O’ZBEKISTON RESPUBLIKASI

OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

ALISHER NAVOIY NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

MEXANIKA-MATEMATIKA FAKULTETI

”NAZARIY VA AMALIY MEXANIKA” KAFEDRASI

BOBONAZAROVA NIGORA

DEFORMATSIYALANUVCHI MUHITDA JOYLASHGAN DOIRAVIY SILINDRIK QOBIQNING BO’YLAMA-RADIAL TEBRANISHLARINI SONLI TADQIQ ETISH

”5140300 -mexanika” ta’lim yunalishi bo’yicha

Bakalavr darajasini olish uchun

BITIRUV MALAKAVIY ISHI

Ilmiy rahbar: dots. Berdiyev Sh.

2016 yil “____” ______________

Bitiruv malakaviy ishi ”Mexanika” kafedrasida bajarildi.

Kafedraning 2016 yil ”29” maydagi majlisida muhokama qilindi

va himoyaga tavsiya etildi (10-bayonnoma)

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Bitiruv malakaviy ishi YaDAKning 2016 yil ”___” iyundagi majlisida himoya qilindi va _____ ball bilan baholandi. (___- bayonnoma)

YaDAK raisi: ____________________

A’zolar: ____________________

____

Samarqand 2016

Mexanika-matematika fakulteti Amaliy matematika, informatika va mexanika bo'limi 4-kurs talabasi Bobonazarova Nigoraning

”Deformatsiyalanuvchi muhitda joylashgan doiraviy silindrik

qobiqning bo'ylama-radial tebranishlarini sonli tadqiq etish”

nomli bitiruv malakaviy ishiga

TAQRIZ

Bitiruv malakaviy ishi ”5140300-Mexanika ta'lim yo'nalishi bo'yicha bakalavr darajasini olish uchun bajarilgan.

Bitiruv malakaviy ishi 2 ta bobdan iborat bo'lib birinchi bob doiraviy elastik slindrik qobiqning bo'ylama-radial tebranish tenglamalariga bag'ishlangan bo'lib, unda doiraviy silindrik elastik qobiqning bo'ylama-radial tebranishlari umumiy tenglamalari, doiraviy elastik silnidrik qobiqning bo'ylama-radial tebranishlari uchun aylanish inersiyasi va ko'ndalang siljish deformatsiyasi ta'sirini hisobga olgan holda umumiy tenglamalarda ko'chishlarni aniqlashga bag'ishlangan.

Ikkinchi bob doiraviy silindrik elastik qobiqning bo'ylama-radial tebranishlari aniqlastirilgan va klassik tenglama asosida sonli tadqiq etishga bag'ishlangan. Bunda differensial tenglamalarni yechishning chekli ayirmalar usuli va xususiy hosilali differensial tenglamalarni taqribiy yechish uchun umumiy mulohazalari bayon etilgan. Doiraviy silindrik qobiqning bo'ylama-radial tebranishlarida ko'chishlarni hisobga olish, doiraviy silindrik qobiqning bo'ylama-radialuralma tebranishlarida o'rab turuvchi muhit ta'sirini hisobga olish kabi amaliy masalalarga chekli ayirmalar usuli qo'llanilgan. Bunda oshkormas ko'rinishdagi chekli ayirmali sxema asosida tenglamalar aproksimatsiya qilingan. Hosil qilingan algebraik tenglamalar sistemasini yechish uchun ”Maple-9.5” dasturlash tilida programma tuzilgan hamda sonli natijalar olingan. Olingan natijalar grafiklarda tasvirlangan hamda xulosalar chiqarilgan.

Umuman olganda talaba Bobonazarova Nigoraning ”Deformatsiyalanuvchi muhitda joylashgan doiraviy elastik silindrik qobiqning bo'ylama-radial tebranishlarini sonli tadqiq etish” nomli malakaviy bitiruv ishi qo'yilgan barcha talablarga javob beradi, hamda ishini 86 ball bilan hisoblash mumkin.

Taqrizchi: SamDU Mintaqaviy malaka oshirish

markazi o'qituvchisi f.-m.f.n. Bobojonov A.

Mexanika-matematika fakulteti Amaliy matematika, informatika va mexanika bo'limi 4-kurs talabasi Bobonazarova Nigoraning ”Deformatsiyalanuvchi muhitda joylashgan doiraviy silindrik qobiqning bo'ylama-radial tebranishlarini sonli tadqiq etish” nomli bitiruv malakaviy ishiga ilmiy rahbar

MULOHAZASI

Bitiruv malakaviy ishi 2 ta bobdan iborat bo'lib, birinchi bob doiraviy elastik slindrik qobiqning bo'ylama-radial tebranish tenglamalariga bag'ishlangan bo'lib, unda doiraviy silindrik elastik qobiqning bo'ylama-radial tebranishlari umumiy tenglamalari keltirib chiqarilgan.

Ikkinchi bob doiraviy silindrik elastik qobiqning bo'ylama-radial tebranishlari aniqlastirilgan va klassik tenglama asosida sonli tadqiq etishga bag'ishlangan. Bunda differensial tenglamalarni yechishning chekli ayirmalar usuli va xususiy hosilali differensial tenglamalarni taqribiy yechish uchun umumiy mulohazalari bayon etilgan. Doiraviy silindrik qobiqning bo'ylama-radial tebranishlarida ko'chishlarni hisobga olish, doiraviy silindrik qobiqning bo'ylama-radial tebranishlarida o'rab turuvchi muhit ta'sirini hisobga olish kabi amaliy masalalarga chekli ayirmalar usuli qo'llanilgan. Hosil qilingan algebraik tenglamalar sistemasini yechish uchun ”Maple-9.5” dasturlash tilida programma tuzilgan hamda sonli natijalar olingan. Olingan natijalar grafiklarda tasvirlangan hamda xulosalar chiqarilgan.

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Ilmiy tahbar, t.f.n. Berdiyev Sh.

MUNDARIJA

Kirish

So'nggi yillarda fan va texnikaning rivojlanishi turli xil yangi turdagi konstruksiya, har xil apparatlar va qurilmalarni yangi texnologiyalar asosida yaratishga olib kelmoqda. Hozirgi zamon texnikasining juda tez sur'atlar bilan rivojlanishi deformatsiyalanuvchi jismlar mexanikasi oldiga yangidan-yangi amaliy masalalarni yechishni qo'ymoqda. Shu paytgacha materiallar yuqori bosimli va yuqori haroratli o'ta murakkab sharoitlarda ishlatilmoqda, yangi-yangi materiallar har xil yuqori haroratlarga chidamli qotishmalar, o'ta mustahkam va yaroqli modulli tolalar amaliyotda qo'llanilmoqda.

Bunday o'zgarishlar jismning elastik modeli bilan bir qatorda deformatsiyalanuvchi qattiq jismning boshqa, mukammalroq modellarini ham yaratishga, muhandislik qurilmalari hisobida ishlab chiqilganiga ancha bo'lgan. Lekin shu vaqtgacha foydalanilmagan usullardan, xususan plastiklik, qayishqoq-elastiklik, polzuchest nazariyalari usullaridan foydalanishga olib kelmoqda.

Keyingi bir necha o'n yillar davomida deformatsiyalanuvchi qattiq jismlar mexanikasining "yemirilish mexanikasi", "kompozit materiallar mexanikasi", "Nanomexanika" va shu kabi qator yangi yo'nalishlar paydo bo'lishiga ham ana shu yangi talablar taqozo qildi. Ushbu yo'nalishlar rivoji ham, eng avvalo elastik jism modeliga tayanadi.

Bitiruv ishining tadqiqot predmeti muhandislik qurilmalari elementlari sterjenlar, plastina va qobiqlarda, ularga berilgan tashqi ta'sir natijasida vujudga keladigan tebranishlarni o'rganish. Tebranishlar nostatsionar xarakterga ega bo'lgan hollarda bunday elementlarda paydo bo'ladigan nostatsionar to'lqinlar tarqalish jarayonlarini, ularning o'zlariga xos xususiyatlarini hisobga olgan holda o'zgarish va tadqiq qilishdan iborat. Nostatsionar tebranishlarini o'rganishda ularning xususiy chastotalarini topish, xususiy amplitudalarini aniqlash va tebranish shakllarini topish masalalarini qo'yish ularni hal qilish va ilmiy xulosalar chiqarish.

Erkin tebranishlarning topilgan fizik-mexanik xarakteristikalaridan nostatsionar tebranishlar biror vaqt davomida ta'sir etuvchi tashqi dinamik ta'sir natijasida uyg'otilgan hollar uchun tadbiq etish, ulardan foydalana bilish ham dissertatsiya ishining predmetini tashkil etadi.

Bitiruv ishining tadqiqot ob'ekti yuqorida aytilganlardan kelib chiqqan holda, ko'ndalang kesimi doiraviy bo'lgan, chekli uzunlidagi elastik materialdan yasalgan qatlamlardir. Bunda, bunday qobiqlarning buralma tebranishlarida vujudga keladigan kuchlanganlik-deformatsiyalanganlik holatini qatlam materialining elastiklik xususiyatini hisobga olgan holda aniqlash mumkin. Bundan tashqari qatlamni urab turuvchi ob'ekt sifatida tashqi deformatsiyalanuvchi muhit ta'sirini ham qarash mumkin.

Mavzuning dolzarbligi doiraviy elastik qobiqlar juda ko'p va xilma-xil muhandislik qurilmalarining tarkibiy qismlarini tashkil etadilar. Bundan tashqari bunday qobiq va qatlamlar ko'plab mashina va mexanizmlarning elementlari hamdir. Shunday holda bu qobiq va qatlamlar turli xil dinamik tashqi ta'sirlar ostida ishlaydilar va ularning kesmlarida turli xil yuklanishlar vujudga keladi. Qobiqlardagi bunday yuklanishlarni aniqlash masalasi deformatsiyalanuvchi qattiq jismlar mexanikasining dolzarb masalalarida bunday yuklar ta'siri ostidagi qatlamlar ko'ndalang kesimlaridagi kuchlanganlik-deformatsiyalanganlik holatlarini aniqlashdan iboratdir. Ammo, qatlam nuqtalaridagi, tashqi dinamik ta'sirlar natijasida vujudga keladigan kuchlanganlik-deformatsiyalanganlik holatlarini aniqlash analitik usullar bilan topish hamma vaqt ham mumkin bo'lavermaydi. Bunday holda masalani yechish uchun sonli usullardan foydalanishga to'g'ri keladi. Shu sababli bitiruv ishida ko'rilgan masala dolzarb masalalar qatoriga kiradi deb hisoblash mumkin.

Hozirgi kunda deformatsiyalanuvchi qattiq jismlar mexanikasi masalalarini yechishda qo'llanilib kelinayotgan sonli usullardan chekli elementlar, chegaraviy elementlar, chekli ayirmalar usullarini keltirishimiz mumkin. Biz quyida bitiruv malakaviy ishi doirasida chekli ayirmalar usulidan foydalanamiz. Bundan tashqari masalalarni yechish uchun ularning matematik modelini yaratishda asosiy rol o'ynovchi tebranish tenglamalarini [2, 3] ishning natijalaridan foydalanib keltirib chiqaramiz. Mana shularni hisobga olgan holda bitiruv ishining maqsad va vazifalari belgilanadi.

