V.I. ROMANOVSKIY NOMIDAGI MATEATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

JALILOV ALISHER AKBAROVICH

AYLANA GOMEOMORFIZMLARI BILAN BOG‘LIQ DANJUA TENGLIGI VA CHEKSIZ IKKILIK KETMA-KETLIKLARI

01.01.01 – Matematik analiz

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATISYASI AVTOREFERATI

Toshkent – 2021

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Оглавление автореферата доктора философии (PhD) по физико-математическим наукам

Content of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan koʻplab ilmiy-amaliy tadqiqotlarda bir oʻlchovli va simvolli dinamik sistemalar keng oʻrin egallaydi. Dinamik sistemalar matematikaning bir qancha boshqa sohalarida keng qoʻllaniladi. Undan tashqari, meditsina, biologiya, axborot texnologiyalari va boshqa tabiiy fanlarda masalalarni yechishda aylana akslantirishlari va simvolli dinamikadan izchil foydalanib kelinmoqda. Chiziqli boʻlmagan dinamik sistemalar koʻrinishida modellashtirilgan muayyan jarayonni uzoq muddatli holatini bashorat qilishda dinamik sistemalarning tushish vaqtlari, tasodifiy siljishlari va cheksiz simvolli ketma-ketliklar murakkablik funksiyalarining xossalari muhim rol oʻynaydi. Shu sababdan dinamik sistemalarning tushish vaqtlari uchun limitik teoremalar, chiziqli va nochiziqli akslantirishlarni tafsiflovchi ketma-ketliklarning xossalarini tadqiq qilish dinamik sistemalar nazariyasining muhim va dolzarb masalalaridan biri boʻlib kelmoqda.

Hozirgi kunda olib borilayotgan nochiziqli dinamik sistemalarni uzoq vaqtdagi holatini bashorat qilish, xususan aylana akslantirishlari bilan bogʻliq boʻlgan muammolar muhim ahamiyat kasb etmoqda. Aylana akslantirishlarining muhim sinfi hisoblangan aylana diffeomorfizmlari masalalari chuqur va toʻliq hal qilingan boʻlib, aylana diffeomorfizmlarining tabiiy umumlashmasi boʻlgan boʻlakli silliq aylana akslantirishlari uchun Danjua tengligi hozirgi kungacha topilmagan. Soʻnggi yillarda maxsuslikka ega aylana akslantirishlari nazariyasi uchun Danjua tengsizligi, qattiqlik muammosi masalalari yechilgan. Bu borada: ikkita sinish nuqtasiga ega aylana akslantirishlari uchun Danjua tengligini topish, kiritik aylana akslantirishlarining tushish vaqtlari nolmallangan taqsimot funksiyalari yaqinlashishini isbotlash va cheksiz ketma-ketliklarning murakkablik funskiyalarini topish maqsadli ilmiy tadqiqotlardan hisoblanadi.

Mamlakatimizda amaliy va fundamental fanlarda qoʻllaniladigan, chiziqli boʻlmagan va simvolli dinamik sistemalari nazariyasining muhim yoʻnalishlarini ishlab chiqishga katta eʻtibor qaratildi. «Funksional analiz, matematik fizika va statistik fizika» kabi muhim sohalarda xalqaro statndartlar darajasida tadqiqotlar olib borish fundamental tadqiqotlarning asosiy vazifasi etib belgilandi[1]. Mazkur qaror ijrosini ta’minlashda ilm-fanning turdosh sohalarida foydalanish maqsadida boʻlakli silliq dinamik sistemalar nazariyasini rivojlantirish muhim ahamiyatga ega.

Oʻzbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi PF-4947-son «Oʻzbekiston Respublikasini yanada rivojlantirish boʻyicha xarakatlar strategiyasi toʻgʻrisida»gi Farmoni, 2019 yil 9 iyuldagi PQ-4387-son «Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo’llab-quvvatlash, shuningdek, O’zbekiston Respublikasi Fanlar Akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari toʻgʻrisida»gi va 2020 yil 7 maydagi PQ-4708-son «Matematika sohasidagi ta’lim

[1] Oʻzbekiston Respublikasi Vazirlar mahkamasi 2017 yil 18 maydagi «Oʻzbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish toʻgʻrisida»gi 292-sonli qarori.

sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida»gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ–huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalar rivojlanishining ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. «Matematika, mexanika va informatika» ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Simvolli dinamika sohasi umumiy dinamik sistemalarni tahlil qilish vositasi sifatida rivojlandi. Adamar 1898 yilda salbiy egilish yuzalaridagi geodezik oqimlarni tahlil qilishda simvolli dinamika usullaridan birinchi marta muvaffaqiyatli foydalangan. Qirq yil o‘tgach, M. Mors va G. Hedlund fundamental maqolasida simvolli dinamikadan birinchi bo‘lib foydalangan va unga nom bergan. Keyinchalik S.E.Shennon tomonidan axborotlar nazariyasida simvolli dinamika qo‘llanilgan va boshqa olimlar tomonidan keng qo‘llanila boshlangan. Simvolli dinamikada cheksiz ikkilik ketma-ketliklari juda ko‘p soxalarda tatbiqga ega muhim tushuncha hisoblanadi va bunda murakkablik funksiyasi asosiy rol o‘ynaydi. Murakkablik funksiyasi minimal bo‘lgan cheksiz nodavriy ikkilik ketma-ketliklar Shturm ketma-ketlaridir. Bu ketma-ketliklar juda keng o‘rganilgan bo‘lib, aylana akslantirishlari bilan uzviy bog‘langan.

Ya. Sinay o‘zining muhim ishida Anosov oqimlarini tadqiq qilish uchun simvolli dinamikadan foydalangan. E. Vul, Ya. Sinay va K. Xanin Feigenbaum asklantirishi uchun simvolli dinamika tuzgan va termodinamik formalizmni qurgan. Simvolli dinamika aylana akslantirishlari nazariyasida ham muhim rol o‘ynaydi. Ma’lumki, aylananing har bir irratsional burish akslantirishlari yordamida cheksiz ikkilik ketma-ketliklar fazosi qurish mumkin. Ikkilik ketma-ketligining murakkablik funksiyalarini o‘rganish simvolli dinamika nazariyasining asosiy muammolaridan biridir. Z. Koelo, E. De Faria irratsional burishlar uchun qayta tushish vaqtlarini o‘rgangan. A. Djalilov kritik aylana akslantirishlari uchun termodinamik formalizmni qurgan.

Danjua tengsizligi aylana dinamikasida muhim o‘rin egallaydi. Dastlab bu tengsizlik Danjua tomonidan aylana diffeomorfizmlari uchun isbotlangan. Bu muhim tengsizlikdan foydalanib berilgan aylana gomeomorfizmini chiziqli irratsional burish akslantirishiga topologik ekvivalentligi ko‘rsatilgan. Ya. Sinay va K. Xanin, Y. Katsnelson va D. Ornsteyn renormalizatsiya metodidan foydalanib Danjua tengsizligini kuchaytirilgan ko‘rinishda isbotlagan. Bu olingan natija esa qo‘shma akslantirishining silliqligini isbotlashda muhim rol o‘ynaydi. Bo‘lakli-silliq aylana akslantirishlari diffeomorfizmlarning tabiiy umumlashmasidir. M. Erman o‘zining fundamental ishida bo‘lakli-chiziqli ikkita sinishga ega bo‘lgan, ergodik aylana gomeomorfizmlari uchun invariant o‘lchovlarni faqat ikkala sinish nuqtalari bitta orbitada yotgan holda absolyut uzluksiz bo‘lishini isbotlagan. Maxsuslikka ega bo‘lgan f aylana akslantirishlari uchun “qattiqlik” muommosi ya’ni qo‘shma gomeomorfizmning silliqligi bilan bog‘liq masalalarda Df^{q_n} hosilalarini baholash juda katta ahamiyatga ega.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan ilmiy tekshirish institutining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti V.I. Romanovskiy nomidagi Matematika institutining OT-F4-82 «Operatorli va noassotsiativ algebralar lokal differensiallashlari va avtomorfizmlari, chiziqli bo‘lmagan dinamik sistemalarda fazali o‘tish va tartibsizlik» (2017-2020 yillar) va YoFA-Ftex-2018-78 «Amenabel bo‘lmagan graflarda dinamik va termodinamik sistemalar» (2018-2019 yillar) mavzusidagi ilmiy tadqiqot loyihalari doirasida bajarilgan.

Tadqiqotning maqsadi aylana akslantirishlari bilan bog‘liq cheksiz ikkilik ketma-ketliklarining murakkablik funksiyalarini qurish va Erman akslantirishlari uchun Danjua tengligini topishdan iborat.

Tadqiqotning vazifalari:

aylananing irratsional burish akslantirishlari bilan bog‘liq cheksiz ikkilik ketma-ketliklarning barcha asosiy xossalarini va murakkablik funksiyalarini o‘rganish;

Erman akslantirishlari va uning yuqori iteratsiyalari tomonidan hosil qilingan cheksiz ketma-ketliklarning murakkablik funksiyalarini o‘rganish;

Erman akslantirishlari uchun sinish nuqtalari orbitasining aylanada joylashuvini o‘rganish;

Erman akslantirishlarining Danjua ko‘paytmasi uchun sinish nuqtalarining sinish kattaliklari va invariant o‘lchov orqali aniq ifodasini topish.

Tadqiqotning obyekti cheksiz ikkilik ketma-ketliklari, aylananing irratsional burish akslantirishi, Erman akslantirishi, kritik aylana gomeomorfizmlari va tushish vaqtlari.

Tadqiqotning predmeti simvolli dinamikalar fazosi, aylana akslantirishlari sinfi, ehtimollik inavariant o‘lchovlari nazariyasi.

Tadqiqotning usullari. Dissertatsiyada matematik analiz, funksional analiz, simvolli dinamika, ergodiklik nazariyasi va ehtimollar nazariyalari usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

aylananing irratsional burish akslantirishlari bilan bog‘langan cheksiz ikkilik ketma-ketliklarining murakkablik funksiyalari topilgan;

bitta kubik kritik nuqtaga va irratsional burish soniga ega bo‘lgan silliq aylana akslantirishlarining normallangan tushish vaqtlari taqsimot funksiyalarining yaqinlashishi va limit taqsimot funksiyasining singularligi isbotlangan;

ikkita sinish nuqtasiga ega va burish soni irratsional bo‘lgan bo‘lakli-chiziqli aylana gomeomorfizmlarining sinish nuqtalari joylashuvi topilgan va bunday akslantirishlar uchun Danjua tengligi isbotlangan;

ikkita sinish nuqtalariga ega bo‘lgan bo‘lakli-chiziqli va burish soni irratsional bo‘lgan aylana akslantirishlarining birinchi qaytish vaqti funksiyasining normallangan hosilalari bilan bog‘langan ikkilik ketma-ketliklari Shturm ketma-ketligi ekanligi isbotlangan.

Tadqiqotning amaliy natijasi cheksiz ikkilik ketma-ketliklarining xossalari va murakkablik funksiyalari, singulyar invariant o‘lchovning sonli ko‘rsatkichlari va taqsimot funksiyalari ketma-ketligining sust yaqinlashishini isbotlashda qo‘llanilgan usullarni bayon qilinganligidan iborat.

Tadqiqot natijalarining ishonchliligi funksional analiz, stoxastik jarayonlar va diskret vaqtli dinamik sistemalar nazariyasi usullaridan foydalanilgani hamda matematik mulohazalarning qat’iyligi bilan asoslangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati aylana gomeomorfizmlari bilan bog‘liq cheksiz ikkillik ketma-ketliklarning murakkablik funksiyalarini topish, termodinamik formalizm metodi yordamida invariant o‘lchovning singulyarlik ko‘rdatkichlarini topish va taqsimot funksiyalarining yaqinlashishini isbotlash bilan izohlanadi.

