В строительном, дорожном и коммунальном машиностроении необходимо реализовать мероприятия по изготовлению машин, механизмов, инструментов и другой продукции, позволяющих существенно повысить технический уровень строительного производства, резко сократить применение ручного труда; обеспечить производство систем и оборудования, необходимых для комплексной механизации строительных процессов, более совершенной и экономичной землеройной техники, малогабаритных машин с наборами сменного оборудования. Важной проблемой является сокращение трудоемких, утомительных и непрестижных операций, которые до сих пор выполняется вручную, на основании внедрения новых высокопроизводительных машин и разнообразных методов интенсификации производства работ.

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“BIR XIL TAQSIMLANGAN TASODIFIY MIQDORLAR UCHUN LOKAL TEOREMADAGI NOTEKIS BAHO” mavzusida

5A460104-“Ehtimollar nazariyasi va matematik statistika” mutaxassisligi bo‘yicha

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MAGISTRLIK DISSERTATSIYASI

Ilmiy rahbar: Fizika-matematika
fanlari nomzodi, dotsent Dedamirza Otaqo‘ziyev

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M U N D A R I J A

ANNOTATSIYA

Ushbu magistrlik dissertatsiyasi "Bir xil taqsimlangan tasodifiy miqdorlar uchun lokal teoremadagi notekis baho" deb nomlangan va ehtimollar nazariyasi-ning limit teoremalarining tatbiqlariga bag'ishlangan .

K I R I S H

Ushbu magistrlik dissertatsiyasi ehtimolliklar nazariyasining asosiy bo'limlaridan biri – lokal limit teoremalar nazariyasiga bag'ishlangan bo'lib, unda panjarasimon taqsimlangan va uzluksiz tasodifiy miqdorlarning oddiy yig'indilari uchun lokal limit teoremalar ko'rilgan, bir xil taqsimlangan va har xil taqsimlangan tasodifiy miqdorlar ketma-ketligi uchun mavjud natijalar batafsil yoritilgan.

Ma'lumki, ehtimollar nazariyasi va matematik statistikaning ko'plab masalalarini hal etishda tasodifiy miqdorlarning yig'indilari, qo'shiluvchilar soni kattalashib borganda, qanday qonunga intilishini aniqlash muhim masala bo'lib qoladi; shunday masalalarni hal etish jarayonida, masalaning mohiyatiga qarab turlicha – markaziy limit teoremalar, lokal limit teoremalar va global limit teoremalar isbot etiladi. Biroq, bunday teoremalarning isbotlanishi bilan muammolar to'la hal bo'lib qolmaydi, chunki bu teoremalarda qo'shiluvchilar soni n - qanday bo'lganda bu teoremalarni qo'llash samarali bulishini oydinlashtirish lozim bo'ladi.

Bunday savollarga javob berish uchun, isbotlangan lokal limit teoremalarda yaqinlashish tezligini aniqlash, olingan baholarning tasodifiy miqdorlarning sonli xarakteristikalariga va o'zgaruvchi
x - ga qanday aloqadaligini ko'rsatish, qolaversa olingan tengsizliklarda qatnashuvchi o'zgarmas S – konstantalarning optimal qiymatlarini topish eng muhim masalalardir.

$\xi_1, \xi_2, \xi_3, ..., \xi_n, ...$ tasodifiy miqdorlar ketma - ketligidagi har bir qo'shiluvchining zichlik funksiyasi chegaralangan ($P(x) \le A$) hol mavjud adabiyotlarda har tomonlama asoslangan.

Ikkinchi bobda ko'p o'lchovli hol, ya'ni $\mathbf{R}^K$ fazodagi tasodifiy vektorlar ketma-ketligi uchun lokal limit teoremalar ko'rib o'tildi, tabiiyki, bu natijalar birinchi bobdagi natijalarning umumlashmasidan iborat.

$n \ge n_0 \ge 1$ bo'lgan holda tasodifiy miqdorlarning yig'indisi uchun umumiy holda lokal limit teoremalardagi yaqinlashish tezligini aniqlash masalasi aktual masaladir, ishning 3 bobida shu yo'nalishda olingan tekis va notekis baholar, global teoremalardagi yaqinlashish tezligini aniqlash masalalari bayon etilgan.

Ma'lumki, Vatanimiz istiqlol g'alabasidan so'ng shahdam odimlar bilan olg'a bormoqda, ilm – fan va texnikaning zamonaviy sohalari rivojlanmoqda va bu rivojlanish ilm ahli oldiga ko'plab zamonaviy muammolarni hal etishni ko'ndalang qilib qo'ymoqda. Ushbu fikrimizni Prezidentimiz Islom Abdug'anievich Karimovning "O'zbekiston XXI asr bo'sag'asida: xavfsizlikka tahdid, barqarorlik shartlari va taraqqiyot kafolatlari" nomli kitoblarida keltirilgan quyidagi
so'zlardan ham bilib olsak bo'ladi:

"Respublikamizda quyidagi yo'nalishlar bo'yicha jahon dara-jasidagi ilmiy maktablar yaratilgan bo'lib, ularda tadqiqotlar muvafaqqiyatli olib borilmoqda. Jumladan, matematika, ehtimollar nazariyasi, tabiiy va ijtimoiy jarayonlarni matematik modellash, informatika va hisoblash texnikasi sohasidagi tadqiqotlar.Matematika fanining ehtimollar nazariyasi va matematik statistika, differensial tenglamalar va matematik fizika, funksional tahlil sohasidagi yutuqlari respublikadan ancha uzoqda ham mashhur".

Magistrlik ishining maqsad va vazifalari.

Magistrlik ishining asosiy maqsadi matematik tahlildagi funksiyalarni qatorlarga yoyish, qoldiq hadni baholash, absolyut integrallanuvchi funksiyalar va ayrim klassik tengsizliklarni ehtimolar nazariyasi va matematik statistikaning bo‘limlaridagi muhim muamolarni hal etishga tadbiq etishni ko‘rsatishdan iborat.

Bizning vazifamiz matematik tahlil aparatidan foydalanib extimollar nazariyasi va matematik statistikadagi muhim yo‘nalish lokal limit teoremalar sohasida olingan natijalarni kengaytirishdan iboratdir.

Amaliy ahamiyati. Ehtimollar nazariyasining muhim qismi – limit teoremalar bo‘limida erishilgan yutuqlarni xalq xo‘jaligi, zamonaviy texnika, avtomatik liniyalar, kosmik tadqiqotlarga oid masalalarda qo‘llashning imkoniy va foydali yo‘nalishlarini ko‘rsatishdan iborat.

Bu sohada dunyoning mashhur matematiklaridan tortib yosh izlanuvchilarning hissalari ham muhim. Akademik B. V. Gnedenko va uning shogirdlari, Akademik S. H. Sirojiddinov va uning shogirdlari – M. Mamatov, T. Azlarov, N. Shohaydarova, M. G‘ofurov, A. Jomirzaev, R. Ibragimov, V.Hojiboev, A. Mashrabbayev, D. Otaqo‘ziyev, M. Xolmuradov va boshqa yuzlab matematiklar bu sohada izlanishlar olib borganlar va juda muhim natijalarga erishganlar, xozirgi kunlarda ham izlanishlarni davom ettirmoqdalar.

Ilmiy yangiligi. Magistrlik dissertatsiyasida tanlangan sohaga oid $n \geq n_0 \geq 1$ hol uchun lokal va global teoremalardagi yaqinlashish tezligini aniqlovchi teoremalardagi tekis va notekis baholar olingan.

Tadqiqot ob'yekti va predmeti. Ehtimollar nazariyasi va matematik statistika va unga qo‘shni boshqa sohalarga oid ilmiy izlanishlarni davom ettirish.

Ushbu magistrlik dissertatsiyasi Kirish, Asosiy qism, Xulosa, Foydalanilgan adabiyotlar va Ilovadan tashkil topgan. Asosiy qism uchta bobdan iborat bulib, birinchi va ikkinchi boblar tegishli mavzularning obzoriga bag’ishlangan, uchinchi bobda esa asosiy natijalar va ularning isbotlari keltirilgan. Internet ma'lumotlarida dissertatsiya mavzusiga oid eng yangi natijalardan namunalar keltirilgan.

Magistrlik dissertatsiyasining birinchi bobida uzluksiz tasodifiy miqdorlarning oddiy yig‘indisi uchun har bir qo’shiluvchining zichlik funksiyasi chegaralangan holda klassik lokal limit teoremalar va bu teoremalarga oid turli xil tekis va notekis baholar keltirilgan.

Ikkinchi bobda $\mathbf{R}^K$ fazodagi tasodifiy vektorlar ketma-ketligi uchun lokal limit teoremalarni isbotlash va bu teoremadagi yaqinlashish tezligini aniqlash masalalari bayon etilib, mos isbotlangan jumlalar keltirilgan.

Uchinchi bobda tasodifiy miqdorlarning zichlik funksiyasi chegaralanmagan holda umumiy lokal limit teoremalar batafsilroq o’rganilgan, tekis va notekis baholar, global teoremalarga oid natijalar keltirilgan. Magistrant tomonidan olingan natijalar ham shu bobdan joy olgan.

Magistrant tomonidan olingan natijalar matematika fakultetida o’tkazilgan ilmiy anjumanlarda bayon etilgan.

I BOB
ZICHLIK FUNKSIYASI CHEGARALANGAN TASODIFIY MIQDORLAR YIG‘INDISI UCHUN LOKAL LIMIT TEOREMALAR

1-§. Masalaning qo‘yilishi

Ma'lumki, juda ko‘p masalalarni hal etishda ehtimollar nazariyasidan foydalaniladi. Ko‘plab nazariy va amaliy masalalarni yechish jarayonida tasodifiy miqdorlarning yig‘indisi sifatida ifodalanadigan xarakteristikalar bilan ish ko‘rishga to‘g‘ri keladi. Aksariyat hollarda qo‘shiluvchilar soni katta bo‘lgani sababli bunday yig‘indilar uchun limitik taqsimotni izlanadi.

Masalan, tajribalar soni n unchalik katta bo‘lmaganda biror hodisaning n marta tajriba o‘tkazilganda k marta ro‘y berish ehtimoli quyidagi binomial qonun bilan topiladi:

$$P_n(k)=C_n^k p^k q^{n-k}, \qquad (1.1.1)$$

bu yerda p - qaralayotgan hodisaning ro‘y berish ehtimoli,

$$q=1-p.$$

Ammo n va k lar yetarlicha katta bo‘lganda (1.1.1) formuladan foydalanish juda ham murakkab hisob-kitoblarni talab etib, amaliyotda qo‘llanishi samarali natijalarga olib kelmaydi.

(1.1.1) formuladagi n va k lar yetarlicha katta bo‘lgan hol uchun tegishli asimptotik formula topilib, bunga Muavr – Laplasning lokal teoremasi orqali erishilgandir.

Teorema. A hodisaning har bir tajribada ro‘y berish ehtimoli p ga teng bo‘lsa, u holda A hodisaning n ta tajribada aniq k marta ro‘y berish ehtimoli quyidagicha ifodalanadi:

$$P_n(k)=\frac{1}{\sqrt{2\pi npq}}\cdot e^{-\frac{x^2}{2}}\cdot(1+\varepsilon_n), \qquad (1.1.2)$$

bu yerda

$$q=1-p,\quad x=\frac{k-np}{\sqrt{npq}} \text{ va } \lim_{n\to\infty}\varepsilon_n=0.$$

n yetarlichi katta bo‘lganda (1.1.2) formuladan quyidagi taqribiy formulaga o‘tiladi:

$$P_n(k)\approx\frac{1}{\sqrt{2\pi npq}}e^{-\frac{x^2}{2}}. \qquad (1.1.3)$$

(1.1.3) formuladan foydalanish uchun standart normal qonunining zichlik funksiyasi deb ataluvchi quyidagi funksiyaning qiymatlarini bilish talab etiladi.

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}.$$

Bu funksiyaning muhimligini e'tiborga olib, uning qiymatlarini topish uchun maxsus jadvallar tuzilgan bo‘lib ehtimollar nazariyasi va matematik statistikaga oid adabiyotlarda ilova sifatida keltiriladi.

Ehtimollar nazariyasi va matematik statistikaning keng qamrovli tadbiqlarini amalga oshirish uchun masalani umumiyroq tarzda qo‘yib, zarur natijalarga ega bo‘lish muhimdir. Ehtimollar nazariyasining ushbu yo‘nalishdagi masalalar har tomonlama o‘rganilgan, ya'ni bir xil taqsimlangan o‘zaro bog‘liqmas tasodifiy miqdorlar ketma-ketligi uchun, o‘zaro bog‘liq tasodifiy miqdorlar ketma-ketligi uchun ma'lum shartlar o‘rinli bo‘lganda tasodifiy miqdorlarning momentlariga turli xil shartlar qo‘yib lokal limit teoremalar isbotlangan va ulardagi yaqinlashish tezliklarini aniqlangandir.

Juda ko‘plab mualliflarning ishlarida ξ tasodifiy miqdorning zichlik funksiyasi $P(x)$ ning chegaralangan bo‘lishi, ya'ni $P(x) < A < \infty$ shartning bajarilishi talab etiladi, bu esa qaralayotgan masalalar sinfining torayishiga sabab bo‘ladi. Biroq, lokal limit teoremaning o‘rinli bo‘lishi uchun $P(x) < A$ shartning bajarilishi zarur bo‘lmasdan, biror $n_0 > 1$ nomerdan boshlab

$$\xi_1 + \xi_2 + \ldots + \xi_{n_0}$$

yig‘indining zichlik funksiyasi $P_{n_0}(x)$ ning chegaralangan bo‘lishi zarur bo‘ladi. Bu yo‘nalishdagi teoremalar ham o‘rganilgan, tegishli baholar olingan. Tabiiyki, bu yo‘nalishdagi natijalar o‘rganiladigan tasodifiy miqdorlar ketma-ketligi sinfining kengayishiga olib keladi va $n_0 = 1$ bo‘lganda avvalgi natijalar hosil bo‘ladi.

Texnikada, meditsinada, statistikaning turli masalalarini yechishda, xalq xo‘jaligining xilma-xil muammolarini hal etish jarayonida olingan limitik teoremalardan n ning qanday qiymatlaridan boshlab foydalanish yetarlicha effekt berishini bilish muhimdir. Ushbu savolga javob berish uchun esa lokal limit teoremalardagi yaqinlashish tezligini aniqlashga imkon beruvchi formulalarni topishning ahamiyati kattadir.

Yaqinlashish tezligini aniqlash masalasi ham ko‘p qirrali bo‘lib, bular – x ga nisbatan notekis baholar, momentlarga nisbatan tekis baholar, olingan baholardagi C - koeffitsient qiymatini minimallashtirish, momentlar tartibini pasaytirish kabi o‘nlab yo‘na-lishlar mavjud.

Endi shu yo‘nalishlarda erishilgan natijalar bilan tanishib chiqamiz.

2-§. PANJARASIMON TAQSIMLANGAN TASODIFIY MIQDORLAR UCHUN LOKAL LIMIT TEOREMA

Ta'rif. Agar shunday a va $h > 0$ sonlar mavjud bo‘lib,

$$\sum_{k=-\infty}^{\infty} P(\xi = a + kh) = 1$$

tenglik bajarilsa, u o‘olda ξ tasodifiy miqdor h qadamli panjarasimon taqsimotga ega deyiladi. ξ esa panjarasimon taqsimlangan tasodifiy miqdor deyiladi.

$$P_n(k) = C_n^k p^k q^{n-k}$$ - binomial qonun

va

$$P_n(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$ - Puasson qonun

bilan taqsimlangan tasodifiy miqdorlar panjarasimon taqsimlangan tasodifiy miqdorlarga misol bo‘la oladi.

Lemma. ξ panjarasimon taqsimlangan tasodifiy miqdor bo‘lishi uchun biror $t \neq 0$ da ξ tasodifiy miqdor xarakteristik funksiyasining moduli birga teng bo‘lishi zarur va yetarlidir.

Agar hech bir $b\ (-\infty < b < \infty)$ va $h_1 > h$ da ξ tasodifiy miqdorning barcha qiymatlarini $b + kh_1$ ko‘rinishda ifodalab bo‘lmasa, u holda h son taqsimotning maksimal qadami deyiladi.

$\xi_1, \xi_2, \ldots, \xi_n, \ldots$ o‘zaro bog‘liqmas panjarasimon taqsimotga ega bo‘lgan va bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligi bo‘lsin.

U holda

$$S_n = \xi_1 + \xi_2 + \ldots + \xi_n$$

yig‘indisi ham panjarasimon taqsimotga ega bo‘lib, uning qiymatlarini $\{na + kh\}$ ko‘rinishda ifodalash mumkin.

Aytaylik,

$$P_n(x) = P(S_n = na + kh)$$

bo‘lsin.

Ushbu belgilashlarni kiritamiz.

$$Z_{n,k} = \frac{an + kh - A_n}{B_n}, \qquad A_n = MS_n, \quad B_n^2 = DS_n = nD\xi_1$$

.

