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Mexanika – matematika fakulteti

«Hisoblash usullari» kafedrasi

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ODDIY DIFFERENSIAL TENGLAMALAR SISTEMASINI MAPLE VA MATHCAD MATEMATIK PAKETLARI YORDAMIDA TAQRIBIY YECHISH

«5480100 – Amaliy matematika va informatika» ta’lim yo‘nalishi
bo‘yicha bakalavr darajasini olish uchun

BITIRUV MALAKAVIY ISH

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Bitiruv malakaviy ish «Hisoblash usullari» kafedrasida bajarildi, kafedraning 2013 yil «__» _________dagi majlisida muhokama qilindi va himoyaga tavsiya etildi (____ -son bayonnoma).

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Prezidentimiz I.A.Karimovning “Jahon moliyaviy-iqtisodiy inqirozi, O`zbekiston sharoitida uni bartaraf etishning yo`llari va choralari” asarida “…korxonalarni modernizatsiya qilish, texnik va texnologik qayta jihozlashni yanada jadallashtirish, zamonaviy, moslashuvchan texnologiyalarni keng joriy etish” inqirozga qarshi choralardan biri sifatida ko’rsatilgan [1]. Prezidentimiz I.A.Karimovning «Vatanimizning kelajagi, xalqimizning ertangi kuni, mamlakatimizning jahon hamjamiyatidagi obro’-e’tibori, avvalambor farzandlarimizning o’nib-o’sib, ulg’ayib, qanday inson bo’lib hayotga kirib borishiga bog’liqdir. Biz bunday o’tkir haqiqatni hech qachon unutmasligimiz kerak» degan oqilona gaplariga amal qilmog’imiz lozim. Bu vazifalarni zamonaviy kompyuter texnologiyalarining tadbiqisiz bajarish mumkin emas. Ilmiy asoslangan rejalar tuzish, ularni amaliyotga joriy etish eng ilg`or axborot- kommunikatsiya texnologiyalardan foydalanishni taqozo etadi [2, 3]. Prezidentimizning 2002 yil 30 mayda qabul qilingan “Kompyuterlashtirishni yanada rivojlantirish va axborot-kommunikatsiya texnologiyalarini joriy etish” to‘g‘risidagi farmonida kompyuter texnologiyalaridan foydalanishning samaradorligini oshirish yo`nalishlari belgilab berilgan [4, 5].

Mavzuning dolzarbligi. Kompyuterning qo’llanilish sohalaridan biri texnik va texnologik jarayonlarni va ob’ektlarni matematik modellashtirish orqali tadqiq etish bo`lib qolmoqda. Matematik modellashtirish usullari va kompyuterlarning zamonaviy imkoniyatlari birgalikda fizik jarayonlar va ob`yektlarning shu paytgacha noma`lum xususiyatlarini ochishga va, shu asnoda, texnologik jarayonlarni takomillashtirishga xizmat qilmoqda. Ushbu bitiruv malakaviy ishning mavzusi ham hisoblash matematikasi usullaridan foydalanib, matematik modellashtirish, kompyuterning va undagi dasturiy vositalarning ilmiy tadqiqot ishlarida qo’llanilishiga bog`liq bo’lib, ilmiy va amaliy jihatdan dolzarbdir [6, 14].

Hozirgi kunda fan-texnika rivojlanib borgan sari hisoblash matematikasining roli ortib bormoqda. Shu jumladan hisoblash matematikasi usullaridan fizika, mexanika va astronomiya hamda iqtisodiy masalalarni yechishda, biologik jarayonlarni tahlil etishda va boshqa ko'p sohalarda foydalaniladi. Bu sohalardagi jarayonlarning matematik modeli differensial tenglamalar nomi bilan yuritiladi, ularni yechish esa bevosita taqribiy usullarga va kompyuter dasturiy vositalariga borib taqaladi.

Ushbu ishda oddiy differensial tenglamalar sistemasini Maple va Mathcad dasturlari yordamida analitik va taqribiy yechish masalasi qaraladi. Quyida masalaning qo'yilishi va uni yechishning ketma-ket algoritmi keltirilgan. Differensial tenglamalar sistemasini yechish uchun zarur bo'lgan hisoblash usullari tavsiflanadi.

Masalaning qo'yilishi. Quyida ana shunga erishish uchun avvalo differensial tenglamalar sistemasi, chegaraviy masala, ularning umumiy va xususiy yechimlari, ularni analitik usulda topish, qay hollarda matematik paketlardan qanday foydalanish mumkinligi haqida so'z yuritish. Nochiziqli differensial tenglamalar sistemasidan iborat bo'lgan Mak-Artur, Bazikin, Lodka-Volterr va Xolling-Tenner modeli differensial tenglamalari sistemasini taqribiy yechish masalasi qaralib, taqribiy hisob usuli Runge-Kutta usuli bo'yicha aniq amaliy masalalar yechish.

Ishning maqsadi va vazifalari. Ushbu bitiruv malakaviy ishini yozishda oddiy differensial tenglamalar sistemasini, chegaraviy masalalarni analitik va sonli yechish usullari yordamida Maple va Mathcad matematik paketidan foydalanib, yechish, aniq amaliy masalalarda bu jarayonni ko'rsatish, masalani sonli yechishning algoritmi va dasturini yaratish ko'zda tutilgan.

Muammoning ishlab chiqilish darajasi. Bitiruv malakaviy ishida nochiziqli differensial tenglamalar sistemasidan iborat bo'lgan Bazikin, Lodka-Volterr va Xolling-Tenner modellari differensial tenglamalari sistemasini sonli yechish masalasi qaralib, taqribiy hisob usuli Runge-Kutta usuli bo'yicha aniq amaliy masalalar yechish. Tadqiqotlar aniq misollarda bajarildi, ular uchun zarur algoritm va dasturlar tuzildi.

Ishning ilmiy yangiligi. Oddiy differensial tenglamalar sistemasi, Koshi masalasi va chegaraviy masalalardagi masalalarni matematik paketlardan foydalanib yechishda bu bo'limlarda qo'llaniladigan (ba'zaviy) metodlarni bilish zarur. Ular oliy matematikaning asosiy bo'limlarida qo'llaniladigan elementar almashtirishlar va hisoblashlarning buyruqlaridan (operatorlaridan) foydalanish imkonini beradi.

Amalda ixtiyoriy matematik paket yordamida amalga oshirish mumkin bo'lgan "elementar" hisoblashlar va almashtirishlar zanjiri murakkab masalalarni ham yechish imkonini beradi (masalan, oddiy differensial tenglama va tenglamalar sistemasi, chegaraviy masalalarni yechish).

Tadqiqot predmeti. Maple va Mathcad dasturiy paketlari oliy matematikaning maxsus bo'limlaridagi ko'pgina masalalarning yechimlarini topishga imkon beradi. Maple va Mathcad muhitida ishlash texnologiyasi bilan [6, 9, 10, 11, 13] larda tanishish mumkin.

Ushbu bitiruv malakaviy ishida Maple va Mathcad matematik paketlarning oddiy differensial tenglamalar sistemasi, chegaraviy masalalarning ba'zi turlarini yechish uchun qo'llash uslubi keltirilgan.

Tadqiqot obyekti. Oddiy differensial tenglamalar sistemasi, chegaraviy masalalar bitiruv malakaviy ishining tadqiqot obyektidir. Oddiy differensial tenglamalar sistemasi, chegaraviy masalalarni aniq usullar bilan yechish [7, 8, 12, 15] adabiyotlarda va ularni taqribiy yechish usullari esa [6, 14] adabiyotlarda keltirilgan.

Ishning ilmiy ahamiyati. Bu bitiruv malakaviy ishida oddiy differensial tenglamalar sistemasi, chegaraviy masalalarni Maple va Mathcad matematik paket yordamida analitik va taqribiy yechish usullari ko'rsatilgan.

Ishning amaliy ahamiyati. Bitiruv malakaviy ishidan «Differensial tenglamalar» fanidan bo'ladigan amaliy mashg'ulotlarda, seminar mashg'ulotlarida, oddiy differensial tenglamalar sistemasi, chegaraviy masalalarni sonli yechish bo'yicha tanlov fanlari mashg'ulotlarida foydalanish mumkin.

Ishning tuzilishi. Bitiruv malakaviy ishi Kirish qismi, Asosiy qism, Xulosa va foydalanilgan adabiyotlar ro'yxatidan iborat bo'lib, jami 54 betdan iborat. Asosiy qism 3 ta bobdan iborat bo'lib, 1-bobda oddiy differensial tenglamalar sistemasini Maple va Mathcad paketi yordamida analitik yechish berilgan va buning qo’llanilishi misollarda ko’rsatilgan. 2-bobda oddiy differensial tenglamalar sistemasini Maple va Mathcad paketi yordamida taqribiy yechish usullari ko’rsatilgan. 3-bobda esa nochiziqli differensial tenglamalar sistemasidan iborat bo’lgan Mak-Artur, Bazikin, Lodka-Volterr va Xolling-Tenner modeli differensial tenglamalari sistemasini taqribiy yechish masalasi qaralib, taqribiy hisob usuli Runge-Kutta usuli bo’yicha aniq amaliy masalalar yechilgan. Xulosa qismida bitiruv ishining asosiy natijalari va uning amaliy tadbiqlari bayon qilingan. Foydalanilgan adabiyotlar ro’yxati 24 ta adabiyotdan iborat.

Olingan natijalarning qisqacha mazmuni (annotatsiyasi). Bu ishda oddiy differensial tenglamalar sistemasini Maple va Mathcad matematik paketi yordamida analitik va taqribiy yechish hisob ketma-ketligi keltirilgan. oddiy differensial tenglamalar sistemasi tadbiqlarining, masalan, biologik jarayonlar masalalarida qo’llanilishi ko’rsatilgan. Nochiziqli differensial tenglamalar sistemasidan iborat bo’lgan Mak-Artur, Bazikin, Lodka-Volterr va Xolling-Tenner modeli differensial tenglamalari sistemasini taqribiy yechish masalasi qaraladi. Differensial tenglamalarni yechishning bir qator taqribiy hisob usullar (Eyler usuli, Runge-Kutta usuli va boshqa usullar)dan iborat. Shulardan Runge-Kutta usuli bo’yicha aniq amaliy masalalar yechilgan, hisob algoritmi va blok-sxemasi tuzilgan, shunga ko’ra Maple va Mathcad matematik paketida dastur ishlab chiqilgan. Olingan natijalar analitik yechim funksiyasi ko’rinishida, grafiklarda, fazoviy portretlarda ifodalanib, tegishli xulosalar chiqarilgan.

1-BOB.
ODDIY DIFFERENSIAL TENGLAMALAR SISTEMASINING ANALITIK YECHIMINI MAPLE VA MATHCAD DASTURLARI YORDAMIDA TOPISH

Matematika va har xil fan sohalari (masalan, fizika, kimyo, biologiya, tibbiyot, texnika va hokazo)ning turli masalalarini o'rganish ko'p hollarda oddiy differensial tenglamalar yoki tenglamalar sistemasini yechishga olib kelinadi. Aniq amaliy masala esa ixtiyoriy tartibli differensial tenglama yoki har xil tartibli differensial tenglamalar sistemasini yechishni talab etadi. Bunday masalalarni ko'p hollarda analitik usullar bilan yechib bo'lmaydi. Ana shunday hollarda biz sonli usullarga murojaat qilamiz. Sonli usullar yordamida taqribiy yechim quriladi va tegishli xulosalar siqariladi. Mazkur ishda ana shunday masalalarni Maple va MathCad matematik paketlari yordamida taqribiy yechish masalalari qaraladi.

1.1. Differensial tenglamalar sistemasi va uning yechimi tushunchasi

Masalaning qo'yilishi. Ixtiyoriy n-tartibli differensial tenglama deb quyidagi tenglamaga aytiladi:

$$x^{(n)}(t) = f\left(t, x', x'', \ldots, x^{(n-1)}\right).$$

Bunda $x_i(t) = x^{(i)}(t)$ almashtirish olib, uni quyidagi n ta birinchi tartibli differensial tenglamalar sistemasiga keltirish mumkin:

$$\begin{cases} x_i'(t) = x_{i+1}, \quad i = 0,1,2,\ldots,n-2, \\ x_{n-1}'(t) = f(t, x_0, x_1, \ldots, x_{n-1}), \end{cases}$$

bu yerda $x_0(t) = x(t)$.

Xuddi shunday, ixtiyoriy n-tartibli differensial tenglamalar sistemasi deb quyidagi sistemaga aytiladi:

$$x_i'(t) = f_i\left(t, x_1, x_2, \ldots, x_n\right) \qquad i = 1,2,\ldots,n.$$

Buni, soddalik uchun, vektor shaklida yozaylik:

$$x'(t) = f(t, x(t)),$$

bu yerda x va f ustun vektorlar:

$$x = (x_1, x_2, \ldots, x_n), \qquad f = (f_1, f_2, \ldots, f_n)$$

Ixtiyoriy n-tartibli chiziqli differensial tenglamalar sistemasi deb quyidagi sistemaga aytiladi:

$$x_i' = \sum_{j=1}^{n} a_{i,j} x_j + b_i, \quad i = 1,2,\ldots,n.$$

Yuqoridagi tenglamalar sistemasi, umumiy holda n ta $c = (c_1, c_2, \ldots, c_n)$ parametrlardan bog'liq yechimlar to'plamiga ega va bu umumiy yechimni vektor shaklida $x = x(t,c)$ kabi yozish mumkin.

Berilgan differensial tenglamalar sistemasining talab qilingan yechimini topish uchun $x_i(t)$ funksiyalarga qo'shimcha n ta shartlar qo'yishni talab qiladi.