Tadqiqot natijalarning amaliy ahamiyati bir nechta sinish nuqtalariga va kritik nuqtalarga ega aylana gomeomorfizmlarida “qattiqlik muammosi”ni o‘rganishda tatbiq etish bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Aylana gomeomorfizmlari bilan bog‘liq Danjua tengligi va cheksiz ikkilik ketma-ketliklari bo‘yicha olingan natijalar asosida:

ikkita sinish nuqtasiga ega va burish soni irratsional bo‘lgan bo‘lakli-chiziqli aylana gomeomorfizmlari sinish nuqtalari joylashuvidan OT-F-4-03 raqamli «Uzluksiz hamda diskret vaqtli aniq dinamik sistemalar, qismiy integral operatorlar spektrlari» mavzusidagi fundamental ilmiy loyihada maxsuslikka ega aylana akslantirishlariga mos transfer operatorning darajalari asimptotikasini va spektrini topishda foydalanilgan (Qarshi davlat universitetining 2021 yil 12 iyundagi № 04/1900-sonli ma’lumotnomasi). Ilmiy natijaning qo‘llanilishi diskret vaqtli dinamik sistemalar uchun orbitalar va ularning asimptotik xossalarini to‘liq tasniflash imkonini bergan;

kritik nuqtaga va irratsional burish soniga ega bo‘lgan silliq aylana akslantirishlarining normallangan tushish vaqtlari taqsimoq funksiyalarining yaqinlashishidan OT-F-4-40 raqamli «O‘lchovli funksiyalar sinfida indekslangan integral empirik protsesslarning asimptotik xossalarini tadqiq qilish» (O‘zbekiston Milliy universitetining 2021 yil 16 sentabrdagi № 04/11-4794 sonli ma’lumotnomasi) mavzusidagi fundamental loyihada o‘lchovli funksiyalar sinfida indekslangan integral empirik protseslarning asimptotik xossalarini aniqlash jarayonida foydalanilgan. Ilmiy natijaning qo‘llanilishi kuchli bog‘langan tasodifiy miqdorlar taqsimot funksiyalarining absolyut uzluksizligini isbotlash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 2 ta xalqaro va 3 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 10 ta ilmiy ish chop etilgan, shulardan, O‘zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktori dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 5 ta maqola, jumladan, 2 tasi xorijiy va 3 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning hajmi 96 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zaruriyati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, ob’ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Simvolli dinamika va aylana akslantirishlari**” deb nomlangan birinchi bobida simvolli dinamik sistemalardan, xususan, cheksiz ikkilik so‘zlar va Shturm so‘zlariga doir kerakli ta’riflar va teoremalar bayon qilingan.

Shuningdek, aylana gomoemorfizmlari va simvolli dinamika o‘rtasidagi bog‘liqlik o‘rganilgan. Bundan tashqari, aylana akslantirishlari nazariyasiga oid asosiy tushunchalar va tasdiqlar keltirilgan: yo‘nalishni saqlovchi aylana gomeomorfizmlarining burish soni, ehtimollik invariant o‘lchovi, kritik aylana akslantirishlari, chekli sondagi sinish nuqtalariga ega bo‘lgan bo‘lakli-silliq aylana gomeomorfizmlari va tushish vaqti tushunchalari keltirilgan.

Birinchi bobning birinchi paragrafida cheksiz ikkilik so‘zlarga doir kerakli faktlar, xususan, ikkilik so‘zlarining tili, davriyligi, murakkabligi, so‘zlarning takrorlanishi tushunchalari keltirilgan. Shuningdek, cheksiz ikkilik so‘zlarning alohida turlari o‘rganilgan.

Agar biror u chekli so‘z va y cheksiz so‘z uchun $\omega = uwy$ o‘rinli bo‘lsa, u holda w chekli so‘z chekli A to‘plamda aniqlangan ω cheksiz so‘zining **qismso‘zi** deyiladi.

1-ta’rif. *Cheksiz so‘z ω ning* ***murakkabligi*** *deb ω ning uzunligi n ga teng barcha qismso‘zlari sonini aniqlaydigan $p_\omega(n)$ funksiyaga aytiladi.*

Bu funksiya quyidagi xossalarga ega:

- kamaymaydigan funksiya, ya‘ni, $p_\omega(n+1) \geq p_\omega(n)$.
- $p(1) = \#\text{A}$.
- davriy so‘z uchun shunday $C \in \mathbb{N}$ topiladiki, barcha $n \in \mathbb{N}$ larda $p_\omega(n) \leq C$ shartni qanoatlantiradi.

2-ta’rif. *Agar barcha n natural sonlar uchun $p_s(n) = n+1$ bajarilsa, u holda s cheksiz so‘*z ***Shturm so‘zi*** *deyiladi.*

Aylananing irratsioanal burishi akslantirishi bilan bog‘langan $\omega(x) = \{\omega_1, \omega_2, ...\}$ cheksiz so‘z **irratsional mexanik so‘z** deyiladi.

Birinchi bobning birinchi paragrafida Shturm so‘zlarining fundamental ekvivalentlik teoremasi keltirib o‘tilgan va bu uchinchi bobning uchunchi paragrafidagi asosiy tasdiqni isbotlash uchun juda muhim sanaladi.

1-teorema. *Aytaylik ω cheksiz ikkilik so‘zi bo‘lsin. Quyidagi tasdiqlar ekvivalentdir:*

(1) ω Shturm so‘zi.

(2) ω irratsional mexanik so‘z.

(3) ω muvozanatli va davriy emas.

Ikkinchi paragrafda aylana gomeomorfizmlari nazariyasi, kritik aylana akslantirishlari, dinamik bo‘linishlar va ular bilan bog‘liq simvolli dinamika haqidagi dastlabki zarur tushunchalar o‘rganilgan.

3-ta’rif. *Agar f funksiya biror ε - atrof $U_{\varepsilon}(x_{cr})=(x_{cr}-\varepsilon, x_{cr}+\varepsilon)$ uchun $C^{2d+1}(U_{\varepsilon}(x_{cr}))$ sinfga tegishli bo‘lsa va quyidagi shart bajarilsa*

$$\frac{df}{dx}(x_{cr})=\frac{d^2 f}{dx^2}(x_{cr})=...=\frac{d^{2d} f}{dx^{2d}}(x_{cr})=0, \quad \frac{d^{2d+1} f}{dx^{2d+1}}(x_{cr})\neq 0,$$

u holda $x_{cr}\in S^1$ nuqta f gomeomorfizmning $(2d+1), d\geq 1$ tartibli kritik nuqtasi deyiladi.

4-ta’rif. *Agar f akslantirish yagona toq tartibli kritik nuqtaga ega bo‘lsa, u holda f akslantirish* ***kritik akslantirish*** *deyiladi.*

So‘nggi 20-25 yillarda ko‘plab mualliflar kritik akslantirishlar uchun “qattiqlik” muammosini o‘rgandilar.

Ostlund va boshqa olimlar kritik aylana akslantirishlari sinfidagi renormgruppa almashtirishini o‘rganganlar. Ostlund va boshqalarning ishidan foydalanib, uchinchi tartibli haqiqiy-analitik kritik aylana gomeomorfizmlar to‘plamiga mos keladigan haqiqiy-analitik juftliklarni aniqlaymiz. Haqiqiy sonlar to‘plamida qat’iy o‘suvchi va quyidagi shartlarni qanoatlantiradigan haqiqiy-analitik juftliklar (ξ,η) to‘plami $\mathcal{X}_{cr}$ ni qaraymiz:

(c_1) $0<\xi(0)<1, \ \xi(0)=\eta(0)+1;$

(c_2) $\xi(\eta(0))=\eta(\xi(0))>0;$

(c_3) $\xi'(0)=\eta'(0)=\xi''(0)=\eta''(0)=0,$ lekin $\xi'''(0)\neq 0$ va $\eta'''(0)\neq 0$;

(c_4) $(\xi\circ\eta)'''(0)=(\eta\circ\xi)'''(0).$

(c_1) va (c_2) shartlar birlik aylanada $f=f_{\xi,\eta}$ gomeomorfizmni har bir $(\xi,\eta)\in\mathcal{X}_{cr}$ juftlik bilan bog‘lash imkonini beradi. Ya’ni, $[\eta(0),0]$ oraliqda $f=\xi$ va $[0,\xi(0)]$ oraliqda $f=\eta$ deb aniqlaymiz va $[\eta(0),\xi(0)]$ birlik intervalni chetki nuqtalari bilan ayniylanuvchi birlik aylana bilan bog‘laymiz. $f_{\xi,\eta}$ akslantirishning $\rho=\rho(f_{\xi,\eta})$ burish soni odatiy yo‘l bilan aniqlanadi.

Burish soni “oltin kesimga”, ya‘ni $\rho(f_{\xi,\eta})=\bar{\rho}=\frac{\sqrt{5}-1}{2}$ ga teng $\mathcal{X}_{cr}$ ning qism to‘plami bo‘lgan (ξ,η) juftliklarni $\mathcal{X}_{cr}(\bar{\rho})$ bilan belgilaymiz.

Quyidagi renormgruppa almashtirishini aniqlaymiz: $\Re:\mathcal{X}_{cr}(\bar{\rho})\to\mathcal{X}_{cr}(\bar{\rho})$, $\Re(\xi,\eta)=(\alpha\eta(\alpha^{-1}x),\alpha\eta(\xi(\alpha^{-1}x))$, bu yerda $\alpha:=\alpha_{\xi,\eta}=[\eta(0)-\eta(\xi(0))]^{-1}$.

(c_1), (c_2) shartlaridan $\alpha<-1$ ekanligi kelib chiqadi. $\Re$ renormgruppa almashtirishi $\mathcal{X}_{cr}(\bar{\rho})$ qism fazoda yagona giperbolik qo‘zg‘almas (ξ_0,η_0) nuqtaga ega. Shuningdek, $\xi_0(x)$, $\eta_0(x)$ funksiyalar x^3 ning haqiqiy-analitik funksiyalari va $\alpha_0:=\alpha_{\xi_0,\eta_0}\approx-1,2886$. (ξ_0,η_0) juftlik bilan bog‘langan aylana akslantirishlarini $f_{cr}:=f_{\xi_0,\eta_0}$ orqali belgilaymiz.

f_{cr} ga C^1-qo‘shma bo‘lgan barcha aylana gomemomorfizmlari to‘plamini $Cr(\bar{\rho})$ orqali belgilaymiz. Ma’lumki, ixtiyoriy ikkita topologik qo‘shma gomeomorfizmlar bir xil burish soniga ega. Bundan tashqari, $Cr(\bar{\rho})$ ga tegishli gomeomorfizmlarning burish sonlari bir xil va $\bar{\rho}$ ga teng bo‘ladi.

$Cr(\bar{\rho})$ sinfdan bo‘lgan kritik aylana akslantirishlari uchun termodinamik formalizm A. Djalilov tomonidan qurilgan.

Uchinchi paragrafda kritik aylana akslantirishlarining tushish vaqtlari uchun kerakli bo‘lgan ta’riflar va teoremalar keltirilgan.