1.2.**1-teorema (B.V.Gnedenko).** Agar $0 < D\xi_1 < \infty$ bo‘lsa, u holda $n \to \infty$ da

$$\frac{B_n}{n} P_n(k) - \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{Z_{n,k}^2}{2}} \to 0$$

munosabat k ga nisbatan tekis bajarilishi uchun h ning maksimal qadam bo‘lishi zarur va yetarlidir.

Eslatma. Ushbu bobda olingan natijalarni isbotsiz keltiramiz. Ularning hammasi xarakteristik funksiyalar metodi bilan isbotlangan. Yetarlicha ma'lumot olish uchun tavsiya etilgan adabiyotlarga murojaat etish mumkin.

Endi ushbu teoremadagi yaqinlashish tezligini aniqlashga doir teoremalarni keltiramiz. Umumiylikka halal bermagan holda, $h=1$ deb olamiz.

$\xi_1, \xi_2, \xi_3, \dots, \xi_n, \dots$ o‘zaro bog‘liqmas butun qiymatli, maksimal qadami 1 ga teng bo‘lgan tasodifiy miqdorlar ketma-ketligi bo‘lsin.

U holda

$$S_n = \xi_1 + \xi_2 + \dots + \xi_n$$

yig‘indi ham butun qiymatli bo‘ladi.

Faraz qilaylik

$$M\xi_1 = a,\ D\xi_1 = \sigma^2,\ B_n = n\sigma^2,\ \xi_n = (S_n - an)/\sigma\sqrt{n}.$$

Quyidagi belgilashlarni kiritamiz.

$$y = Y_{n,k} = \frac{k-ah}{\sigma\sqrt{n}},\ \varphi(y) = \frac{1}{\sqrt{2n}} e^{-\frac{y^2}{2}},$$

$$P\left\{\xi_n = \frac{k-an}{\sigma\sqrt{n}}\right\} = P\{S_n = k\} = P_n(k).$$

1.2.2-Teorema. Agar $M|\xi_1|^3 = \beta_3 < \infty$ bo‘lsa, u holda

$$\left| B_n P_n(k) - \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{y^2}{2}} \right| = O\left(\frac{1}{\sqrt{n}}\right).$$

Quyidagi teoremada $\beta_3 < \infty$ talab o‘rniga $\beta_{2+\delta} < \infty$ $(0 < \delta \le 1)$ deb olib, natija umumlashtirilgan:

1.2.3-Teorema. (M.Mamatov). Agar ξ_i tasodifiy miqdorlar $2+\delta$ $(0 < \delta \le 1)$ tartibli absolyut markaziy momentga ega bo‘lsa, u holda

$$\left| B_n P_n(k) - \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{y^2}{2}} \right| = O\left(\frac{1}{n^{\frac{\delta}{2}}}\right).$$

1.2.4-Teorema. (M.Mamatov). Agar $M|\xi_1|^{2+\delta} < \infty$ $(0 < \delta < 1)$ bo‘lsa, quyidagi munosabat o‘rinli

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1-\delta}{2}}} \left| P\{S_n = k\} - \frac{1}{\sqrt{2\pi n\sigma}} \cdot e^{\frac{-(k-na)^2}{2n\sigma^2}} \right| < \infty.$$

Bu teoremadan, yetarlicha katta n larda quyidagi ifoda kelib chiqadi:

$$\sigma\sqrt{n}P\{S_n = k\} - \frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{Z_{n,k}^2}{2}} = o\left(n^{-\frac{\delta}{2}}\right).$$

Yuqorida keltirilgan teoremalarda ξ_1 tasodifiy miqdorning momentlari chekli bo'lishi talab etiladi. Bu natijalar quyidagi teoremalarda yanada umumlashtirilgandir.

$g(x)$ $[0, \infty)$ da nomanfiy, juft va kamaymovchi funksiya bo'lib, $\lim_{x\to\infty} g(x) = \infty$ bo'lsin.

1.2.5-Teorema. (N.Shohaydarova, T.Zuparov). Ushbu

$$\max_x \left|\frac{\sigma\sqrt{n}}{h}P_n(k) - \varphi\left(\frac{an+kh}{\sigma\sqrt{n}}\right)\right| = O\left(g^{-1}\left(\sqrt{n}\right)\right)$$

munosabat o'rinli bo'lishi uchun

1) $\int_{|x|\geq Z} x^2 dF(x) = O\left(g^{-1}(Z)\right)$;

2) h - maksimal qadam shartlarning bajarilishi zarur va yetarlidir.

Yuqorida keltirilgan teoremalar uchun ξ_1 - tasodifiy miqdorning momentlashga nisbatan tekis baholar olingan bo'lib, ushbu yo'nalishi juda mukammal o'rganilgandir.

Tegishli ma'lumotlar olish uchun ro'yxatda keltirilgan adabiyotlarga murojaat etishni tavsiya etamiz.

3-§. UZLUKSIZ TASODIFIY MIQDORLAR YIG‘INDISI UCHUN LOKAL LIMIT TEOREMALAR

$$\xi_1,\ \xi_2,\ \xi_3,\ \dots,\ \xi_n,\ \dots$$

uzluksiz taqsimlangan tasodifiy miqdorlar ketma-ketligi bo‘lib, ular $P(x)$ umumiy zichlik funksiyaga va chekli o‘rta qiymat hamda dispersiyaga ega bo‘lsinlar.

1.3.1-Teorema. (B.V.Gnedenko). $\{\xi_n\}$ bog‘liq bo‘lmagan tasodifiy miqdorlar ketma-ketligi umumiy $F(x)$ taqsimot funksiyaga ega bo‘lsin va ularning o‘rta qiymatlari hamda dispersiyalari chekli bo‘lib, biror n_0 - nomerdan boshlab $\frac{S_n - A_n}{\sqrt{nD\xi_1}}$ yig‘indi $\overline{P}_n(x)$ zichlik funksiyaga ega bo‘lsin.

$$\overline{P}_n(x) - \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \to 0$$

munosabat $n \to \infty$ da $x\ (-\infty < x < \infty)$ ga nisbatan tekis bajarilishi uchun $P_{n_0}(x) < \infty$ tengsizlikni qanoatlantiruvchi n_0 sonning topilishi zarur va yetarlidir.

$\{\xi_n\}$ – bog‘liqmas, bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligi bo‘lib, $F(x)$ ularning taqsimot funksiyasi va $M\xi_1 = 0,\ \ D\xi_1 = \sigma^2 < \infty$ bo‘lsin.

Quyidagi yig‘indilarni tuzamiz.

$$S_n = \xi_1 + \xi_2 + \xi_3 + \dots + \xi_n; \qquad \xi_n = \frac{S_n}{\sigma\sqrt{n}}.$$

S_n va ξ_n yig‘indilarning zichlik funksiyalari (agar ular mavjud bo‘lsa) $P_n(x)$ va $\overline{P}_n(x)$ kabi belgilaymiz.

1.3.2-Teorema. (D.Otaqo‘ziyev). $\{\xi_n\}$ bog‘liqmas bir xil taqsimlangan tasodifiy miqdorlar ketma - ketligi bo‘lib, $F(x)$ ularning taqsimot funksiyasi bo‘lsin. Agar shunday n_0 son topilsinki, $\sup\limits_x P_{n_0}(x) \le A$ o‘rinli bo‘lsa, u holda $n \ge 2n_0$ nomerlar uchun quyidagi tengsizlik o‘rinli:

$$\sup_x \left|\overline{P}_n(x) - \varphi(x)\right| \le \frac{C \cdot \sqrt{n_0} \cdot \beta_3 \max\left[1;\ \left(\sigma\sqrt{n_0}A\right)^3\right]}{\sigma^3\sqrt{n}},$$

bu yerda $\beta_z = M|\xi_1|^3$.

$n_0 = 1$ va $n_0 > 1$ bo‘lgan hollarni alohida – alohida ko‘rib o‘tamiz.

4-§. $n_0 = 1$ HOLDA YAQINLASHISH TEZLIGINI BAHOLASH

Ushbu paragrfda zichlik funksiyalari chegaralangan tasodifiy miqdorlar uchun lokal – limit teoremalardagi yaqinlashish tezligini aniqlashga doir teoremalarni keltiramiz.

$\xi_1, \xi_2, \dots, \xi_n, \dots$ bog‘liqmas bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligi bo‘lib, bir xil $P(x)$ zichlik funksiyaga ega bo‘lsin, hamda

$$M\xi_1 = 0, \quad D\xi_1 = 1, \quad M|\xi|^3 < \infty, \quad \sup_x P(x) \le A$$

shartlar o‘rinli bo‘lsin.

$$\xi_n = \frac{(\xi_1 + \xi_2 + \dots + \xi_n)}{\sqrt{n}}$$

tasodifiy yig‘indining zichlik funksiyasi $\overline{P}_n(x)$ bo‘lsin.

$\sup_x P(x) \le A < \infty$ shart ancha og‘ir shart bo‘lib, barcha zichlik funksiyalar ham bunday shartni qanoatlantiravermaydi.

Quyida bu shartga bo‘ysunuvchi tasodifiy miqdorlarga misol keltiramiz.

1) $M\xi_1 = 0, \; M\xi_1^2 = 1$ shartni qanoatlantiruvchi normal qonun bilan taqsimlangan tasodifiy miqdorlar uchun

$$P(x) = \varphi(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}.$$

Bu funksiya uchun

$$\sup_x |P(x)| = \sup_x \left| \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} \right| \le \frac{1}{\sqrt{2\pi}} = A.$$

2) ξ tasodifiy miqdor Koshi taqsimotiga ega bo‘lsa uning zichlik funksiyasi quyidagi ko‘rinishga ega:

$$P(x) = \frac{1}{\pi(1+x^2)}, \qquad (-\infty < x < \infty).$$

U holda

$$\sup_x |P(x)| = \sup_x \frac{1}{\pi(1+x^2)} \le \frac{1}{\pi} = A.$$

3) Agar ξ tasodifiy miqdor ikki yoqlama Laplas qonuni bilan taqsimlangan bo‘lsa unga mos zichlik funksiya

$$P(x) = \frac{\lambda}{2} e^{-\lambda|x|}$$

ko‘rinishda bo‘ladi.

Bu hol uchun:

$$\sup_x P(x) = \sup_x \frac{\lambda}{2} e^{-\lambda|x|} = \frac{\lambda}{2} \cdot e^{-\lambda \cdot 0} = \frac{\lambda}{2} = A,$$

bu yerda λ - taqsimot parametri.

Bunday misollarni ko‘plab keltirishimiz mumkin.

$\sup_x P(x) \le A$ shart bajarilmaydigan, ya'ni $\sup_x P(x) = \infty$ bo‘lgan holni keyingi II , III boblarda ko‘rib o‘tamiz.

Endi $\sup_x P(x) \le A$ bo‘lgan hol uchun olingan natijalarni keltiramiz:

1.4.**1-Teorema** (S.H.Sirojiddinov, M.Mamatov). $\{\xi_n\}$ bog‘liqmas bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligi $P(x)$ zichlik funksiyaga ega bo‘lib,

$$M\xi_1 = 0, \quad M\xi_1^2 = 1, \qquad M\xi_1^3 = \alpha_3 < \infty$$

bo‘lsin.

$$\int_{-\infty}^{\infty} \left| \overline{P}_n(x) - \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \right| dx = \frac{|\alpha_3|}{3\sqrt{2\pi n}} \cdot \left(1 + 4e^{-\frac{3}{2}}\right) + o\left(\frac{1}{\sqrt{n}}\right).$$

1.4.2-Teorema. (N.Shohaydarova). $\{\xi_n\}$ bog‘liqmas bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligi $P(x)$ zichlik funksiyaga ega bo‘lib,

$$M\xi_1 = 0, \qquad M\xi_1^2 = 1, \quad M|\xi_1|^3 = \beta_3 < \infty, \quad \sup_x P(x) \le A$$

bo‘lsin.

U holda

$$\sup_x \left| \overline{P}_n(x) - \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \right| \le \frac{C\beta_3}{\sqrt{n}} \cdot \max(i;\ A^3);$$

va barcha x lar uchun

$$\left| \overline{P}_n(x) - \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \right| \le \frac{C \cdot \beta_3^{2m-1} \max(1;\quad A^{2m+1})}{\sqrt{n}(1 + |x|^m)},$$

bu yerda $m = 2;\ 3, \quad C$ - absolyut o‘zgarmas son.

5-§. HAR XIL TAQSIMLANGAN TASODIFIY MIQDORLAR YIG‘INDISI UCHUN LOKAL LIMIT TEOREMALAR

Endi umumiyroq holga o‘tamiz. $\{\xi_n\}$ - bog‘liqmas har xil taqsimlangan tasodifiy miqdorlar ketma-ketligi bo‘lsin.

$M\xi_k = 0,\ D\xi_k = \sigma_k^2$ va ξ_k ning zichlik funksiyasi $P_k(x),\ k = 1,2,3,\ldots$ bo‘lsin. Har bir tasodifiy miqdorning uchinchi tartibli momenti $\beta_{3k} = M|\xi_k|^3$ mavjud bo‘lsin deb faraz qilamiz.

Quyidagi yig‘indini tuzamiz

$$\frac{\xi_1 + \xi_2 + \ldots + \xi_n}{S_n},$$

bu yerda

$$S_n = \sum_{k=1}^{n} \sigma_k^2,$$

$\overline{P}_n(x)$ - orqali yuqoridagi yig‘indining zichlik funksiyasini belgilaymiz.

$$B_{3n} = \sum_{k=1}^{n} \beta_{3k}; \qquad T_{3n} = \frac{S_n^3}{4B_{3n}}$$

deb olamiz.

$$P_k(x) \le A_k < \infty, \quad \varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

bo‘lsin.

Umumiylikka halal bermasdan quyidagicha yozish mumkin:

$$A_1 \le A_2 \le \ldots \le A_n \le \ldots$$

Ushbu belgilashlarni kiritamiz:

I. $A_i < \infty, \qquad i = 1,2,\ \ldots$

II. $\displaystyle\sum_{k=1}^{n} \frac{1}{A_k^2 \sigma_k^2} \ge C_1 \cdot \frac{\max_k \beta_{3k}^2}{\sigma_{k_n}^6} \cdot \ln S_n$,

bu yerda C_1 qandaydir musbat son $\left(C_1 \ge 16\pi^2 / C_k\right)$, $\sigma_{k_n}^2$ - esa barcha dispersiyalarning eng kichigi, $\sigma_{k_n}^2 \ne 0$.

1.5.1-Teorema. (N.Shohaydarova). $\{\xi_n\}$ tasodifiy miqdorlar ketma-ketligi uchun I va II shartlar o‘rinli bo‘lsin. U holda $n \to \infty$ da x - ga nisbatan tekis holda quyidagi tengsizlik o‘rinli bo‘ladi:

$$\left|\overline{P}_n(x)-\varphi(x)\right|\le C\cdot\left[\frac{\sum_1^n \beta_{3k}}{S_n^3}+\frac{A_2}{S_n}\right],\quad x\in(-\infty;\ \infty)$$

.

1.5.2-**Teorema.** 1.5.1-teoremaning shartlari biror musbat C son bilan o‘rinli bo‘lib, $|\xi_k|\le L=\lambda_n S_n$ tengsizlik k ga nisbatan tekis bajarilsa, u holda $0<\lambda_n x<\left(3-\sqrt{5}\right)/4$ tengsizlik o‘rinli bo‘lganda

$$\frac{\overline{P}_n(x)}{\varphi(x)}=e^{\frac{x^3}{\sqrt{n}}\cdot\lambda\left(\frac{x}{\sqrt{n}}\right)}\left[1+\theta_1\lambda_n x\right]\left[1+\theta_2\left(\frac{\sum_1^n \beta_{3k}}{S_n^3}+\frac{A_2}{S_n}\right)\right],$$

munosabat o‘rinli bo‘ladi, bu yerda $\lambda_n\to 0,\quad \lambda\left(\frac{x}{\sqrt{n}}\right)$ - qaralayotgan intervalda yaqinlashuvchi qator, θ_1 va θ_2 lar absolyut sonlar bilan chegaralangan miqdorlar.

Agar $\{\xi_n\}$ - bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligi bo‘lsa, 4-teoremadan quyidagi natija kelib chiqadi:

Natija. Agar $C_3A^2\beta^2\le\frac{n}{\ln\sqrt{n}}$ bo‘lsa, u holda $n\to\infty$ da

$$\left|\overline{P}_n(x)-\varphi(x)\right|\le\frac{\beta}{\sqrt{n}}\max(1;\ A),\qquad x\in(-\infty;\ \infty).$$

$\{\xi_n\}$ - tasodifiy miqdorlarning momentlari o‘rniga quyidagi “kesilgan” momentlarni qarab, yuqorida olingan natijalarni yaxshilash mumkin. Bu yo‘nalishda M. G‘ofurov tomonidan olingan quyidagi natijalarni ko‘rib o‘tish mumkin.

Quyidagi belgilashlarni kiritamiz:

$$S_n^2=\sum_{k=1}^n\sigma_k^2;\qquad \lambda_k=\sup_{z\ge 0}\int_{|x|>z}x^2P_k(x)dx;$$

$$\rho_k=\sup_{z\ge 0}\left\{\left|\int_{-z}^{z}x^3P_k(x)dx\right|+z\int_{|x|>z}x^2P_k(x)dx\right\}.$$

Faraz qilaylik, $P_k(x)\le A_k<\infty$ bo‘lsin.