Ishning maqsad va vazifalari magistrlik dissertatsiya ishining asosiy maqsadi ko'ndalang kesimi doiraviy bo'lgan elastik silindrik qobiqlarning nostatsionar buylama-radial tebranishlarini tadqiq qilish, aylanish inersiyasi va ko'ndalang siljish deformatsiyasi ta'sirlarini, o'rab turuvchi muhit ta'sirini hisobga olish, qilib belgilangan. Bunda tadqiqotni klassik va aniqlashtirilgan [9] tebranish tenglamalari asosida olib borish va masalalarni sonli usullar yordamida yechish talab etiladi. Ana shulardan kelib chiqqan holda bitiruv ishining asosiy vazifalari qilib quyidagilar belgilangan:

1. Elastiklik nazariyasi asosiy munosabatlarini o'rganish;
2. Ko'ndalang kesimi doiraviy elastik qobiqning buylama-radial tebranishlari umumiy tenglamalarini keltirib chiqarish va undan xususiy hollarda klassik va aniqlashtirilgan tenglamalarni keltirish;
3. Differensial tenglamalarni yechishning sonli usullarini, xususan, chekli ayirmalar va progonka usullarini o'rganish va amaliy masalalar yechishga tadbiq etish;
4. Amaliy masalalar yechish;
5. Olingan natijalar asosida ilmiy xulosalar chiqarish.

Tadqiqotning ilmiy yangiligi. Doiraviy elastik silindrik qobiqlarning buylama-radial tebranishlari haqidagi masalalar analitik yechimlar asosida tadqiq etilgan. Bunda tebranish tenglamalari sifatida aniqlashtirilgan tenglamalardan foydalanilgan. Tebranish tenglamalarida tashqi o'rab turuvchi muhit ta'siri hisobga olingan. Ushbu tenglamalarga sonli usullar qullanilgan.

Shu sababli qobiqlarning buylama-radial tebranishlari haqidagi masalalarni sonli tadqiq etish masalasi hozirgi vaqtlarda katta ilmiy va amaliy ahamiyatga ega bo'lmoqda. Bitiruv malakaviy ishida qaralgan va yechilishi uchun sonli usullar tadbiq etilgan masalalarning ilmiy ahamiyati birinchidan elastiklik xususiyatini hisobga olinganligi va ikkinchidan masalani yechish uchun chekli ayirmalar usulining qo'llanilishi ularning shu turdagi masalalarni yechishda asos bo'lishini ko'rsatadi.

Tadqiqotning amaliy ahamiyati. Hozirgi zamon texnikasi, qurilish, yer osti va yer usti inshoatlari, aviatsiya, kemasozlik va boshqa juda ko'plab sohalarda ko'ndalang kesimi doiraviy bo'lgan qobiqlar muhandislik qurilmalarining asosiy elementlaridan biri sifatida ishlatiladi. Qo'llanilish jarayonida bunday qatlamlar intensiv va impulsiv dinamik yuklar ta'siri ostida bo'ladilar va juda ko'p hollarda ularning dinamik chidamlilik darajasini tajribadan emas, balki hisoblashlar yordamida aniqlashga to'g'ri keladi.

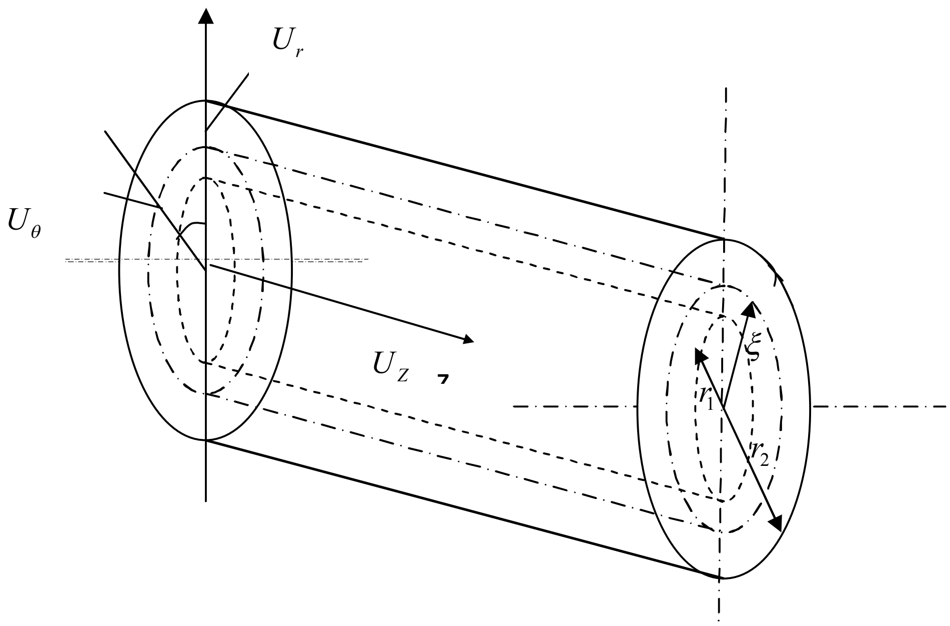
Yuqorida aytilganlar doiraviy elastik qatlamlarning nostatsionar tebranishlarini tadqiq qilish, ularning tebranish chastotasi, amplitudasi, shakli, ko'chisi va kuchlanishi kabi boshqa xarakteristikalarini aniqlash muhim amaliy ahamiyatga ega ekanligini ko'rsatadi. Bitiruv malakaviy ishida qaralgan qatlamning buylama-radial tebranishlari haqidagi masalalar tadqiqotlari ham shunday tadqiqotlar jumlasiga kiradi va muhim amaliy ahamiyatga ega bo'lgan masalalardan biridir.

Bitiruv malakaviy ishining tuzilishi. Ushbu bitiruv malakaviy ishi kirish, ikkita bob, xulosa va foydalanilgan asosiy adabiyotlar ro'yxatidan iborat bo'lib jami 41 betni tashkil qiladi.

1-BOB.DOIRAVIY ELASTIK QOBIQNING BUYLAMA-RADIAL TEBRANISH TENGLAMALARI

§1.1. Doiraviy silindrik elastik qobiqning buylama-radial tebranish tadqiqotlari

Malakaviy bitiruv ishida mateiali transversal-izotrop silindrik qatlamning buralma tebranishlarining deformatsiyalanuvchi muhit bilan o’zaro ta’sirini e’tiborga olgan holda umumiy tenglamalarni keltirib chiqaramiz. Bunda qobiqning uchlari buralishdan mahkamlangan va izotropiya tekisligi qatlamning ko`ndalang kesimi bilan o`stma-o`st tushadi deb hisoblaymiz.



1-rasm

r_1 va r_2 - qatlamning ichki va tashqi radiusi;

ξ - qatlam o`rta sirtining radiusi;

ξ - qatlam o`rta sirtining radiusiquyidagi formula bilan topiladi:

$$\xi = \frac{r_1}{2}\left(\chi - \frac{r_1}{r_2}\right), \qquad 2 + \frac{r_1}{r_2} \le \chi \le 2\frac{r_2}{r_1} + \frac{r_1}{r_2}; \tag{1.15}$$

Noldan farqli qatlamning buylama-radial tebranishlari uchun kuchish va kuchlanishlar formulasini quyidagi kuchishlarda yozish mumkin:

$$U_r^{(0)}(r,z,t) = a_{r1}U_{r,0}^{(0)} + a_{r2}U_{z,0}^{(0)} - \xi a_{r3}U_{r,1}^{(0)} - \xi a_{r4}U_{z,1}^{(0)}, \tag{1.16}$$

$$U_z^{(0)}(r,z,t) = a_{z1}U_{z,0}^{(0)} + a_{z2}U_{r,0}^{(0)} - \xi a_{z3}U_{z,1}^{(0)} - \xi a_{z4}U_{r,1}^{(0)},$$

$$\sigma_{rz}^{(0)}(r,z,t) = M_0\left\{e_1U_{r,0}^{(0)} + e_2U_{z,0}^{(0)} - \xi e_3U_{r,1}^{(0)} - \xi e_4U_{z,1}^{(0)}\right\},$$

$$\sigma_{rr}^{(0)}(r,z,t) = M_0\left\{d_1U_{r,0}^{(0)} - d_2U_{z,0}^{(0)} - \xi d_3U_{r,1}^{(0)} + \xi d_4U_{z,1}^{(0)}\right\},$$

$$\sigma_{\theta\theta}^{(0)}(r,z,t) = M_0\left\{b_{\theta1}U_{z,0}^{(0)} - b_{\theta2}U_{r,0}^{(0)} + \xi b_{\theta3}U_{z,1}^{(0)} - \xi b_{\theta4}U_{r,1}^{(0)}\right\}, \tag{1.17}$$

$$\sigma_{zz}^{(0)}(r,z,t) = M_0\left\{b_{z1}U_{z,0}^{(0)} - b_{z2}U_{r,0}^{(0)} + \xi b_{z3}U_{z,1}^{(0)} - \xi b_{z4}U_{r,1}^{(0)}\right\},$$

bu yerda

$$e_j = e_{ji}\Big|_{r_i=r}; \qquad d_j = d_{ji}\Big|_{r_i=r}; \qquad (j=\overline{1,4}; \qquad i=1,2)$$

$b_{\theta i}, b_{zi}$ $\quad (i=\overline{1,4})$ - integridifferensial operator.

Tebranishlar uchun umumiy tenglamalarni Kirxgof-Lyav va Timoshenko tipidagi tenglamakarda yo'l quyilgan kamchiliklarni o'zida mijassamlashtirgan, slindrik qobiq va qatlamlar uchun aniqlashtirilgan nazariya tebranish tenglamalarini umumiy kuchishlarini quyidagicha yozish mumkin [2].

$$-q_1^{(0)}U_{r,0}^{(0)} - \left(1+q_1^{(0)}\right)\frac{\partial}{\partial z}U_{z,0}^{(0)} + \xi\left[\left(q_2^{(0)}\ln\frac{r_1}{\xi} - \frac{1}{2}\right)\lambda_{02} + \frac{2}{r_1^2}\right]U_{r,1}^{(0)} +$$

$$+\xi\left[\left(1+q_2^{(0)}\right)\ln\frac{r_1}{\xi} + \frac{1}{2}\right]\frac{\partial U_{z,1}^{(0)}}{\partial z} = M_0^{-1}\left[f_r^{(1)}\right],$$

$$\left(-q_1^{(0)} + \frac{r_2}{2}R_1\right)U_{r,0}^{(0)} - \left(1+q_1^{(0)}\right)\frac{\partial}{\partial z}U_{z,0}^{(0)} +$$

$$+\xi\left\{\left[q_2^{(0)}\ln\frac{r_2}{\xi} - \frac{1}{2}\right]\lambda_{02} + \frac{2}{r_2^2} - R_1\left[\frac{r_2}{2}\left(1+q_2^{(0)}\right)\ln\left(\frac{r_2}{\xi}\right)\lambda_{02} + \frac{1}{r_2}\right]\right\}U_{r,1}^{(0)} +$$

$$+\xi\left\{\left[(1+q_2^{(0)})\ln\frac{r_2}{\xi} + \frac{1}{2}\right] - R_1\left[\frac{r_2}{2}q_2^{(0)}\ln\frac{r_2}{r_2\xi}\right]\right\}\frac{\partial U_{z,1}^{(0)}}{\partial z} = F, \quad (1.18)$$

$$(1+q_2^{(0)})\frac{\partial U_{r,0}^{(0)}}{\partial z} + (1-q_1^{(0)})\lambda_{01}U_{z,0}^{(0)} - \xi\left[(1+2q_2^{(0)})\lambda_{02}\ln\frac{r_1}{\xi} + \frac{2}{r_1^2}\right]\frac{\partial}{\partial z}U_{r,1}^{(0)} -$$

$$+\xi\left[\left(\lambda_{02} + 2q_2^{(0)}\frac{\partial^2}{\partial z^2}\right)\ln\frac{r_1}{\xi} + \frac{2}{r_1^2}\right]U_{z,1}^{(0)} = \frac{2}{r_1}M_0^{-1}\left[f_{rz}^{(1)}\right],$$

$$(1+q_2^{(0)})\frac{\partial}{\partial z}U_{r,0}^{(0)} + (1-q_1^{(0)})\lambda_{01}U_{z,0}^{(0)} - \left[(1+2q_2^{(0)})\lambda_{02}\ln\frac{r_2}{\xi} + \frac{2}{r_2^2}\right]\frac{\partial U_{r,1}^{(0)}}{\partial z} -$$

$$-\xi\left[\left(\lambda_{02} + 2q_2^{(0)}\frac{\partial^2}{\partial z^2}\right)\ln\frac{r_2}{\xi} + \frac{2}{r_2^2}\right]U_{z,1}^{(0)} = 0.$$

bu yerda

$$q_1^{(0)} = 1 - L_0M_0^{-1}; \qquad q_2^{(0)} = M_0L_0^{-1} - 1;$$

$$\lambda_{01}^n = \left[\rho_0L_0^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - \frac{\partial^2}{\partial z^2}\right]^n; \qquad \lambda_{02}^n = \left[\rho_0M_0^{-1}\left(\frac{\partial^2}{\partial t^2}\right) - \frac{\partial^2}{\partial z^2}\right]^n;$$

Kuchlanishlarga nisbatan chiqarilgan (1.18) tenglamalar sistemasini masalalar yechishda qullaymiz.