Aytaylik, f irratsional $\rho=\rho(f)$ burish soniga ega bo‘lgan $S^1=\mathbb{R}/\mathbb{Z}\cong[0,1)$ aylanadagi yo‘nalishni saqlovchi gomeomorfizm bo‘lib, $\mu=\mu(f)$ o‘lchov f ning yagona invariant ehtimollik o‘lchovi bo‘lsin. Aylanadan $z\in S^1$ nuqtani fiksirlaymiz va $V_\varepsilon(z)=[z,z+\varepsilon]\subset S^1$ intervalni qaraymiz. Ushbu V_ε intervalga birinchi tushish vaqtini quyidagi formula yordamida aniqlaymiz:

$$E_\varepsilon(t)=\inf\{i\geq 1: T^i(t)\in V_\varepsilon(z)\}.$$

Endi esa, normallangan tushish vaqtini $\bar{E}_\varepsilon(t)=\mu(V_\varepsilon(z))E_\varepsilon(t)$ orqali aniqlaymiz. Tasodifiy miqdor $\bar{E}_\varepsilon(t)$ ning taqsimot funksiyasining yaqinlashishi masalasini qaraymiz, ya’ni quyidagi taqsimot funksiyasining $\varepsilon\to0$ da, limit funksiyasining uzluksizlik nuqtalariga tegishli bo‘lgan har bir t uchun:

$$F_\varepsilon(t)=\mu\left(x\in S^1: E_\varepsilon^{(1)}(x)\leq t\right),\ \forall t\in\mathbb{R}$$

yaqinlashishini o‘rganamiz.

Chiziqli irratsional burish $f_\rho(x)=x+\rho(\mathrm{mod}1)$ uchun $\bar{E}_\varepsilon(t)$ tasodifiy miqdorning yaqinlashishi masalasi Z. Koelo va E. de Faria tomoidan o‘rganilgan. Ma’lumki, f_ρ chiziqli irratsional burishlar uchun yagona invariant o‘lchov bu ℓ Lebeg o‘lchovidir. Agar $\varepsilon=\varepsilon_n$ni Z. Koelo va E. de Faria ishlaridagi kabi tanlab $V_{\varepsilon_n}(z)$ intervallar f_ρ ning renorm intervallari ketma-ketligiga mos bo‘lsa, u holda Lebeg o‘lchovida deyarli barcha ρ burish sonlari uchun $\bar{E}_\varepsilon(t)=\mu(V_\varepsilon(z))E_\varepsilon(t)$ normallangan tushish vaqti ε nolga intilganda taqsimot bo‘yicha (sust) yaqinlashmasligi isbotlangan hamda ε_n ketma-ketlikning barcha qismiy ketma-ketliklari qaralgan. Shuningdek, agar $F_{\varepsilon_n}(t)$ har biri nolga yaqinlashadigan biror

ε_n uchun yaqinlashsa, $F(t)$ ehtimollik taqsimotining limiti ikkita uzilish nuqtasiga ega zinapoya funksiyasi yoki [0,1] oraliqda tekis taqsimot bo‘ladi.

Aytaylik q_n, $n \ge 1$ sonlar f_ρ ning birinchi qaytish vaqtlari bo‘lsin. Biror $\theta \in (0,1)$ ni fiksirlaymiz. Quyidagi formula yordamida barcha $n \ge 1$ lar uchun $c_n(\theta)$ nuqtalarni aniqlaymiz:

$$\mu([x_0, c_n(\theta)]) = \theta \cdot \mu([x_0, f_\rho^{q_n}(x_0)]).$$

$\Delta_{n,c}$ orqali $[x_0, c_n(\theta))$ intervalni belgilaymiz. $\Delta_{n,c}$ intervalga tushish vaqti $E_{n,c}$ ni qaraymiz. Normallangan tushish vaqtini quyidagi ko‘rinishda aniqlaymiz:

$$\bar{E}_{n,c}(x) = \mu(\Delta_{n,c}) E_{n,c}(x).$$

$F_{n,c}(t)$ orqali $\bar{E}_{n,c}(x)$ ning taqsimot funksiyasini belgilaymiz. Ixtiyoriy ρ irratsional son uchun $\bar{E}_{n,c}(x)$ normallangan tushish vaqti ε nolga intilganda taqsimot bo‘yicha yaqinlashmasligi Koelo va de Faria tomonidan isbotlangan.

Ushbu dissertatsiyada, yagona kritik nuqtaga ega bo‘lgan aylana gomeomorfizmlari uchun normallangan tushish vaqtlarini tadqiq qilamiz. Aytaylik, $f \in Cr(\bar{\rho})$ kritik aylana gomeomorfizmi bo‘lsin. Aylanada ikkita tabiiy o‘lchovlar mavjud: μ invariant ehtimollik o‘lchovi va ℓ Lebeg o‘lchovi. $\bar{E}_{n,c}(x)$ normallangan tushish vaqtining mos ravishda μ va ℓ ga nisbatan $F_{n,c}(t)$ va $\Phi_{n,c}(t)$ taqsimot funksiyalarini qaraymiz. f akslantirishning $F_{n,c}(t)$ taqsimot funksiyasi f_ρ chiziqli burishning taqsimot funksiyasi bilan ustma-ust tushadi. Shu bois, Koelo va de Faria ishidagi barcha tasdiqlar f-invariant μ o‘lchovlar uchun ham o‘rinli. μ invariant o‘lchovi Lebeg o‘lchoviga nisbatan singulyar bo‘ladi. E’tiborli jihati shundaki, chekli sondagi sinish nuqtalari va irratsional ρ burish soniga ega bo‘lakli silliq f aylana gomeomorfizmlari Lebeg o‘lchoviga nisbatan ham ergodik bo‘ladi. Ta’rifga ko‘ra

$$\Phi_\varepsilon(t) = \ell\left(x \in S^1 : E_\varepsilon^{(1)}(x) \le t\right), \ \forall t \in \mathbb{R}.$$

A.Djalilov oltin kesim irratsional burish soniga ega kritik aylana akslantirishlari va kritik nuqtaning renorm atrofi uchun taqsimot funksiyalarining Legeb o‘lchoviga nisbatan limit holatlarini o‘rgangan. A. Djalilov va J. Karimov bitta sinish nuqtasi va oltin kesim irratsional burish soniga ega aylana gomeomorfizmlari uchun taqsimot funsiyasining limit holatini tadqiq qilgan.

Birinchi bob to‘rtinchi paragrafda sinishga ega bo‘lgan aylana gomeomorfizmlariga tegishli zarur tasdiqlar keltirilgan.

Sinish nuqtalariga ega bo‘lgan aylana akslantirishlari so‘nggi 20 yil ichida K. Xanin, D. Xmelev, A. Teplinski, D. Smania, K. Kunya, D. Mayer, A. Djalilov, I. Luiz va boshqalar tomonidan muttasil ravishda o‘rganilgan. Sinish nuqtasiga ega bo‘lakli silliq aylana gomeomorfizmlari aylana diffeomorfizmlarining tabiiy umumlashmasidir. Agar aylananing biror nuqtasida birinchi tartibli hosilasida sakrash mavjud bo‘lsa, u holda bu nuqta **sinish nuqtasi** deb ataladi. Bo‘lakli chiziqli aylana gomeomorfizmlari sinishlarga ega eng oddiy gomeomorfizmlardir. M. Erman ikkita sinish nuqtasiga va irratsional burish soniga ega bo‘lgan bo‘lakli chiziqli gomeomorfizmlarning invariant o‘lchovi absolyut uzluksiz bo‘lishi uchun ikkala sinish nuqtasi bir orbitada yotishi zarur va yetarliligini isbotlagan.

Sinishga ega nochiziqli bo‘lakli silliq aylana gomeomorfizmlarining invariant o‘lchovlari K. Xanin, A. Djalilov, A.Teplinski, D. Mayer, I. Luiz, D. Smania va boshqalar tomonidan o‘rganilgan.

Diffeomorfizmlardan farqli o‘laroq, bunday gomeomorfizmlarning o‘zgarmas o‘lchovlari Lebeg o‘lchoviga nisbatan singulyardir (K. Xanin, A. Djalilov, D. Mayer, I. Luiz). Sinishga ega yetarlicha silliq (har bir oraliqda silliq) gomeomorfizmlar uchun ularning renormlari chiziqli bo‘lakli akslantirishlar bilan yaqinlashadi (K. Xanin, D. Xmelev, A. Teplinski, D. Smaniya, K. Kunya).

Dissertansiyaning “**Aylana akslantirishlari bilan bog‘langan murakkablik funksiyalari va tushish vaqtlari**” deb nomlangan ikkinchi bobida aylananing irratsional burishlari va kritik aylana akslantirishlarining tushish vaqtlari asimptotik holatlari bilan bog‘liq bo‘lgan cheksiz ikkilik so‘zlar tadqiq qilingan.

Ikkinchi bobning birinchi paragrafida birlik aylanada irratsional burish bilan bog‘liq bo‘lgan cheksiz ikkilik ketma-ketliklarining asosiy xossalari o‘rganilgan.

Ushbu [0,1) intervalda $f_\rho(x) = x + \rho (\text{mod} 1)$ irratsional burishni qaraymiz va $b \in S^1$ nuqtani fiksirlaymiz. Aylanada $P = \{[0,b),\ [b,1)\}$ bo‘linishni qaraylik. Har bir $n \geq 1$ uchun $S_n(0,b)$ to‘plamni aniqlaymiz:

$$S_n(0,b) := \{0, f_\rho^{-1}(0), ..., f_\rho^{-n+1}(0)\} \cup \{b, f_\rho^{-1}(b), ..., f_\rho^{-n+1}(b)\}.$$

$S_n(0,b)$ to‘plamning nuqtalari P_n bilan belgilangan bo‘linishni tashkil qiladi. $P_1 := P$ bo‘lsin, P_n ni quyidagicha aniqlaymiz,

$$P_n := P \vee f_\rho^{-1}(P) \vee ... \vee f_\rho^{-n+1}(P).$$

Bu yerda $P \vee Q := \{A \cap B : A \subset P, B \subset Q\}$. P_n va P_{n+1} bo‘linishlarning elementlarini hech qanday indekslarsiz $I^{(n)}$ va $I^{(n+1)}$ bilan belgilaymiz.

$P_n,\ n \geq 0$ bo‘linishlar ketma-ketligidan foydalangan holda simvolli dinamikaning ma’lum bir maxsus turini hosil qilamiz. Quyidagi $I_1 := [0,b)$ va $I_0 := [b,1)$ belgilashlarni kiritamiz. $\nu_b : S^1 \to \{0,1\}$ kodlash funksiyasini qaraymiz: barcha $i \geq 0$ lar uchun

$$\nu_b(f_\rho^i(x)) := \begin{cases} 0, \text{ agar } x \in f_\rho^{-i}(I_0), \\ 1, \text{ agar } x \in f_\rho^{-i}(I_1). \end{cases}$$

Ixtiyoriy $x \in S^1$ ni olib, bu nuqtaga mos keluvchi

$$\underline{\omega} = (\omega_0 \omega_1 ... \omega_n ...) := (\nu_b(x) \nu_b(f_\rho(x)) ... \nu_b(f_\rho^n(x)) ...) \qquad (1)$$

nol va birlardan iborat $\underline{\omega} := \underline{\omega}(x)$ cheksiz ketma-ketlikni aniqlaymiz.

Yuqorida keltirilgan (1) formula orqali aniqlangan barcha maqbul so‘zlar to‘plamini $L_\omega(\rho,b)$ orqali belgilaymiz:

$$L_\omega(\rho,b)=\{\underline{\omega}(x), x\in S^1\}.$$

Quyidagi belgilashlarni kiritamiz:

- $\omega_m^{m+s}=(\omega_m\omega_{m+1}...\omega_{m+s}),\ m,\ s\geq 0,\ \underline{\omega}$ cheksiz so‘zning qismso‘zi,
- $\omega_m^{\infty}=(\omega_m\omega_{m+1}...),\ m\geq 0\ \ \underline{\omega}$ ning dum qismi.

