OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

ABDULLA QODIRIY NOMIDAGI
JIZZAX DAVLAT PEDAGOGIKA INSTITUTI
FIZIKA-MATEMATIKA FAKULTETI

Qo’lyozma huquqida
UDK:517.91/93

PIRIMQULOVA NAFISA MARATOVNA

Parametrga bog’liq bo’lgan yuqori tartibli chiziqli bo’lmagan differensial tenglamalarni entegrallash

5A-110101-MATEMATIKA O’QITISh METODIKASI

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Ilmiy rahbar: **dots. Alishev A.**

Jizzax-2017 yil

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Annotatsiya.

Ushbu ishda yuqori tartibli chiziqli bo'lmagan differenseal tengalamalar sistemasining xarakterestek tenglamasining ildizlariga bog'liq bo'lgan formal xususiy yechimlarini tuzish masalasi qaraladi.

Annotation.

In this work, it is paid attention to the solution of a formal private problem dependent on the reference equation is roots of not being high regularlylined differential equations systems.

Mundarija

Kirish

Mustaqillikning dastlabki yillaridanoq Prezidentimiz raxnomoligida ta'lim sohasini isloh qilish, har tomonlama yetuk mustaqil fikrlaydigan shaxsni shakllantirish, yuqori malakali mutaxassislar tayyorlash masalalariga jiddiy e'tibor qaratildi. Jumladan mamlakatimiz tarixida "Inson manfatlari yili" 1997-yil 29-avgust alohida sanalardan biri bo'lib hisoblandi. Oliy majlisning to'qqizinchi sessiyasida "Ta'lim to'g'risida" va "Kadrlar tayyorlash milliy dasturi" to'g'risida qonunlarni qabul qilindi. Bu qonunlar uzluksiz ta'lim tizimining o'zbek modelini aniqlab berdi.[1]

Ana shunday ta'lim tizimidagi islohatlarni amalga oshirish masalasi Prezidentimiz muallifligida O'zbekistonda kadrlar tayyorlash milliy modelida to'la qonli aks etgani, milliy model asosida kadrlar tayyorlash milliy dasturi yaratilgani e'tirof etilmoqda.

"Kadrlar tayyorlash milliy dasturi" ning asosiy maqsadi, ta'lim sohasini tubdan isloh qilish, yuksak va ma'naviy va ahloqiy talablarga javob beruvchi yuqori malakali kadrlar tayyorlash milliy tizimini yaratishdir. Ta'lim tizimida tayyorlanayotgan mutaxassis yuqori sifat me'zonlariga javob berishi lozim.[2]

O'zbekistondagi innovasiyalar ko'rsatkichi bo'yicha 75,38 ball bilan dunyoda ikkinchi o'rinni egalladi. Bu esa yurtimizda yosh avlodning chuqur ta'lim sifatini takomillashtirish ishlari jadal olib borilayotganiga berilgan holis bahodir.

Respublika ta'lim sohasini tubdan isloh qilish ishlarini amalga oshirish ko'zda tutilgan. Yurtimizda rivojlangan demokratik davlatlar talablar darajasida, kadrlar imkonini beradigan ta'lim tizimini yaratish davom etmoqda. [3]

O'zbekiston Respublikasi Prezidentining 2012-yil 24-iyuldagi PF-4456-son "Oliy malakali ilmiy va ilmiy pedagog kadrlar tayyorlash va attestasiyadan o'tkazish tizimini yanada takomillashtirish to'g'risida" gi Farmoniga va Vazirlar Mahkamasining 2007-yil 10-sentayabrdagi 190-son "O'zbekiston Respublikasi oliy ta’lim tizimida magistratura faoliyatini yanada takomillashtirish uning samaradorligini oshirish chora-tadbirlari to’g’risida” gi qaror qabul qilindi.

Magistraturaga qabul qilish Vazirlar Mahkamasining 2010 yil 118-son qarori (O’zbekiston Respublikasi qonun xujjatlari to’plami, 2010 y., 24-25-son 195-modda) bilan tasdiqlangan Oliy ta’lim muassasalarining qabul qilish tartibi to’g’risidagi nizomiga muvofiq amalga oshiriladi.

Tegishli mutaxassisliklar bo’yicha magistrlarni tayyorlash kadrlar buyurtmachilarning takliflari asosida, davlat akkreditasiyasidan o’tgan oliy ta’lim muassasalarida amalga oshirildi.[4]

Magistratura aniq mutaxassislik bo’yicha fundamental va amaliy bilim beradigan, bakalavriat negizida ta’lim muddati kamida 2 yil davom etadigan oliy ta’limdir. Meni ham magistrlikdagi ilmiy ishim mavzusi “Parametrli differentsial tenglamalar sistemasining yechimlarini tadbiqi”.

Differensial tenglamalar tabiatda uchraydigan jarayonlarni o’rganishda, texnika va amaliy masalalarni yechishda qo’llaniladigan matematikaning universal apparatidir.

Bunda asosiy muammo differensial tenglamalarni yechimini topish yoki ularni integrallashdan iborat.

Differensial tenglamalarni integrallash ancha murakkab jarayon. Agar haqiqiy o’zgaruvchili koeffisiyentli differensial tenglamalar sistemasi bo’lsa yanada murakkablashadi, chunki bunday tenglamalar integrallanmaydi. Bunday hollarda har xil yaqinlashuvchi metodlardan foydalanishga to’g’ri keladi.

Yaqinlashuvchi metodlardan biri asimptotik metod bo’lib, dastlab J.Fur’ye, J.Liuvillya, J.Shturm, A.Puankare, V.A.Steklov ishlarida qo’llanilgan.

Fizikaning ko’pgina masalalari differesial tenglamalar, xususan xususiy hosilali differesial tenglamalar orqali ifodalanadi. Ularni aniq yechib bo’lmaydi. Agar bunday tenglamalarda o’zgaruvchilarni ajratish mumkin bo’lsa, u holda unga (1807 y) Fur’ye tomonidan taklif etilgan metodni qo’llash mumkin. Bu metod chegaraviy shartlarni qanoatlantiruvchi parametrga bog’liq bo’lgan oddiy differensial tenglamaga keltiriladi.

Bunday masalaga misol qilib bir jinsli chegaraviy

$$y(a) = y(b) = 0 \qquad (1)$$

shartlarni qanoatlantiruvchi

$$\frac{d^2y}{dx^2} + (\lambda g(x) - r(x)) = 0 \qquad (2)$$

ikkinchi tartibli differensial tenglamani qarash mumkin, bunda $\lambda -$ katta parametri, *g(x),r(x),[a,b]* kesmada berilgan funksiyalar.

(2)- differesial tenglamaga Shturm-Liuvilli tenglamasi deyiladi.

Fur’ye metodini qo’llab o’zgaruvchi koeffisiyentli oddiy differensial tenglama hosil qilamiz. Bunday tenglama umumiy holda integrallanmaydi.

Faqat xususiy hollarda (1), (2) masalalar uchun Fur’ye tomonidan yechimlar aniqlangan. Bu yechimlar fundamental funksiyalar deyiladi.

Fur’yening natijalari (1828 y) Liuvilli tomonidan umumlashtirilgan. U ixtiyoriy *f(x)*funksiyani (1), (2) masalalarning $y_k(x,\lambda)$, (k=1,2,…) fundamental funksiyalari orqali

$$f(x) = \sum_{k=1}^{\infty} C_k y_k(x,\lambda) \qquad (3)$$

qatorga yoyish metodini taklif etdi. Bu funksiyalar λ parametrning katta qiymatlarida ortogonallik xossasiga ega. Shuning uchun (3) qatorning $c_k, (k = 1,2, ...)$koeffisiyetlari

$$c_k = \int_a^b f(x) y_k(x,\lambda) dx, \; k = 1,2, ... \qquad (4)$$

formuladan aniqlanadi.

(3) qator *[a,b]* kesmada yaqinlashuvchi bo'lib uning yig'indisi *f(x)* funksiya ekanligi isbotlandi. (3) qatorning yaqinlashishini Liuvilli $y_k(x,\lambda)$, $k = 1,2,\ldots$ funksiyalar uchun λ parametr orqali asimptotik formula kiritish yordamida isbotlagan. (3) tenglikni isbotlash uchun Shturmning ilmiy xulosalaridan foydalanib, $n-$ tartibli differesial tenglamaning fundamental funksiyalari uchun asimptotik formulani hosil qiladi.

Fur'ye, Shturm, Liuvilli ishlaridan keyin yechimlarni $\lambda \to \infty$ da λ parametr orqali asimptotik formula koa'rinishida ifodalash yoki asimptotik integrallash nazariyasi jadal rivojlandi.

Liuvilli tomonidan ishlab chiqilgan asimptotik nazariya ko'plab amaliy masalalarni yechishda qo'llanildi.

Fauller, Lakka va Sparre ishlarida snaryad harakatining ifodalovchi tenglamalarini yaqinlashuvchi yechimlarini aniqlash uchun bu metod tadbiq etildi.

Differesial tenglamalarining yechimlarini asimptotik ifodalash nazariyasida A.Puankarening (1883 y), "Samoviy (osmon) mexanikasining yangi metodlari" ishi muhim ahamiyatga ega. Bu ishda birinchi bo'lib differesial tenglamalarning yechimlarini asimptotik ifodalashda qo'llanilgan formal qatorlarning amaliy qo'llanishi ko'rsatilgan

$$\frac{d^2y}{dx^2} + \left(1 - \frac{a}{t^2}\right)y = 0 \qquad (5)$$

Differesial tenglamani qaraymiz, bunda *a*-kompleks son, $t > 0$ haqiqiy o'zgaruvchi. (5)-tenglama o'zgaruvchi koeffisiyentli uni integrallab bo'lmaydi. Bu tenglamaning yechimini

$$y(t) = e^{it}\,(c_0 + c_{-1}\, t^{-1} + \cdots + c_{-n}\, t^{-n} + \cdots \qquad (6)$$

ko'rinishda izlaymiz. U holda

$$\sum_{k=1}^{\infty} c_{-k} t^{-k} \quad (7)$$

qatorning c_{-k} koeffisiyentlarini tanlash natijasida funksiya (5) tenglamaning formal qanoatlantirishi mumkin. (formal deb shu ma'noda olinadiki, t^{-m} ning har qanday darajalari oldidagi koeffisiyentlarni yiqqanimizda nolga teng bo'lgan ayniyat hosil bo'ladi).

(6)-darajali qator *t* ning katta qiymatlarida uzoqlashuvchi bo'lishini ko'rsatish qiyin emas (buni Puankare ko'rsatgan). (5)-tenglamani aniq yechimi

$$y(t) = e^{it}(S_n(t) + \theta(t^{-n-1})), \qquad t \to \infty \quad (8)$$

ko'rinishda asimptotik formula orqali ifodalanadi, bunda $S_n(t)$ - (7)-qatorning xususiy yig'indisi.

(7)-xossaga ega bo'lgan (6) formal yechimga asimptotik yechim, asimptotik yechimni tuzish metodlariga asimptotik metod deyiladi.

(8) formulada y(t) funksiya t erkli o'zgaruvchining asimptotik ko'rinishini ifodalaydi. Oldingi tadqiqotlarning metodlari har qanday $x \epsilon [a, b]$ va λ parametr bo'yicha asimptotik yechimlarni tuzish imkonini beradi.

Asimptotik metodni rivojlanishiga rus matematigi V.A.Steklov (1864-1926) katta xissa qo'shgan.

(2) tenglama uchun tuzilgan fundamental funksiyalarning yopiqlik teoremasini isbotladi. V.A.Steklov funksiyaning (2) tenglamaning fundamental yechimlari orqali qatorga yoyishning Furyening oddiy trigonometrik qatori darajasida aniqlab ikkala masalani ham hal qildi.

Yuqorida qayd etilgan barcha tadqiqotlar Liuvillining ishlarini umumlashtirishdan iborat bo'lib o'zaro qo'shma differesial tenglama va tenglamalar sistemasi keltirilgan.

Bu muhim cheklanishlar L.Shlezinger, G.Birkgof, Ya.D.Tamarkin ishlarida yo'qotilgan.

Birkgof,

$$[\quad y^{(n)} + \rho a_1(x,\rho)\, y^{(n-1)} + \cdots + \rho^n a_0(x,\rho) y = 0 \qquad (9)$$

differesial tenglamaning yechimini tuzish uchun asimptotik formulani kiritadi, bunda $a_i(x,\rho), i = \overline{0, n-1}$, $\rho -$ kompleks parametr analitik funksiyalar bo'lib [a,b] kesmada *x* haqiqiy o'zgaruvchi bo'yicha cheksiz differesiallanuvchi Shlezinger yechimning asimptotik xossasini katta $|\rho|$ uchun belgilangan $arg\rho = \lambda$ o'qda isbotlaydi. Birkgof undan farqli asimptotik xossani kompleks tekisligining $\theta \le arg\rho \le 2$ sektori uchun isbotlaydi.

Shlezinger va Birkgof ishlarini Tamarkin

$$\frac{dy_i}{dx} = \sum_{j=0}^{n} a_{ij}(x,\rho) y_j, \qquad i = \overline{1,n} \qquad (10)$$

chiziqli differesial tenglamalar sistemasi uchun umumlashtirgan, bunda $a_{ij}(x,\rho), i,j = \overline{1,n}$ haqiqiy o'zgaruvchi bo'yicha cheksiz differesiallanuvchi va $\rho = \infty$ nuqta atrofida kompleks parametr bo'yicha analitik funksiyalar bo'lib, bu nuqtada o'ziga xos xususiyatga ($r \ge 1$ tartibli qutbga) ega.

Shlezinger, Birkgof, Tamarkinlarning ilmiy natijalari

$$\det\left(a_{ij}^{(0)}(x) - \lambda\delta_{ij}\right) = 0, \qquad i,j = \overline{1,n} \qquad (11)$$

xarakteristik tenglama har qanday $x \in [a,b]$ uchun oddiy ildizlarga ega bo'lgandagina o'rinli, bunda $a_{ij}^{(0)}(x), a_{ij}(x,\rho)$ funksiyani ρ parametrning darajalari bo'yicha qatorga yoyganda hosil bo'lgan ozod had, $\delta_{ij} -$ Kronekera simvoli. Faqat ikkinchi tartibli

$$y'' + \rho a_1(x,\rho)y' + \rho^2 a_2(x,\rho)y = 0$$

tenglama uchun (1) tenglamaning ildizlari [a,b] kesmada bir-biriga aynan teng bo'lgan hol [1] ishda qaralgan. Yuqori tartibli differesial tenglamalar, shuningdek (10) ko'rinishdagi sistemalar uchun xarakteristik tenglama karrali ildizlarga ega bo'lgan hol uzoq muddat o'rganilmay kelingan. O'tgan asrning 60-70 yillariga kelib xarakteristik tenglama karrali ildizlarga ega bo'lgan hol ustida tadqiqot ishlari olib borilgan.

Katta ρ parametrga bog'liq bo'lgan differensial tenglamalar sistemasining formal yechimlarini ifodalash uchun Eylerning metodiga o'xshash o'zgarmas koeffisiyentli chiziqli differensial tenglamasiga qo'llash uchun

$$\exp\left(\rho^r \int_a^x \sum_{s=0}^{r-1} \rho^{-s}\lambda_s(\tau)d\tau\right)\sum_{s=0}^{\infty} \rho^{-s} y_s(x)$$

ifodadan foydalanilgan, natijada izlanayotgan yechim uchun

$$\mathbf{y(x,\rho)} = \exp\left(\rho^r \int_a^x \sum_{s=0}^{r-1} \rho^{-s}\lambda_s(\tau)d\tau\right)\left(\sum_{s=0}^{\infty} \rho^{-s} y_s(\tau) + O[(\rho]^{-m})\right)$$

asimptotik formula hosil qilingan.

Yechimning asimptotik ko'rinishda umumiyroq ifodalashning

$$\mathbf{y_i(x,\rho)} = \sum_{i=1}^{n} [[z_i(x,\rho)\left(\sum_{j=1}^{m} y_{ij}(x)\rho^{-j}] + O[[(\rho]^{-m})\right)$$

ko’rinishga ega bo’lgan holi, bunda $z_i(x,\rho), i=\overline{1,n}$ funksiyalar, ba’zi chiziqli differensial tenglamalar sistemasini ifodalashi V.S.Pugachev ishlarida berilgan.

V.S.Pugachevning ishlaridan I.M.Rapoport [6] differensial tenglamalar sinfini *L*-diagonal ko’rinishdagi differensial tenglamalar sistemasiga keltiruvchi chiziqli almashtirishni tuzishda foydalangan. Bunday sistemalarni yechish uchun $t=+\infty$ nuqta atrofida *t* erkli o’zgaruvchiga nisbatan asimptotik formula hosil qilingan. V.S.Pugachevning ilmiy xulosalaridan S.F.Feshenko sekin o’zgaruvchi koeffisiyentli chiziqli differensial tenglamalar sistemasini asimptotik qismlarga ajratishda foydalangan.