Quyida chetlari sharnirli tayangan doiraviy elastik slindrik qatlamning buylama-radial tebranishlariga ko'ndalang siljish deformatsiyasi ta'sirini qarab chiqamiz.

Asosiy tebranish tenglamalari sifatida aniqlashtirilgan tebranish tenglamalari (1.18) dan foydalanamiz, bunda qobiq materiali elastik, qobiq sirtida tashqi ta'sirlar yo'q va tashqi muhit ta'siri hisobga olinmagan holda (1.18) tenglamalar sistemasi quyidagi kurinishga keladi

$$\begin{gathered} N_3 U_{r,0} + N_4 \frac{\partial U_{z,0}}{\partial z} + \xi\left[a_1(r_i)\partial_2 + \frac{2}{r_i^2}\right]U_{r,1} + \xi a_2(r_i)\frac{\partial U_{z,1}}{\partial z} = 0; \qquad i = 1,2 \\ -N_4 \frac{\partial U_{r,0}}{\partial z} + N_5 \partial_1 U_{z,0} - \xi\left[a_3(r_i)\partial_2 + \frac{2}{r_i^2}\right]\frac{\partial U_{r,1}}{\partial z} - \\ -\xi\left[\left(\partial_2 + 2N_1\frac{\partial^2}{\partial z^2}\right)\ln\frac{r_i}{\xi} + \frac{2}{r_i^2}\right]U_{z,1} = 0, \qquad i = 0,1 \end{gathered} \tag{1.19}$$

bu yerda $U_{r,0}, U_{r,1}$ va $U_{z,0}, U_{z,1}$-qatlam radial U_r va buylama U_z kuchishlarining bosh qismlari;

$$N_1 = \frac{1-4\nu_0}{1-\nu_0}, \qquad N_2 = \frac{1-5\nu_0}{1-\nu_0}, \qquad N_3 = \frac{2(1-\nu_0)}{1-2\nu_0}, \qquad N_4 = \frac{1}{1-2\nu_0},$$

$$N_5 = \frac{3-4\nu_0}{1-2\nu_0}, \qquad N_6 = \frac{3(1-3\nu_0)}{1-\nu_0}, \qquad a_1(r_i) = N_1 \ln\frac{r_i}{\xi} - \frac{1}{2},$$

$$a_2(r_i) = N_2 \ln\frac{r_i}{\xi} + \frac{1}{2}, \qquad a_3(r_i) = N_6 \ln\frac{r_i}{\xi},$$

$$\partial_1 = \frac{\rho_0}{\lambda_0 + 2\mu_0}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}, \qquad \partial_2 = \frac{\rho_0}{\mu_0}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}$$

ν_0-Puasson koeffisienti, λ_0, μ_0 - Lame koeffisientlari, ρ_0- qatlam materiali zichligi,

Qaralayotgan (1.19) tenglamalar to'rt nomalumli to'rtta tenglamalar sistemasidir. Ushbu tenglamalar sistemasidan $U_{r,0}, U_{z,0}, U_{r,1}$ va $U_{z,1}$ funksiyalarni ajratish uchun [3] ishlaridan hamda Kramer qoidasidan foydalanib $U_{z,0}$ ga nisbatan yechsak u holda (1.19) tenglamalar sistemasi quyidagicha bo'ladi

$$\left[N_2^2\frac{\partial^2}{\partial z^2} - N_3N_5\partial_1\right]\left\{-S_1^2 + S_1\left[N_2\partial_2 - (1-N_1)\frac{\partial^2}{\partial z^2}\right] + \right.$$
$$\left. + \partial_2\left[N_1\partial_2 - (N_1+N_6)\frac{\partial^2}{\partial z^2}\right]\right\}U_{z,0} = 0, \tag{1.20}$$

bu yerda

$$S_1 = 2\frac{r_2^2 - r_1^2}{r_1^2 r_2^2 \ln\frac{r_2}{r_1}}; \qquad \partial_1 = \frac{\rho}{\lambda + 2\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}; \qquad \partial_2 = \frac{\rho}{\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}.$$

(1.20) tenglamalar sistemasini T. Boggio [14] teoremasiga kura yechib quyidagi tenglamalarni hosil qilamiz:

$$\left(N_4^2\frac{\partial^2}{\partial z^2} - N_3N_5\partial_1\right)U_{z,0} = 0, \tag{1.21}$$

$$\left\{-S_1^2+S_1\left[N_2\partial_2-(1-N_1)\frac{\partial^2}{\partial z^2}\right]++\partial_2\left[N_1\partial_2-(N_1+N_6)\frac{\partial^2}{\partial z^2}\right]\right\}U_{z,0}=0. \qquad (1.22)$$

Ushbu (1.21) tenglamada ∂_1-ni hisobga olsak qatlam tebranishlarining klassik holiga mos keladi (sterjenlar nazariyasiga ko'ra). Shuning uchun keyingi qiladigan ishlarimizda (1.21) tenglamalar sistemasidagi ikkinchi tenglamadan foydalanamiz. Hosil qilingan tenglamani $-\mu J/k_2$, ga ko'paytiramiz, bu yerda

$J=\dfrac{\pi(r_2^2-r_1^2)}{2}$ - qatlam kundalang kesimi inersiya momenti;

$$k_1=\frac{\mu_0}{\lambda_0+2\mu_0}; \qquad k_2=\frac{\lambda_0+\mu_0}{\lambda_0+2\mu_0}; \qquad k_3=\frac{\nu_0}{1+\nu_0},$$

∂_2 operatorni teskari kurinishda yozsak (1.22) ni ba'zi bir almashtirishlardan so'ng quyidagicha yozamiz

$$EJ\frac{\partial^4 U_{z,0}}{\partial z^4}+\rho_0^2\frac{J}{k_3\mu_0}\frac{\partial^4 U_{z,0}}{\partial t^4}-\left(\rho_0 J+EJ\frac{\rho_0}{k_4\mu_0}\right)\frac{\partial^4 U_{z,0}}{\partial t^2\partial z^2}+$$

$$+S_3\rho_0 F\frac{\partial^2 U_{z,0}}{\partial t^2}-EJ\frac{S_4}{r_2^2}\frac{\partial^2 U_{z,0}}{\partial z^2}+S_5 EJU_{z,0}=0, \qquad (1.23)$$

bu yerda E - qatlam materiali elastiklik moduli; F - qatlam kundalang kesimi yuzasi;

$$k_4 = \frac{2\nu_0(1+\nu_0)^2}{2\nu_0^2 + 2\nu_0 + \nu_0}; \;\; k_5 = \frac{1-\nu_0 - 2\nu_0}{\nu_0^2}; \;\; S_3 = k_5 \frac{r_2^4 - r_1^4}{2r_1^2 r_2^2 \ln \frac{r_2}{r_1}};$$

$$S_4 = \frac{5-4\nu_0}{\nu_0} \frac{r_2^4 - r_1^4}{r_1^2 \ln \frac{r_2}{r_1}}; \qquad S_5 = \frac{1-\nu_0}{\nu_0} S_1^2;$$

(1.23) tenglamalar sistemasida k_3 va k_4 koeffisentlar ko'ndalang siljish deformatsiyasi ta'sirini hisobga oladi, $\rho_0 J \frac{\partial^4 U_{z,0}}{\partial t^2 \partial z^2}$ - aylanish inersiyasi ta'sirini; $S_3 \rho_0 F \frac{\partial^2 U_{z,0}}{\partial t^2}$- bo'ylama kuch inersiyasi ta'sirini; $EJ\left(\frac{S_4}{r_2^2}\frac{\partial^2 U_{z,0}}{\partial z^2} - S_5 U_{z,0}\right)$ - reaktiv moment, $EJ\frac{\partial^4 U_{z,0}}{\partial z^4}$ - bo'ylama tekis taqsimlangan kuchni hisobga oladi.

Ushbu (1.23) tenglamada a_1 parametrni kiritamiz. Bu parametr "nol" va "bir" qiymatlarni qabul qiladi, ya'ni ko'ndalang siljish deformatsiyasi ta'siri bor yoki yo'q ekanligini bildiradi. (1.23) tenglamada ulchamsiz koordinataga utamiz, ya'ni $z = \xi z^*$; $r_{1,2} = \xi r_{1,2}^*$; $U_{z,0} = \xi U_{z,0}^*$; $t = \frac{\xi}{b} t^*$ bu yerda $b = \sqrt{\frac{\mu_0}{\rho_0}}$ va yozish osonlashish uchun keyingi yozuvlarda yo'duzchalarni tashlab yozamiz

$$\frac{E}{\mu_0}\frac{\partial^4 U_{z,0}}{\partial z^4} + a_1 \frac{1}{k_3}\frac{\partial^4 U_{z,0}}{\partial t^4} - \left(a_1 \frac{E}{k_4 \mu_0}\right)\frac{\partial^4 U_{z,0}}{\partial t^2 \partial z^2} +$$

$$+ \frac{S_3 F}{J}\frac{\partial^2 U_{z,0}}{\partial t^2} - \frac{S_4}{\mu_0 J r_2^2}\frac{\partial^2 U_{z,0}}{\partial z^2} + \frac{E}{\mu_0} S_5 EJU_{z,0} = 0. \qquad (1.24)$$

§1.2. Doiraviy silindrik elastik qobiqning xususiy bo'ylama-radial tebtanishlarida deformatsiyalanuvchi muhit ta'sirini hisobga olish

Quyida uzunligi l ga teng bo'lgan va chetlari qistirib maxkamlangan qoiraviy qobiqning xususiy bo'ylama-radial tebranishlarini qarab chiqamiz.