5-ta’rif. *Agar barcha $n\geq N$ lar uchun $\omega_{n+T}=\omega_n$ shatrni qanoatlantiruvchi shunday $N,T\in\mathbb{N}$ topilsa, u holda $\underline{\omega}$ cheksiz so‘z* ***shartli davriy*** *deyiladi. Aks holda $\underline{\omega}$* ***davriy emas*** *deyiladi.*

Simvolli kodlash ta’rifidan quyidagi foydali faktlar osongina kelib chiqadi:

a) *aytaylik, $I^{(n)}$ interval P_n bo‘linishning ixtiyoriy intervali bo‘lsin. U holda ixtiyoriy ikkita $x,y\in I^{(n)}$ nuqtalar uchun ularning $n+1$ uzunlikdagi prefikslari ustma-ust tushadi, ya‘ni $\omega_0^n(x)=\omega_0^n(y)$. Shu sababli, $\bigcap_{i=0}^{m-1} f_\rho^{-i}(P)$ ning har bir intervaliga uzunligi m bo‘lgan bitta so‘z mos keladi.*

b) *$u_0u_1...u_{m-1}$ so‘zi biror $\underline{\omega}=(\omega_0\omega_1...\omega_n...)$ cheksiz so‘zning qismso‘zi bo‘lishi uchun, ya‘ni $i=0,...,(m-1)$ larda $\omega_{n+i}=u_i$ bajarilishi uchun, $f_\rho^n(x)\in I(u_0u_1...u_{m-1})$ bo‘lishi zarur va yetarli, bu yerda*

$$I(u_0u_1...u_{m-1})=\bigcap_{i=0}^{m-1} f_\rho^{-i}(I_{b_i})\in P_m.$$

c) *$I(u_0u_1...u_{m-1})$ interval bo‘sh bo‘lmasligi uchun $u_0u_1...u_{m-1}$ so‘zi $\underline{\omega}$ ning qismso‘zi bo‘lishi zarur va yetarli.*

Quyidagi belgilashni kiritamiz: $d_n=\max_{I^{(n)}\in P_n}|I^{(n)}|$.

1-lemma. *Barcha $n\geq 1$ lar uchun $d_n\geq d_{n+1}$ va $\lim\limits_{n\to\infty} d_n=0$ bo‘ladi.*

$I^{(n)}(x)$ orqali x nuqtani o‘z ichiga olgan $n-$bo‘linish P_n ning intervalini va $W_n(\underline{\omega}(x))$ orqali $\underline{\omega}(x)$ ikkilik cheksiz so‘zining uzunligi n ga teng barcha qism so‘zlari to‘plamini belgilaymiz.

2-teorema. *Aytaylik $x,y\in S^1$ va $\underline{\omega}(x),\ \underline{\omega}(y)$ cheksiz so‘zlar ularning mos ravishda (1) formula yordamida aniqlangan cheksiz ketma-ketliklari bo‘lsin. U holda quyidagilar o‘rinli:*

1. *Barcha $n\geq 0$ lar uchun $W_n(\underline{\omega}(x))=W_n(\underline{\omega}(y))$ bo‘ladi.*
2. *$\underline{\omega}(x)$ tekis takrorlanuvchi.*
3. *Ixtiyoriy $x\in S^1$ uchun $\underline{\omega}(x)$ davriy emas.*

Ikkinchi bob ikkinchi paragrafida birlik aylananing irratsional burishi bilan bog‘liq bo‘lgan cheksiz ikkilik ketma-ketliklarining murakkablik funksiyalari o‘rganilgan.

Ixtiyoriy $b\in[0,1)$ uchun $L_{\omega}(\rho,b)$ ning cheksiz so‘zlari murakkablik funsiyalarini o‘rganamiz. $b\neq\{\rho,1-\rho\}$ bo‘lsa, u holda barcha $n\geq1$ lar uchun $p_{\underline{\omega}}(n)\geq n+1$ bo‘ladi.

Agar $a,b\in A$ uchun ua va ub so‘zlar ham ω so‘zining qism so‘zi bo‘lsa, u holda u chekli so‘z ω cheksiz ikkilik so‘zining **o‘ng maxsus qismso‘zi** deyiladi.

$r(n)$ orqali n uzunlikdagi o‘ng maxsus qismso‘zlar sonini belgilaymiz va aytaylik $k_0=min\{n\geq1: r(n)=2\}$ bo‘lsin.

A ikkilik alifboda $r(n)$ o‘ng maxsus qismso‘zlar sonidan quyidagi formula yordamida murakkablik funksiyalarini aniqlashda foydalaniladi:

$$p_{\underline{\omega}}(n+1)=p_{\underline{\omega}}(n)+r(n).$$

Endi esa, ikkinchi paragrafning ikkita asosiy teoremalarini keltiramiz. Birinchi teoremada b nuqta 0 nuqtaning orbitasiga tegishli bo‘lmagan holat o‘rganilgan.

3-teorema. *Aytaylik, f_{ρ} akslantirish $\rho\in(0,1)$ irratsional burchakka chiziqli burish bo‘lsin va $b+n\rho\notin\mathbb{Z}$ bajarilsin. U holda quyidagilar o‘rinli:*

1. Agar $0<\rho<b$ bo‘lsa, u holda barcha $n\geq1$ uchun $p_{\underline{\omega}}(n)=2n$.

2. Agar $0<b<\rho$ bo‘lsa, u holda

$$p_{\underline{\omega}}(n):=\begin{cases} n+1, & agar\ \ n<k_0,\\ 2n-k_0+1, & agar\ \ n\geq k_0.\end{cases}$$

Quyidagi teoremada b nuqta 0 nuqtaning orbitasida yotadigan holat o‘rganilgan.

4-teorema. *Aytaylik, f_{ρ} akslantirish 3-teoremadagi shartlarni qanoatlantirsin va biror $d\in\mathbb{Z}\setminus\{0\}$ son uchun $b+d\rho\in\mathbb{Z}$ bo‘lsin. U holda quyidagilar o‘rinli:*

1. Agar $0<\rho<b$ bo‘lsa, u holda $p_{\underline{\omega}}(n):=\begin{cases} 2n, & agar\ \ n<|d|,\\ n+d, & agar\ \ n\geq|d|.\end{cases}$

2. Agar $0<b<\rho$ bo‘lsa. u holda $p_{\underline{\omega}}(n):=\begin{cases} n+1, & agar\ n<k_0,\\ 2n-k_0+1, & agar\ k_0\leq n<|d|\\ n+d-k_0-1, & agar\ n\geq|d|.\end{cases}$

Ikkinchi bobning uchinchi paragrafida yagona kubik kritik nuqtaga ega bo‘lgan $f\in Cr(\overline{\rho})$ kritik aylana akslantirishlari uchun normallangan tushish vaqtlarining asimptotik holatlari o‘rganilgan. Bu paragrafda quyidagi teorema isbotlangan va isbotlash davomida $E^{(1)}_{n,\theta}(x)$ tasodifiy miqdorlar uchun $\Phi_{n,\theta}(t)$ taqsimot funksiyalarining aniq formulasi topilgan.

Uchinchi paragrafning asosiy natijasini keltiramiz.

5-teorema. *Aytaylik* $\bar{\rho}=\dfrac{\sqrt{5}-1}{2}$ *va* $f\in Cr(\bar{\rho})$ *kritik akslantirish bo'lsin.* $\theta\in(0,1)$ *da* $[x_c,c_n(\theta)]$ *intervalga normallangan birinchi tushish vaqti* $E^{(1)}_{n,\theta}(x)$ *ga aylanada Lebeg o'lchoviga nisbatan* $\left\{\Phi_{n,\theta}(t)\right\}_{n=1}^{\infty}$ *taqsimot funksiyalari ketma-ketligini qaraymiz. U holda*

1) *Barcha* $t\in\mathbb{R}$ *lar uchun quyidagi chekli limit,* $\lim\limits_{n\to\infty}\Phi_{n,\theta}(t)=\Phi_\theta(t)$ *mavjud, undan tashqari* $t\le 0$ *da* $\Phi_\theta(t)=0$ *va* $t>1$ *da* $\Phi_\theta(t)=1$.

2) $\Phi_\theta(t)$ *limit funksiya [0,1] da qat'iy o'suvchi va* $\mathbb{R}$ da *uzluksiz taqsimot funksiyasi.*

3) $\Phi_\theta(t)$ *limit funksiya [0,1] da singulyar, ya'ni deyarli hamma yerda (aylanada* ℓ *Lebeg o'lchovi bo'yicha)* $\dfrac{\Phi_\theta(t)}{dt}=0$.

Dissertatsiyaning "**Danjua tengligi va P-gomeomorfizmlar uchun Shturm so'zlari**" deb nomlangan uchinchi bobi P-gomeomorfizmlarining sinish nuqtalari joylashuvi hamda h^{q_n} Erman akslantirishlarining normallangan hosilalari holatini o'rganishga bag'ishlangan.

$x_0\in S^1$ nuqta va uning $O(x_0)=\{x_i=f^i(x_0),\ i\in\mathbb{Z}\}$ orbitasini qaraymiz. Eslatib o'tamiz, $\mathcal{P}_n(x_0)=\{I_i^{(n-1)}(x_0),0\le i<q_n\}\cup\{I_j^{(n)}(x_0),0\le j<q_{n-1}\}$ intervallar sistemasi ixtiyoriy n uchun aylananing bo'linishini aniqlaydi, bu x_0 nuqtaning n**-dinamik bo'linishi** deyiladi.

Irratsional burish soni ρ_f va bitta orbitada yotmaydigan ikkita a_0 va c_0 sinish nuqtalariga ega bo'lgan f P-gomeomorfizmni qaraymiz. $\dfrac{p_n}{q_n}$ orqali ρ_f ning n-yaqinlashuvchi munosib kasrini belgilaymiz. S^1 aylanada f^{q_n} ning sinish nuqtalari joylashuvini va Df^{q_n} hosilani aniqlaymiz. Ravshanki, f^{q_n} akslantirish $BP_f^n:=BP_f^n(a_0)\cup BP_f^n(c_0)$ orqali ifodalangan $2q_n$ ta sinish nuqtasiga ega, ya'ni mos ravishda $a^*_{-i}=f^{-i}(a_0)$ va $c^*_{-i}=f^{-i}(c_0)$, $0\le i\le q_n-1$ sinish nuqtalaridan iborat. f^{q_n} akslantirishning ushbu sinish nuqtalari S^1 aylanada o'zaro kesishmaydigan $2q_n$ ta intervallarga bo'linadigan $B_n(f)$ bo'linishni aniqlaydi.

Ikkinchi sinish nuqtasi c_0 ning $B_n(f)$ bo'linish intervallaridagi joylashuvini quyida keltirilgan 4 ta farqli holatlarga bo'lib o 'rganamiz.

Aytaylik, $\mathcal{P}_n(a_0^*)$ bo'linish f akslantirishga nisbatan $a_0^*=a_0$ sinish nuqtasi orqali aniqlangan n-dinamik bo'linish bo'lsin. Faraz qilaylik, a_0^* va c_0^* sinish nuqtalari bitta orbitada yotmasin, u holda ikkinchi sinish nuqtasi uchun quyidagi uchta holatlar o'rinli bo'ladi:

I hol. $c_0^*\in I_{i_0}^{(n)}(a_0)$ biror $0\le i_0<q_{n-1}$ uchun.

II hol. $c_0^* \in f^{j_0}((a_0, a_{-q_n}))$ biror $0 \le j_0 < q_n$ uchun.

III hol. $c_0^* \in f^{j_0}((a_{-q_n}, a_{q_{n-1}}))$ biror $0 \le j_0 < q_n$ uchun.

Agar a_0^* va c_0^* sinish nuqtalari bitta orbitada yotsa, u holda:

IV hol. $c_0^* = f^{i_0}(a_0^*)$ biror $0 \le i_0 < q_n$ uchun.