Berilgan differensial tenglamalar sistemasini aynan nolga aylantiruvchi $x = (x_1(t), x_2(t), \ldots, x_n(t))$ vektor shu tenglamalar sistemasining aniq yechimi deb ataladi.

Oddiy differensial tenglama yoki tenglamalar sistemasi uchun 3 ta turdagi masalalar qo'yiladi: Koshi masalasi; chegaraviy masala; xos qiymat masalasi.

Shulardan Koshi masalasi (boshlang'ich shartli masala)ni qaraylik. Unga ko'ra yuqoridagi tenglamalar sistemasi $t = t_0$ da quyidagi shartlar bilan to'ldiriladi:

$$x_i(t_0) = x_{i0}, \; i = 1,2, \ldots, n$$

Hosil bo'lgan Koshi masalasining yechimini biror $[a,b]$ kesmada topish talab etiladi, bundan $t = t_0$ shu kesmadagi nuqta, masalan $t = t_0 = a$, olinadi.

Izlanayotgan yechim shu kesmada mavjud va yaqona bo'lishi uchun bunda $f = (f_1, f_2, \ldots, f_n)$ funksiyalar shu sohada uzluksiz va chegaralangan ($|f_i| \le M$, bunda M – biror chekli musbat son) bo'lishi, m-tartibgacha hosilalarga ega bo'lishi hamda ular Libshits shartini (ya'ni shu sohadan olingan ikkita nuqtadagi qiymatlari farqi chekli: $|f_i(t,u) - f_i(t,x)| \le L|u-x|$) qanoatlantirishi lozom.

Masalani yechish usullari. Masalani yechish usullarini 3 turga bo'lish mumkin, bular: aniq usullar, taqribiy-aniq usullar, sonli usullar.

Aniq usullar bu yechimni elementar funksiyalar orqali ifodalash yoki elementar funksiyalar integrali orqali ifodalashdan iborat bo'lib, bu usullar oddiy differensial tenglamalar kursida o'rganilgan. Aniq usullar amaliyotda uchraydigan masalalarning ba'zi bir turlarinigina yechish imkonini beradi. Masalan, ushbu

$$\frac{dx(t)}{dt} = t^2 + x^2(t)$$

Differensial tenglamaning yechimini elementar funksiyalar orqali ifodalab bo'lmaydi. Ushbu

$$\frac{dx(t)}{dt} = \frac{x(t) - t}{x(t) + t}$$

tenglamaning umumiy yechimi

$$0{,}5\ln\left(t^2 + x^2\right) + \operatorname{arctg}\left(\frac{x}{t}\right) = C$$

Ko'rinishda, ammo bu transendent formulali yechimdan $x(t)$ ning t dan bog'liq ifodasini chiqarish berilgan tenglamani yechisdanda oson emas.

Taqribiy-analitik usullarga $x(t)$ yechim funksiyani biror $x_k(t)$ – funksiyalar ketma-ketligi orqali ifodalash kiradi, bunda $x_k(t)$ lar elementar funksiyalar yoki ularning integrallari orqali ifodalangan bo'ladi. Shunday qilib, cheklangan k qiymat uchun $x(t)$ yechimga ega bo'lamiz. Bu usullarga misol qilib yechimni umumlashgan darajali qatorlarga yoyish usuli, Pikar usuli, kichik parametrlar usulini keltirish mumkin. Bu usullardan masalani yechishning boshlang'ich katta qismini aniq amalga oshirish mumkin bo'lgandagina foydalanish mumkin. Bunga faqat soddaroq masalalarni yechishdagina erishish mukin.

Sonli usullarda esa $x(t)$ yechim funksiyaning taqribiy qiymatlari to'r tugunlari deb ataluvchi t_1, t_2, ..., t_N nuqtalarda taqribiy hisoblanadi. Bunda yechimlar jadval ko'rinishida olinadi.

Sonli usullar berilgan sistemaning umumiy yechimini topish imkonini bermaydi, balki qo'yilgan masalaning, masalan, Koshi masalasining, qaysidir bir xususiy yechiminigina topib beradi. Ana shu holar sonli usullarning asosiy kamchiligi hisoblanadi. Shunga qaramasdan bu usullar juda keng sinfdagi masalalarni yechish imkonini beradiki, keyingi paytlarda amaliyotda bu usullar samarali qo'llanilib kelinmoqda.

Shularni e'tiborga olib, mazkur ishda sonli usullardan biri Runge-Kutta yordamida oddiy differensial tenglamalar sistemasini yechishni Maple va Mathcad matematik paketlari yordamida amalga oshirishni ko'ramiz.

Runge-Kutta usulining g'oyasi:

Faraz qilaylik, ushbu

$$x_i'(t) = f_i(t, x_1, x_2, ..., x_n), \quad t \in [a,b], \quad x_i(t_0) = x_{i0}, \; i = 1,2, ..., n$$

oddiy differensial tenglamalar sistemasi va boshlang'ich shartlar bilan berilgan masalaning taqribiy yechimini h - teng qadamli ushbu

$$\omega_h = \{t_j = a + jh, \; j = 0,1,..., N\}, \quad h = (b-a)/N$$

to'r tugunlarida topish talab etilsin. To'rning j nomerli nuqtasida masalaning aniq yechimini $x_{i,j} = x_{i,j}(t_j)$ orqali, taqribiy yechimini esa $y_{i,j}$ orqali belgilaylik.

Runge-Kutta usulining o'zgarmas h qadam bilan 4-tartibli aniqlikdagi taqribiy hisob formulasi quyidagicha:

$$y_{i,j+1} = y_{i,j} + h(k_{1,j} + 2k_{2,j} + 2k_{3,j} + k_{4,j})$$

bu yerda

$$k_{1,j} = f(t_j, y_{i,j}),$$

$$k_{2,j} = f(t_j + \frac{h}{2}, y_{i,j} + \frac{h}{2}k_{1,j}),$$

$$k_{3,j} = f(t_j + \frac{h}{2}, y_{i,j} + \frac{h}{2}k_{2,j}),$$

$$k_{4,j} = f(t_j + h, y_{i,j} + hk_{3,j}),$$

Runge-Kutta usulining qo'llanilish dasturi matematik paketda ko'zda tutilgan, shuning uchun undan dasturda ko'rsatilgan tartibda foydalanilgan.

1.2. Oddiy differensial tenglamalar sistemasining umumiy va xususiy yechimini Maple dasturida topish

Oddiy differensial tenglamalar sistemasini Maple dasturida yechish. Differensial tenglamalar sistemasining umumiy yechimini **dsolve** komanda bilan topish mumkin, agarda unda quyidagilar ko'rsatilsa:

dsolve({sys},{x(t),y(t),…}),

bu yerda **sys** – differensial tenglamalar sistemasi, **x(t),y(t),…** – noaniq funksiyalar ketma – ketligi.

Differensial tenglamalar sistemasining yoki Koshi masalasining xususiy yechimini **dsolve** komanda bilan topish mumkin, agarda unda quyidagilar ko'rsatilsa:

dsolve({sys, ic},{x(t),y(t),…},extra_args),

bu yerda **sys** – differensial tenglamalar sistemasi, **x(t),y(t),…** – noaniq funksiyalar ketma – ketligi; **ic** – boshlang'ich yoki chegaraviy shartlar; **extra_args** – masalani yechish usulini aniqlovchi parametr, masalan, masala sonli yechilsa, u holda u **type=numeric** deb yoziladi.

Muammoni oydinlashtirishni mashqlarda bajarib ko'raylik va quyidagi tadbiqlarni bajaraylik:

1-misol. Quyidagi differensial tenglamalar sistemasining umumiy yechimini toping:

$$\begin{cases} x' = -4x - 2y + \dfrac{2}{e^t - 1}, \\ y' = 6x + 3y - \dfrac{3}{e^t - 1} \end{cases}$$

Yechish. Berilgan differensial tenglamalar sistemasining umumiy yechimini Maple paketi yordamida analitik usulda topamiz:

Berilgan differensial tenglamalar sistemasini tuzish quyidagicha:

```
> sys:=diff(x(t),t)=-4*x(t)-2*y(t)+2/(exp(t)-1),
     diff(y(t),t)=6*x(t)+3*y(t)-3/(exp(t)-1);
```

$$sys := \frac{\partial}{\partial t}\,x(t) = -4\,x(t) - 2\,y(t) + \frac{2}{e^{t}-1},\ \frac{\partial}{\partial t}\,y(t) = 6\,x(t) + 3\,y(t) - \frac{3}{e^{t}-1}$$

Berilgan sistemaning umumiy yechimi quyidagicha:

```
> dsolve({sys},{x(t),y(t)});
```

$$\{y(t) = -2\,\_C2 - 3\,e^{(-t)} - 3\,e^{(-t)}\ln(e^{t}-1) + \frac{3}{2}\,e^{(-t)}\,\_C1,$$

$$x(t) = \frac{2 + 2\ln(e^{t}-1) - \_C1}{e^{t}} + \_C2\}$$

Bu yerda $\_C1$ va $\_C2$ ixtiyoriy o'zgarmaslarga bog'liq bo'lgan $x(t)$ va $y(t)$ funksiyalar topilgan.

2-misol. Ushbu

$$\begin{cases} x' = y, \\ y' = -4y - 13x + e^{\sin t} \end{cases}$$

differensial tenglamalar sistemasining yechimini quyidagi

$$x(0,25)=-1,\ y(0,25)=1.$$

boshlang'ich shartlarda toping:

Yechish. Berilgan differensial tenglamalar sistemasining ko'rsatilgan boshlang'ich shartlardagi xususiy yechimini Maple paketi yordamida analitik usulda topamiz:

Tenglamalar sistemasi va boshlang'ich shartlarning berilishi quyidagicha:

```
> Eq2:=diff(x(t),t)=y(t), diff(y(t),t)=-4*y(t)-13*x(t)+exp(t);
init_2:=y(0,25)=1,x(0,25)=-1;
```

$$Eq2 := \frac{\partial}{\partial t}\,x(t) = y(t),\ \frac{\partial}{\partial t}\,y(t) = -4\,y(t) - 13\,x(t) + e^{t}$$

$$init\_2 := y(0, 25) = 1,\ x(0, 25) = -1$$

Berilgan chegaraviy masalaning yechimi quyidagicha:

```
>dsolve({Eq2,init_2},{x(t),y(t)});
```

$$\{ y(t) = \frac{71}{18} e^{(-2t)} \sin(3t) + \frac{17}{18} e^{(-2t)} \cos(3t) + \frac{1}{18} e^{t},$$
$$x(t) = -\frac{7}{18} e^{(-2t)} \sin(3t) - \frac{19}{18} e^{(-2t)} \cos(3t) + \frac{1}{18} e^{t} \}$$

3-misol. Ushbu

$$\begin{cases} \dfrac{dx}{dt} = x + 3y, \\ \dfrac{dy}{dt} = -x + 5y \end{cases}$$

differensial tenglamalar sistemasining $y(x)$ yechimini quyidagi

$$x(0)=3,\ y(0)=1.$$

boshlang'ich shartlarda toping. $y(x)$ yechimning grafigini quring.

Yechish. Berilgan differensial tenglamalar sistemasining ko'rsatilgan boshlang'ich shartlardagi $y(x)$ xususiy yechimini Maple paketi yordamida analitik usulda topamiz:

Tenglamalar sistemasi va boshlang'ich shartlarning berilishi quyidagicha:

```
> Eq2:=diff(x(t),t)=x(t)+3*y(t),  diff(y(t),t)=-x(t)+5*y(t);
init_2:=x(0)=3,y(0)=1;
```

$$Eq2 := \frac{\partial}{\partial t} x(t) = x(t) + 3\,y(t),\ \frac{\partial}{\partial t} y(t) = -x(t) + 5\,y(t)$$

$$init\_2 := x(0) = 3,\ y(0) = 1$$

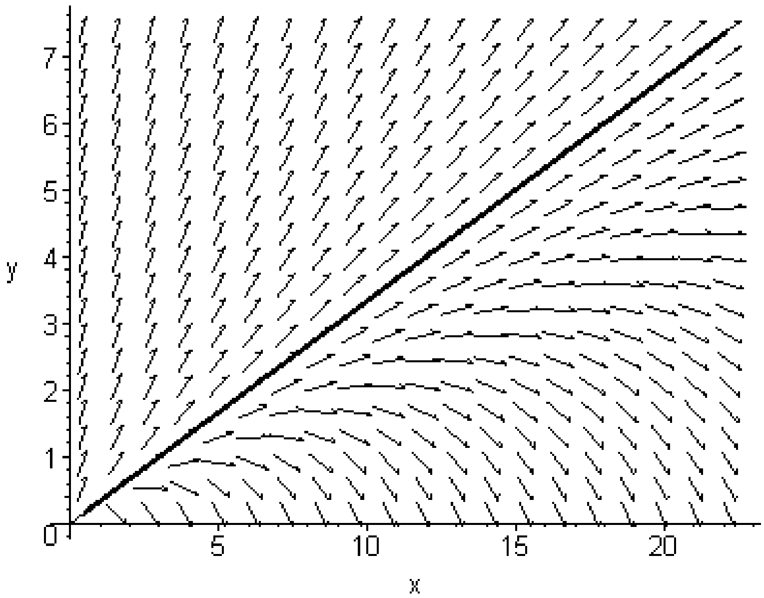
Berilgan chegaraviy masalaning yechimi quyidagicha:

```
> dsolve({Eq2,init_2},{x(t),y(t)});
```

$$\{ x(t) = 3\,\mathbf{e}^{(2\,t)},\ y(t) = \mathbf{e}^{(2\,t)} \}$$

Izlanayothan $y(x)$ integral egri chiziqning grafigi quyidagicha chiziladi:

```
> with(DEtools):
DEplot({Eq2},[x(t),y(t)],t=-1..1,[[init_2]], linecolor=black, stepsize=0.05,
color=black);
```



1-rasm. Izlanayothan $y(x)$ integral egri chiziqning grafigi.

