi. Funksiya hosilasini kritik nuqtalar atrofidagi ishorasiga qarab funksiya ekstremumlarini izlaymiz. Funksiya ekstremumining yetarlilik shartiga ko‘ra, agar $y = f(x)$ funksiya hosilasining ishorasi $x = x_0$ nuqtadan o‘tishda “+” dan “−“ ga o‘zgarsa, funksiya maksimumga, “−“dan “+” ga o‘zgarsa, shu nuqtada funksiya minimumga erishadi. Chizmadan ko‘rinib turibdiki, funksiya hosilasining ishorasi $x = \pm 4$ nuqtadan o‘tishda “−“dan “+” ga o‘zgaradi. Demak $f(x)$ funksiya shu nuqtalarda minimumga erishadi. Funksiya hosilasining ishorasi $x = 1$ nuqtadan o‘tishda “+” dan “−“ ga o‘zgarganligi uchun $f(x)$ funksiya bu nuqtada maksimumga erishadi. Funksiya hosilasining ishorasi $x = -2$ nuqtadan o‘zgarmaganligi uchun bu nuqtada $f(x)$ funksiya ekstremumga ega emas.

Algebra kursida berilgan tasvirli masalani qaraymiz.

4 – masala. Ushbu uchburchakda $<AEF = <ACB$, $AB = c$, $AC = b$, $BC = a$, $AE = m$, $AF = n$, $EF = p$ bo‘lsa, $\begin{vmatrix} 3 & p & a \\ 0 & m & b \\ -2 & n & c \end{vmatrix} = ?$



Izoh: Bu tasvirli masala o‘quvchidan bir vaqtda o‘xshash uchburchaklar va determinantning xossalarini bilishni talab etadi.

Masalaning yechilishi: Masala shartiga ko‘ra, $\angle AEF = \angle ACB$ va $\angle A$ umumiy ekanligidan, ABC va AEF uchburchaklar o‘xshash. Uchburchaklar o‘xshashligidan

$$\frac{AB}{AF} = \frac{BC}{EF} = \frac{AC}{AE}, \qquad \frac{c}{n} = \frac{a}{p} = \frac{b}{m} = k$$

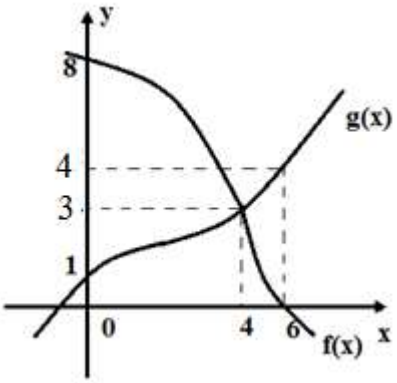
kelib chiqadi. U holda berilgan determinant

$$\begin{vmatrix} 3 & p & kp \\ 0 & m & km \\ -2 & n & kn \end{vmatrix}$$

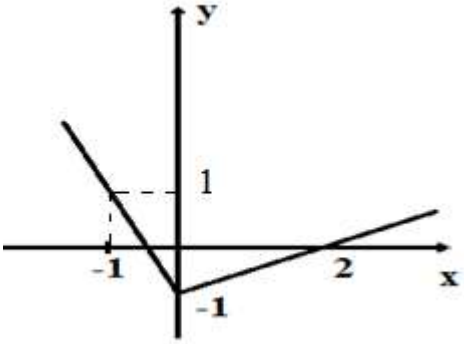
ko‘rinishiga keladi. Ma’lumki, determinantning ikki satri yoki ustuni elementlari o‘zaro mos ravishda proporsional bo‘lsa, uning qiymati nolga teng bo‘ladi. Demak, berilgan determinantning qiymati nolga teng.

Algebra va analizga doir tasvirli masalalarga namunalar.

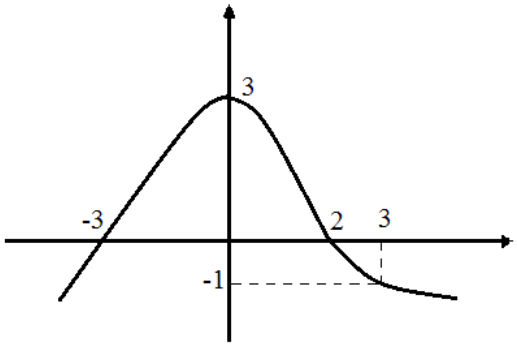
1. Chizmaga ko‘ra, $f\left(g^{-1}(3)\right) = ?$



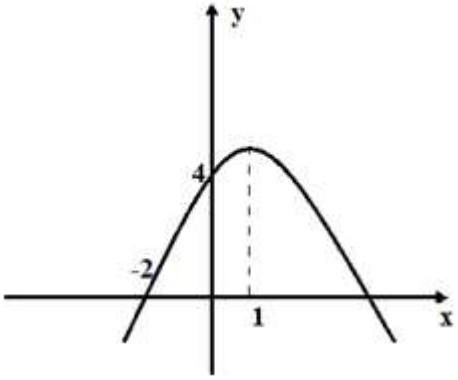
2. Grafigi quyida berilgan funksiyaga ko‘ra, $f\left(f\left(f(4)\right)\right) = ?$



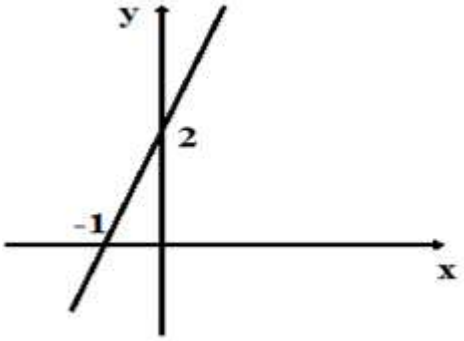
3. $f\left(f(2)\right) = ?$



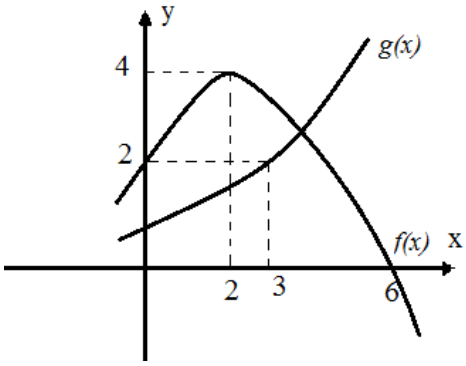
4. Grafigi quyida berilgan parabola uchining ordinatasini toping.



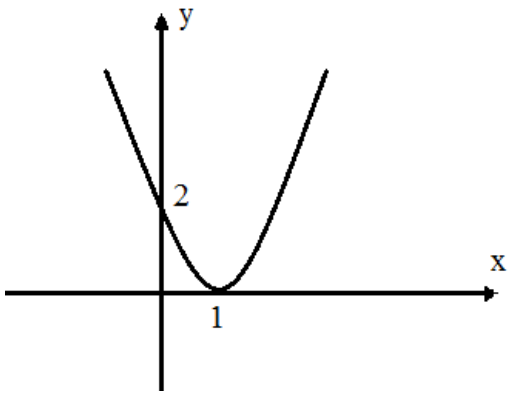
5. Grafigi quyida berilgan funksiyaga teskari funksiya tenglamasini toping.



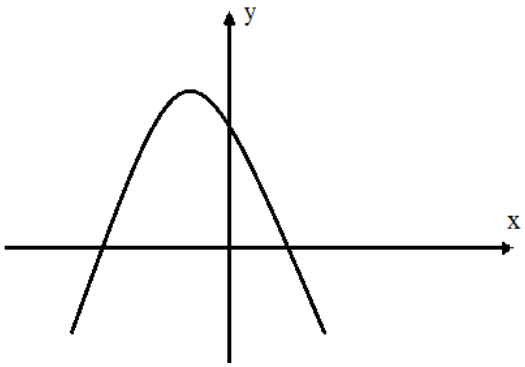
6. Berilgan $f(x)$ va $g(x)$ funksiyalari uchun $h(x) = x \cdot f(g(x))$ bo‘lsa, $h(3) = ?$



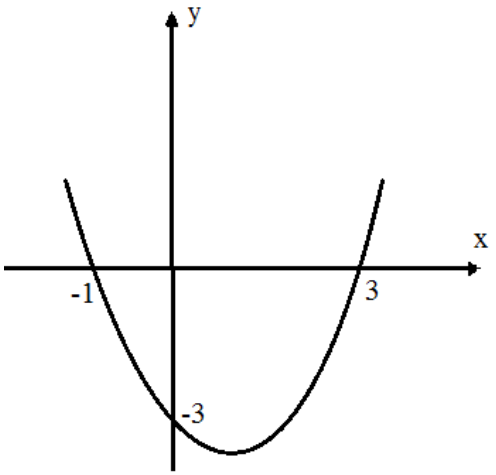
7. Shaklda $y = ax^2 + bx + c$ funksiyaning grafigi berilgan. $a + b = ?$



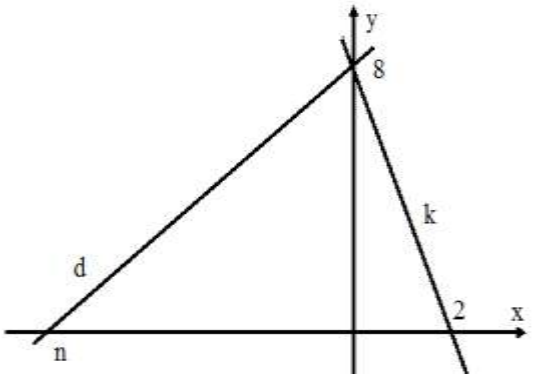
8. Shaklda $y = a(x - r)^2 + k$ funksiyaning grafigi berilgan. a, r, k larning ishorasini aniqlang.



