

O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS  
TA‘LIM VAZIRLIGI



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5130100-MATEMATIKA TA‘LIM YO‘NALISHI

301-GURUHI TALABASI

Qo‘shnazarova Nafisaning Kompleks o‘zgaruvchili funksiyalar nazariyasi  
fanidan

*Aniq integralni chegirmalar yordamida hisoblash*  
mavzusida yozgan

# ***REFERATI***

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## **Kirish**

O‘zbekiston Respublikasining “Ta’lim to‘g‘risida“ gi Qonuni va “Qadrlar tayyorlash Milliy dasturi“ mazmunida barkamol shaxs va malakali mutaxassisni tarbiyalab voyaga yetkazish jarayonining mohiyati to‘laqonli ochib berilgandir. Malakali qadrlar tayyorlash jarayonining har bir bosqichi o‘zida ta’lim jarayonini samarali tashkil etish, uni yuqori bosqichlarga ko‘tarish, shu bilan birga jaxon ta’limi darajasiga yetkazish borasida muayyan maqsad va vazifalarni amalga oshirish lozim.

Yuqoridagi vazifalardan kelib chiqqan xolda biz yoshlar yurtimizda yaratilayotgan katta imkoniyatlardan to‘laqonli foydalangan xolda matematika soxasida qilingan yutuqlarni to‘la o‘zlashtirishimiz va ularni rivojlantirishga katta xissa qo‘shmog‘imiz zarur.

## 1-§. Chegirmalar va uni hisoblash.

Faraz qilaylik  $f(z)$  funksiya  $\{0 \leq |z-a| \leq \delta\}$  da golomorf bo'lsin, ya'ni  $a$  bu funksiyaning yakkalangan maxsus nuqtasi bo'lsin.

**1-ta'rif.** Ushbu

$$\frac{1}{2\pi i} \oint_{|z-a|=\rho} f(z) dz \quad (0 \leq \rho \leq \delta)$$

Integral  $f(z)$  funksiyaning  $a$  nuqtadagi chegirmasi deyiladi va

$\operatorname{res}_{z=a} f(z)$  kabi belgilanadi:

$$\operatorname{res}_{z=a} f(z) = \frac{1}{2\pi i} \oint_{|z-a|=\rho} f(z) dz.$$

Ravshanki,  $f(z)$  funksiya  $a$  nuqtada golomorf bo'lsa,

$\operatorname{res}_{z=a} f(z) = 0$  bo'ladi.

Aytaylik,  $f(z)$  funksiya  $\{r < |z| < \infty\}$  da golomorf bo'lsin.

**2-ta'rif.** Ushbu

$$-\frac{1}{2\pi i} \oint_{|z|=\rho} f(z) dz \quad (r < \rho)$$

Integral  $f(z)$  funksiyaning  $z = \infty$  nuqtadagi chegirmasi deyiladi va  $\operatorname{res}_{z=\infty} f(z)$  kabi belgilanadi:

$$\operatorname{res}_{z=\infty} f(z) = -\frac{1}{2\pi i} \int_{|z|=\rho} f(z) dz.$$

**1-teorema.** Agar  $f(z)$  funksiya  $\{0 < |z-a| < r\}$  sohada- halqada Loran qatori

$f(z) = \sum_{n=-\infty}^{\infty} c_n (z-a)^n$  ga yoyilgan bo'lsa, u holda  $f(z)$  funksiyaning  $z=a$

nuqtadagi chegirmasi  $c_{-1}$  ga teng. Yani

$\operatorname{res}_{z=a} f(z) = c_{-1}$  bo'ladi.

Agar  $f(z)$  funksiya  $\{r < |z| < \infty\}$  halqada Loran qatori

$f(z) = \sum_{n=-\infty}^{\infty} c_n z^n$  ga yoyilgan bo'lsa, u holda  $f(z)$  funksiyaning  $z=\infty$  nuqtadagi

chegirmasi  $-c_1$  ga teng, ya'ni

$\operatorname{res}_{z=\infty} f(z) = -c_1$  bo'ladi.

