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MAXSUS INTEGRAL TENGSIZLIKLAR VA ULARNING TADBIQLARI

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DISSERTATSIYA

Ilmiy rahbar:

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Mundarija.

Kirish

Integral tengsizlik va uni difFerensial tenglama yechimini o'rganishga tadbiq qilish g'oyasi birinchi bo'lib Gronuoll tomonidan 1919-yilda amalga oshirilgan.

Gronuoll tomonidan integral tengsizlik haqidagi quyidagi teorema isbotlangan.

Teorema. Agar $\mathrm{I}(a \le t < \infty)$ oraliqda aniqlangan uzluksiz va manfiy bo'lmagan $u(t)$ funksiya va o'zgarmas $c \ge 0,\ k \ge 0$ sonlar uchun

$$u(t) \le c + k\int_a^t u(s)ds$$

tengsizlik o'rinli bo'lsa, u holda shu I yarim o'qda

$$u(t) \le ce^{k(t-a)}$$

tengsizlik o'rinli bo'ladi.

Xususiy holda, agar $c = 0$ bo'lsa, $u(t) \equiv 0$ bo'ladi.

Bu teorema yordamida $y' = f(t, y)$ differensial tenglama uchun $y(a) = c$ boshlang'ich shartni qanoatlantiruvchi $y(t)$ yechimining yagonaligi va boshlang'ich shartdan uzluksiz bog'liqligi isbotlangan.

R. Bellman tomonidan bu tengsizlik

$$u(t) \le c + \int_a^t K(s)u(s)ds \qquad (*)$$

ko'rinishda umumlashtirilib uning yechimi

$$u(t) \le ce^{\int_a^t K(s)ds}$$

ko'rinishda topilgan va differensial tenglama uchun Koshi masalasi yechimining turg'unligi haqidagi teoremalarni isbotlashga qo'llanilgan. Gronuoll-Bellman tengsizligi turg'unlik nazariyasining fundamental natijasi sifatida tan olindi va u turli yo'nalishlarda umumlashtirildi. Shunday qilib integral tengsizliklar ko'plab olimlarning va tadqiqotchilarning diqqat e'tiborini o'ziga tortdi va bu yo'nalish mustaqil nazariya bo'lib rivojlandi.

Integral tengsizliklar haqida ko’plab teoremalar O’zbekistonlik va chet el olimlari tomonidan isbotlandi.

Bunday teoremalar T.Nurimov, A.N.Filatov va L.V.Sharovalarning “Интегральные неравенства” (1990 й) nomli monografiyasida jamlangan va turli umumlashmalari keltirilgan[2].

Mazkur magistrlik dissertatsiyasida integral tengsizliklar haqida yangi teoremalar isbotlangan bo’lib ular BuxoroDU va SamDU ilmiy tadqiqot axborotnomalarida chop etildi va ularning tadbiqlari haqida yangi teoremalar isbotlandi.

-tadqiqot mavzusining dolzarbligi: Integral tengsizliklar integral, differensial, integro-differensial va xususiy hosilali differensial tenglamalar nazariyasida muhim tadbiqlarga ega bo’lishini e’tiborga olgan holda shu yo’nalishda yangi va uni to’ldirishga xizmat qiluvchi

1. $u(t) \le \alpha + \beta \int_{-\infty}^{t} \left[\lambda(s)u(s) + \int_{-\infty}^{s} \mu(s,\tau)u(\tau)d\tau \right] ds$

2. $u(t) \le ct^{-\alpha} + m \int_{-\infty}^{t} \left[\frac{au(s)}{s} + \int_{-\infty}^{s} \frac{bu(\tau)}{s\tau} d\tau \right] ds$

3. $u(t) \le f(t) + \int_{0}^{t} \frac{m}{(t-s)^{\alpha}} u(s) ds \quad (0 < \alpha < 1)$

ko’rinishdagi uchta maxsus tengsizliklar haqidagi teoremalar isbotlanadi. So’ngra bu teoremalarning integro-differensial tenglama va Vol’terra integral tenglamasi yechimlarini tadqiq qilishga qo’llab yangi natijalarga erishish dolzarb izlanishlardan hisoblanadi.

-magistrlik dissertatsiyasining maqsadi va vazifalari: Magistrlik dissertatsiyasining maqsadi integral tengsizliklar nazariyasining chiziqli maxsus integral tengsizliklarga oid natijalarni batafsil o’rgangan holda bu nazariyaga yangi tengsizliklar haqidagi yangi natijalarni qo’shish orqali uni kengaytirishga urinishlar olib borib ma’lum qo’shimchalar kiritishdan iborat va bu tengsizliklar yordamida chiziqli bo’lmagan integro-differensial va chiziqli bo’lmagan Vol’terra tenglamalari yechimlarining mavjudligi va yagonaligi hamda ba'zi hossalarini o'rganishga tadbiq etish asosiy maqsad qilib olingan.

-tadqiqot ob'ekti va predmeti: Tadqiqot ob'ekti integral tenglamalar va tengsizliklar haqidagi hozirgi kunda mavjud bo'lgan ilmiy xulosalarni o'rganib ular haqidagi natijalarni rivojlantirib yangi xulosalarga kelishdan iborat. Bunga maxsus integral tengsizliklarni integral va integro-differensial tenglamalarga qo'llab yangi teoremalar isbotlashga erishish.

-tadqiqot usullari: Ma'lumki integral tengsizliklar haqidagi teoremalarni isbotlashning uchta usuli 1) belgilash kiritish usuli, 2) ketma-ket o'rniga qo'yish usuli, 3) ketma-ket yaqinlashish usuli mavjud. Mazkur ishda bu usullardan foydalaniladi. Isbotlangan integral tengsizliklar esa integro-differensial va integral tenglamalarni tadqiq qilishda eng qulay apparat vazifasini bajaradi.

-tadqiqotning ilmiy yangiligi: Integral va integro-differensial tenglamalarning yadrolari o'zgarmas koeffitsentli Lipshis shartini qanoatlantirgan xollar batafsil o'rganilgan. Ishda Lipshis shartidagi koeffitsent $(-\infty, t_0]$ oraliqda integrallanuvchi, $(-\infty, t_0]$ oraliqda integrallanmaydigan va $K(t,s)$ koeffitsent $\frac{m}{(t-s)^{\alpha}}$ $(0<\alpha<1)$ ko'rinishda bo'lgan xollar ilgari o'rganilmagan xollar sarasiga kiradi. Bunday xollarni o'rganishda yuqorida aytib o'tilgan integral tengsizliklar muhim ahamiyatga ega bo'lib yangi natijalarga olib keladi.

-muammoning ishlab chiqilish darajasi: Mazkur ishda

$$\frac{dx(t)}{dt} = F(t, x(t)) + \int_{-\infty}^{t} K(t, s, x(s))ds \qquad (1)$$

$$\lim_{t \to -\infty} x(t) = c \qquad (2)$$

va

$$x(t) = f(t) + \int_{0}^{t} K(t, s, x(s))ds \qquad (3)$$

ko'rinishdagi tenglamalar yechimlarining mavjudligi, yagonaligi, aniq va taqribiy yechimlari orasidagi farqni baholash, yechimlarning boshlang'ich shartlar va tenglamada qatnashgan yadrolardan uzluksiz bog'liqligi, yechimning parametrga uzluksiz bog'liqligi va parametr bo'yicha differensiallanuvchanligi haqidagi teoremalar isbotlanadi. Bu teoremalarni isbotlash integral tengsizliklarni tadbiq etish orqali amalga oshiriladi.

-tadqiqot natijalarining ilmiy va amaliy ahamiyati: Tadqiqot ishi fundamental ilmiy ishlariga talluqli bo'lib bunga $(1)-(2)$ masala va (3) tenglamalarning yechimlarini o'rganish uchun 1) 2) 3) integral tengsizliklardan foydalaniladi. Bunga chekli oraliqda o'rganilgan tengsizliklar chegaralari cheksiz bo'lgan 1) 2) holga umumlashtiriladi, yadrosi uzluksiz funksiyadan iborat bo'lgan $(*)$ ko'rinishdagi integral tengsizlik yadrosi $\frac{m}{(t-s)^{\alpha}}$ $(0<\alpha<1)$ ko'rinishda bo'lgan 3) holga ko'chiriladi. Bu holda ketma-ket o'rniga qo'yish usuli ishlatiladi va bundan tashqari Gamma va Betta funksiyalarning hossalaridan unumli foydalaniladi.

-natijalarning joriy qilinishi: Tadqiqot ishining natijalari universitet talabalarining 2013 va 2014 yillarda o'tkazilgan ilmiy konferensiyalarida ma'ruza qilingan. Asosiy natijalar

1. Singulyar integral tengsizlik va uning tadbiqlari.
2. Gronuooll tipidagi singulyar integral tengsizlik.
3. Integro-differensial tenglama yechimining parametrga nisbatan uzluksizligi va differensiallanuvchanligi.

-dissertatsiyaning tuzilishi va hajmi: Dissertatsiya kirish va uch bobdan iborat bo'lib, kirish qismida integral tengsizliklar nazariyasining qisqacha rivojlanish tarixi, birinchi bobda integral tenglama va tengsizlik haqida boshlang'ich ma'lumotlar, ikkinchi bobda esa maxsus integral tengsizliklar haqidagi teoremalar va ularning isbotlari, uchinchi bobda ikkinchi bobda keltirilgan asosiy natijalarning tadbiqlari keltirilgan. Ya'ni, chiziqli bo'lmagan integro-differensial tenglamalar uchun Koshining limitik masalasi va chiziqli bo'lmagan Vol'terra tenglamasi o'rganilgan. Har bir bob uchun xulosalar keltirilgan. Dissertatsiya xulosa bilan birga 72 betdan iborat.

-dissertatsiyaning asosiy mazmuni: Maxsus integral tengsizliklar haqida yangi teoremalar isbot qilish va ular yordamida chiziqli bo'lmagan integro-differensial tenglama va Vol'terra tenglamasi yechimlari haqida yangi teoremalar isbot qilishdan iborat.

III-bobda quyidagi uchta maxsus integral tengsizlikka oid yangi teoremalar isbotlangan.

1-teorema. Faraz qilaylik $I(-\infty < t \le t_0)$ oraliqda aniqlangan, uzluksiz $u(t) \ge 0$, $\lambda(t) \ge 0$, $P(-\infty < t \le t_0, -\infty < s \le t)$ sohada aniqlangan uzluksiz $\mu(t,s) \ge 0$ funksiya hamda o'zgarmas $\alpha \ge 0$, $\beta \ge 0$ sonlar uchun

$$u(t) \le \alpha + \beta \int_{-\infty}^{t} \left[\lambda(s)u(s) + \int_{-\infty}^{s} \mu(s,\tau)u(\tau) \right] ds$$

tengsizlik o'rinli bo'lsin, bu yerda

$$\int_{-\infty}^{t} \lambda(s)ds < \infty, \qquad \int_{-\infty}^{t} \lambda(s) \int_{-\infty}^{s} \mu(s,\tau)d\tau ds < \infty.$$

U holda shu I oraliqda

$$u(t) \le \alpha \exp\left\{ \beta \int_{-\infty}^{t} \left[\lambda(s) + \int_{-\infty}^{s} \mu(s,\tau)d\tau \right] ds \right\}$$

tengsizlik ham o'rinli bo'ladi.

2-teorema. Agar $I(-\infty < t < 0)$ oraliqda aniqlangan uzluksiz $0 \le u(t) \le kt^{-\alpha}, (k \ge 1)$ funksiya va o'zgarmas $m \ge 0$ $a < 0, b > 0$ $\alpha = \dfrac{\lambda}{\mu} > 1, (\lambda,\mu) = 1$ (λ-juft son) sonlar uchun

$$u(t) \le ct^{-\alpha} + m \int_{-\infty}^{t} \left[\frac{au(s)}{s} + \int_{-\infty}^{s} \frac{bu(\tau)}{s\tau} d\tau \right] ds$$

tengsizlik o'rinli bo'lsa, u holda

$$m\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right) < 1$$

bo'lganda ushbu

$$u(t) \le \frac{ct^{-\alpha}}{1 - m(\frac{b}{\alpha^2} - \frac{a}{\alpha})}$$

tengsizlik o'rinli bo'ladi. Agar $c = 0$ bo'lsa $u(t) \equiv 0$ bo'ladi.

3-Teorema. Faraz qilaylik $u(t) \ge 0$ funksiya $[0, t_0]$ segmentda aniqlangan uzluksiz funksiya bo'lib, ushbu

$$u(t) \le f(t) + \int_0^t \frac{m}{(t-s)^\alpha} u(s) ds$$

integral tengsizlikni qanoatlantirsin.

Bu yerda $m \ge 0$ $0 < \alpha < 1$ sonlar, $f(t) \ge 0$ $[0, t_0]$ segmentda uzluksiz va

$$\max_{t \in [0, t_0]} f(t) = c \quad c \ge 0 \quad \text{son}$$

shartni qanoatlantirsin, u holda quyidagi tengsizlik o'rinli bo'ladi.

$$u(t) \le f(t) + C \sum_{n=1}^{\infty} \frac{\left[t^{1-\alpha} m \Gamma(1-\alpha) \right]^n}{n(1-\alpha)\Gamma(n(1-\alpha))}$$

va $f(t) = 0$ bo'lsa $u(t) = 0$ bo'ladi.

Uchinchi bob ikkinchi bobda isbotlangan integral tengsizliklar haqidagi yangi teoremalarning tadbiqlariga bag'ishlanadi.

Ushbu

$$\frac{dx}{dt} = F(t, x) + \int_{-\infty}^{t} K(t, s, x) ds \tag{1}$$

yarim o'qda chiziqli bo'lmagan integro-differensial tenglamani qaraymiz. Bu tenglamaning

$$\lim_{t \to -\infty} x(t) = c \tag{2}$$

boshlang'ich shartni qanoatlantiruvchi yechimini topish bilan shug'illanamiz.

1-teorema. Faraz qilaylik

1) $F(t, x)$ funksiya $D = \{(t, x) : -\infty < t < t_0, |x| < \infty\}$ sohada $K(t, s, x)$ funksiya esa $Q = \{(t, s, x) : -\infty < t < t_0, -\infty < s < t, |x| < \infty\}$ sohada aniqlangan va uzluksiz funksiyalar bo'lib, bundan tashqari

$$\int_{-\infty}^{t} |F(s,c)|\,ds < \infty, \qquad \int_{-\infty}^{t}\int_{-\infty}^{s} |K(s,\tau,c)|\,d\tau ds < \infty$$

shartlar bajarilsin.

2) $F(t,x)$ va $K(t,s,x)$ funksiyalar x argumenti bo'yicha Lipshis shartini qanoatlantiradi, ya'ni

$$|F(t,x_1) - F(t,x_2)| \le \lambda(t)|x_1 - x_2|$$

$$|K(t,s,x_1) - K(t,s,x_2)| \le \mu(t,s)|x_1 - x_2|$$

bu yerda

$$\int_{-\infty}^{t} \lambda(t)ds < \infty, \qquad \int_{-\infty}^{t}\int_{-\infty}^{s} \mu(s,\tau)d\tau ds < \infty$$

deb hisoblanadi. U holda (1) va

$$x(t) = c + \int_{-\infty}^{t} F(t,x(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s,\tau,x(\tau))d\tau ds \qquad (3)$$

masala $I(-\infty,t_0)$ oraliqda yagona uzluksiz yechimga ega bo'ladi. Bundan tashqari aniq va taqribiy yechimlar orasidagi farq quyidagicha baholanadi.

$$|x(t) - x_n(t)| \le \frac{a(t_0)b^{n-1}(t_0)}{n!}\exp\left\{\int_{-\infty}^{t}\left[\lambda(t) + \int_{-\infty}^{t}\mu(t,s)d\tau\right]ds\right\}$$

2-teorema. Faraz qilaylik (3) tenglamada $F(t,x)$ va $K(t,s,x)$ funksiyalar quyidagi shartlarni qanoatlantirsin:

1) $F(t,x)$ funksiya

$$D = \{(t,x): -\infty < t \le t_0 < 0 \quad |x| < \infty\}$$

sohada, $K(t,s,x)$ funksiya esa

$$Q = \{(t,s,x): -\infty < t \le t_0 < 0, -\infty < s \le t, \quad |x| < \infty\}$$

sohada aniqlangan va uzluksiz funksiyalar bo'lib, bundan tashqari

$$\left(\int_{-\infty}^{t} |s|^q |F(s,c)|^q ds\right)^{\frac{1}{q}} \le Mt^{-\frac{1}{q}}, \quad \left(\int_{-\infty}^{t}\int_{-\infty}^{s} |s|^q |\tau|^q |K(s,\tau,c)|^q d\tau ds\right)^{\frac{1}{q}} \le N \quad M > 0,\ N > 0$$

shartlar bajarilsin, bu yerda $\frac{1}{q} + \frac{1}{p} = 1$ bo'lib, $p > 1$ toq son deb hisoblaymiz.

2) $|F(t,x_1) - F(t,x_2)| \le \frac{a}{t}|x_1 - x_2|, \quad a < 0$;

$$\left|K(t,s,x_1)-K(t,s,x_2)\right|\le \frac{b}{ts}\left|x_1-x_2\right|,\quad b>0$$

U holda (3) integral tenglama yoki bari bir $(1)-(2)$ masala yagona uzluksiz yechimga ega bo'ladi.

Ushbu

$$\varphi(t)=f(t)+\int_0^t K(t,s,\varphi(s))ds \qquad (4)$$

chiziqli bo'lmagan Vol'terra tenglamasini qaraymiz. Bu yerda $f(t)$ $[0,t_0]$ segmentda $K(t,s,\varphi)$ esa $D(0\le t\le t_0, 0\le s\le t, |\varphi|<\infty)$ sohada aniqlangan va uzluksiz funksiyalar deb hisoblaymiz. Ma'lumki $K(t,s,\varphi)$ funksiya D sohada φ argumenti bo'yicha $|K(t,s,\varphi_1)-K(t,s,\varphi_2)|\le L(t,s)|\varphi_1-\varphi_2|$ Lipshis shartini qanoatlantirib $L(t,s)$ $D_0(0\le t\le t_0, 0\le s\le t)$ sohada uzluksiz bo'lsa (4) tenglama $[0,t_0]$ segmentda uzluksiz yechimga ega bo'ladi. Biz quyida $L(t,s)=\frac{m}{(t-s)^\alpha}$ $0<\alpha<1$, holda (4) tenglama yechimining mavjudligi, yagonaligi va ba'zi xossalarini o'rganamiz. Bunda $L(t,s)$ funksiya $t=s$ to'g'ri chiziq bo'ylab uzulishga ega, lekin,

$$\int_0^t \frac{m}{(t-s)^\alpha}=\frac{m}{1-\alpha}t^{1-\alpha}$$

ya'ni $[0,t_0]$ segmentda integrallanuvchi.

3-Teorema. Faraz qilaylik $f(t)$ funksiya $[0,t_0]$ segmentda aniqlangan va uzluksiz, $K(t,s,\varphi)$ funksiya esa $D(0\le t\le s,\quad 0\le s\le t,\quad |\varphi|<\infty)$ sohada aniqlangan bo'lib φ argumenti bo'yicha Lipshis shartini qanoatlantirsin:

$$|K(t,s,\varphi)-K(t,s,\psi)|\le \frac{m}{(t-s)^\alpha}|\varphi-\psi|,\quad 0<\alpha<1,\quad m\ge 0. \qquad (5)$$

Bundan tashqari

$$\int_0^{t_0}|K(t,s,f(s))|ds\le C, \qquad (6)$$

$C\ge 0$ son.

U holda (4) tenglama $[0,t_0]$ segmentda yagona uzluksiz yechimga ega bo'ladi.

4-teorema. Faraz qilaylik

1) $F(t,x)$ funksiya

$$D=\{(t,x):-\infty<s\le t\le t_0,|x|<\infty\}$$

sohada, $K(t,s,x)$ funksiya esa

$$Q=\{(t,s,x):-\infty<t\le t_0,-\infty<s\le t\le t_0,|x|<\infty\}$$

sohada aniqlangan va

$$\int_{-\infty}^{t}|F(s,c)|ds<\infty,\int_{-\infty}^{t}\int_{-\infty}^{s}|K(s,\tau,c)|d\tau ds<\infty$$

shartlar bajarilsin.

2) $F(t,x)$ va $K(t,s,x)$ funksiyalar x argumenti bo'yicha Lipshis shartini qanoatlantirsin

$$|F(t,x_1)-F(t,x_2)|\le\lambda(t)|x_1-x_2|$$

$$|K(t,s,x_1)-K(t,s,x_2)|\le\mu(t,s)|x_1-x_2|$$

bu yerda

$$\int_{-\infty}^{t}\lambda(s)ds<\infty,\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(t,s)d\tau ds<\infty$$

deb hisoblanadi. U holda shunday $\delta>0$ va $\varepsilon>0$ sonlar mavjud bo'ladiki, $(-\infty,t_0]$ oraliqda

$$|c-\rho|<\delta$$

bo'lganda

$$|x(t)-y(t)|<\varepsilon$$

tengsizlik ham o'rinli bo'ladi.