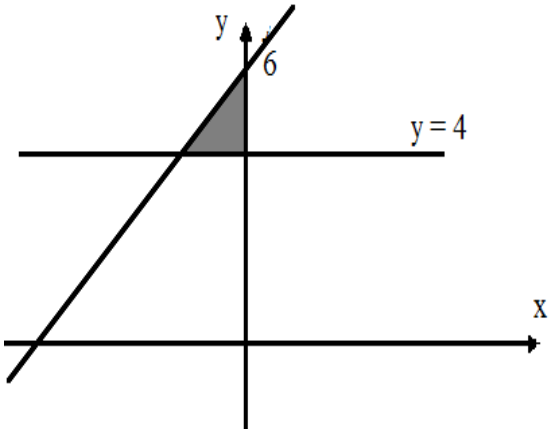
9. Shakldagi parabola uchining koordinatasini toping.



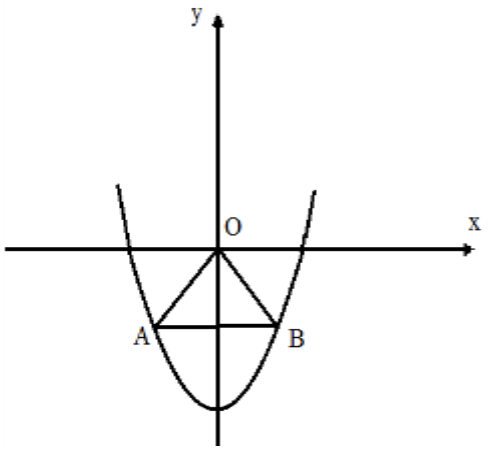
10. Shaklga ko‘ra $d \perp k$ bo‘lsa, n = ?



11. Shaklga ko‘ra bo‘yalgan sohaning yuzi 3 birlik kvadrat bo‘lsa, to‘g‘ri chiziq tenglamasini tuzing.

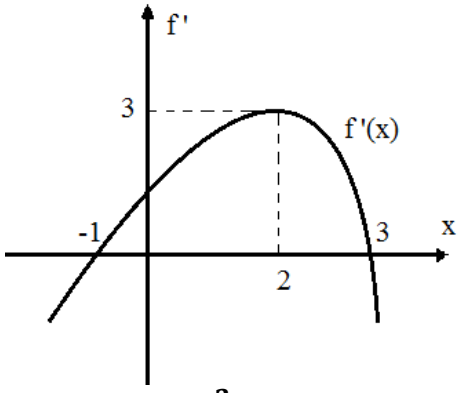


12. Shaklda A va B nuqtalar $y = 2x^2 - 3$ parabola ustida yotadi. OAB uchburchak yuzining eng katta qiymatini toping.

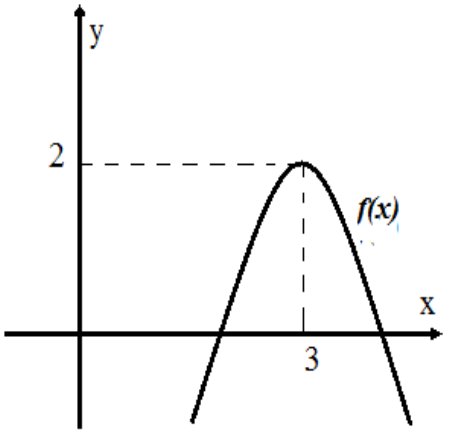


13. Hosilasining grafigi shaklda berilgan funksiya uchun quyidagi tengsizliklarni tekshiring.

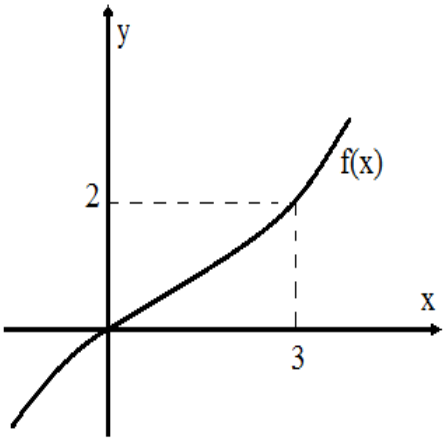
1. $f(-2) > f(-1)$ 2. $f(2) > f(1)$,
3. $f(3) > f(-1)$, 4. $f(0) < f(3)$
5. $f(2) < f(3)$



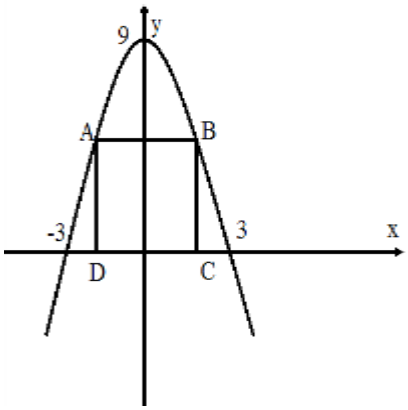
14. $\boldsymbol{g(x) = [f(x)]^2}$ bo‘lsa, $g'(3) = ?$



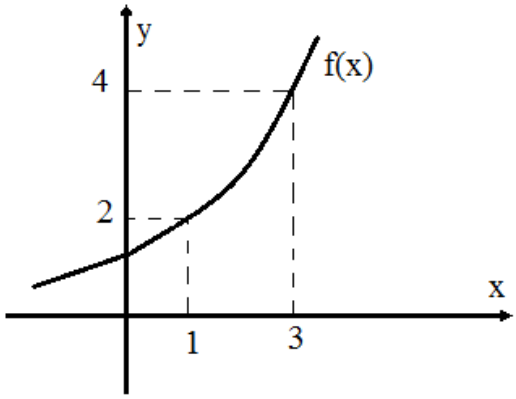
15. $\int_0^3 f^2(x) \cdot f'(x)dx = ?$



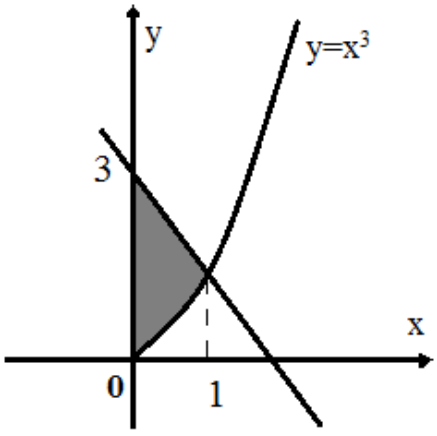
16. *ABCD* to‘g‘ri to‘rtburchakning ikki uchi $y = 9 - x^2$ parabola ustida qolgan ikki uchi *Ox* oqida yotadi. Shunday to‘g‘ri to‘rtburchaklar ichida eng katta peremtrlisini toping.



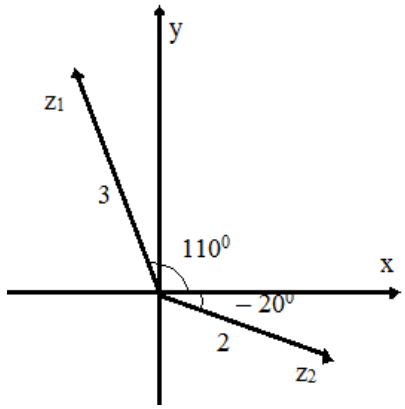
17. $\int_1^3 \frac{f'(x)dx}{f(x)} = ?$



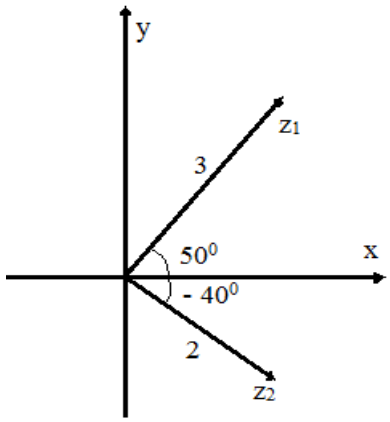
18. Shaklda bo‘yalgan sohaning yuzini toping.



