THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Computer Simulation of the Coupled Dynamic Thermoelasticity Problem for a Two-Dimensional Isotropic Bodies

Qalandarov Aziz Abduqayumovich
Senior Lecturer, Department of Information Technologies,
Gulistan State University, Uzbekistan
Khaldjigitov Abduvali Abdisamatovich
Professor, Tashkent University of Information Technologies, Uzbekistan

Abstract:
The two-dimensional coupled thermodynamic boundary problem for isotropic bodies is formulated. The explicit and implicit two-dimensional finite difference equations are constructed. The received explicit and implicit schemes are solved using the elimination method and recurrence formulas, respectively. The comparison of the obtained numerical results by the two methods shows a good coincidence.

Keywords:
Thermo-elasticity, coupled problem, thermal conductivity, difference equations, explicit scheme, implicit scheme, grid method, elimination method

1. Introduction

Investigation the deformation process of elastic and plastic bodies, taking into account the temperature fields plays an important role in many applications of science and engineering problems associated with the heating of various structures and their elements under the influence of thermomechanical loads.
The aforementioned thermomechanical deformation process of solids may be described by the coupled and uncoupled thermoelastic or thermoplastic boundary value problems.
In recent years, researchers paid special attention to the adequate mathematical modeling of the coupled thermomechanical deformation processes of isotropic and anisotropic materials for related tasks. The coupled thermodynamic problem, firstly was considered by Biot [1] in 1956.
Further, these studies were continued in the works of Lord-Shulman (1967) [2], Muller (1972) [3], Green-Laws (1972) [4], Youssef (2006) [5], Aboudi (1985) [6] and others.
In general, the thermodynamic linear coupled boundary value problem for elastic bodies consists of the motion equations in conjunction with the heat equation.

Coupled thermoelastic boundary problem, generally consisting of three hyperbolic and a parabolic heat equation depending on the three components of the displacement vector and temperature is a complex and may be solved analytically only in some one-dimensional particular cases or for specially selected shapes of solids and coordinate systems. This paper deals with the numerical solution of two-dimensional dynamical coupled problems of thermoelasticity for isotropic bodies.
Using epy finite difference methods are constructed explicit and implicit schemes. For the numerical solution of the finite difference equations the elimination method and recurrence formulas, in the case of explicit schemes, are applied. Comparison, of the obtained using two methods numerical results and graphs provides the reliability and validity of the received results.
Problem. The coupled thermodynamic thermoelastic boundary value problem consists of the motion equations problem for isotropic materials [7]

$$\sigma_{ij,j} + X_i = \rho \ddot{u}_i \qquad (1)$$

Duhamel- Neuman thermoelasticity constitutive relations

$$\sigma_{ij} = \lambda\theta\delta_{ij} + \mu\varepsilon_{ij} - \alpha(3\lambda+2\mu)(T-T_0)\delta_{ij} \qquad (2)$$

Cauchy relations

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

heat equation for isotropic materials

$$\lambda_0 T_{,ii} - c_\varepsilon \dot{T} - \alpha(3\lambda + 2\mu)T_0\dot{\varepsilon}_{ii} = 0 \tag{4}$$

with corresponding initial

$$u_i\big|_{t=t_0} = \varphi_i \ , \ \dot{u}_i\big|_{t=t_0} = \psi_i \ , \ T\big|_{t=t_0} = T_0 \tag{5}$$

and boundary conditions

$$u_i\big|_{\Sigma_1} = u_i^0 \ , \ T\big|_{\Sigma} = \overline{T}_0 \ , \ \sigma_{ij} n_j\big|_{\Sigma_2} = S_i^0 \tag{6}$$

where, c_ε – heat at a constant deformation, α – thermal expansion coefficient, λ_0 – the heat flow coefficient, σ_{ij} – stress tensor, ε_{ij} – strain tensor, u_i – displacement, T – temperature, X_i – volume force, λ, μ – Lame constants, θ – spherical part of strain tensor, ρ – density of the body, δ_{ij} – kronecker symbol.

We consider equations eqs. (1 - 6) in two-dimensional case.