5-Teorema. Agar $F(s,x)$, $\bar{F}(s,x)$, $K(t,s,x)$, $\bar{K}(t,s,x)$ funksiyalar $D=\{(s,x),-\infty<t\le t_0,|x|<\infty$ va $Q(t,s,x):-\infty<t\le t_0,-\infty<s\le t,|x|<\infty\}$ sohada aniqlangan va uzluksiz funksiyalar bo'lib, ular uchun yechimning mavjudligi va yagonaligi haqidagi 3-teoremaning barcha shartlari bajarilsin. U holda shunday $\varepsilon>0$, $\delta>0$ sonlar mavjud bo'ladiki D va Q sohalarda

$$|F(t,\varphi)-\bar{F}(t,\psi)|<\delta a(t)$$

$$|K(t,s,\varphi)-\bar{K}(t,s,\psi)|<\delta b(t,s)$$

tengsizliklar bajarilganda $(-\infty, t_0]$ oraliqda

$$|\varphi(t) - \psi(t)| < \varepsilon$$

tengsizlik o'rinli bo'ladi. Bu yerda

$$\int_{-\infty}^{t} a(s)ds < +\infty$$

va

$$\int_{-\infty}^{t} \int_{-\infty}^{s} b(s,\tau)d\tau ds < +\infty.$$

Ushbu

$$\frac{dx}{dt} = F(t,x,\lambda) + \int_{-\infty}^{t} K(t,s,x,\lambda)ds \qquad (7)$$

yarim o'qda chiziqli bo'lmagan integro-differensial tenglamani qaraymiz. Bu tenglamaning $x = x(t,\lambda)$ yechimi mavjudligi va yagonaligi haqidagi teorema 3-§ da isbotlangan. Osongina ko'rsatish mumkinki (7) tenglamaning

$$\lim_{t \to -\infty} x(t,\lambda) = c \qquad (8)$$

shartni qanoatlantiruvchi $x(t,\lambda)$ yechimini topish masalasi ushbu

$$x(t) = \int_{-\infty}^{t} F(s,x(s),\lambda)ds + \int_{-\infty}^{t} \int_{-\infty}^{s} K(s,\tau,x(\tau),\lambda)d\tau ds \qquad (9)$$

integral tenglamaning $x = \varphi(t,\lambda)$ yechimini topishga keltiriladi. Shuning uchun quyidagi (9) tenglama yechimini tekshirish bilan shug'illanamiz. Bu tenglamada qatnashayotgan F va K funksiyalar quyidagi shartlarni qanoatlantirsin deb hisoblaymiz:

a) $F(s,x,\lambda)$ funksiya

$$D_\lambda = \{(s,x,\lambda): -\infty < s < t_0, -\infty < \tau < s, |x| < \infty, \lambda_1 \le \lambda \le \lambda_2\},$$

$K(s,\tau,x,\lambda)$ funksiya esa

$$Q_\lambda = \{(s,\tau,x,\lambda): -\infty < s \prec t_0, -\infty < \tau \le s \prec t_0, |x| < \infty, \lambda_1 \le \lambda \le \lambda_2\}$$

sohada aniqlangan va uzluksiz.

b) $F(s,x,\lambda)$ funksiya D_λ sohada $F_x^{'}$ va $F_\lambda^{'}$ uzluksiz hosilalarga ega hamda

$$|F_x^{'}(s,x,\lambda)| \le \lambda(s), \qquad |F_\lambda^{'}(s,x,\lambda)| \le \lambda(s)$$

bundan tashqari

v) $\left|F_x^{'}(s,x_1,\lambda)-F_x^{'}(s,x_2,\lambda)\right|\le\lambda(s)\left|x_1-x_2\right|,\quad \left|F_x^{'}(s,x,\lambda_1)-F_x^{'}(s,x,\lambda_2)\right|\le\lambda(s)\left|\lambda_1-\lambda_2\right|$

g) $K(s,\tau,x,\lambda)$ funksiya

$$Q_\lambda=\left\{(s,\tau,x,\lambda):-\infty\le s\le a,-\infty\le\tau\le s,|x|\prec\infty,\lambda_1\le\lambda_\xi\le\lambda_2\right\}$$

sohada $K_x^{'}$, $K_\lambda^{'}$ o'zgarmas hosilalarga ega hamda

$$\left|K_x^{'}(s,\tau,x,\lambda)\right|\le\mu(s,\tau),\qquad \left|K_\lambda^{'}(s,\tau,x,\lambda)\right|\le\mu(s,\tau).$$

Bundan tashqari

$$\left|K_x^{'}(s,\tau,x_1,\lambda)-K_x^{'}(s,\tau,x_2,\lambda)\right|\le\mu(s,\tau)\left|x_1-x_2\right|$$

$$\left|K_\lambda^{'}(s,\tau,x,\lambda_1)-K_x^{'}(s,\tau,x,\lambda_2)\right|\le\mu(s,\tau)\left|\lambda_1-\lambda_2\right|$$

d) $\int\limits_{-\infty}^{t}\left\{\lambda(s)+\int\limits_{-\infty}^{s}\mu(s,\tau)d\tau\right\}ds\le M.$

Ushbu

$$U(t)=\int\limits_{-\infty}^{t}\left\{F_x^{'}(s,x,\lambda)+\int\limits_{-\infty}^{s}K_x^{'}(s,\tau,x,\lambda)d\tau\right\}U(s)ds+\int\limits_{-\infty}^{t}\left\{F_\lambda^{'}(s,x,\lambda)+\int\limits_{-\infty}^{s}K_\lambda^{'}(s,\tau,x,\lambda)d\tau\right\}ds \quad (2.4.4)$$

yordamchi integral tenglamani qaraymiz. Osongina ko'rsatish mumkinki yuqoridagi shartlar bajarilsa bu tenglama yagona $u=u(t,\lambda)$ yechimga ega bo'ladi.

6-Teorema. Agar $F(s,x,\lambda)$, $K(s,\tau,x,\lambda)$ funksiyalar a)-d) shartlarni qanoatlantirsa (7) tenglamaning yechimi λ parametr bo'yicha uzluksiz va u λ parametr bo'yicha uzluksiz hosilaga ega bo'ladi, bu hosila (9) tenglamaning yechimidan iborat bo'ladi.

I-BOB. Integral tenglamalar va tengsizliklar haqida boshlang'ich ma'lumotlar.

Hozirgi davrda ko'plab O'zbekiston va chet el matematik olimlarning e'tibori integral tenglamalar nazariyasiga qaratilgan bo'lib, bu sohada katta ilmiy izlanishlar olib borilmoqda. Integral tenglamalar qatorida Vol'terra integral tenglamalari alohida o'rin egallaydi.

Integral tenglamalar nazariyasi G.Myunts, I.I.Privalov, U.B.Lovitt, S,G.Mixlin, M.L.Krasnov, K.T.Axmedov, Ya.D.Mamedov, S.A.Ashirov$[1-17]$ va boshqa avtorlarning ishlarida va ilmiy maqolalarida bayon etilgan va rivojlantirilgan.

1-§. Integral tenglamalar haqida boshlang'ich ma'lumotlar.

1.1 Integral tenglamalarning umumiy ko'rinishlari.

Integral tenglamalar nazariyasini o'rganish muhim ahamiyat kasb etadi. Hammaga ma'lumki, hozirgi vaqtda mexanika, fizika, ekanomika, ximya, bialogiya, meditsina va boshqa sohalarda uchraydigan ko'plab masalalarning matematik modellarini o'rganish integral tenglamalarni tekshirishga olib keladi.

1.1.1-ta'rif. Agar tenglamada noma'lum funksiya shu funksiyaning argumeneti bo'yicha olinadigan integral belgisi ostida albatta qatnashsa, bunday tenglama *integral tenglama* deb ataladi.

1.1.2-ta'rif. Agar integral tenglamada integral belgisi ostidagi funksiya va tenglamaning boshqa qismlari (hadlari) noma'lum funksiyaga nisbatan chiziqli bo'lsa, u holda tegishli tenglama *chiziqli integral tenglama* deyiladi.

1.1.3-ta'rif. Ushbu

$$f(t) = \lambda \int_a^t K(t,s)\varphi(s)ds \qquad (1.1.1)$$

ko'rinishdagi tenglama Vol'terraning *birinchi tur chiziqli integral tenglamasi* deyiladi. Unda integral ostidagi funksiya noma'lum $\varphi(t)$ funksiyaga nisbatan chiziqli. (1.1.1) tenglamada $f(t)$ funksiya $I(a \le t \le b)$ segmentda, $K(t,s)$ funksiya esa $R(a \le t \le b, a \le s \le t)$ sohada berilgan hamda o'z argumentlari bo'yicha uzluksiz funksiyalardir. Undan tashqari, $\lambda-$ o'zgarmas son (parametr), $a,b-$ berilgan haqiqiy o'zgarmas sonlar, $K(t,s)$ funksiya $(1.1.1)$ tenglamaning *yadrosi*, $f(t)$ esa *ozod hadi* deyiladi.

1.1.4-ta'rif. Ushbu

$$\varphi(t)=f(t)+\lambda\int_a^t K(t,s)\varphi(s)ds \qquad (1.1.2)$$

ko'rinishdagi tenglama Vol'terraning *ikkinchi tur chiziqli integral tenglamasi* deb ataladi. Bunda $\varphi(t)$ noma'lum funksiya integral belgisidan tashqarida ham alohida va chiziqli bo'lib qatnashmoqda.

Agar $(1.1.2)$ tenglamada ixtiyoriy $t\in I$ uchun $f(t)\equiv 0$ bo'lsa, $(1.1.2)$ dan

$$\varphi(t)=\lambda\int_a^t K(t,s)\varphi(s)ds \qquad (1.1.3)$$

tenglama hosil bo'ladi va u Vol'terraning *bir jinsli chiziqli integral tenglamasi* deb ataladi.

1.1.5-ta'rif. Ushbu

$$\psi(t)\varphi(t)=f(t)+\lambda\int_a^t K(t,s)\varphi(s)ds \qquad (1.1.4)$$

ko'rinishdagi tenglama Vol'terraning *uchunchi tur chiziqli integral tenglamasi* deb ataladi.

Agar ixtiyoriy $t\in I$ uchun $\psi(t)\equiv 0$ bo'lsa, $(1.1.4)$ dan $(1.1.1)$ tenglama, $\psi(t)=1$ bo'lsa, $(1.1.4)$ dan $(1.1.2)$ tenglama kelib chiqadi.

Ba'zi hollarda $K(t,s)$ yadro xususan $K(t-s)$, ya'ni $t-s$ ayirmaning funksiyasi ko'rinishiga ega bo'lishi mumkin. Bu holda, jumladan, ushbu

$$\varphi(t)=f(t)+\lambda\int_a^t K(t-s)\varphi(s)ds \qquad (1.1.5)$$

tenglama Vol'terraning *o'ram (yig'ma) tipidagi integral tenglamasi* deb yuritiladi.

Agar Vol'terra tenglamalarida $a\to-\infty$ yoki $b\to\infty$ bo'lsa, ushbu

$$\varphi(t)=f(t)+\lambda\int_t^\infty K(t,s)\varphi(s)ds \qquad (1.1.6)$$

$$\varphi(t) = f(t) + \lambda \int_{-\infty}^{t} K(t,s)\varphi(s)ds \qquad (1.1.7)$$

tenglamalarga, agar $K(t,s)$ yadro qaralayotgan sohaning bir yoki bir nechta nuqtasida cheksizlikka aylansa, masalan ushbu

$$\varphi(t) = f(t) + \lambda \int_{a}^{t} \frac{\varphi(s)}{s-a} ds, \qquad (1.1.8)$$

$$\varphi(t) = f(t) + \lambda \int_{a}^{t} \frac{H(t,s)}{(s-a)^{\alpha}} \varphi(s)ds, \quad 0 < \alpha < 1 \qquad (1.1.9)$$

tenglamalarga ega bo'lamiz. Ularga Vol'terraning *maxsus integral tenglamalari* deyiladi.

Vol'terra tenglamalarida integrallash chegaralaridan biri yoki ikkalasi ham funksiyadan iborat bo'lishi mumkin. Ushbu

$$\varphi(t) = f(t) + \int_{a}^{g(t)} K(t,s)\varphi(s)ds, \qquad (1.1.10)$$

$$\varphi(t) = f(t) + \int_{t-a}^{t} K(t,s)\varphi(s)ds, \alpha > 0, \qquad (1.1.11)$$

$$\varphi(t) = f(t) + \int_{pt}^{t} K(t,s)\varphi(s)ds, 0 < p < 1 \qquad (1.1.12)$$

tenglamalar ana shunday tenglamalardir.

1.2 Integral tenglamalarni yechish usullari.

Integral tenglamalar nazariyasi matematikaning eng rivojlangan sohalaridan biri bo'lib, fan va texnikaning rivojlanishida kata rol o'ynaydi. Bunda Vol'terra tenglamalari muhum o'rin egallaydi. Biz bu yerda Vol'terra tenglamalariga oid dastlabki ma'lumotlardan ba'zilarini keltiramiz.

Ketma-ket o'rniga qo'yish usuli.

Quyidagi

$$\varphi(t) = f(t) + \lambda \int_{a}^{t} K(t,s)\varphi(s)ds \qquad (1.1.13)$$

ikkinchi tur Vol'terra tenglamasini qaraymiz. Bu yerda va kelgusida uchraydigan barcha Vol'terra tenglamalarining ozod hadi va yadrosi haqiqiy o'zgaruvchili noldan farqli funksiyalardan, λ parametr esa haqiqiy sondan iborat deb faraz qilamiz.

1.1.1-teorema. Agar (1.1.13) tenglamada $f(t)$ funksiya $I(a \le t \le b)$ segmentda, $K(t,s)$ funksiya esa $R(a \le t \le b, a \le s \le t)$ sohada uzluksiz bo'lib, $f(t) \ne 0$, $K(t,s) \ne 0$, $\lambda = const$ bo'lsa, u holda (1.1.13) tenglama I segmentda yagona uzluksiz yechimga ega bo'ladi.

Isbot. Bu teoremani isbotlash usuli ketma-ket o'rniga qo'yish usuli deb yuritiladi. Bu usul bilan (1.1.13) tenglamaning yagona uzluksiz yechimi quriladi. Izlanuvchi $\varphi(t)$ funksiyaning (1.1.13) tenglamadagi ifodasini shu tenglamaning o'ng tomoniga qo'yib, quyidagini hosil qilamiz:

$$\varphi(t) = f(t) + \lambda \int_a^t K(t,s)\left[f(s) + \lambda \int_a^s K(s,s_1)ds_1 \right] ds = f(t) + \lambda \int_a^t K(t,s) f(s) ds +$$

$$+\lambda^2 \int_a^t K(t,s) \int_a^s K(s,s_1)\varphi(s_1) ds_1 ds.$$

Bu yerdagi $\varphi(s_1)$ o'rniga yana uning (1.1.13) tenglamadagi ifodasini qo'ysak, quyidagiga ega bo'lamiz:

$$\varphi(t) = f(t) + \lambda \int_a^t K(t,s) f(s) ds + \lambda^2 \int_a^t K(t,s) \int_a^s K(s,s_1)\left[f(s_1) + \lambda \int_a^{s_1} K(s_1,s_2)\varphi(s_2) ds_2 \right] ds_1 ds =$$

$$= f(t) + \lambda \int_a^t K(t,s) f(s) ds + \lambda^2 \int_a^t K(t,s) \int_a^s K(s,s_1) f(s_1) ds_1 ds +$$

$$+\lambda^3 \int_a^t K(t,s) \int_a^s K(s,s_1) \int_a^{s_1} K(s_1,s_2) u(s_1) ds_2 ds_1 ds.$$

Bu jarayonni davom ettirib, n marta o'rniga qo'yishni bajarsak, quyidagiga ega bo'lamiz:

$$\varphi(t) = f(t) + \lambda \int_a^t K(t,s) f(s) ds + \lambda^2 \int_a^t K(t,s) \int_a^s K(s,s_1) f(s_1) ds_1 ds + \ldots +$$

$$+\lambda^n \int_a^t K(t,s) \int_a^s K(s,s_1) \ldots\ldots \int_a^{s_{n-2}} K(s_{n-2},s_{n-1}) f(s_{n-1}) ds_{n-1} \ldots ds + R_{n+1}(t) \qquad (1.1.14)$$

bu yerda

$$R_{n+1}(t)=\lambda^{n+1}\int_a^t K(t,s)\int_a^s K(s,s_1)\dots\int_a^{s_{n-1}} K(s_{n-1},s_n)\varphi(s_n)ds_n\dots ds.$$

Bunga asosan quyidagi qatorni qaraymiz:

$$f(t)+\lambda\int_a^t K(t,s)f(s)ds+\dots+\lambda^n\int_a^t K(t,s)\int_a^s K(s,s_1)\dots\int_a^{s_{n-2}} K(s_{n-2},s_{n-1})f(s_{n-1})ds_{n-1}\dots ds+\dots \quad (1.1.15)$$

Teoremaning shartlariga asosan, bu qatorning har bir hadi t ning I segmentda uzluksiz funksiyasidan iborat. Demak, agar bu qator I segmentda tekis yaqinlashuvchi bo'lsa, uning yig'indisi ham shu segmentda aniqlangan uzluksiz funksiyadan iborat bo'ladi.

$K(t,s)$ va $f(t)$ funksiyalar mos ravishda yopiq R va I sohalarda uzluksiz ekanidan quyidagini yozish mumkin:

$$\max_{(t,s)\in R}|K(t,s)|=M,(yoki\,|K(t,s)|<M);$$

$$\max_{t\in I}|f(t)|=N,(yoki\,|f(t)|<N).$$

Bularga asosan $(1.1.15)$ qatorning $n-$hadi ($f(t)$ 0-had)

$$V_n(t)=\lambda^n\int_a^t K(t,s)\int_a^s K(s,s_1)\dots\int_a^{s_{n-2}} K(s_{n-2},s_{n-1})f(s_{n-1})ds_{n-1}\dots ds \quad (1.1.16)$$

quyidagicha baholanadi:

$$|V_n(t)|\le|\lambda^n|NM\frac{(t-a)^n}{n!}\le|\lambda|^n N\frac{[M(b-a)]^n}{n!},t\in I.$$

Bundan ko'rinadiki, umumiy hadi ($n-$hadi) ushbu

$$|\lambda|^n N\frac{[M(b-a)]^n}{n!}$$

ko'rinishda bo'lgan musbat hadli sonli qator λ, M, N va $b-a$ larning har qanday musbat chekli qiymatlarida yaqinlashuvchi ekani ravshan (bunga Dalamber alomatini bevosita qo'llanish yordamida ishonch hosil qilish mumkin). Shuning uchun Veyershtrass teoremasiga asosan umumiy hadi $(1.1.16)$ dan iborat bo'lgan $(1.1.15)$ qator I da absolyut va tekis yaqinlashuvchi bo'ladi.

Agar $(1.1.13)$ tenglama uzluksiz yechimga ega bo'lsa, bu yechim $(1.1.14)$ tenglamaning ham yechimi bo'lishi kerak, ya'ni bu yechim qoldiq hadi $R_{n+1}(t)$ dan iborat bo'lgan (1.1.14) qator bilan ifodalanishi kerak. Agar (1.1.13) tenglama uzluksiz yechimga ega bo'lsa, o'sha yechim (1.1.14) ko'rinishda yozilishi mumkin. $\varphi(t)$ ning I da uzluksizligi esa ushbu munosabatni yozishga asos bo'ladi: $\max_{t\in I}|\varphi(t)| = Q.$ Shuning uchun ushbuga egamiz:

$$|R_{n+1}(t)| \le |\lambda^{n+1}| QM^{n+1}\frac{(t-a)^{n+1}}{(n+1)!} \le |\lambda|^{n+1} Q\frac{[M(b-a)]^{n+1}}{(n+1)!}.$$

Demak, $\lim_{n\to\infty} R_{n+1}(t) = 0.$

Shunday qilib, ixtiyoriy n uchun (1.1.13) tenglamaning yechimi (bu yechim mavjud bo'lganda) (1.1.15) qatorga yoyildi.

Endi (1.1.15) qatorning yig'indisidan iborat uzluksiz $\varphi(t)$ funksiya (1.1.13) tenglamaning yechimi ekanini ko'rsatamiz. Bunga ushbu

$$\varphi(t) = f(t) + \lambda\int_a^t K(t,s) f(s)ds + \lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1) f(s_1)ds_1 ds + \ldots \qquad (1.1.17)$$

qatorni (1.1.13) tenglamadagi $\varphi(t)$ ning o'rniga bevosit qo'yish natijasida ishonch hosil qilish mumkin. Haqiqatan, yuqoridagi (1.1.17) tenglikning har ikkala tomonini $\lambda K(t,s)$ ga ko'paytirib, uning har ikkala tomonini a dan t gacha integrallaymiz. U holda quyidagiga ega bo'lamiz:

$$\lambda\int_a^t K(t,s)\varphi(s)ds = \lambda\int_a^t K(t,s)\left[f(s) + \lambda\int_a^s K(s,s_1) f(s_1)ds_1 + \ldots \right]ds =$$

$$= \lambda\int_a^t K(t,s) f(s)ds + \lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1) f(s_1)ds_1 ds + \ldots = \varphi(t) - f(t).$$

Bu esa (1.1.17) tenglik bilan aniqlanuvchi $\varphi(t)$ funksiya (1.1.13) tenglikning yechimi ekanini ko'rsatadi.