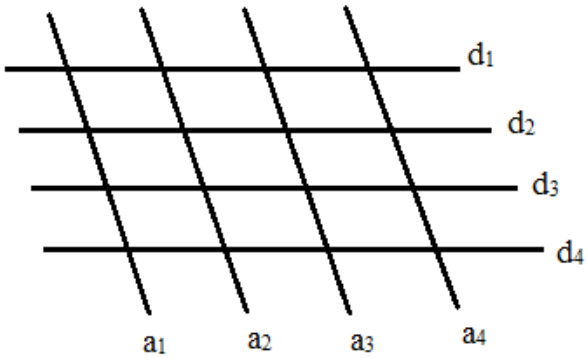
19. Shaklga ko‘ra, $z_1 \cdot z_2 = ?$



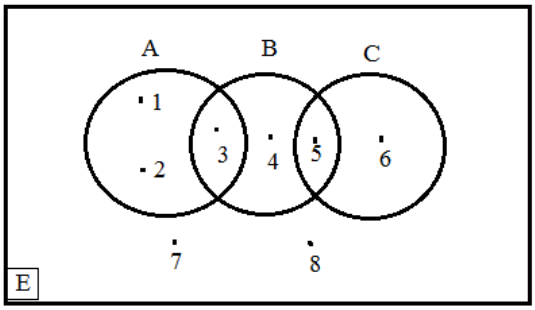
20. Shaklga ko‘ra, $\frac{z_1}{z_2} = ?$



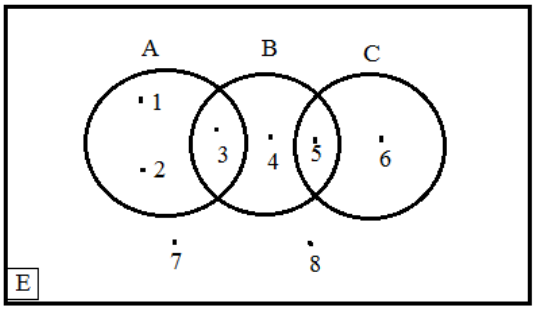
21. Shaklda berilgan parallel to‘g‘ri chiziqlardan nechta bir-biridan farqli parallelogramm hosil qilish mumkin.



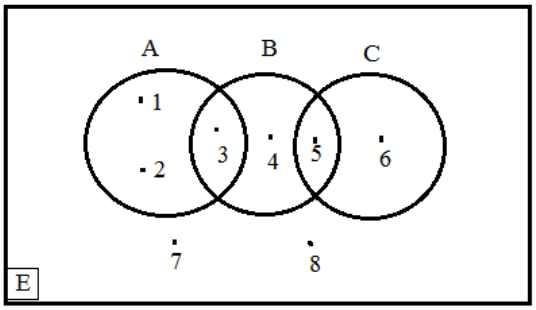
22. Berilgan shaklga ko‘ra, $\bar{B} \setminus (A \cup B) = ?$



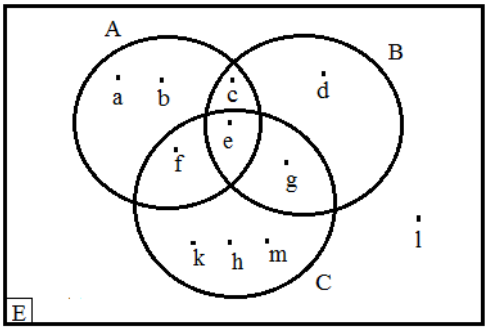
23. Berilgan shaklga ko‘ra, $\bar{B} \setminus (A \cap C) = ?$



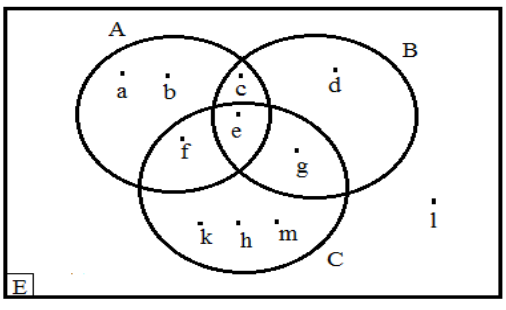
24. Berilgan shaklga ko‘ra, $\bar{B} \setminus (\bar{A} \cup C) = ?$



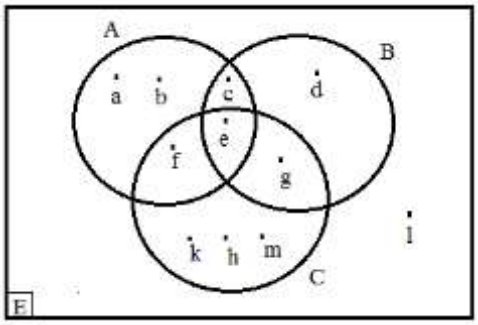
25. Berilgan shaklga ko‘ra, $(A \cap B) \setminus (C \cup \bar{A}) = ?$



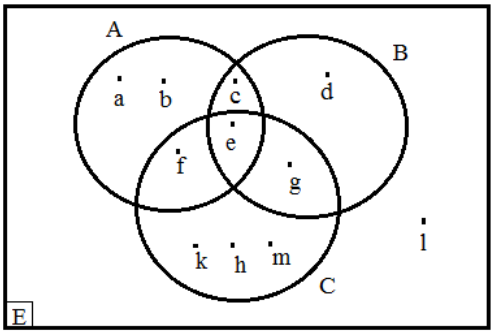
26. Berilgan shaklga ko‘ra, $(A \cap C) \setminus (\bar{B} \cap C) = ?$



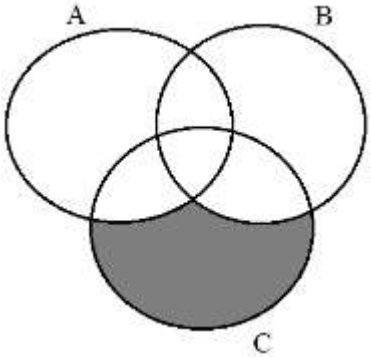
27. Berilgan shaklga ko‘ra, $A \cap (B \backslash C) = ?$



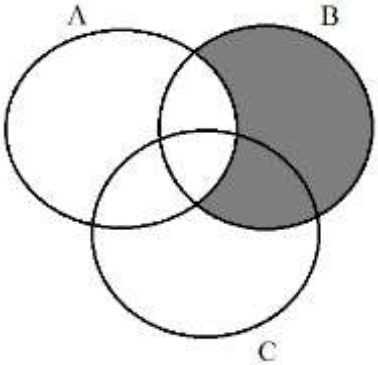
28. Berilgan shaklga ko‘ra, $(A \cap B) \cup (\bar{C} \cup \bar{A}) = ?$



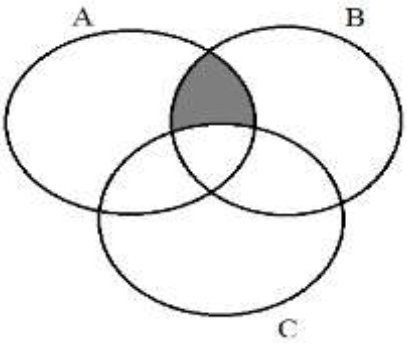
29. Berilgan shaklga ko‘ra, shtrixlangan qismni ifodalang.



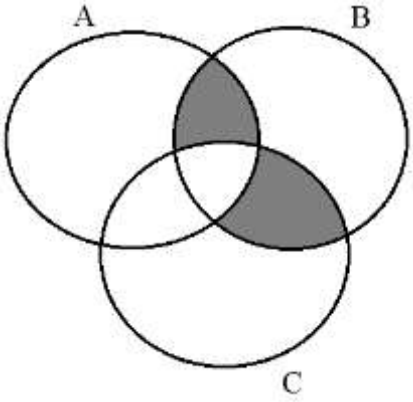
30. Berilgan shaklga ko‘ra, shtrixlangan qismni ifodalang.



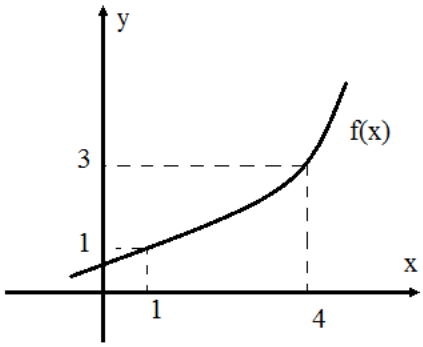
31. Berilgan shaklga ko‘ra, shtrixlangan qismni ifodalang.



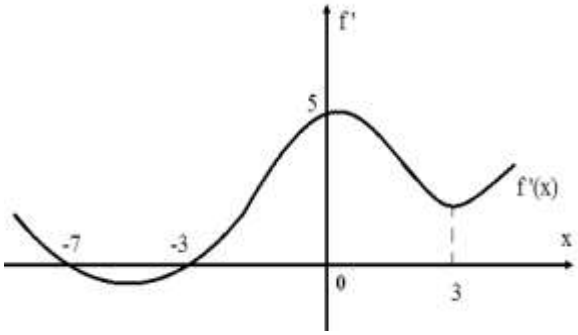
32. Berilgan shaklga ko‘ra, shtrixlangan qismni ifodalang.