1.3. Oddiy differensial tenglamalar sistemasining umumiy va xususiy yechimini Mathcad dasturida topish

Mathcad dasturida oddiy differensial tenglamalar sistemasini yechish uchun **Given** blokiga tegishli **odesolve** funksiyasi mavjud bo'lib, u quyidagicha yoziladi:

$$y = \text{odesolve}(x, t, b)$$

bunda x – integrallanuvchi tenglamalar; t – integrallash o'zgaruvchilari; b – integrallash intervalining oxirgi nuqtasi; boshlang'ich shartlar quyidgicha ifodalanadi:

$$x(a)=x_0 \text{ va } y(a)=y_0$$

Oddiy differensial tenglamalar sistemasining yechimi $[a,b]$ kesmada aniqlangan y funksiyalar ko'rinishida tiklanadi.

1-misol. Ushbu

$$\frac{dx}{dt} = \cos(x(t)y(t)),$$
$$\frac{dy}{dt} = \sin(x(t) + ty(t))$$

oddiy differensial tenglamalar sistemasini $x(0)=0$ va $y(0)=0$ boshlang'ich shart uchun [0;10] intervalda Mathcad paketi yordamida yeching.

Yechish.

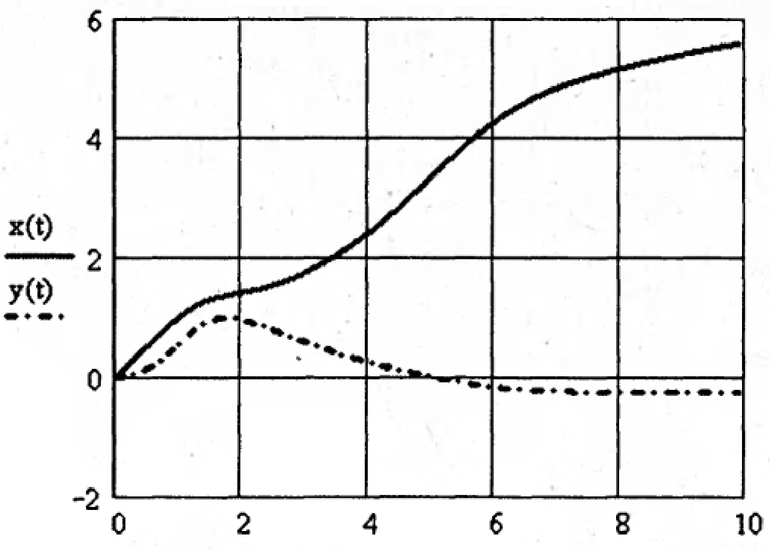
a:=0; b:= 10 ;

Given

$$x'(t) = \cos(x(t)\cdot y(t)) \qquad y'(t) = \sin(x(t) + t\cdot y(t))$$

$$x(a) = 0 \qquad y(a) = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} x \\ y \end{pmatrix}, t, b\right]$$



2-BOB.
ODDIY DIFFERENSIAL TENGLAMALAR SISTEMASINI MAPLE VA MATHCAD DASTURLARI YORDAMIDA SONLI YECHISH

2.1. Oddiy differensial tenglamalar sistemasini Maple dasturidagi dsolve komandasi yordamida sonli yechish va uning yechimi grafigini odeplot komandasi yordamida qurish

1-Misol. Oddiy differensial tenglamalar sistemasi bilan berilgan quyidagi Koshi masalasining analitik va sonli yechimlarini toping:

$$x'(t)=y(t),$$
$$5y'(t)+13x(t)=e^t,$$
$$x(0{,}25)=-1,\ y(0{,}25)=1.$$

Yechish: a) Bu chegaraviy masalani analitik yechish dasturi va uning natijalari quyidagicha:

```
> Eq2:=diff(x(t),t)=y(t),5*diff(y(t),t)+13*x(t)=exp(t);
iq_2:=x(0,25)=-1,y(0,25)=1;  dsolve({Eq2,iq_2},{x(t),y(t)});
```

$$Eq2 := \frac{\partial}{\partial t}\mathrm{x}(t)=\mathrm{y}(t),\ 5\left(\frac{\partial}{\partial t}\mathrm{y}(t)\right)+13\,\mathrm{x}(t)=\mathbf{e}^t$$

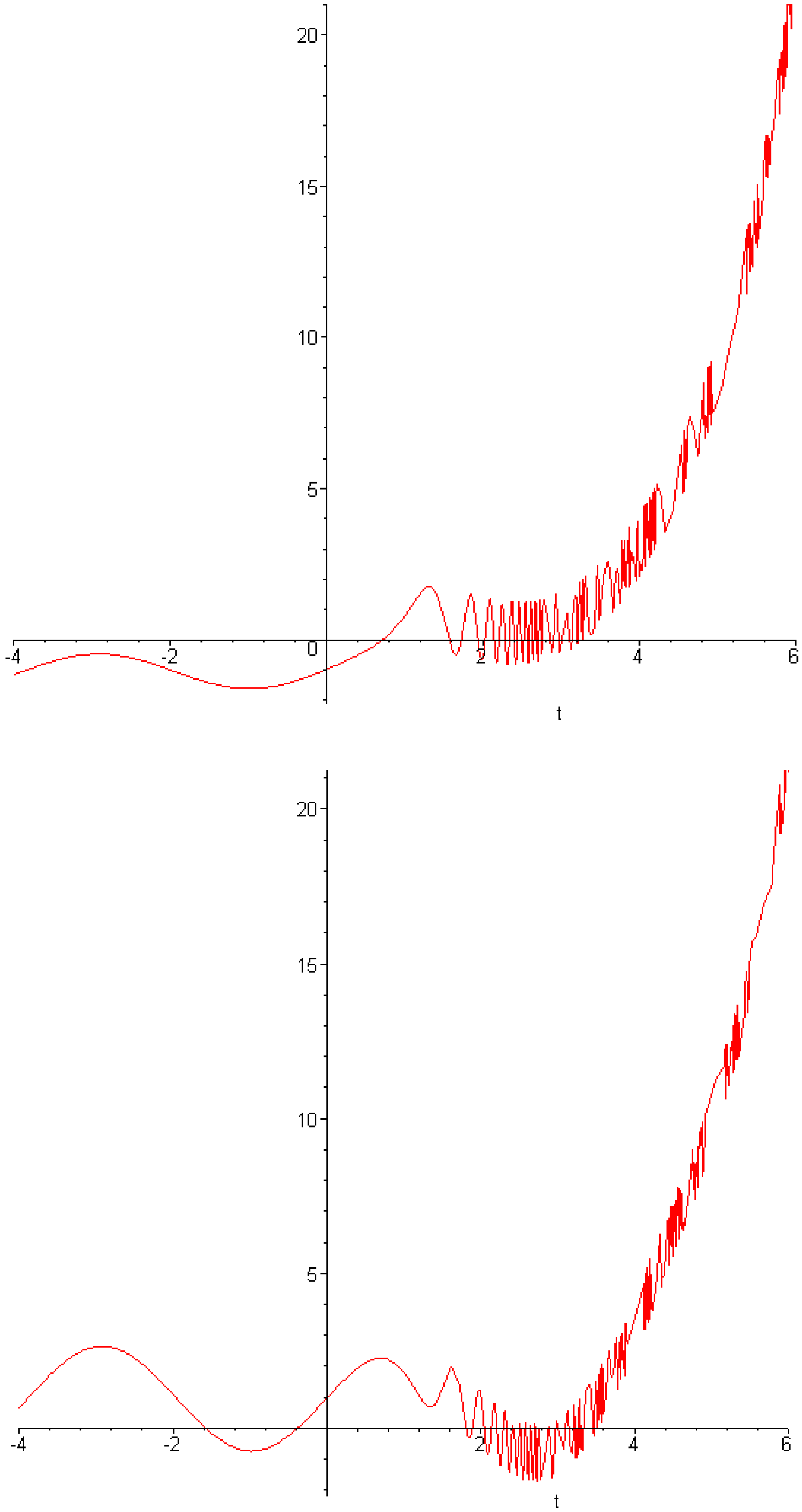
$$iq\_2 := \mathrm{x}(0,25)=-1,\ \mathrm{y}(0,25)=1$$

$$\{\mathrm{y}(t)=\frac{17}{18}\cos\left(\frac{1}{5}\sqrt{65}\,t\right)+\frac{19}{90}\sin\left(\frac{1}{5}\sqrt{65}\,t\right)\sqrt{65}+\frac{1}{18}\mathbf{e}^t,$$
$$\mathrm{x}(t)=\frac{17}{234}\sin\left(\frac{1}{5}\sqrt{65}\,t\right)\sqrt{65}-\frac{19}{18}\cos\left(\frac{1}{5}\sqrt{65}\,t\right)+\frac{1}{18}\mathbf{e}^t\}$$

Endi bu funksiyalarning grafiklarini chiqaraylik (2.1-rasm):

```
> with(plots):
plot((17/234)*sqrt(65)*sin(sqrt(65)*t/5)-(19/18)*cos(sqrt(65)^t/5)+exp(t)/18,t=-
4..6);
```

```
plot((19/90)*sqrt(65)*sin(sqrt(65)*t/5)+(17/18)*cos(sqrt(65)^t/5)+exp(t)/18,t=-4..6);
```



2.1-rasm. Izlanayotgan masala yechimining grafiklari.

b) Bu chegaraviy masalani sonli yechish dasturi va uning natijalari quyidagicha:

```
restart; cond := x(0.25) =-1, y(0.25) = 1;
 sys := diff (x(t), t) = y(t),
diff (y(t), t) = 0.2·(-13·x(t) + exp(t));
 F := dsolve({sys, cond}, [x(t), y(t)], numeric) :
 F(0.25); F(1); F(2);
with(plots) :
 p1 := odeplot(F, [t, x(t)],-4..6, color = black, thickness = 2,
       linestyle = 4) :
 p2 := odeplot(F, [t, y(t)],-4..6, color = green, thickness = 2) :
 p3 := textplot([4, 10, "x(t)"], font = [TIMES, ITALIC, 12]) :
 p4 := textplot([6, 7, "y(t)"], font = [TIMES, ITALIC, 12]) :
 display(p1, p2, p3, p4);
```

Chegaraviy masalaning berilishi:

$$x(0.25) = -1, y(0.25) = 1$$

$$\frac{d}{dt} x(t) = y(t), \frac{d}{dt} y(t) = -2.6\, x(t) + 0.2\, e^{t}$$

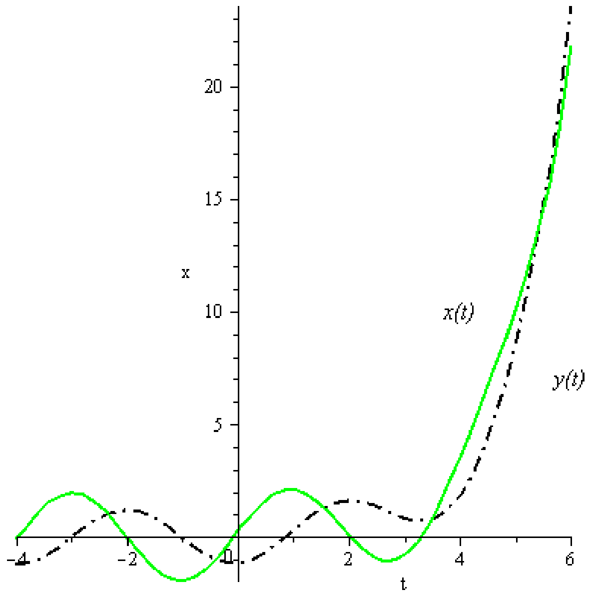
Izlanayotgan yechimning kerakli nuqtalardagi qiymati:

$$[t = 0.250000000000000000, x(t) = -1., y(t) = 1.]$$

$$[t = 1., x(t) = 0.310869154159726435, y(t) = 2.09527725645919460]$$

$$[t = 2., x(t) = 1.60858046024177370, y(t) = 0.0720054729515650220]$$

Natijalarning grafik shakldа ifodasi (2.2-rasm), buni yuqoridagi natija bilan taqqoslash mumkin.



2.1-rasm. Chegaraviy masala sonli yechimining grafigi.

2-Misol. Oddiy differensial tenglamalar sistemasi bilan berilgan quyidagi Koshi masalasining sonli yechimlari grafiklarini quring:

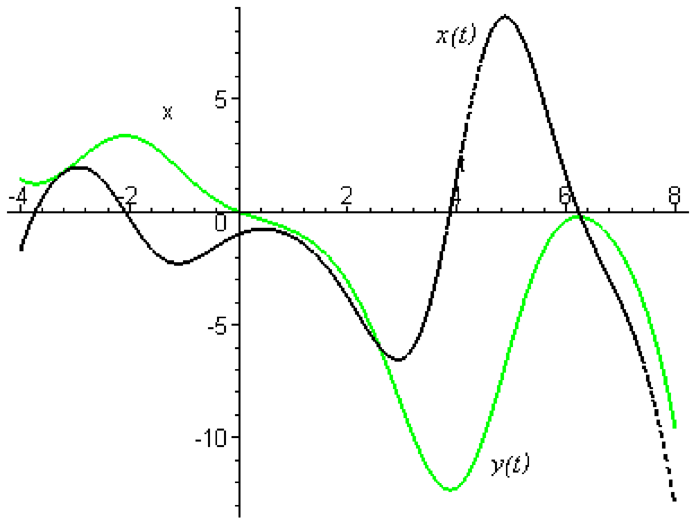
$$x'(t)=2y(t)\sin(t)-x(t)-t,$$

$$y'(t)=x(t),$$

$$x(0)=-1,\ y(0)=0.$$

Yechish: Bu chegaraviy masalani yechish dasturi va uning natijalari quyidagicha (2.2-rasm):

```
> restart; cond:=x(0)=-1,y(0)=0:
 sys:=diff(x(t),t)=2*y(t)*sin(t)-x(t)-t,
diff(y(t),t)=x(t):
 F:=dsolve({sys,cond},[x(t),y(t)], numeric):
 with(plots):
 p1:=odeplot(F,[t,x(t)],-4..8, color=black, thickness=2, linestyle=4):
 p2:=odeplot(F,[t,y(t)],-4..8, color=green, thickness=2):
 p3:=textplot([4,8,"x(t)"], font=[TIMES,ITALIC, 12]):
 p4:=textplot([5,-11,"y(t)"], font=[TIMES,ITALIC, 12]):
 display(p1,p2,p3,p4);
```



2.2-rasm. Chegaraviy masala sonli yechimining grafigi.