Uchinchi bobning birinchi paragrafida f^{q_n} akslantirish sinish nuqtalari joylashuvini I va III hollarda oʻrganamiz va Erman akslantirishlari uchun Danjua tengligini I va III holatlarda isbotlaymiz.

Ikkita sinish nuqtasiga ega boʻlakli-chiziqli aylana akslantirishlari, Erman akslantirishlarini, h orqali belgilaymiz. Irratsional burish soni ρ ga ega boʻlgan ixtiyoriy Erman akslantirishlari Danjua teoremasi shartlarini qanoatlantiradi. Demak h akslantirish f_ρ chiziqli burish bilan topologik ekvivalent.

Biz h^{q_n} akslantirishning normallangan hosilalarini $\sigma_h(a_0)$ sinish kattaligi hamda h^{q_n} ning sinish nuqtalaridan hosil qilingan $B_n(h)$ boʻlinish intervallarining μ_h-oʻlchovi yordamida ifodalaymiz.

I va III hollarda $a_0^* = 0$ dan hosil boʻladigan $PB_h(a_0^*)$ sinish nuqtalari va $c_0^* = c_0$ dan hosil boʻladigan $PB_h(c_0^*)$ sinish nuqtalari S^1 aylanada navbatma-navbat joylashadilar.

Aytaylik, n toq boʻlsin. h^{q_n} akslantirishning sinish nuqtalaridan foydalanib quyidagi subintervallarni aniqlaymiz:

$$A_n := \bigcup_{s=1}^{i_0} [c^*_{-i_0+s},\, a^*_{-q_n+s}], \quad B_n := \bigcup_{s=i_0+1}^{q_n} [c^*_{-i_0-q_n+s},\, a^*_{-q_n+s}].$$

III holda keltirilgan farazimizda subintervallar mos ravishda $[a^*_{-q_n+s}, c^*_{-i_0+s}],\ 1 \le s \le i_0$ va $[a^*_{-q_n+s}, c^*_{-i_0-q_n+s}],\ i_0+1 \le s \le q_n$ koʻrinishida berilgan. Bu subintervallarni quyidagi qism toʻplamlarga birlashtiramiz:

$$A_n := \bigcup_{s=1}^{i_0} [a^*_{-q_n+s}, c^*_{-i_0+s}], \quad B_n := \bigcup_{s=i_0+1}^{q_n} [a^*_{-q_n+s}, c^*_{-i_0-q_n+s}].$$

n soni juft boʻlganda yuqoridagi intervallarning chetki nuqtalari oʻrin almashgan holda boʻladi. I hol va n juft boʻlganda va mos ravishda III hol va n toq boʻlganda A_n va B_n qism toʻplamlarini avvalgidek aniqlash mumkin. Yuqoridagilar shuni koʻrsatadiki, A_n va B_n qism toʻplamlaridagi har bir intervalning chetki nuqtalari mos ravishda $PB_h(a_0^*)$ va $PB_h(c_0^*)$ dan olingan sinish nuqtalaridan iboratdir. $\sigma = \sigma_h(a_0)$ orqali h akslantirishning $x = 0$ nuqtadagi sinish kattaligini belgilaymiz va birinchi paragrafning asosiy natijasini keltiramiz.

6-teorema. *Aytaylik, h akslantirish ρ_h irratsional burish soniga, turli orbitalarda yotuvchi ikkita $a_0^* = 0$ va $c_0^* := c_0$ sinish nuqtalariga ega va umumiy sinish kattaligi $\sigma_h = 1$ boʻlgan boʻlakli-chiziqli aylana gomeomorfizmi boʻlsin.*

Faraz qilaylik, c_0^* *biror* i_0 *,* $0 \le i_0 < q_{n-1}$ *uchun mos ravishda I va III hollardagi shartlarni qanoatlantirsin. U holda I holda*

$$(Dh^{q_n}(x))^{(-1)^n} = \begin{cases} \sigma^{\mu_h(A_n \cup B_n)-1}, & agar\ x \in A_n \cup B_n \\ \sigma^{\mu_h(A_n \cup B_n)}, & agar\ x \in S^1 \setminus (A_n \cup B_n); \end{cases}$$

va mos ravishda III holda

$$(Dh^{q_n}(x))^{(-1)^{n+1}} = \begin{cases} \sigma^{\mu_h(A_n \cup B_n)-1}, & agar\ x \in A_n \cup B_n \\ \sigma^{\mu_h(A_n \cup B_n)}, & agar\ x \in S^1 \setminus (A_n \cup B_n). \end{cases}$$

Ikkinchi paragrafida II va IV hollar uchun h^{q_n} ning sinish nuqtalari joylashuvi va Erman akslantirishlari h^{q_n} ning normallangan hosilalari uchun Danjua tengligi isbotlangan.

Uchinchi bobning uchinchi paragrafida h Erman akslantirishlari bilan bog‘liq cheksiz ketma-ketliklarning murakkablik funksiyalari o‘rganilgan. h^{q_n}, $n > 0$ ning normallangan hosilalari bilan bog‘liq cheksiz ikkilik ketma-ketliklari Shturm ketma-ketliklari bo‘lishi isbotlangan.

XULOSA

Ushbu dissertatsiya aylanada irratsional burish bilan bog‘liq cheksiz ikkilik ketma-ketliklarining murakkablik funksiyalarini, kritik aylana akslatirishlari uchun tushish vaqtlari holatlarini va bo‘lakli-chiziqli akslantirishlar uchun Danjua funksiyasini baholashni tadqiq qilishga bag‘ishlangan.

Tadqiqot ishining asosiy natijalari quyidagilar:

1. Irratsional aylana burishi bilan bog‘liq cheksiz ikkilik ketma-ketliklarning murakkablik funksiyalari topilgan.
2. Bitta kubik kritik nuqtaga va “oltin kesim” burish soniga ega bo‘lganaylana akslantirishlarining nomallangan umumlashgan tushish vaqtlari taqsimot funksiyalari bo‘yicha yaqinlashishi va limit taqsimot funksiyasi singular, ya’ni uzluksiz, qat’iy o‘suvchi va Lebeg o‘lchovi bo‘yicha deyarli barcha yerda hosilasi nolga teng ekanligi isbotlangan.
3. Ikkita sinish nuqtasiga ega bo‘lakli-chiziqli aylana gomeomorfizmlarining sinish nuqtalari joylashuvi topilgan.
4. h^{q_n} Erman akslantirishlari uchun Danjua tengligi isbotlangan.
5. h^{q_n} ning normallangan hosilalari bilan bog‘liq cheksiz ikkilik ketma-ketliklar Shturm ketma-ketligi ekanligi ko‘rsatilgan.

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INSTITUTE OF MATHEMATICS

JALILOV ALISHER AKBAROVICH

DENJOY EQUALITY AND INFINITE BINARY SEQUENCES ASSOCIATED WITH CIRCLE HOMEOMORPHISMS

01.01.01-Mathematical analysis

ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PHD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2021

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INTRODUCTION

Actuality and demand of the theme of the thesis. In the world, many scientific and applied researchs are reduced to the study of one dimensional and symbolic dynamical systems. Dynamic systems are widely used in many other mathematical fields. Moreover, circle maps and symbolic dynamics are used in solving problems of medicine, biology, information technology and other exact science. Hitting times, random shifts and properties of the complexity functions of infinite symbolic sequences play an important role in predicting the long-term state of a particular process modeled in the form of nonlinear dynamic systems. For this reason, the study of the properties of limit theorems for hitting times of dynamic systems, the properties of sequences describing linear and nonlinear maps has become one of the most important and topical issues in the theory of dynamic systems.

Nowadays. many scientific studies are concerned with predicting the long-term state of nonlinear dynamic systems, especially those with circle maps are very important. Circle diffeomrphisms, an important class of circle maps, have been fully and deeply studied. Denjoy equality for piece-wise smooth maps, the natural generalization of circle diffeomorphisms, was not showen yet. Recent years, Denjoy inequality, rigidity problems for circle maps wth singularities were studied intensively. Finding Denjoy equality, convergence in law of the distributions of rescaled hitting times and complexity functions of infinite binary sequences are considered as aimed research.

In our country, much attention has been paid to develop important directions of one-dimensional dynamical systems and symbolic dynamics which have applications to the applied and fundamental sciences. Investigations on the international level in such important areas as the functional analysis, mathematical physics, theory of probability and theory of dynamical systems considered as the main task of fundamental research. Circle maps and symbolic dynamical systems are important not only for natural sciences, but also for economics, information theory, biology, for the study of various heart diseases, for blood tests, etc. Significant results were obtained on the dynamical systems generated by discontinuous function. Investigations on the international level in such important areas as the functional analysis and dynamical systems theory has been considered the main task of fundamental research[1].

The subject and object of research of this dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan of February 7, 2017, PF-4947, «On the strategy of action for the further development of the Republic of Uzbekistan», PQ-4387 dated July 9, 2019 «On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovskiy of the Academy of Sciences of the Republic of

[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017-year 18 May « On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan» № 292 dated May 17, 2017.

Uzbekistan», PQ-4708 of May 7, 2020 «On measures to improve the quality of education and research in the field of mathematics» as well as in other regulations related to basic sciences.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of the Republic of Uzbekistan IV, «Mathematics, Mechanics and Computer Science».

The degree of scrutiny of the problem. The field of symbolic dynamics evolved as a tool for analyzing general dynamical systems. Hadamard first successfully used the symbolic dynamics techniques in his analysis of geodesic flows on surfaces of negative curvature in 1898. Forty years later the subject got its first systematic study, and its name, in the foundational paper of Marsten Morse and Gustav Hedlund. In the future the symbolic dynamics used in information theory by C.E. Shannon.

Ya. Sinai in his fundamental work used the symbolic dynamis for investigating Anosov’s flows. E. Vul, Ya. Sinai and k. Khanin constructed the symbolic dynamics for Feigenbaum’s map and build the thermodynamic formalism. The symbolic dynamics plays important role in the theory circle maps. It is well known that every irrational rotation of the circle produced the space of dual sequences X. To study the complexity functions of dual sequences is one of main problem of symbolic dynamical systems. Coelho and De Faria studied the behaviour of rescaled hitting times for irrational rotations. A. Dzhalilov builds the thermodynamic formalism for critical circle maps.

Denjoe inequalities play an important role in circle dynamics. First Denjoy proved the inequality for diffeomorphisms. Using this important inequality was proven topologically conjugacy of circle homeomorphism to irrational rotation. Using renormalization method, Y. Sinai and K. Khanin, Y. Katznelson and D. Ornstein proved stronger version of Denjoy inequality. That result was very important to prove smoothness of the conjugacy map. Piece-wise smooth circle maps are natural generalizations of diffeomorphisms. M. Herman proved that the invariant measure is absolute continuous when both break points of piece-wise smooth circle maps are in the same orbit. For circle maps f with singularities “rigidity” problem, i.e. problems related to the smoothness of the conjugacy, estimates for the derivatives Df^{q_n} are very important.

The connection of the theme of the thesis with the research plans of the higher education institute, where the research on the thesis is carried out. The dissertation research is done in accordance with the planned theme of scientific research OT-F4-82 + OT-F4-87 «Local derivations and automorphisms of operator and nonassociative algebras, phase transitions and chaos in nonlinear dynamical systems» + «The theory of global invariants of curves and surfaces in Euclidean and pseudo-Euclidean spaces and its applications in mechanics» (2017-2020) and scientific research «YoOT-Ftex- 2018-78, Dynamical and thermodynamical systems on non-amenable graphs» (2018–2019) at the Institute of Mathematics after named V.I. Romanovskiy.