Ketma-ket yaqinlashish usuli

Vol'terra tenglamasi yechimining mavjudligi va yagonaligi ketma-ket yaqinlashish usuli yordamida ham ko'rsatiladi.

Ushbu

$$\varphi(t) = f(t) + \lambda\int_a^t K(t,s)\varphi(s)ds \qquad (1.1.18)$$

Vol'terraning chiziqli tenglamasini qaraymiz.

1.1.2-teorema. Agar $f(t)$ funksiya $I(a \le t \le b)$ segmentda, $K(t,s)$ funksiya (yadro) esa $R(a \le t \le b, a \le s \le t)$ sohada uzluksiz bo'lib, $f(t) \ne 0,$ $K(t,s) \ne 0,$ $\lambda = const$ bo'lsa, u holda (1.1.18) tenglama I segmentda yagona uzluksiz yechimga ega bo'ladi va bu yechim $\varphi_0(t)$ ixtiyoriy uzluksiz funksiya bo'lganda ushbu

$$\varphi_n(t) = f(t) + \lambda \int_a^t K(t,s)\varphi_{n-1}(s)ds \qquad (n = 1, 2, 3, \ldots)$$

recurrent formula yordamida aniqlanuvchi $\{\varphi_n(t)\}$ ketma-ketlikning $n \to \infty$ dagi limitidan iborat bo'ladi.

Isbot. Teoremaning yechimi mavjudligi haqidagi qismi yangi tasdiq emas. Biz uning ikkinchi qismini isbotlashimiz lozim. I segmentda uzluksiz bo'lgan ixtiyoriy $\varphi_0(t)$ funksiyani tanlaymiz. Bu funksiyani (1.1.13) tenglamaning o'ng tomonini $\varphi(s)$ ning o'rniga qo'yib,

$$\varphi_1(t) = f(t) + \lambda \int_a^t K(t,s)\varphi_0(s)ds$$

tenglikni hosil qilamiz. Shu tarzda topilgan $\varphi_1(t)$ funksiya I segmentda uzluksiz funksiyadan iborat.

Endi topilgan $\varphi_1(t)$ funksiyani yana (1.1.18) tenglamaning o'ng tomonidagi $\varphi(s)$ ning o'rniga qo'yib,

$$\varphi_2(t) = f(t) + \lambda \int_a^t K(t,s)\varphi_1(s)ds$$

tenglikni hosil qilamiz. Ko'rinib turibdiki, $\varphi_2(t)$ funksiya ham I segmentda uzluksizdir.

Bu jarayonni davom ettirib,

$$\varphi_0(t), \ \varphi_1(t), \ \varphi_2(t), \ldots, \ \varphi_n(t), \ldots$$

funksiyalar ketma-ketligini hosil qilamiz. Bu yerda $\varphi_i(t), i = 1, 2, \ldots$ funksiyalar quyidagi tengliklarni qanoatlantiradi:

$$\varphi_1(t) = f(t) + \lambda\int_a^t K(t,s)\varphi_0(s)ds,$$

$$\varphi_2(t) = f(t) + \lambda\int_a^t K(t,s)\varphi_1(s)ds, \qquad (1.1.19)$$

……………………………………………

$$\varphi_n(t) = f(t) + \lambda\int_a^t K(t,s)\varphi_{n-1}(s)ds.$$

Yuqorida $f(t)$ va $K(t,s)$ funksiyalar uchun qo'yilgan shartlarga asoslanib, n cheksizlikka intilganda $\{\varphi_n(t)\}$ ketma-ketlik $(1.1.19)$ tenglamaning $\varphi(t)$ yechimiga yaqinlashishini ko'rsatamiz. Shu maqsadda $\varphi_i(t)$ lar uchun yozilgan $(1.1.19)$ ifodalarni yuqoridan boshlab birin-ketin o'zidan keyingisiga qo'yib chiqamiz. Natijada quyidagi ifodalar hosil bo'ladi:

$$\varphi_1(t) = f(t) + \lambda\int_a^t K(t,s)\varphi_0(s)ds,$$

$$\varphi_2(t) = f(t) + \lambda\int_a^t K(t,s)f(s)ds + \lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1)\varphi_0(s_1)ds_1ds$$

……………………………………………………………

$$\varphi_n(t) = f(t) + \lambda\int_a^t K(t,s)f(s)ds + \lambda^2\int_a^t K(t,s)\int_a^s K(s,s_1)\varphi_0(s_1)ds_1ds + \ldots +$$

$$+\lambda^{n-1}\int_a^t K(t,s)\int_a^s K(s,s_1)\ldots\int_a^{s_{n-3}} K(s_{n-3},s_{n-2})f(s_{n-2})ds_{n-2}\ldots ds_1ds + R_n(t), \qquad (1.1.20)$$

bu yerda

$$R_n(t) = \lambda^n\int_a^t K(t,s)\int_a^s K(s,s_1)\ldots\int_a^{s_{n-3}} K(s_{n-2},s_{n-1})\varphi_0(s_{n-1})ds_{n-1}\ldots ds_1ds.$$

Shartga ko'ra $K(t,s)$ va $\varphi_0(t)$ funksiyalar mos ravishda R va I larda uzluksiz, demak,

$$|K(t,s)| \le M, |\mu_0(t)| \le Q.$$

Bularga asosan quyidagiga ega bo'lamiz:

$$|R_n(t)| \le QM^n|\lambda|^n\int_a^t ds\int_a^s ds_1\ldots\int_a^{s_{n-1}} ds_{n-2} = QM^n|\lambda|^n\frac{(t-a)^n}{n!},$$

ya'ni

$$|R_n(t)| \le QM^n |\lambda|^n \frac{(b-a)^n}{n!}.$$

Shuning uchun ravshanki, $\lim\limits_{n\to\infty} R_n(t) = 0.$

Buni nazarda tutib, (1.1.20) tenglikning har ikkala tomonida $n \to \infty$ da limitga o'tsak, $\varphi_n(t)$ funksiyalarning limiti

$$f(t) + \lambda \int_a^t K(t,s) f(s)ds + \ldots + \lambda^n \int_a^t K(t,s) \int_a^s K(s,s_1) \ldots \int_a^{s_{n-1}} K(s_{n-2}, s_{n-1}) f(s_{n-1}) ds_{n-1} \ldots ds + \ldots \quad (1.1.21)$$

qatorning yig'indisiga teng ekanligi va uning (1.1.18) tenglamaning yechimi ekaniga ishonch hosil qilamiz. Demak,

$$\lim_{n\to\infty} \varphi_n(t) = \varphi(t).$$

Ko'rinib turibdiki, topilgan $\varphi(t)$ yechim I segmentda uzluksiz funksiyadir.

Yuqorida bayon etilgan usul Vol'terraning chiziqli integral tenglamasini yechishning ketma-ket yaqinlashish usuli deyiladi.

Bu usul har bir $\varphi_n(t)$ funksiya tanlangan $\varphi_0(t)$ uzluksiz boshlang'ich funksiyaga bog'liq, lekin $\varphi(t)$ yechim esa $\varphi_0(t)$ ning tanlanishiga bog'liq emas.

Xususiy holda $\varphi_0(t)$ sifatida $f(t)$ funksiya olinsa, $\varphi_n(t)$ lar (1.1.21) qatorning xususiy yig'indilaridan iborat bo'ladi.

Boshlang'ich $\varphi_0(t)$ funksiyani qulay qilib tanlash $|\varphi_n(t)|$ ketma-ketlikning tenglama yechimiga tezroq yaqinlashishini taminlaydi.

Abel tenglamasi

Dastlab Abel tomonidan qo'yigan masala

$$\int_0^h \frac{\varphi(y)dy}{\sqrt{h-y}} = \psi(h) \quad (1.1.22)$$

birinchi tur Vol'terra tenglamasi yechimini topishga keltirilishini ko'rib o'tamiz.

Abel masalasi. Moddiy nuqta o'zining og'irlik kuchi ta'siri ostida vertical tekislikda joylashgan biror silliq egri chiziq bo'ylab uning ordinatasi h ga teng bo'lgan ixtiyoriy M nuqtasidan ordinatasi 0 ga teng bo'lgan eng quyi O

nuqtasiga boshlang’ich tezliksiz $T=\psi(t)$ ($\psi(h)-$berilgan funksiya) vaqt ichida yetib kelgan bo’lsin. Shu egri chiziqni toping.

Yechish. Izlangan egri chiziqning eng quyi O nuqtasini koordinatalar boshi uchun qabul qilamiz va Ox o’qni gorizantal, Oy o’qni esa vertikal yo’naltiramiz. Bu egri chiziqning tenglamasini $x=\xi(y)$ ko’rinishda izlaymiz.

Agar egri chiziqning yoy elementini ds bilan belgilasak,

$$ds=\sqrt{1+\left[\xi'(y)\right]^2}\,dy$$

bo’ladi.

h balandlikdan tushayotgan nuqtaning massasini m bilan, tezligini v bilan belgilasak, moddiy nuqta boshlang’ich M nuqtadan ixtiyoriy N nuqtagacha harakatlanganda uning kinetik energiyasining o’zgarishi og’irlik kuchining bajargan ishiga teng bo’ladi, ya’ni

$$\frac{1}{2}mv^2=mg(h-y)$$

tenglikka ega bo’lamiz, bunda $g-$erkin tushish tezlanishi, mg esa og’irlik kuchidir. Agar $v=ds/dt$ ekanini nnazarga olsak,oxirgi tenglikdan quyidagiga ega bo’lamiz:

$$\frac{1}{2}\left(\frac{ds}{dt}\right)^2=g(h-y).$$

Bundan

$$dt=\frac{-ds}{\sqrt{2g(h-y)}}=\frac{1}{2g}\frac{-\sqrt{1+\left[\xi'(y)\right]^2}}{\sqrt{h-y}}dy.$$

(Bu tenglikda t o’sishi bilan nuqtaning ordinatasi y ning kamayishi minus ishora bilan hisobga olingan.)

M nuqtadan O nuqtaga tushish vaqti y ning h dan O gacha o’zgarishiga mos kelganligi sababli oxirgi tenglikdan quyidagiga ega bo’lamiz:

$$\psi(h)=T=\frac{1}{\sqrt{2g}}\int_0^h\frac{\sqrt{1+\left[\xi'(y)\right]^2}}{\sqrt{h-y}}dy.$$

Bu yerda

$$\varphi(y)=\frac{\sqrt{1+[\xi'(y)]^2}}{\sqrt{2g}}$$

deb belgilasak, masala ushbu

$$\int_0^h \frac{\varphi(y)}{\sqrt{h-y}}dy=\psi(h)$$

Birinchi tur chiziqli integral tenglamaning yechimini topishga keltiriladi. Bu tenglama Abel nomi bilan yuritiladi. Bu yerda (1.1.22) tenglamani Abel usuli bilan yechamiz. Bu usul Vol'terra tenglamalarini yechish usullaridan birmuncha farq qiladi.

Berilgan tenglamadagi h ni s ga almashtiramiz va hosil bo'lgan tenglikning ikkala tomonini $(h-s)^{-\frac{1}{2}}ds$ ga ko'paytirib, s bo'yicha 0 dan h gacha integrallaymiz:

$$\int_0^h \frac{ds}{\sqrt{h-y}}\int_0^s \frac{\varphi(y)}{\sqrt{s-y}}dy=\int_0^h \frac{\psi(s)ds}{\sqrt{h-s}}.$$

Chap tomondagi integralga Dirixle formulasini qo'llasak,

$$\int_0^h \varphi(y)dy\int_y^h \frac{ds}{\sqrt{(h-s)(s-y)}}=\int_0^h \frac{\psi(s)ds}{\sqrt{h-y}}$$

tenglik hosil bo'ladi. Chap tomonda hosil bolgan ichki integral

$$s=\frac{h+y}{2}+\frac{h-y}{2}\sin\theta$$

almashtirish yordamida osongina hisoblanadi, ya'ni

$$\int_y^h \frac{ds}{\sqrt{(h-s)(s-y)}}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta=\pi.$$

Shuning uchun quyidagiga egamiz:

$$\int_0^h \varphi(y)dy=\frac{1}{\pi}\int_0^h \frac{\psi(s)ds}{\sqrt{h-s}}.$$

Demak,

$$\varphi(y)=\frac{d}{dh}\left[\int_0^h \frac{\psi(s)ds}{\sqrt{h-s}}\right] \text{ yoki } \varphi(y)=\frac{d}{dy}\left[\int_0^y \frac{\psi(h)dh}{\sqrt{y-h}}\right], y\succ 0.$$

Bu esa Abel tenglamasining yechimi bo'ladi. Bunga asosan Abel masalasida izlangan $x=\xi(y)$ egri chiziq

$$\varphi(y)=\sqrt{1+\xi'^2}\Big/\sqrt{2g}$$

tenglikdan topiladi.

Xususan, $\psi(h)=C=const$ bo'lsa, Abel masalasining yechimi sikloiddan iborat bo'lishini ko'rsatamiz. Bu holda quyidagiga egabo'lamiz:

$$\varphi(y)=\frac{d}{dy}\int_0^y\frac{cdh}{\pi\sqrt{y-h}}=\frac{d}{dy}\int_0^{\sqrt{y}}\frac{2C}{\pi}dt=\frac{d}{dy}\left(\frac{2C}{\pi}\sqrt{y}\right)=\frac{C}{\pi\sqrt{y}}.$$

Demak,

$$\sqrt{1+\xi'^2(y)}=\sqrt{2g}\varphi(y)=\frac{C}{\pi}\sqrt{\frac{2g}{y}}.$$

Bundan $\xi'(y)$ ni topamiz:

$$\xi'(y)=\sqrt{\frac{2gC^2}{\pi^2 y}-1},\quad 0\prec y\le\frac{2gC^2}{\pi^2}.$$

Bu tenglikni 0 dan y gacha integrallaymiz, u holda

$$x=\xi(y)=\int_0^y\sqrt{\frac{2gC^2}{\pi^2\eta}-1d\eta}$$

hosil bo'ladi.

Oxirgi integral ushbu

$$\eta=\frac{2gC^2}{\pi^2}\sin^2\frac{\theta}{2}=\frac{gC^2}{\pi^2}(1-\cos\theta)$$

almashtirish yordamida osongina hisoblanadi:

$$x=\frac{gC^2}{\pi^2}(\sin y+y).$$

Ushbu

$$x=\frac{gC^2}{\pi^2}(\sin y+y)$$

munosabat Abel masalasining yechimi bo'lib, bu chiziqsikloiddan iboratdir.

Endi $\psi(h)=C\sqrt{h}$ bo'lgan holda Abel masalasining yechimi to'g'ri chiziqdan iborat ekanligini ko'rsatamiz.

Bu holda

$$\varphi(y)=\frac{d}{dy}\int_0^y\frac{C\sqrt{h}dh}{\pi\sqrt{y-h}}$$

tenglikka ega bo'lamiz. Integralni

$$h=y\sin^2\frac{\theta}{2}$$

almashtirish yordamida hisoblab, quyidagiga ega bo'lamiz:

$$\varphi(y)=\frac{d}{dy}\left(\frac{Cy}{2}\right)=\frac{C}{2}.$$

Bunga asosan

$$1+\xi'^2(y)=\frac{1}{2}gC^2 \quad \text{va} \quad \frac{1}{2}gC^2\succ 1$$

munosabatlar o'rinli. Bundan:

$$\xi'^2(y)=\frac{1}{2}gC^2-1=k^2.$$

Demak,

$$x=\xi(y)ky,$$

bu esa koordinata boshidan o'tadigan to'g'ri chiziqdir.

2-§. Integral tengsizliklar haqida boshlang'ich ma'lumotlar.

2.1 Integral tengsizliklar haqida boshlang'ich ma'lumotlar.

Integral tengsizlik va uning differensial tenglama yechimini o'rganishga tadbiq qilish g'oyasi birinchi bo'lib Gronual tomonidan 1919-yilda amalgam oshirilgan bo'lib, hozirgi kunda integral tengsizliklar ko'plab olimlarning ilmiy tadqiqot ob'ektiga aylanadi va mustaqil nazaqriya shaklida fanda o'z o'rnini egalladi. Bu nazariya Rossiya, Germaniya, Ozorboyjon, Qirg'iziston, O'zbekiston va boshqa chet ellik olimlar tomonidan rivojlantirilib, ko'plab ilmiy maqolalar va monografiyalarda bayon etildi.

Integral tengsizliklarning ahamiyati shundan iboratki, ular yordamida integral, differensial, integro-differensial va xususiy hosilali differensial tenglamalar yechimlarining mavjudligi va yagonaligi, boshlang'ich shartlar va parametrlardan uzluksiz bog'liqligi, turg'unligi va boshqa ko'plab xossalarni o’rganishda juda qulay apparat bo’lib xizmat qiladi. Bundan tashqari tenglamalarning aniq va taqribiy yechimlari orasidagi farqni baholashda ham integral tengsizliklardan foydalanish mumkin.

Gronuoll tengsizligi va uni isbotlash usullari. Gronuoll-Bellman tengsizligi.

1.2.1-teorema. (Gronuoll.) [2]. Agar $I(a \le t < \infty)$ yarim o’qda aniqlangan, uzluksiz $u(t) \ge 0$ funksiya va o’zgarmas $C \ge 0, \quad K \ge 0$ sonlar uchun

$$u(t) \le C + K\int_a^t u(s)ds \qquad (1.2.1)$$

tengsizlik o’rinli bo’lsa, u holda shu I yarim o’qda

$$u(t) \le Ce^{K(t-a)} \qquad (1.2.2)$$

tengsizlik ham o’rinli bo’ladi.

Xususiy holda, agar $C = 0$ bo’sa, $u(t) \equiv 0$ bo’ladi.

Isbot. 1-hol. $C > 0$ bo’lsin. Berilgan $(1.2.1)$ tengsizlikning o’ng tomonini $v(t)$ bilan belgilaymiz, ya’ni

$$v(t) = C + K\int_0^t u(s)ds \qquad (1.2.3)$$

Bundan $v(a) = C > 0$ va teoremaning shartiga ko’rib $v(t) \ge 0.$ endi $t > 0$ uchun $(1.2.1)$ tengsizlikni ushbu

$$u(t) \le v(t) \qquad (1.2.4)$$

ko’rinishda yozsa bo’ladi. $(1.2.3)$ tenglikning ikki tomonini t bo’yicha differensiallaymiz:

$$v'(t) = Ku(t).$$

Yuqoridagi $(1.2.4)$ tengsizlikka ko’ra quyidagi

$$v'(t) \le Kv(t).$$

tengsizlik hosil bo’ladi. Bu tengsizlikning ikki tomonini musbat $v(t)$ (chunki $C > 0, \quad u(t) \ge 0, \quad K \ge 0$) funksiyaga bo’lamiz:

$$\frac{v'(t)}{v(t)} \le K.$$

Agar $v(a)=C$ ekanini nazarga olib, oxirgi tengsizlikni t bo'yicha a dan t gacha integrallasak,

$$v(t)\le Ce^{K(t-a)}$$

tengsizlikni hosil qilamiz. Bundan (1.2.4) tengsizlikka ko'ra (1.2.2) tengsizlikning to'g'riligi kelib chiqadi.

2-hol. $C=0$ bo'lsin. Agar (1.2.1) tengsizlik $C=0$ uchun o'rinli bo'lsa, $u(t)\equiv 0$ ayniyat o'rinli bo'ladi. Buni isbotlash uchun (1.2.1) va (1.2.2) larda $C\to 0$ da limitga o'tish yetarli. Teorema isbot bo'ldi.

Biz quyida Gronuoll tengsizliging turli ko'rinishlari bilan tanishamiz.

1.2.2-teorema. (Gronuoll-Bellman.)[2]**.** Agar $I(a\le t<\infty)$ yarim o'qda aniqlangan, uzluksiz $u(t)\ge 0,\ g(t)\ge 0$ funksiya va o'zgarmas $C\ge 0$ son uchun

$$u(t)\le C+\int_a^t g(s)u(s)ds \qquad (1.2.5)$$

tengsizlik o'rinli bo'lsa, u holda shu I yarim o'qda

$$u(t)\le C\exp\left(\int_a^t g(s)ds\right) \qquad (1.2.6)$$

tengsizlik ham o'rinli bo'ladi.

Isbot. $C>0$ bo'lganda $v(t)=C+\int_a^t g(s)u(s)ds$ deb $v'(t)=g(t)u(t)$ va $u(t)\le v(t)$ munosabatlarni hosil qilamiz. Ulardan $v'(t)\le g(t)v(t)$ kelib chiqadi. $v(t)>0$ bo'lgani uchun oxirgi tengsizlikning ikki tomonini $v(t)$ ga bo'lib, a dan t gacha integrallaymiz:

$$\ln v(t)\Big/_{a}^{t}\le\int_a^t g(s)$$

yoki $v(a)=C$ ekanini nazarda tutib, izlangan (1.2.6) tengsizlikni hosil qilamiz. Endi Gronuoll-Belman tengsizligining umumlashmasini keltiramiz.