Substituting eq. (3) into eq. (2) and obtained in eq. (1) we have the equation of motion for displacement

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} - (3\lambda + 2\mu)\alpha\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \tag{7}$$

$$(\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + \mu\frac{\partial^2 v}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} - (3\lambda + 2\mu)\alpha\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2} \tag{8}$$

and 2D heat equations

$$\lambda_0\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - c_\varepsilon\frac{\partial T}{\partial t} - \alpha(3\lambda + 2\mu)T_0\left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 v}{\partial y \partial t}\right) = 0 \tag{9}$$

with initial

$$u(x,y,t)\big|_{t=0} = \phi_1, \ \frac{\partial u}{\partial t}\bigg|_{t=0} = \psi_1, \ v(x,y,t)\big|_{t=0} = \phi_2, \ \frac{\partial v}{\partial t}\bigg|_{t=0} = \psi_2, \ T(x,y,t)\big|_{t=0} = T_0$$

and boundary conditions in 2D case

$$u(x,y,t)\big|_{x=0} = u_0 \ , \ u(x,y,t)\big|_{x=\ell_1} = \overline{u}_0 \ , \ u(x,y,t)\big|_{y=0} = u_0' \ , \ u(x,y,t)\big|_{y=\ell_2} = \overline{u}_0'$$

$$v(x,y,t)\big|_{x=0} = v_0 \ , \ v(x,y,t)\big|_{x=\ell 1} = \overline{v}_0 \ , \ v(x,y,t)\big|_{y=0} = v_0' \ , \ v(x,y,t)\big|_{y=\ell_2} = \overline{v}_0'$$

$$T(x,y,t)\big|_{x=0} = T_1(t) \ , \ T(x,y,t)\big|_{x=\ell 1} = T_2(t) \ , \ T(x,y,t)\big|_{y=0} = T_1'(t) \ , \ T(x,y,t)\big|_{y=\ell_2} = T_2'(t)$$

2. Finite Difference Equations

Considering in the area $t \ge 0$, $0 \le x \le l_1$, $0 \le y \le l_2$ two sets of parallel lines $x = ih_1$ $(i = \overline{0,n})$, $y = jh_2$ $(j = \overline{0,n})$, $t = k\tau$ $(k = 0,1,2,\ldots)$ and replacing the derivatives in eqs.(7)-(9) by difference quotients, we obtain [8]

$$\left.\begin{aligned}
&(\lambda+2\mu)\frac{u_{i+1,j}^k - 2u_{i,j}^k + u_{i-1,j}^k}{h_1^2} + (\lambda+\mu)\frac{v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k}{4h_1h_2} + \\
&\mu\frac{u_{i,j+1}^k - 2u_{i,j}^k + u_{i,j-1}^k}{h_2^2} - \gamma\frac{T_{i+1,j}^k - T_{i-1,j}^k}{2h_1} = \rho\frac{u_{i,j}^{k+1} - 2u_{i,j}^k + u_{i,j}^{k-1}}{\tau^2} \\
&(\lambda+2\mu)\frac{v_{i,j+1}^k - 2v_{i,j}^k + v_{i,j-1}^k}{h_1^2} + (\lambda+\mu)\frac{u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k}{4h_1h_2} + \\
&\mu\frac{v_{i+1,j}^k - 2v_{i,j}^k + v_{i-1,j}^k}{h_2^2} - \gamma\frac{T_{i,j+1}^k - T_{i,j-1}^k}{2h_2} = \rho\frac{v_{i,j}^{k+1} - 2v_{i,j}^k + v_{i,j}^{k-1}}{\tau^2}
\end{aligned}\right\} \tag{10}$$

and heat equation

$$\lambda_0\left(\frac{T^k_{i+1,j}-Tu^k_{i,j}+T^k_{i-1,j}}{h_1^2}+\frac{T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1}}{h_2^2}\right)-c_\varepsilon\frac{T^{k+1}_{i,j}-T^k_{i,j}}{\tau}-$$
$$-\gamma T^k_{i,j}\left(\frac{u^{k+1}_{i+1,j}-u^{k+1}_{i-1,j}-u^{k-1}_{i+1,j}+u^{k-1}_{i-1,j}}{4h_1\tau}+\frac{v^{k+1}_{i,j+1}-v^{k+1}_{i,j-1}-v^{k-1}_{i,j+1}+v^{k-1}_{i,j-1}}{4h_2\tau}\right)=0 \quad (11)$$