**2-teorema.** Agar  $f(z)$  funksiya  $C \setminus \{a_1, a_2, \dots, a_n\}$  to'plamda golomorf bo'lsa, u holda

$$\sum_{k=1}^n \operatorname{res}_{z=a_k} f(z) + \operatorname{res}_{z=\infty} f(z) = 0 \quad \text{bo'ladi.}$$

Endi funksiyaning chegirmalarini hisoblashda foydalanadigan formulalarni keltiramiz:

1) Agar  $z=a$  nuqta  $f(z)$  funksiyaning birinchi tartibli qutb nuqtasi bo'lsa,

$$\operatorname{res}_{z=a} f(z) = \lim_{z \rightarrow a} (z-a)f(z) \quad (1)$$

bo'ladi.

2) Agar  $f(z) = \frac{\varphi(z)}{\psi(z)}$  uchun  $\varphi(z)$  va  $\psi(z)$  funksiyalar  $a$  nuqtada golomorf bo'lib,  $\psi(a) = 0$ ,  $\psi'(a) \neq 0$  bo'lsa, u holda

$$\operatorname{res}_{z=a} f(z) = \frac{\varphi(a)}{\psi'(a)} \quad (2)$$

bo'ladi.

3) Agar  $z=a$  nuqta  $f(z)$  funksiyaning  $n$ -tartibli qutb nuqtasi bo'lsa,

$$\operatorname{res}_{z=a} f(z) = \frac{1}{(n-1)!} \lim_{z \rightarrow a} \frac{d^{n-1} [(z-a)^n f(z)]}{dz^{n-1}} \quad (3)$$

bo'ladi.

4) Agar  $z=\infty$  nuqtada  $f(z)$  funksiya golomorf bo'lsa,

$$\operatorname{res}_{z=\infty} f(z) = \lim_{z \rightarrow \infty} z[f(\infty) - f(z)] \quad (4)$$

bo'ladi.

5) Agar  $f(z) = \varphi\left(\frac{1}{z}\right)$  bo'lib,  $\varphi(z)$  funksiya  $z=0$  nuqtada golomorf bo'lsa,

$$\operatorname{res}_{z=\infty} f(z) = -\varphi'(0) \quad (5)$$

bo'ladi.

**1-misol.** Ushbu

$$f(z) = \frac{e^z}{(z-1)^2}$$

funksiyaning  $z=1$  nuqtadagi chegirmasini toping.

Berilgan funksiyaning  $z=1$  nuqtaning teshik atrofi  $0 < |z-1| < \varepsilon$  da  $(z-1)$  ning darajalari bo'yicha Loran qatoriga yoyamiz:

$$f(z) = \frac{e^z}{(z-1)^2} = \frac{e}{(z-1)^2} e^{z-1} = \frac{e}{(z-1)^2} \left[ 1 + (z-1) + \frac{(z-1)^2}{2!} + \frac{(z-1)^3}{3!} + \dots \right] =$$

$$= \frac{e}{(z-1)^2} + \frac{e}{z-1} + \frac{e}{2} + \frac{e(z-1)}{3!} + \dots$$

Bu yoyilmadan  $\tilde{n}_{-1} = e$  bo'lishi kelib chiqadi.

1-teoremadan foydalanib, berilgan funksiyaning  $z=1$  nuqtadagi chegirmasi

$$\operatorname{res}_{z=1} f(z) = \operatorname{res}_{z=1} \frac{e^z}{(z-1)^2} = e \quad \text{bo'lishini topamiz.}$$

**2-misol.** Ushbu

$$f(z) = z^2 \sin \frac{\pi}{z} \quad \text{funksiyaning } z = \infty \text{ nuqtadagi chegirmasini toping.}$$

Berilgan funksiyani  $z$  ning darajalari bo'yicha Loran qatoriga yoyamiz:

$$f(z) = z^2 \sin \frac{\pi}{z} = z^2 \left[ \frac{\pi}{z} - \frac{\left(\frac{\pi}{z}\right)^3}{3!} + \frac{\left(\frac{\pi}{z}\right)^5}{5!} - \dots \right] =$$

$$= \pi z - \frac{\pi^3}{6} \cdot \frac{1}{z} + \frac{\pi^5}{5!} \cdot \frac{1}{z^3} - \dots$$

Demak,  $\tilde{n}_{-1} = -\frac{\pi^3}{6}$  va funksiyaning  $z = \infty$  nuqtadagi chegirmasi

$$\operatorname{res}_{z=\infty} z^2 \sin \frac{\pi}{z} = \frac{\pi^3}{6} \quad \text{bo'ladi.}$$

## 2-§. Integrallarni chegirma yordamida hisoblash

Chegirmalar yordamida turli integrallarni hisoblash mumkin. Bunda quyida keltiriladigan teorema muhim rol o'ynaydi.