Yechimlarni asimptotik ko’rinishda ifodalash F.M.Xukuxara, G.Territin ishlarida ham qaralgan, ularning ishlarida katta parametrga bog’liq bo’lgan chiziqli differensial tenglamalar sistemasi asimptotik parchalash bilan kichik tartibdagi qism sistemalarga keltirilgan. Differensial tenglamalarni asimptotik integrallash masalalari bo’yicha V.Vazov, L.Chezari, R.Larger ishlarida muhim ilmiy natijalar olingan.

Chiziqli bo’lmagan mexanikaning asimptotik metodlari sobiq ittifoqning olimlari N.M.Krilov va N.N.Bogolyubov, Yu.M.Mitropolskiy tomonidan yaratilgan. Tebranish jarayonlarini tadqiq qilishda yaratilgan universal apparat barcha yaqinlashuvchi metodlardan.

Bu metodlar dastlab Krilov va Bogolyubov tomonidan

$$\frac{d^2x}{dt^2}+\omega^2x=\varepsilon f\left(x,\frac{dx}{dt},\varepsilon,t\right) \qquad (12)$$

chiziqli bo’lmagan differensial tenglamalarni taqribiy integrallash uchun ishlab chiqilgan,bunda ε - kichik parametr.

(12) tenglama yuqorida qaralgan tenglamalardan tubdan farq qiladi. Haqiqatdan ham $\varepsilon=0$ bo’lganja (12) tenglamadan sodda integrallanadigan o’zgarmas koeffisiyentli ikkinchi tartibli chiziqli tenglamaga kelamiz. Shuning uchun (12) tenglamaga Puankare tomonidan taklif etilgan izlanayotgan yechimini ε ning darajalari bo’yicha

qatorga yoyish metodini qo'llash mumkin. Buning natijasida yaqinlashuvchi yechim o'zida sekulyar hadlarni saqlab qoladi (erkli o'zgaruvchi t trigonometrik funksiyalar oldida ko'paytma ko'rinishida keluvchi hadlar), bu kuzatiladigan tebranish jarayoni vaqt oralig'ini ancha qisqartiradi.

Krilov va Bogolyubov metodlari asosida hosil qilingan yaqinlashuvchi formulalar sekulyar hadlarni saqlamaydi, bu erkli o'zgaruvchi t ning yetarli katta oralig'ida chekli tebranish jarayonlarini o'rganishda tadqiqot ishlarini olib borishga imkon beradi.

Krilov va Bogolyubov metodlaridan Yu.A.Mitropolskiy foydalanib stasionar bo'lmagan tebranish jarayonlarni o'rganish bo'yicha tadqiqot olib bordi [7]. U birinchi bo'lib kichik aynituvchi davriy kuch ta'sirida sistema ichki va tashqi rezonans holati yuz beradigan murakkab hol ustida tadqiqot olib borgan. U shuningdek ikkinchi tartibli differensial operatorning chiziqli qismi sekin o'zgaruvchi koeffisiyentli hadlarni saqlagan holni ham qaragan.

Mitropolskiy yaratgan asimptotik metodlari yordamida chiziqli va chiziqli bo'lmagan differensial tenglamalarning sifat nazariyasining muhim masalalari o'rganilishi bilan birgalikda yangi texnikaning ko'pgina masalalari yechildi.

Yu.A.Mitropolskiy ideyasi B.I.Maissenko, V.I.Fodchuk, A.M.Samaylenko, O.B.Likova, D.I.Martinyuk, V.G.Kolomiyi ishlarida rivojlantirildi.

Sekin o'zgaruvchi koeffisientli differensial tenglamalar uchun qo'llaniladigan metodlar bilan batafsil tanishib chiqamiz.

Differensial tenglamalarning keng sinfi sekin o'zgaruvchi koeffisiyentli differensial tenglamalarga keltiriladi. Ko'pgina amaliy masalalarning [8] shunday tenglamalarni yechishga kelitirilishi haqida aytmasak ham bo'ladi. Liuvilli, Shlezinger, Bmrkgof, Tamarkinlar tomonidan o'rganilgan katta parametrli tenglama erkli o'zgaruvchini almashtirish yordamida sekin o'zgaruvchi koeffisiyentli differensial tenglamalarga keltiriladi.

Buning uchun (2) ko'rinishdagi Shturm-Liuvilli tenglamasidagi erkli o'zgaruvchini almashtiramiz:

$$x = \tau = \varepsilon t, \qquad \varepsilon = \frac{1}{\sqrt{\lambda}}$$

U holda (2) tenglama

$$\frac{d^2y}{dt^2} + \left(g(\tau) - \varepsilon^2 r(\tau)\right)y = 0$$

ko'rinishga keladi, bunda $g(\tau), r(\tau)$-koeffisiyentli sekin o'zgaruvchili funksiyalar bo'ladi. Shunga o'xshash almashtirish yordamida (9) tenglama va (10) sistemani, sekin o'zgaruvchi koeffisiyentli tenglamaga (sistemaga) keltirish mumkin.

Sekin o'zgaruvchi koeffisiyentli chiziqli differensial tenglamalar sistemasini o'rganish o'tgan asrning 50-yillarida S.F.Feshenkoning ishlari paydo bo'lgandan keyin boshlangan S.F.Feshenko tomonidan

$$\frac{d^2y}{dt^2} + \varepsilon\rho(\tau,\varepsilon)\frac{dy}{dt} + q(\tau,\varepsilon)y = \varepsilon f(\tau,\varepsilon)e^{i\theta(\tau,\varepsilon)}$$

ko'rinishdagi ikkinchi tartibli differensial tenglamaning asimptotik yechimni tuzish metodi taklif etilgan, bunda $\rho(\tau,\varepsilon), q(\tau,\varepsilon), \frac{d\theta(t,\varepsilon)}{dt}$ -sekin o'zgaruvchi funksiyalar, $\tau = \varepsilon t, \quad \varepsilon > 0$ kichik parametr. Bu ishda $k(\tau) = \frac{d\theta}{dt}$ - (tashqi chastota) funksiya ba'zi $\tau\epsilon[0, L]$ uchun berilgan tenglamaning xarakteristik tenglamasining oddiy ildizlaridan (xususiy chastotasiga) birortasiga teng bo'lgan hol qaralgan. Bu hol nazariy va amaliy jihatdan muhim bo'lib matematika fizikaning masalalariga tadbiq etilgan. Mexanikada uni "rezonans" deyiladi. Shuningdek $k(\tau)$ funksiya $\forall\tau\epsilon[0, L]$ uchun xarakteristik tenglamani birorta ham ildiziga teng bo'lmagan hol qaralgan.

S.F. Feshenko tuzilgan yechimlarning Krilov-Bogolyubov ma'nosida asimptotik xarakterga ega ekanligini isbotlagan, xususan tuzilgan yechimning *m*-ta xususiy yig'indisi aniq yechimga intilishini *m*-ning ortishi bilan emas belgilangan $m \geq 1$ va $\varepsilon \to 0$ ko'rsatgan.

Feshenkoning ilmiy natijalari G.N.Savin, V.N.Shevelo, A.I.Kuji va O.A.Gareshkolar tomonidan shaxtaning ko'tarish qanotida yuz beradigan zo'riqish orqali ifodalanuvchi differensial tenglamalarning yaqinlashuvchi yechimini aniqlashda foydalanganlar.

Shunga o'xshash S.F. Feshenko tomonidan

$$A(\tau,\varepsilon)\frac{d^2x}{dt^2} + \varepsilon C(\tau,\varepsilon)\frac{dx}{dt} + B(\tau,\varepsilon)x = f(\tau,\varepsilon)e^{i\theta(t,\varepsilon)}$$

differensial tenglamalar sistemasi uchun olib borilgan, bunda $A(\tau,\varepsilon), C(\tau,\varepsilon), B(\tau,\varepsilon)$matrisalar va $f(\tau,\varepsilon)$ vektor ε parametrning darajalari bo'yicha qatorga yoyiladi. Bunda$A_0(\tau), B_0(\tau), C_0(\tau)$ozod hadlari simmetrik matrisalar degan shart qo'yilgan. Bunday kuchli shart keyinchalik N.I.Shkilning ishlarida olib tashlangan.

Shuningdek S.F. Feshenko

$$\frac{dx}{dt} = A(\tau,\varepsilon)x \qquad (13)$$

ko'rinishdagi chiziqli differensial tenglamalar sistemasini kichik tartibdagi qism sistemaga parchalashga doir ko'pgina teoremalarni isbotlagan.

Bu ishlardan xususiy holda xarakteristik tenglamaning oddiy ildizlari uchun differensial tenglamalarni yechimlari asimptotik ifodalashga doir Shlezinger, Birkgof, Tamarkinlarning ishlari kelib chiqadi.

Asimptotik parchalashga doir yaratilgan teoremalar yordamida berilgan sistemani taxminan tartibini pasaytirish mumkin. Umumiy holda, masalan xarakteristik tenglamaning karrali ildizlari uchun bu teoremalar yordamida berilgan differensial tenglamalar sistemasining yechimlarini topib bo'lmaydi.

Bu hol amaliy masalalarni yechish va nazariy tadqiqotlarda ko'plab uchraydi. Hatto Shturm-Liuvilli tenglamalariga o'xshash oddiy tenglamalarni o'rganishda karrali ildizlar bilan ish yuritishga to'g'ri keladi.

Hosila oldida kichik parametr qatnashgan differensial tenglamalar sistemasini tadqiq qilish, optimal boshqaruv masalalari va boshqa masalalarda karrali ildizlar bilan ishlashga to'g'ri keladi. Karrali ildizlarga, karrali elementar bo'luvchilar mos kelgan hol yetarlicha murakkab ekanligini alohida qayd etish mumkin. Bu holda berilgan differensial tenglamalar sistemasi yechimlarini ε parametrning butun darajalari bo'yicha qatorga yoyib bo'lmaydi. Bunday yechimlar oddiy ildizlardan farqli holda parametrning har xil kasr darajalari orqali formal qator ko'rinishida ifodalanadi, daraja ko'rsatkichlar nafaqat xarakteristik tenglamaning karrali ildizlariga bog'liq bo'lib qolmasdan elementar bo'luvchilarining karralilariga ham bog'liq bo'ladi.

Sekin o'zgaruvchi koeffisiyentli differensial tenglamalar sistemasining xarakteristik tenglamasining karrali ildizlariga ega bo'lgan hollarni N.I.Shkil [5] ishlarida o'rgangan. U tomonidan berilgan differensial tenglamalar sistemasining yechimlarini asimptotik ko'rinishda ifodalash yuzasidan matematik nuqtai nazardan yangi ilmiy natijalar olingan. Ularning ba'zilari bilan tanishamiz.

Faraz qilaylik

$$\det(A_0(\tau) - \lambda E) = 0 \qquad (14)$$

xarakteristik tenglama hyech bo'lmasa bitta $\lambda = \lambda_0(\tau)$ o'zgarmas *k* karrali $2 \le k < n$ ildizga ega bo'lib, unga shunday karrali elementar bo'luvchi mos kelsin.

1-Teorema. Agar $A(\tau, \varepsilon)$ matrisa [*0*,*L*] kesmada τ o'zgaruvchi bo'yicha istalgan tartibli hosilaga ega bo'lib, $\forall \tau \epsilon [0, L]$ uchun

$$S(\tau) = T^{\uparrow}(-1)\ (\tau)(dT(\tau)/d\tau - A_{\downarrow}1\ (\tau)T(\tau) \qquad (15)$$

matrisaning elementi

$c_{k_1}(\tau) \neq 0$

bo’lsa, u holda (13) differensial tenglamalar sistemasi

$$x = U(\tau,\mu)\exp\left(\int_0^t \lambda(\tau,\mu)dt\right) \qquad (16)$$

ko’rinishdagi formal yechimlariga ega, bunda *n*-o’lchovli $U(\tau,\mu)$ vektor va $\lambda(\tau,\mu)$ skalyar funksiya

$$U(\tau,\mu) = \sum_{s=0}^{\infty} \mu^s u_s(\tau), \qquad \lambda(\tau,\mu) = \sum_{s=0}^{\infty} \mu^s \lambda_s(\tau), \qquad \mu = \varepsilon^{\frac{1}{k}} \qquad (17)$$

darajali qatorga yoyiladi.

Ushbu magistrlik ishida quyidagi masalalar qaraladi.
I-bob sekin o’zgaruvchi koeffisiyentli yuqori tartibli chiziqli bo’lmagan differensial tenglamalar sistemasining xarakteristik tenglamasi oddiy, karrali ildizlarga oddiy elementar bo’luvchilar mos kelgan hol va karrali ildizlar aynan shunday elementar bo’luvchilar mos kelgan hollar uchun asimptotik formal xususiy yechimlar tuzilib, tuzilgan yechimlarning asimptotik xossaga ega ekanligi baholanadi.

$$\frac{d^k x}{dt^k} + A(\tau,\varepsilon)x = \varepsilon f(\tau,x,\varepsilon), \ k \geq 3 \qquad (18)$$

ko’rinishdagi tenglama qaraladi, bunda $\varepsilon > 0$ kichik parametrlar $\tau = \varepsilon t \in [0,L]$ sekin o’zgaruvchi vaqt, $A(\tau,\varepsilon) - n -$tartibli kvadrat matrisa, $f(\tau,x,\varepsilon) - n -$o’lchovli vektor, $(\tau, x_0, 0)$ nuqta atrofida Teylor qatoriga yoyiladi.
II-bobda kasr rangli yuqori tartibli chiziqli bo’lmagan differensial tenglamalar sistemasining xarakteristik tenglamasi yuqorida qayd etilgan hollar uchun formal xususiy yechimlari tuzilib, tuzilgan formal yechimlarning asimptotik xossaga ega ekanligi isbotlanadi. Bu bobda qaralayotgan sistemada sekin o’zgaruvchi vaqt $\tau = \varepsilon^{p/q} t \in [0,L]$ ko’rinishga ega.
Agar differensial tenglamaning yuqori tartibli hosilasi oldida kasr darajali parametr ko’paytuvchi sifatida qatnashsa unga kasr rangli differensial tenflamadir. *p* va *q* sonlari o’zaro tub sonlar. Shu bobda *p* va *q* sonlari orasidagi munosabatga bog’liq. $p>q$ va $p<q$ hollar alohida-alohida qaraladi. Xarakteristik tenglama karrali ildizga ega

bo'lsa, uning karraligi tartibi bilan p va q sonlar orasidagi munosabatga bog'liq formal xususiy yechimlar tuzish masalasi yoritilgan.

III-bobda yuqori tartibli kechikuvchi argumentli chiziqli bo'lmagan differensial tenglamalar sistemasi asimptotik yechimlarini tuzish masalasi qaralib, qaralayotgan sistemaning xarakteristik tenglamasining ildizlariga bog'liq xususiy formal yechimlarini ko'rsatuvchi teoremalarni isboti keltirilib, bu yechimlarini asimptotik xarakterga ega ekanligini ko'rsatuvchi teorema isbotlanib, aniq yechim bilan yaqinlashuvchi yechim orasidagi farqning bahosi kelitirilgan.

Ishda olingan ilmiy natijalar [17,18] ishlarda e'lon qilingan.

I-bob. Yuqori tartibli differinsial tenglamalar sistemasining formal xususiy yechimlarini tuzish.

1-§. Masalaning qo'yilishi.

$$\frac{d^k x}{dt^k} + A(\tau,\varepsilon)x = \varepsilon f(\tau, x, \varepsilon) \qquad (1.1)$$

ko'rinishdagi kichik parametrga bog'liq bo'lgan $\kappa \geq 3$ tartibli chiziqli bo'lmagan differensial tenglamalar sistemasini formal xususiy yechimlarini tuzish masalasini qaraymiz, bunda $x, f - n -$o'lchovli vektorlar. $x -$noma'lum vektor, $A(\tau,\varepsilon)$ n- tartibli kvadrat matritsa, $\tau = \varepsilon^r t \in [0, L] -$ sekin o'zgaruvchi vaqt, $\varepsilon > 0$ kichik parametr, $L > 0$ berilgan son, $k \in \mathrm{N}$

(1.1)-tenglamani koeffesenti $A(\tau,\varepsilon)$ matritsa ε parametrning darajalari bo'yicha yaqinlashuvchi

$$A(\tau,\varepsilon) = \sum_{S=0}^{\infty} \varepsilon^S A_S(\tau) \qquad (1.2)$$

Darajali qatorga yoyiladi.Berilgan tenglamaning o'n tomonida $f(\tau, x, \varepsilon)$ vector $(\tau, x_0(\tau), 0)$ nuqta atrofida Teylor qatoriga yoyiladi. Bu vektorning qatorga yoyilmasini har bir qaralayotgan hollar uchun alohida keltiramiz.
1.1 tenglamaning formal xususiy yechimlari asosan uning

$$\det[A_0(\tau) + \lambda E] = 0 \qquad (1.3)$$

ko'rinishdagi xarakteristik tenglamasining ildizlariga bog'liq, bunda ***E-n*** – o'lchovli birlik matritsa

Ushbu ishda asosan ilgari adabiyotlarda o'rganilmagan kretik hol, ya'ni (1.3) tenglamaning ildizlaridan ba'zilari aynan nolga teng bo'lgan hollar uchun (1.1) sistemaning xususiy yechimlari tuzish masalasi bilan bog'liq quyidagi hollar alohida qaraladi.