Ushbu tengsizlik o‘rinli:

$$\sigma_k^2 A_k^2 \geq \frac{1}{12}.$$

Tabiiyki, $\rho_k < \beta_{3k}$ o‘rinli bo‘lgani uchun, ushbu shartlar bilan olingan teoremalar avvalgi natijalarni yaxshilaydi.

$$\min_{1\leq j\leq n}{}^{(k)} A_j = A_k^{(n)}$$

orqali $A_1, A_2, ..., A_n$ sonlar ichida k - chi eng kichigini belgilaymiz.

Xususan, $\min\limits_{1\leq j\leq n}{}^{(1)} A_j = \min\limits_{1\leq j\leq n} A_j$.

Quyidagi shartlarni kiritamiz:

A) $A_2^{(n)} < \infty$;

B) $A_4^{(n)} < \infty$;

V) shunday α va β o‘zgarmas sonlar mavjud bo‘ladiki, barcha n lar uchun

$$\sum_{k=1}^{n} \rho_k \leq \alpha n, \qquad \bar{\sigma}_n^2 = \min_{1\leq k\leq n} \sigma_k^2 \geq \beta_k > 0.$$

Teoremalardagi C lar bir xil bo‘lmagan o‘zgarmas sonlar bo‘lishi mumkin.

1.**5.3-teorema.** (M.O‘.G‘ofurov). Agar $\{\xi_n\}$ - bog‘liqmas tasodifiy miqdorlar ketma-ketligi uchun A, B shartlar o‘rinli bo‘lsa, u holda quyidagi tengsizlik x- ga nisbatan tekis bajariladi.

$$\left|\bar{P}_n(x) - \varphi(x)\right| \leq C \cdot \left\{\lambda(n) + \frac{\alpha \cdot \left(w_k + A_2^n\right)}{\beta\lambda(n) \cdot \Lambda^k(n)}\right\},$$

bu yerda

$$w_k = \frac{\beta}{\alpha}\left(\frac{\alpha^2}{\beta^3}\right)^k, \quad \lambda(n) = \sum_{k=1}^{n} \rho_k / S_n^3, \quad \Lambda(n) = \sum_{k=1}^{n} \frac{1}{A_k^2 \sigma_k^2},$$

k -ixtiyoriy musbat son.

Ko‘rinib turibdiki, yuqoridagi natija $\Lambda(n)$ ga bog‘liq. Buni e'tiborga olsak quyidagi teorema o‘rinlidir:

1.5.4-teorema. Agar biror absolyut C_0 konstanta uchun
$$\Lambda(n) \geq C_0 \frac{\max\limits_{1\leq k\leq n} \rho_k^2}{\bar{\sigma}_n^6} \log S_n$$
va A shart o‘rinli bo‘lsa, u holda quyidagi baho $x-$ga nisbatan tekis bajariladi

$$\left|\overline{P}_n(h)-\alpha(x)\right| \le C\left(\lambda(n)+\frac{A_2^{(n)}}{S_n}\right).$$

Ushbu teorema 3-teoremani yaxshilaydi, chunki 3-teoremada 3-chi tartibli momentning chekli bo‘lishi talab etilgan.

1.5.5.-teorema. $\{\xi_n\}$ tasodifiy miqdorlar ketma-ketligi uchun δ, B shartlar o‘rinli bo‘lsa, u holda

$$\left|\overline{P}_n(x)-\varphi(x)\right| \le \frac{C}{1+x^2}\left[\lambda(n)+\frac{\alpha^3\left(w_k+A_k^{(n)}\right)}{\beta^4\lambda^3(n)\Lambda^k(n)}\right].$$

Bu natijalarda $\{\xi_n\}$ - bir xil taqsimlangan tasodifiy miqdorlar bo‘lib, ularning zichlik funksiyasi $P(x)$ bo‘lsa, va

$$M\xi_1=0,\quad D\xi_1=1, \qquad \rho=\sup_{z>0}\left\{\left|\int_{-z}^{z}x^3P(x)dx\right|+z\int_{|x|>z}x^2P(x)dx\right\}<\infty$$

shartlar o‘rinli bo‘lsin. U holda quyidagi teorema o‘rinli:

1.5.6-teorema. Agar $P(x)\le A$ bo‘lsa, quyidagi tengsizliklar o‘rinli bo‘ladi:

$$\left|\overline{P}_n(x)-\varphi(x)\right| \le C\cdot\frac{\rho\max(1;\,A^3)}{\sqrt{n}},\qquad -\infty<x<\infty,$$

$$\left|\overline{P}_n(x)-\varphi(x)\right| \le C\cdot\frac{\rho^3\max(1;\,A^5)}{(1+x^2)\sqrt{n}},\qquad -\infty<x<\infty.$$

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$$f_k(t)=Me^{it\xi_k},\qquad \bar{f}_n(t)=Me^{it\xi_n}.$$

1.5.1–Lemma. $|t|\le\frac{1}{94\lambda(n)}$ da

$$\left|\bar{f}_n(x)\right|<e^{-\frac{t^2}{4}}$$

va $|t|\le\frac{1}{3\lambda^{\frac{1}{3}}(n)}$ holda

$$\left|\bar{f}_n(t)-e^{-\frac{t^2}{2}}\right|\le 4\lambda(n)|t|^3 e^{-\frac{t^2}{2}}.$$

1**.5.2. – Lemma.** Agar $\sigma^2 = M\xi^2,\ \lambda=\sup\limits_{z>0} z\int\limits_{|x|>z} x^2 dF(x)$ bo‘lsa,

u holda $\dfrac{\sigma^2}{\lambda^{\frac{2}{3}}}<2$.

1**.5.3. – Lemma.** $|t|\le\dfrac{1}{94\lambda(n)}$ quyidagi tengsizlik o‘rinli:

$$\left|\bar{f}_n'(t)\right|\le c'\left(1+|t|\right)e^{-\frac{t^2}{4}},\quad \left|\bar{f}_n''(t)\right|\le x'\left(1+t^2\right)e^{-\frac{t^2}{4}},$$

bu yerda

$$c'=e^{\frac{32}{2209}}.$$

1**.5.4. – Lemma.** $|t|\le\dfrac{1}{3\lambda^{\frac{1}{3}}(n)}$ va $\rho_k<\infty,\ k=1,2,...$ holda quyidagini hosil qilamiz:

$$\left|\bar{f}_n'(t)-\left(e^{-\frac{t^2}{2}}\right)'\right|\le 9\lambda(n)t^2\left(1+t^2\right)e^{-\frac{t^2}{2}},$$

$$\left|\bar{f}_n''(t)-\left(e^{-\frac{t^2}{2}}\right)''\right|\le 19\lambda(n)|t|\left(1+|t|+t^2+t^4\right)e^{-\frac{t^2}{2}}.$$

1.5.3 – teoremaning isboti.

Teorema shartlaridagi quyidagiga egamiz

$$\overline{P}_n(x)-\varphi(x)=\frac{1}{2\pi}\int\limits_{\infty}^{\infty} e^{-itx}\left[\bar{f}_n(t)-e^{-\frac{t^2}{2}}\right]dt \qquad (1.5.1)$$

$\dfrac{1}{3\lambda^{\frac{1}{3}}(n)}\le\dfrac{1}{94\lambda(n)}$ bo‘lsin.

(1.5.1) dan quyidagini hosil qilamiz:

$$\left|\overline{P}_n(x)-\varphi(x)\right| \le \int\limits_{|t|\le \frac{1}{3\lambda^{\frac{1}{3}}(n)}} \left|\bar{f}_n(t)-e^{-\frac{t^2}{2}}\right| dt + \int\limits_{|t|> \frac{1}{3\lambda^{\frac{1}{3}}(n)}} e^{-\frac{t^2}{2}} dt +$$

$$+ \int\limits_{\frac{1}{3\lambda^{\frac{1}{3}}(n)} \le |t| \le \frac{1}{94\lambda(n)}} \bar{f}_n(t)dt + \int\limits_{|t|>\frac{1}{94\lambda(n)}} \left|\bar{f}_n(t)\right| dt = I_1 + I_2 + I_3 + I_4 \quad (1.5.2)$$

1.5.1. – Lemmaga ko‘ra

$$I_1 = \int\limits_{|t|\le \frac{1}{3\lambda^{\frac{1}{3}}(n)}} \left|\bar{f}_n(t)-e^{-\frac{t^2}{2}}\right| dt \le c\lambda(n) \quad (1.5.3)$$

Ko‘rinib turibdiki

$$I_2 = \int\limits_{|t|> \frac{1}{3\lambda^{\frac{1}{3}}(n)}} e^{-\frac{t^2}{2}} dt \le c\lambda(n) \quad (1.5.4)$$

1.5..1.-Lemmaning birinchi qismidan quyidagini hosil qilamiz:

$$I_3 = \int\limits_{\frac{1}{3\lambda^{\frac{1}{3}}(n)} \le t \le \frac{1}{94\lambda(n)}} \bar{f}_n(t)dt \le \int\limits_{|t|\ge \frac{1}{3\lambda^{\frac{1}{3}}(n)}} e^{-\frac{t^2}{4}} dt \le c\lambda(n) \quad (1.5.6)$$

Yuqoridagi kabi:

$$I_4 = \int\limits_{|t|>\frac{1}{94\lambda(n)}} \left|\bar{f}_n(t)\right| dt = \int\limits_{|t|>\frac{1}{94\lambda(n)}} \prod_1^n \left| f_n\left(\frac{t}{S_n}\right)\right| dt = S_n \int\limits_{|t|>\frac{1}{94 S_n \lambda(n)}} \prod_1^n \left| f_k(t)\right| dt$$

bo‘ladi.

Quyidagi tasodifiy hollarni ko‘ramiz:

1) $\dfrac{1}{94S_n\lambda(n)} \le \dfrac{\pi}{\bar{\sigma}_n}$,

2) $\dfrac{1}{94S_n\lambda(n)} > \dfrac{\pi}{\bar{\sigma}_n}$.

2-holat oson baholanishini e'tiborga olib, faqat 1 holning isbotiga to‘xtalamiz. Quyidagini yozamiz:

$$I_4 = S_n \int\limits_{\frac{1}{94 S_n \lambda(n)} < |t| \le \frac{\pi}{\bar{\sigma}_n}} \prod_1^n |f_k(t)| dt + S_n \int\limits_{|t| > \frac{\pi}{\bar{\sigma}_n}} \prod_1^n |f_k(t)| dt = I_4^1 + I_4^{11}$$

.

$$\min_{1 \le j \le n}{}^{(1)} A_j = A_{n_1} = A_1^{(n)}, \qquad \min_{1 \le j \le n}{}^{(2)} A_j = A_{n_2} = A_2^{(n)}$$

bo‘lsin, bu yerda $1 \le n_1 \le n, \quad 1 \le n_2 \le n$.

U holda [5] ishning 1.5.1.-lemmasiga ko‘ra quyidagiga egamiz

$$I_4 = S_n \int\limits_{|t| > \frac{\pi}{\bar{\sigma}_n}} \prod_1^n |f_k(t)| dt \le S_n e^{-c \sum\limits_{\substack{k=1 \\ k \ne n_1 \ne n_2}} \frac{1}{A_k^2 \sigma_k^2}} \int\limits_{-\infty}^{\infty} |f_n(t)| \, |f_{n_2}(t)| dt$$

Oxirgi integral quyidagidan oshmaydi.

$$\int\limits_{-\infty}^{\infty} \left| f_{n_1}(t) \right|^2 dt \int\limits_{-\infty}^{\infty} \left| f_{n_2}(t) \right|^2 dt \le 2\pi \sqrt{A_1^{(n)}} \cdot \sqrt{A_2^{(n)}} < 2\pi A_2^{(n)}$$

Bundan 1.5.2.-lemmaga asosan quyidagini hosil qilamiz:

$$I_4^{11} \le 2\pi S_n A_n^{(n)} e^{-c\Lambda(n)} e^{c\left[\frac{1}{A_{n_1}^2 \sigma_{n_1}^2} + \frac{1}{A_{n_2}^2 \sigma_{n_2}^2}\right]} \le c S_n A_2^{(n)} e^{-c\Lambda(n)} \le c \frac{\alpha A_2^{(n)}}{\beta \lambda(n) \Lambda^k(n)}$$

(1.5.7)

I_4^1 integralni baholaymiz.

1.5.3.-Lemmadan $|t| \le \frac{\pi}{\bar{\sigma}_n}$ bo‘lgan holda quyidagiga egamiz:

$$\left| f_j(t) \right| \le 1 - \left(1 - k^2\right) \frac{\bar{\sigma}_n^2}{8\pi} t^2 ,$$

bu yerda $k^2 = 1 - \frac{c}{A_j^2 \sigma_j^2}$.

Bundan quyidagi tengsizlik kelib chiqadi.

$$\left| f_j(t) \right| \le 1 - c \frac{\bar{\sigma}_n^2 t^2}{8\pi^2 A_j^2 \sigma_j^2} \le e^{-c \frac{\bar{\sigma}_n^2 t^2}{8\pi^2 A_j^2 \sigma_j^2}} \qquad (1.5.8)$$

(1.5.8) ni e'tiborga olib va o‘zgaruvchilarni $u = \frac{\bar{\sigma}_n}{2\pi} \sqrt{c\Lambda(n)} t$ almashtirib, quyidagini hosil qilamiz:

$$I_4^1 \le S_n \int\limits_{\frac{1}{94 S_n \lambda(n)} \le |t| \le \frac{\pi}{\bar{\sigma}_n}} e^{-c\frac{\bar{\sigma}_n^2 \cdot t^2}{8\pi^2}} \Lambda(n) dt = \frac{2\pi S}{\bar{\sigma}_n \sqrt{c\Lambda(n)}} \int\limits_{|t| > \frac{\bar{\sigma}_n}{188\pi S_n \lambda(n)}} e^{-\frac{u^2}{4}} du \le c \frac{S_n^2 \lambda(n)}{\bar{\sigma}_n^2 \Lambda(n)} e^{-c\frac{\bar{\sigma}_n^2 \Lambda(n)}{8836 S_n^2 \lambda^2(n)}} \le$$

$$\le c \frac{\beta^2}{\alpha^2} \left(\frac{\alpha^2}{\beta^3} \right)^k \frac{\lambda(n) S_n^2}{\Lambda^k(n)} \le c \left(\frac{\alpha^2}{\beta^3} \right)^k \frac{1}{\lambda(n) \Lambda^k(n)} \tag{1.5.9}$$

Oxirgi tengsizlik quyidagi munosabatni ifodalaydi:

$$S_n^2 \lambda(n) \le \frac{\alpha^2}{\beta^2 \lambda(n)}.$$

(1.5.3), (1.5.7) va (1.5.9) ga ko‘ra 1.5.3-teorema kelib chiqadi.

1.5.5-teoremaning isboti.