**3-teorema.** (Koshi teoremasi). Faraz qilaylik,

1)  $f(z)$  funksiya

$$D \setminus \{a_1, a_2, \dots, a_n\}$$

Sohada golomorf ( $D \subset C, a_1, a_2, \dots, a_n \in D$ )

2)  $f(z)$  funksiya sohani chegarasigacha aniqlangan va  $\bar{D} \setminus \{a_1, a_2, \dots, a_n\}$  da uzluksiz.

3)  $\partial D$  to'g'irlanuvchi yopiq kontur bo'lsin. U holda

$$\int_{\partial D} f(z) dz = 2\pi i \sum_{k=1}^n \operatorname{res} f(z) \quad (6)$$

formula o'rinlidir.

**Izoh:** (6) formula  $\infty \in D$  bo'lgan hol uchun o'rinlidir. Faqat bu holda  $z = \infty$  ni  $f(z)$  uchun maxsus nuqta deb hisoblash hamda  $\partial D$  chiziq orientatsiyani soat strelkasi yo'nalishida olish kifoyadir.

Yuqorida keltirilgan Koshi teoremasidan foydalanib yopiq kontur bo'yicha olingan integrallarni hisoblaymiz.

**3-misol.** Ushbu

$$\oint_{|z|=3} \frac{1}{z^3 + 4z} dz \quad \text{integralni hisoblang.}$$

Bu holda integral ostidagi funksiya

$$f(z) = \frac{1}{z^3 + 4z}$$

Integrallash konturi  $\{z \in C : |z| = 3\}$  aylana, D soha esa  $D = \{z \in C : |z| < 3\}$  doiradan iborat.  $f(z)$  funksiyani

$$f(z) = \frac{1}{z^3 + 4z} = \frac{1}{z(z^2 + 4z)} = \frac{1}{z(z + 2i)(z - 2i)}$$

ko'rinishda yozib,  $a_1 = 0, a_2 = -2i, a_3 = 2i$  lar funksiyaning 1-tartibli qutb nuqtalari ekanini aniqlaymiz. Ravshanki,  $a_1, a_2, a_3$

mahsus nuqtalar  $D$  sohaga tegishli bo'ladi. 3-teoremani barcha shartlari bajarilib, shu teoremaga ko'ra

$$\oint_{|z|=3} \frac{1}{z^3+4z} dz = 2\pi i \sum_{n=1}^3 \operatorname{res}_{z=a_n} \frac{1}{z^3+4z}$$

bo'ladi.

O'ng tamondagi chegirmalarni (1) formulaga ko'ra hisoblaymiz:

$$\operatorname{res}_{z=0} f(z) = \operatorname{res}_{z=0} \frac{1}{z^3+4z} = \lim_{z \rightarrow 0} z \cdot \frac{1}{z^3+4z} = \frac{1}{4},$$

$$\operatorname{res}_{z=2i} f(z) = \operatorname{res}_{z=2i} \frac{1}{z^3+4z} = \lim_{z \rightarrow 2i} \frac{1}{z(z-2i)} = -\frac{1}{8},$$

$$\operatorname{res}_{z=2i} f(z) = \operatorname{res}_{z=2i} \frac{1}{z^3+4z} = \lim_{z \rightarrow 2i} \frac{1}{z(z+2i)} = -\frac{1}{8}.$$

Natijada

$$\oint_{|z|=3} \frac{1}{z^3+4z} dz = 2\pi i \left( \frac{1}{4} - \frac{1}{8} - \frac{1}{8} \right) = 0$$

bo'lishini topamiz.

**4-misol.** Ushbu

$$\oint_{\gamma} \frac{1}{z^4+1} dz$$

integralni hisoblang, bunda  $\gamma: x^2+y^2=2x$  aylanadan iborat.

Ravshanki,

$$x^2+y^2=2x \Rightarrow (x-1)^2+y^2=1 \Rightarrow \{z \in C: |z-1|=1\},$$

Dsoha esa  $D=\{|z-1|<1\}$  doiradir.