Solving the discreet (10) and (11) about $u^{k+1}_{i,j}$, $v^{k+1}_{i,j}$, $T^{k+1}_{i,j}$ respectively, we get [9]

$$u^{k+1}_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u^k_{i+1,j}-2u^k_{i,j}+u^k_{i-1,j}}{h_1^2}+\mu\frac{u^k_{i,j+1}-2u^k_{i,j}+u^k_{i,j-1}}{h_2^2}+\right.$$
$$\left.+(\lambda+\mu)\frac{v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1}}{4h_1h_2}-\gamma\frac{T^k_{i+1,j}-T^k_{i-1,j}}{2h_1}\right)+2u^k_{i,j}-u^{k-1}_{i,j} \quad (12)$$

$$v^{k+1}_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v^k_{i,j+1}-2v^k_{i,j}+u^k_{i,j-1}}{h_2^2}+\mu\frac{v^k_{i+1,j}-2v^k_{i,j}+v^k_{i-1,j}}{h_1^2}+\right.$$
$$\left.+(\lambda+\mu)\frac{u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1}}{4h_1h_2}-\gamma\frac{T^k_{i,j+1}-T^k_{i,j-1}}{2h_2}\right)+2v^k_{i,j}-v^{k-1}_{i,j} \quad (13)$$

$$T^{k+1}_{i,j}=\frac{\tau}{c_\varepsilon}\left(\lambda_0\left(\frac{T^k_{i+1,j}-Tu^k_{i,j}+T^k_{i-1,j}}{h_1^2}+\frac{T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1}}{h_2^2}\right)-\right.$$
$$\left.-\gamma T^k_{i,j}\left(\frac{u^{k+1}_{i+1,j}-u^{k+1}_{i-1,j}-u^{k-1}_{i+1,j}+u^{k-1}_{i-1,j}}{4h_1\tau}+\frac{v^{k+1}_{i,j+1}-v^{k+1}_{i,j-1}-v^{k-1}_{i,j+1}+v^{k-1}_{i,j-1}}{4h_2\tau}\right)\right)+T^k_{i,j} \quad (14)$$

As can be seen, eq. (12,13) and eq. (14) allow to find the function values $u(x,y,t)$, $v(x,y,t)$, $T(x,y,t)$ at the layer t^{j+1} using the given values of these functions at the two previous layers. The values of $u(x,y,t)$ and $v(x,y,t)$ on two primary layers $k=0$ and $k=1$ we can find from the initial conditions

$u^0_{i,j}=\varphi_1(x_i,y_j)$, $v^0_{i,j}=\varphi_2(x_i,y_j)$, $T^0_{i,j}=T_0$

Rewriting eq. (13) for k=0

$$u^1_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u^0_{i+1,j}-2u^0_{i,j}+u^0_{i-1,j}}{h_1^2}+\mu\frac{u^0_{i,j+1}-2u^0_{i,j}+u^0_{i,j-1}}{h_2^2}+\right.$$
$$\left.+(\lambda+\mu)\frac{v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1}}{4h_1h_2}-\gamma\frac{T^0_{i+1,j}-T^0_{i-1,j}}{2h_1}\right)+2u^0_{i,j}+2\tau\psi_1 \quad (15)$$

and the initial condition $\left.\frac{\partial u}{\partial t}\right|_{t=0}=\psi_1$ in the following form

$$\frac{u^1_{i,j,k}-u^{-1}_{i,j,k}}{2h_1}=\psi_1(x_i,y_j,z_k)$$

or

$$u^1_{i,j,k}=2h_1\psi_1(x_i,y_j,z_k)+u^{-1}_{i,j,k} \quad (16)$$

eliminating $u^{-1}_{i,j,k}$ from the eqs. (15,16) we take

$$u^1_{i,j}=\frac{1}{2}\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u^0_{i+1,j}-2u^0_{i,j}+u^0_{i-1,j}}{h_1^2}+\mu\frac{u^0_{i,j+1}-2u^0_{i,j}+u^0_{i,j-1}}{h_2^2}+\right.\right.$$
$$\left.\left.+(\lambda+\mu)\frac{v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1}}{4h_1h_2}-\gamma\frac{T^0_{i+1,j}-T^0_{i-1,j}}{2h_1}\right)+2u^0_{i,j}+2\tau\psi_1\right) \quad (17)$$