Tebranish tenglamasi sifatida aniqlashtirilgan (1.18) tebranish tenglamalaridan foydalanamiz. Bunda $L_0 = \lambda_0 + 2\mu_0$; $M_0 = \mu_0$ (elastik) ekanligini e'tiborga olamiz. Bundan tashqari qobiqni yupqa devorli deb qabul qilamiz. Ya'ni $\varepsilon < \frac{1}{10}$ yoki $\frac{h}{R} < \frac{1}{10}$ bu yerda h qalinlik. R o'rta sirt radiusi. U holda (1.18) tenglamalar sistemasidan $\ln\frac{r_1}{\xi} = 0$, $\ln\frac{r_2}{\xi} = 0$ ekanligini hisobga olib quyidagi tenglamalarga ega bo'lamiz:

$$
\begin{aligned}
&-qU_{r,0} - (1+q_1)\frac{\partial U_{z,0}}{\partial z} + \xi\left[\frac{2}{r_i^2}\right]U_{r,1} + \frac{\xi}{2}\frac{\partial U_{z,1}}{\partial z} = M_0^{-1}\left[f_r^{91)}\right];\\
&\left(-q_1 + \frac{r_1^2}{2}R_1\right)U_{r,0} - (1+q_1)\frac{\partial U_{z,0}}{\partial z} + \xi\left[\frac{2}{r_2^2} - R_1\frac{1}{r_1^2}\right]U_{r,1} + \frac{\xi}{2}\frac{\partial U_{z,1}}{\partial z} = F,\\
&(1+q_1)\frac{\partial U_{z,0}}{\partial z} + (1-q_1)\lambda_{01}U_{z,0} - \frac{2\xi}{r_1^2}\frac{\partial U_{z,1}}{\partial z} - \frac{2\xi}{r_1^2}U_{z,1} = \frac{2}{r_1}M_0^{-1}\left[f_{rz}^{91)}\right],\\
&(1+q_1)\frac{\partial U_{r,0}}{\partial z} + (1-q_1)\lambda_{01}U_{z,1} - \frac{2\xi}{r_2^2}\frac{\partial U_{r,1}}{\partial z} - \frac{2\xi}{r_2^2}U_{z,1} = 0.
\end{aligned}
\tag{1.25}
$$

(1.25) tenglamalar sistemasini no'malum $U_{z,0}$ ga nisbatan yechamiz, bimda $U_{z,0}$ bo'ylama kochish U_z ning bosh qismi hisoblanadi.

(1.25) tenglamalar sistemasininh uch va tortinchi tenglamalaridan quyidagini hosil qilamiz.

$$-2\xi\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)\left[\frac{\partial U_{r,1}}{\partial z}+U_{z,1}\right]=\frac{2}{r_1}\mu_0^{-1}\left[f_{rz}^{(1)}\right]$$

Bu yeran

$$U_{z,1}=-\frac{1}{r_1\xi\left(\frac{1}{r_2^2}-\frac{1}{r_1^2}\right)\mu_0}f_{rz}^{(1)}-\frac{\partial U_{r,1}}{\partial z} \qquad (1.26)$$

(1.18) tenglamalar sistemasining ikkinchi va birinchi tenglamalaridan quyidagi tenglikni hosil qilamiz

$$U_{r,0}-\left[\frac{1}{r_2}R_1-\left(\frac{2}{r_2^2}-\frac{2}{r_1^2}\right)\right]U_{r,1}+\frac{r_2}{2}R_1U_{r,0}=F-\frac{1}{\mu_0}f_r^{(1)}$$

Bundan $U_{r,1}$ funksiyani $U_{r,0}$ orqali yozib olamiz

$$U_{r,1}=G^{-1}1\left[\frac{r_2}{2}R_1U_{r,0}-\frac{1}{\mu_0}f_r^{(1)}\right], \qquad (1.27)$$

Bu yerda $G=\xi\left[\frac{1}{r_2}R_1-\left(\frac{2}{r_2^2}-\frac{2}{r_1^2}\right)\right].$

(1.27) ni (1.21) ga quyamiz va $U_{z,1}$ ni $U_{r,0}$ orqali ifodalab olib quyidagiga ega bo'lamiz

$$U_{z,1} = -\frac{1}{r_1\xi\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)} f_{rz}^{(1)} - G^{-1}\left\{\frac{1}{2}R_1\left(\frac{\partial U_{r,0}}{\partial z}\right) - \frac{\partial F}{\partial z} + \frac{1}{\mu_0}\frac{\partial f_r^{(1)}}{\partial z}\right\} \tag{1.28}$$

(1.18) tenflamalar sistemasining uchinchi tenglamasini $\frac{1}{r_2^2}$ to’rtinchi tenglamasini $\frac{1}{r_1^2}$ ga ko’paytiramiz va quyidagi tenglamaga rga bo’lamiz.

$$(1+q_1)\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)\frac{\partial U_{r,0}}{\partial z} + (1+q_1)\lambda_{01}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)U_{z,0} = \frac{2}{r_1 r_2^2 \mu_0} f_{rz}^{(1)}$$

Ushbu tenglama bizga $U_{r,0}$ funksiyani $U_{z,0}$ va $f_{rz}^{(1)}$ orqali yozish imkonini beradi.

$$\frac{\partial U_{r,0}}{\partial z} = \frac{q_1 - 1}{q_1 + 1}\lambda_{01}U_{z,0} + \frac{2}{r_1 r_2^2}\frac{f_{rz}^{(1)}}{(1+q_1)\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)} \tag{1.29}$$

(1.27) ni hisobga olib z koordinata bo’yicha (1.29) dan

$$\frac{\partial U_{r,1}}{\partial z} = G^{-1}\left\{\frac{r_2}{2}\left[R_1\lambda_{01}\frac{q_1 - 1}{q_1 + 1}U_{z,0} + \frac{2}{r_1 r_2^2 \mu_0}\frac{f_{rz}^{(1)}}{(1+q_1)\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)}R_1\right] - \frac{\partial F}{\partial z} + \frac{1}{\mu_0}\frac{\partial f_{rz}^{(1)}}{\partial z}\right\} \tag{1.30}$$

(1.29) ifodani (1.28) ga qo’yib quyidagini hosil qilamiz

$$U_{z,1} = -\frac{f_{rz}^{(1)}}{\xi r_1\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)} - G^{-1}\left\{\frac{r_2}{2}\left[R_1\lambda_{01}\frac{q_1 - 1}{q_1 + 1}U_{z,0} + \frac{2}{r_1 r_2^2 \mu_0}\frac{f_{rz}^{(1)}}{(1+q_1)\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)}R_1\right] - \frac{\partial F}{\partial z} + \frac{1}{\mu_0}\frac{\partial f_r^{(1)}}{\partial z}\right\} \tag{1.31}$$

(1.25) tenglamalar sistemasidagi birinchi tenglamani z koordinatasi bo'yicha didderensiallasak

$$-q_1\frac{\partial U_{r,0}}{\partial z}-(1+q_1)\frac{\partial^2 U_{z,0}}{\partial z^2}+\frac{2\xi}{r_1^2}\frac{\partial U_{r,1}}{\partial z}+\frac{\xi}{2}\frac{\partial^2 U_{z,1}}{\partial z^2}=\frac{1}{\mu_0}\frac{\partial f_r^{(1)}}{\partial} \tag{1.32}$$

Hosil bo'lgan ifodalar $U_{r,0}, U_{r,1}$ va $U_{z,1}$ larni $U_{z,0}$ funksiya orqali (1.29), (1.30) va (1.31) lardan foydalanib quyidagicha yozamiz:

$$\begin{aligned}
&-q_1G\left[\frac{q_1-1}{q_1+1}\lambda_0U_{z,0}+\frac{2}{r_1r_2^2}\frac{\mu_0^{-1}(f_{rz}^{(1)})}{(1+q_1)\left(\frac{1}{r_2^2}-\frac{1}{r_1^2}\right)}-(1+q_1)G\frac{\partial^2U_{z,0}}{\partial z^2}\right]+\\
&+\frac{2\xi}{r_1^2}\left\{\frac{r_2}{2}\left[\frac{q_1-1}{q_1+1}\lambda_{01}U_{z,0}+\frac{2}{r_1r_2^2}\frac{\mu_0^{-1}(f_{rz}^{(1)})R_1}{(1+q_1)\left(\frac{1}{r_2^2}-\frac{1}{r_1^2}\right)}\right]-\frac{\partial F}{\partial z}+\frac{1}{\mu_0}\left(\frac{\partial f_{rz}^{(1)}}{\partial z}\right)\right\}+\\
&+\frac{\xi}{2}\left\{-\frac{\mu_0^{-1}(f_{rz}^{(1)})G}{\xi r_1\left(\frac{1}{r_2^2}-\frac{1}{r_1^2}\right)}\left(\frac{\partial^2 f_{rz}^{(1)}}{\partial z^2}\right)-\frac{r_2}{2}\left[\frac{q_1-1}{q_1+1}\lambda_{01}R_1\frac{\partial^2U_{z,0}}{\partial z^2}\right]+\frac{2}{r_1r_2^2}\frac{\mu_0^{-1}R_1}{(1+q_1)\left(\frac{1}{r_2^2}-\frac{1}{r_1^2}\right)}\left(\frac{\partial^2 f_{rz}^{(1)}}{\partial z^2}\right)\right\}-\\
&-\frac{\partial^3F}{\partial z^3}+\frac{1}{\mu_0}\left(\frac{\partial^3 f_{rz}^{(1)}}{\partial z^3}\right)=\frac{1}{\mu_0}\left(\frac{\partial f_{rz}^{(1)}}{\partial z}\right)
\end{aligned} \tag{1.33}$$

(1.33) da ba'zi bir matematik amallarni bajarib quyidagi tenglamani hosil qilamiz

$$f_1\lambda_{01}\left(-q_1G+\frac{\xi r_2}{r_1^2}R_1\right)U_{z,0}-\left[(1+q_1)G+\frac{\xi r_2}{4}f_1R_1\lambda_{01}\right]\frac{\partial^2U_{z,0}}{\partial z^2}-q_1Gf_2+$$

$$+\frac{\xi r_2}{r_1^2}f_2R_1-\frac{2\xi}{r_1^2}\frac{\partial F}{\partial z}+\frac{2\xi}{r_1^2}\left[\frac{\partial f_r^{(1)}}{\partial z}\right]-\frac{\xi r_2}{4}f_2R_1-\frac{\partial^2}{\partial z^2}-\frac{\mu_0^{-1}G}{2r_1\left(\frac{1}{r_2^2}-\frac{1}{r_1^2}\right)}\left[\frac{\partial^2 f_r^{(1)}}{\partial z^2}\right]- \quad (1.34)$$

$$-\frac{\xi}{2}\frac{\partial^3F}{\partial z^3}+\frac{\xi}{2}-\frac{1}{\mu_0}\left[\frac{\partial^3 f_r^{(1)}}{\partial z^3}\right]=\frac{1}{\mu_0}\left[\frac{\partial f_r^{(1)}}{\partial z}\right],$$

Bu yerda

$$f_1=\frac{q_1-1}{q_1+1};\quad f_2=\frac{2}{r_1r_2^2}\frac{\mu_0^{-1}(f_{rz}^{(1)})}{(1+q_1)\left(\frac{1}{r_2^2}-\frac{1}{r_1^2}\right)};\ G=\xi\left[\frac{1}{r_2}R_1-\left(\frac{2}{r_2^2}-\frac{2}{r_1^2}\right)\right].$$

$$\lambda_{01}=\left[\frac{\rho_0}{\lambda_0+2\mu_0}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right];\quad \lambda_0=\frac{\nu E_0}{(1+\nu_0)(1-2\nu_0)};\quad \mu_0=\frac{E_0\mu}{2(1+\nu_0)};$$
$$\mu_1=\frac{E_1\mu}{2(1+\nu_1)};\qquad q_1=1-\frac{\lambda_0+2\mu_0}{\mu_0};\quad \lambda_1=\frac{\nu E_1}{(1+\nu_1)(1-2\nu_1)}. \qquad (1.35)$$

r_1 va r_2 qobiqning ichki va tashqi radiuslari;

ρ_0 va ρ_1 qobiq materiali va muhitning zichligi;

ν_0 va ν_1 qobiq va muhit uchun Puasson koeffitsienti;

E_0 va E_1 qobiq va muhit uchun Yung moduli;

ξ qatlam oraliq sirti radiusi.