Endi  $f(z)=\frac{1}{z^4+1}$  funksiyaning  $D$  sohaga tegishli bo'lgan mahsus nuqtalarini topamiz:

$$z^4+1=0 \Rightarrow z = \sqrt[4]{-1} = \sqrt[4]{1 \cdot (\cos \pi + i \sin \pi)} \Rightarrow z_k = \cos \frac{\pi+2k\pi}{4} + i \sin \frac{\pi+2k\pi}{4}$$

$$(k=0, 1, 2, 3). \quad z_0, z_1, z_2, z_3 \quad \text{mahsus nuqtalardan}$$

$$z_0 = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}(1+i),$$

$$z_3 = \cos \frac{\pi+6\pi}{4} + i \sin \frac{\pi+6\pi}{4} = \frac{\sqrt{2}}{2}(1-i)$$

lar  $D$  sohaga tegishli bo'ladi. Shuni e'tiborga olib, (6) formuladan

$$\int_{\lambda} \frac{1}{z^4+1} dz = 2\pi i \left( \operatorname{res}_{z=z_0} \frac{1}{z^4+1} + \operatorname{res}_{z=z_3} \frac{1}{z^4+1} \right)$$

bo'lishini topamiz. Bu tenglikning o'ng tomonidan chegirmalrni hisoblaymiz:

$$\operatorname{res}_{z=z_0} \frac{1}{z^4+1} = \frac{1}{(z^4+1)'}\Big|_{z=z_0} = \frac{1}{4z_0^3},$$

$$\operatorname{res}_{z=z_3} \frac{1}{(z^4+1)'}\Big|_{z=z_3} = \frac{1}{4z_3^3}.$$

Natijada

$$\oint_{\gamma} \frac{1}{z^4+1} dz = 2\pi i \frac{1}{4} \left( \frac{1}{z_0^3} + \frac{1}{z_3^3} \right)$$

bo'ladi.

Agar

$$\frac{1}{z_0^3} + \frac{1}{z_3^3} = \frac{1}{\left[\frac{\sqrt{2}}{2}(1+i)\right]^3} + \frac{1}{\left[\frac{\sqrt{2}}{2}(1-i)\right]^3} = \frac{4}{\sqrt{2}} \left[ \frac{(1-i)^3 + (1+i)^3}{((1+i)(1-i))^3} \right] = -\sqrt{2}$$

bo'lishini hisobga olsak, unda

$$\oint_{\gamma} \frac{1}{z^4+1} dz = -\frac{\sqrt{2}}{2} \pi i$$

ekanini topamiz.

**5-misol.** Ushbu

$$\oint_{|z|=2} \frac{1}{(z-3)(z^5-1)} dz$$

integralni hisoblang.

(6) formulaga ko'ra  $f(z) = \frac{1}{(z-3)(z^5-1)}$  uchun

$$\oint_{z=2} f(z) dz = -2\pi i [\operatorname{res}_{z=3} f(z) + \operatorname{res}_{z=\infty} f(z)]$$

Bo'ladi. Bu tenglikning o'ng tomonidagi chegirmalarni xisoblaymiz:

$$\operatorname{res}_{z=3} f(z) = \lim_{z \rightarrow 3} (z-3)f(z) = \lim_{z \rightarrow 3} \frac{1}{z^5-1} = \frac{1}{242}$$

Agar

$$f(z) = \frac{1}{(z-3)(z^5-1)} = \frac{1}{z^6} \frac{1}{\left(1-\frac{3}{z}\right)\left(1-\frac{1}{z^5}\right)}$$

Ekanlikini etiborga olsak unda  $z = \infty$  nuqta  $f(z)$  funksiyaning 6-tartibli noli bo'lishini aniqlaymiz. Bu funksiyaning Loran qatori

$$f(z) = \frac{1}{z^6} + \frac{c_{-7}}{z^7} + \frac{c_{-8}}{z^8} + \dots$$

Bo'lib  $c_{-1} = 0$  bo'ladi. Demak,

$$\operatorname{res}_{z=\infty} f(z) = 0$$

Shunday qilib

$$\oint_{z=2} \frac{1}{(z-3)(z^5-1)} dz = 2\pi i \left( \frac{1}{242} + 0 \right) = -\frac{\pi i}{121}$$

**6-misol:** Ushbu

$$\frac{1}{2\pi i} \oint_{|z|=r} z^k e^{\frac{z}{2}} dz \quad (k \text{ butun son, } r > 0)$$

Integralni xisoblang.