Faraz qilaylik: $g(t) = e^{-\frac{t^2}{2}}$.
Quyidagi tengsizlikka ega bo‘lamiz.

$$x^2 \left| \overline{P}_n(x) - \varphi(x) \right| \le \int\limits_{-\infty}^{\infty} \left| \bar{f}_n''(t) - g''(t) \right| dt \le \int\limits_{|t| \le \frac{1}{3\lambda^{\frac{1}{3}}(n)}} \left| \bar{f}_n''(t) - g''(t) \right| dt + \int\limits_{|t| > \frac{1}{3\lambda^{\frac{1}{3}}(n)}} \left| g''(t) \right| dt +$$

$$+ \int\limits_{\frac{1}{3\lambda^{\frac{1}{3}}(n)} < |t| \le \frac{1}{94\lambda(n)}} \left| f_n''(t) \right| dt + \int\limits_{|t| > \frac{1}{94\lambda(n)}} \left| \bar{f}_n''(t) \right| dt = \Omega_1 + \Omega_2 + \Omega_3 + \Omega_4 .$$

1.5.4.-lemmadan quyidagini hosil qilamiz.

$$\Omega_1 = \int\limits_{|t| \le \frac{1}{3\lambda^{\frac{1}{3}}(n)}} \left| \bar{f}_n''(t) - g''(t) \right| dt \le 19\lambda(n) \int\limits_{-\infty}^{\infty} |t| \left(1 + |t| + t^2 + t^4 \right) e^{-\frac{t^2}{2}} dt \le c\lambda(n) \tag{1.5.10}$$

Ko‘rinib turibdiki,

$$\Omega_2 = \int\limits_{|t| \ge \frac{1}{3\lambda^{\frac{1}{3}}(n)}} \left| t^2 - 1 \right| e^{-\frac{t^2}{2}} dt \le c\lambda(n) \tag{1.5.11}$$

1.5.3.-lemmadan quyidagiga egamiz:

$$\Omega_3 = \int\limits_{\frac{1}{3\lambda^{\frac{1}{3}}(n)}<|t|<\frac{1}{94\lambda(n)}} \left|\bar{f}_n''(t)\right|dt \le c \int\limits_{|t|>\frac{1}{3\lambda^{\frac{1}{3}}(n)}} \left(1+t^2\right)e^{-\frac{t^2}{4}}dt \le c\lambda(n) \qquad (1.5.12)$$

Oxirgi integralni baholaymiz.

$$\Omega_4 = \int\limits_{|t|>\frac{1}{94\lambda(n)}} \left|\bar{f}_n''(t)\right|dt$$

Ω_4 ni quyidagicha ifodalaymiz.

$$\Omega_4 = \int\limits_{\frac{1}{94\lambda(n)}<|t|<\frac{\pi S_n}{\bar{\sigma}_n}} \left|\bar{f}_n''(t)\right|dt + \int\limits_{|t|>\frac{\pi S_n}{\bar{\sigma}_n}} \left|\bar{f}_n''(t)\right|dt = \Omega_4^1 + \Omega_4^{11}.$$

Keyinchalik quyidagi tengsizlikdan foydalanamiz.

$$\left|\bar{f}_n''(t)\right| < (n+1)\max_{1\le k\ne j\le n} \prod_{\substack{r=1\\ r\ne k\ne j}} \left|f_r\left(\frac{t}{S_n}\right)\right| \qquad (1.5.13)$$

Quyidagiga ega bo‘lamiz.

$$\Omega_4^{11} = \int\limits_{|t|>\frac{\pi S_n}{\bar{\sigma}_n}} \left|\bar{f}_n''(t)\right|dt \le (n+1)\int\limits_{|t|>\frac{\pi S_n}{\bar{\sigma}_n}} \max_{1\le k\ne j\le n} \prod_{\substack{r=1\\ r\ne k\ne j}}^{n} \left|f_2\left(\frac{t}{S_n}\right)\right|dt \le 2nS_n \int\limits_{|t|>\frac{\pi}{\bar{\sigma}_n}} \max_{1\le k\ne j\le n} \prod_{\substack{r=1\\ r\ne k\ne j}}^{n} \left|f_r(t)\right|dt.$$

[5] dagi 1.5.1.-lemmaga va $\sigma_k^2 A_k^2 \ge \frac{1}{12}$ tengsizlikka ko‘ra oxirgi ifoda quyidagidan oshmaydi.

$$nS_n \exp\left\{-c\sum_{r\ne k\ne j\ne\lambda\ne 0}\frac{1}{A_r^2\sigma_r^2}\right\}\int\limits_{|t|>\frac{\pi}{\bar{\sigma}_n}}\left|f_\lambda(t)\right|\left|f_\nu(t)\right|dt \le c_n S_n e^{-c\Lambda(n)}\int\limits_{-\infty}^{\infty}\left|f_\lambda(t)\right|\left|f_\nu(t)\right|dt \le$$

$$\le c_n S_n A_4^{(n)} e^{-c\Lambda(n)} \le c\frac{\alpha^3 A_4^{(n)}}{\beta^4\lambda^3(n)\Lambda^k(n)} \qquad (1.5.14)$$

Ω_4^1 bahoga kelamiz. (1.5.13) dan foydalanib, quyidagiga ega bo‘lamiz:

$$\Omega_4^1 \le \int\limits_{\frac{1}{94\lambda(n)}<|t|<\frac{\pi S_n}{\bar{\sigma}_n}} \left|\bar{f}_n''(t)\right|dt \le \int\limits_{\frac{1}{94\lambda(n)}\le|t|<\frac{\pi S_n}{\bar{\sigma}_n}} \max_{1\le k\ne j\le n}\prod_{\substack{r=1\\ r\ne k\ne j}}^{n}\left|f_r\left(\frac{t}{S_n}\right)\right|dt \le$$

$$\le nS_n \int\limits_{\frac{1}{94\lambda(n)S_n}<|t|<\frac{\pi}{\bar\sigma_n}} \max_{1\le k\ne j\le n} \exp\left\{-c\frac{\bar\sigma_n^2 t^2}{8\pi^2}\cdot\left(\Lambda(n)-\frac{1}{A_k^2}-\frac{1}{A_j^2\sigma_j^2}\right)\right\}dt$$

oraliqdagi t ni ko‘rayotganimiz uchun:

$$\frac{\bar\sigma_n^2 t^2}{\pi^2}\left[\frac{1}{A_k^2\sigma_k^2}+\frac{1}{A_j^2\sigma_j^2}\right]<24$$

oxirgi integral quyidagidan oshmaydi.

$$c_n S_n \int\limits_{\frac{1}{94\lambda(n)S_n}<|t|<\frac{\pi}{\bar\sigma_n}} e^{\frac{\bar\sigma_n^2 t^2}{8\pi^2}\cdot\Lambda(n)}dt$$

va bundan kelib chiqadiki,

$$\Omega_4^1 \le c\left(\frac{\alpha^2}{\beta_3}\right)^{k+1}\frac{1}{\lambda^3(n)\Lambda^k(n)}. \tag{1.5.15}$$

(1.5.10), (1.5.12), (1.5.14) va (1.5.15) ni e'tiborga olib quyidagini topamiz:

$$x^2\left|\bar P_n(x)-\varphi(x)\right|\le c\left[\lambda(n)+\frac{\alpha^3\left(w_k+A_4^{(n)}\right)}{\beta^4\lambda^3(n)\Lambda^k(n)}\right] \tag{1.5.16}$$

(1.5.30 va (1.5.16) ni birlashtirib 1.5.3-teoremaning isbotini olamiz.

II BOB

$\mathbf{R^k}$ FAZODA ZICHLIK TAQSIMOTI UCHUN LOKAL LIMIT TEOREMALAR.

1-§. Masalaning qo’yilishi.

Ushbu bobda ikkinchi bobdagi masalalarni ko’p o’lchovli holga umumlashtirishga oid masalalar bilan shug’ullanamiz.

$\vec{x}=(x_1,x_2,...,x_k)$, $\vec{y}=(y_1,y_2,...,y_k)$ vektorlar R^k evklid fazosidagi k – o’lchovli vektorlar bo’lsin. $\vec{x}_1$ - transponirlangan $|\vec{x}|-\vec{x}$ vektorning uzunligi vektor, $(\vec{x},\vec{y})$ -

skalyar ko'paytma bo'lsin. $\vec{0}$ - nol vector, $\vec{e}_m$ - esa R^k dagi birlik vektor, ya'ni uning *m* – chi kordinatasi birga, qolganlari nolga teng:

$$\vec{\xi}_j = \left(\vec{\xi}_{j1}, \vec{\xi}_{j2}, ..., \vec{\xi}_{jk},\right), \qquad j = 1,2,... \qquad (2.1)$$

-*k* o'lchovli bog'liqmas tasodifiy vektorlar (t.v) ketma-ketligini qaraymiz, ular uchun quyidagilar o'rinli bo'lsin:

$$M\vec{\xi}_j = \left(M\vec{\xi}_{j1}, M\vec{\xi}_{j2}, ..., M\vec{\xi}_{jk}\right) \equiv 0,$$

$$\bar{\sigma}_j^2 = M\left|\vec{\xi}_j\right|^2 < \infty, \qquad j = 1,2,...$$

$\vec{\xi}_j$ - tasodifiy vektor $A_j,\ j = 1,2,...$ kanstanta bilan chegaralangan $p_j(\vec{x})$ - zichlik taqsimotiga ega bo'lsin deb faraz qilamiz.

Quyidagi belgilashlarni kiritamiz:

$$\vec{S}_n = \sum_{j=1}^{n} \vec{\xi}_j, \qquad D_n = M\vec{S}_n\vec{S}_n, \qquad B_n^2 = \sum_{j=1}^{n} \vec{\sigma}_j^2,$$

$$\vec{Y}_n = \vec{S}_n K_n, \qquad \vec{Z}_n = \vec{S}_n \vec{M}_n$$

Bu yerdagi K_n matritsa shundayki, $K_n^{'} D_n K_n = I$ tenglik bajariladi. Unda K_n - transponirlangan matritsa, I-esa birlik matritsa, M_n - esa *m* – chi diagonal elementi B_{nm}^{-1} bo'lgan diagonal matritsa, bu yerda B_{nm}^2 D_n matritsaning *m* – chi diagonal elementi, $m = 1,2,..., k$.

$\vec{\xi}_j$ - t.v harakteristik funksiyasini (x.f) $f_j(\vec{t}), \vec{Y}_n$ - t.v. ning t.f, z.f. va x.f. larini

$V_n(\vec{x}),\ U_n(\vec{x}),\ va\ \ f_{\vec{Y}_n}(\vec{x}),$ kabi, $\vec{Z}_n$ - t.v. ning t.f, z.f. va x.f. larinibelgilaymiz

$\Phi(\vec{X}, \vec{T}),\quad \varphi(\vec{X}, \vec{T}),$ va $h(\vec{t}, \vec{T}),$ kabi belgilaymiz.

2-§. ASOSIY TEOREMALAR.

1 – teorema. Agar (2.1.) ketma-ketlik uchun

1) $$\sup_{\vec{x}} |U_n(\vec{x}) - \Phi(\vec{x}, \vec{I})| \to 0,\ \ n \to \infty,$$

2) Barcha $j=1,2,...$ lar uchun shunday *M* kostanta mavjudki

$$\vec{\sigma}_j^k A_j \le M$$

3) $\max\limits_{1\le j\le n} \sigma_j \le \beta_n^p, \qquad \min\limits_{1\le j\le n} \sigma_j \ge B_n^{-p}$

bu yerda *p* va *R* lar musbat sonlar, bundan tashqari $\rho<1,\ R$ esa chekli, shartlar o'rinli bo'lsa, u holda $n\to\infty$ da

$$\sup_{\vec{x}}\left(1+|\vec{x}|^2\right)\left|U_n(\vec{x})-\varphi(\vec{x};I)\right|\to 0$$

2 – teorema. Agar (2.1) ketma-ketlik uchun 1-teoremaning 2) va 3) shartlari o'rinli va

$$1')\qquad \sup_{\vec{x}}|V_n(\vec{x})-\Phi(\vec{x};\vec{T})\to 0, \qquad n\to\infty$$

shart bajarilsa, bu yerda *T* – aynimagan, u holda $n\to\infty$ da

$$\sup_{\vec{x}}(1+|\vec{x}|)|V_n(\vec{x})-\varphi(\vec{x};T)|\to 0, \qquad n\to\infty$$

1 (2) – teoremalarning tasdiqlari quyidagi tasdiqlarga teng kuchli bo'lishini ta'kidlaymiz:

$$\sup_{\vec{x}}|\vec{x}|^m\,|U_n(\vec{x})-\varphi(\vec{x};I)|\to 0, \qquad n\to\infty \quad da,$$

$$\left(\sup_{\vec{x}}|\vec{x}|^m\,|V_n(\vec{x})-\varphi(\vec{x};\vec{T})|\to 0, \qquad n\to\infty \quad da\right).$$

$k=1$ va $m=0$ bo'lganda ikkala teorema $[26]$ ishda bayont etilgan va ancha umumiyrq formada $[22]$ ishda isbotlangan. Eng umumiy natija $[27]$ishda olingan.

$k>1$ va $m=0$ da 1 va 2 teoremalar $[29]$ ishda keltirigan va bunda 3 shart o'rniga kamroq cheklashlarga ega $k>1$

$$\frac{\max\limits_{1\le j\le n}\sigma_j}{B_n}\to 0, \qquad n\to\infty \qquad (2.2)$$

Shartdan foydalanilgan.

$k=1$ va $0\le m\le 2$ da teoremalar $[25]$ da isbotlangan bitta teoremaga aylanadi.

Teoremalarni isbotlash uchun bir nechta lemma kerak bo'ladi.

3-§. YORDAMCHI TEOREMALAR.

1 – lemma. Agar *k* – o'lchovli $\vec{\xi}$ t.v. *A* – konstanta bilan chegaralangan $p(\vec{x})$ z.f ga ega bo'lsa va $\sigma^2 = M\left|\vec{\xi}\right|^2 < \infty$ bajarilsa, u holda $f(\vec{t})$ h.f. uchun quyidagi tengsizlik o'rinli:

$$\sup_{|\vec{t}|>\frac{\pi}{\sigma}} \left| f(\vec{t}) \right| \le 1 - \frac{C_k}{\sigma^{2k} A^2}$$

bu yerda $C_k = C' Q_2^{k-1}, \quad C'$ - abs. konstanta, Q_{k-1} - esa $(k-1)$ - o'lchovli birlik sfera hajmi. (isboti $[30]$ da 37-bet).

Quyidagi lemma Kramerning ($[30]$, 37 bet) ma'lum lemmasining ko'p o'lchovli analogidir.

2 – lemma. Agar $f(\vec{t})$ ko'p o'lchovli h.f. $H| \ge b$ da $\left| f(\vec{t}) \right| \le p < 1$ shartni qanoatlantirsa, u holda $|t| < b$ da

$$| \; f(t) \le \exp\left\{ -\frac{1-p^2}{8b^2} \left|\vec{t}\right|^2 \right\}$$

(isboti $[29]$ da).

3 – lemma. Agar $n \to \infty$ da

$$\sup_{\vec{x}} \left| U_n(\vec{x}) - \varphi(\vec{x}; I) \right| \to 0$$

tekis bajarilsa, u holda $n \to \infty$ da

$$\frac{\partial^2 f_{\vec{Y}_n}(\vec{t})}{\partial t^2 m} \to \frac{\partial^2 h(\vec{t}; I)}{\partial t^2 m}$$

shart tekis bajariladi, $m = 1,2,..., k.$

Isboti. $U_n(\vec{x})$ va $\varphi(\vec{x}, \vec{I})$ larning ta'rifidan quyidagilar kelib chiqadi:

$$\int_{R^k} (\vec{x};\vec{e}_m) U_n(\vec{x}) d\vec{x} = 0, \quad \int_{R^k} (\vec{x};\vec{e}_m)^2 U_n(\vec{x}) d\vec{x} = 1,$$

$$\int_{R^k} (\vec{x};\vec{e}_m) \varphi(\vec{x};I) d\vec{x} = 0.$$

$$\int_{R^k} (\vec{x};\vec{e}_m)^2 \varphi(\vec{x};I) d\vec{x} = 1, \qquad m = 1,2,\ldots,k$$

Har bir $\varepsilon > 0$ uchun shunday $R = R(\varepsilon)$ topiladiki,

$$\int_{R^k} (\vec{x};\vec{e}_m)^2 \varphi(\vec{x};I) d\vec{x} > 1 - \frac{\varepsilon}{2},$$

Endi fiksirlangan $R = R(\varepsilon)$ uchun shunday $n_0(R) = n_0(\varepsilon)$ nomer topiladiki, barcha $n > n_0(\varepsilon)$ lar uchun

$$| U_n(\vec{x}) - \varphi(\vec{x};I) | < \frac{3\varepsilon}{2^{k+1} R^{k+2}}$$

U holda

$$\int_{|\vec{x}| \le R} (\vec{x};\vec{e}_m)^2 U_n(\vec{x}) d\vec{x} \ge \int_{|\vec{x}| \le R} (\vec{x};\vec{e}_m)^2 \varphi(\vec{x};I) d\vec{x} -$$

$$- \frac{3\varepsilon}{2^{k+1} R^{k+2}} \int_{|\vec{x}| \le R} (\vec{x};\vec{e}_m)^2 d\vec{x} \ge \int_{|\vec{x}| \le R} (\vec{x};\vec{e}_m)^2 \varphi(\vec{x};I) d\vec{x} - \frac{\varepsilon}{2} >$$

$$> 1 - \frac{\varepsilon}{2} - \frac{\varepsilon}{2} = 1 - \varepsilon$$

Bundan barcha $n > n_0(\varepsilon)$ va $m = 1,2,\ldots,k$ lar uchun

$$\int_{|\vec{x}| \le R} (\vec{x};\vec{e}_m)^2 U_n(\vec{x}) d\vec{x} < \varepsilon$$

So’ngra quyidagilarga ega bo’lamiz

$$\frac{\partial \int_{\vec{Y}_n}(\vec{t})}{\partial t^2 m} - \frac{\partial^2 h(\vec{t};I)}{\partial \vec{t}^{\,2} m} = -\int_{R^k} e^{i(\vec{t};\vec{x})}(\vec{x},\vec{e}_m)^2 \left(U_n(\vec{x}) - \varphi(\vec{x};I)\right) d\vec{x}$$

Demak, barcha $n > n_0(\varepsilon)$ va $m = 1,2,\ldots,k$ lar uchun

$$\left| \frac{\partial \int_{\vec{Y}_n}(\vec{t})}{\partial \vec{t}^{\,2} m} - \frac{\partial^2 h(\vec{t};I)}{\partial \vec{t}^{\,2} m} \right| \le \int_{|\vec{x}|\le R} (\vec{x},\vec{e}_m)^2 \, |U_n(\vec{x}) - \varphi(\vec{x};I) d\vec{x} +$$

$$+\frac{\varepsilon}{2} < \frac{3\varepsilon}{2^{k+1} R^{k+2}} \int_{|\vec{x}|\le R} (\vec{x},\vec{e}_m)^2 d\vec{x} + \frac{3}{2}\varepsilon = \frac{\varepsilon}{2} + \frac{3}{2}\varepsilon = 2\varepsilon$$

Lemma isbot bo’ldi.