2.2. Oddiy differensial tenglamalar sistemasining fazoviy portretini qurish

DEplot komandasi birinchi tartibdan yuqori tartibli differensial tenglamalarning faqat yechimlari egri chizig'ini chizadi, birinchi tartibli differensial tenglamalar sistemasi uchun esa fazoviy portretlarini chizadi.

DEplot komandasi yordamida, agar uning **scene=[x,y]** parametri ko'rsatilgan bo'lsa, ushbu ikkita $\frac{dx}{dt}=f(x,y,t)$, $\frac{dy}{dt}=g(x,y,t)$ differensial tenglamalar sistemasining (x, y) tekislikdagi fazoviy portretini chizish mumkin.

Agar differensial tenglamalar sistemasi avtonom bo'lsa, u holda fazoviy portret strelkalar bilan ko'rsatilgan yo'nalishlar maydonida quriladi. Strelkalarning o'lchamlari ushbu **arrows**=**SMALL**, **MEDIUM**, **LARGE**, **LINE** или **NONE** parametrlar bilan boshqariladi.

Barcha fazoviy portretni chizish uchun har bir fazoviy traektoriyaning boshlang'ich shartini ko'rsatish lozim: masalan, birinchi tartibli ikkita differensial tenglamalar sistemasi uchun bir nechta boshlang'ich shartlar **DEplots** komandada ***t*** erkli o'zgaruvchining o'zgarish diapazonidan keyin ko'rsatiladi:

[[x(0)=x1, y(0)=y1], [x(0)=x2, y(0)=y2],…, [x(0)=xn, y(0)=yn]].

Boshlang'ich shartlarni juda ixcham ko'rinishda berish mumkin:

[t0, x0, y0],

bu yerda **t0** – boshlang'ich shartlar berilgan nuqta; **x0** и **y0** – berilgan **t0** nuqtada izlanayotgan funksiyalarning qiymatlari.

Birinchi tartibli ikkita differensial tenglamalar sistemasi uchun fazoviy portretni

phaseportrait(sys, [x,y],x1..x2,[[cond]]),

komanda yordamida ham qurish mumkin, bunda **sys** – birinchi tartibli ikkita differensial tenglamalar sistemasi; **[x,y]** – izlanayotgan funksiyalar nomi; **x1..x2** – fazoviy portret chiziladigan interval; figurali qavslarda boshlang'ich shartlar ko'rsatiladi. Bu komanda **DEtools** paketda joylashgan bo'lib, avvalo bu paket yuklangan bo'lishi zarur.

1-misol. Ushbu

$$\begin{cases} x' = y \\ y' = x - x^3 \end{cases}$$

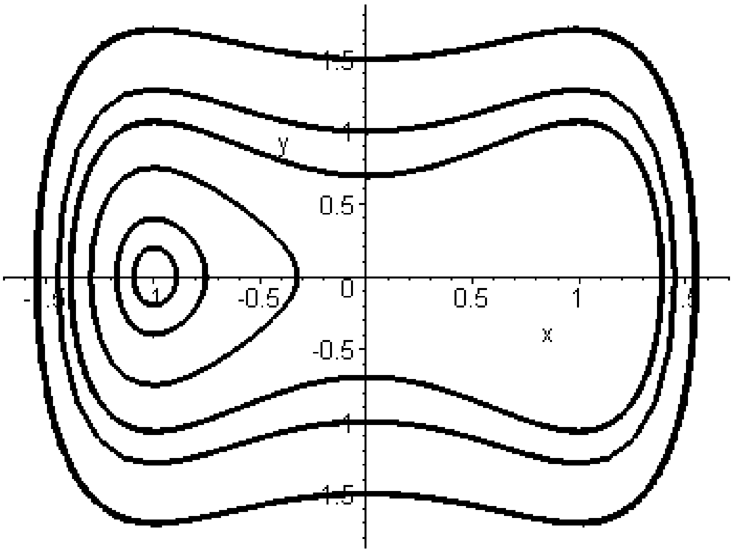
differensial tenglamalar sistemasining fazoviy portretlarini quyidagi har xil boshlanchish shartlar uchun quring:

- $x(0)=1$, $y(0)=0.2$;
- $x(0)=0$, $y(0)=1$;
- $x(0)=1$, $y(0)=0.4$;
- $x(0)=1$, $y(0)=0.75$;
- $x(0)=0$, $y(0)=1.5$;
- $x(0)=-0.1$, $y(0)=0.7$.

Yechish: differensial tenglamalar sistemasining fazoviy portretlarini bitta grafikda ifodalaylik (2.3-rasm):

```
> restart;  with(DEtools):
 DEplot({diff(x(t),t)=y, diff(y(t),t)=x-x^5},
[x(t),y(t)], t=0..20, [[0,-1,0.2], [0,0,1],
[0,-1,0.4], [0,-1,0.75], [0,-0,1.5], [0,-0.1,0.7]],
```

stepsize=0.1, arrows=none, linecolor=black);



2.3-rasm. Differensial tenglamalar sistemasining fazoviy portretlari.

2-misil. Ushbu

$$\begin{cases} x' = y \\ y' = \sin x \end{cases}$$

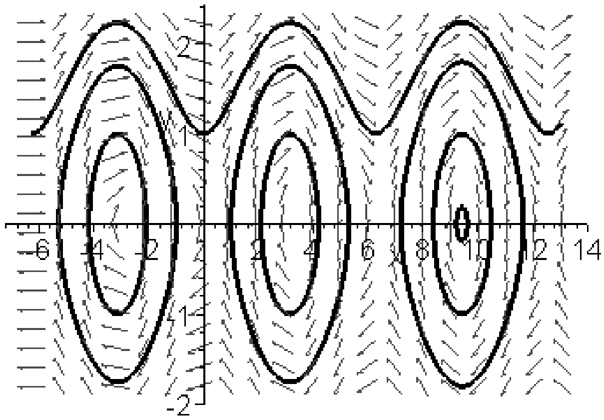
avtonom sistemaning fazoviy portretlarini quyidagi har xil boshlang'ich shartlarda yo'nalishlar maydoni bilan tasvirlang:

- $x(0)=1$, $y(0)=0$;
- $x(0)=-1$, $y(0)=0$;
- $x(0)=\pi$, $y(0)=1$;
- $x(0)=-\pi$, $y(0)=1$;
- $x(0)=3\pi$, $y(0)=0.2$;
- $x(0)=3\pi$, $y(0)=1$;
- $x(0)=3\pi$, $y(0)=1.8$;
- $x(0)=-2\pi$, $y(0)=1$;.

Yechish: differensial tenglamalar sistemasining fazoviy portretlarini bitta grafikda yo'nalishlar maydoni bilan tasvirlaylik (2.4-rasm):

```
> restart; with(DEtools):
```

```
> sys:=diff(x(t),t)=y, diff(y(t),t)=sin(x):
> DEplot({sys},[x(t),y(t)], t=0..4*Pi, [[0,1,0],
  [0,-1,0], [0,Pi,1], [0,-Pi,1], [0,3*Pi,0.2],
  [0,3*Pi,1], [0,3*Pi,1.8], [0,-2*Pi,1]],
  stepsize=0.1, linecolor=black);
```



2.4-rasm. Differensial tenglamalar sistemasining fazoviy portretlari.

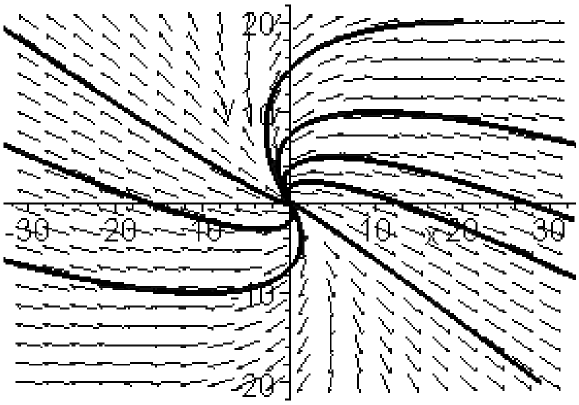
3-misol. Ushbu

$$\begin{cases} x' = 3x + y \\ y' = y - x \end{cases}$$

differensial tenglamalar sistemasining fazoviy portretlarini quring. Boshlang'ich shartlarni, o'zgaruvchilarning o'zgarish intervalini va koordinat o'qlarining o'lchamlarini fazoviy portretning xarakteridan kelib chiqib, mustaqil tanlansin.

Yechish. Masalani yechish dasturi quyidagicha, uning natijalari esa 2.5-rasmda tasvirlangan:

```
> restart; with(DEtools):
> sys:=diff(x(t),t)=3*x+y, diff(y(t),t)=-x+y:
> phaseportrait([sys],[x(t),y(t)],t=-10..10,
[[0,1,-2], [0,-3,-3], [0,-2,4], [0,5,5], [0,5,-3],
[0,-5,2], [0,5,2], [0,-1,2]], x=-30..30,y=-20..20,
stepsize=.1, colour=blue,linecolor=black);
```



2.5-rasm. Differensial tenglamalar sistemasining fazoviy portreti.

4-misol. Ushbu

$$\begin{cases} x' = -y \\ y' = -x \end{cases}$$

differensial tenglamalar sistemasining fazoviy portretlarini ushbu

x(0)=5, y(0)=10

boshlang'ich shartlarda quring. Boshlang'ich shartlarni, o'zgaruvchilarning o'zgarish intervalini va koordinat o'qlarining o'lchamlarini fazoviy portretning xarakteridan kelib chiqib, mustaqil tanlansin.

Yechish. Masalani yechish dasturi quyidagicha, uning natijalari esa 2.6-rasmda tasvirlangan:

```
> restart;
> with(DEtools):
> sys:=diff(x(t),t)=-y(t), diff(y(t),t)=-x(t);
```

$$sys := \frac{d}{dt}\,\mathrm{x}(t) = -\mathrm{y}(t),\ \frac{d}{dt}\,\mathrm{y}(t) = -\mathrm{x}(t)$$

```
> cond:=y(0)=10, x(0)=5;
```

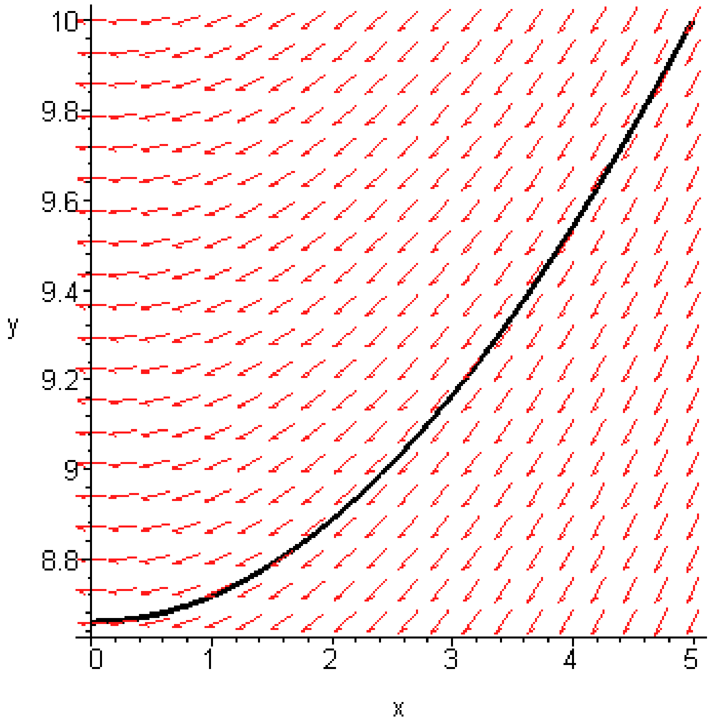
$$cond := \mathrm{y}(0) = 10,\ \mathrm{x}(0) = 5$$

```
> dsolve({sys, cond}, [y(t),x(t)]);
```

$$\{\mathrm{x}(t) = \frac{15}{2}\,\mathbf{e}^{(-t)} - \frac{5}{2}\,\mathbf{e}^{t},\ \mathrm{y}(t) = \frac{15}{2}\,\mathbf{e}^{(-t)} + \frac{5}{2}\,\mathbf{e}^{t}\}$$

```
> ini:=[[x(0)=5,y(0)=10]]:
```

```
> phaseportrait([sys],[x(t),y(t)],t=0..ln(3)/2,  ini,stepsize=0.5,  linecolor=black);
```