1.2.3-teorema[2]**.** Agar $I(a\le t<\infty)$ yarim o'qda aniqlangan va uzluksiz $u(t)\ge 0,\quad f(t)\ge 0,\quad \alpha(t)\ge 0,\quad \beta(t)\ge 0$ funksiyalar uchun

$$u(t) \le f(t) + \alpha(t)\int_a^t \beta(s)u(s)ds \qquad (1.2.7)$$

tengsizlik o'rinli bo'lsa, u holda shu I yarim o'qda

$$u(t) \le f(t) + \alpha(t)\int_a^t f(s)\beta(s)\exp\left(\int_s^t \alpha(\tau)\beta(\tau)d\tau\right) \qquad (1.2.8)$$

tengsizlik ham o'rinli bo'ladi. Agar I yarim o'qda uzluksiz $f(t),$ $\alpha(t)$ funksiya uchun $f(t) > 0,$ $f'(t) \ge 0$ va $\alpha(t) \ge 1$ bo'lsa,

$$u(t) \le f(t)\alpha(t)\exp\left(\int_a^t \beta(s)\alpha(s)ds\right) \qquad (1.2.9)$$

tengsizlik o'rinli bo'ladi.

Agar $f(t) = 0$ bo'lsa, u holda $u(t) \equiv 0$ bo'ladi.

Isbot. Quyidagi

$$v(t) = \int_a^t \beta(s)u(s)ds. \qquad (1.2.10)$$

belgilashni kiritamiz. Bundan, ravshanki,

$$v(a) = 0, \quad dv/dt \le \beta(t)u(t). \qquad (1.3.11)$$

Berilgan $(1.2.7)$ tengsizlikning har ikkala tomonini $\beta(t) > 0$ ga ko'paytirib, $(1.2.10)$ belgilash va $(1.2.11)$ munosabatni nazarda tutib,

$$dv/dt \le f(t)\beta(t) + \alpha(t)\beta(t)v \qquad (*)$$

tengsizlikni hosil qilamiz.

Endi

$$v(t) = \omega(t)\exp\left(\int_a^t a(\tau)\beta(\tau)d\tau\right). \qquad (1.2.12)$$

Tenglik yordamida $\omega(t)$ funksiyani kiritamiz. Bundan ikki tomonini diferensiallab, $\frac{dv}{dt}$ uchun tegishli $(*)$ tengsizlikdan foydalansak, quyidagi tengsizlik hosil bo'ladi:

$$d\omega/dt\exp\left(\int_a^t \alpha(\tau)\beta(\tau)d\tau\right) \le f(t)\beta(t)$$

yoki

$$\omega(t) \le \int_a^t f(s)\beta(s)\exp\left(-\int_a^s \alpha(\tau)\beta(\tau)d\tau\right)ds. \qquad (1.2.13)$$

Shunday qilib, (1.2.10), (1.2.12), (1.2.13) munosabatlardan

$$v(t) = \int_a^t \beta(s)u(s)ds \le \int_a^t f(s)\beta(s)\exp\left(\int_s^t \alpha(\tau)\beta(\tau)d\tau\right)ds$$

tengsizlikka ega bo'lamiz. Bunga asosan (1.2.7) dan isbot etish talab qilingan (1.2.8) tengsizlik kelib chiqadi.

Endi teoremaning ikkinchi qismini isbotlaymiz. Ma'lumki, $f(t) > 0,$ $f'(t) \ge 0,$ $\alpha(t) \ge 1$ tengsizliklar o'rinli. Shuning uchun (1.2.7) ning ikki tomonini $f(t) \succ 0$ ga bo'lib, ushbuga ega bo'lamiz:

$$\frac{u(t)}{f(t)} \le 1 + \frac{\alpha(t)}{f(t)}\int_a^t \beta(s)u(s)ds \le \alpha(t)\left[1 + \frac{1}{f(t)}\int_a^t \beta(s)u(s)ds\right]. \qquad (1.2.14)$$

Quyidagicha belgilab olamiz:

$$R(t) = 1 + \frac{1}{f(t)}\int_a^t \beta(s)u(s)ds.$$

Ko'rinib turibdiki, $R(a) = 1.$ Endi $\frac{dR}{dt}$ ni hisoblaymiz:

$$\frac{dR}{dt} = \frac{u(t)\beta(t)}{f(t)} - \frac{f^1(t)}{f^2(t)}\int_a^t \beta(s)u(s)ds \le \frac{u(t)\beta(t)}{f(t)}.$$

Shunday qilib,

$$\frac{dR(t)}{dt} \le \beta(t)\frac{u(t)}{f(t)}.$$

Endi (1.2.14) tengsizlikni nazarga olib, oxirgi tengsizlikdan quyidagini topamiz:

$$dR/dt \le \beta(t)\alpha(t)R$$

yoki

$$dR/R \le \beta(t)\alpha(t)dt.$$

Bu tengsizlikni a dan t gacha integrallab

$$\ln R(t) - \ln R(a) \le \int_a^t \beta(s)\alpha(s)ds$$

yoki

$$R(t) \le \exp\int_a^t \beta(s)\alpha(s)ds$$

tengsizlikka ega bo’lamiz.

Buni nazarga olib (1.2.14) tengsizlikdan isbotlash talab etilgan (1.2.9) tengsizlikni hosil qilamiz.

Nihoyat, $f(t)=0$ bo’lsa, (1.2.8) yoki (1.2.9) tengsizliklardan $u(t)\le 0$ kelib chiqadi. Bundan esa $u(t)$ ning manfiy emasligiga asosan $u(t)\equiv 0$ ekaniga ishonch hosil qilamiz.

Karrali integralni saqlovchi quyidagi teorema o’rinli [1].

1.2.4-teorema[2]**.** Agar $I(a\le t<\infty)$ yarim o’qda aniqlangan, uzluksiz $u(t)\ge 0,\ \upsilon(t)\ge 0,\ P(a\le t<\infty, a\le s\le t)$ sohada aniqlangan, uzluksiz $\mu(t,s)\ge 0$ funksiya hamda o’zgarmas $\alpha\ge 0, \beta\ge 0$ sonlar uchun

$$u(t)\le \alpha+\beta\int_a^t\left[\upsilon(s)u(s)+\int_a^s \mu(s,\tau)u(\tau)d\tau\right]ds$$

tengsizlik o’rinli bo’lsa, u holda shu I yarim o’qda

$$u(t)\le \alpha\exp\left\{\beta\int_a^t\left[\upsilon(s)u(s)+\int_a^s \mu(s,\tau)u(\tau)d\tau\right]ds\right\}$$

tengsizlik ham o’rinli bo’ladi.

Isbot. Quyidagicha belgilash kiritamiz:

$$w(t)=\alpha+\beta\int_a^t\left[\upsilon(s)u(s)+\int_a^s \mu(s,\tau)u(\tau)d\tau\right]ds.$$

Bundan quyidagiga ega bo’lamiz:

$$\frac{w'(t)}{w(t)}=\beta\left[\vartheta(t)\frac{u(t)}{w(t)}+\int_0^t \mu(t,\tau)\frac{u(\tau)}{w(t)}d\tau\right]\le \beta v(t)+\beta\int_0^t \mu(t,\tau)d\tau,$$

chunki $w'(t)\ge 0$ bo’lgani sababli $t\ge s$ bo’lganda $w(t)\ge w(s)$ va teoremaning shartiga ko’ra $w(s)\ge u(s),$ ya’ni $w(t)\ge u(s).$ hosil bo’lgan tengsizlikni a dan t gacha integrallaymiz:

$$\ln w(t)-\ln a\le \beta\int_a^t\left[v(s)+\int_a^s \mu(s,\tau)d\tau\right]ds$$

yoki

$$u(t)\le w(t)\le \alpha\exp\left[\beta\int_a^t\left(v(s)+\int_a^s\mu(s,\tau)d\tau\right)ds\right].$$

Teorema to'liq isbot bo'ldi.

2.2 Yadrosi integrallanmaydigan Vo'lterra tipidagi integral tengsizlik.

1.2.5-teorema.[2] Agar $I(a\le t<\infty)$ yarim o'qda aniqlangan, uzluksiz $u(t)\ge 0$ funksiya va o'zgarmas $c\ge 0,\alpha\ge 0,\beta\ge 0$ sonlar uchun

$$u(t)\le c(t-a)^{\alpha}+m(t-a)^{\beta}\int_a^t\frac{u(s)}{s-a}ds \qquad (1.2.15)$$

tengsizlik o'rinli bo'lsa, u holda shu I yarim o'qda

$$u(t)\le c(t-a)^{\alpha}\left\{1+\sum_{n=1}^{\infty}\frac{m^n(t-a)^{n\beta}}{\alpha(\alpha+\beta)\ldots[\alpha+(n-1)\beta]}\right\} \qquad (1.2.16)$$

tengsizlik o'rinli bo'ladi.

Xususiy holda, agar $u(t)\le c(t-a)^{\alpha},\beta=0,m<\alpha$ bo'lsa,

$$u(t)\le\frac{ca}{\alpha-m}(t-a)^{\alpha} \qquad (1.2.17)$$

tengsizlik o'rinli bo'ladi.

Isbot. Teoremani ketma-ket o'rniga qo'yish usuli bilan isbotlaymiz. (1.2.16) tengsizlikdan quyidagiga ega bo'lamiz:

$$u(t)\le c(t-a)^{\alpha}+m(t-a)^{\beta}\int_a^t\frac{1}{s-a}\left\{c(s-a)^{\alpha}+m(s-a)^{\beta}\int_0^t\frac{u(\tau)}{\tau-a}d\tau\right\}ds\le$$

$$\le c(t-a)^{\alpha}+\frac{cm}{\alpha}(t-a)^{\alpha+\beta}+m^2(t-a)^{\beta}\int_0^t(s-a)^{\beta-1}\int_a^s\frac{1}{\tau-a}\left\{c(\tau-a)^{\alpha}+m(\tau-a)^{\beta}\int_a^t\frac{u(\tau_1)}{\tau_1-a}d\tau_1\right\}d\tau ds\le$$

$$\le c(t-a)^{\alpha}+\frac{cm}{\alpha}(t-a)^{\alpha+\beta}+\frac{cm^2}{\alpha(\alpha+\beta)}(t-a)^{\alpha+2\beta}+m^3(t-a)^{\beta}\int_a^t(s-a)^{\beta-1}\int_a^s(\tau-a)^{\beta-1}\int_a^{\tau}\frac{1}{\tau_1-a}\cdots$$

$$\cdots\left\{c(\tau_1-a)^{\beta}\int_a^{\tau}\frac{u(\tau_2)}{\tau_2-a}\right\}d\tau_1 d\tau ds\le\ldots=c(t-a)^{\alpha}\left\{1+\sum_{n=1}^{\infty}\frac{m^n(t-a)^{n\beta}}{\alpha(\alpha+\beta)\ldots[\alpha+(n-1)\beta]}\right\}$$

Bu esa (1.2.16) tengsizlikning o'rinli ekanini ko'rsatadi.

Agar $\beta=0$ bo'lsa, (1.2.16) tengsizlik quyidagi ko'rinishga ega bo'ladi:

$$u(t)\le c(t-a)^{\alpha}\left[1+\frac{m}{a}+\left(\frac{m}{a}\right)^{2}+\left(\frac{m}{a}\right)^{3}+\ldots+\left(\frac{m}{a}\right)^{n}+\ldots\right].$$

Bundan, $m<\alpha$ bo'lganda (1.2.17) tengsizlik kelib chiqadi.

2.3 Integrallash chegaralaridan biri cheksiz bo'lgan integral tengsizlik.

1.2.6-Teorema[2]**.** Faraz qilaylik $I(-\infty<t\le t_0)$ oraliqda aniqlangan uzluksiz $u(t)\ge 0$ funksiya

$$u(t)\le C+\int_{-\infty}^{t}\varphi(s)u(s)ds$$

Integral tengsizlikni qanoatlantirsin, bu yerda $C\ge 0$ son, $\varphi(t)\ge 0$ va $\int_{-\infty}^{t}\varphi(s)ds\le+\infty.$

U holda quyidagi tengsizlik ham o'rinli bo'ladi:

$$u(t)\le Ce^{\int_{-\infty}^{t}\varphi(s)ds}$$

Isbot. Teoremani ketma-ket o'rniga qo'yish usuli bilan isbotlaymiz.

$$u(t)\le C+C\int_{-\infty}^{t}\varphi(s)ds+C\int_{-\infty}^{t}\varphi(s)\int_{-\infty}^{s}\varphi(s_1)ds_1ds+C\int_{-\infty}^{t}\varphi(s)\int_{-\infty}^{s}\varphi(s_1)\int_{-\infty}^{s_1}\varphi(s_2)ds_2ds_1ds+$$

$$+\ldots\ldots+C\int_{-\infty}^{t}\varphi(s)\int_{-\infty}^{s}\varphi(s_1)\int_{-\infty}^{s_1}\varphi(s_2)\ldots\ldots\int_{-\infty}^{s_{n-1}}\varphi(s_n)ds_nds_{\,n-1}\ldots\ldots ds_1ds+$$

$$+\int_{-\infty}^{t}\varphi(s)\int_{-\infty}^{s}\varphi(s_1)\ldots\ldots\int_{-\infty}^{s_n}\varphi(s_{n+1})u(s_{n+1})ds_{n+1}\ldots\ldots ds_1ds.$$

yoki

$$u(t)\le C+C\int_{-\infty}^{t}\varphi(s)ds+\frac{C}{2!}\left(\int_{-\infty}^{t}\varphi(s)ds\right)^{2}+\frac{C}{3!}\left(\int_{-\infty}^{t}\varphi(s)ds\right)^{3}+\ldots\ldots+\frac{C}{n!}\left(\int_{-\infty}^{t}\varphi(s)ds\right)^{n}+$$

$$+\int_{-\infty}^{t}\varphi(s)\int_{-\infty}^{s}\varphi(s_1)\ldots\ldots\int_{-\infty}^{s_n}\varphi(s_{n+1})u(s_{n+1})ds_{n+1}ds_n\ldots\ldots ds$$

Bu jarayonni cheksiz davom ettirsak, ya'ni $n\to\infty$desak oxirgi had nolga intiladi va

$$u(t)\le C\left[1+\int_{-\infty}^{t}\varphi(s)ds+\ldots+\frac{1}{n!}\left(\int_{-\infty}^{t}\varphi(s)ds\right)^{n}+\ldots\right]=C\exp\left(\int_{-\infty}^{t}\varphi(s)ds\right)$$

tenglikka ega bo'lamiz. Shuni isbot qilish kerak edi.

I bob bo'yicha xulosa.

Dissertatsiya ishining birinchi bobida asosan dissertatsiya ishini yozishda kerak bo'ladigan asosiy boshlang'ich tushunchalar berilgan.

Birinchi bob integral tenglama va tengsizliklar haqida boshlang'ich ma'lumotlar deb nomlanib, bunda integral tenglamaning umumiy ko'rinishlari, yechish usullari, integral tengsizliklar haqida boshlang'ich ma'lumotlar, yadrosi integrallanmaydigan Vol'terra tipidagi integral tengsizlik va integrallash chegaralaridan biri cheksiz bo'lgan integral tengsizliklar haqida boshlang'ich ma'lumotlar keltirilgan

.

II-BOB. Maxsus integral tengsizliklar.

1-§. Chegaralaridan biri cheksiz bo’lgan karrali integral tengsizliklar.

1.1 Chegaralaridan biri cheksiz bo’lgan karrali integrallarni saqlovchi integral tengsizlik.

Bu paragrafda kelajak uchun bizga zarur bo’lgan maxsus integral tengsizliklar haqidagi teoremalarni isbotlaymiz.

2.1.1-teorema[14]**.** Faraz qilaylik $I(-\infty < t \le t_0)$ oraliqda aniqlangan, uzluksiz $u(t) \ge 0,\ \lambda(t) \ge 0,\ P(-\infty < t \le t_0, -\infty < s \le t)$ sohada aniqlangan uzluksiz $\mu(t,s) \ge 0$ funksiya hamda o’zgarmas $\alpha \ge 0,\ \beta \ge 0$ sonlar uchun

$$u(t) \le \alpha + \beta \int_{-\infty}^{t} \left[\lambda(s)u(s) + \int_{-\infty}^{s} \mu(s,\tau)u(\tau)d\tau \right] ds \qquad (2.1.1)$$

Tengsizlik o’rinli bo’lsin, bu yerda

$$\int_{-\infty}^{t} \lambda(s)ds < \infty, \qquad \int_{-\infty}^{t} \lambda(s) \int_{-\infty}^{s} \mu(s,\tau)d\tau ds < \infty.$$

U holda shu I oraliqda

$$u(t) \le \alpha \exp\left\{ \beta \int_{-\infty}^{t} \left[\lambda(s) + \int_{-\infty}^{s} \mu(s,\tau)d\tau \right] ds \right\} \qquad (2.1.2)$$

tengsizlik ham o’rinli bo’ladi.

Isbot. Quyidagicha belgilash kiritamiz

$$w(t) = \alpha + \beta \int_{-\infty}^{t} \left[\lambda(s)u(s) + \int_{-\infty}^{s} \mu(s,\tau)u(\tau)d\tau \right] ds$$

Bu tenglikni t bo’yicha differensiallaymiz, u holda

$$w'(t) = \beta \left[\lambda(t)u(t) + \int_{-\infty}^{t} \mu(t,\tau)u(\tau)d\tau \right]$$

hosil bo’ladi. Bu tenglikning har ikkala tomonini $w(t) \ne 0$ ga bo’lamiz:

$$\frac{w'(t)}{w(t)} = \beta \left[\lambda(t)\frac{u(t)}{w(t)} + \int_{-\infty}^{t} \mu(t,\tau)\frac{u(\tau)}{w(\tau)}d\tau \right]$$

Agar $t \ge s$ bo’lganda $w(t) \ge w(s)$ va $w(t) \ge u(t)$ ekanligini nazarga olsak

$$\frac{w'(t)}{w(t)} \le \beta\lambda(t) + \beta\int_{-\infty}^{t} \mu(t,\tau)d\tau$$

tengsizlikka ega bo'lamiz. Bu tengsizliklarni $-\infty$ dan t gacha integrallab, quyidagini topamiz.

$$\ln w(t) - \ln\alpha = \beta\int_{-\infty}^{t}\left[\lambda(s) + \int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds$$

yoki

$$\ln\frac{w(t)}{\alpha} = \beta\int_{-\infty}^{t}\left[\lambda(s) + \int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds$$

bundan

$$w(t) \le \alpha\exp\left\{\beta\int_{-\infty}^{t}\left[\lambda(s) + \int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds\right\}$$

$u(t) \le w(t)$ ekanligini nazarga olsak oxirgi tengsizlikdan isbot talab etilgan tengsizlik kelib chiqadi.

1.2 Chegaralaridan biri cheksiz va yadrolari integrallanmaydigan bo'lgan karrali integrallarni saqlovchi integral tengsizlik.

Endi chegaralaridan biri cheksiz va yadrolari integrallanmaydigan maxsus integral tengsizlikni o'rganamiz.

2.1.2-teorema[15]. Agar $I(-\infty < t < 0)$ oraliqda aniqlangan uzluksiz $0 \le u(t) \le kt^{-\alpha}, (k \ge 1)$ funksiya va o'zgarmas $m \ge 0$ $a < 0, b > 0$ $\alpha = \frac{\lambda}{\mu} > 1, (\lambda,\mu) = 1$ (λ-juft son) sonlar uchun

$$u(t) \le ct^{-\alpha} + m\int_{-\infty}^{t}\left[\frac{au(s)}{s} + \int_{-\infty}^{s}\frac{bu(\tau)}{s\tau}d\tau\right]ds \qquad (2.1.3)$$

tengsizlik o'rinli bo'lsa, u holda

$$m\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right) < 1 \qquad (2.1.4)$$

bo'lganda ushbu

$$u(t) \le \frac{ct^{-\alpha}}{1 - m(\frac{b}{\alpha^2} - \frac{a}{\alpha})} \qquad (2.1.5)$$

tengsizlik o'rinli bo'ladi.

Agar $c = 0$ bo'lsa $u(t) \equiv 0$ bo'ladi.