1) **(**1.3) tenglamaning ildizlardan bittasi nolga teng bo'lib qolganlari noldan farqli bo'lgan hol, ya'ni $\forall\tau\in[0,L]$ uchun

$$\lambda_1(\tau)\equiv 0,\ \lambda_j(\tau)\neq 0,\ \lambda_j(\tau)\neq\lambda_i(\tau)\ i\neq j,\ i,j=\overline{1,n}, \qquad (1.4)$$

shart bajarilgan hol.

2) (1.4) – tenglamaning ildizlaridan ***p*** tasi nolga teng bo'lib qolganlari noldan farqli, ya'ni

$$\forall\tau\in[0,L]\text{uchun}\,\lambda_i(\tau)\equiv 0,\ i=\overline{1,p},\ \lambda_j(\tau)\neq 0,\ j=p+1,...,n.$$

3) (1.4) –tenglama bitta ***n*** karrali nol ildizga ega bo'lgan hol

$A_0(\tau)$matritsaning xos qiymatlariga mos keluvchi xos vektorlarini $\varphi(\tau)$ bilan, $A_0(\tau)$matritsaning qo'shma $A_0^*(\tau)$ matritsasining nol- fazoviy elementlarini $\psi(\tau)$ vektor bilan belgilaymiz, ya'ni $\varphi(\tau)\in N(A_0^*(\tau))$ Bu matritsalarni tuzish (*) ishda ko'rsatilgan

2-§.*k*-chi tartibli chiziqli bo'lmagan differensial tenglamalar sistemasi formal xususiy yechimlarini tuzish.

Ushbu parametrda (1.3) tenglamaning ildizlari (1.4) shartni qanoatlantiradigan hol uchun xususiy yechimda tuzish masalasini qaraymiz.

2.1- **Teorema.** Agar $A_S(\tau)(S=0,1,...)$ matritsalar τ o'zgaruvchi bo'yicha $[0,L]$ kesmada va $f(\tau,x,\varepsilon)$ vektor-funksiya $Q(\tau,x,\varepsilon)=Q(\tau,x)$. $(0<\varepsilon\leq\varepsilon_0)$ sohada $(Q(\tau,x))$ τva$x$ o'zgaruvchilarning fazoviy sohasi cheksiz differensiallanuvchi bo'lsa yetarlicha kichik $\varepsilon\in(0,\varepsilon_0]$ va $\forall\tau\in[0,L]$ uchun (1.1) sistema

$$x(t,\varepsilon)=\sum_{S=0}^{\infty}\varepsilon^S u_S(\tau) \qquad (2.1)$$

ko'rinishdagi formal xususiy yechimga ega bo'ladi.

Isbot. 2.1 qatorni 1.1 sistemaga qo'yish va $f(\tau,u(\tau,\varepsilon),\varepsilon)$ vector-funksiani $(\tau,x_0(\tau),0)$nuqta atrofida Teylor qatoriga yoyilmasini e'tiborga olib

$$\varepsilon^k u^{(k)}(\tau,\varepsilon) + A(\tau,\varepsilon)u(\tau,\varepsilon) = \varepsilon\{f(\tau,u_0) + \varepsilon[f_u(\tau)u_1(\tau) + f_1(\tau)] + ...\varepsilon^S[f_u(\tau)u_S(\tau) - f_S(\tau)] + ...\}$$

ayniyatga ega bo'lamiz, bunda $f_u(\tau) = \frac{\partial f^i}{\partial u^j}$ matritsaning elementlari va $f_1(\tau) = \frac{\partial f}{\partial \varepsilon}$ vektorning koordinatalari $(\tau, u_0(\tau), 0)$ nuqtaga hisoblanadi, $f_S(\tau)$ vector $u_q(\tau)$ $q = (0,1,2,..., S-1)$ orqali ifodalanadi.

(2.2) tenglamadan bir xil darajali ε parametrning oldidagi koeffisientlarini tenglashtirib quyidagi tenglamalar sistemasini xosil qilamiz.

$$A_0 u_0 = 0 \tag{2.3}$$

$$A_0 u_1 = -A_1 u_0 + f(\tau, u_0) \tag{2.4}$$

$$A_0 u_S = [f_u(\tau) - A_1(\tau)]u_{S-1} + b_S \tag{2.5}$$

bunda

$$b_S = -\sum_{i=2}^{S} A_i u_{S-i} + f_{S-1}(i) - u_{S-k}^{(k)}, \qquad S = 2,3,...$$

Agar (2.1) vector-funksiya (1.1) sistemaning formal yechimi bo'lsa (2.1) qatorning koeffesientlari (2.3)-(2.5) tenglamalar sistemaasini qanoatlantiradi. Teskarisi ham o'rinli, agar (2.1) qatorning koeffesientlari (2.3)-(2.5) tenglamalar sistemasini qanoatlantirsa, u holda (2.1) vektor-funksiya (1.1) sistemaning formal yechimi bo'ladi. Shu sababli teoremani isbot qilish uchun (2.3)-(2.5) tenglamalrdan ketma-ket (2.1) tnglikni koeffesentlarini aniqlash yetarli.

(2.3) tenglamadan [12] ishga asosan

$$u_0(\tau) = \varphi(\tau)\eta_0(\tau) \tag{2.6}$$

vektorni aniqlaymiz, bunda $\varphi(\tau), A_0(\tau)$ matritsaning nol ildizga mos keluvchi xos vektori $\eta_0(\tau)-$ keyingi qadamda aniqlanadigan noldan farqli funksiya.(2.5) ni (2.4) tenglamaga qo'yib

$$A_0 u_1 = -A_1 \varphi \eta_0(\tau) + f(\tau, \varphi \eta_0(\tau)) \qquad (2.7)$$

tenglamani hosil qilamiz. (2.7) tenglama yechimga ega bo'ladi, qachonki uning o'ng tomoni $\psi(\tau)$ vektorga ortogonal bo'lsa

$$\left((f(\tau, \varphi \eta_0(\tau)) - A_1 \varphi \eta_0(\tau)) + \varphi\right) = 0 \qquad (2.8)$$

Bu tenglamani

$$F(\tau, \eta_0(\tau)) = 0 \qquad (2.9)$$

ko'rinishda yozamiz. Natijada no'malum $\eta_0(\tau)$ funksiyaga nisbatan oshkormas funksiyaga ega bo'ldik. Bu funksiya [14] ishga asosan hamma vaqt $\eta_0(\tau)$ funksyaga nisbatan yechijadi.Demak $\eta_0(\tau)$ noma'lum funksiya aniq, (2.7) tenglama uchun yechim mavjud bo'lish sharti bajarildi.U holda (2.7) tenglamadan vektor funksiyani aniqlaymiz:

$$u_1(\tau) = A_0^+(\tau)\left[f(\tau, \varphi \eta_0(\tau)) - A_1 \varphi \eta_0(\tau)\right] + \varphi \eta_1(\tau) \qquad (2.10)$$

bunda $\eta_1(\tau)$ keyingi qadamda aniqlanadigan noma'lum funksiya $A_0^+(\tau)$, $A_0(\tau)$ matritsaning umulashgan teskari matritsasi

$$A_0^+(\tau) = \left[A_0(\tau) + (\psi \otimes \varphi)\right]^{-1} - (\varphi \otimes \psi) \qquad (2.11)$$

($\otimes -$ tenzor ko'paytma belgisi)

S =2 bo'lganda (2.5) tenglamadan

$$A_0 u_2 = \left[f_u(\tau) - A_1(\tau)\right] u_1(\tau) + f_1(\tau) \qquad (2.12)$$

Tenglamaga ega bo'lamiz. Bu tenglamani (2.10) tenglamani e'tiborga olib

$$A_0u_2 = \left[f_u(\tau) - A_1(\tau)\right]\varphi\eta_1(\tau) + f_1(\tau), \qquad (2.12)$$

ko'rinishda yozamiz, bunda

$$f_1(\tau) = \left[f_u(\tau) - A_1(u)\right]A_0^+(\tau)[+f(\tau,\varphi\eta_0(\tau)) - A_1\varphi\eta_0] + f_1(\tau).$$

(2.12) bir jinsli bo'lmagan tenglama uchun yechim bo'lish sharti

$$([f_u(\tau) - A_1(\tau)]\varphi,\psi)\eta_1(\tau) + (f_1(\tau),\psi) = 0 \qquad (2.13)$$

ko'rinishga ega. Teoremaning shartiga asosan [*0,L*] kesmada

$$(f_u(\tau)\varphi,\psi) \neq (A_1\varphi,\psi) \qquad (2.14)$$

bo'ladi. U holda (2.13) tenglamadan noma'lum $\eta_1(\tau)$ funksiyani quyidagicha aniqlaymiz:

$$\eta_1(\tau) = \frac{(f_1(\tau),\psi)}{(A,\varphi,\psi) - (f_u(\tau)\varphi,\psi)} \qquad (2.15)$$

(2.12) tenglama uchun yechim mavjud bo'lish sharti bajarilganligi uchun undan noma'lum $u_2(\tau)$ vektorni aniqlaymiz:

$$u_2(\tau) = A_0^+(\tau)[f_u(\tau)\psi\eta_1(\tau) + f_1(\tau)] + \varphi\eta_2(\tau) \qquad (2.16)$$

Bunda $\eta_2(\tau)$ keyingi qadamda aniqlanadigan noma'lum funksiya. Shu jarayonni davom ettirib (2.5) tenglamada $u_{S-1}(\tau)$ vektor va $\eta_{S-2}(\tau)$ noma'lum funksiyalar aniqlangan deb $U_S(\tau)$ vektorni aniqlash uchun

$$A_0u_S = \left[f_u(\tau) - A_1(\tau)\right]\varphi\eta_{S-1}(\tau) + b_S, \quad S = 3,4..., \qquad (2.17)$$

Tenglamani hosil qilamiz. Bu tenglama uchun yechim bo'lish sharti

$$((f_u(\tau) - A_1(\tau)]\varphi,\psi)\eta_{S-1}(\tau) + (b_S,\psi) = 0 \qquad (2.18)$$

ko'rinishga ega bo'ladi. (2.14) shartni e'tiborga olib (2.18) tenglikdan $\eta_{S-1}(\tau)$ funksiyani aniqlaymiz:

$$\eta_{S-1}(\tau) = \frac{(b_S, \psi)}{([f_u(\tau) - (A_1(\tau)]\varphi, \psi)} \quad S = 3,4..., \tag{2.19}$$

U holda (2.17) tenglamadan (2.18) shartni bajarilganligini e'tiborga olib $u_S(\tau)$ vektorni

$$u_2(\tau) = A_0^+(\tau)[f_u(\tau)\psi\eta_{S-1}(\tau) + b_S(\tau)] + \varphi\eta_S(\tau), \; S = 3,4..., \tag{2.20}$$

ko'rinishda aniqlaymiz.

Yuqorida bajarilgan algarotim yordamida (2.1) ko'rinishning istalgan noma'lum hadini aniqlash mumkin. Teorem isbotlandi.

Endi (1.3) tenglamaning ildizlaridan ***p*** tasi nolga teng bo'lib qolganlari noldan farqli bir biriga teng bo'lmagan holni qaraymiz.

2.2- Teorema. Agar 2.1 teoremaning shartlari bajarilib (1.3) tenglamaning ildizlari

$$\lambda_i(\tau) \equiv 0, \; \lambda_i = \overline{1,p}, \; \lambda_i(\tau) \neq \lambda_j(\tau) \; i \neq j, \; j = p+1,...,n. \tag{2.21}$$

Shartni qanoatlantiradi. U holda (1.1) sistema (2.1) ko'rinishdagi formal xususiy yechimga ega bo'ladi.

Isbot. Bu teoremani isbot qilish uchun (2.3)-(2.5) tenglamalaridan foydalanamiz. (2.3) tenglamadan [12] ishga asosan $u_0(\tau)$ noma'lum vektorni

$$u_0(\tau) = \sum_{i=1}^{p} \varphi_i \eta_{0i}(\tau) \tag{2.22}$$

ko'rinishda aniqlaymiz, bunda $\varphi_i \in N(A_0(\tau))\; \eta_{0i}(\tau) - i = \overline{1,p}$ formulalar noldan farqli bo'lgan noma'lum funksiyalar bo'lib keyingi qadamda aniqlanadi. U holda (2.4) tenglama (2.22) tenglamaga asosan

$$A_0 u_1 = -A_1 \sum_{i=1}^{p} \varphi_i \eta_{io}(\tau) + f(\tau, \sum_{i=1}^{p} \varphi_i \eta_{oi}(\tau)) \qquad (2.23)$$

ko'rinishga keladi. Bu jinsli bo'lmagan tenglama bo'lgani uchun uning yechimi o'ng tomondagi yig'indining $\psi_i (i = \overline{1, p})$ vektorga skalyar ko'paytmasi ortoganal bo'lishi zarur va yetarlidir; ya'ni $\forall \tau \in [o, L]$ uchun

$$((-A_1 \sum_{i=1}^{p} \varphi_i \eta_{io}(\tau) + f(\tau, \sum_{i=1}^{p} \varphi_i \eta_{oi}(\tau)), \psi_i) = 0 \qquad (2.24)$$

tenglik o'rinli.

(2.24) tenglamani

$$F(\tau, \eta_{oi}(\tau)) = 0, \quad i = \overline{1, p} \qquad (2.25)$$

ko'rinishda yozamiz. Natijada noma'lum $\eta_{oi}(\tau)(i = \overline{1, p})$ funksiyalar uchun oshkormas funksiyalar sistemasini hosil qildik. [14] ishga asosan bu tenglik yechimga ega., undan $\eta_{oi}(\tau)(i = \overline{1, p})$ funksiyalar aniqlanadi. Demak (2.23)-tenglama uchun (2.24) ko'rinishdagi yechimmavjud bo'lish sharti o'rinli, u holda (2.23) tenglamadan noma'lum vektorni quyidagicha aniqlaymiz:

$$u_1(\tau) = A_0^+(\tau) \left[f(\tau), \sum_{i}^{p} \varphi_i \eta_{oi}(\tau) \right] + \sum_{i=1}^{p} \varphi_i \eta_{1i}(\tau) \qquad (2.26)$$

bunda $\eta_{1i}(\tau)(i = \overline{1, p})$ keyingi qadamda aniqlanadigan noma'lum funksiyalar, $A_0^+(\tau)$ umumlashgan teskari matretsa

$$A_0^+(\tau) = \left[A_0^+(\tau) + (\psi_j \otimes \sum_{i=1}^{p} \varphi_i) \right]^{-1} - \left(\sum_{i=1}^{p} \varphi_i \otimes \psi_j \right) (j = \overline{1, p}) \qquad (2.27)$$

ko'rinishga ega.

s=1 bo'lganda (2.5) tenglamadan hosil bo'ladigan tenglamani (2.26)-tenglamani etiborga olib

$$A_0 u_2 = [f_u(\tau) - A_1(\tau)]\sum_{i=1}^{p}\varphi_i \eta_{1i}(\tau) + b_0(\tau) \qquad (2.28)$$

ko'rinishda yozamiz, bunda

$$b_0(\tau) = f_1(\tau) - A_2 u_0$$

(2.28) tenglama uchun yechim mavjud bo'lish sharti $\forall\tau \in [0, L]$ uchun

$$(([f_u(\tau) - A_1(\tau)]\sum_{i=1}^{p}\varphi_i \eta_{oi}(\tau) + b_0(\tau)), \psi_j) = 0 \;\; (j = \overline{1, p}) \qquad (2.29)$$

ko'rinishda bo'ladi.
(2.29) tenglamaning skalyar ko'paytmani bajarib

$$\Pi(\tau) h_0(\tau) = q(\tau) \qquad (2.30)$$

ko'rinishda yozamiz, bunda

$$\Pi(\tau) = \begin{pmatrix} (C_0(\tau)\varphi_1, \psi_1) & (C_0(\tau)\varphi_1, \psi_2) & ,..., & (C_0(\tau)\varphi_1, \psi_k) \\ (C_0(\tau)\varphi_2, \psi_1) & (C_0(\tau)\varphi_2, \psi_2) & ,..., & (C_0(\tau)\varphi_2, \psi_k) \\ ... & ... & ... & ... \\ (C_0(\tau)\varphi_k, \psi_1) & (C_0(\tau)\varphi_k, \psi_2) & ,..., & (C_0(\tau)\varphi_k, \psi_k) \end{pmatrix},$$

$$h_0(\tau) = \begin{pmatrix} \eta_{11}(\tau) \\ \eta_{12}(\tau) \\ \vdots \\ \eta_{1k}(\tau) \end{pmatrix}, \quad q(\tau) = \begin{pmatrix} (b_0(\tau), \psi_1) \\ (b_0(\tau), \psi_2) \\ \vdots \\ (b_0(\tau), \psi_k) \end{pmatrix}.$$

$$C_0(\tau) = f_u(\tau) - A_1(\tau).$$

(2.14)-shartga asosan $\forall \tau \in [0, L]$ Shuning uchun ***det*** $\Pi(\tau) \neq 0$.Shuning uchun $[0, L]$ kesmada unga teskari bo'lgan $\Pi^{-1}(\tau)$ matretsa mavjud. U holda (2.30) tenglamadan noma'lium $h_0(\tau)$ vektor aniqlanadi:

$$h_0(\tau) = \Pi^{-1}(\tau)q(\tau). \tag{2.31}$$

Shu bilan $\eta_{1i}(\tau)$ $(i = \overline{1.p})$ noma'lum funksiyalar aniqlandi. (2.28) tenglama uchun yechim bo'lish sharti bajarilganligini etiborga olib undan

$$u_2(\tau) = A_0^+(\tau)\left[f_u(\tau)\sum_{i=1}^{p}\varphi_i\eta_{1i}(\tau) + b_0(\tau)\right] + \sum_{i=1}^{p}\varphi_i\eta_{2i}(\tau) \tag{2.32}$$

yechimni aniqlaymiz, bunda $\eta_{2i}(\tau)$ yuqoridagidek keyingi qadamda aniqlanadigan noma'lum funksiyalar.