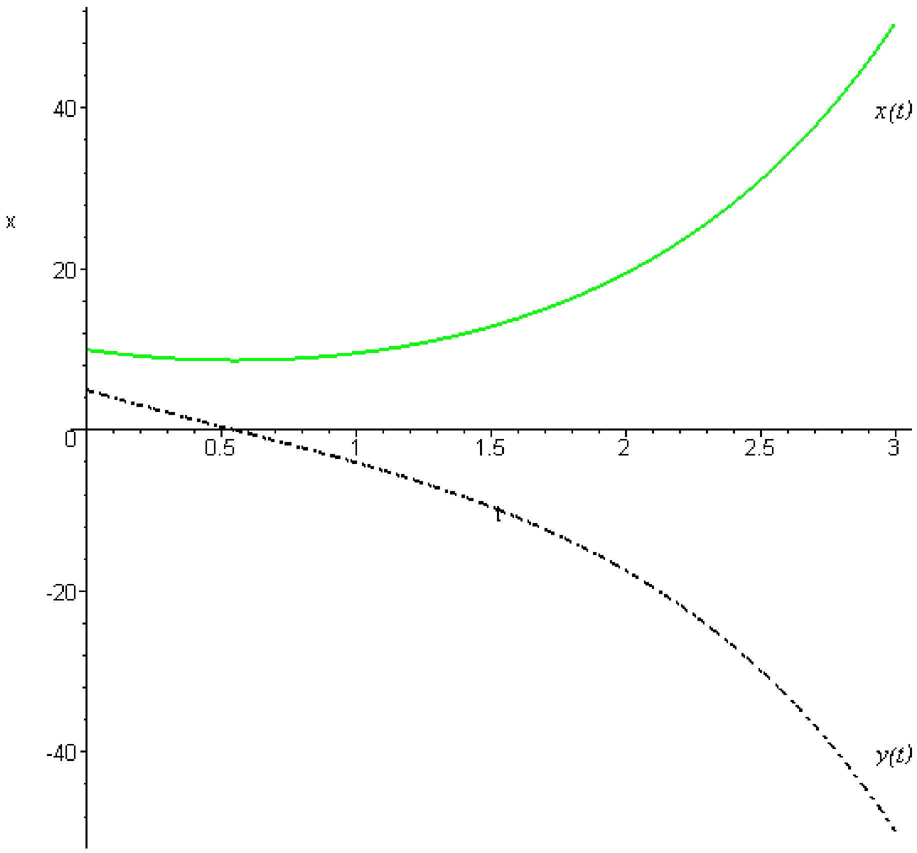
2.6-rasm. Differensial tenglamalar sistemasining fazoviy portreti.

Bu sistemaning sonly yechimi quyidagi natijalarni beradi (2.7-rasm):

```
> restart; cond:=x(0)=5,y(0)=10;
 sys:=diff(x(t),t)=-y(t),diff(y(t),t)=-x(t);
 F:=dsolve({sys,cond},[x(t),y(t)], numeric):
 with(plots):
 p1:=odeplot(F,[t,x(t)],0..3, color=black, thickness=2, linestyle=4):
 p2:=odeplot(F,[t,y(t)],0..3, color=green, thickness=2):
 p3:=textplot([3,40,"x(t)"],  font=[TIMES,ITALIC, 12]):
 p4:=textplot([3,-40,"y(t)"],  font=[TIMES,ITALIC, 12]):
 display(p1,p2,p3,p4);
```

$$cond := x(0) = 5,\ y(0) = 10$$

$$sys := \frac{d}{dt} x(t) = -y(t),\ \frac{d}{dt} y(t) = -x(t)$$



2.7-rasm. Differensial tenglamalar sistemasi sonli yechimining grafigi.

2.3. Oddiy differensial tenglamalar sistemasini Mathcad dasturi yordamida sonli yechish

Birinchi tartibli hosilaga nisbatan yechilgan oddiy differensial tenglamalar sistemasini yechish uchun rkfixed funksiyadan foydalaniladi, bu funksiya yozilishining umumiy ko'rinishi quyidagicha:

rkfixed(y, x1, x2, npoints, D)

bu yerda *y* – boshlang'ich shartlar vektori; [*x*1, *x*2] – integrallah intervali; npoints – hisoblanadigan nuqtalar soni (boshlang'ich nuqta bunga kirmaydi); D – vektor (tenglamalar sistemasi o'ng tomonining vektor-funksiyasi).

1-misol. Ushbu

$$\frac{dy_1}{dx} = y_1 + xy_2,$$
$$\frac{dy_2}{dx} = x - y_1 - y_2$$

oddiy differensial tenglamani $y_1(-3)=1$ va $y_2(-3)=-1$ boshlang'ich shartlar uchun [-3;6] intervalda Mathcad paketi yordamida Runge-Kutta usuli bilan taqribiy yeching.

Yechish.

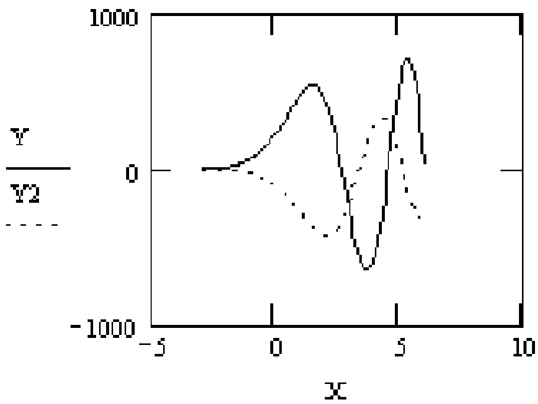
ORIGIN := 1

$$y := \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$D(x,y) := \begin{pmatrix} y_1 + x \cdot y_2 \\ x - y_1 - y_2 \end{pmatrix}$$

Z := rkfixed(y,−3,6,100,D)

$X := Z^{\langle 1 \rangle}$ $\quad Y := Z^{\langle 2 \rangle}$ $\quad Y2 := Z^{\langle 3 \rangle}$



2-misol. Ushbu

$$y'' + y' + y = 0$$

oddiy differensial tenglamani $y(0)=1$ va $y'(0)=0,5$ boshlang'ich shartlar uchun Mathcad paketi yordamida Runge-Kutta usuli bilan taqribiy yeching.

Yechish. Avvalo bu tenglamani $y_0 = y$ va $y_1 = y'$ almashtirish olib, uni quyidagi

$$\frac{dy_0}{dx} = y_1,$$
$$\frac{dy_1}{dx} = -y_1 - y_0$$

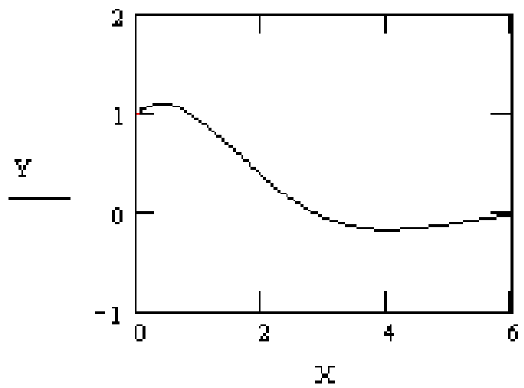
oddiy differensial tenglamalar sistemasiga keltiramiz va keyin ularni $y_0(0)=1$ va $y_1(0)=0{,}5$ boshlang'ich shartlar uchun Mathcad paketi yordamida Runge-Kutta usuli bilan taqribiy yechamiz:

$$y := \begin{pmatrix} 1 \\ .5 \end{pmatrix}$$

$$D(x, y) := \begin{pmatrix} y_1 \\ -y_1 - y_0 \end{pmatrix}$$

$$Z := \mathrm{rkfixed}(y, 0, 6, 100, D)$$

$$X := Z^{\langle 0 \rangle} \qquad Y := Z^{\langle 1 \rangle}$$



$$X_{50} = 3 \qquad Y_{50} = -0.058$$

$$X_{100} = 6 \qquad Y_{100} = -0.028$$

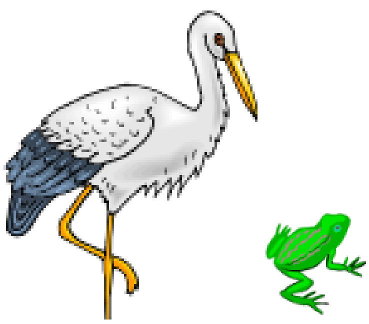
3. BIOLOGIK JARAYONLARGA OID BA'ZI AMALIY MASALALARNING DIFFERENSIAL TENGLAMAL SISTEMASIINI TAQRIBIY YECHISH

Ko'pgina biologik yoki kimyiviyk jarayonlar oddiy differensial tenglamalar sistemasini yechishga oid chegaraviy masalalarga olib kelinadi. Bunday jarayonlarning matematik modelini qurish avvaldan ma'lum deb, quyidagi bir qator yirtqich – o'lja turning dinamik ko'payishi haqidagi modellarini, ularga oid jarayonlar oddiy differensial tenglamalar sistemasini berilgan boshlang'ich shartlardagi yechimini matematik paketlar yordamida taqribiy topamiz.

3.1. Yirtqich – o'lja turi bo'yicha Bazikin modeli

Masalaning qo'yilishi. Ko'payishi o'zaro ta'sir ostida bo'lgan ikki biologik yirtqich – o'lja turning dinamik ko'payishi haqidagi masalani ularning oziq-ovqat manbalari cheklangan va o'ljaning nochiziqli ko'payuvchi tur zichligi kam bo'lgan ichki kurashi sharoitida yeching. Agar $x(t)$ – o'ljaning va $y(t)$ – yirtqichning t vaqt momentidagi ko'payish zichligi deb belgilasak, u holda bu turlarning ko'payish dinamikasini quyidagi differensial tenglamalar sistemasi bilan ifodalash mumkin:

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{ax^2}{N+x}\dfrac{K-x}{K} - bxy, \\ \dfrac{dy}{dt} = -cy + dxy, \end{cases}$$



bu yerda a, b, c, d, N, K – manfiy bo'lmagan sonlar.

Tenglamalarning tuzilishi quyidagicha:

- $\dfrac{ax^2}{N+x}\dfrac{K-x}{K}$ - yirtqichlar bo'lmagan holda o'ljaning ko'payish tezligi. Agar x ning qiymatlari kichik bo'lsa, tezlik $\dfrac{ax}{N}$ (giperbolik qonuniyat) miqdordan aniqlanadi. Zichlikning katta qiymatlarida (K miqdorgacha) ko'payish o'zadi, $x > K$ larda esa kamayadi (tezlik manfiy bo'ladi). Shunday qilib, bu had manbalarning cheklanganligini ifodalaydi: K dan kichik bo'lgan zichlikdagina atrof muhit turning ko'payishini ta'minlaydi.

- bxy – yirtqichning o'lja ko'payishiga ta'sirini ifodalaydi. Bunda bx – birlik vaqt ichida bitta yirtqich tomonidan o'ldirilgan o'ljalar sonini ifodalaydi. Bu esa modeldagi yirtqichlarning juda ashaddiy qonxo'r ekanligini bildiradi.
- Ikkinchi tenglama yirtqichlarning ko'payishi o'zgarishini ifodalaydi. Tenglamadagi o'zgarmas yirtqichlarning tabbiy holdagi o'limini ifodalaydi. Ikkinchi had dx – yirtqichlarning o'lja zichligidan bog'liq holda ko'payishini ifodalaydi. Bu esa model bo'yicha yirtqichlarning ko'payishi juda ham jadallik bilan borishini bildiradi.

Berilgan tenglamalar sistemasini o'lchamsiz holatga keltiramiz, buning uchun quyidagi o'lchamsiz miqdorlarni kiritamiz:

$$X = \frac{x}{K}; \quad Y = \left(\frac{b}{a}\right) y; \quad \tau = at; \quad n = \frac{N}{K}; \quad m = \frac{c}{dK}; \quad \gamma = \frac{dK}{a}.$$

Natijada berilgan sistema quyidagi ko'rinishga keladi:

$$\begin{cases} \dfrac{dX}{dt} = \dfrac{(1-X)X^2}{N+X} - XY, \\ \dfrac{dY}{dt} = \gamma(X-m)Y. \end{cases}$$

Bu differensial tenglamalar sistemasini quyidagi boshlang'ich shartlar bilan to'ldiramiz:

$$X(0) = X_0, \qquad Y(0) = Y_0 .$$

Xususiy hol. Quyidagi differensial tenglamalar sistemasini [0;4] kesmada yeching:

$$\begin{cases} \dfrac{dx}{dt} = -\alpha x - \beta y + (\alpha + \beta - 1)e^{-t}, \\ \dfrac{dy}{dt} = \beta x - \alpha y + (\alpha + \beta - 1)e^{-t}, \end{cases}$$

bunda $\alpha = 2$; $\beta = 3$; $x(0) = X_0 = 1$; $y(0) = Y_0 = 1$.

Olingan natijani quyidagi analitik yechim bilan taqqoslang: $x = y = e^{-t}$.