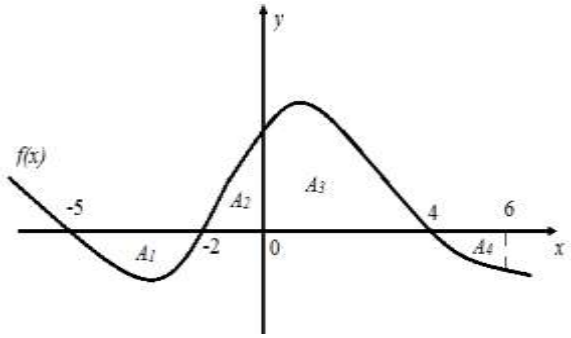
33. $\int_1^2 f(3x-2) \cdot f'(3x-2)dx = ?$



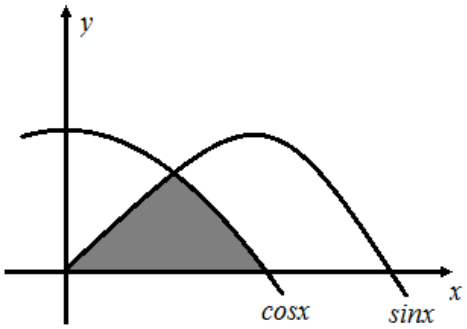
34. Hosilasining grafigi berilgan *f(x)* funksiya *x* ning qanday qiymatida maksimumga erishadi.



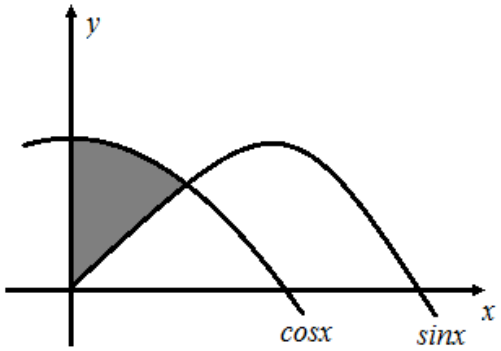
35. Chizmada $A_1 = 3\ br^2$, $A_2 = 1\ br^2$, $A_3 = 4\ br^2$, $A_4 = 2\ br^2$ bo‘lsa, $\int_{-5}^{6} f(x)dx = ?$



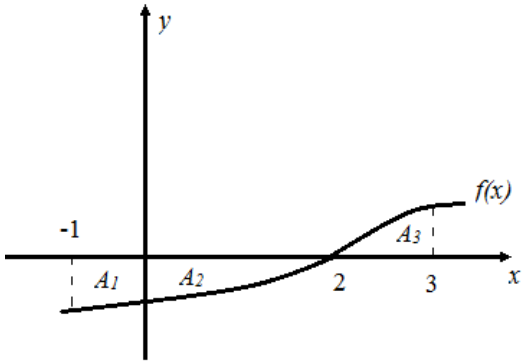
36. Shaklda shtixlangan yuzani toping.



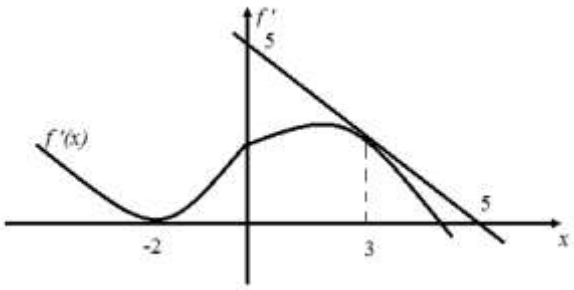
37. Shaklda shtixlangan yuzani toping.



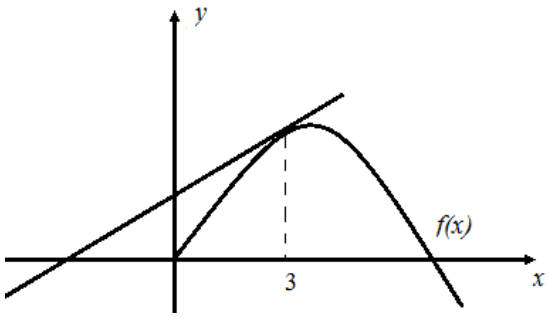
38. Chizmada $a > 0$, $A_1 = 2a$, $A_2 = 3a$, $A_3 = a$ bo‘lsa, $\int_{-1}^{3} f(x)dx = ?$



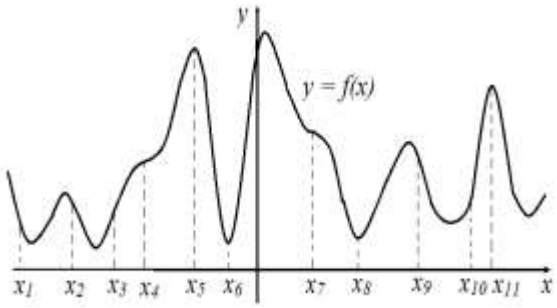
39. Hosilasining grafigi berilgan *f(x)* funksiya uchun $\int_{-2}^{3} f'(x) \cdot f''(x)dx = ?$



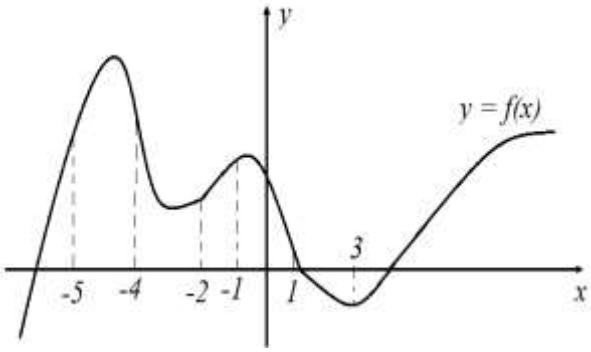
40. $f(x) = ax^2 + bx + c$ funksiyaning $T(3; f(3))$ nuqtasidagi urinma *Ox* o‘qi bilan 45° burchak tashkil qilsa, $6a + b + c = ?$



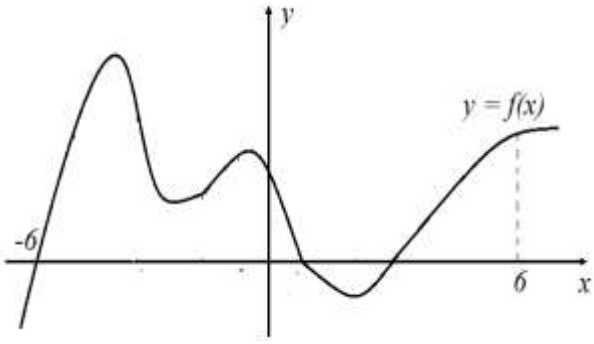
41. Tasvirda $y = f(x)$ funksiya grafigi tasvirlangan. Abssisalar o‘qining berilgan $x_1, x_2, \dots, x_{11}$ nuqtalar bilan chegaralangan qaysi oraliqlarida funksiyaning hosilasi musbat bo‘ladi?



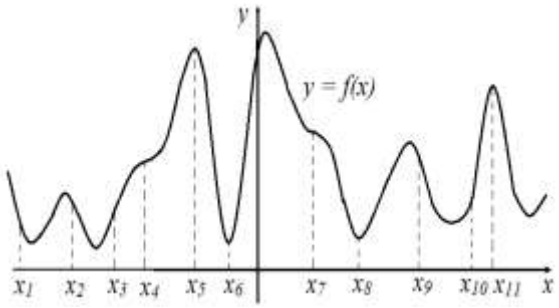
42. Rasmda $y = f(x)$ funksiyaning grafigi tasvirlangan. Quyidagi berilgan nuqtalardan qaysi birida funksiyaning hosilasi eng katta bo‘ladi?



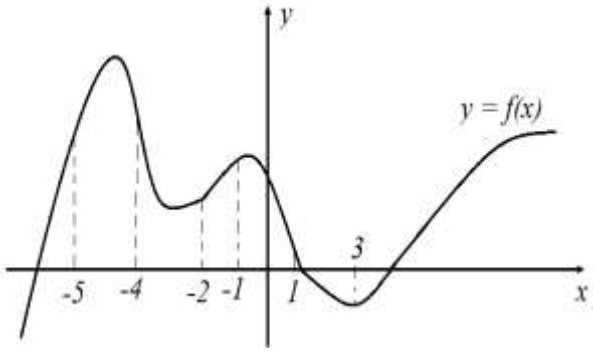
43. Rasmda $y = f(x)$ funksiyaning grafigi tasvirlangan. Unga ko‘ra $f'(x) = 0$ tenglama [-6;6] kesmada nechta yechimga ega?