4-§. ASOSIY TEOREMALARNING ISBOTI.

1 – teoremaning isboti. $m = 0$ uchun teoremaning to’g’riligi oson kelib chiqadi, chunki $[30]$ ga ko’ra $n \to \infty$ da

$$\frac{\max\limits_{1\le j\le n} \sigma_j}{B_n} \to 0$$

ni ko’rsatish kifoya.

Haqiqatan ham 3) – shartdan quyidagi kelib chiqadi.

$$\frac{\max\limits_{1\le j\le n} \sigma_j}{B_n} \le B_n^{\rho-1}, \qquad 0 < \rho < 1. \qquad (2.3)$$

So’ngra, $B_n \to C$ bo’lsin, *C* – qandaydir o’zgarmas.

U holda

$\min_{1\le j\le n}\sigma_j \ge B_n^{-R}$ shartga ko’ra $B_n \to \infty$.

Hosil qilingan zidlik $B_n \to \infty$ deb ta’kidlashga imkon tug’diradi. Natijada (2.3) tengsizlikka qaytib, (2.2) munosabatga ega bo’lamiz.

Teoremani $m=2$ uchun isbatlash qoladi. (2.1) ketma-ketlikning $\vec{\xi}_j$ t.v. lari chegaralangan zichlikka ega bo’lgani uchun, u holda $\vec{Y}_n$ t.v. ham zichlikga ega

$$U_n(\vec{x}) = \frac{1}{(2\pi)^k}\int_{R^k} e^{-i(\vec{t},\vec{x})} \underset{\vec{Y}_n}{f}(\vec{t})d\vec{t}$$

Bundan quyidagini topamiz

$$(2\pi)^k |\vec{x}|^2 |U_n(\vec{x}) - \varphi(\vec{x};I)| \le \sum_{m=1}^{k}\int_{R^k}\left|\frac{\partial^2 \underset{\vec{Y}_n}{f}(\vec{t})}{\partial \vec{t}_m^{\,2}} - \right.$$

$$\left. - \frac{\partial^2 h(\vec{t})}{\partial \vec{t}_m^{\,2}}\right| d\vec{t} \le \sum_{m=1}^{k}\left(I_m^{(1)} + I_m^{(2)} + I_m^{(3)} + I_m^{(4)} + I_m^{(5)}\right) \qquad (2.4)$$

Bu yerda

$$\varphi(\vec{x};I) = \frac{1}{(2\pi)^{\frac{k}{2}}|I|^{\frac{1}{2}}}\exp\left\{-\frac{\vec{x}I\vec{x}'}{2}\right\} = \frac{1}{(2\pi)^{\frac{k}{2}}}\exp\left\{-\frac{|\vec{x}|^2}{2}\right\} \qquad (2.5)$$

limitik normal qonunning zichligi,

$$I_m^{(1)} = \int_{|\vec{t}|\le A}\left|\frac{\partial^2 \underset{\vec{Y}_n}{f}(\vec{t})}{\partial \vec{t}_m^{\,2}} - \frac{\partial^2 h(\vec{t};I)}{\partial \vec{t}_m^{\,2}}\right| d\vec{t},$$

$$I_m^{(2)}=\int_{|\vec{t}|\le A}\left|\frac{\partial^2 h(\vec{t};I)}{\partial \vec{t}_m^{\,2}}\right|d\vec{t}\,,\qquad I_m^{(3)}=\int_{|\vec{t}|\le A\le \frac{IB_n}{\max\limits_{1\le j\le n}\sigma_j}}\left|\frac{\partial^2 \oint_{\vec{Y}_n}(\vec{t})}{\partial \vec{t}_m^{\,2}}\right|d\vec{t}\,,$$

$$I_m^{(4)}=\int_{\frac{\pi B_n}{\max\limits_{1\le j\le n}\sigma_j}<|\vec{t}|\le \frac{\pi B_n}{\min\limits_{1\le j\le n}\sigma_j}}\left|\frac{\partial^2 \oint_{\vec{Y}_n}(\vec{t})}{\partial \vec{t}_m^{\,2}}\right|d\vec{t}\,,$$

$$I_m^{(5)}=\int_{|\vec{t}|>\frac{\pi B_n}{\min\limits_{1\le j\le n}\sigma_j}}\left|\frac{\partial^2 \oint_{\vec{Y}_n}(\vec{t})}{\partial \vec{t}_m^{\,2}}\right|d\vec{t}$$

$$h(\vec{t};I)=\exp\left\{-\frac{\vec{t}I\vec{t}\,'}{2}\right\}=\exp\left\{-\frac{|\vec{t}|^2}{2}\right\} \qquad (2.6)$$

Integrallarni baholashga o’tamiz. Teoremaning shartlari va 3-lemmaga ko’ra $n\to\infty$ da

$$I_m^{(1)}\to 0$$

So’ngra, $v=0,1,2\ va\ m=1,2,...,k$ lar uchun quyidagini tekshirish qiyin emas

$$\left|\frac{\partial^v \vec{t}I\vec{t}\,'}{\partial \vec{t}_m^{\,v}}\right|\le 2k\left(\vec{t}I\vec{t}\,'\right)^{\frac{2-v}{2}}=2k|\vec{t}|^{2-v} \qquad (2.7)$$

$h(\vec{t};I)$ ning ta’rifidan quyidagini olamiz

$$\frac{\partial^2 h(\vec{t};I)}{\partial \vec{t}_m^{\,2}} = e^{-\frac{\vec{t}I\vec{t}'}{2}}\left[\frac{1}{4}\left(\frac{\partial}{\partial t_m}(\vec{t}I\vec{t}')^2 - \frac{1}{2}\frac{\partial^2}{\partial t_m^2}(\vec{t}I\vec{t}')\right)\right]$$

Bundan, (2.7) bahoga ko'ra

$$\left|\frac{\partial^2 h(\vec{t};I)}{\partial \vec{t}_m^{\,2}}\right| \le e^{-\frac{\vec{t}I\vec{t}'}{2}}\left(k + k^2(\vec{t}I\vec{t}')\right) \qquad (2.8)$$

Natijada, ixtiyoriy $\varepsilon > 0$ uchun A yetarlicha katta bo'lganda

$$I_m^{(2)}\int_{|\vec{t}|>A} e^{-\frac{|\vec{t}|^2}{2}}\left(k + k^2|\vec{t}|^2\right)dt \le \varepsilon$$

Quyidagilar o'rinli bo'lishiga e'tibor qaratamiz

$$f_{\vec{Y}>A}(\vec{t}) = f_{\vec{S}_n}(\vec{t}K_n') = \prod_{j=1}^{n} f_j(\vec{t}K_n'), \qquad (2.9)$$

$$\frac{\partial f_j(\vec{t}K_n')}{\partial t_m} = i\int_{R^k} e^{i(\vec{t}K_n',\vec{x})}\frac{\partial(\vec{t}K_n',\vec{x})}{\partial t_m}p_j(\vec{x})d\vec{x} =$$

$$= i\int_{R^k}\left(1 + i(\vec{t}K_n',\vec{x})e^{i\theta(\vec{t}K_n',\vec{x})}\right)(\vec{x}K_n',\vec{e}_m)p_j(\vec{x})d\vec{x},$$

bu yerda θ qandaydir miqdor, $|\theta| \le 1$. Bulardan quyidagilarga ega bo'lamiz

$$\left|\frac{\partial f_j(\vec{t}K_n')}{\partial t_m}\right| \le \int_{R^k}\left|(\vec{x}K_n,\vec{t})\right|\left|(\vec{x}K_n,\vec{e}_m)\right|p_j(\vec{x})d\vec{x} \le$$

$$\le |\vec{t}|\int_{R^k}|\vec{x}K_n|^2 p_j(\vec{x})d\vec{x} = |\vec{t}|M|\xi_j K_n|^2,$$

$$\left|\frac{\partial^2 \oint_j \left(\vec{t} K_n'\right)}{\partial t_m}\right| \leq M\left|\xi_j K_n\right|^2$$

Natijada, $K_n' D_n K_n = I$ dan foydalanib, topamiz

$$\sum_{j=1}^{n}\left|\frac{\partial \oint_i \left(\vec{\mathrm{t}} \mathrm{K_n}'\right)}{\partial t_m}\right| \leq \left|\vec{t}\right| \sum_{j=1}^{n} M\left|\vec{\xi}_j K_n\right|^2 =$$

$$= \left|\vec{t}\right| \sum_{j=1}^{n} M\left(\vec{\xi}_j, \vec{\xi}_j K_n K_n'\right) = \left|t\right| \sum_{j=1}^{n} M\left(\vec{\xi}_j, \vec{\xi}_j D_n^{-1}\right) = k\left|\vec{t}\right| \qquad (2.10)$$

va

$$\frac{\partial^2 \oint_{S_n} \left(\vec{t} K_n'\right)}{\partial t_m^2} = \sum_{e=1}^{n} \frac{\partial \oint_i \left(\vec{\mathrm{t}} \mathrm{K_n}'\right)}{\partial t_m} \prod_{\substack{j=1 \\ j \neq l}} \oint_i \left(\vec{\mathrm{t}} \mathrm{K_n}'\right) +$$

$$+ \sum_{l=1}^{n} \frac{\partial \oint_l \left(\vec{\mathrm{t}} \mathrm{K_n}'\right)}{\partial t_m} \sum_{\substack{j=1 \\ j \neq l}}^{n} \frac{\partial \oint_l \left(\vec{\mathrm{t}} \mathrm{K_n}'\right)}{\partial t_m} \prod_{\substack{r=1 \\ r \neq j \neq l}}^{n} \oint_l \left(\vec{\mathrm{t}} \mathrm{K_n}'\right) \qquad (2.10)$$

bo’lgani uchun, (2.10) va (2.11) tengsizliklarga asosan:

$$\left|\frac{\partial^2 \int_{\vec{Y}_n} \left(\vec{t}\right)}{\partial t_m^2}\right| = \left|\frac{\partial^2 \int_{\vec{S}_n} \left(\vec{t} K_n'\right)}{\partial t_m^2}\right| \leq$$

$$\leq \max_{1 \leq j \neq l \leq n} \prod_{r=1} \left|\oint_r \left(\vec{t} K_n'\right)\right| \left(K + K^2 \left|\vec{t}\right|^2\right) \qquad (2.13)$$

1 – lemmaga ko’ra

$$\sup_{|\vec{t}|>\frac{\pi}{\sigma_j}} \left| \phi_j(\vec{t}) \right| \le 1 - \frac{C_k}{\sigma_j^{2K} A_j^2}, \qquad (2.14)$$

va 2-lemma va teoremaning ikkinchi shartiga binoan

$$\sup_{|\vec{t}|<\frac{\pi}{\sigma_j}} \left| \phi_j(\vec{t}) \right| \le \exp\left\{ -\frac{C_k \sigma_j^2}{8\pi^2 \sigma_j^{2K} A_j^2} |\vec{t}| \right\} \le \exp\left\{ -\frac{C_k \sigma_j^2}{8\pi^2 M^2} |\vec{t}|^2 \right\} \qquad (2.15)$$

Yetarlicha katta n lar uchun (2.2) munosabatni eslab, $|\vec{t}| \le \frac{\pi}{\max\limits_{1\le j\le n} \sigma_j}$ intervalda quyidagicha bo'lishini topamiz

$$\max_{1\le j\ne l\le n} \prod_{\substack{r=1 \\ r\ne j\ne l}} \left| \phi_r(\vec{t}) \right| \le \exp\left\{ -\frac{C_k B_n^2}{16\pi^2 M^2} |\vec{t}|^2 \right\} \qquad (2.16)$$

(2.13) tengsizlikdan quyidagi kelib chiqadi

$$I_m^{(3)} \le \int_{l<|\vec{t}|<\frac{\pi B_n}{\max\limits_{1\le j\le n}}} \max_{1\le j\ne l\le n} \prod_{\substack{r=1 \\ r\ne j\ne l}}^{n} \left| \phi_r(\vec{t} K_n') \right| \left(K + K^2 |\vec{t}|^2 \right) d\vec{t}$$

$\vec{t} K_n' = \vec{t}\,'$ o'zgaruvchi almashtiramiz. Ko'rish osonki almashtirishning yakobiani

$$\left| K_n^{-1} \right| = \sqrt{|D_n|} = \sqrt{\lambda_1 \lambda_2 ... \lambda_k} \le \prod_{m=1}^{k} B_{nm} \le B_n^k$$

va

$$\left| \vec{t}\,'(K_n^{-1}) \right|^2 \le \lambda_1\left((K_n^{-1})'(K_n^{-1}) \right) |\vec{t}|^2 = \lambda_1(D_n) |\vec{t}\,'|^2 \le B_n^2 |\vec{t}\,'|^2$$

Chunki matritsaning xos qiymatlari musbat va

$$\sum_{m=1}^{k}\lambda_m = \sum_{m=1}^{k} B_{nm}^2 = B_n^2$$

(2.16) tengsizlikdan foydalanib, topamiz

$$I_m^{(3)} \le B_n^k \int_{A<\left|\vec{t}\left(K_n^{-1}\right)\right|<\frac{\pi B_n}{\max\limits_{1\le j\le n}}} \max_{1\le j\ne l\le n} \prod_{\substack{r=1\\ r\ne j\ne l}}^{n} \left|\oint_r \left(\vec{t}\right)\right| \left(K + K^2 B_n^2 \left|\vec{t}\right|^2\right) d\vec{t} \le$$

$$\le \int_{\left|\vec{t}\right|>\frac{A}{}} \exp\left\{-\frac{C_k B_n^2 \left|\vec{t}\right|^2}{16\pi^2 M}\right\}\left(K + K^2 B_n^2 \left|\vec{t}\right|^2\right) d\vec{t} \le$$

$$\le B_n^k \exp\left\{-\frac{C_k A^2}{326\pi^2 M}\right\} \int_{R^k} \exp\left\{-\frac{C_k B_n^2 \left|\vec{t}\right|^2}{32\pi^2 M}\right\} \cdot$$

$$\cdot \left(K + K^2 B_n^2 \left|\vec{t}\right|^2\right) d\vec{t} \le \left(\frac{3\sqrt{2M}\pi}{\sqrt{C_k}}\right)^k \exp\left\{-\frac{C_k A^2}{32\pi^2 M}\right\} \cdot$$

$$\cdot \int_{R^k} \exp\left\{-\frac{|t|^2}{2}\right\}\left(K + \frac{18\pi^2 M}{C_k} K^2 \left|\vec{t}\right|^2\right) d\vec{t} \to 0,$$

$A \to \infty$ bo'lganda, chunki $\left|\vec{t}\right|^2 \exp\left\{-\frac{\left|\vec{t}\right|^2}{2}\right\}$ funksiya R^k da integrallanuvchi.