Bu integral (6) formulaga ko'ra

$$\frac{1}{2\pi i} \oint_{|z|=r} z^k e^{\frac{z}{2}} dz = \operatorname{res}_{z=0} z^k e^{\frac{z}{2}}$$

Bo'ladi. Chegirmani hisoblash uchun  $f(z) = z^k e^{\frac{z}{2}}$  funksiyani  $z=0$  nuqtaning o'yilgan atrofida loran qatoriga yoyamiz:

$$\begin{aligned} f(z) &= z^k e^{\frac{z}{2}} = z^k \left[ 1 + \frac{z}{2} + \frac{1}{2!} \cdot \frac{z^2}{z^2} + \dots + \frac{1}{(k+1)!} \cdot \frac{z^{k+1}}{z^{k+1}} + \dots \right] = \\ &= z^k + 2z^{k-1} + \frac{2^2}{2!} z^{k-2} + \dots + \frac{2^{k+1}}{(k+1)!} \cdot \frac{1}{z} + \frac{2^{k+2}}{(k+2)!} \cdot \frac{1}{z^2} + \dots \end{aligned}$$

Bu tenglikdan

$$c_{-1} = \begin{cases} \frac{2^{k+1}}{(k+1)!}, & \text{agar } k \geq -1 \text{ bo'lsa} \\ 0, & \text{agar } k < -1 \text{ bo'lsa} \end{cases}$$

bo'lishi kelib chiqadi.

Endi

$$c_{-1} = \operatorname{res}_{z=0} z^k e^{\frac{z}{2}}$$

bo'lishini e'tiborga olib,

$$\frac{1}{2\pi i} \oint_{|z|=r} z^k e^{\frac{1}{z}} dz = \begin{cases} \frac{2^{k+1}}{(k+1)!}, \text{ agar } k \geq -1. \text{ bo'lsa} \\ 0, \text{ agar } k < -1. \text{ bo'lsa} \end{cases}$$

bo'lishini topamiz.

### 3-§. Aniq integrallarni chegirmalar yordamida hisoblash

Aniq integrallarni chegirmalar yordamida hisoblash mumkin. Bunda aniq integrallar kompleks o'zgaruvchili funksiyaning kontur bo'yicha olingan integraliga keltirib topiladi.

1)  $\int_0^{2\pi} R(\cos x, \sin x) dx$  ko'rinishdagi integrallarni hisoblash.

Ushbu

$$I = \int_0^{2\pi} R(\cos x, \sin x) dx \quad (7)$$

Integral berilgan bo'lib, uni hisoblash talab etilsin, bunda  $R(\cos x, \sin x)$ -  
 $\cos x$  va  $\sin x$  larning ratsional funksiyasi va u  $[0, 2\pi]$  da uzluksiz.

Eyler formulasiga ko'ra

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}, \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

bo'lishini e'tiborga olib, so'ng

$$z = e^{ix}$$

deb belgilashni kiritsak, unda

$$x \in [0, 2\pi] \Rightarrow \{z \in \mathbb{C} : |z|=1\},$$

$$\cos x = \frac{1}{2} \left( z + \frac{1}{z} \right), \quad \sin x = \frac{1}{2i} \left( z - \frac{1}{z} \right),$$

$$dx = \frac{1}{iz} dz$$

bo'lib, berilgan (7) integral quyidagicha

$$I = \int_0^{2\pi} R(\cos x, \sin x) dx = \oint_{|z|=1} \tilde{R}(z) dz$$

bo'ladi, bunda

$$\tilde{R}(z) = \frac{1}{iz} = R\left(\frac{1}{2}\left(z + \frac{1}{z}\right), \frac{1}{2i}\left(z - \frac{1}{z}\right)\right).$$

**7-misol.** Ushbu

$$\int_0^{2\pi} \frac{dx}{\cos x - 2}$$

integralni hisoblang.