Shu jarayonni davom ettirib $u_{S-1}(\tau)$ vektor bu $\eta_{S-2i}(\tau)$ $i = \overline{1, p}$ funksiyalar ma'lum qiymatga ega deb (2.5) tenglamani

$$A_0 u_S = C_0(\tau)\sum_{i=1}^{p}\varphi_i\eta_{S-1i}(\tau) + b_S, \quad S = 2,3,... \tag{2.33}$$

ko'rinishda yozamiz. Bu tenglama uchun [*0*,*L*] kesmada yechim mavjud bo'lish sharti

$$((C_0(\tau)\sum_{i=1}^{p}\varphi_i\eta_{S-1i}(\tau) + b_S), \psi_j) = 0 \; (j = \overline{1, p}) \tag{2.34}$$

ko'rinishga ega.

(2.34) tenglamadan noma'lum $\eta_{S-1i}(\tau)$ $(i = \overline{1.p})$
funksiyalar

$$\eta_{S-1i}(\tau) = \Pi^{-1}(\tau)q_2(\tau) \tag{2.35}$$

Tenglik orqali aniqlanadi, bunda

$$q_2(\tau)=\begin{pmatrix}(b_S(\tau),\psi_1)\\(b_S(\tau),\psi_2)\\\vdots\\(b_S(\tau),\psi_k)\end{pmatrix},\ S=2,3,...$$

(2.33) tenglamadan $u_S(\tau)$ vektorni

$$u_S(\tau)=A_0(\tau)\left[f_u(\tau)\sum_{i=1}^{p}\varphi_i\eta_{S-1i}(\tau)+b_S(\tau)\right]+\sum_{i=1}^{p}\varphi_i\eta_{Si}(\tau) \qquad (2.36)$$

ko'rinishda aniqlaymiz, bunda $\eta_{Si}(\tau)$ keyingi qadamda aniqlanadigan no'ma'lum funksiya.

3-§. Karrali ildizlar uchun yuqori tartibli differensial tenglamalar sistemasining xususiy yechimini tuzish.

Faraz qilaylik 1.1 sistemaning 1.3 tenglamasi bitta n karrali nol ildizga ega bo'lib unga shunday o'lchovli ildizlarning qism fazosi mos kelsin.

3.1. Teorema. Agar $A_0(\tau)$ matritsa n karrali xos qiymatga ega bo'lib 2.1-teoremaning shartlari bajarilsa, u holda (1.1) sistema

$$\chi(t_1,\varepsilon)=\sum_{S=0}^{\infty}\mu^S u_S(\tau),\ \mu=\sqrt[n]{\varepsilon} \qquad (3.1)$$

ko'rinishdagi yechimga ega bo'ladi.

Isbot. Bu teoremani isbot qilish uchun yuqoridagidek (3.1) tenglamani (1.1) sistemaga qo'yib noma'lum $U_S(\tau)$ $(S=0,1,...,)$ vektorlarni aniqlaymiz. (3.1) qatorni (1.1) sistemaga qo'yib va $f(\tau,U(\tau,\mu),\varepsilon)$ vektorni $(\tau,U_0(\tau),0)$ nuqta atrofida Teylor qatoriga yoyilmasini e'tiborga olib

$$\begin{aligned}&\mu^{nk}u^{(k)}(\tau,\mu)+A(\tau,\varepsilon)u(\tau,\mu)=\mu^n\{f(\tau,u_0(\tau))+\mu fu(\tau)u_1+\\&...+\mu^S[f_u(\tau)u_S(\tau)+f_{S-n}+f_{S-(n(+1)}(\tau)]+...\}\end{aligned}\quad S=2,3,... \qquad (3.2)$$

ayniyatga ega bo'lamiz, bunda $fu(\tau)=\dfrac{\partial f^i}{\partial u^j}$ matritsaning elementlari va $f_{S-n-1}(\tau)$ vektorning koordinatalari nuqtada hisoblanadi, $f_{S-1}(\tau)$ vektor $u_i(\tau)$ $i=1,2,...,S-1$ orqali ifodalanadi. (3.2) ayniyatdan $\mu^S(S=0,1,...,)$ parametrlar oldidagi koeffisentlarni tenglashtirib $u_0(\tau),u_1(\tau),...,u_S(\tau),...$ noma'lum vektorlarni aniqlash uchun quyidagi algebraic tenglamalar sistemasini hosil qilamiz.

$$A_0(\tau)u_0(\tau)=0, \tag{3.3}$$

$$A_0(\tau)u_q(\tau)=0, \; q=\overline{1,n-1} \tag{3.4}$$

$$A_0(\tau)u_n(\tau)=-A_1(\tau)u_0(\tau)+f(\tau,u_0(\tau)), \tag{3.5}$$

. . . .

$$A_0(\tau)u_S(\tau)=\left[\left[f_u(\tau)-A_1(\tau)\right]u_{S-n}(\tau)\right]+b(\tau), \; S=n+1,n+2,..., \tag{3.6}$$

bunda

$$b_S(\tau)=f_{S-n-1}(\tau)+f_{S-2n-1}-\sum_{i=2}^{\left[\frac{S}{n}\right]}A_i(\tau)u_{S-ni}(\tau)\frac{(k)}{S-n(k-0)}u_{S-nk}^{(k)}(\tau)\left[\frac{S}{n}\right],$$

$\dfrac{S}{n}$ sonning butun qismi.

(3.3), (3.4) tenglamalardan $u_0(\tau)$ va $u_q(\tau)$ vektorlarni aniqlaymiz:

$$\begin{cases} u_0(\tau)=\varphi_1\eta_0(\tau); \\ u_q(\tau)=\varphi_1\eta_q(\tau), q=\overline{1,(n-1)}, \end{cases} \tag{3.7}$$

bunda $\eta_0(\tau),\eta_q(\tau)$ $q=\overline{1,(n-1)}$ noldan farqli noma'lum funksiyalar keyingi qadamlarda aniqlanadi $\varphi_1\in N(A_0(\tau))$ birinchi koordinatasi birga teng bo'lib qolgan koordinatalari nolga teng bo'lgan vector. (3.5) tenglamani (3.7) sistemaning birinchi tenglamasiga asosan

$$A_0(\tau)u_n(\tau) = -A_1\varphi_1\eta_0(\tau) + f(\tau,\varphi_1\eta_0(\tau)), \qquad (3.8)$$

ko'rinishda yozamiz. Bu tenglama uchun yechim mavjud bo'lish sharti

$$\big((f(\tau,\varphi_1\eta_0(\tau)\big) - A_1\psi_1\eta_0(\tau))\psi_1) = 0 \qquad (3.9)$$

ko'rinishga ega, bunda $\psi_1 \in N(A_0^*(\tau))$ oxirgi koordinatasi birga teng bo'lib boshqa koordinatalari nolga teng bo'lgan n-o'lchovli vector.

(3.9) tenglikni

$$\big((f(\tau,\varphi_1\eta_0(\tau)\big),\psi_1 - (A_1\varphi_1\psi_1)\eta_0(\tau)) = 0$$

yoki

$$\big(F(\tau,\varphi_1\eta_0(\tau)\big) = 0 \qquad (3.10)$$

ko'rinishda yozamiz. (3.10)- tenglama noma'lum $\eta_0(\tau)$ funksiyaga nisbatan oshkormas funksiyani ifodalaydi.

Bunday funksiyalar [10] ishga asosan hamma vaqt $\eta_0(\tau)$ funksiyani aniqlash mumkin. Demak (3.8) tenglama uchun yechim mavjud bo'lish sharti o'rinli, u holdan undan no;malum $u_n(\tau)$ vektorni

$$u_n(\tau) = A_0^+(\tau) * f(\tau,\varphi_1\eta_0(\tau)) + \varphi_1\eta_n(\tau) \qquad (3.11)$$

Tenglama orqali aniqlaymiz, bunda $\eta_n(\tau)$, $[0,L]$ kesmada noldan farqli bo'lgan noma'lum funksiya, keyingi qadamlarda aniqlanadi.

S=n+1 bo'lganda (3.6) tenglamadan

$$A_0u_{n+1} = \left[f_n(\tau) - A_1(\tau)\right]u_1(\tau) + f_0(\tau) \qquad (3.12)$$

tenglamaga ega bo'lamiz. Bu tenglamani (3.7) tenglikka asosan q=1 bo'lganda

$$A_0 u_{n+1} = \left[f_n(\tau) - A_1(\tau) \right] \varphi_1 \eta_1(\tau) + f_0(\tau), \tag{3.13}$$

ko'rinishda yozamiz, (3.13) tenglama uchun yechim mavjud bo'lish sharti

$$(\left[f_n(\tau) - A_1(\tau) \right] \varphi_1, \psi_1) \eta_1(\tau) + (f_0(\tau), \psi_1) = 0 \tag{3.14}$$

ko'rinishga ega.

Bu tenglamadan $\eta_1(\tau)$ noma'lum funksiyani aniqlaymiz.

$$\eta_1(\tau) = \frac{(f_0(\tau), \psi_1)}{((f_u(\tau) - A_1(\tau))\varphi_1, \psi_1)} \tag{3.15}$$

(3.13) tenglama uchun (3.14) ko'rinishdagi yechimmavjud bo'lish shartini bajarilishini etiborga olib noma'lum $u_{n+1}(\tau)$ vektorni aniqlaymiz

$$u_{n+1}(\tau) = A_0^+(\tau) \left[f_u(\tau) \varphi_1 \eta_1(\tau) + f_0(\tau) \right] + \varphi_1 \eta_{n+1}(\tau), \tag{3.16}$$

bunda $\eta_{n+1}(\tau)$ keyingi qadamlarda aniqlanadigan noma'lum funksiya.

Shu jarayonni davom ettirib (3.6) tenglamani (3.8), (3.12) tenglamarga o'xshash.

$$A_0 u_S = \left[f_u(\tau) - A_1(\tau) \right] \varphi_1 \eta_{S-n}(\tau) + b_S(\tau) \quad S = n+2, n+3, ... \tag{3.17}$$

ko'rinishda yozamiz. Bu tenglama uchun yechim mavjud bo'lish sharti

$$(\left[f_u(\tau) - A_1(\tau) \right] \varphi_1, \psi_1) \eta_{S-n}(\tau) + (b_S(\tau), \psi_1)) = 0 \quad S = n+2, n+3, ... \tag{3.18}$$

ko'rinishda bo'ladi. (3.12) shartni noma'lum $\eta_{S-n}(\tau)$ ($S = n+2, n+3,...$) funksiyani

$$\eta_{S-n}(\tau) = \frac{(b_S, \psi_1)}{([A_1(\tau) - f_u(\tau)]\varphi_1, \psi_1)}, \quad S = n+2, n+3,... \tag{3.19}$$

ko'rinishda aniqlaymiz. U holda (3.17) tenglamadan noma'lum $U_S(\tau)$ vector quyidagicha aniqlanadi:

$$u_S(\tau) = A_0^+(\tau)\left[f_u(\tau)\varphi_1\eta_{S-n}(\tau) + b_S(\tau)\right] + \varphi_1\eta_S(\tau) \quad S = n+2, n+3,... \tag{3.20}$$

bunda $\eta_S(\tau)$ keyingiqadamda aniqlanadigan noma'lum funksiya.

Demak keltirilgan algaretm yordamida (3.1) qatorning istalgan hadlarini aniqlash mumkin.

Teorema isbotlandi.

4-§. Formal yechimlarning asimptotek xarakteri.

Bu paragrifda tuzilgan (2.1) yechimning Krilov- Bogolyubov ma'nosidagi asimptotik [arakterga ega ekanligini isbotlaymiz. Buning uchun avvalo (1.1)deffirenseal tenglamalar sistemasini birinchi tartibli tanglamalar sistemasiga keltirib olamiz. Buning uchun (1.1) sistemada

$$x = Z_1, \ \frac{dx}{dt} = Z_2, \ \ldots, \ \frac{d^{k-1}x}{dt^{k-1}} = Z_k \tag{4.1}$$

almashtirib olib,

$$\frac{dz}{dt} + \tilde{A}(\tau, \varepsilon) = \varepsilon \tilde{f}(\tau, z, \varepsilon) \qquad (4.2)$$

Ko'rinishdagi birinchi tartibli chiziqli bo'lmagan differensial tenglamalar sistemasiga ega bo'lamiz, bunda (nkxnk)- o'lchovli matretsa $\tilde{A}(\tau,\varepsilon)$ va nk-o'lchovli $f(\tau, z, \varepsilon)$ vektor

$$\tilde{A}(\tau,\varepsilon) = \begin{pmatrix} 0 & E & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ A(\tau,\varepsilon) & 0 & 0 & \dots & 0 \end{pmatrix}, \qquad f(\tau,z,\varepsilon) = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ f(\tau,z,\varepsilon) \end{pmatrix}$$

Ko'rinishga ega, $z = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_k \end{pmatrix}$, 0-nol; E-birlik (nxn) ko'rinishdagi matretsalar.

(4.2)-tenglamaning xarakterestek tenglamasi

$$\det\left\| \tilde{A}_0(\tau) + \omega(\tau)E \right\| = 0 \qquad (4.3)$$

nk ta ildizga ega bo'ladi. U holda (2.1) yechim quyidagi ko'rinishga ega

$$z(t_1\varepsilon) = \sum_{S=0}^{\infty} \varepsilon^S u_S(\tau), \qquad (4.4)$$

bunda $u(\tau,\varepsilon)$

(4.2) tenglamaning (4.4) yechimi asimptotik xarakterga ega ekanligini ko'rsatamiz. (4.2) tenglamani m – yaqinlashuvchi yechimini

$$z_m(\tau,\varepsilon)\sum_{S=0}^{m}\varepsilon^s\tilde{u}(\tau,\varepsilon) \qquad (4.5)$$

ko’rinishda olamiz.

(4.2) sistemaning aniq yechimi bilan m - yaqinlanuvchi yechim orasidagi farqni baholash uchun dastlab quyidagi lemmani keltiramiz.

4.1 Lemma. Agar 2.1-teoremaning shartlari bajarilsa va (4.3) tenglama oddiy ildizlarga ega bo’lsa, u xolda m-yaqinlashuvchi yechim

$$\frac{dzm}{dt}+\tilde{A}(\tau,\varepsilon)z_m=\varepsilon\tilde{f}(\tau,z_m,\varepsilon)+\varepsilon^{m+1}R_m(\tau,\varepsilon), \qquad (4.6)$$

tenglamani qanoatlantiradi, bunda $R_m(\tau,\varepsilon)$, $[0,L]$ kesmada chegaralangan vector - funksiya.

Bu lemmani isboti ishda keltirilganligi sababli isbotga to’xtalmaymiz.