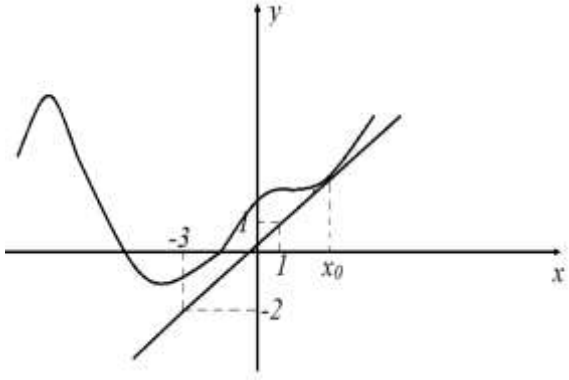
44. Tasvirda $y = f(x)$ funksiya grafigi tasvirlangan. Abssisalar o‘qining berilgan $x_1, x_2, \dots, x_{11}$ nuqtalar bilan chegaralangan qaysi oraliqlarida funksiyaning hosilasi manfiy bo‘ladi?



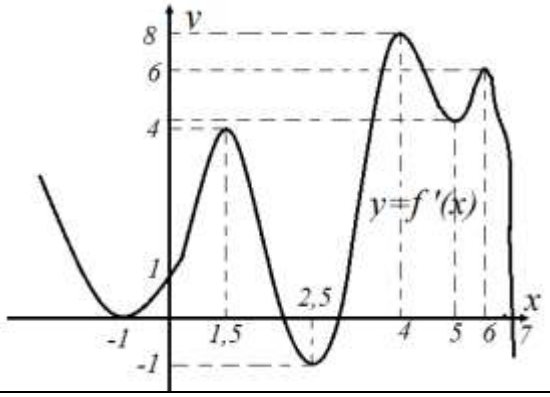
45. Rasmda $y = f(x)$ funksiyaning grafigi tasvirlangan. Quyidagi berilgan nuqtalardan qaysi birida funksiyaning hosilasi eng kichik bo‘ladi?



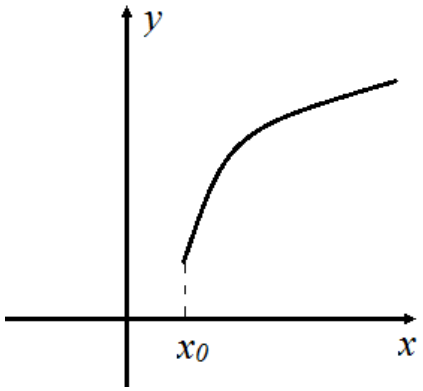
46. Rasmda $y = f(x)$ funksiyaning grafigi va uning x_0 nuqtasiga o‘tkazilgan urinma tasvirlangan. $f'(x_0)$ ni toping.



47. Quyida $y = f(x)$ funksiya hosilasining grafigi tasvirlangan. Unga ko‘ra $y = f(x)$ funksiyaning nechta statsionar nuqtalari bor?.

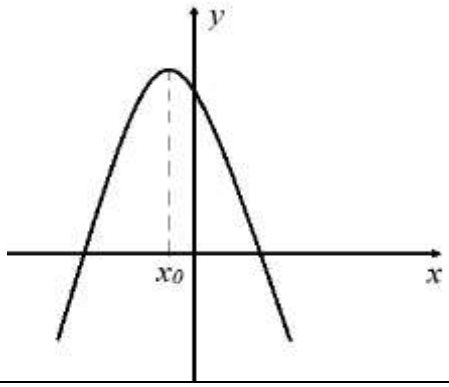


48. Chizmada $y = a\sqrt{bx + c} + d$ funksiyaning grafigi tasvirlangan. *a,b,c,d* larning ishorasini aniqlang (y_0 – funksiyaning eng kichik qiymati).



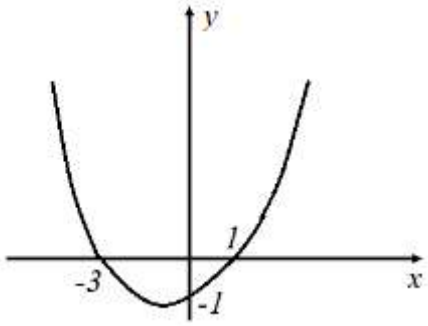
49. Chizmada $y = ax^2 + bx + c$ kvadrat uchhad grafigi tasvirlangan. Quyidagi tengsizliklardan qaysi biri to‘g‘ri? ($D = b^2 - 4ac$)

1) $a^2D^3 - c^4b^5 > 0$, 2) $ac - b^2D > 0$,
3) $b^2a^3 - c^4D < 0$,
4) $a^3b^4c^5 - D^6 < 0$

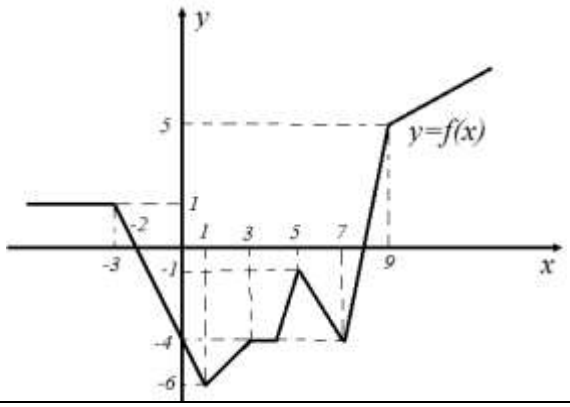


50. Chizmada $y = ax^2 + bx + c$ kvadrat uchhad grafigi tasvirlangan. Unga ko‘ra,

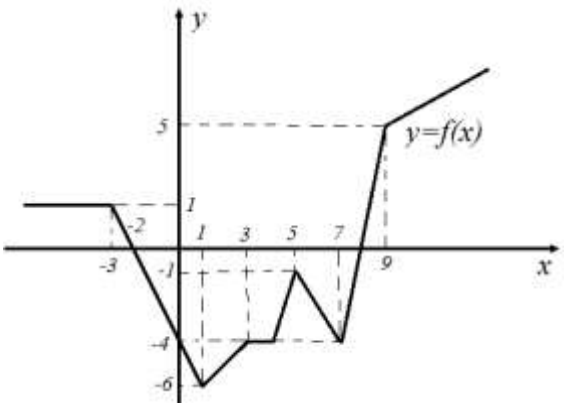
$$3a + 6b + 2c = ?$$



51. Chizmada $y = f(x)$ funksiyaning grafigi tasvirlangan bo‘lib, $F(x)$ uning boshlang‘ich funksiyasi. Chizmadan foydalanib, $F(9) - F(-3) = ?$



52. Chizmada $y = f(x)$ funksiyaning grafigi tasvirlangan bo‘lib, $F(x)$ uning boshlang‘ich funksiyasi. Chizmadan foydalanib, $F(9) - F(1) = ?$

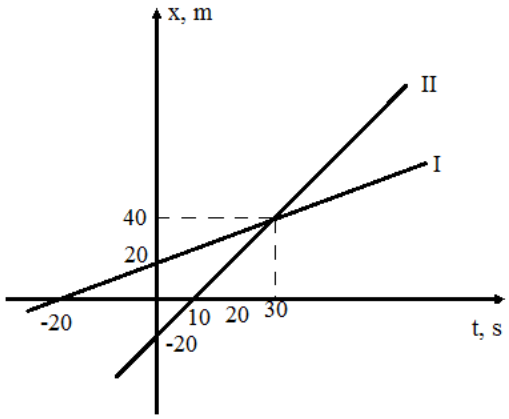


§7. Fanlar uzviyligiga doir tasvirli masalalar.

Yoshlar tarbiyasida fanlarning uzluksizligi va uzviyligi muhum ahamiyatga ega. Bunda o‘quvchi bir fandan olgan bilim va ko‘nikmalarini boshqa fan masalalarini yechishga qo‘llay olishi fanlarning uzviyligini ta’minlaydi. Geometriya fani bilan bir vaqtda o‘quvchilar fizika, kimyo, va geografiya fanlari bilan ham shug‘ullanadilar. Bu fanlardagi ko‘pchilik tushunchalar geometriya fanining asosiy tushunchalari bilan mohiyati jihatidan bir-biriga o‘xshashdir. Masalan, moddiy nuqta, harakat, yadro, atom va haritadagi belgilar kabi. Shuning uchun ham geometriya fanidagi tasvirli masalalardan bir vaqtda o‘qitilayotgan boshqa fanlarga doir masalalarni hal qilishda foydalanish mumkin.

Biz quyida shu usulga doir masalalar namunasini keltiramiz.

1 – masala. Ikki jismning harakat grafiklari quyida keltirilgan. $x = x(t)$ harakat tenglamalarini toping.

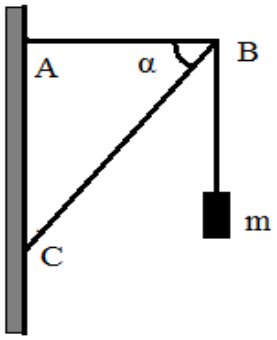


Masalaning yechilishi: Har ikkala jismning ham harakati to‘g‘ri chiziqli bo‘lganligi uchun ularning tenglamasi ham to‘g‘ri chiziqdan iborat bo‘ladi. Birinchisi (0;20) va (30;40) ikkinchisi esa, (10;0) va (30;40) nuqtalardan o‘tadi.

Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzishga ko‘ra, har ikkala jism harakat tenglamasini tuzamiz:

$$\begin{cases} x_1 = \frac{2}{3} \cdot t + 20 \\ x_2 = 2 \cdot t - 20 \end{cases}.$$

2 – masala. Rasmda $\alpha = 60°$, jism massasi 3 *kg* bo‘lsa, *AB* va *BC* sterjenga ta’sir qiluvchi kuchlarni toping. $g = 10\ {}^{m}/_{s^2}$ deb oling.

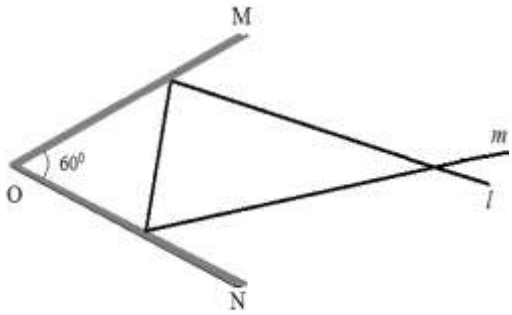


Masalaning yechilishi: Jismga ta’sir etuvchi og‘irlik kuchi *P = mg = 30 N* ga teng. Bu kuchni *AC* tomonga proyeksiyalaymiz. To‘g‘ri burchakli uchburchakda o‘tkir burchakning tangensi va sinusidan foydalanib,

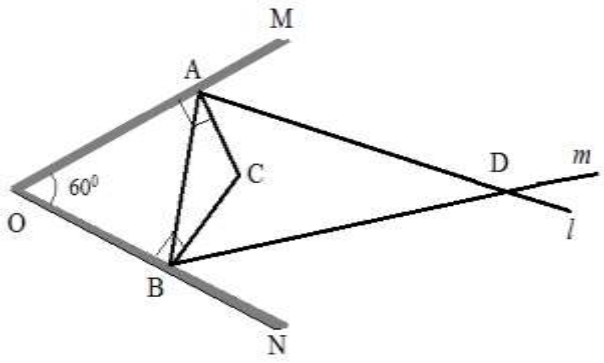
$$tg\alpha = \frac{P}{F_{AB}} \Rightarrow F_{AB} = 10\sqrt{3}\ N \quad \text{va} \quad sin\alpha = \frac{P}{F_{BC}} \Rightarrow F_{BC} = 20\sqrt{3}\ N$$

natijalarga ega bo‘lamiz.

3 – masala. Rasmda *MON* yassi ko‘zgu bo‘lsa, *l* tushayotgan nur va *m* ko‘zgulardan qaytgan nur orasidagi burchakni toping.



Masalaning yechilishi: Nur tushgan nuqtalarga normal o‘tkazamiz va kesishish nuqtalarini belgilaymiz. *OBCA* to‘rtburchakdan $\angle C = 120^0$ ekanligi ko‘rinib turibdi. Bizga optika qismidan ma’lumki, yassi ko‘zguga tushgan va qaytgan nur bir xil burchakda bo‘ladi, ya’ni, $\angle DAC = \angle CAB$ va $\angle ABC = \angle CBD$.

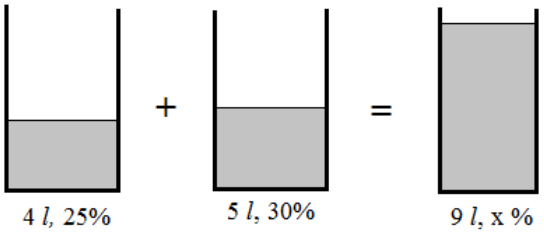


ABC va *ABD* uchburchakdan,

$$\angle C + \angle CAB + \angle ABC = 180^0, \quad \angle CAB + \angle ABC = 60^0 \text{ va}$$

$2 \cdot \angle CAB + 2 \cdot \angle ABD + \angle D = 180°$, $\angle D = 60°$ natijaga erishamiz.

4 – masala. Noma'lum *x* ni toping.



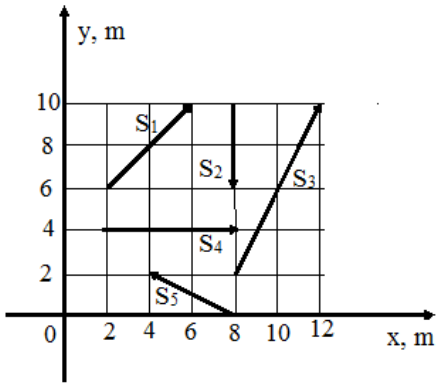
Masalaning yechilishi: Aralashma formulasidan foydalanib,

$$x = \frac{4 \cdot 25\% + 5 \cdot 30\%}{4 + 5} = 27\frac{7}{9}\ \%$$

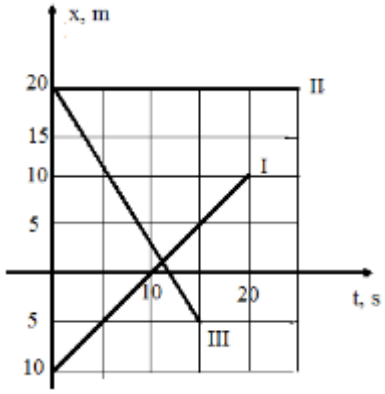
natijani olamiz.

Fanlar uzviyligiga bog‘liq tasvirli masalalarga namunalar.

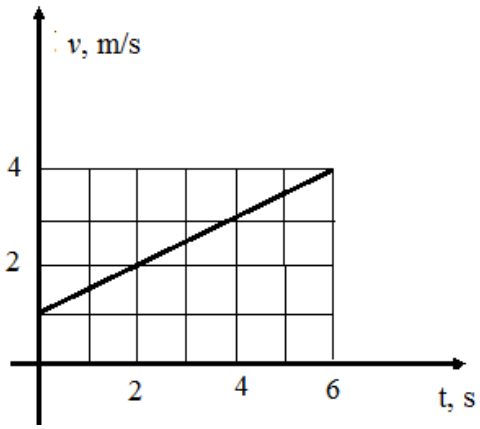
1. Tasvirda beshta moddiy nuqtaning ko‘chishi ko‘rsatilgan. Ko‘chish vektorlarining koordinata o‘qlariga proyeksiyalarini toping.



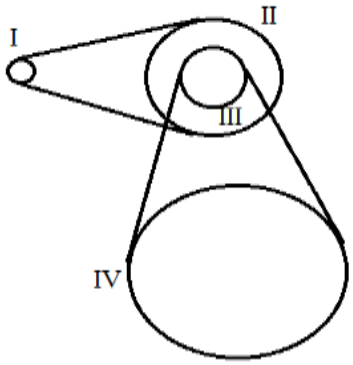
2. Berilgan grafikdan moddiy nuqta harakat tezliklarini toping.



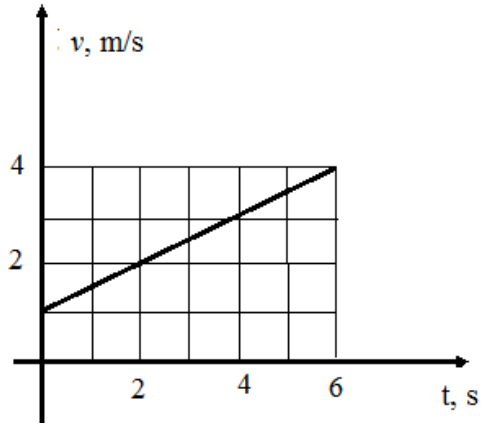
3. Tezlik grafigidan foydalanib, $t = 6s$ dagi moddiy nuqta ko‘chishini toping.



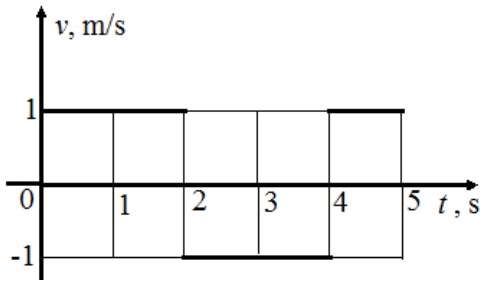
4. Harakat I shkivdan IV shkivga rasmdagidek tasmali uzatiladi. Agar I shkiv minutiga 1200 marta aylansa va $r_1 = 8\ sm,\ r_2 = 32\ sm,\ r_3 = 11\ sm,$ $r_4 = 55\ sm$ bo‘lsa, IV shkivning aylanish chastotasini toping.



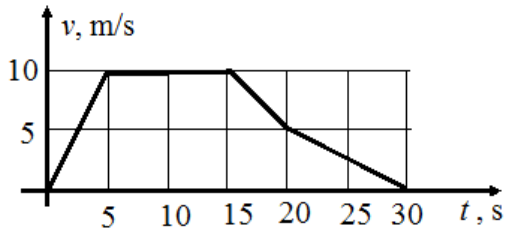
5. Tezlik grafigidan foydalanib tezlanishni toping.