The aim of the research work is to study the complexity functions for binary infinite sequences associated by irrational rotations of the circle and Denjoy equality for Herman's maps.

Research problems:

to investigate the complexity functions of infinite binary sequences associated by irrational circle rotations;

to study the complexity functions of infinite sequences produced by Denjoy functions and its high iterations;

to study the structure of orbits of break points for Herman's map;

to find an explicit expression for Denjoy products of Herman's map by the size of break point and invariant measure.

The research object infinite binary sequences, irrational rotations of the circle, Herman's map, critical circle maps, hitting times for circle maps.

The research subject: symbolic dynamical systems, the theory of circle maps, invariant measures.

Research methods. In the work used the methods of mathematical analysis, functional analysis, symbolic dynamics, ergodic theory and probability theory.

Scientific novelty of the research work is as follows:

It is found the complexity functions of infinite binary sequences associated by irrational circle rotations.

It is proved that the rescailed generalized hitting times of critical circle maps with one cubic critical point and irational rotation number converges in distribution and the limit distribution is singular function.

It is found the location of break points of piecewise-smooth circle homeomorphisms with two break points and it is proved the Denjoy equality.

It is proved that the binary sequences associated by rescaled derivatives of piecewise-smooth circle homeomorphisms with two break points and irrational rotation number is Sturmian.

Practical results of the research are: properties and complexity functions of binary sequences were showen, singularity of invariant measure and convergence in law of sequence of distributions were proved.

The reliability of the results of the study. The results have been obtained by using the methods of functional analysis, theory of discrete time dynamical systems and stochastic processes. The obtained results are mathematically strongly proved.

Scientific and practical significance of the research results. The scientific significance of the research results is explained by the facts that the complexity function of binary sequences associated by irrational rotations was found, by thermodynamic formalism method was showen singularity of the invariant measure and convergence in law of the sequence of distributions was showen.

The practical significance of the research is determined by applications to chaotic mapping problems of intervals by determining the piecewise-smooth maps with several break points and critical circle maps.

Implementation of the research results. Results related to piecewise-smooth dynamical systems with discrete-time were used in the following research projects:

the locations of break points of piece-wise smooth circle maps with two break points have been used in the research project No. OT-Ф-4-03 «Continuous and discrete time exact dynamic systems, spectras of partial integral operators» for describing the asymptotics of degrees and spector of transfer operators (Reference No. 04/1900 of Karshi State University dated June 12, 2021). The application of the scientific result allowed to classify the orbits and their asymptotic properties of discrete time dynamic systems;

the convergence in law of the sequence of distributions of rescaled hitting times of critical circle maps were used in the fundamental research OT-F-4-40 «Research on asymptotic properties of the integral empiric processes indexed in measurable functions class» to describe the properties of integral empiric processes indexed in measurable functions class (Reference № 04/11-4794 of NUUz dated September 16, 2021). The application of the scientific result allowed to prove that distribution functions of strongly depended random varianbles are absolute continuous.

Approbation of the research results. The main results of the research have been discussed at 2 international and 3 national scientific conferences.

Publications of the research results. On the topic of the dissertation 5 research papers have been published in the scientific journals, three of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis; in addition, two of them were published in international journals.

The structure and volume of the thesis. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The general volume of the thesis is 96 pages.

THE MAIN CONTENT OF THE THESIS

The introduction we gave the motivation of research theme an correspondence to the priority research areas of science and technology of the Republic. We showed the degree of study of the problem, formulates goals and objectives, identifies the object and subject of research, sets out scientific novelty and practical the results of the research, the theoretical and practical significance of the results obtained is disclosed, information is given on the implementation of the research results, on the published works and on the structure of the dissertation

In the first chapter of the thesis, titled "**Symbolic dynamics and circle maps**" we gave necessary definitions and theorems from the symbolic dynamical systems, especially, infinite binary words and Sturmian words.

Also, we considered relation between circle homeomorphisms and symbolic dynamics. Moreover, we gave the main notions and facts from circle dynamics: rotation number of the orientation preserving circle homeomorphism, probability invariant measure, critical circle maps, piecewise-smooth circle homeomorphisms with finite number of break points and hitting times.

In section 1.1, we gave the necessary definitions and facts on infinite binary words such as, language, periodicity and complexity, recurrence of a binary word. We have learned some special types of the infinite binary words.

A finite word w is a **factor** of an infinite word ω over a finite set A if $\omega = uwy$ for some finite word u and infinite word y.

Definition 1. *The **factor complexity** of an infinite word ω is the function $p_\omega(n)$ counting the number of its factors of length n.*

And it satisfies the following properties:

• non-decreasing function, i.e., $p_\omega(n+1) \geq p_\omega(n)$

• $p(1) = \#\mathrm{A}$

• for a periodic word ω there exists $C \in \mathbb{N}$ such that $p_\omega(n) \leq C$ for all $n \in \mathbb{N}$.

Definition 2. *An infinite word s is called **Sturmian** if $p_s(n) = n+1$ for every natural n.*

The infinite word $\omega(x) = \{\omega_1, \omega_2, ...\}$ associated by irrational rotation is called **irrational mechanical word**.

In particular, we provided fundamental equivalence theorem of Sturmian words which is very important to prove main fact in section 3.3.

Theorem 1. *Let ω be an infinite binary word. The followings are equivalent:*

(1) ω is Sturmian.

(2) ω is irrational mechanical.

(3) ω is balanced and aperiodic.

In section 1.2 we studied prelimianry informations on the theory of circle homeomorphisms, critical circle maps, dynamical partitions and symbolic dynamics.

Definition 3. *A point $x_{cr} \in S^1$ is called **critical point** of order $(2d+1)$, $d \geq 1$ for homeomorphism f , if for some ε- neighborhood $U_\varepsilon(x_{cr}) = (x_{cr} - \varepsilon, x_{cr} + \varepsilon)$, the function f belongs to $C^{2d+1}(U_\varepsilon(x_{cr}))$ and*

$$\frac{df}{dx}(x_{cr}) = \frac{d^2 f}{dx^2}(x_{cr}) = ... = \frac{d^{2d} f}{dx^{2d}}(x_{cr}) = 0, \quad \frac{d^{2d+1} f}{dx^{2d+1}}(x_{cr}) \neq 0.$$

Definition 4. *The map f is called **critical map**, if it has unique critical point of the odd order.*

During the last 20-25 years the rigidity problem for critical maps was studied by many authors.

Ostlund and others investigated the renormalization transformation in the class of critical circle maps. Following to the work of Ostlund and others we define a set of real-analytic commuting pairs that corresponds to a set of real-analytic critical circle homeomorphisms the order of three. Consider the set $\mathcal{X}_{cr}$ of pairs (ξ,η) of real-analytic, strictly increasing on real line and satisfying the following conditions:

(c_1) $0<\xi(0)<1, \xi(0)=\eta(0)+1;$

(c_2) $\xi(\eta(0))=\eta(\xi(0))>0;$

(c_3) $\xi'(0)=\eta'(0)=\xi''(0)=\eta''(0)=0,$ but $\xi'''(0)\neq 0$;

(c_4) $(\xi\circ\eta)'''(0)=(\eta\circ\xi)'''(0).$

Conditions (c_1) and (c_2) permit us to associate a homeomorphism $f=f_{\xi,\eta}$ on the unit circle with each $(\xi,\eta)\in\mathcal{X}_{cr}$. Define $f=\xi$ on $[\eta(0),0]$ and $f=\eta$ on $[0,\xi(0)]$ and associate the unit interval $[\eta(0),\xi(0)]$ with the circle by identifying endpoints. A rotation number $\rho=\rho(f_{\xi,\eta})$ can be defined for $f_{\xi,\eta}$ in the usual way.

We denote by $\mathcal{X}_{cr}(\bar{\rho})$ the subset $\mathcal{X}_{cr}$ of pairs (ξ,η) for which the rotation number $\rho(f_{\xi,\eta})=\bar{\rho}=\dfrac{\sqrt{5}-1}{2}$ i.e. it is equal to the golden mean.

Next, we define the renormalization group transformation $\Re:\mathcal{X}_{cr}(\bar{\rho})\to\mathcal{X}_{cr}(\bar{\rho})$ by

$$\Re(\xi,\eta)=(\alpha\eta(\alpha^{-1}x),\alpha\eta(\xi(\alpha^{-1}x)),$$

where $\alpha:=\alpha_{\xi,\eta}=[\eta(0)-\eta(\xi(0))]^{-1}.$

The conditions $(c_1),(c_2)$ imply that $\alpha<-1.$ The renormalization group transformation $\Re$ has a single hyperbolic fixed point (ξ_0,η_0) in the subspace $\mathcal{X}_{cr}(\bar{\rho})$. Notice that $\xi_0(x),\eta_0(x)$ are real analytic functions of x^3 and the constant $\alpha_0:=\alpha_{\xi_0,\eta_0}\approx-1,2886$. Denote by $f_{cr}:=f_{\xi_0,\eta_0}$ the circle map associated to (ξ_0,η_0).

Denote by $Cr(\bar{\rho})$ the set of all circle homeomorphisms whose are C^1- conjugated to f_{cr}. It is well known that any two topological conjugated homeomorphisms have the same rotation number. Therefore, the rotation numbers of homeomorphisms of $Cr(\bar{\rho})$ are the same and equal to $\bar{\rho}$.

A. Dzhalilov build the thermodynamic formalism for the critical circle maps from $Cr(\bar{\rho})$.

Section 1.3 cointains the necessary definitions and theorems on hitting times for critical circle maps.

Let f be an orientation preserving homeomorphism of the circle $S^1=\mathbb{R}/\mathbb{Z}\cong[0,1)$ with irrational rotation number $\rho=\rho(f)$. Let $\mu=\mu(f)$ be the unique invariant probability measure of f. Fix a point $z\in S^1$ and consider the interval $V_\varepsilon(z)=[z,z+\varepsilon]\subset S^1$. Consider the first entrance time to the interval V_ε by

$$E_\varepsilon(t)=\inf\{i\geq 1: T^i(t)\in V_\varepsilon(z)\}.$$

Next, we define rescaled entrance time by $\bar{E}_\varepsilon(t)=\mu(V_\varepsilon(z))E_\varepsilon(t)$. We are interested in the converges of the distribution function of the random variable $\bar{E}_\varepsilon(t)$ i.e. in the convergence of the distribution function

$$F_\varepsilon(t)=\mu\left(x\in S^1: E_\varepsilon^{(1)}(x)\leq t\right), \forall t\in\mathbb{R},$$

as $\varepsilon\to 0$, for every t belonging to the continuity points of the limit function.

Coelho and de Faria investigated the problem of convergence of random variables $\bar{E}_\varepsilon(t)$ for linear irrational rotations $f_\rho(x)=x+\rho(\text{mod}1)$. It is known that for linear irrational rotation f_ρ unique invariant measure is Lebesgue measure ℓ. If $\varepsilon=\varepsilon_n$ is chosen such that $V_{\varepsilon_n}(z)$ corresponds to a sequence of renormalisation intervals for f_ρ as done in Coelho and de Faria's work, it is proved that for Lebesgue almost every rotation number ρ, the rescaled entrance times $\bar{E}_\varepsilon(t)=\mu(V_\varepsilon(z))E_\varepsilon(t)$. do not converge in law as ε tends to zero, and all possible limit laws under a subsequence of ε_n are obtained. Notice that if $F_{\varepsilon_n}(t)$ converges for some ε_n converging to zero every limit probability distribution $F(t)$ either step function with two discontinuity points or uniform distribution on interval [0,1].