Agar (1.34) tenglamada tashqi ta'sir funksiyalarining qiymati nolga teng deb olsak quyidagi tenglamaga ega bo'lamiz:

$$f_1\lambda_{01}\left(-q_1G+\frac{\xi r_2}{r_1^2}R_1\right)U_{z,0}-\left[(1+q_1)G+\frac{\xi r_2}{4}R_1f_1\lambda_{01}\right]\frac{\partial U_{z,0}}{\partial z^2}=0 \tag{1.36}$$

Bu yerda R_1 - urab turuvchi muhit reaksiyasi bo'lib quyidagiga teng

$$R_1=\sqrt{\rho_1}\frac{\sqrt{\lambda_1+2\mu_1}}{\mu_0}\frac{\partial}{\partial t}$$

(1.34) ifodalarni hisobga olib (1.35) tenglamani quyidagicha yozamiz:

$$\begin{aligned}
&f_1f_3\frac{r_2}{4}\frac{\partial^5U_{z,0}}{\partial z^4\partial t}-f_1f_3f_4\frac{r_2}{4}\frac{\partial^5U_{z,0}}{\partial z^2\partial t^3}=f_1f_3f_4\left(\frac{N_1}{r_2}-\frac{r_2}{r_1^2}\right)-\frac{\partial^3U_{z,0}}{\partial t^3}+\\
&+f_3\left(-f_1\frac{N_1}{r_2}-f_1\frac{r_2}{r_1^2}+\frac{N_2}{r_2}\right)\frac{\partial^3U_{z,0}}{\partial z^2\partial t}-f_1f_4N_1\left(\frac{2}{r_2^2}-\frac{2}{r_1^2}\right)\frac{\partial^2U_{z,0}}{\partial t^2}+\\
&+\left(f_1N_1-N_2\right)\left(\frac{2}{r_2^2}-\frac{2}{r_1^2}\right)\frac{\partial^2U_{z,0}}{\partial z^2}=0.
\end{aligned} \tag{1.37}$$

Bu yerda

$$f_3=\frac{\sqrt{\rho_1(\lambda_1+2\mu_1)}}{\mu_0};\quad f_4=\frac{\rho_0}{\lambda_0+2\mu_0};$$
$$N_1=\frac{1-4\nu_0}{1-\nu_0};\qquad N_2=\frac{3-4\nu_0}{1-\nu_0}.$$

(1.37) tenglamada o'lchamsiz kattaliklarga o'tamiz buning uchun quyidagi almashtirishlardah foydalanamiz:

$$z=\xi z^*,\qquad r_{1,2}=\xi r_{1,2}^*,\qquad t=\frac{\xi}{a_0^{'}}t^*;$$

Yozuv qulay bo'lishi uchun keyinchalik yuldo'zchalarni tashlab yozamiz

$$
f_1 \frac{a_0^{'}}{a_1^{'}} \frac{r_2}{4} \frac{\partial^5 U_{z,0}}{\partial z^4 \partial t} - f_1 \frac{a_0^{'}}{a_1^{'}} \frac{r_2}{4} \frac{\partial^5 U_{z,0}}{\partial z^2 \partial t^3} + f_1 \frac{a_0^{'}}{a_1^{'}} \left(\frac{N_1}{r_2} - \frac{r_2}{r_1^2} \right) - \frac{\partial^3 U_{z,0}}{\partial t^3} +
$$

$$
+ f_3 \left(- f_1 \frac{N_1}{r_2} - f_1 \frac{r_2}{r_1^2} + \frac{N_2}{r_2} \right) \frac{\partial^3 U_{z,0}}{\partial z^2 \partial t} - f_1 f_4 N_1 \left(\frac{2}{r_2^2} - \frac{2}{r_1^2} \right) \frac{\partial^2 U_{z,0}}{\partial t^2} + \qquad (1.38)
$$

$$
+ \left(f_1 N_1 - N_2 \right) \left(\frac{2}{r_2^2} - \frac{2}{r_1^2} \right) \frac{\partial^2 U_{z,0}}{\partial z^2} = 0.
$$

Bu yerda $a_0^{'}$ va $a_1^{'}$ qobiq va muhit materialida bo'ylama to'lqinlarning tarqalish tezligi bo'lib quyidagiga teng

$$
a_0^{'} = \frac{\sqrt{\lambda_0 + 2\mu_0)}}{\rho_0}; \qquad a_1^{'} = \frac{\sqrt{\lambda_1 + 2\mu_1)}}{\rho_0}.
$$

II-BOB. DEFORMATSIYALANUVCHI MUHITDAGI DOIRAVIY SILINDRIK ELASTIK QOBIQNING BUYLAMA-RADIAL TEBRANISHLARINI SONLI TADQIQ ETISH.

§2.1 Differentsial tenglamalarni yechishning chekli ayirmalar usuli

Plastinkalar va qobiqlar nazariyasining qator masalalari berilgan chegaraviy shartli xususiy xosilali deferensial tenglamalarga keltiriladi. Bu tenglamalar oddiy masalalar hamda chegaralar va tashqi yuklarning oddiy shakllaridagina aniq yechiladilar. Ko'p hollarda aniq yechimlarni topish qiyin bo'ladi. Shuning uchun sonli usullarga murojaat qilinadi. Xususan shunday sonli usullardan biri differensial tenglamalarni tegishli chekli ayirmalar qatnashgan tenglamalar bilan almashtirishdan iboratdir.

Chekli ayirmalar metodining imkoniyati EHM paydo bo'lishi bilan oshib ketdi. Plastinkalar va qobiqlar nazariyasi masalalarini chekli ayirmalar usuli yordamida yechishga doir ilmiy ishlar keskin ko'paydi [7].

Biz quyida [7] ishidan foydalangan holda chekli ayirmalar usulining asosiy mohiyatini bayon qilamiz.

Xususiy xosilali differensial tenglamalarni taqribiy yechishda to'rlar yoki chekli ayirmalar metodining asosiy g'oyasi quyidagilardan iborat:

– yechim qidirilayotgan G soha to'rli G_λ soha bilan almashtiriladi;

– berilgan differensial tenglama sohaning ichki nuqtalarida tegishli chekli ayirmalar qatnashgan tenglamalar bilan almashtiriladi;

– chegaraviy shartlar asosida qidirilayotgan yechimning qiymatlari G_λ soha tugunlarida topiladi.

Bunday almashtirish natijasida ko'p noma'lumli algebraik tenglamalar sistemasiga kelamiz. Bunda noma'lumlar soni tenglamalr soni va tugunlar soni ko'paytmasiga teng bo'ladi. Bu sistemani yechib izlanayotgan funksiyaning nuqtalardagi sonli qiymatlarini hosil qilamiz. Funksiyaning boshqa nuqtalaridagi qiymatlari interpolyasion formulalardan topiladi.

To’rli G_λ sohani tanlash har bir aniq masalaga qarab amalga oshiriladi, biroq hamma vaqt G_λ kontur G ni yaxshiroq approksimatsiya qilishga intiladi.

To’rli soha kvadrat, to’g’ri to’rtburchak, uchburchak, oltiburchak va boshqa elementlardan iborat bo’lishi mumkin.

R_n qoldiq hadning qiymatlari alohida elementning o’lchamlaridan bog’liq bo’ladi. Ya’ni o’lchamlar qancha kichik olinsa R_n qoldiq had shuncha kichik bo’ladi. Ammo tugunlar soni ortib ketishi natijasida tenglamalar soni ko’payadi va uni EHM da yechish amalda mumkin bo’lmay qoladi.

Ma’lumki $y = f(x)$ funksiuaning x_0 nuqtadagi hosilasi quyidagicha topiladi:

$$y'_\lambda(x_0) = \lim_{\lambda \to 0} \frac{f(x_0+\lambda) - f(x_0)}{\lambda}, \text{ yoki } y'_\lambda(x_0) = \lim_{\lambda \to 0} \frac{f(x_0) - f(x_0-\lambda)}{\lambda}$$

Bu yerda λ - argumentning chekli orttirmasi, $\lambda > 0$ bo’lganda birinchi formulaga o’ng hosila, ikkinchi formulaga esa chap hosila deyiladi.

Yuqorida berilgan formulada limit simvolini tushurib qoldirsak o’ng va chap hosilalar uchun ushbu taqribiy formulalarni olamiz.

$$y'_\lambda(x_0) = \frac{f(x_0+\lambda) - f(x_0)}{\lambda}, \text{ yoki } y'_\lambda(x_0) = \frac{f(x_0) - f(x_0-\lambda)}{\lambda} \tag{2.1}$$

Amalda x_0 nuqtada markaziy ayirmalardan ham foydalanadi

$$y'_\lambda(x_0) = \frac{f(x_0+\lambda) - f(x_0-\lambda)}{2\lambda}, \tag{2.2}$$

Ikkinchi tartibli chekli ayirma xosila uchun

$$y'_\lambda(x_0) = \frac{f(x_0+\lambda) - 2f(x_0) + f(x_0-\lambda)}{\lambda^2}, \tag{2.3}$$

formuladan foydalanish qulay. Yuqoridagi (2.2) va (2.3) ifodalar asosida yuqori tartibli chekli ayirmali xosilalar uchun formulani keltirish mumkin

$$y'''_\lambda(x_0) = \frac{d}{dx}\left[y''_\lambda(x_0)\right] = \frac{f(x_0+2\lambda) - 2f(x_0+\lambda) + 2f(x_0-\lambda) - f(x_0-2\lambda)}{2\lambda^3},$$

Hosilalarning almashtirishning bu usuli uning aniqligini baholashga imkon bermaydi.

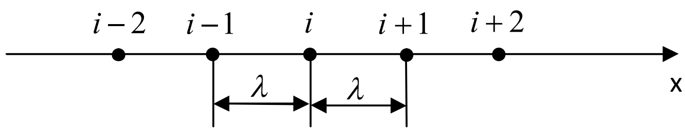
Funksiya hosilalarining uning diskret nuqtalaridagi qiymatlari orqali almashtirib approksimatsiya xatosining bahosini ham beruvchi turli usullar mavjud. Quyida Teylor formulalariga asoslanuvchi usuldan foydalanamiz.

O'ng chekli ayirmali hosila

Teng uzoqlikda joylashgan $x_i = x_0 + i\lambda$ nuqtalar uchun $y_i = f(x_i) = f(x_0 + i\lambda)$, $(i = 0,1,2,3,...,n)$ qiymatlari ma'lum bo'lsin, bunda λ - ox o'qi bo'yicha qadam (1-rasm).

$$\Delta y_i = y_{i+1} - y_i \,,\ (i = 0,1,2,...,n); \tag{2.4}$$

kattalikka $f(x)$ funksiyaning x_i nuqtadagi o'ng ayirmasi deyiladi.