Faraz qilaylik $v(t,\varepsilon)$

$$\frac{dv}{dt}=\tilde{A}_0(\tau)V, \qquad Vt,\varepsilon|_{t=0}=V_0(\varepsilon) \qquad (4.7)$$

masalaning yechimi bo’lib , $\forall\tau\epsilon[0,L]$ uchun

$$\|V(t,\varepsilon)\|\leq M \qquad (4.8)$$

tengsizlikni qanoatlantirsin. Endi (4.4) yechimining asimptotik xarakterga ega ekanligini ko'rsatuvchi quyidagi teoremani isbotlaymiz.

4.1- Teorema. Agar (4.2) sistema uchun quyidagi shartlar bajarilsa;

1) 2.1- teoremaning shartlari bajarilsa;
2) $\tilde{f}(x, z, \varepsilon)$ vektor- funksiya uchun ℓ o'zgarmas bo'yicha Lipshis sharti o'rinli bo'lsa;

$$\|\tilde{f}(\tau, z, \varepsilon) - \tilde{f}(\tau, z_m, \varepsilon)\| = \ell \|z - z_m\|; \quad (4.9)$$

3) (4.8) tengsizlik bajarilsa va shuningdek

$$z(t_1, \varepsilon) |_{(t=0)} = z_m(t, \varepsilon) |_{(t=0)} \quad (4.10)$$

tenglik o'rinli bo'lsa. U holda har qanday katta $L > 0$ va ε ga bog'liq bo'lgan o'zgarmas $C > 0$ uchun shunday kichik $\varepsilon \epsilon (0, \varepsilon_0]$ ni mos qo'yish mumkinki oraliqda

$$\| z(t_1 \varepsilon) - z_m(t_1 \varepsilon) \| \leq C \varepsilon^m \quad (4.11)$$

tengsizlik o'rinli bo'ladi.
Isbot.(4.2) sistemadan (4.6) sistemani ayirib quyidagi tenglamani hosil qilamiz:

$$\frac{dy}{dt} = -\tilde{A}_0(\tau)y - \varepsilon \tilde{A}_1(\tau, \varepsilon)y + \varepsilon\left[\tilde{f}(\tau, z, \varepsilon) - \tilde{f}(\tau, z_m, \varepsilon)\right] - \varepsilon^{m+1} R_m(\tau, \varepsilon) \quad (4,12)$$

bunda

$$y(t, \varepsilon) = z(t, \varepsilon) - z_m(t, \varepsilon), \quad (4.13)$$

$$\tilde{A}_1(\tau, \varepsilon) = \sum_{S=1}^{m} \varepsilon^S \tilde{A}_S(\tau, \varepsilon)$$

(4.14)

(4.12) tenglama quyidagi integral tenglamaga ekvivalent

$$= -\varepsilon \int_0^t v$$

$y(t,\varepsilon)$ $(t,\varepsilon)v^{\uparrow}(-1)$ $[(t]_{\downarrow}1, \varepsilon) F_1(\tau_1,\varepsilon) y(t,\varepsilon) dt_1 + \varepsilon \int_0^t v (t,\varepsilon) v^{-1}$
$(t_1,\varepsilon)[\tilde{f}(\tau, z, \varepsilon) -$

$$\tilde{f}(\tau, z z_1 m, \varepsilon \vdash)] \, dt + \varepsilon^{m+1} \int_0^t v(t\varepsilon) v^{-1}(t_1,\varepsilon) R_m(\tau,\varepsilon) dt \qquad (4.15)$$

bu tenglamadan norma olib

$$\|y(t,\varepsilon)\| \varepsilon \int_0^t \| v(t,\varepsilon) \| \, \|v^{-1}(t_1,\varepsilon)\| \, \|F_1(\tau_1,\varepsilon)\| + \varepsilon \int_0^t \| v(t,\varepsilon) \| \, \|v^{-1}(t_1,\varepsilon)\| \qquad (4.16)$$

tengsizlikka ega bo'lamiz, bunda $\forall \tau \in [0, L]$ uchun

$$\| F_1(\tau,\varepsilon) \| \le N, \ \| R_m(\tau,\varepsilon) \| \le R. \qquad (4.17)$$

(4.16) tengsizlikka Gronuolla – Bellman tengsizligini qo'llab

$$\| y(t,\varepsilon) \| \le z z(t,\varepsilon) - z z_m(t,\varepsilon) \le C \varepsilon^m, \qquad (4.18)$$

tengsizlikni hosil qilamiz, bunda

$C = MLR_* \exp(N + \ell M) L$

$z(t,\varepsilon)$ vektorning $z(t,\varepsilon) = colon\left(x; \frac{dx}{dt}; \dots; \frac{d^{(k-1)}}{dt^k}\right)$ ekanligini e'tiborga olib (1.1) sistemaning aniq yechimi $x(t,\varepsilon)$ bilan m-chi yaqinlashuvchi $x_m(t_1\varepsilon)$ orasidagi farq

$$\|x(t,\varepsilon) - x_m(t,\varepsilon)\| \le C\varepsilon^m$$

$$\left\| \frac{d^r x(t,\varepsilon)}{dt^r} - \frac{d^r x_m(t,\varepsilon)}{dt^r} \right\| \le C\varepsilon^m, r = \overline{1, k-1} \tag{4.19}$$

tengsizlikni qanoatlantirishiga ishonch hosil qilamiz. Teorema isbotlandi.

II-bob.Yuqori tartibli hosila oldida parametr qatnashgan differensial tenglamalar sistemasining yaqinlashuvchi yechimlari:

2.1. Kasr rangli sekin o'zgaruvchi koeffitsiyentli differensial tenglamalar sistemaning formal xususiy yechimlari.

Ushbu punktda yuqori tartibli hosila oldida kichikparametr kasr darajasi bilan qatnashga k-tartibli chiziqli bo'lmagan differensial tenglamalar sistemasining assimptotik yechimlarini tuzish masalasi bilan tanishamiz. Bunday tenglamalarga singulyar tenglamalar deyiladi. Kasr rangli sekin o'zgaruvchi koeffitsiyentli chiziqli differensional tenglamalar sistemasining assimptotik yechimlarini tuzish masalasi bilan ShkilA.I va uning o'quvchilari shug'ullangan. Kasr rangli birinchi va ikkinchi tartibli chiziqli bo'lmagan differensial tenglamalar sistemasining assimptotik yechimlarini tuzish masalasi [9,12] ishlarida yoritilgan.

Ushbu bobda

$$\varepsilon^{p/q}\frac{d^k x}{dt^k}+A(\tau,\varepsilon)x=\varepsilon f(\tau,x,\varepsilon) \qquad (2.1)$$

ko'rinishidegi diffetsional tenglamalar sistemasini assimptotik yechimlarini tuzish masalasi bilan tanishamish bunda A(τ,ε)-n tartibli kvadrat matritsa (1.2)qatorga yoyiladi, $f(\tau,x,\varepsilon$)-n- o'lchovli vektor (τ,x_0,v)nuqta atrofida Teylar qatoriga yoyiladi, p va qbutun sonlar (p,q)=1 shartni qanoatlantiradi.

Yuqorida qayd etganimizdek (2.1) sistemaning yechimi uning

$$\det[A_0(\tau)+\lambda E]=0 \qquad (2.2)$$

xarakteristik tenglamasining $\lambda_1(\tau),\lambda_2(\tau),\dots,\lambda_n(\tau)$ ildizlariga bog'liq. Xarakteristik tenglamaning ildizlaridan bittasi nol,qolganlari noldan farqli, r tasi
(r< n) nol qolganlari noldan farqli va n-ta nol qarorli bo'lgan kritik hollar uchun asimptotik yechimlari tuzish masalasiga to'xtalamiz.

Yuqoridagi tanishganimizdek[12] ishga asosan faqat nol ildizlarga mos keluvchi asimptotik yechimlarini tuzish masalasiga to'xtalamiz. Noldan farqli ildizlarga mos keluvchi yechimlari [9,13]ishga asosan tuziladi:

Faraz qilaylik (2.2) tenglamaning ildizlari.

$$\lambda_1^{(\alpha)}(\tau) \equiv 0, \qquad \lambda_1^{(\tau)}(\tau) \neq 0, \qquad \alpha = \overline{2,k}, \ \lambda_j^{(k)}(\tau) \neq 0, \qquad j = \overline{2,n} \qquad (2.3)$$

shartni qanoatlantirsin.

2.1 Teorema. Agar (2.1) tenglamani $A_1s\ (\tau)\ (S = 0,1,\dots)$koefistientlari τ erkli o'zgaruvchi bo'yicha [0.L] kesmada, $f(\tau, x, \varepsilon)$ vektor P(τ, x, ε) = P($\tau, x,$)x $< \varepsilon \leq \varepsilon_0$

x(0) sohada cheksiz differensionallanuvchi bo'lsa, u holda (2.1) sistema
[0.4] kesmada

$$\mathbf{X}(t_1\varepsilon) = \sum_{s=0}^{\infty} \mu^s u_s(\tau), \mu = \sqrt[k]{\varepsilon} \qquad (2.4)$$

ko'rinishidagi farmal xususiy yechimga ega bo'ladi.

Isbot. (2.4) qatorni (2.1) sistemaga qo'yish uchun □$\mu = \sqrt[kq]{\varepsilon}$ deb olib, uni k-chi tartibli xosilasini olib va $f(r, u, (\tau, \mu), \mu^{nq})$ vektorni ($\tau, u_0, (\tau)_0$)
nuqta atrofidagi Teylor yoyilmasini e'tiborga olib quyidagi ayniyatga ega bo'lamiz:

$$\mu^{nq} u^{(k)}(\tau,\mu) + A(\tau,\mu^{nq})u(\tau,\mu) = \mu^{nq}\ \{f(r, u_{10}\ (\tau)) +$$

$$+\mu f_u(\tau)u_1(\tau) + \ \dots + \mu^s\ [f_u(\tau)u_s(\tau) + f_{s-1}(\tau) +$$

$$\tilde{f}_{s+1-q}(\tau)] + \dots\}, \ s=q,\ q+1\dots, \qquad (2.5)$$

Bunda $f_u(\tau) = \dfrac{\partial^i f}{\partial u^j}$ matritsaning elementlari va $\tilde{f}_{s+1-q}(\tau)$vektorning koordinatalari ($\tau, u_0, (\tau), 0$) nuqtada hisoblanadi, $f_{s-1}(\tau)$vektor $u_i(\tau)(i = 1,2,\dots,s-1)$ orqali ifodalanadi.

Agar (2.5) tenglamadan$\mu^s(s = 0,1,\dots)$ parametrlarni oldidagi koeffisientlarnitenglashtirsak p>q bo'lgan hol uchun

$u(\tau), u(\tau), \dots u_n(\tau), \dots$ no’malum vektorlarni aniqlash uchun

$$A_0(\tau)u_0(\tau) = 0;$$
$$A_0(\tau)U_r(\tau) = 0, r = \overline{1, q-1} \quad ; \qquad (2.6)$$

$$A_0(\tau)u_q(\tau) = -A_1 u_0 + f(\tau, u_0); \qquad (2.7)$$

$$A_0(\tau)u(\tau) = -A_1 u + f_u(\tau)u_1; \qquad (2.8)$$

$$A_0(\tau)u_{q+2}(\tau) = -A_1 u_2 + f_u(\tau)u_2 + f_1(\tau); \qquad (2.9)$$

…………………………………………………………..

$$A_0(\tau)u_s(\tau) = -A_1 u_{s-q} + f_u(\tau)u_{s-q} + b_s(\tau), s = q+3, \dots \qquad (2.10)$$

algebraik tinglamalar sistemasini hosil qilamiz.
bunda

$$b_s(\tau) = -\sum_{i=2}^{\left[\frac{s}{q}\right]} A_i u_{s-iq} f_{s-k}(i) + \tilde{f}_{s-kq}(\tau) - u\,\frac{k(\tau)}{s-kp}\, s = q+3, q+4 \dots$$

(2.6) sistemadan

$$-u_r(\tau) = \varphi\eta(\tau), \qquad r = \overline{0, q}\text{-}1 \qquad (2.11)$$

Yechimlarnianiqlaymiz,bunda$\eta_r(\tau)(r = \overline{0, q-1})$, [0.L]kesmadanoldanfarqlibo’lganix tiyoriynoma’lumfunksiyalar keyingiqadamlardaaniqlanadi, $\varphi, A_0(\tau)$tenglamaningnolildizigamosxosvektori, uningbirinchikoordintasibirgatengqolganlarinolgatengbo’lganvektor
(2.7) tenglama(2.11) tenglikkaasosan

$$A_0 u_q(\tau) = -A_0 \varphi \eta_0(\tau) + f\left(\tau, \varphi \eta_0(\tau)\right) \quad (2.12)$$

ko'rinishigaega (2.12) tenglamabirjinslibo'lmagani uchun uning yechimini bo'lishi sharti

$$((-A_1(\tau)\varphi\eta_1(0)(\tau) + f(\tau, \varphi\eta_1(0)(\tau)), \psi) = 0 \quad (2.13)$$

dan iborat, bunda ψ vektor $A_0(\tau)$ matrissaning qo'shmasi bo'lgan $A_0^*(\tau)$ matrissaning xos qiymati bo'lib uning oxirgi koordinatasi birga, qolganlari nolga teng (2.13) tenglamani

$$((f(\tau, \varphi\eta_0(\tau), \psi) - (A_1(\tau)\varphi, \psi)\eta_0(\tau) = 0$$

yoki

$$F(\tau, \eta_0(\tau)) = 0 \quad (2.14)$$

ko'rinishida yozamiz.

Natijada noma'lum $\eta_0(\tau)$ funksiyasiga nisbatan oshkormas funksiya hosil qildik. So'ngi tenglama uchun oshkormas funksiya haqidagi [14] teoremaning barcha shartlari bajariladi,undan $\eta_0(\tau)$ funksiya aniqlanadi (2.12) tenglama uchun yechim mavjud bo'lish sharti bajarilgani uchun undan $U_0(i)$ vektorni

$$u_q(\tau) = A_0^+(\tau)[f(\tau, \varphi\eta_1(0)(\tau)] + \varphi\eta_q(\tau) \quad (2.15)$$

ko'rinishida aniqlaymiz, bunda $\eta_q(\tau)$ keyingi qadamda aniqlanadigan noma'lum funksiya, $A_0^+(\tau), A_0(\tau)$ matrissaning umumlashgan teskari matrissasi.

$$A_0^+(\tau) = [A_0^-(\tau) + (\psi \otimes \varphi]^{(-1)} - (\varphi \otimes \psi)$$

(2.8) tenglamani (2.11) tenglamaga asosan

$$A_0(\tau)u_{q+1}(\tau) = -A_1\varphi\eta_1(\tau) + f_u(\tau)\varphi, \eta_1(\tau) \quad (2.16)$$

ko'rinishidagi yozamiz. Bu tenglama uchun yechim mavjud bo'lishi sharti

$$[(f_u(\tau)\varphi,\psi)-(A,\varphi,\psi)]\eta_1(\tau)=0 \qquad (2.17)$$

tenglikdan iborat. Teoremaning shartiga asosan $\forall\tau\epsilon[0,L]$ uchun

$$\psi(f_u(\tau)\varphi,\psi\ )\neq A,\varphi,\psi \qquad (2.18)$$

bo'ladi,u holda (2.17) tenglamadan $\eta_1(\tau)$ noma'lum funksiyasini aniqlaymiz:

$$\eta_1(\tau)=0 \qquad (2.19)$$

u holda (2.16) tenglamani yechimi

$$u_{q+1}(\tau)=\varphi\eta_{q+1}(\tau) \qquad (2.20)$$

bo'ladi, bunda $\eta_{q+1}(\tau)$ keyingi qadamlarida aniqlanadigan noma'lum funksiya. shu jarayonni davom ettirib (2.10) tenglamani

$$A_0u_s(\tau)=-A,\varphi\eta_{s-iq}(\tau)+f_u(\tau)\varphi\eta_{s-iq}+b_s(\tau),s=q+2,q+3,\ldots \qquad (2.22)$$

ko'rinishida yozamiz.
(2.22) tenglamauchun yechim mavjud bo'lish sharti

$$[(f_u(\tau)\varphi,\psi)-(A,\varphi,\psi)]\eta_{s-iq}(\tau)+(b_s(\tau),\psi)=0,s=q+2,q+3,\ldots, \qquad (2.23)$$

ko'rinish bo'ladi. Bu tenglamada noma'lum $\eta_{s-iq}(\tau)$ funksiyasini (2.18) e'tiborga olib

$$\eta_{s-iq}(\tau)=\frac{(b_s(\tau),\psi)}{(A,\varphi,\psi)-(f_u(\tau),\varphi\psi)} \qquad (2.24)$$

ko'rinishida aniqlaymiz. U holda noma'lum vektor $u_s(\tau)$, (2.22) tenglamadan quyidagicha aniqlanadi:

$$u_s(\tau)=A_0^+(\tau)\left[f_u(\tau)\varphi\eta_{s-iq}+b_s(\tau)\right]+\ \varphi\eta_s(\tau) \qquad (2.25)$$

bunda $\eta_s(\tau)$ keying qadamda aniqlanadigan noma'lum funksiya.Teorema isbotlandi. Faraz qilaylik (2.1) sistemaning (2.2) xarakteristik tenglamasining ildizlari

$$\lambda_r(\tau) \equiv 0, r = 1,2,\dots,d, d < n, \lambda_i(\tau) \neq \lambda_j(\tau), i \neq j, i,j = \overline{d+1,n}; \qquad (2.26)$$

shartni qanoatlantirsin. Bu ildizlarga oddiy elementar bo'luvchilar mos kelsin. U holda quyidagi teorema o'rinli.