Bu integralda  $e^{ix} = z$  belgilashni kiritamiz. Unda

$$\int_0^{2\pi} \frac{dx}{\cos x - 2} = \frac{1}{i} \oint_{|z|=1} \frac{\frac{1}{z} dz}{\frac{1}{2}\left(z + \frac{1}{z}\right)} = \frac{2}{i} \oint_{|z|=1} \frac{dz}{z^2 - 4z + 1}$$

bo'ladi. Integral ostidagi

$$f(z) = \frac{1}{z^2 - 4z + 1}$$

funksiya uchun  $z_1 = 2 + \sqrt{3}$  va  $z_2 = 2 - \sqrt{3}$  nuqtalar 1-tartibli qutb nuqtalari

bo'lib, ulardan  $z_2 = 2 - \sqrt{3}$  nuqta  $D = \{z \in C : |z| < 1\}$  sohaga tegishli bo'ladi:

$z_2 = 2 - \sqrt{3} \in D$ . Unda Koshining chegirmalar haqidagi teoremasiga asosan

$$\oint_{|z|=1} f(z) dz = \oint_{|z|=1} \frac{1}{z^2 - 4z + 1} dz = 2\pi i \operatorname{res}_{z=z_2} f(z).$$

Funksiya chegirmasini hisoblaymiz:

$$\begin{aligned} \operatorname{res}_{z=z_2} f(z) &= \lim_{z \rightarrow z_2} (z - z_1) f(z) = \lim_{z \rightarrow z_2} (z - z_2) \cdot \frac{1}{(z - z_1)(z - z_2)} = \\ &= \frac{1}{z_2 - z_1} = -\frac{1}{2\sqrt{3}}. \end{aligned}$$

Demak,

$$\int_0^{2\pi} \frac{1}{\cos x - 2} dx = \frac{2}{i} \oint_{|z|=1} \frac{1}{z^2 - 4z + 1} dz = 2\pi i \frac{2}{i} \left(-\frac{1}{2\sqrt{3}}\right) = -\frac{2\pi}{\sqrt{3}}.$$

2) Xosmas integrallarni hisoblash.

Chegirmalar nazariyasidan foydalanib xosmas integrallarni ham hisoblash mumkin. Bu quyidagi teoremaga asoslangan.

**4-teorema.**  $f(z)$  funksiya  $\{z \in C : \text{Im } z > 0\}$  sohaning chekli sondagi mahsus nuqtalardan tashqari barcha nuqtalarida golomorf bo'lib, uning chegarasida uzluksiz bo'lsin. Agar

$$\lim_{r \rightarrow \infty} \int_{\gamma_r} f(z) dz = 0 \quad (\gamma_r = \{|z| = r, 0 \leq \arg z \leq \pi\}) \quad (8)$$

bo'lsa, u holda

$$\int_{-\infty}^{\infty} f(x) dx = 2\pi i \sum_{\text{Im } z_k > 0} \text{res } f(z) \quad (9)$$

bo'ladi.

Bu teoremadagi (8) shartning bajarilishini ko'rsatishda quyidagi lemmalardan foydalaniladi.

**1-lemma.** (Jordan lemmasi). Agar

$$\lim_{r \rightarrow \infty} r \cdot \max_{z \in \gamma_r} |f(z)| = 0 \quad (10)$$

bo'lsa,

$$\lim_{r \rightarrow \infty} \int_{\gamma_r} f(z) dz = 0 \quad (11)$$

bo'ladi.

**2-lemma.** (Jordan lemmasi). Agar

$$\lim_{r \rightarrow \infty} \max_{z \in \gamma_r} |f(z)| = 0 \quad (12)$$

bo'lsa, u holda  $\forall \lambda > 0$  uchun

$$\lim_{r \rightarrow \infty} \int_{\gamma_r} f(z) e^{i\lambda z} dz = 0 \quad (13)$$

bo'ladi.

## Foydalanilgan adabiyotlar

1. Б.В.Шабат. Введение в комплексный анализ. Часть 1,2. М.Наука.1985.
2. Ю.В. Сидоров, М.В Федорюн, М.И. Шабукин. Лекции по теории функции комплексного переменного. М.Наука. 1982.
3. И.И. Привалов. Введение в теорию функции комплексного переменного. М.Госиздат. физ-мат литературы. 1977.
4. Sh. Maqsudov, M.Salohiddinov, S.Sirojiddinov. // Kompleks o`zgaruvchining funksiyalari nazariyasi. “O`QITUVCHI” nashriyoti.Toshkent-1979.
5. Sadullaev A.S Ko`p argumentli golomorf funksiyalar. Urganch 2004
6. <http://www.roman.by/>,<http://www.vargen.mephi.ru/>,  
<http://www.ziyonet.uz/>