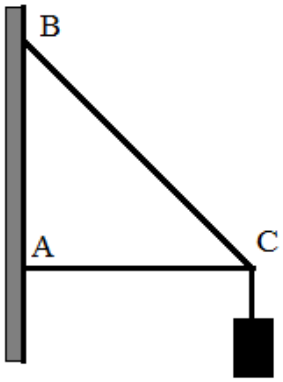
6. Berilgan $a = a(t)$ grafik berilgan. Grafikdan $t = 5\ s$ vaqtdagi tezlikni toping. Boshlang‘ich tezlik nolga teng.



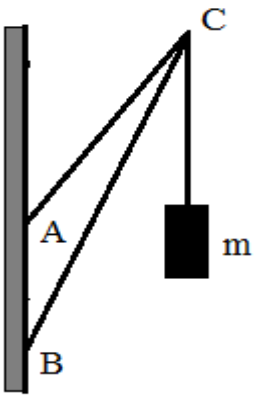
7. Massasi 2 *kg* jism tezligining o‘zgarish grafigi berilgan. Harakatning har bir bosqichida jismga ta’sir etayotgan kuchni toping.



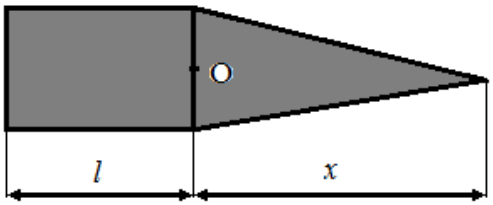
8. $m = 60\ kg$ massali yuk devorga rasmdagidek osib qo‘yilgan. *AC* va *BC* sterjenga ta’sir etuvchi kuchlarni toping. Bunda $AC = 2\ m$, $BC = 2{,}5\ m$. $g = 10\ {}^{m}/_{s^2}$ deb oling.



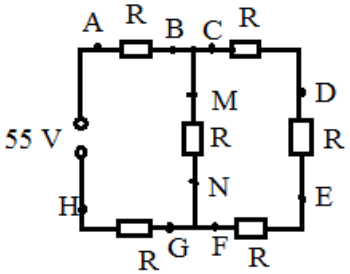
9. Agar $AB = 1{,}5\ m$, $AC = 3\ m$, $BC = 4m$ va yukning massasi 200 *kg* bo‘lsa, *BC* sterjenga va *AC* trosga ta’sir qiluvchi kuchni toping. $g = 10\ {}^{m}/_{s^2}$ deb oling.



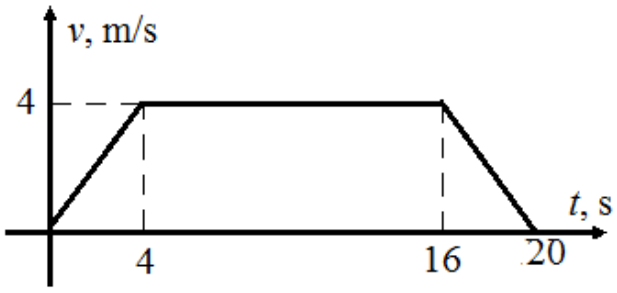
10. Yupqa bir jinsli sterjenning og‘irlik markazi *O* nuqtada bo‘lsa, $\frac{x}{l} = ?$



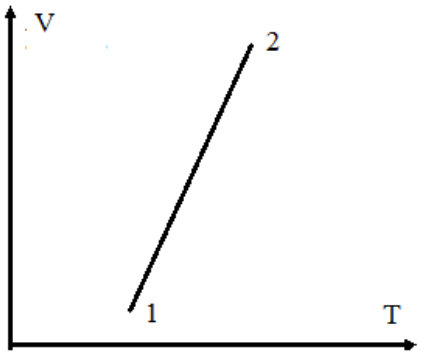
11. Sxemasi quyida tasvirlangan zanjirda $R = 2\ \Omega$ ga teng. Sxemaning *AB* qismidagi tok kuchi va kuchlanishning taqsimlanishini toping.



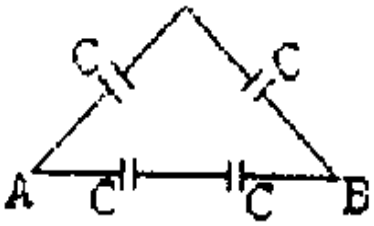
12. Liftning harakati tasvirlangan grafikdan foydalanib, uning qanday balandlikka ko‘tarilganligini aniqlang (*m*).



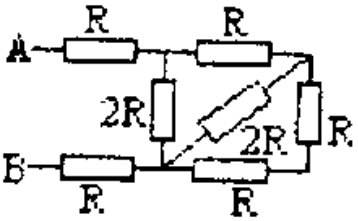
13. Ideal gaz 1 – holatdan 2 – holatga o‘tdi. Bunda gaz bosimi qanday o‘zgaradi?



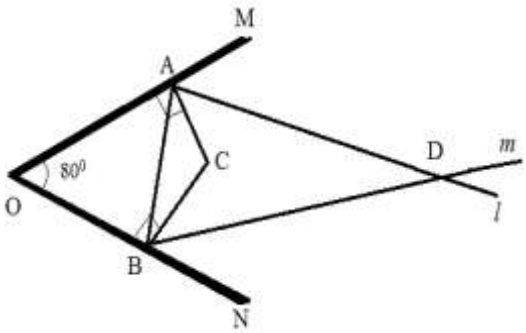
14. *A* va *B* nuqtalar orasidagi umumiy sig‘imni toping.



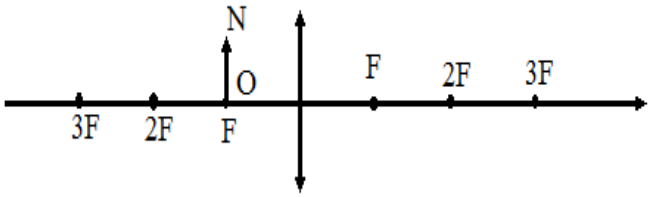
15. *A* va *B* nuqtalar orasidagi qarshilikni toping.



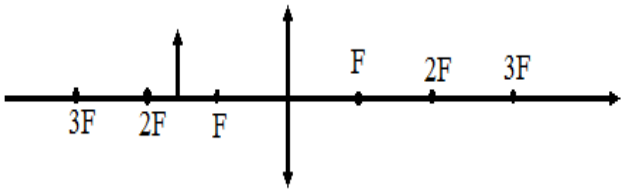
16. Rasmda *MON* yassi ko‘zgu bo‘lsa, *l* tushayotgan nur va *m* ko‘zgulardan qaytgan nur orasidagi burchakni toping.



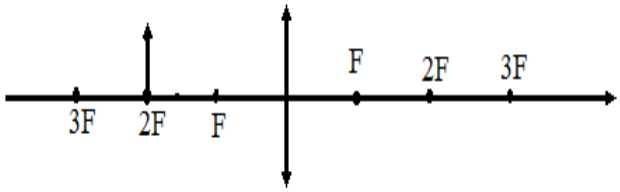
17. Rasmdagi *ON* buyumning tasvirini aniqlang.



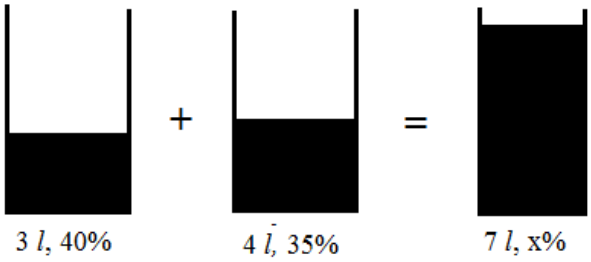
18. Rasmdagi buyumning tasvirini aniqlang.



19. Rasmdagi buyumning tasvirini aniqlang.



20. Noma’lum *x* ni toping.



Javoblar.

Sodda tasvirli masalalar.

1. $\sqrt{39}$ **2.** $130°$ **3.** $145°$ **4.** $\frac{3\cdot(\sqrt{18}+\sqrt{6})}{6}$ **5.** 2 **6.** $135°$ **7.** $2\cdot(\sqrt{13}+1)$

8. 7 **9.** $110°$ **10.** $80°$ **11.** $15°$ **12.** $2m+180°$ **13.** $20°$ **14.** $80°$

15. 40 **16.** $50°$ **17.** $120°$ **18.** 3 **19.** 3 **20.** 9 **21.** $100°$ **22.** $6\frac{2}{3}$

23. $9\pi-18$ **24.** 3 **25.** $120°$ **26.** $75°$ **27.** $105°$ **28.** $75°$ **29.** $6°$

30. $40°$ **31.** $\frac{47\sqrt{2}}{2}$ **32.** $\sqrt{73}$

Tasvirdagi shakl yetarli bo‘lgan masalalar.