2.2. Teorema. Agar 2.1 –teoremaning shartlari bajarilib (2.26) munosabat o'rinli bo'lsa, u holda (2.1)tenglama (2.4) ko'rinishidagi formula xususiy yechimga ega bo'ladi.

Isbot. Bu teoremani isbot qilish uchun ham (2.4)qatorining (2.1) sistemani qanoatlantiruvchi noma'lum hadlarni aniqlashimiz kerak. U holda (2.4) qatorni (2.1) sistemaga qo'yib, (2.26) shartni e'tiborga olib va hosil qilingan natijadan μ parametrining bir xil darajalari oldidagi koeffisentilarini tenglaashtirib, (2.6)-(2.9) algebraik tenglamalar sistemasini hosil qilamiz.

(2.6) sistemasidan qaralayotgan hol uchun

$$u_\alpha(\tau) = \sum_{i=1}^{\alpha} \varphi_i \eta_{\alpha i} \quad \alpha = \overline{0, q-1}, \qquad (2.27)$$

Bunda η_1 (i=)keyingi qatorlarda aniqlanadigan noma'lum funksiyalar. (2.7) tenglikka asosan

$$A_0(\tau)u_q(\tau) = -A_1 \sum_{i=1}^{\alpha} \varphi_i \eta_{\alpha i}(\tau) + f\left(\tau, \sum_{i=1}^{\alpha} \varphi_i \eta_{\alpha i}(\tau)\right) \qquad (2.28)$$

ko'rinishida yozamiz. Bu tenglama uchun yechim mavjud bo'lishi sharti

$$\left(\left(f\left(\tau, \sum_{i=1}^{\alpha} \varphi_i \eta_{\alpha i}(\tau)\right) - A_1 \sum_{i=1}^{\alpha} \varphi_i \eta_{\alpha i} \tau\right), \Psi_j\right) = 0 \qquad (2.29)$$

ko’rinishiga ega bo’ladi. Bu tenglamani

$$f(\tau \sum_{i=1}^{\alpha} [\varphi_i \eta_{\alpha i}] (\tau)), \Psi_j) - ((A_1 \sum_{i=1}^{\rho} [\varphi_i), \Psi_j) \eta_{\alpha i}(\tau) = 0, j = \overline{(1, \alpha)} \qquad (2.30)$$

ko’rinishda yozamiz. (2.30) tenglamani

$$F(\tau, \eta_{\alpha i}(\tau)) = 0, i =$$

shaklida yozamiz. Natijada noma’lum $\eta_{\alpha i}(\tau)$, $\alpha = \overline{0, q-1}$, funksiyalarga nisbatan

oshkormas funksiyalar sistemasiga ega bo’ldik. ishga asosan bunday sistema yechimga ega,undan har bir aniq qolib uchun yechim mavjud bo’lish sharti o’rinli u holda undanda noma’lum $u_q(\tau)$ vektor quyidagicha aniqlanadi:

$$u_q(\tau) = A_0^+(\tau) f\left(\tau, \sum_{i=1}^{p} [\varphi_i] \eta_{\alpha i}(\tau)) + \sum_{i=1}^{p} \varphi_i] \eta_{\alpha i}(\tau)]\right. \qquad (2.31)$$

bunda $\eta_{\alpha i}(\tau)$(i=) keyingi qadamlarda aniqlanadigan noma’lum funksiya (2.28) tenglama (2.27) – tenglamaga asosan $\alpha = 1$ bo’lmaganda

$$A_0(\tau) u_{q+1}(\tau) = -A_1 \sum_{i=1}^{p} \varphi_i \eta_{1i}(\tau) + f_u(\tau) \sum_{i=1}^{p} \varphi_i \eta_{1i}(\tau) \qquad (2.32)$$

tenglamaga ega bo’lamiz. Bu tenglama uchun yechim mavjud bo’lishi sharti

$$\left[\left(\left(f_u(\tau)\sum_{i}^{p}[\![\varphi_i),\Psi_j-]\!]\left(A_1(\tau)\sum_{i=1}^{p}[\![\varphi_i),\Psi_j\right)]\!]\right)\right]\eta_i(\tau)=0, j=\overline{1,p} \qquad (2.33)$$

ifodadan iborat. 2.1-Teoremaning shartiga asosan $\forall\tau\epsilon[0,L]$ uchun

$$[\![(f]\!]\downarrow u\ (\tau)\ \Sigma\downarrow(i=1)\uparrow p\equiv\varphi_\downarrow i\ ).\Psi_\downarrow j\neq((A_\downarrow 1\ (\tau)\ )\ \Sigma\downarrow(i=1)\uparrow p\equiv\varphi_\downarrow i\ ).\Psi_\downarrow j).j=\overline{(1.q)}$$

munosabat o'rinli, u holda (2.33) tenglamadan

$$\eta_{1i}(\tau)=0,\ i= \qquad (2.35)$$

bo'lishini aniqlaymiz.(2.32) tenglama uchun yechim mavjud bo'lish shartini bajarilganligini e'tiborga olib undan noma'lum

$$u_{q+1}(\tau)=A_0^{+}(\tau)f_u(\tau)\sum_{i=1}^{p}\varphi_i\eta_{1i}(\tau)+\sum_{i=1}^{p}\varphi_i\eta_{q+1i}(\tau) \qquad (2.36)$$

vektorini aniuqlaymiz,bunda $\eta_{q+1i}(\tau)(i=1,p)$ keyingi qadamlarda aniqlanadigan noma'lum funksiyalar (2.9) tenglamadagi $u_2(\tau)$ vektorni o'rniga

$$\sum_{i=0}^{p}\varphi_i\eta_{2i}(\tau)$$

niqo'yib

$$A_0(\tau)u_{q+2}(\tau)=-A_1(\tau)\sum_{i=1}^{p}\varphi_i\eta_{\,2i}(\tau)+f_u(\tau)\sum_{i=1}^{p}\varphi_i\eta_{\,2i}(\tau)+f_1(\tau) \qquad (2.37)$$

tenglamaga ega bo'lamiz. (2.37) tenglama uchun yechimi mavjud bo'lish sharti ko'rinishida bo'ladi. Bu tenglamadan $\eta_{2i}(\tau)$ noma'lum funksiyalar

$$\eta_{2i}(\tau) = \frac{f_1(\tau), \psi_j}{\left(A_1(\tau)\sum_{i=1}^{p} [\varphi_1, \psi_j\right) - \left(f_u(\tau)\sum_{i=1}^{p} \varphi_i, \psi_j\right)]}, \text{ j=} \qquad (2.39)$$

tenglik orqali aniqlanadi.(2.38) tenglikni bajarilishini e'tiborga olib,(2.37) tenglamadan

$$u_{q+2}(\tau) = A_0^{+}(\tau)\left[f_u(\tau)\sum_{i=1}^{p}[\varphi_i\eta_{2i}(\tau) + f_1(\tau)] + \sum_{i=1}^{p}\varphi_i\eta_{q+2}(\tau)\right] \qquad (2.40)$$

yechimini aniqlaymiz.

Shu jarayonni davom ettirib, (2.10) tenglamani

$$A_0(\tau)u_s(\tau) = -A_1(\tau)\sum_{i=1}^{p}\varphi_i\eta_{(s-q)i}(\tau) + f_u(\tau)\sum_{i=1}^{p}\varphi_i\eta_{(s-q)i}(\tau) + b_s(\tau),$$

$$s = q + 3, \dots \qquad (2.41)$$

ko'rinishida yozamiz. Bu tenglama uchun (2.38) ko'rinishidagi yechim mavjud bo'lish sharti bajariladi deb noma'lum $\eta_{(s-q)i}(\tau)$ (s = q+3,..) funksiyalarni

$$\eta_{(s-q)i}(\tau) = \frac{[(b]_s(\tau), \psi_j)}{(f]_u(\tau)\sum_{i=1}^{p}[\varphi_i, \psi_j) - (A_1(\tau)\sum_{i=1}^{p}\varphi_i, \psi_j)}, \text{ s=q+3,… j=j} \qquad (2.42)$$

tenglik orqali aniqlaymiz, u holda (2.41)tenglamadan $u_s(\tau)$ noma'lum vektor

$$u_s(\tau) = A_0^+(\tau)\left[f_u(\tau)\sum_{i=1}^{p}[\![\varphi_i\eta_{s-qi}(\tau) + b_s(\tau)\right] + \sum_{i=1}^{p}\varphi_i\eta_{si}(\tau),]\!] \qquad (2.43)$$

S=q+3,...,j=

tenglamadan aniqlanadi,bunda $\eta_{si}(\tau)$noma’lum funksiyalar keyingi qadamlardan topiladi. 2.2. teorema isbotlandi.

2.2-§. Tuzilgan formal xususiy yechimning asimptotik xossasi

Endi o'tganparagrifning 2.1-teoremasi asosida tuzilgan formal xususiy yechimning assimptotik xosaga ega ekanligini ko'rsatamiz. Buning uchun (2.1) sistemaning m-yaqinlashuvchi yechimini

$$x_m(t,\varepsilon)=\sum_{s=0}^{m}\mu^s u_s(\tau) \tag{3.1}$$

ko'rinishida olamiz.

(2.1) sistemada

$$x=z_1,\ \frac{dx}{dt}=z_2,\dots,\frac{d^{k-1}x}{dt^{k-1}}=z_k, \qquad z=colon(z_1,z_2,\dots,z_k)$$

almashtirish olib

$$\frac{dz}{dx}=\widetilde{A}(\tau,\mu)z+\mu^q f(\tau,z,\mu) \tag{3.2}$$

ko'rinishidagi birinchi tartibli differinsial tenglamalar sistemasini hosil qilamiz, bunda

$$\widetilde{A}(\tau,\mu)=\begin{pmatrix}0 & E\ \dots 0\\ 0 & 0\ \dots 0\\ -\varepsilon^{-p/q}A(\tau,\varepsilon) & 0\ \dots 0\end{pmatrix}, f(\tau,z,\mu)=\begin{pmatrix}0\\0\\-\varepsilon^{-p/q}f(\tau_1,z_1,\mu_2)\end{pmatrix}$$

0-nol, E – birlik matritsialar (n x n) tartibli $\widetilde{A}(\tau,\mu)$(kn x kn) tartibli matritsiya f (τ,z,μ) – kn- o'lchovchi vektor. (3.2) tenglama uchun m-yaqinlashuvchi yechim $z_m(t,\varepsilon)$dan iborat.

Dastlab quyidagi yordamchi lemmani isboti keltiriladi:

Lemma. Agar 2.1- teoremaning shartlari bajarilsa,u holda m- yaqinlashuvchi $z_m(t,\mu)$ yechim

$$\frac{dz_m}{dt} = [\widetilde{A}_0(\tau) + \mu\widetilde{A}_1(\tau,\mu)]z_m + \mu^q f(\tau, z_m, \mu) + \mu^{m+1}R_m(\tau,\mu) \qquad (3.3)$$

differinsial tenglamani qanoatlantiradi, bundavektor -funksiya [0,L] kesmada tekis chegaralngan funksiya.

$$\widetilde{A}_1(\tau,\mu) = \sum_{i=1}^{\infty} \mu^{i-1}\widetilde{A}_i(\tau) \qquad (3.4)$$

Lemmani isboti

$$z_m(t,\mu) = \sum_{s=1}^{m} \mu^s z_s(t) \qquad (3.5)$$

ifodani (3.2) differinsial tenglamaga quyib undan keyingi yechimimizga qadar bo'ladi [3]

$$y(t,\mu) = z(t,\mu) - z_m(t,\mu) \qquad (3.6)$$

ayirmani qaraymiz, bunda $z_m(t,\mu)$ va z(t,μ) (3.2) sistemaning yaqinlashuvchi va aniq yechimi bo'lib, bir xil boshlang'ich sharti qanoatlantiradi.
U holda y(t,μ) vektor – funksiya

$$y(0,\mu) = 0 \qquad (3.7)$$

boshlang'ich shartni qanoatlantiruvchi

$$\frac{dy}{dt} = \widetilde{A}_0 y + \mu\widetilde{A}_1(\tau,\mu) + \mu^q[f(\tau, z_m, \mu) - f(\tau, z_m, \mu)] + +\mu^{m+1}R_m(\tau,\mu) \qquad (3.8)$$

differinsial tenglamani qanoatlantiradi.

2.3. Teorema. Faraz qilaylik 2.1- teoremaning shartlari bajarilsin va $f(\tau, z, \mu)$ vektor – funksiya ℓ o'zgarmas bog'liq

$$\| f(\tau, z_{m,}\mu) - f(\tau, z, \mu,)\| \leq \ell \| z_m - z \| \tag{3.9}$$

tengsizlikni qanoatlantirsin. U holda shunday musbat o'zgarmas C>0 va $\varepsilon\epsilon(0, \varepsilon_0]$ topiladiki $\varepsilon < \varepsilon_1$ bo'lganda $0 \leq t \leq L/_\varepsilon$ oraliqda

$$\| z_m(t,\mu) - z(t,\mu) \| \leq \mu^{m+1-q} C \tag{3.10}$$

tengsizlik o'rinli bo'ladi.

Isbot. Ma'lumki (3.7) shartni qanoatlantiruvchi (3.8) tenglama quyidagi integral tenglamalar sistemasiga ekvivalent:

$$y(t,\varepsilon) = \mu \int_0^t v(t,\mu) v^{-1}(t_1,\mu) A_1(\tau_1,\mu) y(t_{1,}\mu) dt + \mu^q \int_0^t v(t,\mu) v^{-1}(t_{1,}\mu)$$

$$[f(\tau_1, z_{m,}\mu) - f(\tau_1, z, \mu)] dt_1 + \mu^{m+1} \int_0^t v(t,\mu) v^{-1}(t_1,\mu) R_m(\tau_1,\mu) dt_1 \tag{3.11}$$

Bunda $v(t,\mu)$ matritsia

$$\frac{dv}{dt} = \widetilde{A}_0(\tau) V \quad . V(t,\mu)/_{t=0} = V_0 \tag{3.12}$$

masalaning fundamental yechimi bo'lib $\forall\tau[0.L]$ uchun

$$\| v(t,\mu) \| \leq \mu \tag{3.13}$$

tengsizlikni qanoatlantiradi.

(3.11) tenglamadan norma olib va (3.10),(3.13) tengsizliklarni e'tiborga olib

$$\|y(t,\mu)\| \leq \mu\int_0^t \|v(t,\mu)\| \|v^{-1}(t_1,\mu)\| \|y(t,\mu)\| + \|v(t,\mu)\| \|v^{-1}(t_1,\mu)\mu\| \ell \|z_m - z\| dt_1 + \mu^{m+1}\int_0^t \|v(t,\mu)\| \|v^{-1}(t_1,\mu)\| \|R_m(t_1,\mu)\| \mathbf{dt}$$

(3.14)

tengsizlikni hosil qilamiz.

[0,L]kesmada $\widetilde{A}(\tau_1,\mu)$ va $R_m(\tau,\mu)$ larni tekis chegaralanganligiga asosan

$$\|\widetilde{A}_1(\tau_1,\mu)\| \leq K, \quad R_* = \max_{0\leq\tau\leq L} \|R_m(\tau_1,\mu)\| \tag{3.15}$$

bo'lishini e'tiborga olib (3.14) tengsizlikdan

$$\|y(\tau,\mu)\| \leq \mu(K+\ell M)\int_0^t \|y_1(t,\mu)\| \mathbf{dt_1} + \mu^{m+1} M R_* \int_0^t dt_1 \tag{3.16}$$

tengsizlikka ega bo'lamiz.(3.16) tengsizlikka Gronuolla- Belman tengsizligini qo'llab

$$\|y(t,\mu)\| \leq \mu^{m+1-q} M R_* \exp\left(\mu_1(k+\ell M)\right)$$

yoki

$$\|y(t,\mu)\| \leq \mu^{m+1-q}\, C \tag{3.17}$$

tengsizlikni hosil qilamiz, bunda

$$C = M R_* \exp\left(\mu_1(k+\ell M)\right)$$

(3.17)- tengsizlikni (3.6) tenglikni e'tiborga olib

$$\|z_m(t,\mu) - z(t,\mu)\| \leq \mu^{m+1-q} C \tag{3.18}$$

ko'rinishida yozamiz. U holda (2.1) sistema uchun yuqorida olingan almashtirishni e'tiborga olib (2.1) sistemaning formal yechimi

$$\| x_m(t,\varepsilon) - x(t,\varepsilon) \| \le \mu^{m+1-q} C \quad ,$$

$$\left\| \frac{d^r x_m}{dt^r} - \frac{d^r x}{dt^r} \right\| \le \mu^{m+1-q} C \ , r = \overline{1, k-1} \quad (3.19)$$

tengsizliklar sistemasini qanoatlantirishini aniqlaymiz. 2.3 teorema isbotlandi.