In the same way from the eq. (13) we can find the values of the function $v(x,y,t)$

$$v_{i,j}^{1}=\frac{1}{2}\left(\frac{\tau^{2}}{\rho}\left((\lambda+2\mu)\frac{v_{i,j+1}^{0}-2v_{i,j}^{0}+u_{i,j-1}^{0}}{h_{2}^{2}}+\mu\frac{v_{i+1,j}^{0}-2v_{i,j}^{0}+v_{i-1,j}^{0}}{h_{1}^{2}}+\right.\right.$$
$$\left.\left.+(\lambda+\mu)\frac{u_{i+1,j+1}^{0}-u_{i-1,j+1}^{0}-u_{i+1,j-1}^{0}+u_{i-1,j-1}^{0}}{4h_{1}h_{2}}-\gamma\frac{T_{i,j+1}^{0}-T_{i,j-1}^{0}}{2h_{2}}\right)+2v_{i,j}^{0}+2\tau\psi_{2}\right) \quad (18)$$

Replacing mixed derivatives in eq. (11) with another finite difference we can get relation for values of $T(x,y,t)$ on the first layer

$$T_{i,j}^{1}=\frac{\tau}{c_{\varepsilon}}\left(\lambda_{0}\left(\frac{T_{i+1,j}^{0}-2T_{i,j}^{0}+T_{i-1,j}^{0}}{h_{1}^{2}}+\frac{T_{i,j+1}^{0}-2T_{i,j}^{0}+T_{i,j-1}^{0}}{h_{2}^{2}}\right)-\right.$$
$$\left.-\gamma T_{i,j}^{0}\left(\frac{u_{i+1,j}^{1}-u_{i-1,j}^{1}-u_{i+1,j}^{0}+u_{i-1,j}^{0}}{2h_{1}\tau}+\frac{v_{i,j+1}^{1}-v_{i,j-1}^{1}-v_{i,j+1}^{0}+v_{i,j-1}^{0}}{2h_{2}\tau}\right)\right)+T_{i,j}^{0} \quad (19)$$

In the above, the explicit schemes have been used. Now consider the implicit schemes for the numerical solution of the coupled thermoelasticity problem. In this case, the eqs. (7-9) take the following form

$$(\lambda+2\mu)\frac{u_{i+1,j}^{k+1}-2u_{i,j}^{k+1}+u_{i-1,j}^{k+1}}{h_{1}^{2}}+\mu\frac{u_{i,j+1}^{k}-2u_{i,j}^{k}+u_{i,j-1}^{k}}{h_{2}^{2}}+$$
$$+(\lambda+\mu)\frac{v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k}}{4h_{1}h_{2}}- \quad (20)$$
$$-(3\lambda+2\mu)\alpha_{T}\frac{T_{i+1,j}^{k}-T_{i-1,j}^{k}}{2h_{1}}=\rho\frac{u_{i,j}^{k+1}-2u_{i,j}^{k}+u_{i,j}^{k-1}}{\tau^{2}}$$

$$(\lambda+2\mu)\frac{v_{i,j+1}^{k}-2v_{i,j}^{k}+u_{i,j-1}^{k}}{h_{2}^{2}}+\mu\frac{v_{i+1,j}^{k+1}-2v_{i,j}^{k+1}+v_{i-1,j}^{k+1}}{h_{1}^{2}}+$$
$$+(\lambda+\mu)\frac{u_{i+1,j+1}^{k}-u_{i-1,j+1}^{k}-u_{i+1,j-1}^{k}+u_{i-1,j-1}^{k}}{4h_{1}h_{2}}- \quad (21)$$
$$-(3\lambda+2\mu)\alpha_{T}\frac{T_{i,j+1}^{k}-T_{i,j-1}^{k}}{2h_{2}}=\rho\frac{v_{i,j}^{k+1}-2v_{i,j}^{k}+v_{i,j}^{k-1}}{\tau^{2}}$$