Endi $I_m^{(4)}$ ni baholaymiz. (2.13) tengsizlikni e'tiborga olib va $I_m^{(3)}$ ni baholashdagi kabi o'zgaruvchini almashtirib, topamiz

$$I_m^{(4)} \le \left(K + K^2 \frac{\pi^2 B_n^2}{\min\limits_{1\le j\le n} \sigma_j^2}\right) B_n^k \cdot$$

$$\cdot \int_{\frac{\pi B_n}{\max\limits_{1\le j\le n} \sigma_j} \le \left|\vec{t}\left(K_n'\right)^{-1}\right| \le \frac{\pi B_n}{\min\limits_{1\le j\le n} \sigma_j}} \max_{1\le j\ne l\le n} \prod_{\substack{r=1\\ r\ne j\ne l}}^{n} \left|\oint_r \left(\vec{t}\right)\right| d\vec{t}. \qquad (2.17)$$

(2.14) va (2.15) tengsizliklardan

$$\frac{\pi B_n}{\max\limits_{1\le j\le n}\sigma_j} \le \left|\vec{t}\left(K_n'\right)^{-1}\right| \le \frac{\pi B_n}{\min\limits_{1\le j\le n}\sigma_j}$$

Intervalda

$$\left|\oint_r\left(\vec{t}\right)\right| \le \exp\left\{-\frac{C_k}{8M}\frac{\sigma_r^2}{\max\limits_{1\le j\le n}\sigma_j}\right\}$$

Natijada,

$$\max_{1\le j\ne l\le n}\prod_{\substack{r=1\\ r\ne j\ne l}}^{n}\left|\oint_r\left(\vec{t}\right)\right| \le \left|\oint_{L_1}\left(\vec{t}\right)\right|\left|\oint_{L_2}\left(\vec{t}\right)\right|\cdot$$

$$\cdot\exp\left\{-\frac{C_k}{8M}\sum_{\substack{r=1\\ r\ne j\ne l\\ r\ne l_1\ne L_2}}\frac{\sigma_r^2}{\max\limits_{1\le j\le n}s_j^2}\right\} \le \left|\oint_{L_1}\left(\vec{t}\right)\right|\left|\oint_{L_2}\left(\vec{t}\right)\right|\cdot$$

$$\cdot\exp\left\{\frac{4C_k}{8M}\right\}\exp\left\{-\frac{C_k B_n^2}{8M\max\limits_{1\le j\le n}\sigma_j^2}\right\}$$

Bundan, (2.17) tengsizlik ko’p o’lchovli Parseval tengligi va teoremasining uchinchi shartiga ko’ra $n\to\infty$ da quyidagiga ega bo’lamiz:

$$I_m^{(4)} \le \left(K + K^2 \frac{\pi^2 B_n^2}{\min\limits_{1\le j\le n} \sigma_j^2} \right) \exp\left\{ \frac{4C_k}{8M} \right\} \cdot$$

$$\cdot B_n^k \exp\left\{ - \frac{C_k B_n^2}{8M \max\limits_{1\le j\le n} \sigma_j^2} \right\} \int_{R^k} \left| \oint_{L_1} (\vec{t}) \right| \left| \oint_{L_2} (\vec{t}) \right| d\vec{t} \le$$

$$\le (2\pi)^k \sqrt{A_{L_1} A_{L_2}} \exp\left\{ \frac{4C_k}{8M} \right\} \left(K + K^2 \pi^2 B_n^{2(R+1)} \right) \cdot$$

$$\cdot B_n^k \exp\left\{ - \frac{C_k}{8M} B_n^{2(1-\rho)} \right\} \to 0$$

$I_m^{(5)}$ integralni baholash qoldi. Gyolderning umumlashgan tengsizligidan foydalanib, topamiz ($[30]$, 197 bet):

$$\left| \frac{\partial}{\partial t_m} \oint_j (\vec{t} K_n') \right| = \left| i \int_{R^k} e^{i(\vec{t} K_n', \vec{x})} \frac{\partial}{\partial t_m} (\vec{t} K_n', \vec{x}) p_j(\vec{x}) d\vec{x} \right| \le$$

$$\le \int_{R^k} \left| (\vec{x} K_n, \vec{e}_m) \right| p_j(\vec{x}) d\vec{x} \le \left(\int_{R^k} \left| \vec{x} K_n \right|^2 p_j(\vec{x}) d\vec{x} \right)^{\frac{1}{2}} = \left(M \left| \xi_j K_n \right|^2 \right)^{\frac{1}{2}}.$$

Bundan,

$$\sum_{i=1}^{n} \left| \frac{\partial}{\partial t_m} \oint_l (\vec{t} K_n') \right| \le \sum_{l=1}^{n} \left(M \left| \vec{\xi}_l K_n \right|^2 \right)^{\frac{1}{2}} \le$$

$$\le \sqrt{n} \left(\sum_{l=1}^{n} M \left| \vec{\xi}_l K_n \right|^2 \right)^{\frac{1}{2}} = \sqrt{Kn} \qquad (2.18)$$

(2.11) va (2.18) tengsizliklardan foydalanib (2.12) munosabatdan quyidagini topamiz

$$\left|\frac{\partial}{\partial t_m}\oint_{\vec{Y}_n}\left(\vec{t}\right)\right| \le \left(n+1\right)\max_{1\le j\ne l\le n}\prod_{\substack{r=1\\ r\ne j\ne l}}^{n}\left|\oint_r\left(\vec{t}K_n'\right)\right| \qquad (2.19)$$

$|t| \ge \pi \Big/ \min\limits_{l\le j\le n}\sigma_j$ bo'lganda 1-lemma va teoremaning ikkinchi shartida ko'ra,

$$\max_{1\le j\ne l\le n}\prod_{\substack{r=1\\ r\ne j\ne l}}^{n}\left|\oint_r\left(\vec{t}\right)\right| \le \left|\oint_{L_1}\left(\vec{t}\right)\right|\left|\oint_{L_2}\left(\vec{t}\right)\right|\exp\left\{-\frac{C_K}{M}(n-4)\right\} \qquad (2.20)$$

Natijada $I_m^{(4)}$ integrallarni baholashdagi kabi, (2.19) va (2.20) tengsizliklardan foydalanib, quyidagilarga ega bo'lamiz

$$I_m^{(5)} \le K(n+1)B_n^K\int_{\left|\vec{t}\left(K_n^{-1}\right)'\right|\ge \frac{\pi B_n}{\min\limits_{1\le j\le n}\sigma_j}}\max_{1\le j\ne l\le n}\prod_{\substack{r=1\\ r\ne j\ne l}}\left|\oint_r\left(\vec{t}\right)\right|d\vec{t} \le$$

$$\le K(n+1)B_n^K\exp\left\{-\frac{C_k}{M}(n-4)\right\}\int_{R^K}\left|\oint_{L_1}\left(\vec{t}\right)\right|\left|\oint_{L_2}\left(\vec{t}\right)\right|d\vec{t} \le$$

$$\le (2\pi)^K\left(A_{L_1}A_{L_2}\right)^{\frac{1}{2}}K(n+1)B_n^K\exp\left\{-\frac{C_k}{M}(n-4)\right\}.$$

$B_n = 0\left(e^{\beta(K)n}\right),\ \beta(K) > 0$ munosabatga asosan, $n\to\infty$ da

$$I_m^{(5)} \to 0$$

ga ega bo'lamiz. Teorema isbot bo'ldi.

2 – teoremaning isboti ham xuddi 1 – teoremaning isboti kabi bajariladi, faqat prinsipial qiymatlariga olib kelmaydigan ayrim o'zgartirishlar kiritiladi ($[25]$ga qarang)

III - BOB

ZICHLIK FUNKSIYASI CHEGARALANMAGAN TASODIFIY MIQDORLAR KETMA KETLIGI UCHUN LOKAL LIMIT TEOREMALAR

1-§. $n_0 > 1$ HOL UCHUN LOKAL TEOREMADA YAQINLASHISH TEZLIGINI BAHOLASH

$$\xi_1,\ \xi_2,\ \xi_3,\ \dots,\ \xi_n,\ \dots$$

uzluksiz taqsimlangan tasodifiy miqdorlar ketma-ketligi bo‘lib, ular $P(x)$ umumiy zichlik funksiyaga va chekli o‘rta qiymat hamda dispersiyaga ega bo‘lsinlar.

3.1.1-Teorema. (B.V.Gnedenko). $\{\xi_n\}$ bog‘liq bo‘lmagan tasodifiy miqdorlar ketma-ketligi umumiy $F(x)$ taqsimot funksiyaga ega bo‘lsin va ularning o‘rta qiymatlari hamda dispersiyalari chekli bo‘lib, biror n_0 - nomerdan boshlab $\frac{S_n - A_n}{\sqrt{nD\xi_1}}$ yig‘indi $\overline{P}_n(x)$ zichlik funksiyaga ega bo‘lsin.

$$\overline{P}_n(x) - \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \to 0$$

munosabat $n \to \infty$ da $x\ (-\infty < x < \infty)$ ga nisbatan tekis bajarilishi uchun $P_{n_0}(x) < \infty$ tengsizlikni qanoatlantiruvchi n_0 sonning topilishi zarur va yetarlidir.

$\xi_1, \xi_2, \dots, \xi_n, \dots$ bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligidagi har bir $\xi_j\,(j = 1,2,\dots)$ qo’shiluvchining zichlik funksiyasi chegaralanmagan, ya’ni $\sup\limits_x p(x) = \infty$ shart bajariluvchi, lekin biror $n_0 > 1$ nomerdan boshlab $S_{n_0} = \xi_1 + \dots + \xi_n$ yig’indining zichlik funksiyasi $p_{n_0}(x)$ biror son bilan chegaralangan bo’ladi.

Masalan, zichlik funksiyasi

$$p(x) = \begin{cases} 0, & agar\ |x| > 1 \quad bo'lsa, \\ \frac{1-\alpha}{2}\frac{1}{|x|^a} & agar\ |x| > 1 \quad bo'lsa, \end{cases}$$

Bunda $0 < \alpha < 1$ ko’rinishda bo’lgan tasodifiy miqdorlar ketma-ketligini qaraydigan bo’lsak, $x = 0$ nuqtada bu funksiya chegaralangan emas, ya’ni

$$\sup_{x=0} p(x) = \frac{1-\alpha}{2} \cdot \frac{1}{o^{\alpha}} = \infty$$

Biroq, $n_0 = \left[\frac{1}{1-\alpha}\right] + 1$ nomerlardan boshlab

$$S_{n_0} = \xi_1 + \xi_2 + \dots + \xi_{n_0}$$

yig'indining zichlik funksiyasi chegaralangan bo'ladi.

Zichlik funksiyasi yuqoridagi ko'rinishda bo'lgan tasodifiy miqdorning barcha momentlari chekli bo'lishini bevosita hisoblab ko'rish qiyin emas.

Yana umumiy holga qaytadigan bo'lsak, zichlik funksiyalari chegaralangan

$$\xi_1, \xi_2, \dots, \xi_n, \dots$$

tasodifiy miqdorlar o'rniga, zichlik funksiyalari chegaralangan

$$\xi_{1n_0}, \xi_{2n_0}, \dots, \xi_{kn_0}, \dots \qquad (*)$$

tasodifiy miqdorlar ketma-ketligiga o'tib olish lozim bo'ladi, bu yerda

$$\xi_{kn_0} = \xi_{(k-1)n_0+1} + \dots + \xi_{kn_0}.$$

Bo'lgani uchun $(*)$ ketma-ketlikka nisbatan mazkur ishning I – bobida keltirilgan teoremalarga o'xshash teoremalarni isbotlash mumkin, tabiiyki, bu holda olingan baholarda $n_0 > 1$ son u yoki bu darajada ishtirok etadi.

Masalan,

$$\sup\left|\overline{P}_n(x) - \varphi(x)\right| \le \frac{c}{\sqrt{n}}$$

ko'rinishda olingan *x* ga nisbatan tekis baholarga nisbatan, *x* ga nisbatan notekis bo'lgan

$$\left|\overline{P}_n(x) - \varphi(x)\right| \le \frac{C_1}{\sqrt{n}\left(1 + |x|^m\right)}, \qquad m = 1,2$$

ko'rinishdagi baholarda aniqlik yuqori bo'ladi. Mazkur ishda *x* ga nisbatan notekis va n_0 ning darajasi aniq bo'lgan baholarni olish maqsad qilib qo'yilgan.

$\{\xi_n\}$ – bog'liqmas, bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligi bo'lib, $F(x)$ ularning taqsimot funksiyasi va $M\xi_1 = 0, \ \ D\xi_1 = \sigma^2 < \infty$ bo'lsin.

Quyidagi yig'indilarni tuzamiz.

$$S_n = \xi_1 + \xi_2 + \xi_3 + \dots + \xi_n; \qquad \xi_n = \frac{S_n}{\sigma\sqrt{n}}.$$

S_n va ξ_n yig'indilarning zichlik funksiyalari (agar ular mavjud bo'lsa) $P_n(x)$ va $\overline{P}_n(x)$ kabi belgilaymiz.

3.1.2-Teorema. $\{\xi_n\}$ bog'liqmas bir xil taqsimlangan tasodifiy miqdorlar ketma - ketligi bo'lib, $F(x)$ ularning taqsimot funksiyasi bo'lsin. Agar shunday n_0 son topilsinki, $\sup\limits_x P_{n_0}(x) \le A$ o'rinli bo'lsa, u holda $n \ge 2n_0$ nomerlar uchun quyidagi tengsizlik o'rinli:

$$\sup_x\left|\overline{P}_n(x)-\varphi(x)\right|\le\frac{C\cdot\sqrt{n_0}\cdot\beta_3\max\left[1;\left(\sigma\sqrt{n_0}A\right)^3\right]}{\sigma^3\sqrt{n}},$$

bu yerda $\beta_z = M\left|\xi_1\right|^3$.

3.1.3 – teorema. $\{\xi_n\}$ - bog'liqmas, bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligi bo'lib, $F(x)$ - ularning taqsimot funksiyasi bo'lsin. Agar shunday n_0 nomer topilsaki, $\sup_x P_{n_0}(x)\le A<\infty$ shart bajarilsa, u holda $n\ge 2n_0$ nomerlar uchun

$$\left|\overline{P}_n(x)-\varphi(x)\right|\le\frac{Cn_0\max\left[\beta_3,(a\beta_3)^3,\left(n_0^2,A^n\right)\right]}{(1+|x|)\sqrt{n}}$$

tengsizlik o'rinli bo'ladi, bu yerda va keyingi hisoblashlarda C – lar turli xil absolyut sonlarni ifodalaydi, $\beta_3 = M\left|\xi_1\right|^3$.

3.1.2-teoremaning isboti.

Yuqorida keltirilgan teoremalarning isbotlari haqida tasavvurga ega bo'lish uchun, misol tariqasida,3.1.2 va 3.1.3 - teoremalarning isbotlarini keltiramiz. Isbot jarayonida quyidagi lemmalar kerak bo'ladi.

Quyidagi belgilashlarni kiritamiz:

$$f(t)=\int_{-\infty}^{\infty}e^{itx}P(x)dx,\qquad f_n(t)=f^n\left(\frac{t}{\sigma\sqrt{n}}\right),$$

$$\frac{d^k}{dt^k}f_n(t)=f_n^{(k)}(t),\qquad g(t)=e^{-\frac{t^2}{2}}.$$

3.1.1-Lemma. $|t|\le T_{1n}=\left(\frac{\sigma^2\sqrt{n}}{2\sqrt{2\beta_4}}\right)$ lar uchun quyidagi tengsizliklar o'rinli:

I. $$\left|f_n(t)-g(t)\right|\le\frac{Ct^4}{(1-\alpha)^n}\cdot e^{-\frac{t^2}{4}};$$

II. $$\left|f_n'(t)-g'(t)\right|\le\frac{C\left(|t|^3+|t|^5\right)}{(1-\alpha)n}\cdot e^{-\frac{t^2}{4}};$$

III. $$\left|f_n''(t)-g''(t)\right|\le\frac{C}{(1-\alpha)n}\cdot\left(t^2+t^4+t^6\right)\cdot e^{-\frac{t^2}{4}}.$$

Isboti. I – tengsizlikni isbotlaymiz, II, III – larning isboti shunga o‘xshash bo‘ladi. $f\left(\frac{t}{\sigma\sqrt{n}}\right)$ xarakteristik funksiyani quyidagicha yoyib yozamiz:

$$f\left(\frac{t}{\sigma\sqrt{n}}\right)=1-\frac{t^2}{2n}+\frac{t^4}{24\sigma^4n^2}\cdot\int_{-\infty}^{\infty}e^{i\frac{t}{\sigma\sqrt{n}}x}x^4dF(x).$$

Bu yoyilmani quyidan baholaymiz:

$$\left|f\left(\frac{t}{\sigma\sqrt{n}}\right)\right|\geq 1-\frac{t^2}{2n}-\frac{\beta_4t^4}{24\sigma^4n^2}\geq 1-\frac{T_{1n}^2}{2n}-\frac{\beta_4T_{1n}^4}{24\sigma^4n^2}=1-\frac{97\sigma^4}{96\cdot 16\beta_4},$$

$\sigma^4=\beta_2^2\leq\beta_4$ ni e'tiborga olsak

$$\left|f\left(\frac{t}{\sigma\sqrt{n}}\right)\right|>\frac{89}{96},$$

demak, $|t|\leq T_{1n}$ sohada $f\left(\frac{t}{\sigma\sqrt{n}}\right)$ funksiya noldan farqli.

Shuning uchun $|t|\leq T_{1n}$ sohada quyidagicha yozishimiz mumkin:

$$f_n(t)=e^{n\ln f\left(\frac{t}{\sigma\sqrt{n}}\right)},$$

so‘ngra $n\ln f\left(\frac{t}{\sigma\sqrt{n}}\right)$ ifodani quyidagicha yozamiz:

$$n\ln f\left(\frac{t}{\sigma\sqrt{n}}\right)=-\frac{t^2}{2}+\frac{\alpha t^4}{24n\sigma^4},$$

bu yerda

$$\alpha=\left(\frac{d^4}{dz^4}\ln f(z)\right)_z=\theta\frac{t}{\sigma\sqrt{n}}.$$

Topilgan ifodani I -ga qo‘yamiz:

$$|f_n(t)-g(t)|=\left|e^{-\frac{t^2}{2}+\frac{\alpha t^4}{24n\sigma^4}}-e^{-\frac{t^2}{2}}\right|=e^{-\frac{t^2}{2}}\cdot\left|e^{\frac{\alpha t^4}{24n\sigma^4}}-1\right|. \quad (3.1.1)$$

Ma'lumki,

$$\left|e^{\beta}-1\right|\le|\beta|\cdot e^{|\beta|}. \quad (3.1.2)$$

Endi α ni yuqoridan baholaymiz:

$$|\alpha|=\left|\left(f^3 f^{IV}-ff'f''-3f^2 f''^2+\right.\right.$$
$$\left.\left.+3f^2 f' f'''-6ff'^2 f''+6f'^4\right)\cdot f^{-4}\right|<40\beta_4 \quad (3.1.3)$$

(3.1.2) va (3.1.3) tengsizliklarni (3.1.1) da e'tiborga olsak, ushbu tengsizlik hosil bo‘ladi:

$$|f_n(t)-g(t)|\le e^{-\frac{t^2}{2}}\cdot\frac{40\beta_4 t^4}{24n\sigma^4}e^{\frac{40\beta_4 t^4}{24n\sigma^4}}. \quad (3.1.4)$$

$|t|\le T_{1n}$ sohada ushbu tengsizlik o‘rinli:

$$\frac{t^2}{4}-\frac{5\beta_4 t^4}{3n\sigma^4}=\frac{t^2}{4}\left(1-\frac{20}{3}\frac{\beta_4}{\sigma^4}\cdot\frac{t^2}{n}\right)>\frac{t^2}{24}\ge 0.$$

Buni e'tiborga olsak:

$$|f_n(t)-g(t)|\le\frac{2\beta_4 t^4}{n\sigma^4}\cdot e^{-\frac{t^2}{4}}. \quad (3.1.5)$$

Lemmaning I - qismi isbot bo‘ladi.