Let q_n, $n\geq 1$be the first return times for f_ρ. Fix $\theta\in(0,1)$. For every $n\geq 1$, we define the points $c_n(\theta)$ by:

$$\mu([x_0,c_n(\theta)])=\theta\cdot\mu([x_0,f_\rho^{q_n}(x_0)]).$$

We denote by $\Delta_{n,c}$ the interval $[x_0,c_n(\theta))$. Consider the entrance time $E_{n,c}$ to the interval $\Delta_{n,c}$. Define rescaled entrance time by

$$\bar{E}_{n,c}(x)=\mu(\Delta_{n,c})E_{n,c}(x).$$

Denote by $F_{n,c}(t)$ the distribution function of $\bar{E}_{n,c}(x)$. It is proved that for any irrational number ρ the rescaled hitting times $\bar{E}_{n,c}(x)$ do not converge in law as ε tends to zero.

In this thesis, we investigate the rescaled entrance times for circle homeomorphisms with a single critical point. Let $f\in Cr(\bar{\rho})$ be critical circle homeomorphism. There are two natural measures on the circle: invariant probability measure μ and Lebesgue measure ℓ. Now, we consider two distribution functions $F_{n,c}(t)$ and $\Phi_{n,c}(t)$ of rescaled entrance time $\bar{E}_{n,c}(x)$ with respect to measures μ and ℓ, respectively. The distribution function $F_{n,c}(t)$ of f coincide with distribution function of linear rotation f_ρ. Therefore, all statements of Coelho and de Faria's work are true for f invariant measure μ. The invariant measure μ is singular with respect to Lebesgue measure ℓ. Notice that for piecewise smooth circle homeomorphisms f with finite number of break points and irrational rotation number the map ρ is ergodic w.r.t. Lebesgue measure also. By definition

$$\Phi_{\varepsilon}(t)=\ell\left(x\in S^{1}:E_{\varepsilon}^{(1)}(x)\leq t\right),\forall t\in\mathbb{R}.$$

A. Dzhalilov studied the limit behaviour of distribution function with respect to Lebesgue measure for critical circle maps with “golden mean” irrational rotation number and for renormalized neighborhood of critical point. A. Dzhalilov and J. Karimov investigated the limit behavior of distribution function with respect to Lebesgue measure for circle homeomorphism with single break point and with golden mean irrational rotation number.

Section 1.4 cointains the necessary fact on circle homeomorphisms with break points.

Circle maps with break points have been intensively studied last 20 years in the works of K. Khanin, D. Khmelev, A. Teplinsky, D. Smania, K. Cunha, D. Mayer, A. Dzhalilov, I. Liousse and others. Piecewise smooth homeomorphisms with breaks are the natural generalization of circle diffeomorphisms. The point is called a breakpoint if there is a jump of the first derivative in it. The simplest homeomorphisms with breaks are piecewise linear homeomorphisms of the circle. M. Herman showed that the invariant measure of a piecewise linear homeomorphism with two breaks and an irrational number of rotations is absolutely continuous if and only if both break points lie on the same orbit.

Invariant measures of nonlinear piecewise smooth homeomorphisms of a circle with breaks were studied by K. Khanin, A. Dzhalilov, A. Teplinsky, D. Mayer, I. Liousse, D. Smania, and others.

In contrary to diffeomorphisms, the invariant measures of such homeomorphisms are singular with respect to the Lebesgue measure (K. Khanin, A. Dzhalilov, D. Mayer, I. Liousse). For sufficiently smooth (smoothness on each interval) homeomorphisms with breaks, their renormalizations are approximated by linear fractional maps (K. Khanin, D. Khmelev, A. Teplinsky, D. Smania, k. Cunha, S. Kosich).

In chapter two, titled “**Complexity functions and hitting times associated by circle maps**” we investigated the infinite binary words associated by irrational rotations of the circle and the asymptotic behavior of hitting times of the critical circle maps.

In the first section of chapter 2, we study main properties of infinite binary sequences associated by irrational rotation of the unit circle.

We consider an irrational rotation $f_{\rho}(x)=x+\rho \ (\mathrm{mod}1)$ on the interval [0,1). Fix a point $b\in S^{1}$. Consider the partition $P=\{[0,b),[b,1)\}$ of the circle. For each $n\geq 1$ we define the set $S_n(0,b)$:

$$S_{n}(0,b):=\{0,f_{\rho}^{-1}(0),...,f_{\rho}^{-n+1}(0)\}\cup\{b,f_{\rho}^{-1}(b),...,f_{\rho}^{-n+1}(b)\}.$$

The points of the set $S_n(0,b)$ determine a partition denoted by P_n. Put $P_1 := P$.

In fact,

$$P_n := P \vee f_\rho^{-1}(P) \vee ... \vee f_\rho^{-n+1}(P)$$

where $P \vee Q := \{A \cap B : A \subset P, B \subset Q\}$. Here we denoted elements of P_n and P_{n+1} by just $I^{(n)}$ and $I^{(n+1)}$ without any indexes.

Next, using the sequence of partitions P_n, $n \geq 0$ we construct some special kind of symbolic dynamics. Put $I_1 := [0,b)$ and $I_0 := [b,1)$. Define the coding function $\nu_b : S^1 \to \{0,1\}$: for all $i \geq 0$

$$\nu_b(f_\rho^i(x)) := \begin{cases} 0, & \text{if } \ x \in f_\rho^{-i}(I_0), \\ 1, & \text{if } \ x \in f_\rho^{-i}(I_1). \end{cases}$$

Take any $x \in S^1$. The corresponding infinite sequence $\underline{\omega} := \underline{\omega}(x)$ of zeros and ones we define as

$$\underline{\omega} = (\omega_0 \omega_1 ... \omega_n ...) := (\nu_b(x) \nu_b(f_\rho(x)) ... \nu_b(f_\rho^n(x)) ...) . \qquad (1)$$

Denote the collection of such admissible infinite words $L_\omega(\rho,b)$ i.e.

$$L_\omega(\rho,b) = \{\underline{\omega}(x), x \in S^1\}.$$

We introduce the following notations:

- $\omega_m^{m+s} = (\omega_m \omega_{m+1} ... \omega_{m+s})$, $m,s \geq 0$ - the factor of $\underline{\omega}$;
- $\omega_m^\infty = (\omega_m \omega_{m+1} ...)$, $m \geq 0$ - the tale of $\underline{\omega}$.

Definition 5. *An infinite word $\underline{\omega}$ is called an* ***ultimately periodic*** *if there exist $N,T \in \mathbb{N}$ such that $\omega_{n+T} = \omega_n$ for each $n \geq N$. Otherwise $\underline{\omega}$ is called* ***aperiodic****.*

From the definition of symbolic coding easily follows the following useful **facts**:

a) *Let $I^{(n)}$ be any interval of the partition P_n. Then for any two points $x, y \in I^n$ their prefixes of length $n+1$ are coincides i.e. $\omega_0^n(x) = \omega_0^n(y)$. Hence, for each interval of $\bigcap_{i=0}^{m-1} f_\rho^{-i}(P)$ corresponds one word of length m.*

b) *The word $u_0 u_1 ... u_{m-1}$ is a factor of some admissible infinite word $\underline{\omega} = (\omega_0 \omega_1 ... \omega_n ...)$ i.e. $\omega_{n+i} = u_i$ for $i = 0,...,(m-1)$ if and only if $f_\rho^n(x) \in I(u_0 u_1 ... u_{m-1})$, where $I(u_0 u_1 ... u_{m-1}) = \bigcap_{i=0}^{m-1} f_\rho^{-i}(I_{b_i}) \in P_m$.*

c) *In particular, the interval $I(u_0 u_1 ... u_{m-1})$ is nonempty if and only if $u_0 u_1 ... u_{m-1}$ is a factor of $\underline{\omega}$.*

Denote by $d_n = \max_{I^{(n)} \in P_n} |I^{(n)}|$.

Lemma 1. *For all* $n \geq 1$, $d_n \geq d_{n+1}$ *and* $\lim_{n\to\infty} d_n = 0$.

Denote by $I^{(n)}(x)$ the interval of $n-$th partition P_n containing x and let $W_n(\underline{\omega}(x))$ be the set of subwords of the length n of the infinite binary word $.\underline{\omega}(x)$..

Theorem 2. *Let* $x, y \in S^1$ *and* $\underline{\omega}(x), \underline{\omega}(y)$ *are their infinite coding sequences defined by (1). Then followings are hold*

1. *for all* $n \geq 0$, *we have* $W_n(\underline{\omega}(x)) = W_n(\underline{\omega}(y))$.
2. $\underline{\omega}(x)$ *is uniformly recurrent.*
3. $\underline{\omega}(x)$ *is aperiodic for any* $x \in S^1$.

In section 2.2, we studied the complexity functions of infinite binary sequences associated by irrational rotation on the unit circle.

Recall that, the complexity of an infinite word $\underline{\omega}$ is the function $p_{\underline{\omega}}(n)$ counting the number of its factors of length n.

Further, for simplicity, instead of factor complexity we write complexity function.

We study the complexity functions of infinite words of $L_\omega(\rho, b)$ for all $b \in [0,1)$. We know that,

i) If $b = \{\rho, 1-\rho\}$, then $p_{\underline{\omega}}(n) = n+1$ for all $n \geq 1$.

ii) If $b \neq \{\rho, 1-\rho\}$, then $p_{\underline{\omega}}(n) \geq n+1$ for all $n \geq 1$.

We consider the latter case.

Recall, that a word u is called a **right special factor** of a infinite binary word ω if ua and ub are also factors of ω, for $a, b \in \mathrm{A}$.

Denote by $r(n)$ the **number of right special factors of length** n and let

$$k_0 = min\{n \geq 1 : r(n) = 2\}.$$

On a binary alphabet A the number of right special factors $r(n)$ is used to determine the complexity function by following formula:

$$p_{\underline{\omega}}(n+1) = p_{\underline{\omega}}(n) + r(n).$$

Now we formulate two main theorems of the section 2.2.

The first theorem corresponds to the case when the point b is not in the orbit of 0.

Theorem 3. *Let* f_ρ *be the linear rotation to the irrational angle* ρ, $\rho \in (0,1)$ *and let* $b + n\rho \notin \mathbb{Z}$. *Then the followings are hold*

1. *If* $0 < \rho < b$, *then* $p_{\underline{\omega}}(n) = 2n$ *for all* $n \geq 1$.
2. *If* $0 < b < \rho$, *then*

$$p_{\underline{\omega}}(n) := \begin{cases} n+1, \text{ if } n < k_0, \\ 2n - k_0 + 1, \text{ if } n \geq k_0. \end{cases}$$

The following theorem corresponds to the case when b lies in the orbit of 0.

Theorem 4. *Let f_ρ be same as theorem 3 and $b+d\rho\in\mathbb{Z}$, for some $d\in\mathbb{Z}\setminus\{0\}$. Then the followings are hold*

1. *If* $0<\rho<b,$ *then* $p_{\underline{\omega}}(n):=\begin{cases}2n, & \text{if } n<|d|,\\ n+d, & \text{if } n\geq|d|.\end{cases}$

2. *If* $0<b<\rho,$ *then* $p_{\underline{\omega}}(n):=\begin{cases}n+1, \text{ if } n<k_0,\\ 2n-k_0+1, \text{ if } k_0\leq n<|d|\\ n+d-k_0-1, \text{ if } n\geq|d|.\end{cases}$

In the section 2.3, we investigate asymptotic behavior of the rescaled hitting times for critical circle maps $f\in Cr(\bar{\rho})$ with a single cubic critical point.

It is proved in the theorem 5, where we found the explicit formula for distribution functions $\Phi_{n,\theta}(t)$ of random variables $E^{(1)}_{n,\theta}(x)$.

We formulate the main result of the section 2.3.