$$\lambda_{0}\left(\frac{T_{i+1,j}^{k+1}-2T_{i,j}^{k+1}+T_{i-1,j}^{k+1}}{h_{1}^{2}}+\frac{T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k}}{h_{2}^{2}}\right)-$$
$$-C_{\varepsilon}\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{\tau}-T_{0}(3\lambda+2\mu)\alpha_{T}* \quad (22)$$
$$*\left(\frac{u_{i+1,j}^{k+1}-u_{i-1,j}^{k+1}-u_{i+1,j}^{k-1}+u_{i-1,j}^{k-1}}{4h_{1}\tau}+\frac{v_{i,j+1}^{k+1}-v_{i,j-1}^{k+1}-v_{i,j+1}^{k-1}+v_{i,j-1}^{k-1}}{4h_{2}\tau}\right)=0$$

The difference eq. (20) can be written as follows

$$a_{i}u_{i+1,j}^{k+1}+b_{i}u_{i,j}^{k+1}+c_{i}u_{i-1,j}^{k+1}=f_{i} \quad (23)$$

where $a_{i}=\frac{\lambda+2\mu}{h_{1}^{2}}$, $b_{i}=-\frac{\rho}{\tau^{2}}-\frac{2(\lambda+2\mu)}{h_{1}^{2}}$, $c_{i}=\frac{\lambda+2\mu}{h_{1}^{2}}$,

$$f_{i}=\rho\frac{-2u_{i,j}^{k}+u_{i,j}^{k-1}}{\tau^{2}}-\left(\mu\frac{u_{i,j+1}^{k}-2u_{i,j}^{k}+u_{i,j-1}^{k}}{h_{2}^{2}}+\right.$$
$$+(\lambda+\mu)\frac{v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k}}{4h_{1}h_{2}}-$$
$$\left.-(3\lambda+2\mu)\alpha_{T}\frac{T_{i+1,j}^{k}-T_{i-1,j}^{k}}{2h_{1}}\right)$$

The finite difference eq. (21) may also be reduced to the following form

$$a_i v_{i+1,j}^{k+1} + b_i v_{i,j}^{k+1} + c_i v_{i-1,j}^{k+1} = f_i \tag{24}$$

where $a_i = \frac{\mu}{h_1^2}$, $b_i = -\frac{\rho}{\tau^2} - \frac{2\mu}{h_1^2}$, $c_i = \frac{\mu}{h_1^2}$,

$$f_i = \rho \frac{-2v_{i,j}^k + v_{i,j}^{k-1}}{\tau^2} - ((\lambda + 2\mu)\frac{v_{i,j+1}^k - 2v_{i,j}^k + v_{i,j-1}^k}{h_2^2} +$$
$$+(\lambda+\mu)\frac{u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k}{4h_1h_2} -$$
$$-(3\lambda + 2\mu)\alpha_T \frac{T_{i,j+1}^k - T_{i,j-1}^k}{2h_2})$$

and the discreet heat eq. (22) takes following form

$$a_i T_{i+1,j}^{k+1} + b_i T_{i,j}^{k+1} + c_i T_{i-1,j}^{k+1} = f_i \tag{25}$$

where

$$a_i = \frac{\lambda_0}{h_1^2}, b_i = -\frac{2\lambda_0}{h_1^2} - \frac{C_\varepsilon}{\tau}, \; c_i = \frac{\lambda_0}{h_1^2},$$
$$f_i = T_0(3\lambda + 2\mu)\alpha_T \left(\frac{u_{i+1,j}^{k+1} - u_{i-1,j}^{k+1} - u_{i+1,j}^{k-1} + u_{i-1,j}^{k-1}}{4h_1\tau} + \right.$$
$$\left. + \frac{v_{i,j+1}^{k+1} - v_{i,j-1}^{k+1} - v_{i,j+1}^{k-1} + v_{i,j-1}^{k-1}}{4h_2\tau} \right) -$$
$$-C_\varepsilon \frac{T_{i,j}^k}{\tau} - \lambda_0 \frac{T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k}{h_2^2}$$

The values of the unknown functions $u(x,y,t)$, $v(x,y,t)$ and $T(x,y,t)$ on the first two layers, can be find using the initial conditions and the formulas (17)-(19). The values of the unknowns in the subsequent layers may be find solving the eqs. (23-25) by the elimination method with account the given initial and boundary conditions.