2.3-§. Karrali ildizlar uchun formal xususiy yechimni tuzish.

Ushbu paragrifda (2.1) sistemaning (2.2) tenglamasi bitta n - karrali ildizga ega unga shunday o’lchovdagi elementar bo’luvchi mos kelgan hol uchun formal yechimni tuzish masalasini qaraymiz.

Faraz qilaylik (2.1) sistemaning (2.2) tenglamasi bitta n-karrali nol ildizga ega bo’lib unga shunday o’lchovli ildizlarining qism fazosi mos kelsin, ya’ni [0,L] kesmada

$$\lambda_n(\tau) \equiv 0, \det A_0(\tau) = 0$$

bo’lsin.

2.4. Teorema. Agar $A_0(\tau)$ matritsia bitta n –karrali nol xos qiymatga ega bo’lib,2.1 teoremaning shartlari bajarilsa, u holda (2.1) sistema

$$\mathbf{x}(\mathbf{t},\varepsilon) = \sum_{s=0}^{\infty} \mu_1^s u_s(\tau), \ \mu_1 = \sqrt[nq]{\varepsilon} \quad (4.1)$$

ko’rinishida formal xuxusiy yechimga ega bo’ladi.

Isbot.n,p va q sonlar orasidagi munosabatlarga qarab 6-ta holni qarash mumkin. Lekin bu munosabatlar asosan ikki holga keltiriladi. Shuning uchun teoremani sibotini n=q, p>q, n=q,p<q hollarning faqat n=q, p>q munosabat uchun keltiramiz.

Kelgan hollar uchun teoremani isboti shunga o’xshash bo’ladi.

(4.1) qatorni (2.1) sistemaga quyish

$$\mu^{nkp} u^{(k)}(\tau,\mu_1) = A\left(\tau,\mu_1^{nq}\right) u(\tau,\mu_1) + \mu_1^{nq} f\left(\tau, u(\tau,\mu), \mu_1^{nq}\right) \quad (4.2)$$

tenglikni hosil qilamiz. Bu tenglamadan $\mu_1^s (s = 0,1, \dots)$ parametrlarning bir xil darajalari oldidagi koeffisentilarni tenglashtirib va $f(\tau, u(r, \mu_1), \mu_1^{nq})$ vektorni $(\tau, u_0(\tau), 0)$ nuqtalar atrofida Teylor qatoriga yoyilmasini e'tiborga olib quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$A_0(\tau)u_0(\tau) = 0;$$
$$A_0(\tau)U_\alpha(\tau) = 0, \alpha = \overline{1, nq-1}$$

; (4.3)

$$A_{0(\tau)}u_{nq}(\tau) = -A_1(\tau)u_0(\tau) - f\left(\tau, u_0(\tau)\right), \quad (4.4)$$

…………………………………………………………………………………

$$A_{0(\tau)}u_s(\tau) = -A_1(\tau)u_{s-nq}(\tau) - f_u(\tau)u_{s-nq}(\tau) + T_{s-nq-1}(\tau), \quad s = nq + 1, \dots, \quad (4.5)$$

bunda

$$T_{s-nq-1}(\tau) = u_{s-nkp}^{(k)}(\tau) - \sum_{i=2}^{\left[\frac{s}{nq}\right]} A_i(\tau)u_{s-inq}(\tau)\tilde{f}_{s-2nq+1}(\tau)$$

$f_u(\tilde{\tau}) = \dfrac{\partial f^i}{\partial U^j}$ matritsiyaning elementlari va $\tilde{f}_{s-2nq+1}(\tau)$ vektorning koordinatalari $(\tau, u_0(\tau), 0)$ nuqtada hisoblanadi, $f_{s-nq-1}(\tau)$ vektorlar $U_i(\tau)$ (i=1,2,…,s-nq-1) lar orqali ifodalanadi.

(4.1) qatorning noma'lum hadlarini aniqlash uchun yuqoridagidek (4.3) – (4.5) tenglamalardan foydalanamiz.

(4.3) sistemadan

$$\begin{cases} u_0(\tau) = \varphi_1 C_0(\tau); \\ u_j(\tau) = \varphi_1 C_j(\tau), \alpha = \overline{1, nq-1,} \end{cases} \varphi_1 = \ \text{N}(A_0(\tau))$$

(4.6)

larni aniqlaymiz, $C_\alpha(\tau), \overline{\alpha = (0, nq-1)}$ keyingi qadamda aniqlanadigannoma'lum funksiyalar (4.4) tenglamani (4.6) sistemaning birinchi tenglamasiga asosan

$$A_{0(\tau)} u_{nq}(\tau) = -A_1(\tau)\varphi_0 C_0(\tau) - f_u\left(\tau, \varphi_1, C_0(\tau)\right), \quad (4.7)$$

ko'rinishida yozamiz. Bu tenglama uchun yechim mavjud bo'lish sharti

$$(A,(\tau)\varphi, \Psi_1)C_0(\tau) + \left(f\left(\tau, \varphi_1 C_0(\tau)\right), \Psi_1\right) = 0, \Psi_1 \epsilon N\left(A_0^{*}\left(\tilde{\tau}\right)\right)$$

(4.8)

(4.8) tenglama ham yuqorida tanishib o'tganimizdek $C_0(\tau)$ noma'lum funksiyaga nisbatan oshkormas funksiyaga ega bo'lamiz. (4.8) tenglama uchun oshkormas funksiya haqidagi teoremaning barcha shartlari bajariladi deb $C_0(\tau)$ funksiyani aniqlaymiz.

U holda (4.7) tenglamadan noma'lum $u_{nq}(\tau)$ vektor

$$u_{nq}(\tau) = F_1(\tau) + \varphi_1 C_{nq}(\tau), \quad (2.4.9)$$

ko'rinishda aniqlanadi, bunda

$F_0(\tau) = A_0^{+}(\tau) f(\varphi_1 C_0(\tau), \tau), C_{nq}(\tau)$noma'lum funksiya keyingi qadamda aniqlanadi.

Shu jarayoni davom ettirib s-chi qadamda (4.5) tenglamani

$$(A_1(\tau)\varphi_1, \Psi_1)C_{s-nq}(\tau) + (f_u(\tau)\varphi_1, \Psi_1)c_{s-nq}(\tau) = \left(\tilde{F}_{s-2nq}(\tau), \psi_1\right) \quad (2.4.10)$$

bunda

yechim mavjud bo'lish shartidan noma'lum $C_{s-nq}(\tau)$ funksiyani aniqlangandan keyin, noma’lum $u_s(\tau)$ vektorni

$u_s(\tau) = H_{s-nq-2}(\tau) + \varphi_1 C_s(\tau)$ (2.4.11)

ko’rinishda aniqlaymiz, bunda

$$H_{s-nq-2}(\tau) = P_{s-nq-2}(\tau) + Q_{s-nq-2}(\tau),$$

$P_{s-nq-2}(\tau) = A_0^+(\tau)\left[T_{s-nq-2}(\tau) + f_u(\tau)\varphi_1 C_{s-nq}(\tau)\right]$, $\quad nq + 2 \leq s < 2nq$,

$Q_{s-nq-2}(\tau) = A_0^+(\tau)[\tilde{F}_{s-2nq-2}(\tau) + f_u(\tau)\varphi_1 C_{s-nq}(\tau)$, s=2nq, 2nq+1, …, $C_s(\tau)$- keyingi qadamga aniqlanadigan noma’lum funksiya. 2.4 – teorema isbotlandi.

3-bob. Chiziqli bo'lmagan yuqori tartibli differensial tenglamalar sistemasining yechimlarining asimptotik xususiyatlari.

3.1-§. Kechikuvchi argumentli differensial tenglamalar sistemasining formal yechimlari.

Ushbu paragrifda neytral tipdagi sekin o'zgaruvchi koyeffisiyentli differensial tenglamalar

$$\varepsilon \frac{d^k x}{dt^k} + A(\tau,\varepsilon)x(t,\varepsilon) + B(\tau,\varepsilon)x'(t-\Delta(\tau),\varepsilon) = \varepsilon^2 f(\tau,x,\varepsilon) \qquad (3.1.1)$$

sistemasini o'rganishga bag'ishlangan, bunda x, f -yuqoridagidek n-o'lchovli vektorlar, A(τ,ε), V(τ,ε) – n-tartibli kvadrat matrisalar bo'lib,

$$A(\tau,\varepsilon) = \sum_{s=0}^{\infty} \varepsilon^s A_s(\tau), \quad V(\tau,\varepsilon) = \sum_{s=0}^{\infty} \varepsilon^s B_s(\tau) \qquad (3.1.2)$$

formal qatorlarga yoyiladi, $\forall \tau \in [0,L]$uchun $\det A_0(\tau) \equiv 0,\ 0 \le \tau \le \varepsilon t \le L < +\infty$sekin o'zgaruvchi vaqt, $L > 0$–berilgan son, $\varepsilon > 0$kichik parametr, $\Delta(\tau)\ \forall \tau \in [0,L]$uchun

$$\Delta(\tau) \ge 0 \qquad (3.1.3)$$

shartni qanoatlantiradigan funksiya, $f(\tau,x,\varepsilon)$-vektor$(\tau,x_0,0)$nuqta atrofida Teylor qatoriga yoyiladi.

Bunday tenglamalar muhim nazariy va amaliy ahamiyatga ega. (3.1.1)-sistemasini yechimlarini tuzishda [12] Kato ma'nosidagi kritik holni qaraymiz.

(3.1.1)-sistemani asimptotik yechimlari

$$\det\|A_0(\tau) - \lambda(\tau)E\| = 0 \qquad (3.1.4)$$

(Ye-birlik matrisa) xarakteristik tenglamasining ildizlariga bog'liq.

Bu paragrifda asosan $\tau \in [0,L]$ lar uchun (3.1.4) tenglamaning ildizlari oddiy ya'ni bir-biriga teng bo'lmagan holni qaraymiz.

Bu hol uchun (3.1.1) sistemani yechimini ko'rsatuvchi quyidagi teorema o'rinli.

1-Teorema. Quyidagi shartlar bajarilsin:

a) $A_s(\tau), B_s(\tau)(s = 0,1,......)$ matrisalar $\tau \in [0,L]$ kesmada, $f(\tau,x,\varepsilon)$ vektor $P(\tau,x,\varepsilon) = P(\tau,x) \times (0 < \varepsilon \le \varepsilon_0)$ sohada, bunda $P(\tau,x)$, τ, x o'zsharuvchilar ba'zi sohaning fazosi bo'lib s^k sinfga tegishli (*k*-ning yetarlicha katta qiymatlari uchun);

b) $\tau \in [0,L]$ kesmada $\lambda_P(\tau) \equiv 0$, $(p=1, p<n)$ $\mathrm{Re}\lambda_P(\tau) \le 0,\ p=\overline{2,n}$, ya'ni $A_0(\tau)$ oddiy xos qiymatlarga ega;
v) $\forall \tau \in [0,L]$ uchun $B_0(\tau)$ matrisaning $b_{n1}(\tau)$ elementi noldan farqli

$$b_{n1}(\tau) \neq 0 \tag{3.1.5}$$

U holda (3.1.1) tenglama

$$x(t,\varepsilon) = \sum_{s=0}^{\infty} \varepsilon^s u_s(\tau) \tag{3.1.6}$$

ko'rinishdagi formal xususiy yechimga ega bo'ladi.
Isbot. 1-teoremani isbot qilish uchun (3.1.6) tenglikni (3.1.1) sistemaga quyamiz, bunda $f(\tau,u(\tau,\varepsilon),\varepsilon)$ vektorni $(\tau,u_0(\tau),0)$ nuqta atrofida Teylor qatoriga yoyilmasini e'tiborga olib

$$\varepsilon^k u^k(\tau,\varepsilon) + A(\tau,\varepsilon)u(\tau,\varepsilon) + \varepsilon B(\tau,\varepsilon)u'(\tau-\varepsilon\Delta(\tau),\varepsilon) =$$

$$= \varepsilon^2\left\{f_0(\tau,u_0(\tau),0) + \varepsilon[f_x(\tau)u_1(\tau)+f_1(\tau)] + \ldots + \varepsilon^S[f_x(\tau)u_S(\tau)+f_S(\tau)] + \ldots\ldots\right\} \tag{3.1.7}$$

bunda $f_x(\tau) = \dfrac{\partial f^i}{\partial x^j}$ matrisaning elementlari va $f_x(\tau) = \dfrac{\partial f}{\partial \varepsilon}$ vektorning koordinatalari $(\tau,u_0(\tau),0)$, nuqta atrofida hisoblanadi, $f_S(\tau)$ vektor $x_q(\tau)$ ($q=0,1,\ldots s-1$) lar orqali ifodalanadi.
(3.1.6)-qatorning hadlarini aniqlashdan oldin $u(\tau-\varepsilon\Delta(\tau),\varepsilon)$ vektorni ε parametrning darajalari bo'yicha formal qatorga yoyamiz.

$$u(\tau-\varepsilon\Delta(\tau),\varepsilon) = \sum_{s=0}^{\infty}\varepsilon^s \sum_{r=0}^{s} \frac{(-\Delta(\tau))^{s-r}}{s-r}\frac{d^{s-r}u_r(\tau)}{d\tau^{s-r}} \tag{3.1.8}$$

(3.1.8)-qatorni e'tiborga olib (3.1.7) tenglamada ε parametrning bir xil darajalari oldidagi koyeffisiyentlarni tenglashtirib (3.1.6)-qatorning noma'lum elementlarini aniqlaymiz.
ε^0 parametr oldidagi koyeffisiyentlardan

$$A_0(\tau)u_0(\tau) = 0 \tag{3.1.9}$$

tenglamani hosil qilamiz. Bu tenglamalardan [10] ishga asosan

$$u_0(\tau) = \varphi\eta_0^{(\tau)}, \tag{3.1.10}$$

ni aniqlaymiz, bunda $\varphi \in N(A_0(\tau)) - n$-o'lchchovli vektor bo'lib, uning birinchi koordinatasi birga teng qolganlari nolga teng, $\eta_0(\tau)$, barcha $\forall \tau \in [0, L]$ uchun noldan farqli bo'lgan ixtiyoriy funksiya, keyingi qadamda aniqlanadi.
ε parametr oldidagi koyeffisiyentlarni tenglashtirib bir jinsli bo'lmagan

$$A_0(\tau)u_1(\tau) = -A_1(\tau)u_0(\tau) - B_0(\tau)\frac{du_0(\tau)}{d\tau} \tag{3.1.11}$$

tenglamani hosil qilamiz. (3.1.10) tenglamani e'tiborga olib (3.1.11) tenglamani

$$A_0(\tau)u_1(\tau) = -A_1(\tau)\varphi\eta_0(\tau) - B_0(\tau)\varphi\frac{d\eta_0(\tau)}{d\tau}. \tag{3.1.12}$$

ko'rinishda yozamiz.
(3.1.12) tenglama yechimga ega bo'lish uchun uning o'ng tomoni bir jinsli qismiga bog'liq bo'lgan tenglamaning yechimiga ortogonal bo'lishi zarur va yetarlidir, ya'ni $\forall \tau \in [0, L]$ uchun

$$(\psi, (A_1(\tau)\varphi\eta_0(\tau) + B_0(\tau)\varphi\frac{d\eta_0(\tau)}{d\tau})) = 0 \tag{3.1.13}$$

tenglik o'rinli bo'lsin, bunda $\psi \in N(A_0^*(\tau))$ n-o'lchovli vektor, uning n-chi koordinatasi birga teng qolganlari nolga teng, $A_0^*(\tau)$, $A_0(\tau)$-matrisaning qo'shma kompleks matrisasi.
(3.1.13) tenglamani quyidagi ko'rinishda yozamiz.

$$(\psi, B_o(\tau)\varphi)\frac{d\eta_0(\tau)}{d\tau} = -(\psi, A_1(\tau)\varphi)\eta_0(\tau) \tag{3.1.14}$$

(3.1.14)-tenglamani integrallab va (3.1.5) ni e'tiborga olib noma'lum $\eta_0(\tau)$ funksiyani aniqlaymiz.

$$\eta_0(\tau) = ae^{\int_0^\tau \frac{a_{n1}(\tau)}{b_{n1}(\tau)}d\tau} \tag{3.1.15}$$

(3.1.15)-tenglikdagi integrallovchi ko'paytuvchini birga teng a=1 deb olamiz. U holda noma'lum funksiyani

$$\eta_0(\tau) = e^{\int_0^\tau \frac{a_{n1}(\tau)}{b_{n1}(\tau)}d\tau}. \tag{3.1.16}$$

ko'rinishda yozamiz.
(3.1.12) tenglama uchun yechim mavjud bo'lish sharti o'rinli bo'ldi, u holda undan noma'lum $u_1(\tau)$ vektorni aniqlaymiz:

$$u_1(\tau) = -A_0^+(\tau)[A_1(\tau)\varphi\eta_0(\tau) - B_0(\tau)\varphi\frac{d\eta_0(\tau)}{d\tau}] + \varphi\eta_1(\tau) \qquad (3.1.17)$$

bunda $\eta_1(\tau)$, yuqoridagidek keyingi qadamda aniqlanadigan noma'lum funksiya, $A_0^+(\tau)$ - $A_0(\tau)$-matrisaning umumlashgan teskari matrisasi.
ε^2 parametr oldidagi koyeffisiyentlarini yig'ib

$$A_0u_2 = F_1(\tau) - (A_1(\tau) + B_0(\tau))u_1(\tau) \qquad (3.1.18)$$

tenglamani hosil qilamiz, bunda
$F_1(\tau) = B_0(\tau)\Delta(\tau)\frac{du_0(\tau)}{d\tau} + f(\tau, u_0(\tau)) - u_0'(\tau) - [A_2(\tau)B_1(\tau)]u_0(\tau)$.
(3.1.18)-tenglama uchun (3.1.17) tenglikni e'tiborga olib yechim mavjud bo'lish shartini barcha $\forall\tau\in[0,L]$ uchun

$$(\psi, (\tilde{F}_1(\tau) - (A_1(\tau) + B_0(\tau)\varphi\eta_1(\tau))) = 0, \ \forall\tau\in[0,L] \qquad (3.1.19)$$

ko'rinishda yozamiz.