Masalani Maple dasturida analitik va sonli yechish. Chegaraviy masalaning $x(t)$ va $y(x)$ yechim funksiyalari va ularning grafiklari (3.1-rasm):

1) Masalaning analitik yechimi:

$$restart;\ cond := x(0) = 1, y(0) = 1;$$
$$sys := diff(x(t), t) = -2\cdot x(t) - 3\cdot y(t) + 4\cdot \exp(-t),$$
$$diff(y(t), t) = 3\cdot x(t) + 2\cdot y(t) - 6\cdot \exp(-t);$$
$$F := dsolve(\{sys, cond\}, \{x(t), y(t)\});$$

$$x(0) = 1, y(0) = 1$$

$$\frac{d}{dt} x(t) = -2\,x(t) - 3\,y(t) + 4\,e^{-t}, \frac{d}{dt} y(t) = 3\,x(t) + 2\,y(t) - 6\,e^{-t}$$

$$\{x(t) = e^{-t}, y(t) = e^{-t}\}$$

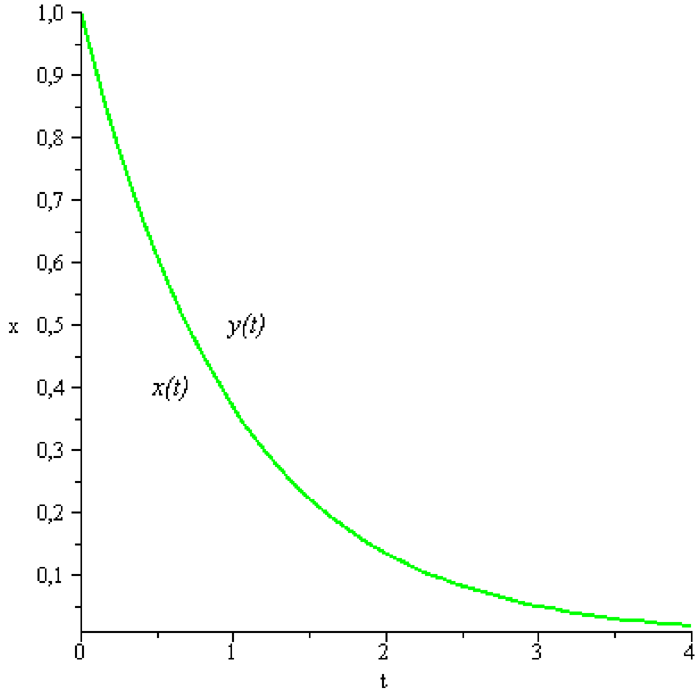
2) Masalaning Runge-Kutta usuli yordamidagi sonli yechimi:

$$diff(y(t), t) = 3\cdot x(t) + 2\cdot y(t) - 6\cdot \exp(-t);$$
$$F := dsolve(\{sys, cond\}, [x(t), y(t)], numeric);$$
$$with(plots):$$
$$p1 := odeplot(F, [t, x(t)], 0..4, color = black, thickness = 2, linestyle = 4):$$
$$p2 := odeplot(F, [t, y(t)], 0..4, color = green, thickness = 2):$$
$$p3 := textplot([0.6, 0.4, \text{"x(t)"}], font = [TIMES, ITALIC, 12]):$$
$$p4 := textplot([1.1, 0.5, \text{"y(t)"}], font = [TIMES, ITALIC, 12]):$$
$$display(p1, p2, p3, p4);$$

$$x(0) = 1, y(0) = 1$$

$$\frac{d}{dt} x(t) = -2\,x(t) - 3\,y(t) + 4\,e^{-t}, \frac{d}{dt} y(t) = 3\,x(t) + 2\,y(t) - 6\,e^{-t}$$

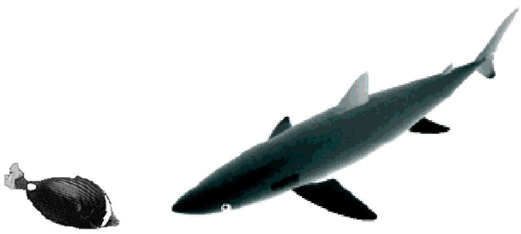
proc(x_rkf45) ... **end proc**



3.1-rasm. Chegaraviy masalaning $x(t)$ va $y(x)$ yechim funksiyalari grafiklari.

3.2. Yirtqich – o‘lja turi bo‘yicha Lotka-Volter modeli

Masalaning qo’yilishi. Yirtqich – o‘lja turi bo‘yicha o‘zaro bog‘langan ikki turning maxsus ko‘payishlar soni *x* va *y*. Bu ko‘payishning vaqt bo‘yicha o‘zgarishi ushbu



$$\begin{cases} \dfrac{dx}{dt} = ax - bxy, \\ \dfrac{dy}{dt} = -cy + dxy, \end{cases}$$

differensial tenglamalar sistemasi (Lotka-Volter modeli) bilan ifodalanadi, bunda

t – vaqt;

ax – o‘ljalarning ko‘payish tezligi;

bxy – o‘ljalarning yirtqichlar bilan to‘qnash kelish chastotasi hisobga olingandagi qirilishi tezligi;

cy – yirtqichlarning qirilish tezligi;

dxy – o‘ljalar mavjud bo‘lganda yirtqichlarning ko‘payish tezligi.

Masalani berilgan *a*, *b*, *c*, *d* parametrlarda va $t = 0$ dagi x_0, y_0 boshlang‘ich shartlarda yeching. Turlarning ko‘payishlari tebranishlari qaytarilishini hisobga olib, shu oraliq uchun yechimni aniqlang. Bularga mos $x(t)$ va $y(x)$ yechim funksiyalarining grafiklarini hamda ularning fazoviy portretlarini chizing.

Xususiy hol. Xususan, ushbu

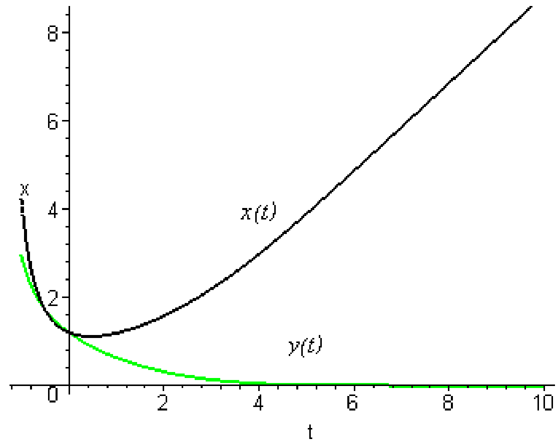
$$\begin{cases} \dfrac{dx}{dt} = 1 - xy, \\ \dfrac{dy}{dt} = -0{,}3y + 0{,}3xy \end{cases}$$

differensial tenglamalar sistemasi bilan berilgan Lotka-Volterr modelining yechimini quyidagi boshlang’ich shartlarda toping:

- x(0)=1,2; y(0)=1,2.
- x(0)=1; y(0)=0,7.

Masalani Maple dasturida sonli yechish. 1) Chegaraviy masalaning $x(t)$ va $y(x)$ yechim funksiyalari va ularning grafiklari (3.2-rasm):

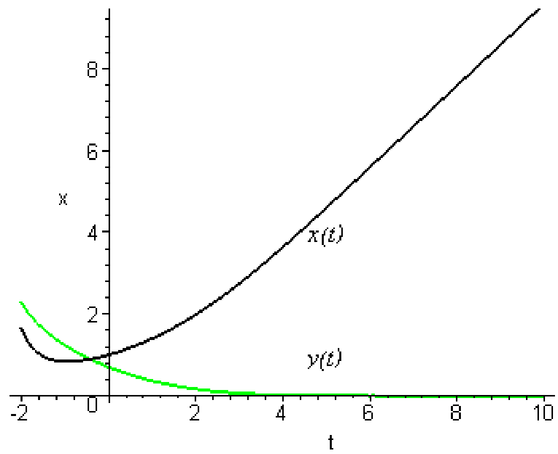
```
> restart; cond:=x(0)=1.2,y(0)=1.2:
 sys:=diff(x(t),t)=1-x(t)*y(t),
diff(y(t),t)=-0.3*y(t)-0.3*x(t)*y(t):
 F:=dsolve({sys,cond},[x(t),y(t)], numeric):
 with(plots):
 p1:=odeplot(F,[t,x(t)],-1..10, color=black, thickness=2, linestyle=4):
 p2:=odeplot(F,[t,y(t)],-1..10, color=green, thickness=2):
 p3:=textplot([4,4,"x(t)"], font=[TIMES,ITALIC, 12]):
 p4:=textplot([5,1,"y(t)"], font=[TIMES,ITALIC, 12]):
 display(p1,p2,p3,p4);
```



3.2-rasm. Chegaraviy masalaning $x(t)$ va $y(x)$ yechim funksiyalari grafiklari.

Xuddi shunday ikkinchi chegaraviy shartlar uchun quyidagi yechimga kelamiz (3.2-rasm):

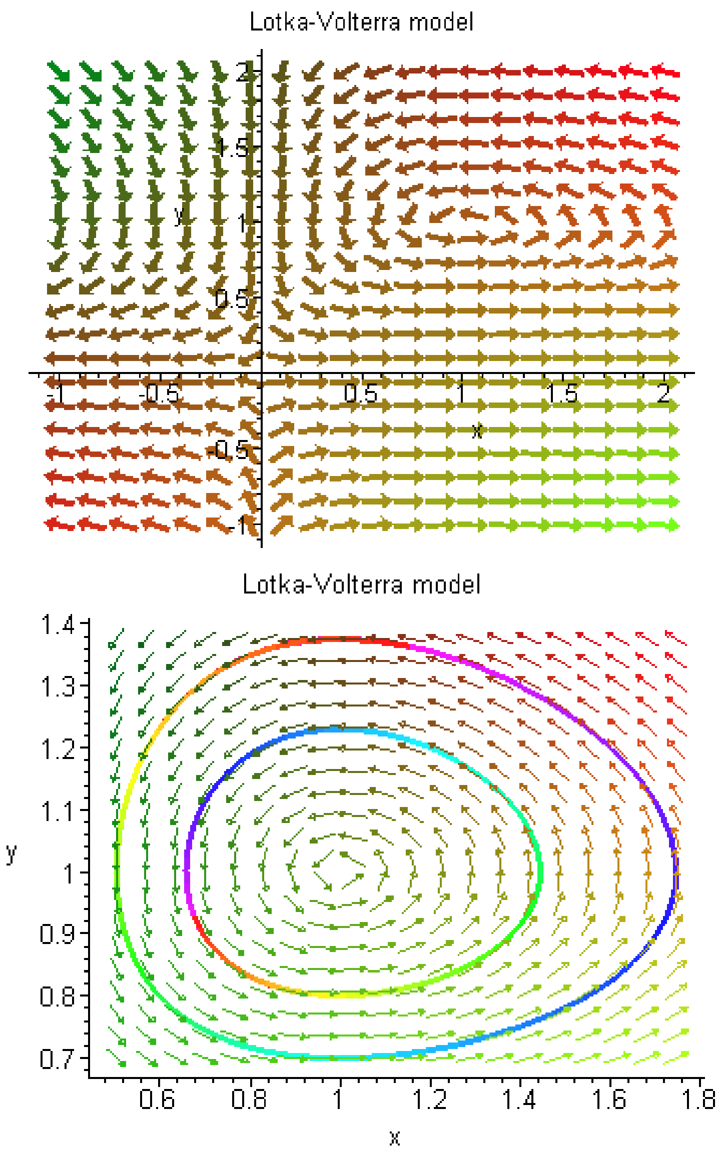
```
restart; cond:=x(0)=1,y(0)=0.7:
sys:=diff(x(t),t)=1-x(t)*y(t),
diff(y(t),t)=-0.3*y(t)-0.3*x(t)*y(t):
F:=dsolve({sys,cond},[x(t),y(t)], numeric):
with(plots):
p1:=odeplot(F,[t,x(t)],-2..10, color=black, thickness=2, linestyle=4):
p2:=odeplot(F,[t,y(t)],-2..10, color=green, thickness=2):
p3:=textplot([5,4,"x(t)"], font=[TIMES,ITALIC, 12]):
p4:=textplot([5,1,"y(t)"], font=[TIMES,ITALIC, 12]):
display(p1,p2,p3,p4);
```



3.2-rasm. Chegaraviy masalaning $x(t)$ va $y(x)$ yechim funksiyalari grafiklari (*xulosa*: bu turlarning ko'payishidagi mutanosiblik faqat t=0 atrofda ekan).

2) Har ikkala hol uchun *x*(*t*) va *y*(*x*) yechim funksiyalarining fazoviy portretlari (3.3-rasm):

```
> DEplot([diff(x(t),t)=x(t)*(1-y(t)),diff(y(t),t)=.3*y(t)*(x(t)-1)],
[x(t),y(t)],t=-2..2,x=-1..2,y=-1..2,arrows=LARGE,
title=`Lotka-Volterra model`,color=[.3*y(t)*(x(t)-1),x(t)*(1-y(t)),.1]);
> DEplot([diff(x(t),t)=x(t)*(1-y(t)),diff(y(t),t)=.3*y(t)*(x(t)-1)],
[x(t),y(t)],t=-7..7,[[x(0)=1.2,y(0)=1.2],[x(0)=1,y(0)=.7]],stepsize=.2,
title=`Lotka-Volterra model`,color=[.3*y(t)*(x(t)-1),x(t)*(1-y(t)),.1],
linecolor=t/2,arrows=MEDIUM,method=rkf45);
```



3.3-rasm. *x*(*t*) va *y*(*x*) yechim funksiyalarining fazoviy portretlari.

3.3. Yirtqich – o‘lja turi bo‘yicha Xolling-Tenner modeli

Masalaning qo’yilishi. Yirtqichlar va o‘ljalarning ko‘payishi ushbu

$$\begin{cases} \dfrac{dx}{dt} = a\left(1 - \dfrac{x}{k1}\right)x - \dfrac{b}{k2 + x}xy, \\ \dfrac{dy}{dt} = \left(c - d\dfrac{y}{x}\right)y, \end{cases}$$

differensial tenglamalar sistemasi (Xolling-Tenner modeli) bilan tavsiflanadi, bunda

x va y – o‘ljalar va yirtqichlarning mos nisbiy soni;

t – vaqt;

ax – o‘ljalarning ko‘payish tezligi;

$ax/k1$ – o‘ljalarning ovqat uchun kurashishini hisobga oluvchi had;

$bxy/(k2+x)$ – yirtqichlar zichligining ko‘payishi bilan o‘ljalarning qirilish tezligi;

cy – yirtqichlarning ko‘payish tezligi;

cdy^2/x – yemish yetishmasligidan yirtqichlarning qirilishii hisobga oluvchi had.