3.1.2-lemma. Agar $M|\xi_1|^m<\infty$ bo‘lsa, u holda

$$\left|f_n^m(t)\right|\le m!\left(\sqrt{n}\right)^m\rho_m\left|f\left(\frac{t}{\sigma\sqrt{n}}\right)\right|^{n-m}, \quad m=1,2,\ldots;\ \rho_m=\frac{\beta_m}{\sigma^m}.$$

3.1.3-lemma. Xarakteristik funksiyasi $f(t)$ bo‘lgan ξ tasodifiy miqdor uchun quyidagi shartlar o‘rinli bo‘lsin:

$$P(x) \le A < \infty, \quad x \in (-\infty,\ \infty); \quad M|\xi|^2 = \beta_\alpha < \infty, \quad \alpha \in (0;\ 2].$$

U holda quyidagi tengsizliklar o‘rinli:

$$|f(t)| \le \exp\left(-\frac{C_1(\alpha)}{A^2 \beta_2^{\frac{2}{\alpha}}} \right), \qquad |t| \ge \frac{\pi}{\beta_\alpha^{\frac{1}{\alpha}}};$$

$$|f(t)| \le \exp\left(-\frac{C_2(\alpha) t^2}{A^2} \right), \qquad |t| < \frac{\pi}{\beta_\alpha^{\frac{1}{\alpha}}}$$ bo‘lsa.

Eslatma. $\alpha = 2$ da P.Survila lemmasi hosil bo‘ladi (q. [20]). 2 – 3 lemmalarning isbotini [23], 51 – betdan topish mumkin.

3.1.2 Teoremaning isboti.

$n \ge n_0$ bo‘lganda quyidagicha yozish mumkin:

$$\overline{P}_n(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} f_n(t) dt,$$

$$\varphi(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(t) dt.$$

Bularni e'tiborga olsak:

$$2\pi \left| \overline{P}_n(x) - \varphi(x) \right| \le \int_{-\infty}^{\infty} | f_n(t) - g(t) | dt = I_1 + I_2 + I_3, \tag{3.1.6}$$

bu yerda

$$I_1 = \int\limits_{|t|\le T_n} |f_n(t) - g(t)| dt,$$

$$I_2 = \int\limits_{|t|\ge T_n} |f_n(t)| dt,$$

$$I_3 = \int\limits_{|t|\ge T_n} e^{-\frac{t^2}{2}} dt, \qquad T_n = \frac{\sigma^3\sqrt{n}}{5\beta_3}.$$

Teorema shartlari o‘rinli bo‘lganda $|t| \le T_n$ sohada quyidagi tengsizlik o‘rinli (q. [ ], III, 2-§.):

$$|f_n(t) - g(t)| \le \frac{7}{6} \cdot \frac{\beta_3 |t|^3}{\sigma^3\sqrt{n}} \cdot e^{-\frac{t^2}{4}} \tag{3.1.7}$$

I_1 integralni (3.1.7) yordamida baholaymiz:

$$I_1 \le \frac{7\beta_3}{6\sqrt{n}\sigma^3} \int\limits_{|t|\le T_n} |t|^3 e^{-\frac{t^2}{4}} dt \le \frac{C\beta_3}{\sigma^3\sqrt{n}}. \tag{3.1.8}$$

I_2 integralni quyidagicha ikkita integral yig‘indisi sifatida yozib olamiz:

$$I_2 = \sigma\sqrt{n} \int\limits_{|t| > \frac{\sigma^2}{5\beta_3}} |f(t)|^4 dt = \sigma\sqrt{n} \int\limits_{\frac{\sigma^2}{5\beta_3} \le |t| \le \frac{\pi}{\sigma\sqrt{n_0}}} |f(t)|^n dt +$$
$$+ \sigma\sqrt{n} \int\limits_{|t| > \frac{\pi}{\sigma\sqrt{n_0}}} |f(t)|^{2n_0} \cdot |f(t)|^{n-2n_0} dt = I_2'' + I_3''. \tag{3.1.9}$$

I_2' integralni 3.1.3 – lemma yordamida baholaymiz:

$$I_2' \le 5\sqrt{n} \cdot \frac{\beta_3}{\sigma} \int\limits_{\frac{\sigma^2}{5\beta_3} \le |t| \le \frac{\pi}{\sigma\sqrt{n_0}}} |t| \cdot e^{-\frac{Cn}{n_0 A^2} t^2} dt \le \frac{C n_0 \beta_3 A^2}{\sigma\sqrt{n}}. \tag{3.1.10}$$

I_2'' integralni baholashda 3.1.3-lemma va Parseval tengsizligidan foydalanamiz:

$$I_2'' \le \sigma\sqrt{\pi}\exp\left(-\frac{C(n-2n_0)}{n_0^2\sigma^2A^2}\right)\int_{|t|>\frac{\pi}{\sigma\sqrt{n_0}}}|f(t)|^{2n_0}dt \le$$

$$\le \sigma\sqrt{n}\exp\left(-\frac{C(n-2n_0)}{n_0^2\sigma^2A^2}\right)\int_{-\infty}^{\infty}P_{n_0}^2(x)dx \le \frac{Cn_0^2\sigma^3A^3}{\sqrt{n}}. \tag{3.1.11}$$

(3.1.9), (3.1.10) larni (3.1.11) ga qo‘yamiz:

$$I_2 \le \frac{C\sqrt{n_0}\beta_3\max\left[\left(\sigma\sqrt{n_0}A\right)^2,\ \left(\sigma\sqrt{n_0}A\right)^3\right]}{\sigma^3\sqrt{n}} \tag{3.1.12}$$

I_3 integralni baholash qiyin emas, tabiiyki:

$$I_3 \le \frac{C\beta_3}{\sigma^3\sqrt{n}} \tag{3.1.13}$$

(3.1.8), (3.1.12) va (3.1.13) larni (3.1.6) ga qo‘yamiz:

$$\sup_x\left|\overline{P}_n(x)-\varphi(x)\right| \le \frac{C\beta_3}{\sigma^3\sqrt{n}} + \frac{C\sqrt{n_0}\beta_3\max\left[\left(\sigma\sqrt{n_0}A\right)^2,\ \left(\sigma\sqrt{n_0}A\right)^3\right]}{\sigma^3\sqrt{n}} +$$

$$+\frac{C\beta_3}{\sigma^3\sqrt{n}} \le \frac{C\sqrt{n_0}\beta_3\max\left(1;\ \left(\sigma\sqrt{n_0}A\right)^3\right)}{\sigma^3\sqrt{n}}.$$

Teorema isbot bo‘ldi.

3.1.3 – teoremaning isboti.

Ma’lumki, tasodifiy miqdorlarning funksiyalari o’rtasida quyidagi munosabatlar o’rinli:

$$f_n(t) = \int_{-\infty}^{\infty} e^{itx}\overline{P}_n(x)dx,$$

$$g(t) = \int_{-\infty}^{\infty} e^{itx}\varphi(x)dx,$$

bu yerda $\varphi(x)$ - normal taqsimotning zichlik funksiyasi.

Yuqoridagi funksiyalarni *t* – bo'yicha differensiallaymiz:

$$f_n'(t) = \int_{-\infty}^{\infty} e^{itx}\left(ix\overline{P}_n(x)\right)dx,$$

$$g'(t) = \int_{-\infty}^{\infty} e^{itx}\left(ix\varphi(x)\right)dx,$$

Hosil bo'lgan ifodalarda quyidagi almashtirishlarni bajarish mumkin:

$$ix\overline{P}_n(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx} f_n'(t)dt,$$

$$ix\varphi(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx} g'(t)dt,$$

Hosil qilingan ifodalardan quyidagi munosabatga o'tamiz:

$$ix\left(\overline{P}_n(x) - \varphi(x)\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\left(f_n'(t) - g'(t)\right)dt,$$

Tenglikni ikkala tomonini absolyut qiymatlarini olib, tengsizlikning o'ng tomonini kuchaytirib boramiz:

$$|x|\left|\overline{P}_n(x) - \varphi(x)\right| \le \int_{-\infty}^{\infty} \left|f_n'(t) - g'(t)\right|dt$$

O'ng tomonidagi integralni quyidagi uchta integralga ajratamiz:

$$|x|\left|\overline{P}_n(x) - \varphi(x)\right| \le I_1 + I_2 + I_3 \qquad (5)$$

Bu yerda

$$I_1 = \frac{1}{2\pi}\int_{|t| \ge T_h} \left|f_n'(t) - g'(t)\right|dt,$$

$$I_1 = \frac{1}{2\pi}\int_{|t| \ge T_h} \left|f_n'(t)\right|dt,$$

$$I_1 = \frac{1}{2\pi}\int_{|t| \ge T_h} \left|g'(t)\right|dt,$$

Ketma-ket I_1, I_2, I_3 integralni baholaymiz.

1 – lemmaga asosan:

$$I_1 = \frac{1}{2\pi}\int_{|t|\geq T_h} |f_n'(t) - g'(t)| dt \leq \frac{CB_3}{\sqrt{n}}\int_{|t|\geq T_h} \left(t^2 + t^4\right) e^{-\frac{t^2}{4}} dt$$

O'ng tomonidagi integral chekli son bo'lgani uchun

$$I_1 \leq \frac{C\beta_3}{\sqrt{n}} \qquad (6)$$

I_2 integralni baholashda quyidagi

$$f_n(t) = \left(f\left(\frac{t}{\sqrt{n}}\right)\right)^2, \qquad f_n'(t) = \sqrt{n} f'\left(\frac{t}{\sqrt{n}}\right)\left(f\left(\frac{t}{\sqrt{n}}\right)\right)^{n-1}$$

Ifodalardan foydalanamiz.

$$I_2 \leq \frac{\sqrt{n}}{2\pi}\int_{|t|>T_n} \left| f\left(\frac{t}{\sqrt{n}}\right)\right|^{n-1} dt = \frac{n}{\pi}\int_{C\sqrt{n}/\beta_3}^{\infty} f_n\left(\frac{t}{\sqrt{n}}\right)^{n-1} d\left(\frac{t}{\sqrt{n}}\right) =$$

$$= Cn\left(\int_{C/\beta_3 < t < \frac{\pi}{n_c}}^{\infty} \left| f_n\left(\frac{t}{\sqrt{n}}\right)\right|^{n-1} dt + \int_{t\geq \frac{\pi}{\sqrt{n}}}^{\infty} |f(t)|^{n-1} dt\right) =$$

$$I_2' + I_2'' \qquad (7)$$

I_2' integralda quyidagi o'zgartirishni bajaramiz:

$$I_2' = Cn\int_{C/\beta_3}^{\pi/n_0} \left| f^{n_0}(t)\right|^{\frac{n-1}{n_0}} dt$$

I_2' da 2- lemmaning 2-chi tengsizligidan foydalanamiz:

$$I_2' \le Cn\int_{C/\beta_3}^{\pi/\sqrt{n_0}} \left(\exp\left(-\frac{Ct^2}{A^2}\right)\right)^{\frac{n-1}{n_0}} dt =$$

$$Cn\int_{C/\beta_3}^{\pi/\sqrt{n_0}} e^{-\frac{C(n-1)}{n_0 A^2}t^2} dt \le Cne^{-\frac{C(n-1)}{n_0 A^2 \beta_3^2}} \int_{C/\beta_3}^{\pi/\sqrt{n_0}} dt =$$

$$= \frac{Cn}{e^{\frac{C(n-1)}{n_0 A^2 \beta_3^2}}} t\int_{C/\beta_3}^{\pi/\sqrt{n_0}} \le \frac{Cn}{\left(\frac{C(n-1)}{n_0 A^2 \beta_3^2}\right)^{3/2}} \frac{\pi}{n_0} \le \frac{Cn_0 A^3 \beta_3^3}{\sqrt{n}} \qquad (8)$$

I_2'' ning ko'rinishi quyidagicha o'zgartiramiz:

$$I_2'' = Cn\int_{\pi/n_0}^{+\infty} (f(t))^{2n_0} \left|f^{n_0}(t)\right|^{\frac{n-2n_0-1}{n_0}} dt$$

Hosil bo'lgan integralda 2-lemmaning 1-chi tengsizligidan foydalanamiz:

$$I_2'' \le Cne^{\frac{C(n-2n_0-1)}{n_0^2 A^2}} \int_{\pi/\sqrt{n_0}}^{+\infty} \left|f^{n_0}(t)\right|^2 dt \le$$

$$\le \frac{Cn_0^3 A^3}{\sqrt{n}} \int \left|f^{n_0}(t)\right|^2 dt \qquad (9)$$

Ma'lumki:

$$\int \left|f^{n_0}(t)\right|^2 dt = \int P_{n_0}^2(x)dx$$

$P_{n_0}(x) \le A$ ni e'tiborga olsak:

$$\int \left|f^{n_0}(t)\right|^2 dt \le A\int Pn_0(x)dx = A \qquad (10)$$

U holda (10) ni (9) ga qo'ysak:

$$I_2'' \le \frac{C_{n_0}^3 A^4}{\sqrt{n}} \qquad (11)$$

(8) ni (11) larni (7) ga qo'ysak :

$$I_2 \le \frac{C_{n_0} A^2 \max\left(\beta_3^3, n_0^2 A\right)}{\sqrt{n}}$$

I_3 ni baholashda

$$g(t) = e^{-\frac{t^2}{2}}, \qquad g'(t) = -te^{-\frac{t^2}{2}}$$

larni e'tiborga olamiz :

$$I_3 \le \frac{1}{2\pi}\int_{|t|>T_n} |t| e^{-\frac{t^2}{2}} dt \le \frac{C}{T_n}\int_{T_n}^{\infty} t^2 e^{-\frac{t^2}{2}} dt \le \frac{C\beta_3}{\sqrt{n}} \qquad (13)$$

(6), (12) va (13) ifodalarni (5) ga qo'ysak:

$$|x|\left|\overline{P}_n(x) - \varphi(x)\right| \le \frac{C_{n_0} \max\left(\beta_3; (A\beta_3)^3; (n_0^2 A^4)\right)}{\sqrt{n}} \qquad (14)$$

(3) va (14) tengsizliklarni mos ravishda qo'yamiz :

$$(1+|x|)\left|\overline{P}_n(x) - \varphi(x)\right| \le \frac{C_{n_0} \max\left(\beta_3; (A\beta_3)^3; (n_0^2 A^4)\right)}{\sqrt{n}}$$

Ikkala tomonni $1+|x|$ ga bo'lsak :

$$\left|\overline{P}_n(x) - \varphi(x)\right| \le \frac{C_{n_0} \max\left(\beta_3; (A\beta_3)^3; (n_0^2 A^4)\right)}{(1+|x|)\sqrt{n}}$$

Teorema to'la isbot bo'ldi.

2-§. GLOBAL LOKAL TEOREMALAR

Ushbu magistrlik dissertatsiyasida – mazkur yo‘nalishda yetarlicha tasavvurga ega bo‘lish, 1 – teoremaga asoslangan global teoremalardagi baholarda n_0 qanday bo‘lishini ochish maqsad qilib qo‘yilgan.

Bu masalani hal etishda (1) ketma-ketlik o‘rniga

$$\xi_{1n_0},\ \xi_{2n_0},\ \dots,\xi_{kn_0},\ \dots \qquad (3.2.1)$$

tasodifiy miqdorlar ketma-ketligini qarash lozim bo‘ladi, chunki (3.2.1) dagi tasodifiy miqdorlar uchun $P_{n_0}(x) \le A$ shart bajariladi, bu yerda

$$\xi_{kn_0} = \xi_{(k-1)n_0+1} + \xi_{(k-1)n_0+2} + \dots + \xi_{kn_0}, \qquad k = 1,2,3,\dots$$

(3.2.1) tasodifiy miqdorlar ketma-ketligi uchun quyidagi teoremani isbotlaymiz:

3.2.1 Teorema. Agar $P_{n_0}(x) \le A$ tengsizlik o‘rinli bo‘lsa, u holda $n \ge 2n_0$ uchun quyidagi munosabat o‘rinli:

$$\int_{-\infty}^{\infty} \left|\overline{P}_n(x) - \varphi(x)\right|^2 dx \le \frac{cn_0^{\frac{3}{2}} \cdot \beta_3^3 \cdot A^4 \cdot e^{\frac{c}{A^2}}}{n}$$

bu yerda va butun ish davomida S orqali turli xil musbat absolyut sonlar belgilangan.