Barcha $\forall\tau\in[0,L]$ uchun $\det[A_1(\tau) + B_0(\tau)] \neq 0$ ekanligini e'tiborga olib (3.1.19) tenglamadan $\eta_1(\tau)$ funksiyani aniqlaymiz:

$$\eta_1(\tau) = \frac{(\psi, \tilde{F}_1(\tau))}{(\psi, (A_1(\tau) + B_1(\tau))\varphi)} \ . \qquad (3.1.20)$$

(3.1.19) shart (3.1.18) tenglama uchun bajarilganligini e'tiborga olib undan (3.1.5) qatorning noma'lum $u_2(\tau)$ vektorini aniqlaymiz:

$$u_2(\tau) = A_0^+(\tau)\big[F_1(\tau) - (A_1(\tau) + B_0(\tau))\varphi\eta_1(\tau)\big] + \varphi\eta_2(\tau), \qquad (3.1.21)$$

bunda $\eta_2(\tau)$-keyingi qadamda aniqlanadigan noma'lum funksiya.
Shu jarayonni davom ettirib, (3.1.5) qatorning noma'lum $u_S(\tau)$ $(s = 3,4,...)$ elementlarini aniqlash mumkin. Teorema isbotlandi.

3.2-§. Yechimning asimptotik xarakteri.

Yuqorida keltirilgan algoritm (3.1.5) qatorning istalgan elementini aniqlash imkoniyatini beradi. Ammo amaliyotda hisoblashlarni kengayib ketishini e'tiborga olib (3.1.5) qatorning dastlabki *m*-ta hadini aniqlash bilan chegaralanadi.
Shuning uchun (1)-sistemaning izlanayotgan yechimiga

$$x_m(t,\varepsilon) = \sum_{S=0}^{m} \varepsilon^s u_s(\tau) \qquad (3.2.1)$$

formula orqali keltirilgan *m*-chi yaqinlashuvchi yechimini qarash kifoya.
Faraz qilaylik$x(t,\varepsilon)$(3.1.1)-sistemaning aniq yechimi bo'lsin va $t=0$bo'lganda

$$x(t,e)\big|_{t=0} = x_m(t,e)\big|_{t=0} \qquad (3.2.2)$$

shartni qanoatlantirsin. U holda yaqinlashuvchi $x_m(t,\varepsilon)$ aniq $x(t,\varepsilon)$ yechimga asimptotik yaqinlashishni ko'rsatish mumkin.
Yaqinlashuvchi yechim bilan aniq yechim orasidagi farqning normasini baholash uchun

$$y(t,\varepsilon) = x_m(t,\varepsilon) - x(t,\varepsilon) \qquad (3.2.3)$$

tenglikdan foydalanamiz.
Lemma 1. Agar $A\ (\tau,\varepsilon)$ matrisa

$$A\ (\tau,\varepsilon) = A_0(\tau) + \varepsilon A_1(t,\varepsilon), \qquad (3.2.4)$$

yig'indiga ega bo'lsa, bunda $A_s(\tau), s = 0,1,...,$ matriqalar $[0,L]$ kesmada cheksiz differensiallanuvchi bo'lsa va bundan tashqari har qanday *n*-o'lchovli *z*-vektor uchun

$$\mathrm{Re}(A_0(\tau)z,z) \leq 0, \qquad (3.2.5)$$

tengsizlik o'rinli bo'lsa, u holda $x(0,\varepsilon) = x_0$ boshlang'ich shartni qanoatlantiruvchi

$$\varepsilon\frac{dx}{dt} = [A_0(\tau) + \varepsilon A_1(t,e)]x \qquad (3.2.6)$$

tenglamani yechish uchun $\left[0,\frac{L}{\varepsilon}\right] \times (0 < \varepsilon \leq \varepsilon_0)$ kesmada

$$\|x(t,\varepsilon)\| \le c\|x_0\| \qquad (3.2.7)$$

tengsizlik o'rinli bo'ladi, bunda s, ε -ga bog'liq bo'lmagan musbat o'zgarmas.
Bu yechimni isbotini [15] ishdan topish mumkin.
Berilgan sistemaning formal yechimini asimptotik xarakterga ega ekanligini quyidagi teorema ko'rsatadi..
2 Teorema. Faraz qilaylik (3.1.1) differensial tenglamalarsistemasi uchun 1-teoremaning shartlari va quyidagi shartlar bajarilsin:
1) (3.1.4) tenglama oddiy ildizga ega bo'lib, uning ildizlari $\forall \tau \in [0, L]$ uchun

$$\operatorname{Re} \lambda(\tau) \le 0, \qquad (3.2.8)$$

2) (3.2.1) tenglik orqali aniqlanadigan $x_m(t,\varepsilon)$ vektor boshlang'ich $(t \in \varepsilon\Delta(\tau))$ to'plamda (3.1.1) sistemaning aniq yechimiga teng bo'lsin

$$x_m(t,\varepsilon) = x(t,\varepsilon),\ t \in E_\Delta, \qquad (3.2.9)$$

3) $\left[0, \frac{L}{\varepsilon}\right]$ kesmada $t - \Delta(\tau)$ funksiya qat'iy monotom o'suvchi bo'lsin:

$$\Delta(\tau) > 0 \quad \forall \tau \in [0, L]. \qquad (3.2.10)$$

4) $$\operatorname{Re}(A_0(\tau)x(t,\varepsilon), x(t,\varepsilon) \le 0,\ \forall \tau \in [0, L]. \qquad (3.2.11)$$

5) $f(\tau, x, \varepsilon)$ vektor funksiyao'zgarmas *l* uchun Lepщis shartini qanoatlantirsin*:*

$$\|f(\tau, x_m \varepsilon) - f(\tau, x, \varepsilon)\| \le l\|x_m - x\| \qquad (3.2.12)$$

U holda ε parametrga bog'liq bo'lmagan shunday o'zgarmas c_r, p_r majud bo'ladiki, barcha $t \in [(r-1)d, rd]$, $1 \le r \le \left[\frac{L}{\varepsilon d}\right]$ va

$$d = \min\ \Delta(\tau),\ \tau \in [0, L] \qquad (3.2.13)$$

lar uchun

$$\|x_m(t,\varepsilon) - x(t,\varepsilon)\| \le \varepsilon^{m+1-r} c_r; \qquad (3.2.14)$$

$$\left\| \frac{dx_m(t,\varepsilon)}{dt} - \frac{dx(t,\varepsilon)}{dt} \right\| \le \varepsilon^{m+1-r} p_r$$

tengsizlik o'rinli bo'ladi.

Isbot. (3.1.1) differensial tenglamaga (22)-vektorni qo'yib bunda (3.2.8) ifodani e'tiborga olib.

$$\frac{d^k x_m}{dt^k} = -A(\tau,\varepsilon)x_m(t,\varepsilon) - B(t,\varepsilon)\frac{dx(t-\Delta(\tau),\varepsilon)}{d\tau} + \varepsilon^2 f(\tau, x_m, \varepsilon) + \varepsilon^{m+1} r_m(\tau,\varepsilon), \quad (3.2.15)$$

tenglamani hosil qilamiz, bunda $r_m(\tau,\varepsilon)$, $\varepsilon = 0$ nuqta atrofida chegaralangan vektor. U holda (3.2.3) vektor

$$\frac{d^k y(t,\varepsilon)}{dt^k} = A(\tau,\varepsilon)y - B(\tau,\varepsilon)\frac{dy(t-\Delta(\tau),\varepsilon)}{dt} + \varepsilon\left[f(\tau, x_m, \varepsilon) - f(\tau, x, \varepsilon)\right] + \varepsilon^{m+1} r_m(\tau,\varepsilon),$$
(3.2.16)

Differensial tenglamani qanoatlantiradi. (3.2.9) shartga asosan

$$y(t,\varepsilon) = 0, \quad \forall \tau \in E_\Delta \quad (3.2.17)$$

$Y(t,\varepsilon)$ orqali

$$\frac{d^k y}{dt^k} = A_0(\tau)Y \quad (3.2.18)$$

sistemaning normallangan fundamental matrisasini belgilaymiz, ya'ni u qarayotgan sistemaning $Y(0,\varepsilon) = E$ shartni qanoatlantiradigan yechimi. (3.2.16) ni e'tiborga olib ixtiyoriy r-chi qadamda $((\tau-1)d \le t \le rd)$ quyidagi integral tenglamaga yekvivalent:

$$y^{(k)}(t,\varepsilon) = y^{(k)}((r-1)d,\varepsilon) + \int_{(r-1)d}^{t} Y(t,\varepsilon)Y^{-1}(s,\varepsilon)B(\tau,\varepsilon)\dot{y}(s-\Delta(\tau),\varepsilon ds +$$

$$+\varepsilon^2 \int_{(r-1)d}^{t} Y(t,\varepsilon)Y^{-1}(s,\varepsilon)[f(\tau,x_m,\varepsilon) - f(\tau,x,\varepsilon)]ds + \varepsilon^{m+1}\int_{(r-1)d}^{t} Y(t,\varepsilon)Y^{-1}(s,\varepsilon)r_m(\sigma,\varepsilon)ds$$
(3.2.19)

bunda $\sigma = \varepsilon$ s.Agar (3.2.11) tengsizlikni 1-lemmani qo'llasak $\forall t \in [0, \frac{L}{\varepsilon}]$ u $\varepsilon \in (0, \varepsilon_0]$ uchun

$$\left\| Y(t,\varepsilon)Y^{-1}(t,\varepsilon) \right\| \le M \quad (3.2.20)$$

tengsizlikni hosil qilamiz, bundaM,ε-parametrga bog'liq bo'lmagan o'zgarmas. (3.2.19) tenglamadagi vektorlar va matriqalarni norma bo'yicha baholab hamda (3.2.12)–ni e'tiborga olib

$$\left\|y^{(k)}(t,\varepsilon)\right\| \le \left\|y(r-1)d,\varepsilon)\right\| + \varepsilon MlL\left\|\dot{y}(r-1)d,\varepsilon)\right\|(1+\varepsilon_0 MlL)\left\|y(r-1)d,\varepsilon)\right\| +$$

$$a_1 \int_{(r-1)d}^{t} \dot{y}(s-\Delta(\tau),\varepsilon ds + a_2\varepsilon^m \qquad (3.2.21)$$

tengsizlikni hosil qilamiz, bunda $a_1 = M\sup_{\substack{\tau\in[o,L]\\ \varepsilon\in[0,\varepsilon_0]}}\left\|B(\tau,\varepsilon)\right\|, \quad a_2 = M\sup_{\substack{\tau\in[o,L]\\ \varepsilon\in[0,\varepsilon_0]}}\left\|r_m(\tau,\varepsilon)\right\|.$

Bunda birinchi $(0\le t\le d)$ qadam uchun teorema o'rinli. Haqiqatdan ham (3.2.21) tengsizlikdan (3.2.17) ifodaga asosan

$$\left\|y^{(k)}(t,\varepsilon)\right\| \le \varepsilon^m c_1, t\in[0,d], \qquad (3.2.22)$$

tengsiszlik kelib chiqadi, bunda $c_1 = a_2$. (3.2.16) ifodadan

$$\left\|\frac{d^k y(t,\varepsilon)}{dt^k}\right\| \le b_1\left\|y(t,\varepsilon)\right\| + b_2\left\|\dot{y}(t-\Delta(\tau),\varepsilon)\right\| + b_3\left\|y(t,\varepsilon)\right\| + b_4\varepsilon^{m+1} =$$

$$= (b_1+b_3)\left\|y(t,\varepsilon)\right\| + b_2\left\|\dot{y}(t-\Delta(\tau),\varepsilon)\right\| + \varepsilon^{m+1}b_4 \qquad (3.2.23)$$

ni aniqlaymiz, bunda

$$b_1 = \sup_{\substack{\tau\in[o,L]\\ \varepsilon\in[0,\varepsilon_0]}}\left\|A(\tau,\varepsilon)\right\|, b_2 = a_1 M^{-1}, \ b_4 = a_3(ML)^{-1} \qquad (3.2.24)$$

(3.2.22) va (3.2.23) larga asosan $\frac{dy(t,\varepsilon)}{dt}$ hosila uchun birinchi qadamda

$$\left\|\frac{dy^k(t,\varepsilon)}{dt^k}\right\| \le \varepsilon^m p_1 , \qquad (3.2.25)$$

bahoni olamiz, bunda $\kappa_1 = (b_1+b_3)a_1 + \varepsilon b_4$.

Yendi teoremani to'g'riligini ikkinchi $(d\le t\le 2d)$ qadam uchun isbolaymiz. (3.2.23) va (3.2.25) tengsizliklarni e'tiborga olib (3.2.21) ifodadan ikkinchi qadam uchun

$$\left\|y^k(t,\varepsilon)\right\| \le \varepsilon^{m-1}a_2 \qquad (3.2.26)$$

tengsizlikni hosil qilamiz, bunda $a_2 = (a_1L+\varepsilon_0)c_1 + \varepsilon_0 c_2$. Shunga o'xshash (3.2.23) ifodadan ikkinchi qadam uchun hosilani baholaymiz.

$$\left\| \frac{d^k y(t,\varepsilon)}{dt^k} \right\| \le \varepsilon^{m-1} p_1 \qquad (3.2.27)$$

bunda $k_2 = (b_1 + b_3)a_2 + \varepsilon_0^2 b_4 + \varepsilon_0 b_2 c_1$.

Matematik induksiya metodi asosida teoremaning tasdig'i ixtiyoriy qadam uchun umumlashtiriladi. Haqiqatdan ham, faraz qilaylik teorema *r*-chi $(r-1)d \le t \le rd$ qadam uchun o'rinli bo'lsin ya'ni ε parametrga bog'liq bo'lmagan c_2 va p_2 o'zgarmaslar mavjud bo'lsinki, ular uchun

$$\left\| y^k(t,\varepsilon) \right\| \le \varepsilon^{m+1-r} c_r$$

$$\left\| \frac{d^k y(t,\varepsilon)}{dt^k} \right\| \le \varepsilon^{m+1-r} p_r, \quad t \in [(r-1)d, > d] \qquad (3.2.28)$$

tengsizlik o'rinli bo'lsin. U holda *(r+1)*-chi $(rd \le t \le (r+1)d)$ qadam uchun (3.2.22) va (3.2.23) ifodalardan

$$\left\| y^k(t,\varepsilon) \right\| \le \varepsilon^{m-r} c_r;$$

$$\left\| \frac{d^k y(t,\varepsilon)}{dt^k} \right\| \le \varepsilon^{m-r} p_r, \; r = \overline{1, k-1} \qquad (3.2.29)$$

tengsizliklarni hosil qilamiz.

Bundan teorema ixtiyoriy $r(1 \le r \le [\frac{L}{\varepsilon a}])$ uchun o'rinli ekanligiga ishonch hosil qilamiz. 2-teorema isbotlandi.

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