Quyidagi jadvalda keltirilgan parametrlar qiymatlarida va x_0, y_0 boshlang‘ich shartlarda tebranishlarni ifodalovchi $x(t)$ va $y(t)$ masala yechimlarini aniqlang. Bularga mos $x(t)$ va $y(x)$ yechim funksiyalarining grafiklarini hamda ularning fazoviy portretlarini chizing.

Xususiy hol. Xususan, ushbu

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{3}{2}\left(1 - \dfrac{x}{8}\right)x - \dfrac{3}{2(1+x)}xy, \\ \dfrac{dy}{dt} = \left(\dfrac{1}{10} - \dfrac{y}{10x}\right)y \end{cases}$$

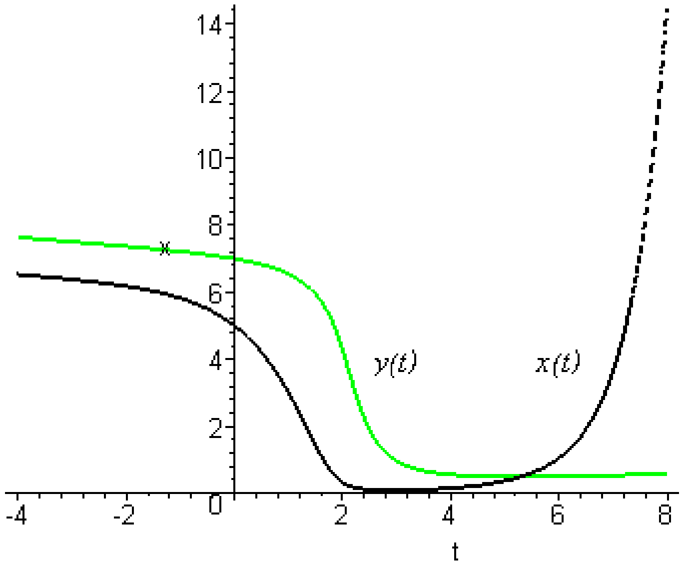
differensial tenglamalar sistemasi bilan berilgan Xolling-Tenner modelining yechimini quyidagi boshlang’ich shartlarda toping:

- x(0)=2; y(0)=0.5.
- x(0)=7; y(0)=3.

Masalani Maple dasturida sonli yechish.

1) Chegaraviy masalaning *x*(*t*) va *y*(*x*) yechim funksiyalari va ularning grafiklari (3.4-rasm):

```
> restart; cond:=x(0)=5,y(0)=7:
 sys:=diff(x(t),t)=1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),
diff(y(t),t)=(1/10-y(t)/(10*x(t)))*y(t):
 F:=dsolve({sys,cond},[x(t),y(t)], numeric):
 with(plots):  p1:=odeplot(F,[t,x(t)],-4..8, color=black, thickness=2, linestyle=4):
 p2:=odeplot(F,[t,y(t)],-4..8, color=green, thickness=2):
 p3:=textplot([6,4,"x(t)"], font=[TIMES,ITALIC, 12]):
 p4:=textplot([3,4,"y(t)"], font=[TIMES,ITALIC, 12]):
 display(p1,p2,p3,p4);
```

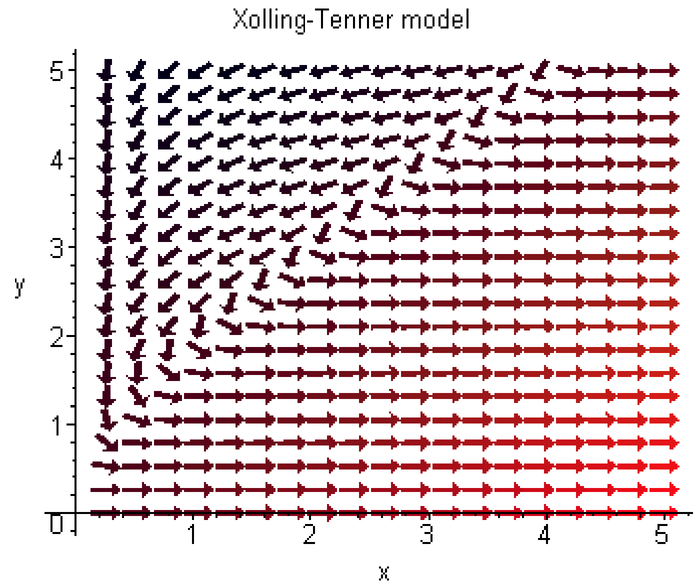


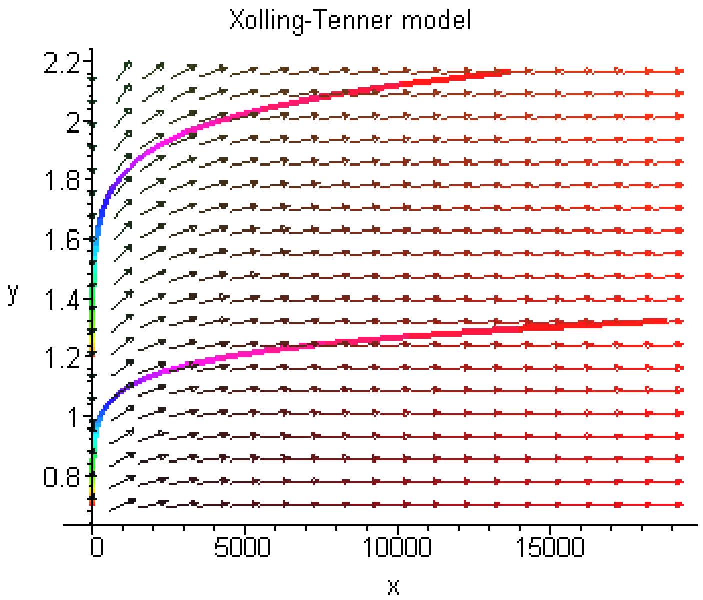
3.4-rasm. Chegaraviy masalaning *x*(*t*) va *y*(*x*) yechim funksiyalari grafiklari (*xulosa*: turlarning ko'payish mutanosibligi $t < 6$ da ekan).

2) *x*(*t*) va *y*(*x*) yechim funksiyalarining fazoviy portretlari (3.5-rasm):

```
> DEplot([diff(x(t),t)= 1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),diff(y(t),t)=
(1/10-y(t)/(10*x(t)))*y(t)],
```

```
[x(t),y(t)],t=0..7,x=0..5,y=0..5,arrows=LARGE,
title=`Xolling-Tenner  model`,color=[1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),
(1/10-y(t)/(10*x(t)))*y(t),.1]);
 DEplot([diff(x(t),t)= 1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),diff(y(t),t)= (1/10-
y(t)/(10*x(t)))*y(t)],
[x(t),y(t)],t=0..7,[[x(0)=1.2,y(0)=1.2],[x(0)=1,y(0)=.7]],stepsize=.2,
title=`Xolling-Tenner  model`,color=[1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),
(1/10-y(t)/(10*x(t)))*y(t),.1],
linecolor=t/2,arrows=MEDIUM,method=rkf45);
```

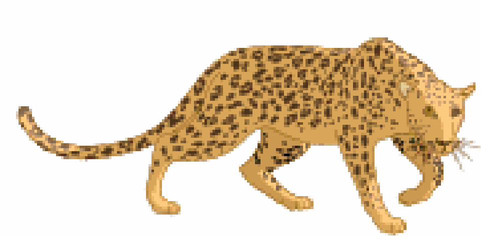


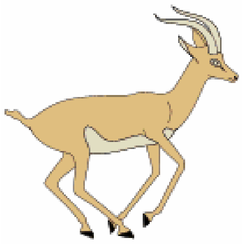


3.5-rasm. $x(t)$ va $y(x)$ yechim funksiyalarining fazoviy portretlari.

3.4. Yirtqich – o‘lja turi bo‘yicha Mak-Artur modeli

Masalaning qo’yilishi. Yirtqichlar va o‘ljalarning ko‘payishi ushbu





$$\begin{cases} \dfrac{dx}{dt} = a\left(1 - \dfrac{x}{K}\right)x - \dfrac{b}{1+Ax}xy, \\ \dfrac{dy}{dt} = -cy + \dfrac{d}{1+Ax}xy \end{cases}$$

differensial tenglamalar sistemasi (Mak-Artur modeli) bilan tavsiflanadi, bunda

a, b, c, d, A, K – manfiy bo’lmagan sonlar;

x va y – o‘ljalar va yirtqichlar ko’payishining mos nisbiy soni;

t – vaqt.

Tenglama hadlari tuzilishi quyidagicha:

$a(1–x/K)$ – yirtqich bo’lmagan holda o‘ljalarning ko‘payish tezligi;

$bxy/(1+Ax)$ – yirtqichlar zichligining ko‘payishi bilan o‘ljalarning qirilish tezligi;

cy – yirtqichlarning ko‘payish tezligi;

$dxy/(1+Ax)$ – o’ljaning ko’payishi bilan yirtqichlar zichligining ko‘payishi tezligi;

Quyidagi jadvalda keltirilgan parametrlar qiymatlarida va x_0, y_0 boshlang‘ich shartlarda tebranishlarni ifodalovchi $x(t)$ va $y(t)$ masala yechimlarini aniqlang. Bularga mos $x(t)$ va $y(x)$ yechim funksiyalarining grafiklarini hamda ularning fazoviy portretlarini chizing.

Xususiy hol. Xususan, ushbu

$$\begin{cases} \dfrac{dx}{dt} = -\dfrac{\sin t}{\sqrt{1+e^{2t}}} - x(x^2+y^2-1), \\ \dfrac{dy}{dt} = \dfrac{\cos t}{\sqrt{1+e^{2t}}} - y(x^2+y^2-1) \end{cases}$$

differensial tenglamalar sistemasi bilan berilgan Xolling-Tenner modelining yechimini [0;5] oraliqda quyidagi boshlang'ich shartlarda toping:

$$x(0)=3;\ y(0)=1.$$

Masalani Maple dasturida sonli yechish.

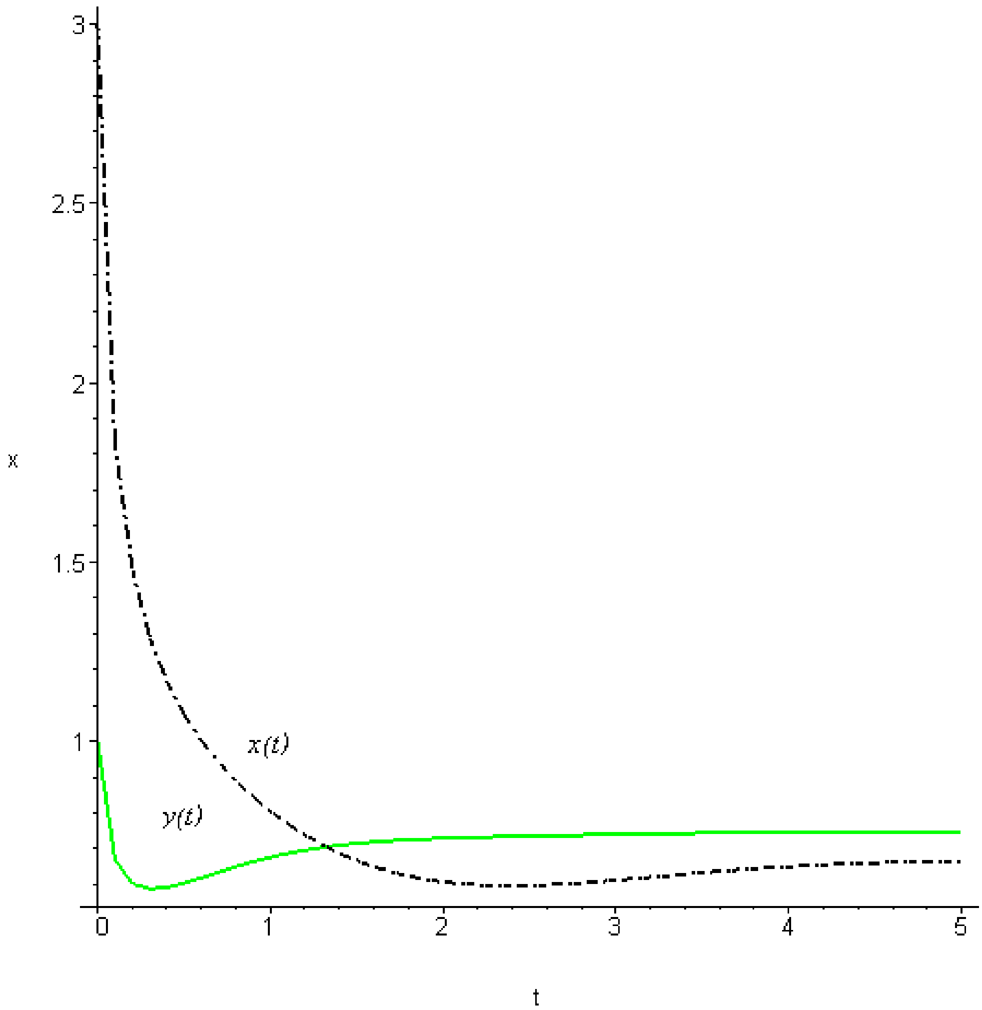
Chegaraviy masalaning $x(t)$ va $y(x)$ yechim funksiyalari va ularning grafiklari (3.6-rasm):

```
> restart; cond:=x(0)=3,y(0)=1;
 sys:=diff(x(t),t)=-sin(t)/sqrt(1+exp(2*t))-x(t)*(x(t)^2+y(t)^2-1),
diff(y(t),t)=cos(t)/sqrt(1+exp(2*t))-y(t)*(x(t)^2+y(t)^2-1);
 F:=dsolve({sys,cond},[x(t),y(t)], numeric);
 with(plots):
 p1:=odeplot(F,[t,x(t)],0..5, color=black, thickness=2, linestyle=4):
 p2:=odeplot(F,[t,y(t)],0..5, color=green, thickness=2):
 p3:=textplot([1,1,"x(t)"], font=[TIMES,ITALIC, 12]):
 p4:=textplot([0.5,0.8,"y(t)"], font=[TIMES,ITALIC, 12]):
 display(p1,p2,p3,p4);
```

$$cond := x(0) = 3,\ y(0) = 1$$

$$sys := \frac{d}{dt}x(t) = -\frac{\sin(t)}{\sqrt{1+e^{(2t)}}} - x(t)\,(x(t)^2 + y(t)^2 - 1),$$
$$\frac{d}{dt}y(t) = \frac{\cos(t)}{\sqrt{1+e^{(2t)}}} - y(t)\,(x(t)^2 + y(t)^2 - 1)$$

$$F := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc}$$



3.6-rasm. Chegaraviy masalaning $x(t)$ va $y(x)$ yechim funksiyalari grafiklari.