3. Numerical Tests

As an example, is solved the coupled thermoelasticity problem (1-6) using the recurrence formulas and elimination method under the below constants, initial and boundary conditions:

$\lambda_0 = 0.06$, $\lambda = 0.78$, $\mu = 0.5$, $\alpha = 0.05$, $C_\varepsilon = 3.4$, $T_0 = 15$, $h_1 = 0.1$, $h_2 = 0.1$, $\tau = 0.01$,

$\rho = 0.86$, $\ell_1 = \ell_2 = 1$

$u(x,y,t)\big|_{t=0} = 0$, $v(x,y,t)\big|_{t=0} = 0$, $T(x,y,t)\big|_{t=0} = T_0 + T_0\sin(\pi x(i))\sin(\pi y(j))$, $u(x,y,t)\big|_{\Gamma} = 0$, $v(x,y,t)\big|_{\Gamma} = 0$

, $T(x,y,t)\big|_{\Gamma} = T_0$

where Γ- boundary of the body.

The following Figures 1-8 show the distribution of the displacement components $u(x,y,t)$, $v(x,y,t)$ and temperature $T(x,y,t)$ along the different coordinate axes. Notice, that every unknown value is found using the elimination method (green line) and recurrence formulas (red line)

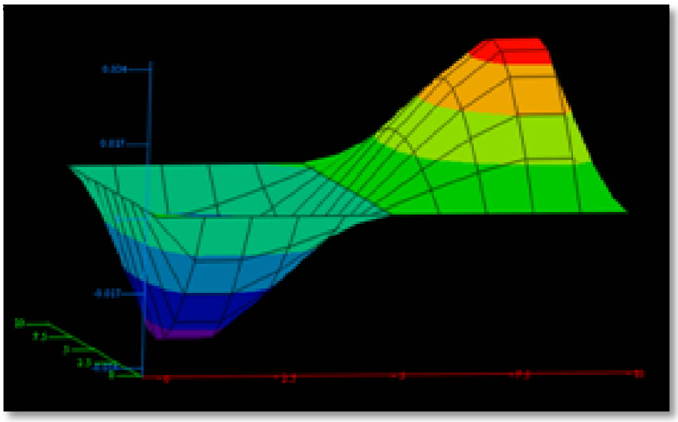

Figure 1: Distribution of the Displacement $u(x,y,t)$ *Along The X-Y-Axis at* $t=0.1$ *(Recurrence Formula)*

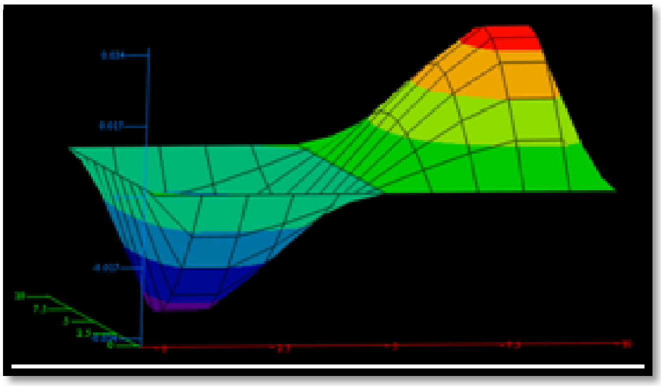

Figure 2: Distribution of the Displacement $u(x,y,t)$ *Along the X-Y-Axis at* $t=0.1$ *(Elimination Method)*