3.2.2-Teorema. $\{\xi_n\}$- bir xil taqsimlangan tasodifiy miqdorlar ketma-ketligi uchun $P_{n_0}(x) \le A$ o‘rinli bo‘lsin. U holda barcha $n \ge 2\ (n_0 + 1)$ nomerlar uchun $P \ge 2$ va $0 \le q \le 2p$ bo‘lganda quyidagi tengsizlik o‘rinli:

$$\int_{-\infty}^{\infty} |x|^q \left|\overline{p_n}(x) - \varphi(x)\right|^p dx \le \frac{C}{n^p}\left[n_0^2 + \frac{1}{n_0^2}\exp\left(cn_0^2\right)\right]^{p-1}.$$

3-§. YORDAMCHI TEOREMALAR

3.2.1–Lemma. $M|\xi_1|^3 = \beta_3 < \infty$ bo‘lsa, $|t| \le \frac{c\sqrt{n}}{\beta_3}$ lar uchun ushbu tengsizlik bajariladi:

$$|f_n(t) - g(t)| \le \frac{c\beta_3|t|^3}{\sqrt{n}} \cdot e^{-\frac{t^2}{4}}.$$

Lemmaning isboti [5] – da

3.2.2–Lemma. Xarakteristik funksiyasi $f_1(t)$ bo‘lgan tasodifiy miqdor uchun $D\xi = I,\ P(x) = A_1$ bo‘lsa $(x \in (-\infty; \infty))$, u holda

$|t| \ge \pi$ bo‘lganda $\qquad |f_1(t)| \le \exp\left(-\frac{c}{A_1^2}\right)$,

$|t| \le \pi$ bo‘lganda $\qquad \left|f_1(t) \le \exp\left(-\frac{ct^2}{A_1^2}\right)\right|$.

Lemmaning isboti [23] da.

4-§. GLOBAL TEOREMALARNING ISBOTI

3.2.1.Teoremaning isboti.

Ma'lumki, zichlik funksiyasi bilan xarakteristik funksiyalar orasida quyidagi munosabatlar o‘rinlidir:

$$f_n(t)=\int_{-\infty}^{\infty} e^{itx}\cdot \overline{P}_n(x)dx$$

$$g(t)=\int_{-\infty}^{\infty} e^{itx}\varphi(x)dx$$

$$f_n(t)-g(t)=\int_{-\infty}^{\infty} e^{itx}\left(\overline{P}_n(x)-\varphi(x)\right)dx$$

$\overline{P}_n(x)-\varphi(x)$ va $f_n(t)-g(t)$ funksiyalar uchun quyidagi munosabat o‘rinligidan foydalanamiz:

$$\int_{-\infty}^{\infty}\left|\overline{P}_n(x)-\varphi(x)\right|^2 dx=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|f_n(t)-g(t)\right|^2 dt \qquad (3.2.2)$$

Yuqoridagi lemmalardan foydalanish uchun (3.2.2) ning o‘ng tomonidagi integralni 3 ta integral yig‘indisi sifatida yozib olamiz:

$$\int_{-\infty}^{\infty}\left|\overline{P}_n(x)-\varphi(x)\right|^2 dx \le I_1+I_2+I_3$$

$$I_1=\frac{1}{\pi}\int_{|t|\le T_n}\left|f_n(t)-g(t)\right|^2 dt$$

$$I_2=\frac{1}{\pi}\int_{|t|>T_n}\left|f_n(t)\right|^2 dt$$

$$I_3=\frac{1}{2\pi}\int_{|t|>T_n} e^{-t^2}dt \qquad (3.2.3)$$

I_1 ni baholashda 1 – lemmadan foydalanamiz:

I_1

$$I_1\le\frac{1}{\pi}\int_{|t|\le T_n}\left|f_n(t)-g(t)\right|^2 dt\le\frac{c\beta_3^2}{n}\int_{|t|\le T_n} t^6\cdot e^{-\frac{t^2}{2}}dt$$

$\int_{|t|\le T_n} t^6\cdot e^{-\frac{t^2}{2}}dt\le c$ bo‘lgani uchun

$$I_1\le\frac{c\beta_3^2}{n} \qquad (3.2.4)$$

Endi I_3 ni baholaymiz:

$$I_3 = \frac{1}{2\pi}\int_{|t|>T_n} e^{-t^2}dt \le \frac{c}{T_n^2}\int t^2 \cdot e^{-\frac{t^2}{2}}dt \le \frac{c\beta_3^2}{n} \tag{3.2.5}$$

I_2 integralni baholashda quyidagicha shakl almashtirishlar bajaramiz:

$$I_2 = \frac{1}{\pi}\int_{|t|>T_n}\left|f_n(t)\right|^2 dt = \frac{1}{\pi}\int_{|t|>\frac{c\sqrt{n}}{\beta_3}}\left|f\left(\frac{t}{\sqrt{n}}\right)\right|^{2n} dt = \frac{\sqrt{n}}{\pi}\int_{|t|>\frac{c}{\beta_3}}\left|f(t)\right|^2 dt = c\sqrt{n}\cdot\int_{\frac{c}{\beta_3}<|t|<\pi}\left|f^{n_0}(t)\right|^{\frac{2n_1}{n_0}-2}\cdot\left|f^{n_0}(t)\right|^2 dt +$$

$$+c\sqrt{n}\int_{|t|>\pi}\left|f^{n_0}(t)\right|^{\frac{2n}{n_0}-2}\cdot\left|f^{n_0}(t)\right|^2 dt = I_1^2 + I_2^2 \tag{3.2.6}$$

I_2^1 - ni baholashda 2 – Lemmaning ikkinchi tengsizligidan foydalanamiz:

$$I_2^1 \le c\sqrt{n}\int_{\frac{c}{\beta_3}<|t|<\delta} e^{-\frac{ct^2}{A_1^2}\left(\frac{2n}{n_0}-2\right)}\cdot\left|f^{n_0}(t)\right|^2 dt \le \frac{c\sqrt{n}\cdot e^{\frac{c}{\beta_3^2\cdot A_1^2}}}{\dfrac{n^{\frac{3}{2}}}{\left(\beta_3^2\cdot A_1^2 n_0\right)^{\frac{3}{2}}}}\cdot\int\left|f^{n_0}(t)\right|^2 dt \le$$

$$\le \frac{c\beta_3^2\cdot A_1^3\cdot n_0^{\frac{3}{2}}\cdot e^{\frac{c}{\beta_3^2\cdot A_1^2}}}{n}\cdot\int\left|f^{n_0}(t)\right|^2 dt \tag{3.2.7}$$

Parseval tengligiga asosan:

$$\int\left|f^{n_0}(t)\right|^2 dt\int P_{n_0}^2(x)dx \le c\cdot A_1\int P_{n_0}(x)dx = c\cdot A_1 \tag{3.2.8}$$

(3.2.8) ni (3.2.7) ga qo‘ysak

$$I_2^1 \le \frac{c\beta_3^3 A_1^4\cdot e^{\frac{c}{(\beta_3\cdot A_1)^2}}\cdot n_0^{\frac{3}{2}}}{n} \tag{3.2.9}$$

I_2^2 ni baholashda 2 – lemmaning birinchi tengsizligidan foydalanamiz:

$$I_2^2 \le c\sqrt{n}\int_{|t|>\pi} e^{-\frac{c}{A_1^2}\left(\frac{2n}{n_0}-2\right)}\cdot\left|f^{n_0}(t)\right|^2 dt \le \frac{\sqrt{n}\cdot e^{\frac{c}{A_1^2}}}{e^{\frac{cn}{n_0 A_1^2}}}\cdot\int\left|f^{n_0}(t)\right|^2 dt \le \frac{cn_0^{\frac{3}{2}}\cdot A_1^3\cdot e^{\frac{c}{A_1^2}}}{n}\cdot\int\left|f^{n_0}(t)\right|^2 dt$$

(3.2.10)

(3.2.10) da (3.2.8) ni e'tiborga olsak:

$$I_2^2 \le \frac{cn_0^{\frac{3}{2}} \cdot A_1^4 \cdot e^{\frac{c}{A_1^2}}}{n} \qquad (3.2.10)$$

(3.2.9) va (3.2.10) larni (3.2.6) ga qo‘yamiz:

$$I_2 \le \frac{c \cdot n_0^{\frac{3}{2}} \cdot \beta_3^3 \cdot A_1^4 \cdot e^{\frac{c}{A_1^2}}}{n} \qquad (3.2.11)$$

(3.2.4), (3.2.5), (3.2.11) ifodalarni (3.2.3) ga qo‘yib, teorema isbotiga ega bo‘lamiz.

$$\int_{-\infty}^{\infty} \left|\overline{P}_n(x) - \varphi(x)\right|^2 dx \le \frac{c\beta_3^2}{n} + \frac{c \cdot n_0^{\frac{3}{2}} \cdot \beta_3^3 \cdot A_1^4 \cdot e^{\frac{c}{A_1^2}}}{n} \le \frac{c \cdot n_0^{\frac{3}{2}} \cdot \beta_3^3 \cdot A_1^4 \cdot e^{\frac{c}{A_1^2}}}{n}.$$

3.2.2-teoremaning isboti.

$n \ge n_0 + 1$ bo‘lganda quyidagi munosabat o‘rinli:

$$2\pi |x| \left|\overline{P}_n(x) - \varphi(x)\right| \le \int_{-\infty}^{\infty} \left| f_n'(t) - g'(t) \right| dt = I_7 + I_8 + I_9, \qquad (3.2.12)$$

bu yerda

$$I_7 = \int_{|t| \le T_{1n}} \left| f_n'(t) - g'(t) \right| dt;$$

$$I_8 = \int_{|t| > T_{1n}} \left| f_n'(t) \right| dt;$$

$$I_9 = \int_{|t| > T_{1n}} \left| \frac{d}{dt} \cdot e^{-\frac{t^2}{2}} \right| dt.$$

I_7 integralni 3.1.1 – Lemmadan foydalanib baholaymiz:

$$I_7 \le \frac{c}{(1-\alpha)n} \int_{|t| \le T_{1n}} \left(|t|^3 + |t|^5 \right) e^{-\frac{t^2}{2}} dt \le \frac{cn_0}{n}. \qquad (3.2.13)$$

so‘ngra, 3.1.3-Lemmadan foydalanib, I_8 integralni quyidagicha yozib olamiz:

$$I_8 \le \frac{\sqrt{n}}{\sigma} \int\limits_{|t|>T_{1n}} \left| f\left(\frac{t}{\sigma\sqrt{n}}\right)\right|^{n-1} dt = 2n \left(\int\limits_{\frac{\sigma}{2\sqrt{2\beta_4}} < t \le \frac{\pi}{\sigma\sqrt{n_0}}} + \int\limits_{t>\frac{\pi}{\sigma\sqrt{n_0}}} |f(t)|^{n-1} dt \right)$$

3.1.3-Lemmadan foydalanib, quyidagini hosil qilamiz:

$$I_8 \le 2n \left(\exp\left(-\frac{c\sigma^2 (n-1-2n_0)}{n_0 \beta_4 A^2} \right) \right) + \exp\left(-\frac{c(n-1-2n_0)}{n_0^2 \sigma^2 A^2} \right) \cdot \int\limits_{-\infty}^{\infty} |f(t)|^{2n_0} dt .$$

bu yerdagi A son $P_{n_0}(x)$ ni chegaralovchi son.

Endi quyidagi Parseval formulasidan foydalanamiz:

$$\int\limits_{-\infty}^{\infty} |f(t)|^{2n_0} dt = 2\pi \int\limits_{-\infty}^{\infty} \left|P_{n_0}(x)\right|^2 dx .$$

So‘ngra A-konstantani e'tiborga olib, ushbu tengsizlikni hosil qilamiz:

$$I_8 \le \frac{c}{nn_0^2} \cdot 2^{cn_0} . \qquad (3.1.14)$$

I_9 integral uchun quyidagi tengsizlikning o‘rinli bo‘lishi ravshan:

$$I_9 \le \frac{cn_0}{n} . \qquad (3.1.15)$$

(3.1.12) – (3.1.14) munosabatlardan quyidagi tengsizlikni hosil qilamiz:

$$|x|\left|\overline{P}_n(x) - \varphi(x)\right| \le \frac{c}{n}\left(n_0 + \frac{1}{n_0^2} \cdot 2^{cn_0} \right) . \qquad (3.1.16)$$

Hosil qilingan tengsizlikdan foydalanib, teoremaning isbotini bajaramiz.

1-hol. $p \ge 2, \ q = 2p$. U holda

$$\int\limits_{-\infty}^{\infty} |x|^q \left|\overline{P}_n(x) - \varphi(x)\right|^p dx = \int\limits_{-\infty}^{\infty} |x|^{2p} \left|\overline{P}_n(x) - \varphi(x)\right|^p dx \le$$

$$\le \sup_x \left(x^2\left|\overline{P}_n(x)-\varphi(x)\right|\right)^{p-2}\int_{-\infty}^{\infty} x^4\left|\overline{P}_n(x)-\varphi(x)\right|^2 dx$$
.

Bu munosabatda (3.1.16) va (3.1.9)-ni e'tiborga olsak:

$$\int_{-\infty}^{\infty}|x|^q\left|\overline{P}_n(x)-\varphi(x)\right|^p dx \le \frac{c(p)}{n^p}\left(n_0+\frac{1}{n_0^2}\exp cn_0\right)^{p-1} . \quad (3.1.17)$$

2-hol. $p \ge 2,\ 0 \le q \le 2p$. Ushbu holda

$$\int_{-\infty}^{\infty}|x|^q\left|\overline{P}_n(x)-\varphi(x)\right|^p dx \le \int_{-1}^{1}\left|\overline{P}_n(x)-\varphi(x)\right|^p dx + 2\int_{1}^{\infty}|x|^{2p}\left|\overline{P}_n(x)-\varphi(x)\right|^p dx \le$$

$$\le \int_{-\infty}^{\infty}\left|\overline{P}_n(x)-\varphi(x)\right|^p dx + \int_{-\infty}^{\infty}|x|^{2p}\left|\overline{P}_n(x)-\varphi(x)\right|^p dx \le$$

$$\le \sup_x\left(\left|\overline{P}_n(x)-\varphi(x)\right|\right)^{p-2}\int_{-\infty}^{\infty}\left|\overline{P}_n(x)-\varphi(x)\right|^2 dx + \int_{-\infty}^{\infty}|x|^{2p}\left|\overline{P}_n(x)-\varphi(x)\right|^p dx = I_{10}+I_{11} .$$

I_{10} baholashda 3.2.1-teorema va (3.1.3) dan foydalanamiz:

$$I_{10} \le \frac{c(p)}{n^p}\left(n_0^2+\frac{1}{n_0^2}\exp cn_0\right)^{p-1} .$$

Oxirgi tengsizlik va (3.1.17) lardan quyidagi tengsizlikka ega bo‘lamiz:

$$\int_{-\infty}^{\infty}|x|^q\left|\overline{P}_n(x)-\varphi(x)\right|^p dx \le \frac{c(p)}{n^p}\left(n_0^2+\frac{1}{n_0^2}\exp cn_0\right)^{p-1} . \quad (3.1.18)$$

(3.1.17) va (3.1.18) munosabatlardan teoremaning isboti kelib chiqadi.

XULOSA

Ko‘rib o‘tilgan mavzulardan ma'lum bo‘ldiki, Limit teoremalar va ularning baholarini o‘rganish juda ham dolzarb muammolardan biri ekan.

Bunday xulosaga kelishda tasodifiy miqdorlarning yig‘indilari uchun isbotlanadigan teoremalarning kelib chiqish mavzulari juda ham xilma-xil ekanligi va olingan natijalarning ko‘plab sohalarga – Ommaviy xizmat ko‘rsatish nazariyasiga, Ishonchlilik nazariyasiga, Zahiralar nazariyasiga, Jadvallar tuzish nazariyasiga va hokazo, barcha tasodifiy sondagi tasodifiy miqdorlarning tasodifiy yig‘indisiga keladigan masalalar uchun muhimligidadir.

Ko‘rilgan masalalarning tadbiqlari muhimligi uchun olingan teoremalardagi yaqinlashish tezliklarini aniqlash aktual masaladir, shu sababli baholarni aniqlash va isbotlangan teoremalarda qatnashadigan C o‘zgarmas miqdorlarning aniq qiymatini topish va bu qiymatni minimallashtirish masalasi juda ham ahamiyatga ega. Buni e'tiborga olib, olingan teoremalardagi konstantalarning aniq qiymatlarini topish va ularni kichiklashtirish masalalari bilan shug‘ullanish ham aktual hisoblanadi.

Bundan tashqari bu magistrlik dissertatsiyasida bir xil taqsimlangan tasodifiy miqdorlar qaralganligini e'tiborga olsak, bu natijalarni har xil taqsimlangan tasodifiy miqdorlarning yig‘indilari uchun umumlashtirish muammolari bilan shug‘ullanish ham muhimdir.

Yuqoridagilardan xulosa qilib shuni aytish mumkinki, tanlangan mavzu juda ham dolzarb va u bilan keyinchalik batafsilroq hamda chuqurroq shug‘ullanish zarur, bu bilan juda ko‘plab amaliy ahamiyatga ega bo‘lgan muammolarning hal etilishiga munosib xissa qo‘shilgan bo‘ladi.

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