Berilgan chegaraviy masalaning analitik yechimi quyidagicha

$$\begin{cases} x(t) = -\dfrac{\sin t}{\sqrt{1+e^{2t}}}, \\ y(t) = \dfrac{\cos t}{\sqrt{1+e^{2t}}}. \end{cases}$$

Buni rasmdan ham taqqoslab ko'rsa bo'ladi.

Xulosa sifatida shuni aytish mumkinki, $t > 1$ da bu turlarning ko'payishi orasida mutanosiblik mavjud ekan, buni rasmdan ham ko'rish mumkin.

XULOSA

Mazkur bitiruv malakaviy ishining muhim natijalari quyidagilar:

- amaliy masalalarni matematik modellashtirish jarayonida oddiy differensial tenglamalar sistemasi, ularni Maple va Mathcad paketi yordamida yechishning muammolari o'rganildi, uni amalga oshirishning bosqichlari ishlab chiqildi;
- oddiy differensial tenglamalar sistemasining analitik yechimini Maple va Mathcad paketi yordamida yechish o'rganildi, hisob algoritmiga oid tushunchalar bilan tanishildi, amaliy masalalar yechildi;
- oddiy differensial tenglamalar sistemasining sonli yechimini Maple va Mathcad paketi yordamida yechish o'rganildi, hisob algoritmiga oid tushunchalar bilan tanishildi, amaliy masalalar yechildi;
- oddiy differensial tenglamalar sistemasining sonli yechimining Maple va Mathcad paketi yordamida grafiklarini qurish, fazoviy portretlarini chizish, har xil boshlang'ich shartlarda chegaraviy masalani yechishning algoritmi, dasturi, matematik paketlardan foydalanish bosqichlari bajarildi, har xil amaliy masalalar yechildi;
- yirtqich-o'ljaning ko'payishi haqidagi amaliy masalalar (oddiy differensial tenglamalar sistemasi bilan berilgan Mak-Artur, Bazikin, Lotka-Volterr va Xolling-Tenner modellari) sonli yechildi, yechimlar grafiklari chizilda, sistemalarning berilgan boshlang'ich shartlardagi fazoviy portretlari qurildi, biologik turlar ko'payishining o'zaro bog'liqlik holatlari grafiklarda ko'rsatildi;
- qo'yilgan masalani matematik paketlar yordamida samarali yechishga oid tavsiyalar ishlab chiqildi, undan foydalanishning mumkin bo'lgan imkoniyatlari ketma-ket tahlil qilindi;
- Olingan sonli yechimlar analitik yechimlar bilan taqqoslandi, hisob jarayonining to'g'ri ekanligi, algoritm va dasturdan samarali foydalanish mumkinligi ko'rsatildi;
- ishlab chiqilgan hisob metodikasi va yaratilgan hisob dasturiy vositasidan har xil oddiy differensial tenglamalar va differensial tenglamalar sistemasiga oid amaliy masalalarini yechishda samarali foydalanish mumkin.

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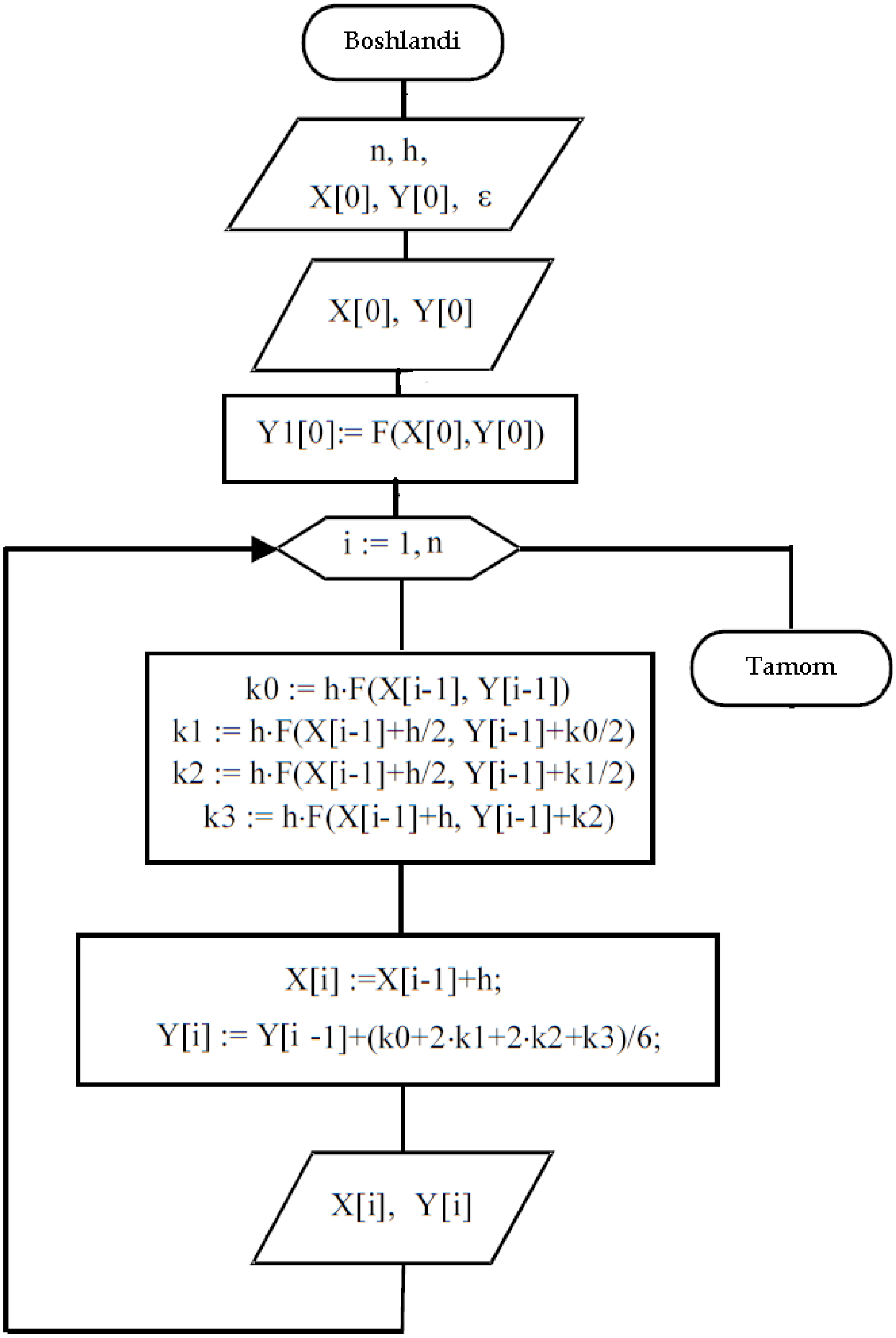
ILOVALAR

1-ilova.
Runge-Kutta sonli hisob usuli blok-sxemasi



2-ilova.

Bazikin modeli Maple hisob dasturi:

$diff(y(t), t) = 3 \cdot x(t) + 2 \cdot y(t) - 6 \cdot \exp(-t);$
$F := dsolve(\{sys, cond\}, [x(t), y(t)], numeric);$
$with(plots):$
$p1 := odeplot(F, [t, x(t)], 0..4, color = black, thickness = 2,$
$linestyle = 4):$
$p2 := odeplot(F, [t, y(t)], 0..4, color = green, thickness = 2):$
$p3 := textplot([0.6, 0.4, \text{"x(t)"}], font = [TIMES, ITALIC, 12]):$
$p4 := textplot([1.1, 0.5, \text{"y(t)"}], font = [TIMES, ITALIC, 12]):$
$display(p1, p2, p3, p4);$

3-ilova.

Lotka-Volterr modeli Maple hisob dasturi:

```
1) > restart; cond:=x(0)=1.2,y(0)=1.2:
 sys:=diff(x(t),t)=1-x(t)*y(t),
diff(y(t),t)=-0.3*y(t)-0.3*x(t)*y(t):
 F:=dsolve({sys,cond},[x(t),y(t)], numeric):
 with(plots):
 p1:=odeplot(F,[t,x(t)],-1..10, color=black, thickness=2, linestyle=4):
 p2:=odeplot(F,[t,y(t)],-1..10, color=green, thickness=2):
 p3:=textplot([4,4,"x(t)"], font=[TIMES,ITALIC, 12]):
 p4:=textplot([5,1,"y(t)"], font=[TIMES,ITALIC, 12]):
 display(p1,p2,p3,p4);
2) restart; cond:=x(0)=1,y(0)=0.7:
 sys:=diff(x(t),t)=1-x(t)*y(t),
diff(y(t),t)=-0.3*y(t)-0.3*x(t)*y(t):
 F:=dsolve({sys,cond},[x(t),y(t)], numeric):
 with(plots):
 p1:=odeplot(F,[t,x(t)],-2..10, color=black, thickness=2, linestyle=4):
```

```
 p2:=odeplot(F,[t,y(t)],-2..10, color=green, thickness=2):
 p3:=textplot([5,4,"x(t)"], font=[TIMES,ITALIC, 12]):
 p4:=textplot([5,1,"y(t)"], font=[TIMES,ITALIC, 12]):
 display(p1,p2,p3,p4);
3) > DEplot([diff(x(t),t)=x(t)*(1-y(t)),diff(y(t),t)=.3*y(t)*(x(t)-1)],
[x(t),y(t)],t=-2..2,x=-1..2,y=-1..2,arrows=LARGE,
title=`Lotka-Volterra  model`,color=[.3*y(t)*(x(t)-1),x(t)*(1-y(t)),.1]);
> DEplot([diff(x(t),t)=x(t)*(1-y(t)),diff(y(t),t)=.3*y(t)*(x(t)-1)],
[x(t),y(t)],t=-7..7,[[x(0)=1.2,y(0)=1.2],[x(0)=1,y(0)=.7]],stepsize=.2,
title=`Lotka-Volterra  model`,color=[.3*y(t)*(x(t)-1),x(t)*(1-y(t)),.1],
linecolor=t/2,arrows=MEDIUM,method=rkf45);
```

4-ilova.

Xolling-Tenner modeli Maple hisob dasturi:

```
1) > restart; cond:=x(0)=5,y(0)=7:
 sys:=diff(x(t),t)=1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),
diff(y(t),t)=(1/10-y(t)/(10*x(t)))*y(t):
 F:=dsolve({sys,cond},[x(t),y(t)], numeric):
 with(plots):
 p1:=odeplot(F,[t,x(t)],-4..8, color=black, thickness=2, linestyle=4):
 p2:=odeplot(F,[t,y(t)],-4..8, color=green, thickness=2):
 p3:=textplot([6,4,"x(t)"], font=[TIMES,ITALIC, 12]):
 p4:=textplot([3,4,"y(t)"], font=[TIMES,ITALIC, 12]):
 display(p1,p2,p3,p4);

2) > DEplot([diff(x(t),t)=  1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),diff(y(t),t)=
(1/10-y(t)/(10*x(t)))*y(t)],
[x(t),y(t)],t=0..7,x=0..5,y=0..5,arrows=LARGE,
```

```
title=`Xolling-Tenner  model`,color=[1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),
(1/10-y(t)/(10*x(t)))*y(t),.1]);
 DEplot([diff(x(t),t)= 1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),diff(y(t),t)= (1/10-
y(t)/(10*x(t)))*y(t)],
[x(t),y(t)],t=0..7,[[x(0)=1.2,y(0)=1.2],[x(0)=1,y(0)=.7]],stepsize=.2,
title=`Xolling-Tenner  model`,color=[1.5(1-x(t)/8)*x(t)-3*x(t)*y(t)/(2*(1+x(t))),
(1/10-y(t)/(10*x(t)))*y(t),.1],
linecolor=t/2,arrows=MEDIUM,method=rkf45);
```

5-ilova.
Mak-Artur modeli Maple hisob dasturi:

```
>  restart; cond:=x(0)=3,y(0)=1;
 sys:=diff(x(t),t)=-sin(t)/sqrt(1+exp(2*t))-x(t)*(x(t)^2+y(t)^2-1),
diff(y(t),t)=cos(t)/sqrt(1+exp(2*t))-y(t)*(x(t)^2+y(t)^2-1);
 F:=dsolve({sys,cond},[x(t),y(t)], numeric):
 with(plots):
 p1:=odeplot(F,[t,x(t)],0..5, color=black, thickness=2, linestyle=4):
 p2:=odeplot(F,[t,y(t)],0..5, color=green, thickness=2):
 p3:=textplot([1,1,"x(t)"], font=[TIMES,ITALIC, 12]):
 p4:=textplot([0.5,0.8,"y(t)"], font=[TIMES,ITALIC, 12]):
 display(p1,p2,p3,p4);
```