Isbot. Teoremani ketma-ket o'rniga qo'yish usuli yordamida isbotlaymiz. (2.1.3) tengsizlikning o'ng tomonidagi $u(s)$ va $u(\tau)$ lar o'rniga ulardan kichchik bo'lmagan (2.1.3) tengsizlikning o'ng tomonidagi ifodani qo'yamiz. So'ngra hosil bo'lgan tengsizlik uchun shu ishni bajaramiz va h.k. natijada quyidagi tengsizlik hosil bo'ladi:

$$u(t) \le ct^{-\alpha} + f_1(t) + f_2(t) + \cdots + f_n(t) + R_{n+1} \qquad (2.1.6)$$

bu yerda

$$f_1(t) = m\int_{-\infty}^{t} [a\frac{f_0(s)}{s} + \int_{-\infty}^{s} \frac{bf_0(\tau)d\tau}{s\tau} d\tau]ds \qquad f_0 = ct^{-\alpha}.$$

$$f_2(t) = m\int_{-\infty}^{t}\left[a\frac{f_1(s)}{s} + \int_{-\infty}^{s} b\frac{f_1(\tau)}{s\tau}d\tau\right]ds,$$

……………………………………

$$f_n(t) = m\int_{-\infty}^{t}\left[a\frac{f_{n-1}(s)}{s} + \int_{-\infty}^{s} b\frac{f_{n-1}(\tau)}{s\tau}d\tau\right]ds,$$

……………………………………

$$R_1(t) = m\int_{-\infty}^{t}\left[a\frac{u(s)}{s} + \int_{-\infty}^{s} b\frac{u(\tau)}{s\tau}d\tau\right]ds,$$

$$R_2(t) = m\int_{-\infty}^{t}\left[a\frac{R_1(s)}{s} + \int_{-\infty}^{s} b\frac{R_1(\tau)}{s\tau}d\tau\right]ds,$$

……………………………………

$$R_{n+1}(t) = m\int_{-\infty}^{t}\left[a\frac{R_n(s)}{s} + \int_{-\infty}^{s} b\frac{R_n(\tau)}{s\tau}d\tau\right]ds$$

……………………………………

Endi $f_1(s),\ f_2(s),\ f_3(x)\ \ldots,\ f_n(s)$ va $R_{n+1}(t)$ larni ketma-ket hisoblab chiqamiz.

$$f_1(t) = mc\int_{-\infty}^{t}\left[as^{-\alpha-1} + bs^{-1}\int_{-\infty}^{s}\tau^{-\alpha-1}d\tau\right]ds =$$

$$= mc\int_{-\infty}^{t}\left[as^{-\alpha-1} - \frac{b}{\alpha}s^{-\alpha-1}\right]ds = mc\left[-\frac{at^{-\alpha}}{\alpha} + \frac{bt^{-\alpha}}{\alpha^2}\right]$$

Demak, $f_1(t) = cm\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)t^{-\alpha}$ xuddi shuningdek

$$f_n(t) = c\left[m\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)\right]^n t^{-\alpha}.$$

ga ega bo'lamiz.

Endi $u(t) \le kt^{-\alpha}$ ekanligini nazarga olib $R_{n+1}(t)$ uchun baho topamiz. Ko'rinib turibdiki

$$R_1(t) \le km\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)t^{-\alpha}$$

$$R_{n+1}(t) \le k\left[m\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)\right]^{n+1} t^{-\alpha}$$

Endi ketma-ket qo'yish protsessini cheksiz davom ettirishga krishamiz. Buning uchun (2.1.6) tengsizlikda $n \to \infty$ da limitga o'tish yetarli. Bu holda

$$\lim_{n\to\infty} R_{n+1}(t) = 0$$

va ushbu tengsizlik hosil bo'ladi:

$$u(t) \le ct^{-\alpha} + cm\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)t^{-\alpha} + \ldots\ldots + c\left[m\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)\right]^{n+1} t^{-\alpha}\cdots$$

yoki

$$u(t) \le \frac{ct^{-\alpha}}{1 - m\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)}$$

Bu esa isbot talab etilgan tengsizlikdir.

2-§. Abel yadroli Vol'terra tipidagi integral tengsizlik.

Ma'lumki integral, differensial va integro-differensial tenglamalar yechimlari va ularning xossalarini o'rganishda integral tengsizliklar muhim ahamiyatga ega. Hozirgi kunda integral tengsizliklar nazariyasi matematika fanining chuqur rivojlangan sohalaridan biri bo'lib, bu yo'nalishda O'zbekistonlik va chet el olimlari ko'plab ilmiy izlanishlar olib borishmoqda. [1,2,3]

Gronuoll tomonidan o'rganilgan

$$u(t) \le C + m\int_0^t u(s)ds$$

integral tengsizlik va uning ko'plab umumlashmalari differensial tenglamalar yechimlarining mavjudligi, yagonaligi, turg'unligi va boshqa hossalarini o'rganishga tadbiq etilgan va bu g'oya asosida ko'plab ilmiy natijalar olingan.

Gronuoll tengsizligining umumlashgan ko'rinishlaridan biri

$$u(t) \le f(t) + \int_a^t K(t,s)u(s)ds$$

bo'lib, bunda $u(t) \ge 0$, $f(t) \ge 0$, $[a,b]$ oraliqda, $K(t,s)$, $D(0 \le t \le t_0,\ 0 \le s \le t)$ sohada aniqlangan uzluksiz funksiyalar deb qaraladi. Bu holda uning yechimi

$$u(t) \le f(t) + \int_a^t R(t,s) f(s)ds$$

ko'rinishda topiladi. Bu yerda $R(t,s)$

$$v(t) = f(t) + \int_a^t K(t,s)v(s)ds$$

tenglamaning rezolventasi.

Mazkur ishda $f(t) = C$ va yadro

$$K(t,s) = \frac{m}{\sqrt{t-s}}$$

ko’rinishda olinadi va u Abel yadrosi deb ataladi. Bu yadro $t = s$ chiziq bo’ylab uzulishga ega. Bunda quyidagi teorema o’rinli bo’ladi.

2.2.1-Teorema. Faraz qilaylik $[o, t_0]$ oraliqda aniqlangan uzluksiz $u(t) \geq 0$ funksiya

$$u(t) \leq C + m\int_0^t \frac{u(s)ds}{\sqrt{t-s}} \tag{2.2.1}$$

Integral tengsizlikni qanoatlantirsin, bu yerda $C \geq 0$, $m \geq 0$ sonlar. U holda quyidagi tengsizlik o’rinli bo’ladi.

$$u(t) \leq \frac{C}{m}\left[\sum_{n=0}^{\infty}\frac{m(2\pi m^2 t)^n}{(2n)!!} + \sum_{n=0}^{\infty}\frac{(2\pi m^2 t)^{n+1}}{\pi\sqrt{t}(2n+1)!!}\right] \tag{2.2.2}$$

Bu teoremaning shartlari bajarilganda oxirgi tengsizlikni quyidagicha yozish mumkin

$$u(t) \leq \frac{C}{\sqrt{2t}m\pi}\int_0^{+\infty} e^{\sqrt{2}x} e^{-\frac{x^2}{2\pi m^2 t}}dx \tag{2.2.3}$$

Bu tasdiqning to’g’riligini qatorlar va xosmas integrallar orasidagi bog’lanishlarga asoslanib ko’rsatish mumkin.

Biz quyida yadroning ko’rinishi $(t-s)^{-\alpha}$ $(o < \alpha < 1)$ bo’lgan hol uchun umumlashtirish mumkinligini ko’rsatamiz.

Ma’lumki Gronuoll tengsizligi differensial, integral va integro-differensial tenglamalar nazariyasida muhim tadbiqlarga ega. Gronuoll tengsizligining ko’plab umumlashmalari [3,4,5] ishda keltirilgan bo’lib ularning eng umumiy ko’rinishi

$$u(t) \leq f(t) + \int_a^t K(t,s)u(s)ds$$

dan iborat.

Biz bu yerda

$$K(t,s) = \frac{m}{(t-s)^{\alpha}} \qquad 0 \prec \alpha \prec 1, \quad m \succ 0$$

singulyar yadrodan iborat bo’lgan holni qaraymiz. Bu funksiya to’g’ri chiziq bo’ylab cheksiz uzulishga ega.

2.2.2-Teorema[16]**.** Faraz qilaylik $u(t)\geq 0$ funksiya $[0,t_0]$ segmentda aniqlangan uzluksiz funksiya bo'lib, ushbu

$$u(t)\leq f(t)+\int_0^t \frac{m}{(t-s)^{\alpha}}u(s)ds \qquad (2.2.4)$$

integral tengsizlikni qanoatlantirsin.

Bu yerda $m\geq 0$ $0<\alpha<1$ sonlar, $f(t)\geq 0$ $[0,t_0]$ segmentda uzluksiz va

$$\max_{t\in[0,t_0]} f(t)=C \quad C\geq 0 \text{ son}$$

shartni qanoatlantirsin, u holda quyidagi tengsizlik o'rinli bo'ladi.

$$u(t)\leq f(t)+C\sum_{n=1}^{\infty}\frac{\left[t^{1-\alpha}m\Gamma(1-\alpha)\right]^n}{n(1-\alpha)\Gamma(n(1-\alpha))} \qquad (2.2.5)$$

va $f(t)=0$ bo'lsa $u(t)=0$ bo'ladi.

Isbot. Teoremani ketma-ket o'rniga qo'yish usuli yordamida isbotlaymiz. (2.2.4) tengsizlikning o'ng tomonidagi $U(s)$ o'rniga tengsizlikning o'ng tomonida turgan ifodani ketma-ket qo'yib quyidagiga ega bo'lamiz.

$$u(t)\leq f(t)+m\int_0^t \frac{f(s)}{(t-s)^{\alpha}}ds+m^2\int_0^t\frac{ds}{(t-s)^{\alpha}}\int_0^s\frac{f(s_1)ds_1}{(t-s_1)^{\alpha}}+$$

$$+m^3\int_0^t\frac{ds}{(t-s)^{\alpha}}\int_0^s\frac{ds}{(s-s_1)^{\alpha}}\int_0^{s_1}\frac{f(s_2)ds_2}{(s_1-s_2)^{\alpha}}+\ldots+m^n\int_0^t\frac{ds}{(t-s)^{\alpha}}\int_0^s\frac{ds_1}{(s-s_1)}\ldots\int_0^{s_{n-2}}\frac{f(s_{n-1})ds_{n-1}}{(s_{n-2}-s_{n-1})^{\alpha}}+$$

$$+m^{n+1}\int_0^t\frac{ds}{(t-s)^{\alpha}}\int_0^s\frac{ds_1}{(s-s_1)^{\alpha}}\ldots\int_0^{s_{n-1}}\frac{u(s_n)ds_n}{(s_{n-1}-s_n)^{\alpha}}$$

yoki

$$u(t)\leq f(t)+mf_1(t)+m^2f_2(t)+m^3f_3(t)+\ldots+m^nf_n(t)+m^{n+1}R_{n+1}(t).$$

Endi $f_1(t),\quad f_2(t),\quad f_3(t),\quad \ldots,\quad f_n(t),\quad R_{n+1}(t)$ larni baholaymiz.

$$f_1(t)=\int_0^t\frac{f(s)ds}{(t-s)^{\alpha}}\leq C\int_0^t\frac{ds}{(t-s)^{\alpha}}=\frac{C}{1-\alpha}t^{1-\alpha}.$$

$$f_2(t)=\int_0^t\frac{f_1(s)ds}{\left(t-s\right)^{\alpha}}\leq\frac{C}{1-\alpha}\int_0^t\frac{s^{1-\alpha}ds}{\left(t-s\right)^{\alpha}}$$

Bu integralda $s=t\sin^2\frac{\theta}{2}$ almashtirish olamiz. U holda $ds=t\sin\frac{\theta}{2}\cos\frac{\theta}{2}$, $s=0$ da $\theta=0$, $s=t$ da $\theta=\pi$ bo'lib

$$f_3(t) \le \frac{C}{1-\alpha}\int_0^{\pi} t^{2-2\alpha}\sin^{3-2\alpha}\frac{\theta}{2}\cos^{1-2\alpha}\frac{\theta}{2}d\theta = \frac{2C}{1-\alpha}\int_0^{\frac{\pi}{2}} t^{2-2\alpha}\sin^{3-2\alpha}\tau\cos^{1-2\alpha}\tau d\tau$$

Endi oxirgi integralda $x=\sin\tau$ almashtirish olamiz. U holda $dx=\cos\tau d\tau$ $d\tau = \frac{dx}{\cos\tau} = \frac{dx}{\sqrt{1-x^2}}$, $\tau=0$ da $x=0$, $\tau=\frac{\pi}{2}$ da $x=1$ bo'lib

$$f_2(t) \le \frac{2C}{1-\alpha}t^{2-2\alpha}\int_0^1 x^{3-2\alpha}(1-x^2)^{-\alpha}dx$$

ga ega bo'lamiz.

Bu integralda $x^2=y$, $dx=\frac{dy}{2\sqrt{y}}$ deb olsak

$$f_2(t) \le \frac{C}{1-\alpha}t^{2-2\alpha}\int_0^1 y^{\frac{3-2\alpha}{2}}(1-y)^{-\alpha}y^{-\frac{1}{2}}dy = \frac{C}{1-\alpha}t^{2-2\alpha}\int_0^1 y^{1-\alpha}(1-y)^{-\alpha}dy =$$

$$= \frac{C}{1-\alpha}t^{2-2\alpha}\int_0^1 y^{(2-\alpha)-1}(1-y)^{(1-\alpha)-1} = \frac{C}{1-\alpha}t^{2-2\alpha}B(2-\alpha,1-\alpha) =$$

$$= \frac{C}{1-\alpha}t^{2-2\alpha}\frac{(1-\alpha)\Gamma^2(1-\alpha)}{2(1-\alpha)\Gamma(2(1-\alpha))} = \frac{Ct^{2(1-\alpha)}\Gamma^2(1-\alpha)}{2(1-\alpha)\Gamma(2(1-\alpha))}$$

Demak,

$$f_2(t) \le \frac{Ct^{2(1-\alpha)}\Gamma^2(1-\alpha)}{2(1-\alpha)\Gamma\left(2(1-\alpha)\right)}$$

$$f_3(t) = \int_0^t \frac{f_2(s)ds}{(t-s)^{\alpha}} \le \frac{C\Gamma^2(1-\alpha)}{2(1-\alpha)\Gamma(2(1-\alpha))}\int_0^t \frac{s^{2(1-\alpha)}}{(t-s)^{\alpha}}ds$$

Huddi yuqoridagidek hisoblashlarni takrorlab

$$f_3(t) \le \frac{C\Gamma^2(1-\alpha)t^{3(1-\alpha)}}{2(1-\alpha)\Gamma(2(1-\alpha))}\mathrm{B}(3-2\alpha,1-\alpha)$$

ni topamiz. Buni quyidagicha yozish mumkin

$$f_3(t) \le \frac{Ct^{3(1-\alpha)}\Gamma^3(1-\alpha)}{3(1-\alpha)\Gamma(3(1-\alpha))}$$

Bu jarayonni davom ettirib

$$f_n(t) \le \frac{C(t^{1-\alpha}\Gamma(1-\alpha))^n}{n(1-\alpha)\Gamma(n(1-\alpha))}$$

ga ega bo'lamiz. Osongina ko'rsatish mumkinki $n\to\infty$ da $R_{n+1}(t)\to 0$ bo'ladi va quyidagiga ega bo'lamiz

$$u(t) \le f(t) + C\sum_{n=1}^{\infty} \frac{(mt^{1-\alpha}\Gamma(1-\alpha))^n}{n(1-\alpha)\Gamma(n(1-\alpha))}. \qquad (2.2.6)$$

Endi oxirgi qatorning $[0,t_0]$ qismida tekis yaqinlashishini ko'ramiz. Soddalik uchun $a = 1-\alpha$ deb olamiz, $a < 1$, va quyidagi sonli qatorni qaraymiz

$$\sum_{n=1}^{\infty} \frac{\left[ml^a\Gamma(a)\right]^n}{na\Gamma(na)} \qquad l = \max_{t\in[0,t_0]} t \qquad (2.2.7)$$

Bu yerdagi $\Gamma(na)$ ni Gauss formulasi yordamida almashtiramiz.

$$\Gamma(na) = \frac{n^{na-\frac{1}{2}}}{(2\pi)^{\frac{n-1}{2}}}\Gamma(a)\Gamma(a+\frac{1}{n})\cdots\Gamma(a+\frac{n}{n+1})$$

Dalamber alomatiga ko'ra

$$D = \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\left[ml^a\Gamma(a)\right]^{n+1}}{(n+1)\Gamma((n+1)a)} : \frac{\left[ml^a\Gamma(a)\right]^n}{na\Gamma(na)} = \lim_{n\to\infty}\frac{ml^a\Gamma(a)n\Gamma(na)}{(n+1)\Gamma((n+1)a)} =$$

$$= ml^a\Gamma(a)\lim_{n\to\infty}\frac{n}{n+1}\cdot\frac{n^{na-\frac{1}{2}}(2\pi)^{\frac{n}{2}}\Gamma(a)\Gamma(a+\frac{1}{2})\cdots\Gamma(a+\frac{n-1}{n})}{(2\pi)^{\frac{n-1}{2}}(n+1)^{(n+1)a-\frac{1}{2}}\Gamma(a)\Gamma(a+\frac{1}{n+1})\cdots\Gamma(a+\frac{n}{n+1})}$$

Ba'zi bir elementar hisoblashlardan keyin $D = 0$ ekanligini ko'rsatish mumkin. Bu esa (2.2.7) qatorning yaqinlashishini ko'rsatadi. Bundan esa (2.2.6) tengsizlikdagi qatorning $[0,t_0]$ kesmada tekis yaqinlashishi kelib chiqadi. Shu bilan teorema to'liq isbot bo'ldi.

II-bob bo’yicha xulosa.

Bu dissertatsiya ishining ikkinchi bobida maxsus integral tengsizliklar qaralgan.

Bu bob ikki paragrafdan iborat bo’lib, birinchi paragraf chegaralaridan biri cheksiz bo’lgan karrali integrallarni saqlovchi integral tengsizliklar va chegaralaridan biri va yadrolari integrallanmaydigan karrali integrallarni saqlovchi integral tengsizlik.

Ikkinchi paragrafda esa Abel yadroli Vol’terra tipidagi integral tengsizlik isbotlangan. Bunda

1. $u(t) \le \alpha + \beta \int_{-\infty}^{t} \left[\lambda(s)u(s) + \int_{-\infty}^{s} \mu(s,\tau)u(\tau)d\tau \right] ds$

2. $u(t) \le ct^{-\alpha} + m \int_{-\infty}^{t} \left[\frac{au(s)}{s} + \int_{-\infty}^{s} \frac{bu(\tau)}{s\tau} d\tau \right] ds$

3. $u(t) \le f(t) + \int_{0}^{t} \frac{m}{(t-s)^{\alpha}} u(s) ds \qquad (0 < \alpha < 1)$

Maxsus integral tengsizliklar isbotlangan.

III-BOB. Maxsus integral tengsizliklarning integral va integro-differensial tenglamalar yechimlarini tadqiq qilishga tadbiqlari.

Ma'lumki Gronuoll tomonidan 1919 yilda

$$u(t) \le C + k\int_0^t u(s)ds$$

ko'rinishidagi integral tengsizlik qaralib, uning yechimi $u(t) \le Ce^{kt}$ ko'rinishida topilgan va u $y' = f(x,y),\ \ y(0) = y_0$ Koshi masalasi yechimining yagonaligini isbotlashga tadbiq etilgan. Bu g'oya ko'plab chet el va O'zbekistonlik olimlarning diqqatini o'ziga tortdi va bu tengsizlik turli yo'nalishlarda umumlashtirilib mustaqil nazariyani vujudga kelishiga sabab bo'ldi. Integral tengsizliklar nazariyasi hozirgi kunda keng rivojlandi va ularga oid ma'lumotlar [1,2,3,4,5] adabiyotlarda jamlangan.

Integral tengsizliklar hozirgi kunda integral, differensial, integro-differensial, xususiy hosilali differensial tenglamalar va boshqa ko'pgina tenglamalar yechimlarining mavjudligi, yagonaligi, boshlang'ich shartlardan uzluksiz bog'liqligi, parametrlar bo'yicha uzluksizligi va differesianllanuvchanligi, turg'unligi, chegaralanganligi va boshqa ko'pgina xossalarini o`rganishda qulay apparat bo'lib xizmat qilmoqda. Bulardan tashqari tenglamalarning aniq va taqribiy yechimlari orasidagi farqni baholashda ham tengsizliklardan foydalanish mumkin [1,2,3,4,5].

1-§. Integro-differensial tenglama uchun Koshining limitik masalasi yechimini tadqiq qilish.

Biz bu yerda yarim o'qdagi chiziqli bo'lmagan integro-differensial tenglama yechimining mavjudligi va yagonaligi haqidagi teoremani isbotlaymiz.

Ushbu

$$\frac{dx}{dt} = F(t,x) + \int_{-\infty}^{t} K(t,s,x)ds \qquad (3.1.1)$$

yarim o'qda chiziqli bo'lmagan integro-differensial tenglamani qaraymiz. Bu tenglamaning

$$\lim_{t\to-\infty} x(t) = c \qquad (3.1.2)$$

boshlang'ich shartni qanoatlantiruvchi yechimini toppish bilan shug'illanamiz. $(3.1.1)-(3.1.2)$ birgalikda Koshining limitik masalasi deb aytiladi.