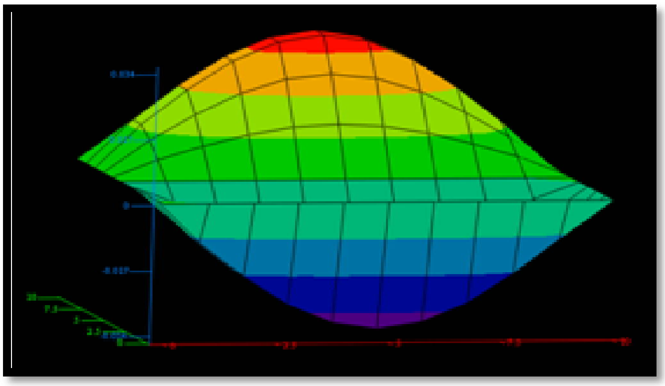

Figure 3: Distribution of the Displacement $v(x,y,t)$ *Along The X-Y-Axis at* $t=0.1$ *(Recurrence Formula)*

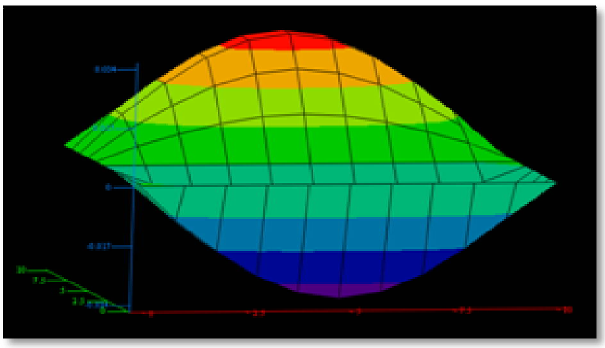

Figure 4: Distribution of the Displacement $v(x,y,t)$ *Along The X-Y-Axis at* $t=0.1$ *(Elimination Method)*

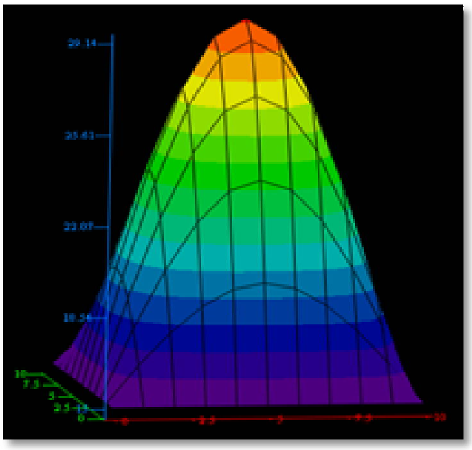

Figure 5: Distribution of the Displacement $T(x, y, t)$ *Along The X-Y-Axis at* $t = 0.1$ *(Recurrence Formula)*

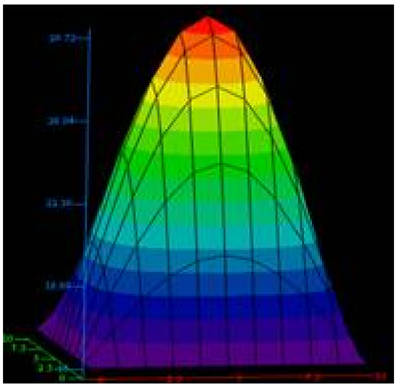

Figure 6: Distribution of the Displacement $T(x, y, t)$ *Along The X-Y-Axis at* $t = 0.1$ *(Elimination Method)*

x / y	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	-0.0114	-0.0113	-0.0083	-0.0044
0.2	0	-0.0202	-0.0201	-0.0148	-0.0078
0.3	0	-0.0277	-0.0276	-0.0204	-0.0107
0.4	0	-0.0325	-0.0324	-0.0239	-0.0126
0.5	0	-0.0342	-0.0341	-0.0251	-0.0132

Table 1: The Values of the Displacement Component $u(x, y, t)$ *on the Time Layer K = 10 Found by Elimination Method*

x / y	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	-0.0107	-0.0110	-0.0082	-0.0044
0.2	0	-0.0190	-0.0198	-0.0148	-0.0078
0.3	0	-0.0262	-0.0272	-0.0203	-0.0107
0.4	0	-0.0308	-0.0320	-0.0239	-0.0126
0.5	0	-0.0324	-0.0337	-0.0252	-0.0133