Theorem 5. *Let* $\bar{\rho}=\dfrac{\sqrt{5}-1}{2}$ *and let* $f\in Cr(\bar{\rho})$ *be critical circle map. Consider for* $\theta\in(0,1)$ *the sequence of distribution functions* $\left\{\Phi_{n,\theta}(t)\right\}_{n=1}^{\infty}$ *with respect to Lebesgue measure on circle corresponding to the first rescaled hitting times* $E^{(1)}_{n,\theta}(x)$ *to interval* $[x_c,c_n(\theta)]$. *Then*

1) *for all* $t\in\mathbb{R}$ *there exists the finite limit* $\lim\limits_{n\to\infty}\Phi_{n,\theta}(t)=\Phi_\theta(t),$ *where* $\Phi_\theta(t)=0,$ *if* $t\leq 0,$ *and* $\Phi_\theta(t)=1,$ *if* $t>1$*;*
2) *the limit function* $\Phi_\theta(t)$ *is a strictly increasing on [0,1] and continuous distribution function on* $\mathbb{R}$*;*
3) $\Phi_\theta(t)$ *is singular on [0,1] i.e.* $\dfrac{\Phi_\theta(t)}{dt}=0$ *a.e. with respect to Lebesgue measure* ℓ *on the circle.*

In chapter three, titled “**Denjoy equality and sturmian words for p-homeomorphisms**” we studied the location of break points of piecewise-smooth circle homeomorphisms with two break points and the behavior of rescaled derivatives of Herman maps h^{q_n}. Also, we investigated the complexity function of infinite binary sequences associated by rescaled derivatives of h^{q_n}.

Take some $x_0\in S^1$ and consider its orbit $O(x_0)=\{x_i=f^i(x_0),\ i\in\mathbb{Z}\}$. Recall that the system of intervals

$$\mathcal{P}_n(x_0)=\{I_i^{(n-1)}(x_0),\ 0\leq i<q_n\}\cup\{I_j^{(n)}(x_0),\ 0\leq j<q_{n-1}\}$$

determines for any n a partition of the circle, which is called the n**-th dynamic partition** of the point x_0.

Consider, an arbitrary P-homeomorphism f with irrational rotation number ρ_f and two break points a_0 and c_0, which are not on the same orbit. Denote by $\frac{p_n}{q_n}$ the n-convergent of ρ_f. We will next determine the location of the break points of f^{q_n} and the derivative Df^{q_n} on S^1. Obviously, the map f^{q_n} has $2q_n$ break points denoted by $BP_f^n := BP_f^n(a_0) \cup BP_f^n(c_0)$ with $BP_f^n(a_0) := \{a_0^*, a_{-1}^*, ..., a_{-q_n+1}^*\}$, respectively $BP_f^n(c_0) := \{c_0^*, c_{-1}^*, ..., c_{-q_n+1}^*\}$, where $a_{-i}^* = f^{-i}(a_0)$, respectively $c_{-i}^* = f^{-i}(c_0)$, $0 \le i \le q_n - 1$. It is clear, that these break points of the map f^{q_n} define a partition $B_n(f)$ of the circle S^1 into $2q_n$ intervals with pairwise non-intersecting interior.

We divided the location of the second break point c_0 in intervals of partition $B_n(f)$ to four nonintersecting cases.

Let $\mathcal{P}_n(a_0^*)$ be the n-th dynamical partition determined by the break point $a_0^* = a_0$ with respect to the map. Assume that breaks points c_0^* and a_0^* are in different orbits, then the following three cases hold for the second break point c_0^*:

Case I. $c_0^* \in I_{i_0}^{(n)}(a_0)$ for some $0 \le i_0 < q_{n-1}$;

Case II. $c_0^* \in f^{j_0}((a_0, a_{-q_n}])$ for some $0 \le j_0 < q_n$;

Case III. $c_0^* \in f^{j_0}((a_{-q_n}, a_{q_{n-1}}))$ for some $0 \le j_0 < q_n$;

If both break points c_0^* and a_0^* are in the same orbit, then

Case IV. $c_0^* = f^{i_0}(a_0^*)$ for some $0 \le i_0 < q_n$.

In section 3.1 we studied the location of break points of f^{q_n} in Case I and Case III. We also proved Denjoy equality for Herman’s map in the Cases I and III.

We denote by h Herman’s maps i.e. PL circle homeomorphisms with two break points. Each Herman’s map with irrational rotation number ρ satisfies the conditions of Denjoy’s theorem. Hence h topologically conjugated by linear rotation f_ρ.

We expressed the rescaled derivative of h^{q_n} by the jump ratio $\sigma_h(a_0)$ and the μ_h-measures of intervals of the partition $B_n(h)$ produced by break points of h^{q_n}.

In Case I and Case III the break points $PB_h(a_0^*)$ originating from $a_0^* = 0$ and the break points $PB_h(c_0^*)$ originating from $c_0^* = c_0$ alternate in their order along the circle S^1.

Let n be odd. Using the break points of h^{q_n}, we define the following subintervals:

$$A_n := \bigcup_{s=1}^{i_0} [c_{-i_0+s}^*, a_{-q_n+s}^*], \quad B_n := \bigcup_{s=i_0+1}^{q_n} [c_{-i_0-q_n+s}^*, a_{-q_n+s}^*].$$

In Case III the subintervals are given by $[a^*_{-q_n+s}, c^*_{-i_0+s}], 1 \le s \le i_0$, respectively $[a^*_{-q_n+s}, c^*_{-i_0-q_n+s}], i_0+1 \le s \le q_n$, which we combine to the subsets

$$A_n := \bigcup_{s=1}^{i_0} [a^*_{-q_n+s}, c^*_{-i_0+s}], \quad B_n := \bigcup_{s=i_0+1}^{q_n} [a^*_{-q_n+s}, c^*_{-i_0-q_n+s}].$$

For n even, the orientation of the above intervals are to be reversed. Therefore, in Case I we have the following system of disjoint intervals $[a^*_{-q_n+s}, c^*_{-i_0+s}]$, $1 \le s \le i_0$, respectively, $[a^*_{-q_n+s}, c^*_{-i_0-q_n+s}]$, $i_0+1 \le s \le q_n$.

In Case III one finds $[c^*_{-i_0+s}, a^*_{-q_n+s}]$, $1 \le s \le i_0$, respectively $[c^*_{-i_0-q_n+s}, a^*_{-q_n+s}]$, $i_0+1 \le s \le q_n$. In Case I and n even, respectively in Case II and n odd, the subsets A_n and B_n can be defined as before. The above constructions show, that the boundaries of every interval in the subsets A_n and B_n is an interval whose boundaries consist of break points from $PB_n(a_0^*)$ respectively, $PB_n(c_0^*)$.

We put $\sigma = \sigma_h(a_0)$ and formulate the main result of the section 3.1.

Theorem 6. *Let h be a PL circle homeomorphism with irrational rotation number* ρ_h *and two break points* $a_0^* = 0$ *and* $c_0^* := c_0$, *whose total jump ratio* $\sigma_h = 1$, *and which lie on different orbits. Assume* c_0^* *fulfills the assumptions of Case I respectively Case II for some* i_0 *with* $0 \le i_0 < q_{n-1}$. *Then in Case I*

$$(Dh^{q_n}(x))^{(-1)^n} = \begin{cases} \sigma^{\mu_h(A_n \cup B_n)-1}, & if \ x \in A_n \cup B_n \\ \sigma^{\mu_h(A_n \cup B_n)}, & if \ x \in S^1 \setminus (A_n \cup B_n); \end{cases}$$

respectively, in Case III,

$$(Dh^{q_n}(x))^{(-1)^{n+1}} = \begin{cases} \sigma^{\mu_h(A_n \cup B_n)-1}, & if \ x \in A_n \cup B_n \\ \sigma^{\mu_h(A_n \cup B_n)}, & if \ x \in S^1 \setminus (A_n \cup B_n). \end{cases}$$

In section 3.2, we investigated the location of break points of h^{q_n} in the Cases II and IV. It was proved Denjoy equality for rescaled derivative of h^{q_n} for Herman's map in the Cases II and IV.

In section 3.3, we studied the complexity functions of infinite sequences associated by Herman's map h. We showed that the binary sequence associated by rescaled derivatives of h^{q_n}, $n > 0$ is Sturmian.

CONCLUSION

The thesis is devoted to investigation of the complexity functions for infinite binary sequences associated by irrational rotations of the circle, the behavour of hitting times for critical circle maps and the estimate of Denjoy function for piecewise-linear maps.

The basic results of the research work are follows:

1. It is found the complexity functions of infinite binary sequences associated by irrational circle rotations.
2. It is proved that the rescaled generalized hitting times of critical circle maps with one cubic critical point and golden mean rotation number converges in distribution and the limit distribution is singular function i.e. it is continuous, strictly increasing and has zero derivative almost everywhere w.r.t. Lebesgue measure.
3. It is found the location of break points of piecewise-smooth circle homeomorphisms with two break points.
4. It is proved the Denjoe equality for Herman's map h^{q_n}.
5. It is showed that the binary sequences associated by rescaled derivatives h^{q_n} is Sturmian sequences.

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ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО**

ИНСТИТУТ МАТЕМАТИКИ

ЖАЛИЛОВ АЛИШЕР АКБАРОВИЧ

РАВЕНСТВО ДАНЖУА И БЕСКОНЕЧНЫЕ ДВОИЧНЫЕ ПОСЛЕДОВАТЕЛЬНОСТИ СВЯЗАННЫЕ С ГОМЕОМОРФИЗМАМИ ОКРУЖНОСТИ

01.01.01 –Математический анализ

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

ТАШКЕНТ-2021

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является изучение функций сложности для двоичных бесконечных последовательностей ассоцированных с иррациональными вращениями окружности и равенства Данжуа для отображения Эрмана.

Объект исследования: функция сложности бесконечных последовательностей, иррациональные повороты окружности, отображение Эрмана, критические отображения окружностей, времена попадания для отображения окружности.

Научная новизна исследования состоит в следующем:

полностью найдены функции сложности бесконечных двоичных последовательностей ассоциированная с иррациональными вращениями окружности;

доказано, что нормированные время попадания критического отображения окружности с одной кубической критической точкой и с числом вращения равным золотому сечению сходится по распределению и предельное распределение является сингулярной функцией.

полностью найдено расположение точек излома кусочно-гладких гомеоморфизмов окружности с двумя точками излома.

доказано равенство Данжуа для отображения Эрмана. Показано, что двоичная последовательность ассоциированная с перенормированными производными кусочно-гладких отображений окружности с двумя изломами является штурмовской.

Внедрение результатов исследования. Результаты, связанные с кусочно-гладкими динамическими системами с дискретным временем, были использованы в следующих исследовательских проектах:

расположение точек особенности отображений окружности с двумя изломами использовались в исследовательском проекте № ОТ-Ф-4-03 «Точные динамические системы с непрерывным и дискретным временем, спектры частных интегральных операторов» для описания асимптотики степеней и спектра операторов переноса (Справка Каршинского государственного университета № 04/1900 от 7 сентября 2021 г.). Применение научного результата позволило классифицировать орбиты и их асимптотические свойства динамических систем с дискретным временем

сходимость по закону последовательности распределений перенормированных времен попадания критического отображения окружности использовалась фундаментальном научном проекте ОТ-Ф-4-40 «Исследование асимптотических свойств интегральных эмпирических процессов, индексированных в классе измеримых функций» для описания свойств интегральных эмпирических процессов, индексированных в классе измеримых функций (Справка Национальный Университет Узбекистана № 04/11-4794 от 16 сентября 2021 г.). Применение научного результата позволило доказать, что функции распределения сильно зависимых случайных величин абсолютно непрерывны.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав, заключения и списка использованной литературы. Объем диссертации составляет 96 страниц.

E‘LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo ‘lim (part I;I часть)

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