Biz avvalo $(3.1.1)-(3.1.2)$ masalaning yechimini topishni unga ekvivalent bo'lgan integral tenglamaning yechimini toppish masalasiga keltiramiz. Haqiqatdan $(3.1.1)$ tenglamaning $(3.1.2)$ shartni qanoatlantiruvchi yechimi bo'lsa, u holda

$$\frac{dx(t)}{dt} = F(t, x(t)) + \int_{-\infty}^{t} K(t, s, x(t))ds \qquad (3.1.3)$$

ayniyatga ega bo'lamiz. Bu tenglikning har ikkala tomonini $-\infty$ dan t gacha integrallab, $(3.1.2)$ ni hisobga olsak ushbu integral tenglamaga kelamiz:

$$x(t) = c + \int_{-\infty}^{t} F(t, x(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s, \tau, x(\tau))d\tau ds \qquad (3.1.4)$$

Bu tenglamani ketma-ket yaqinlashish usuli yordamida yechamiz. Yechimga ketma-ket yaqinlashishlarni quyidagicha tuzamiz:

$$\begin{cases} x_0(t) = c \\ x_1(t) = c + \int_{-\infty}^{t} F(s, x_0(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s, \tau, x_0(\tau))d\tau ds \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_n(t) = c + \int_{-\infty}^{t} F(s, x_{n-1}(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s, \tau, x_{n-1}(\tau))d\tau ds \end{cases} \qquad (3.1.5)$$

Quyida shu $x_0(t), x_1(t), \cdots, x_n(t), \cdots$ ketma-ketlikning biror $x(t)$ funksiyaga tekis yaqinlashishini, $x(t)$ funksiyaning uzluksizligini va uning $(3.1.4)$ tenglamani qanoatlantirishini ko'rsatamiz. $(3.1.5)$ tengliklardagi $x_n(t)$ funksiyaning $(3.1.4)$ tenglamaning taqribiy yechimi ham bo'ladi. Uning aniq yechim bilan farqini ham baholash mumkinligini ko'ramiz. So'ngra ma'lum shartlar bajarilganda yechimning yagonaligini ham ko'rsatamiz.

3.1.1-teorema. Faraz qilaylik

1) $F(t,x)$ funksiya $D=\{(t,x):-\infty<t<t_0,|x|<\infty\}$ sohada $K(t,s,x)$ funksiya esa $Q=\{(t,s,x):-\infty<t<t_0,-\infty<s<t,|x|<\infty\}$ sohada aniqlangan va uzluksiz funksiyalar bo'lib, bundan tashqari

$$\int_{-\infty}^{t}|F(s,c)|ds<\infty, \qquad \int_{-\infty}^{t}\int_{-\infty}^{s}|K(s,\tau,c)|d\tau ds<\infty$$

shartlar bajarilsin.

2) $F(t,x)$ va $K(t,s,x)$ funksiyalar x argumenti bo'yicha Lipshis shartini qanoatlantiradi, ya'ni

$$|F(t,x_1)-F(t,x_2)|\le\lambda(t)|x_1-x_2|$$

$$|K(t,s,x_1)-K(t,s,x_2)|\le\mu(t,s)|x_1-x_2|$$

bu yerda

$$\int_{-\infty}^{t}\lambda(t)ds<\infty, \qquad \int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)d\tau ds<\infty$$

deb hisoblanadi. U holda (3.1.3)–(3.1.4) masala $I(-\infty,t_0)$ oraliqda yagona uzluksiz yechimga ega bo'ladi. Bundan tashqari aniq va taqribiy yechimlar orasidagi farq quyidagicha baholanadi.

$$|x(t)-x_n(t)|\le\frac{a(t_0)b^{n-1}(t_0)}{n!}\exp\left\{\int_{-\infty}^{t}\left[\lambda(t)+\int_{-\infty}^{t}\mu(t,s)d\tau\right]ds\right\}$$

Isbot. Avvalo yechimning mavjudligini isbotlaymiz. Shu maqsadda (3.1.5) tenglikdan ketma-ket quyidagilarni topamiz:

$$|x_1(t)-x_0(t)|=\left|\int_{-\infty}^{t}F(s,x_0(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x_0(\tau))d\tau ds\right|\le$$

$$\le\int_{-\infty}^{t}|F(s,c)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}|K(s,\tau,c)|d\tau ds=a(t)$$

$$|x_2(t)-x_1(t)|=\int_{-\infty}^{t}|F(t,x_1(s))-F(t,x_0(s))|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}|K(s,\tau,x_1(\tau))-K(s,\tau,x_0(\tau))|d\tau ds\le$$

$$\le\int_{-\infty}^{t}\lambda(s)|x_1(s)-x_0(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x_1(s)-x_0(\tau)|d\tau ds\le$$

$$\le\int_{-\infty}^{t}\lambda(s)\left[|x_1(s)-x_0(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x_1(\tau)-x_0(\tau)|d\tau\right]ds\le$$

$$\le a(t)\int_{-\infty}^{t}\lambda(s)\left[1+\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds=a(t)b(t)$$

bu yerda

$$a(t)=\int_{-\infty}^{t}|F(s,c)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}|K(s,\tau,c)|d\tau ds$$

$$b(t)=\int_{-\infty}^{t}\lambda(s)\left[1+\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds.$$

Bu jarayonni n marta takrorlasak natijada ushbu

$$|x_n(t)-x_{n-1}(t)|\le a(t)\frac{b^{n-1}(t)}{(n-1)!} \qquad (3.1.6)$$

tenglikni hosil qilamiz.

Matematik analiz kursidan ma'lumki $x_0(t),x_1(t),\cdots,x_n(t),\cdots$ ketma-ketlikning tekis yaqinlashishini ko'rsatish uchun ushbu

$$x_0(t)+(x_1(t)-x_0(t))+\cdots+(x_n(t)-x_{n-1}(t))\cdots \qquad (3.1.7)$$

qatorning tekis yaqinlashishini ko'rsatish yetarli. Biz yuqorida $(3.1.7)$ qatorning har bir hadi uchun $(3.1.6)$ ko'rinishdagi baholarni topgan edik. Shu sababli $(3.1.7)$ qator uchun ushbu

$$a(t_0)+a(t_0)\frac{b(t_0)}{1!}+a(t_0)\frac{b^2(t_0)}{2!}+a(t_0)\frac{b^3(t_0)}{3!}+\cdots+a(t_0)\frac{b^{n-1}(t_0)}{(n-1)!}+\cdots$$

Qator majorant qator vazifasini bajaradi. Bu yaqinlashuvchi sonli qator bo'lganligi uchun $(3.1.7)$ qator absolyut va tekis yaqinlashuvchi bo'ladi (Veyershtras alomatiga asosan). Demak $\{x_n(t)\}_{n=1}^{\infty}$ ketma-ketlik ham $(-\infty,t_0)$ da tekis yaqinlashadi, ya'ni

$$\lim_{n\to\infty}x_n(t)=x(t),\qquad t\in(-\infty,t_0),\qquad (t_0\le\infty)$$

F va K funksiyalarning aniqlanish sohalarida uzluksiz funksiyalar bo'lganligi uchun $(3.1.5)$ tengliklardan ko'rinadiki, $\{x_n(t)\}$ funksiyalar $(-\infty,t_0)$ oraliqda uzluksiz funksiyalarni aniqlaydi. Demak $x(t)$ funksiya ham uzluksiz va $(3.1.4)$ tenglamani qanoatlantiradi. Haqiqatdan, $F(t,x)$ va $K(t,s,x)$ funksiyaning o'z aniqlanish sohasida uzluksiz bo'lganliklaridan (3.1.5) tengliklarning n-chisidan $n \to \infty$ da limitga o'tsak

$$\lim_{n\to\infty} x_n(t) = x(t)$$

bo'lganligi uchun,

$$x(t) = c + \int_{-\infty}^{t} F(s, x(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s,\tau,x(\tau))d\tau ds$$

tenglikka ega bo'lamiz. Bunda integral ostida limitga o'tish mumkinligi haqidagi teoremadan foydalandik. Endi yechimning yagonaligini ko'ramiz. Faraz qilaylik (3.1.4) tenglama $(-\infty, t_0)$ oraliqda $x(t)$ dan boshqa yana bir $y(t)$ yechimga ham ega bo'lsa, u holda

$$x(t) = c + \int_{-\infty}^{t} F(s, x(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s,\tau,x(\tau))d\tau ds$$

va

$$y(t) = c + \int_{-\infty}^{t} F(s, y(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s,\tau,y(\tau))d\tau ds$$

tengliklarga ega bo'lamiz. Bularning biridan ikkinchisini ayirib, hosil bo'lgan tenglikning har ikkala tomonidan modul olsak quyidagi tenglik hosil bo'ladi:

$$|x(t) - y(t)| \le \int_{-\infty}^{t} \lambda(s)|x(s) - y(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \mu(s,\tau)|x(\tau) - y(\tau)|d\tau ds$$

ixtiyoriy o'zgarmas $a \succ 0$ soni uchun quyidagi tengsizlikni yozish mumkin

$$|x(t) - y(t)| \le a + \int_{-\infty}^{t} \lambda(s)|x(s) - y(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \mu(s,\tau)|x(\tau) - y(\tau)|d\tau ds$$

Integral tengsizliklar haqidagi 1.2.4-teoremaga asosan bu yerdan quyidagiga ega bo'lamiz

$$|x(t) - y(t)| \le a \exp\left\{ \int_{-\infty}^{t} \left[\lambda(s) + \int_{-\infty}^{t} \mu(s,\tau)d\tau \right] ds \right\}$$

$a \succ 0$ ning ixtiyoriyligiga ko'ra, uni nolga intiltirib quyidagi tengsizlikni hosil qilamiz

$$|x(t) - y(t)| \le 0$$

Bundan esa $x(t)-y(t)\equiv 0$ yoki $x(t)\equiv y(t)$ degan hulosaga kelamiz. Demak, (3.1.4) tenglamaning yechimi yagona ekan. Endi aniq va taqribiy yechimlar orasidagi farqlarni baholashga kirishamiz. Yuqorida qayd qilinganidek (3.1.4) tenglamaning aniq yechimi $x(t)$, taqribiy yechimi esa $x_n(t)$ bo'lsin:

$$x(t)=c+\int_{-\infty}^{t}F(s,x(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x(\tau))d\tau ds$$

$$x_n(t)=c+\int_{-\infty}^{t}F(s,x_{n-1}(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x_{n-1}(\tau))d\tau ds$$

Bularning biridan ikkinchisini ayirib hosil bo'lgan tenglikning har ikkala tomonidan modul olamiz:

$$|x(t)-x_n(t)|\le a+\int_{-\infty}^{t}\lambda(s)|x(s)-x_{n-1}(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_{n-1}(\tau)|d\tau ds$$

O'ng tomondagi modullar ostidagi ifodaga $x_n(t)$ ni qo'shib va ayirib quyidagi tengsizlikni hosil qilamiz

$$|x(t)-x_n(t)|\le \int_{-\infty}^{t}\lambda(s)|x_n(s)-x_{n-1}(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x_n(\tau)-x_{n-1}(\tau)|d\tau ds+$$
$$+\int_{-\infty}^{t}\lambda(s)|x(s)-x_n(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_n(\tau)|d\tau ds$$

Bu tengsizlikdagi $|x_n(\tau)-x_{n-1}(s)|$ ayirma modulini (3.1.6) formula yordamida baholaymiz: u holda

$$|x(t)-x_n(t)|\le \int_{-\infty}^{t}\lambda(s)a(s)\frac{b^{n-1}(s)}{(n-1)!}+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)a(\tau)\frac{b^{n-1}(s)}{(n-1)!}d\tau ds+$$
$$+\int_{-\infty}^{t}\lambda(s)|x(s)-x_n(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_n(t)|d\tau ds$$

yoki

$$|x(t)-x_n(t)|\le \frac{a(t)}{(n-1)!}\int_{-\infty}^{t}b^{n-1}\left[\lambda(s)\left(1+\int_{-\infty}^{t}\mu(s,\tau)d\tau\right)\right]ds+$$
$$+\int_{-\infty}^{t}\lambda(s)|x(s)-x_n(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_n(\tau)d\tau|ds$$

bu yerdan quyidagiga ega bo'lamiz:

$$|x(t)-x_n(t)|\le a(t)\frac{b^n(t)}{(n)!}+\int_{-\infty}^{t}\lambda(s)|x(s)-x_n(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_n(\tau)|d\tau ds$$

$a(t)\le a(t_0),\quad b(t)\le b(t_0),\quad (t\le t_0)$ ekanligini nazarga olib oxirgi tengsizlikdan integral tengsizliklar haqidagi 1.2.4-teoremaga ko'ra quyidagiga ega bo'lamiz

$$\left|x(t)-x_n(t)\right|\le \frac{a(t_0)b^n(t_0)}{(n)!}\exp\left\{\int_{-\infty}^{t}\left[\lambda(s)+\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds\right\}$$

bu esa izlangan bahodir. Shu bilan teorema to'liq isbotlandi.

3.1.2-teorema. Faraz qilaylik (3.1.4) tenglamada $F(t,x)$ va $K(t,s,x)$ funksiyalar quyidagi shartlarni qanoatlantirsin:

1) $F(t,x)$ funksiya

$$D=\left\{(t,x):-\infty<t\le t_0<0\quad |x|<\infty\right\}$$

sohada, $K(t,s,x)$ funksiya esa

$$Q=\left\{(t,s,x):-\infty<t\le t_0<0,-\infty<s\le t,\quad |x|<\infty\right\}$$

sohada aniqlangan va uzluksiz funksiyalar bo'lib, bundan tashqari

$$\left(\int_{-\infty}^{t}|s|^q|F(s,c)|^q ds\right)^{\frac{1}{q}}\le Mt^{-\frac{1}{q}},\ \left(\int_{-\infty}^{t}\int_{-\infty}^{s}|s|^q|\tau|^q|K(s,\tau,c)|^q d\tau ds\right)^{\frac{1}{q}}\le N\quad M>0,\ N>0$$

shartlar bajarilsin, bu yerda $\frac{1}{q}+\frac{1}{p}=1$ bo'lib, $p>1$ toq son deb hisoblaymiz.

2) $\left|F(t,x_1)-F(t,x_2)\right|\le\frac{a}{t}|x_1-x_2|,\quad a<0$;

$$\left|K(t,s,x_1)-K(t,s,x_2)\right|\le\frac{b}{ts}|x_1-x_2|,\quad b>0$$

U holda (3.1.4) integral tenglama yoki bari bir (3.1.1) – (3.1.2) masala yagona uzluksiz yechimga ega bo'ladi.

Isbot. (3.1.5) tengliklardan ketma-ket quyidagilarni topamiz:

$$\left|x_1(t)-x_0(t)\right|\le\int_{-\infty}^{t}\left|F(s,c)\right|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\left|K(s,\tau,c)\right|d\tau ds\le$$

$$\le\left(\int_{-\infty}^{t}|s|^{-p}ds\right)^{\frac{1}{p}}\left(\int_{-\infty}^{t}|s|^q|F(s,c)|^q ds\right)^{\frac{1}{q}}+\left(\int_{-\infty}^{t}\int_{-\infty}^{s}|s|^{-p}|\tau|^{-p}d\tau ds\right)^{\frac{1}{p}}\left(\int_{-\infty}^{t}\int_{-\infty}^{s}|s|^q|\tau|^q|K(s,\tau,c|^q d\tau ds\right)^{\frac{1}{q}}\le$$

$$\le Mt^{-\frac{1}{q}}\left(\frac{t^{1-p}}{1-p}\right)^{\frac{1}{p}}+N\left(\frac{t^{2(1-p)}}{2(1-p)^2}\right)^{\frac{1}{p}}\le Qt^{\frac{2}{p}-2},$$

bu yerda $Q = \dfrac{M+N}{\left(2(1-p)^2\right)^{\frac{1}{p}}} > 0$

Demak,

$$|x_1(t) - x_0(t)| \le Qt^{\frac{2}{p}-2}$$

Agar $\alpha = 2 - \dfrac{2}{p} \ge 1$ deb belgilab olsak, $|x_1(t) - x_2(t)| \le Qt^{-\alpha}$

tengsizlikka ega bo'lamiz.

Bunga ko'ra qiyidagilarni topamiz.

$$|x_2(t) - x_1(t)| \le \int_{-\infty}^{t} \frac{a}{s}|x_1(s) - x_0(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \frac{b}{s\tau}|x_1(\tau) - x_0(\tau)|d\tau ds \le Q\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)t^{-\alpha}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.1.8)$$

$$|x_n(t) - x_{n-1}(t)| \le Q\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)^n t^{-\alpha}.$$

Bu tengsizliklardan ko'rinadiki

$$\sum_{n=1}^{\infty}\left[x_n(t) - x_{n-1}(t)\right]$$

qator $(-\infty; t_0)$ oraliqda tekis yaqinlashadi, chunki

$$Q\sum_{n=1}^{\infty}\left(\frac{b}{\alpha^2} - \frac{a}{\alpha}\right)^n t_0^{-\alpha} \quad (t_0 < 0)$$

Sonli qator $\dfrac{b}{\alpha^2} - \dfrac{a}{\alpha} < 1$ bo'lganda yaqinlashuvchi.

Demak, $\lim\limits_{n\to\infty} x_n(t) = x(t) \quad (-\infty, t_0)$ oraliqdagi barcha t lar uchun tekis bajariladi. Osongina ko'rsatish mumkinki $x(t)$ funksiya $(3.1.4)$ va demak $(3.1.2)-(3.1.3)$ masalaning yechimi bo'ladi. Endi topilgan yechimning yagonaligini ko'rsatamiz.

Faraz qilaylik $(3.1.4)$ tenglama ikkita $x(t)$ va $y(t)$ yechimlarga ega bo'lsin. U holda ularning har biri $(3.1.4)$ tenglamani qanoatlantirishi kerak. Teorema shartiga ko'ra

$$|x(t)-y(t)| \le \int_{-\infty}^{t} \frac{a}{s}|x(s)-y(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \frac{b}{s\tau}|x(s)-y(s)|dsd\tau$$

Bu esa 3.1.1-teoremaga ko'ra $|x(t)-y(t)| \le 0$ yoki $x(t) \equiv y(t)$. Demak yechim yagona ekan.

Endi aniq va taqribiy yechimlar orasidagi farqni baholaymiz. $x(t)$ $(3.1.1)-(3.1.2)$ masalaning aniq yechimi va $x_n(t)$ esa uning taqribiy yechimi (ya'ni unga $n-$chi yaqinlashish bo'lsin).

$(3.1.4)$ va $(3.1.5)$ larni o'zaro ayirib quyidagiga ega bo'lamiz:

$$|x(t)-x_n(t)| \le \int_{-\infty}^{t} \frac{a}{s}|x(s)-x_{n-1}(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \frac{b}{s\tau}|x(\tau)-x_{n-1}(\tau)|d\tau ds$$

O'ng tomidagi modullar orasidagi ifodaga $x_n(t)$ ni qo'shib va ayirib quyidagi tengsizlikni hosil qilamiz:

$$|x(t)-x_n(t)| \le \int_{-\infty}^{t} \frac{a}{s}|x_n(s)-x_{n-1}(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \frac{b}{s\tau}|x_n(\tau)-x_{n-1}(\tau)|d\tau ds +$$
$$+ \int_{-\infty}^{t} \frac{a}{s}|x(s)-x_n(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \frac{b}{s\tau}|x(\tau)-x_{n-1}(\tau)|d\tau ds$$

Bu tengsizlikdagi $|x_n(s)-x_{n-1}(s)|$ ayirma modulini $(2.2.8)$ formula yordamida baholaymiz

$$|x(t)-x_n(t)| \le Qa\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^{n-1} \int_{-\infty}^{t} s^{-1-\alpha}ds +$$
$$+Qb\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^{n-1} \int_{-\infty}^{t}\int_{-\infty}^{s} s^{-1-\alpha}\tau^{-1-\alpha}d\tau ds +$$
$$+\int_{-\infty}^{t} \frac{a}{s}|x(s)-x_n(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \frac{b}{s\tau}|x(\tau)-x_n(\tau)|d\tau ds$$

yoki

$$|x(t)-x_n(t)| \le Q\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)t^{-\alpha} + \int_{-\infty}^{t} \frac{a}{s}|x(s)-x_n(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \frac{b}{s\tau}|x(s)-x_n(s)|d\tau ds.$$

Bu tengsizlikda $C = Q\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^n$ $m=1$, desak

$$|x(t)-x_n(t)|\le\frac{Q\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^n t^{-\alpha}}{1-\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)}$$

tengsizlik hosil bo'ladi. Shunday qilib bu ishda $(3.1.1)-(3.1.2)$ masalaning yechimini tadqiq qilishda integral tengsizliklar haqida teorema isbotladik va uning tadbiqlarini ko'rib chiqdik.

2-§. Chiziqli bo'lmagan Vol'terra tenglamasi yechimining mavjudligi va yagonaligi haqida.