2.1-rasm

Ikkinchi o'ng ayirma esa birinchi ayirmadan olingan ayirma hisoblanadi.

$$\Delta^2 y_i = \Delta(\Delta y_i) = \Delta(y_{i+1} - y_i) = (y_{i+2} - y_{i+1}) - (y_{i+1} - y_i) = y_{i+2} - 2y_{i+1} + y_i;$$

Xuddi shu yo'l bilan yuqori tartibli ayirmalarni ham topish mumkin. Shunday qilib x_i nuqtada m-tartibli o'ng ayirmani topish uchun ushbu formuladan foydalanamiz.

$$\Delta^m y_i = \Delta^{m-1}(y_{i+1} - y_i) = ... = \sum_{j=0}^{m} (-1)^j C_m^j y_{i+m-j} \tag{2.5}$$

Endi x_i nuqtada Δ ayirmali operator va $\frac{d}{dx} = D$ differensial operatorlar orasidagi bog'lanishni aniqlaymiz. Buning uchun $f(x)$ va $e^{\alpha x}$ funksiyalar Teylor qatoriga yoyiladi

$$\begin{aligned} f(x+\Delta x) &= f(x) + \frac{f'(x)}{1!}\Delta x + \frac{f''(x)}{2!}\Delta x^2 + ...; \\ e^{\alpha x} &= 1 + \frac{\alpha x}{1!} + \frac{\alpha^2 x^2}{2!} + ...; \end{aligned} \tag{2.6}$$

Ushbu belgilashlarni olamiz $\Delta x = \lambda,\ x = x_i\,,\ f^n(x_i) = D^n f(x_i),$ u holda

$$y_{i+1} = f(x_i + \lambda) = f(x_i) + \frac{\lambda}{1!} Df(x_i) + \frac{\lambda^2}{2!} D^2 f(x_i) + \ldots = \left(1 + \frac{\lambda}{1!} D + \frac{\lambda^2}{2!} D^2 + \ldots\right) f(x_i)$$

Hosil qilingan bu ifodaga (2.6) formulani ikkinchisini qo'llab quyidagicha yozish mumkin

$$y_{i+1} = e^{\lambda D} y_i; \tag{2.7}$$

(2.7) ni (2.4) ga qo'yib quyidagini olamiz

$$\Delta y_i = y_{i+1} - y_i = e^{\lambda D} y_i - y_i = \left(e^{\lambda D} - 1\right) y_i;$$

Bu yerdan Δ, D operatorlar orasidagi quyidagi bog'lanishga ega bo'lamiz.

$$\Delta = \left(e^{\lambda D} - 1\right) \text{ yoki}$$

$$\lambda D = \ln(1 + \Delta); \tag{2.8}$$

(2.9) ni n-darajaga ko'tarib n-tartibli hosila operatori orasida bog'lanishni topamiz

$$(\lambda D)^n = \ln^n (1 + \Delta); \tag{2.9}$$

(2.9), (2.10) formulalarni amalda tadbiq etish uchun $\ln(1+\Delta)$ ni Δ bo'yicha qatorga yoyamiz.

$$\ln(1 + \Delta) = \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \ldots \tag{2.10}$$

Shunday qilib

$$D^n = \frac{1}{\lambda^n}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \ldots\right], \ (n = 1,2,3,\ldots) \tag{2.11}$$

Bu formula yordamida $f(x)$ funksiyaning x_i nuqtadagi xosilasi ixtiyoriy aniqlikda o'ng ayirmalar orqali ifodalash mumkin.

Chap ayirmali hosilalar

$y = f(x)$ funksiyaning x_i nuqtadagi birinchi chap ayirmasi yoki chap ayirmasi deb ushbuga aytamiz.

$$\nabla y_i = y_i - y_{i-1}, \ (i = 0,1,2,\ldots,n); \tag{2.12}$$

Bu yerda chap ayirma o'ng ayirmadan farqli ravishda ∇ (nabla) bilan belgilandi, yuqori tartibli ayirmalar

$$\nabla^2 y_i = \nabla(\nabla y_i) = \nabla(y_i - y_{i-1}) = (y_i - y_{i-1}) - (y_{i-1} - y_{i-2}) = y_i - 2y_{i-1} + y_{i-2};$$

$$\nabla^m y_i = \nabla^{m-1}(\nabla y_i) = \nabla^{m-1}(y_i - y_{i-1}) = \ldots = \sum_{j=0}^{m} (-1)^j C_m^j y_{i-m+j}; \tag{2.13}$$

Yuqoridagi (2.6) yoyilmani $\Delta x = -\lambda,\ x = x_i,\ f^n(x_i) = D^n f(x_i)$ lar uchun ishlatsak:

$$y_{i-1} = f(x_{i-1}) = e^{-\lambda D} y_i; \tag{2.14}$$

hosil qilingan (2.14) ni (2.12) ga qo'ysak

$$\nabla y_i = y_i - y_{i-1} = y_i - y_i e^{-\lambda D} = \left(1 - e^{-\lambda D}\right) y_i$$

U holda ∇ va D operatorlari orasida bog'lanish topiladi:

$$\nabla = 1 - e^{-\lambda D} \text{ yoki } \lambda D = -\ln(1-\nabla); \tag{2.15}$$

$\ln(1-\nabla)$ ni ∇ ning darajalari bo'yicha qatorga yoyib, hosila operatorini chap ayirmalar operatori orqali ifodalaymiz:

$$\lambda D = \nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \ldots \tag{2.16}$$

n-tartibli chap ayirmali hosilalarni chap ayirmalar orqali (2.15) ni *m*-darajaga ko'tartish yo'li bilan hosil qilamiz:

$$(\lambda D)^m = [-\ln(1-\nabla)]^m = \left[\nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \ldots\right]^m, m = 1,2,3,\ldots \tag{2.17}$$

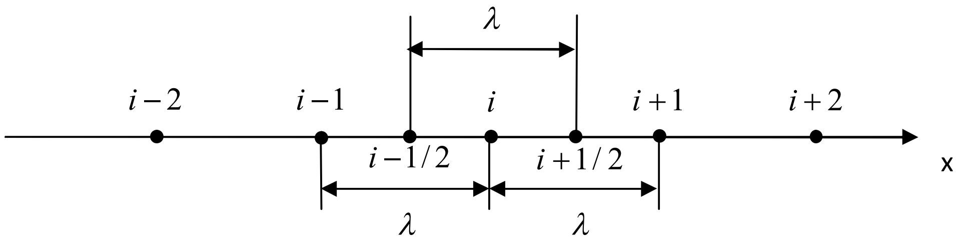
Bu yerda $\nabla^m y_i$ ayirma λ^m tartibga ega ekanligini ko'ramiz. Δ va ∇ operatorlar orasida ushbu bog'lanish mavjud

$$1 + \Delta = (1-\nabla)^{-1}. \tag{2.18}$$

Markaziy ayirmali hosilalar

$y = f(x)$ funksiyaning x_i nuqtadagi birinchi markaziy ayirmasi yoki markaziy ayirmasi deb ushbu ifodagi aytiladi

$$\delta y_i = f\left(x_i + \frac{\lambda}{2}\right) - f\left(x_i - \frac{\lambda}{2}\right) = y_{i+\frac{1}{2}} - y_{i-\frac{1}{2}} \tag{2.19}$$



2.2-rasm

Yuqoridagi kabi yuqori tartibli ayirmalarni ham kiritish mumkin.

Masalan,

$$\delta^2 y_i = \delta(\delta y_i) = \delta\left(y_{i+\frac{1}{2}} - y_{i-\frac{1}{2}}\right) = (y_{i+1} - y_i) - (y_i - y_{i-1}) = y_{i+1} - 2y_i + y_{i-1}, \qquad (2.20)$$

$$\delta^3 y_i = \delta(\delta^2 y_i) = \delta(y_{i+1} - 2y_i + y_{i-1}) = \left(y_{i+\frac{3}{2}} - y_{i+\frac{1}{2}}\right) - 2\left(y_{i+\frac{1}{2}} - y_{i-\frac{1}{2}}\right) + \left(y_{i-\frac{1}{2}} - y_{i-\frac{3}{2}}\right) =$$

$$= y_{i+\frac{3}{2}} - 3y_{i+\frac{1}{2}} + 3y_{i-\frac{1}{2}} - y_{i-\frac{3}{2}}.$$

Barcha toq tartibli markaziy ayirmalar funksiyaning $x_{i+\frac{1}{2}}$ nuqtalardagi qiymatlari orqali, barcha juft tartibli markaziy ayirmalar esa funksiyaning x_i nuqtalaridagi qiymatlari orqali hisoblanadi. Bunday yozuvdan qutulish uchun o'rtalashgan markaziy ayirma tushunchasi kiritiladi.

O'rtalashtirish operatorini quyidagicha kiritamiz

$$\mu y_i = \frac{1}{2}\left(y_{i+\frac{1}{2}} + y_{i-\frac{1}{2}}\right) \qquad (2.21)$$

Buning yordamida birinchi tartibli o'rtalashgan ayirma quyidagicha yoziladi

$$\mu\delta y_i = \frac{1}{2}(y_{i+1} - y_{i-1}); \qquad (2.22)$$

O'rtalashgan uchinchi ayirma (1.21) va (1.22) ga asosan

$$\mu\delta^3 y_i = \frac{1}{2}(y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}). \qquad (2.23)$$

Barcha toq o'rtalashgan ayirmalar ayirma yuqoridagi kabi topiladi.

μ va δ operatorlarni o'zaro bog'lash mumkin

$$\mu^2 y_i = \mu(\mu y_i) = \frac{\mu}{2}\left(y_{i+\frac{1}{2}} + y_{i-\frac{1}{2}}\right) = \frac{1}{4}[(y_{i+1} + y_i) + (y_i + y_{i-1})] =$$

$$= \frac{1}{4}[(y_{i+1} - 2y_i + y_{i-1}) + 4y_i] = \frac{1}{4}(\delta^2 + 4)y_i$$

bundan

$$\mu^2 = \frac{\delta}{4} + 1 \qquad (2.24)$$

Ikkinchi tomondan $y = f(x)$ funksiyani $x_{i+\frac{1}{2}}$ va $x_{i-\frac{1}{2}}$ nuqtalarda Teylor qatoriga yoyib ushbuni olamiz:

$$\delta\, y_i = \left[e^{\frac{\lambda D}{2}} - e^{-\frac{\lambda D}{2}} \right] y_i = 2sh\frac{\lambda D}{2} y_i; \qquad (2.25)$$

$$\mu\, y_i = \frac{1}{2}\left[e^{\frac{\lambda D}{2}} + e^{-\frac{\lambda D}{2}} \right] y_i = ch\frac{\lambda D}{2} y_i;$$

$$\delta\,\mu\, y_i = 2sh\frac{\lambda D}{2} ch\frac{\lambda D}{2} = sh\lambda D.$$

Birinchi tartibli hosila operatori o'rtallashgan markaziy ayirma orqali quyidagicha ifodalanadi

$$\lambda D = arsh(\mu\delta) \qquad (2.26)$$

bunda ushbu yoyilmadan foydalanamiz

$$arsh\, x = x - \frac{x^3}{2\cdot 3} + \frac{1\cdot 3x^3}{2\cdot 4\cdot 5} - \frac{1\cdot 3\cdot 5x^3}{2\cdot 4\cdot 6\cdot 7} - ... + (-1)^n \frac{(2n-1)!!}{(2n)!!(2n+1)} + ... \ ,$$

$$\lambda D = \mu\delta - \frac{1}{6}(\mu\delta)^3 + \frac{3}{40}(\mu\delta)^5 - \frac{5}{112}(\mu\delta)^7 + ...$$

(2.25) tenglikni hisobga olsak bu ifodani ko'rinishi quyidagicha bo'ladi

$$\lambda D = \mu\delta\left(1 - \frac{1}{6}\left(1 + \frac{\delta^2}{4}\right)\delta^2 + \frac{3}{40}\left(1 + \frac{\delta^2}{4}\right)^2 \delta^4 - \frac{5}{112}\left(1 + \frac{\delta^2}{4}\right)^3 \delta^6 + ... \right)$$

Bu ifodani δ ni darajasini o'sib borish tartibida yozamiz

$$\lambda D = \mu\delta\left(1 - \frac{\delta^2}{6} + \frac{\delta^4}{30} - \frac{\delta^6}{140} - \frac{129\delta^8}{4480} + ... \right) \qquad (2.27)$$

Oxirgi yoyilmadan foydalanib ixtiyoriy tartibli markaziy ayirmani topish mumkin. (2.27) dan ko'rinadiki yoyilmaning bitta hadining qo'shilishi approksimatsiya aniqligini λ^2 ga oshiradi. Bundan tashqari markaziy ayirmali hosilalar bir tomonli hosilalarga nisbatan aniqlik darajasi katta .