1. 8 **2.** $\sqrt{29}$ **3.** 36 **4.** $\frac{12}{35}$ **5.** $\frac{\sqrt{2}}{2}$ **6.** 5 **7.** $\frac{40}{13}$ **8.** $2(2+\sqrt{3})$ **9.** $3+\sqrt{3}$

10. $\sqrt{2}$ **11.** 6 **12.** $\frac{1}{4}$ **13.** $b>c>a$ **14.** $12\sqrt{2}$ **15.** 6 **16.** 20

17. $2(1+\sqrt{13}+\sqrt{7})$ **18.** 3 **19.** $8\sqrt{3}$ **20.** $\sqrt{5}$ **21.** $\frac{\sqrt{3}}{2}$ **22.** $3\sqrt{3}$ **23.** 18

24. $2\sqrt{3}-3$ **25.** $2\sqrt{13}$ **26.** 8 **27.** $\sqrt{73}$ **28.** 4 **29.** 4 **30.** $2(\sqrt{2}+1)$

31. $6\sqrt{6}$ **32.** $5\sqrt{13}$ **33.** $\frac{25}{4}$ **34.** $\sqrt{7}$ **35.** 4 **36.** 58 **39.** $\sqrt{10}$ **40.** 2

41. 4 **42.** 13 **43.** 27 **44.** $\frac{21}{4}$ **45.** $\frac{3}{4}$ **46.** $12\sqrt{3}$ **47.** $6\sqrt{5}$ **48.** 2 **49.** $6\sqrt{3}$

50. $4{,}5$ **51.** 8 **52.** 5 **53.** 4 **54.** 8 **55.** $6\sqrt{3}$ **56.** $8\sqrt{3}$ **57.** 8 **58.** 13

59. 4 **60.** 28 **61.** $\frac{4\sqrt{3}}{3}$ **62.** 6 **63.** 18 **64.** 48 **65.** 16 **66.** 3 **67.** $7{,}5$

68. 12 **69.** 10 **70.** 3 **71.** $\frac{4}{9}$ **72.** $(a-b)^2$ **73.** 4 **74.** $\frac{2}{3}$ **75.** 6 **76.** 8

77. 25 **78.** 18 **79.** $2(2+\sqrt{3})$ **80.** $\frac{25}{4}$ **81.** $2\sqrt{3}$ **82.** 18 **83.** $\frac{14}{5}$ **84.** 16

85. $\frac{5\sqrt{3}}{4}$ **86.** 12 **87.** 12 **88.** $3\sqrt{2}$ **89.** $\frac{8}{5}$ **90.** $2\sqrt{7}$ **91.** $\frac{5}{4}$ **92.** 2 **93.** 3

94. 160

Tasvirdagi shaklni to‘ldirish yordamida yechiladigan masalalar.

1. 8 **2.** 2,5 **3.** $\frac{43}{4}$ **4.** $\frac{1}{24}$ **5.** 3 **6.** 8 **7.** $\frac{7}{8}$ **8.** 6 **9.** $\frac{35}{2}$ **10.** $\frac{432}{25}$ **11.** 8

12. 3 **13.** $\frac{18}{13}$ **14.** $4\sqrt{2}$ **15.** 15 **16.** 8 **17.** 4 **18.** $9\sqrt{3}$ **19.** 6 **20.** 2,5

21. 0,5 **22.** 1,5 **23.** $12(\sqrt{17}+1)$ **24.** 5 **25.** $\frac{8}{3}$ **26.** $3\sqrt{3}$ **27.** $\frac{14}{3}$ **28.** 8

29. $120°$ **30.** $\frac{69}{176}$ **31.** 15 **32.** 4 **33.** $\frac{96}{7}$ **34.** 2 **35.** 3 **36.** $4(\sqrt{3}+1)$

37. 28 **38.** 10 **39.** $\frac{9}{15}$ **40.** 60 **41.** $40°$ **42.** $30°$ **43.** $115°$ **44.** $90°$

45. $\frac{3}{4}$ **46.** $\frac{14}{5}$ **47.** $4(\sqrt{2}-1)$ **48.** $6(3\sqrt{3}-\pi)$ **49.** $\overrightarrow{AB}$ **50.** $\overrightarrow{BA}$ **51.** -2

52. -36 **53.** 1 **54.** $\overrightarrow{BC}$ **55.** 84,5 **56.** -100

Nuqtaning vaziyatiga bog‘liq tasvirli masalalar.

1. 32 **2.** 6 **3.** $6\sqrt{3}$ **4.** 0,5 **5.** $20\sqrt{3}$ **6.** $\frac{10}{7}$ **7.** 18 **8.** $\frac{2\sqrt{3}-3}{6}$ **9. 1** **10.** 3

11. 18 **12.** 2

Streometriyaga oid tasvirli masalalar.

1. 128π **2.** $\frac{16}{\pi}$ **3.** $\sqrt{114}$ **4.** 4 **5.** $\frac{3\sqrt{2}}{8}$ **6.** 8 **7.** 264π **8.** 36 **9.** $\frac{140\pi}{3}$

10. 392π **11.** $\frac{32\pi}{3}$ **12.** $\frac{3\pi}{2}$ **13.** $18\sqrt{2}$ **14.** 72π **15.** 8 **16.** $21\frac{1}{3}$ **17.** 48

18. $\frac{5\sqrt{6}}{18}$ **19.** 110π **20.** 16π **21.** 30π **22.** $\frac{896\pi}{5}$ **23.** 96π **24.** 3π **25.** 24

26. 6 **27.** 172 **28.** 55 **29.** 94 **30.** 48 **31.** $16\sqrt{3}$ **32.** $2\sqrt{3}$ **33.** 120

34. 92 **35.** 56 **36.** 18

Algebra va analizga doir tasvirli masalalar.

1. 3 **2.** 0 **3.** 3 **4.** 4,5 **5.** $y = \frac{x}{2} - 1$ **6.** 12 **7.** -2 **8.** $a < 0, r < 0, k > 0$

9. - 4 **10.** $n = -32$ **11.** $y = \frac{2x}{3} + 6$ **12.** $\sqrt{2}$ **13.** Barchasi to‘g‘ri **14.** 0

15. $\frac{8}{3}$ **16.** 20 **17.** $ln2$ **18.** $\frac{7}{4}$ **19.** $-6i$ **20.** $-1{,}5i$ **21.** 60 **22.** {6,7,8}

23. {1,2,6,7,8} **24.** {1,2} **25.** {c} **26.** {e} **27.** {a,b,c,d,e,f} **28.** E\{f**}**

29. C\(A∪B) **30.** B\A **31.** (A∩B)\C **32.** (A∩B)∪(B∩C) **33.** 4 **34.** $x = -7$

35. 0 **36.** $2 - \sqrt{2}$ **37.** $\sqrt{2} - 1$ **38.** $-4a$ **39.** 0,5 **40.** 1 **41.** x_3, x_4

42. $x = -5$ **43.** 4 ta **44.** x_1, x_2, x_7, x_9 **45.** $x = -2$ **46.** $\frac{3}{4}$ **47.** 4 ta

48. $a > 0, b > 0, c < 0, d > 0$ **49.** 1, 3, 4 **50.** 3 **51.** 40 **52.** 30,5

Fanlar uzviyligiga doir tasvirli masalalar.

1. $\begin{cases} S_{x_1} = 4\ m \\ S_{y_1} = 4\ m \end{cases}$, $\begin{cases} S_{x_2} = 0 \\ S_{y_2} = -4\ m \end{cases}$, $\begin{cases} S_{x_3} = 4\ m \\ S_{y_3} = 8\ m \end{cases}$, $\begin{cases} S_{x_4} = 6\ m \\ S_{y_4} = 0 \end{cases}$, $\begin{cases} S_{x_5} = -4\ m \\ S_{y_5} = 4\ m \end{cases}$

2. $\vartheta_I = 1\ {}^{m}/_{s}$, $\vartheta_{II} = 0$, $\vartheta_{III} = \frac{5}{3}\ {}^{m}/_{s}$ **3.** 15 m **4.** $\gamma_4 = 1\ Hz$

5. 0,5 ${}^{m}/_{s^2}$ **6.** 1 ${}^{m}/_{s}$ **7.** $F_1 = 4\ N$, $F_2 = 0$, $F_3 = -2\ N$, $F_4 = -1\ N$

8. $F_{BC} = 1000\ N$, $F_{AC} = 800\ N$ **9.** $F_{AC} = 4000\ N$, $F_{BC} = \frac{16000}{3}\ N$

10. 3 **11.** $I_{AB} = 10\ A$, $U_{AB} = 20\ V$ **12.** 64 m **13.** Kamayadi **14.** $C_{AB} = C$

15. $R_{AB} = 3R$ **16.** 20^0 **17.** Cheksizlikda hosil bo‘ladi. **18.** d > 2F

19. d = 2F **20.** $x = 37\frac{1}{7}$ %

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