Ushbu

$$\varphi(t)=f(t)+\int_0^t K(t,s,\varphi(s))ds \qquad (3.2.1)$$

chiziqli bo'lmagan Vol'terra tenglamasini qaraymiz. Bu yerda $f(t)$ $[0,t_0]$ segmentda $K(t,s,\varphi)$ esa $D(0\le t\le t_0, 0\le s\le t, |\varphi|<\infty)$ sohada aniqlangan va uzluksiz funksiyalar deb hisoblaymiz. Ma'lumki $K(t,s,\varphi)$ funksiya D sohada φ argumenti bo'yicha $|K(t,s,\varphi_1)-K(t,s,\varphi_2)|\le L(t,s)|\varphi_1-\varphi_2|$ Lipshis shartini qanoatlantirib $L(t,s)$ $D_0(0\le t\le t_0, 0\le s\le t)$ sohada uzluksiz bo'lsa $(3.2.1)$ tenglama $[0,t_0]$ segmentda uzluksiz yechimga ega bo'ladi. Biz quyida $L(t,s)=\frac{m}{(t-s)^\alpha}$ $0<\alpha<1$, holda $(3.2.1)$ tenglama yechimining mavjudligi, yagonaligi va ba'zi xossalarini o'rganamiz. Bunda $L(t,s)$ funksiya $t=s$ to'g'ri chiziq bo'ylab uzulishga ega, lekin,

$$\int_0^t \frac{m}{(t-s)^\alpha}=\frac{m}{1-\alpha}t^{1-\alpha}$$

ya'ni $[0,t_0]$ segmentda integrallanuvchi.

3.2.1-Teorema. Faraz qilaylik $f(t)$ funksiya $[0,t_0]$ segmentda aniqlangan va uzluksiz, $K(t,s,\varphi)$ funksiya esa $D(0\le t\le s,\quad 0\le s\le t,\quad |\varphi|<\infty)$ sohada aniqlangan bo'lib φ argumenti bo'yicha Lipshis shartini qanoatlantirsin:

$$|K(t,s,\varphi)-K(t,s,\psi)|\le\frac{m}{(t-s)^\alpha}|\varphi-\psi|,\quad 0<\alpha<1,\quad m\ge 0. \qquad (3.2.2)$$

Bundan tashqari

$$\int_0^{t_0} |K(t,s,f(s))|ds \le C, \qquad (3.2.3)$$

$C \ge 0$ son.

U holda $(3.2.1)$ tenglama $[0,t_0]$ segmentda yagona uzluksiz yechimga ega bo'ladi.

Isbot. Teoremani ketma-ket yaqinlashish usuli yordamida isbotlaymiz. Ketma-ket yaqinlashishlarni quyidagicha tuzamiz:

$$\varphi_0(t) = f(t)$$

$$\varphi_1(t) = f(t) + \int_0^t K(t,s,\varphi_0(s))ds$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.2.4)$$

$$\varphi_n(t) = f(t) + \int_0^t K(t,s,\varphi_{n-1}(s))ds$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$(3.2.2)$ va $(3.2.3)$ ga ko'ra $(3.2.4)$ tengliklardan ketma-ket quyidagilarni topamiz:

$$|\varphi_1(t) - \varphi_0(t)| = \left|\int_0^t K(t,s,f(s))ds\right| \le \int_0^t |K(t,s,f(s))|ds \le C$$

$$|\varphi_2(t) - \varphi_1(t)| \le \int_0^t |K(t,s,\varphi_1(s)) - K(t,s,\varphi_0(s))|ds \le m\int_0^t \frac{|\varphi_1(s) - \varphi_0(s)|ds}{(t-s)^\alpha} \le$$

$$\le mc\int_0^t \frac{ds}{(t-s)^\alpha} = mc\frac{t^{1-\alpha}}{1-\alpha}$$

$$|\varphi_3(t) - \varphi_2(t)| \le m\int_0^t \frac{|\varphi_2(s) - \varphi_1(s)|}{(t-s)^\alpha}ds \le \frac{cm^2}{1-\alpha}\int_0^t \frac{s^{1-\alpha}}{(t-s)^\alpha}$$

Oxirgi integralni hisoblash uchun $s = t\sin^2\frac{\theta}{2}$ almashtirish olamiz. Bu holda

$$ds = t\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

Bunda $s = 0$ bo'lsa $\theta = 0,$ $s = t$ bo'lsa $\theta = \pi$ bo'lgani uchun quyidagiga ega bo'lamiz:

$$\int_0^t \frac{s^{1-\alpha}}{(t-s)^\alpha} = \int_0^\pi \frac{t^{1-\alpha}\sin^{2(1-\alpha)}\frac{\theta}{2} t\sin\frac{\theta}{2}\cos\frac{\theta}{2}d\theta}{(t-t\sin^2\frac{\theta}{2})^2} = t^{2(1-\alpha)}\int_0^{\frac{\pi}{2}} \sin^{3-2\alpha}\tau\cos^{1-2\alpha}d\tau$$

Endi oxirgi tenglikda $x=\sin\tau$ almashtirish olamiz. Bu holda $\tau=0$ desak, $x=0$; $\tau=\frac{\pi}{2}$ desak $x=1$ bo'lib, $dx=\cos\tau d\tau$ $d\tau=\frac{dx}{\cos\tau}=\frac{dx}{\sqrt{1-\sin^2\tau}}=\frac{dx}{\sqrt{1-x^2}}$ bo'ladi.

Demak,

$$\int_0^t \frac{s^{1-\alpha}ds}{(t-s)^\alpha} = t^{2(1-\alpha)}\int_0^1 x^{3-2\alpha}(1-x^2)^{\frac{1-2\alpha}{2}}(1-x^2)^{-\frac{1}{2}}dx = t^{2-2\alpha}\int_0^1 x^{3-2\alpha}(1-x^2)^{-\alpha}dx$$

endi bu integralda $x=\sqrt{y}$, $dx=\frac{dy}{2\sqrt{y}}$ deymiz. U holda

$$\int_0^t \frac{s^{1-\alpha}ds}{(t-s)^\alpha} = t^{2(1-\alpha)}\int_0^1 y^{\frac{3-2\alpha}{2}}(1-y)^{-\alpha}y^{-\frac{1}{2}}dy = t^{2(1-\alpha)}\int_0^1 y^{1-\alpha}(1-y)^{-\alpha}dy = t^{2(1-\alpha)}\int_0^1 y^{(2-\alpha)-1}(1-y)^{(1-\alpha)-1} =$$

$$= t^{2(1-\alpha)}B(2-\alpha,1-\alpha) = t^{2(1-\alpha)}\frac{\Gamma(2-\alpha)\Gamma(1-\alpha)}{\Gamma(3-2\alpha)} = t^{2(1-\alpha)}\frac{(1-\alpha)\Gamma^2(1-\alpha)}{2(1-\alpha)\Gamma(2(1-\alpha))}\frac{t^{2(1-\alpha)}\Gamma^2(1-\alpha)}{2\Gamma(2(1-\alpha))}$$

Shunday qilib

$$|\varphi_3(t)-\varphi_2(t)| \le \frac{cm^2}{1-\alpha}\frac{\Gamma^2(1-\alpha)}{2\Gamma(2(1-\alpha))}t^{2(1-\alpha)}.$$

Endi xuddi yuqoridagiga o'xshash amallarni bajarib $|\varphi_4(t)-\varphi_3(t)|$ ni baholaymiz:

$$|\varphi_4(t)-\varphi_3(t)| \le m\int_0^t \frac{|\varphi_3(s)-\varphi_2(s)|ds}{(t-s)^\alpha} \le \frac{cm^3}{1-\alpha}\frac{\Gamma^2(1-\alpha)}{2\Gamma(2(1-\alpha))}\int_0^t \frac{s^{2-2\alpha}}{(t-s)^\alpha}ds =$$

$$= \frac{cm^3\Gamma^2(1-\alpha)}{(1-\alpha)2\Gamma(2(1-\alpha))}t^{3-3\alpha}B(3-2\alpha,1-\alpha).$$

Agar

$$B(3-2\alpha,1-\alpha) = \frac{\Gamma(3-2\alpha)\Gamma(1-\alpha)}{\Gamma(4-3\alpha)} = \frac{2(1-\alpha)\Gamma(2(1-\alpha))\Gamma(1-\alpha)}{3(1-\alpha)\Gamma(3(1-\alpha))} = \frac{2\Gamma(2(1-\alpha))\Gamma(1-\alpha)}{3\Gamma(3(1-\alpha))}$$

ekanligini nazarga olsak

$$|\varphi_4(t)-\varphi_3(t)| \le \frac{cm^3}{1-\alpha}\frac{\Gamma^2(1-\alpha)t^{3-3\alpha}}{2\Gamma(2(1-\alpha))}\frac{2\Gamma(2(1-\alpha))\Gamma(1-\alpha)}{3\Gamma(3(1-\alpha))}$$

yoki

$$|\varphi_4(t)-\varphi_3(t)| \le \frac{cm^3}{1-\alpha}\frac{\Gamma^3(1-\alpha)}{3\Gamma(3(1-\alpha))}t^{3(1-\alpha)}$$

ga ega bo'lamiz.

Bu jarayonni $n+1$ marta takrorlab quyidagini topamiz

$$|\varphi_{n+1}(t)-\varphi_n(t)|\leq\frac{c}{1-\alpha}\frac{(m\Gamma(1-\alpha))^n}{n(1-\alpha)\Gamma(n(1-\alpha))}t^{n(1-\alpha)}$$

Shunday qilib

$$\varphi_0(t)+(\varphi_1(t)-\varphi_0(t))+\ldots+(\varphi_{n+1}(t)-\varphi_n(t))+\ldots \qquad (3.2.5)$$

funksional qator uchun majorant qator rolini o'ynovchi

$$c+\frac{c}{1-\alpha}\sum_{n=1}^{\infty}\frac{(mt_0^{1-\alpha}\Gamma(1-\alpha))^n}{n(1-\alpha)\Gamma(n(1-\alpha))}$$

sonli qatorga ega bo'lamiz $1-\alpha=a \quad 0\prec a\prec 1$ desak u ushbu

$$c+\frac{c}{a}\sum_{n=1}^{\infty}\frac{(mt_0^a\Gamma(a))^n}{na\Gamma(n\alpha)}$$

ko'rinishga keladi. Bu esa yaqinlashuvchi sonli qatordan iborat.

Demak $(3.2.5)$ qator $[0,t_0]$ oraliqda absolyut va tekis yaqinlashuvchi. Bundan esa $\lim\limits_{n\to\infty}\varphi_n(t)=\varphi(t), \quad t\in[0,t_0]$ kelib chiqadi. Osongina ko'rsatish mumkinki $\varphi(t)$ funksiya $(3.2.1)$ tenglamani qanoatlantiradi. Endi yechimning yagonaligini ko'rsatamiz. Faraz qilaylik $(3.2.1)$ tenglama ikkitta $\varphi(t)$ va $\psi(t)$ yechimlarga ega bo'lsin. U holda quyidagi tengsizlikni yozish mumkin:

$$|\varphi(t)-\psi(t)|\leq\int_0^t m\cdot\frac{|\varphi(s)-\psi(s)|}{(t-s)^{\alpha}}ds$$

Bundan $(3.1.4)$ ga ko'ra $\varphi(t)=\psi(t)$ kelib chiqadi.

3-§. Integro-differensial tenglama uchun Koshining limitik masalasi yechimining boshlang'ich shartdan va yadrodan uzluksiz bog'liqligi.

Ushbu

$$\frac{dx(t)}{dt}=F(t,x(t))+\int_{-\infty}^{t}K(t,s,x(s))ds \qquad (3.3.1)$$

Integro-differensial tenglama uchun Koshining limitik masalasini qaraymiz:

$$\lim_{t\to-\infty}x(t)=C \qquad (3.3.2)$$

Ko'rinib turibdiki $(3.3.1)-(3.3.2)$ masala ushbu

$$x(t) = C + \int_{-\infty}^{t} F(s, x(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s, \tau, x(\tau))d\tau ds \qquad (3.3.3)$$

integral tenglamaga teng kuchli.

Biz yuqorida bu tenglamaning yechimi mavjudligi va yagonaligi haqidagi teoremalarni isbotlagan edik. Bu yerda yechimning boshlang'ich shartga uzluksiz bog'liqligi haqidagi teoremani isbotlaymiz.

Faraz qilaylik $x(t)$ funksiya (3.3.3) tenglamaning $(-\infty, t_0]$ oraliqdagi yechimi, $y(t)$ funksiya esa

$$y(t) = P + \int_{-\infty}^{t} F(s, y(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s, \tau, y(\tau))d\tau ds \qquad (3.3.4)$$

tenglamaning $(-\infty, t_0]$ oraliqdagi yechimi bo'lsin.

3.3.1-teorema. Faraz qilaylik

1) $F(t, x)$ funksiya

$$D = \{(t, x) : -\infty < s \le t \le t_0, |x| < \infty\}$$

sohada, $K(t, s, x)$ funksiya esa

$$Q = \{(t, s, x) : -\infty < t \le t_0, -\infty < s \le t \le t_0, |x| < \infty\}$$

sohada aniqlangan va

$$\int_{-\infty}^{t} |F(s, c)| ds < \infty, \int_{-\infty}^{t}\int_{-\infty}^{s} |K(s, \tau, c)| d\tau ds < \infty$$

shartlar bajarilsin.

2) $F(t, x)$ va $K(t, s, x)$ funksiyalar x argumenti bo'yicha Lipshis shartini qanoatlantirsin

$$|F(t, x_1) - F(t, x_2)| \le \lambda(t)|x_1 - x_2|$$

$$|K(t, s, x_1) - K(t, s, x_2)| \le \mu(t, s)|x_1 - x_2|$$

bu yerda

$$\int_{-\infty}^{t} \lambda(s)ds < \infty, \int_{-\infty}^{t}\int_{-\infty}^{s} \mu(t, s)d\tau ds < \infty$$

deb hisoblanadi. U holda shunday $\delta > 0$ va $\varepsilon > 0$ sonlar mavjud bo'ladiki, $(-\infty, t_0]$ oraliqda

$$|c-\rho|<\delta$$

bo'lganda

$$|x(t)-y(t)|<\varepsilon$$

tengsizlik ham o'rinli bo'ladi.

Isbot. (3.3.3) va (3.3.4) tengsizliklardan quyidagini yozib olamiz:

$$|x(t)-y(t)|=|c-\rho|+\int_{-\infty}^{t}|F(s,x(\tau))-F(s,y(\tau))|ds+$$

$$+\int_{-\infty}^{t}\int_{-\infty}^{s}|K(s,\tau,x(\tau))-K(x,\tau,y(\tau))|d\tau ds$$

yoki

$$|x(t)-y(t)|\le\delta+\int_{-\infty}^{t}\left[\lambda(s)|x(s)-y(s)|+\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-y(\tau)|d\tau\right]ds$$

integral tengsizliklar haqidagi teoremaga ko'ra

$$|x(t)-y(t)|\le\delta\exp\left\{\int_{-\infty}^{t}\left[\lambda(s)+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds\right\}$$

Agar

$$\exp\left\{\int_{-\infty}^{t}\left[\lambda(s)+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds\right\}\le M$$

desak

$$|x(t)-y(t)|\le M\delta=\varepsilon$$

tengsizlikka ega bo'lamiz. Bu esa (3.3.1)–(3.3.2) masala yechimining ozod hadga uzluksiz bog'liqligini bildiradi, ya'ni boshlang'ich shartning ozgina o'zgarishiga yechimning ham ozgina o'zgarishi mos keladi.

Endi

$$\frac{dx(t)}{dt}=F(t,x(t))+\int_{-\infty}^{t}K(t,s,x(s))ds$$

$$\lim_{t\to-\infty}x(t)=c$$

masala yechimining F va K funksiyalardan uzluksiz bo'liqliqligi masalasini o'rganamiz. Buning uchun bu masalani unga ekvivalent bo'lgan integral tenglama bilan almashtiramiz:

$$x(t)=c+\int_{-\infty}^{t}F(s,x(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x(\tau))d\tau ds.$$

Faraz qilaylik $\varphi(t)$ funksiya

$$\varphi(t)=c+\int_{-\infty}^{t}F(s,\varphi(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,\varphi(\tau))d\tau ds \qquad (3.3.5)$$

tenglamaning, $\psi(t)$ esa

$$\psi(t)=c+\int_{-\infty}^{t}F(s,\psi(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,\psi(\tau))d\tau ds \qquad (3.3.6)$$

tenglamaning $(-\infty,t_0]$ oraliqdagi yechimi bo'lsin.

3.3.2-Teorema. Agar $F(s,x),\ \bar{F}(s,x),\ K(t,s,x),\ \bar{K}(t,s,x)$ funksiyalar $D=\{(s,x),-\infty<t\le t_0,|x|<\infty$ va $Q(t,s,x):-\infty<t\le t_0,-\infty<s\le t,|x|<\infty\}$ sohada aniqlangan va uzluksiz funksiyalar bo'lib, ular uchun yechimning mavjudligi va yagonaligi haqidagi 3.2.1-teoremaning barcha shartlari bajarilsin. U holda shunday $\varepsilon>0,\ \delta>0$ sonlar mavjud bo'ladiki D va Q sohalarda

$$\left|F(t,\varphi)-\bar{F}(t,\psi)\right|<\delta a(t)$$

$$\left|K(t,s,\varphi)-\bar{K}(t,s,\psi)\right|<\delta b(t,s)$$

tengsizliklar bajarilganda $(-\infty,t_0]$ oraliqda

$$|\varphi(t)-\psi(t)|<\varepsilon$$

tengsizlik o'rinli bo'ladi. Bu yerda

$$\int_{-\infty}^{t}a(s)ds<+\infty$$

va

$$\int_{-\infty}^{t}\int_{-\infty}^{s}b(s,\tau)d\tau ds<+\infty.$$

Isbot. $(3.3.5)$ va $(3.3.6)$ tengliklardan quyidagilarga ega bo'lamiz:

$$|\varphi(t)-\psi(t)|\le\int_{-\infty}^{t}\left|F(s,\varphi(s))-\bar{F}(s,\psi(s))\right|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\left|K(s,\tau,\varphi(\tau))-\bar{K}(s,\tau,\psi(\tau))\right|d\tau ds\le$$

$$\le\int_{-\infty}^{t}\left|F(s,\varphi(s))-F(s,\psi(s))\right|+\int_{-\infty}^{t}\int_{-\infty}^{s}\left|K(s,\tau,\varphi(s))-K(s,\tau,\psi(s))\right|d\tau ds+$$

$$+\int_{-\infty}^{t}\int_{-\infty}^{s}\left|K(s,\tau,\varphi(s))-\bar{K}(s,\tau,\varphi(s))\right|d\tau ds\le\int_{-\infty}^{t}\lambda(s)\left|\varphi(s)-\psi(s)\right|ds+$$

$$+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)\left|\varphi(s)-\psi(s)\right|d\tau ds+\delta\left(\int_{-\infty}^{t}a(s)ds+\int_{-\infty}^{t}\int_{-\infty}^{s}b(s,\tau)d\tau ds\right)$$

Demak,

$$\left|\varphi(t)-\psi(t)\right|\le\delta M+\int_{-\infty}^{t}\lambda(s)\left|\varphi(s)-\psi(s)\right|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)\left|\varphi(s)-\psi(s)\right|d\tau ds$$

bu yerda

$$\max_{t\in(-\infty,t_0]}\left[\int_{-\infty}^{t}a(s)ds+\int_{-\infty}^{t}\int_{-\infty}^{s}b(s,\tau)d\tau ds\right]$$

Oxirgi tengsizlikka integral tengsizliklar haqidagi 3.2.1-teoremani tadbiq etsak

$$\left|\varphi(t)-\psi(t)\right|\le\delta M\exp\left\{\int_{-\infty}^{t}\left[\lambda(s)+\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds\right\}$$

tengsizlik hosil bo’ladi. Yoki

$$\left|\varphi(t)-\psi(t)\right|\le\delta MN$$

bu yerda

$$N=\exp\left\{\int_{-\infty}^{t_0}\left[\lambda(s)+\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds\right\}$$

Bunda $\delta=\dfrac{\varepsilon}{MN}$ deb olinsa

$$\left|\varphi(t)-\psi(t)\right|<\varepsilon$$

bo’ladi. Teorema isbot bo’ldi.

4-§. Integro-differensial tenglama uchun Koshining limitik masalasi yechimining parametr bo’yicha uzluksizligi va differensiallanuvchanligi.

Ma’lumki differensial va integral tenglamalar yechimlarining parametrga nisbatan uzluksizligi va differensiallanuvchanligini o’rganish muhim ahamiyatga ega. Ular [1,3] ishda keltirilgan. Bunda izlanishlar chekli oraliqda olib borilgan.

Bu ishda yarim o’qdagi chiziqli bo’lmagan integro-differensial tenglama yechimining parametrga nisbatan uzluksiz bog’liqligi va differensiallanuvchanligi haqidagi teoremani isbotlaymiz.