Yuqori tartibli hosilalarni (2.27) dan uning o'ng va chap tomonlarini n-darajaga ko'tarish yo'li bilan hosil qilamiz, bunda juft darajalarda (2.24) ni hisobga olamiz.

$$(\lambda D)^n = (\mu\delta)^n\left(1-\frac{\delta^2}{6}+\frac{\delta^4}{30}-\frac{\delta^6}{140}+\frac{129\delta^8}{4480}-\dots\right)^n \qquad (2.28)$$

Oxirgi formulada $n = 2,3,4$ bo'lganda

$$(\lambda D)^2 = \delta^2\left(1-\frac{\delta^2}{12}+\frac{\delta^4}{90}-\frac{\delta^6}{560}+\dots\right);$$

$$(\lambda D)^3 = \mu\delta^3\left(1-\frac{\delta^2}{4}-\frac{\delta^4}{120}+\dots\right);$$

$$(\lambda D)^4 = \delta^4\left(1-\frac{\delta^2}{8}+\frac{376\delta^4}{720}+\dots\right);$$

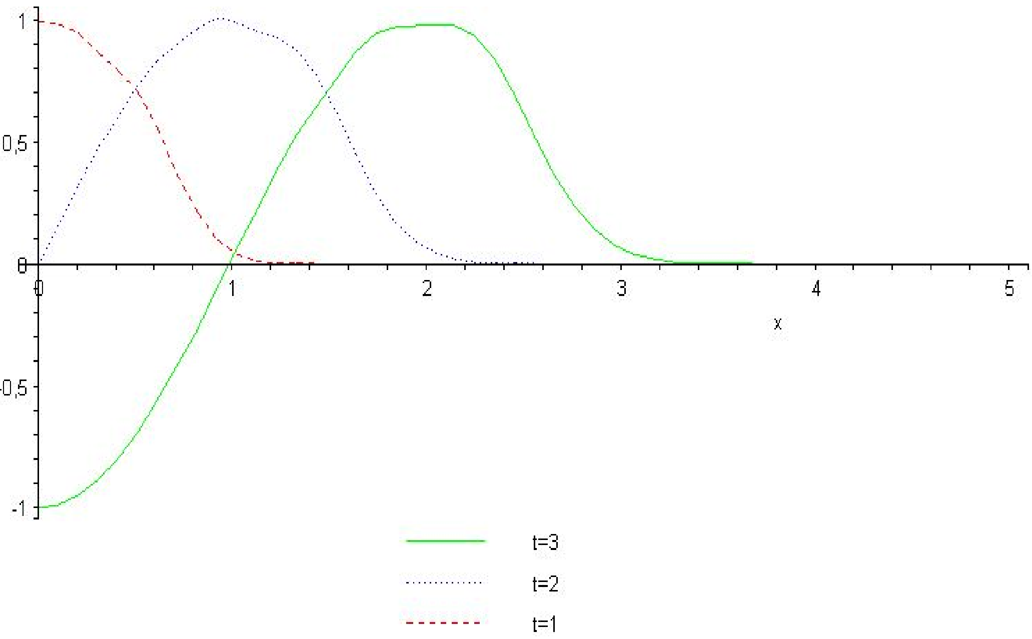
Keltirib chiqarilgan (2.28) formuladan ixtiyoriy tartibli markaziy ayirmali hosilani aniqlashimiz mumkin.

§2.2 Doiraviy silindrik qobiqning buralma-radial tebranishlarida o'rab turuvchi muhit ta'sirini hisobga olish

Biz yo'qorida keltirgan (1.38) tenglamada yuqori tartibli hosilalarni tashlab quyidagi tenglamaga ega bo'lamiz:

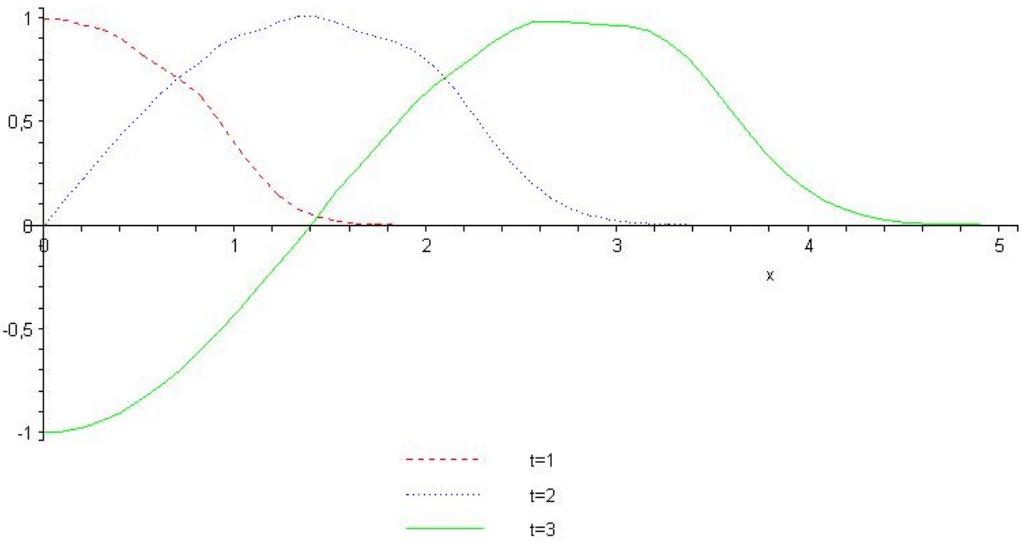
$$f_3\left(-f_1\frac{N_1}{r_2}-f_1\frac{r_2}{r_1^2}+\frac{N_2}{r_2}\right)\frac{\partial^3 U_{z,0}}{\partial z^2\partial t}-f_1 f_4 N_1\left(\frac{2}{r_2^2}-\frac{2}{r_1^2}\right)\frac{\partial^2 U_{z,0}}{\partial t^2}+$$
$$+\left(f_1 N_1-N_2\right)\left(\frac{2}{r_2^2}-\frac{2}{r_1^2}\right)\frac{\partial^2 U_{z,0}}{\partial z^2}=0. \qquad (2.37)$$

Ushbu tenglamaga differentsial tenglamalarni yechishning chekli ayirmalar usulining Krank-Nikolson sxemasidan foydalanamiz. (2.37) tenglamaga bu sxemani qo'llash natijasida hosil qilingan algebraik tenglamalar sistemasini Maple -9.5 dasturi yordamida yechib sonli natijalar olingan. Olingan natijalar asosida ko'chishlarning vaqtdan bog'liq o'zgarishi grafiklari tasvirlangan. Tenglamada o'rab turuvchi muhit sifatida suglinok va glina qabul qilingan. Hisob ishlari uchun materialni po'lat deb olib $E=2\cdot 10^5\ MPa$, $\nu=0{,}3$, $\rho_0=7850$, $r_2=1.1$ $r_1=1.0$ o'rab turuvchi muhit suglinok bo'lganda $E=19\,MPa$, $\nu=0{,}35$, $\rho_1=1750n/m^3$, glina bo'lganda $E=21\,MPa$, $\nu=0{,}42$, $\rho_1=1930n/m^3$ ko'rinishida tanlaymiz.



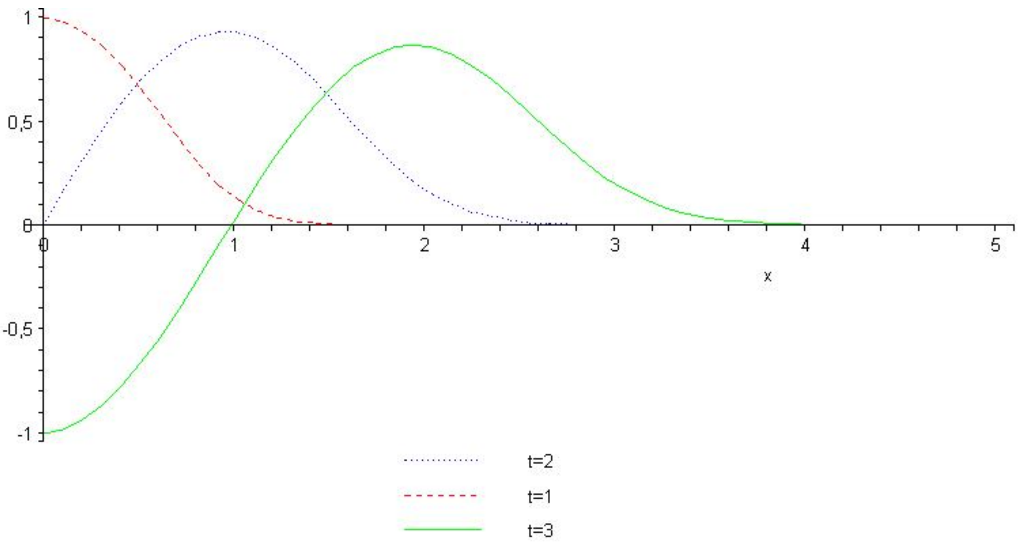
h=0,01; Po'lat

2.1-rasm.