Table 2: The Values of the Displacement Component $u(x, y, t)$ *on the Time Layer K = 10 Found by Recurrence Formulas*

y \ x	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	-0.0114	-0.0202	-0.0277	-0.0325
0.2	0	-0.0113	-0.0201	-0.0276	-0.0325
0.3	0	-0.0083	-0.0148	-0.0204	-0.0239
0.4	0	-0.0043	-0.0078	-0.0107	-0.0126
0.5	0	0	0	0	0

Table 3: The Values of the Displacement Component $v(x, y, t)$ On The Time Layer K = 9 Found By Elimination Method

y \ x	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	-0.0114	-0.0203	-0.0279	-0.0327
0.2	0	-0.0113	-0.0204	-0.0279	-0.0328
0.3	0	-0.0084	-0.0150	-0.0206	-0.0242
0.4	0	-0.0044	-0.0079	-0.0109	-0.0128
0.5	0	0	0	0	0

Table 4: The Values of the Displacement Component $v(x, y, t)$ on the Time Layer K = 9 Found By Recurrence Formulas

y \ x	0	0.1	0.2	0.3	0.4
0	15	15	15	15	15
0.1	15	16.4575	17.6921	18.6966	19.3473
0.2	15	17.6921	19.9623	21.7972	22.9824
0.3	15	18.6966	21.7972	24.3000	25.9154
0.4	15	19.3473	22.9824	25.9154	27.8080
0.5	15	19.5720	23.3908	26.4717	28.4596

Table 5: Temperature Values $T(x, y, t)$ on the Time Layer K = 10 Found by Elimination Method

y \ x	0	0.1	0.2	0.3	0.4
0	15	15	15	15	15
0.1	15	16.4187	17.6320	18.6103	19.2426
0.2	15	17.7020	20.0221	21.8895	23.0962
0.3	15	18.7630	21.9928	24.5929	26.2731
0.4	15	19.4593	23.2857	26.3666	28.3574
0.5	15	19.7195	23.7684	27.0287	29.1354

Table 6: Temperature Values $T(x, y, t)$ on the Time Layer K = 10 Found by Recurrence Formulas

Comparison of the numerical values of displacement components and temperature obtained by the elimination method (Table N«a») and recurrence formulas (Table N«b») numbers shows a good coincidence.

4. Conclusion

For two-dimensional coupled thermodynamic boundary value problem, the explicit and implicit finite difference equations are constructed. The received explicit and implicit schemes are solved using the recurrence formulas and elimination methods, respectively. The comparison of the obtained numerical results by the two methods shows a good coincidence. The above-mentioned methods may be easily applied for numerical solving of the three-dimensional thermodynamic coupled problems for elastic and plastic bodies.

5. References

i. Biot M.A. Thermoelasticity and Irreversible thermodynamics. J. of Appl. Physics. Vol. 27, №3, p.240-253, 1956
ii. Lord H.W. and Shulman Y., (1967). A generalized dynamical theory of thermoelasticity,J. Mech. Phys. Solids, Vol. 15 (5), pp. 299-309.
iii. Muller, I.M.: The coldness, a universal function in thermoelastic bodies. ¨ Arch. Rational Mech. Anal., 319, (1971), 41.

iv. Green A.E., Laws N.: On the entropy production inequality. Arch. Rat. Mech. Anal., 45, (1972), 47-53.
v. Youssef H.M. Theory of two-temperature-generalized thermoelasticity. IMA J. of APPL. Math. 2006, 71, p.383-390
vi. Aboudi J. The effective thermomechanical behavior of inelastic fiber-reinforced materials. Int. J. Engng. Sci., 1985. 23, No 7. - P. 773-787,
vii. Nowacki W., Dynamic problems of thermoelasticity, M.Mir, 1970, 256 p. (in Russian)
viii. Samarskii A., Nikolaev E. Numerical methods of grid equation, Birkhauser Verlag, Berlin, 1989.
ix. Khaldjigitov A., Qalandarov A., Nik M.A.Asri Long, Eshquvatov Z. Numerical solution of 1D and 2D thermoelastic coupled problems. International journal of modern physics. Vol. 9, pp. 503-510, 2012.