Ushbu

$$\frac{dx}{dt} = F(t,x,\lambda) + \int_{-\infty}^{t} K(t,s,x,\lambda)ds \qquad (3.4.1)$$

yarim o’qda chiziqli bo’lmagan integro-differensial tenglamani qaraymiz. Bu tenglamaning $x = x(t,\lambda)$ yechimi mavjudligi va yagonaligi haqidagi teorema 3-§ da isbotlangan. Osongina ko’rsatish mumkinki (3.4.1) tenglamaning

$$\lim_{t\to-\infty} x(t,\lambda) = c \qquad (3.4.2)$$

shartni qanoatlantiruvchi $x(t,\lambda)$ yechimini topish masalasi ushbu

$$x(t) = \int_{-\infty}^{t} F(s,x(s),\lambda)ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s,\tau,x(\tau),\lambda)d\tau ds \qquad (3.4.3)$$

integral tenglamaning $x = \varphi(t,\lambda)$ yechimini topishga keltiriladi. Shuning uchun quyidagi (3.4.3) tenglama yechimini tekshirish bilan shug’illanamiz. Bu tenglamada qatnashayotgan F va K funksiyalar quyidagi shartlarni qanoatlantirsin deb hisoblaymiz:

a) $F(s,x,\lambda)$ funksiya

$$D_\lambda = \{(s,x,\lambda): -\infty < s < t_0, -\infty < \tau < s, |x| < \infty, \lambda_1 \le \lambda \le \lambda_2\},$$

$K(s,\tau,x,\lambda)$ funksiya esa

$$Q_\lambda = \{(s,\tau,x,\lambda): -\infty < s \prec t_0, -\infty < \tau \le s \prec t_0, |x| < \infty, \lambda_1 \le \lambda \le \lambda_2\}$$

sohada aniqlangan va uzluksiz.

b) $F(s,x,\lambda)$ funksiya D_λ sohada $F_x^{'}$ va $F_\lambda^{'}$ uzluksiz hosilalarga ega hamda

$$|F_x^{'}(s,x,\lambda)| \le \lambda(s), \qquad |F_\lambda^{'}(s,x,\lambda)| \le \lambda(s)$$

bundan tashqari

v) $|F_x^{'}(s,x_1,\lambda) - F_x^{'}(s,x_2,\lambda)| \le \lambda(s)|x_1 - x_2|$, $|F_x^{'}(s,x,\lambda_1) - F_x^{'}(s,x,\lambda_2)| \le \lambda(s)|\lambda_1 - \lambda_2|$

g) $K(s,\tau,x,\lambda)$ funksiya

$$Q_\lambda = \{(s,\tau,x,\lambda): -\infty \le s \le a, -\infty \le \tau \le s, |x| \prec \infty, \lambda_1 \le \lambda_\xi \le \lambda_2\}$$

sohada $K_x^{'}$, $K_\lambda^{'}$ o'zgarmas hosilalarga ega hamda

$$|K_x^{'}(s,\tau,x,\lambda)| \le \mu(s,\tau), \quad |K_\lambda^{'}(s,\tau,x,\lambda)| \le \mu(s,\tau).$$

Bundan tashqari

$$|K_x^{'}(s,\tau,x_1,\lambda) - K_x^{'}(s,\tau,x_2,\lambda)| \le \mu(s,\tau)|x_1 - x_2|$$

$$|K_\lambda^{'}(s,\tau,x,\lambda_1) - K_x^{'}(s,\tau,x,\lambda_2)| \le \mu(s,\tau)|\lambda_1 - \lambda_2|$$

d) $\int\limits_{-\infty}^{t} \left\{ \lambda(s) + \int\limits_{-\infty}^{s} \mu(s,\tau)d\tau \right\} ds \le M.$

Ushbu

$$U(t) = \int\limits_{-\infty}^{t} \left\{ F_x^{'}(s,x,\lambda) + \int\limits_{-\infty}^{s} K_x^{'}(s,\tau,x,\lambda)d\tau \right\} U(s)ds + \int\limits_{-\infty}^{t} \left\{ F_\lambda^{'}(s,x,\lambda) + \int\limits_{-\infty}^{s} K_\lambda^{'}(s,\tau,x,\lambda)d\tau \right\} ds \quad (3.4.4)$$

yordamchi integral tenglamani qaraymiz. Osongina ko'rsatish mumkinki yuqoridagi shartlar bajarilsa bu tenglama yagona $u = u(t,\lambda)$ yechimga ega bo'ladi.

3.4.1-Teorema. Agar $F(s,x,\lambda)$, $K(s,\tau,x,\lambda)$ funksiyalar a)-d) shartlarni qanoatlantirsa (3.4.1) tenglamaning yechimi λ parametr bo'yicha uzluksiz va u λ parametr bo'yicha uzluksiz hosilaga ega bo'ladi, bu hosila (3.4.4) tenglamaning yechimidan iborat bo'ladi.

Isbot. (3.4.1) tenglamaning $\lambda = \lambda_0$ ga mos yechimini $x(t,\lambda_0)$ bilan belgilaymiz:

$$x(t,\lambda_0) = c + \int\limits_{-\infty}^{t} F(s,x(s,\lambda_0),\lambda_0)ds + \int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s} K(s,\tau,x(\tau,\lambda_0),\lambda_0)d\tau ds \quad (3.4.5)$$

endi (3.4.3) va (3.4.5) tengliklardan

$$x(t,\lambda) - x(t,\lambda_0) = \int\limits_{-\infty}^{t} [F(s,x(s,\lambda),\lambda) - F(s,x(s,\lambda_0),\lambda_0)]ds +$$

$$+ \int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s} [K(s,\tau,x(\tau,\lambda),\lambda) - K(s,\tau,x(\tau,\lambda_0),\lambda_0)]d\tau ds$$

o'ng tomondagi integrallar ostidagi ayirmalarga ikki argumentli funksiya uchun Lagranj teoremasini qo'llab tenglikning ikkala tomonidan modul olamiz

$$|x(t,\lambda)-x(t,\lambda_0)|\le\int_{-\infty}^{t}|F_x^{'}(s,x^*,\lambda)||x(s,\lambda)-x(s,\lambda_0)|ds+\int_{-\infty}^{t}|F_\lambda^{'}(s,x(s,\lambda),\lambda^*)||\lambda-\lambda_0|ds+$$

$$+\int_{-\infty}^{t}\int_{-\infty}^{s}|K_x^{'}(s,\tau,x^*,\lambda)||x(s,\lambda)-x(s,\lambda_0)|d\tau ds+\int_{-\infty}^{t}\int_{-\infty}^{s}|K_\lambda^{'}(s,\tau,x(s,\lambda),\lambda^*)||\lambda-\lambda_0|ds,$$

bu yerda

$$\varphi^*=\varphi(s,\lambda_0)+\theta_1(\varphi(s,\lambda)-\varphi(s,\lambda_0))$$

$$\lambda^*=\lambda_0+\theta_2(\lambda-\lambda_0)$$

$$0<\theta_1,\theta_2<1.$$

Teoremaning shartlariga asosan

$$|x(t,\lambda)-x(t,\lambda_0)|\le|\lambda-\lambda_0|\left[\int_{-\infty}^{t}\lambda(s)ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)d\tau ds\right]+\int_{-\infty}^{t}\lambda(s)|x(s,\lambda)-x(s,\lambda_0)|ds+$$

$$+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau,\lambda)-x(\tau,\lambda_0)|d\tau ds\le M|\lambda-\lambda_0|+\int_{-\infty}^{t}\lambda(s)|x(s,\lambda)-x(s,\lambda_0)|ds+$$

$$+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(s,\lambda)-x(s,\lambda_0)|ds.$$

Yuqorida keltirilgan teoremaga ko'ra

$$|x(t,\lambda)-x(t,\lambda_0)|\le M|\lambda-\lambda_0|\exp\left\{\int_{-\infty}^{t}\left[\lambda(s)+\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds\right\}$$

yoki

$$|x(t,\tau)-x(t,\lambda_0)|\le Me^M|\lambda-\lambda_0|$$

bu yerda

$$M=\int_{-\infty}^{t}\left[\lambda(s)+\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds.$$

Demak, $|\lambda-\lambda_0|\prec\delta$ bo'lganda $|x(t,\lambda)-x(t,\lambda_0)|\prec\varepsilon$ tengsizlik bajariladi, ya'ni $\varphi(t,\lambda)$ funksiya λ ning uzluksiz funksiyasi bo'ladi.

Endi (3.4.3) tenglamaning yechimi $\varphi(t,\lambda)$ λ bo'yicha differensiallanuvchi va uning hosilasi (3.4.4) tenglamaning $u(t)$ yechimdan iborat ekanligini ko'rsatamiz. Shu maqsadda (3.4.2),(3.4.4),(3.4.5) tengliklardan quyidagini yozib olamiz.

$$x(t,\lambda)-x(t,\lambda_0)-(\lambda-\lambda_0)U(t)=\int\limits_{-\infty}^{t}\left[F(s,x(s,\lambda),\lambda)-F(s,x(\tau,\lambda_0),\lambda_0)\right]+$$

$$+\int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s}\left[K(s,\tau,x(s,\lambda),\lambda)-K(s,\tau,x(s,\lambda_0),\lambda_0)\right]ds+$$

$$+(\lambda-\lambda_0)\int\limits_{-\infty}^{t}\left\{F_x^{'}(s,x,\lambda)-\int\limits_{-\infty}^{s}K_x^{'}(s,\tau,x,\lambda)d\tau\right\}U(s)ds-$$

$$-(\lambda-\lambda_0)\int\limits_{-\infty}^{t}\left\{F_x^{'}(s,x,\lambda)-\int\limits_{-\infty}^{s}K_x^{'}(s,\tau,x,\lambda)d\tau\right\}ds=$$

$$=\int\limits_{-\infty}^{t}\left\{F_x^{'}(s,x^*,\lambda)\left[x(s,\lambda)-x(s,\lambda_0)\right]+F_\lambda^{'}(s,x(s,\lambda),\lambda^*)(\lambda-\lambda_0)\right\}ds+$$

$$+\int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s}\left\{K_x^{'}(s,\tau,x^*,\lambda)\left[x(\tau,\lambda)-x(\tau,\lambda_0)\right]+K_x^{'}(s,\tau,x(\tau,\lambda),\lambda^*)(\lambda-\lambda_0)\right\}d\tau ds+$$

$$+(\lambda-\lambda_0)\int\limits_{-\infty}^{t}F_x^{'}(s,x,\lambda)U(s,\lambda)ds-(\lambda-\lambda_0)\int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s}K_x^{'}(s,\tau,x,\lambda)U(s,\lambda)d\tau ds+$$

$$+(\lambda-\lambda_0)\int\limits_{-\infty}^{t}F_\lambda^{'}(s,x(s,\lambda),\lambda)ds-(\lambda-\lambda_0)\int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s}K_\lambda^{'}(s,\tau,x(s,\lambda),\lambda)d\tau ds+$$

$$+(\lambda-\lambda_0)\int\limits_{-\infty}^{t}F_x^{'}(s,x^*,\lambda)U(s,\lambda)ds+(\lambda-\lambda_0)\int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s}K_x^{'}(s,x^*,\lambda)U(s,\lambda)ds-$$

$$-(\lambda-\lambda_0)\int\limits_{-\infty}^{t}F_x^{'}(s,x^*,\lambda)U(s,\lambda)-(\lambda-\lambda_0)\int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s}K_x^{'}(s,x^*,\lambda)U(s,\lambda)ds.$$

Bundan

$$\frac{x(t,\lambda)-x(t,\lambda_0)}{\lambda-\lambda_0}-U(t,\lambda)=\int\limits_{-\infty}^{t}F_x^{'}(s,x^*,\lambda)\left\{\frac{x(t,\lambda)-x(t,\lambda_0)}{\lambda-\lambda_0}-U(s,\lambda)\right\}ds+$$

$$+\int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s}K_x^{'}(s,\tau,x^*,\lambda)\left\{\frac{x(\tau,\lambda)-x(\tau,\lambda_0)}{\lambda-\lambda_0}-U(\tau,\lambda)\right\}d\tau ds+$$

$$+\int\limits_{-\infty}^{t}\left\{F_x^{'}(s,x^*,\lambda)-F_x^{'}(s,x(s,\lambda),\lambda)\right\}U(s,\lambda)ds+$$

$$+\int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s}\left\{F_x^{'}(s,\tau,x^*,\lambda)-F_x^{'}(s,\tau,x(s,\lambda),\lambda)\right\}U(\tau,\lambda)d\tau ds+$$

$$+\int\limits_{-\infty}^{t}\left\{F_\lambda^{'}(s,x(s,\lambda_0),\lambda^*)-F_\lambda^{'}(s,x(s,\lambda),\lambda)\right\}ds+$$

$$+\int\limits_{-\infty}^{t}\int\limits_{-\infty}^{s}\left\{\left\{K_\lambda^{'}(s,\tau,x(\tau,\lambda_0),\lambda^*)-K_\lambda^{'}(s,\tau,x(\tau,\lambda),\lambda)\right\}d\tau ds\right\} \qquad (3.4.6)$$

Endi $\varepsilon > 0$ berilgan bo'lsin. U holda shunday $\delta > 0$ son topiladiki, $|\lambda - \lambda_0| < \delta$ bo'lganda

$$\left|F_x^{'}(s, x^*, \lambda) - F_x^{'}(s, x(s,\lambda), \lambda)\right| < \lambda(s)\left|x^* - x(s,\lambda)\right| < \varepsilon$$

$$\left|F_\lambda^{'}(s, x(s,\lambda_0), \lambda^*) - F_\lambda^{'}(s, x(s,\lambda), \lambda)\right| < \lambda(s)\left|\lambda^* - \lambda\right| < \varepsilon$$

$$\left|K_x^{'}(s, \tau, x^*, \lambda) - K_x^{'}(s, \tau, x(s,\lambda), \lambda)\right| < \mu(s,\tau)\left|x^* - x(s,\lambda)\right| < \varepsilon$$

$$\left|K_\lambda^{'}(s, \tau, x(\tau,\lambda_0), \lambda^*) - K_\lambda^{'}(s, \tau, x(\tau,\lambda), \lambda)\right| < \mu(s,\tau)\left|x^* - x(s,\lambda)\right| < \varepsilon$$

tengsizliklarga ega bo'lamiz, bu yerda

$$x^* = x(s,\lambda_0) + \theta_1(\varphi(s,\lambda) - \varphi(s,\lambda_0))$$

$$\lambda^* = \lambda_0 + \theta_2(\lambda - \lambda_0), \qquad 0 < \theta_1, \theta_2 < 1$$

Bularga asosan (2.5.6) dan quyidagikelib chiqadi:

$$\left|\frac{x(t,\lambda) - x(t,\lambda_0)}{\lambda - \lambda_0} - U(t,\lambda)\right| \le \varepsilon \int_{-\infty}^{t} \left[\lambda(s)\left|U(s)\right| + \lambda(s)\right] ds +$$

$$+\varepsilon \int_{-\infty}^{t}\int_{-\infty}^{s} \left[\mu(s,\tau)\left|U(s,\lambda)\right| + \mu(s,\tau)\right] d\tau ds + + \int_{-\infty}^{t} F_x^{'}(s, x^*, \lambda)\left|\frac{x(s,\lambda) - x(s,\lambda_0)}{\lambda - \lambda_0} - U(s,\lambda)\right| ds +$$

$$+ \int_{-\infty}^{t}\int_{-\infty}^{s} K_x^{'}(s,\tau,x^*,\lambda)\left|\frac{x(\tau,\lambda) - x(\tau,\lambda_0)}{\lambda - \lambda_0} - U(\tau,\lambda)\right| d\tau ds$$

yoki $|U(s,\lambda)| \le U_0$ ekannini hisobga olsak,

$$\left|\frac{x(t,\lambda) - x(t,\lambda_0)}{\lambda - \lambda_0} - U(t,\lambda)\right| \le (U_0 + 1)\varepsilon \int_{-\infty}^{t} \left[\lambda(s) + \int_{-\infty}^{s} \mu(s,\tau) d\tau\right] ds +$$

$$+ \int_{-\infty}^{t} \lambda(s)\left|\frac{x(s,\lambda) - x(s,\lambda_0)}{\lambda - \lambda_0} - U(s,\lambda)\right| ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \mu(s,\tau)\left|\frac{x(\tau,\lambda) - x(\tau,\lambda_0)}{\lambda - \lambda_0} - U(\tau,\lambda)\right| d\tau ds$$

Bu tengsizlikka integral tengsizliklar haqidagi yuqoridagi teoremani qo'llasak

$$\left|\frac{x(t,\lambda) - x(t,\lambda_0)}{\lambda - \lambda_0} - U(t,\lambda)\right| \le \varepsilon(U_0 + 1)Me^M,$$

bu yerda

$$\int_{-\infty}^{t} \left[\lambda(s) + \int_{-\infty}^{s} \mu(s,\tau) d\tau\right] ds \le M$$

Bundan

$$\lim_{\lambda \to \lambda_0} \left| \frac{x(t,\lambda) - x(t,\lambda_0)}{\lambda - \lambda_0} - U(t,\lambda) \right| = 0$$

Demak,

$$\frac{\partial x(t,\lambda)}{\partial \lambda} = U(t,\lambda)$$

o’rinli bo’lar ekan. Shu bilan teorema to’liq isbotlandi.

III bob bo'yicha xulosa.

III bob uchta paragrafdan iborat bo'lib, u ikkinchi bobda isbotlangan integral tengsizliklar haqidagi yangi teoremalarning tadbiqlariga bag'ishlanadi.

Bunga asosan

$$\frac{dx}{dt} = F(t,x) + \int_{-\infty}^{t} K(t,s,x)ds$$

tenglama uchun Koshining limitik masalasi yechimining mavjudligi, yagonaligi hamda aniq va taqribiy yechimlari orasidagi farq, yechimning boshlang'ich shart va tenglamada qatnashgan F va K funksiyalardan uzluksiz bog'liqligi va yechimning parametr bo'yicha uzluksizligi va differensiallanuvchanligi haqidagi xossalari o'rganiladi.

Bulardan tashqari Abel yadroli integral tengsizlik haqidagi teoremadan foydalanib

$$x(t) = f(t) + \int_{0}^{t} K(t,s,x(t))ds$$

Vol'terraning chiziqli bo'lmagan tenglamasi tadqiq qilinadi. Bunda $K(t,s,x)$ funksiya $L = \frac{m}{(t-s)^{\alpha}}$ $(0 < \alpha < 1)$ ko'rinishdagi Lipshis shartini qanoatlantirishi talab etiladi.

Xulosa

Magistrlik dissertatsiyalariga qo'yiladigan talablarga javob beraoladigan ilmiy tadqiqot ishlarini bajarish zamon talabidir. Bunday talablarga javob berish uchun albatta tanlangan yo'nalishda shu nazariyaning barcha natijalari talab darajada egallagan holda ma'lum bir yangi natijalarni qo'lga kiritish asosiy vazifa hisoblanadi.

1. Tadqiqot natijalari bo'yicha xulosalar va ularning asoslanishi.

Mazkur dissertatsiyada qo'yilgan masala maxsus integral tengsizliklar nazariyasini ma'lum ma'noda to'ldirish, ya'ni Gronuoll-Bellman tengsizligini maxsus xollarga umumlashtirish orqali yangi tengsizliklar isbotlashdan va bu olingan natijalarning tadbiqlarini ko'rsatishdan iborat.

2. Erishilgan asosiy natijalar.

Bu tadbiqlar asosan

$$\frac{dx(t)}{dt} = F(t, x(t)) + \int_{-\infty}^{t} K(t, s, x(s))ds$$

$$\lim_{t \to -\infty} x(t) = c$$

masala yechimining mavjudligi, yagonaligi, aniq va taqribiy yechimlarining bir-biridan farqi yechimning ozod had va yadrodan uzluksiz bog'liqligi, hamda yechimning parametrdan uzluksiz bog'liqligi va parametr bo'yicha differensiallanuvchanligi kabi xossalarni o'rganishga bag'ishlanadi. Bu borada F va K funksiyalarga Lipshis shartini qanoatlantirishi talab etilib Lipshis koeffitsenti $(-\infty, t_0]$ da integrallanuvchi va integrallanmaydigan hollar alohida qaralgan.

So'ngra

$$x(t) = f(t) + \int_{0}^{t} K(t, s, x(s))ds$$

chiziqli bo'lmagan Vol'terra tenglamasi yechimlari tadqiq etiladi. Bunda K funksiya uchun Lipshis koeffitsenti

$$\frac{m}{(t-s)^{\alpha}} \quad (0 < \alpha < 1)$$

ko’rinishda bo’lganda o’rganilmaganini hisobga olib uning yechimining mavjudligi, yagonaligi, taqribiy yechimning aniq yechimdan farqini baholash bo’yicha yangi natijalarga erishilgan.

3. Magistr shaxsan erishgan yutuqlari.

Mazkur ishda chegaralaridan biri cheksiz bo’lgan, yadrolari integrallanuvchi va integrallanmaydigan hollar uchun yangi integral tengsizliklar haqidagi teoremalar isbotlanadi. Bu teoremalar integral tengsizliklar nazariyasini to’ldirishda alohida ahamiyatga ega.

Bu tengsizliklarni integro-differensial va Vol’terra tenglamasi yechimlarini tadqiq etishga tadbiqlari haqida yangi teoremalar isbotlanadi.

Shunday qilib mazkur dissertatsiya ishida olingan natijalar integral tengsizliklar, integral va integro-differensial tenglamalar nazariyasini ma’lum ma’noda to’ldirishga xizmat qiladi.

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