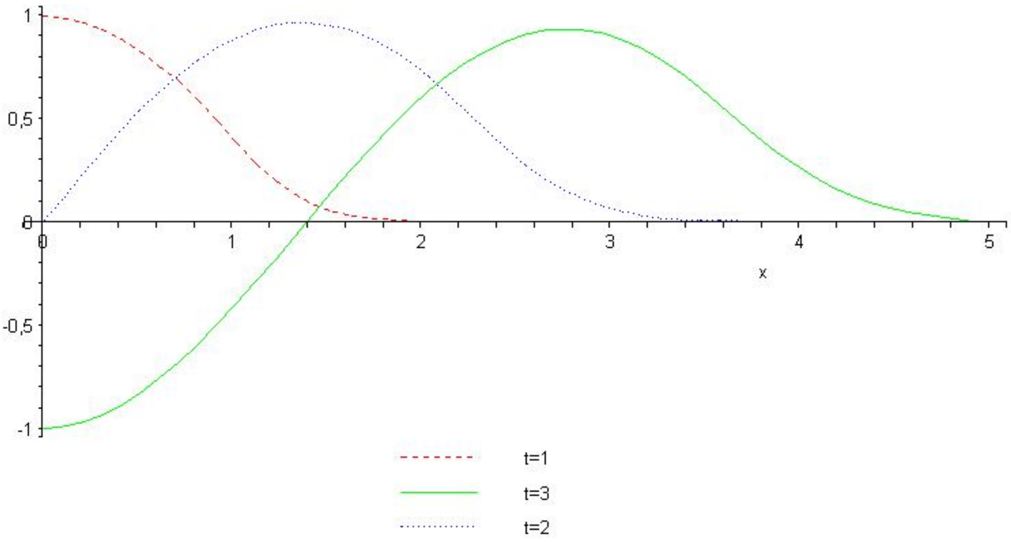
h=0,02; Po'lat

2.2-rasm



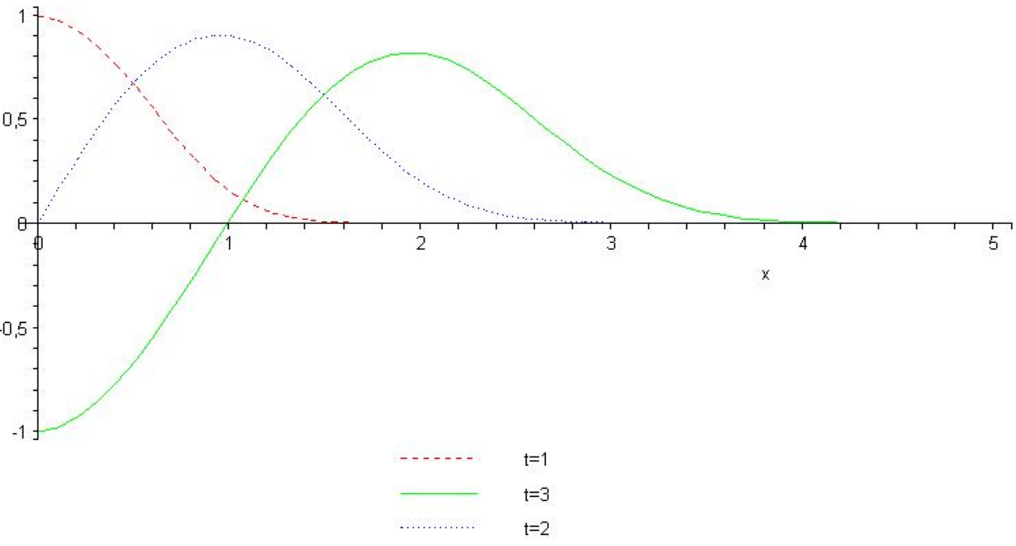
h=0,01; Po'lat-Suglinok

2.3-rasm



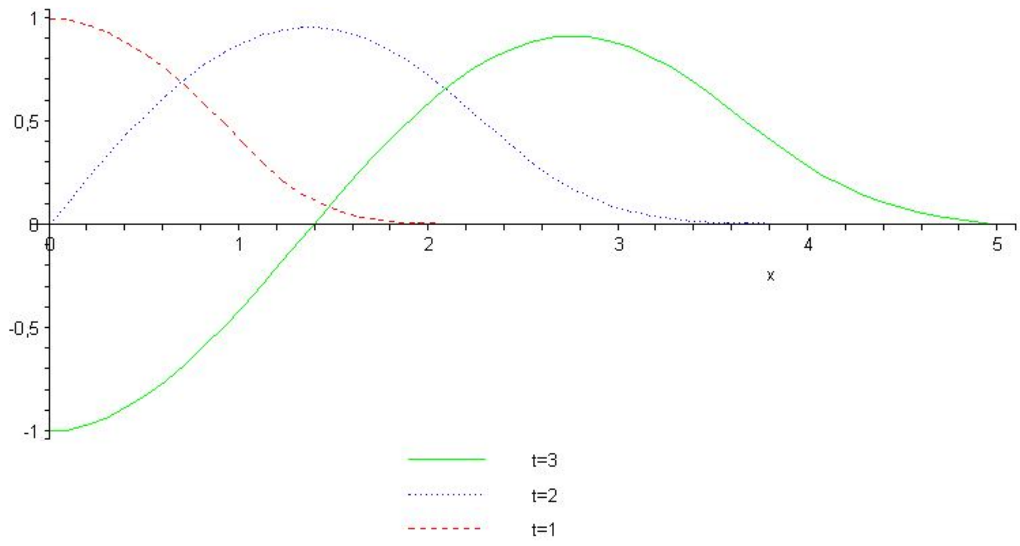
h=0,02; Po’lat-Suglinok

2.4-rasm



h=0,01; Po’lat-Glina

2.5-rasm



h=0,02; Po’lat-Glina

2.6-rasm

2.1-rasmda qalinligi $h = 0.01$ bo’lgan po’lat qobirda bo’ylama ko’chishlarning koordinatadan bog’liq o’zgarishi grafigi vaqtning turli momentlari uchun keltirilgan.

2.2-rasmda qalinligi $h = 0.02$ bo’lgan po’lat qobirda bo’ylama ko’chishlarning koordinatadan bog’liq o’zgarishi grafigi vaqtning turli momentlari uchun keltirilgan.

2.3-rasmda qalinligi $h = 0.01$ bo’lgan qobiqqa o’rab turuvchi muhit (suglinok) ta’sirida bo’ylama ko’chishlarning koordinatadan bog’liq o’zgarishi grafigi vaqtning turli momentlari uchun keltirilgan.

2.4-rasmda qalinligi $h = 0.02$ bo’lgan qobiqqa o’rab turuvchi muhit (suglinok) ta’sirida bo’ylama ko’chishlarning koordinatadan bog’liq o’zgarishi grafigi vaqtning turli momentlari uchun keltirilgan.

2.5-rasmda qalinligi $h = 0.01$ bo’lgan qobiqqa o’rab turuvchi muhit (glina) ta’sirida bo’ylama ko’chishlarning koordinatadan bog’liq o’zgarishi grafigi vaqtning turli momentlari uchun keltirilgan.

2.6-rasmda qalinligi $h = 0.02$ bo’lgan qobiqqa o’rab turuvchi muhit (glina) ta’sirida bo’ylama ko’chishlarning koordinatadan bog’liq o’zgarishi grafigi vaqtning turli momentlari uchun keltirilgan.

Xulosa

2.1 va 2.2 chizmalardan ko'rinadiki sirtlariga berilgan ko'chish uzunlik bo'ylab xuddi shunday qonuniyat bo'yicha amplitudasi o'zgarmasdan tarqaladi.

Qatlam qalinligi oshib borishi bilan bo'ylama to'lqin tarqalish tezligi ham oshishi kuzatiladi. Qalinlikning o'zgarishi amplitudaning o'zgarishiga ta'sir ko'rsatmaydi. 2.3 va 2.4 chizmalarda silindrik qobiqning tashqi sirti suglinok bilan o'zaro ta'sirlashganda qatlam qalinligi $h = 0.01$; $h = 0.02$ bo'lgan hollar uchun vaqtning turli momentlarida bo'ylama ko'chishining koordinatadan bog'liq o'zgarishi grafiklari keltirilgan. Grafiklardan ko'rinadiki tashqi muhitning ta'siri tebranish amplitudasining pasayishiga olib keladi. Bu qalinlik kichik bo'lgan hollarda yanada sezilarli bo'ladi. Qalinlik $h = 0.01$, $t=3$ bo'lganda tebranish amplitudasi muhit ta'sirida 12% gacha kamaymoqda.

2.5 va 2.6 chizmalarda silindrik qobiqning tashqi sirti glina bilan o'zaro ta'sirlashganda qatlam qalinligi $h = 0.01$; $h = 0.02$ bo'lgan hollar uchun vaqtning turli momentlarda bo'ylama ko'chishining koordinatadan bog'liq o'zgarishi grafiklari keltirilgan. Grafiklardan ko'rinadiki tashqi muhitning ta'siri tebranish amplitudasining pasayishiga olib keladi. Bu qalinlik kichik bo'lgan hollarda yanada sezilarli bo'ladi. Qalinlik $h = 0.01$, $t=3$ bo'lganda tebranish amplitudasi muhit ta'sirida 23% gacha kamaymoqda.

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16. http.www.ruscommech.ru

17. http:lib.ru

ILOVA

```
> restart;
r1:=0.99; r2:=1.01; h:=r2-r1;
nu:=0.3; E:=2e11; rho:=7.85e;
mu:=E/(2*(1+nu)); lambda:=nu*E/((1+nu)*(1-2*nu));t1:=2;
a:=sqrt((lambda+2*mu)/rho); b:=sqrt(mu/rho);
q1:=-(lambda+mu)/mu; q2:=-(lambda+mu)/(lambda+2*mu);
E1:=2.1e7; rho1:=1.93e; nu1:=0.42;
mu1:=E1/(2*(1+nu1)); lambda1:=nu1*E1/((1+nu1)*(1-2*nu1));
f1:=(q1-1)/(q1+1);
f3:=sqrt(rho1*(lambda1+2*mu1))/mu;
f4:=rho/(lambda+2*mu);
N1:=(1-4*nu)/(1-nu);
N2:=(3-4*nu)/(1-nu);
aa:=1;
PDE:= -a^2*f1*f4*N1*(2/r2^2+2/r1^2)*diff(u(x,t),t,t)-20*(f1*N1-
N2)*(2/r2^2-2/r1^2)*diff(u(x,t),x,x)-        10*aa*a*f3*(-f1*N1/r2-
f1*r2/r1^2+N2/r2)*diff(u(x,t),x,x,t)=0;
```

$$r1 := 0.99$$

$$r2 := 1.01$$

$$h := 0.02$$

$$n := 0.3$$

$$E := 2\ 10^{11}$$

$$r := 7.85$$

$$m := 7.692307692\ 10^{10}$$

$l := 1.153846154\ 10^{11}$

$t1 := 2$

$a := 1.85194258210^{5}$

$b := 98990.49489$

$q1 := -2.500000000$

$q2 := -0.7142857143$

$E1 := 2.1\ 10^{7}$

$r1 := 1.93$

$n1 := 0.42$

$m1 := 7.394366197\ 10^{6}$

$l1 := 3.882042254\ 10^{7}$

$f1 := 2.333333333$

$f3 := 1.32233498310^{-7}$

$f4 := 2.91571428610^{-11}$

$N1 := -0.2857142857$

$N2 := 2.571428571$

$aa := 1$

$$PDE := 2.667466800 \left(\frac{\partial^2}{\partial t^2} u(x,t) \right) - 5.181988766 \left(\frac{\partial^2}{\partial x^2} u(x,t) \right) - 0.1962829023 \left(\frac{\partial^3}{\partial x^2\, \partial t} u(x,t) \right) = 0$$

```
>  IBC := {u(x,0)=0,D[2](u)(x,0)=0, u(0,t)=sin(Pi*t/t1), u(5,t)=0};
```

```
IBC1 := {u(x,0)=0,D[2](u)(x,0)=0, u(0,t)=sin(Pi*t/t1), u(5,t)=0};
```

$IBC := ? u(0, t) = \sin ? \frac{1}{2} \text{p } t ? , u(x, 0) = 0, D_2(u)(x, 0) = 0, u(5, t) = 0?$

$IBC1 := ? u(0, t) = \sin ? \frac{1}{2} \text{p } t ? , u(x, 0) = 0, D_2(u)(x, 0) = 0, u(5, t) = 0?$

```
> pds := pdsolve(PDE,IBC,numeric,spacestep=1/10);
```

pds := module () local INFO; export plot, plot3d, animate, value, settings; option `Copyright (c) 2001 b

Waterloo Maple Inc. All rights reserved.`; end module

```
> p:=pds:-plot(t=1,numpoints=50,color=red,linestyle=3,legend=["t=1
"]):
q:=pds:-plot(t=2,numpoints=50,color=blue, linestyle=2,legend=["t=2
"]):
r:=pds:-plot(t=3,numpoints=50,color=green,linestyle=1,legend=["t=3
"]):
plots[display]({p,q,r});
```